

A Review of Mathematical Models and Algorithms in Block-Caving Scheduling

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Abstract

Production scheduling is one of the most important problems in a mining operation, as it has a significant impact on a project's profitability. As open-pit mines go deeper, because of the massive amount of waste removal which is required to extract the ore as well as high operational costs per tonne, underground mining has become more attractive. Among underground mining methods, block caving could be a good alternative because its high rate of production is similar to that of open-pit mining, and it has the added advantage of low operational costs. Relying only on manual planning methods or computer software based on heuristic algorithms will lead to mine schedules that are not the optimal global solution. Block-caving scheduling has been the subject of a lot of research. Most studies have applied mathematical programming, simulation, and stochastic approaches. This paper reviews mathematical programming applications in block-caving scheduling, highlights, and suggestions for future works.

1. Introduction

These days, most surface mines work in a higher stripping ratio than in the past. In the following conditions, a surface mine can be less attractive to operate and underground mining is used instead. These conditions are (i) too much waste has to be removed in order to access the ore (high stripping ratio), (ii) waste storage space is limited, (iii) pit walls fail, or (iv) environmental considerations could be more important than exploitation profits (Newman et al., 2010).

Among underground methods, block-cave mining, because of its high production rate and low operation cost, could be considered an appropriate alternative. Projections show that 25 percent of global copper production will come from underground mines by 2020. Mining companies are looking for an underground method with a high rate of production, similar to that of open-pit mining. Therefore, there is an increased interest in using block-cave mining to access deep and low-grade ore bodies.

Production scheduling is one of the most important steps in the block-caving design process. Optimum production scheduling could add significant value to a mining project. The goal of long-term mine production scheduling is to determine the mining sequence, which optimizes the company's strategic objectives while honoring the operational limitations over the mine life. The

production schedule defines the management investment strategy. An optimal plan in mining projects will reduce costs; increase equipment use; and lead to optimum recovery of marginal ores, steady production rates, and consistent product quality (Chanda, 1990; Chanda and Dagdelen, 1995; Dagdelen and Johnson, 1986; Winkler, 1996; Wooller, 1992). Although manual planning methods or computer software based on heuristic algorithms are generally used to generate a good solution in a reasonable time, they cannot guarantee mine schedules that are the optimal global solution.

Mathematical programming with exact solution methods is considered a practical tool to model block-caving production scheduling problems; this tool makes it possible to search for the optimum values while considering all of the constraints involved in the operation. Solving these models with exact solution methodologies results in solutions within known limits of optimality.

In this paper, block caving and production scheduling in cave mining have been introduced, and mathematical programming methods which can be used as a tool for production scheduling have been discussed. Finally, the related research in this area is presented, as are some conclusions and new ideas for future studies.

2. Block caving

Generally speaking, underground mining methods can be classified in three categories: (i) caving methods such as block caving, sublevel caving, and longwall mining; (ii) stoping methods such as room-and-pillar, sublevel stoping, and shrinkage; and (iii) other methods such as postpillar cut-and-fill, and Avoca (Carter, 2011).

Block caving is usually appropriate for low grade and massive ore bodies in which natural caving could occur after an undercut layer is made under the ore-body. Laubscher (1994) refers block caving “to all mining operations in which the ore-body caves naturally after under cutting its base. The caved material is recovered using drawpoints.”

Depending on the ore-body dimensions, inclination, and rock characteristics, block caving could be implemented as block caving, panel caving, inclined drawpoint caving, and front caving. The low-cost operation could be understood from the natural caving. In other words, during the extraction period, there is no cost for caving unless some small blasting is needed to deal with hang-ups. In block caving (Fig 1), the pre-development period can last for more than five years. This is a significant period of time with no cash back. But when the production starts, the extraction network can be used for the life of the drawpoint, so the operating cost is low and production rate can be remarkable. To sum up, block caving has the lowest operating cost of all underground mining methods. In some cases its cost is comparable to that of open-pit mining.

There are three methods of block caving. In the grizzly or gravity system, the ore from the drawpoints flows directly to the transfer raises after sizing at the grizzly, and then is gravity-loaded into ore cars. In the slusher system, slusher scrapers are used for the main production unit. In the load-haul-dump (LHD) system, rubber tired equipment are used for ore handling in production level (Hustrulid, 2001). Table 1 shows some examples for each method. Caterpillar jointly with the Chilean mining company Codelco has developed a continuous haulage technology for block caving operation. In this method, the LHD at the drawpoint is replaced by a rock feeder. This device pushes the rock into the haulage access, where it drops onto a hard rock production conveyor.

The size of the caved material, the mine site location, availability of labor, and economics are some aspects which determine the block-caving system (Julin, 1992). Factors that have to be considered in block caving include caveability, fragmentation, draw patterns for different types of ore, drawpoint or drawzone spacing, layout design, undercutting sequence, and support design (Laubscher, 2011). Some large-scale open-pit mines will be transferred to underground mining as they go deeper; they need to produce in a similar rate to open-pit mines to provide their processing plants with feed, so block caving with a high production rate could be an attractive alternative. Around the world, more

than 60 mines have been closed, are operating or are planned to be mined by block caving (Woo et al., 2009).

