Block-Cave Production Scheduling Using Mathematical Programming

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Abstract

As the mineral resources near the surface are being exploited, the mining operation goes deeper into the ground, waste removal rates increase, capital and operation costs become higher, and environmental impacts are more evident. In such a situation, underground mining with lower waste removal and less environmental impact are becoming more attractive. Among underground methods, block-cave mining with its high rate of production, low operational cost, and automated systems can be one of the best choices instead of surface mining or block-cave mining can be considered as part of production after surface mining during the life of mine. Production scheduling is one of the critical steps in the block-caving design process so that an optimum production scheduling could add significant value to a mining project. Block-cave mining operations can be complicated and behave as a non-linear phenomenon. So, production scheduling for this kind of operations with lots of involving dynamic parameters could be a big size problem in non-linear environment. This research uses mining background and its parameters, with the help of mathematical programming and computer science, to model the production scheduling in block-cave mining to maximize the net present value of the project using MILP and also implement MIQP as non-linear tool to minimize the difference between the objective and the target tonnage of the mining project considering the related constraints of the operations.

1. Introduction

In block cave mining, the gravity of the material is simply used for extraction. It means that compare to other underground methods, extraction is easier and cheaper. Theoretically it is simple but practically it is complicated because many constraints are involved. Omitting any of them can result in inefficiency or even failure in operations. For each drawpoint, its draw rate can affect the draw rate and even grade of other drawpoints which are located in its neighborhood. This nonlinear relationship makes the production scheduling complicated. An ununiformed shape of extraction from drawpoints can result in a high amount of mixing between the draw columns which are located in its neighborhood. This research proposes a methodology to model a block cave operation using mathematical programming. The aim is to minimize the gap between certain amount of production (as an initial expectation) and the results from the optimization. Quadratic programming as a strong tool is used to model this non-linear relationship in the block-cave mining. In this research, two methodologies with different objective functions are proposed to model the block

Pourrahimian, Y., Ben-Awuah, E., and Askari-Nasab, H. (2015), Mining Optimization Laboratory (MOL), Report Six, © MOL, University of Alberta, Edmonton, Canada, Pages 250 , ISBN: 978-1-55195-356-4, pp. 189-204.

cave operations: mixed integer linear programming (MILP) and mixed integer quadratic programming (MIQP). The proposed models are tested for a real case block cave mine.

2. Summary of literature review

Production scheduling in caving means determining how much of material should be extracted from each drawpoint in each period during the life of the mine. Having an optimum extraction of drawpoints can add a significant value to the mining project. There are many constraints limiting the production: geotechnical, economic, environmental, and operational. Mathematical programming is a useful tool to model such a problem in order to find the best solutions for catching the goals while considering the related constraints. Like open pit mining, many researchers have already worked on production scheduling for block cave mining. They have mostly used LP (Guest, Van Hout, & Von Johannides, 2000; Hannweg & Van Hout, 2001; Winkler, 1996), MILP (Alonso-Ayuso, et al., 2014; Chanda, 1990; Epstein, et al., 2012; Guest, et al., 2000; Parkinson, 2012; Pourrahimian, 2013; Rahal, 2008; Rahal, Dudley, & Hout, 2008; Rahal, Smith, Van Hout, & Von Johannides, 2003; Rubio, 2002; Rubio & Diering, 2004; Smoljanovic, Rubio, & Morales, 2011; Song, 1989; Weintraub, Pereira, & Schultz, 2008; Winkler, 1996), and QP (Diering, 2012; Rubio & Diering, 2004). A detailed literature review can be find in (Khodayari & Pourrahimian, 2015b).

3. Methodology

We model the production scheduling of a block-cave mining operations using two different types of mathematical programming: mixed-integer linear programming (MILP) and mixed-integer quadratic programming (MIQP). The models carry same constraints with different objective functions. Models, the related indices, variables, and parameters are discussed in this section.

3.1. Notation

Indices

$t \in \{1,...,T\}$	Index for scheduling periods
$n \in \{1,...,N\}$	Index for drawpoints
g_n	Average grade of draw column associated with drawpoint n
$tonnage_n$	Ore tonnage of draw column associated with drawpoint n

Variables

tarton_n^t	Target tonnage of extraction for the drawpoint n at period t based on the solution of the production scheduling problem (the optimum tonnages that we are looking for, considering the problem's constraints)
$obj\ \text{ton}_n^t$	Objective tonnage of extraction for the drawpoint n at period t based on the production goals
$X_n^t \in [0,1]$	Continues decision variable that represents the portion of draw column n which is extracted in period t
$(Y1)_n^t \in [0,1]$	Binary variable which determines whether drawpoint "n" in period "t" is active [$(Y1)_n^t = 1$] or not [$(Y1)_n^t = 0$]
$(Y2)_n^t \in [0,1]$	Binary variable which determines whether drawpoint "n" till period "t" (periods 1, 2,..,t) has started extraction [$(Y2)_n^t = 1$] or not [$(Y2)_n^t = 0$]

$(Y3)_n^t \in [0,1]$	Binary variable which determines whether the Depletion Percentage (DP) of drawpoint "n" in period "t" is less than Draw Control Factor (DCF) or not (based on the draw rate curve): $if\ DP \le DCF \ \rightarrow\ (Y3)_n^t = 0$ $if\ DP \ge DCF \ \rightarrow\ (Y3)_n^t = 1$
$(Y4)_n^t \in [0,1]$	Binary variable which determines whether the material which has remained from drawpoint "n" in period "t" is greater than maximum draw rate (DRMax) based on the draw rate curve [$(Y4)_n^t = 1$] or not [$(Y4)_n^t = 0$]

Parameters

price	Metal price
ir	Interest rate of return
rec	Metal recovery in the processing plant
cost	Operating cost per ton of ore (including mining and processing)
DRMin	Minimum production rate based on the draw rate curve
DRMax	Maximum production rate based on the draw rate curve
M_{min}	Minimum mining capacity base on the capacity of mining equipment
M_{max}	Maximum mining capacity base on the capacity of mining equipment
G_{min}	Minimum production grade
G_{max}	Maximum production grade
ActMin	Minimum number of active drawpoints in each period
ActMax	Maximum number of active drawpoints in each period
$DP_n^t \in [0,1]$	Depletion Percentage which is the portion of draw column "n" which has been extracted till period "t"
$(DP4)_n^t$	Depletion Percentage which is the portion of draw column "n" which has been extracted till period "t-1"
$DCF \in [0,1]$	Draw Control Factor which is the turning point at the draw rate curve
M	An arbitrary big number

