A Mathematical Programming Model for Mine Planning in the Presence of Grade Uncertainty

Behrang Koushavand and Hooman Askari-Nasab
Center for Computational Geostatistics (CCG)
and Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

Abstract

The optimality of an open pit production scheduling problem is affected dramatically by grade uncertainty. Recent research initiatives have attempted to consider the effect of grade uncertainty on production schedules. These methods are aimed either at minimizing the risk using grade uncertainty or at maximizing the net present value (NPV) without taking into account grade uncertainty explicitly. Another major problem in open pit production scheduling is the size of the optimization problem. The mathematical programming formulation of real size long-term open pit production schedules is beyond the capacity of current hardware and optimization software. In this paper a mathematical programming formulism is presented to find a sequence in which ore and waste blocks should be removed from a predefined open pit outline and their respective destinations, over the life of mine, so that the net present value of the operation is maximized and the deviations from the annual target ore production is minimized in the presence of grade uncertainty. Two main methods are presented: (1) without a stockpile and (2) with a stockpile. The new parameters that are controlling the uncertainty part of the optimization are studied. At the end, an oil sand deposit in northern Alberta is used to generate an optimum schedule.

1. Introduction

Mine planning is the process of finding a feasible block extraction schedule that maximizes net present value (NPV) and is one of the critical processes in mining engineering. It entails consideration of some technical, financial and environmental constraints. In the case of open pit mines, as Whittle (1989) notes, planning involves: "Specifying the sequence of blocks extraction from the mine to give the highest NPV, subject to variety of production, grade blending and pit slope constraints".

In this context, the uncertainty of ore grade may cause some shortfalls in the designed production and discrepancies between planning expectations and actual production (Koushavand and Askari-Nasab, 2009; Osanloo et al., 2008; Vallee, 2000). Therefore using only one block model would not be optimal. Various authors present methodologies to employ grade uncertainty, and show its impact, in mine planning.

Dowd (1994) propose a risk-based algorithm for surface mine planning. Ravenscroft (1992) and Koushavand and Askari-Nasab (2009) uses conditional simulated orebodies to show the impact of grade uncertainty on production scheduling. Dowd (1994) and Ravenscroft (Ravenscroft, 1992) use stochastic orebody models sequentially in traditional optimization methods. However the sequential process cannot produce an optimal schedule which takes uncertainty into account.

Godoy and Dimitrakopoulos (2003) and Leite and Dimitrakopoulos (2007) present a new risk-inclusive Long Term Production Plan(LTPP) approach based on simulated annealing. A multistage heuristic framework is presented to generate a final schedule, which considers geological uncertainty so as to minimize the risk of deviations from production targets. A basic input to this framework is a set of equally probable scenarios of the orebody, generated by conditional simulation. The authors report a significant improvement in NPV in the presence of uncertainty. Heuristic methods do not guarantee the optimality of the results. Also these techniques are sometimes very complex, and many parameters need to be chosen carefully in order to get reasonable results.

Dimitrakopoulos and Ramazan (2004) propose a probabilistic method for long-term mine planning based on linear programming. This method uses probabilities of being above or below a cut-off to deal with uncertainty. The LP model is used to minimize the deviation from target production. This method does not directly and explicitly account for grade uncertainty and also does not maximize the NPV.

Leite and Dimitrakopoulos (2007) present a technique that generates an optimal schedule for each realization. Simulated annealing is subsequently used to generate a single schedule based on all schedules, such that the deviation from target production is minimized. For each conditional simulation, an optimum schedule is generated. Using simulated annealing, a single schedule is generated based on all schedules such that deviation from target production is minimized. The main drawback of this method is that it does not necessarily find the optimum solution.

Dimitrakopoulos and Ramazan (2008) present a linear integer programming (LIP) model to generate the optimal production schedule. Equally probable simulated block models are used as input. This model has a penalty function that is the cost of deviation from the target production and is calculated based on the geological risk discount rate (GDR), which is the discounted unit cost of deviation from a target production. They use linear programming to maximize a new function that is NPV minus penalty costs. With this method, it is not clear how to define the GDR parameter. For different GDR values there are different optimal solutions. In the presented model, mixed integer programming has been used. A variable is defined for each block. Adding constraints increases the complexity and CPU time to solve the optimization. This method is not tractable with a real case study. In addition, there is no stockpile defined in this method.

2. Background

Geological characteristics of each point (grade) are assigned using available estimation techniques. Kriging (Deutsch and Journel, 1998a; Goovaerts, 1997) is the most common estimation method used in industry; however, Kriging results do not capture uncertainty and may leading to conditional biased reserve estimates (Isaaks, 2005). Also, mine plans that are generated based on one input block model fail to quantify the geological uncertainty and its impact on the future cash flows and production targets.

Geostatistical simulation algorithms are widely used to quantify and assess uncertainty. The generated realizations are equally probable and represent possible outcomes (Deutsch and Journel, 1998b; Goovaerts, 1997; Journel and Huijbregts, 1981). Choosing one of these realizations will not be objective to fair uncertainty assessment. Also, generated final pit limit and production schedule based on one block model would not necessarily be the optimum one. Therefore to get robust and optimum long-term production planning (LTPP), a sufficient number of realizations should be used simultaneously.

