Production Scheduling Optimization with Stockpiling

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ABSTRACT

Strategic mine planning is a complicated process that is usually broken down into smaller problems in order to get more practical solutions in shorter time. In this paper, we present a multi-step approach that starts form a pushback design procedure that is able to create pushbacks with controlled ore and waste tonnage. This is a hybrid algorithm that incorporates mathematical programming with heuristics to tackle with large-scale problems. It benefits from the special structures of the mathematical formulation to calculate relaxation bounds and uses heuristics to provide near-optimal solutions. Next, we propose a hierarchical clustering algorithm that creates mining units with minable shapes and homogeneity in rock-type and grade within the boundaries of generated pushbacks. The third step of our solution procedure is to form a mathematical model that provides a long-term mining schedule based on the generated mining units as well as the created pushbacks. Our proposed formulation considers various mining and processing constraints and is able to include stockpiling in long-term plans to improve the blending. Finally, we use the idea of piecewise linearization to modify the model to be able to solve it with mixed integer linear programming solvers. The model is tested on a synthetic dataset to evaluate the performance of the model and show the errors introduced by linearization.

1. Introduction

Open-pit mining is the most common and the oldest method of mining valuable material from the ground. It has attracted many researchers to study various aspects of the operation such as production planning, truck-shovel allocations, risk analysis and grade blending. Various heuristic, meta-heuristic and mathematical programming techniques have been implemented on these areas to improve the operation. Their goals are to maximize profit, minimize costs and to optimize the utilization of the resources or the outcome of the mining operation. The mine planning problem has also been studied in different time frames. Long- to short-term plans are usually determined based on different levels of details. Long-term plans usually deal with larger units of production and decide when to extract material and where to send them. Short-term plans, on the other hand, deal with smaller units and make more detailed decisions on the production levels, blending, truck-shovel allocations etc. In this paper, we present a multi-step hybrid approach to deal with the long-term multi-destination open-pit production planning problem by creating controlled pushbacks and

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Some parts of this paper are direct excerpts from the paper presented in APCOM 2015 (Tabesh M., et al., 2015) aggregated mining units and using mathematical programming to solve the problem. We incorporate the blending constraints and stockpiling in the long-term mine planning decisions to improve the operation and help the mine planners decide if they want to use stockpiles in the operation.

Mathematical programming is not new to mine planning researchers. Johnson (1969) introduced mathematical programming and in particular linear programming to the mine planning research area. He proposed a linear programming model for the long-term multi-destination open-pit production planning problem along with a decomposition approach to solve the problem. However, this initial model was using continuous variables to control precedence constraints which would result in partial extraction of blocks and infeasible solutions (Gershon, 1983). Although, introducing binary variables to control the block extraction precedence can solve this problem, it will create another obstacle on the way: curse of dimensionality. In other words, introducing binary variables will make the problem NP-Hard and impossible to solve for real size block models. Therefore, the focus of mine planners in the past few decades has been on breaking the problem into smaller problems, reducing the size of the problem or finding near-optimal solutions to the problem. Interested readers are referred to Osanloo et. al (2008) and Newman et. al (2010) for a complete review on the applications of operations research and mathematical programming on the mine planning problem. On the other hand, most of the proposed models incorporate mining, processing and precedence constraints and do not include grade blending and stockpiling constraints.

2. Pushback design

Pushback design is an important step in LTOPP in which the phases of production, pushbacks, are determined. The intersection of pushbacks and mining benches are called bench-phases. These are the most common units of long-term planning in open-pit mines. From manual methods such as fixed lead to more advanced heuristics such as Milawa (Geovia, 2012) use the bench-phases to optimize the long-term open-pit production plan. Therefore, how the pushbacks are defined can significantly affect the output. In this paper, we are using a hybrid heuristic-binary programming approach from Mieth (2012) to create the pushbacks. The pushback design procedure is explained in details in Mieth (2012) and Tabesh et al. (2014). The generated bench-phases are then used as units of mining and as boundaries for clustering.

3. Clustering

Clustering is the process of grouping similar objects together in a way that maximizes the similarity between the objects of the same cluster and the dissimilarity between the objects of different clusters. However, the clustering algorithm we used here not only accounts for the similarities but also respects the size and shape constraints. The clustering algorithm mentioned is a variation of hierarchical agglomerative clustering and is thoroughly explained in Tabesh and Askari-Nasab (2011) and Tabesh and Askari-Nasab (2013). We use the clustering algorithm to create processing units within the boundaries of bench-phases. Therefore, the bench-phases are divided into smaller units with similar rock-type and grade which are the basis for making processing and stockpiling decisions.

4. Mathematical Formulations

As mentioned earlier, various LTOPP mathematical models have been proposed in the literature. However, none of them incorporate stockpiling in long-term planning. One major reason is that calculating the reclamation grade of the stockpiles introduces non-linearity into the model. Bley, Boland, Froyland, & Zuckerberg (2012) model the LTOPP with stockpiling by adding the non-linear constraints and proposing a problem-specific solution method. In this paper, we tried to avoid the non-linear constraints by benefiting from the piecewise linearization technique. We introduce multiple stockpiles with different acceptable grades to be able to assign fixed reclamation grades to each stockpile. These input grade ranges, as well as reclamation grades, are determined based on histograms of grades to be representative of data.

4.1. Original Model

We first present the original LTOPP mathematical model without the stockpile. The model is a multi-destination LTOPP which uses two different sets of units for making mining and processing decisions. Two sets of variables are defined for bench-phases: $y_m^t \in [0,1]$ is the portion of bench-phase extracted in each period and $b_m^t \in \{0,1\}$ is the binary variable to control the precedence. Since the number of bench-phases is less than number of blocks and clusters, controlling the precedence with bench-phases results in less binary variables and less resource consumption for solving the model. Moreover, using bench-phases as mining units is the common practice in the mining industry. However, making material destination decisions requires more accurate units with distinction between ore and waste. This is achieved by dividing every bench-phase into smaller units using clustering algorithm.

- ***Sets***

S^m	For each bench-phase m, there is a set of bench-phases (S^m) that have to be extracted prior to extracting bench-phase m to respect slope and precedence constraints
U^m	Each bench-phase m is divided into a set of clusters. U^m is the set of clusters that are contained in bench-phase m

- ***Indices***

$d \in \{1,...,D\}$	Index for material destinations
$m \in \{1,...,M\}$	Index for bench-phases
$p \in \{1,...,P\}$	Index for clusters
$c \in \{1,...,C\}$	Index for processing plants
$e \in \{1,...,E\}$	Index for elements
$t \in \{1,...,T\}$	Index for scheduling periods

- ***Parameters***

D	Number of material destinations (including processing plants and waste dumps)
M	Total number of bench-phases
P	Total number of clusters
E	Number of elements in the block model

T	Number of scheduling periods
$\overline{MC}^t$	Upper bound on the mining capacity in period t
$\underline{MC}^t$	Lower bound on the mining capacity in period t
$\overline{PC}_c^t$	Maximum tonnage allowed to be sent to plant c in period t
$\underline{PC}_c^t$	Minimum tonnage allowed to be sent to plant c in period t
$\overline{G}_c^{t,e}$	Upper limit on the allowable average grade of element e at processing plant c in period t
$\underline{G}_c^{t,e}$	Lower limit on the allowable average grade of element e at processing plant c in period t
s_m	Number of predecessors of bench-phase m (members of S^m)
o_m	Total ore tonnage in bench-phase m
w_m	Total waste tonnage in bench-phase m
o_p	Total waste tonnage in cluster p
w_p	Total waste tonnage in cluster p
c_m^t	Unit discounted cost of mining material from bench-phase m in period t
$r_{p,c}^t$	Unit discounted revenue of sending material from processing unit p to processing destination c in period t minus the processing costs
g_p^e	Average grade of element e in cluster p

- ***Decision Variables***

$y_m^t \in [0,1]$	Continuous decision variable representing the portion of bench-phase m extracted in period t
$x_{p,c}^t \in [0,1]$	Continuous decision variable representing the portion of ore tonnage in cluster p extracted in period t and sent to processing plant c
$b_m^t \in \{0,1\}$	Binary decision variable indicating if all the predecessors of bench-phase m are completely extracted by or in period t

- ***Objective Function***

$$\max \sum_{t=1}^{T} \left(\sum_{p=1}^{P} \sum_{c=1}^{C} (r_{p,c}^t \times o_p \times x_{p,c}^t) - \sum_{m=1}^{M} (c_m^t \times (o_m + w_m) \times y_m^t) \right) \qquad (1)$$

- ***Constraints***

$$\underline{MC}^t \leq \sum_{m=1}^{M}\left((o_m + w_m)\times y_m^t\right) \leq \overline{MC}^t \qquad \forall t \in \{1,...,T\} \qquad (2)$$

$$\underline{PC}_c^t \leq \sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right) \leq \overline{PC}_c^t \qquad \forall t \in \{1,...,T\}, \forall c \in \{1,...,C\} \qquad (3)$$

$$\sum_{p\in U^m}\sum_{d=1}^{D}\left(o_p \times x_{p,d}^t\right) \leq \left(o_m + w_m\right)\times y_m^t \qquad \forall t \in \{1,...,T\}, \forall m \in \{1,...,M\} \qquad (4)$$

$$\underline{G}_c^{t,e} \leq \frac{\sum_{p=1}^{P}\left(o_p \times g_p^e \times x_{p,c}^t\right)}{\sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right)} \leq \overline{G}_c^{t,e} \qquad \forall t \in \{1,...,T\}, \forall c \in \{1,...,C\}, \forall e \in \{1,...,E\} \qquad (5)$$

$$\sum_{t=1}^{T} y_m^t = 1 \qquad \forall m \in \{1,...,M\} \qquad (6)$$

$$\sum_{i=1}^{t} y_m^i \leq b_m^t \qquad \forall m \in \{1,...,M\}, \forall t \in \{1,...,T\} \qquad (7)$$

$$s_m \times b_m^t \leq \sum_{i\in S^m}\sum_{j=1}^{t} y_i^j \qquad \forall m \in \{1,...,M\}, \forall t \in \{1,...,T\} \qquad (8)$$

$$b_m^t \leq b_m^{t+1} \qquad \forall m \in \{1,...,M\}, \forall t \in \{1,...,T-1\} \qquad (9)$$

The objective function (Eq. (1)) is summation of discounted revenue made from sending material to the processing plants minus the total cost of mining material from the ground. Eqs.(2) and (3) are responsible for controlling the minimum and maximum extraction and processing capacity in each period. Eq. (4) controls the relation between the tonnage mined from each bench-phase and the tonnage processed from the clusters within that bench-phase. Note that the difference between the tonnage extracted and the tonnage processed is the waste extracted and sent to the waste dump. However, if we have a waste dump with an extra haulage cost the dump can be defined as a destination with negative revenue. Eq. (5) controls the average head grade of material sent to processing plants in each period. However, to avoid non-linearity the equations are rearranged before putting into matrix format. Eq. (6) ensures that all the material within the ultimate pit is extracted during mine life. Eqs. (7) to (9) are the precedence control constraints with the binary variables.

