Transfer of grade uncertainty into mine planning

Behrang Koushavand and Hooman Askari-Nasab
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

Abstract

Uncertainty is always present because of sparse geological data. Conditional simulation algorithms such as Sequential Gaussian Simulation (SGS) and Sequential Indicator Simulation (SIS) are used to assess uncertainty in the spatial distribution of grades. Long-term mine planning and the management of future cash flows are vital for surface mining operations. Traditionally the long-term mine plans are generated based on an estimated input geological block model. Estimated or kriged models do not capture uncertainty and must be tuned to avoid biases. Mine plans that are generated based on one input block model fail to account for the uncertainty and its impact on the future cash flows and production targets. A method is presented to transfer grade uncertainty into mine planning. First, Sequential Gaussian Simulation is used to generate fifty realizations of an oil sands deposit. An optimum final pit limits design is carried out for each SGS realization while fixing all other technical and economic input parameters. Afterwards, the long-term schedule of each final pit shell is generated. Uncertainty in the final pit outline, net present value, production targets, and the head grade are assessed and presented. The results show that there is significant uncertainty in the long-term production schedules. In addition, the long-term schedule based on one particular simulated ore body model is not optimal for other simulated geological models. The mine planning procedure is not a linear process and the mine plan generated based on the krig estimate is not the expected result from all of the simulated realizations. The probability of each block being extracted in each planning period and the probability that the block would be treated as ore or waste in the respective period are calculated and can be used to assist in long range mine planning. Finally a stochastic linear programming model is presented to use all simulation realization in mine planning. This model tries to minimize negative effects of geological uncertainty and maximize the NPV simultaneously.

1. Introduction

Open pit production scheduling is the process of defining a feasible block extraction sequence that maximizes the net present value (NPV) of mining operation while meeting technical and economic constraints. There are three time ranges for production scheduling: long-term, medium-term and short-term. Long-term can be in the range of 20 – 30 years. This period is divided into several medium-term periods between 1 to 5 years. Medium-term schedules provide detailed information that allows for an accurate design of ore extraction from a special area of the mine, or information that would allow for necessary equipment substitution or the purchase of essential equipment. The medium-term schedule is also divided into 1 to 6 month periods (Osanloo *et al.*, 2008).

In this paper, the main focus will be on long-term production planning (LTPP) in open pit mines. LTPP determines the distribution of cash flow over the mine life, the feasibility of the project, and also it is a prerequisite for medium and short-term scheduling.

Uncertainty is inevitable with sparse geological data. Geostatistical simulation algorithms are widely used to quantify and assess this uncertainty. The generated realizations are equally probable and represent plausible geological outcomes (Journel and Huijbregts, 1981; Deutsch and Journel, 1998). Choosing one or some of these realizations does not realistically account for what might happen in the future. In addition, the uncertainty of ore grade in block models may cause discrepancies between planning expectations and actual production. Traditional production scheduling methods that use an estimated block model as the input into the scheduling process cannot capture the risk associated with production schedules caused by the grade variability. The majority of the current production scheduling methods used in industry have two major shortcomings: (i) the production scheduling methods are either heuristic based, or derived from the mine planner's experience; most of the current scheduling tools do not use global optimization methods, therefore the generated schedules are not necessarily optimal; and (ii) the inability to account for the grade uncertainty inherent within the production scheduling problem and as a result, there is no measure of the associated risk with the generated mine plans.

Effective open pit design and production scheduling is a critical stage of mine planning. The effects of pit design and scheduling and related predictions have major consequences on cash flows, which are typically on the order of millions of dollars. Open pit push-back design is commonly based on the well known Nested Lerchs-Grossman algorithm. This algorithm provides an optimal scenario of how an orebody should be mined given a set of geological, mining and economic considerations. Since 1965, several types of mathematical formulations have been considered for the LTPP problem: Linear programming (LP), mixed integer programming (MIP), pure integer programming (IP), dynamic programming (DP) and Meta-heuristic techniques.

All of these deterministic algorithms try to solve the LTPP problem without considering grade uncertainty.

Vallee (2000) reported that 60% of the mines surveyed had 70% less production than designed capacity in the early years. Rossi and Parker (1994) reported shortfalls against predictions of mine production in later stages of production. Traditional production scheduling methods that do not consider the risk of not meeting production targets caused by grade variability, cannot produce optimal results. Therefore, the common drawback of all deterministic algorithms is that they do not consider any type of uncertainty during the optimization process.

Dimitrakopoulos et al. (2001) show that there are substantial conceptual and economic differences between risk-based frameworks and traditional approaches. Some authors tried to use stochastic orebody models sequentially in traditional optimization methods. Dowd (1994) proposed a framework for risk integration in surface mine planning. Ravenscroft (1992) discussed risk analysis in mine production scheduling. He used simulated orebodies to show the impact of grade uncertainty on production scheduling. He concluded that conventional mathematical programming models cannot accommodate quantified risk. Dowd (1994) and Ravenscroft (1992) used stochastic orebody models sequentially in traditional optimization methods. However the traditional process cannot produce an optimal schedule considering uncertainty.

Godoy and Dimitrakopoulos (2003) and Leite and Dimitrakopoulos (2007) presented a new risk inclusive LTPP approach based on simulated annealing. A multistage heuristic framework is presented to generate a final schedule, which considers geological uncertainty so as to minimize the risk of deviations from production targets. A basic input to this framework is a set of equally probable scenarios of the orebody, generated by the technique of conditional simulation. They reported significant improvement on NPV in presence of uncertainty.

Leite and Dimitrakopoulos (2007) presented a proposed technique that for each of conditional simulation realizations, an optimum schedule is generated. Afterwards, using simulated annealing technique, a single schedule is generated based on all schedules, such that deviation from target production is minimized. The main drawback of simulated annealing method is that it merely finds an acceptably good solution in a fixed amount of time, rather than the optimum solution.

Dimitrakopoulos and Ramazan (2004) proposed a probabilistic method for long-term mine planning based on linear programming. This method uses probabilities of being above or below a cut-off to deal with uncertainty. The LP model is used to minimize the deviation from target production. This method does not directly and explicitly account for grade uncertainty and also does not maximize the NPV.

Dimitrakopoulos and Ramazan (2008) presented a stochastic integer programming (SIP) model to generate the optimal production schedule using equally probable simulated orebody models as input, without averaging the related grades. This model has a penalty function that is the cost of deviation from the target production and is calculated from geological risk discount rate (GDR) that is discounted unit cost of deviation from a target production. They use linear programming to maximize a function equal to NPV minus penalty costs. They concluded that the generated production schedule is the optimum solution that can produce the maximum achievable discounted total value from the project, given the available orebody uncertainty described through a set of stochastically simulated orebody models. The proposed scheduling approach considers multiple simulated orebody models without increasing the required number of binary variables and thus computational complexity.

The objective of this study is to: (i) present two methodologies to quantify the grade uncertainty transferred into production schedules; (ii) assess the impact of grade uncertainty on output parameters of mine production scheduling such as: NPV, ore tonnage, head grade, stripping ratio, amount of final product, and annual targeted production; and (iii) propose a mixed integer linear programming (MILP) formulation for optimal production scheduling that aims at maximizing the net present value while minimizing the deviations from targeted production, caused by grade uncertainty.

Kriging (Goovaerts, 1997; Deutsch and Journel, 1998) is used to estimate grades and construct the block model. Next, a final pit limit optimization study is performed using Lerchs and Grossmann (LG) (Lerchs and Grossmann, 1965) algorithm. Afterwards, a long-term life-of-mine production schedule is generated using Whittle software (Gemcom Software International, 1998-2008); this is referred to as the krig schedule throughout the paper.

Method 1- We use Sequential Gaussian Simulation (SGS) (Journel and Huijbregts, 1981; Goovaerts, 1997; Deutsch and Journel, 1998) is used to generate equally probable realizations of the orebody. An optimum final pit limit design is carried out for each SGS realization with the same technical and economic input parameters as used for the kriged model. Next, the long-term schedule of each final pit shell is generated. Uncertainty in the final pit outline, net present value, production targets, and the head grade are assessed and compared against the krig schedule. This process is labelled as method number one in Fig. 1.

Method 2- The optimal final pit limit and the krig production schedule are the basis of this approach. The same ultimate pit limit and the same krig schedule are applied to all the SGS realizations. This provides an assessment of the uncertainty in the production schedule. This process is labelled as method number two in Fig. 1.