Mine planning is a process that defines a sequence of extraction of blocks with the objective of net present value (NPV) maximization. The mine production scheduling can be formulated as an optimization problem. NPV is the discounted revenue that discounted cost has been deducted from it:

$$\text{discountedprofit} = \text{discounted revenue} - \text{discounted costs} \tag{1}$$

Askari-Nasab and Awuah-Offei (2009) have presented the objective functions of the LP formulations that maximize the net present value of the mining operation. It is important to define a clear concept of economic block value based on ore parcels which could be mined selectively. The profit from mining a block depends on the value of the block and the costs incurred in mining and processing. The cost of mining a block is a function of its spatial location, which characterizes how deep the block is located relative to the surface and how far it is relative to its final dump. The spatial factor can be applied as a mining cost adjustment factor for each block according to its location on the surface. The discounted profit from a block is equal to the discounted revenue generated by selling the final product contained in block n minus all the discounted costs involved in extracting the block; this is presented at Eq.(1) The discounted cost can rewrite as Eq. (2) and Eq. (3):

$$v_n^t = o_n \times \left(g_n \times r^t \times \mathrm{P}^t - cp^t \right) \tag{2}$$

$$q_n^t = \left(o_n + w_n \right) \times cm^t \tag{3}$$

Where n is the id number of the block, and v_n^t and q_n^t are discounted revenue and cost of extraction from block n at period t respectively. o_n and w_n are the tonnage of ore and waste for block n, and cp^t and cm^t are cost of processing and mining at period t per ton respectively. r^t is processing recovery; P^t is the price of the final product. If there is more than one valuable element in the final product, the revenue of the block will be added up for each element. In addition, if there are contaminants that are to be processed and eliminated from final product, the cost of processing will be deducted from the revenue of that block.

The objective function is to maximize the summation profit (Eq. (1)) of all blocks at all periods with two separate decision variables for each block in each period. First, q_n^t is the portion of the block n to be extracted at period t, and second, z_n^t, is the portion of block n to be processed (if it is ore) at period t. Therefore the mathematical form of the optimal mining schedule is presented in Eq. (4):

$$Max \ \sum_{t=1}^{T} \sum_{n=1}^{N} \left(v_n^t \times z_n^t - q_n^t \times y_n^t \right) \tag{4}$$

Subject to:

$$gl^t \le \frac{\sum_{n=1}^{N} g_n \times o_n \times z_n^t}{\sum_{n=1}^{N} o_n \times z_n^t} \le gu^t \qquad \forall t = 1, 2, \ldots, T \tag{5}$$

$$pl^t \le \sum_{n=1}^{N} o_n \times z_n^t \le pu^t \qquad \forall t = 1, 2, \ldots, T \tag{6}$$

$$ml^t \le \sum_{n=1}^{N} \left(o_n + w_n \right) \times z_n^t \le mu^t \qquad \forall t = 1, 2, \ldots, T \tag{7}$$

$$z_n^t \le y_n^t \qquad \forall t = 1, 2, \ldots, T \ , n = 1, 2, \ldots, N \tag{8}$$

$$a_n^t - \sum_{i=1}^{t} y_l^i \le 0 \qquad \forall t = 1,2,\ldots,T \ , n = 1,2,\ldots,N \ , \ l = 1,2,\ldots,C(L) \qquad (9)$$

$$\sum_{i=1}^{t} y_n^i - a_n^t \le 0 \qquad \forall t = 1,2,\ldots,T \ , n = 1,2,\ldots,N \qquad (10)$$

$$a_n^t - a_n^{t+1} \le 0 \qquad \forall t = 1,2,\ldots,T-1 \ , \ n = 1,2,\ldots,N \qquad (11)$$

Where Eq. (5) is grade blending constraints; these inequalities ensure that the head grade is within the desired range in each period. g_n is the estimated grade of block n, gl^t and gu^t are the allowable lower limit and upper limit of the input grade at period t. There will be separate constraints for each element of interest and any contaminants in each period. There are two equations (upper bound and lower bound) per element per scheduling period in Eq.(5). Eq. (6) is the processing capacity constraints, where pl^t and pu^t are the lower limit and upper limit (target production) for the designed processing plan; these inequalities ensure that the total ore processed in each period is within the acceptable range of the processing plant capacity. There are two equations (upper bound and lower) per period per ore type. Eq. (7) is the mining constraints where ml^t and mu^t are lower and upper limit for mining limits; these inequalities ensure that the total tonnage of material mined (ore, waste, overburden, and undefined waste) in each period is within the acceptable range of mining equipment capacity in that period. There are two equations (upper bound and lower bound) per period. Eq. (8) represents inequalities that ensure that the amount of ore of any block which is processed in any given period is less than or equal to the amount of rock extracted in the considered time period.

Eqs. (9) to (11) control the relationship of block extraction precedence by binary integer variables a_n^t, which is equal to one if the extraction of block n has started by or in period t (otherwise it is zero), and i, which is the index for set of the blocks, C(L), that need to be extracted prior to the extraction of block n. This model only requires the set of immediate predecessors' blocks on top of each block to model the order of block extraction. This is presented by set C(L) in Eq. (9).

The amount of ore processed and amount of material mined are controlled by two separate continuous variables rather than by binary integer variables. In this model, there is T (number of periods) multiplied by N (number of blocks) integer variables.

The estimated block model is usually used to maximize NPV; therefore, NPV_{es} is calculated from Eq. (12):

$$NPV_{es} = \sum_{t=1}^{T} \sum_{n=1}^{N} \left(v_n^t \times z_n^t - q_n^t \times y_n^t \right) \qquad (12)$$

In this model, the number of decision variables equals two times of the number of blocks multiplied by the number of periods. Therefore, it would be a time-consuming process to solve this using linear programming. Boland et.al. (2009) and Askari-Nasab and Awuah-Offei (2009) tried to solve this problem by clustering the blocks in order to reduce the number of variables. Using some grade aggregation methodology and based on lithological information, similar blocks were summarized to a group and are dealt with as one variable which will be extracted in the same period. Each group of blocks is called a mining cut. Grouping the blocks into mining cuts is done without sacrificing the accuracy of the estimated (or simulated) values and to model a more realistic equipment movement strategy. The mining-cut clustering algorithm developed uses fuzzy logic clustering (Kaufman and Rousseeuw, 1990). The coordinates of each mining-cut are represented by the center of the cut and its spatial location.

The proposed linear programming was formulated in a MATLAB environment (MathWorks Inc., 2007). TOMLAB/CPLEX (Holmström, 1989-2009) was used as the Linear programming Solver. TOMLAB/CPLEX efficiently integrates the solver package CPLEX (ILOG Inc, 2007) with MATLAB.

