An Integer Programming Model to Integrate Mine Planning with Composite Tailings Production Plan

Mohammad Mahdi Badiozamani and Hooman Askari-Nasab
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

Abstract

Long-term mine planning models maximize the net present value of the extracted ore material over the mine life-time, subject to a number of technical constraints, such as spatial precedence and equipment capacities. In oil sands mining, further processing of extracted material results in massive volumes of slurry, known as tailings. One of the common technologies for tailings dewatering is adding sand and fluctuant to the slurry and making composite tailings. In this paper, a tailings model is developed to calculate the volume of tailings slurry and CT produced from the hot water extraction process. An integrated mixed integer linear programming (MILP) model is developed to optimize the long-term mine planning model, with respect to CT production and deposition. Two techniques are implemented to reduce the problem size. The model is verified by carrying out a case study on an oil sands dataset and has resulted in an integer solution within less than a 1% optimality gap.

Keywords: open-pit mining, production scheduling, integrated mine planning, integer programming, tailings management, composite tailings.

1. Introduction

The most important byproduct of processing oil sands is tailings slurry. The slurry is pumped into a number of prepared ponds, either in the already-extracted pits as in-pit tailings facilities, or external tailings facilities. In practice, tailings management is critical to mine planning, as the tailings containments can be formed only within the limited lease areas and will cover extensive landscapes. In order to make the tailings ponds trafficable and ready for reclamation, several technologies have been used to accelerate the natural dewatering time for the tailings, one of which is composite tailings (CT) production from mature fine tailings (MFT). CT is a non-segregating material that releases water and consolidates more rapidly than MFT (Guo and Wells, 2010). In the current practice in oil sands surface mining, there is not any direct linkage between the long-term mine plans and the final non-segregating tailings production plans (Kalantari et al., 2012).

The focus of this study is on development of an integrated long-term mine planning model that includes a CT production plan. The proposed MILP model will maximize the net present value (NPV), including the CT deposition costs, subject to a number of constraints, including the CT deposition capacities. Review of the related literature reveals that the merger between mine planning and tailings management is missing. Therefore, the objective of this paper is to develop,

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implement, and verify an integrated optimization framework for long-term oil sands open-pit mine and tailings production planning. In this study, it is assumed that a block model of the ore body is available, as an input to the production planning. The workflow of the study is as follows:

1. Development of a tailings model

   The Clark hot water extraction (CHWE) process (Clark and Pasternack, 1932; Clark, 1939) has been the main method of bitumen extraction, including crushing the extracted material, adding hot water and separating the blend through a number of cells. Extracting and processing bitumen will result in downstream slurry production, which is pumped into the tailings ponds and will stay there for segregation. Coagulants such as gypsum will be added to the MFT to catalyze the dewatering. The resulting non-segregating material is called CT. In this paper, a tailings model has been developed that calculates the volume of total tailings slurry and its components (water, sand, and fine material) produced from ore-feed processing. The tailings model in this paper is based on the tailings flow process of Suncor Energy (Suncor, 2009) and has the same basis as the tailings calculations presented by (Kalantari et al., 2012).

2. Development of an integrated mine and tailings scheduling MILP model

   The order of extracting material in the long-term is optimized in the strategic mine production scheduling model. The objective is to maximize the NPV over the mine life-time, subject to a number of constraints such as mining and processing capacities. In this paper, CT and MFT production and CT deposition sequencing are added into the MILP model. The new objective function includes the cost of CT deposition. A number of decision variables and constraints are introduced to control the CT production and its deposition in CT cells, which have resulted in an integrated MILP model for mine planning and tailings management.

3. Making the schedule more practical

   Most of the production-scheduling formulations presented in the literature are based on block sequencing. However, in most of the cases, block resolution production scheduling does not result in a practical production plan. It is desirable to consider larger mining units as the selective mine scheduling units. Two techniques are used in this paper to make the results practical and reduce the solution time: (1) using aggregates of mining blocks, as mining cuts and mining panels, to guarantee the practicality of the results, and (2) preprocessing, which is fixing the value of some decision variables into either zero or one by considering earliest and latest start times for extraction (Bley et al., 2010; Tabesh and Askari-Nasab, 2011b).

   The integration of tailings management, specifically CT deposition, with the long–term mine planning model makes the proposed model significantly important in oil sands' surface mining. The model optimizes the NPV over the mine-life, while it considers CT production and deposition in CT cells. The other contribution of this study is the implementation of two techniques to reduce the size of the problem and make it practical for real instances. The proposed model can be considered as one of the building blocks towards a complete integration of mine planning with dyke construction and tailings planning.

   This paper is organized in the following order: Section 2 includes a brief definition of the problem. The related literature to the research, including tailings management and mine planning optimization, are reviewed in section 3. The proposed tailings model and equations, the mathematical model, and the implemented techniques for problem-size reduction are discussed under the theoretical framework in section 4. To verify the proposed model, a case study is presented in section 5, followed by the numerical results and discussion in section 6. The paper is concluded in section 7, by mentioning the potential next steps of the research.

2. Problem definition

Typically, MILP has been used to model long-term scheduling problems. The basis in all MILP models consists of an objective function and a list of constraints. The objective function maximizes the NPV, subject to limited capacities for mining and processing operations, processing plant requirements, and extraction precedence. On the other hand, tailings slurry is the most important byproduct of bitumen extraction. It has a significant influence on mine planning, since the available area for storage of tailings slurry is limited to the lease area.

The total volume of tailings slurry and its composition is directly related to the amount of extracted and processed oil sands material. This fact indicates that any change in mine planning will change the total volume of tailings consequently. Thus, a tailings plan must be considered as part of long-term mine planning so that the available capacity of a tailings facility and tailings deposition cost can be included in the mine planning model. In a few works the authors have tried to link the tailings production planning with oil sands mine planning. (Kalantari et al., 2012) map the ore-feed tonnage, considering the composition of ore-feed, with the volume of produced CT downstream, by calculating the volume of CT as a function of a mine production plan. However, a more complete version of mathematical programming for integrated mine planning with CT production and deposition planning can be constructed as follows:

Maximize (NPV – CT deposition costs)

Subject to:

Processing plant constraints

Mining capacity constraints

Extraction precedence constraints

Tailings and MFT capacity constraints

CT deposition constraints

This model will be discussed, solved and verified on a real dataset in the following sections of this paper.

3. Literature review

The early use of mathematical programming in open-pit mine planning goes back to 1960 (Johnson, 1969). Since then, many models have been developed to incorporate different related concepts in mine planning. A comprehensive review for the usage of operations research tools in the natural resource industry, including mining, can be found in (Bjorndal et al., 2011).

Operations research and mine planning

Strategic mine planning refers to the group of works that consider long-term aspects in scheduling, such as the total production and processing capacities, over the mine life-time. The related literature on the applications of operations research techniques in modeling and solving of the mine production scheduling problems is comprehensively reviewed by (Newman et al., 2010). Examples of operations research applications in open-pit mining are: ultimate pit limit design and mine layout models, equipment allocation models, and tactical block-sequencing models. In many early models, the whole mine sequencing problem has been considered at once, which resulted in a fixed mining sequence (Bjorndal et al., 2011). However, as the problem is so large due to the large number of periods and blocks, researchers have considered the discrete nature of the problem and have used a number of techniques to solve the problem more efficiently. Examples include using Lagrangian relaxation as a decomposition technique (Dagdelen and Johnson, 1986), using dynamic programming (Onur and Dowd, 1993), using genetic algorithm (Denby and Schofield, 1994), using branch and cut technique (Cacceta and Hill, 2003), and decomposing the problem and reducing it to a knapsack problem (Chicoisne et al., 2012).

Mine planning models are NP-hard for real size problems (Gleixner, 2008). That is because there are numerous integer and continuous variables in the models to control the precedence of extraction and material destinations for each mining block over the mine life-time. The other shortcoming of considering mining blocks in mine planning is that the generated schedules are not practical, as in many cases the blocks that must be extracted in one period are scattered all over the bench. In order to generate a practical production schedule, a number of techniques are developed to aggregate the mining blocks into larger units.

Over the last decade, the concept of block aggregation has been implemented to generate a more practical production schedule and also to reduce the number of decision variables. Block aggregation has been used in mine planning models (Samanta et al., 2005; Zhang, 2006; Boland et al., 2009). An example of the block aggregation method is developed by Ramazan as the fundamental tree algorithm (Ramazan et al., 2005; Ramazan, 2007). Another example is the implementation of clustering algorithms, as proposed and used by (Tabesh and Askari-Nasab, 2011a), to aggregate the blocks based on a similarity index. The index measures the similarity between each pair of blocks based on the rock type, grade, and distance between the blocks. The authors propose that using aggregate units generates a more practical production schedule considering selective mining units. The aggregate units have been used in an MILP model to find the optimal production schedule. A larger unit for material extraction can be defined as a mining panel, which is more compatible for oil sands surface mining as a shallow deposit. Panels are the intersections of pushbacks and mining benches. (Ben-Awuah, 2013) uses different resolutions for mining and processing. The author uses mining panels for mining operations, while the mining cuts are used for determining the destinations for extracted material. Other techniques, such as upper and lower cones variable reduction, have been developed and used to reduce the number of decision variables (Bley et al., 2010; Tabesh and Askari-Nasab, 2011b).

The Lagrangian relaxation is the other operations research technique that is used in mine-planning optimization. The core assumption in Lagrangian relaxation is that the constraints can be divided into two groups of soft versus hard constraints, and the problem is easily solvable in the presence of only soft constraints. Penalty functions will be considered for any violation from the relaxed hard constraints. (Dagdelen and Johnson, 1986) use Lagrangian relaxation to solve the open-pit production planning problem. The authors relax the capacity constraints for mining per period and solve the model with precedence constraints. For small cases, the sub-gradient algorithm updates the Lagrangian multipliers. (Akaike and Dagdelen, 1999) continue the previous work by using an iterative procedure to update the Lagrangian multipliers and run it until a feasible solution is found for the original MILP problem. (Cullenbine et al., 2011) solve the block sequencing integer programming problem through a sliding time window heuristic that uses Lagrangian relaxation for computational efficiency in solving the sub-models defined over time windows. The proposed heuristic algorithm solves a restricted Lagrangian sub-problem in each of its iterations.

