In-Pit and External Oil Sands Dyke Construction Scheduling using Goal Programming

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Abstract

Historical analysis of mineral resource evaluations has demonstrated the sensitivity of project's profitability to decisions based on long-term mine production planning. In oil sands mining, providing processable ore and tailings containment with less environmental footprints at the right time are the main drivers for profitability and sustainability. Recent environmental and regulatory requirements makes waste management an integral part of mine planning in the oil sands industry (Directive 074). This requires the development of a well integrated strategy of directional mining and tailings dyke construction for in-pit and ex-pit tailings storage management systems. The objective of this paper is to: 1) determine the order and time of extraction of ore, dyke material and waste to be removed from a predefined final pit limit over the mine life that maximizes the net present value of the operation; and 2) determine the destination of dyke material that minimizes construction cost depending on the construction requirements of the various dykes as per their designs. We have developed, implemented, and verified a theoretical optimization framework based on mixed integer linear goal programming (MILGP) to address this objective. The research introduced a MILGP mine planning model for multiple material types and destinations. This study also presents an integration of mixed integer linear programming and goal programming in solving large scale mine planning optimization problems using clustering and pushback techniques. Application of the MILGP model was presented with an oil sands mining case. The MILGP model generated a smooth and uniform mining schedule that generates value and provides a robust framework for effective waste disposal planning.

1. Introduction

Open-pit mining involves the process of extracting blocks of earth from the surface to retrieve the ore contained in them. This mining process causes the surface of the land to be continuously excavated causing an increasingly deeper pit to be formed until the end of the mine life (Hochbaum and Chen, 2000; Newman et al., 2010). Prior to the mining operation, the complex strategy of displacement of ore, waste, overburden, and tailings over the mine life need to be decided and this is known as mine planning. Open-pit mine planning can be defined as the process of finding a feasible block extraction sequence that generates the highest net present value (NPV) subject to operational and technical constraints (Whittle, 1989). Mine planning is done for different time horizons and these include short-term, medium-term, and long-term production scheduling plans. This paper focuses on the long-term production scheduling optimization process which is the backbone of the entire mining operation. In mining projects, deviations from optimal mine plans will result in significant financial losses, future financial liabilities, delayed reclamation, and resource sterilization.

The objective of this study is to develop a theoretical framework that maximizes the NPV of an oil sands mining operation and minimizes dyke construction cost for tailings containment using a mixed integer linear goal programming (MILGP) model. The MILGP model incorporates multiple material types with multiple elements for multiple destinations in long-term production scheduling. Though operation research methods have been applied in mine production scheduling, very little work has been done in terms of oil sands mine planning which has a unique scenario in terms of waste management. Oil sands mining profitability depends on a carefully planned and integrated mine planning and waste management strategy that generates value and sustainability by maximizing NPV and creating timely tailings storage areas with less environmental footprints. Recent mining regulations by Alberta Energy Resources and Conservation Board (Directive 074) (McFadyen, 2008) requires that oil sands mining companies develop an integrated mine planning and waste management strategy for their in-pit and external tailings facilities. This requires a new and more systematic approach in looking at the planning of oil sands mining operations.

The next section of this paper presents the problem definition and section 3 is on our conceptual mining model. Section 4 covers a literature review on goal programming (GP), mixed integer programming (MIP) and mixed integer linear programming (MILP). The application of MILGP to the long-term production planning (LTPP) problem is formulated in section 5. The formulation is applied to an oil sands mine planning and waste management case with an example and the results discussed in sections 6 and 7 respectively. Section 8 outlines the conclusions and future research direction.

2. Problem definition

Mine management is always faced with the problem of achieving multiple goals with the available limited resources. In oil sands mining, due to the limitation of lease area, the pit phase advancement is carried out simultaneously with the construction of tailings dykes in the mined out areas of the pit and designated areas outside the pit. These dykes are constructed to hold tailings that are produced during the processing of the oil sands. Dykes with different configurations are required during the construction. Most of the materials used in constructing these dykes come from the oil sands mining operation. The dyke materials are comprised of overburden and interburden (OI) dyke material and tailings coarse sand (TCS) dyke material. It is assumed that the material sent to the processing plant (ore) must have a specified amount of bitumen and percentage fines as well as the material sent for dyke construction (dyke material). Any other material that does not meet the requirements of ore or dyke material is sent to the waste dump.

The main problem here has been categorized in two parts: 1) determining the order and time of extraction of ore, dyke material and waste to be removed from a predefined ultimate pit limit over the mine life that maximizes the net present value of the operation; 2) determining the destination of dyke material that minimizes construction cost depending on the construction requirements of the various dykes as per their designs.

Fig. 1 illustrates the scheduling of an oil sands ultimate pit block model containing K mining-cuts. Mining-cuts are clusters of blocks within the same level or mining bench that are grouped based on the attributes; location, rocktype and grade distribution (Askari-Nasab and Awuah-Offei, 2009; Ben-Awuah and Askari-Nasab, 2011). Each mining-cut k, is made up of ore o_k, OI dyke material d_k, and waste w_k. The material in each mining-cut is to be scheduled over T periods depending on the goals and constraints associated with the mining operation. The OI dyke material scheduled, d_k^T and the TCS dyke material from the processed ore, l_k^T must further be assigned to the dyke construction sites based on the construction requirements. For period t_1, the dyke construction material required by site 1 is $dyke_1$, the dyke construction material required by site 2 is $dyke_2$ and the dyke construction material required by site i is $dyke_i$.

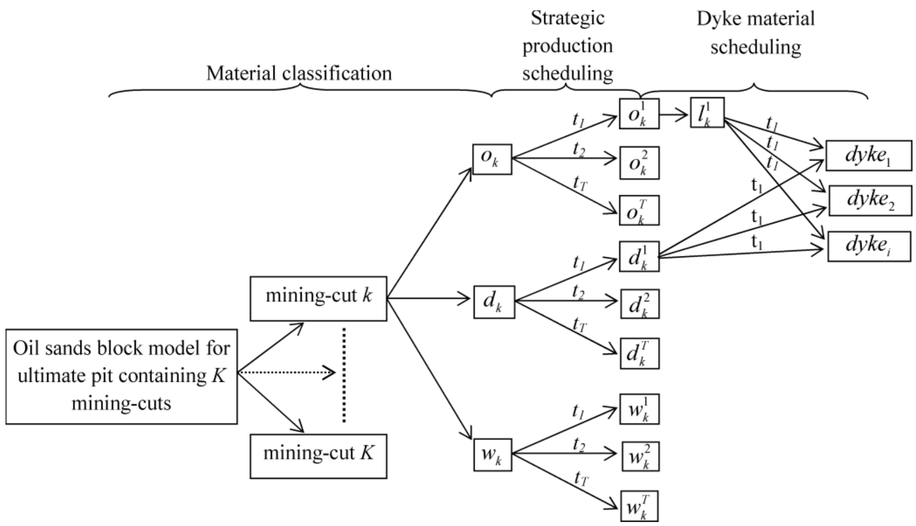

Fig. 1. Schematic representation of the problem definition showing strategic and dyke material production scheduling modified after Ben-Awuah and Askari-Nasab (2011)

The strategic and dyke material production schedules to be developed are subject to a variety of economic, technical, and physical constraints. The constraints control the mining extraction sequence, ore and dyke material blending requirements and mining, processing, and dyke material goals. The mining, processing, and dyke material goals specify the quantities of material allowed for the mining operation, processing plant, and dyke construction respectively.

The strategic and dyke material production schedules are the main drivers for the profitability of oil sands mining operation. The schedules control the NPV of the operation and enable a robust waste management planning strategy. Improper waste management planning can lead to environmental issues causing immediate mine closure by regulatory agencies and major financial liabilities.

3. Conceptual mining model

The key drivers for oil sands mine planning are the provision of a processable blend of ore at the required grade and the provision of tailings containment at the right time. A conceptual mining model that is consistent with practical oil sands mining and waste management was set up to illustrate how the MILGP model can be used to generate a strategic and dyke material production schedules. As shown in Fig. 2, the mining model is made up of an oil sands deposit area which is to be mined and simultaneously used as an in-pit tailings storage area as mining progresses in a specified direction and the in-pit tailings dyke footprints are released. Each oil sands mining-cut is made up of ore, OI dyke material and waste. After processing the ore to extract bitumen, two main types of tailings are produced; fine and coarse tailings. The coarse tailings also referred to as TCS dyke material and OI dyke material are used in the construction of dykes for tailings facilities. The fine tailings form the slurry which needs to be contained in the tailings facilities.

3.1. Tailings storage management strategy

Each tonne of ore is made up of bitumen, fines, sand, and water. Using the oil sands extraction process volume changes on the path from ore to waste as outlined in a report for Alberta Energy Research Institute (Devenny, 2009), the volume of tailings to be produced can be calculated and an appropriate storage management strategy planned. In the conceptual mining model, using the tailings storage volume required and the total in-pit tailings facilities volume available, an external tailings facility (ETF) volume required to support the mining operation can be calculated.

