A mathematical model for short-term open pit mine planning

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Abstract

In this paper, a mixed integer model for the short-term scheduling of an open pit mine is formulated. The model includes a number of buffer stockpiles, waste dumps, and processes. The buffer stockpiles assist to smooth the process plant feed. Also, as blocks have different rock types, various processes and waste dumps are considered. In other words, extracted materials of blocks are sent to different destinations (waste dumps, processes, and stockpiles) based on their rock types. For validation of proposed formulation, a case study including 150 cuts are solved by CPLEX.

1. Introduction

One of the most important aspects of mine production planning is the short-term production planning/scheduling (Dimitrakopoulos and Ramazan, 2004). The objective of the short-term production scheduling is to determine the optimum sequence of extraction of blocks to meet the goals imposed by the long-term scheduling in each short-term period (output of long-term scheduling). In fact, the mine short-term production scheduling problem is the sequencing of each block's removal from the mine with respect to a variety of physical and economic constraints. Typically, these limitations are about to: the mining extraction sequence, mining, milling and refinery capacities, grades of mill feed and concentrates, and different operational requirements like minimum mining width (Caccetta and Hill, 2003).

On the other hand, one of the most challenging issues in the mine scheduling problems is the choice of solution methods. Typically, the short-term mine scheduling problems are complex. The complexities arise from the involvement of several continuous and binary variables and the large number of blocks to be extracted in the real-world problems. There are many research related to solution methods of the mine scheduling problems in the literature. These methods are not limited to but include:

1. Heuristics: Caccetta & Hill (2003) developed and implemented a graph-theoretic technique originally proposed by Lerchs and Grossman. They implemented a strategy including the application of a dynamic programming technique to "bound" the optimum. Also, Gershon (1987) proposed two heuristic methods; the first method is applicable in surface and underground mining with decisions on blending issues. Another method is applicable in general situations but mostly efficient in open pit mining problem.
2. Dynamic programming techniques: Tolwinski & Underwood (1996) proposed a model for the mine scheduling problem as a sequential optimization. Tolwinski & Underwood (1996) developed an algorithm for its solution. To overcome the difficulty related to large number states in the problem, they used a method based on the dynamic programming.

3. Mixed integer linear programming: Caccetta & Hill (2003) proposed a mix integer programming model for open pit mine scheduling problem. Also, Gershon (1983) developed a mixed-integer formulation, modeling the mine scheduling problem to optimize both the mine production sequencing and the mill blending and processing problems, simultaneously. Carlyle & Eaves (2001) formulated a maximization integer programming model for mining platinum and palladium at Stillwater Mining Company, which is able to reach near-optimal solutions without applying any special methods to lessen solution time.
4. And the applications of artificial intelligence algorithms: these algorithms include simulated annealing, genetic algorithms, and neural networks. For example, (Denby & Schefield (1995) applied the neural network algorithm in the mining context. Also, Denby et al. (1991) implemented the genetic algorithm in the underground mine scheduling.

Smith et al. (2003) created a production scheduling model for a copper and zinc underground mine at Mount Isa, Australia. The decision variables in Smith et al. (2003) model represent the time at which to mine each extractable production block with the objective of net present value maximization. The constraints were the operational constraints, e.g., ore availability, concentrator capacity, mine infrastructure production capacity, grade (mineral quality) limits, continuous production rules, and precedence relationships between production blocks. Also, Shu-xing and Peter (2009) surveyed the application of real options in the mining industry and proposed a framework developed at the University of Queensland, Australia, for integrating real options into medium/short-term mine planning and production scheduling. Their work introduced a new form of real option or mine planning flexibility, into medium/short-term mine planning and production scheduling processes. Also, Shu-xing and Peter (2009) demonstrated the method for valuation of these flexibilities based on the concept of real options. Basically, the uncertainty parameter considered in their paper is fuel price uncertainty. In addition, they used the approaches to estimate the values of the real options.

Classical methods are not successful to consider the risk of not meeting production targets caused by the uncertainty in estimated grades. Gap between planned expectations and actual production may occur in any steps of mining (Dimitrakopoulos and Ramazan, 2004). Vallee (2000) reported that 60% of the examined mines had an average rate of production that was less than 70% of the nominal capacity in early years. Rossi and Parker (1994) reported shortcomings against predictions of mine production in the later years of production. Dimitrakopoulos and Ramazan (2004) propose a method to integrate the grade uncertainties in optimization models. Their integrated model considers quantification of risk, equipment access, and other operational requirements. These operational requirements are blending, mill capacity, and mine production capacity. Godoy and Dimitrakopoulos (2004) proposed a method for risk-inclusive cutback designs, which yield fundamental NPV increases.

The organization of current paper is as follows: in section 2, the problem is defined in details. The theoretical framework of the proposed formulation, including definition of parameters, variables, and mathematical models are explained in section 3. In section 4 and 5, the proposed model is implemented and a case study for a year of mine production with 3089 blocks which have been clustered in 150 mining-cuts is presented. The conclusions and future work directions are presented in the section 6.

2. Problem definition

As mentioned in the introduction section, the problem is the short-term scheduling of an open pit mine. Fig. 1 shows the general schematic view of the problem. The problem is defined for an open pit mine including *S* buffer stockpiles, *P* processes, and *W* waste dumps. The mine has *E* elements, which one of them is considered as the major product. The role of buffer stockpiles is to hold excess amount of ore extracted from the mine. The shortage of feed for the processes in different periods is balanced by reclaiming ore from buffer stockpiles, whenever required. In fact, in each period, an amount of ore is extracted from the mine with respect to the mining capacity and precedence limitations. A portion of this extracted ore is delivered to the processes directly. But, all processes cannot accept ore with all rock types. The remaining ore is delivered to the stockpiles. Also, the same procedure is valid for the waste dumps. Each waste dump accepts waste materials with specific rock-types. Therefore, the short-term scheduling of the open pit mine is making a decision about: (i) the amount of material (ore and waste) that must be extracted from the mine in each period, (ii) the destinations (waste dumps, stockpiles, and processes) that the mined materials are sent to, and (iii) also, the decision about the amount of ore reclaimed from the stockpiles to the processes in each period.