3. MILP formulation based on grade uncertainty without stockpile

A Mixed Integer Linear Programming (MILP) model for optimizing long term production scheduling in open pit mines is developed with an objective function that maximizes the total NPV of the project under a managed grade risk profile. Grade uncertainty causes shortfalls from target productions. Therefore, to obtain an optimal solution, NPV must be maximized and deviation from target production must be minimized simultaneously among all simulation realizations:

$$\begin{cases} \text{Max. NPV} \\ \text{Min. Deviationfrom target production} \end{cases}$$

Two main assumptions are made to model this optimization problem:

1. There is no stockpile to store any possible overproduced ore. Most of the time not having a stockpile is very unlikely in real life. However the assumption made here is a hypothetical case.
2. Long term scheduling is a dynamic process. This means that it changes during mine life. There are many situations that may occur at the operational level that management needs to change the extraction schedule such as misclassification of ore and waste, so called as information effect, failures at equipment, and price changes at final production. In addition, during every period, the generated schedule is updated with new information such as blasthole data and new exploration drill holes. Therefore no long term schedule is followed from the first year until the end of the mine life. However the goal here is to find a robust long term schedule using all useful information. In the case of violation from target production in operational level, the optimization should run again with new information to find the new optimum schedule.

The proposed method tries to postpone the extraction of uncertain blocks to later years when there will be new information and less uncertainty. The main idea is to minimize the risk of not meeting the target production, because the uncertainty may impose costs to the project. The cost of uncertainty is considered in paper 104. The schedule generated using the method proposed here poses less risk in the early years of production.

The objective function has two components. As in Eq.(4), only one block model is used to maximize NPV; this is usually an estimated block model such as Kriging. The generated schedule is such that in all periods except the last one, the plant is fully fed by this block model and there is no deviation from target production. The second objective is applied to realizations. There is a probability that any schedule may not meet the target production because of grade uncertainty. These probabilities can be calculated using simulation values. The method presented here tries to minimize these probabilities in the early years of mine production. Two new variables, op_l^t and up_l^t, represent the amount of over-production and under production for realization l at period t. Each of these variables is multiplied by the discounted cost for over and under production, called $c_{op}{}^t$ and $c_{up}{}^t$. These two parameters are chosen by the user. They are the discounted penalty dollar values per ton for probable over and under production, and they are discounted based on the defined discount rate.

It is also important to note that in most cases, there is not enough ore to feed the plant during the final year of mine life. Therefore, as discussed in paper number 104, the cost of underproduction for final year is assumed to be equal to zero: ${c_{up}}^{T} = 0$.

Therefore, the mathematical form of the optimal mining schedule in the presence of grade uncertainty is shown in Eq.(13).

$$Max \sum_{t=1}^{T} \left\{ \underbrace{\sum_{n=1}^{N} \left(v_n^t \times z_n^t - q_n^t \times y_n^t \right)}_{first\ part: MAX\ NPV} - \underbrace{\frac{1}{L} \sum_{l=1}^{L} \left({c_{op}}^{t} \times op_l^t + {c_{up}}^{t} \times up_l^t \right)}_{second\ part: MIN\ Risk.} \right\} \tag{13}$$

The first part of this model maximizes the NPV using one block model which usually is estimate values, and the second part tries to minimize risk by deferring high uncertain blocks to the later years.

There are two more constraints for each period and each realization. These constraints control two new variables: op_l^t and up_l^t. The other constraints of this model are the same as those in Eq. (5) to (11).

Two new constraints are defined by Eq.(14) and (15):

$$\sum_{n=1}^{N} \left(o_{n,l} \times z_n^t - op_l^t \right) \le P_u^t \qquad \forall\, t = 1,2,\ldots,T\ ,\ l = 1,2,\ldots,L \tag{14}$$

$$\sum_{n=1}^{N} \left(-o_{n,l} \times z_n^t - up_l^t \right) \le -P_l^t \qquad \forall\, t = 1,2,\ldots,T\ ,\ l = 1,2,\ldots,L \tag{15}$$

Where $o_{n,l}$ is the tonnage of ore at block n in realization l. The number of decision variables is: $2 \times N \times T + 2 \times L \times T$. The Numbers of binary variable is the same as in Eq. (4). This model was implemented in MATLAB (MathWorks Inc., 2007) and solved by the CPLEX TOMLAB (Holmström, 1989-2009) library. Because the size of the problem is too large to handle with current hardware and software, the fuzzy clustering technique (Askari-Nasab and Awuah-Offeri, 2009) was used to aggregate similar blocks within a group called mining cut.

4. MILP formulation based on grade uncertainty with stockpile

The assumption that a plant does not have a stockpile is a very severe assumption. In most plants, there is a stockpile for storing over produced ore when there is enough material to feed the plant. These surplus ores are used when there is some problem with feeding the plant, such as failure of the extraction and hauling system, or when there is a grade blending problem with input material to the mill. Therefore any possible over produced ore would be processed in later years, and the penalty value defined in Eq.(13) for over production would be less than it would be in the presence of stockpile. This means that any plausible over production based on a realization will be kept in a stockpile and will be used in the next period of extraction.

The costs of over production are:

- The cost of re-handling materials from a stockpile
- The loss of discounted value of ore transferred ore to the next period

In Eq.(13), the cost of over production for each period is deducted from the cost of over production in the next period, which means that for each period the penalty value is only the loss of the discounted value of ore that is transferred to the next period. Meanwhile, any re-handling costs are added to the cost of over production in each period. Therefore, a new optimization model for long term mine planning in the presence of grade uncertainty and of a stockpile is presented at Eq. (16):

$$Max \sum_{t=1}^{T} \left\{ \underbrace{\sum_{n=1}^{N} \left(v_n^t \times z_n^t - q_n^t \times y_n^t \right)}_{first\ part: MAX\ NPV} - \underbrace{\frac{1}{L} \sum_{l=1}^{L} \left[\left(c_{op}{}^{t} - c_{op}{}^{t+1} \right) \times op_l^t + c_{up}{}^{t} \times up_l^t \right]}_{second\ part: MIN\ Risk.} \right\} \tag{16}$$

The only difference between Eqs. (13) and (16) is in the cost of over production. Having a stockpile reduces the cost of possible over production.