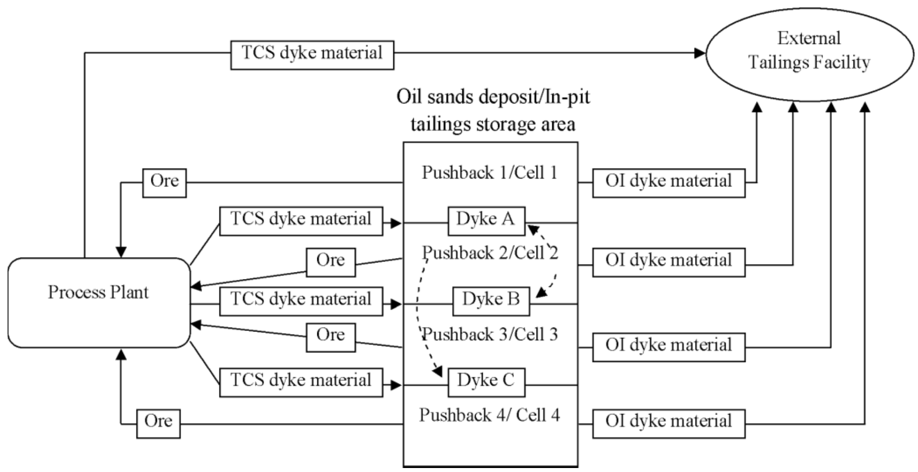

Fig. 2. Conceptual mining model showing mining and waste management strategy (Askari-Nasab and Ben-Awuah, 2011)

The oil sands deposit area was divided into pushbacks which coincide with the areas required by tailings dam engineers to set up in-pit tailings facility cells. In the case of our illustrative example in Fig. 2, the deposit covers an area of 8 km x 4 km with an average height of 75 m. Based on literature on oil sands mining operations with regards to standard sizes of ex-pit and in-pit tailings facility cells (Fort Hills Energy Corporation, 2009; Jackpine Mine, 2009; Kearl Oil Sands Project, 2009; Muskeg River Mine, 2009; Suncor Energy Incorporated Oil Sands, 2009; Syncrude Aurora North, 2009; Syncrude Aurora South, 2009; Syncrude Mildred Lake, 2009), it was decided to divide the mining area into four pushbacks which will result in four in-pit cells as shown in Fig. 2. Each cell will have approximate dimensions of 2 km x 4 km x 75 m except cells 1 and 4. The mining operation will stay ahead of dyke construction by about 100 m resulting in cell 1 having a size of 1.9 km x 4 km x 75 m and cell 4 having a size of 2.1 km x 4 km x 75 m. It is assumed that mining will start in pushback 1 and progress in a north-south direction. During the mining of pushback 1, all IO and TCS dyke material will be sent to the ETF for the construction of the ETF dyke. Fluid fine tailings produced from pushback 1 will be sent to the ETF after the key trench and starter dyke construction is completed. Once mining of pushback 1 is completed, the dyke ‘A’ footprint required to construct cell 1 becomes available. OI and TCS dyke material from pushback 2 will be used for the construction of dyke ‘A’ to enable in-pit tailings storage to start in cell 1.

As mining progresses to pushbacks 3 and 4, the OI and TCS dyke material produced can be used to construct dykes ‘B’ and ‘C’ to make available cells 2 and 3 respectively for tailings storage. Any excess OI and TCS dyke material can be used for other purposes like shelling dumps, road construction, sand capping, and fines trapping as in non-segregating tailings. It is assumed that cell 4 will not be available for tailings storage until the end of the mine life; therefore it was not used for the volume balance calculations in the tailings storage management strategy. Table 1 shows estimates from the balancing of tailings storage requirements for the conceptual mining model. From the in-pit cell volumes generated for cells 1, 2, and 3, the required capacity of the ETF can be calculated and designed. The ETF was designed to cover an area of 16 Mm^2 with a height of 60 m resulting in a 13% excess containment capacity. The freeboard used for the designs is 5 m.

This tailings storage management strategy is based on the assumption that, all the available ore will be mined and processed. After the optimization of the production schedule, the actual mined ore tonnes can be used to reassess the tailings storage management strategy and appropriate modifications made. Further analysis of the conceptual mining model was done by starting the mining operation in pushback 4 and progressing in a south-north direction.

Table 1. Estimates for tailings storage requirements for the conceptual mining model

Material type	Oil sands deposit (Mtonnes)	Available dyke material (Mm^3)	Tailings/Waste produced (Mm^3)	Cells/ETF designed capacity (Mm^3)
Ore	2792.5	-	2251.1	Cell 1: 532
OI dyke material	1697.8	797.6	-	Cell 2: 560
TCS dyke material	2110.0	975.0	-	Cell 3: 560
Waste	375.9	-	179.0	ETF: 880

3.2. Conceptual dykes' designs

Simplified conceptual dykes' designs were made for all the dykes and used as the basis for OI and TCS dyke material scheduling in all pushbacks. It was assumed that each dyke is made up of a key trench, a starter dyke and the main dyke as shown in Fig. 3. The key trench and starter dyke will be constructed using OI dyke material and the main dyke will be constructed using TCS dyke material. Once construction of the key trench and starter dyke is complete, the tailings facility can be used whiles construction of the main dyke progresses. In line with the geology of the McMurray formation, it was assumed that the ETF dyke will be constructed possibly on a weak foundation and the in-pit cell dykes will be constructed on a good foundation, thus requiring different side slopes. Table 2 shows the designed material requirements for the main dyke, starter dyke, and key trench at various destinations. The estimates are the minimum material required at the various destinations for dyke construction and any excess material can be used for other purposes.

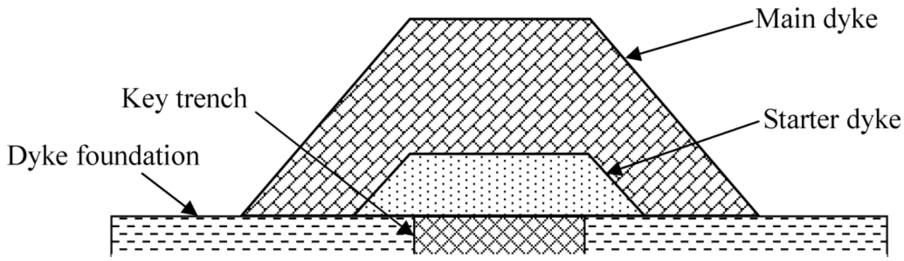

Fig. 3. Schematic diagram showing cross section of a dyke (Askari-Nasab and Ben-Awuah, 2011)

Table 2. Material requirements for dykes at different locations

Dyke location	OI and TCS dyke material required (Mm^3)		
	Key trench	Starter dyke	Main dyke
ETF dyke	1.96	20.58	507.63
Dykes A+B+C	1.38	10.80	304.95

4. Literature review

Mining is the process of extracting a beneficial natural resource from the earth (Newman et al., 2010) and historical analysis of mineral resource evaluations has demonstrated the sensitivity of project's profitability to decisions based on long-term mine production schedules. LTPP problems have been a major research area for some time now and though major improvements have been made, the current dynamic mining environment brings about new and complex problems. Effective LTPP can increase the profitability and life of mine considerably.

Using mathematical programming models with exact optimization methods to solve the LTPP problem have proved to be robust. Mathematical programming models including GP, LP, MIP, and MILP have the capability of considering multiple material types, elements, and destinations. Solving them with exact optimization methods result in solutions with known extent of optimality. As the solution gets closer to optimality, it leads to production schedules that generate higher NPV than those obtained from heuristic optimization methods. GP allows for flexible formulation, specification of priorities among goals, and some level of interaction between the decision maker and the optimization process (Zeleny, 1980; Hannan, 1985). Zhang et al. (1993) applied GP to a production scheduling problem using multiple criteria decision-making formulation. This formulation was developed for one ore type process and multiple goals were considered based on their priorities. The model was implemented for a surface coal mine production schedule. A 0-1 non-linear GP model was used by Esfandiri et al. (2004) in defining a mineral dressing criterion for an iron ore mine. The GP model was defined based on multiple criteria decision making and the deviations from the goals for economics, mining, and mineral dressing functions were minimized. The model was found to have limitations and constraints that were numerous for practical applications. The scheduling of multiple maintenance projects for a mineral processing equipment at a copper mine was developed by Chen (1994) using a 0-1 GP model. Chen used 0-1 binary integer decision variables and multiple scheduling periods, to schedule 4 projects, 40 jobs and 9 resource types. Comparing the results to a heuristic method that was used by the mine, the GP model reduced the total project cost, project duration, and overall workload. Some mine production related problems have been tackled using modified forms of GP. Chanda and Dagdelen (1995) used GP and an interactive graphics system for optimal blending in a coal mine production. A fuzzy GP model was developed by Oraee and Asi (2004) for optimizing a haulage system in an open pit mine. Other industrial production planning and project selection decision-making problems that have been solved making use of the advantages of GP formulations includes the works of Jääskeläinen (1969), Mukherjee and Bera (1995), Leung et al. (2003), and Lee et al. (2010). These GP applications can be considered as small-scale optimization problems in comparison with mine production scheduling optimization problems which involve a large number of decision variables and constraints.