4.2. Non-linear Model

We can modify the original LTOPP model to account for stockpiling by adding stockpiles as material destinations and introducing $f_c^t \geq 0$ variables. These variables are the tonnages reclaimed from the stockpile and sent to processing plants in each period. The stockpile is added as a destination with the index of c'. $G^{t,e}$ is the reclamation grade of element e in period t and $r_c^{t,e}$ is the unit discounted revenue of processing one unit of element e from stockpile in processing destination c in period t minus the processing and re-handling costs. Accordingly, we can rewrite the LTOPP model be replacing Eqs.(1), (3) and (5) with Eqs.(10), (11) and (12) respectively. Note that the objective function is not linear anymore. Moreover, we have to add a constraint for calculating $G^{t,e}$ as in Eq. (13) which has a nonlinear term. Eq. (14) ensures that the summation of tonnages reclaimed from stockpile from the first period to the current period does not exceed the summation of tonnages sent to the stockpile by the current period.

$$\max \sum_{t=1}^{T}\left(\sum_{p=1}^{P}\sum_{c=1}^{C}(r_{p,c}^{t}\times o_p\times x_{p,c}^{t})-\sum_{m=1}^{M}(c_m^t\times\left(o_m+w_m\right)\times y_m^t)+\sum_{e=1}^{E}\sum_{c=1}^{C}\left(f_c^t\times G^{t,e}\times r_c^{t,e}\right)\right) \quad (10)$$

$$\underline{PC}_c^t \le \sum_{p=1}^{P}\left(o_p\times x_{p,c}^t\right)+f_c^t \le \overline{PC}_c^t \qquad \forall t\in\{1,...,T\}, \forall c\in\{1,...,C\} \quad (11)$$

$$\underline{G}_c^{t,e} \le \frac{\sum_{p=1}^{P}\left(o_p\times g_p^e\times x_{p,c}^t\right)+f_c^t\times G^{t,e}}{\sum_{p=1}^{P}\left(o_p\times x_{p,c}^t\right)+f_c^t} \le \overline{G}_c^{t,e} \qquad \forall t\in\{1,...,T\}, \forall c\in\{1,...,C\}, \forall e\in\{1,...,E\} \quad (12)$$

$$G^{t,e}=\frac{\sum_{p=1}^{P}\left(o_p\times g_p^e\times x_{p,c'}^t\right)-\sum_{t'=1}^{t-1}\sum_{c=1}^{C}f_c^{t'}\times G^{t',e}}{\sum_{p=1}^{P}\left(o_p\times x_{p,c'}^t\right)+\sum_{t'=1}^{t-1}\sum_{c=1}^{C}f_c^{t'}} \qquad \forall t\in\{1,...,T\}, \forall e\in\{1,...,E\} \quad (13)$$

$$\sum_{t'=1}^{t}\sum_{c=1}^{C}f_c^{t'} \le \sum_{t'=1}^{t-1}\sum_{p=1}^{P}\left(o_p\times x_{p,c'}^{t'}\right) \qquad \forall t\in\{2,...,T\} \quad (14)$$

4.3. Linearized Model

In order to have a linear LTOPP model with stockpiling, we assume that there are multiple stockpiles with tight ranges for the acceptable element grades. Therefore, we can assign an average reclamation grade and the corresponding reclamation revenue to each stockpile. The more stockpiles defined the smaller error is introduced into the model. However, more stockpiles sacrifices the complete blending assumption present in most stockpiling scenarios. Therefore, making reasonable assumptions regarding the number of stockpiles to define and the acceptable element grade ranges is crucial to obtaining reasonable results.

In order to create the linear LTOPP model with stockpiling we define S stockpiles. G_s^e is the average reclamation grade of element e from stockpile s and $r_{s,c}^t$ is the unit discounted revenue of reclaiming material from stockpile s with the average grade and processing them in plant c in period t minus the processing and re-handling costs. $\underline{G}_s^e$ and $\overline{G}_s^e$ are the lower and upper bounds on the acceptable element grade e for stockpile s. $f_{s,c}^t \ge 0$ is the set of variables representing the tonnage of material reclaimed from stockpile s in period t and sent to processing destination c. Now we can rewrite the model by replacing the objective function with Eq. (15) and Eqs.(11) to (14) with Eqs.(16) to (19) respectively.

$$\max \sum_{t=1}^{T}\left(\sum_{p=1}^{P}\sum_{c=1}^{C}(r_{p,c}^{t}\times o_{p}\times x_{p,c}^{t})-\sum_{m=1}^{M}(c_{m}^{t}\times\left(o_{m}+w_{m}\right)\times y_{m}^{t})+\sum_{s=1}^{S}\sum_{c=1}^{C}\left(f_{s,c}^{t}\times r_{s,c}^{t}\right)\right) \quad (15)$$

$$\underline{PC}_{c}^{t}\leq\sum_{p=1}^{P}\left(o_{p}\times x_{p,c}^{t}\right)+\sum_{s=1}^{S}f_{s,c}^{t}\leq\overline{PC}_{c}^{t} \qquad \forall t\in\{1,...,T\},\forall c\in\{1,...,C\} \quad (16)$$

$$\underline{G}_{c}^{t,e}\leq\frac{\sum_{p=1}^{P}\left(o_{p}\times g_{p}^{e}\times x_{p,c}^{t}\right)+\sum_{s=1}^{S}f_{s,c}^{t}\times G_{s}^{e}}{\sum_{p=1}^{P}\left(o_{p}\times x_{p,c}^{t}\right)+\sum_{s=1}^{S}f_{s,c}^{t}}\leq\overline{G}_{c}^{t,e} \qquad \forall t\in\{1,...,T\},\forall c\in\{1,...,C\},\forall e\in\{1,...,E\} \quad (17)$$

$$\underline{G}_{s}^{e}\leq\frac{\sum_{p=1}^{P}\left(o_{p}\times g_{p}^{e}\times x_{p,s}^{t}\right)}{\sum_{p=1}^{P}\left(o_{p}\times x_{p,s}^{t}\right)}\leq\overline{G}_{s}^{e} \qquad \forall t\in\{1,...,T\},\forall s\in\{1,...,S\},\forall e\in\{1,...,E\} \quad (18)$$

$$\sum_{t'=1}^{t}\sum_{c=1}^{C}f_{s,c}^{t'}\leq\sum_{t'=1}^{t-1}\sum_{p=1}^{P}\left(o_{p}\times x_{p,s}^{t'}\right) \qquad \forall t\in\{2,...,T\},\forall s\in\{1,...,S\} \quad (19)$$

5. Case Study

Marvin is a well-known test dataset used as the demo dataset in Whittle™ mine planning software (Geovia, 2012) and presented as a standard dataset in MineLib (Espinoza et al., 2013). Marvin dataset consists of 53,271 blocks with four different rock-types and two element grades. Each block is 30 meters in all dimensions. The main rock-types are Mixed (MX), Oxide (OX) and Primary (PM) along with the undefined waste denoted here with UND. Marvin mine is a synthetic gold and copper mine with a thin layer of overburden which makes planning easier and provides access to ore in the very first periods of extraction.

As mentioned earlier, Marvin dataset exists in both Whittle™ and MineLib (Espinoza et al., 2013). Despite using the same cost and profit parameters the optimum pit determined by Whittle™ is different from the optimum pit determined in MineLib (Espinoza et al., 2013). The former has 9,381 blocks (575 million tons) in the final pit compared to the latter with 8,516 blocks (527 million tons) in the final pit. We have used the outputs from Whittle™ in our case studies to be able to compare our results to commercial software used by many companies in the mining industry.

In all cases, the block economics are calculated based on a mining cost of \$1.5 per ton and a processing cost of \$6.25 per ton for all rock-types. The gold and copper recoveries in the processing plant, selling costs and prices are summarized in Table 1. Sample plan views of rock-type and grade distribution within the final pit are presented in Fig 1 to Fig 6. For the purpose of calculating clustering measures, we use the initial destinations determined by Milawa NPV in Whittle™ as the destination of blocks in the clustering step.

First, we compare our MILP schedules based on processing blocks and clusters against Whittle™ Milawa algorithms. Next, we add stockpiles and compare the stockpiling scenarios and present the actual versus estimated cash flows for our model. Moreover, we show how we can decrease the error by restricting stockpiles to specified grade values which is not an uncommon practice in real mining operations. Finally, we use the stockpiles and restrict the head grade of material sent to the processing plant to show the flexibilities that come with using MILP models instead of heuristics for scheduling.

Table 1. Marvin Element Economics

Element	Unit	Recovery	Selling Cost	Price
Au	gram	0.6	4.80	38.6
Cu	%m	0.8	11.03	33.1