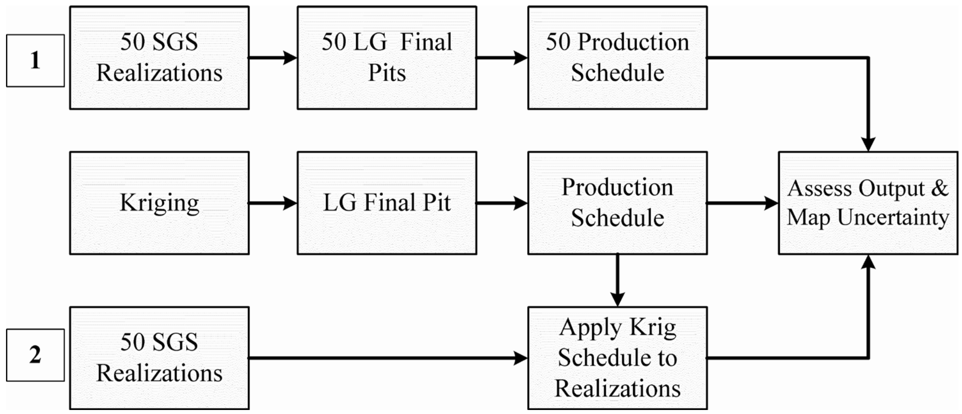

Fig. 1. Flow diagram of the study.

The results show that there is significant uncertainty in the long-term production schedules. In addition, the long-term schedule based on one particular simulated ore body model is not optimal for other simulated geological models. The mine planning procedure is not a linear process and the mine plan generated based on the krig estimate is not the expected result from all of the simulated realizations. The probability of each block being extracted in each planning period and the probability that the block would be treated as ore or waste in the respective period are calculated and can be used to assist in long range mine planning.

2. Methodology

The comparison of Method 1 and 2 and the respective results will be illustrated through a case study corresponding to an oil sands deposit in Fort McMurray, Alberta, Canada. In order to quantify the grade uncertainty transferred into the mine plans using Methods 1 and 2 the following steps are followed:

2.1. Geostatistical Modeling

A sufficient number of realizations must be considered for the purpose of mine planing; otherwise, there may be undue reliance on some stochastic features. The steps presented by Leuangthong *et al.* (2004) are followed for geostatistical modelling of an oil sand deposit using GSLIB (Deutsch and Journel, 1998) software catalogue to create conditional simulated realizations. The steps presented by Leuangthong et al. (2004) (see Fig. 2) are:

1. Analyze of correlation structure- This investigates whether a transformation of the vertical coordinate system is required, in order to determine the true continuity structure of the deposit. Determination of the correct grid is dependent on the correlation grid that yields the maximum horizontal continuity.
2. *Decluster drillhole data distribution-* The relevant statistics must be deemed representative of the deposit prior to modelling. Declustering may be employed to determine the summary statistics that are representative of the field.
3. *Variography-* Model the spatial continuity of the normal scores of the bitumen grade using variograms. Directional experimental variograms are calculated and fit. The Azimuths of major and minor directions are 50 and 140 degrees. Figure 3 shows the experimental and the fitted variogram models in major (Fig. 3a), minor (Fig. 3b) and vertical (Fig. 3c) directions.

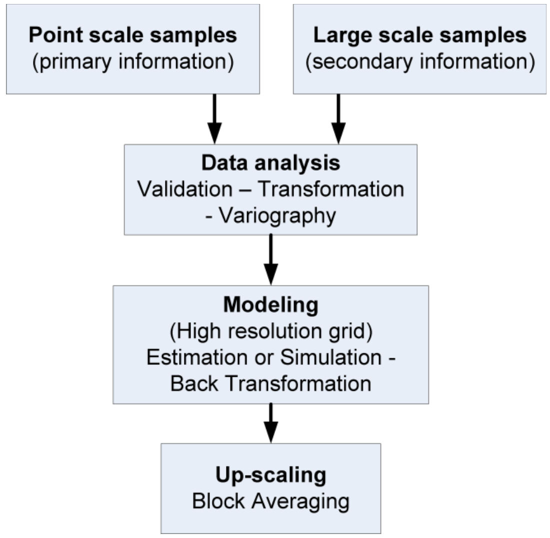

Fig. 2. Geostatistical modeling stages.

4. *Estimation*- Perform estimation and cross validation using kriging as checks against simulation results. Ordinary kriging is used to estimate the bitumen grade (with no normal score transform) at each block location.
5. Fig. 4a and 4b illustrate the map of the bitumen grade for the kriged and the E-type models. As expected, the E-type model is smoother than the kriged model. Multiple realizations of the bitumen grade are generated using Sequential Gaussian Simulation (SGS) (Isaaks and Srivastava, 1989) at a very high resolution three-dimensional grid at the point scale, this method is the means of constructing uncertainty models of bitumen grades.

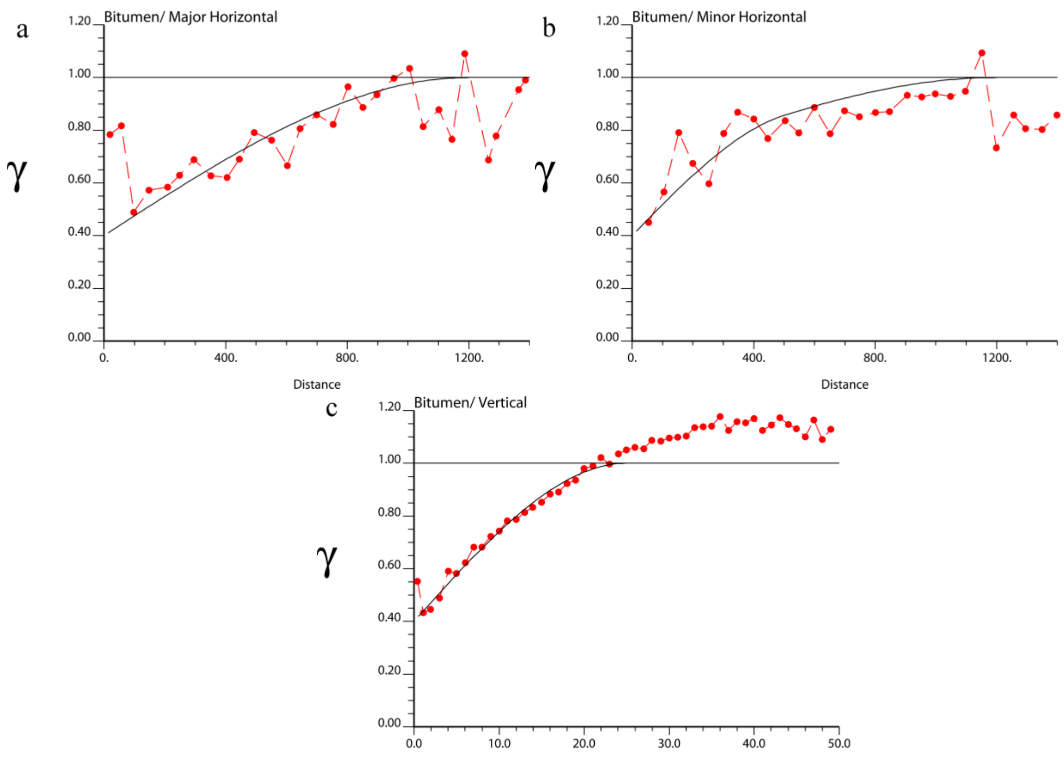

Fig. 3. Experimental directional variograms (dots) and the fitted variogram models (solid lines), distance units in meters.

6. Check simulation results against the input data and compare results against the Kriging model. We check the quality of geo-model by histogram and variogram reproduction. Fig. 5a to 5c show the variogram reproduction at major and minor horizontal, and vertical directions.
7. The block dimensions for mine planning are selected according to the exploration drilling pattern, ore body geology, mine equipment and anticipated operating conditions. The sizes of the blocks used in mine planning are a function of the selective mining unit (SMU). The high resolution grid is up-scaled to get the correct block scale values. Arithmetic averaging of point scale grades provides the up-scaled SMU grades (Fig. 2).