The mathematical programming and operations research techniques are proved to be powerful means for modeling and solving the mine planning models. Integer programming, goal programming, Lagrangian relaxation, LP relaxation, and branch-and-cut algorithm are instances of operations research applications in mine production scheduling.

Tailings management

The dominant method for extraction of bitumen out of the oil sands is the CHWE process (Clark and Pasternack, 1932; Clark, 1939), which results in huge volumes of slurry, known as tailings. The traditional approach has been pumping the tailings slurry into tailings ponds. A number of environmental issues are tied to the tailings ponds. A complete list of environmental impacts associated with oil sands tailings can be found in (Rodriguez, 2007; Singh, 2008; Woynillowicz and Severson-Baker, 2009). Tailings slurry needs to be treated in a way to reduce the environmental problems. Different technologies have been developed for dewatering tailings slurry (Longo et al., 2010) and change the tailings ponds to a trafficable landscape, ready for reclamation and re-vegetation (Sobcowicz and Morgenstern, 2010). Transforming tailings to CT is one of these technologies (Guo and Wells, 2010).

Some authors have addressed tailings planning and management in the oil sands (Mikula et al., 1998; Chalaturnyk et al., 2002; Soane et al., 2010). However, a literature review reveals that there are not many academic publications on tailings planning. Directive 074 mandates that Alberta oil sands operators publish their tailings management plans (McFadyen, 2008). Under the Directive, oil sands operators are supposed to report their short-term plans and update long-term plans for treating their produced tailings.

So far, many works have been published covering different aspects of mine planning. There is a need to consider tailings management in mine planning, as decisions about tailings are critical to mine plans. A literature review shows that there are few significant works that include tailings planning in the same framework as mine planning. Despite the few works such as (Kalantari et al., 2012) that have targeted the integration of mine planning and CT planning, the merger between tailings management, specifically CT production and deposition planning, and long-term mine planning is missing in the current academic literature. Therefore, the main objective in this paper is to develop an integrated mine planning model that includes CT production and deposition planning. The results of a comprehensive tailings model are used as inputs to the mine-planning model to make an integrated mine-planning framework. Two variable reduction techniques are used to make the problem tractable for large-scale cases.

4. Theoretical framework

Three main categories are used in this paper to develop an integrated mine planning model: (1) developing an integrated MILP model for mine planning and CT production, (2) developing a tailings model, and (3) variable reduction techniques for improving the solution time.

4.1. The MILP model

An MILP model is developed to optimize the mine production scheduling problem. The MILP model integrates tailings and CT deposition planning with long-term mine planning. The model includes an objective function and a number of constraints. The proposed model maximizes the profit that is the revenue made from selling the final product, minus the mining cost and the cost of CT deposition. The profit from mining and processing of each mining panel is calculated through Eq. (1).

$$d_p^{a,u,t} = \sum_{k \in p_a} r_k^{u,t} - q_p^{a,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\},\ p \in \{1,...,P\},\ a \in \{1,...,A\} \tag{1}$$

Where:

$$r_k^{u,t} = \sum_{e=1}^{E} o_k \times g_k^e \times r^{u,e} \times \left(p^{e,t} - cs^{e,t}\right) - \sum_{e=1}^{E} o_k \times cp^{u,e,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{2}$$

$$q_p^{a,t} = \sum_{k \in p} \left(o_k + w_k\right) \times cm^{a,t} \qquad \forall t \in \{1,..,T\}, p \in \{1,..,P\}, a \in \{1,..,A\} \tag{3}$$

The total cost of CT deposition will be deducted from the calculated revenue in Eq. (1). For each CT cell, the deposition cost is the cost of sending a volume unit of CT for deposition, multiplied by the total volume of the CT cell. The cost of CT deposition is calculated as in Eq.(4).

$$i_c^t = h_c \times ct^{c,t} \qquad \forall t \in \{1,..,T\}, c \in \{1,..,C\} \tag{4}$$

4.1.1. The MILP formulation

$$Max \sum_{t=1}^{T} \left(\sum_{u=1}^{U} \sum_{a=1}^{A} \sum_{j=1}^{J} \sum_{\substack{p \in B_j \\ k \in B_p}} \left[r_k^{u,t} \times x_k^{u,t} - q_p^t \times y_p^{a,t} \right] - \sum_{c=1}^{C} i_c^t \times z_c^t \right) \tag{5}$$

$$T_{Ml}^{a,t} \le \sum_{j=1}^{J} \left(\sum_{p \in B_j} \sum_{k \in B_p} \left(o_k + w_k \right) \times y_p^{a,t} \right) \le T_{Mu}^{a,t} \qquad \forall t \in \{1,...,T\}, \forall a \in \{1,...,A\} \tag{6}$$

$$T_{Pl}^{u,t} \le \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(o_k \times x_k^{u,t} \right) \right) \le T_{Pu}^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{7}$$

$$\underline{g}^{u,t,e} \sum_{j=1}^{J} \sum_{k \in B_j} \left(o_k \times x_k^{u,t} \right) \le \sum_{j=1}^{J} \sum_{k \in B_j} \left(g_k^e \times o_k \times x_k^{u,t} \right) \qquad \forall t \in \{1,..,T\},\ u \in \{1,..,U\}, e \in \{1,..,E\} \tag{8}$$

$$\sum_{j=1}^{J} \sum_{k \in B_j} \left(g_k^e \times o_k \times x_k^{u,t} \right) \le \overline{g}^{u,t,e} \sum_{j=1}^{J} \sum_{k \in B_j} \left(o_k \times x_k^{u,t} \right) \qquad \forall t \in \{1,..,T\},\ u \in \{1,..,U\}, e \in \{1,..,E\} \tag{9}$$

$$\underline{f}^{u,t,o} \sum_{j=1}^{J} \sum_{k \in B_j} \left(o_k \times x_k^{u,t} \right) \le \sum_{j=1}^{J} \sum_{k \in B_j} \left(f_k^o \times o_k \times x_k^{u,t} \right) \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\} \tag{10}$$

$$\sum_{j=1}^{J} \sum_{k \in B_j} \left(f_k^o \times o_k \times x_k^{u,t} \right) \le \overline{f}^{u,t,o} \sum_{j=1}^{J} \sum_{k \in B_j} \left(o_k \times x_k^{u,t} \right) \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\} \tag{11}$$

$$T_{Tl}^{u,t} \le \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(t_k \times x_k^{u,t} \right) \right) \le T_{Tu}^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{12}$$

$$T_{Fl}^{u,t} \le \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(f_k \times x_k^{u,t} \right) \right) \le T_{Fu}^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{13}$$

$$T_{Sl}^{u,t} \le \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(s_k \times x_k^{u,t} \right) \right) \le T_{Su}^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{14}$$

$$T_{Wl}^{u,t} \le \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(r_k \times x_k^{u,t} \right) \right) \le T_{Wu}^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{15}$$

$$T_{Xl}^{u,t} \le \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(h_k \times x_k^{u,t} \right) \right) \le T_{Xu}^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{16}$$

$$T_{Yl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(p_k \times x_k^{u,t}\right)\right) \le T_{Yu}^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{17}$$

$$\sum_{c=1}^{C}\left(h_c \times z_c^t\right) = \sum_{j=1}^{J}\sum_{k\in B_j}\left(p_k \times x_k^{u,t}\right) \qquad \forall t \in \{1,...,T\} \tag{18}$$

$$\sum_{u=1}^{U}\sum_{k\in B_p}\left(o_k \times x_k^{u,t}\right) \le \sum_{a=1}^{A}\sum_{k\in B_p}\left(o_k + w_k\right)\times y_p^{a,t} \qquad \forall t \in \{1,..,T\},\ p \in \{1,..,P\} \tag{19}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} x_k^{u,t} \le 1 \qquad \forall k \in \{1,..,K\} \tag{20}$$

$$b_p^t - \sum_{a=1}^{A}\sum_{i=1}^{t} y_s^{a,i} \le 0 \qquad \forall t \in \{1,...,T\},\ p \in \{1,...,P\},\ s \in N_p(L) \tag{21}$$

$$b_p^t - \sum_{a=1}^{A}\sum_{i=1}^{t} y_r^{a,i} \le 0 \qquad \forall t \in \{1,...,T\},\ p \in \{1,...,P\},\ r \in O_p(L) \tag{22}$$

$$\sum_{a=1}^{A}\sum_{i=1}^{t} y_p^{a,i} - b_p^t \le 0 \qquad \forall t \in \{1,...,T\},\ p \in \{1,...,P\} \tag{23}$$

$$b_p^t - b_p^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\},\quad p \in \{1,...,P\} \tag{24}$$

$$a_c^t - \sum_{i=1}^{t} z_r^i \le 0 \qquad \forall t \in \{1,...,T\},\ c \in \{1,...,C\},\ r \in Q_c(R) \tag{25}$$

$$\sum_{i=1}^{t} z_c^i - a_c^t \le 0 \qquad \forall t \in \{1,...,T\},\ c \in \{1,...,C\} \tag{26}$$

$$a_c^t - a_c^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\},\quad c \in \{1,...,C\} \tag{27}$$

$$H \times c_j^t - \sum_{a=1}^{A}\sum_{i=1}^{t} y_h^{a,i} \le 0 \qquad \forall t \in \{1,...,T\},\ j \in \{1,...,J\},\ h \in B_j(H) \tag{28}$$

$$\sum_{a=1}^{A}\sum_{i=1}^{t} y_h^{a,i} - H \times c_j^t \le 0 \qquad \forall t \in \{1,...,T\},\ j \in \{1,...,J\},\ h \in B_{j+1}(H) \tag{29}$$

$$c_j^t - c_j^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\},\ j \in \{1,...,J\} \tag{30}$$

$$\sum_{t=1}^{T} y_p^{a,t} = 1 \qquad \forall p \in \{1,...,P\}, a \in \{1,...,A\} \tag{31}$$

Eq. (5) defines the objective function, which maximizes the NPV and minimizes CT deposition costs. Eq. (6) shows the total mining capacity constraint. Eq. (7) defines capacity constraint for the processing plant. Eqs. (8), (9), (10), and (11) specify the upper and lower limits of requirements for the grade of bitumen and fines in the material that are sent to the processing plant. Eqs. (12), (13), (14), (15), (16), and (17) define the capacities for the volume of total tailings, volume of tailings fines, sand and water, and the volume of produced MFT and CT, respectively. Eq. (18) ensures that the total volume of produced CT in each period is deposited in CT cells in the same period. Eq. (19) defines the mass balance between the tonnages of total mined material and the ore that is sent to the processing plant. Eq. (20) ensures that the total fractions of ore from mining cut k in all periods add up to one. Eqs. (21), (22), (23), and (24) control the vertical and horizontal precedence in extraction of mining panels. Eqs. (25), (26), and (27) define the precedence order for filling the

CT cells. Eqs. (28), (29), and (30) control the vertical and horizontal precedence of extraction for pit phases (pushbacks). Finally, Eq. (31) ensures that the whole mining panels within the optimal pit are completely extracted.