It is very unlikely that in the last period of mine life, one realization will create over production, because in most cases there is not enough ore to feed the plant in the final year of mine life. But if, in a hypothetical case, a realization does generate over production in the final year, the ore will not be processed and the cost of over production will be the same as in the previous model. This means that there will be no deduction from over production costs for the final year. This can be imposed to mathematical form as: $c_{op}{}^{T+1} = 0$. There are some modifications to be made in the constraints which control the two variables op_l^t and up_l^t. Because any possible over produced ore will be used in the next period, this should be considered by two constraints as shown in Eqs. (14) and (15). Modified versions of these two constrains are shown in Eqs. (17) and (18):

$$\sum_{n=1}^{N} \left[o_{n,l} \times z_n^t - \left(op_l^{t-1} + op_l^t \right) \right] \leq P_u^t \qquad \forall t = 1,2,\ldots,T \;,\; l = 1,2,\ldots,L \tag{17}$$

$$\sum_{n=1}^{N} \left[-o_{n,l} \times z_n^t - \left(op_l^{t-1} + up_l^t \right) \right] \leq -P_l^t \qquad \forall t = 1,2,\ldots,T \;,\; l = 1,2,\ldots,L \tag{18}$$

Note that there is no overproduction in period 0: $op_l^0 = 0$.

The number of decision variables and binary variables are the same as in the previous model. Matlab and Tom lab are used to solve this optimization problem. Clustering method is used as before to reduce number of variables.

5. Discussion

In the early stages of production, the cost of uncertainty is higher than in later years because of the new information and the reduced uncertainty that later emerges. It is a reasonable decision to postpone the extraction of a highly uncertain block to later years, which will reduce the probability of obtaining deviations from target production. Higher penalty values mean that in the early years of production, less uncertain blocks are preferred. The optimizer generates a schedule that maximizes the NPV using the Kriging block model and minimizes the average penalty value calculated from L realizations. There is trade off in choosing high grade and low uncertainty blocks in the early years of production. Conversely, a higher $c_{op}{}^{t}$ and $c_{up}{}^{t}$ result in a lower NPV generated from the first part of Eqs. (13) and (16). Therefore it is very important to find the optimum values for these parameters. In order to do so, two techniques are proposed.

a) Numerical method: In this method, different $c_{op}{}^{t}$ and $c_{up}{}^{t}$ are used. In addition, over production and under production costs are considered to be equal and are called c. The optimization is run with different cost values. For each number, NPV_{es}^{c}, which is the NPV derived from the first part of Eqs. (13) and (16), is calculated. The cost of uncertainty is calculated (based on notation in paper 301), as are the differences:

$$Delta = NPV_{es}^{c} - Unc.Cost \tag{19}$$

Fig. 1 shows the expected results for this method. As is shown in this graph, NPV_{es}^{c} decreases slowly with higher c factors, while the cost of uncertainty decreases rapidly. The optimum c value is chosen where the delta value exceeds to its maximum value.

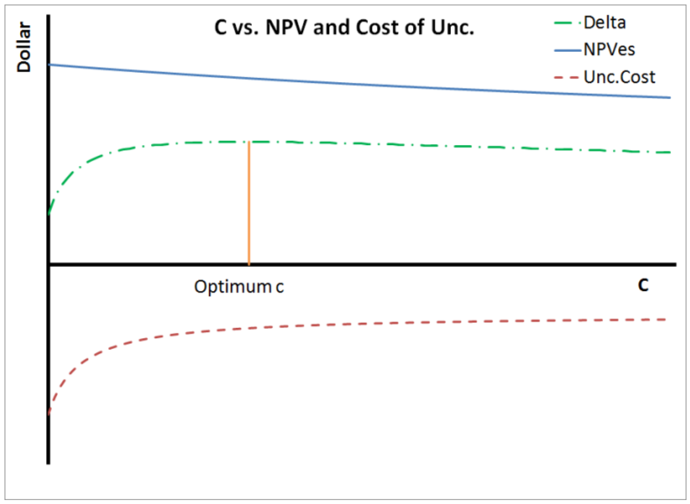

Fig. 1. Expected graph for optimum C factor based on numerical method.

b) The second method is based on the cost of uncertainty that is presented in paper 104. With equal $c_{op}{}^{t}$ and $c_{up}{}^{t}$ values, the second part of Eq. (13) is changed to the similar notation that was presented in paper 104.

$$\begin{aligned} &\frac{1}{L}\sum_{l=1}^{L}\left[c^t \times op_l^t + c^t \times up_l^t\right] = \frac{1}{L}\sum_{l=1}^{L}\left[c^t \times \left(op_l^t + up_l^t\right)\right] \\ &= \frac{1}{L}\sum_{l=1}^{L}\left[c^t \times \left(op_l^t + up_l^t\right)\right] = \frac{1}{L}\sum_{l=1}^{L}\left[c^t \times \left|P_l^t - \text{Target}_l^t\right|\right] \end{aligned} \tag{20}$$

On the other hand the cost of uncertainty is:

$$CoU = \frac{1}{L}\sum_{l=1}^{L}\left[\left(\bar{g}^t \times P_r^t \times \text{Price}^t \text{ - } P_c^t\right) \times \left|P_l^t - \text{Target}_l^t\right|\right] \tag{21}$$

By comparing Eqs. (20) and (21), $\bar{C}^t$, the average penalty cost for over and under production can be calculated from Eq. (22):

$$\Rightarrow \bar{C}^t = \frac{c^t}{L} = \frac{\bar{g}^t \times P_r^t \times \text{Price}^t \text{ - } P_c^t}{L} \tag{22}$$

Where P^t is the input ore to the mill in period t, Target^t is the target production for period t, $\bar{g}^t$ is the average grade of input ore in period t, P_r^t is the processing recovery in period t, Price^t is the selling price of the final product in each period, P_c^t is the processing cost and L is the number of realizations.