3.2. MILP objective function

The MILP objective function is going to maximize the net present value (NPV) of the mining project during the life of mine:

$$Maximize \quad NPV = \sum_{t=1}^{T}\sum_{n=1}^{N}\frac{DEV_n^t}{(1+i)^t}\times X_n^t = \sum_{t=1}^{T}\sum_{n=1}^{N}\frac{[(price\times g\times ton\times rec)-\cos t]_n^t}{(1+ir)^t}\times X_n^t \qquad (1)$$

3.3. MIQP objective function

Production goals determine the required tonnage of extraction in a mining project. But there are always some constraints that control the goals. In this research, the optimization problem is looking for the best solution to reduce the gap between the expected production and the practical production considering the related constraints. The objective function is going to minimize the difference between the objective and the target tonnage:

$$Minimize \ \sum_{t=1}^{T}\sum_{n=1}^{N}(\mathrm{tarton}_n^t - objton_n^t)^2 \tag{2}$$

$$= \sum_{t=1}^{T}\sum_{n=1}^{N}(\mathrm{tarton}_n^t)^2 - (2*\mathrm{objton}_n^t)*\mathrm{tarton}_n^t$$

$$= \sum_{t=1}^{T}\sum_{n=1}^{N}(tonnage_n * X_n^t)^2 - (2*\mathrm{objton}_n^t * tonnage_n)*X_n^t$$

Extraction from drawpoints while having a uniform extraction surface is one of the most important concerns in block cave mining, to minimize the dilution, which can be improved by solving this optimization problem.

3.4. Constraints

There are lots of geotechnical, operational, and economical constraints related to mining projects which limit the whole system in achieving the operational and strategic plans. This research will try to make sure that related constraints are considered so that the model's results can be applicable in real case block-cave mining.

3.4.1. Binary variables

These sets of constrains define the required binary variables. Totally 4 sets of binary variables are defined in order to be able to apply the related constraints for the:

Set 1 $((Y1)_n^t \in [0,1], \begin{Bmatrix} n \in N \\ t \in T \end{Bmatrix})$:

This set contains N*T variables, it means for each drawpoint there is one variable per each period. Variables (N*T)+1 to (2*N*T) in the model are allocated to this set. This set determines whether drawpoint "n" is active in period "t" or not; if any extraction from drawpoint "n" at period "t" occurs it means the drawpoint is active (x>0) then Y1=1 and if there is no any extraction (x=0) it means it is not active then Y1=0. The mathematical formulation of this set of constraint includes two parts of equations:

$$Y1 - Mx \le 0 \tag{3}$$

$$x - Y1 \le 0 \tag{4}$$

Set 2 $((Y2)_n^t \in [0,1], \begin{Bmatrix} n \in N \\ t \in T \end{Bmatrix})$:

This set contains variables N*T variables, it means for each drawpoint there is one variable per each period. Variables (2*N*T)+1 to 3*N*T in the model are allocated to this set. This set determines whether the depletion percentage of drawpoint "n" in period "t" is 0 or not. Depletion percentage (DP) is the summation of the x values for drawpoint "n" from period "1" till period "t" based on the draw rate curve.

$$DP = \sum_{t=1}^{t} x_n^t \tag{5}$$

If the depletion percentage is 0 (DP=0) then Y2=0 and if depletion percentage is greater than 0 (DP>0) then Y2=1. Two equations are defined for this set:

$$DP - Y2 \le 0 \tag{6}$$

$$Y2 - M * DP \leq 0 \tag{7}$$

Set 3 $\left((Y3)_n^t \in [0,1], \begin{Bmatrix} n \in N \\ t \in T \end{Bmatrix} \right)$:

This set contains variables N*T variables, it means for each drawpoint there is one variable per each period. Variables (3*N*T)+1 to 4*N*T in the model are allocated to this set. This set determines whether the depletion percentage (DP) is in the second area of the depletion curve or it is in the third area of the depletion curve. If the depletion percentage (DP) is in the second area of the depletion curve (DP < DCF) then Y3=0 and if it is in the third area of the depletion curve (DP > DCF) then Y3=1. DCF is Draw Control Factor in the draw rate curve (DCF∈[0,1]). This set contains 2 equations:

$$DP - Y3 \leq DCF \tag{8}$$

$$Y3 - DP \leq 1 - DCF \tag{9}$$

Set 4 $\left((Y4)_n^t \in [0,1], \begin{Bmatrix} n \in N \\ t \in T \end{Bmatrix} \right)$:

This set contains variables N*T variables, it means for each drawpoint there is one variable per each period. Variables (4*N*T)+1 to 5*N*T in the model are allocated to this set. This set determines whether the remained material in draw column "n" at period "t" is less than maximum allowable draw rate (DRMax) or not. If the remained material in draw column "n" at period "t" is less than DRMax then Y4=0 if it is greater than DRMax then Y4=1. Two equations in the constraints define this set:

$$(ton - DP4 * ton) - DRMax \leq M * Y4 \rightarrow -(DP4 * ton) - M * Y4 \leq DRMax - ton \tag{10}$$

$$DRMax - (ton - DP4 * ton) \leq M * Y4 \rightarrow (DP4 * ton) + M * Y4 \leq M + ton - DRMax \tag{11}$$

3.4.2. Mining capacity

This constraint defines the whole production from all drawpoints for each period of time. It can be determined based on the whole operations system capacity. It helps to make sure that the system is working optimally.

$$\forall t \in T \rightarrow M_{\min} \leq \sum_{n=1}^{N} ton_n \times X_n^t \leq M_{\max} \tag{12}$$

3.4.3. Average grade of production

The average grade of the extracted material should be in an acceptable range. This constraint helps to have a uniform extraction of the ore during the mine life and can be determined based on processing plant requirements.

