An Application of Mixed Integer Linear Programming for Block Cave Production Scheduling

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Abstract

Planning of caving operations poses complexities in different areas such as safety, environment, ground control, and production scheduling. As the mining industry is faced with more marginal resources, it is becoming essential to generate production schedules that will provide optimal operating strategies while meeting practical, technical, and environmental constraints. Production scheduling of any mining system has an enormous effect on the economics of the operation. The scheduling problems are complex due to the nature and variety of the constraints acting upon the system. Relying only on manual planning methods or computer software based on heuristic algorithms will lead to mine schedules that are not the optimal global solution. The objective of this paper is to develop a practical optimization framework to schedule production for caving operations. We present two mixed integer linear programming (MILP) formulations for the long-term production scheduling of block caving. First we solve the problem at the drawpoint level. Then, we aggregate drawpoints into larger units referred to as clusters. The formulations are developed, implemented, and verified in the TOMLAB/CPLEX environment. The production scheduler aims to maximize the net present value of the mining operation while the mine planner has control over the development rate, vertical mining rate, lateral mining rate, mining capacity, maximum number of active drawpoints, and advancement direction. To support a given production target, the production scheduler defines the opening and closing time for each drawpoint and cluster, the draw rate from each drawpoint and cluster, the number of new drawpoints and clusters that need to be constructed, and the sequence of extraction from the drawpoints and clusters. Application and comparison of the models for production scheduling using real mine data over 15 periods are presented.

1. Introduction

The goal of long-term mine production scheduling is to determine the mining sequence, which optimizes the company's strategic objective, while honoring the operational limitations over the mine life. There are a number of strategic objectives, which are common in the industry. Usually, the target is to maximize the net present value of mining operations within the existing economic, technical, and environmental constraints. However, other objectives such as cost minimization or reserve maximization could also be considered. The production schedule defines the management investment strategy. An optimal plan in mining projects will result in cost reduction, increasing

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equipment utilization, optimum recovery of marginal ores, steady production rates, and consistent product quality.

The optimization methods currently used to generate production schedules in block cave mines are not just limited to, but can be classified into two main categories: heuristic and exact methods. Heuristic methods refer to experience-based techniques for problem solving, learning, and discovery. Where an exhaustive search is impractical, heuristic methods are used to speed up the process of finding a satisfactory solution. Exact algorithms are guaranteed to find an optimal solution and to prove its optimality for every instance of an optimization problem. Mixed Integer Linear Programming (MILP) has been used by various researchers to model mine production scheduling. The MILP models have the capability to model complex real world problems. The advantage of MILP models solved by exact algorithms is that an optimal solution is guaranteed. The main shortcoming is MILP's inability to generate a global optimal large-scale life-of-mine production schedule within a reasonable CPU time.

This study's objective is to develop, implement, and verify a theoretical framework to address the long-term production scheduling problem of block cave mines. The objective of the theoretical framework is to maximize the net present value (NPV) of the mining operation, while controlling the planning parameters. The planning parameters considered in this study are: (i) mining capacity, (ii) draw rate, (iii) mining precedence, (iv) maximum number of active drawpoints, (v) number of new drawpoints in each period, (vi) continuous mining, and (vii) reserves.

The production scheduler defines the opening and closing time of each drawpoint and cluster, the draw rate from each drawpoint and cluster, the number of new drawpoints and clusters that need to be constructed, and the sequence of extraction from the drawpoints and clusters to support a given production target. In the proposed formulation, the orebody is represented by a geological block model, which is a three-dimensional array of rectangular or cubical blocks. The draw column, which is vertical, is created based on the block model, and the total column is divided into slices which match the vertical spacing of the geological block model. Numerical data are used to represent each attribute of the orebody. It is assumed that the physical layout of the production level is offset herringbone (Brown, 2003). All the material in the draw column must be extracted. MILP formulation is implemented for eight advancement directions.

In a typical block cave long-term scheduling problem, the number of integer and continuous decision variables, and the number of constraints formulating a problem exceeds the capacity of current state of hardware and software to solve the problem in a reasonable time. To overcome the size problem of mathematical programming models and to generate a robust practical near optimal schedule, the following general workflow for block cave operation is proposed:

1. Aggregate drawpoints into practical scheduling units using clustering algorithms. These automatically aggregated drawpoints are referred to as drawpoint groups. A hierarchical agglomerative clustering algorithm by Tabesh and Askari-Nasab (2011) is used to create mining-cuts based on a similarity index between drawpoints. Drawpoint aggregation is required for two reasons: (i) to generate a practical mining schedule that follows a selective mining unit, (ii) to reduce the number of variables, especially binary variables in the MILP formulation to make it computationally tractable. Aggregation is necessary to generate near-optimal realistic mine plans in a reasonable CPU time that could be considered useful in real-world mine planning.
2. Generate the optimal life-of-mine multi-period block cave production schedule at the aggregated drawpoint level. This is the strategic yearly production schedule with the objective of net present value (NPV) maximization. The strategic plan honors mining capacity and uniform feed to the processing plant. This is the MILP formulation at cluster level presented in this paper.

3. Generate the optimal long-term block cave production schedule at the drawpoint level. The output of the cluster level life-of-mine plan is the input into this stage. The time horizon for this model can vary as a subset of the life-of-mine to control the size of the MILP to be solved. This model is presented in this paper as MILP formulation at drawpoint level.
4. Generate the optimal medium-term plan at drawpoint level including slices. The output of the drawpoint level is the input into this stage. The time horizon for this detailed 3D model could vary as a subset of the time horizons chosen in the previous step. This model has been developed and is under verification and testing. The results will be published in the future publications.

In this paper, we present two independent MILP formulations at two different levels of resolution: (i) drawpoint level, and (ii) aggregated drawpoints (cluster level). We aggregated drawpoints using clustering algorithms. For a review of block aggregation and clustering techniques used in mines production scheduling, see Tabesh and Askari-Nasab(2011) and Weintraub et al.(2008).

We implemented the optimization formulation in the TOMLAB/CPLEX (Holmstrom, 2011) environment. A scheduling case study with real mine data was carried out over fifteen periods to verify the MILP model.

The next section of the paper covers relevant literature about the block cave production scheduling problem. Section three defines the problem and assumptions, while section four and five present mixed integer linear programming formulations of the problem and problem solving techniques. Section six presents an example with results and discussion. Finally, the last section presents the conclusions, and future work followed by a reference list.

2. Literature review

The majority of the scheduling publications to date have been concerned with open-pit mining applications. Underground mining is more complex in nature than surface mining (Kuchta, Newman, & Topal, 2004). Underground mining is less flexible than surface mining due to the geotechnical, equipment, and space constraints (Topal, 2008). This complexity is especially applicable when considering caving operations. An effective draw control plan is difficult to implement because several complex mechanisms control gravity flow. These mechanisms include caveability, stress, fragmentation, and the flow and mixing of broken material.

In spite of the difficulties associated with applying mathematical programming to production scheduling in underground mines, authors have attempted to develop methodologies to optimize production schedules. Newman et al. (2010) presented a comprehensive review of operations research in mine planning. Many types of scheduling methods have been applied to underground mine scheduling. These can be divided into subcategories which include alternative scheduling methods, simulation and heuristics, and linear programming and mixed integer linear programming (Rahal, 2008).

Alternative scheduling methods include queuing theory, network analysis, and dynamic programming. Queuing theory manages the system with waiting lines. It is suitable for modeling trucking and LHD arrival and departure from different locations in the mining system. In the network analysis, a graph of the mining system is developed as a series of nodes with interconnecting arcs. Several authors have used the mentioned methods successfully (Brazil, et al., 2003; Huang & Kumar, 1994; Muge, Santos, Vierira, & Cortez, 1992; Su, 1986).

Heuristic methods are generally used to generate a good solution in a reasonable time. These methods are used when there is no known method to find an optimal solution under the given constraints. Despite shortcomings such as lack of a way to prove optimality and frequently required intervention, simulation and heuristics are able to handle nonlinear relationships as a part of the scheduling procedure.

Scheduling underground mining operations is primarily characterized by discrete decisions to mine blocks of ore, along with complex sequencing relationships between blocks. Since Linear Programming (LP) models cannot capture the discrete decisions required for scheduling, MIPs are generally the appropriate mathematical programming approach to scheduling. The advantage of using these methods for production scheduling is that they can provide a mathematically provable optimum schedule. These methods are able to approximate some nonlinear systems, but they are not as flexible as simulation.

Many authors have combined mathematical programming with simulation in an attempt to improve the scheduling process (Chanda, 1990; Song, 1989; Winkler, 1998).

The original heuristic methods were the manual draw charts used at the beginning of block caving. These methods evolved through use at the Henderson mine, where a way to avoid early dilution entry was described by constraining the draw profile to 45-degree angle of draw (E. Rubio, 2006). Heslop et al. (1981) described a volumetric algorithm to simulate the mixing along the draw cone. Diering (2000) presented the principles behind the commercial tool PCBC to compute production schedules using several case studies with different draw methods.

Riddle (1977) described the application of operation research methods and applied dynamic programming to the problem of stope planning and layout in a block cave mine. The final algorithm worked with the block model directly. It did not use the concept of a draw cone as an individual entity of the optimization process.