Ramazan and Dimitrakopoulos (2004) developed MIP formulations where they attempt to decrease the number of binary variables and solution times by setting some variables as binary and others as continuous. The results showed partial mining of blocks with the same ore value, thus affecting the NPV generated. LP and MIP models that were subsequently developed by Akaike and Dagdelen (1999), Caccetta and Hill (2003), Ramazan et al. (2005), Ramazan (2007), and Boland et al. (2009) were either not able to generate a global optimum solution for large-scale LTPP problems or there were not enough information to assess the practicality of the generated schedules from mining operation point of view. Recent applications of MILP models to the LTPP problem by Askari-Nasab et al. (2010) has lead to the development of models that use block clustering techniques to reduce the number of decision variables. The formulation was implemented for an iron ore mine case study where long-term production scheduling was done for 21 periods. This model does not consider multiple material types or destinations.

In summary, these GP, LP, MIP, and MILP applications lack the framework that can be used in solving the oil sands mine production planning and waste management problem. They are limited to either single ore, element, or destination, small-scale optimization problems or no consideration for directional mining, and integration of mine production and waste disposal planning. Some efforts have been made to combine GP, MIP, and MILP models to solve some industrial problems because of the advantages of such hybrids. This model referred to as MILGP, has been successfully applied to scheduling and budgeting problems in nursing, business administration, and manufacturing industries (Selen and Hott, 1986; Ferland et al., 2001; Liang and Lawrence, 2007; Nja and Udofia, 2009). The application of MILGP to the oil sands mine production planning and waste management problem as outlined in this paper has been setup in an optimization framework that integrates multiple material types, elements, and destinations. It includes large-scale optimization, directional mining, and integration of mine production planning and waste management. The practical implementation of the MILGP model and the generated production schedules are also discussed.

5. MILGP model for open pit production scheduling

The long-term mine production scheduling problem will be formulated using a combination of mixed integer and goal programming. Using goal programming is appropriate in this context because the structure enables the optimization solution to try achieving a set of goals where some goals can be traded off against one another depending on their priority. Hard constraints can also be converted to soft constraints which otherwise could lead to infeasible solutions. In simple terms, goal programming allows for flexible formulation and the specification of priorities among goals (Liang and Lawrence, 2007). The formulated model for the strategic production and dyke material scheduling problem has an objective function, goal functions and constraints. The goal objectives are mining, processing and dyke construction (Ferland et al., 2001; Esfandiri et al., 2004; Liang and Lawrence, 2007).

5.1. Notations

The notations used in the formulation of the oil sands strategic and dyke material production scheduling problem has been classified as sets, indices, subscripts, superscripts, parameters, and decision variables. Details of these notations can be found in Appendix 1. In general, the MILGP formulation is for multiple material types and destinations as well as pushbacks which ties into the waste management strategy. The MILGP formulation framework was developed based on mining-cuts. This MILGP model is an extension of the oil sands mine planning formulation by Ben-Awuah and Askari-Nasab (2011).

5.2. Modeling of economic mining-cut value

The objective function of the MILGP model for LTPP is to maximize the net present value of the mining operation and minimize the dyke construction cost and deviations from the mining goal, processing goal, OI dyke material goal, and TCS dyke material goal for all destinations. The concept of economic mining-cut value is based on ore parcels within mining-cuts which could be mined selectively. The profit from mining a mining-cut is a function of the value of the mining-cut based on the processing destination and the costs incurred in mining, processing, and dyke construction at a specified destination. The cost of dyke construction is also a function of the location of the tailings facility being constructed and the type and quantity of dyke material needed. The discounted profit from mining-cut *k* is equal to the discounted revenue obtained by selling the final product contained in mining-cut *k* minus the discounted cost involved in mining mining-cut *k* as waste minus the extra discounted cost of mining OI dyke material minus the extra discounted cost of mining TCS dyke material. This has been simplified into Eqs. (1) to (5).

$$d_k^{u,t} = v_k^{u,t} - q_k^{u,t} - p_k^{u,t} - h_k^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\},\ k \in \{1,...,K\} \tag{1}$$

Where:

$$v_k^{u,t} = \sum_{e=1}^{E} o_k \times g_k^e \times r^{u,e} \times \left(p^{e,t} - cs^{e,t}\right) - \sum_{e=1}^{E} o_k \times cp^{u,e,t} \quad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{2}$$

$$q_k^{u,t} = \left(o_k + d_k + w_k\right) \times cm^{u,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{3}$$

$$p_k^{u,t} = d_k \times ck^{u,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{4}$$

$$h_k^{u,t} = l_k \times ct^{u,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{5}$$

5.3. The MILGP model

Using multiple criteria decision making analysis, the objective functions of the MILGP model for strategic and dyke material LTPP as applied in oil sands mining, can be formulated as: i) maximizing the NPV, ii) minimizing the dyke construction cost, and iii) minimizing deviations from the goals. These are represented by Eqs. (6), (7), and (8) respectively.

$$\sum_{j=1}^{J}\left(Max\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{k\in B_j}\left(v_k^{u,t}\times x_k^{u,t}-q_k^{u,t}\times y_k^{u,t}\right)\right)\right) \tag{6}$$

$$\sum_{j=1}^{J}\left(Min\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{k\in B_j}\left(p_k^{u,t}\times z_k^{u,t}+h_k^{u,t}\times s_k^{u,t}\right)\right)\right) \tag{7}$$

$$\sum_{j=1}^{J}\left(Min\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{k\in B_j}\begin{bmatrix}P_1\left(a_1d_1^{-,u,t}\right)+P_2\left(a_2d_2^{-,u,t}\right)+\\ P_3\left(a_3d_3^{-,u,t}+a_3d_3^{+,u,t}\right)+P_4\left(a_4d_4^{-,u,t}+a_4d_4^{+,u,t}\right)\end{bmatrix}\right)\right) \tag{8}$$

Eqs. (6) to (8) can be combined as a single objective function formulated as in Eq. (9).

$$\sum_{j=1}^{J}\left(Max\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{k\in B_j}\begin{bmatrix}\left(v_k^{u,t}\times x_k^{u,t}-q_k^{u,t}\times y_k^{u,t}\right)-\left(p_k^{u,t}\times z_k^{u,t}+h_k^{u,t}\times s_k^{u,t}\right)-\\ \begin{pmatrix}P_1\left(a_1d_1^{-,u,t}\right)+P_2\left(a_2d_2^{-,u,t}\right)+\\ P_3\left(a_3d_3^{-,u,t}+a_3d_3^{+,u,t}\right)+P_4\left(a_4d_4^{-,u,t}+a_4d_4^{+,u,t}\right)\end{pmatrix}\end{bmatrix}\right)\right) \tag{9}$$

The complete MILGP model comprising of the objective function, goal functions and constraints can be formulated as;

Objective function:

$$\sum_{j=1}^{J}\left(Max\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{k\in B_j}\begin{bmatrix}\left(v_k^{u,t}\times x_k^{u,t}-q_k^{u,t}\times y_k^{u,t}\right)-\left(p_k^{u,t}\times z_k^{u,t}+h_k^{u,t}\times s_k^{u,t}\right)-\\ \begin{pmatrix}P_1\left(a_1d_1^{-,u,t}\right)+P_2\left(a_2d_2^{-,u,t}\right)+\\ P_3\left(a_3d_3^{-,u,t}+a_3d_3^{+,u,t}\right)+P_4\left(a_4d_4^{-,u,t}+a_4d_4^{+,u,t}\right)\end{pmatrix}\end{bmatrix}\right)\right) \tag{10}$$

Goal functions:

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(o_k+w_k+d_k\right)\times y_k^{u,t}\right)+d_1^{-,u,t}=T_m^{u,t}\qquad \forall t\in\{1,...,T\},\ u\in\{1,...,U\} \tag{11}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(o_k\times x_k^{u,t}\right)\right)+d_2^{-,u,t}=T_p^{u,t}\qquad \forall t\in\{1,...,T\},\ u\in\{1,...,U\} \tag{12}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(d_k\times z_k^{u,t}\right)\right)+d_3^{-,u,t}-d_3^{+,u,t}=T_d^{u,t}\qquad \forall t\in\{1,...,T\},\ u\in\{1,...,U\} \tag{13}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(l_k\times s_k^{u,t}\right)\right)+d_4^{-,u,t}-d_4^{+,u,t}=T_l^{u,t}\qquad \forall t\in\{1,...,T\},\ u\in\{1,...,U\} \tag{14}$$

Constraints:

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} g_k^e \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \le \overline{g}^{u,t,e} \qquad \forall t\in\{1,..,T\},\, u\in\{1,..,U\}, e\in\{1,..,E\} \tag{15}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} g_k^e \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \ge \underline{g}^{u,t,e} \qquad \forall t\in\{1,..,T\},\, u\in\{1,..,U\}, e\in\{1,..,E\} \tag{16}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^e \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \le \overline{f}^{u,t,e} \qquad \forall t\in\{1,..,T\}, u\in\{1,..,U\}, e\in\{1,..,E\} \tag{17}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^e \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \ge \underline{f}^{u,t,e} \qquad \forall t\in\{1,..,T\}, u\in\{1,..,U\}, e\in\{1,..,E\} \tag{18}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^d \times d_k \times z_k^{u,t} \Big/ \sum_{k\in B_j} d_k \times z_k^{u,t}\right) \le \overline{f}^{u,t,d} \qquad \forall t\in\{1,..,T\}, u\in\{1,..,U\}, e\in\{1,..,E\} \tag{19}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^d \times d_k \times z_k^{u,t} \Big/ \sum_{k\in B_j} d_k \times z_k^{u,t}\right) \ge \underline{f}^{u,t,d} \qquad \forall t\in\{1,..,T\}, u\in\{1,..,U\}, e\in\{1,..,E\} \tag{20}$$

$$\sum_{u=1}^{U}\left(x_k^{u,t} + z_k^{u,t}\right) \le \sum_{u=1}^{U} y_k^{u,t} \qquad \forall t\in\{1,..,T\},\, k\in\{1,..,K\} \tag{21}$$

$$\sum_{u=1}^{U} s_k^{u,t} \le \sum_{u=1}^{U} x_k^{u,t} \qquad \forall t\in\{1,..,T\},\, k\in\{1,..,K\} \tag{22}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} x_k^{u,t} \le 1 \qquad \forall k\in\{1,..,K\} \tag{23}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} z_k^{u,t} \le 1 \qquad \forall k\in\{1,..,K\} \tag{24}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} s_k^{u,t} \le 1 \qquad \forall k\in\{1,..,K\} \tag{25}$$

$$b_k^t - \sum_{u=1}^{U}\sum_{i=1}^{t} y_s^{u,i} \le 0 \qquad \forall t\in\{1,...,T\},\, k\in\{1,...,K\},\, s\in C_k(L) \tag{26}$$

$$b_k^t - \sum_{u=1}^{U}\sum_{i=1}^{t} y_r^{u,i} \le 0 \qquad \forall t\in\{1,...,T\},\, k\in\{1,...,K\},\, r\in M_k(P) \tag{27}$$

$$\sum_{u=1}^{U}\sum_{i=1}^{t} y_k^{u,i} - b_k^t \le 0 \qquad \forall t\in\{1,...,T\},\, k\in\{1,...,K\} \tag{28}$$

$$b_k^t - b_k^{t+1} \le 0 \qquad \forall t\in\{1,...,T-1\},\quad k\in\{1,...,K\} \tag{29}$$

$$c_j^t - \sum_{u=1}^{U}\sum_{i=1}^{t} y_h^{u,i} \le 0 \qquad \forall t \in \{1,...,T\},\ j \in \{1,...,J\},\ h \in B_j(H) \tag{30}$$

$$\sum_{u=1}^{U}\sum_{i=1}^{t} y_h^{u,i} - c_j^t \le 0 \qquad \forall t \in \{1,...,T\},\ j \in \{1,...,J\},\ h \in B_{j+1}(H) \tag{31}$$

$$c_j^t - c_j^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\},\ j \in \{1,...,J\} \tag{32}$$

$$d_1^{-,t},\ d_2^{-,u,t},\ d_3^{-,u,t},\ d_3^{+,u,t},\ d_4^{-,u,t},\ d_4^{+,u,t} \ge 0 \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{33}$$

$$P_1 > P_2 > P_3 > P_4 \tag{34}$$

Eq. (10) is the objective function of the formulation which seeks to i) maximize the NPV, ii) minimize the dyke construction cost, and iii) minimize deviations from the goals. Eqs. (11), (12), (13), and (14) are the goal functions which define the mining, processing, OI dyke material, and TCS dyke material goals that are required for all destinations. Eqs. (15) to (20) specify the limiting grade requirements for ore bitumen, ore fines, and OI dyke material fines for all destinations. Eq. (21) ensures that the total material mined in each period for all destinations does not exceed the sum of the ore and OI dyke material mined. Eq. (22) states that the fraction of TCS dyke material mined in each period should be less or equal to the fraction of ore material mined for all destinations. Eqs. (23), (24), and (25) ensures that the total fractions of mining-cut k sent to all destinations in all periods is less or equal to one. Eqs. (26), (27), (28), and (29) check the set of immediate predecessor mining-cuts that must be mined prior to mining mining-cut k for all periods and destinations. These equations control the vertical and horizontal block extraction sequence. They ensure that mining proceeds in the specified mining direction as the mine goes deeper. Eqs. (30), (31), and (32) check the set of immediate predecessor pit phase that must be mined prior to mining phase j in all periods for all destinations. Eq. (33) ensures that the negative and positive deviations from the targeted mining, processing, OI dyke material, and TCS dyke material goals are always positive for all periods and destinations. Eq. (34) states the order of prioritization associated with achieving the goals. The model assumes that there exists a pre-emptive priority structure among the goals and this can be changed depending on the mining operation and aim of optimization.

Using mathematical models like the MILGP formulation for mine optimization usually results in large-scale optimization problems. A commercial optimization solver capable of handling such problems is ILOG CPLEX (ILOG Inc., 2007). This optimization solver uses branch and cut algorithm and makes the solving of large-scale problems possible for the MILGP model. Branch and cut is a method of combinatorial optimization for solving integer programming problems. This algorithm is a hybrid of branch-and-bound and cutting plane methods (Horst and Hoang, 1996; Wolsey, 1998).

The MILGP model solver in this research is TOMLAB/CPLEX (Holmström, 2009). The user sets an optimization termination criterion in CPLEX known as the gap tolerance (EPGAP). The EPGAP which is a measure of optimality sets an absolute tolerance on the gap between the best integer objective and the objective of the best node remaining in the branch and cut algorithm. It instructs CPLEX to terminate once a feasible integer solution which is within the set EPGAP has been found (ILOG Inc., 2007).

6. Implementing the MILGP model for production scheduling and waste disposal planning

The MILGP model for open pit strategic and dyke material production scheduling has the objective of maximizing the NPV of the mining operation, minimizing the dyke construction cost for the tailings management plan, and minimizing the deviations from the set goals. The goals are the mining, processing, OI dyke material, and TCS dyke material targets in tonnes. The size of the mining-cuts used for production scheduling must be carefully selected to ensure that it is comparable to the selective mining units of the operation in practice. The proposed MILGP model uses continuous decision variables, $y_k^{u,t}$, $x_k^{u,t}$, $z_k^{u,t}$, and $s_k^{u,t}$ to model mining, processing, OI dyke material and TCS dyke material requirements respectively for all destinations. Binary integer decision variables, b_k^t and c_j^t are used to control precedence of mining-cuts and pushback extraction. Continuous deviational variables, $d_1^{-,t}$, $d_2^{-,u,t}$, $d_3^{-,u,t}$, $d_3^{+,u,t}$, $d_4^{-,u,t}$, and $d_4^{+,u,t}$ have been defined to support the goal functions that control mining, processing, OI and TCS dyke material for all destinations. The deviational variables make available a continuous range of units (tonnes) that the optimizer chooses from to satisfy the set goals and these deviational variables are minimized in the objective function. The objective function also contains deviational penalty cost and priority parameters which are important aspects of this formulation. The deviational penalty cost parameters defined by a_1, a_2, a_3, and a_4 penalizes the NPV for any deviation from the set goals. This parameter forces the optimizer to meet the set goals to avoid penalizing the NPV. The priority parameters P_1, P_2, P_3, and P_4 are used to place emphasis on the goals that are more important. This parameter is also set up to penalize the NPV more if the most important set goal is not met.

In setting up these parameters, the modeler needs to monitor how smooth the mining proceeds from one period to another and the uniformity of tonnages mined per period; as well as the corresponding NPV generated in other to keep track of the impact of any parameter change on these key performance indicators. In some cases, the extent of setting the priority or penalty cost depends on the extent to which the modeler wants to trade off NPV to meet the set goals. A higher priority or penalty may enforce a goal to be met whilst reducing the NPV of the operation. A case showing this trend has been analyzed.

7. Results and discussions

The developed MILGP model was implemented and tested in TOMLAB/CPLEX environment (Holmström, 2009). The performance of the proposed model was analyzed based on NPV, mining production goals, smoothness and practicality of the generated schedules and the availability of tailings containment areas at the required time. The proposed formulation was verified by numerical experiments on a synthetic and an oil sands data set. The model was implemented on a Dell Precision T3500 computer at 2.4 GHz, with 3GB of RAM.

Further implementation of the MILGP model was done for a large scale oil sands deposit covering an area of 8 km x 4 km which is similar to that used in the conceptual mining model. 864 drillholes with an average depth of 82 m were sampled in this area. The drillhole data were used in developing the rock types and grade models for the oil sands deposit using Gemcom GEMS software (Gemcom Software International, 2008). The modeled rock types are made up of the Pleistocene, Clearwater, Upper McMurray, Middle McMurray and Lower McMurray formations. Whittle software and Gemcom GEMS (Gemcom Software International, 2008) were used in determining and designing the final pit which contains 61490 blocks over five 15 m mining benches ranging from 265 m to 325 m. Each block represents a volume of rock equal to 50 m x 50 m x 15 m. The model contains 4866.2 million tonnes of material with 2792.5 million tonnes of ore, 1697.8 million tonnes of OI dyke material, 2110.0 million tonnes of TCS dyke material and 375.9 million tonnes of waste. The deposit is to be scheduled over 20 periods.