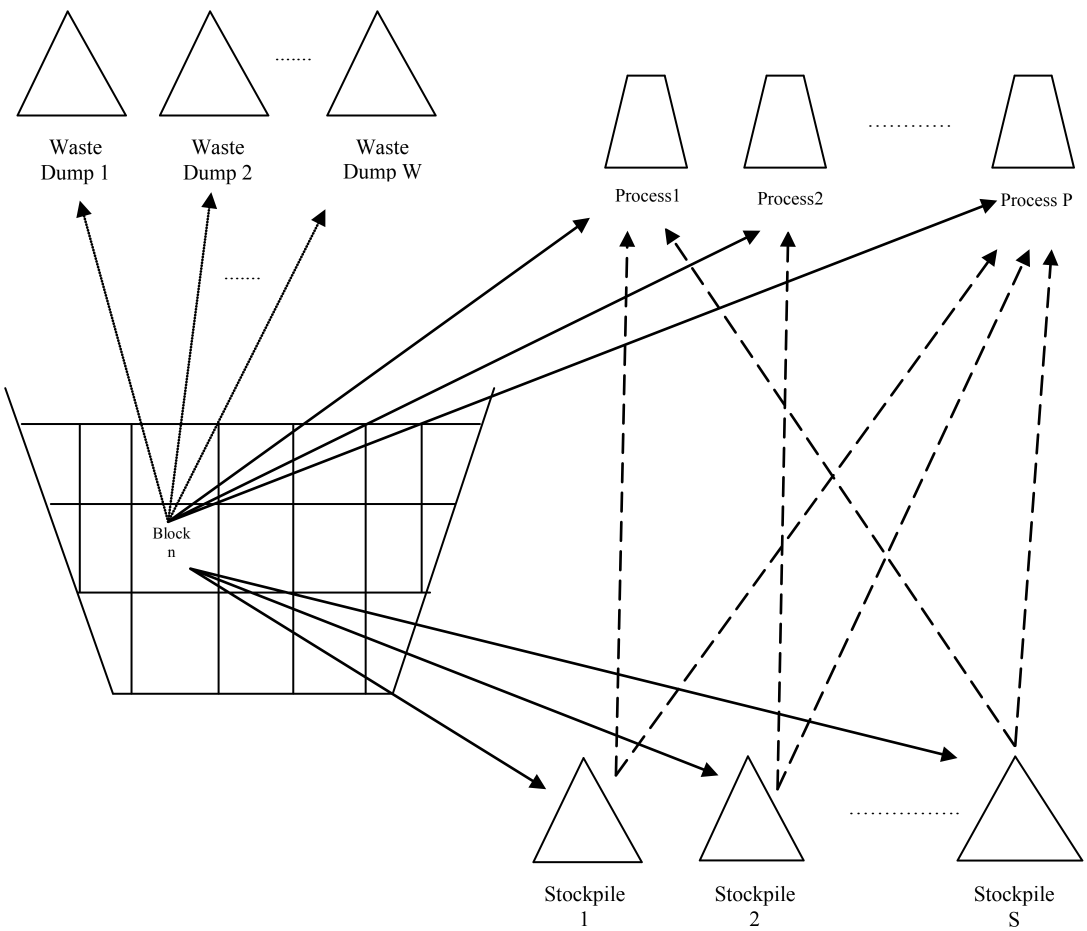

Fig. 1. Schematic view of the problem.

We assume that in the short-term horizon we have K periods with span of t. Also, from the long-term scheduling it has been determined that during K periods which blocks should be extracted from the mine. In the short-term scheduling we should decide about the amount and the time of extraction of each block in each period. There are some main assumptions in the problem. Briefly, these imposed assumptions are as follows:

- Each stockpile accepts ore from the mine with certain ranges of grade for different elements. In other words, the ore sent to each stockpile must have grades of elements within predefined ranges for that stockpile. We define stockpiles based on grade range bins.

- The grade of ore reclaimed from each stockpile is equal to the average grade of each element in that stockpile.
- Depending on the rock type of each block, the waste material of that block is sent to a certain waste dump.
- It is possible that some processes accept ore from specific stockpiles not from all of them.
- Stockpiles are classified based on rock types and grade ranges.

3. Theoretical framework and models

The proposed model is a short-term scheduling model with the objective of costs minimization with respect to physical and economic constraints. There are many parameters and variables in the model. Before presenting the formulation of the proposed model, it would be better to define these parameters and decision variables.

