Open Pit Mine Planning and Waste Management Optimization: A Review of Models

Ahlam Maremi and Eugene Ben-Awuah
Mining Optimization Laboratory (MOL)
Laurentian University, Sudbury, Canada

Abstract

Open pit mine planning involves extracting the mining blocks in specific and strategic sequence from the mine in order to obtain the highest net present value. Open pit planning takes into consideration variety of constraints such as production, grade blending and pit slope limitations. Long-term production planning (LTPP) and scheduling are key factors in deciding whether mining projects should continue or be suspended. Since the 1960s literature has supported the application of mathematical programming algorithms for optimization of LTPP and scheduling in open pit mine operations. In this paper, heuristic, meta-heuristic and deterministic optimization approaches, as well as application of artificial intelligence and uncertainty-based approaches to mine planning and waste management have been reviewed and discussed. Limitations of current mine planning models have been outlined.

1. Introduction

The five main steps in the process of recovering valuable material from the earth's crust are: prospecting, exploration, development, exploitation and reclamation. Visual and physiochemical evaluations are used by geologists to discover the valuable mineral deposits. Then drillholes are drilled and samples taken to determine the mineral concentration and the variability of the deposit. Based on the economic parameters, and after representative tonnage-grade curves, the prospective profits from extracting the ore are determined using different interpolation and simulation techniques. The overburden is removed, geometrical preparations of infrastructure and production capacity is calculated, and detailed engineering design is implemented in the development stage. Ore is recovered using open pit or underground techniques in the exploitation stage. Processing plant, stockpiles and waste dumps are the destinations of the extracted ore and waste. Finally, the mining site is reclaimed as close as possible to its original state (Hochbaum and Chen, 2000; Newman et al., 2010).

In order to guarantee mining industry's goal which is maximization of profits, each of the mining steps should be planned and carried out carefully to find the feasible plan for ore extraction. To maximize the overall profitability of the mining project and to minimize deviation from target production, optimization models are being introduced in the mine planning process primarily for development and exploitation stages.

This paper will review mathematical programming models such as linear programming, integer programming, mixed integer linear programming, dynamic programming and goal programming. It will focus mainly on long term mine planning and waste management of oil sands resources. It will be organized as follows: open pit mining, open pit mine planning and scheduling, waste management, integrating open pit mine planning and waste management, oil sands mining and different approaches of modeling the optimization problem. Finally, the limitations of current mine planning and waste management models will be discussed.

2. Open Pit Mining

Mines could be shallow, deep and long depending on the natural occurrence of the deposit. Open pit or surface mining is defined as the method of recovering valuable minerals from deposits fairly near the earth's surface. Open pit mining is the most common, productive and oldest method of mining ore from the ground (Newman et al., 2010). The mining rate includes both mining the ore and removing the waste. The production rate in open pit mines could be 20,000 to 100,000 tonnes per day (Scott Dunbar, 2012). There are several factors which determine whether ore will be extracted through surface or underground mining operations. These are: 1) amount of the overburden, 2) limited area for dumping the waste, 3) unstable pit walls and 4) environmental considerations (Newman et al., 2010). The overburden (the material that covers the deposit and contains no economic quantity of minerals) in addition to waste rock within the deposit (interburden that currently contains no economic quantity of minerals) must be removed to gain access to the mineralized zone. The mineral content that distinguishes ore from waste (cut-off grade) can change depending on the market conditions and the availability of extraction technology. It is possible that material which has been considered waste becomes a potential reserve (Scott Dunbar, 2012).

The surface of the land is continuously excavated by mining processes until the end of the mine life resulting in a deep pit. Benches are used for extracting the ore (Hochbaum and Chen, 2000; Ben-Awuah and Askari-Nasab, 2011). The ore is taken to the processing plant and the result of a mineral separation process in the concentrator (known as tailings) is taken to tailings containments or ponds Fig. 1 Economic and technical factors, in addition to production constraints determine the size and shape of the pit. There are a series of intermediate pits sometimes referred to as pushbacks before the ultimate pit that exists at the end of the mining process (Lerchs and Grossmann, 1965; Askari-Nasab, 2006).

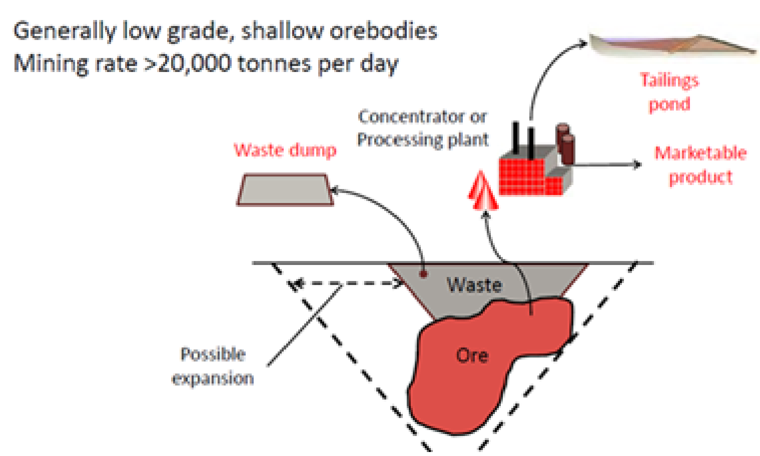

Fig. 1. Open pit mining operations (Scott Dunbar, 2012)

3. Open Pit Mine Planning and Scheduling

Extracting the blocks from the mine in specific sequence to give the highest net present value (NPV) is known as open pit mine planning and scheduling. This is subject to a variety of production, grade blending and pit slope constraints (Whittle, 1989). Minimizing the mining costs and maximizing the production considering the quality and the operational requirements are the goals of mine production planning (Newman et al., 2010).

According to Jardine and Evans (1989), mine production planning and scheduling includes six repeated tasks during life of mine: 1) extracting point data such as drillhole data and then 3-D geological modeling of deposit, 2) designing the mine pit limits, 3) building mining blocks and assigning reserve features to each block, 4) determining the extraction rate and sequence of blocks, 5) simulating the block extraction, and finally, 6) reporting the results of extraction sequence.