4.2. Tailings model

The CHWE method has been introduced and deployed as an efficient method of bitumen extraction in oil sands surface mining (Clark and Pasternack, 1932; Clark, 1939). All of the active oil sands operators follow almost the same steps of CHWE for bitumen extraction from oil sands. Figure 1 illustrates a brief flow process of the CHWE method that is used by Suncor Energy. The total volume of the produced slurry at the end points of the process is a summation of volumes of fine material, sand, and water captured in the three streams of overflow, underflow, and bitumen froth treatment. Tailings, in its slurry form, cannot be left in the tailings pond without further processing. It must be changed to a trafficable mass, so that the pond can be capped and re-vegetated in a closure phase. The most important technologies that are being used currently for dewatering the tailings slurry are CT production, non-saturated tailings (NST) and atmospheric fines drying (AFD). CT production is the current dominant technology for tailings treatment in an in-pit facility. CT is a mixture of tailings coarse sand, MFT, and gypsum acting as coagulant. It is a non-segregating material that releases water and consolidates more rapidly than MFT.

The tailings model proposed here is developed based on the hot water extraction method that is used by Suncor. The basis for the mathematical equations is the mass balance relations between the ore feed from one side, and CT, MFT, and the released water being produced downstream at the end points of the process. The slurry produced in bitumen froth and overflow streams is sent to beaches 1 and 2 and forms MFT, while the underflow slurry is sent to beaches 3, 4, and 5 as MFT and to the CT plant to produce CT.

The total volume of the tailings slurry, produced in all three streams, is calculated as in Eq. (32).

$$V_{Tailings}^{Slurry} = V_{Overflow}^{Slurry} + V_{Underflow}^{Slurry} + V_{Bitumen\ froth}^{Slurry} \tag{32}$$

The total volume of fines, sand and water are calculated as in Eqs. (33), (34), and (35).

$$V_{Fines} = V_{Fines}^{Overflow} + V_{Fines}^{Underflow} \tag{33}$$

$$V_{Sand} = V_{Sand}^{Overflow} + V_{Sand}^{Underflow} \tag{34}$$

$$V_{Water} = V_{Water}^{Overflow} + V_{Water}^{Underflow} + V_{Water}^{Bitumenfroth} \tag{35}$$

The total tonnage of released produced CT resulting from processing 1000 tonnes of ore feed is calculated as in Eq. (36). Considering the average densities of CT, MFT and water, all the tonnages are transferred to volumetric values before being used in the mathematical model.

$$CT = CT\%_{Tremie}^{Spec} \times \left\{ T_{UnderFlow}^{Sand\ to\ CT} \times \frac{1+SFR}{SFR} + T_{CT}^{Water} + T_{CT}^{Make\ up\ Water} \right\} \tag{36}$$

The notation used in tailings calculations and more details of tailings equations, including the formulations for water and MFT volumes, are presented in appendix A.

Continuous variables ($z_c^t \in [0,1]$) are defined to control the deposition of CT in CT cells. A negative term ($-\sum_{c=1}^{C} i_c^t \times z_c^t$) is added to the objective function in Eq. (5) to apply the CT deposition cost in the NPV calculation. Integer variables ($a_c^t \in \{0,1\}$) are also introduced to control the precedence order for filling the CT cells. The set of constraints presented in (25), (26), and (27) control the feeling precedence of the CT cells.

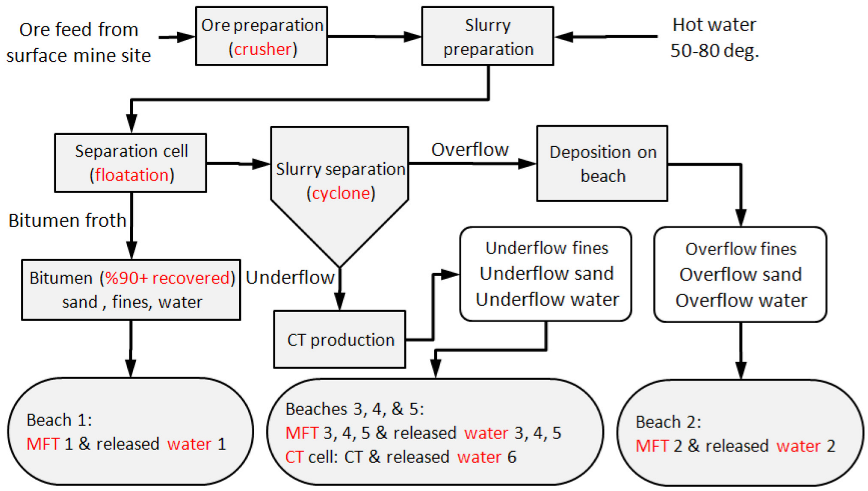

Figure 1. Process flow of hot water extraction method at Suncor.

4.3. Improvement of results and solution time

In order to generate a production schedule that is minable in practice, the mining blocks are aggregated into mining panels. Aggregation will also reduce the number of integer variables and improve the solution time. A conceptual block model can be divided into a number of nested pit shells that will form the mining phases, also known as pushbacks. Mining panels are the intersections of pushbacks and mining benches. As an illustrative example, a mining pit with three pushbacks in four benches will have 12 mining panels (Figure 2). In this paper, the ore processing is done at the mining cut level (Askari-Nasab et al., 2010; Tabesh and Askari-Nasab, 2011a), while mining operations are considered to be done at the mining panel level. In the dataset that is used as a case study, there are 98745 mining blocks, aggregated into 2153 mining cuts, which then form 164 panels in 20 pushbacks.

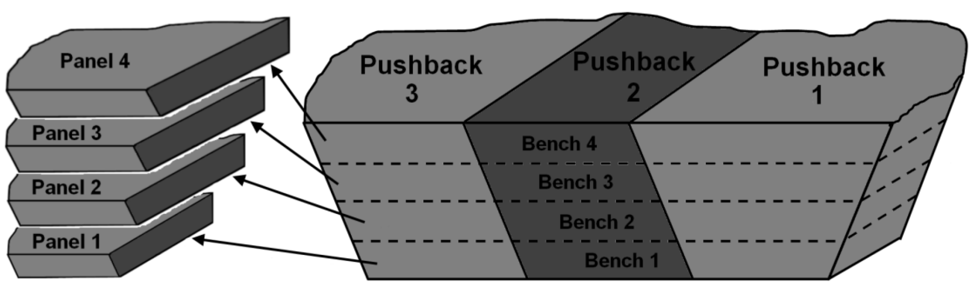

Figure 2. Conceptual illustration for mining panels.

Preprocessing refers to the step of eliminating some of the integer variables by pre-fixing their values to either zero or one. The mining capacity in each period is such that some benches will not be accessible for extraction in a specific period. The idea of preprocessing is proposed by (Bley et al., 2010) and has been used in a number of works since 2010. The authors define a set including all of the predecessor blocks for each mining block "*x*". The set has a total tonnage for mining block "*x*" that must be extracted prior to extraction of the block (A_x). This total tonnage is compared to the accumulated mining capacity in each period (B_t). If $A_x > B_t$, it means that mining block "*x*" cannot be extracted until after period "*t*", and all of the continuous and integer decision variables related to mining block *x* between periods 1 and *t-1* are fixed to zero. We call this "upper-cone variable reduction." With the same logic, more decision variables can be eliminated from the model, by considering "lower-cone variable reduction" (Tabesh and Askari-Nasab, 2011b). In this case, it is assumed that the total tonnage of the lower cone for block x is C_x, and the accumulated mining capacity from period t until the end of mine-life is D_t. The assumption is that all the mining blocks within the final pit must be extracted. If $C_x > D_t$, then mining block x must be extracted by period t and all the corresponding variables to block x for periods t+1 and after are fixed to one. The concept of upper and lower cones in 2D is illustrated in Figure 3. After fixing the decision variables, some of constraint lines will contain only zero values and can be ignored. Preprocessing has reduced the number of integer variables in the case study from 4,225 to 576 (86% reduction), and the number of constraints from 16,983 to 11,189 (34% reduction).

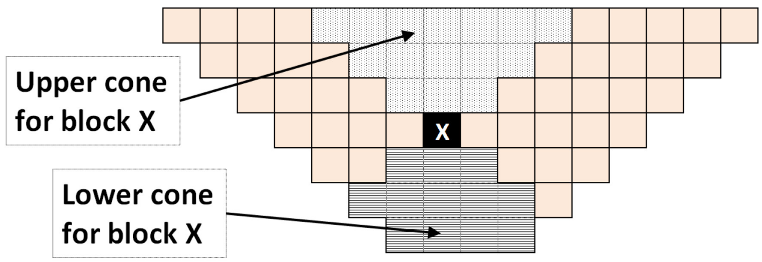

Figure 3. Illustration for upper & lower cones (as a variable reduction technique).

5. The case study

In order to verify the performance of the proposed integrated mine planning model and CT production plan, an oil sands dataset is used. The dataset includes 98,745 mining blocks. The blocks' dimensions are 50m by 50m by 15m. Three different pushback configurations are proposed, generating three scenarios with 10, 16, and 20 pushbacks, to investigate the effect of pushback design on generated NPV. A river divides the mine site into two separated pits in the south and north. Two destinations are considered for the extracted material: the processing plant and the waste dump. Table 1 presents an overview of the case dataset.

Table 1. Dataset information and input parameters used in case study (20 pushbacks).