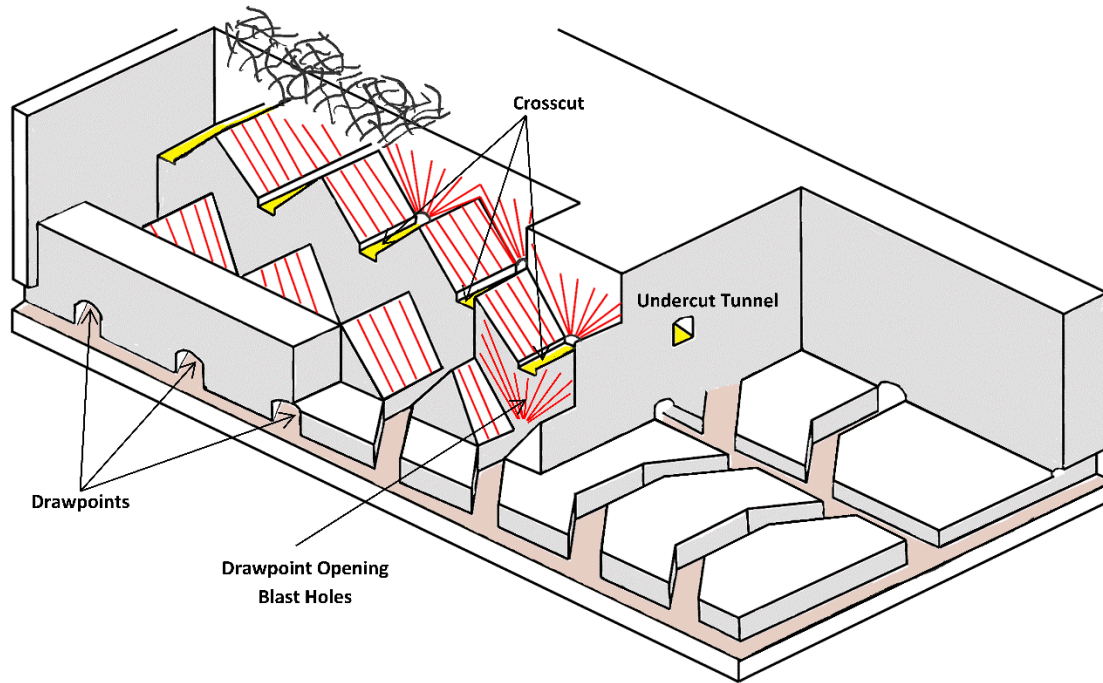


Fig 1. Block-caving mining

Table 1. Some real cases for different block-caving methods (Bergen et al., 2009; Inc., 2012; Julin, 1992; Song, 1989)

Method	Mine	Ore Type	Location
Gravity (Grizzly)	San Manuel	Copper	Arizona
	Andina	Copper	Chile
Slusher	Climax	Molybdenum	Colorado
	Tong Kuang Yu	Copper	China
LHD	Henderson	Molybdenum	Colorado
	Ertsberg	Copper	Indonesia
	El Teniente	Copper	Chile
	New Afton	Copper-Gold	Canada

Laubscher (2000) identified 10 different horizontal LHD layouts as having been use in block caving mines. Fig 2 and Fig 3 illustrate two of them. Fig 2 shows offset Herringbone. In this layout, the drawpoints on opposite sides of a production drift are offset. This helps improve both the stability conditions and the operational efficiency. This layout was used initially at the Henderson Mine, USA, and Bell Mine, Canada. Fig 3 shows the layout developed at the El Teniente Mine, Chile. In this layout, the drawpoint drifts are developed in straight lines oriented at 60° to the production drift (Brown, 2003).

One of the most critical processes in block-cave mining is undercutting. The undercutting strategy can have a significant influence on cave propagation and on the stresses induced in, and the performance of the extraction level installations (Brown, 2003; 2007). The three mostly used undercutting strategies are post, pre, and advanced undercutting. In the post-undercutting strategy, undercut drilling and blasting takes place after the production level has been developed. In the pre-undercutting method, no development or construction takes place on the production level before the undercut has been blasted. In the advanced-undercutting strategy, the production level is developed in advance of the blasting of the undercut. This method was introduced to reduce the drawpoints' exposure to the abutment stress zones, which were induced as a result of the undercutting process. The next section presents production scheduling in mining operations, and particularly in block-cave mining.

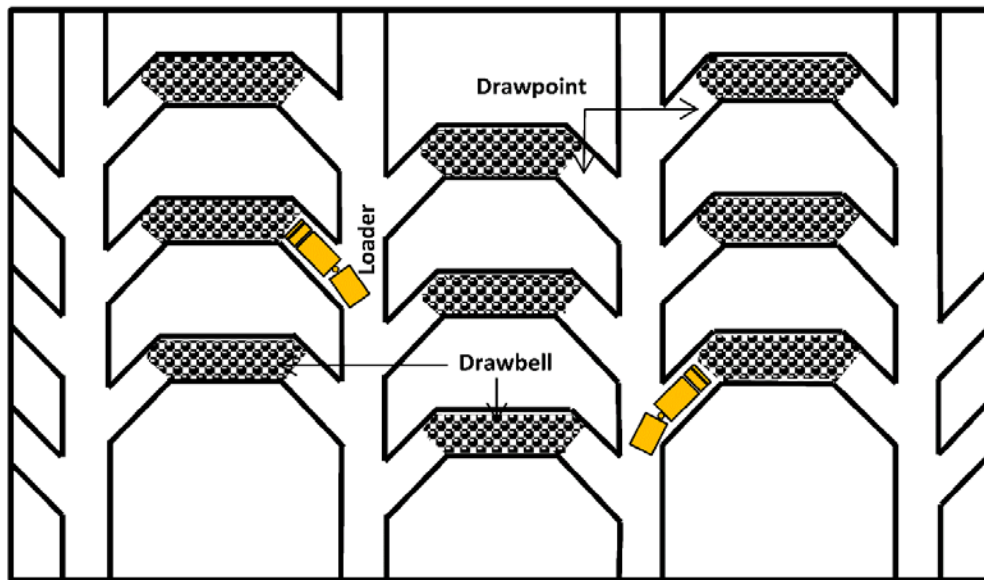


Fig 2. Typical offset Herringbone layout (after Brown, 2007)