3.1. Parameters and decision variables

3.1.1 Parameters

1- t : period of scheduling (*t=1,…,K*)
2- e : element e (*e=1,..,E*)
3- $J(n)$: set of blocks that must be extracted during the short-term time horizon
4- $P(n)$: set of processes that can receive ore from block *n*
5- $SP(n)$: set of stockpiles that can receive ore from block *n*
6- $W(n)$: set of waste dumps that can receive waste from block *n*
7- N : number of blocks in *J(n)*
8- C_m^t : unit mining cost in period *t*
9- $C_{wr}^{t,i}$: unit waste rehabilitation cost in period t for waste dump *i*
10- $C_p^{t,i}$: unit processing cost in period *t* for plant *i*
11- $RH_{SP,i}^t$: unit rehandling cost of stockpile *i* in period *t*
12- MU^t : upper bound of mining capacity use in period *t*
13- ML^t : lower bound of mining capacity use in period *t*
14- PU_i^t : upper bound of capacity of process *i* in period *t*
15- PL_i^t : upper bound of capacity of process *i* in period *t*
16- O_n : mineral zone tonnage of block *n*
17- R_n : rock tonnage of block *n*
18- g_n^e : grade of element e in block *n*
19- dE_n^i : distance of block n to exit passing from ramp *i*
20- $gu_e^{t,i}$: upper bound on acceptable grade of element e for process i in period *t*
21- $gl_e^{t,i}$: lower bound on acceptable grade of element e for process i in period *t*
22- $PB(n)$: set of precedent blocks of block *n*
23- $N_{PB(n)}$: number of blocks in set *PB(n)*

24- $R(n)$: set of ramps for block *n*

25- ME^t : minimum extraction of blocks in period *t*

26- $OG^e_{SP,i}$: grade of element e in the output ore of stockpile *i*

27- $UG^e_{SP,i}$: upper bound of grade of element e in received ore from mine for stockpile *i*

28- $LG^e_{SP,i}$: lower bound of grade of element e in received ore from mine for stockpile *i*

29- $C_{SP,i}$: Capacity of stockpile *i*

30- P(SP(i)) : set of processes that receive ore from stockpile *i* (destination for stockpile *i)*

31- SP(P(j)) : set of stockpile that send ore to process *j* (source for process *j)*

3.1.2 Decision variables

Decision variables are as follows:

1- u^t_n : fraction of block *n* extracted in period *t* $(n \in J(n))$

2- $u^{t,i}_{n,W}$: fraction of block *n* extracted in period *t* and sent to waste dump *i* $(i \in W(n))$

3- $u^{t,i}_{n,SP}$: fraction of block *n* extracted in period *t* and sent to stockpile *i* $(i \in SP(n))$

4- $u^{t,i}_{n,P}$: fraction of block *n* extracted in period *t* and sent to process *i* $(i \in P(n))$

5- b^t_n : binary variable, if block *n* is extracted in period *t* it gets value of 1 otherwise 0

6- $x^{t,i}_{n,r}$: amount of extracted material of block *n* in period *t* exit from ramp *i*

7- $b^{t,i}_{n,r}$: binary variable, if ramp *i* is selected for handling of amount of extracted material from block *n* in period *t*, it gets value of 1, otherwise 0

8- $y^{t,P,j}_{SP,i}$: amount of ore sent from stockpile *i* to process *j* in period *t* $(i \in SP(P(j))$ which is the set of stockpiles that send ore to process *j*)

9- $I^t_{SP,i}$: Inventory of stockpile *i* in period *t*

3.2. Formulation

After defining the parameters and variables of the model, the mathematical formulation of the proposed model is presented.

$$\text{Min } Z = \underbrace{\sum_{t=1}^{k}\sum_{n=1}^{N} R_n \cdot u^t_n \cdot C^t_m}_{\text{total mining cost}} + \underbrace{\sum_{t=1}^{k}\sum_{n=1}^{N}\sum_{i\in P(n)} u^{t,i}_{n,p} \cdot R_n \cdot C^{t,i}_p + \sum_{t=1}^{k}\sum_{j=1}^{P}\sum_{i\in SP(P(j))} y^{t,p,j}_{SP,i} \cdot C^{t,j}_p}_{\text{total processing cost}} +$$

$$\underbrace{\sum_{t=1}^{k}\sum_{n=1}^{N}\sum_{i\in W(n)} u^{t,i}_{n,w} \cdot R_n \cdot C^{t,i}_{wr}}_{\text{total waste rehabilitation cost}} + \underbrace{\sum_{t=1}^{k}\sum_{i=1}^{S}\sum_{j\in P(SP(i))} y^{t,p,j}_{SP,i} \cdot RH^{t,j}_{SP,i}}_{\text{total rehandling cost}} \tag{1}$$

$$+\underbrace{\sum_{t=1}^{k}\sum_{n=1}^{N}\sum_{i\in R(n)} x^{t,i}_{n,r} \cdot dE^i_n \cdot H^t}_{\text{total haulage cost}}$$

Subject to:

$$\sum_{t=1}^{k} u_n^t = 1,\ \forall n = 1...N \tag{2}$$

$$\sum_{i\in P(n)} u_{n,P}^{t,i} + \sum_{i\in SP(n)} u_{n,SP}^{t,i} + \sum_{i\in W(n)} u_{n,w}^{t,i} = u_n^t,\ \forall n = 1...N, \forall t = 1...k \tag{3}$$

$$R_n \cdot \sum_{t=1}^{k}\Big(\sum_{i\in P(n)} u_{n,P}^{t,i} + \sum_{i\in SP(n)} u_{n,SP}^{t,i}\Big) = O_n,\ \forall n = 1...N \tag{4}$$

$$u_n^t \le b_n^t,\ \forall n = 1...N, \forall t = 1...k \tag{5}$$

$$ME^t \cdot b_n^t \le u_n^t,\ \forall n = 1...N, \forall t = 1...k \tag{6}$$

$$ML^t \le \sum_{n=1}^{N} R_n \cdot u_n^t \le MU^t,\ \forall t = 1...k \tag{7}$$

$$PL_j^t \le \sum_{\forall n:P(n)=j} R_n \cdot u_{n,P}^{t,j} + \sum_{i\in SP(P(j))} y_{SP,i}^{t,P,j} \le PU_j^t,\ \forall t = 1...k, \forall j = 1...P \tag{8}$$

$$N_{PB(n)} \cdot b_n^t \le \sum_{\tau=1}^{t} \sum_{i\in PB(n)} u_i^{\tau},\ \forall n = 1...N, \forall t = 1...k \tag{9}$$