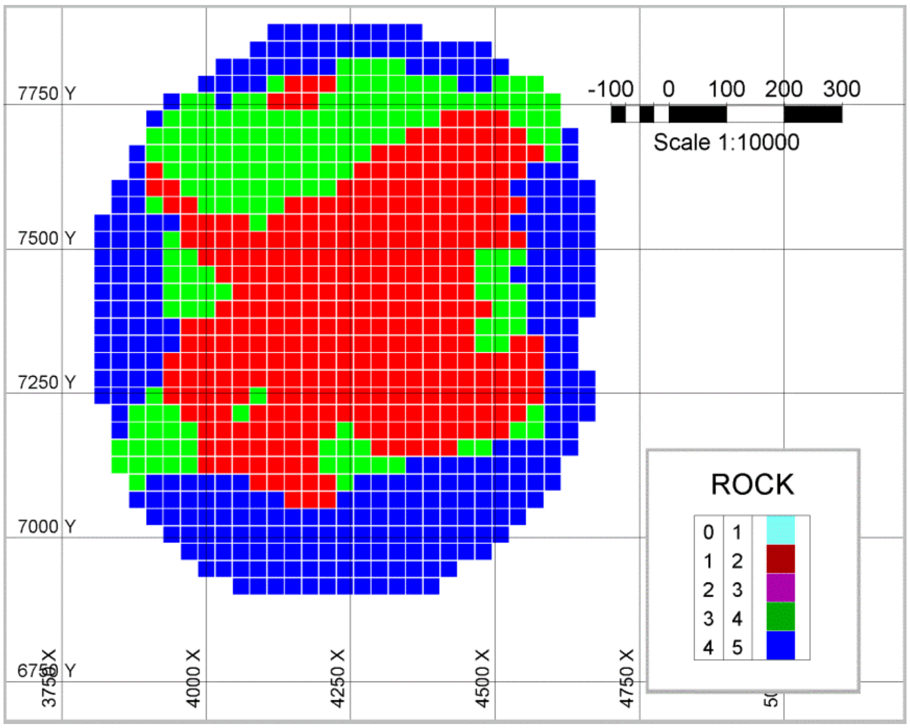

Fig 1. Rock-type Distribution Plan View at 600m Elevation

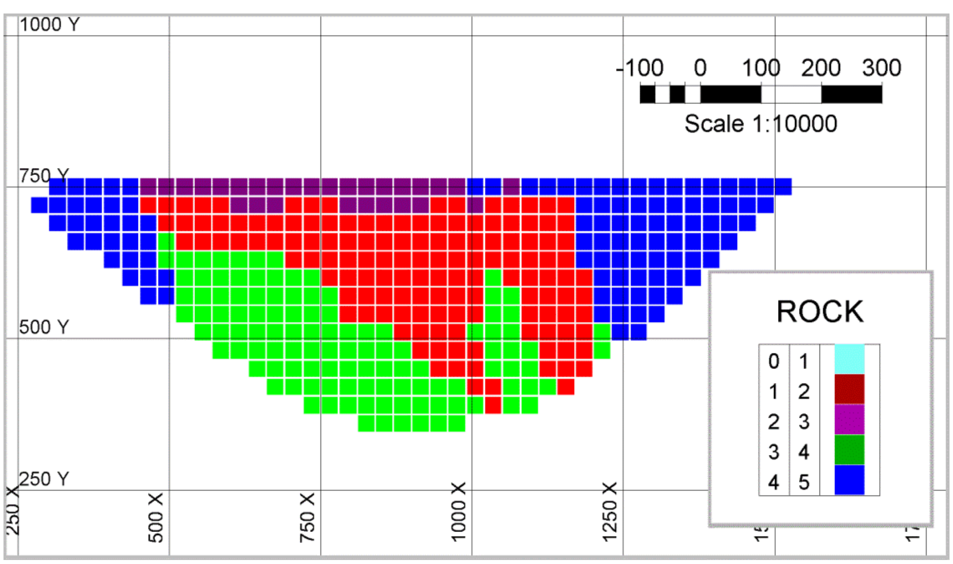

Fig 2. Rock-type Distribution at 4100m Easting

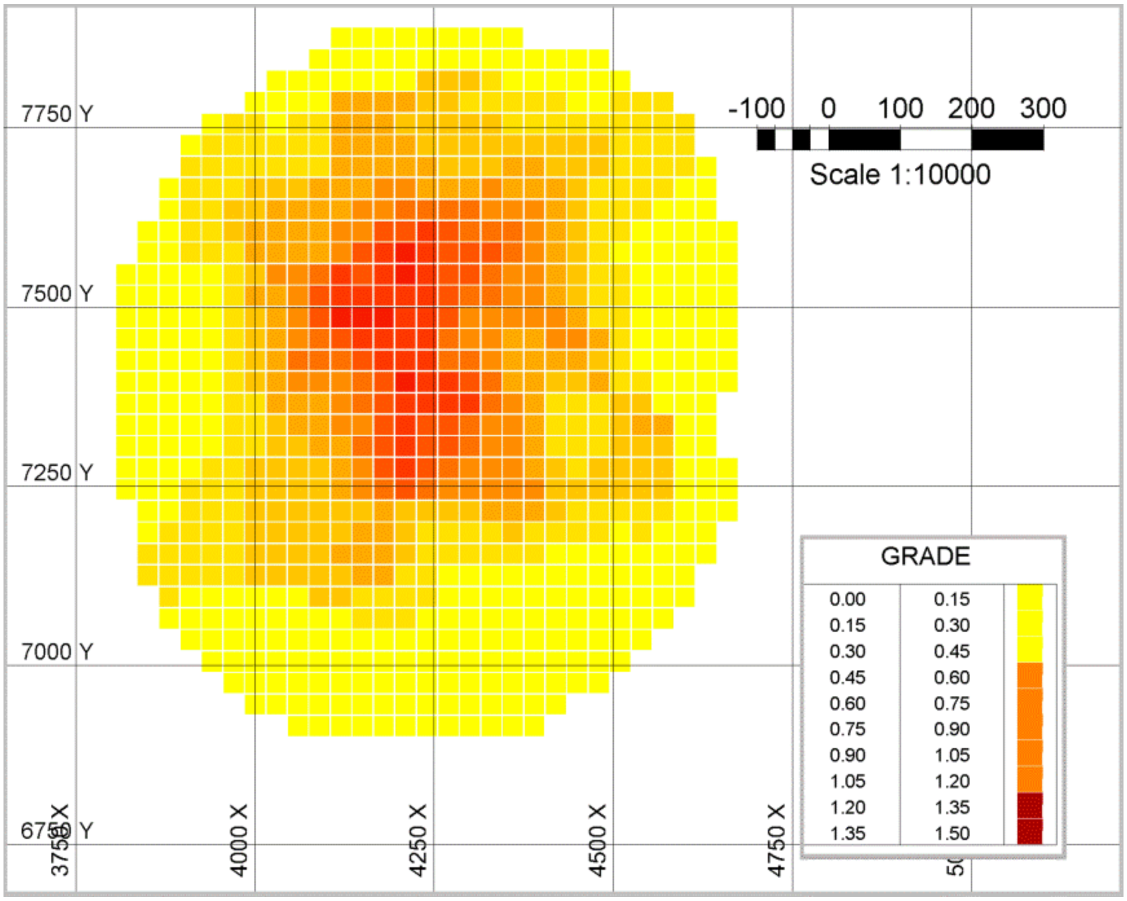

Fig 3. Gold Grade Distribution Plan View at 600m Elevation

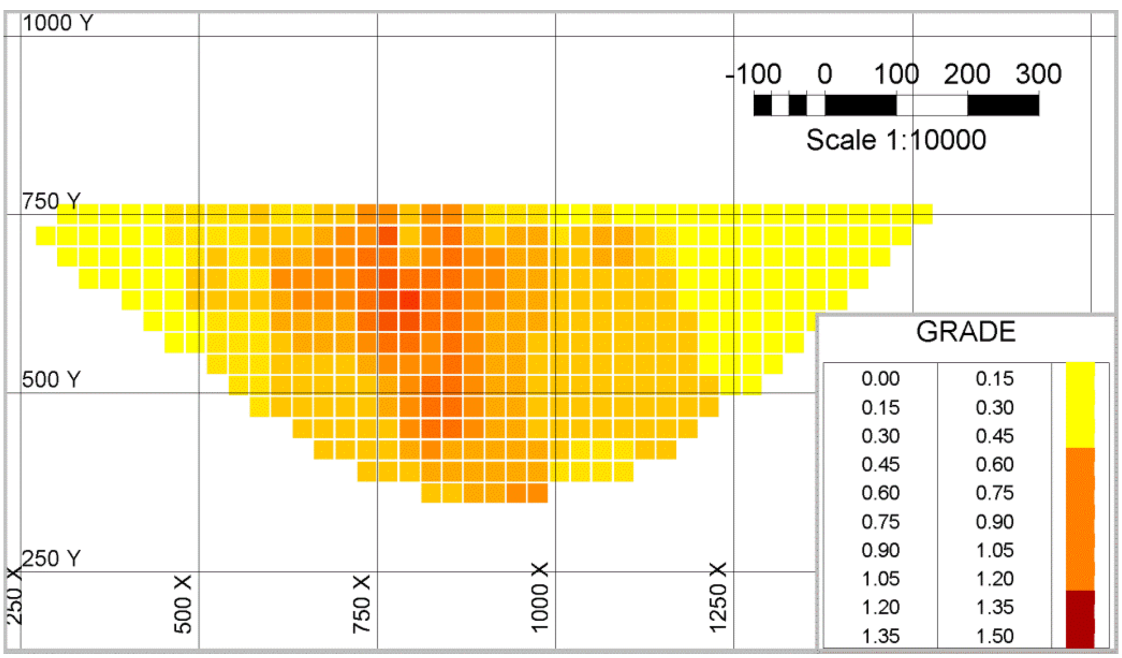

Fig 4. Gold Grade Distribution at 4100m Easting

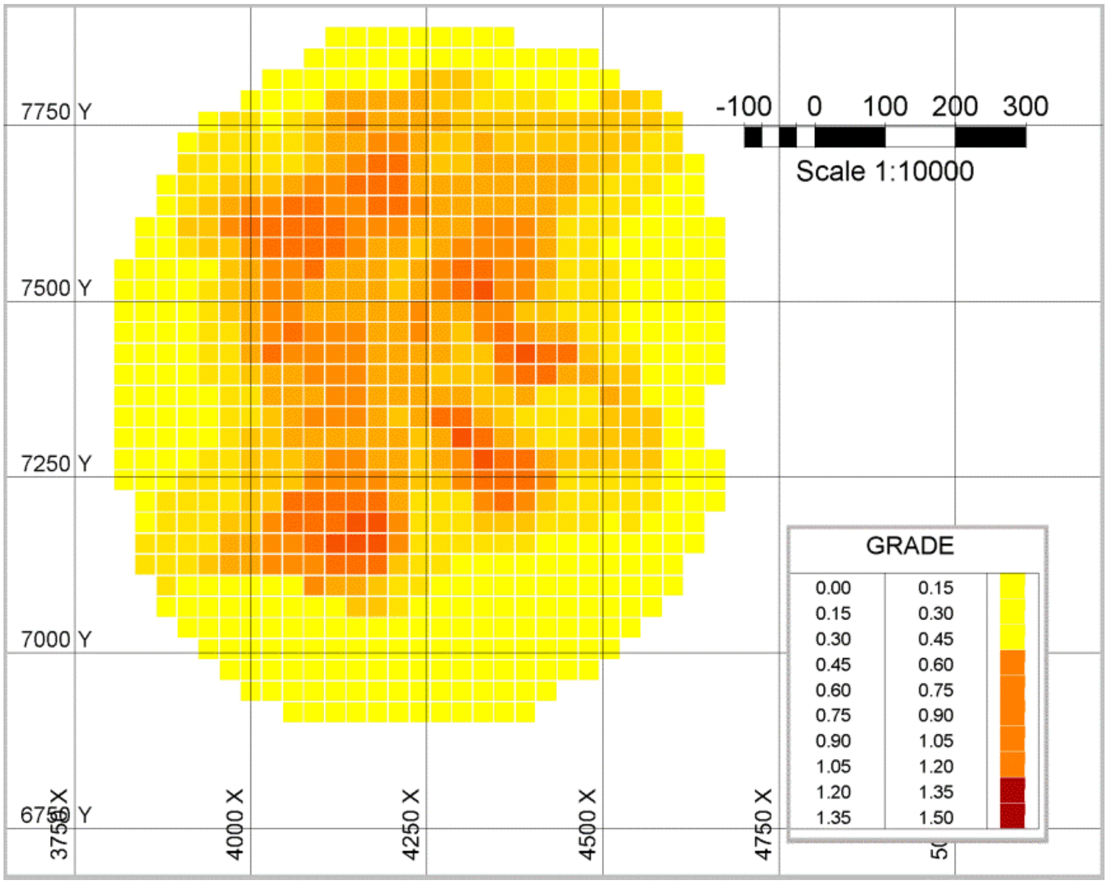

Fig 5. Copper Grade Distribution Plan View at 600m Elevation

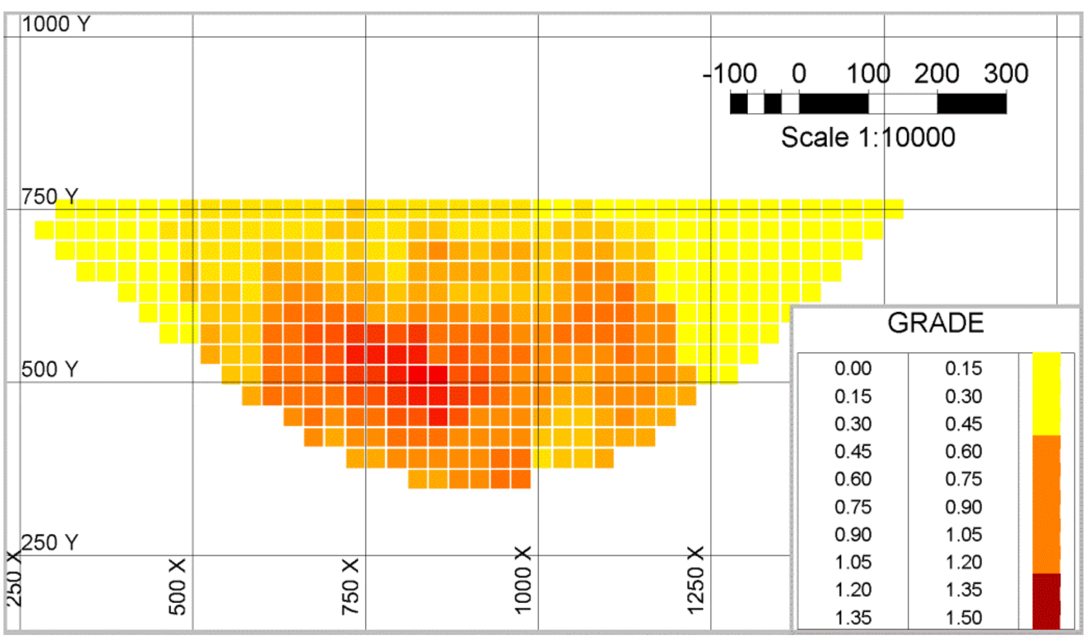

Fig 6. Copper Grade Distribution at 4100m Easting

5.1. GEOVIA Whittle™ Schedule

We obtained two production schedules based on four pushbacks from Whittle™. The first schedule is based on Milawa NPV algorithm and the second one is based on Milawa Balanced. These algorithms are heuristics deigned to use pushbacks as mining units and maximize NPV of the operation. Milawa NPV focuses on maximizing NPV where Milawa Balanced looks into maximizing NPV and having a balanced schedule at the same time. The Milawa balanced algorithm results in $2,166M of NPV and the schedule in Fig 10. The Milawa NPV algorithm results in $2,240M of NPV and the schedule in Fig 7. As expected, the Milawa NPV algorithm resulted in higher NPV by sacrificing the balance in utilizing resources.