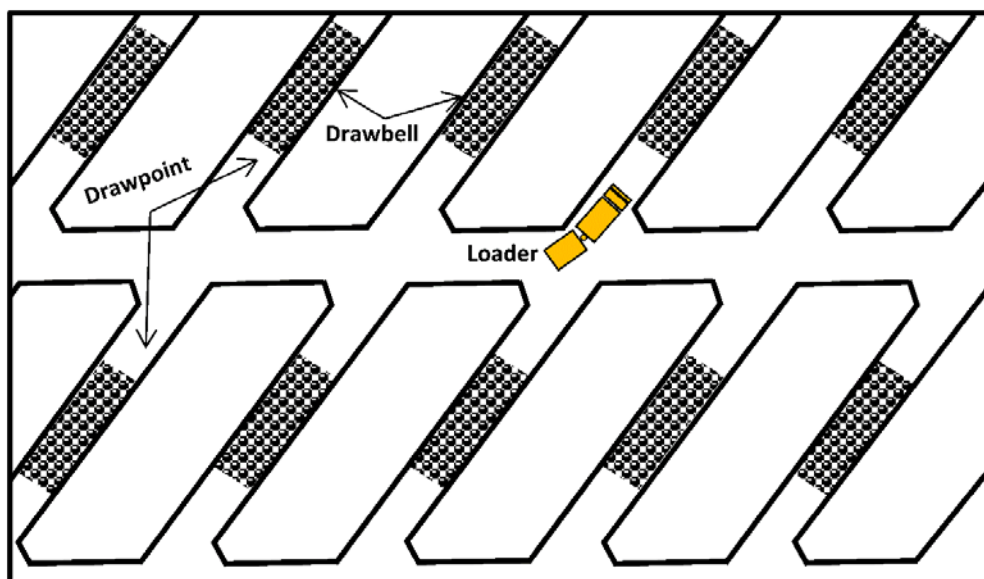


Fig 3. Typical El Teniente layout (after Brown, 2007)

3. Underground mines production scheduling

The production scheduling in mining operations requires determining which blocks should be extracted and the time of their extraction during the life of mine, while considering geomechanical, operational, economic, and environmental constraints. Production scheduling for any mining system has an enormous effect on the operation's economics. Some of the benefits expected from better production schedules include increased equipment use, optimum recovery of marginal ores, reduced costs, steady production rates, and consistent product quality (Chanda, 1990; Chanda and Dagdelen, 1995; Dagdelen and Johnson, 1986; Winkler, 1996; Wooller, 1992).

There are three time horizons for production scheduling: long-, medium-, and short-term. Long-term mine-production scheduling provides a strategic plan for mining operations, whereas medium-term scheduling provides a monthly operational scheme for mining while tracking the strategic plan. Medium-term schedules include more detailed information that allows for a more accurate design of ore extraction from a special area of the mine, or information that allows for necessary equipment substitution or the purchase of necessary equipment and machinery. The medium-term schedule is also divided into short-term periods (Osanloo et al., 2008).

The majority of scheduling publications to date have been concerned with open-pit mining applications. As a result, the software development for underground operations has been delayed and many of the scheduling concepts and algorithms developed for surface mining have found their way into underground mining. Underground mining methods are characterized by complex decision combinations, conflicting goals and interaction between production constraints.

Current practice in underground-mine scheduling has tended toward using simulation and heuristic software to determine feasible, rather than optimal, schedules. A compromise between schedule quality and problem size has forced the use of mine design and planning models, which incorporate the essential characteristics of the mining system while remaining mathematically tractable. Different types of methods have been applied to underground mine scheduling. Similar to open-pit mines, production scheduling algorithms and formulations in literature can be divided into two main research areas: (i) heuristic methods and (ii) exact solution methods for optimization. Heuristic methods are generally used to generate a good solution in a reasonable amount of time. These methods are used when there is no known method to find an optimal solution under the given constraints. Despite shortcomings such as frequently required intervention, and the lack of a way to prove optimality, simulation and heuristics are able to handle non-linear relationships as part of the scheduling procedure.

In addition to these categories, other methods such as queuing theory, network analysis, and dynamic programming have been used to schedule production and/or material transport. This paper reviews mathematical programming applications in block-caving production scheduling. In block-cave mining, production scheduling determines the amount of material which should be mined from each drawpoint in each period of production, the number of new drawpoints that need to be constructed, and their sequence during the life of mine (Pourrahimian, 2013). The same concerns in deep open-pit mining can be applied to block-cave mining; possibility of value changes of the project through scheduling is remarkable.

4. Mathematical programming methods

Mathematical programming (MP) is the use of mathematical models, particularly optimizing models, to assist in making decisions. The MP model comprises an objective function that should be maximized or minimized while meeting some constraints which determine the solution space and a set of decision variables whose values are to be determined. Objectives and constraints are functions of the variables and problem data. Mathematically, a MP problem can be stated as,

$$\begin{aligned}
 & \text{Maximize} && f(X) \\
 & \text{Subject to} && g_i(X) \leq 0 \quad i = 1, 2, \dots, m. \\
 & && X \geq 0
 \end{aligned}$$

Where $X = (x_1, x_2, \dots, x_n)^T \in R^n$, $f(X), g_i(X), i = 1, 2, \dots, m$ are real valued functions of X . If the functions $f(X)$ and $g_i(X)$ are all linear, the problem is known as a linear programming problem (LP). Otherwise it is said to be a non-linear programming (NLP) problem (Sinha, 2006).