Number of blocks:	98,745	Default Mining cap.	300M tonne/period
Number of cuts:	5583	Maximum Mining cap.	360M tonne/period
Number of panels:	164	Processing cap. (t: 1 to 7):	linear increase
Acceptable bitumen grade in ore	7% – 16%	Processing cap. (t: 8 to 20):	60M tonne/period
Acceptable fines grade in ore	0% – 30%	Total Tailings capacity:	130M m3/period
Total material tonnage:	7,507M tonne	MFT production capacity:	20M m3/period
Total mineralized tonnage (%7+):	2,311M tonne	CT production capacity:	55M m3/period

The default mining capacity is increased in the first nine periods from 300 to 330 million tonnes to accelerate the access to the bituminous sand beneath the overburden. Then, in period 12, the capacity has been increased to 360 million tonnes to generate a uniform processing rate. The setting for the mining capacity has been the same for three cases.

The grade distribution for a sample bench is illustrated in Figure 4. The dark color patches show the high grade oil sands with bitumen above 7%, which will be sent to the processing plant. High grade material is mostly located on left side (west) of the mine site. Therefore, it is reasonable to consider the west-east as the best mining direction for this case to end up in a higher NPV value. This direction has been the best one, as reported by Whittle (Gemcom-Software-International, 2012), too.

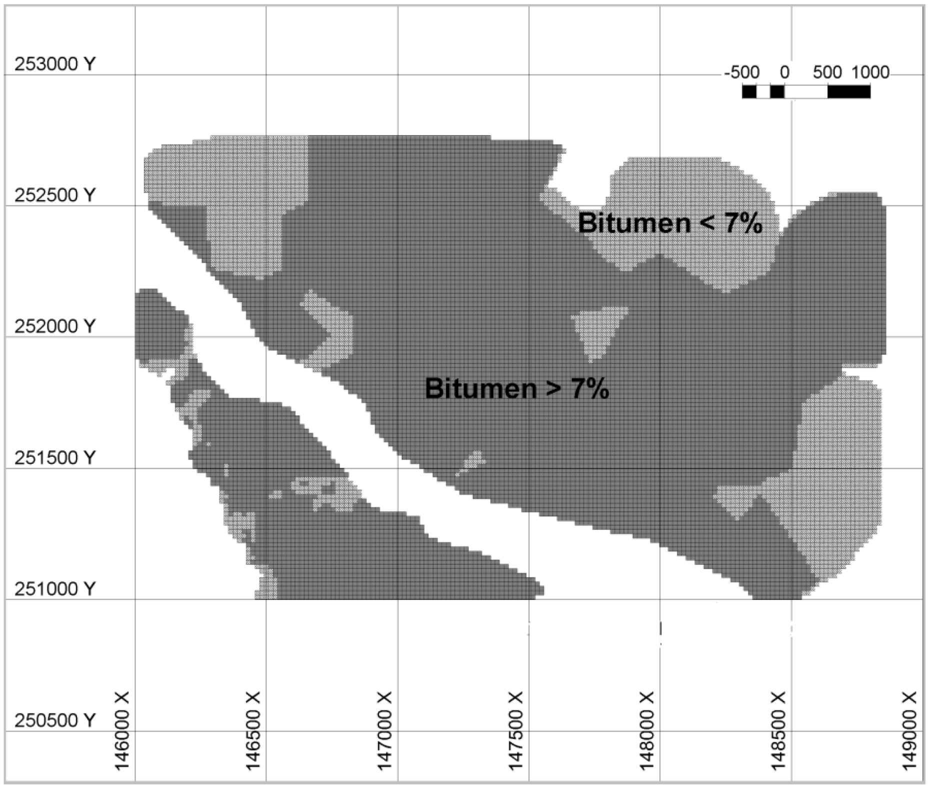

Figure 4. Grade distribution in a sample plan view.

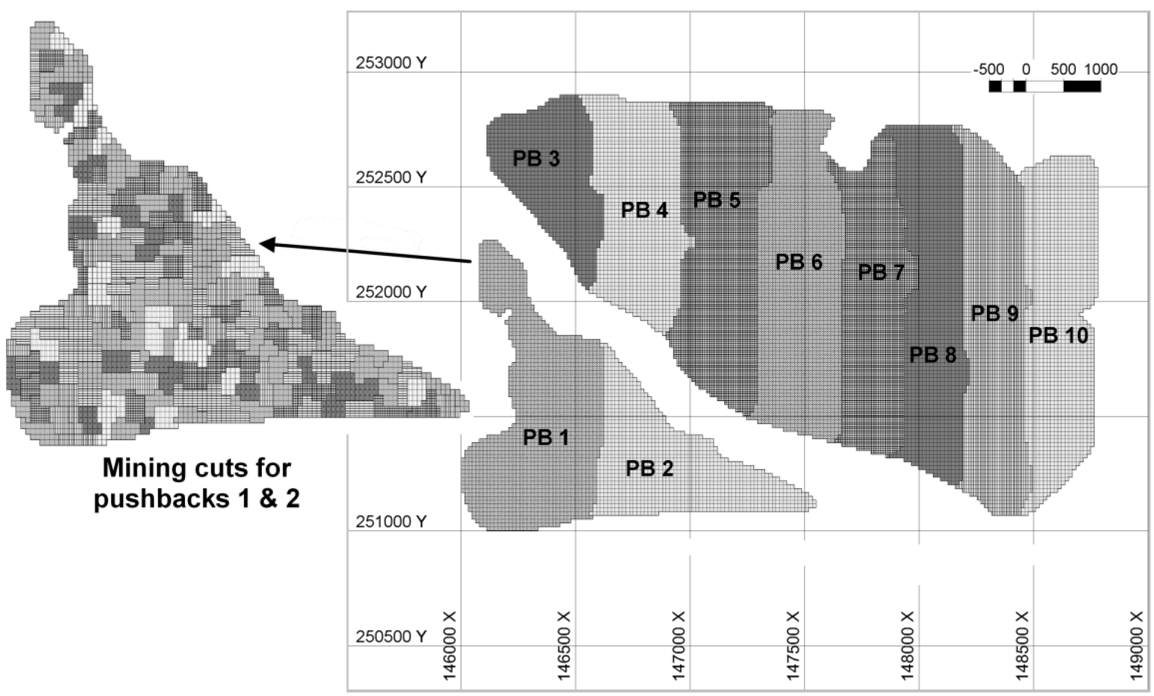

Figure 5. A sample plan view, showing pushbacks, panels, and cuts.

The plan view of the same bench, showing mining panels and mining cuts, is illustrated in Figure 5. Mining cuts are formed within the boundaries of panels. In this case, mining panels have been used for mining operation in the MILP model, while mining cuts are used in sending the material to different destinations (either the processing plant or the waste dump). Defining panels will result in a more practical production schedule with regards to selective mining units. Also, as can be seen in Figure 5, defining mining panels has drastically reduced the number of mining units and decision variables.

In the studied case it is assumed that the produced CT is sent to the CT cells that are formed by the construction of dyke walls, as illustrated in Figure 6. During the first periods, when there is not enough in-pit space, the CT is stored in an external tailings facility (ETF). The CT will be sent to the cells with the order of cell 1, cell 2, cell 3, cell 4, and cell 5. The CT cells have enough volumetric capacity to store the potential volume of CT produced in later periods. A schematic view of showing the footprint of dyke walls and CT cells is illustrated in Figure 6.

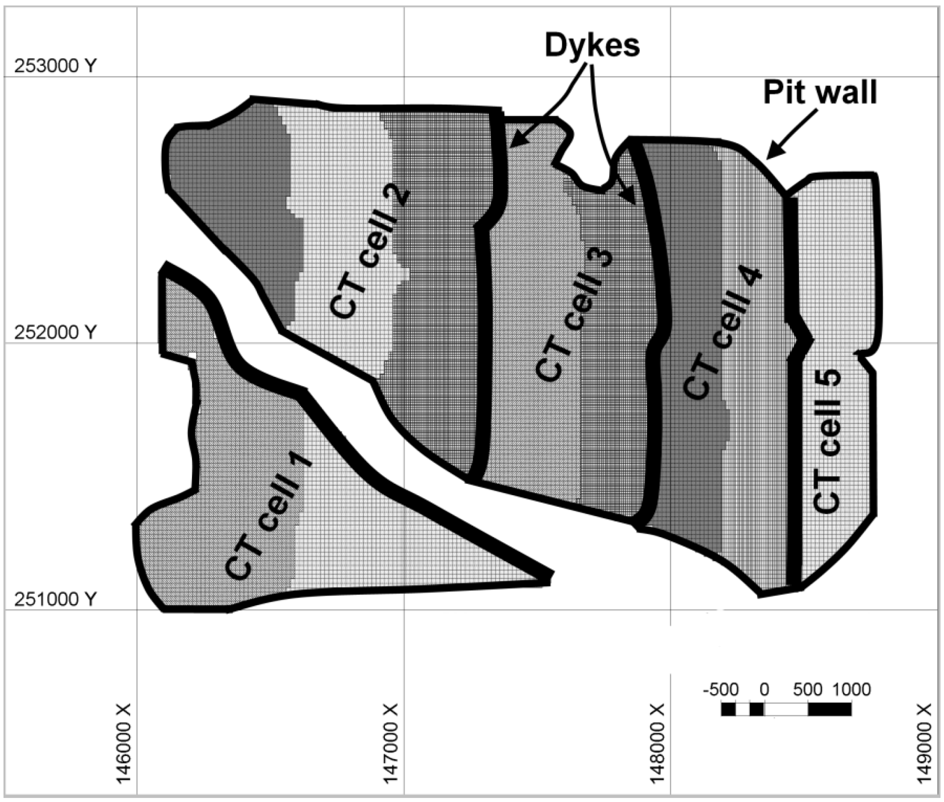

Figure 6. A schematic view of CT deposition cells.

6. Results and discussions

Matlab (MathWorksInc., 2011) has been used to prepare the required matrices for constraints, objective function, and the bounds for constraints and variables. Matlab calls TOMLAB/CPLEX (TOMLAB, 2010) to run the code and solve the mathematical model. The code is executed on a Dual quad-core Dell Precision T7500 computer at 2.8 GHz, with 24GB of RAM.

The problem is solved for 25 periods. The value of the objective function is compared for three different cases with 10, 16, and 20 pushbacks. After determining pushbacks and mining panels, the mining blocks within mining panels are aggregated through hierarchical clustering algorithm to form the mining cuts. The distance between blocks and the blocks' bitumen grade are used to measure the similarity between mining blocks for aggregation. At the maximum, each mining cut includes 25 mining blocks. Table 2 compares the results of solving the MILP model with different resolutions.