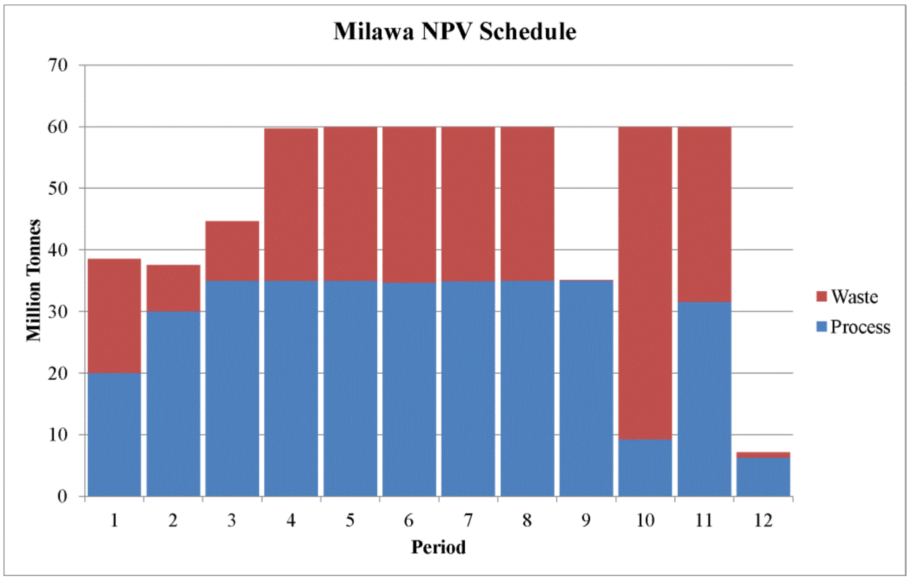

Fig 7. Milawa NPV Schedule

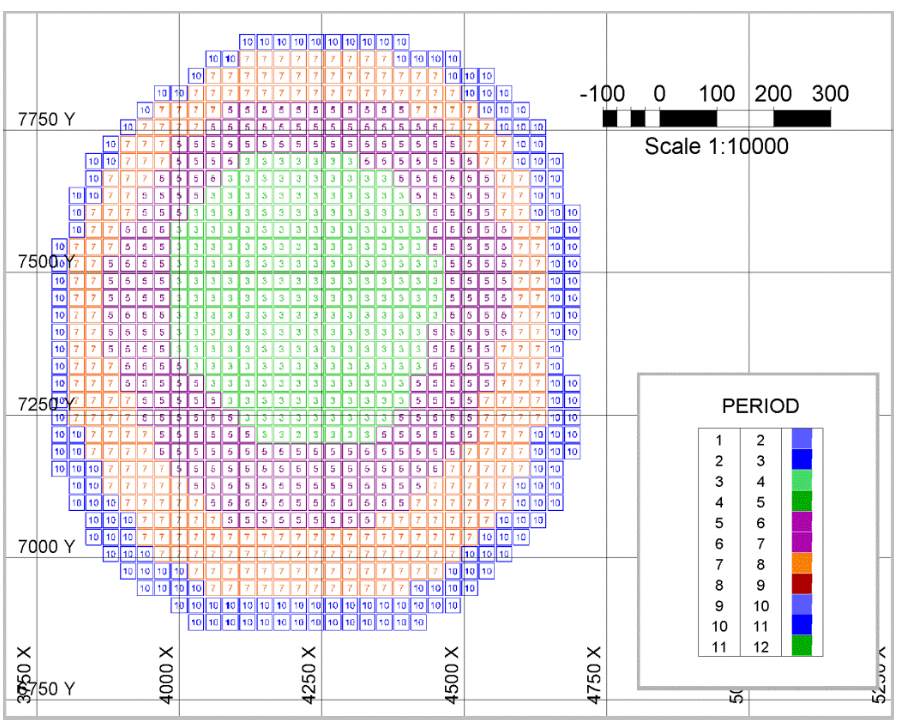

Fig 8. Milawa NPV Schedule Plan View at 600m Elevation

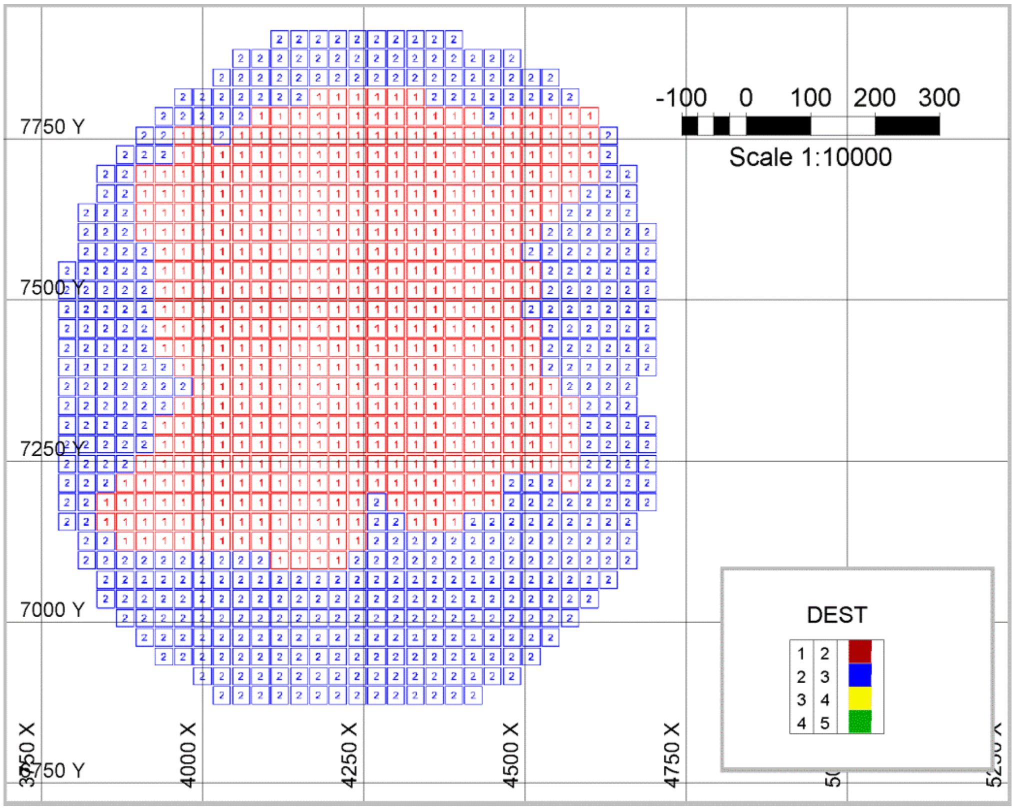

Fig 9. Milawa NPV Destination Plan View at 600m Elevation

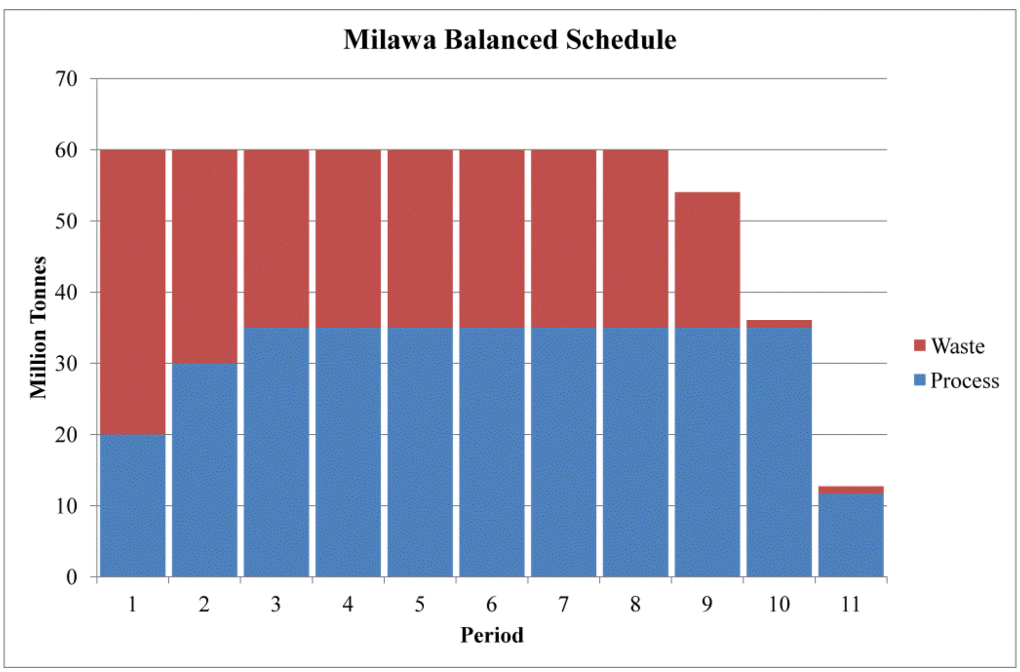

Fig 10. Milawa Balanced Schedule

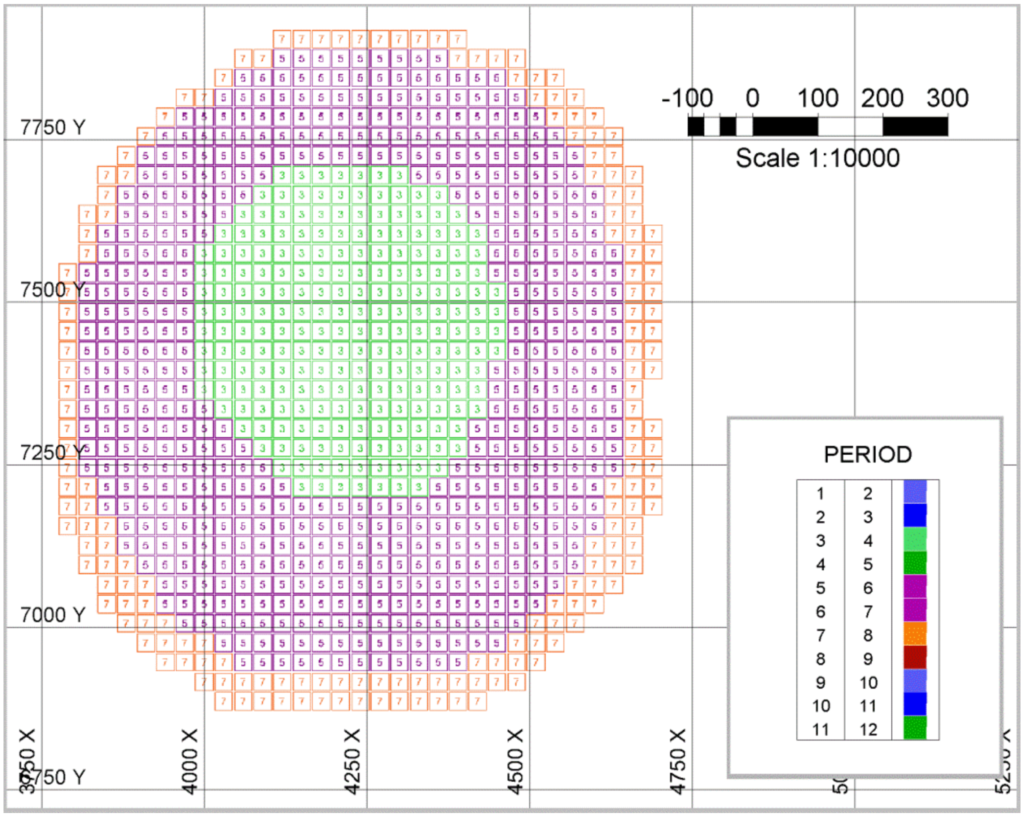

Fig 11. Milawa Balanced Schedule Plan View at 600m Elevation

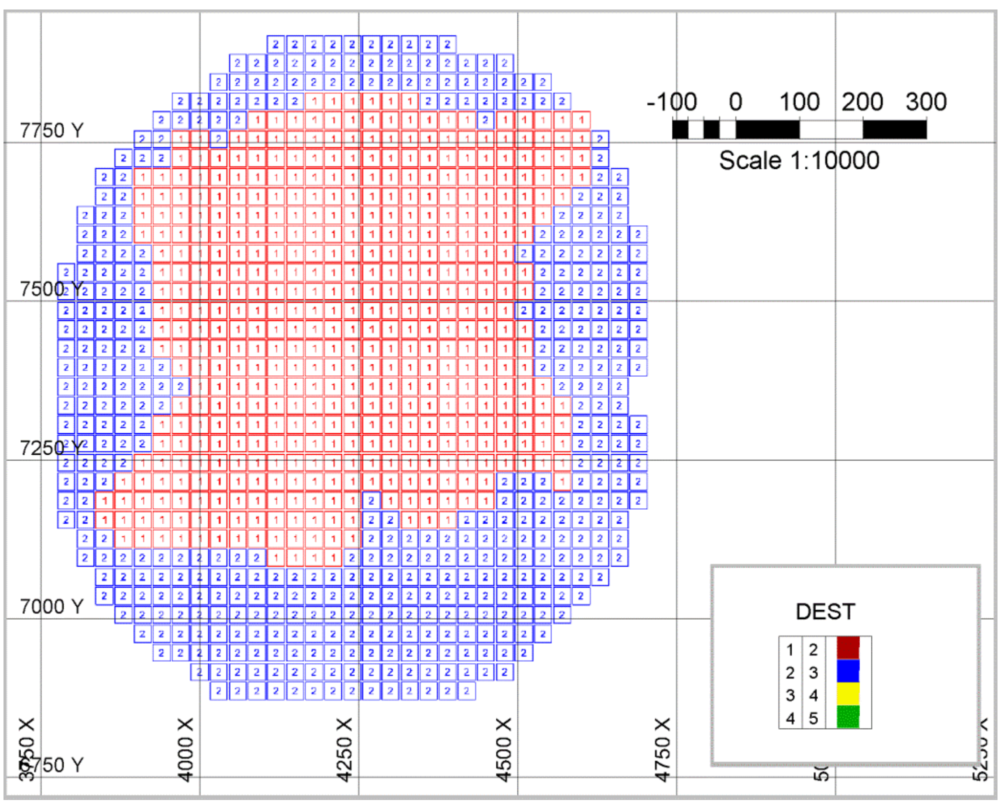

Fig 12. Milawa Balanced Destination Plan View at 600m Elevation

5.2. MILP Schedule, Mining Units: Bench-phases, Processing Units: Blocks

Since the Whittle™ scheduling is performed based on using bench-phases as mining units and blocks as processing units, we used the same resolution for the first case-study. We first ran the MILP model to 5% optimality gap. The results were obtained in 7 seconds and an NPV of $2,653M is reached. Since the runtime was short we ran the model to optimality and obtained the optimal solution in this resolution in 322 seconds. The optimal NPV is $2,660M and the corresponding schedule graph and plan views are presented in Fig 13 to Fig 15. Although the block destinations are very similar to the Whittle™ results, the NPV of the operation shows 18.8% improvement over the highest NPV obtained by implementing Mila NPV scheduling algorithm.