$$gl_{e,P}^{t,j} \le \frac{\sum_{i\in SP(P(j))} y_{SP,i}^{t,P,j} \cdot OG_{SP,i}^{e} + \sum_{\forall n:P(n)=j} u_{n,p}^{t,j} \cdot R_n \cdot g_n^e}{\sum_{i\in SP(P(j))} y_{SP,i}^{t,P,j} + \sum_{\forall n:P(n)=j} u_{n,p}^{t,j} \cdot R_n} \le gu_{e,P}^{t,j},\ \forall t = 1...k, \forall j = 1...P, \forall e = 1...E \tag{10}$$

$$\sum_{\forall n:SP(n)=i} R_n \cdot u_{n,SP}^{t,i} - \sum_{j\in P(SP(i))} y_{SP,i}^{t,P,j} + I_{SP,i}^{t-1} = I_{SP,i}^{t},\ \forall t = 1...k, \forall i = 1...S \tag{11}$$

$$\sum_{j\in P(SP(i))} y_{SP,i}^{t,P,j} \le I_{SP,i}^{t-1},\ \forall t = 1...k, \forall i = 1...S \tag{12}$$

$$LG_{SP,i}^{e} \le \frac{\sum_{\forall n:SP(n)=i} u_{n,SP}^{t,i} \cdot R_n \cdot g_n^e}{\sum_{\forall n:SP(n)=i} u_{n,SP}^{t,i} \cdot R_n} \le UG_{SP,i}^{e},\ \forall t = 1...k, \forall i = 1...S, \forall e = 1...E \tag{13}$$

$$\sum_{j\in P(SP(i))} y_{SP,i}^{t,P,j} \le \frac{N_{SP,i}^{e,t-1} - LG_{SP,i}^{e} \cdot I_{SP,i}^{t-1}}{OG_{SP,i}^{e} - LG_{SP,i}^{e}},\ \forall t = 1...k, \forall i = 1...S, \forall e = 1...E \tag{14}$$

$$\sum_{j\in P(SP(i))} y_{SP,i}^{t,P,j} \le \frac{-N_{SP,i}^{e,t-1} + UG_{SP,i}^{e} \cdot I_{SP,i}^{t-1}}{-OG_{SP,i}^{e} + UG_{SP,i}^{e}},\ \forall t = 1...k, \forall i = 1...S, \forall e = 1...E \tag{15}$$

$$\sum_{i\in R(n)} x_{n,r}^{t,i} = R_n \cdot u_n^t,\ \forall n = 1...N, \forall t = 1...k \tag{16}$$

$$x_{n,r}^{t,i} \le M.b_{n,r}^{t,i},\ \forall n = 1...N, \forall t = 1...k, \forall i = 1...R(n) \quad \text{(M is a large number)} \tag{17}$$

$$\sum_{i\in R(n)} b_{n,r}^{t,i} = b_n^t, \ \forall n = 1...N, \forall t = 1...k \tag{18}$$

$$0 \le u_n^t \le 1, \forall n = 1...N, \forall t = 1...k \tag{19}$$

$$0 \le u_{n,W}^{t,i} \le 1, \forall n = 1...N, \forall t = 1...k, \forall i \in W(n) \tag{20}$$

$$0 \le u_{n,P}^{t,i} \le 1, \forall n = 1...N, \forall t = 1...k, \forall i \in P(n) \tag{21}$$

$$0 \le u_{n,SP}^{t,i} \le 1, \forall n = 1...N, \forall t = 1...k, \forall i \in SP(n) \tag{22}$$

$$0 \le x_{n,r}^{t,i}, \quad \forall n = 1...N, \forall t = 1...k, \forall i \in R(n) \tag{23}$$

$$0 \le y_{SP,i}^{t,P,j}, \quad \forall t = 1...k, \forall i = 1...S, \forall j \in P(SP(i)) \tag{24}$$

$$0 \le I_{SP,i}^t \le C_{SP,i} \quad \forall t = 1...k, \forall i = 1...S \tag{25}$$

$$b_n^t \text{ and } b_{n,r}^{t,i} = 0/1, \forall n = 1...N, \forall t = 1...k, \forall i \in R(n) \tag{26}$$

Eq. (1) indicates the minimization objective function. This function includes the cost terms as follows:

1. Total mining cost: this cost contains the mining costs such as drilling, blasting and loading of material throughout *K* periods.
2. Total processing cost: this cost is the summation of processing costs of ore sent from mine directly or from stockpiles indirectly throughout *K* periods.
3. Total waste rehabilitation cost: this cost indicates the summation of costs for rehabilitation of waste dumps during *K* periods.
4. Total rehandling cost: this is the total cost incurred for handling of ore from stockpiles to different processes.
5. Total haulage cost: this is the total cost of haulage of materials from blocks to the exit of the mine.

Minimization of the abovementioned costs is under some constraints. These constraints have been reflected in Eqs. (2) to (26). Eq. (2) shows the constraints regarding to the complete extraction of all blocks. All blocks should be mined during *K* periods. Eq. (3) refers to this fact that the fraction of each block that is mined in each period is sent to different destinations. In other words, total extracted material from block *n* in each period goes to processes, stockpiles, and waste dumps. So far, decision variables u_n^t, $u_{n,w}^{t,i}$, $u_{n,SP}^{t,i}$ and $u_{n,P}^{t,i}$ can get every value. Constraint 3 imposes the relationship between these decision variables. Eq. (4) indicates that at the end of *K* periods, all of the mineralized material and waste material of each block should be mined completely. Eq. (5) defines the relation between the binary variable b_n^t and the continuous variable u_n^t. Whenever a portion of a typical block *n* is going to be mined, the respected binary variable b_n^t is set to 1. Eq. (6) refers to minimum extraction of blocks in each period. Based on this constraint, when the shovel extracts a block, it cannot extract a small portion of the block. In fact, it is not practical to extract a small fraction of a block. Thus, to avoid the very small extraction amounts, constraint 5 is used. Eq. (7) indicates the mining capacity limitation in each period. In each period the total mined rock tonnage should be in an acceptable certain range of values. This range reflects the capacity of mining by available equipments. Eq. (8) is the constraint of process capacity in each period.