$$\forall t \in T \rightarrow G_{\min} \times \left(\sum_{n=1}^{N} ton_n \times X_n^t\right) \leq \sum_{n=1}^{N} g_n \times ton_n \times X_n^t \tag{13}$$

$$\forall t \in T \rightarrow \sum_{n=1}^{N} g_n \times ton_n \times X_n^t \leq G_{\max} \times \left(\sum_{n=1}^{N} ton_n \times X_n^t\right) \tag{14}$$

3.4.4. Reserve

The BHOD is calculated before applying the mathematical model. This constraint controls the amount of resource that is going to be extracted during the life of mine (based on the BHOD).

$$\forall n \in N \rightarrow \sum_{t=1}^{T} X_n^t \leq 1$$
$$\text{or} \sum_{t=1}^{T} X_n^t = 1 \qquad (15)$$

3.4.5. Number of allowable active drawpoints at each period of time

This constraint controls maximum and minimum number of active drawpoints at each period of time.

$$\forall t \in T \rightarrow \sum_{n=1}^{N} (\mathrm{Y}1)_n^t \leq ActMax \qquad (16)$$

$$\forall t \in T \rightarrow \sum_{n=1}^{N} (\mathrm{Y}1)_n^t \geq ActMin \qquad (17)$$

3.4.6. Development direction and mining precedence

Extraction of each drawpoint can be started if the previous drawpoints (based on the defined precedence) have been started before. Two of the key steps in block-caving operation scheduling are development direction and drawpoints' precedence determination. More details about this constraint can be found in (Khodayari & Pourrahimian, 2015a). The priorities for development direction and precedence of extraction of the case study of this research is shown layout in Fig. 1.

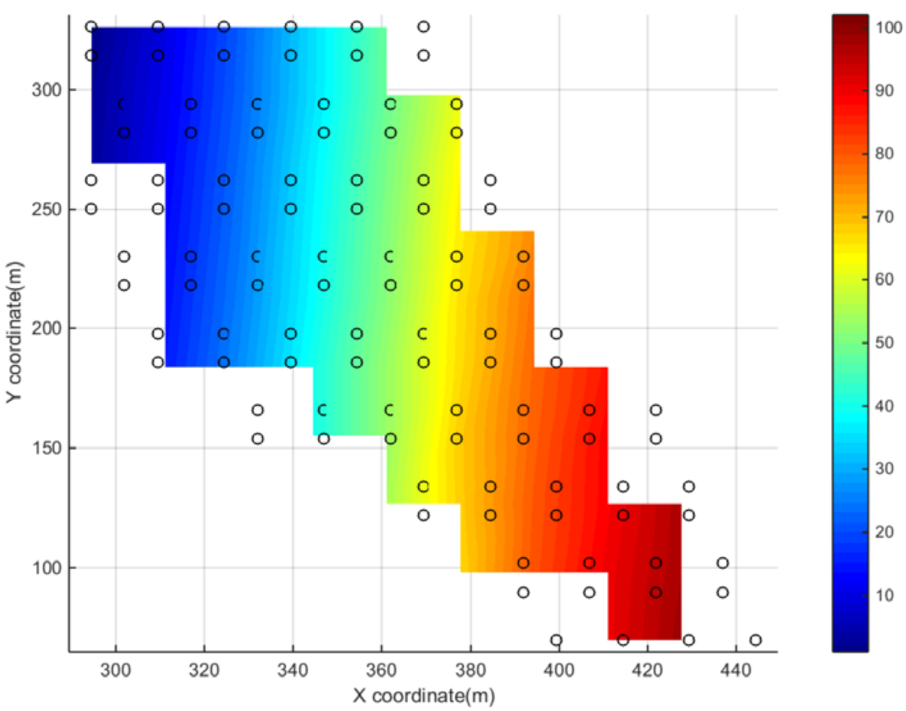

Fig. 1. Mining direction determination for block-cave layout

The precedent constraint is defined by the following function:

$$\forall n \in N \;\&\; t \in T \rightarrow (Y2)_n^t \leq (Y2)_{n-1}^t \tag{18}$$

Y2 is the second set of binary variables (section 3.4.1). It means that i[th] drawpoint can start its production if only if drawpoint i-1 has already started its production in previous periods or they can be started at the same period. i represents the precedence of that specific drawpoint.

3.4.7. Continuous mining

This constraint makes sure that if extraction from a drawpoint starts in a period, then the extraction will continue till end of its life. It means that there is no gap between extractions for drawpoints in their lives of production.

$$\forall n \in N \;\&\; t \in T \rightarrow (Y1)_n^t + (DP4)_n^t \geq (Y1)_n^{t-1} \tag{19}$$

Y1 is the first set of binary variable (section 3.4.1). DP4 represents the total of draw percentage for periods 1,…,t-1 which doesn't include the current period (t).

3.4.8. Draw rate

This constraint controls the production rate for each drawpoint based on the draw rate curve. Draw rate curve is a function of the material that has been already extracted during the previous periods of production in a drawpoint. It makes sure that the neighboring ratio is considered to have an uniform extraction profile during the production which results in low mixing and dilution. The draw rate curve is divided to different area based on the amount of extraction (depletion percentage). Fig. 2 Presents draw rate curve and its different area.

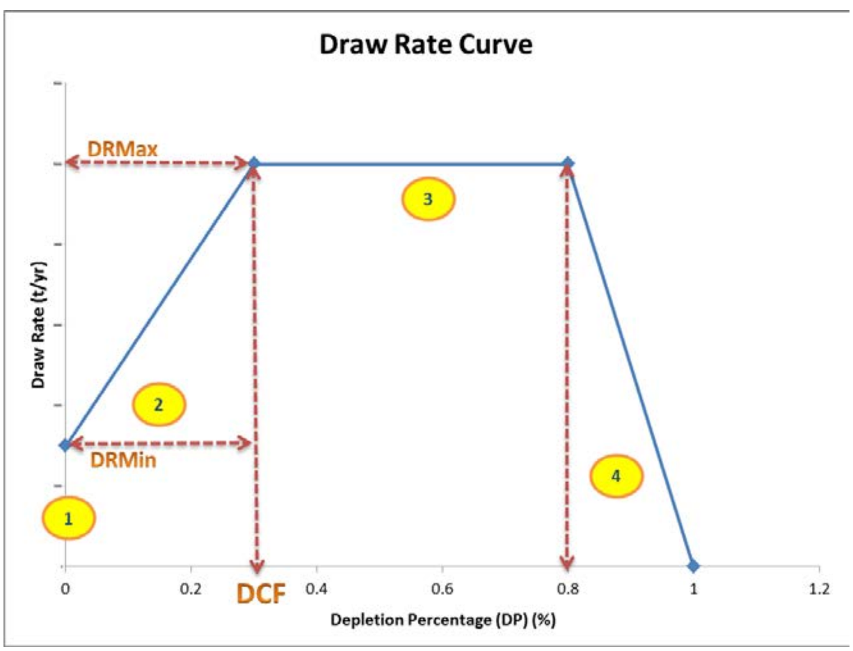

Fig. 2. Dividing draw rate curve to model it using binary variables

For each area (1, 2, 3, 4) in the draw rate curve, some formulations will be added to the model to have the production based on the draw rate curve. The current model doesn't include this constraint and we are still working on that.