Table 2. The numerical results of the MILP model.

# of Push backs	# of Panels	# of Cuts	# of constraints Initial Reduced	# of Vars. Initial Reduced	# of Int. vars. Initial Reduced	Solution gap (%)	Run time (sec)	NPV (M$)	Improvement in NPV
10	87	5534	12,019 8,854	281,300 79,835	2,300 344	0.9	17	3,595	-
16	134	5552	14,971 10,410	284,550 80,691	3,475 489	1.0	18	3,757	4.5%
20	164	5583	16,983 11,189	287,600 80,922	4,225 576	1.0	22	3,810	6.0%

Comparison of the results shows that increasing the number of pushbacks results in higher NPV values. This happens because mining is more flexible when the mining units are smaller, unlike what happens in the larger panels. Further, by increasing the number of mining panels the number of mining cuts also increases slightly from 5534 to 5583, which means the mining blocks can be clustered more reasonably based on their bitumen content, and less dilution will enter into the CHWE process.

Three sample plan views, showing the production schedules for scenarios of 10, 16 and 20 pushbacks, are illustrated in Figure 7, Figure 8, and Figure 9, respectively. The numbers on the figures are the number of periods in which the portion is extracted.

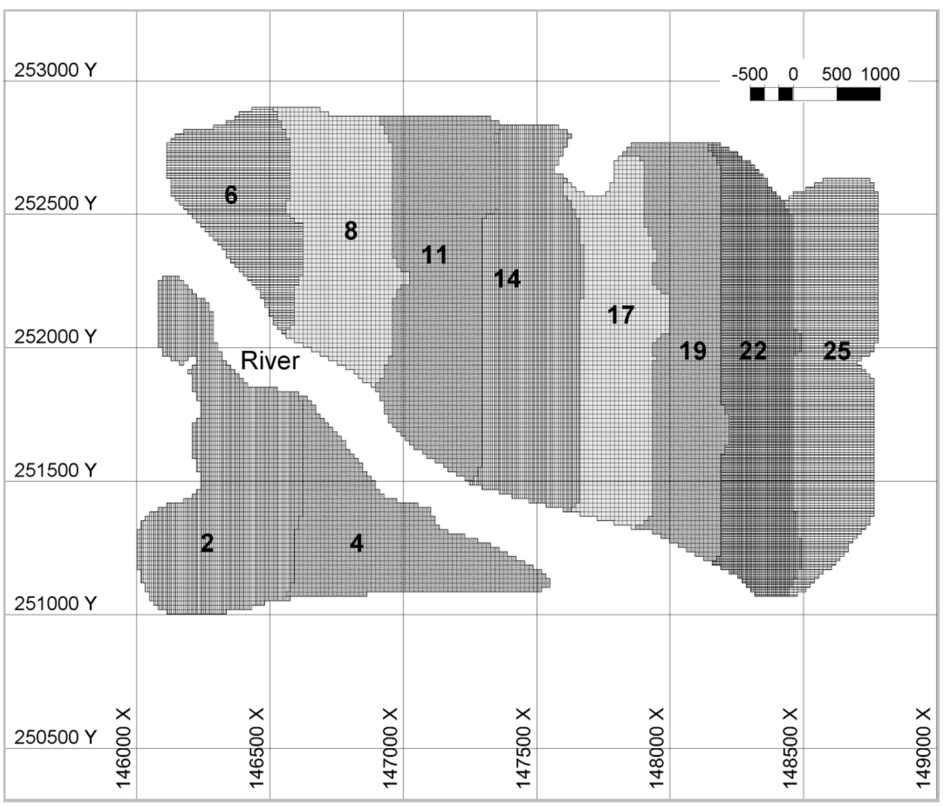

Figure 7. Production schedule (for the 10-pushbacks case).

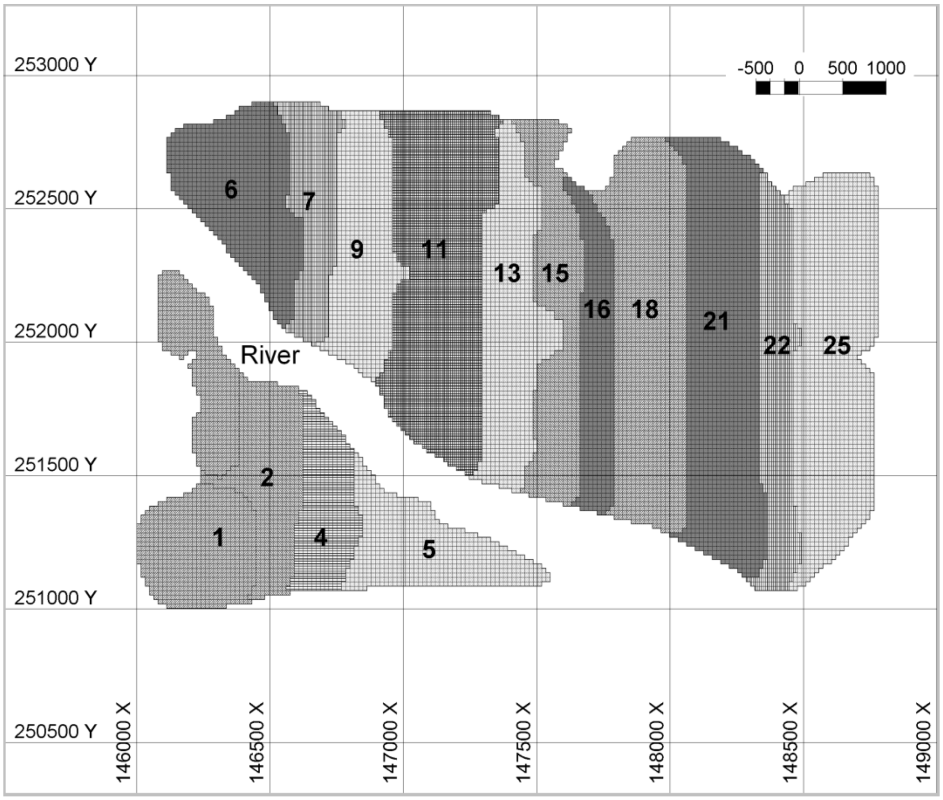

Figure 8. Production schedule (for the 16-pushbacks case).

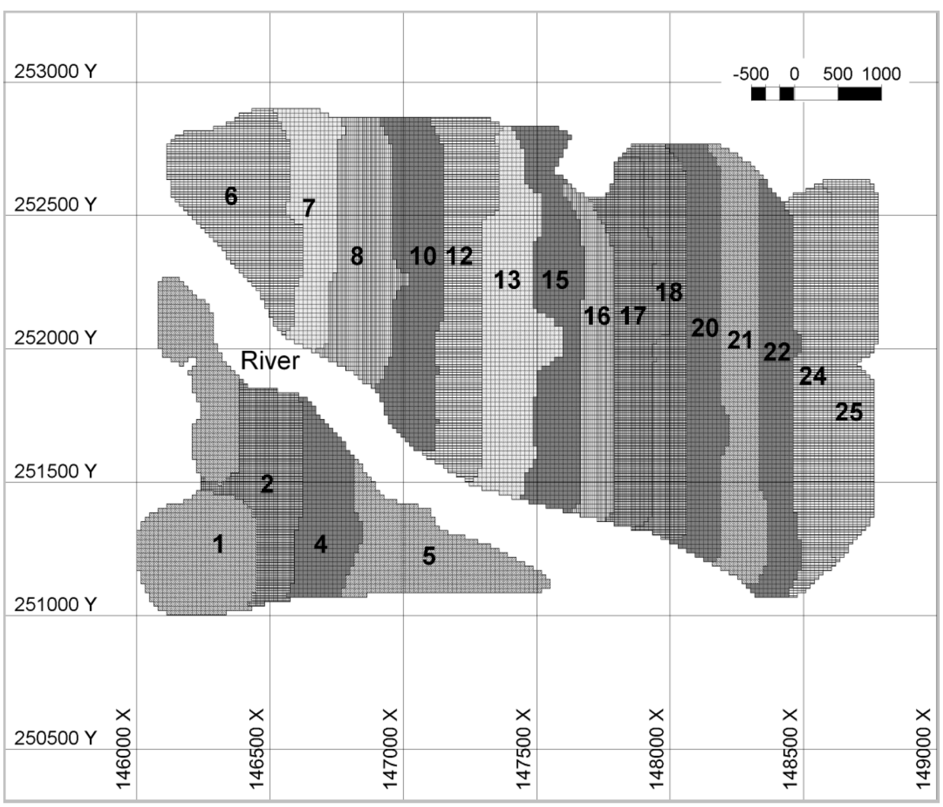

Figure 9. Production schedule (for the 20-pushbacks case).

The horizontal direction of mining has been west-to-east (left to right). Pushback precedence has been in place to prepare enough empty space for construction of CT cells as in-pit tailings facilities. The resulted schedules follow the directional mining constraints. The effect of increasing the number of pushbacks (having smaller panels) is that the mining operations will reach to the bottom of the pit in earlier periods, but the emptied portion is narrower, compared to the cases with larger pushbacks and panels. The minimum mining width statistics for three cases are reported in Table 3.

Table 3. Minimum mining width for three cases.

10 pushbacks	Min: 788 m	Max: 2490 m	Average: 1258 m
16 pushbacks	Min: 372 m	Max: 1855 m	Average: 901 m
20 pushbacks	Min: 388 m	Max: 1790 m	Average: 662 m

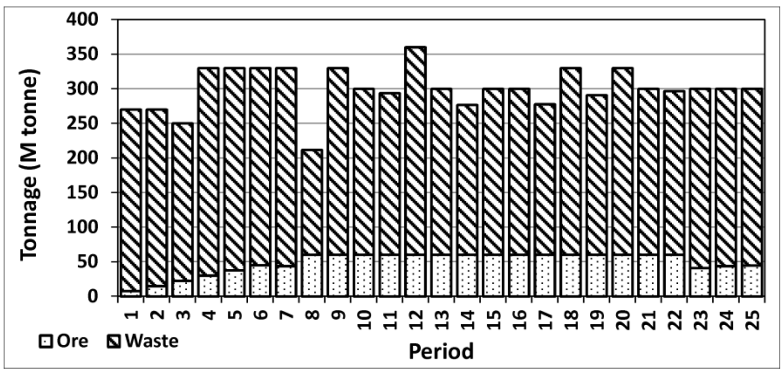

Figure 10. Production and processing schedule - case of 10 pushbacks.