According to this constraint, total ore sent to each process from mine and stockpiles should be in a certain range. Like mining constraint, this range is assigned by available capacity of processes in each period. Eq. (9) forces the precedence or slope constraints in which the extraction of each block can be start only when all blocks which are placed above that block directly are mined completely. To explain this constraint, suppose block (x) with 9 blocks above. According to precedence constraint concept, in each period the extraction of block x can be performed only when all of these 9 blocks have been extracted completely through the previous periods and current period. The right-hand side of Eq. (9) shows the summation of portions of all of these 9 blocks through the previous periods and current period. The value of this summation can equal to a value less than 9 or exactly 9. Less than 9 value means that all of 9 blocks above are not mined completely. Therefore, the extraction of block x cannot be started. By dividing the value of summation by 9, a value less than 1 would be obtained. This less than 1 value forces b_n^t to be 0. On the other hand, if the value of summation equals to 9, by dividing by 9, the value of 1 would be obtained which indicates that mining of block x can be start from this period. Eq. (10) indicates the head grade constraint of each process in each period. Based on this constraint, the average grade of each element in ore sent to each process in each period should be in a certain range which is determined by processes. Eq. (11) is the balancing of stockpiles inventory during time horizon. Total material that each stockpile receives in each period plus the current inventory minus the ore tonnage it sends to processes determines the inventory for the next period. Eq. (12) shows that the total ore sent from each stockpile in each period should be less than the inventory of that stockpile at the beginning of that period. Here, the assumption is that the ore is going to be sent from each stockpile to processes at the beginning of each period. As each stockpile stores ore with a certain grade for each element, Eq. (13) is the constraint on the values of grade of elements in ore sent to each stockpile. Eqs. (14) and (15) guarantee that after sending ore from each stockpile, the average grade of elements of ore inside that stockpile remains in the acceptable range. The following theorem shows that Eqs. (14) and (15) result in keeping the grade of elements in stockpiles between lower and upper bounds.

Theorem 1:

Fig. 2 indicates the procedure of reclaiming ore from a typical stockpile *i* in period *t.* As mentioned, the assumption is that reclaiming ore from stockpiles is done at the beginning of each period. Suppose the metal content for element *e* in stockpile *i* at the beginning of period *t* (or at the end of period *t*-1) is $\mathrm{N}_{\mathrm{SP,i}}^{\mathrm{e,t-1}}$. Since the ore is sent with average grade $\mathrm{OG}_{\mathrm{SP,i}}^{\mathrm{e}}$, the remaining metal content for element *e* is as follows:

$$\mathrm{N}_{\mathrm{SP,i}}^{\mathrm{e,t-1}} - \mathrm{OG}_{\mathrm{SP,i}}^{\mathrm{e}} \cdot \sum_{\mathrm{j \in P(SP(i))}} \mathrm{y}_{\mathrm{SP,i}}^{\mathrm{t,P,j}}, \quad \forall \mathrm{e} = 1 \ldots \mathrm{E} \tag{27}$$

Also, the remaining ore in stockpile *i* is as follows:

$$\mathrm{I}_{\mathrm{SP,i}}^{\mathrm{t-1}} - \sum_{\mathrm{j \in P(SP(i))}} \mathrm{y}_{\mathrm{SP,i}}^{\mathrm{t,P,j}} \tag{28}$$

As the average grade of element *e* in ore stored inside the stockpile *i* should be between its predefined lower and upper bound, then

$$\mathrm{LG}_{\mathrm{SP,i}}^{\mathrm{e}} \le \frac{\mathrm{N}_{\mathrm{SP,i}}^{\mathrm{e,t-1}} - \mathrm{OG}_{\mathrm{SP,i}}^{\mathrm{e}} \cdot \sum_{\mathrm{j \in P(SP(i))}} \mathrm{y}_{\mathrm{SP,i}}^{\mathrm{t,P,j}}}{\mathrm{I}_{\mathrm{SP,i}}^{\mathrm{t-1}} - \sum_{\mathrm{j \in P(SP(i))}} \mathrm{y}_{\mathrm{SP,i}}^{\mathrm{t,P,j}}} \le \mathrm{UG}_{\mathrm{SP,i}}^{\mathrm{e}}, \quad \forall \mathrm{e} = 1 \ldots \mathrm{E} \tag{29}$$

After reorganization of Eq. (29), Eqs.(14) and (15) are obtained. It should be mentioned that the metal content of element *e* in stockpile *i* is calculated by Eq. (30).

$$N_{SP,i}^{e,t-1} = \sum_{\tau=1}^{t-1} \sum_{\forall n:SP(n)=i} u_{n,SP}^{\tau,i} \cdot R_n \cdot g_n^e + IMC_{SP,i}^e - \sum_{\tau=1}^{t-1} \sum_{j \in P(SP(i))} y_{SP,i}^{\tau,P,j} \cdot OG_{SP,i}^e \,, \quad \forall e = 1...E, \forall t = 1...K, \forall i = 1...S \qquad (30)$$