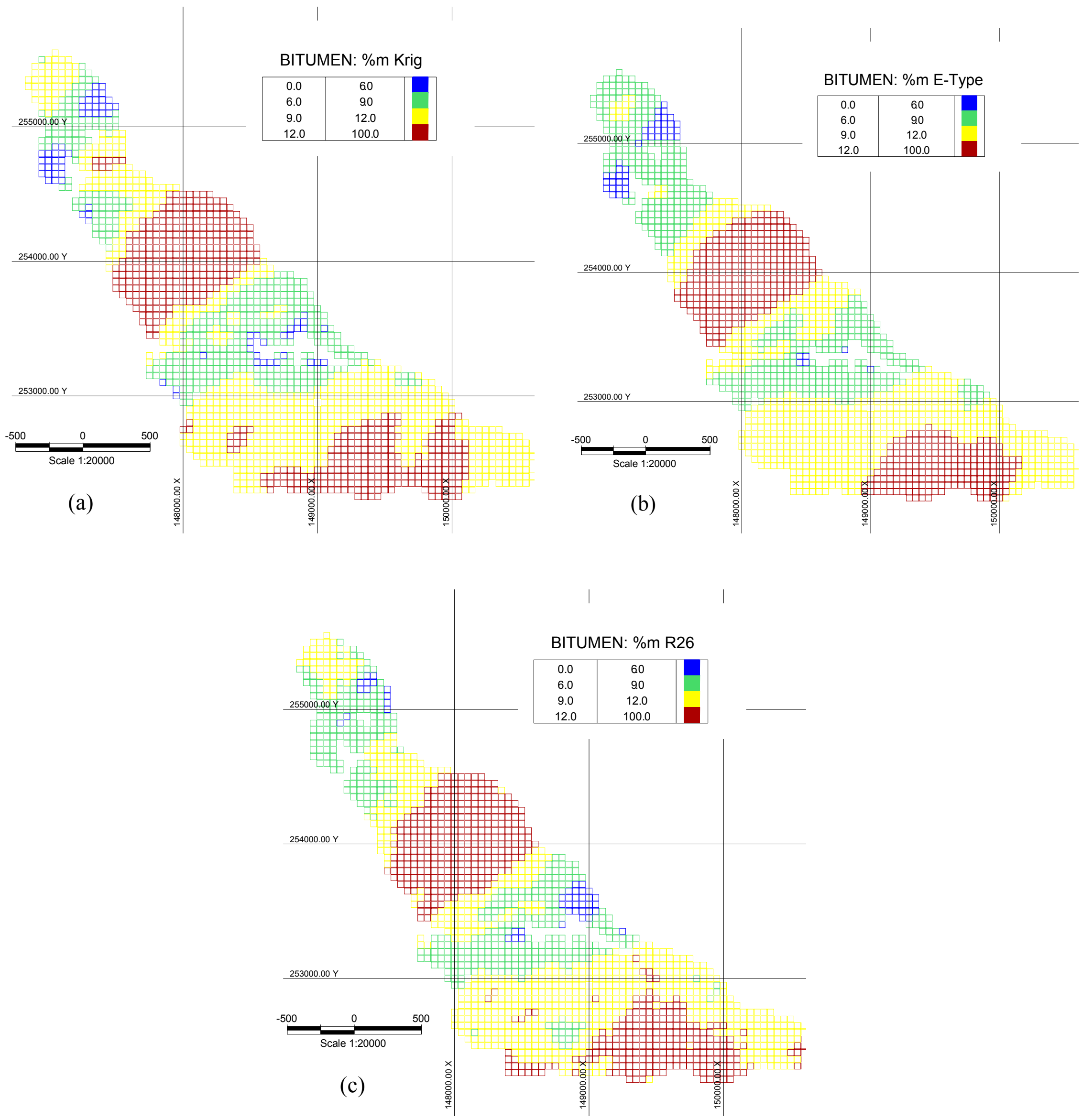

Fig. 4. Plan view at 260m; (a) Kriging model, (b) E-type model, (c) realization 26.

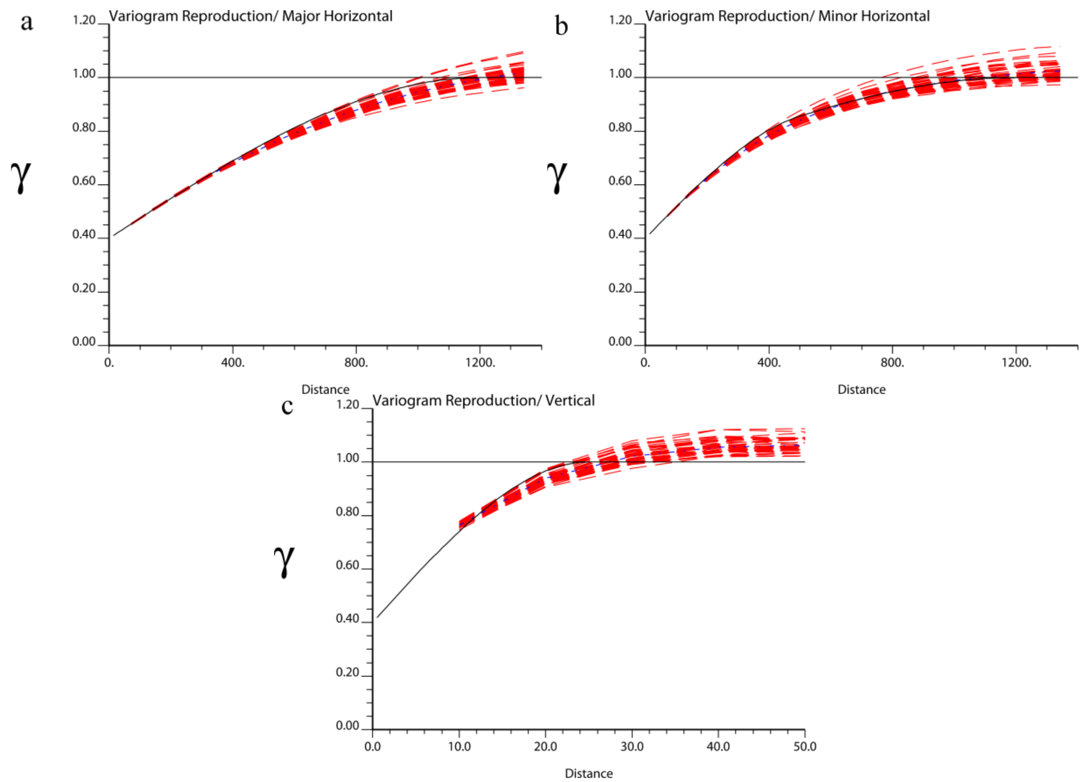

Fig. 5. Variogram reproduction of simulation realizations (red dash lines) and reference variogram model (black line).

2.2. Optimal final pit outline design

The final pit limit design is carried out based on the industry standard Lerchs and Grossmann algorithm (Lerchs and Grossmann, 1965) using the Whittle strategic mine planning software (Gemcom Software International, 1998-2008). The kriged, E-type, and fifty SGS realizations models are imported into the Whittle software. The ultimate pit limit design is carried out based on the Syncrude's costs in CAN$/bbl of sweet blend for the third quarter of 2008 (Jaremko 2009). Price of oil was considered US$45 with an exchange rate of 1.25:1 equal to CAN $56.25/bbl SSB for the same time period. We assume that every two tonnes of oil sands with an average grade of 10% mass will produce one barrel of sweet blend, which is approximately 200 kg. We also assume a density of 2.16 tonne/m³ for oil sands, and a density of 2.1 tonne/m³ for waste material, including clay and sand. Table 1 summarizes the costs used in the pit limit design. The mining cost of $12.18 is per tonne of oil sands ore, we assumed a stripping ratio of 1.8:1, and this would lead to a cost of $4.6/tonne of extracted material (ore and waste).

Table 1. Summary of costs used in pit limit design.

Description	Value	Description	Value
Mining Costs (CAN $/ bbl SSB)	24.35	Mining Costs (CAN $/tonne)	12.18
Upgrading Costs (CAN $/ bbl SSB)	10.05	Upgrading Costs (CAN $/tonne)	5.025
Others (CAN $/bbl SSB)	1.5	Others (CAN $/tonne)	0.75
Total Costs (CAN $/ bbl SSB)	35.9	Total Costs (CAN $/tonne)	17.28

Table 2. Final pit limit and mine planning input parameters.

Description	Value	Description	Value
Cutoff grade (%mass bitumen)	6	Processing limit (M tonne/year)	20
Mining recovery fraction	0.88	Mining limit (M tonne/year)	35
Processing recovery factor	0.95	Overall slope (degrees)	20
Minimum mining width (m)	150	Pre-stripping (years)	5

Table 2 shows the pit design and production scheduling input parameters. Thirty three pit shells are generated using 49 fixed revenue factors ranging between 0.1 to 2.5, based on the Kriging block model. The number of pit shells is reduced to 14 after applying the minimum mining width of 150 meters for the final pit and the intermediate pits. Table 3 summarizes the information related to the Kriging final pit limit at 6% bitumen cut-off grade. Fig. 6 illustrates the histogram of total tonnage of material within the optimal final pit limits. Ultimate pits are generated for each SGS realization. E-type and Kriging results are marked by a solid circle and a hollow circle respectively.

Table 3. Material in the final pit using the Kriging block model.