4. Implementation of the models

A real case data for a block-cave mining operation is implemented for testing both the MILP and MIQP models. The resource estimation shows that the main element of the ore body is Copper. Mine development has been finished and the life of mine is 5 years. The mine has been designed and the production is going to be based on 102 drawpoints. Fig. 3 shows the designed lay out.

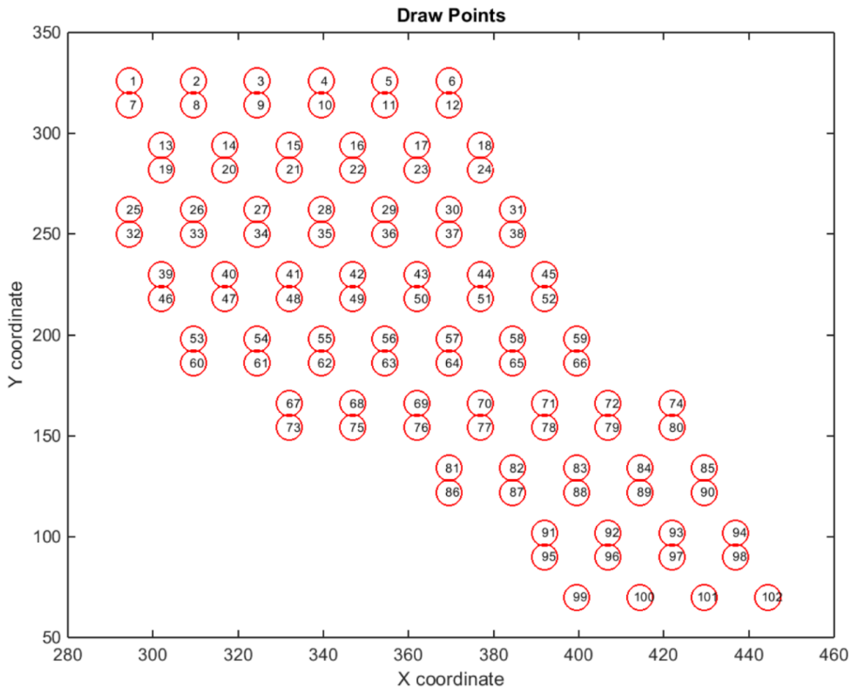

Fig. 3. The layout of drawpoints (the numbers inside drawpoints are just initial record numbers)

Based on the reserve estimation, total tonnage is 13.4 million tonne with the average weighted grade of %1.33 of Cu (the grade range is %0.5 to %1.61). The constraints are the same for both models. The scheduling parameters are presented in table 1.

Table 1. The scheduling parameters

Parameter	Value	unit
G_{min}	0.9	%
G_{max}	1.6	%
M_{min}	2.5	Mt
M_{max}	3	Mt
ActMin	50	-
ActMax	90	-
Price	4,318	USD/tonne
Cost	18	USD/tonne
Recovery	85	%
Interest rate	10	%
EGap	5	%

4.1. MILP model

We tested the MILP model for the case study, the model contains 5*N*T decision variables in which the first N*T variables are continuous and the rest are binary variables. As it was mentioned, the objective function is going to maximize net present value of the project considering the related constraints. The resulted production during the life of mine is shown in Fig. 4.

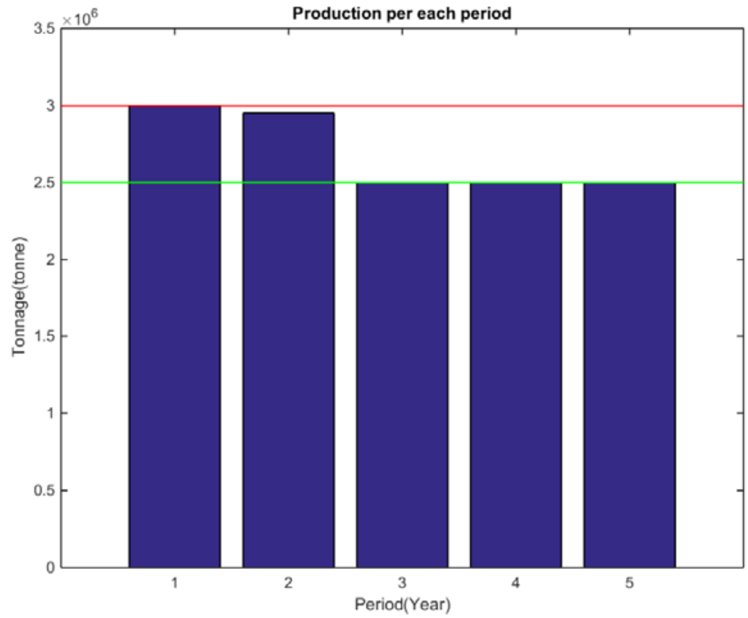

Fig. 4. Total production during the life of mine (the green and red lines are the minimum and maximum mining capacity respectively) (MILP results)

It can be seen that the MILP model tries to produce more during the first years of production to maximize the NPV while satisfying the mining capacity constraint. The average production grade is presented in Fig. 5.

