Numerical modelling of the MILP formulation for open pit production scheduling

Hooman Askari-Nasab and Kwame Awuah-Offei[1]
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

Abstract

Development of new optimization techniques and uncertainty quantification for long-term mine planning plays a vital role in reducing environmental footprint and financial risk of mining projects. Deviations from optimal plans in mega mining projects will result in huge financial losses, delayed reclamation, and resource sterilization. In this research we developed a mathematical methodology for optimal large-scale open pit mine production scheduling. The deterministic mathematical models developed in this project will pave the way for the uncertainty quantification associated with mine plans by means of stochastic mathematical programming. We have developed a mixed integer linear programming model based on mining-cuts with reduced number of binary integer variables. The numerical modeling techniques are illustrated in details and a mining case study with around twenty thousand blocks over seventeen periods have been scheduled.

1. Introduction

Mixed integer linear programming mathematical optimization have been used by different researchers to tackle the long-term open-pit scheduling problem (2003; Ramazan and Dimitrakopoulos, 2004; Dagdelen and Kawahata, 2007). The MILP models theoretically have the capability to consider diverse mining constraints such as multiple ore processors, multiple material stockpiles, and blending strategies. The applications of MILP models result in production schedules generating near theoretical optimal net present values for mining ventures. The number of binary variables required in formulations presented by Caccetta and Hill (2003) and Ramazan and Dimitrakopoulos (2004) is equal to the number of blocks in the block model multiplied by the total number of scheduling periods. For a typical real size open pit scheduling problem number of blocks is in the order of couple of hundred thousands to millions and the number of scheduling periods usually varies between twenty to thirty years for a life-of-mine schedule. Evidently, a problem of this size is numerically intractable with current state of hardware and commercial optimization solvers. Ramazan and Dimitrakopoulos (2004) presented a method to reduce the number of

---

[1] Assistant Professor, Department of Mining & Nuclear Engineering, University of Missouri-Rolla, USA,

binary integer variables by setting waste blocks as linear variables. Setting waste blocks as linear variables will cause a block to be extracted in multiple periods, generating a schedule which is not feasible from practical equipment access point of view. Also, notable is work by Dagdelen, who applied the Lagrangian relaxation technique and sub-gradient methods to solve the mine production scheduling MILP problem (Dagdelen and Kawahata, 2007).

Boland et al. (2007) extended the formulation of Caccetta and Hill (2003) based on strict temporal sequence of blocks rather than assigning blocks to periods of extraction. Boland et al. (2007) reduced the number of decision variables by eliminating a number of variables presented in Caccetta and Hill (2003) formulation prior to optimization. This was achieved by combining the block precedence constraints with the production constraints, aggregated over a sequence of time periods. The numerical results illustrated a decrease in computational requirements to obtain the optimal integer solution. Boland et al. (2009) have demonstrated an iterative disaggregation approach to using a finer spatial resolution for processing decisions to be made based on the small blocks, while allowing the order of extraction decisions be made at an aggregate level. Boland et al. (2009) reported notable improvements on the convergence time of their algorithm. Boland et al. (2009) id not present enough information on their method of aggregation and assumed that some aggregation technique already exist.

We have used Boland et al. (2009) general formulation as our starting point of analysis. We divided the major decision variables into two categories, continuous variables representing the portion of a block that is going to be extracted in each period and binary integer variables controlling the order of extraction of blocks through a dependency graph using depth-first-search algorithm. We have implemented our new optimization formulation in TOMLAB/CPLEX environment (Holmström, 1989-2009). The models were verified and validated through synthetic data and a mining case study on an iron ore mine.

1.1. Economic block value modeling

Assumption: a general parameter f can take four indices in the format of $f_{k,n}^{e,t}$. Where:

$t \in \{1,....,T\}$	index for scheduling periods.
$k \in \{1,...,K\}$	index for mining-cuts.
$n \in \{1,...,N\}$	index for blocks.
$e \in \{1,...,E\}$	index for elements of interest in each block.

The objective functions of the MILP formulations are to maximize the net present value of the mining operation. Hence, we need to define a clear concept of economic block value based on ore parcels which could be mined selectively. The profit from mining a block depends on the value of that block and the costs incurred in mining and processing it. The cost of mining a block is a function of its spatial location, which characterizes how deep the block is located relative to the surface and how far it is relative to its final dump. The spatial factor can be applied as a mining cost adjustment factor for each block according to its location to the surface. The discounted profit from block n is equal to the discounted revenue generated by selling the final product contained in block n minus all the discounted costs involved in extracting block n, this is presented by Eqs. (1) and (2).