Description	Value
Total tonnage of material (M tonne)	653.61
Tonnage of ore (M tonne)	280.5
Tonnage of material below cutoff (M tonne)	37.4
Tonnage of waste (M tonne)	335.71
Bitumen recovered (M tonne)	27.52
Stripping ratio (waste:ore)	1.33

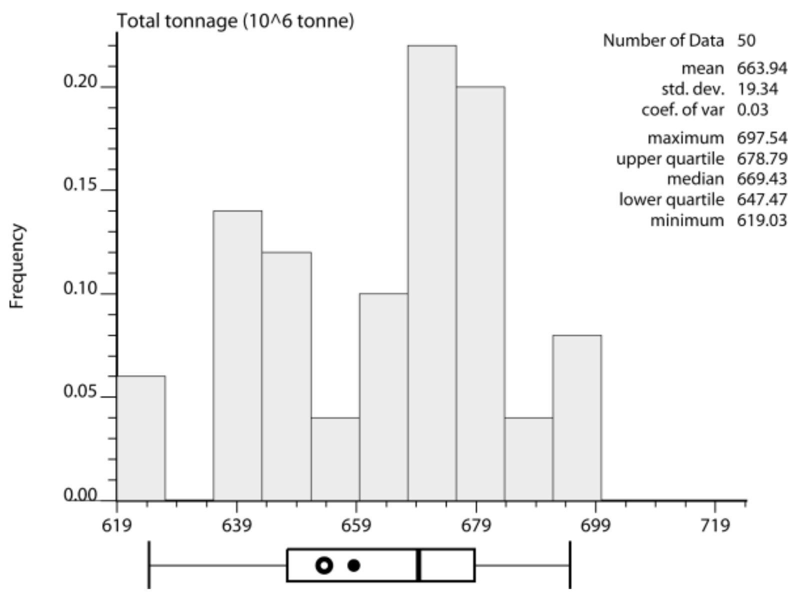

Fig. 6. Histogram of total tonnage of material within the final pit limits.

2.3. Production Scheduling

The kriged model is the basis for production scheduling. The aim is to maintain a uniform processing feed throughout the mine life. Four push backs are defined with a fixed lead of three benches between pushbacks. Five years of pre-stripping is considered to provide enough operating space and ore availability. No stockpile is defined and the target production is set to 20 million tonnes of ore per year with a mining capacity of 35 million tonnes per year. Figure 8 illustrates the kriged block model schedule over 21 years of mine-life. This schedule is the basis of Method 2. The schedule is applied to all the 50 SGS realizations within the krig fixed optimal pit limit. In Method 1, the final pit limits is designed for E-type model and all the fifty realizations with the exact same input variables.

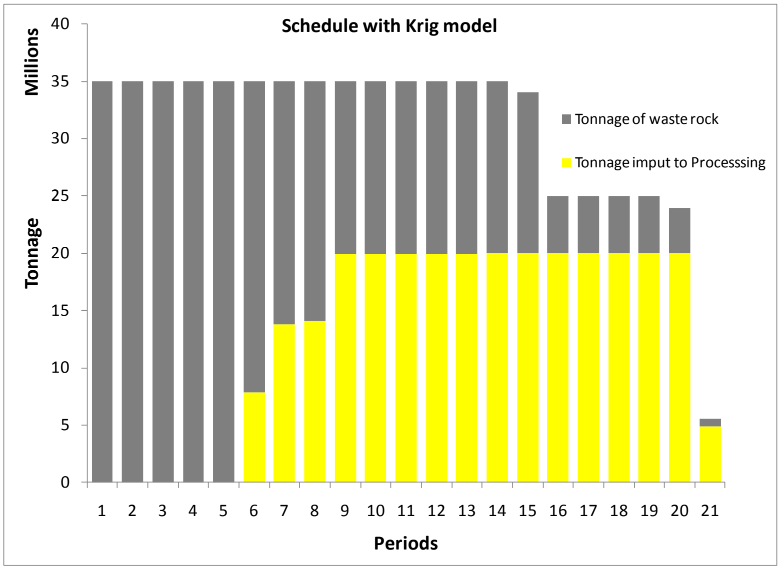

Fig. 7. Production schedule based on the Kriging block model.

3. Discussion of results

Conditional simulation enables us to provide a set of production scenarios, which capture and asses the uncertainty in the final pit outline, net present value, production targets, and the head grades. The realizations provide equally probable scenarios to calculate different outcomes in terms of NPV and production. The probability of each block being extracted in each planning period and the probability that the block would be treated as ore or waste in the respective period is calculated. The histograms and box plots for the following production schedule output variables are presented and discussed for Methods 1 and 2:

1. Overall stripping ratio.
2. Total tonnage of ore.
3. Average grade of ore.
4. Net present value.

Fig. 6 illustrates the uncertainty within the final pit limits. The krig final pit (the hollow circle) has 653 million tonnes, which is less than the average amount of 663 million tonnes, recorded for the simulation realizations. The inter quartile limits are 647 and 678 million tonnes. The stripping ratio of the krig pit is 1.33 (Fig. 8b), which is near the 1.35 stripping ratio of the lower quartile of the simulation realizations. As expected, Method 1 has a larger standard deviation compared against Method 2 (Fig. 8).

Fig. 9 shows the histogram and box plot of ore at final pit. The amount of ore in the kriged and Etype models is close when different schedules are generated for each of them (Fig. 9a). The tonnage of ore in the kriged model is almost the same as Etype where the same schedule is followed (Fig. 9b).

The histogram of average input grade to the mill for kiging (solid circle), Etype (hollow circle) and SGS realizations are presented in Fig. 10. The average grade of the kriged model is more than the Etype model and also it is higher than the third quartile of the realizations of both methods. The grade uncertainty is clearly illustrated in this figure. The average grade of the Etype is less than kriged model and even less than lower quartile of average grade of realizations.

Fig. 11 Figure 12 shows the input average head grade for each period. Kriged model (bold solid line), Etype model (bold dashed line) and the simulation realizations (dashed lines). As expected, the grade fluctuations of the first method (Fig. 11a) are greater than second method (Fig. 11b).

Histogram and box plot of produced ore is showed in Figure 13. 27.52 million tonnes of bitumen is produced by kriged block model. 26.86 and 26.97 million tonnes of bitumen was produced by Etype model for method 1 and 2 respectively. The kriged block model produced more bitumen than third quartile of realizations at both method.

Fig. 13 shows the tonnage of feed to the plant. The deviation from target production is also presented in Fig. 14. There are under productions at first years for each of two methods. In addition, in first method, we have hard constraint for upper limit of plant feed, there is no over production for each of realizations (Fig. 13a). When the same schedule is followed for each of realizations, at some realizations, there are overproduction between years 8 to 16 (Fig. 13b). One should take into account that the shortfall in production has happened although we have used five years of pre-stripping. The effect of the grade uncertainty on the production targets would be more severe if the pre-stripping strategy was not adopted.

Fig. 14 illustrates the box plot for the plant feed, the percentage deviation from the target production, and the probability that we would not meet the target production. There is a relatively high probability to not meeting the target production at first and last years of mine life. If the krig schedule would be followed, the probability of not meeting the target production for some middle age of mine life will be increase. There is a 1.2 and 2.1 percent probability of not meeting the target production for years 13 and 14 respectively.

Fig. 15 shows the histogram of cumulated discounted cash flow. E-type and krig results are marked by solid circle and hollow circle respectively. NPV of kriged block model is 847 million dollars, where the NPV of Etype method is 724 and 780 million dollars for method 1 and 2 respectively. The NPV of kriged model is also more that third quartile of realizations.

Fig. 16 illustrates the discounted cash flow over the years. Cumulative discounted cash flow for 50 realizations and the kriged and Etype models are showed by dashed lines, solid line and dashed blue line respectively. The optimum final pit limit is calculated for each realization; therefore, there are some realizations that the mine life is 22 years, where for most realizations, kriged and the Etype model the mine life is 21 years (Fig. 16a). In method 2, the same schedule as kriged model is followed. Therefore the mine life is 21 years for all realizations and Etype model (Fig. 16b).

Fig. 17 shows the box plot of discounted cash flow for all realizations. Kriging model is showed by solid line and dashed line is used for E-type model. The probability of meeting the NPV from Kriging model is also presented. As it is clearly illustrated, based on 50 realizations, there is very low chance to reach the NPV of Kriging model for each period. There is not any realization at first method to exceed the NPV of Kriging model. Where, there is only one realization (2%) that exceeds the final NPV of Kriging model for second method (Fig. 17b). This graph shows generated schedule based on the kriging model and generally based on only one block model can produce unrealistic and unachievable NPV in present of grade uncertainty.

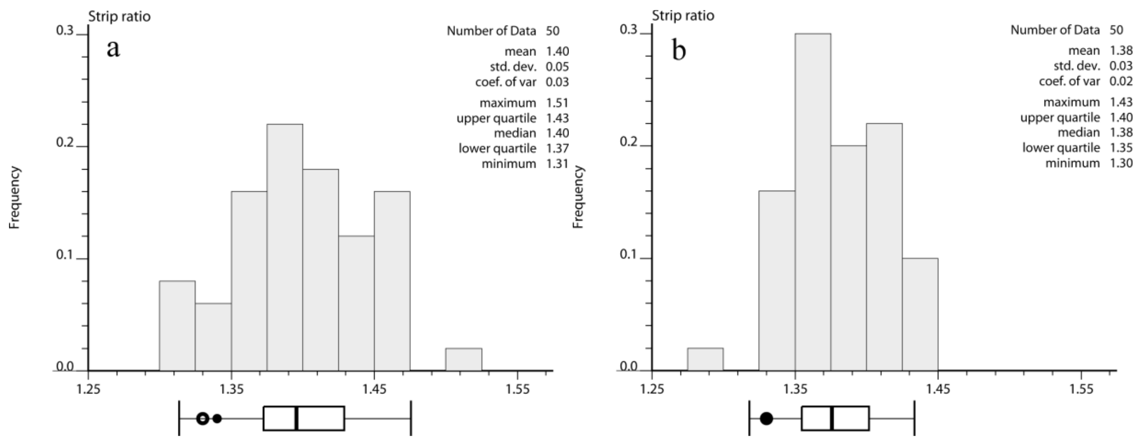

Fig. 8. Histograms and box plots of overall stripping ratio. (a) schedules are generated for each block model separately. (b) Kriging schedule applied to all realizations. E-type and Kriging results are marked by solid circle and hollow circle respectively.