The modeling process in mathematical programming has eight steps (Eiselt and Sandblom, 2010): problem recognition, authorization to model, model building and data collection, model solution, model validation, model presentation, implementation, and monitoring and control. The mathematical programming models which are considered for production scheduling are linear programming (LP), mixed-integer linear programming (MILP), non-linear programming (NLP), dynamic programming (DP), multi-criteria optimization, network optimization, and stochastic programming (Shapiro, 1993). In a LP problem, when some or all of the variables must be integers, the problem is called pure integer (IP) and mixed-integer programming (MIP, MILP) respectively. A linearly constrained optimization problem with a quadratic objective function is called a quadratic program (QP).

The tractability of the mathematical models depends on the size of the problem, in terms of the number of variables and constraints, and the structure of the constraint sets. In the integer programming, as the size of an integer program grows, the time required for solving the problem increases exponentially. The most common problem in the MILP formulation is size of the branch and cut tree. The tree becomes so large that insufficient memory remains to solve the LP sub-problems. The size of the branch and cut tree can actually be affected by the specific approach one takes in performing the branching and by the structure of each problem. So, there is no way to determine the size of the tree before solving the problem (Pourrahimian et al., 2012). For instance, Pourrahimian et al (2013) presented three MILP formulations at three different levels of resolution: (i) aggregated drawpoints level; (ii) drawpoint level, and (iii) drawpoint-and-slice level. Table 2 summarize the number of decision variables and constraints if the proposed formulations are applied on a mine with 3000 drawpoints over 25 years.

Table 2. Number of decision variables and constraints for the models presented by Pourrahimian et al (Pourrahimian et al., 2013)

Level of resolution	Description	Number of continuous variables	Number of binary variables	Number of constraints
Aggregated DPs	3000 DPs aggregated into 200 clusters	5000	10000	20,675
DPs	3000 DPs	75000	150000	309,075
DP-and-slice	3000 DPs and 60530 slices	1,513,250	1,663,250	640,185

5. Production scheduling optimization in block-cave mining

Using mathematical programming optimization with exact solution methods to solve the long-term production planning problem has proved to be robust, and results in answers within known limits of optimality (Pourrahimian, 2013). Lerchs and Grossmann (1965) applied mathematical programming in mine planning (open-pit mining) for the first time. Since the 1960's, considerable research has been done in mine planning using mathematical programming, both in open-pit and underground mining. Newman et al. (2010) and Osanloo et al. (2008) have mentioned many of the studies related

to open-pit mining. Alford (1995) listed problems which have the potential of being considered optimization problems in underground mining. These problems are: (i) primary development (shaft and decline location); (ii) selection from alternative mining methods; (iii) mine layout (i.e., sublevel location and spacing, stope envelope); (iv) production sequencing; (v) product quality control (material blending); (vi) mine ventilation; and (vii) production scheduling (ore transportation and activity scheduling).

Among these problems, product quality control and production scheduling have received the greatest consideration for optimization (Rahal, 2008). Production scheduling optimization is so important because its impact on a project's net present value (NPV) is critical. Therefore, it should be updated periodically. Scheduling underground mining operations is primarily characterized by discrete decisions regarding mine blocks of ore, along with complex sequencing relationships between blocks. To optimize block-caving scheduling, most researchers have used mathematical programming, LP (Guest et al., 2000; Hannweg and Van Hout, 2001; Winkler, 1996), MILP (Alonso-Ayuso et al., 2014; Chanda, 1990; Epstein et al., 2012; Guest et al., 2000; Parkinson, 2012; Pourrahimian, 2013; Rahal, 2008; Rahal et al., 2008; Rahal et al., 2003; Rubio, 2002; Rubio and Diering, 2004; Smoljanovic et al., 2011; Song, 1989; Weintraub et al., 2008; Winkler, 1996), and QP (Diering, 2012; Rubio and Diering, 2004). LP is the simplest method for modeling and solving. Since LP models cannot capture the discrete decisions required for scheduling, MIP is generally the appropriate MP approach to scheduling (Pourrahimian, 2013). Solving a MILP problem can be difficult when production system is large, but MILP is a useful methodology for underground scheduling (Rahal, 2008). This section includes reviews of MP applications in block-caving scheduling and some features for each methodology.

Song (1989) used simulation and a MILP model to find the optimal mining sequence in the block-cave operations at the Tong Kuang Yu mine in China. To obtain an optimal mining sequence, Song first simulated the caving process dependent on undercut parameters. Then, he determined ore-draw spacing and pressure distribution during ore-draw. Finally, he used caving simulation and analysis results to obtain the optimal mining sequence. He optimized the production schedule using total mining cost minimization while considering the geometrical and operational limitations which guarantee caveability and stability demands. Defining linear functions was an advantage of his methodology. The disadvantage, especially in long-term scheduling, was the solution time.

Chanda (1990) combined a simulation with MIP to model the problem of scheduling drawpoints for production at the Chingola Mine, in Zambia. He computerized a model for short-term production scheduling in a block-caving mine. The model used MIP to determine the production rate in finger raises in each production drift considering some quality and quantity constraints. The objective was to minimize the deviation in the average production grade between operating shifts.

Guest et al. (2000) developed LP and MILP models to maximize the NPV of block-caving scheduling (long-term scheduling) over the mine life of a diamond mine in South Africa. This model tried to consider, as constraints, related aspects of mining: mining capacity, metallurgical issues, economic parameters, grades and geotechnical limitations. Applying this wide range of constraints is a remarkable advantage of this model. However, there were two problems with this approach; maximizing tonnage or mining reserves will not necessarily lead to maximum NPV; and draw control is a planning constraint and not an objective function. The objective function in this case would be to maximize tonnage, minimize dilution or maximize mine life (Rubio, 2002).