Chanda (1990) implemented an algorithm to write daily orders and developed the interface between mathematical programming and simulation by integrating the two into a short-term planning system for a continuous block cave. The objective function was defined to minimize the fluctuation in the average grade drawn between shifts.

Guest et al. (2000) also applied mathematical programming to long term scheduling in block caving. In this case, the objective function was explicitly defined to maximize draw control behavior. However, the author stated that the implicit objective was to optimize net present value (NPV). There are two problems with this approach. The first is that maximizing tonnage or mining reserves will not necessarily lead to maximum NPV. The second is that draw control is a planning constraint and not an objective function. The objective function in this case would be to maximize tonnage, minimize dilution, or maximize mine life.

Rubio (2002) developed a methodology that would enable mine planners to compute production schedules in block cave mining. He proposed new production process integration and formulated two main planning concepts as potential goals to optimize the long-term planning process, thereby maximizing NPV and mine life.

Rahal (2008) described a MILP goal program. The model had the dual objectives of minimizing the deviation from the ideal draw. This algorithm assumes that the optimal draw strategy is known.

Diering (2004) presented a non-linear optimization method to minimize the deviation between a current draw profile and the target defined by the mine planner. He emphasized that this algorithm could also be used to link the short-term plan with the long-term plan. The long-term plan is represented by a set of surfaces that are used as a target to be achieved based on the current extraction profile when running the short-term plans.

Rubio and Diering (2004) described the application of mathematical programming to formulate optimization problems in block cave production planning. They formulated two main planning strategies, maximization of net present value and maximization of mine life. They used the operational constraints presented by Rubio (2002).

Most of the methods presented above do not integrate the technical uncertainty and geotechnical constraints to the method. Also, production scheduling optimization techniques are still not widely used in the mining industry, particularly in underground mining. There is a need to show how optimization methods create a practical production schedule. In the available block cave scheduling software programs such as PCBC (T Diering, 2000) the mining sequence is controlled manually and, consequently, we would not have an optimum solution for the problem.

3. Problem definition and assumptions

In the case of a block cave mine, the production schedule mainly defines the amount of the material to be mined from the drawpoints in every period of production to achieve a given planning objective. The mine plan also defines the number of new drawpoints that need to be constructed and their sequence to support a given production target. In other words, scheduling a block cave mine is a matter of finding the goal that better represents the strategic planning vision subject to several mine design, geomechanical, operational, and environmental constraints. The production schedule is subject to a variety of physical and economic constraints. The constraints enforce the production target, draw rate, mining precedence, maximum number of active drawpoints, number of new drawpoints in each period, continuous mining, and complete extraction of reserves.

Several assumptions are used in the proposed MILP formulation. The orebody is represented by a geological block model, which is a three-dimensional array of rectangular or cubical blocks. The column of rock above each drawpoint, which is referred to as a draw column, is simulated and stored in a slice file. The draw columns, which are vertical, are created based on the block model. The total column is divided into slices which match the vertical spacing of the geological block model. Numerical data are used to represent each attribute of the orebody, such as tonnage, density, grade of elements, elevation, percentage of dilution, and economic data for each slice. It is assumed that the physical layout of the production level is offset herringbone (Brown, 2003).The source model is assumed to be static with time. There is no selective mining, meaning that all the material in the draw column must be extracted. All stages before scheduling, from creating a block model to converting slice files, are done using GEMS and PCBC (GemcomSoftwareInternational, 2011). The first step is to create a block model, using GEMS, to provide a quantitative description of the rockmass including and surrounding the cave zone. Then, drawpoint locations are defined and PCBC is used to convert block model data into drawpoint based data. Afterwards, the slices are constructed for each drawpoint. These slices represent the draw column above each drawpoint before any extraction begins. The best height of draw (BHOD) for each draw column is estimated. The BHOD is the height that produces the best economic value. Usually it is not discounted with time. After applying the BHOD, the final height of draw is obtained for each draw column. Then, slices within the same draw column are generated and the total tonnage, draw column economic value, and average weighted grade are calculated. Afterwards, the production schedule of a block cave mine can be optimized using the MILP formulation presented in this paper. It must be mentioned that the precedence between drawpoints or clusters is controlled in horizontal direction. In other words, we treat the problem in darwpoint or cluster level as strategic long-term plan and the slices are not used in the presented formulations.

To solve the problem, three decision variables are employed: one continuous decision variable and two binary integer variables. These decision variables are used in two different levels: drawpoint level and cluster level. When the problem is solved at the drawpoint level, the continuous decision variable indicates the portion of extraction from each draw column in each period. Two binary integer variables control the number of active drawpoints, precedence of extraction between drawpoints, the opening and closing time of each drawpoint, the draw rate from each drawpoint, and the number of new drawpoints that need to be constructed in each period. These variables perform similar roles for clusters when the problem is solved at the cluster level.

To create the clusters, the drawpoints are grouped into clusters based on similarities between their physical location and tonnage of draw column above each drawpoint. Similar to drawpoints, each cluster has coordinates representing the center of the cluster and its coordinates. We assume that the portion scheduled to be extracted from each cluster is taken from all the drawpoints based on the ratio of each draw column's tonnage in the cluster.

Clustering algorithms are usually performed by defining a measure of similarity or dissimilarity between the objects. Attempts have been made to use clustering on mine production planning (H. Askari-Nasab, Awuah-Offei, & Eivazy, 2010; Busnach, Mehrez, & Sinuany-Stern, 1985; Zhang, 2006). In this paper the Fuzzy Logic Clustering (Bezdek, 1981) is used to group the drawpoints into clusters. This topic does not fall within the scope of this paper and, thus, will not be addressed. We will disseminate the clustering approach in another publication.

Because of the continuous advancement of the cave front in block caving, the production schedules using MILP formulation is implemented for eight advancement directions. Fig 1 illustrates these advancement directions. According to the advancement direction, for each drawpoint d there is a set S^d, which defines the predecessor drawpoints among adjacent drawpoints that must be started before drawpoint d is extracted. Based on the search direction, eight different predecessor data sets can be defined for each drawpoint. Fig 2 shows that in the offset herringbone layout (Brown, 2003), each drawpoint is surrounded by a maximum of seven drawpoints. The members of set S^d in each direction are determined using an imaginary line perpendicular to the desired advancement direction at the location of the considered drawpoint. All located adjacent drawpoints behind the imaginary line are defined as members of set S^d in the considered advancement direction. For instance, Fig 3a shows that adjacent drawpoints for drawpoint d4 include d1, d2, d3, d5, d6, d7, and d8. In the advancement direction of north to south (NS), the extraction of drawpoints d1, d2, and d3 has to be started prior to drawpoint d4. Fig 3b shows that in the southwest to northeast (SW to NE) direction, the extraction of drawpoints d1, d6, and d7 has to be started prior to drawpoint d4.

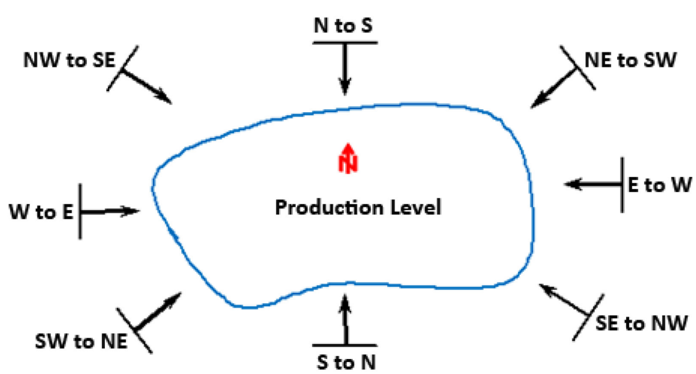

Fig 1. Alternative cave advancement directions.

4. Mixed integer linear programming model for block cave production scheduling

A mixed integer linear programming model contains both integer and continuous variables. There are no quadratic terms in the objective function.

As a general assumption for our formulations, we define that a parameter f can take two indices in the format of $f_{d,t}$ or $f_{cl,t}$. Where:

$t \in \{1,....,T\}$ index for scheduling periods.

$d \in \{1,...,N_d\}$ index for drawpoints.

$cl \in \{1,...,N_{cl}\}$ index for clusters.

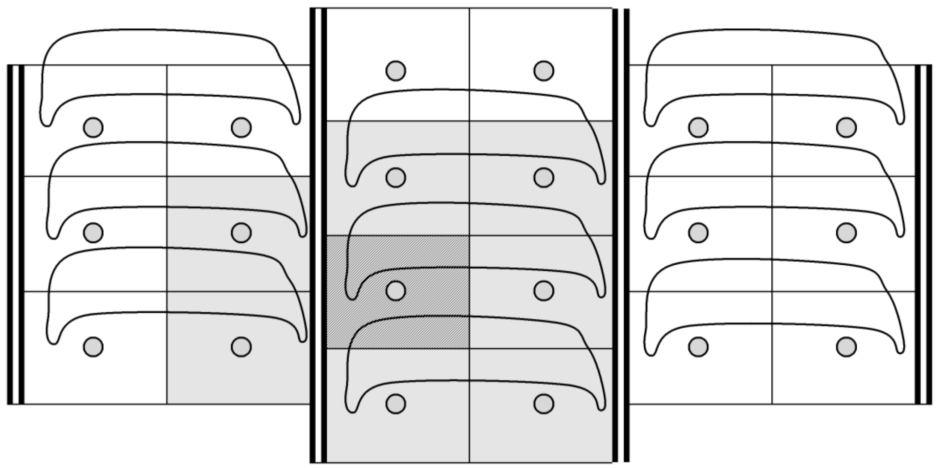

Fig 2. Offset herringbone extraction level layout (Brown, 2003).