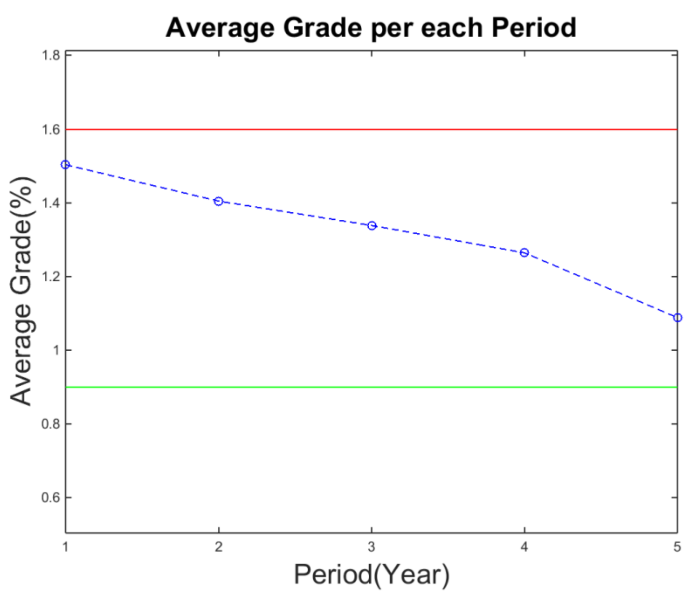

Fig. 5. Average production grade (%Cu)–the green and red lines show the acceptable range of grade for production (MILP results)

The MILP model tries to extract higher grades first while satisfying the grade constraint. The precedence of the drawpoints is determined based on the direction on Fig. 1. The considered precedence is presented in Fig. 6.

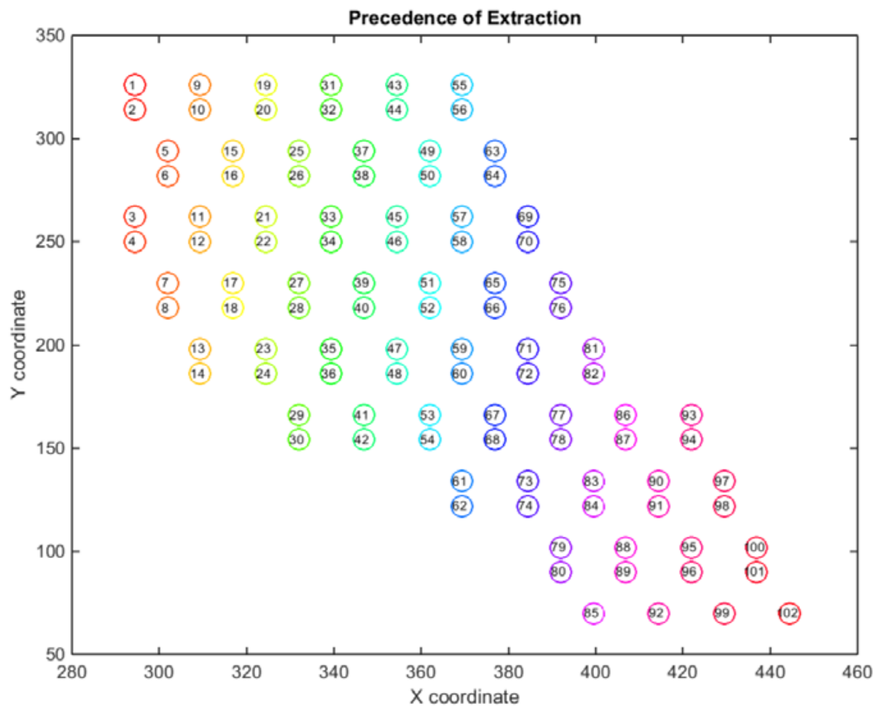

Fig. 6. Precedence of the drawpoints based on the defined direction

Considering the precedence, the extraction of drawpoints starts from number 1 to 102. Fig. 7 shows the starting period for drawpoints during the life of mine.

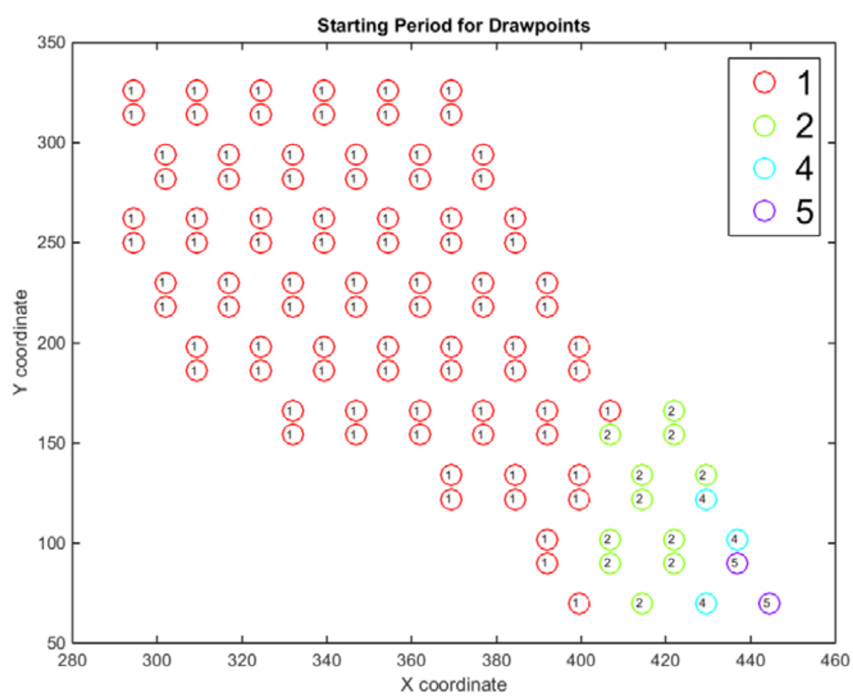

Fig. 7. Starting period for drawpoints during the life of mine (MILP results)

It can be seen that the results follow the defined precedence. Draw rates of the drawpoints during their production life doesn't follow a specific order or trend (Fig. 8). It needs more research to add the draw rate constraint to the optimization model.