In mining projects, Chicoisne et al. (2012) describe the mine planning phase as: 1) Block model determination: this step consists of drilling in different locations and depths of the mine. Obtaining samples of material for grade and densities interpolation, dividing the orebody into blocks of equal size and an estimated tonnage and mineral grades are assigned to each block. As a result, compute the estimated extraction profit for each block in the model. The economic block model then is a block model with profit attributes. 2) Ultimate pit limit is defined as the area in which extraction will take place. Before any block can be extracted, all blocks immediately above and at certain angles must also be removed Fig. 2. To determine the ultimate pit limit, it is necessary to determine slope angle. This depends on the structural composition of the rocks and the location and depth of each block. 3) Production scheduling involving the decision of which blocks and when and how they should be extracted. First, determine a set of pushbacks. Pushbacks are subdivided into groups of blocks at the same vertical level (or bench) known as bench-phases. Finally, bench-phases are scheduled a time of extraction (Chicoisne et al., 2012). Deviations from optimal mine plans may result in significant financial losses, future financial liabilities, delayed reclamation, and resource sterilization (Ben-Awuah and Askari-Nasab, 2011).

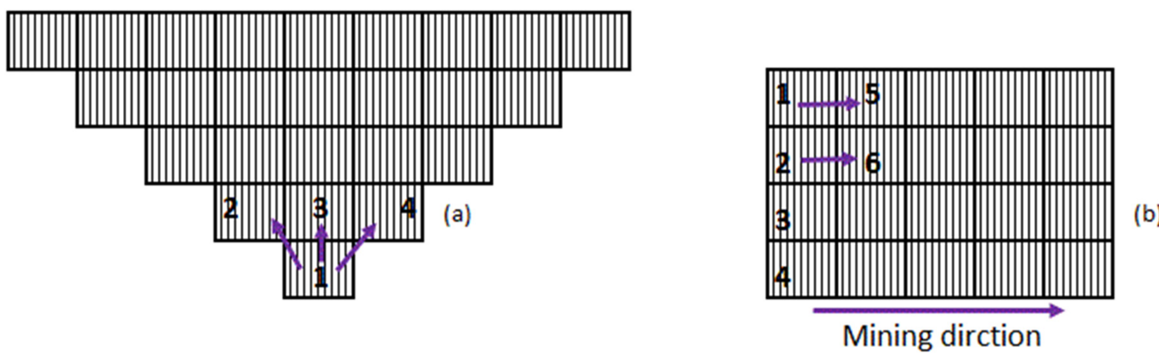

Fig. 2. Block extraction precedence: (a) cross sectional view and (b) plan view modified after Ben-Awuah and Askari-Nasab (2011)

The final pit limit defines the size and shape of the open pit mine at the end of the mine life subject to economic, technical and operational constraints. Cut-off grade or the grade that distinguishes ore from waste is required. It depends on the current and future states of the mine simultaneously. The pit limits are used in determining the boundary layouts and location of mine infrastructure such as processing plants, tailings facilities, waste dumps and mine offices. The location and type of haulage ramps and other infrastructure are other required aspects of designing the open-pit mine, in addition to long-term decisions regarding the size and location of production and processing facilities.

There are many techniques used to find the ultimate pit limits such as the heuristic floating cone technique (Osanloo et al., 2008), the 2D algorithm based on dynamic programming, the 3D algorithm which uses graph theory that is widely accepted in the mining industry (Lerchs and Grossmann, 1965). These are in addition to maximum network flow algorithm (Johnson, 1969) and transportation algorithms (Osanloo et al., 2008). Without the final or ultimate pit limit, the open pit mining of a given deposit will be uneconomic. The optimum final pit limits therefore define the pit outline containing the material extracted to give the total maximum profit whilst satisfying all operational constraints (Caccetta and Giannini, 1990; Ben-Awuah, 2013).

Long-term production planning focuses mainly on ore reserves, stripping ratio (the quantity of waste to be removed in order to obtain one unit of ore), and major annual investment plans (Newman et al., 2010). Three time ranges are included in production planning; i) short-term, between a month and a year, ii) medium-term, between one to five years and iii) long-term, up to 30 years which is the focus for this paper.

4. Waste Management

For the mining industry, managing waste is challenging. Mining operations result in significant amounts of waste material with different types, such as overburden, waste rock (low grade ore), tailings, slags (non-metallic by-products from metal smelting), mine water, water treatment sludge, and gaseous wastes (Scott Dunbar, 2012). Efforts have been made by mining companies to reduce the environmental impact of mining activities and minimize their footprint. Thanks to technological developments and changes in management methods many of the adverse effects of mining affecting its surrounding environment are now avoidable.

Waste management plans are usually developed before the mine is constructed. There is integration between reclamation of waste rock dumps and tailings ponds, and the designs of new mines. The main concern of mining environmental management is the adverse effect of waste materials and tailings disposal on the surface, i.e. tailings containments and waste dumps. Depending on the composition of ore type being mined and the processing techniques used at the mine site the mine waste is different in type, amount and properties. In other words, every mine produces different waste that requires its own classification, estimation, monitoring and treatment. Some of these materials may be acid generating which must be properly managed to protect the environment (Rashidinejad et al., 2008).

In the LTPP, preventing pollution is easier and less expensive than after pollution has been created. Pollution problems result in high capital and operating costs and long-term liabilities. In other words, waste management practices should focus on "prevention" rather than "treatment". Researchers have concentrated on promising prevention techniques such as layering and blending through strategic mine planning. The best way to eliminate these liabilities is minimization of the waste and pollution at the source in the first place (Cheremisinoff, 2003). Large quantities of acid generating waste material and tailings have been inherited from past mining operations. Mine design processes still focuses mainly on technical mining, financial considerations and potential impacts of waste disposal on the environment (Rashidinejad et al., 2008). There are two major categories of environmental impacts from mine sites waste disposal: the loss of productive land after being used as waste storage area, and contamination of adjacent surface and groundwater (Scott Dunbar, 2012). Continuous research in minimizing waste and finding alternative uses for mineralized mine waste is essential for effective waste management.

5. Integrating Open Pit Mine Planning and Waste Management

An important discussion on surface mining operations is mine planning and waste disposal management. Mine closure or future financial liabilities can result from poorly planned mining operations. Modelling an integrated mine planning system adds more difficulty to the LTPP problem (for instance, incorporation of waste disposal planning). According to McFadyen (2008), oil sands waste management is predominately a post-production scheduling optimization activity. Scheduling of dyke material is carried out after mining has started and this may result in inconsistent production of dyke materials at different periods during the mine life. Ben-Awuah (2013) developed an incorporated mine planning and waste management strategy for in-pit and external tailings facilities for oil sands mining operations. His approach requires a new and more systematic method for planning of oil sands mining operations.