discounted profit = discounted revenue - discounted costs (1)

$$d_n^t = \underbrace{[\sum_{e=1}^{E} o_n \times g_n^e \times r^{e,t} \times (p^{e,t} - cs^{e,t})}_{discounted\ revenues} - \underbrace{\sum_{e=1}^{E} o_n \times cp^{e,t}] - [(o_n + w_n) \times cm^t]}_{discounted\ costs} \quad (2)$$

Where

- d_n^t is the discounted profit generated by extracting block n in period t,
- o_n is the ore tonnage in block n and ore tonnage in mining-cut k,
- w_n is the waste tonnage in block n,
- g_n^e is the average grade of element e in ore portion of block n,
- $r^{e,t}$ is the processing recovery, which is the proportion of element e recovered in time period t,
- $p^{e,t}$ is the price in present value terms obtainable per unit of product (element e),
- $cs^{e,t}$ is the selling cost in present value terms per unit of product (element e),
- $cp^{e,t}$ is the extra cost in present value terms per tonne of ore for mining and processing,
- cm^t is the cost in present value terms of mining a tonne of waste in period t.

For simplification purposes we denote:

$$v_n^t = [\sum_{e=1}^{E} o_n \times g_n^e \times r^{e,t} \times (p^{e,t} - cs^{e,t}) - \sum_{e=1}^{E} o_n \times cp^{e,t}] \quad (3)$$

$$q_n^t = (o_n + w_n) \times cm^t \quad (4)$$

Where

- v_n^t is the discounted revenue generated by selling the final product within block n in period t minus the extra discounted cost of mining all the material in block n as ore and processing it; and
- q_n^t is the discounted cost of mining all the material in block n as waste.

2. Mixed integer linear programming model for open pit production scheduling

We present two different formulations for the open pit production scheduling problem, with the objective function to maximize the NPV of the mining operation. We extended our models based on concepts presented in Boland et al. (2009) as the starting point of our research.

2.1. Extraction at mining-cut level and processing at block level

In the proposed model processing is at block level and extraction is at mining-cut level. The amount of ore processed is controlled by the continuous variable x_n^t, and the amount of material mined is controlled by the continuous variable y_k^t. Using continuous decision variables allows fractional extraction of blocks in different periods. $b_k^t \in \{0,1\}$, is the binary integer variable controlling the precedence of extraction of mining- cuts. $b_k^t \in \{0,1\}$ is equal to one if extraction of mining-cut has started by or in period t, otherwise it is zero.

Objective function:

$$\max \sum_{t=1}^{T}\sum_{k=1}^{K}\left(\sum_{n\in c_k} v_n^t \times x_n^t - (\sum_{n\in c_k} q_n^t)\times y_k^t\right) \tag{5}$$

Where

- T is the maximum number of scheduling periods, where $\mathcal{T} = \{1,...,T\}$ is the set of all the scheduling time periods in the model,
- K is the total number of mining-cuts to be scheduled, where $\mathcal{K} = \{1,...,K\}$ is the set of all the mining-cuts in the model,
- c_k represents mining-cut k,
- $x_n^t \in [0,1]$ is a continuous decision variable, representing the portion of block n to be extracted as ore and processed in period t,
- $y_k^t \in [0,1]$ is a continuous decision variable, representing the portion of mining-cut c_k to be mined in period t, fraction of y characterizes both ore and waste included in the mining-cut.

It should be mentioned that in the objective function given by Eq. (5), mining is controlled at the mining-cut level, whereas the processing is at the higher resolution of block level. The objective function is subject to the following constraints.

Mining capacity constraints:

$$\sum_{k=1}^{K}\left(\sum_{n\in c_k}(o_n + w_n)\right)\times y_k^t \le mu^t \qquad \forall t \in \{1,...,T\} \tag{6}$$

$$\sum_{k=1}^{K}\left(\sum_{n\in c_k}(o_n + w_n)\right)\times y_k^t \ge ml^t \qquad \forall t \in \{1,...,T\} \tag{7}$$

Where

- mu^t is the upper bound on mining capacity in period t (tonnes),
- ml^t is the lower bound on mining capacity in period t (tonnes).

Eq. (6) controls that the total amount of ore and waste mined in each period to be within the targeted maximum mining capacity of equipment. The constraints are controlled by the continuous variable y_k^t at the mining-cut level. Eq. (7) controls the minimum amount of material that needs to be mined; Eq. (7) is useful in achieving a constant stripping ratio over the mine life. A production schedule with an invariable stripping ratio would have significant savings potential by ensuring that fleet size required is matched to targets for material movement. The decision of the proper production rate which leads to the boundaries on mining capacity is an important stage of the production scheduling of open pit mines. Different scenarios of annual ore production rates must be examined and the one with highest NPV and uniform mill feed must be chosen. The mining capacity boundaries are function of the ore reserve, overall stripping ratio, designed processing capacity, targeted mine-life, and the capital investment available for purchasing equipment. As it is illustrated by Eqs. (6) and (7), the upper and lower bounds of mining capacity could vary by scheduling periods, this flexibility allows the designer to target on replacing the fleet with different mining capacities at different stages of mine-life. The shortage of equipment in specific periods could be compensated with contract mining. Eqs. (6) and (7) will generate one constraints per period.