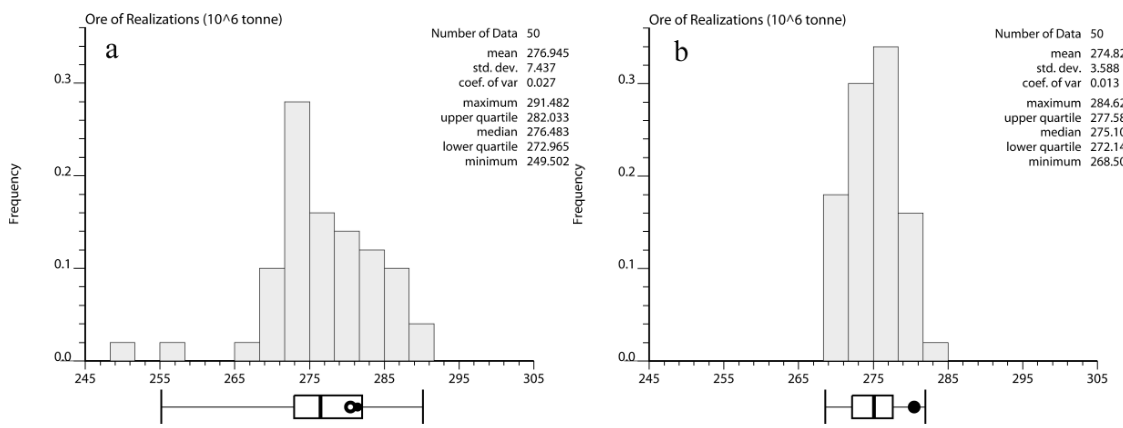

Fig. 9. Histograms and box plots of total tonnage of ore. (a) schedules are generated for each block model separately. (b) Kriging schedule applied to all realizations. E-type and Kriging results are marked by solid circle and hollow circle respectively.

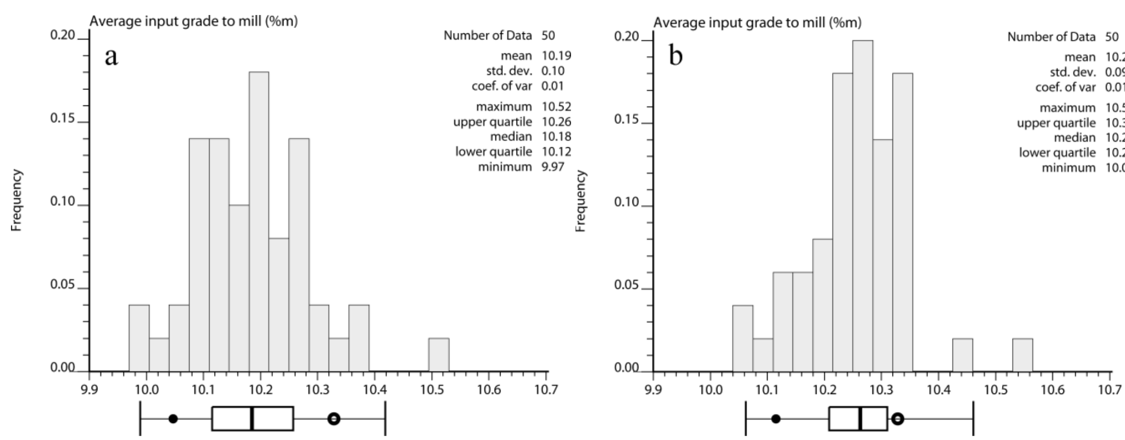

Fig. 10. Histograms and box plots of average head grade in bitumen %mass. (a) schedules are generated for each block model separately. (b) Kriging schedule applied to all realizations. E-type and Kriging results are marked by solid circle and hollow circle respectively.

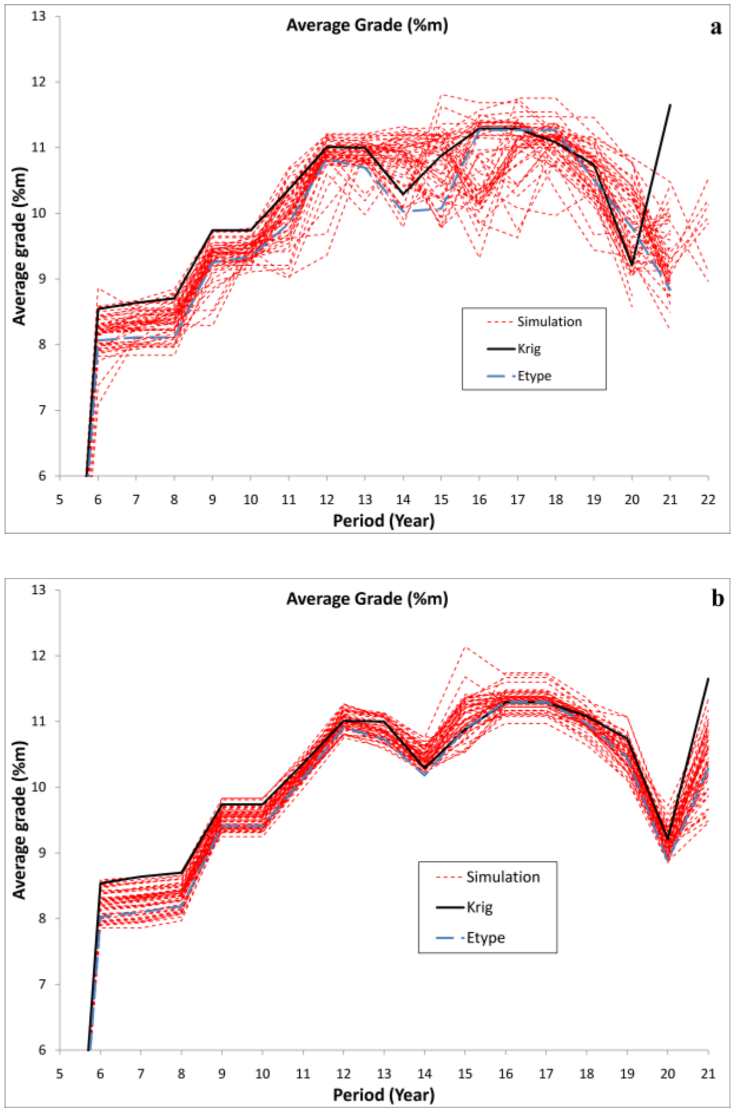

Fig. 11. Head grade simulation realizations, Kriging, and E-type models. (a) schedules are generated for each block model separately. (b) Kriging schedule applied to all realizations. Kriging result is marked by solid line.

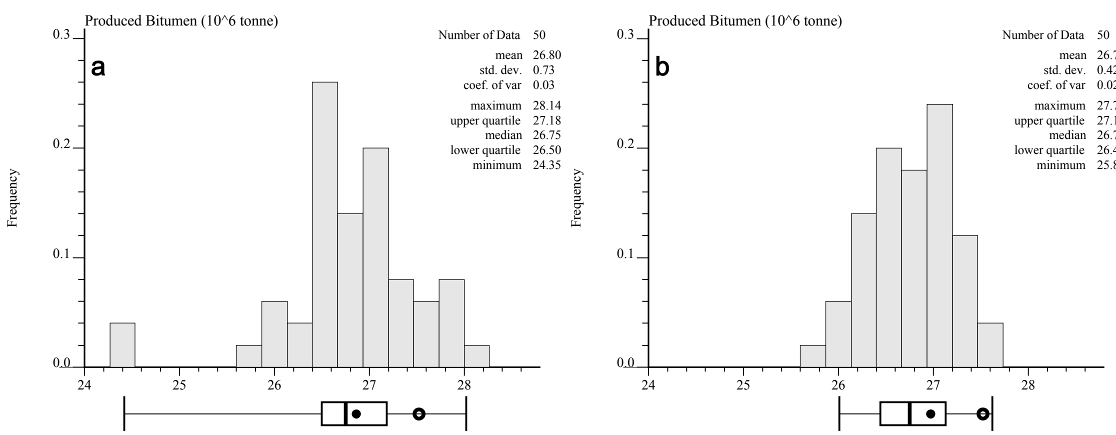

Fig. 12. Histograms and box plots of tonnage of bitumen produced. (a) schedules are generated for each block model separately. (b) Kriging schedule applied to all realizations. E-type and Kriging results are marked by solid circle and hollow circle respectively.