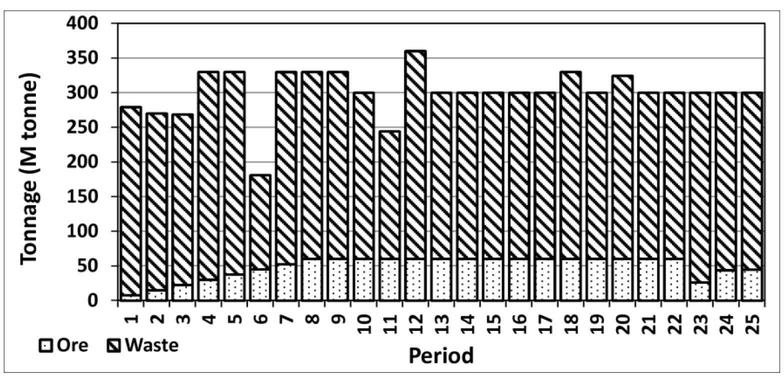

Figure 11. Production and processing schedule - case of 16 pushbacks.

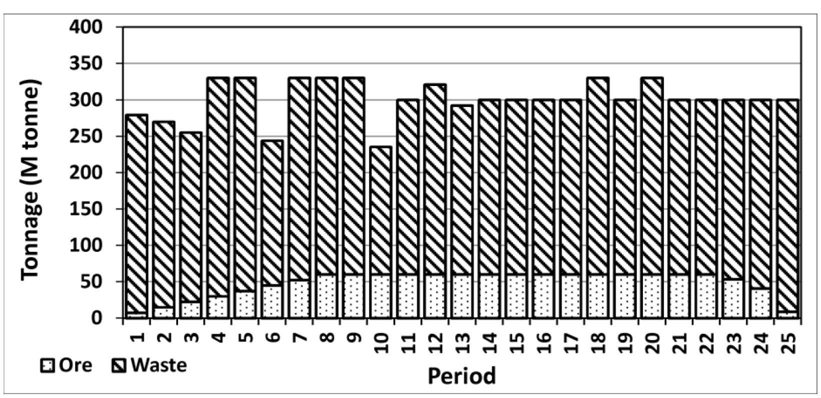

Figure 12. Production and processing schedule - case of 20 pushbacks.

The optimal production plans for different numbers of pushbacks are illustrated in Figures 10 to 12, which present the production of bituminous oil sands and waste material in different periods. The production schedule is uniformly capped to 300 million tonnes, except for some of the periods when it has been changed to generate a uniform processing rate. To reach its maximum rate, the processing capacity has increased linearly during the first eight periods. That is because in early periods, a large portion from the extracted material is overburden and, hence, there is not enough extracted bitumen to enter the HWEP. A slight linear increase in the processing rate in eight periods allows the bitumen processing to reach its maximum capacity of 60 million tonne per period.

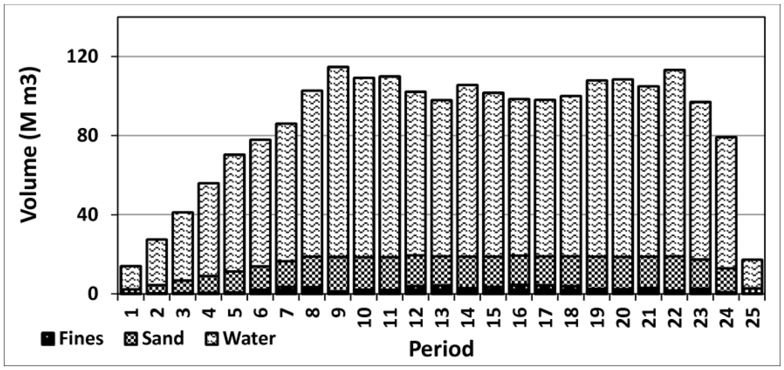

Figure 13. Tailings production schedule – case of 20 pushbacks.

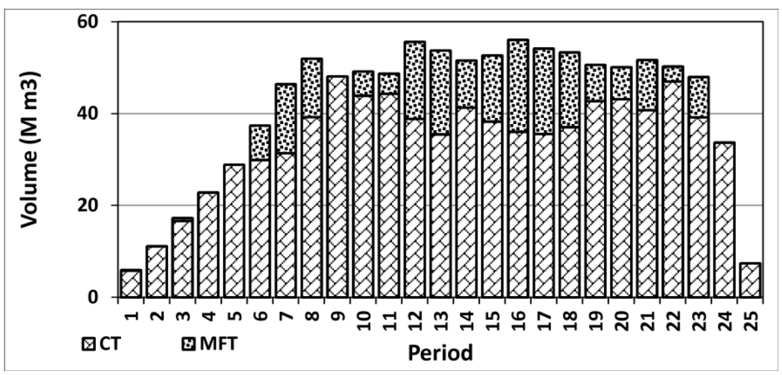

Figure 14. CT and MFT production schedule – case of 20 pushbacks.

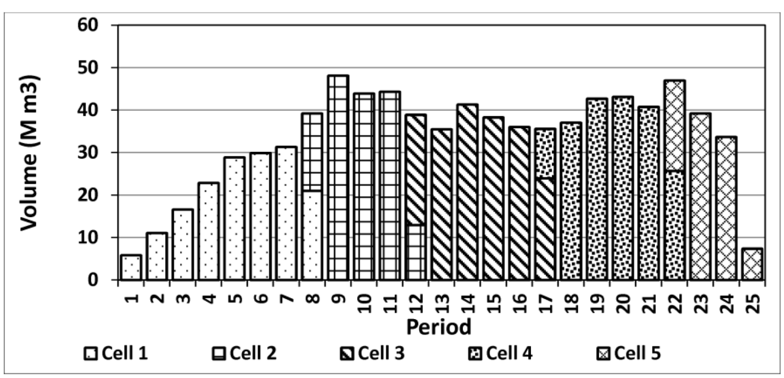

Figure 15. Schedule for deposition of CT in CT cells – case of 20 pushbacks.

Figure 13 shows the resulted tailings plan in periods for the case of 20 pushbacks. Tailings production follows the pattern of bitumen production, as the total volume of produced tailings is directly related to the rate of ore feed entering the processing plant. On average, 82% of the tailings slurry is water, 15% is sands and 3% is fine material. The production of CT and MFT is presented in Figure 14. The CT and MFT production rates follow the ore processing and total tailings production rate too, as the volumes of CT and MFT are proportional to the volume of produced slurry. According to Suncor's tailings flow sheet, most of the produced tailings has been treated as CT and only a small portion of that (average 16%) has been left as MFT. Finally, the deposition of CT in CT cells is presented in Figure 15. The CT deposition facility in this case consists of five separate cells, located within the excavated pit, plus an external tailings facility for first periods. Figure 15 illustrates the schedule for the ETF and four in-pit cells.

7. Conclusions

Over decades, operations research has been used to optimize mine production scheduling that determines the best order of extraction of mining blocks and the destination of the extracted material. In typical modeling of the long-term mine planning problem, the objective function maximizes the NPV over the mine life-time, subject to a number of constraints corresponding to the precedence of extraction, mining, and processing capacities. As a typical practice in oil sands surface mining, mine planners calculate the expected tailings volume produced from processing the oil sands, and plan for its deposition after the mine plan is prepared. The tailings slurry will be dewatered through different technologies, such as CT production, to be prepared for long-term deposition. However, as the available capacities for holding tailings slurry and deposition of CT are limited to the lease area, the volumes of produced tailings and CT are, practically speaking, a key factor in determining the production rate. In this research, CT production and deposition planning is included in the long-term mine planning optimization. A detailed tailings model, based on the CHWE process, has been developed to map the tonnage and specifications of oil sands feed to the volume of total tailings, MFT and CT. The resulted integrated mathematical model is an MILP model, which is NP-hard for real case problems. The problem size has been reduced through two variable reduction techniques, as block-aggregation and preprocessing. The MILP model is solved by CPLEX and has resulted in integer solutions within a 1% optimality gap in seconds. The resulting production pattern follows the directional mining that creates enough space for an in-pit tailings facility in later periods. The next step of this research is to improve CT deposition modeling and integrating mine planning, tailings management and dyke construction planning in a comprehensive integrated model.

8. Appendix A

The notation used in tailings equations is as follows:

Parameters

$Sd\%_{UF}$: Sand content of the underflow

$Sl_{solid\%}$: Solid percent of slurry sent to cyclone

$UF_{Sd\%}$: Sand percent in cyclone underflow

$UF_{F\%}$: Fine percent in cyclone underflow

$UF_{W\%}$: Water percent in cyclone underflow

R : SET recovery percent

$B\%_{SET}$: SET bitumen percent

$F\%_{SET}$: SET fine percent

$Sd\%_{SET}$: SET sand percent

$W\%_{SET}$: SET water percent

$Rj\%$: Reject percent

$Rj\%_{F}$: Fines reject percent

$Rj\%_{Sd}$: Sand reject percent

$Rj\%_{W}$: Water reject percent

$Rj\%_{B}$: Bitumen reject percent

HPW : HPW

SG_{f} : Fines specific gravity

SG_{s} : Sand specific gravity

$F\%_{Beach}$: Fines content in beach solids (%)

BDD_{SET} : Beach dry density (SET)

$S\%_{MFT}$: MFT solid content (%)

$S\%_{DT}$: DT solid content (%)

CDD : Cell dry density

$F\%_{Cell}$: Cell % fines in solids

FC_{Cell} : Cell physical capture

E_{Cell} : Cell Efficiency

BDD_{RunOff} : Beach dry density (Run off)

BDD_{DT} : Beach dry density (DT)

$S\%_{DT}$: Solids% (DT)

$W_{Make-up}$: Make up water

V_{Cell} : Cell volume (m3)

SFR : Sand to fine ratio (target)

$S\%_{CT}$: Solid % (CT)

$CT\%_{Tremie}^{Spec}$: On spec CT to Tremie (%)