The designed final pit block model was divided into 4 pushbacks that are consistent with the conceptual mining model. The sizes of the pushbacks are determined in consultation with tailings dam engineers and are based on the required cell capacities and the timeliness required in making the cell areas available for tailings containment. The blocks within each pushback are clustered into mining-cuts using fuzzy logic clustering algorithm (Kaufman and Rousseeuw, 1990) to reduce the number of decision variables required in the MILGP model. Clustering of blocks into mining-cuts ensures the MILGP scheduler generates a mining schedule at a selective mining unit that is practical from mining operation point of view. The material in the designed final pit is to be scheduled for the processing plant and four dyke construction destinations with the objective of maximizing the NPV of the mining operation and minimizing the dyke construction cost. An EPGAP of 2% was set for the optimization of all pushbacks. A summary of the details for each pushback to be used for production scheduling are shown in Table 3.

For processing plant feed and dyke construction, bitumen grade and fines percent need to be controlled within an acceptable range for all pushbacks and destinations. It is required to keep an average processing plant head grade with bitumen content between 7 and 16% and fines content less than 30%. The OI dyke material is required to have bitumen content less than 7% and fines content less than 50%. Mining will proceed in a north-south direction starting from pushback 1 to 4. When mining of pushback 1 starts, the OI and TCS dyke material will be used in constructing the key trench, starter dyke, and main dyke of the ETF where the initial fluid fine tailings will be stored. When pushback 1 is completely mined, cell 1 area becomes available and OI and TCS dyke material from pushback 2 can be used in constructing dyke ‘A’ about 100 m from the mine face to create cell 1 for in-pit tailings containment to start. This mining and tailings storage management strategy similar to the conceptual mining model will be utilized until all pushbacks are mined.

Table 3. Details for each pushback to be used for production scheduling

Description	**Value**			
	Pushback 1	Pushback 2	Pushback 3	Pushback 4
Number of blocks	14,535	16,433	16,559	13,963
Number of mining-cuts	971	970	977	999
Tonnage of rock (Mt)	1,144.6	1,303.9	1313.2	1104.5
Ore tonnage (Mt)	631.1	758.7	775.7	627.0
OI dyke material tonnage (Mt)	432.4	434.2	435.6	395.7
TCS dyke material tonnage (Mt)	479.4	568.0	587.0	475.5
Average ore bitumen grade (%)	11.7	11.5	11.6	11.6
Average ore fines (%)	8.6	9.7	8.9	8.7
Average OI dyke material fines (%)	4.1	5.8	5.1	4.6

Our objectives are to generate a uniform schedule and a smooth mining sequence based on the availability of material, the plant processing capacity, and dyke construction requirements. The dyke construction material scheduled should meet the minimum requirements of material for the specified destination with any excess material being available for other purposes. Further to this, to ensure that the mining equipment capacity is well utilized throughout the mine life, we intend to keep a uniform stripping ratio when the mining of ore starts. Table 4 shows the input mining, processing and dyke material goals for the MILGP model for 20 periods. Table 5 shows the input grade limits for ore and OI dyke material for the MILGP model for 20 periods.

Table 4. Mining, processing, OI and TCS dyke material goals for the MILGP model for 20 periods

Mining goal (Mt)	**Processing goal (Mt)**	**OI dyke material goal (Mt)**	**TCS dyke material goal (Mt)**
244	140	70	106

Table 5. Ore and OI dyke material grades for the MILGP model for 20 periods

Ore bitumen grade (wt%)		Ore fines (wt%)		OI dyke material fines (wt%)	
$\underline{g}^{u,t,e}$	$\overline{g}^{u,t,e}$	$\underline{f}^{u,t,e}$	$\overline{f}^{u,t,e}$	$\underline{f}^{u,t,d}$	$\overline{f}^{u,t,d}$
7	16	0	30	0	50

Some of the important features that make this MILGP formulation a robust and flexible platform for mine planning are that apart from the NPV maximization and dyke construction cost minimization, the planner has control over the setting of goals and their deviational variables and the upper and lower limits of grades in each period for all pushbacks and destinations. The planner can also decide on tradeoffs between NPV maximization or dyke construction cost minimization and goals achievement using the penalty and priority functions. The penalty cost and priority parameters used in the MILGP model for this optimization were: 0 for mining; 20 for processing; 30 for OI dyke material; and 30 for TCS dyke material. These generated the required tonnages at the various production destinations. Table 6 summarizes the results from the MILGP model in terms of the NPV and dyke construction cost generated after optimization. The four pushbacks were optimized separately over a total of 20 periods. The overall NPV generated including the dyke construction cost for all pushbacks and destinations is \$14,237M.

Table 6. Results from the MILGP model in terms of the NPV and dyke construction cost for all pushbacks and destinations

Pushback #	NPV (\$M)	Dyke construction cost (\$M)	EPGAP (%)
Pushback 1	6,493.77	714.44	2.0
Pushback 2	4,695.34	524.20	2.0
Pushback 3	3,184.72	312.74	1.7
Pushback 4	1,588.65	174.39	1.1

Figs. 4a, 4b, 4c, and 4d show the mining sequence at level 295m for all pushbacks with a north-south mining direction. The MILGP model generated a practical mining sequence that is smooth and consistent with the mining of oil sands. Mining proceeds in the specified direction to ensure least mobility and increased utilization of loading equipment. This is very important in the case of oil sands mining where large cable shovels are used. The size of the mining-cuts in each period enables good equipment maneuverability and the number and size of active bench phases in each period also reduces the number of loading equipments required as well as providing alternative loading points if needed. Another strategic aspect of mining in the specified direction within each pushback is to ensure that the dyke footprints are released on time as the mining proceeds to enable in-pit dyke construction for tailings containment to start. This is an important integral part of the waste management strategy for oil sands mining operations, and a key driver for profitability and sustainable operations. This also reduces the environmental footprints of the ETF.

The results from Fig. 5 shows a uniform mining, processing, OI and TCS dyke material schedules which ensures effective utilization of mining fleet and processing plant throughout the mine life. The schedule ensures that apart from meeting the processing plant requirements to maximize NPV, the required quality and quantity of dyke material needed to build the dykes of the ETF, cells ‘A’, ‘B’, and ‘C’ are provided in a timely manner at a minimum cost for tailings containment. The schedule basically ensures that the minimum dyke material requirements of each dyke construction destination as per the conceptual dykes’ designs are met so that any excess material can be used for other purposes.