Processing capacity constraints:

$$\sum_{n=1}^{N} o_n \times x_n^t \le pu^t \qquad \forall t \in \{1,...,T\} \tag{8}$$

$$\sum_{n=1}^{N} o_n \times x_n^t \ge pl^t \qquad \forall t \in \{1,...,T\} \tag{9}$$

Where

- N is the number of blocks in the block model, where $\boldsymbol{N} = \{1,...,N\}$ is a set of all the blocks in the model,
- pu^t is the upper bound on processing capacity of ore in period t (tonnes),
- pl^t is the lower bound on processing capacity of ore in period t (tonnes).

Eqs. (8) and (9) represent inequality constraints controlling the mill feed or processing capacity; These constraints assist the mine planners in achieving an overall mine-to-mill integration by providing a uniform feed throughout the mine-life. Constraints (8) and (9) are at block level, which means the decisions are made based upon the tonnage of ore above the cut-off grade within individual blocks. In practice, the processing capacity constraints must be set within a tight upper and lower bounds to provide a uniform feed to the mill. Depending on the shape of the orebody and distribution of ore grades in the orebody, these constraints could not be honored under some circumstances, which will lead to an infeasible problem. Pre-stripping could be achieved by setting the upper and lower bounds of processing capacity constraints equal to zero for the desired periods; this would enforce the optimizer to only mine waste blocks in the early periods that would open up the orebody for later mining. Eqs. (8) and (9) will generate one constraints per period per ore type.

Grade blending constraints:

$$\sum_{n=1}^{N} g_n^e \times o_n \times x_n^t \Big/ \sum_{n=1}^{N} o_n \times x_n^t \le gu^{e,t} \qquad \forall t \in \{1,...,T\}, \quad e \in \{1,...,E\} \tag{10}$$

$$\sum_{n=1}^{N} g_n^e \times o_n \times x_n^t \Big/ \sum_{n=1}^{N} o_n \times x_n^t \ge gl^{e,t} \qquad \forall t \in \{1,...,T\}, \quad e \in \{1,...,E\} \tag{11}$$

Where

- g_n^e is the average grade of element e in ore portion of block n, where $\mathcal{E} = \{1,...,E\}$ is the set of all the elements of interest in the model,
- $gu^{e,t}$, is the upper bound on acceptable average head grade of element e in period t,
- $gl^{e,t}$, is the lower bound on acceptable average head grade of element e in period t.

Production scheduling is concerned with the inherent task of blending the run-of-mine materials before processing. The objective is to mine in such a way that the resulting mix meets the quality specifications of the processing plant. The blending problem becomes more important as the design moves towards mid-range to short-range planning, where the planner is concerned with reducing the grade variability. Constraints (10) and (11) are at block level and there would be one equation per element per scheduling period for upper and lower bound.

Ore processed and material mined constraints:

$$x_n^t \le y_{k,n}^t \qquad \forall n \in \{1,...,N\}, \quad n \in c_k, \quad t \in \{1,...,T\} \tag{12}$$

Where x_n^t is the portion of block n to be extracted as ore and processed in period t, and $y_{k,n}^t$ is representing the portion of mining-cut c_k to be mined in period t, fraction of y characterizes both ore and waste included in the mining-cut. Eq. (12) demonstrates inequalities that ensure the amount of ore of any block which is processed in any given period is less than or equal to the amount of rock extracted from the mining-cut that the block belongs to in any given scheduling period. A very important assumption in the formulation is that each mining-cut is extracted homogeneously; this means that $y_{k,n}^t$ illustrates the fraction of mining-cut k to be extracted in time period t, all the blocks within the cut, $n \in c_k$ are extracted with the same proportion of $y_{k,n}^t$. This assumption generates production schedules that mimic the real mining operation in the sense that it would minimize the jumping movement of the equipment from one point to another. Eq. (12) generates one equation per block per period.