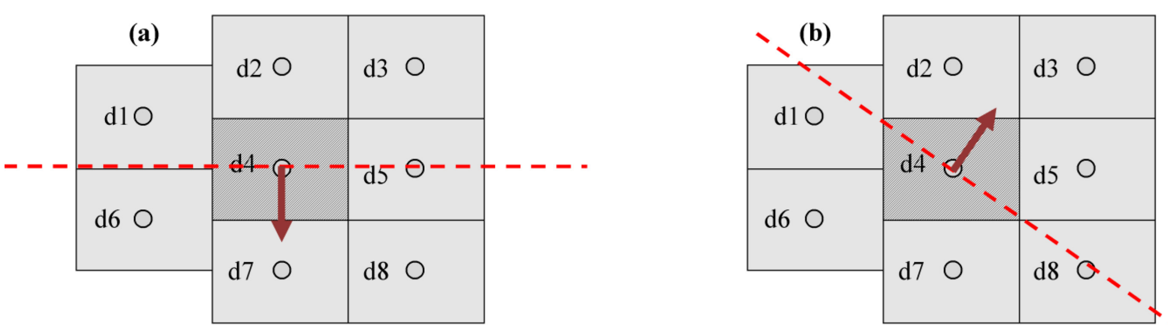

Fig 3. Determination method of members for set S^d in different directions.

4.1. MILP formulation at drawpoint level

4.1.1. Objective function:

The objective function of the MILP formulation is to maximize the net present value of the mining operation. The profit from mining a drawpoint depends on the economic value of the drawpoint and the costs incurred in mining. The objective function, Eq.(1), is composed of the draw column economic value (DEV), discount rate, and a continuous decision variable that indicates the portion of a draw column which is extracted in each period. The most profitable draw columns will be chosen as part of the production in order to optimize the NPV.

$$\text{Maximize} \quad \sum_{t=1}^{T}\sum_{d=1}^{N_d}\left[\frac{DEV_d}{(1+i)^t}\right]U_{d,t} \tag{1}$$

Where

- T is the maximum number of scheduling periods, where $t=\{1,...,T\}$ is the set of all the scheduling time periods in the model,

- N_d is the maximum number of drawpoints in the model, where $d=\{1,...,N_d\}$ is the set of all drawpoints in the model,
- DEV_d is the economic value of the draw column associated with drawpoint d, which is the summation of economic value of slices within the draw column and it is a constant value for each drawpoint.
- i is the discount rate,
- $U_{d,t} \in [0,1]$ is a continuous decision variable, representing the portion of the draw column associated with drawpoint d to be extracted in period t,

The objective function is subject to the following constraints:

4.1.2. Mining capacity:

$$\underline{M_t} \le \sum_{d=1}^{N_d} U_{d,t} \times \left(Ton_d\right) \le \overline{M_t} \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,N_d\} \tag{2}$$

Where

- $\underline{M_t}$ is the lower limit of mining capacity in period t,
- $\overline{M_t}$ is the upper limit of mining capacity in period t,
- Ton_d is the total tonnage of material within the draw column associated with drawpoint d,

These constraints force a mining rate between the desired and maximum mining capacity available. This is applied using inequalities in Eq.(2), which ensures that the total tonnage of material extracted from drawpoints in each period is within the acceptable range that allows flexibility for potential operational variations. The constraints are controlled by the continuous variable $U_{d,t}$. There is one constraint per period.

4.1.3. Maximum number of active drawpoints:

$$A_{d,t} \le L.U_{d,t} \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,N_d\} \tag{3}$$

$$U_{d,t} \le A_{d,t} \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,N_d\} \tag{4}$$

$$\sum_{d=1}^{N_d} A_{d,t} \le N_{Ad,t} \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,N_d\} \tag{5}$$

Where

- $A_{d,t} \in \{0,1\}$ is a binary integer decision variable equal to one if drawpoint d is active in period t, otherwise it is zero,
- $N_{Ad,t}$ is the maximum allowable number of active drawpoints in period t, this number must be given as an input to the algorithm,
- L is a large enough number and it must be greater than $\dfrac{\max\{Ton_d\}}{\text{minimum draw rate}}$,

4.1.4. Precedence of drawpoints:

$$Z_{d,t} - \sum_{j=1}^{t} U_{k,j} \le 1 - \left(\frac{\text{minimum draw rate}}{\max\{Ton_d\}}\right) \qquad \forall d \in \{1,...,N_d\}, \quad t \in \{1,...,T\}, \quad k \in S^d \tag{6}$$

$$\sum_{t=1}^{T} Z_{d,t} = 1 \qquad \forall d \in \{1,...,N_d\}, \quad t \in \{1,...,T\} \tag{7}$$

Where

- $Z_{d,t} \in \{0,1\}$ is a binary integer variable controlling the precedence of extraction of drawpoints. It controls the sequence of opening. It is equal to one if extraction from drawpoint d is started in period t, otherwise it is zero.
- S^d is a set defining the predecessor drawpoints that must be started prior to extraction of drawpoint d,

Based on the advancement direction, extraction from drawpoints belonging to the relevant set S^d must be started prior to extraction of drawpoint d. Set S^d can also be empty, which means drawpoint d can be extracted in any time period in the schedule. To control the precedence of extraction, a binary decision variable, $Z_{d,t}$, is employed. If the extraction of drawpoint d is started in period t, the variable $Z_{d,t}$ has to be 1, otherwise it is zero. Eq.(6) is applied to all members of set S^d. If the summation of extracted portions of each draw column from the respective set until the considered period, t, is equal or greater than the minimum allowable draw rate, then extraction from drawpoint d can be started. Eq.(7) ensures that drawpoint d is opened once during the mine life.

4.1.5. Continuous extraction:

$$A_{d,t} - A_{d,(t-1)} \le Z_{d,t} \qquad \forall d \in \{1,...,N_d\}, \quad t \in \{2,...,T\} \tag{8}$$

$$A_{d,1} - Z_{d,1} \le 0.5 \qquad \forall d \in \{1,...,N_d\} \tag{9}$$

Extraction from each drawpoint must be continuous. Eq.(8) ensures that if extraction from drawpoint d is started from or after period two, at least a portion of the draw column associated with drawpoint d is extracted until all of the material within that drawpoint has been extracted; otherwise the drawpoint must be closed. The minimum portion of extraction must be equal or greater than the minimum allowable draw rate. Eq.(9) is used for period one. It ensures that if extraction from drawpoint d is started in period one, the related variable $A_{d,1}$ for that drawpoint is equal to one.

4.1.6. Draw rate:

$$A_{d,t} \times \underline{DR}_{d,t} \le U_{d,t} \times \left(Ton_d\right) \le \overline{DR}_{d,t} \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,N_d\} \tag{10}$$

Where

- $\underline{DR}_{d,t}$ is the minimum possible draw rate of drawpoint d in period t,
- $\overline{DR}_{d,t}$ is the maximum possible draw rate of drawpoint d in period t,

This constraint controls the maximum and minimum rate of draw and is a function of fragmentation and caveability. This rate should be fast enough to avoid compaction and slow enough to avoid air gaps. The maximum limit to the draw rate is usually determined by the fragmentation process, since time is required to achieve good fragmentation. However, sometimes the maximum rate may be determined by the LHD productivity. When drawpoint d is not active, variable $A_{d,t}$ is equal to zero and this relaxes the lower bound of Eq.(10). It should be mentioned that one method of managing drawpoint production is by establishing a production rate curve, which limits production based on the amount of material that has been drawn previously. This means that production depends on the tonnes mined from a drawpoint. But in the present formulation, we only use lower and upper bounds to control the draw rate.

4.1.7. Number of new drawpoints:

$$\underline{N_{Nd,t}} \leq \sum_{d=1}^{N_d} Z_{d,t} \leq \overline{N_{Nd,t}} \qquad \forall t \in \{2,...,T\}, \quad d \in \{1,...,N_d\} \tag{11}$$

$$\sum_{d=1}^{N_d} Z_{d,1} \leq N_{Ad,1} \qquad \forall d \in \{1,...,N_d\} \tag{12}$$

Where

- $\underline{N_{Nd,t}}$ is the lower limit for the number of new drawpoints, the extraction from which can start in period t,
- $\overline{N_{Nd,t}}$ is the upper limit for the number of new drawpoints, the extraction from which can start in period t,

Variable $Z_{d,t}$ is equal to one if the extraction from drawpoint d is started in period t. Therefore, the summation of this variable for all drawpoints in each period indicates the number of new drawpoints which are opened in that period. At the beginning and in period one, the number of new drawpoints is equal to the maximum number of the active drawpoint.

4.1.8. Reserves

$$\sum_{t=1}^{T} U_{d,t} = 1 \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,N_d\} \tag{13}$$

Eq.(13), ensures that the fractions of draw columns that are extracted over the scheduling periods are going to sum up to one, which means that all the material within the draw column is going to be extracted.