Available literature on oil sands mining lacks the framework for planning of oil sands resources which has a unique scenario for waste management. The material required for construction of dykes come mainly from mining and processing operations; as overburden, interburden, and tailings coarse sand (Fauquier et al., 2009; Ben-Awuah, 2013). Upstream construction, downstream construction, and centerline construction are the main methods for dyke construction. In addition, with respect to the regulatory requirements from Directive 074, waste disposal planning must be considered in a close relation to the oil sands mine planning system (McFadyen, 2008).

Challenges which arise during the integration of oil sands waste disposal and production scheduling optimization include: 1) Intractable size of the optimization problem resulting from scheduling different material types with multiple elements for multiple destinations; 2) The need to integrate the availability of in-pit disposal areas with dyke construction planning on a continual basis throughout the mine life to support the tailings storage plan; 3) The limited lease areas for oil sands operators require the maximum use of in-pit and ex-pit tailings facilities for sustainable mining; 4) NPV derived from production scheduling and sustainable mining derived from waste disposal planning are the two targets that cause difficulty in deciding which must be traded off and at what cost (Fauquier et al., 2009).

Although a team of engineers work on dyke construction planning, there is no guarantee that the developed plan meets all the material requirements for dyke construction in all periods and the resulting NPV is maximized (Fauquier et al., 2009). Ben-Awuah et al. (2012) and Ben-Awuah (2013) have introduced a pioneering effort in developing an integrated mathematical programming model for incorporating oil sands mine planning and waste management using mixed integer linear goal programming (MILGP) in an optimization framework.

Ben-Awuah et al. (2012) have implemented a mathematical model for an integrated oil sands production scheduling and waste disposal planning system. This takes into consideration multiple material types, multiple elements and destinations, directional mining, waste management and sustainable practical mining strategies. Ben-Awuah (2013) reports that the MILGP model is a powerful tool for optimizing LTPP in oil sands mining. The model provides a robust platform for integrating waste disposal planning. It is an efficient production scheduling optimization approach that uses penalty and priority parameters, and goal deviational variables. However, the model implementation results in a large-scale optimization problem, and according to Badiozamani (2014) it does not include tailings slurry, the most important waste in oil sands mining.

Oil sands mines require large tailings containments that will affect the landscape. Reducing the size and need for tailings containments, and increasing the speed with which they can be reclaimed, are challenges for oil sands mining companies. Since the capacity for tailings storage is limited to the lease area, mining cannot be scheduled without considering potential tailings production. An integrated model for LTPP with the concept of tailing management is proposed by Badiozamani (2014). He integrates reclamation material handling and tailings capacity constraints to provide capacity for in-pit tailings facility.

In order to maximize NPV, Badiozamani (2014) determines the destination for each extracted parcel. The selective mining units are followed using mining aggregates. He generates maximum NPV, minimizes the material handling cost of reclamation, and the tailings volume produced downstream meets the tailings capacity constraints in each period. The author integrates mine planning with tailings management in terms of composite tailings (CT) production and deposition, in the mine planning optimization framework. Mine planning, tailings management, waste disposal scheduling and reclamation planning are four areas that should be integrated in order to achieve a more robust schedule (Badiozamani, 2014).

6. Oil Sands Mining

In North America, oil sands mining is one of the most evolving industries. Oil sands mining started in the 1960s with surface mining operations that uses Clark hot water extraction (CHWE) to extract bitumen from the bearing formation. Truck-shovel system is used to extract northern Alberta oil sands reserves which are mostly located near the surface (Clark and Pasternack, 1932; Clark, 1939). Muskeg (the overburden), Pleistocene unit and Clearwater formation (both are waste rocks); McMurray formation (carries the bitumen, the element of interest) and Devonian carbonates (marks the end of the oil sands deposit) are the five main rock types in oil sands formation Fig. 3 (Ben-Awuah, 2013; Badiozamani, 2014).

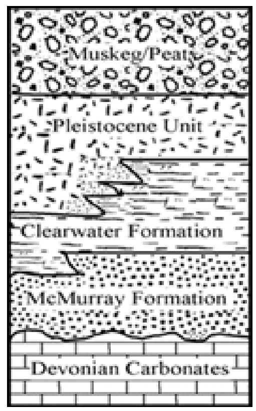

Fig. 3: Vertical soil profile sketch of an oil sands formation (Ben-Awuah, 2013)

In oil sands mining, huge amounts of bituminous sands are sent to the processing plant and these results in a mixture of water, fine materials, sands and residual bitumen - known as tailings which represent more than 80% of the processed ore. The waste material (overburden and interburden) are sent to waste dumps or used for dyke construction. In-pit and ex-pit tailings dykes used for storing tailings are constructed using overburden and interburden seams in addition to tailings coarse sands (TCS) resulting from the processing plant Fig. 4 Accordingly, waste management is a significant part of oil sands mining operations that may lead to economic liabilities if not well managed (Ben-Awuah, 2013; Badiozamani, 2014).

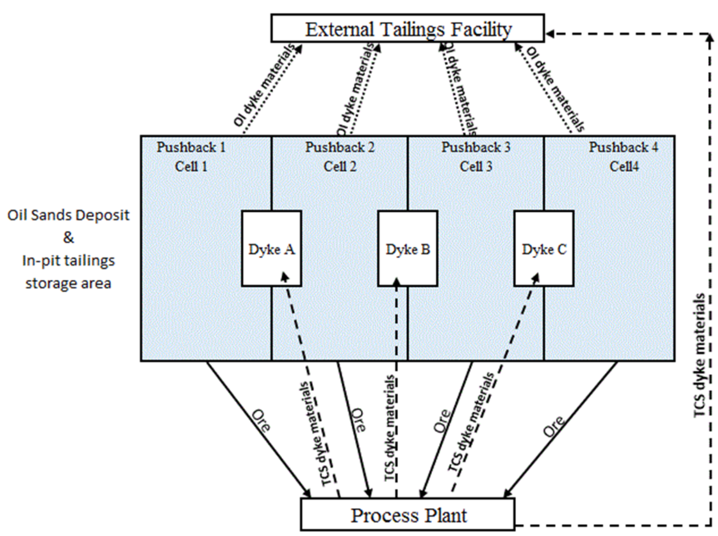

Fig. 4. Conceptual mining model: mining and waste management strategy modified after Ben-Awuah (2013)