Rubio (2002) formulated two strategic goals; maximization of NPV and optimization of the mine life in block caving. As constraints, he considered geomechanical aspects, resource management, the mining system and metallurgical parameters involved in the mining operation. One of the main advantages of his model was that it integrated estimates of mine reserves and the development rate that resulted from the production scheduling. Traditionally these parameters were computed independent of production scheduling. Rubio also formulated a relationship between the draw control

factor and the angle of draw. This relationship was built into the actual draw function to compute schedules with high performance in draw control. Opportunity cost in block caving was defined as the financial cost of delaying production from newer drawpoints; a drawpoint will stay active at any given period of the schedule, if it has enough remaining value to pay the financial cost of delaying production from newer drawpoints that may have a higher remaining value.

Rahal et al. (2003) described a MILP goal program with dual objectives of minimizing deviation from the ideal draw profile while achieving a production target. They performed a schedule optimization using a life-of-mine approach in which all production periods were optimized simultaneously. They assumed that material mixing in the short-term has a minimal effect on the panel's long-term state. The model's constraints were deviation from ideal practice, panel state, material flow conservation, production quality, material flow capacity, and production control. They applied the model to De Beers kimberlite mine. The results showed how different production control constraints regulate production from individual drawpoints, as well as recovery of the ideal panel profile by implementing an optimized draw schedule.

Diering (2004) described the basic problem in block-caving scheduling as trying to determine the best tonnages to extract from a number of drawpoints for various periods of time. Those periods could range a day to the life of the mine. Diering singled out NPV as the overall objective to maximize, subject to some constraints: minimum tonnage per period, maximum tonnage per period, maximum total tonnage per drawpoint, maximum total tonnage per period, ratio of tonnage from current drawpoint compared with neighbors, height of draw of current drawpoint with respect to neighbors, percentage drawn for current drawpoint with respect to neighbors, and maximum tonnage from selected groups of drawpoints in a period (usually the groups of drawpoints are referred to as production blocks or panels). Diering emphasized that it would be better to formulate the problem as a LP instead of a NLP because of solution time and the size of problems. He applied a multi-step non-linear optimization model to minimize the deviation between a current draw profile and a defined target. It was shown that this algorithm could also be used to link the short-term plan with the long-term plan.

Rubio and Diering (2004) applied MP to maximize the NPV, optimize the draw profile and minimize the gap between long- and short-term planning. They integrated the opportunity cost into PC-BC (GEOVIA) for computing the best height of draw in a dynamic manner. To solve their problem, they used different mathematical techniques such as direct iterative methods, LP, a golden section search technique, and integer programming. In their formulation, mining reserves were not part of the set of constraints; the mining reserves were computed as a result of the optimal production schedule. They also used QP to minimize the differences between actual heights of draw versus a desired target.

Rahal (2008) presented a draw control model that indirectly increases resource value by controlling production based on geotechnical constraints. He used MILP to formulate a goal programming model with two strategic targets: total monthly production tonnage and cave shape. This approach increased value by ensuring that reserves are not lost due to poor draw practice. The model's advantage was that it allows any number of processing plants to feed from multiple sources (caves, stockpiles, and dumps). There were three main production control constraints in the MILP: the draw maturity rules, minimum draw rate, and relative draw rate (RDR). Rahal used MILP to quantify production changes caused by varying geotechnical constraints, limiting haulage capacity, and reversing mining direction. He showed that tightening the RDR constraint decreases total cave production. He applied his model for three case studies and illustrated how the MILP can be used by a draw control engineer to analyze production data and develop long-term production targets both before and after a cave is brought into full production. Rahal et al. (2008) used MILP to develop an optimized production schedule for Northparkes E48 mine. They described the system constraints as minimum and maximum draw tonnage, the permissible relative draw rate difference between adjacent drawpoints, drawpoint availability, and the capacity of the materials handling system. The impact of different production constraints on total cave capacity was examined. It was shown that the strength of using

MILP lies in its ability to generate realistic production schedules that require little manual manipulation.

Weintraub et al. (2008) developed an approach to aggregate the reduced models (which have been derived from a global model) using the original data for a MIP mine planning model in a large block-caving mine. The aggregation was based on clustering analysis. The MIP model was developed to support decisions for planning extraction of blocks and the decisions of exact timing for each block in the extraction columns. The final model was developed to integrate all mines for corporate decisions, to determine extraction from each sector, in each mine, for each period (for a five-year horizon). Weintraub et al. used two types of aggregation: *Priori* aggregation and *Posteriori* approach. Comparing the original model with the disaggregation, the first approach reduced execution time by 74% and the model dimension by 90%. The second approach reduced solve time by 88% and the model dimension by 15%.

Smoljanovic et al. (2011) presented a model to optimize the sequence of the drawpoint opening over a given time horizon. They incorporated sequencing and capacity constraints. Their model was based on an open-pit model (BOS2) adapted to underground mining. Binary variables were used to indicate whether or not a specific drawpoint had been opened. The real numbers represent the percentage of the column that was extracted. The model was applied in a panel caving mine in which the studied layout included 332 drawpoints. It was shown that the sequencing can change the value of objective function by as much as 50%. Smoljanovic (2012) applied MILP to optimize NPV and the mining material handling system in a panel caving mine. The model output selected the best sequence after considering different mining systems. Results showed that the out-coming NPV of the objective function for different systems could vary by up to 18%. The importance of the mining system and capacity constraints in the sequencing was shown in comparison to different scenarios.