4.2. Clustering

The proposed MILP formulation according to the Eqs.(1) to (13) requires $(3 \times N_d \times T)$ number of decision variables, where one third are continuous variables and the rest are binary integer variables. For instance, if we apply the proposed formulation on a mine with 3000 drawpoints over 25 years, there will be 225000 decision variables, where 75000 are continuous variables and 150000 are binary integer variables. A problem of this size is computationally intractable with the current state of optimization software.

The most common problem in the MILP formulation is size of the branch and cut tree. The tree becomes so large that insufficient memory remains to solve the LP sub-problems. The size of the branch and cut tree can actually be affected by the specific approach one takes in performing the branching and by the structure of each problem. So, there is no way to determine the size of the tree before solving the problem.

Attempts have been made to overcome the curse of dimensionality which causes the long-term production scheduling combinatorial optimization problems in open pit mines. Askari-Nasab et al. (2011) have proposed a clustering procedure based on a Fuzzy c-means (FCM) algorithm to reduce the number of binary variables in their formulation for open-pit production scheduling. They created clusters based on similarities between blocks regarding the physical location of the block and its grade. Tabesh and Askari-Nasab (2011) have presented a two-stage clustering procedure based on the Hierarchical Clustering and Tabu Search for block aggregation in open pit mines. Weintraub et al. (2008) have developed an approach to reduce model size in MIP mine planning models. The aggregation is done both on the original data of the mine as well as on the MIP original models. They implemented the method in the El Teniente underground mine.

As a general strategy in our formulation, we aimed at reducing the number of binary integer variables. For this purpose we aggregate the drawpoints into clusters based on similarities between draw columns regarding the physical location of the drawpoint and its tonnage using Fuzzy c-means clustering (Bezdek, 1981). Similar to drawpoints, each cluster has coordinates representing the center of the cluster and its location. We assume that the portion scheduled to be extracted from each cluster is taken from all the drawpoints based on the ratio of each draw column's tonnage in the cluster.

Fuzzy c-means is a data clustering technique wherein each data point belongs to a cluster that to some degree is specified by a membership grade. In other words, this algorithm works by assigning membership to each data point corresponding to each cluster center on the basis of the distance between the cluster center and the data point. With Fuzzy c-means, the centroid of a cluster is computed as being the mean of all points weighted by their degree of belonging to the cluster. This technique was originally introduced by Bezdek (1981). It provides a method that shows how to group data points that populate some multidimensional space into a specific number of different clusters.

4.3. MILP formulation at cluster level

The concept of this formulation is similar to the drawpoint level except that here the total tonnage and draw rate of each cluster, and the cluster economic value are all a function of the number of drawpoints within the cluster.

This formulation is implemented for different advancement directions to maximize the NPV. According to the advancement direction, for each cluster cl there is a set S^{cl} which defines the predecessor clusters among adjacent clusters that must be started before cluster cl is extracted. Based on the search direction, eight different predecessor data sets can be defined for each cluster. The set S^{cl} is created in three steps. In step one, the boundary drawpoints of the considered cluster are determined. These boundary drawpoints are located behind an imaginary line perpendicular to the desired advancement direction at the cluster's center point. In step two, all clusters which have at least one adjacent drawpoint with the boundary drawpoints are determined. Afterwards, clusters whose center point is behind the center point of the considered cluster are defined as members of set S^{cl} in the considered advancement direction out of selected clusters in the previous step.

Fig 4 illustrates the above-mentioned method. For advancement from west to east, the boundary drawpoints for cluster 20 are d12, d19, d26, and d32. Only drawpoints d19, d26, and d32 have adjacent drawpoints (d25, d31, and d38) behind an imaginary line perpendicular to the WE direction of advancement. Drawpoints d25 and d31 belong to cluster 18 and drawpoint d38 belongs to cluster 21, so these two clusters can be a member of set S^{20}. Since the cluster 18 center point is behind cluster 20 center point in the considered direction, it is only selected as a member of set S^{20}

4.3.1. Objective function:

$$\text{Maximize} \quad \sum_{t=1}^{T}\sum_{cl=1}^{N_{cl}}\left[\frac{CLEV_{cl}}{(1+i)^{t}}\right]U_{cl,t} \tag{14}$$

Where

- T is the maximum number of scheduling periods, where $t=\{1,...,T\}$ is the set of all the scheduling time periods in the model,

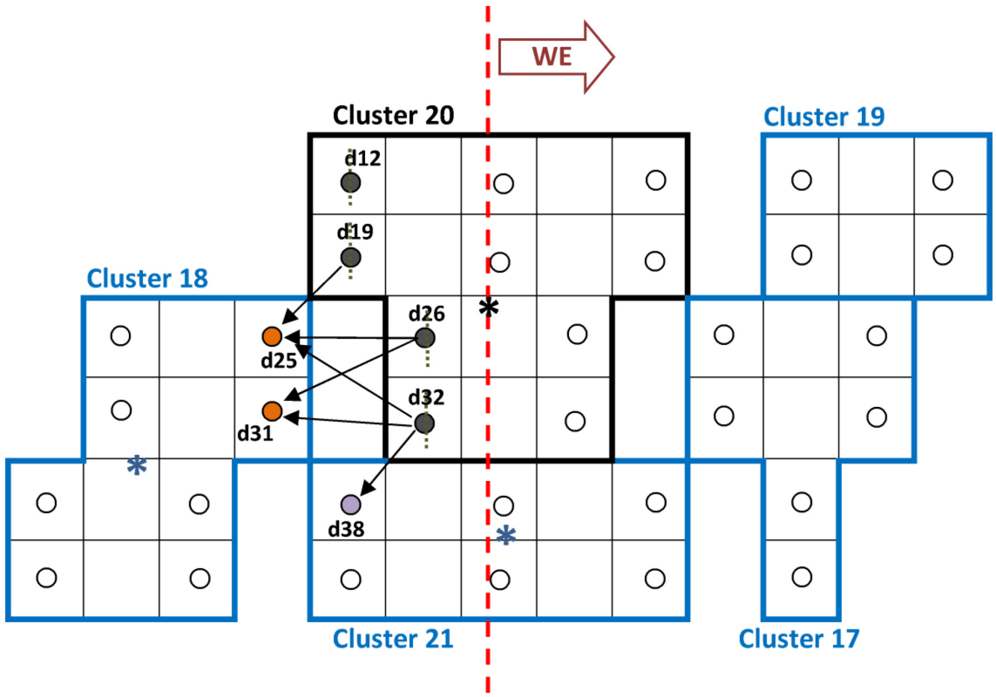

Fig 4. Determination method of members for set S^{cl} in different directions.

- N_{cl} is the maximum number of clusters in the model, where $cl = \{1,...,N_{cl}\}$ is the set of all clusters in the model,
- $CLEV_{cl}$ is the economic value of cluster cl,
- i is the discount rate,
- $U_{cl,t} \in [0,1]$ is a continuous decision variable, representing the portion of cluster cl to be extracted in period t,

The objective function is subject to the following constraints:

4.3.2. Mining capacity:

$$\underline{M_t} \le \sum_{cl=1}^{N_{cl}} U_{cl,t} \times \left(Ton_{cl}\right) \le \overline{M_t} \qquad \forall t \in \{1,...,T\}, \quad cl \in \{1,...,N_{cl}\} \qquad (15)$$

Where

- $\underline{M_t}$ is the lower limit of mining capacity in period t,
- $\overline{M_t}$ is the upper limit of mining capacity in period t,
- Ton_{cl} is the total tonnage of material within cluster cl,

4.3.3. Maximum number of active clusters:

$$A_{cl,t} \le L.U_{cl,t} \qquad \forall t \in \{1,...,T\}, \quad cl \in \{1,...,N_{cl}\} \qquad (16)$$

$$U_{cl,t} \le A_{cl,t} \qquad \forall t \in \{1,...,T\}, \quad cl \in \{1,...,N_{cl}\} \qquad (17)$$

$$\sum_{cl=1}^{N_{cl}} A_{cl,t} \le N_{Acl,t} \qquad \forall t \in \{1,...,T\}, \quad cl \in \{1,...,N_{cl}\} \qquad (18)$$

Where

- $A_{cl,t} \in \{0,1\}$ is a binary integer decision variable equal to one if cluster cl is active in period t, otherwise it is zero,
- $N_{Acl,t}$ is the maximum allowable number of active clusters in period t, this number must be given as an input to the algorithm,
- L is a large enough number and it must be greater than $\left(\dfrac{\max\{Ton_{cl}\}}{\min\{\text{number of drawpoints within a cluster}\}\times \text{minimum draw rate}}\right)$,

4.3.4. Precedence of clusters:

$$Z_{cl,t} - \sum_{j=1}^{t} U_{k,j} \le 1 - \left(\frac{\min\{\text{number of drawpoints within a cluster}\}\times \text{minimum draw rate}}{\max\{Ton_{cl}\}}\right) \quad \forall cl \in \{1,...,N_{cl}\},\quad t\in\{1,...,T\},\quad k \in S^{cl} \tag{19}$$

$$\sum_{t=1}^{T} Z_{cl,t} = 1 \qquad \forall cl \in \{1,...,N_{cl}\},\quad t\in\{1,...,T\} \tag{20}$$