$S\%_{CT}^{Dep}$: Solids in CT deposit (%)

$F\%_{CT}^{Seg}$: Fines in segregated CT (%)

$SCBD$: Density of segregated CT at beach

8.1.1. Input variables

M_{Feed}^{O} : Mass of ore in the feed

B_{Feed} : Bitumen content of the feed (%)

F_{Feed} : Fines content of the feed (%)

W_{Feed} : Water content of the feed (%)

The water content of the ore feed to the processing plant, W_{Feed}, is calculated by Eq. (37) (Masliyah, 2010).

$$W_{Feed} = 0.75 \times F_{Feed} + 2.3 \tag{37}$$

The total tonnage of the overflow slurry is calculated as in Eq. (38).

$$T_{OverFlow} = T_{OverFLow}^{Fines} + T_{OverFLow}^{Sand} + T_{OverFLow}^{Water} \tag{38}$$

Where

$$T_{OverFlow}^{Fines} = M_{Feed}^{O} \times \left[F_{feed} \left(1 - B_{feed} - W_{feed}\right) - Rj\% \times Rj\%_{F} - F\%_{SET} \times X - \frac{G}{M_{Feed}^{O}} \right] \tag{39}$$

$$T_{OverFLow}^{Sand} = M_{Feed}^{O} \left(1 - Sd\%_{UF}\right) \times \left[\left(1 - B_{feed} - W_{feed}\right) \times \left(1 - F_{feed}\right) - Rj\% \times Rj\%_{Sd} - X \right] \tag{40}$$

$$T_{OverFlow}^{Water} = M_{Feed}^{O} \times \left[1 - B_{feed} - W_{feed} - Rj\% \times \left(Rj\%_{F} + Rj\%_{Sd} \right) - X \times \left(F\%_{SET} + Sd\%_{SET} \right) \right] \times \frac{1 - Sl_{solid\%}}{Sl_{solid\%}} - \frac{I}{M_{Feed}^{O}} \tag{41}$$

$$X = Sd\%_{SET} \times R \times \frac{B_{feed} - Rj\% \times Rj\%_{B}}{B\%_{SET}} \tag{42}$$

The total tonnage of the underflow slurry is calculated as in Eq. (43).

$$T_{UnderFlow} = T_{UnderFlow}^{Fines} + T_{UnderFlow}^{Sand} + T_{UnderFlow}^{Water} \tag{43}$$

Where

$$T_{UnderFlow}^{Sand} = Sd\%_{UF} \times M_{Feed}^{O} \left[\begin{array}{l} \left(1 - B_{feed} - W_{feed}\right)\left(1 - F_{feed}\right) - Rj\% \times Rj\%_{Sd} \\ - \dfrac{Sd\%_{SET} \times R\left(B_{feed} - Rj\% \times Rj\%_{B}\right)}{B\%_{SET}} \end{array} \right] \tag{44}$$

$$T_{UnderFlow}^{Fines} = \frac{UF_{F\%}}{UF_{Sd\%}} \times T_{UnderFlow}^{Sand} \tag{45}$$

$$T_{UnderFlow}^{Water} = \frac{UF_{W\%}}{UF_{Sd\%}} \times T_{UnderFlow}^{Sand} \tag{46}$$

The tonnages of MFT and released water produced in bitumen froth treatment stream are calculated as in Eqs. (47) and (48), respectively.

$$MFT\_1 = M_{Feed}^{O} \times \frac{X}{S\%_{MFT} \times Sd\%_{SET}} \times \left[F\%_{SET} - \frac{Sd\%_{SET} \times F\%_{Beach}}{1 - F\%_{Beach}} \right] \tag{47}$$

$$Water\_1 = M_{Feed}^{O} \times \frac{X}{Sd\%_{SET}} \times \left[W\%_{SET} - \frac{Sd\%_{SET}}{BDD_{SET}\left(1 - F\%_{Beach}\right)} \right] + \frac{Sd\%_{SET}}{SG_{s}} + \frac{Sd\%_{SET} \times F\%_{Beach}}{SG_{f}\left(1 - F\%_{Beach}\right)} - \left(F\%_{SET} - \frac{Sd\%_{SET} \times F\%_{Beach}}{1 - F\%_{Beach}} \right) \times \frac{1 - S\%_{MFT}}{S\%_{MFT}} \tag{48}$$

The tonnages of MFT and released water produced in Overflow stream are calculated as in Eqs. (49) and (50) , respectively.

$$MFT\_2 = \frac{1}{S\%_{MFT}} \times \left[T_{OverFlow}^{Fines} - \frac{T_{OverFLow}^{Sand} \times F\%_{Beach}}{1 - F\%_{Beach}} \right] \tag{49}$$

$$Water\_2 = T_{OverFlow}^{Water} - \left[\begin{array}{l} \left(T_{OverFLow}^{Sand} \times F\%_{Beach}\right) \times \left(1 - F\%_{Beach}\right)\left(\dfrac{1}{BDD_{SET}} - \dfrac{1}{SG_f} - \dfrac{1}{S\%_{MFT}} + 1\right) \\ + T_{OverFLow}^{Sand}\left(\dfrac{1}{BDD_{SET}} + \dfrac{1}{SG_s}\right) + T_{OverFlow}^{Fines}\left(\dfrac{1}{S\%_{MFT}} - 1\right) \end{array} \right] \tag{50}$$

The tonnages of MFT and released water produced in an underflow stream (in beaches 3 and 4) are calculated as in Eqs. (51), (52), (53), and (54).

$$MFT\_3 = \frac{Fines_{RunOff} \times \left(1 - F\%_{Beach}\right) - Sand_{RunOff} \times F\%_{Beach}}{S\%_{MFT} \times \left(1 - F\%_{Beach}\right)} \tag{51}$$

$$\begin{aligned} Water\_3 = {} & Water_{RunOff} - Sand_{RunOff} \times \left\{ \frac{\dfrac{1}{BDD_{RunOff}} - \dfrac{F\%_{Beach}}{SG_f}}{1 - F\%_{Beach}} - \frac{1}{SG_s} \right\} - \\ & \left\{ Fines_{RunOff} - \frac{Sand_{RunOff} \times F\%_{Beach}}{1 - F\%_{Beach}} \right\} \times \left(\frac{1}{S\%_{MFT}} - 1 \right) \end{aligned} \tag{52}$$

$$MFT\_4 = \left(\frac{G}{H} - \frac{F\%_{Beach}}{1 - F\%_{Beach}} \right) \times \frac{\left(1 - E_{Cell}\right) \times \left(1 - F\%_{Cell}\right) \times V_{Cell} \times CDD}{FC_{Cell} \times E_{Cell} \times S\%_{MFT}} \tag{53}$$

$$\begin{aligned} Water\_4 = {} & \frac{\left(1 - E_{Cell}\right) \times \left(1 - F\%_{Cell}\right) \times V_{Cell} \times CDD}{FC_{Cell} \times E_{Cell}} \times \\ & \left\{ \begin{array}{l} \dfrac{I}{H} + \dfrac{FC_{Cell} \times E_{Cell} \times W_{Make\_up}}{CDD \times V_{Cell} \times \left(1 - F\%_{Cell}\right)} - \dfrac{F\%_{Beach} \times \left(\dfrac{1}{BDD_{DT}} - \dfrac{1}{SG_f} \right)}{\left(1 - F\%_{Beach}\right)} - \\ \dfrac{1}{BDD_{DT}} + \dfrac{1}{SG_s} + \left(1 - \dfrac{1}{S\%_{MFT}}\right) \times \left(\dfrac{T_{UnderFlow}^{Fines}}{T_{UnderFlow}^{Sand}} - \dfrac{F\%_{Beach}}{1 - F\%_{Beach}} \right) \end{array} \right\} \end{aligned} \tag{54}$$

Where

$$Fines_{RunOff} = V_{Cell} \times CDD \times \left\{ \frac{G \times \left(1 + \left(E_{Cell} - 1\right)\left(1 - F\%_{Cell}\right)\right)}{FC_{Cell} \times E_{Cell} \times H} - F\%_{Cell} \right\} \tag{55}$$

$$Sand_{RunOff} = V_{Cell} \times CDD \times \left(1 - F\%_{Cell}\right) \left\{ \frac{1 - FC_{Cell}}{FC_{Cell}} \right\} \tag{56}$$

$$\begin{aligned} Water_{RunOff} = {} & V_{Cell} \times CDD \times \left(1 - F\%_{Cell}\right) \times \left\{ \frac{1}{SG_s} + \frac{E_{Cell} \times I}{FC_{Cell} \times E_{Cell} \times H} \right\} + \\ & \frac{V_{Cell} \times CDD \times F\%_{Cell}}{SG_f} + E_{Cell} \times W_{Make\_up} - V_{Cell} \end{aligned} \tag{57}$$

The tonnages of MFT and released water produced in an underflow stream (in beach5) are calculated as in Eqs. (58) and (59), respectively.

$$MFT\_5 = T_{MFT\_5}^{Fines} + T_{MFT\_5}^{Water} \tag{58}$$

$$Water\_5 = \left(1 - CT\%_{Tremie}^{Spec}\right) \times \left\{ \begin{bmatrix} T_{CT}^{Water} + T_{CT}^{Make\ up\ Water} - T_{UnderFlow}^{Sand\ to\ CT} \times \\ \dfrac{1}{SCBD} - \dfrac{1}{SG_s} + \\ \left(\dfrac{1}{SCBD} - \dfrac{1}{SG_f}\right) \times \dfrac{F\%_{CT}^{Seg}}{1 - F\%_{CT}^{Seg}} \end{bmatrix} \right\} - T_{MFT\_5}^{Water} \tag{59}$$