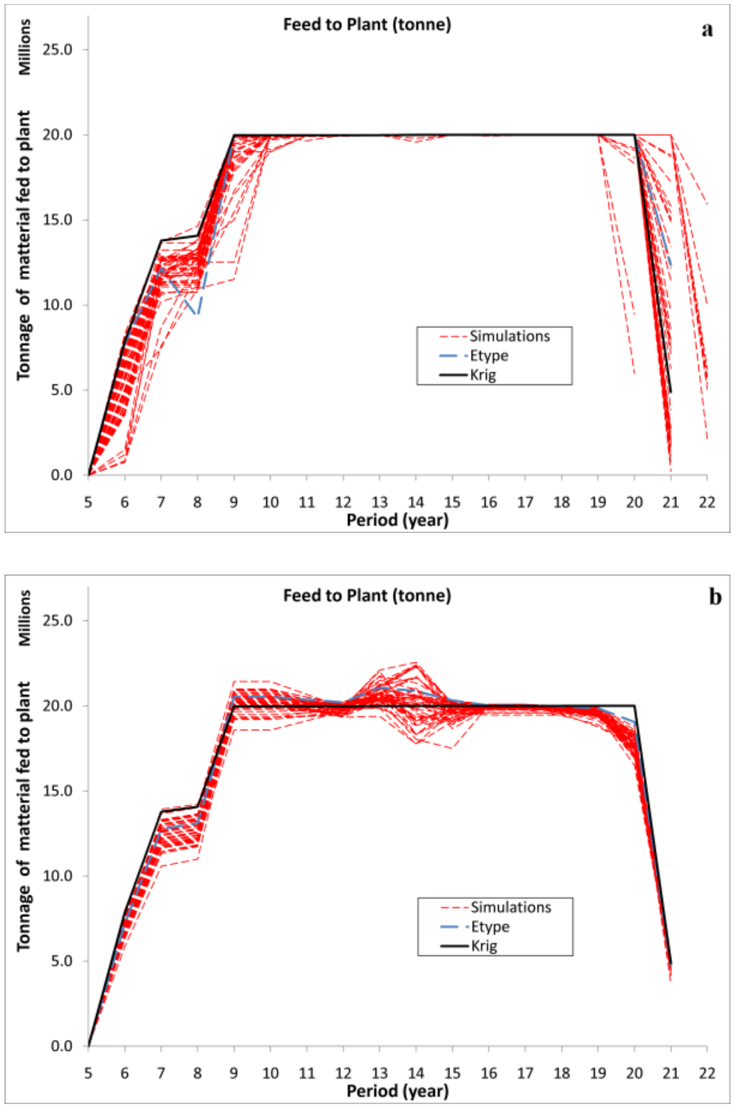

Fig. 13. Plant feed (realizations dash lines), Kriging Model (solid line) and E-type (blue dash line). (a) schedules are generated for each block model separately. (b) Kriging schedule applied to all realizations.

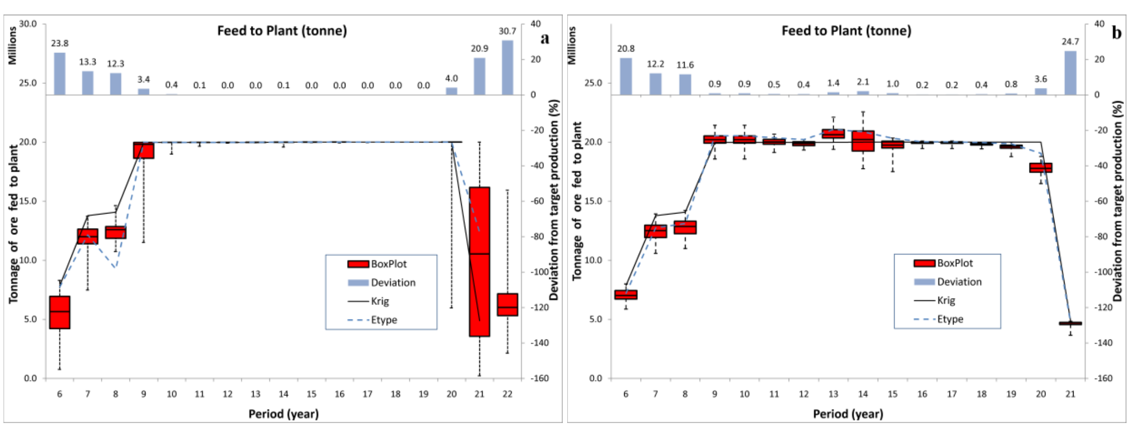

Fig. 14. Box plot of the simulation plant feed, Kriging schedule (solid line) and E-type (blue dash line). Deviations from target production are reported in percentage. (a) schedules are generated for each block model separately. (b) Kriging schedule applied to all realizations.

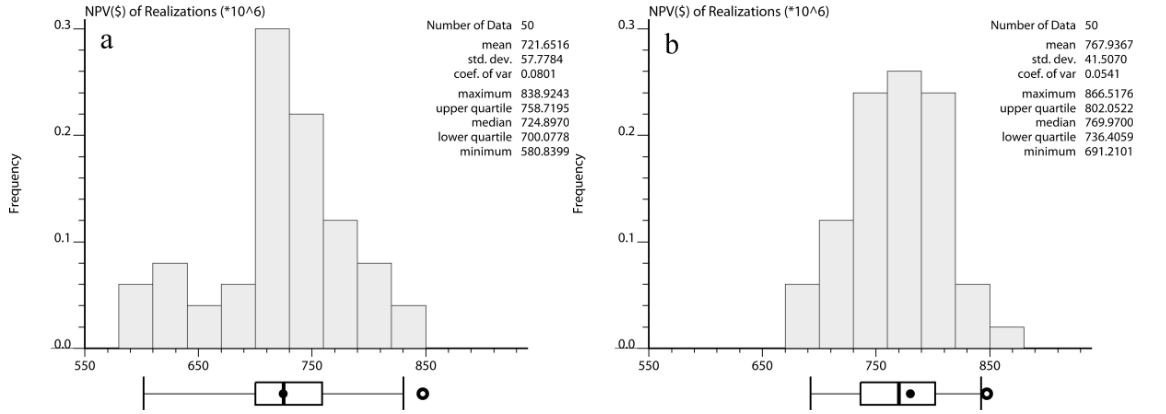

Fig. 15. Histograms and box plots of NPV in billion dollars. (a) schedules are generated for each block model separately. (b) Kriging schedule applied to all realizations. E-type and Kriging results are marked by solid circle and hollow circle respectively.

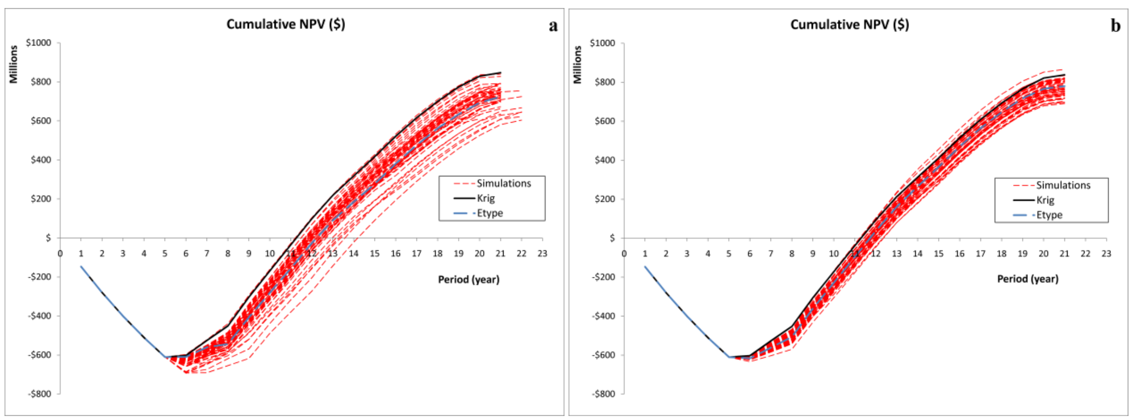

Fig. 16. Cumulative discounted cash flow for 50 realizations (dashed lines) and Kriging model (solid line). (a) schedules are generated for each block model separately. (b) Kriging schedule applied to all realizations.