In oil sands mining, waste and tailings management requires special geotechnical considerations and tailings management techniques (Boratynec, 2003; Ben-Awuah, 2013). Deviations from optimal mine plans will result in significant financial losses, future financial liabilities, and delayed reclamation. There are three significant aspects in dealing with oil sands tailings (the most unwanted by-product of oil sands processing). First, the greenhouse gas emissions resulting from CHWE process. Second, the environmental challenges due to the toxicity of the tailings resulting in contamination of fresh water table by the polluted tailings water leaks. In order to dewater tailings and prepare the tailings containment area for reclamation, composite tailings (CT) production technologies are used. The dewatering technology is based on adding gypsum as a coagulant to aid production of mature fine tailings (MFT) and to increase the dewatering rate of the MFT (Rodriguez, 2007; Singh, 2008). Third, space limitation increases the need for in-pit tailings containment (storage space) since more mining processes lead to a further volume of tailings slurry.

Presently, plans for tailings and reclamation are prepared after the optimization of long-term mine production plans. The optimization of LTPP problems is used as an input to the tailings and reclamation plans (Badiozamani, 2014). The requirements of Directive 074 issued by the Alberta Energy Regulator (AER) mandate oil sands operators to publish their waste disposal and tailings plans (McFadyen, 2008; Ben-Awuah and Askari-Nasab, 2011). Overburden, low-grade interburden (OI), and tailings coarse sand (TCS) (generated from processing of bituminous sands) are used for dyke construction for tailings storage and as a reclamation material at the reclamation stage. This makes it very important to incorporate waste management, tailings planning, and reclamation planning in the long-term mine planning optimization framework. It is important that the sequence of extracting the ore and the supply of material used for dyke construction be continuous to guarantee uniform supply to the plant and for dyke construction throughout the mine life (Fauquier et al., 2009). An integrated oil sands mining operation including material flows(Fig. 5), solid waste and tailings management is provided in detailed description by Ben-Awuah et al. (2012) although in general there is limited research work in this area (Ben-Awuah and Askari-Nasab, 2011; Badiozamani, 2014).

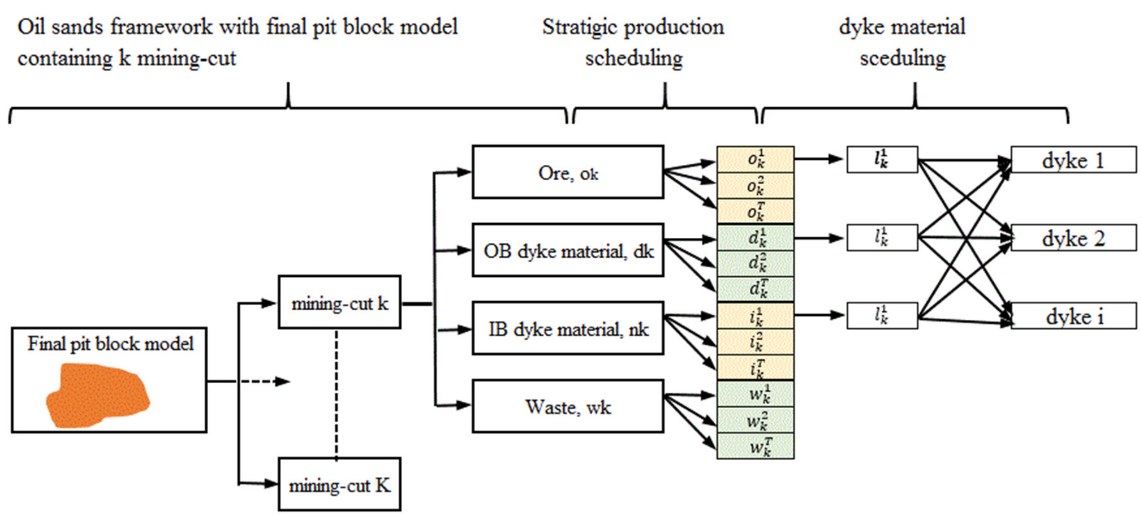

Fig. 5. Schematic representation of oil sands mining and solid waste material flow modified after Ben-Awuah (2013)

7. Mine Production Planning Models and Algorithms

Solving LTPP problems should satisfy the objectives of medium and short-term scheduling otherwise the optimality of the solution might be affected. First, the main input, the geologic block model for open pit mine design and scheduling processes, should be prepared. It is a quantitative definition of the available resource. The deposit is divided into fixed size blocks. The exploration drilling pattern, ore body geology and mine equipment size are the main points that have to be considered in the selection of block dimensions. Next, using available estimation techniques such as inverse distance weighted interpolation technique, weighted moving averages and Kriging geological characteristics of each block (grade) are assigned. Using financial and metallurgical data, the economic value of each block is also calculated. It should be noted that this value excludes the cost of accessing the block. The economic present value of the block can then be obtained by discounting the original value to time zero, using a discounted rate (Osanloo et al., 2008). Based on a deterministic metal price, each block is assigned a value equal to the gross value of its metal content minus the applicable production, processing and refining costs (Abdel Sabour and Dimitrakopoulos, 2011).

In the literature, the LTPP and scheduling methods can be divided into main categories: 1) heuristic algorithms, 2) deterministic algorithms, 3) application of artificial intelligence techniques, and 4) uncertainty based approaches. Deterministic approach assumes the input values and parameters are known and fixed, while uncertainty based approaches considers some input parameters as uncertain.

7.1. Heuristic and Meta- Heuristic Optimization Approach

Some popular mine production scheduling software are developed based on heuristic methods such as XPAC AutoScheduler software (Runge Limited, 2009), Whittle (Gemcom Software International, 2012) and NPV Scheduler (Datamine Corporate Limited, 2008). XPAC AutoScheduler software is developed based on Gershon's proposed heuristic algorithm that generates cones upward from each reference block and determines the possibility of the block to be part of the schedule. According to the importance of mining a block at a certain time, a list of exposed blocks and a ranking of those is updated through the algorithm based on a factor called the positional weight. This weighted function is used to determine the removal sequence. The final pit determination depends on the number of reference blocks that will be used in addition to the sequence of selecting those blocks. The solution is fast and accurate, however, the ultimate pit is not necessarily optimal (Gershon, 1987; Laurich, 1990; Runge Limited, 2009).