In Eq.(13), there is no stockpile. Therefore any possible over produced ore is not processed and will be considered waste. On the other hand modeling the stockpile is a very difficult problem, because it is necessary to have one extra variable to control the average grade of the stockpile and this extra variable is multiplied by the decision variables that control the portion of extraction and processing; this causes the model to be a nonlinear optimization problem. One way to solve this problem is to have the average grade of stockpile as an input. Eq. (16) uses this value as an input parameter. The penalty function that is applied in both Eqs. (13) and (16) is deferent in an over-production situation. Fig. 2 shows the penalty values at different periods; they are discounted for not having any stockpile (at left) and with stockpile (on the right). In each graph, the vertical axis shows the penalty value per tonne of over- and under-production. The horizontal axis shows the under (left side) and over (right side) tonnage of material that is sent to the mill to be processed. The slope of the lines is reduced over time. This means that the penalty value in period 1 for over- and under-production is less than that in period 2. In addition, having a stockpile reduces the penalty value for over production of ore. This cost is related to the re-handling of material at the stockpile and the revenue loss of resulting from the transfer of ore materials to the later years.

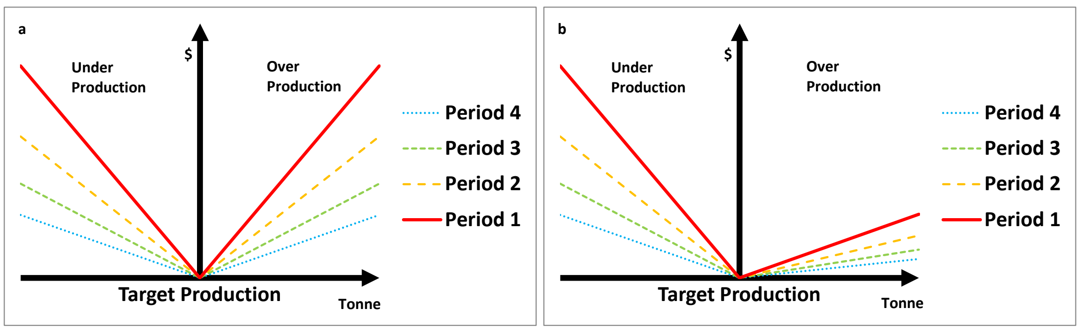

Fig. 2. Penalty function for over and under production at different periods based on a discounting factor, a: no stockpile and b: with stockpile

6. Case study

The same oil sand deposit that is cited in paper 104 is examined in this section. GSLIB (Deutsch and Journel, 1998) programs were used to generate an ordinary Kriging block model and 50 conditional realizations. The Kriging block model was used in the first part of both models to maximize NPV. The realizations were used in the second part to minimize deviation from target production. In this case study a 0.5$ per tonne penalty value was consider for both over-production and under-production ($c_{op}{}^{t}$ and $c_{up}{}^{t}$). Using an average grade 9.5 mass percent bitumen in all periods, a processing recovery of 95 percent, a selling price of \$2.8125 per tonne and a processing cost of \$0.5025 per tonne in Eq. (22), the c factor is calculated with 50 realizations as $\overline{C}^{t} = 0.5$. The gap of 1% was used in CPLEX optimizer. The final gaps for LP without stockpile and with stockpile respectively are 0.98% and 0.68%.

Fig. 3 shows the schedules generated by two methods. In both methods, the optimizer did quite well in feeding the plant over 7 years of production, and the only shortfall, in the last period, resulted from the fact that there was less ore remaining. Fig. 4 shows the cumulative cash flow over periods for Kriging, Etype and simulation realizations for both methods. Note that the generated schedule was followed for each of realizations. Average input grade to the mill in each period is presented in Fig. 5. Having a stockpile relaxes the optimizer that it is able to extract and stock high grade ore in period three, and this increases the average grade in period four. Fig. 6 illustrates input tonnage to the mill using the Kriging, E-type and realization block models for both methods. It is clear that having a stockpile reduced the probability deviation from target production in the early years of production (period 3 and 4). This fact is clear in Fig. 7 which shows the box plot of realizations and deviations from target production. Deviations from target production in period 3 and 4 using the first method (without stockpile) are 2.5 and 5.28 percent respectively. These values for second method (with stockpile) are 0.39 and 3.32. This shows that a stockpile reduce the risk of not meeting the target production.

Table 1 summarizes the results of the two methods when each realization follows the two schedules. The expected NPVs for first and second methods are 2319.18 and 2322.60.

Table 2 shows the statistics for realizations and cumulative cash flow over the periods. The first two periods have a negative cash flow because of pre-stripping. The summary results of the three methods are shown in Table 3. The first method is Kriging without uncertainty and without a stockpile. These results derived from paper 104. The second and third methods involve the optimization of NPV with Kriging and minimization for deviation from target production using simulation realization with and without a stockpile. The NPV_{es}^{c}, which is calculated based on a Kriging block model, is maximized in the first method because there are fewer constraints and more flexibility for optimization: 2461 versus 2449.3 and 2453.8. On the other hand, the cost of uncertainty is higher in this method than in the next two methods: 178.9, 151.6 and 141.9 respectively. The stockpile also reduces the cost of uncertainty. The delta value, which is defined in Eq. (19), is calculated in final column. It is clear that the delta value for the method with a stockpile is higher than two other methods. In addition using realizations creates a higher delta value than the schedule creates using only the Kriging block model.

7. Conclusions

In this paper two methods were presented to generate long term production schedules using a linear programming technique. The net present value was maximized based on the estimate block model, which is usually created using Kriging methods. The second objective was to minimize the deviation from target production or minimize the cost of uncertainty. In both methods, highly uncertain blocks are extracted in later years when more information is provided by new drill holes. This is controlled by two factors, called the cost of over and under production in each period. Having high values for each of these parameters means that less uncertain blocks are preferred in the early years of production. High values for these parameters also reduce the NPV that is calculated with Kriging. Two methods were presented to calculate the optimum values for these parameters.