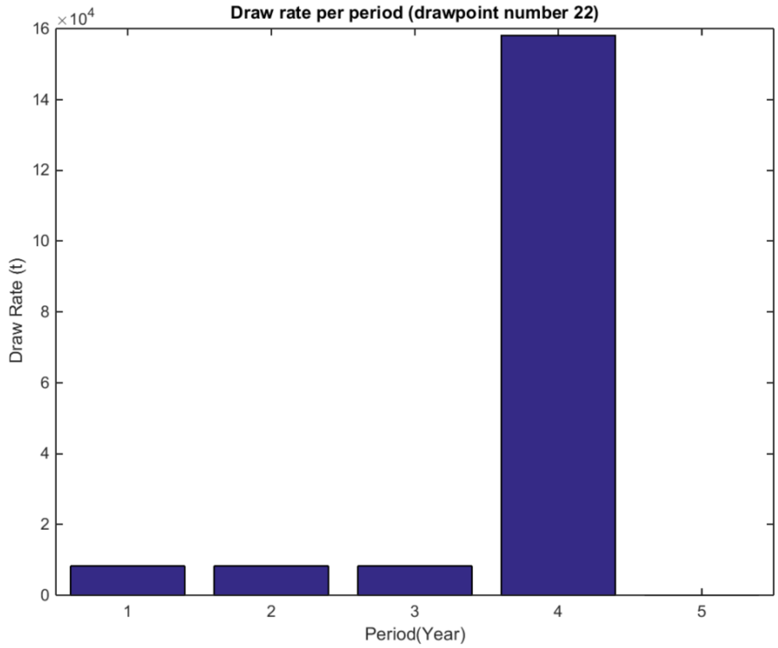

Fig. 8. Draw rate for drawpoint number 22 during the life of mine (MILP results)

The cave surface or the profile of extraction resulted from MILP model shows high fluctuations between drawpoints and their neighbors (Fig. 9). This extraction increases the probability of dilution in the production.

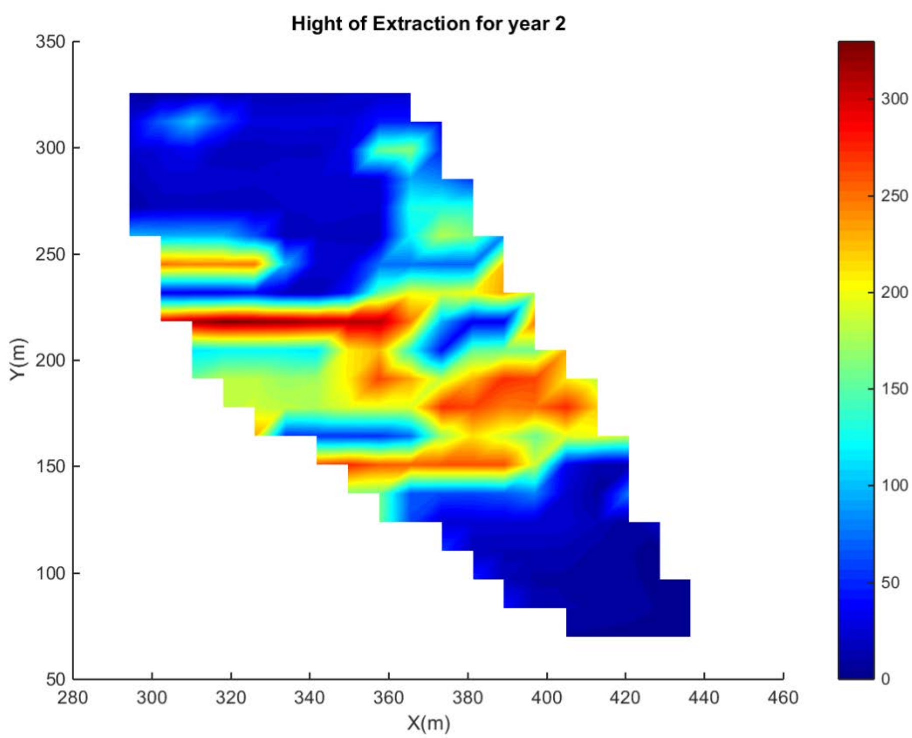

Fig. 9. Extraction profile (MILP results)

For better visualization, the 3-D plot of the extraction surface is presented in Fig. 10. It can be seen that the sharp edges are so common in the MILP model.

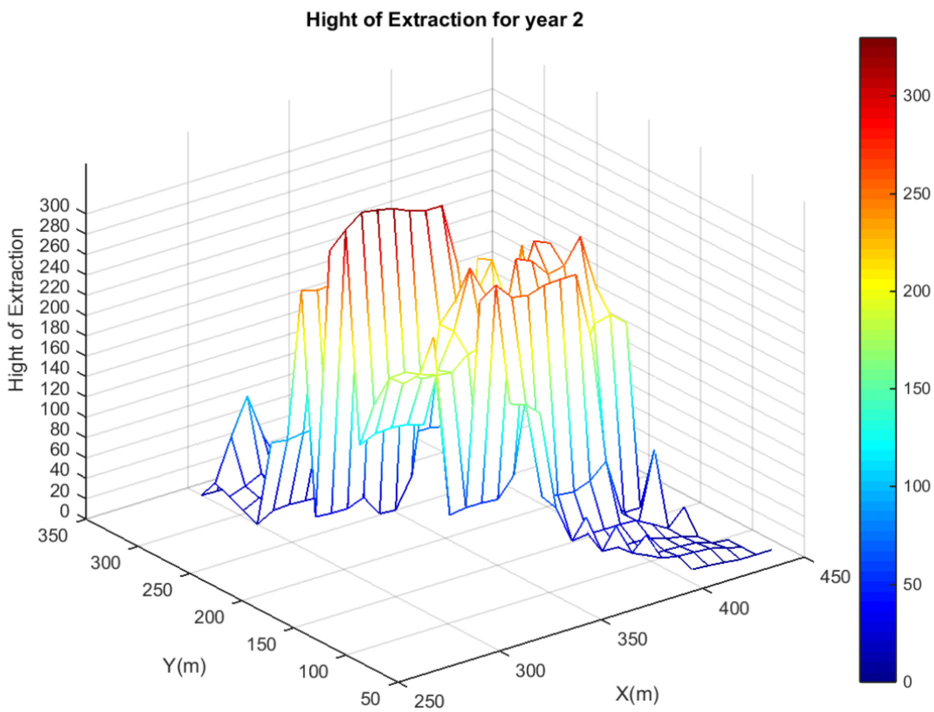

Fig. 10. Extraction profile surface (MILP results)

4.2. MIQP model

We used the same case study for testing the MIQP model. The model contains 5*N*T variables in which the first N*T variables are continuous and the rest are binary variables. The objective function is going to minimize the difference between an initial tonnage of extraction and the tonnage of extraction which is based on the production scheduling. The resulted production during the life of mine is shown in Fig. 11.