Another popular heuristic is introduced by Lerchs and Grossmann (LG) (1965). It has been used in strategic mine planning software, such as Whittle and NPV Scheduler based on the concept of parametric analysis (Osanloo et al., 2008). The LG algorithm provides an optimal solution to the ultimate pit limit. There are many number of strategies with different discounted cash-flows of reaching the ultimate pit. The strategy that would maximize the discounted cash-flow while meeting all the physical and economic constraints is the optimal production schedule. The parametric analysis generates a series of nested pits based on varying the price of the product and finding an optimal pit limit using LG algorithm for that price. The nested pits are grouped into pushbacks, each one associated with similar resource usage. Pushbacks then are used as a guideline to identify clusters of high grade ore and to determine the production schedule. The algorithms are fast to solve and the result is accurate. The main disadvantage of heuristic algorithms is that there is no measure of quality, moreover, the solution will not guarantee optimality which in mega mining projects may cause financial losses (Askari-Nasab and Awuah-offei, 2009; Newman et al., 2010).

On the other hand, solving LTPP problems using meta-heuristic approaches such as genetic algorithms, simulated annealing, etc. have depicted to be effective for large-scale NP-hard problems, especially in the wider field of production planning and scheduling. Denby and Schofield (1994) presented a meta-heuristic model based on genetic algorithm. Their model is able to optimize production planning and ultimate pit limit at the same time. In addition, they achieved an acceptable result in a good time. However, the result changes as they re-run the model and the model do not take into account the effect of pit volume on the unit cost.

7.2. Deterministic Approaches for LTPP

Long term production scheduling problems are very complex to solve. Since the 1960s, many researchers have studied and applied Operations Research (OR) methods in mine production scheduling. Several types of mathematical formulations have been considered for the LTPP problems and have been studied widely in the literature. A variety of deterministic optimization methods including Linear Programming (LP), Integer Programming (IP) and Mixed Integer Linear Programming (MILP) are commonly used, in addition to Dynamic Programming (DP) (Osanloo et al., 2008) and Goal Programming (GP) that have the capability of considering multiple material types, elements, and destinations.

7.2.1. Linear Programming and Mixed Integer Linear Programming Approach for LTPP

Johnson (1969) introduces linear programming as a mathematical programming model to the mine planning research area. He proposes a linear programming model for the long-term, multi-destination and open pit production planning problem along with a decomposition approach to solve the problem. That means a large multi-period model is divided into sub-models considering one period at a time. For each period the sub-model generates optimum results. The author's model considers discounted values of revenues and costs, different processing types and dynamic cut-off grade. After solving all sub-problems, the original problem is relatively simplified. This initial model uses continuous variables to control precedence constraints which would result in fractional extraction of blocks and infeasible solutions. The results are not optimum for the whole multi-period model. Moreover, due to the size of the problem it is computationally intractable. The precedence of block extraction is not satisfied and that results in some percentage of the overlaying blocks being suspended in air (Gershon, 1983).

The initial LP model was subsequently modified by Gershon (1983) to MILP. He considers a set of binary variables to satisfy the precedence of block extraction. He assigned four different decision variables for each block. For a typical open pit long-term scheduling problems, the number of blocks may reach millions, and the number of scheduling periods could be about forty years for a life-of-mine production schedule. The model can handle multiple ore processing options and multiple grades. However, the numbers of binary variables make the model intractable for real size mine planning projects. Although MILP has significant potential for optimizing production planning in open pit mines with the objective of maximizing the discounted cash flow, when it comes to large-scale projects it generates too many binary variables that make it intractable to solve with the current state of hardware and software.

7.2.2. Dynamic Programming Approach for LTPP

A dynamic programming (DP) model that maximizes the NPV, subject to production and processing constraints is presented (Osanloo et al., 2008). This method considers both the time value of money and block sequencing to determine the ultimate pit limit. However, it cannot be applied to large scale problems, and there is no guarantee that mining and milling constraints will be satisfied. Based on a combination of heuristics and DP, Newman et. al.(Newman et al., 2010)(Newman et al., 2010)(Newman et al., 2010)(Newman et al., 2010)(Newman et al., 2010) propose their methodology (Newman et al., 2010). They claim that the ultimate pit limits, the cut-off grade, the mining sequences and production scheduling are related to each other and without the knowledge of one variable the next variable cannot be determined. Their method brings the required simultaneous solution to the problem. Currently, there are some researchers that argue that DP is intractable for large problems and Lagrangian approach is theoretically optimal and suitable for large problems.

7.2.3. Goal Programming Approach for LTPP

Liang and Lawrence (2007) state that goal programming allows for flexible formulation and the specification of priorities among goals. GP is also used by Chanda and Dagdelen (1995). Their model tries to minimize the deviation from goals after setting up the blending problem with multiple goals. The model was tested for a coal mine deposit, but due to some interactions involved in solving the problem, optimal solution cannot always be guaranteed. A mineral dressing criteria was defined by Esfandiri et al. (2004) and used in the optimization of an iron ore mine. A binary non-linear goal programming model was defined based on multiple criteria decision making and the deviations for economics, mining and mineral dressing functions were minimized. This formulation was solved using LINGO software. The model was found to have limitations and constraints that are numerous for practical application.

Ben-Awuah (2013) has formulated the oil sands long-term mine production scheduling and waste disposal planning problem using a combination of mixed integer and goal programming formulations. He claims that using GP is appropriate for his framework because, based on the importance of the goals; the structure will allow the optimizer to achieve some goals while others are traded off. The important goals will be selected according to the impact of a deviation from their targets on the mining operations. The goals to be achieved are the mining and processing targets, and OB, IB and TCS dyke materials targets in tonnes for all mining locations, and processing and dyke construction destinations. The constraints are: grade blending, variables control and mining-panels extraction precedence constraints. This technique is a good choice when there are many goals and some of them need to be chosen among others. It is a flexible technique and allows for some level of interaction between the decision maker and the optimization process (Zeleny, 1980; Hannan, 1985). According to Ben-Awuah (2013), solutions within known optimality limits are expected using exact solution methods for LTPP problems. For the resulting production schedule, a higher NPV is guaranteed as the solution gets closer to optimality.