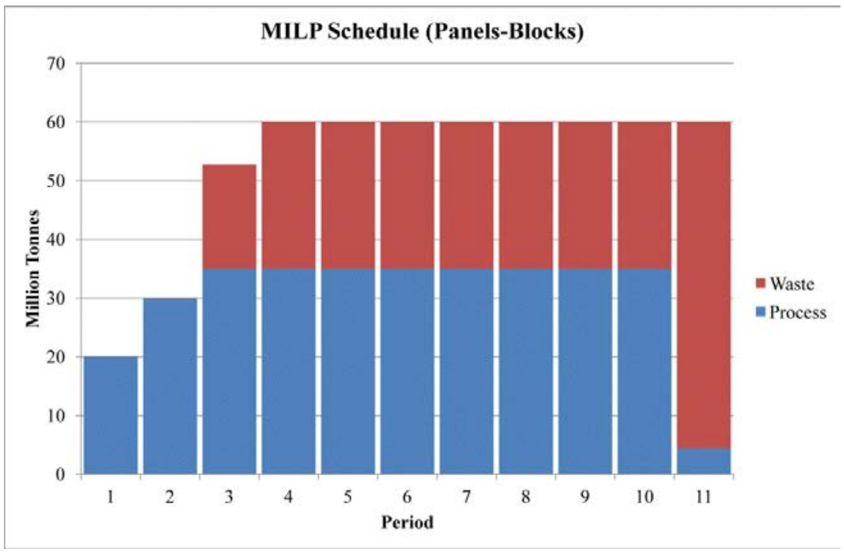

Fig 13. MILP Schedule

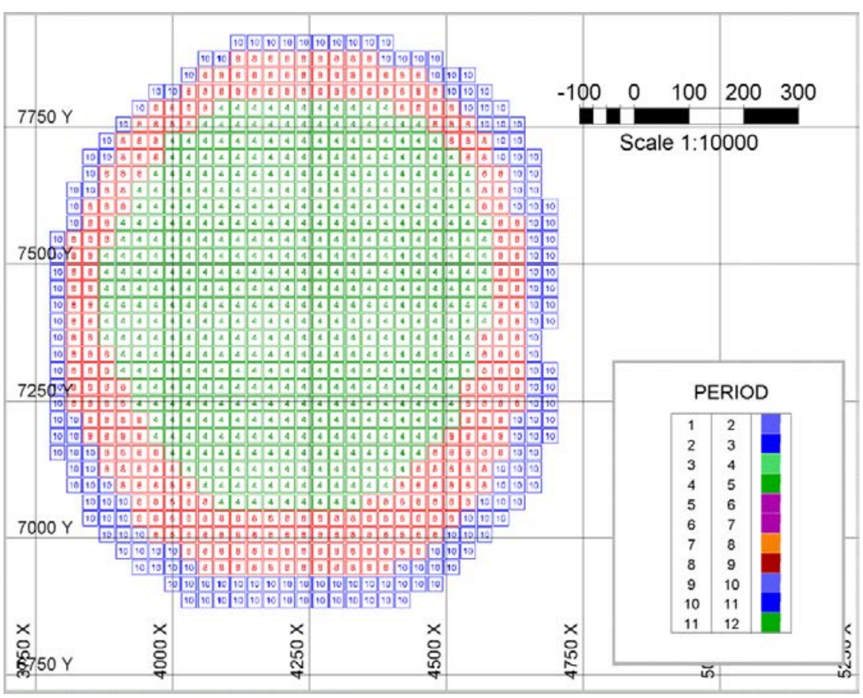

Fig 14. MILP Schedule Plan View at 600m Elevation

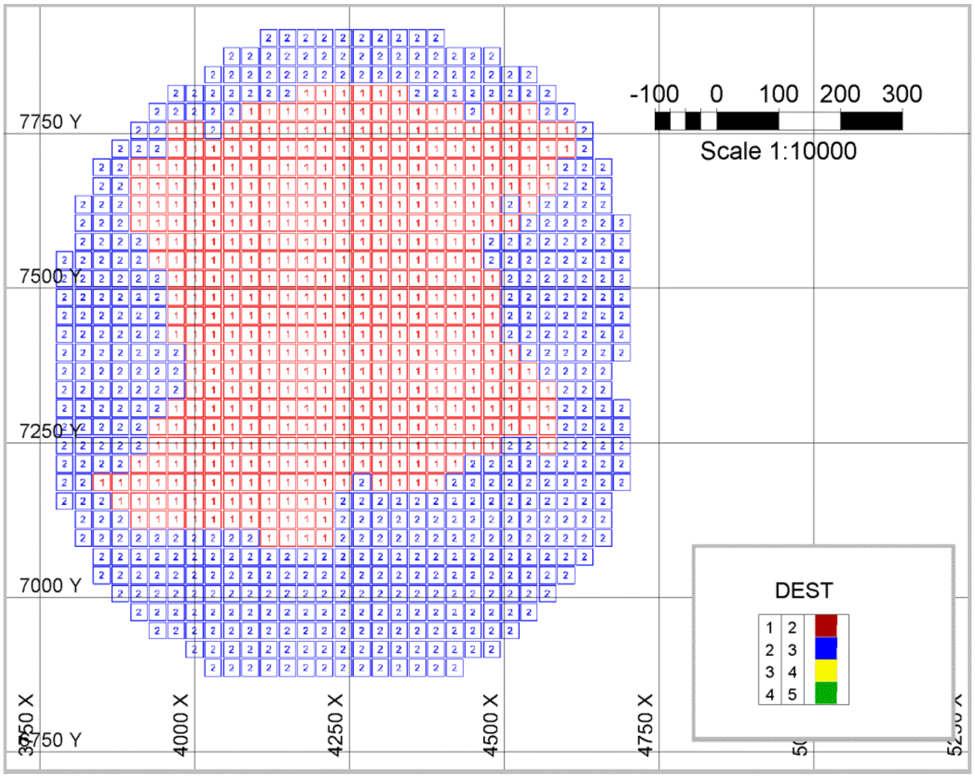

Fig 15. MILP Destination Plan View at 600m Elevation

5.3. MILP Schedule, Mining Units: Bench-phases, Processing Units: Clusters

The focus of this project is on creating clusters of blocks with homogenous grade and rock-type and using the clusters as mining and processing units. Therefore, we use bench-phases as mining units and clusters as processing units in this scenario to evaluate the effects of clustering on the mine planning outcomes. The hierarchical clustering algorithm in this scenario is performed based on the parameters summarized in Table 2. It takes the solver 1.6 seconds to solve the MILP formulation to 10% gap with an NPV of $2,136M. Solving the MILP to optimality takes 30.5 seconds and results in an NPV of $2,185M which is 3.5% less than the Milawa NPV algorithm but with a more balanced schedule. The schedule is presented in Fig 16 and the clusters, extraction periods and destination plan views follow in Fig 17 to Fig 19.