There is not over or under production for the Kriging block model. The probability of deviations from target production in each period is calculated using simulation realizations. The generated schedules are more robust because the probability of not meeting the target production is lower in the early years of production.

There are two types of variables in the proposed optimization models: binary variables that control the precedence of block extraction and decisions variables that indicate the portion of the block that is going to be extracted and sent to the mill in each period. The number of variables for both of these methods is too large to be handled by current commercial software in a real case study. For example, there are 800,000 decision variables for a project with 20,000 blocks and 20 years of mine life. To solve this problem, a clustering technique is used. Blocks in the same level with similar grades are aggregated into mining cuts and the number of variables is reduced. Recently some studies have sought to find better techniques and more complex criteria.

The future work for this study is to use pushbacks to reduce the size of the problem. Pushbacks can also be used to define the large scale strategy of mine development and are used widely in industry. First Lerchs-Grossman algorithm (1965) is used to find the nested pits then the block at each pushbacks are determined. The new constrains are added to the LP model such that the blocks in a certain pushback are extracted before starting to remove the blocks in the next at pushback.

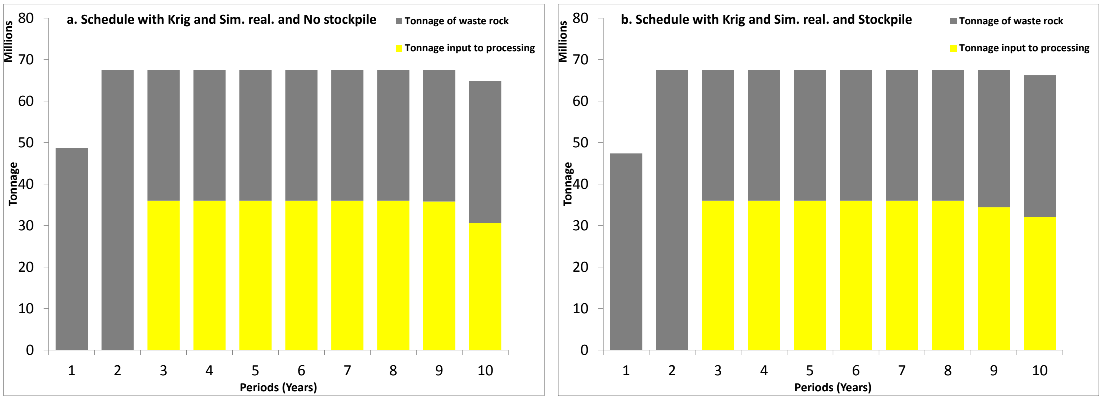

Fig. 3. Schedules generated using krig model and simulation realizations a: without stockpile and b: with stockpile.

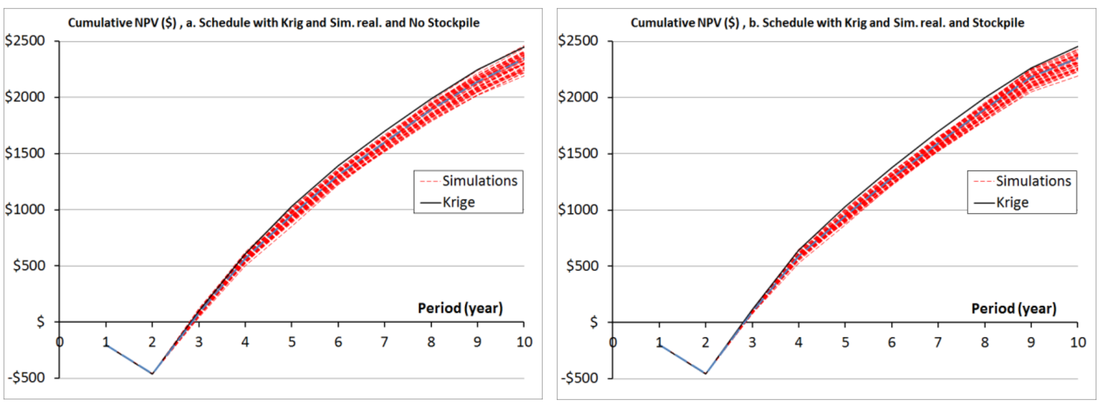

Fig. 4. Comulative NPV over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), a: without stockpile and b: with stockpile.

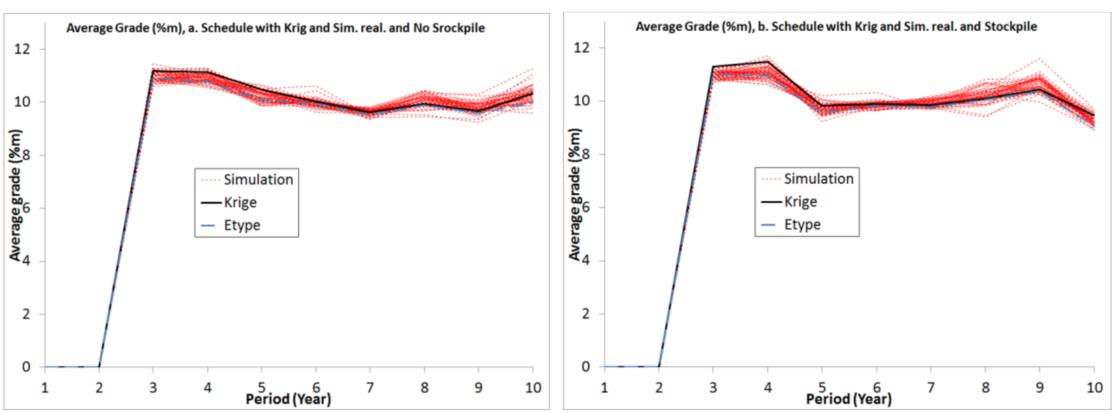

Fig. 5. Input head grade to the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), a: without stockpile and b: with stockpile.