Precedence of mining-cuts extraction and slope constraints

$$b_k^t - \sum_{i=1}^{t} y_s^i \le 0 \qquad \forall k \in \{1,...,K\}, \quad t \in \{1,...,T\}, \quad s \in H(S) \tag{13}$$

$$\sum_{i=1}^{t} y_k^i - b_k^t \le 0 \qquad \forall k \in \{1,...,K\}, \quad t \in \{1,...,T\} \tag{14}$$

$$b_k^t - b_k^{t+1} \le 0 \qquad \forall k \in \{1,...,K\}, \quad t \in \{1,...,T-1\} \qquad (15)$$

Where

- $b_k^t \in \{0,1\}$ is a binary integer decision variable controlling the precedence of extraction of mining- cuts. b_k^t is equal to one if extraction of mining-cut c_k has started by or in period t, otherwise it is zero,
- $H(S)$ is a set $H(S) \subset \mathcal{K}$ for each mining-cut c_k, defining the immediate predecessor cuts that must be extracted prior to extracting mining-cut k, where S is the total number of cuts in set $H(S)$.

For each mining-cut k Eqs. (13) to (15) check the set of immediate predecessor cuts that must be extracted prior to mining-cut k. This precedence relationship ensures that all the blocks above the current mining-cut are extracted prior to extraction of mining-cut. As it could be deduced from Eq. (15), the formulation is based on the temporal sequence of extraction rather than checking for all the periods. For Eqs. (13) to (15), there would be one equation per mining-cut per period.

For each block n there is a set $C(L) \subset \mathcal{N}$, which includes all the blocks that must be extracted prior to mining block n to ensure that block n is exposed for mining with the desired overall pit slopes, where J is the total number of blocks in set $C(L)$. We will use a directed graph to model the precedence of extraction between blocks. We defined a directed graph $G_b(\mathcal{N}, \mathcal{A})$ by the set of vertices, $\mathcal{N}$ (blocks); connected by ordered pairs of elements called arcs, $\mathcal{A}$.

During the clustering of blocks into mining-cuts another directed graph at mining-cut level is constructed, which captures the precedence relationship of mining-cuts. This directed graph is denoted by $G_c(\mathcal{K}, \mathcal{B})$ where $\mathcal{B} = \{1,...,B\}$ is the set of all edges in the mining-cuts precedence directed graph. The directed graph $G_c(\mathcal{K}, \mathcal{B})$ is constructed in a way that while satisfying the order of extraction at mining-cut level, it would also satisfy the relationships defined by the graph $G_b(\mathcal{N}, \mathcal{A})$ at block level. This approach of defining two directed graphs at mining-cut and block level enables us to model variable pit slopes with small acceptable slope errors in the different regions of the open pit. In other words, mining is controlled at the mining-cut level but the slopes are modeled at the block level.

2.2. Alternative MILP formulation

Eqs. (5) to (15) represent the MILP formulation for long-term open pit production scheduling. The proposed formulation requires $(2 \times K + N) \times T$ number of decision variables, where $K \times T$ of these variables are binary integers. One of the major obstacles in using the MILP formulations for mine production scheduling is the sheer size of the problem. The number of blocks, N, in the model is usually between tens of thousands to millions; the numerous number of blocks within the model will lead to a formulation with an objective function with many variables. Moreover, the main physical constraint in open pit mining is the relationship of precedence of extraction of blocks modeled by binary integer variables. The most common difficulty with MILPs is size of the branch and cut tree; the tree becomes so large that insufficient memory remains to solve an LP sub-problem. The number of binary integer variables in the formulations determines the size of the branch and cut tree. As a general strategy in our formulations we aimed at reducing the number of binary integer variables, we also focused on developing formulations that will mainly use continuous optimization techniques rather than discrete optimization. We have reduced the number of the binary integer variables to $K \times T$, where to some extent we have control over the number of mining-cuts K, during the clustering process.

We investigated the effect of using continuous decision variables (x_n^t *and* y_k^t), which leads to fractional block extraction on the quality and practicality of the generated schedules. There is a possibility that block $n \in c_k$ get extracted over multiple periods. Our computational experiments on different orebody models using the formulation presented in section 2.1 revealed that the blocks' fractions are usually scheduled over consecutive periods and in the worst case examined, some blocks were extracted over three periods. We should also emphasize again that blocks are uniformly extracted as part of mining-cuts that means in the worst case observed a mining-cut is extracted over three periods, which is not impractical from mining point of view. The total tonnage of ore processed in the MILP formulation presented in section 2.1 is related to how mining and processing capacities are set in accordance with the ore reserve total tonnage. There is the possibility that quantities of ore above the cut-off grade would not get processed due to the processing capacity limitations; it is feasible to overcome the abovementioned problems by adding reserve and maximum number of fractions constraints to the MILP formulation presented in section 2.1.