Where

$$T_{UnderFlow}^{Sand\ to\ CT} = T_{UnderFlow}^{Sand} \times \left\{ 1 - \frac{V_{Cell} \times CDD \times \left(1 - F\%_{Cell}\right)}{T_{UnderFlow}^{Sand} \times FC_{Cell} \times E_{Cell}} \right\} \tag{60}$$

$$\begin{aligned} T_{CT}^{Water} &= T_{UnderFlow}^{Water} \times \left(1 - V_{Cell} \times CDD \times \frac{1 - F\%_{Cell}}{FC_{Cell} \times E_{Cell} \times T_{UnderFlow}^{Sand}}\right) + \\ & T_{UnderFlow}^{Sand\ to\ CT} \times \frac{1 - S\%_{MFT}}{S\%_{MFT}} \times \frac{T_{UnderFlow}^{Sand} - T_{UnderFlow}^{Fines} \times SFR}{T_{UnderFlow}^{Sand} \times SFR} \end{aligned} \tag{61}$$

$$T_{CT}^{Make\ up\ Water} = MAX \left\{ 0\ ,\ \frac{T_{UnderFlow}^{Sand\ to\ CT} \times \left(1 + SFR\right) \times \left(1 - S\%_{CT}\right)}{SFR \times S\%_{CT}} - T_{CT}^{Water} \right\} \tag{62}$$

$$T_{MFT\_5}^{Fines} = T_{UnderFlow}^{Sand\ to\ CT} \left(1 - CT\%_{Tremie}^{Spec}\right) \left(\frac{1}{SFR} - \frac{F\%_{CT}^{Seg}}{1 - F\%_{CT}^{Seg}} \right) \tag{63}$$

$$T_{MFT\_5}^{Water} = MIN \left\{ \begin{aligned} & T_{MFT\_5}^{Fines} \times \frac{1 - S\%_{MFT}}{S\%_{MFT}}\ ,\left(1 - CT\%_{Tremie}^{Spec}\right) \times \\ & \left(T_{CT}^{Water} + T_{CT}^{Make\ up\ Water} - T_{UnderFlow}^{Sand\ to\ CT} \times \left[\frac{1}{SCBD} - \frac{1}{SG_s} + \left(\frac{1}{SCBD} - \frac{1}{SG_f} \right) \times \frac{F\%_{CT}^{Seg}}{1 - F\%_{CT}^{Seg}} \right] \right) \end{aligned} \right\} \tag{64}$$

Finally, the tonnages of released water produced in a CT plant and the tonnage of produced CT are calculated as in Eqs. (65) and (66), respectively.

$$Water\_6 = CT\%_{Tremie}^{Spec} \times \left\{ T_{UnderFlow}^{Sand\ to\ CT} \times \frac{\left(S\%_{CT}^{Dep} - 1\right)\left(1 + SFR\right)}{S\%_{CT}^{Dep} \times SFR} + T_{CT}^{Water} + T_{CT}^{Make\ up\ Water} \right\} \tag{65}$$

$$CT = CT\%_{Tremie}^{Spec} \times \left\{ T_{UnderFlow}^{Sand\ to\ CT} \times \frac{1 + SFR}{SFR} + T_{CT}^{Water} + T_{CT}^{Make\ up\ Water} \right\} \tag{66}$$

9. Appendix B

The sets, indices, parameters, and decision variables that are used in the MILP model are as follows:

9.1. Sets and indices

$t \in \{1,......,T\}$ index for scheduling periods.

$k \in \{1,.....,K\}$ index for mining cuts.

$p \in \{1,.....,P\}$ index for mining panels.

$e \in \{1,.....,E\}$ index for element of interest in each mining cut.

$j \in \{1,.....,J\}$ index for phases (pushback).

$u \in \{1,.....,U\}$ index for possible destinations for materials.

$a \in \{1,.....,A\}$ index for possible mining locations (pits).

$\boldsymbol{K} = \{1,.....,K\}$ set of all the mining cuts in the model.

$P = \{1,.....,P\}$ set of all the mining panels in the model.

$\boldsymbol{J} = \{1,......,J\}$ set of all the phases (pushbacks) in the model.

$\boldsymbol{U} = \{1,.....,U\}$ set of all the possible destinations for materials in the model.

$A = \{1,.....,A\}$ set of all the possible mining locations (pits) in the model.

$c \in \{1,.....,C\}$ index for CT cells.

$B_p(V)$ for each mining panel "p", there is a set $B_p(V) \subset K$ defining the mining cuts that belongs to the mining panel "p", where V is the total number of mining cuts in the set $B_p(V)$.

$N_p(L)$ For each mining panel "p", there is a set $N_p(L) \subset \boldsymbol{P}$ defining the immediate predecessor mining panels above mining panel "p" that must be extracted prior to extracting mining panel "p", where L is the total number of mining panels in the set $N_p(L)$.

$O_p(L)$ For each mining panel "p", there is a set $O_p(L) \subset \boldsymbol{P}$ defining the immediate predecessor mining panels in a specified horizontal mining direction that must be extracted prior to extracting the mining panel "p" at the specified level, where P is the total number of mining panels in the set $O_p(L)$.

$B_j(H)$ For each phase "j", there is a set $B_j(H) \subset P$ defining the mining panels within the immediate predecessor pit phases (pushbacks) that must be extracted prior to extracting phase “j”, where H is an integer number representing the total number of mining panels in the set $B_j(H)$.

$Q_c(R)$ For each CT cell “c”, there is a set $Q_c(R) \subset \boldsymbol{C}$ defining the immediate predecessor CT cells (in vertical and/or horizontal directions) that must be filled in prior to filling CT cell “c”, where R is the total number of CT cells in the set $Q_c(R)$.

9.2. Parameters

$d_p^{a,u,t}$ Discounted profit obtained by extracting mining panel “p” from location “a” and sending it to destination “u” in period “t”.

$r_k^{u,t}$ Discounted revenue obtained by selling the final products within mining cut “k” in period “t” if it is sent to destination “u”, minus the extra discounted cost of mining all the material in mining cut “k” as ore from location “a” and processing at destination “u”.

$q_p^{a,t}$ Discounted cost of mining all the material in mining panel “p” in period “t” as waste from location “a”.

g_k^e Average grade of element “e” in the ore portion of mining cut “k”.

$\underline{g}^{u,t,e}$ Lower bound on the required average head grade of element “e” in period “t” at processing destination “u”.

$\overline{g}^{u,t,e}$ Upper bound on the required average head grade of element “e” in period "t" at processing destination “u”.

f_k^o Average percentage of fines in the ore portion of mining cut “k”.

$\underline{f}^{u,t,o}$ Lower bound on the required average fines percentage of ore in period “t” at processing destination “u”.

$\overline{f}^{u,t,o}$ Upper bound on the required average fines percentage of ore in period “t” at processing destination “u”.

o_k Ore tonnage in mining cut “k”.

w_k Waste tonnage in mining cut “k”.

t_k Tailings tonnage produced from extracting all of the ore from mining cut “k”.

f_k Fines tonnage produced from extracting all of the ore from mining cut “k”.

s_k Sand tonnage produced from extracting all of the ore from mining cut “k”.

r_k Water tonnage produced from extracting all of the ore from mining cut “k”.

h_k MFT volume produced from extracting all of the ore from mining cut “k”.

p_k CT volume produced from extracting all of the ore from mining cut “k”.

p_p	Mining panel "p".
h_c	Total volume of CT cell "c".
$T_{Mu}^{a,t}$	Upper bound on mining capacity (tonnes) in period "t" at location "a".
$T_{Ml}^{a,t}$	Lower bound on mining capacity (tonnes) in period "t" at location "a".
$T_{Pu}^{u,t}$	Upper bound on processing capacity (tonnes) in period "t" at destination "u".
$T_{Pl}^{u,t}$	Lower bound on processing capacity (tonnes) in period "t" at destination "u".
$T_{Tu}^{u,t}$	Upper bound on capacity of tailings facility (tonnes) in period "t" at destination "u".
$T_{Tl}^{u,t}$	Lower bound on capacity of tailings facility (tonnes) in period "t" at destination "u".
$T_{Fu}^{u,t}$	Upper bound on capacity of fine material (tonnes) in period "t" at destination "u".
$T_{Fl}^{u,t}$	Lower bound on capacity of fine material (tonnes) in period "t" at destination "u".
$T_{Su}^{u,t}$	Upper bound on capacity of tailings sand (tonnes) in period "t" at destination "u".
$T_{Sl}^{u,t}$	Lower bound on capacity of tailings sand (tonnes) in period "t" at destination "u".
$T_{Wu}^{u,t}$	Upper bound on capacity of tailings water (tonnes) in period "t" at destination "u".
$T_{Wl}^{u,t}$	Lower bound on capacity of tailings water (tonnes) in period "t" at destination "u".
$T_{Xu}^{u,t}$	Upper bound on capacity of MFT (tonnes) in period "t" at destination "u".
$T_{Xl}^{u,t}$	Lower bound on capacity of MFT (tonnes) in period "t" at destination "u".
$T_{Yu}^{u,t}$	Upper bound on capacity of CT (tonnes) in period "t" at destination "u".
$T_{Yl}^{u,t}$	Lower bound on capacity of CT (tonnes) in period "t" at destination "u".
$r^{u,e}$	Proportion of element "e" recovered (processing recovery) if it is processed at destination "u".
$p^{e,t}$	Price of element "e" in present value terms per unit of product.
$cs^{e,t}$	Selling cost of element "e" in present value terms per unit of product.
$cp^{u,e,t}$	Extra cost in present value terms per tonne of ore for mining and processing at destination "u".
$cm^{a,t}$	Cost in present value terms of mining a tonne of waste in period t from location a.
$ct^{c,t}$	Cost in present value terms of sending a volume unit of CT in period "t" to CT cell "c".

9.3. Decision variables

$x_k^{u,t} \in [0,1]$ A continuous variable representing the portion of ore from mining cut "k" to be extracted and processed at destination "u" in period "t".

$y_p^{a,t} \in [0,1]$ A continuous variable representing the portion of mining panel "p" to be mined in period "t" from location "a", which includes ore, OI material, tailings sand and waste.

$z_c^t \in [0,1]$ A continuous variable representing the portion of CT cell "c" to be filled with CT in period "t".

$b_p^t \in \{0,1\}$ A binary integer variable controlling the precedence of extraction of mining panels. b_p^t is equal to one if the extraction of mining panel "p" has started by or in period "t"; otherwise it is zero.

$c_j^t \in \{0,1\}$ A binary integer variable controlling the precedence of mining phases. c_j^t is equal to one if the extraction of phase "j" has started by or in period "t"; otherwise it is zero.

$a_c^t \in \{0,1\}$ A binary integer variable controlling the precedence of filling of CT cells. a_c^t is equal to one if the filling of CT cell "c" has started by or in period "t"; otherwise it is zero.

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