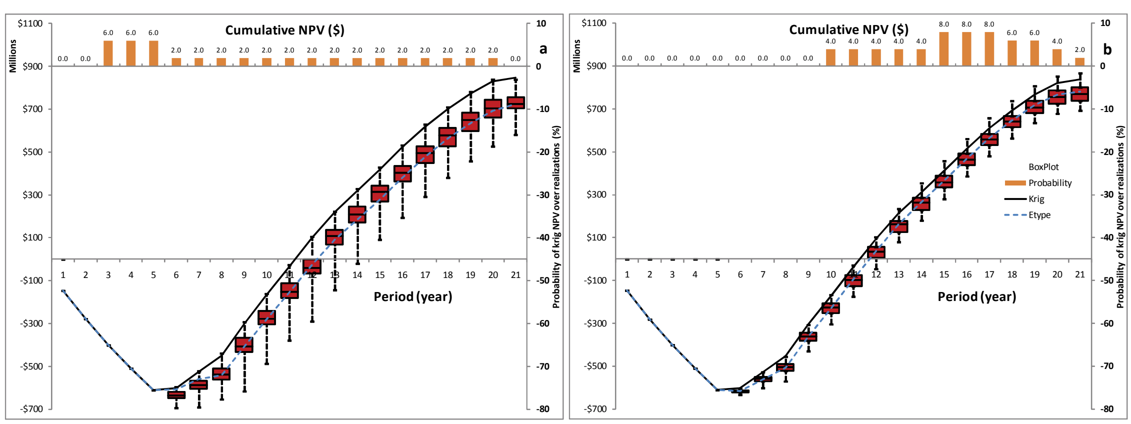

Fig. 17. Box plot of the Cumulative discounted cash flow for 50 realizations (dashed lines) and Kriging model (solid line). Probabilities of reaching the Kriging NPV over realizations are presented in percentage. (a) schedules are generated for each block model separately. (b) Kriging schedule applied to all realizations.

4. MILP formulation

The discounted profit of mining a block is represented by Eq. (1).

$$\text{discountedprofit} = \text{discounted revenue} - \text{discounted costs} \tag{1}$$

Askari-Nasab and Awuah-Offeri (2009) present four mixed integer linear programming models without taking into account the grade uncertainty. The proposed model in this paper uses Askari-Nasab and Awuah-Offeri (2009) fourth model as the starting point.

The profit from mining a block depends on the value of the block and the costs incurred in mining and processing. The cost of mining a block is a function of its location, which characterizes how deep the block is located relative to the surface and how far it is relative to its final dump. The spatial factor can be applied as a mining cost adjustment factor for each block according to its location to the surface. The discounted profit from a block is equal to the discounted revenue generated by selling the final product contained in block n minus all the discounted costs involved in extracting block.

Grade uncertainty causes shortfalls from target production levels. Therefore, to obtain an optimum schedule, NPV must be maximized and the deviation from target productions must be minimized simultaneously among all simulation realizations.

$$\begin{cases} \text{Max. NPV} \\ \text{Min. Deviationfrom target production} \end{cases}$$

A new profit function is defined by Eq. (2).

New discounted profit = discounted profit – penalty cost for over and under production (2)

The objective function of the mathematical programming formulation is presented by Eq (3), All notations are defined in the appendix.

$$Max \sum_{t=1}^{T} \left\{ \sum_{n=1}^{N} \left(V_n^t \times z_n^t - Q_{l,n}^t \times y_n^t \right) - \frac{1}{L} \sum_{l=1}^{L} \left(c_{op}^t \times op_l^t + c_{up}^t \times up_l^t \right) \right\} \tag{3}$$

Subject to:

$$g_l^{t,e} \le \left(\sum_{n=1}^{N} g_n^e \times o_n \Big/ \sum_{n=1}^{N} o_n \right) \times z_n^t \le g_u^{t,e} \qquad \forall\, t = 1,2,\ldots,T \;,\; e = 1,2,\ldots,E \tag{4}$$

$$\sum_{n=1}^{N} \left(o_{n,l} \times z_n^t - op_l^t \right) \le P_u^t \qquad \forall\, t = 1,2,\ldots,T \;,\; l = 1,2,\ldots,L \tag{5}$$

$$\sum_{n=1}^{N} \left(o_{n,l} \times z_n^t - up_l^t \right) \ge P_l^t \qquad \forall\, t = 1,2,\ldots,T \;,\; l = 1,2,\ldots,L \tag{6}$$

$$m_l^t \le \sum_{n=1}^{N} \left(o_n + w_n \right) \times z_n^t \le m_u^t \qquad \forall\, t = 1,2,\ldots,T \tag{7}$$

$$z_n^t \le y_n^t \qquad \forall\, t = 1,2,\ldots,T \;,\; n = 1,2,\ldots,N \tag{8}$$

$$a_n^t - \sum_{i=1}^{t} y_l^i \le 0 \qquad \forall\, t = 1,2,\ldots,T;\; n = 1,2,\ldots,N;\; l = 1,2,\ldots,C_n(L) \tag{9}$$

$$\sum_{i=1}^{t} y_n^i - a_n^t \le 0 \qquad \forall\, t = 1,2,\ldots,T \;\;,\; n = 1,2,\ldots,N \tag{10}$$

Where V_n^t is the expected discounted revenue over all simulation realizations and $Q_{l,n}^t$ is the expected discounted cost over all simulation realizations as is shown by Eq. (11):

$$V_n^t = \frac{1}{L}\sum_{l=1}^{L} v_{n,l}^t \qquad Q_n^t = \frac{1}{L}\sum_{l=1}^{L} q_{n,l}^t \tag{11}$$

The discounted revenue and discounted cost can be rewritten as Eq. (12) and Eq.(13).

$$v_{l,n}^t = \sum_{e=1}^{E} o_{l,n} \times g_n^e \times r^{e,t} \times \left(p^{e,t} - cs^{e,t}\right) - \sum_{e=1}^{E} o_{l,n} \times cp^{e,t} \qquad \forall l = 1,\ldots,L \;\; t = 1,\ldots,T \;\; n = 1,\ldots,N \tag{12}$$

$$q_{l,n}^t = \left(o_{l,n} + w_{l,n}\right) \times cm^t \qquad \forall l = 1,\ldots,L \;\; t = 1,\ldots,T \;\; n = 1,\ldots,N \tag{13}$$

Eq. (4) is grade blending constraints; these inequalities ensure that the head grade of the elements of interest and contaminants are within the desired range in each period. There are two equations (upper bound and lower bound) per element per scheduling period in Eq.(4). Eqs. (5) and (6) are processing capacity constraints; these inequalities ensure that the total ore processed in each period is within the acceptable range of processing plant capacity. There are two equations (upper bound and lower) per period per ore type. Eq. (7) is mining constraints; these inequalities ensure that the total tonnage of material mined (ore, waste, overburden, and undefined waste) in each period is within the acceptable range of mining equipment capacity in that period. There are two equations (upper bound and lower bound) per period. Eq. (8) represents inequalities that ensure the amount of ore of any block which is processed in any given period is less than or equal to the amount of rock extracted in the considered time period.

Eqs. (9) and (10) control the relationship of block extraction precedence by binary integer variables at block level. This model only requires the set of immediate predecessors' blocks on top of each block to model the order of block extraction relationship. This is presented by set $C_n(L)$ in Eq.(9).

In this model, the number of variables is equal to the number of blocks multiplied by the number of periods. Boland *et al.* (2009) and Askari-Nasab and Awuah-Offeri (2009) tried to solve this problem with clustering the blocks into aggregates to reduce the number of variables. Using grade aggregation methodology similar blocks are clustered into one group. Clustering blocks into mining-cuts is done without sacrificing the accuracy of the estimated (or simulated) values. The mining-cut clustering algorithm developed uses fuzzy logic clustering (Kaufman and Rousseeuw, 1990). Coordinates of each mining-cut has been represented by the center of the cut and its location.

5. Conclusion

We used two methods to show the impact of grade uncertainty at mine planning. First, using the kriged model for an oil sand deposit, an optimal final pit limit is generated. Sequential Gaussian Simulation is used to generate fifty realizations. An optimum final pit limits design is carried out for each SGS realization based on same parameters and technique that are fixed with kriged block model. Afterwards, the long-term schedule of each final pit shell is generated. Uncertainty in the final pit outline, net present value, production targets, and the head grade are assessed and presented.

In the second method, for each SGS realization, schedule that generated from kriged model was followed. The results show that there is significant uncertainty in the long-term production schedules. In addition, the long-term schedule based on one particular simulated ore body model is not optimal for other simulated geological models. The mine planning procedure is not a linear process and the mine plan generated based on the kriged estimate is not the expected result from all of the simulated realizations.

One of the main aspects of this study is to show the impact of grade uncertainty on mine planning. The study is not aimed to compare the simulation with kriging, because it is well-known that kriging is conditionally biased (Isaaks, 2005) and on the other hand "there is no conditional bias of simulation when the simulation results are used correctly" (McLennan and Deutsch, 2004). Conditional biasness of kriging can be reduced by tuning estimation parameter but it cannot be eliminated. Grade-Tonnage curve is the good tool to check the impact of kriging biasness. Fig. 18 shows the grade tonnage curve of simulation realization (dashed lines), krig (bold solid line) and Etype (bold dashed line). The systematic biasness of kriging was tried to be minimized but still the there are differences between kriging and simulation results. Also Etype is slightly different than kriging; Theoretically Etype model is identical with simple kriging result at Gaussian space (Journel and Huijbregts, 1981).