7.3. Clustering Technique

To reduce the number of variables some authors try to classify the large amount of data into relatively few classes of similar objects. This classification is known as aggregation or clustering. Newman et al. (2010) proves that nonlinear programming (NP) model is NP-hard. Instead of solving this NP-hard problem, there are some non-deterministic algorithms that have been developed by many authors. Hard and fuzzy clustering are two main clustering algorithms. The first one determines whether each unit belongs to a group or not and the second decides whether each unit belongs to each group to a certain degree. Both clustering algorithms can be organized to hierarchical clustering, partitional clustering or overlapping clustering. Since all blocks must belong to only one cluster, hierarchical and partitional clustering can be used in mine planning. Feng et al. (2010) state that although hierarchical clustering methods generate better results they are computationally expensive.

Boland, et al. (2009) propose a solution procedure based on an aggregate level for the order of extraction decisions while block level is used for processing decisions. They are able to report notable progress in CPU time for their model, however, the degrees of freedom of the optimization problem is reduced by using the aggregated blocks. Askari-Nasab et al. (2010) and Askari-Nasab et al. (2011) have used some block clustering techniques for MILP to reduce the size of the LTPP problem prior to optimization (Fig. 6). They clustered mining-blocks according to their properties such as spatial location, grade and rock type to form larger units known as mining-cuts (Boland et al., 2009; Ben-Awuah, 2013; Tabesh, 2015).

As Barbakh et al. (2009) state, there are many data clustering techniques through heuristic or meta-heuristic algorithm such as hierarchical clustering (Johnson, 1967), k-means clustering (MacQueen, 1967), and fuzzy c-means clustering (Dunn, 1973). In addition, there are some other techniques used to reduce the size of the problem. For example, defining fundamental trees (Ramazan et al., 2005; Ramazan, 2007) and mining-panels that are introduced by Ben-Awuah (2013). In order to decrease the number of binary variables in the IP model, Ramazan et al. (2005) aggregate ore and waste blocks together. Mining panels are generated from the intersections of pushbacks and mining benches, and within the boundaries of mining panels mining-cuts are defined and can be used for destination decisions. Clustering and paneling are powerful techniques that are used to provide larger units that follow the practical selective mining units and reduce the number of decision variables which results in increasing the speed of the algorithm. Clustering reduces the gap factor and it is more accurate because most open pit mines are extracted in mining-cuts not in blocks (Tabesh, 2015).

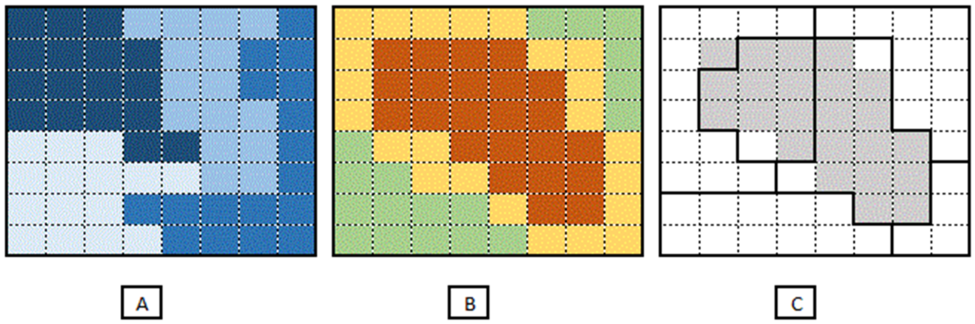

Fig. 6. Schematic view of clustering: A) shows rock types, B) shows grade distribution, C) shows the resulting mining-cuts modified after Badiozamani (2014)

Reducing the number of decision variables before solving the problem is another way of reducing the size of the problem. The idea is that the accumulated mining (and processing) capacities are known and limited for each period in advance, and the total tonnage above any specific mining-block is known based on the precedence order of extraction. Accordingly, it can be decided whether a specific mining-block is accessible in certain periods or not so many variables may be eliminated from the model (Bley et al., 2010; Martinez and Newman, 2011; Tabesh, 2015).

To sum up, although the above mentioned methods significantly decrease the number of binary variables required and enhance the application of MIP in large mineral deposits, in-situ orebody variability is not considered and all inputs are considered without uncertainty. Clustering of mining-blocks are a reasonable assumption for long-term mine planning problems and make the problem tractable, however, they will reduce the accuracy of the solution which might result in losing the optimality of the original problem.

7.4. Application of Artificial Intelligence Techniques and Genetic Algorithm

A method that is a combination of dynamic programming, stochastic optimization and artificial intelligence with heuristic rules to solve LTPP problems is proposed by Underwood and Tolwinski (1998). The method is practical for mining operations since it considers all production constraints. It finds the ultimate pit limit and obtains the production schedule at the same time. However, optimal solution cannot be proven mathematically and a feasible solution for large problems is not guaranteed.

Another method based on genetic algorithm and simulated annealing for production scheduling and ultimate pit limit was proposed by Osanlooet al. (2008). It generates a random pit population and evaluation of a suitability function to obtain the production schedule and ultimate pit limit simultaneously. Although it is flexible, there is no measure of optimality for the solution. To reduce the size of the optimization problem, aggregated blocks and a genetic algorithm are used and compared. Although the practical consequences of aggregation is not mentioned, it has been found that CPLEX reaches the solution in about two to four times longer than the genetic algorithm does.

Osanloo et al. (2008) developed a simulative optimization model to determine the ultimate pit limit using dynamic programming technique. It determines the block extraction sequence and the real unit costs for each new pit condition. After some iterations and according to the nature of the dynamic programming technique, the size of the problem becomes huge. The model solves the ultimate pit limit and production scheduling problems simultaneously. It is unique because of its ability to estimate unit cost for each new pit scenario. The model considers all types of operating constraints, such as transportation, stockpiling, grade blending and plant facilities. However, it is not efficient for medium and large deposits. Also, even for small problems the optimal solution is not guaranteed.

Another method that also generate the ultimate pit limit and production schedule at the same time is the intelligent agent-based mine planning simulator IOPS. Askari-Nasab (2006) developed an intelligent agent-based simulator for surface mine planning that has a component which simulates pushbacks and the intelligent agent learns the optimal pushback using reinforcement learning method. There is no guarantee that the results will reach the theoretical optimum solution. These techniques are based on frameworks that theoretically will reach near optimal solutions, given sufficient number of simulation iterations. The disadvantage of these methods is that there is no quality measure for solutions provided compared against the theoretical optimum.