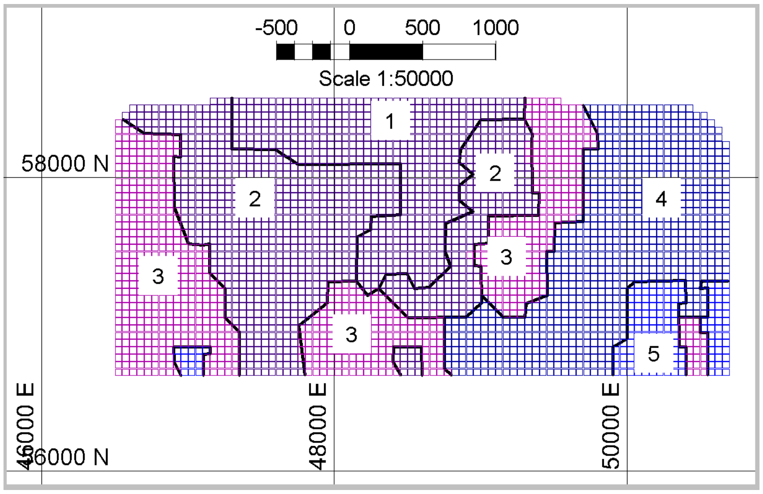

Fig. 4a. Pushback 1 mining sequence at level 295 m

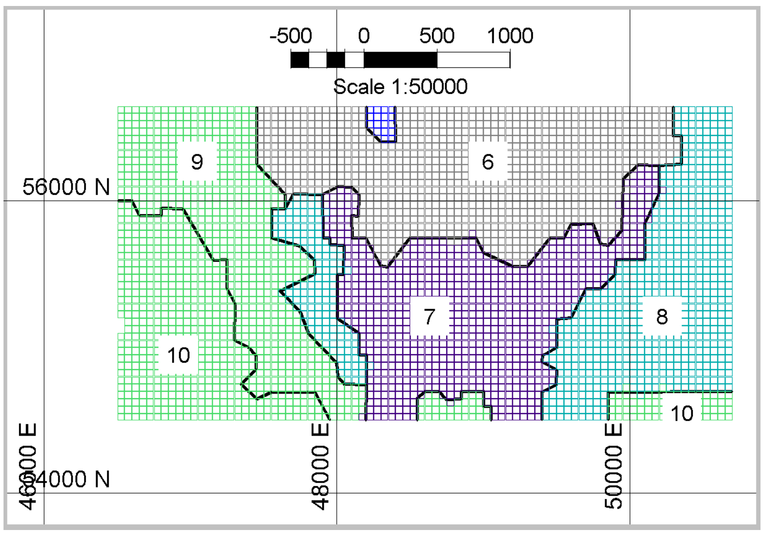

Fig. 4b. Pushback 2 mining sequence at level 295 m

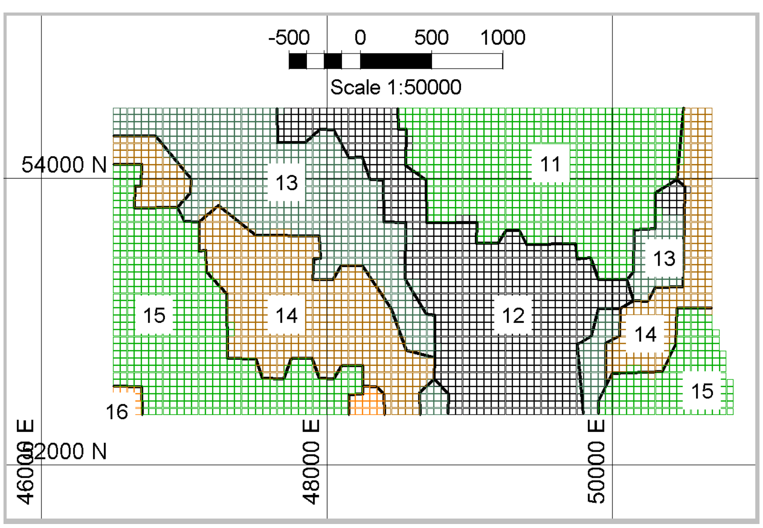

Fig. 4c. Pushback 3 mining sequence at level 295 m

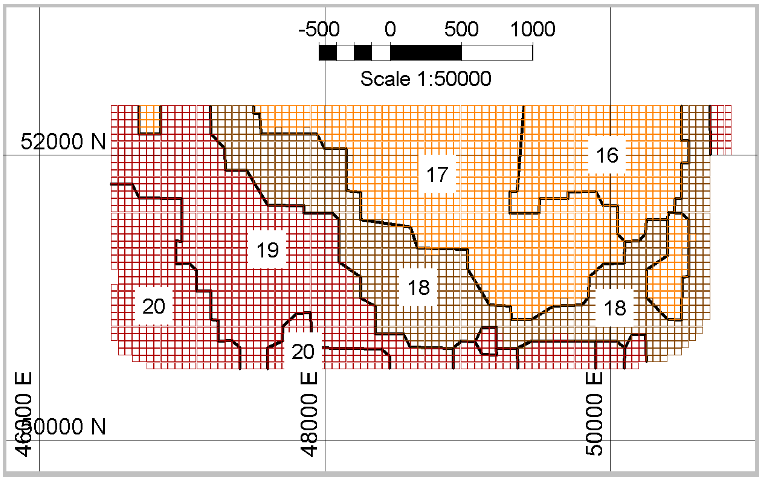

Fig. 4d. Pushback 4 mining sequence at level 295 m

During the first year, due to the requirements of the ETF dyke construction material, less ore is mined and more OI dyke material is mined to facilitate the construction of the key trench and starter dyke and then subsequently, TCS dyke material can be used to continue constructing the main dyke as planned in the conceptual dyke design. This ensures that tailings containment area is created in time for the storage of fluid fine tailings. Ore becomes available at full processing plant capacity from year 2 until the end of the mine life and subsequently TCS dyke material. The OI dyke material supply was also maintained at a uniform rate throughout the mine life. Fig. 5 shows the schedules for ore, OI and TCS dyke material, and waste tonnages generated for 20 periods. Fig. 6 shows the material mined and TCS dyke material tonnage produced in each pushback for 20 periods. Fig. 7 shows the dyke material tonnage sent to the various dyke construction destinations for 20 periods and Fig. 8 shows the OI and TCS dyke material volume scheduled for 20 periods. It can be seen from Table 2 that 23Mm^3 of OI dyke material is required for the ETF key trench and starter dyke construction and this material requirement has been adequately catered for by scheduling 40Mm^3 of OI dyke material in period 1 as shown in Fig. 8.

The total material mined was 4866.2Mt. This is made up of 2720.4Mt of ore and 1386.7Mt of OI dyke material whilst 2055.2Mt of TCS dyke material was generated. A total of 1602.1Mm^3 of dyke material was scheduled. The schedules give the planner good control over dyke material and provides a robust platform for effective dyke construction planning and tailings storage management.

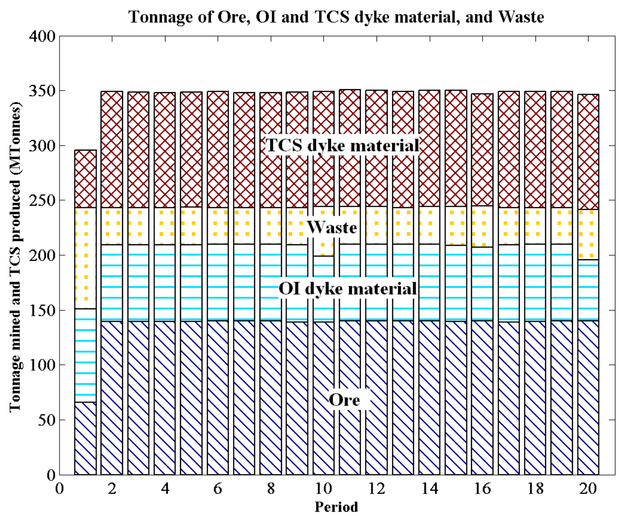

Fig. 5. Schedules for ore, OI and TCS dyke material, and waste tonnages produced over 20 periods

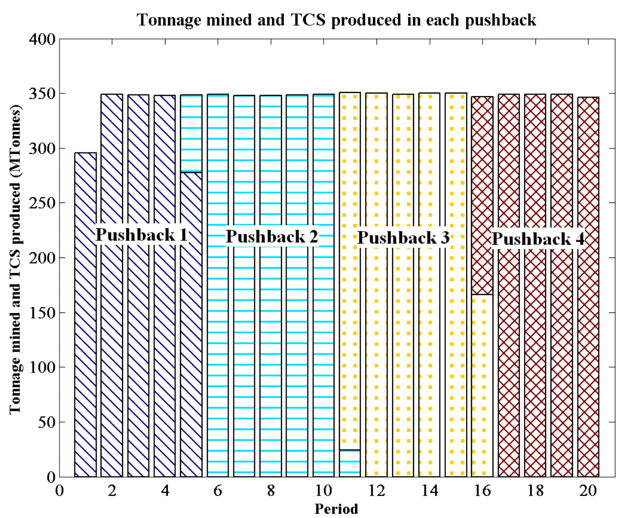

Fig. 6. Material mined and TCS dyke material tonnage produced in each pushback for 20 periods

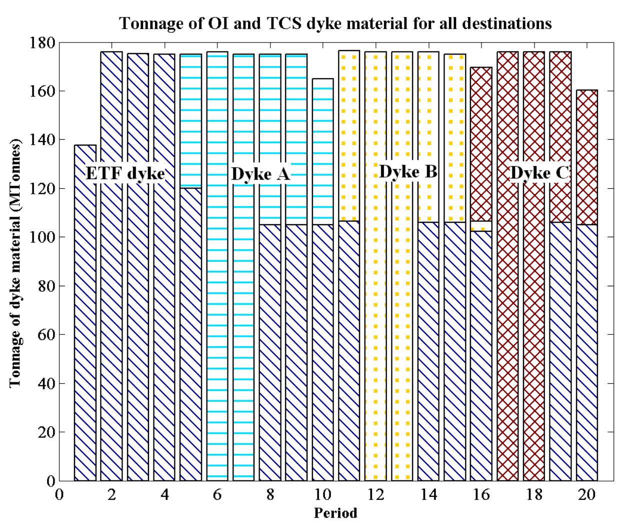

Fig. 7. Dyke material tonnage sent to the various dyke construction destinations for 20 periods

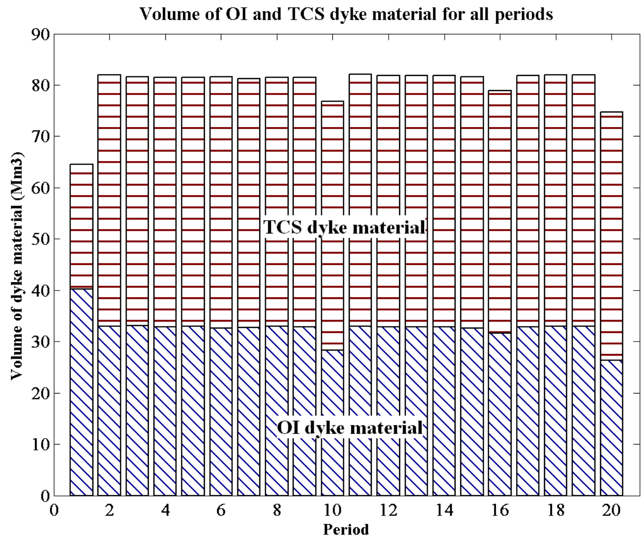

Fig. 8. OI and TCS dyke material volume scheduled for 20 periods

There is also an inherent task of blending the run-of-mine materials to meet the quality and quantity specifications of the processing plant and dyke construction. The blending problem becomes more prominent as more detailed planning is done in the medium to short term. The processing plant head grade and OI dyke material grade that was set were successfully achieved in all periods for all destinations. With the exception of period 1, the scheduled average ore bitumen grade was between 10.9 and 12.2%. The average ore bitumen grade for period 1 was 10.3% basically due to the emphasis placed on mining OI dyke material for the ETF key trench and starter dyke construction. This was required to construct the initial tailings containment when ore processing starts. The average ore and OI dyke material fines percent were between 14 and 30%, and 10 and 23% respectively. Figs. 9 and 10 show the average ore bitumen grade and ore fines percent for all pushbacks respectively. Fig. 11 shows the average OI dyke material fines percent for all pushbacks.