Table 2. Clustering Parameters

Parameter	Value
Distance Weight	0.5
Grade Weight	0
Rock-type Penalty	0.5
Avg. Blocks per Cluster	20
Max. Blocks per cluster	25
Min. Blocks per Cluster	5
Number of Shape Refinement Iterations	3

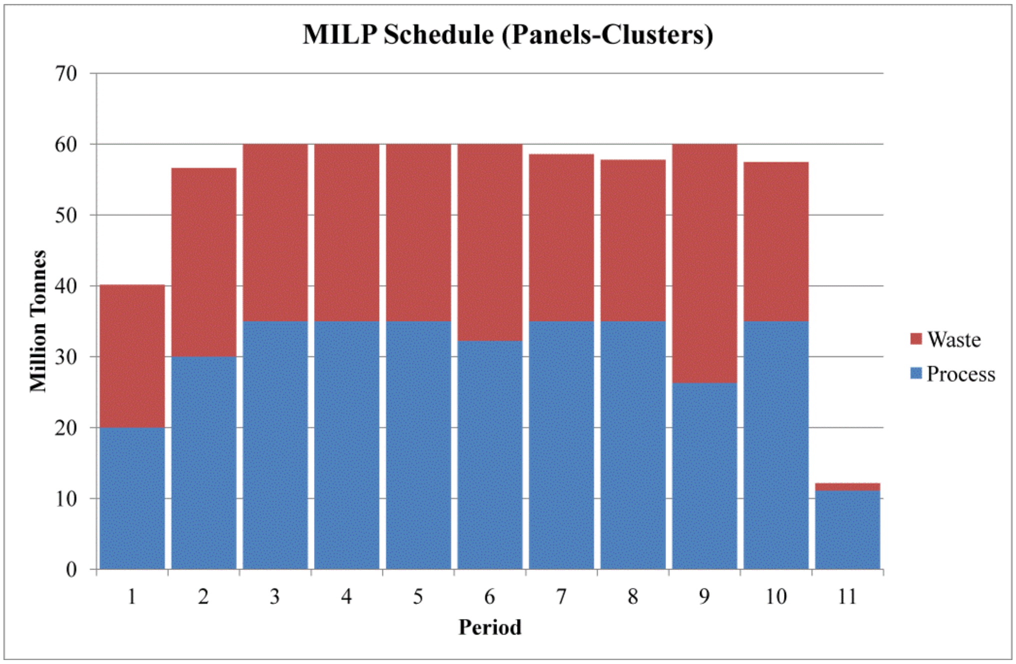

Fig 16. MILP Schedule

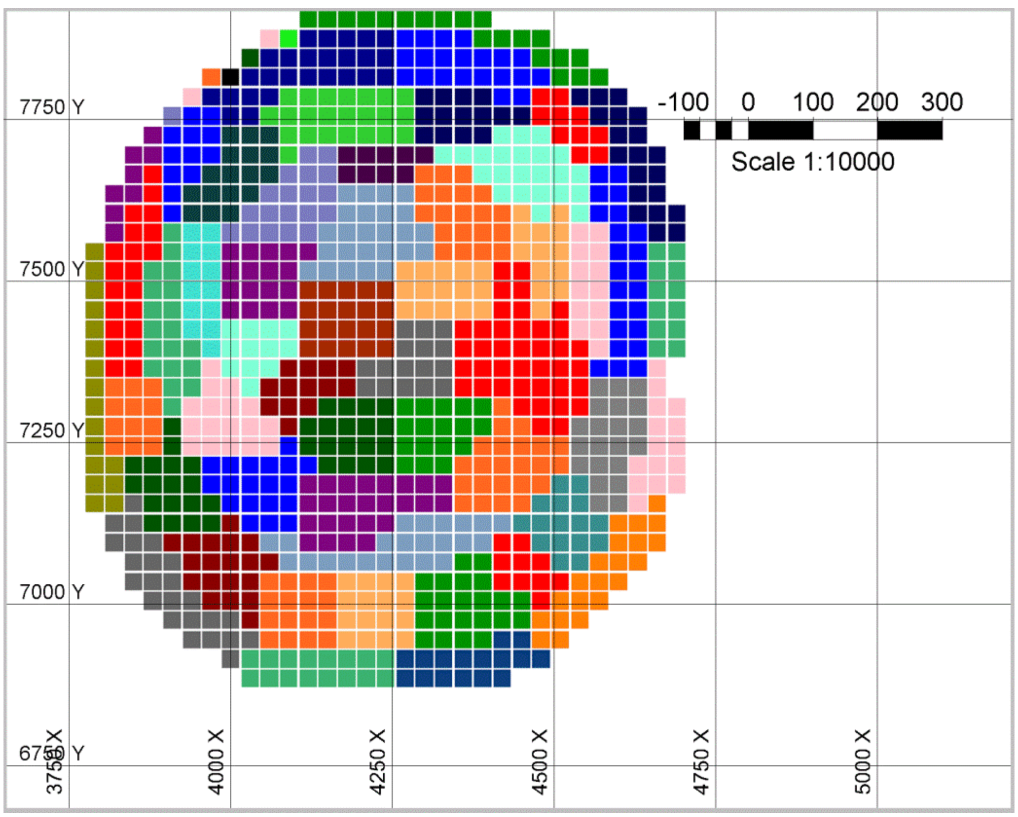

Fig 17. Cluster IDs Plan View at 600m Elevation

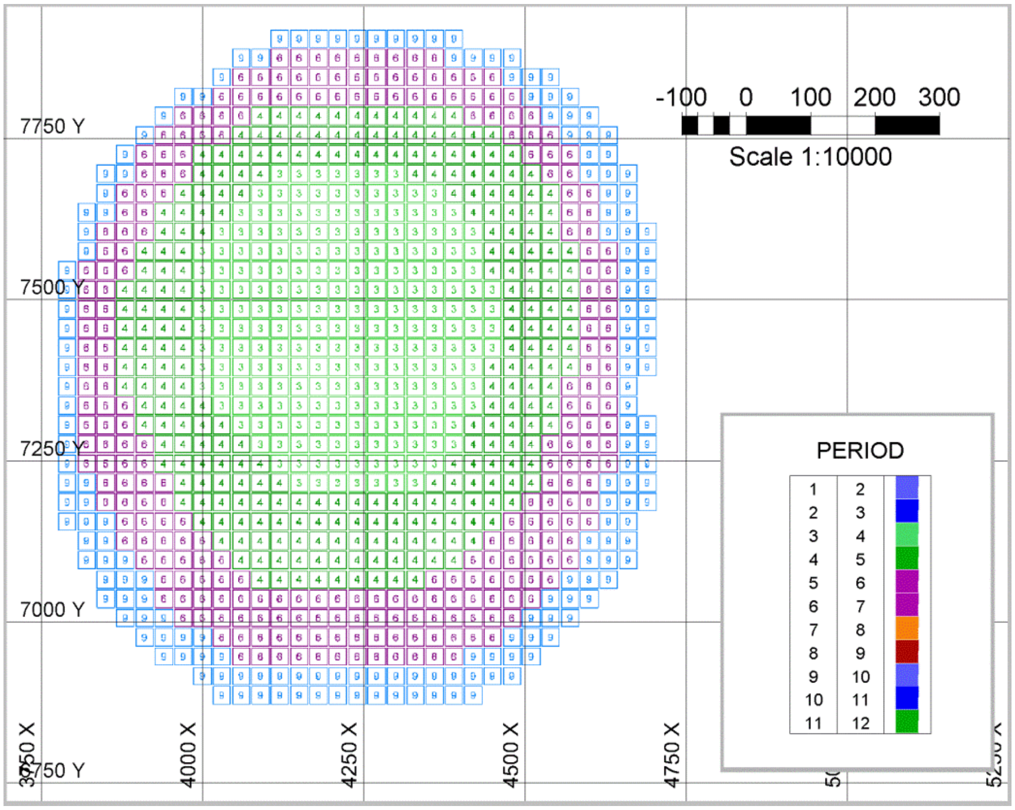

Fig 18. MILP Schedule Plan View at 600m Elevation

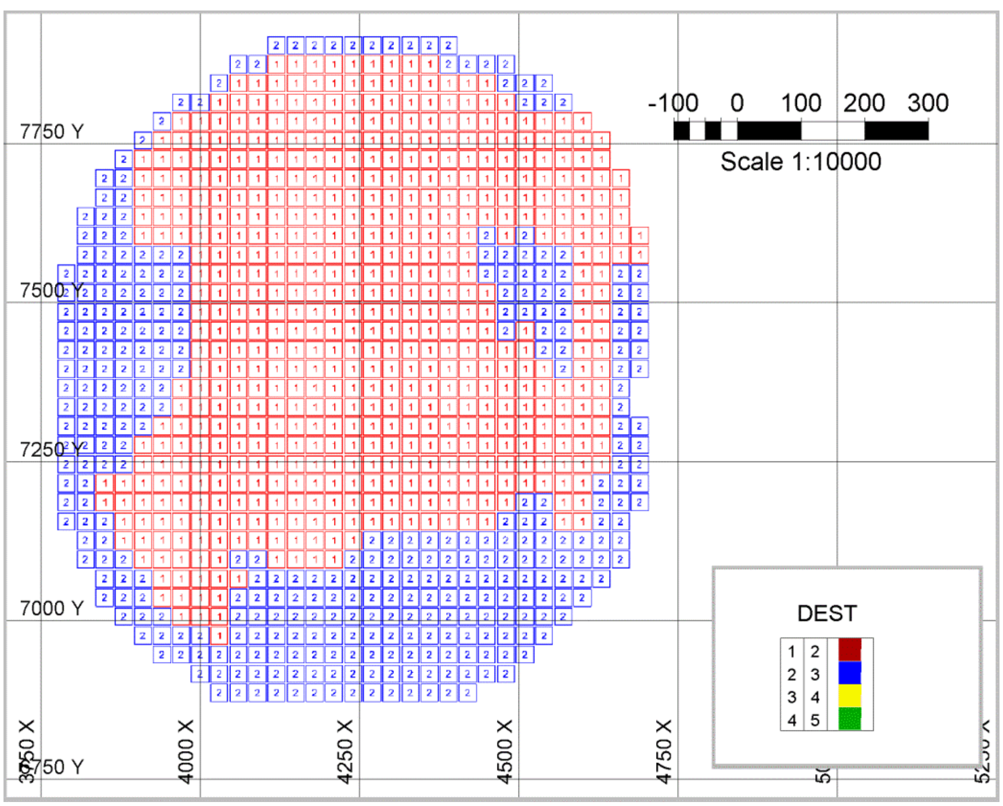

Fig 19. MILP Destination Plan View at 600m Elevation

5.4. Stockpiling

In this section, we study how we can add stockpiles to the case-study. For this purpose, we add three stockpiles with unlimited capacity to Whittle™ scheduler and our own model. Each stockpile is limited to one rock-type. A re-handling cost of 0.4 $/tonne is applied for reclaiming material from stockpiles and sending to the processing plant. Moreover, it is assumed that the recovery of material reclaimed from stockpiles is 2% less than original recoveries. The fleet required to reclaim material from stockpiles and send to the processing plant is considered to be independent of the available mining capacity. Fig 23 shows the schedule generated with Whittle™ based on the listed assumptions. As can be seen in the figure, Whittle™ uses the stockpile to make sure that the plant has enough feed in every period. However, the NPV of the operation based on this schedule is $2,224M which is 0.8% less than the original Milawa NPV algorithm. Fig 24 is the schedule generated from the MILP formulation with the same assumptions. Since the MILP formulation requires a fixed reclamation grade for each stockpile, we averaged the grade values for the clusters with each rock-type and used as the reclamation grade. The reclamation grades presented in Table 3 are used to calculate the revenue generated from reclaiming material from each stockpile and sending to the plant in the MILP formulation.

Table 3. Stockpile Parameters

	Rock-type	**Au Grade (gram/tonne)**			**Cu Grade (%m)**		
		Min	Max	Avg	Min	Max	Avg
SP1	PM	0	0.89	0.45	0	0.42	0.20
SP2	MX	0	1.14	0.58	0	1.29	0.50
SP3	OX	0	1.13	0.42	0	1.29	0.58

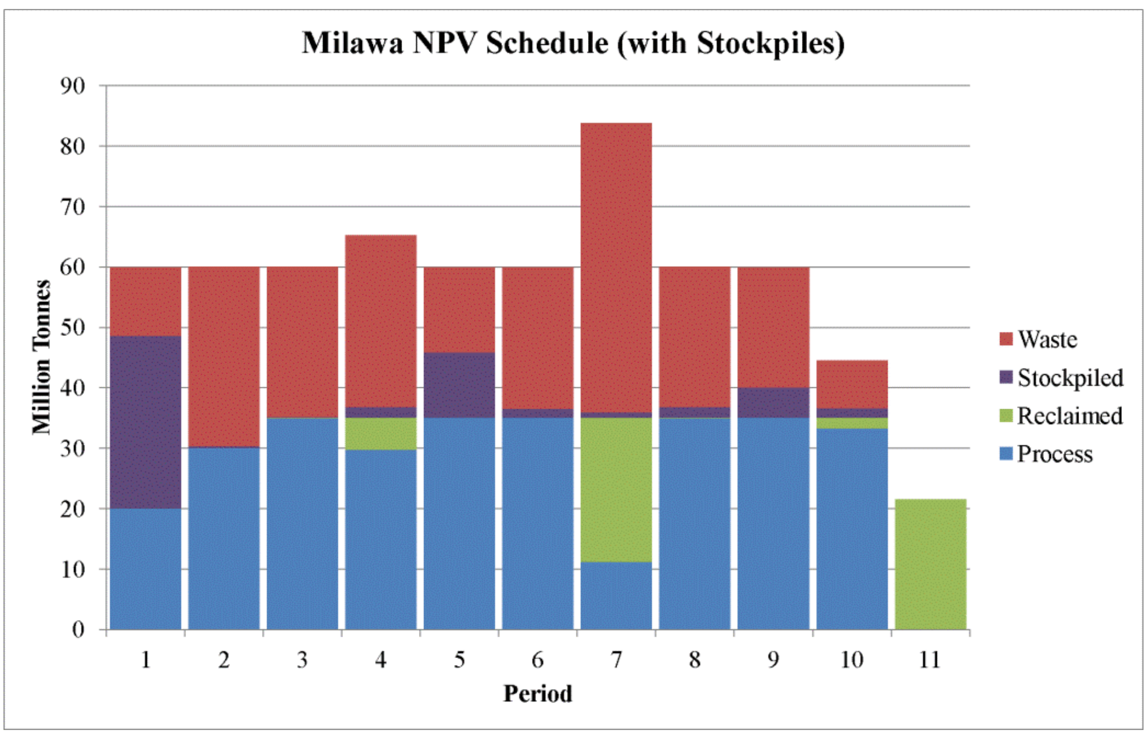

Fig 20. Milawa NPV Schedule (with Unrestricted Stockpiles)