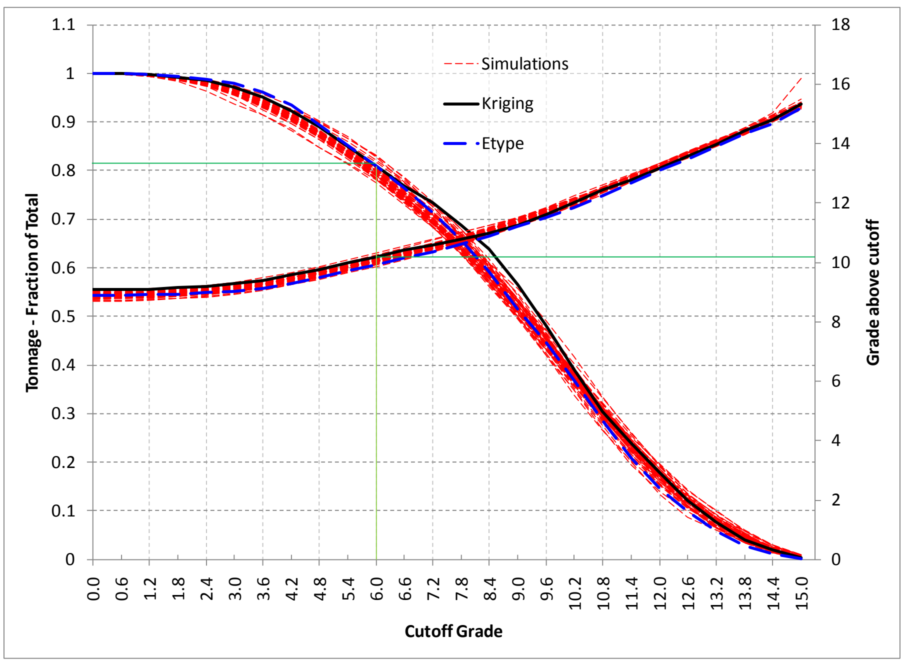

Fig. 18. Grade tonnage curve of Kriging, simulation realizations and E-type

6. Reference:

[1] Askari-Nasab, H. and Awuah-Offeri, K. (2009). Mixed integer programming formulations for open pit production scheduling. *MOL Report one*, *1*, 1-31.

[2] Boland, N., Dumitrescu, I., Froyland, G., and Gleixner, A. M. (2009). LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity. *Comput.Oper.Res.*, *36*,(4), 1064-1089.

[3] Deutsch, C. V. and Journel, A. G. (1998). GSLIB : geostatistical software library and user's guide. in *Applied geostatistics series*, vol. 2. New York: Oxford University Press, pp. 369.

[4] Deutsch, C. V. and Journel, A. G. (1998). *GSLIB geostatistical software library and user's guide.* Oxford University Press, New York.

[5] Dimitrakopoulos, R., Farrelly, C. T., and Godoy, M. (2001). Moving forward from traditional optimization: Grade uncertainty and risk effects in open-pit design. *Transactions of the Institution of Mining and Metallurgy, Section A: Mining Industry*, *111*, 82-88.

[6] Dimitrakopoulos, R. and Ramazan, S. (2004). Uncertainty based production scheduling in open pit mining. 106-112.

[7] Dimitrakopoulos, R. and Ramazan, S. (2008). Stochastic integer programming for optimising long term production schedules of open pit mines: methods, application and value of stochastic solutions. *Mining Technology : IMM Transactions section A*, *117*, 155-160.

[8] Dowd, P. A. (1994). Risk assessment in reserve estimation and open-pit planning. *Transactions of the Institution of Mining and Metallurgy Section a-Mining Industry*, *103*, A148-A154.

[9] Gemcom Software International, I. (1998-2008). Whittle strategic mine planning software. 4.2 ed. Vancouver, B.C.: Gemcom Software International.

[10] Godoy, M. and Dimitrakopoulos, R. (2003). Managing risk and waste mining in long-term production scheduling of open pit mine. *SME Annual Meeting & Exhibition*, *316*, 43-50.

[11] Goovaerts, P. (1997). *Geostatistics for natural resources evaluation.* Oxford University Press, New York, Pages 483.

[12] Isaaks, E. (2005). The Kriging Oxymoron: A conditionally unbiased and accurate predictor (2nd edition). *Geostatistics Banff 2004, Vols 1 and 2*, *14*, 363-374.

[13] Isaaks, E. H. and Srivastava, R. M. (1989). *Applied geostatistics.* Oxford University Press, New York, N.Y. ; Oxford, England, Pages 561.

[14] Journel, A. G. and Huijbregts, C. J. (1981). *Mining geostatistics.* Academic Press, London, Pages 600.

[15] Kaufman, L. and Rousseeuw, P. J. (1990). *Finding groups in data : an introduction to cluster analysis.* Wiley, New York, Pages 342.

[16] Leite, A. and Dimitrakopoulos, R. (2007). Stochastic optimisation model for open pit mine planning: Application and risk analysis at copper deposit. *Transactions of the Institutions of Mining and Metallurgy, Section A: Mining Technology*, *116*,(3), 109-118.

[17] Lerchs, H. and Grossmann, I. F. (1965). Optimum design of open-pit mines. *The Canadian Mining and Metallurgical Bulletin, Transactions*, *LXVIII*, 17-24.

[18] Leuangthong, O., Schnetzler, E., and Deutsch, C. V. (2004). Geostatistical modeling of mcmurray oil sands deposits. 2004 CCG annual report papers, Centre for Computational Geostatistics, Edmonton, September 2004, 309-1 to 309-10.

[19] McLennan, J. A. and Deutsch, C. V. (2004). Conditional non-bias of geostatistical simulation for estimation of recoverable reserves. *CIM bulletin*, *97*,(1080).

[20] Osanloo, M., Gholamnejad, J., and Karimi, B. (2008). Long-term open pit mine production planning: a review of models and algorithms. *International Journal of Mining, Reclamation and Environment*, *22*,(1), 3-35.

[21] Ravenscroft, P. J. (1992). Risk analysis for mine scheduling by conditional simulation. *The Canadian Mining and metallurgical Bulletin, Transactions.(Sec. A: Min. Industry)*, *101*, 82-88.

[22] Rossi, M. E. and Parker, H. M. (1994). *Estimating recoverable reserves: Is it hopeless?, Geostatistics for the Next Century*, in Proceedings of Kluwer Academic, Dordrecht ; Boston.

[23] Vallee, M. (2000). Mineral resource + engineering, economic and legal feasibility = ore reserve. *CIM bulletin*, *90*, 53-61.

7. Appendix

$v_{l,n}^{t}$	the discounted revenue generated by selling the final product within block n in period t at realization number l minus the extra discounted cost of mining all the material in block n as ore and processing.
$q_{l,n}^{t}$	the discounted cost of mining all the material as waste in block n in period t at realization number l.
E	number of element of interests in each block.
T	Total number of periods.
L	Total number of simulation realizations.
$o_{l,n}$	Ore tonnage in block n at realization l.
$g_{l,n}^{e}$	Average grade of element e in one portion of block n at realization l.
$r^{e,t}$	processing recovery, the proportion of element e recovered in time period t.
$p^{e,t}$	Price in present value terms obtainable per unit of product, element e.
$cs^{e,t}$	Selling cost in present value terms obtainable per unit of product, element e.
$cp^{e,t}$	Extra cost in present value terms per unit of production, element e.
$w_{l,n}$	Waste tonnage in block n at realization l.
cm^{t}	Cost in present value terms of mining a tone of waste in period t.
$a_{n}^{t} \in \{0,1\}$	Binary integer variable controlling the precedence of extraction of blocks. It is equal to one if extraction of block n has started by or in period t, otherwise it is zero.
$z_{n}^{t} \in [0,1]$	continues variable, representing the portion of bock n to be extracted as ore and processed in period t.
$y_{n}^{t} \in [0,1]$	continues variable, representing the portion of bock n to be mined in period in period t, fraction of y characterizes both ore and waste.
op_{l}^{t}	is the over produced amount of ore tonnage above a desired tonnage, or upper limit, in period t and realization number l.
up_{l}^{t}	is the under produced amount of ore tonnage bellow a desired tonnage, or upper limit, in period t and realization number l.
c_{op}^{t}	is the discounted unit cost of op^{t} at period t.
c_{up}^{t}	is the discounted unit cost of up_{l}^{t} at period t.
V_{n}^{t}	is the expected discounted revenue over all simulation realizations.
$Q_{l,n}^{t}$	is the expected discounted cost over all simulation realizations.