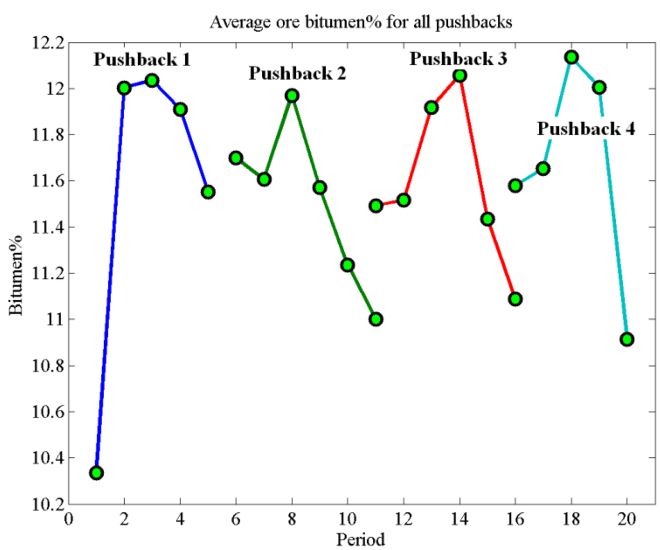

Fig. 9. Average ore bitumen grade for all pushbacks

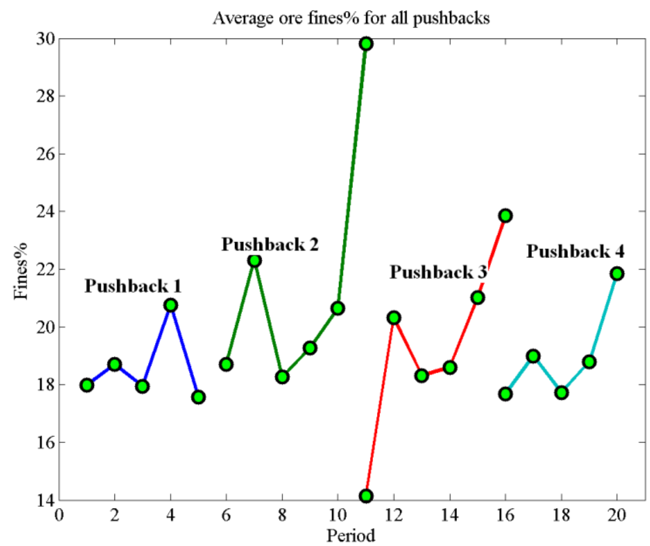

Fig. 10. Average ore fines percent for all pushbacks

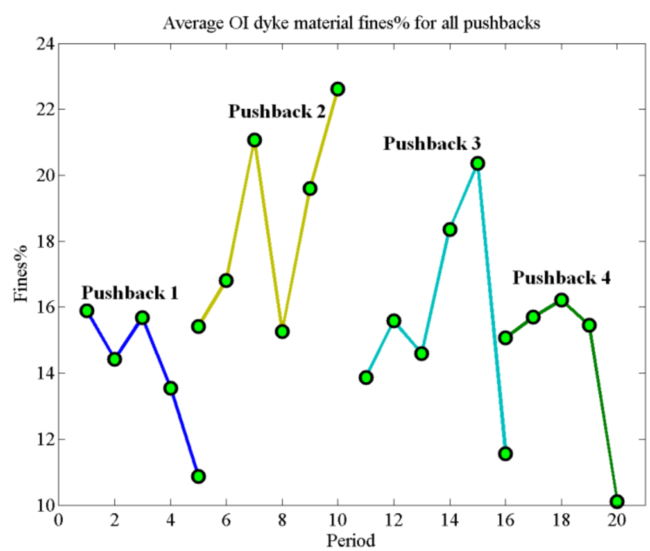

Fig. 11. Average OI dyke material fines percent for all pushbacks

7.1. Supplementary experiments

The data shown in Table 7 represents the summary of results for other optimization experiments that were conducted prior to selecting the illustration presented in this paper. The illustration corresponds to run 3 on the table. The initial optimization experiment conducted was run 1 which schedules for a north-south mining direction. Further work was done by optimizing with a south-north mining direction (run 2) which yielded a lower NPV and a lower dyke material tonnage. The lower NPV results from mining pushbacks with lower economic block values in the early years. Less ore was mined and a less uniform schedule was produced due to the mining direction.

Further investigations were conducted by increasing the number of mining cuts as in run 3. This resulted in an increase in NPV resulting from an increase in the resolution of the optimization problem. The increased resolution increases the flexibility of the problem as well as the number of decision variables thereby increasing the optimization runtime. A smooth and uniform schedule was generated. Another experiment (run 4) was done to test the MILGP model in terms of placing a higher penalty cost and priority (PP) value on one goal as compared to the others. The increased PP value for OI dyke material further constrains the optimization problem decreasing the ore to dyke material ratio and causing a decrease in the overall NPV which includes dyke construction cost. The dyke material tonnes increases and hence the dyke construction cost. As illustrated in Fig. 12, in general within the set mining constraints, as the PP values for dyke material increases, the NPV decreases as a result of a reduction in ore tonnes and/or an increase in dyke material tonnes. This approach is useful when more dyke material is required for tailings containment construction to enable a sustainable mining operation.

Comparing these experiments, run 3 was selected because it generates the best overall NPV as well as a good schedule and the required dyke material tonnage.

Table 7. Results for supplementary experiments showing that run 3 generates the highest NPV and best schedule

Run #	**Total Cuts**	**Mining dxn**	P_1a_1	P_2a_2	P_3a_3	P_4a_4	**Runtime (minutes)**	**Overall NPV ($M)**	**Dyke material (Mt)**	**Schedule uniformity & smoothness ranking**
1	1977	NS	0	20	30	30	105	13,810	3315	3
2	1977	SN	0	20	30	30	17	10,713	3012	4
3	3917	NS	0	20	30	30	288	14,237	3442	1
4	3917	NS	0	20	60	30	59	14,121	3460	2

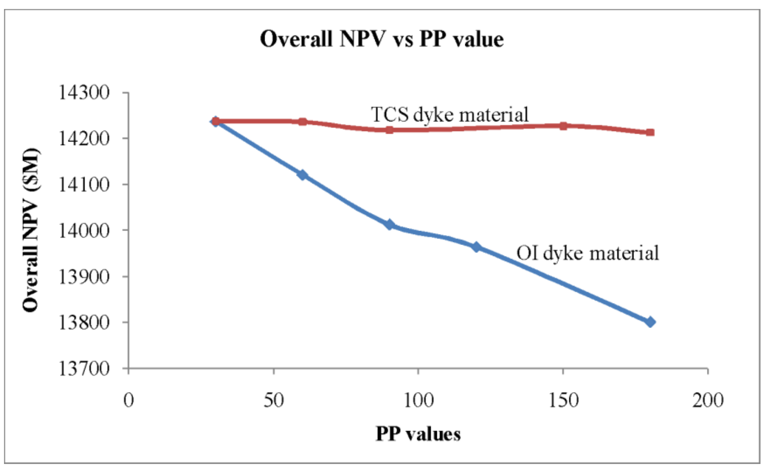

Fig. 12. General trend of overall NPV with PP values of dyke material

8. Conclusions and future work

This paper discussed some of the shortcomings and applications of MIP, MILP and GP to the open pit production scheduling problem. Further to this, the use of MILGP formulations in solving industrial problems due to the advantages derived from this hybrid formulation was reviewed. In this paper, the authors have developed, implemented, and verified a MILGP theoretical framework for open pit production scheduling of multiple material types with multiple elements for multiple destinations. The developed model proved to be able to handle the integration of large-scale mine production planning and waste management problems in the oil sands mining industry.