Parkinson (2012) developed three integer programming models for sequence optimization in block-cave mining: Basic, Malkin, and 2Cone. The research was carried out to help provide a required input to a PC-BC program to find an optimized sequence in which the drawpoints are opened in an automated manner. The models were applied on the two data sets. A simple answer was not found. A combination of the presented models was proposed to help the planner to optimize the sequence. Parkinson demonstrated that integer programming models can generate opening sequences but that the process can be complicated.

Epstein et al. (2012) presented a methodology for long-term mine planning based on a general capacitated multi-commodity network flow formulation. They considered underground and open-pit ore deposits sharing multiple downstream processing plants over a long horizon. The model's target was the optimization of several mines as an integrated problem. LP and IP with a customized procedure were applied to solve the combined model. For the production phase in underground mine, which it was block caving, constraints were production per sector, product and period, production cost, extraction times for each block (at most once), block and period priority, minimum blocks for each column, order of drawpoints, maximum duration of a drawpoint, extraction rate of each column, the column in each period, similarity of heights in neighboring columns, bounds on the area, extracted rock per period, and each sector extraction within its time window. The model developed by Epstein et al. has been implemented at Codelco. Production plans for a single mine and integrating multiple mines increased the NPV.

Diering (2012) used QP techniques for block-caving production scheduling. He focused on single-period formulations. He explained that the block caving process is non-linear (the tons which you mine in later periods will depend on the tons mined in earlier periods), so it would not be appropriate to use LP for production scheduling in block caving. The objective function was the shape of the cave. Three sets of constraints were applied in the model: mandatory, modifying, and grade-related. This formulation omitted the sequence of drawpoint development (interaction between neighboring drawpoints) as a constraint.

Pourrahimian et al. (2012) presented two MILP formulations at two different levels of resolution: (i) drawpoint level, and (ii) aggregated drawpoints (cluster level). The objective function was to maximize the NPV. Pourrahimian et al. used PCBC's slice file as an input into their model, but their models treat the problem in the drawpoint or cluster level as a strategic long-term plan, and the slices are not used in the presented formulations. To reduce the number of binary integer variables, Pourrahimian et al. used Fuzzy c-means clustering to aggregate the drawpoints into clusters based on similarities between draw columns and the physical location of the drawpoint and its tonnage. They used same data for both models and solved the problem for four different advancement directions. The execution time for aggregated drawpoints was reduced by more than 99%.

Pourrahimian et al. (2013) developed a theoretical optimization framework based on a MILP model for block-cave long-term production scheduling. The objective function was to maximize the NPV. Pourrahimian et al. formulated three MILP models for three levels of problem resolution: cluster level, drawpoint level, and drawpoint-and-slice level. They showed that the formulations can be used in both the single-step method, in which each of the formulations is used independently; and as a multi-step method, in which the solution of each step is used to reduce the number of variables in the next level and consequently to generate a practical block-cave schedule in a reasonable amount of CPU runtime for large-scale problems. They considered mining capacity, grade blending, the maximum number of active clusters or drawpoints, the number of new clusters or drawpoints, continuous mining, mining precedence, reserves, and the draw rate as constraints which were involved in the all three levels of resolutions. Using such a flexible formulation is very helpful because depending on the level of studies — prefeasibility studies (PFS), feasibility studies (FS) or detailed feasibility studies (DFS) — a mine planner can use the appropriate level of solution and the related runtime. Pourrahimian et al. developed and tested their methodology in a prototype open-source software application with the graphical user interface DSBC (Drawpoint Scheduling in Block-Caving).

Alonso-Ayuso et al. (2014) considered uncertainty in copper prices along a given time horizon (five years) using a multistage scenario tree to maximize the NPV of a block-cave mine in Chile. The stochastic model then was converted into a MIP model. Alonso-Ayuso et al. applied the stochastic model in both risk-neutral and risk-averse environments. Results showed the advantage of using the risk-neutral strategy over the traditional deterministic approach, as well as the advantage of using any risk-averse strategy over the risk-neutral one.

Rubio (2014) introduced the concept of portfolio optimization for block caving. In this method, every decision related to mine design and mine planning could be a component of a set that defines a feasible portfolio. This set is optimized for different production targets to maximize return subject to a given level of reliability. Using this approach a frontier efficient is proposed as a boundary to display different strategic designs and planning options for the set of variables under study. By this method, the decision makers can define a point along the frontier efficient where they want to place a given project.

Table 3 shows the summary of the aforementioned MP applications in block-caving scheduling.

The available tools for block-cave production scheduling can be divided into two categories: (i) commercial, and (ii) in house tools. One of the commercial software is Geovia PCBC. The program is integrated into a general purpose geological modeling and mine planning system so that it can be used for studies ranging from pre-feasibility to daily draw control. The simulation of mixing is an important part of the program. PCBC simulates the extraction from each active drawpoint period-by-period subject to a range of constraints and inputs (Diering, 2000).