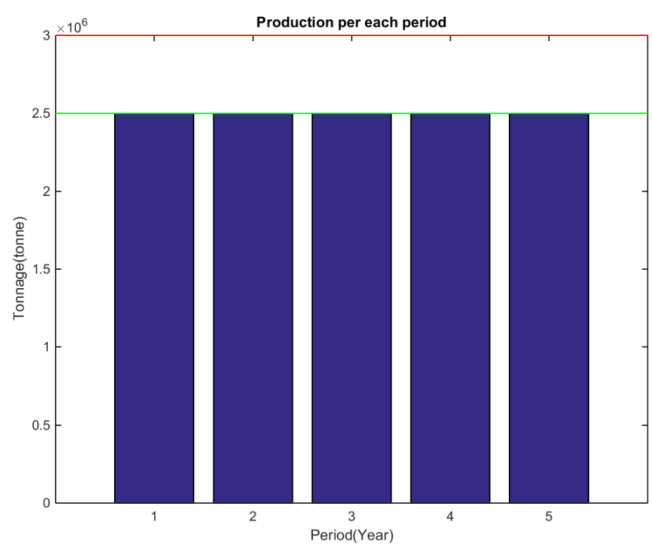

Fig. 11. Total production during the life of mine (the green and red lines are the minimum and maximum mining capacity respectively) (MIQP results)

It can be seen that the MIQP model tries to produce an even amount during the life of the mine while satisfying the mining capacity constraint. The average production grade is presented in Fig. 12.

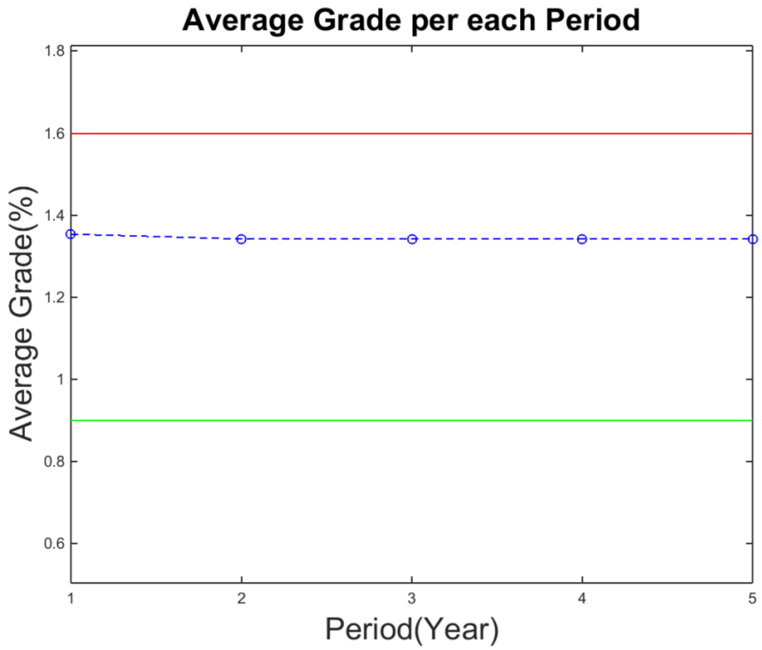

Fig. 12. Average production grade (%Cu)–the green and red lines show the acceptable range of grade for production (MIQP results)

The MIQP model produce an almost fix grade for production. Considering the precedence, the extraction of drawpoints starts from number 1 to 102. Fig. 13 shows the starting period for drawpoints during the life of mine.

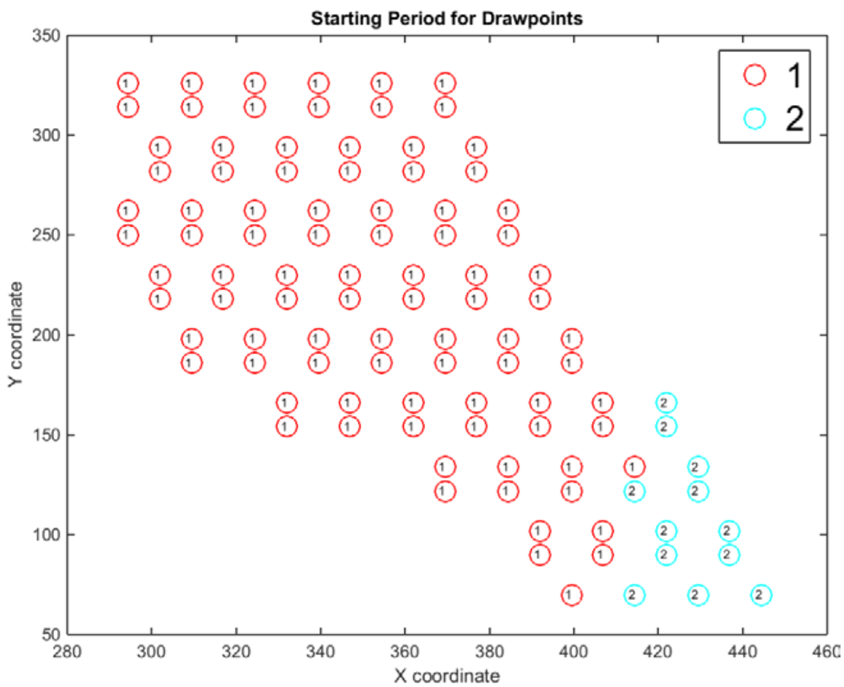

Fig. 13. Starting period for drawpoints during the life of mine (MIQP results)

It can be seen that the results follow the defined precedence. Draw rates of the drawpoints during their production life resulted from MIQP model is reasonable even without adding the draw rate constraint (Fig. 14).