Maximum number of fractions and reserve constraints

$$\sum_{t=1}^{T} x_n^t = 1 \qquad \forall n \in \{1,...,N\} \tag{16}$$

$$\sum_{t=1}^{T} y_k^t = 1 \qquad \forall k \in \{1,...,K\} \tag{17}$$

$$\sum_{t=1}^{T} u_k^t \le m \qquad \forall k \in \{1,...,K\} \tag{18}$$

$$\sum_{t=1}^{T} u_k^t \times y_k^t = 1 \qquad \forall k \in \{1,...,K\} \tag{19}$$

Where

- $u_k^t \in \{0,1\}$ is a binary integer decision variable equal to one if mining-cut c_k is scheduled to be extracted in period t, otherwise zero,
- m is an integer number representing the maximum number of fractions that mining-cuts are allowed to be extracted over.

Equality constraints presented by Eq. (16) would ensure that all the ore within the predefined pit limits or the targeted push-back would be processed during the scheduling. Eq. (16) adds one constraint per block. Eq. (17) would ensure that all the material within the predefined pit outline is going to be mined; this would add one constraint per mining-cut. Eq. (18) and (19) guarantee that the maximum number of fractions of mining-cuts in the solution for y_k^t is not going to exceed m. For large-scale models with many numbers of scheduling periods m could be set equal to two or three fractions maximum. Eq. (19) introduces non-linear constraints into the MILP formulation. The modified model by adding Eqs. (16) to (19) even after linearization, would frame a more complicated MILP formulation comparing to the MILP formulations without Eqs. (16) to (19).

2.3. Extraction and processing at mining-cut level

The formulation of the MILP presented in sections 2.1 and 2.2 has reduced the required numbers of binary integer variables for controlling the precedence relationships drastically. Nevertheless, for large-scale production scheduling models with millions of blocks and decades of mine life the size of the LP sub-problems represented by the x_n^t continuous decision variables would be intractable with the current state of the optimization technology. To be able to overcome this obstacle, we present an MILP formulation which both mining and processing are at the mining-cut level. This approach makes it possible to formulate a tractable MILP model for even very large-scale open pit mines with millions of blocks over decades of mine life. We introduce a continuous processing decision variable at mining-cut level. The blocks are aggregated prior to optimization into mining-cuts with clustering algorithms. the MILP formulation of the model is as follows:

Objective function:

$$\max \sum_{t=1}^{T}\sum_{k=1}^{K}(v_k^t \times s_k^t - q_k^t \times y_k^t) \qquad (20)$$

Subject to:

$$gl^{t,e} \le \sum_{k=1}^{K} g_k^e \times o_k \times s_k^t \Big/ \sum_{k=1}^{K} o_k \times s_k^t \le gu^{t,e} \qquad \forall t \in \{1,...,T\},\quad e \in \{1,...,E\} \qquad (21)$$

$$pl^t \le \sum_{k=1}^{K} o_k \times s_k^t \le pu^t \qquad \forall t \in \{1,...,T\},\quad e \in \{1,...,E\} \qquad (22)$$

$$ml^t \le \sum_{k=1}^{K} (o_k + w_k) \times y_k^t \le mu^t \qquad \forall t \in \{1,...,T\} \qquad (23)$$

$$s_k^t \le y_k^t \qquad \forall k \in \{1,...,K\},\quad t \in \{1,...,T\} \qquad (24)$$

Eqs. (13) to (15).

Where

- $s_k^t \in [0,1]$ is a continuous variable, representing the portion of mining-cut c_k to be extracted as ore and processed in period t,
- o_k is the ore tonnage in mining-cut k,
- w_k is the waste in mining-cut k.

Constraints similar to what is presented by Eqs. (16) to (19) for reserve constraints and maximum number of mining-cuts is optional in this formulation as well.

3. Numerical modeling

In most linear optimization problems, the variables of the objective function are continuous in the mathematical sense, with no gaps between real values. To solve such linear programming problems, ILOG CPLEX implements optimizers based on the simplex algorithms (Winston, 1995) (both primal and dual simplex) as well as primal-dual logarithmic barrier algorithms.

Branch and cut is a method of combinatorial optimization for solving integer linear programs. The method is a hybrid of branch and bound and cutting plane methods (Horst and Hoang, 1996). Refer to Wolsey (1998) for a detailed explanation of branch and cut algorithm, including cutting planes. In recent years there has been significant improvements in mathematical programming optimizers such as ILOG CPLEX (Bixby, 1987-2009). This optimizer uses branch and cut techniques to solve MILP models and it is closing the gap between theory and practice in optimization of large-scale industrial problems. In this study we used TOMLAB/CPLEX version 11.2 (Holmström, 1989-2009) as the MILP solver. TOMLAB/CPLEX efficiently integrates the solver package CPLEX (ILOG Inc, 2007) with MATLAB environment (MathWorks Inc., 2007).