Where

- $Z_{cl,t} \in \{0,1\}$ is a binary integer variable controlling the precedence of extraction of clusters. It controls the sequence of opening. It is equal to one if extraction from cluster cl is started in period t, otherwise it is zero.
- S^{cl} is a set defining the predecessor clusters that must be started before cluster cl is extracted,

4.3.5. Continuous extraction:

$$A_{cl,t} - A_{cl,(t-1)} \le Z_{cl,t} \qquad \forall cl \in \{1,...,N_{cl}\},\quad t\in\{2,...,T\} \tag{21}$$

$$A_{cl,1} - Z_{cl,1} \le 0.5 \qquad \forall cl \in \{1,...,N_{cl}\} \tag{22}$$

4.3.6. Draw rate:

$$A_{cl,t} \times \left(NDP_{cl} \times \underline{DR}_{k,t}\right) \le U_{cl,t} \times \left(Ton_{cl}\right) \le \left(NDP_{cl} \times \overline{DR}_{k,t}\right) \quad \forall cl \in \{1,...,N_{cl}\},\quad t\in\{1,...,T\},\quad k \in S^{cl} \tag{23}$$

Where

- $\underline{DR}_{k,t}$ is the minimum possible draw rate of drawpoints belonging to the set S^{cl} in period t,
- $\overline{DR}_{k,t}$ is the maximum possible draw rate of drawpoints belonging to the set S^{cl} in period t,
- NDP_{cl} is the number of drawpoints within cluster cl,

4.3.7. Number of new clusters:

$$\underline{N_{Ncl,t}} \le \sum_{cl=1}^{N_{cl}} Z_{cl,t} \le \overline{N_{Ncl,t}} \qquad \forall t \in \{2,...,T\},\quad cl \in \{1,...,N_{cl}\} \tag{24}$$

$$\sum_{cl=1}^{N_{cl}} Z_{cl,1} \le N_{Acl,1} \qquad \forall cl \in \{1,...,N_{cl}\} \tag{25}$$

Where

- $\underline{N_{Ncl,t}}$ is the lower limit for the number of new clusters, the extraction from which can start in period t,
- $\overline{N_{Ncl,t}}$ is the upper limit for the number of new clusters, the extraction from which can start in period t,

4.3.8. Reserves

$$\sum_{t=1}^{T} U_{cl,t} = 1 \qquad \forall t \in \{1,...,T\}, \quad cl \in \{1,...,N_{cl}\} \tag{26}$$

5. Solving the optimization problem

One optimization solver capable of handling large scale problems is CPLEX (ILOGInc, 2007). We have developed, implemented, and tested the proposed MILP models in the TOMLAB/CPLEX environment (Holmstrom, 2011). TOMLAB/CPLEX integrates the solver package CPLEX with the MATLAB environment (MathWorksInc, 2011). The algorithms are also coded in MATLAB. We have used the gap tolerance (EPGAP) as an optimization termination criterion. The gap tolerance sets an absolute tolerance on the gap between the best integer objective and the objective of the best node remaining.

6. Results and discussion

We have developed, implemented and tested the proposed MILP formulations presented in section 4 in the TOMLAB/CPLEX environment. The models were tested on a Dell Precision T3500 computer at 2.4 GHz, with 3 GB of RAM. The goal is to maximize the NPV at a discount rate of 12%, while assuring that all constraints are satisfied during the mine life. The performance of the proposed formulations is analyzed based on NPV, satisfaction of constraints, and practicality of the generated schedules from the mining point of view in four main directions. The models are verified by numerical experiments on a real data set containing 102 drawpoints and 3470 slices over 15 periods. Fig 5 illustrates a plan view of the drawpoints' configuration based on the relevant coordinates. The number of slices was reduced to 2058 after applying the BHOD. The total tonnage of material which has to be extracted is almost 13.5 Mt. Fig 6 shows the difference between heights of draw columns before and after applying the BHOD. The top surface in Fig 6 is created based on the initial height of draw columns. The blue bars show the BHOD.

A capacity of 900 kt/yr was considered as the upper bound on the mining capacity for both formulations. The maximum number of active drawpoints and clusters in each period is set to 40 and 5, respectively. The maximum number of new drawpoints and clusters which can be opened in each period for the drawpoint level model and cluster level model is 15 and 2, respectively. The lower bound and upper bound of the draw rate for drawpoints are set to 10 and 40 kt/yr in both of the models.

The drawpoint level formulation has a 4590-decision variable in which 3060 are binary integer variables. We use fuzzy logic clustering to aggregate drawpoints into 17 clusters to reduce the number of variables required in the MILP model. The clustering algorithm aggregates draw columns based on two main criteria: location and tonnage. Draw columns with a higher order of similarity are aggregated into clusters. As the result of clustering, the number of binary integer variables decreases to 510. The minimum and maximum numbers of drawpoints within each cluster are 4 and 10, respectively. The total tonnage of material is calculated for each cluster based on the tonnage of draw columns within the cluster. Table 1shows the value of NPV, CPU time, and optimality gap for the formulation at the drawpoint level. A relative tolerance of 5% on the gap between the best integer objective and the feasible integer solution was set for optimization. The maximum NPV is gained from west to east direction based on the given constraints.

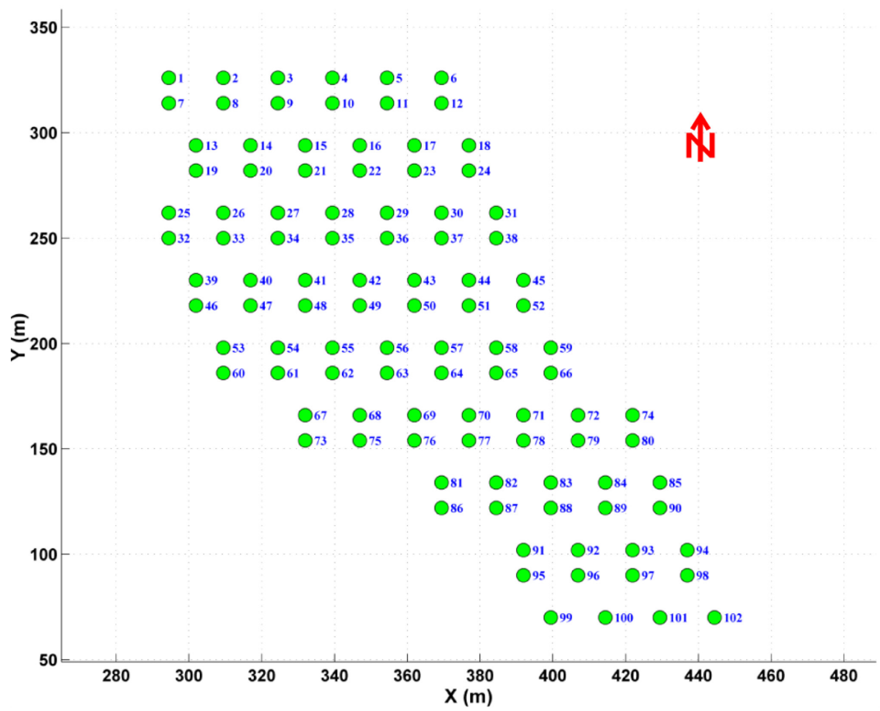

Fig 5. Plan view of drawpoints configuration.

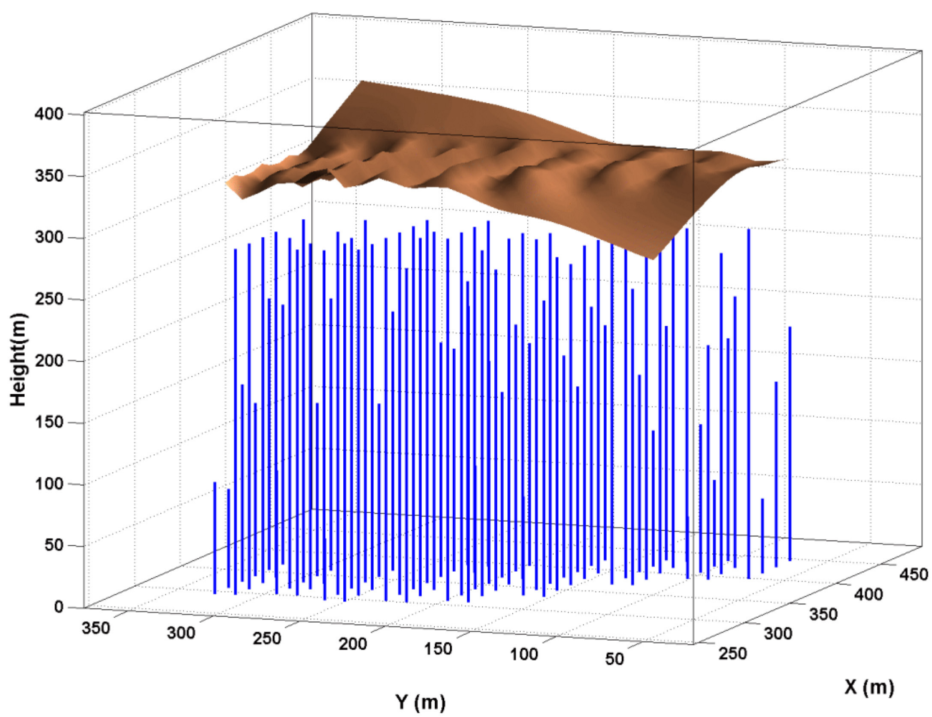

Fig 6. Height of draw columns after applying the BHOD. The surface shows the initial height of draw columns before applying the BHOD.