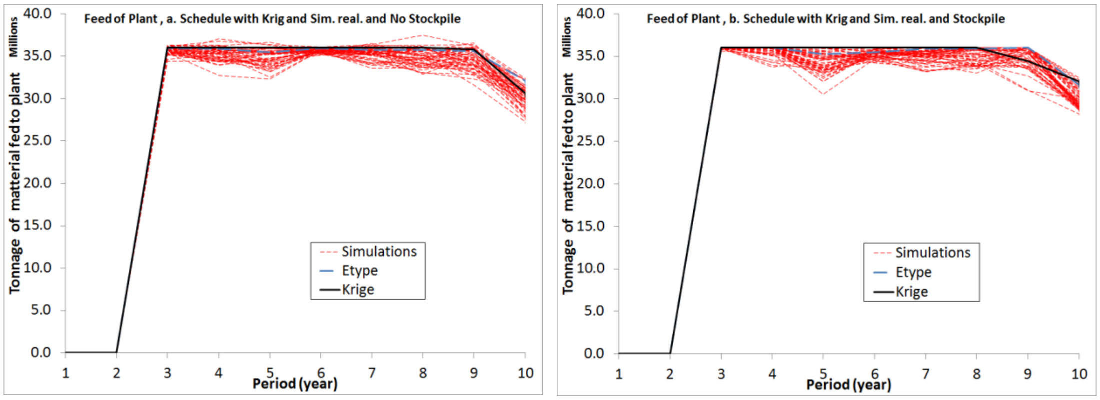

Fig. 6. Feed of the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), a: without stockpile and b: with stockpile.

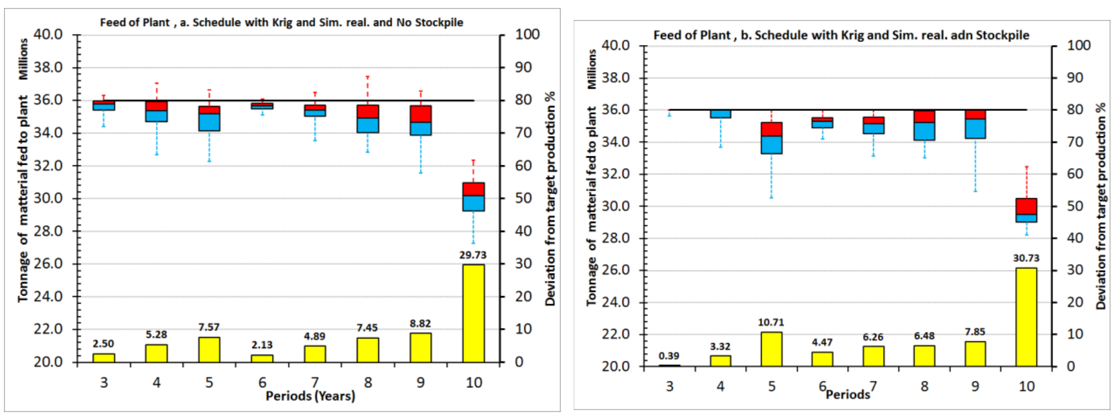

Fig. 7. Boxplot and deviation from target production (yellow bars), calculated using simualtion values, a: without stockpile and b: with stockpile.

Table 1. Summary statistic of realizations when generated schedule with Kriging is followed, at above without stockpile and bottom with stockpile.

a: LP With Krig & Sim. Realizations Without Stockpile	Ore Millions Tonnes	STRO	Input Bitumen Millions Tonnes	Average %	NPV Millions Dollars
Mean	276.14	1.37	28.27	10.24	2319.18
Std. dev	3.60	0.03	0.44	0.09	60.74
Min	269.28	1.29	27.28	10.02	2189.42
Quartile 1	273.31	1.34	27.89	10.19	2269.00
Median	276.50	1.36	28.25	10.24	2316.67
Quartile 2	278.83	1.39	28.61	10.29	2367.12
Max	284.06	1.43	29.12	10.52	2428.65
Krig	282.44	1.31	29.11	10.31	2449.44
Etype	282.22	1.32	28.49	10.10	2346.25

b: LP With Krig & Sim. Realizations With Stockpile	Ore Millions Tonnes	STRO	Input Bitumen Millions Tonnes	Average %	NPV Millions Dollars
Mean	275.92	1.37	28.24	10.24	2322.60
Std. dev	3.61	0.03	0.44	0.09	59.42
Min	269.29	1.29	27.22	10.02	2191.25
Quartile 1	272.97	1.35	27.84	10.19	2269.99
Median	276.37	1.36	28.23	10.24	2318.70
Quartile 2	278.39	1.39	28.56	10.29	2372.09
Max	284.47	1.43	29.10	10.52	2430.31
Krig	282.44	1.31	29.11	10.31	2453.85
Etype	282.07	1.32	28.48	10.10	2349.99

Table 2 Summery statistics of cumulative cash flow at each period, at above without stockpile and bottom with stockpile.

Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Mean	-203.8	-460.4	77.7	555.5	943.5	1,294.5	1,590.8	1,875.0	2,119.8	2,319.2
Std. dev	0.0	0.0	16.9	26.1	35.2	37.4	41.0	47.7	53.7	60.7
Min	-203.8	-460.4	42.8	501.9	851.8	1,219.6	1,516.5	1,784.7	2,017.3	2,189.4
Quartile 1	-203.8	-460.4	66.3	537.5	918.5	1,273.6	1,560.1	1,837.7	2,075.1	2,269.0
Median	-203.8	-460.4	76.6	549.7	943.9	1,289.8	1,588.3	1,883.2	2,125.4	2,316.7
Quartile 2	-203.8	-460.4	90.9	578.2	971.6	1,328.1	1,627.5	1,911.1	2,163.3	2,367.1
Max	-203.8	-460.4	120.5	620.4	1,015.7	1,360.7	1,669.2	1,954.8	2,202.1	2,428.6
Krig	-203.8	-460.4	101.6	609.3	1,032.0	1,391.6	1,698.3	1,991.2	2,244.6	2,449.4
Etype	-203.8	-460.4	80.0	561.5	951.4	1,304.4	1,602.7	1,890.4	2,138.0	2,346.3

Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Mean	-198.1	-454.8	93.2	590.7	939.0	1,277.4	1,585.1	1,878.4	2,162.2	2,322.6
Std. dev	0.0	0.0	9.2	25.6	31.3	32.9	37.7	46.2	54.1	59.4
Min	-198.1	-454.8	76.9	525.7	870.9	1,214.9	1,520.8	1,792.2	2,049.0	2,191.2
Quartile 1	-198.1	-454.8	86.9	573.0	916.6	1,254.4	1,557.1	1,841.2	2,119.9	2,270.0
Median	-198.1	-454.8	93.3	594.0	939.9	1,272.8	1,579.4	1,886.8	2,165.2	2,318.7
Quartile 2	-198.1	-454.8	100.6	607.2	962.5	1,305.2	1,616.9	1,915.1	2,202.6	2,372.1
Max	-198.1	-454.8	114.9	649.2	1,008.3	1,337.5	1,654.8	1,957.1	2,255.8	2,430.3
Krig	-198.1	-454.8	115.1	644.7	1,028.1	1,379.8	1,697.3	1,997.4	2,264.9	2,453.8
Etype	-198.1	-454.8	94.6	591.0	946.4	1,285.8	1,598.3	1,895.0	2,179.9	2,350.0

Table 3. Summary of NPV and Cost of uncertainty at different methods.

Method	NPV_{es}^{c}	**Cost of Unc.**	**Delta**
Kriging Without Unc. No Stockpile	2461.0	178.9	2282.1
Kriging With Sim. No Stockpile	2449.3	151.6	2297.7
Kriging With Sim. and Stockpile	2453.8	141.9	2311.9

8. References

[1] Askari-Nasab, H. and Awuah-Offeri, K. (2009). Mixed integer programming formulations for open pit production scheduling. *MOL Report one*, *1*, 1-31.

[2] Boland, N., Dumitrescu, I., Froyland, G., and Gleixner, A. M. (2009). LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity. *Comput.Oper.Res.*, *36*,(4), 1064-1089.

[3] Deutsch, C. V. and Journel, A. G. (1998). GSLIB : geostatistical software library and user's guide. in *Applied geostatistics series*, vol. 2. New York: Oxford University Press, pp. 369.

[4] Deutsch, C. V. and Journel, A. G. (1998). *GSLIB : geostatistical software library and user's guide.* Oxford University Press, New York, 2nd ed, Pages 369.

[5] Dimitrakopoulos, R. and Ramazan, S. (2004). Uncertainty based production scheduling in open pit mining. 106-112.

[6] Dimitrakopoulos, R. and Ramazan, S. (2008). Stochastic integer programming for optimising long term production schedules of open pit mines: methods, application and value of stochastic solutions. *Mining Technology : IMM Transactions section A*, *117*, 155-160.

[7] Dowd, P. A. (1994). Risk Assessment in Reserve Estimation and Open-Pit Planning. *Transactions of the Institution of Mining and Metallurgy Section a-Mining Industry*, *103*, A148-A154.

[8] Godoy, M. and Dimitrakopoulos, R. (2003). Managing risk and waste mining in long-term production scheduling of open pit mine. *SME Annual Meeting & Exhibition*, *316*, 43-50.

[9] Goovaerts, P. (1997). *Geostatistics for natural resources evaluation.* Oxford University Press, New York, Pages 483.

[10] Holmström, K. (1989-2009). TOMLAB /CPLEX. 11.2 ed. Pullman, WA, USA: Tomlab Optimization.

[11] ILOG Inc. (2007). ILOG CPLEX 11.0 User’s Manual September. 11.0 ed: ILOG S.A. and ILOG, Inc.

[12] Isaaks, E. (2005). The Kriging Oxymoron: A conditionally unbiased and accurate predictor (2nd edition). *Geostatistics Banff 2004, Vols 1 and 2*, *14*, 363-374.

[13] Journel, A. G. and Huijbregts, C. J. (1981). *Mining geostatistics.* Academic Press, London, Pages 600.

[14] Kaufman, L. and Rousseeuw, P. J. (1990). *Finding groups in data : an introduction to cluster analysis.* Wiley, New York, Pages 342.

[15] Koushavand, B. and Askari-Nasab, H. (2009). Transfer Geological Uncertainty through Mine Planning. in *MPES*. MPES - International Symposium of Mine Planning/Equipment Selection

[16] Leite, A. and Dimitrakopoulos, R. (2007). Stochastic optimisation model for open pit mine planning: Application and risk analysis at copper deposit. *Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology*, *116*,(3), 109-118.

[17] Lerchs, H. and Grossmann, I. F. (1965). Optimum design of open-pit mines. *The Canadian Mining and Metallurgical Bulletin, Transactions*, *LXVIII*, 17-24.

[18] MathWorks Inc. (2007). MATLAB Software. 7.4 (R2007a) ed: MathWorks, Inc.

[19] Osanloo, M., Gholamnejad, J., and Karimi, B. (2008). Long-term open pit mine production planning: a review of models and algorithms. *International Journal of Mining, Reclamation and Environment*, *22*,(1), 3-35.

[20] Ravenscroft, P. J. (1992). Risk analysis for mine scheduling by conditional simulation. *The Canadian Mining and metallurgical Bulletin, Transactions.(Sec. A: Min. Industry)*, *101*, 82-88.

[21] Vallee, M. (2000). Mineral resource + engineering, economic and legal feasibility = ore reserve. *CIM bulletin*, *90*, 53-61.

[22] Whittle. (1989). The Facts and Fallacies of Open Pit Optimization. in *Whittle Programming Pty Ltd*