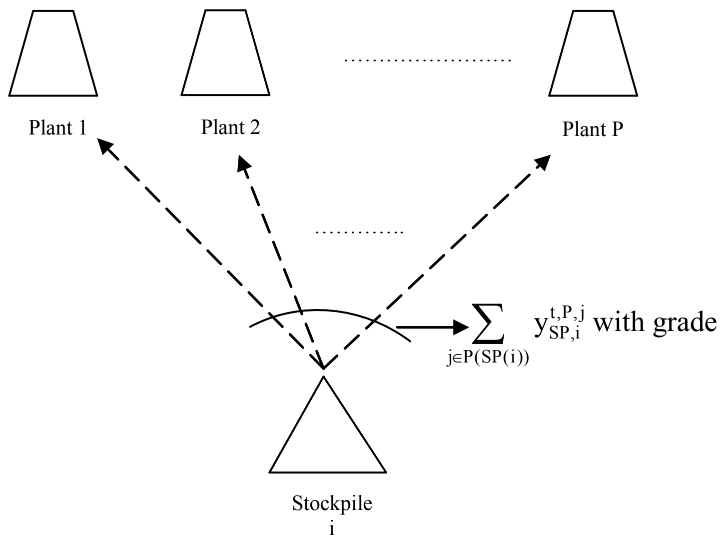

Fig. 2. Procedure of sending ore from stockpile *i* to processes.

In Eq. (30) the metal content of element *e* in period *t* for stockpile *i* is calculated by the summation of all received ore tonnages in the previous periods and the initial metal content of element *e,* minus the total ore tonnages that the stockpile have sent to the processes through the previous periods.

Eqs. (16) to (18) indicate that total extracted material from each block in period *t* is handled to one of the possible ramps. In other words, just one of possible ramp is selected for handling the material of each block. Eq. (16) means summation of material sent from block *n* to different ramps in each period equals to total extracted material from that block in that period. Eqs. (17) and (18) indicate the extracted material can be sent to only one of the possible ramps. Eqs. (19) to (26) represent the sign constraints regarding to decision variables. Among these constraints, Eq. (25) shows the limitation of storage for stockpiles.

4. Discussion and results

In this section, an illustrative example for short-term planning of an open pit mine is presented. The example refers to Gol-E-Gohar mines in south of Iran. The main element of this mine is Iron (Fe). The contaminants present are phosphor and sulfur. Fig. 3 shows the 3D view of one of the designed pit. The open pit has 20 benches. Only blocks of benches 14, 15, 16, and 17 are used for the purpose of short-term planning. These benches are scheduled over 12 months. The total number of blocks in these benches is 3089. These 3089 blocks are clustered into 150 mining-cuts using fuzzy C-mean method. We aggregate blocks for two reasons: (i) to reduce the number of variables in the MILP formulation, and (ii) to generate a schedule that follows a selective mining unit. For this pit, only one exit has been designed. Also, on average, two ramp accesses have been designed for each bench. For the four benches considered in this study, there are 7 ramp accesses. Table 1 represents the general information of the problem. All the blocks can be sent to all destinations. Stockpile 1 only feeds process 1 and Stockpile 2 feeds process 2.

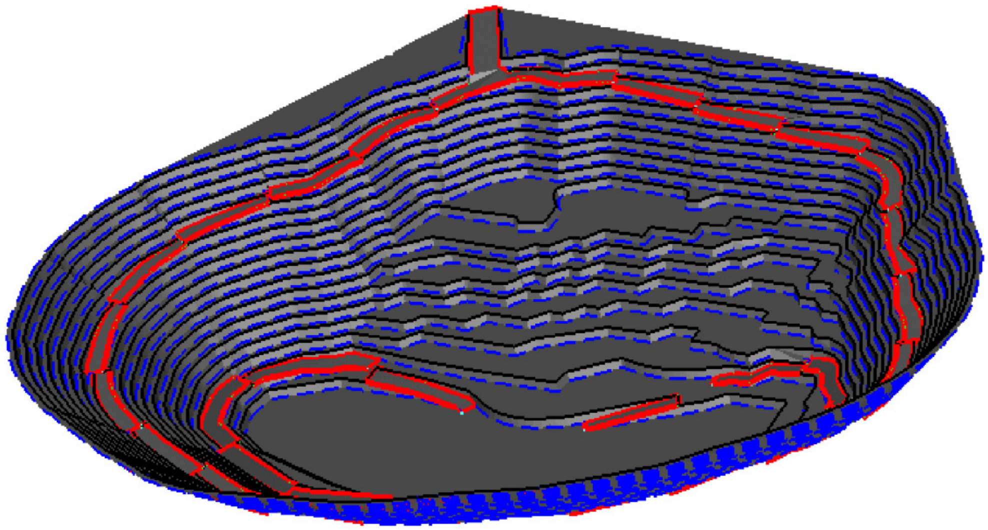

Fig. 3. 3D view of the open pit mine.

Table 1. General information of the problem.

	Bench Number			
	1	2	3	4
Number of blocks	614	726	820	929
Number of Cuts	30	35	40	45
Ramp access #	1 ,2	3, 4	5, 6	7
Total number of blocks	3089			
Total number of cuts	150			
Block size	25×25×15 ( )			
Total rock tonnage	93.8544 Million Tonnes			
Total mineral tonnage	47.4898 Million Tonnes			

For implementation of the proposed mathematical model, 6 destinations including 2 processes, 2 stockpiles, and 2 waste dumps are considered. Table 2 and Table 3 indicate the main specifications of these destinations. Material could be sent to any of the waste dumps.

Table 2. Processes main features.

Process	Lower grade			Upper grade			Min. capacity (tonne)	Max. capacity (tonne)
	MWT	S	P	MWT	S	P		
Process 1	0	0	0	1	1	1	1,000,000	2,000,000
Process 2	0	0	0	1	1	1	1,000,000	1,750,000

Table 3. Stockpiles main features.