Table 1. Numerical results for drawpoint level formulation.

Direction	CPU time	Optimality GAP (%)	NPV ($M)
west to east (WE)	01:52:02	2.73	19.36
east to west (EW)	19:54:36	4.1	19.05
north to south (NS)	00:38:15	4.9	18.89
south to north (SN)	00:55:24	4.0	18.98

Fig 7 to Fig 10 show that all assumed constraints have been satisfied for the drawpoint level model. Fig 7 illustrates tonnage of extraction in different advancement directions. It can be seen clearly that the formulation tries to keep mining capacity at the upper bound and in the period 15; only the material that remains is extracted. Fig 8 illustrates the maximum number of active drawpoints in each period for the different advancement directions. In period one, in the west to east and north to south directions, fewer than 40 drawpoints are active. In all directions, the number of active drawpoints gradually decreases from period 12 to 15. Period 15 has the least number of active drawpoints in all advancement directions. Fig 9 shows the number of new drawpoints, which have to be opened in each period for different advancement directions. The upper bound for the number of new drawpoints in period one is equal to the maximum number of active drawpoints. In all directions, the number of new drawpoints from period 2 to period 15 is less than the amount set. Only in period seven of the east to west direction, seven new drawpoints have to be opened. New drawpoints are opened in most of the periods of the west to east direction, which has the maximum NPV. In this direction, after period 13 there is no new drawpoint, and extraction continues from drawpoints which have been opened in the previous periods.

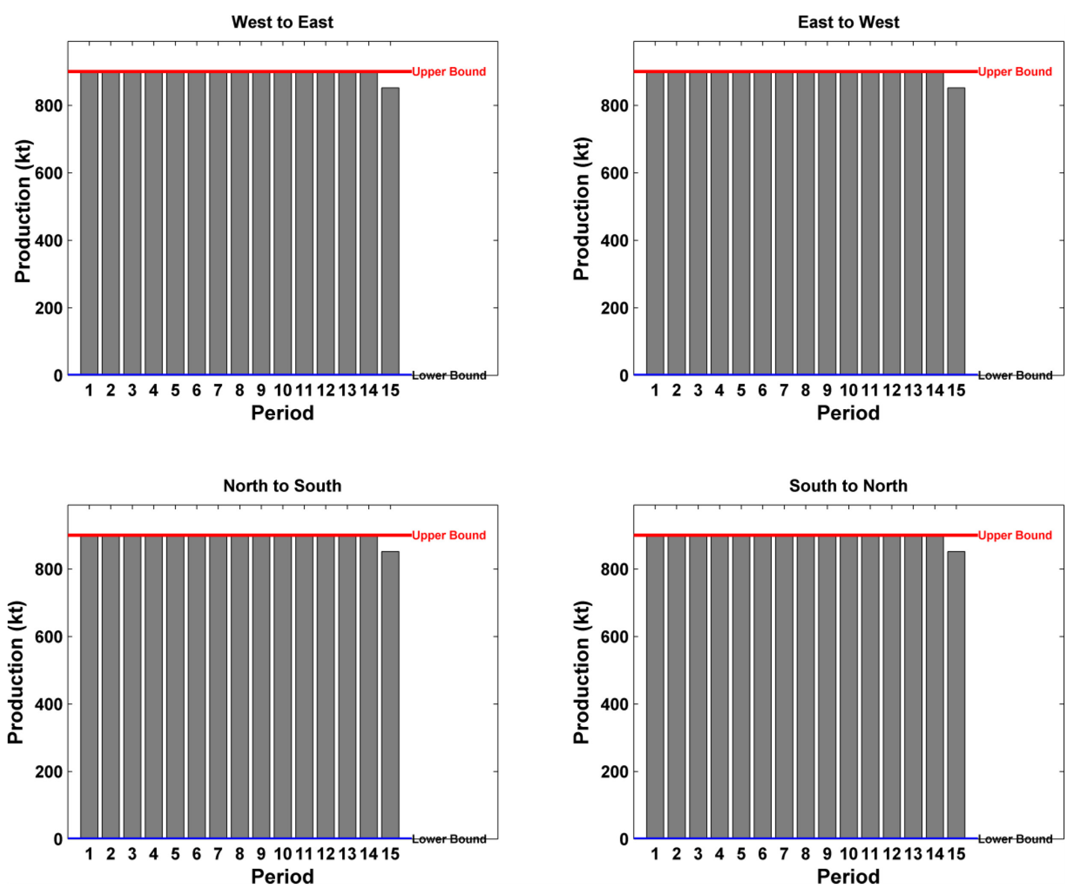

Fig 7. Tonnage of production for different advancement directions over 15 periods (drawpoint level).

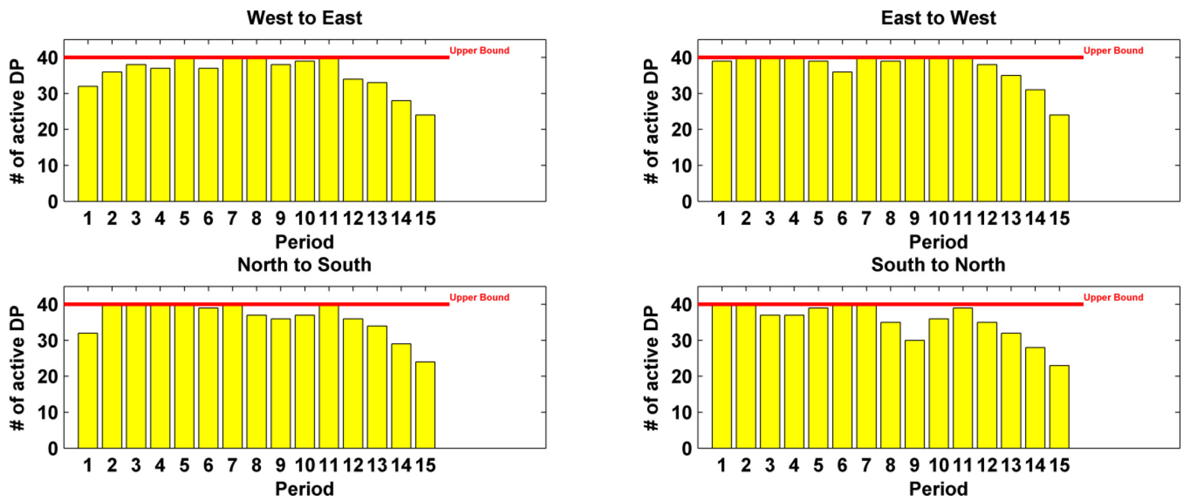

Fig 8. Maximum number of active drawpoints for different advancement directions over 15 periods (drawpoint level).

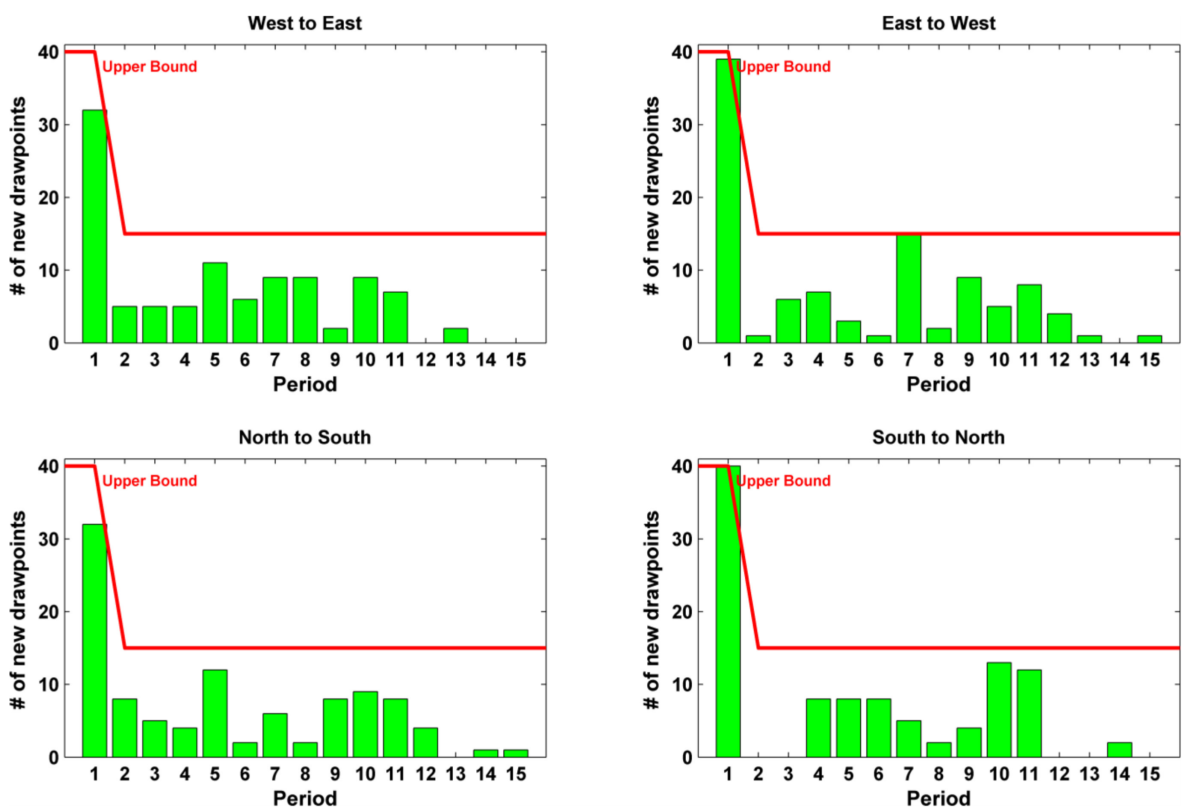

Fig 9. Development rate for different advancement directions over 15 periods (drawpoint level).