Oil sands mining requires a carefully planned and integrated mine planning and waste management strategy that generates value and sustainability. This requires that production schedules are generated for ore, dyke material and waste to ensure that whilst ore is fed to the processing plant, there is enough dyke material available for dyke construction for both the ex-pit and in-pit tailings facilities. This ensures there is adequate storage space for the tailings throughout the mine life whilst reducing the size of the disturbed landscape by making the best use of in-pit tailings facilities and reducing the size of the external tailings facility. The MILGP formulation uses binary integer variables to control mining precedence and continuous variables to control mining of ore and dyke material. There are also goal deviational variables and penalty costs and priorities that must be set up by the planner. The optimization model was implemented in TOMLAB/CPLEX environment.

The developed model was able to create value and a sustainable operation by generating a practical, smooth and uniform schedule for ore and dyke material using mining-cuts from block clustering techniques. The schedule gives the planner good control over dyke material and provides a robust platform for effective dyke construction and waste disposal planning. The schedule ensures that the key drivers for oil sands profitability and sustainability which is maximizing NPV whilst creating timely tailings storage areas are satisfied within an optimization framework. This is in accordance with recent regulatory requirements by Alberta Energy Resources and Conservation Board (Directive 074) that requires oil sands mining companies to develop an integrated life of mine plans and tailings disposal strategies for in-pit and external tailings disposal systems (McFadyen, 2008). The planner also has the flexibility of choosing goal deviational variables, penalty costs and priorities to achieve a uniform schedule and improved NPV. Similarly, tradeoffs between achieving goals and maximizing NPV or minimizing dyke construction cost can be made.

Future research will focus on developing more efficient mathematical formulation techniques for the MILGP model that will reduce the solution time for large-scale open pit production scheduling and waste management problems.

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10. Appendix 1

10.1. Notations

10.1.1. Sets

$\boldsymbol{K} = \{1,.....,K\}$ set of all the mining-cuts in the model.

$\boldsymbol{J} = \{1,......,J\}$ set of all the phases (push-backs) in the model.

$\boldsymbol{U} = \{1,.....,U\}$ set of all the possible destinations for materials in the model.

$C_k(L)$ for each mining-cut *k*, there is a set $C_k(L) \subset \boldsymbol{K}$ defining the immediate predecessor mining-cuts above mining-cut *k* that must be extracted prior to extraction of mining-cut *k*, where *L* is the total number of mining-cuts in the set $C_k(L)$.

$M_k(P)$ for each mining-cut *k*, there is a set $M_k(P) \subset \boldsymbol{K}$ defining the immediate predecessor mining-cuts in a specified horizontal mining direction that must be extracted prior to extraction of mining-cut *k* at the specified level, where *P* is the total number of mining-cuts in the set $M_k(P)$.

$B_j(H)$ for each phase *j*, there is a set $B_j(H) \subset \boldsymbol{K}$ defining the mining-cuts within the immediate predecessor pit phases (push-backs) that must be extracted prior to extracting phase *j*, where *H* is an integer number representing the total number of mining-cuts in the set $B_j(H)$.

10.1.2. Indices, subscripts and superscripts

A parameter, *f*, can take indices, subscripts, and superscripts in the format $f_{k,j}^{u,e,t}$. Where:

$t \in \{1,......,T\}$ index for scheduling periods.

$k \in \{1,.....,K\}$ index for mining-cuts.

$e \in \{1,.....,E\}$ index for element of interest in each mining-cut.

$j \in \{1,.....,J\}$ index for phases.

$u \in \{1,.....,U\}$ index for possible destinations for materials.

d, l, m, p subscripts and superscripts for overburden and interburden dyke material, tailings coarse sand dyke material, mining and processing respectively.

10.1.3. Parameters

$d_k^{u,t}$ the discounted profit obtained by extracting mining-cut *k* and sending it to destination *u* in period *t*.

$v_k^{u,t}$ the discounted revenue obtained by selling the final products within mining-cut k in period t if it is sent to destination u, minus the extra discounted cost of mining all the material in mining-cut k as ore and processing at destination u.

$p_k^{u,t}$ the extra discounted cost of mining all the material in mining-cut k in period t as overburden and interburden dyke material for construction at destination u.

$h_k^{u,t}$ the extra discounted cost of mining all the material in mining-cut k in period t as tailings coarse sand dyke material for construction at destination u.

$q_k^{u,t}$ the discounted cost of mining all the material in mining-cut k in period t as waste and sending it to destination u.

g_k^e the average grade of element e in ore portion of mining-cut k.

$\underline{g}^{u,t,e}$ the lower bound on the required average head grade of element e in period t at processing destination u.

$\overline{g}^{u,t,e}$ the upper bound on the required average head grade of element e in period t at processing destination u.

f_k^e the average percent of fines in ore portion of mining-cut k.

$\underline{f}^{u,t,e}$ the lower bound on the required average fines percent of ore in period t at processing destination u.

$\overline{f}^{u,t,e}$ the upper bound on the required average fines percent of ore in period t at processing destination u.

f_k^d the average percent of fines in overburden and interburden dyke material portion of mining-cut k.

$\underline{f}^{u,t,d}$ the lower bound on the required average fines percent of overburden and interburden dyke material in period t at dyke construction destination u.

$\overline{f}^{u,t,d}$ the upper bound on the required average fines percent of overburden and interburden dyke material in period t at dyke construction destination u.

o_k the ore tonnage in mining-cut k.

w_k the waste tonnage in mining-cut k.

d_k the overburden and interburden dyke material tonnage in mining-cut k.

l_k the tailings coarse sand dyke material tonnage in mining-cut k.

$T_m^{u,t}$ the mining goal (tonnes) in period t at destination u.

$d_1^{-,u,t}$ the negative deviation from the mining goal (tonnes) in period t at destination u.

$d_1^{+,u,t}$ the positive deviation from the mining goal (tonnes) in period t at destination u.

$T_p^{u,t}$ the processing goal in period t at destination u (tonnes).

$d_2^{-,u,t}$	the negative deviation from the processing goal in period t at destination u (tonnes).
$d_2^{+,u,t}$	the positive deviation from the processing goal in period t at destination u (tonnes).
$T_d^{u,t}$	the overburden and interburden dyke material goal in period t at destination u (tonnes).
$d_3^{-,u,t}$	the negative deviation from the overburden and interburden dyke material goal in period t at destination u (tonnes).
$d_3^{+,u,t}$	the positive deviation from the overburden and interburden dyke material goal in period t at destination u (tonnes).
$T_l^{u,t}$	the tailings coarse sand dyke material goal in period t at destination u (tonnes).
$d_4^{-,u,t}$	the negative deviation from the tailings coarse sand dyke material goal in period t at destination u (tonnes).
$d_4^{+,u,t}$	the positive deviation from the tailings coarse sand dyke material goal in period t at destination u (tonnes).
$r^{u,e}$	the proportion of element e recovered (processing recovery) if it is processed at destination u.
$p^{e,t}$	the price of element e in present value terms per unit of product.
$cs^{e,t}$	the selling cost of element e in present value terms per unit of product.
$cp^{u,e,t}$	the extra cost in present value terms per tonne of ore for mining and processing at destination u.
$ck^{u,t}$	the cost in present value terms per tonne of overburden and interburden dyke material for dyke construction at destination u.
$ct^{u,t}$	the cost in present value terms per tonne of tailings coarse sand dyke material for dyke construction at destination u.
$cm^{u,t}$	the cost in present value terms of mining a tonne of waste in period t and sending it to destination u.
P_1	the priority level associated with minimizing the deviations from the mining goal.
P_2	the priority level associated with minimizing the deviations from the processing goal.
P_3	the priority level associated with minimizing the deviations from the overburden and interburden dyke material goal.
P_4	the priority level associated with minimizing the deviations from the tailings coarse sand dyke material goal.
a_1	the penalty paid per tonne in deviating from the mining goal.
a_2	the penalty paid per tonne in deviating from the processing goal.
a_3	the penalty paid per tonne in deviating from the overburden and interburden dyke material goal.

a_4 the penalty paid per tonne in deviating from the tailings coarse sand dyke material goal.

10.1.4. Decision variables

$x_k^{u,t} \in [0,1]$ a continuous variable representing the portion of mining-cut k to be extracted as ore and processed at destination u in period t.

$z_k^{u,t} \in [0,1]$ a continuous variable representing the portion of mining-cut k to be extracted as overburden and interburden dyke material and used for dyke construction at destination u in period t.

$s_k^{u,t} \in [0,1]$ a continuous variable representing the portion of mining-cut k to be extracted as tailings coarse sand dyke material and used for dyke construction at destination u in period t.

$y_k^{u,t} \in [0,1]$ a continuous variable representing the portion of mining-cut k to be mined in period t and sent to destination u, which includes both ore, overburden and interburden dyke material and waste.

$b_k^t \in [0,1]$ a binary integer variable controlling the precedence of extraction of mining-cuts. b_k^t is equal to one if the extraction of mining-cut k has started by or in period t, otherwise it is zero.

$c_j^t \in [0,1]$ a binary integer variable controlling the precedence of mining phases. c_j^t is equal to one if the extraction of phase j has started by or in period t, otherwise it is zero.

11. Appendix 2

HTML documentation of MATLAB code