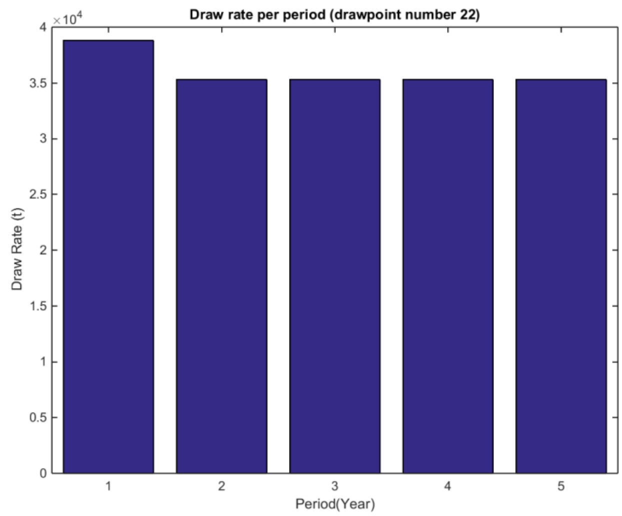

Fig. 14. Draw rate for drawpoint number 22 during the life of mine (MIQP results)

The profile of extraction resulted from MIQP model is more uniformed compare to the MILP model, as we expected (Fig. 15).

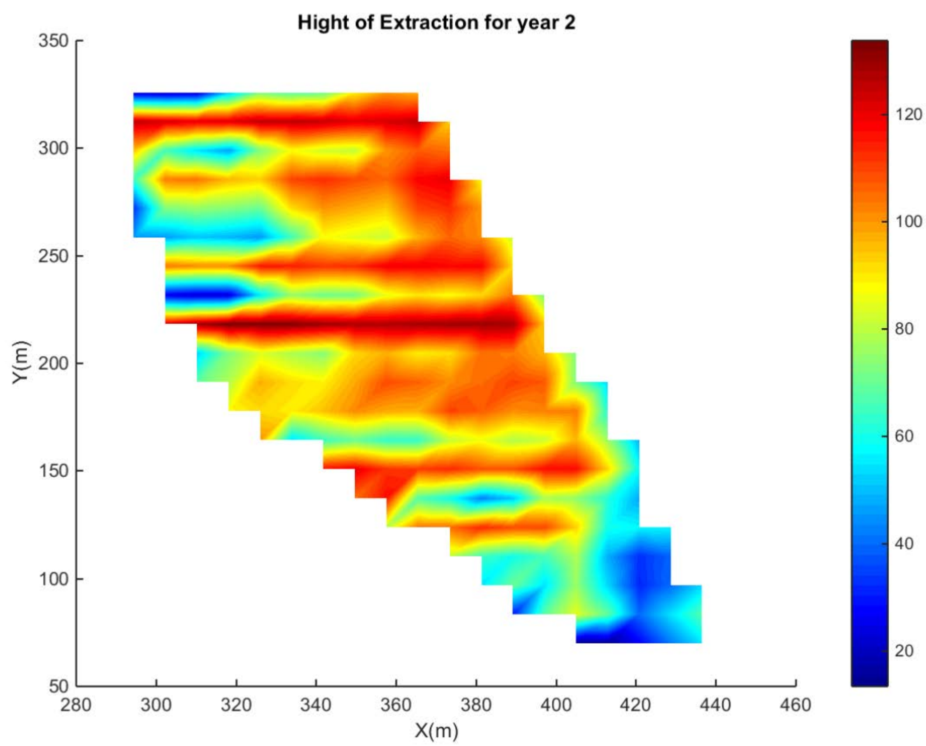

Fig. 15. Extraction profile (MIQP results)

For better visualization, the 3-D plot for extraction profile resulted from MIQP model is presented in Fig. 16. It shows that the MIQP model can generate a more practical profile compare to MILP model.

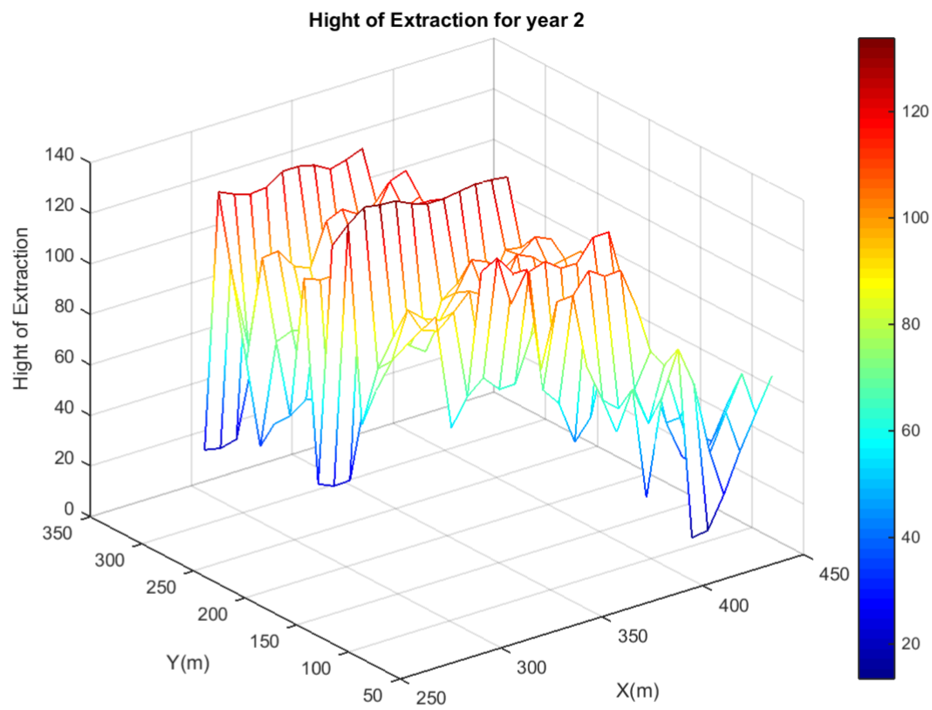

Fig. 16. Extraction profile surface (MIQP results)

5. Conclusion

In this research, we tested both MILP and MIQP models for production scheduling in block-cave mining operation. The MILP model aims to maximize NPV of the project while the MIQP model tries to minimize the tonnage deviation for achieving a uniform extraction profile. We implemented both models with same constraints for one case study. Results show that the MILP model tries to produce more in first years with higher grades. This will result in ununiformed extraction profile with high probability of dilution. The MIQP model extracts from the drawpoints smoothly with a very low fluctuation of tonnage and grade during the life of mine. This will generate uniform extraction profile with low probability of dilution.

6. References

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