Fig 10 illustrates the draw rate for drawpoints d20 and d87 in the west to east direction. It is clear that the defined upper and lower bounds for both selected drawpoints have been satisfied. On the other hand, all material within draw columns associated with drawpoints d20 and d87 is extracted and this shows that the reserves' constraint has been satisfied. It should be mentioned that the formulation tries to extract material from drawpoints with a draw rate within the acceptable range without considering a specific shape.

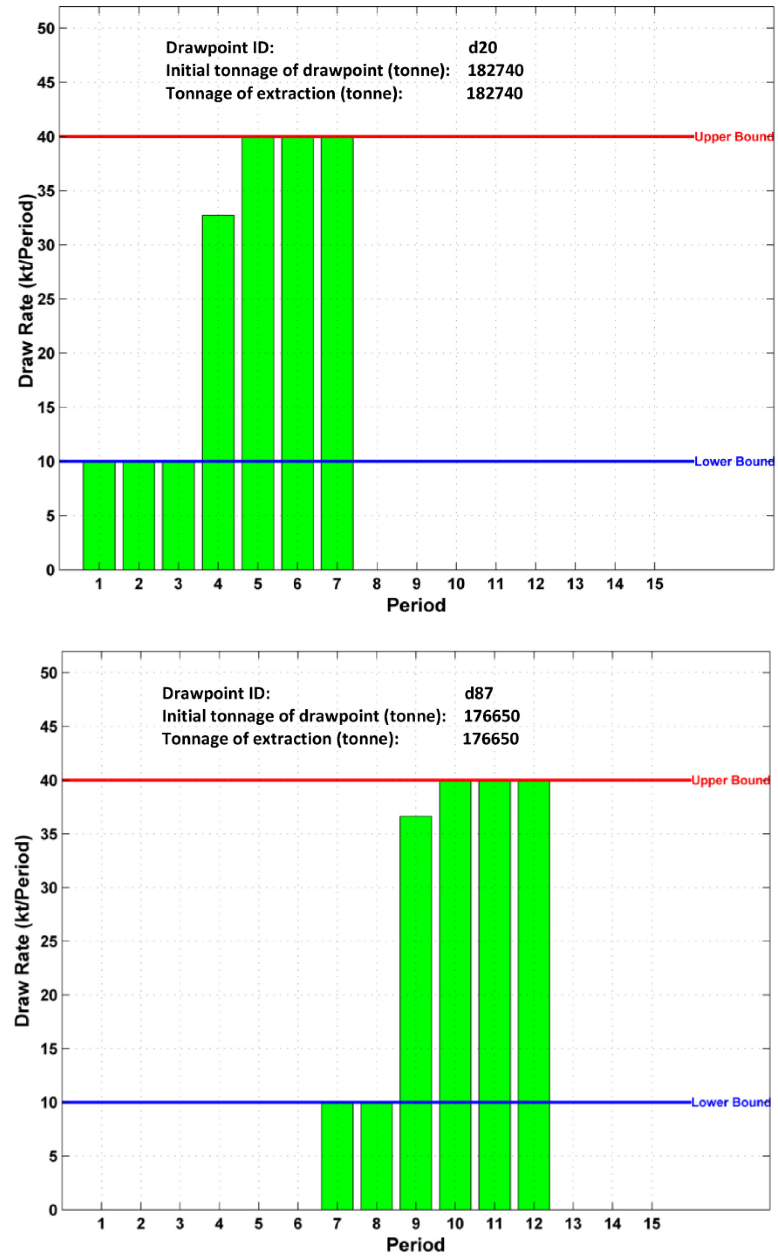

Fig 10. Draw rate of drawpoints d20 and d87 in the West to East direction (drawpoint level).

Because of clustering, the number of binary integer variables decreases from 3060 to 510. Consequently, a small number can be set as a gap between the best integer objective and the feasible integer solution for optimization. Therefore, two relative tolerances of 5% and 1% on the gap between the best integer objective and the feasible integer solution were set for optimization.

Table 2 shows the value of NPV, CPU time, and optimality gap for the formulation at the cluster level. The maximum NPV is gained from the west to east direction based on the given constraints. The best advancement direction for both formulations is alike, but the CPU time of the clustering method is much less than that of the drawpoint level method, and small values of EPGAP can be set for optimization. Table 3 shows a comparison of the execution time and NPV from the two formulations. The reduction of execution time is more than 99% in the three directions except the north to south which is 97.4%. The percent differences of the NPVs are small. These are because of different EPGAPs which were set up for the two formulations.

Table 2. Numerical results for cluster level formulation.

Direction	EPGAP = 5%			EPGAP = 1%		
	CPU time (s)	Optimality GAP (%)	NPV ($M)	CPU time (s)	Optimality GAP (%)	NPV ($M)
west to east (WE)	19	1.12	19.42	54	0.99	19.42
east to west (EW)	30	1.21	19.21	34	0.99	19.21
north to south (NS)	14	3.96	18.20	59	0.99	18.51
south to north (SN)	2	3.15	18.71	14	0.90	18.89

Table 3. Comparison of execution time and NPV obtained from the two different formulations.

Direction	CPU time (s)			NPV ($M)		
	DP level	CL level	Reduction (%)	DP level	CL level	Diff. (%)
west to east (WE)	6721	54	99.2	19.36	19.42	0.31
east to west (EW)	71675	34	99.9	19.05	19.21	0.84
north to south (NS)	2294.6	59	97.4	18.89	18.51	2.05
south to north (SN)	3323.5	14	99.6	18.98	18.89	0.48

Number of the visited nodes for drawpoint level and cluster level formulations is 46379 and 9329, respectively in which size of the branch and cut tree decreases by 98.07% by dropping from 93.01 MB for drawpoint level formulation to 1.79 MB for cluster level formulation in west to east direction.

Fig 11 compares the starting periods of drawpoints at the drawpoint and cluster levels. It is obvious that the cluster level model generates a more practical mining sequence than drawpoint level in the west to east direction. In addition, at the cluster level model, all the drawpoints have been opened until the end of the period 12, while at the drawpoint level this is happened at the end of period 13. The mine can be divided into three areas: north, middle, and south. During the first five periods, at the cluster level, the material is mined from the middle area, whereas at the drawpoint level, it is mined from the middle and north areas. Both models finish the mining in the south area.

7. Conclusions and future work

The economics of today's mining industry are such that the major mining companies are increasing the use of massive mining methods. Of the methods available, caving mines are favored because of their low cost and high production rates.

This paper investigated the development of a mixed integer linear programming (MILP) formulation for block cave production scheduling optimization. We developed, implemented, and tested two MILP formulations for block cave production scheduling in the TOMLAB/CPLEX (Holmstrom, 2011) environment. To reduce the number of continuous and binary variables in the model, we aggregated drawpoints into larger clusters using a Fuzzy c-means clustering algorithm. Both formulations maximize the NPV subject to several constraints such as vertical mining rate (production rate per drawpoint/cluster), lateral mining rate (rate of opening new drawpoints/clusters), mining capacity, and maximum number of active drawpoints or clusters.

The formulations are run on a real data set and the results and CPU times are compared for various methods. The models provide a mine planner with the flexibility of choosing directions for advancement. Both formulations also led to the same advancement direction.