Stockpile	Lower grade			Upper grade			Output grade			Max. capacity (tonne)	Initial inventory (tonne)
	MWT	S	P	MWT	S	P	MWT	S	P		
Stockpile 1	0.55	0	0	0.65	0.3	0.3	0.6	0.04	0.035	2,000,000	0
Stockpile 2	0.651	0	0	0.78	0.3	0.3	0.71	0.04	0.035	2,000,000	0

Unit costs are shown in Table 4. Fig. 5 to Fig. 8 illustrate sample plan views and cross sections of the generated schedule. Fig. 5 shows the plan view of the top level of the considered blocks while Fig. 6 illustrates the plan view of bench 17. Fig. 7 and Fig. 8 show the cross sections looking East and North, respectively. The values inside of these figures indicate the period that the maximum amount of extraction of blocks has occurred. For plan views, each cell has a size of 25m×25m and for Figs. 7 and 8, each cell has a size of 25m×15m. The optimum objective function shows the total costs including mining, haulage, processing, rehandling, and rehabilitation.

Table 4. Unit costs.

Processing 1 ($/tonne)	Processing 2 ($/tonne)	Rehandling 1 ($/tonne.m)	Rehandling 2 ($/tonne.m)	Waste rehabilitation 1 ($/tonne)	Waste rehabilitation 2 ($/tonne)
5.5	5.75	0.5	0.25	1.75	2

Fig. 4 shows the schedule of extraction during 12 periods. Based on this figure, almost the maximum capacities of mining equipments and processes are used with a small deviations.

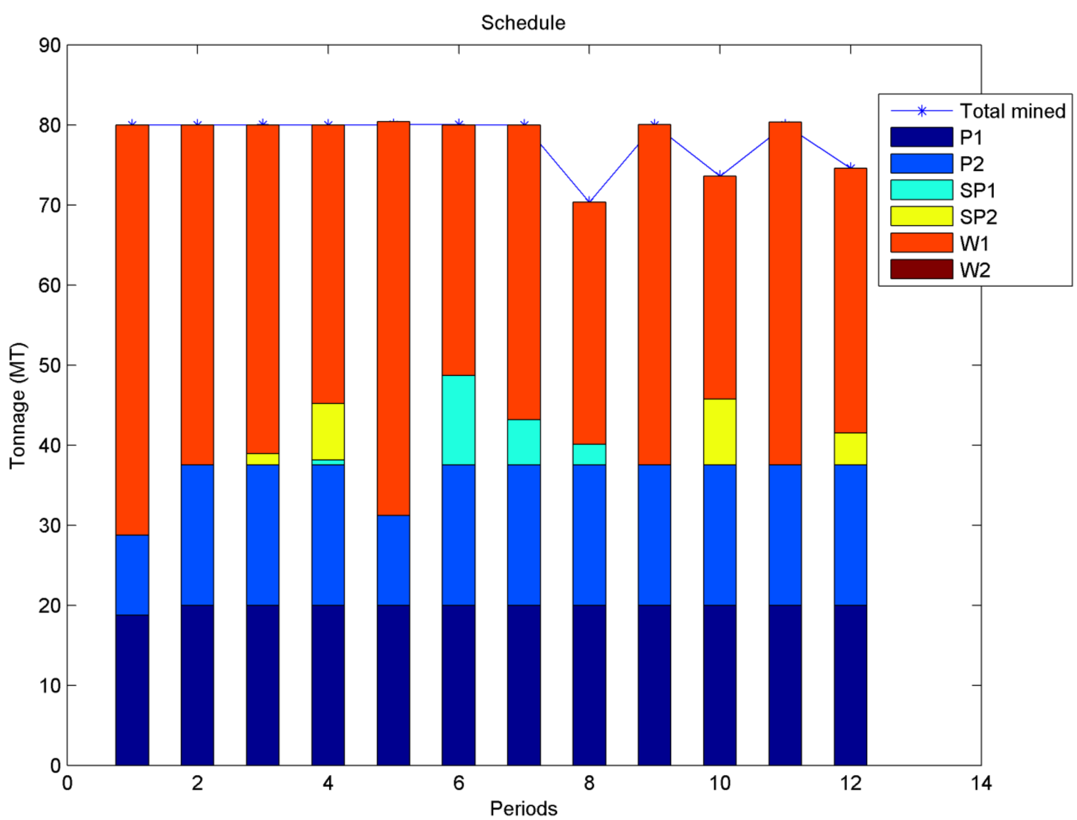

Fig. 4. Schedule of mine production during 12 periods.