7.5. Uncertainty Based Approach

Since deterministic solution methods are incapable of dealing with stochastic variables when solving large scale LTPP problems, many researchers have tried to solve LTPP problems using the uncertainty based approach. Results have shown that some (or significant) differences between actual production and theoretical expectation might exist due to grade and geological uncertainties (the two important sources of risk in mining industries), especially in early years of production. As hardware, software, and solution techniques develop, more accurate models are expected (Osanloo et al., 2008; Newman et al., 2010; Gholamnejad and E., 2012). In literature, there are few works on uncertainty based methods.

The geologic block model is the main input data for optimizing LTPP problems. There are some random input variables such as grades, costs, prices, and recoveries. In deterministic approaches, the best estimated values of these random variables available at the optimization time are used. The optimal solution can be affected by uncertainties related to the input parameters. A rerun is required when new data becomes available. This updating is aligned with the mining industry practices. Uncertainty can be reduced only by getting more data over time. Most research has focused on minimizing the negative impact of grade, geological, and market uncertainties on production schedules.

Dimitrakopoulos and Ramazan (2008) first incorporated geological uncertainty into open pit mine planning. Some researchers have shown the consequences of grade uncertainty and the economic value of each block in production schedules. In the early years, a survey for mining operations show that 60% of mines had 70% less production than designed capacity (Osanloo et al., 2008). The key uncertainty factors are geological and mining, financial and environmental. Geological uncertainty is a major contributor in failing to meet production targets and the financial expectations of a project.

A framework for incorporating risk in open pit mine planning was proposed by Osanlooet al. (2008). He used stochastic orebody models and sequentially generated a production schedule. However, sequential procedures are shown to be inefficient and cannot produce a global optimal schedule considering uncertainty. Osanloo et al. (2008) classifies uncertainties involved in mine planning as: i) orebody model and in-situ grade uncertainty, and material type distribution; ii) extraction capacities and slope consideration, and iii) capital and operating costs uncertainties. Dimitrakopoulos and Ramazan (2008) present a stochastic linear integer programming (SIP) model to generate the optimal production schedule using equally probable stochastically simulated orebody models as inputs. They put a penalty function for the cost of deviation from the target production and use linear programming to maximize NPV minus the penalty costs. They are able to generate an optimum solution that can increase NPV by adding some constraints. However, the difficulty and CPU time required to solve the optimization problem increased. Moreover, the model is not dynamic and flexible to new information that is developed during the mine life.

Some notable research is presented in the area of stochastic pushback design. Osanloo et al. (2008) presents a multi-stage heuristic framework to generate a final schedule which considers geological uncertainty. A basic input to this framework is a set of realizations (equally probable scenarios of the orebody). The author reports significant improvement on NPV in the presence of uncertainty however, there are some disadvantages for the model: (1) it does not consider grade blending, (2) it does not control the risk distribution for the production target, (3) the optimality of the method cannot be guaranteed, (4) the technique is complex and difficult to apply and (5) finally, in order to get reasonable results many parameters need to be chosen carefully.

Net present value, ore tonnage, head grade, stripping ratio, amount of final production and annual target production are output parameters that Koushavand (2014) has evaluated using two different approaches. He evaluated the impact of grade uncertainty on the output parameters of a mine production plan based on an oil sands deposit case study. Usually, drillhole data is used to calculate the values of mining blocks. However, mine planners cannot know with certainty the quantity and quality of ore in the ground. In other words, the ore cannot represent the natural local grade variability within the deposits which might lead to considerable risks if not meeting planned production targets through actual operations. They also cannot predict future metal prices and foreign exchange rates.

Abdel Sabour and Dimitrakopoulos (2011) built their work by quantifying and integrating market uncertainty related to metal prices and exchange rates into mine planning. They developed a system for mine planning selection based on multiple value statistics and cash flow characteristics integrating the value of management flexibility to react to new information. The authors take into account multiple sources of uncertainty simultaneously and integrate the flexibility to revise the ultimate pit limits based on new information. Results show that a significant difference indicating the importance of incorporating uncertainty and operational flexibility into mine designs decisions.

Groeneveld and Topal (2011) evaluated the flexibility of strategic mine design under uncertainty. They used mixed integer programming (MIP) and Monte Carlo simulation (MCS) to maximize NPV and incorporates many design options (mine, stockpile, plant and port) and multiple uncertainties (price, capital cost, operating cost, recoveries and utilization). They claimed the improvement in NPV value could go further by increasing the available flexibilities in the design in addition to other uncertainties in the model. Additionally, the authors recommend further research and model improvements be continued in the following areas: Handling of grade variability through the use of conditional simulation methods will greatly improve the power of the model. If projects are improved in a way that increases flexibility to respond to uncertainties, the mining industry will be more sustainable.

8. Stockpiling

Tabesh (2015) state that most of the proposed models incorporate mining, processing and precedence constraints and do not include grade blending and stockpiling constraints. Usually stockpiling is used in mine operations for many reasons such as blending of material, storage of over produced ore (if there is enough materials to feed the plant), storage of waste material and storage of low grade ore for future production. The stockpiled ore will be processed in later years or at the end of the mine (Groeneveld and Topal, 2011; Koushavand, 2014). At the beginning of stockpiling the material, the grade and tonnage is known but as more material is added to the stockpile the grade and the tonnage become unknown (Groeneveld and Topal, 2011). Koushavand (2014) suggests penalty value for over production which will be less in the presence of stockpiling. This means any reasonable over production based on a realization will be kept in a stockpile and will be used in subsequent periods. Groeneveld and Topal (2011) state that the possibility of a nonlinear constraint increases as the material in the stockpile is unknown prior to optimization. To solve this, simulated grade bins are created in the stockpile. These grade bins have a maximum and minimum grade of material which can enter each bin. When removing material the average grade is taken from the grade or alternatively, the maximum or minimum grade limit of the bin can be used.

Currently, solving large, detailed and realistic optimization problems faster is the aim of the mining industry. According to Newman et al. (2010) recent researchers focus on integrating LTPP optimization problems rather than sub-divided approaches. Developed hardware and software will help researchers solve large-scale, nonlinear problems with uncertainties.