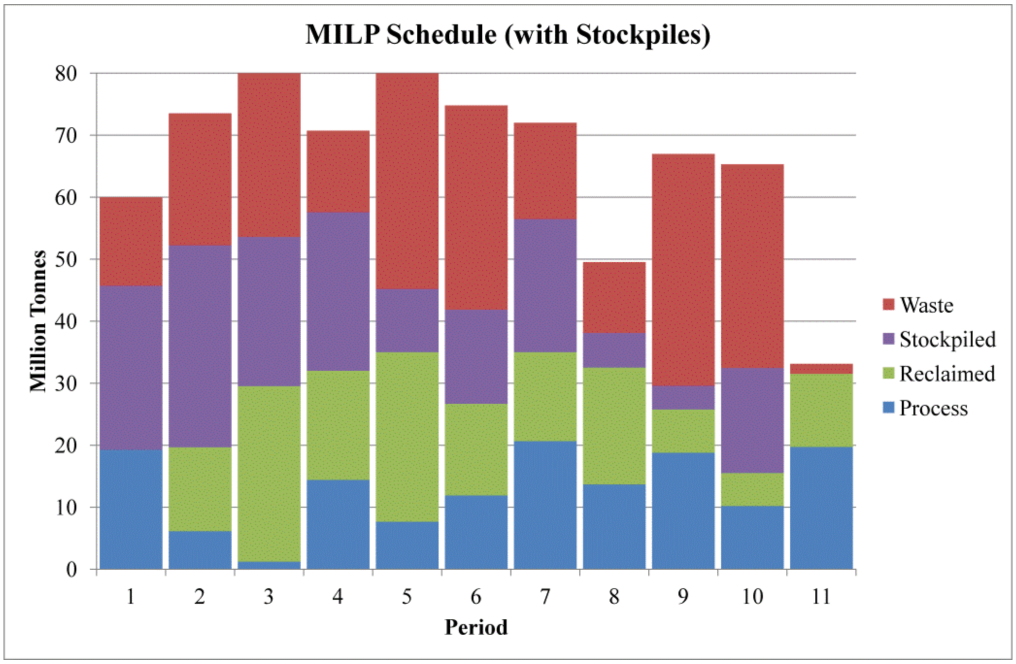

Fig 21. MILP Schedule (with Unrestricted Stockpiles)

The NPV resulted from the MILP schedule is $2,747M which is significantly higher than other scenarios. However, this NPV is resulted from approximating the reclamation grade of the stockpiles with the average grade of the material in the pit. Therefore, we calculated the actual grade of material in the stockpile based on the proposed schedule and the actual revenue generated from reclaiming material from stockpiles and sending to the plant. Fig 22 shows the approximated cash flow based on fixed reclamation grade minus the actual generated revenue for each period. As can be seen in Fig 22, the difference between the approximated revenue and the actual revenue is significant in most of the periods and has resulted in overestimation of the final NPV. In total, the generated NPV is $516M more than the actual NPV that can be generated with this schedule. Therefore, we restricted each stockpile to accept one rock-type with limited grade range to reduce the difference between the assumed average reclamation grade and the actual stockpile grade. The summary of stockpile definitions is provided in Table 4. We calculated the weighted average of grade values in each rock-type within the acceptable ranges and used as the reclamation grades for each stockpile. We used the same ranges for Milawa NPV and compared the outcomes of both schedulers.

As mentioned earlier, Milawa NPV algorithm uses the stockpiles to feed the plant when enough ore cannot be extracted from the mine and extends the mine life to the 11th period in this case. The resulted NPV is $2,155M which is 3.8% less than the original Milawa NPV schedule and 0.6% less than the original Milawa balanced schedule. However, the plant is fully utilized in all periods except than the last period. On the other hand, the MILP model uses stockpiles more frequently and increases the NPV of the operation to $2,432M which is 11.3% more than the original panel-cluster scenario. The plant is also better utilized compared to not using the stockpiles. The generated schedules from Milawa NPV and MILP are presented in Fig 23 and Fig 24 respectively. Similar to the unrestricted stockpile case, we plotted the approximation error in cash flows in Fig 25. The total overestimation in calculating the NPV of the operation is $27M which is a 1.1% error. Moreover, it is possible to decrease the error by using tighter bounds on the stockpile grades or by calibrating the reclamation grades based on the resulted schedule.

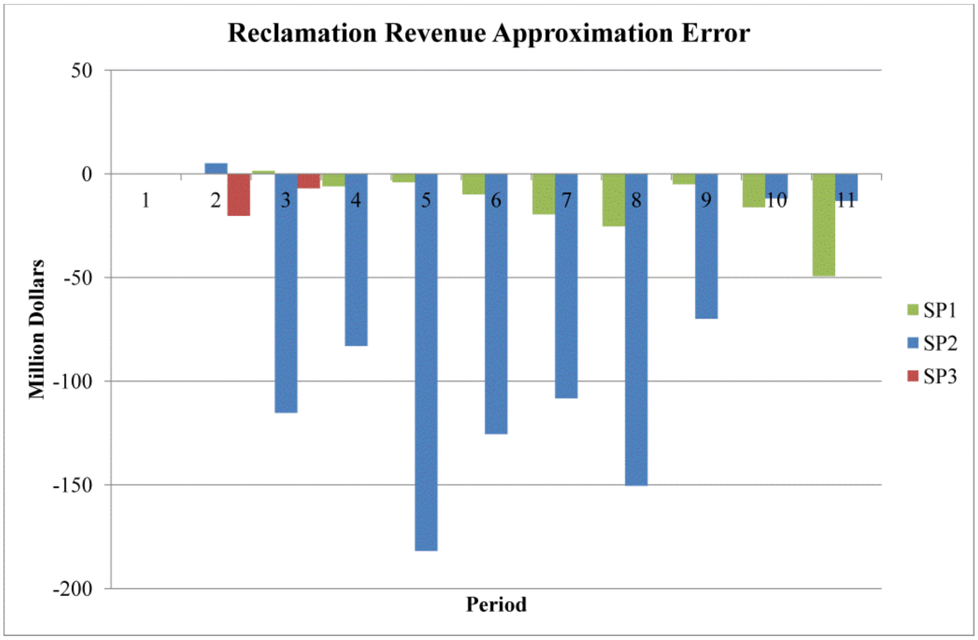

Fig 22. Reclamation Revenue Approximation Error

Table 4. Stockpile Parameters

	Rock-type	Au Grade (gram/tonne)			Cu Grade (%m)		
		Min	Max	Avg	Min	Max	Avg
SP1	PM	0.1	0.3	0.25	0.15	0.45	0.18
SP2	MX	0.2	0.5	0.31	0.1	0.3	0.23
SP3	OX	0.1	0.4	0.15	0.1	0.2	0.17

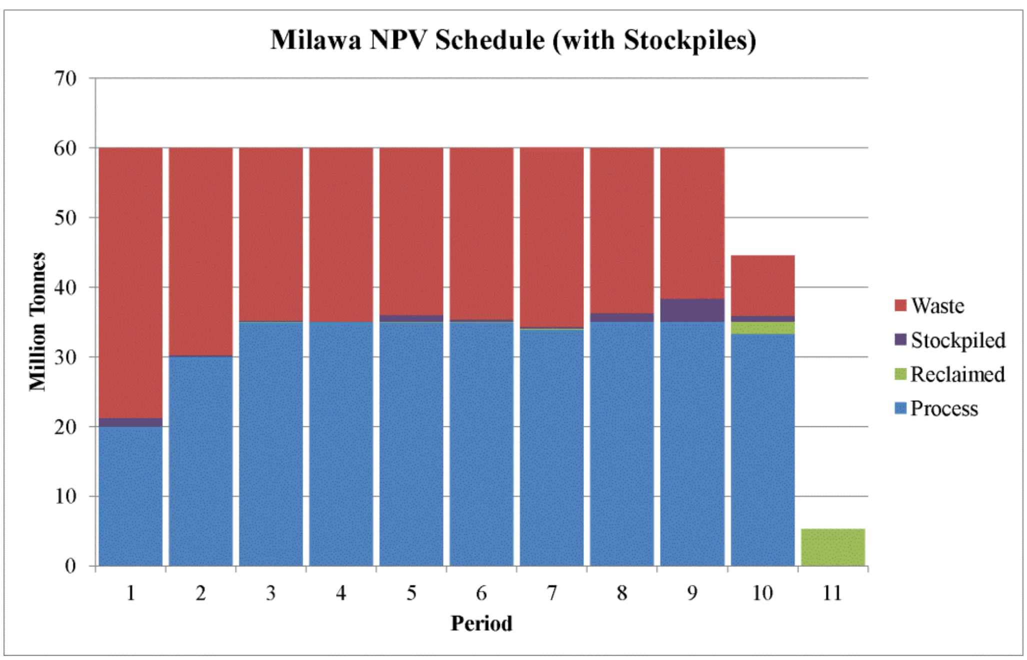

Fig 23. Milawa NPV Schedule (with Stockpiles)

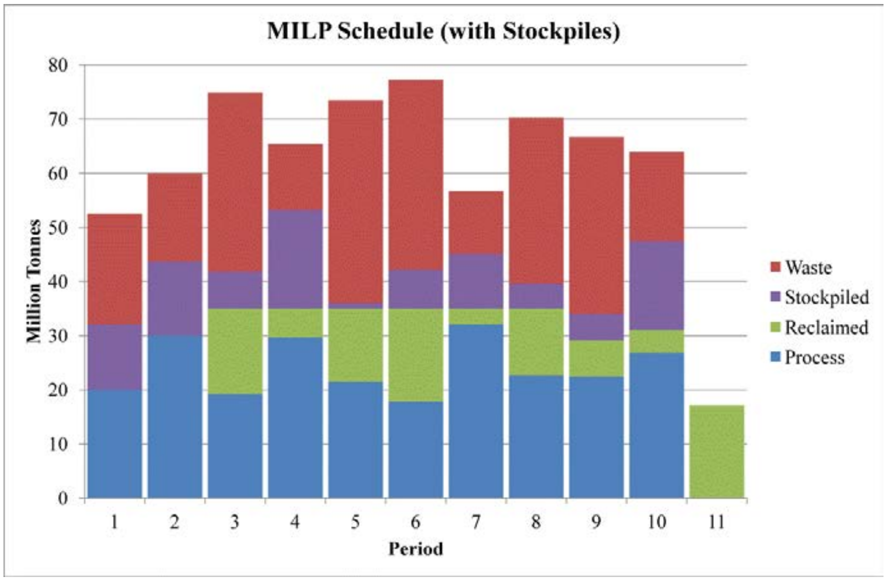

Fig 24. MILP Schedule (with Stockpiles)