An important termination criterion that the user can set explicitly in CPLEX is the MILP gap tolerance. We have used the relative MILP gap tolerance, which indicates to CPLEX to stop when an integer feasible solution has been proved to be within the gap% of optimality.

3.1. General formulation

The general formulation in TOMLAB for a mixed integer linear programming problem is in the form of:

$$\min_{z} f(z) = \mathbf{c}^T.\mathbf{z} \tag{25}$$

Subject to:

$$\mathbf{z}_l \le \mathbf{z} \le \mathbf{z}_u \tag{26}$$

$$\mathbf{b}_l \le \mathbf{A}.\mathbf{z} \le \mathbf{b}_u \tag{27}$$

Where

- $\mathbf{c}$ is a $j \times 1$ vector, the objective function coefficient, and $\mathbf{c}^T$ represents the transpose of $\mathbf{c}$.
- $\mathbf{z}$ is a $j \times 1$ vector, elements of vector $\mathbf{z}$ are the decision variables of the MILP formulation.
- $\mathbf{z}_l$ *and* $\mathbf{z}_u$ are $j \times 1$ vectors, defining the lower and upper bounds on the decision variables.
- $\mathbf{A}$: is a $i \times j$ coefficient matrix, representing the constraints of the MILP formulations.

- $\mathbf{b}_l$ *and* $\mathbf{b}_u$: are $j \times 1$ vectors, defining the lower and upper boundary conditions. Equality constraints are defined by setting the lower bounds equal to the upper bounds, $\mathbf{b}_l = \mathbf{b}_u$.

3.2. Objective function

We will formulate the general MILP model presented in section 2.1 with the maximum number of fractions and reserve constraints demonstrated by Eqs. (16) to (19). The objective of open pit production scheduling problem given by Eq. (5) is to maximize the NPV of the operation. The general format of the MILP formulation in TOMLAB given by Eq. (25) is to minimize the objective function. Therefore the objective function coefficient vector, $\mathbf{c}$, which is a $(4K+N)T \times 1$ vector given by Eq. (28) should be multiplied by a negative sign, as the result Eq. (25) would change to $\min_{z} f(z) = -\mathbf{c}^T.\mathbf{z}$. To simplify the notation, we will use the vertical matrix concatenation operator, ';'. This operator constructs a matrix or vector by concatenating the matrices or vectors along the vertical dimension of the matrix or vector. The objective function coefficient vector, $\mathbf{c}$ is a vector of size $(4K+N)T \times 1$ given by Eq. (28).

$$\mathbf{c}^{(4K+N)T\times 1} = \left[\mathbf{v};\ \mathbf{q};\ \mathbf{0};\ \mathbf{0};\ \mathbf{0}\right] \tag{28}$$

Where

- $\mathbf{v}$ is an $NT \times 1$ vector holding the discounted values defined by Eq. (3) where N is the maximum number of blocks in the model and T is the number of scheduling periods,
- $\mathbf{q}$ is a $KT \times 1$ vector holding the discounted mining costs defined by Eq. (4) where K is the number of mining-cuts in the model and T is the number of scheduling periods,
- $\mathbf{0}$ is $KT \times 1$ zero vector with all elements equal to zero,

The coefficients in the objective function and in the constraints matrix have different units and dissimilar order of magnitude; it is necessary to transform the objective function and the constraints coefficient matrix to unitless matrices and vectors. To do so we normalize the vectors and matrices by dividing them by a norm of its multipliers vectors. Let us define $\overline{\mathbf{v}} = \mathbf{v}/\|\mathbf{v}\|$ and $\overline{\mathbf{q}} = \mathbf{q}/\|\mathbf{q}\|$, where $\|\mathbf{q}\|$ and $\|\mathbf{v}\|$ are norm of $\mathbf{q}$ and $\mathbf{v}$. Therefore, the coefficient vector is going to be in the form of $\mathbf{c} = \left[\overline{\mathbf{v}};\ \overline{\mathbf{q}};\ \mathbf{0};\ \mathbf{0};\ \mathbf{0}\right]$. We will use the notation '$\overline{\ }$' to illustrate a normalized vector in the rest of the paper.