9. Conclusion and Limitations of Current Planning and Optimization Techniques

This paper is a review of optimization models for open pit mine planning and waste management. Many researchers have studied deterministic and uncertainty-based approaches to maximize the NPV of mining operations. There are two main approaches in dealing with the mine production planning problem. First, the ultimate pit limit is determined as well as series of pushbacks using LG algorithm with parametric analysis and then production planning is generated using mathematical programming. This is the most used strategy. Second, using genetic algorithms ultimate pit limits and production planning are determined simultaneously.

Different optimization techniques for LTPP problems are presented including Linear Programming (LP), Mixed Integer Linear Programming (MILP), Integer Programming (IP), Dynamic Programming (DP), and Goal Programming (GP).

9.1. Conclusions

Johnson (1969) introduces a Linear Programming model that considers cut-off grade, however, it has too many constraints and it extracts fractional blocks. Gershon (1983) presentes a practical Mixed Integer Programming model for block sequencing. His model requires one slope constraints per block, however, it does not consider cut-off grade and it is not suitable for large deposits. Askari et al. (2011) introduce MILP model that maximizes the NPV while meeting all operational constraints. The developed model proves to be able to handle deterministic large-scale mine production problems. Badiozamani (2014) presentes a MILP model that integrates production scheduling with waste management and in-pit tailings deposition. The model solves the large-scale problem to optimality. These models do not consider uncertainties associated with variables like grade and metal prices.

Integer Programming models are presented by many researchers. Dagdelen and Johnson (1986) present a model that does not extract fractional blocks which is the main advantage. The model of Akaike and Dagdelen (1999) considers stockpiling and cut-off grade, however, it is not suitable for large deposits and there is a difference between practical and theoretical solution. Ramazan et al. (2005), presented a model that reduces the number of binary variables, minimizes the difference between practical and theoretical solution, and maximizes NPV. The optimal solution depends on pushbacks that should be generated before scheduling and the model is not easy to apply. Caccetta and Hill (2003) introduced a model that considers all operational constraints and is suitable for medium size deposits. It does not take into account dynamic cut-off grade.

Roman, Eleveli, Underwood and Tolwinski are some researchers who presented models based on dynamic programming (Osanloo et al., (2008). Their models depict that they are practical, able to optimize the pit limit and block sequencing at the same time and they consider all operational constraints. However, they do not consider dynamic cut-off grade and are not suitable for large deposits, and most importantly optimal solution is not guaranteed.

Goal Programming technique has the capability of considering multiple material types, multiple elements, and multiple destinations. GP and MILP are suitable mathematical programming models for LTPP problems and some efforts have been made to combine them for solving industrial problems. It is known as Mixed Integer Goal Programming (MIGP). It is flexible since the planner can interact with the optimization process by trading in some goals for others. Ben-Awuah et al. (2012) and Ben-Awuah (2013) have introduced a pioneering effort in developing an integrated mathematical programming model for incorporating oil sands mine planning and waste management using mixed integer linear goal programming (MILGP) in an optimization framework. Clustering technique is used in order to reduce the number of variables used in formulating the mathematical model. Decreasing the number of variables using block clustering methods are considered powerful tools to solve LTPP problems effectively. Tabesh (2015) introduced a MILP model to maximize NPV considering technical and operational constraints in addition to stockpiling schedule. The model is able to determine the optimum stockpiling strategy and the optimum mining and processing schedule in reasonable processing time. The mining-units are block clusters. These models were developed based on deterministic optimization frameworks.

Since mining projects aim at maximizing NPV and minimizing the negative impact of uncertainties, uncertainty-based techniques to solve LTPP problems have been studied by many researchers. Grade and geological uncertainties are expected as the distance between drillholes are relatively wide. Uncertainties are classified into; in-situ grade uncertainty which is the major source of inconsistencies, technical mining specification uncertainty, such as extraction capacities and slope consideration, and economic uncertainties including capital and operating costs. The uncertainty related to input parameters can increase the difference between calculated and realized NPV. Dimitrakopoulos and Ramazan (2008) stated that Rovenscroft introduced a risk analysis model using deterministic algorithms. The model shows the impact of uncertainty, however, it cannot quantify the risk and the solution is not optimal. Dowd (1997) introduced a risk analysis model using dynamic programming. His model is able to quantify the risk, but the solution is not optimal. Dimitrakopoulos and Ramazan (2003) presented a model that use linear goal programming. Although the model generated a schedule that reduces the risk of uncertainty at the early stage of production and considers block access in production planning, it extracts fractional blocks and does not maximize the NPV.

Ramazan and Dimitrakopoulos (2004) use integer programming for their model. The model maximizes NPV considering block access. It is complicated and most importantly, the integration of grade uncertainty and production planning has not been achieved. Gody and Dimitrakopoulos (2003) presented meta-heuristics model that integrates ore body uncertainty, waste management and economic and mining considerations. The model generates optimal mining rates for life of mine. It is complicated and does not guarantee the optimal solution. Koushavand (2014) introduces a MILP model for LTPP based on grade uncertainty and considering stockpiling. He introduces a cost of grade uncertainty as a new term in LTPP problem. The model shows that grade uncertainty has linear and quadratic effects on NPV. The grade uncertainty is reduced by considering stockpiling.

9.2. Limitations

9.2.1. Incorporating Uncertainties

Uncertain input variables (especially grade) should be considered in LTPP problems. That will minimize the difference between theoretical and actual NPV, and will result in a high degree of confidence for mining projects. Comprehenive models that consider the impact of grade uncertainty on stockpiling and cut-off grade optimization in LTTP problems must be developed.

9.2.2. Integrating More Areas in the LTPP

The integration of more areas in open pit mine planning will reduce the gap in the current literature. Maximization of NPV, solid waste management, minimization of dyke construction and tailings disposal costs, minimization of material handling costs for reclamation, and stockpiling materials that are below cut-off grade for limited duration are all areas that can be considered to improve the mine planning process and the performance of the models.

9.2.3. Solving LTPP and Ultimate Pit Limit Problems Simultaneously

Current approaches are still incapable of solving LTPP and ultimate pit limit problems at the same time. The effect of time on ultimate pit limit has not been considered properly. According to some researches meta-heuristic approaches such as genetic algorithms and simulated annealing can be useful for solving these combined LTPP problems.

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