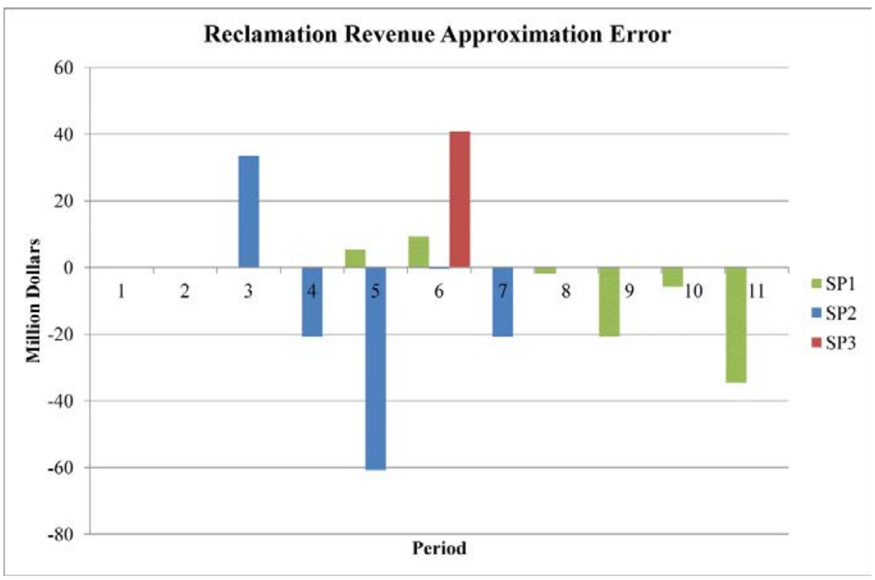

Fig 25. Reclamation Revenue Approximation Error

5.5. Blend Control

Blending is another important aspect of production planning. As mentioned in section 4, our model is capable of controlling the head grade of material sent to processing destinations. This features works for both models with and without stockpiles. In this section, we add lower and upper bounds to the head grade of material sent to the processing plant. The gold and copper grade lower and upper bounds are presented in Table 5. The rest of the parameters are the same as before.

Table 5. Head Grade Control Parameters

Au Grade (gram/tonne)		**Cu Grade (%m)**	
Min	Max	Min	Max
0.3	0.7	0.3	0.7

First, we solve the model by adding grade control constraints and removing stockpiles. The model is solved to optimality in 1,290 seconds and results in an NPV of $2,085M. The production schedule is presented in Fig 26 and the head grade of material sent to the process is presented in Fig 27. Afterwards, we add the same stockpiles as in the previous section to increase the flexibility of the model and test its performance. Although the mathematical formulation uses fixed reclamation grades for stockpiles, we used the actual grade of stockpiles in calculating the head grades. We solved the model by adding the same stockpile settings as the previous section and applying the head grade constraints. Solving the model to optimality takes 613 seconds and results in an NPV of $2,394M which is 1.3% less than not having constraints on the head grades. We expect the NPV to drop more if tighter bounds on the head grade are applied. The production schedule and head grades are plotted in Fig 28 and Fig 29 respectively. As can be seen in Fig 29, the actual head grades violate the upper bounds in only one instance due to approximation of reclamation grade with a fixed number.

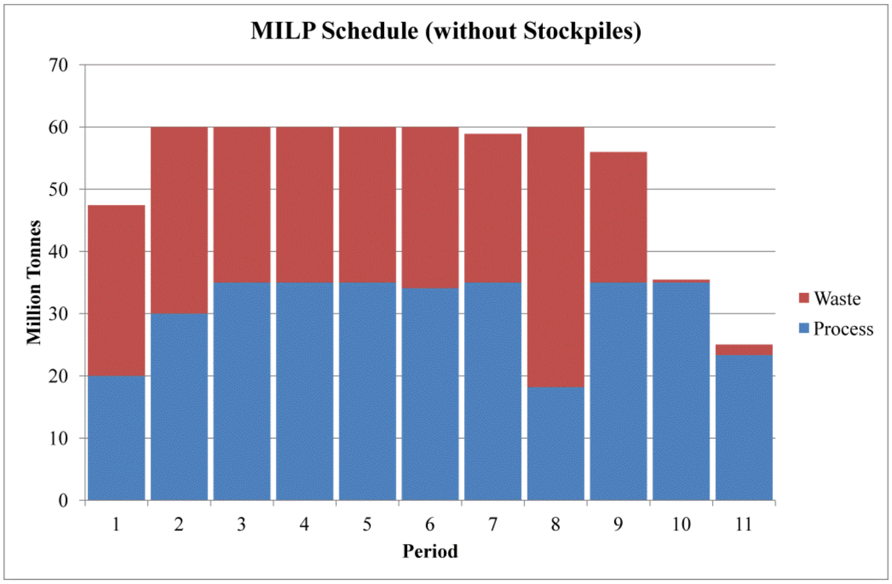

Fig 26. MILP Schedule (without Stockpiles)

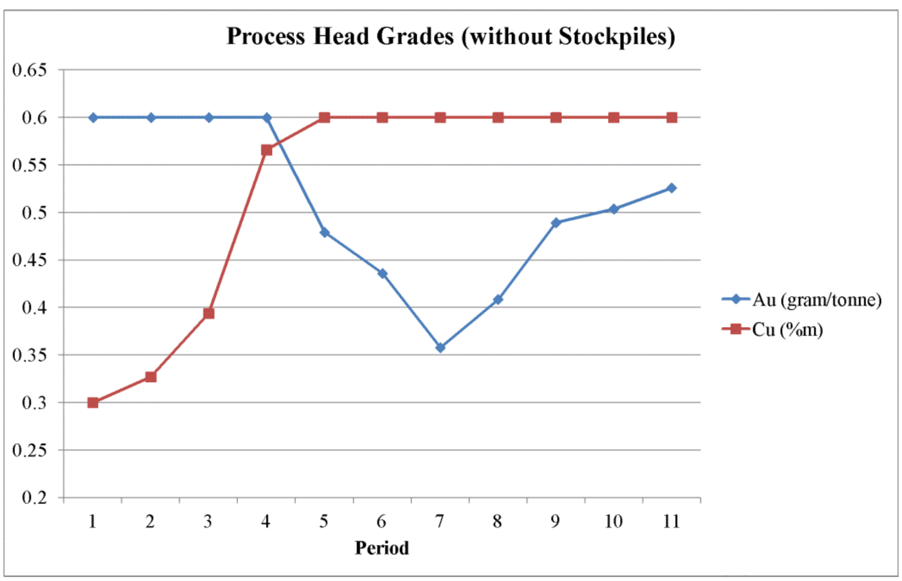

Fig 27. Process Head Grades (without Stockpiles)

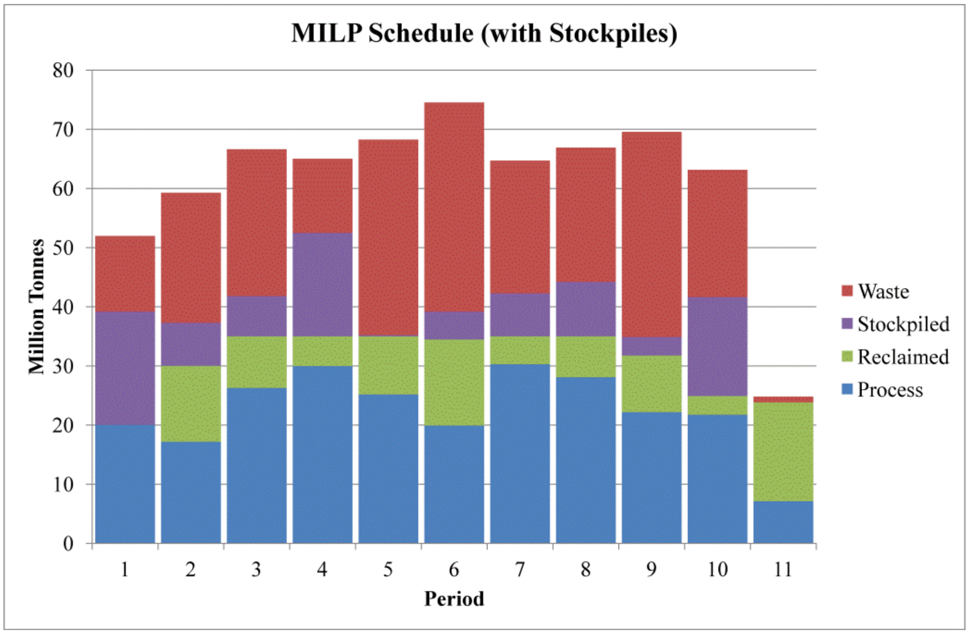

Fig 28. MILP Schedule (with Stockpiles)

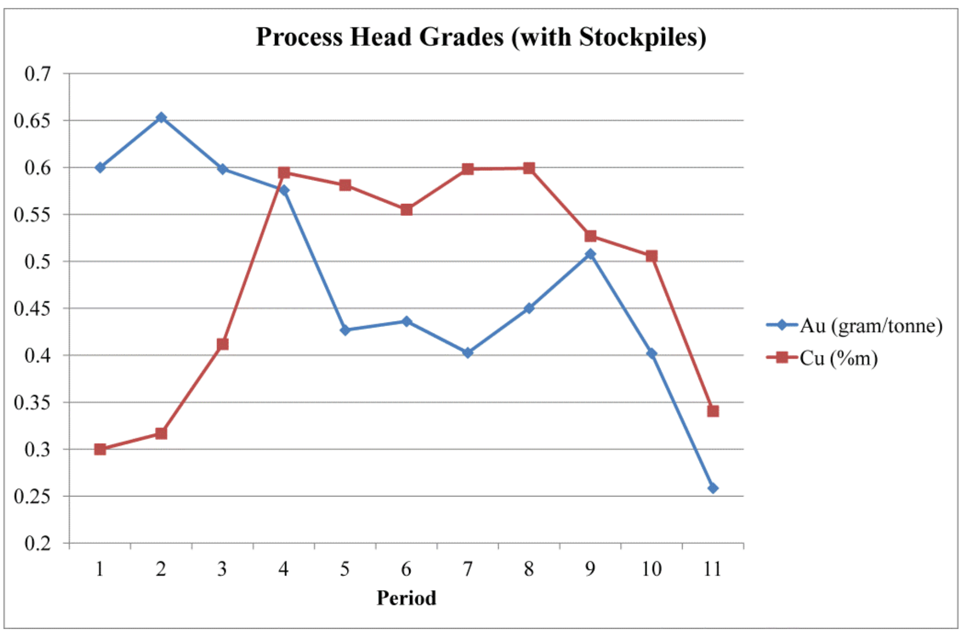

Fig 29. Process Head Grades (with Stockpiles)

6. Conclusion and future work

In this paper, we presented a multi-step approach to long-term open-pit production planning by using different resolutions for making mining and processing decisions. We determine the pushbacks based on a hybrid binary programming-heuristic method and use the intersections of pushbacks and mining benches as mining units. Afterwards, we divide the bench-phases into smaller units with similar rock-type and grade using an agglomerative hierarchical clustering algorithm. These units are then used as processing units. Then, we presented a mathematical model to solve the LTOPP problem with the aggregated units. Finally, we added stockpiling to the model with non-linear and linear objective functions and constraints. In the next step, the linear model was tested on a synthetic small case study to verify the simplification assumptions used for linearization. We concluded that using clusters as processing units results in more practical schedules. Moreover, we showed that we can control the linearization error by restricting stockpiles to predetermined grade values which is aligned with the common practices in mining industry. However, the obtained solution is not optimal for the original non-linear model. Therefore, if we can solve the quadratic model and obtain the optimum solution we can have a better understanding of linearization errors.

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