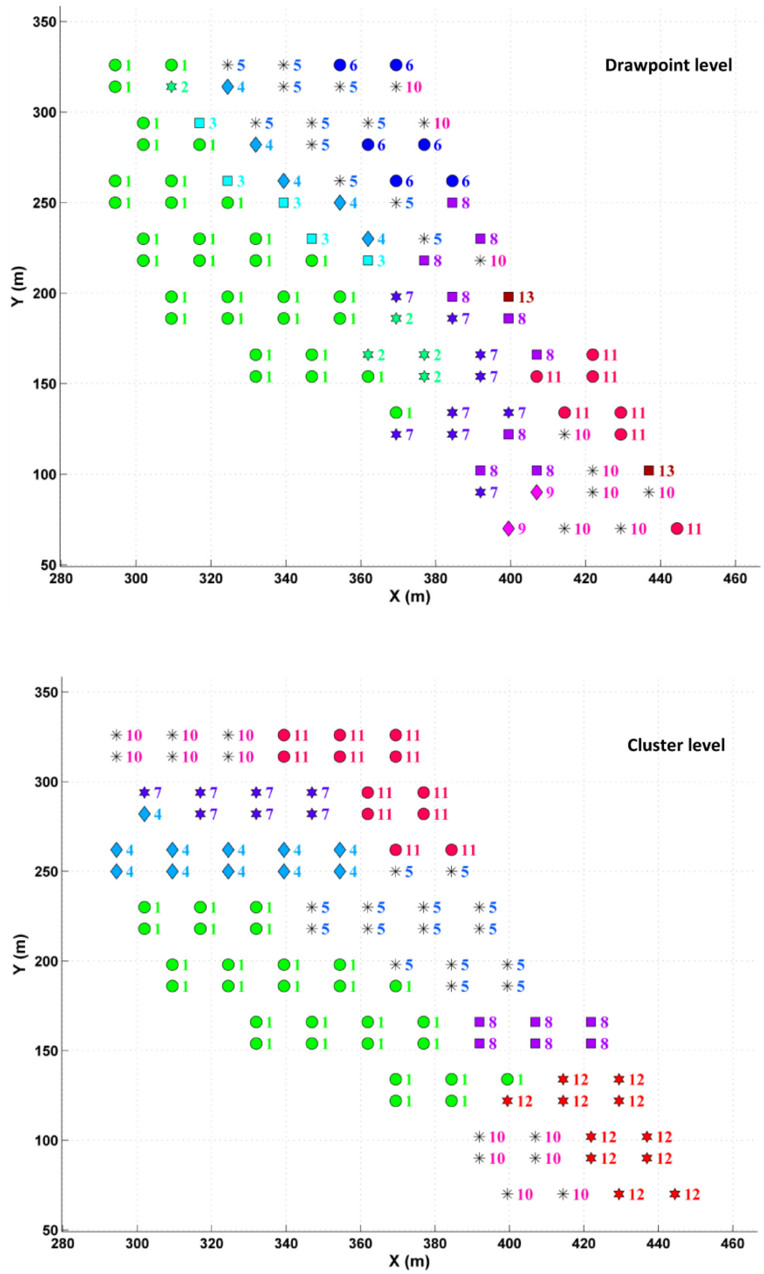

Fig 11. Comparison between starting periods of drawpoints at cluster and drawpoint levels.

The resulting NPV values were almost the same, but the CPU time of the cluster level model was much less than that of the drawpoint level method. It should be mentioned that we assume that the portion scheduled to be extracted from each cluster is taken from all the drawpoints based on the ratio of each draw column’s tonnage in the cluster.

Production scheduling optimization techniques are still not widely used in the mining industry. There is a need to improve the practicality and performance of the current production scheduling optimization tools that mining industry uses. Further focused research is underway to add new capabilities to the models. In the presented models, constant draw rates were used as upper and lower bounds. One method for managing drawpoint production is by establishing a production rate curve, which limits production based on the amount of material that has been drawn previously. This means that production depends on the cumulative tonnes mined from a drawpoint. Using this method, in the new models, a production rate curve (PRC) will be used instead of the constant upper and lower boundaries. Sometimes extraction from drawpoints is started from two or more different areas of the mine; hence we need to have a schedule which considers all mining areas at the same time. For this reason new constraints will be added to the MILP formulations for handling multiple-lift and multiple-mine scenarios. Also, a new model which uses the results of the either drawpoint level or cluster level formulation to solve the optimization based on the slices within the draw columns model has been developed and is under verification and testing. The results will be published in the future publications. In order to avoid non-practical clustering patterns, a new clustering procedure based on hierarchical clustering, in which various properties are taken into account, has been developed and will be added to the models.

8. References

[1] Askari-Nasab, H, Pourrahimian, Y, Ben-Awuah, E, & Kalantari, S. (2011). Mixed integer linear programming formulations for open pit production scheduling. *Journal of MiningScience, © Springer, New York, NY 10013-1578, United States, 47*(3), 338-359.

[2] Askari-Nasab, H., Awuah-Offei, K., & Eivazy, H. (2010). Large-scale open pit production scheduling using mixed integer linear programming. *Int. J. Min. Miner. Eng., 2*(3), 185–214.

[3] Bezdek, J.C. (1981). *Pattern Recognition with Fuzzy Objective Function Algorithms*: Kluwer Academic Publishers Norwell, MA, USA ©1981.

[4] Brazil, M., Lee, D. H., Van Leuven, M., Rubinstein, J. H., Thomas, D. A., & Wormald, N. C. (2003). Optimising declines in underground mines. *Mining Technology, 112*(3), 164-170.

[5] Brown, E.T. (2003). *Block caving geomechanics*. Brisbane: Indooroopilly, Queensland : Julius Kruttschnitt Mineral Research Centre, The University of Queensland.

[6] Busnach, E., Mehrez, A., & Sinuany-Stern, Z. (1985). A production problem in phosphate mining. *Oper. Res. Soc, 36*(4), 285–288.

[7] Chanda, E.C.K. (1990). An application of integer programming and simulation to production planning for a stratiform ore body. *Mining Science and Technology, 11*(2), 165-172.

[8] Diering, T. (2000). *PC-BC: A block cave design and draw control system.* Paper presented at the MassMin 2000, Brisbane.

[9] Diering, T. (2004). *Computational considerations for production scheduling of block cave mines.* Paper presented at the MassMin 2004, Santiago, Chile.

[10] GemcomSoftwareInternational. (2011) (Version 6.2.4). Vancouver, BC, CANADA.

[11] Guest, A., VanHout, G.J., Von, J. A., & Scheepers, L.F. (2000). *An application of linear programming for block cave draw control.* Paper presented at the Massmin2000, Brisbane.

[12] Heslop, T.G., & Laubscher, D.H. (1981). Draw control in caving operations on Southern African Chrpsotile Asbestos mines *Design and operation of caving and sublevel stoping mines* (pp. 755-774). New York: Society of Mining Engineers of AIME.

[13] Holmstrom, K. (2011). TOMLAB/CPLEX, ver. 11.2. Pullman, WA, USA: Tomlab Optimization.

[14] Huang, Y., & Kumar, U. (1994). Optimizing the number of load-haul-dump machines in a Swedish mine by using Queuing theory- A case study. *International Journal of Surface Mining, Reclamation and Environment, 8*(3), 171-174.

[15] ILOGInc. (2007). ILOG CPLEX 11.0 User's Manual (Version 11.0). ILOG S.A. and ILOG, Inc.

[16] Kuchta, M., Newman, A., & Topal, E. (2004). Implementing a production schedule at LKAB 's Kiruna Mine. *Interfaces, 34*(2), 124-134.

[17] MathWorksInc. (2011). MATLAB (R2011b) (Version 7.13.0.564). MathWorks, Inc.

[18] Muge, F. H., Santos, N., Vierira, J. L., & Cortez, L. (1992). *Dynamic programming in mine planning and production scheduling.* Paper presented at the 23rd Application of Computers and Operations Research in the Mineral Industry, Littleton, Colorado, USA.

[19] Newman, A.M., Rubio, E., Caro, R., Weintraub, A., & Eurek, K. (2010). A review of operations research in mine planning. *Interfaces, 40*, 222-245.

[20] Rahal, D. (2008). *The use of mixed integer linear programming for long-term scheduling in block caving mines.* The University of Queensland, Brisbane, Queensland, Australia.

[21] Riddle, J. (1977). *A dynamic programming solution of a block caving mine layout.* Paper presented at the 14th International Symposium on the Application of Computers in the Mineral Industry(APCOM), Pennsylvania.

[22] Rubio, E. (2002). *Long-term planning of block caving operations using mathematical programming tools.* Unpublished MSc, University of British Columbia, Vancouver, Canada.

[23] Rubio, E. (2006). *Block cave mine infrastructure reliability applied to production planning.* Unpublished PhD, University of British Columbia, Vancouver.

[24] Rubio, E., & Diering, T. (2004). *Block cave production planning using operations research tools.* Paper presented at the MassMin 2004, Santiago, Chile.

[25] Song, X. (1989). *Caving process simulation and optimal mining sequence at Tong Kuang Yu mine, China.* Paper presented at the 21st Application of Computers and Operations Research in the Mineral Industry, Las Vegas, NV, USA.

[26] Su, Y. L. (1986). *A computer solution to the Queuing problem at a mine-mill interface.* Paper presented at the 19th Application of Computers and Operations Research in the Mineral Industry, Pennsylvania State University, USA.

[27] Tabesh, M., & Askari-Nasab, H. (2011). Two-stage clustering algorithm for block aggregation in open pit mines. *Mining Technology, 120*(3), 158-169.

[28] Topal, E. (2008). Early start and late start algorithms to improve the solution time for long-term underground mine production scheduling. *Journal of The South African Institute of Mining and Metallurgy, 108*(2), 99-107.

[29] Weintraub, Andres, Pereira, Marianela, & Schultz, Ximena. (2008). A priori and a posteriori aggregation procedures to reduce model size in MIP mine planning models. [doi: 10.1016/j.endm.2008.01.051]. *Electronic Notes in Discrete Mathematics, 30*(0), 297-302.

[30] Winkler, B.M. (1998). *Mine production scheduling using linear programming and virtual reality.* Paper presented at the 27th International Symposium, Application of Computers in the Mineral Industry(APCOM), Royal School of Mines, London, United Kingdom.

[31] Zhang, M. (2006). *Combining genetic algorithms and topological sort to optimise open-pit mine plans.* Paper presented at the 15th Int. Symp. on Mine Planning and Equipment Selection (MPES), Torino, Italy.