Eq. (29) illustrates how the decision variables vector, $\mathbf{z}$ is constructed. $\mathbf{z}$ is a $(4K+N)T \times 1$ vector.

$$\mathbf{z}^{(4K+N)T\times 1} = [\mathbf{x};\ \mathbf{y};\ \mathbf{b};\ \mathbf{u};\ \mathbf{uy}] \tag{29}$$

Where

- $\mathbf{x}$ is an $NT \times 1$ vector with the continuous decision variables, $x_n^t \in [0,1]$ as elements, representing the portion of block n to be extracted as ore and processed in period t,
- $\mathbf{y}$ is a $KT \times 1$ vector with the continuous decision variables, $y_k^t \in [0,1]$ as elements, representing the portion of mining-cut c_k to be mined in period t,
- $\mathbf{b}$ is a $KT \times 1$ vector holding the binary integer decision variables, $b_k^t \in \{0,1\}$, these decision variables control the precedence of extraction of mining- cuts,
- $\mathbf{u}$ is $KT \times 1$ vector holding the binary integer variables $u_k^t \in \{0,1\}$; u_k^t is equal to one if mining-cut c_k is scheduled to be extracted in period t, otherwise zero, u_k^t is defined in Eq. (18),
- $\mathbf{uy}$ is $KT \times 1$ vector holding the continuous variables $uy_k^t \in \{0,1\}$, defining the outcome of $u_k^t \times y_k^t$ defined by Eq. (19).

3.3. Constraints

In this section, we will develop the numerical models for the equality and inequality constraints represented by Eqs. (6) to (19).

<u>*Mining capacity constraints:*</u>

Eqs. (6) and (7) represents the mining capacity constraints, the numerical model is represented by Eq. (30), where $\mathbf{A}_1$ is a $2T \times (4K+N)T$ coefficient matrix and $\mathbf{b}_1$ is a $2T \times 1$ boundary condition vector.

$$\mathbf{A}_1.\mathbf{z} \le \mathbf{b}_1 \tag{30}$$

$$\mathbf{A}_1^{2T\times(4K+N)T} = \begin{bmatrix} \mathbf{0}_1 & \overline{\mathbf{A}}_m & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_1 & -\overline{\mathbf{A}}_m & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \end{bmatrix} \tag{31}$$

$$\mathbf{b}_1^{2T\times 1} = \begin{bmatrix} \overline{\mathbf{m}}_u \\ -\overline{\mathbf{m}}_l \end{bmatrix} \tag{32}$$

Where

- $\mathbf{A}_m$ is a $T \times KT$ matrix with elements holding the total tonnage of material in each mining-cut in each period,
- $\mathbf{m}_u$ is a $T \times 1$ vector of mining capacity upper bounds as defined in Eq. (6),
- $\mathbf{m}_l$ is a $T \times 1$ vector of mining capacity lower bounds as defined in Eq. (7),
- $\mathbf{0}_1$ is a $T \times NT$ zero matrix,
- $\mathbf{0}_2$ is a $T \times KT$ zero matrix,

Processing capacity constraints:

Eqs. (8) and (9) represents the processing capacity constraints, the numerical model is represented by Eq. (33), where $\mathbf{A}_2$ is a $2T \times (4K+N)T$ coefficient matrix and $\mathbf{b}_2$ is a $2T \times 1$ boundary condition vector.

$$\mathbf{A}_2.\mathbf{z} \le \mathbf{b}_2 \tag{33}$$

$$\mathbf{A}_2^{2T\times(4K+N)T} = \begin{bmatrix} \overline{\mathbf{A}}_p & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ -\overline{\mathbf{A}}_p & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \end{bmatrix} \tag{34}$$

$$\mathbf{b}_2^{2T\times 1} = \begin{bmatrix} \overline{\mathbf{p}}_u \\ -\overline{\mathbf{p}}_l \end{bmatrix} \tag{35}$$

Where

- $\mathbf{A}_p$ is a $T \times NT$ matrix with elements holding the total tonnage of ore in each block in each period,
- $\mathbf{p}_u$ is a $T \times 1$ vector of processing capacity upper bounds as defined in Eq. (8),
- $\mathbf{p}_l$ is a $T \times 1$ vector of processing capacity lower bounds as defined in Eq. (9).

Grade blending constraints:

Eqs. (10) and (11) represent the grade blending constraints, the numerical model is represented by Eq. (36), where $\mathbf{A}_3$ is a $2ET \times (4K+N)T$ coefficient matrix and $\mathbf{b}_3$ is a $2ET \times 1$ boundary condition vector.

$$\mathbf{A}_3.\mathbf{z} \le \mathbf{b}_3 \tag{36}$$

$$\mathbf{A}_3^{2ET\times(4K+N)T} = \begin{bmatrix} \overline{\mathbf{A}}_g & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ -\overline{\mathbf{A}}_g & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \end{bmatrix} \tag{37}$$

$$\mathbf{b}_3^{2ET\times 1} = \begin{bmatrix} \overline{\mathbf{g}}_u \\ -\overline{\mathbf{g}}_l \end{bmatrix} \tag{38}$$

Where

- $\mathbf{A}_g$ is an $ET \times NT$ matrix of average grade of each element of interest in each block in each period,
- $\mathbf{g}_u$ is an $ET \times 1$ vector of average grade upper bounds on acceptable average head grade of the elements of interest as defined in Eq. (10),
- $\mathbf{g}_l$ is a $ET \times 1$ vector of average grade lower bounds on acceptable average head grade of the elements of interest as defined in Eq. (11),
- $\mathbf{0}_3$ is an $ET \times KT$ zero matrix.