Table 3. Summary of applied MP models in block-caving production scheduling

Author	Model	Model objective(s)	Constraint
Song (1989)	Simulation and MILP	Minimization of total mining cost	<ul style="list-style-type: none"> • Geometrical limitations • Operational limitations
Chanda (1990)	Simulation and MIP	Minimization of the deviation in the average production grade between operating shifts	<ul style="list-style-type: none"> • Maximum allowable output per shift • Maximum allowable number of working drawpoints per shift • Declaration of exhaustion for exhausted drawpoints • Required grade for each shift (equality) • Tonnage of blended ore in each shift
Guest et al. (2000)	LP and MILP	Maximization of NPV	<ul style="list-style-type: none"> • Geotechnical constraints <ul style="list-style-type: none"> → Column draw rates → Precedence of accumulated tons drawn → Limits in differences of accumulated tons drawn between columns within time horizons → Limits in ratios of tons drawn between columns (neighbors) within time horizons • Mining constraints <ul style="list-style-type: none"> → Ore flow constraints (tunnels, ore passes, haulage, underground accumulation areas, shaft systems) • Metallurgical constraints <ul style="list-style-type: none"> → Treatment plant (capacities per period) • Economic constraints (revenue, costs) • Geological constraints (grade, size)
Rubio (2002)	MILP and NLP	Two models (a) maximization of NPV and (b) optimization of the mine life	<ul style="list-style-type: none"> • Development rate • Undercut sequence • Drawpoint status • Maximum opened production area • Draw rate • Period constraints • Mining reserves
Rubio and Diering (2004)	MILP and QP	Maximization of NPV, optimization of draw profile, and minimization of the gap between long- and short-term planning	<ul style="list-style-type: none"> • Development rate • Undercut sequence • Maximum opened production area • Draw rate • Draw ratio • Period constraints
Diering (2004)	NLP	Maximizing NPV for M periods and minimization of the deviation between a current draw profile and a defined target	<ul style="list-style-type: none"> • Minimum and maximum tonnage per period • Maximum total tonnage per drawpoint and per period • Ratio of tonnage from current drawpoint compared with neighbors. • Height of draw of current drawpoint with respect to neighbors • Percentage drawn for current drawpoint with respect to neighbors • Maximum tonnage from selected groups of drawpoints in a period

Table 3. Summary of applied MP models in block-caving production scheduling (continued)

Author	Model	Model objective(s)	Constraint
Rahal (2008)	MILGP	Minimizing deviation from the ideal draw profile while achieving a production target	<ul style="list-style-type: none"> • Deviation From Ideal Plan <ul style="list-style-type: none"> → Ideal depletion → Panel production rate → Production from external sources • Contents of material sources • Material flow conservation (blocks, tunnels, ore pass, haulage, accumulation, shaft, plant) • Material flow and capacity (source flow, block, externals, ore pass, haulage, accumulation, shaft, plant) • Production Control <ul style="list-style-type: none"> → Block available for draw → Relative draw rate → Block flow bounds → Draw maturity rules (Lower Depletion Bound/Upper Production Bound) • Product quality and quantity • Economics
Weintraub et al. (2008)	MIP	Maximization of profit	<ul style="list-style-type: none"> • Each cluster can be extracted only once • Sequence of extractions • The allowable speed • Capacity of extraction • Conservation of flows and logical relationships between variables
Smoljanovic et al. (2011)	MILP	Optimization of NPV and mining material handling system	<ul style="list-style-type: none"> • Production Constraints <ul style="list-style-type: none"> → Max and min amount of tonnage to be extracted per time period → The overall mine capacity → Total number of drawpoints to be opened at each time period → Capacity per drawpoint → Min percent of extraction for each drawpoint → Lifetime of a drawpoint → Capacity of haulage system • Geometric constraints <ul style="list-style-type: none"> → Connectivity and shape constraints
Epstein et al. (2012)	MIP	Maximization of NPV	<ul style="list-style-type: none"> • Production per sector (product and period) • Production cost • Extraction times for each block • Block and period priority • Minimum blocks for each column • Order of drawpoints • Maximum duration of a drawpoint • Extraction rate of each column • The column in each period • Neighboring columns heights similarity • Bounds on the area • Extracted rock per period • Each sector extraction within its time window

Table 3. Summary of applied MP models in block-caving production scheduling (continued)

Author	Model	Model objective(s)	Constraint
Diering (2012)	QP	Objective tonnage (to optimize the shape of the cave)	<ul style="list-style-type: none"> • Mandatory constraints <ul style="list-style-type: none"> → Production capacity → A maximum tonnage for each drawpoint based on the drawpoint maturity curve → A minimum tonnage for each drawpoint. • Modifying constraints <ul style="list-style-type: none"> → Maximum tonnage from production tunnels. → Maximum tonnage from an orepass or crusher. → Maximum tonnage from an entire sector • Grade-related constraints
Parkinson (2012)	IP	Finding an optimal opening sequence in an automated manner	<ul style="list-style-type: none"> • Each drawpoint starts once • Global capacity (processing plant capacity) • Tunnel development • Additional constraints: <ul style="list-style-type: none"> → Within-tunnel contiguity → Across-tunnel contiguity
Pourrahimian et al. (2013)	MILP	Maximization of NPV	<ul style="list-style-type: none"> • Mining capacity • Grade blending • Maximum number of active clusters or drawpoints (according to the model resolution) • Number of new clusters or drawpoints (according to the model resolution) • Continuous mining • Mining precedence <ul style="list-style-type: none"> → Slice → Drawpoint → Cluster • Reserves • Draw rate <ul style="list-style-type: none"> → Draw column → Cluster
Alonso-Ayuso et al. (2014)	MILP	Maximization of NPV while considering uncertainty in copper price	<ul style="list-style-type: none"> • Each cluster is processed at most once • If a cluster is processed at a given period then all predecessor clusters are also processed by that period • The clusters in each set would be extracted simultaneously in each sector • Number of tons processed in each sector at each period • Flow conservation constraints for the processing stream • Number of tons processed in each period • Upper and lower bounds for the total area processed in each sector • Number of tons processed in each period • Upper bound due to the capacity of processing stream • The maximum increase and decrease of tons in each sector in each period

In mathematical programming, we look for values of variables which are allowed and which do not violate the constraints. This defines what is called a solution space, in which the edges of this space are the constraints. In case of an LP formulation, the solution must be on a boundary of this space. In the case of block-cave scheduling, an LP formulation will always seek to take the maximum tons from the highest value drawpoints and the least tons from the lower valued drawpoints (Diering, 2012). As a result, this kind of scheduling may result in high levels of horizontal mixing between drawpoints because the draw columns have different heights. This is a potential disadvantage of LP application in block-caving scheduling. Table 4 summarizes the advantages and disadvantages of methodologies examined in previous studies.