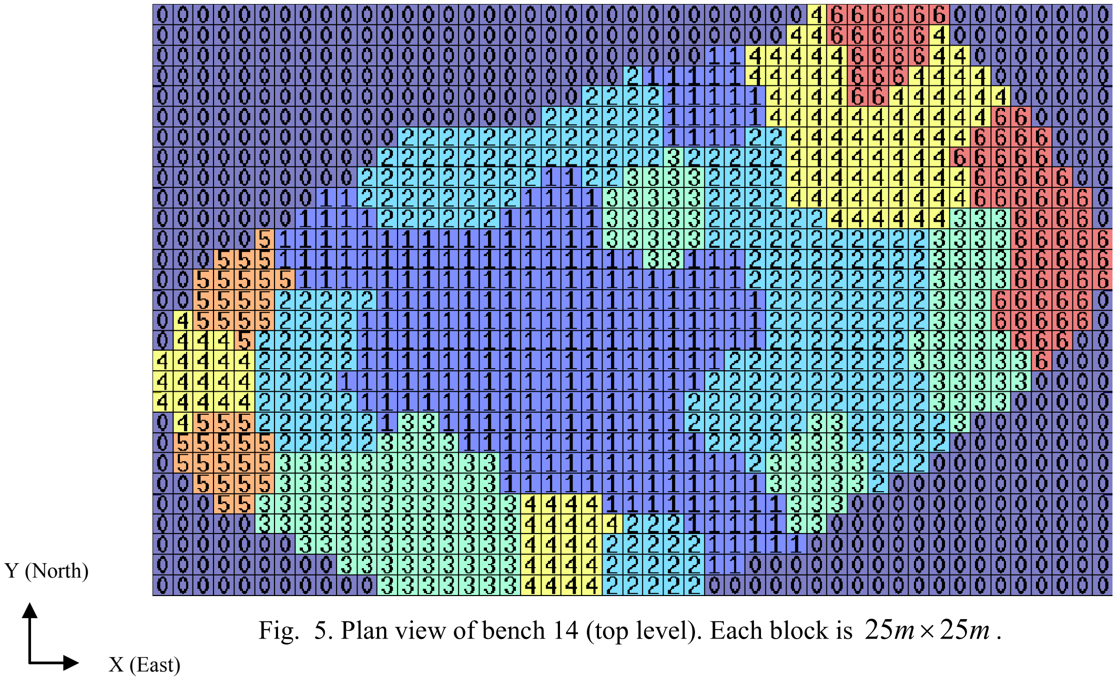

Fig. 5. Plan view of bench 14 (top level). Each block is $25m \times 25m$.

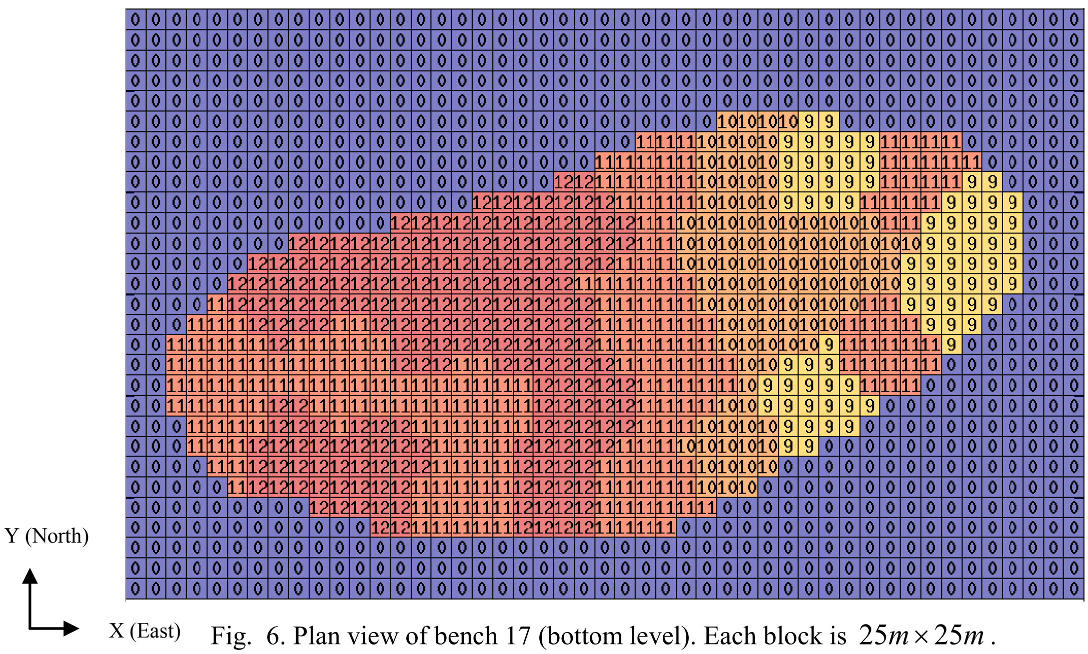

Fig. 6. Plan view of bench 17 (bottom level). Each block is $25m \times 25m$.

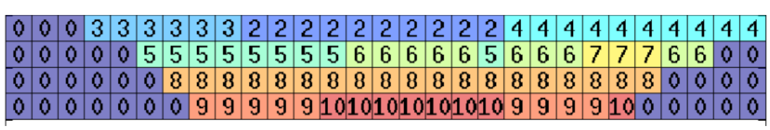

Fig. 7. Sample cross section looking East. Each block is $25m \times 15m$.

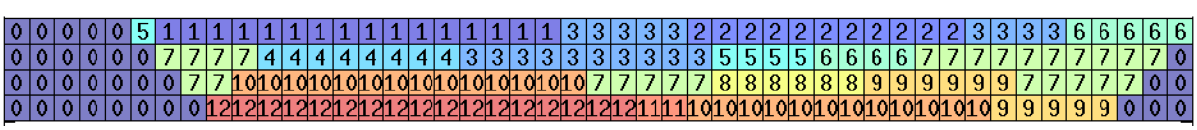

Fig. 8. Sample cross section looking North. Each block is $25m \times 15m$.

5. Conclusions and future work

Mine scheduling is a challenging problem in the mining industry. In this paper, a mixed integer model for the short-term scheduling of open pit mines is formulated and solved. The proposed model includes different destinations, processes, stockpiles, and waste dumps. Each process plant processes the received ore to produce the final product. The stockpiles are considered to keep extra mined ore for using in the next periods. The objective function of model aims to minimize the total costs incurred by mining and haulage, processing, rehandling, and waste rehabilitation. This function is subject to different mining operational constraints such as head grade blending, precedence constraints, etc. The proposed model is solved by CPLEX for a typical set of blocks in 4 benches.

6. References

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7. Appendix

MATLAB and CPLEX Code Documentation