Table 4. Advantages and disadvantages of applied mathematical methodologies in block-caving production scheduling

Methodology	Features
LP	Advantage <ul style="list-style-type: none"> • LP method has been used most extensively (Rahal, 2008) • It can provide a mathematically provable optimum schedule (Rahal, 2008)
	Disadvantage <ul style="list-style-type: none"> • Straight LP lacks the flexibility to directly model complex underground operations which require integer decision variables (Winkler, 1996) • Mine scheduling is too complex to model using LP and the only possible approach is to use some combination of theoretical and heuristic methods to ensure a good, if not optimal schedule (Scheck et al., 1988)
MILP	Advantage <ul style="list-style-type: none"> • Computational ease in solving a MIP problem (and MILP) is dependent upon the formulation structure (Williams, 1974) • MILP could be used to provide a series of schedules which are marginally inferior to a provable optimum (Hajdasinski, 2001) • MILP is superior to simulation when used to generate sub-optimal schedules, because the gap between the MILP feasible solution and the relaxed LP solution provides a measure of solution quality (Rahal, 2008) • MILP can provide a mathematically provable optimum schedule (Rahal, 2008)
	Disadvantage <ul style="list-style-type: none"> • It is often difficult to optimize large production systems using the branch-and-bound search method (Rahal, 2008) • The block-caving process is non-linear (the tons which you mine in later periods will depend on the tons mined in earlier periods), so it would not be appropriate to use LP for production scheduling in block caving (Diering, 2012)
QP	Advantage <ul style="list-style-type: none"> • Since the block-caving process is non-linear, QP could be an appropriate option to model it • It can find solutions in the interior of the solution space, which results in an even height of drawpoints as well as lower horizontal mixing between drawpoints (Diering, 2012)
	Disadvantage <ul style="list-style-type: none"> • Solving this kind of problem could be a challenge. It must be changed to LP and then be solved, to ensure conversion errors

6. Conclusion

Increasing the use of block-cave mining in new-world mining environments has led many researchers to focus on this area to make mining operations as optimal as possible. Production scheduling in block caving, because of its significant impact on the project's value, has been considered a key issue to be improved. The problem is complex, unique for each case, large-scale, and non-linear. Researchers have applied different methods to model production scheduling in block caving, for short-term and long-term periods of mining, some for real case studies as industry projects and others as academic research projects.

Generally speaking, confronting future challenges in block-cave mining can be divided into two categories: (i) operational and (ii) economical. Block caving is known as a low-cost mining method which makes it possible to mine the low grade ore-bodies, therefore, optimal production schedule with lower cost is required. Block-cave mining is one of the best solutions for continuing the operation after shutting down the mine in deep open-pit mines. The new operation (block caving) has to feed the processing plant which used to be fed by the open-pit mine. Therefore, the production rate in the block-cave operation has to be as high as the open-pit mining. Although some semi auto mining equipment has been introduced for block caving, but it is just the starting point to reach the full automated operations. Also, making decisions about the geometry of drawpoints, best height of draw, undercut level, and the production level are critical and challenging. Block-cave mining usually requires much more development compare to other mining methods which needs a long period of time before starting the production, so the high capital cost is needed to run the project. High capital cost increases the risk of project. The operational costs of block cave mining is low but if the rock mass caveability is not achieved as it expected, the costs for additional drilling and blasting can be definitely challenging.

Most of researchers have applied MILP to model production scheduling; it can be useful because both the integer variables (whether a block, slice, or draw column should be extracted) and continues variables (the constraints and mining operation details) can be modeled so that the optimal values can be achieved while considering the system's constraints. Basically, the block-caving operation is non-linear. Therefore, linear programming could be an inappropriate method to model production scheduling. Quadratic programming as a non-linear methodology has been applied by a few researchers for block-cave mining scheduling.

As computer base algorithms are improved, we expect to see the development of more detailed models with more complexity, models that try to be more practical and include all aspects of mining systems, with new algorithms for faster solutions. Using non-linear methodologies for multi-period scheduling with a reasonable solution time would improve block-caving scheduling. In block-caving operations, decisions about current actions are often based on how those actions affect future actions. For this reason, a real options technique can be properly used in production scheduling optimization.

There are some uncertainties in block-cave mining that should be involved in production scheduling. Grade uncertainty is one of the most common, because of the nature of ore-body, but in block-caving operations grade uncertainty is more critical and complicated, due to the vertical and horizontal mixing which occurs during the caving and production processes. Once the rock is fragmented the particles of the rock flow towards the production level in different ways depending on the fragmentation profile and distribution. Price uncertainty is another variable which should be considered when attempting to achieve realistic optimal production scheduling.

Blasting operations in block caving has a critical impact on fragmentation and, as a result, on material flow. The gravity flow of fragmented rock plays an important role in the production rate and grade in block-caving operations. When considering the material flow in the presence of blasting parameters, production scheduling could result in more realistic plans. Since rock mass is broken by caving the actual fragmentation expected at the drawpoints is uncertain. Therefore, drawpoint

productivity is uncertain and the amount of area that needs to be developed and undercut also becomes uncertain.

Geotechnical aspects of ore-body and its surrounding rocks determine caveability and the efficiency of block-caving operations. Using more geotechnical constraints in production scheduling modeling helps the mine planners to have more confidence about scheduling results. If the rock mass is high stress and competent the cave propagation could be uncertain and triggered erratic dilution and non-uniform grade extraction.

Developing accurate clustering methods, with more flexible levels of problem resolution, could lead to better options for mine planners during the different stages of planning.

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