Ore processed and material mined constraints:

Eq. (12) represents the ore processed and material mined constraints, the numerical model is represented by Eq. (39), where $\mathbf{A}_4$ is a $NT \times (4K+N)T$ coefficient matrix and $\mathbf{b}_4$ is an $NT \times 1$ zero boundary condition vector. Eq. (39) inequalities ensure that the amount of ore of any block which is processed in any given period is less than or equal to the amount of rock extracted from the mining-cut that the block belongs to in any given scheduling period.

$$\mathbf{A}_4.\mathbf{z} \leq \mathbf{b}_4 \tag{39}$$

$$\mathbf{A}_4^{NT\times(4K+N)T} = \begin{bmatrix} \mathbf{A}_x & \mathbf{A}_y & \mathbf{0}_4 & \mathbf{0}_4 & \mathbf{0}_4 \end{bmatrix} \tag{40}$$

Where

- $\mathbf{A}_x$ is a $NT \times NT$ matrix with an element of 1 for each block in each period,
- $\mathbf{A}_y$ is a $NT \times KT$ matrix with an element of -1 per mining-cut for each block in each period,
- $\mathbf{0}_4$ is a $NT \times KT$ zero matrix.

Precedence of mining-cuts extraction and slope constraints

Eqs. (13) to (15) represent the precedence of mining-cuts extraction and slope constraints, the numerical model is represented by Eq. (41). We will present the construction of slope constraints matrix, Eq. (41), with an illustrative example.

$$\mathbf{A}_5.\mathbf{z} \leq \mathbf{b}_5 \tag{41}$$

Let's consider a set of mining-cuts to be scheduled. For the sake of discussion we assume that the model consist of five mining-cuts (Fig. 1); the immediate predecessor cuts are labeled with directed-arcs pointing from the parent to the child node. A directed graph constructs the precedence relationship between mining-cuts, the directed graph tags the mining-cuts that must be extracted prior to extracting each mining-cut k. This set is denoted by $H(S) \subset \mathcal{K}$, where S is the total number of mining-cuts in H(S).

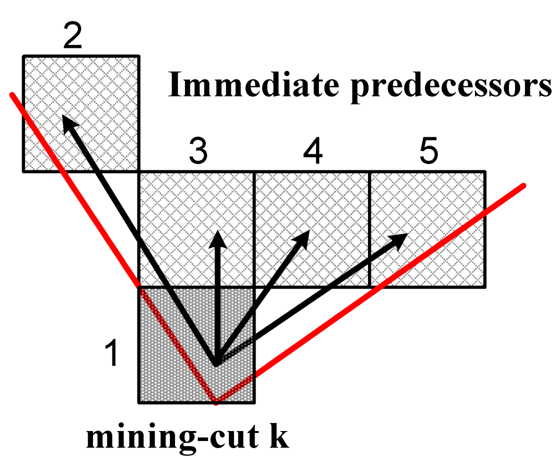

Fig. 1. Schematic view of a mining cut and its predecessors cuts.

We start with constructing the required matrices defining Eq. (13). We assume that the five mining-cuts ($K = 5$) illustrated in Fig. 1, are scheduled to be extracted over four periods ($T = 4$). The immediate predecessors' set $H(S) = \{2,3,4,5\}$; this set represents the mining-cuts that must be extracted prior to extraction of mining-cut labeled as one.

We define $\mathbf{b}1_{vec}$ and $\mathbf{y}1_{vec}$ as $1\times K$ vectors in Eqs. (42) and (43), these vectors are subcomponents used in assembling the matrices required to model Eq. (13). $\mathbf{b}1_{vec}$ and $\mathbf{y}1_{vec}$ are used to assemble matrices $\mathbf{b}1_{mat}$ and $\mathbf{y}1_{mat}$ as presented in Eqs. (44) and (45) for each mining-cut; where $\mathbf{0}_5$ is a $1\times K$ vector of zeros. Then, matrices $\mathbf{A}_{sy1}$ and $\mathbf{A}_{sb1}$, which are $KT\times KT$ matrices are constructed for all the mining-cuts in the model, where $\mathcal{K} = \{1,...,K\}$, this concatenation is demonstrated by Eqs. (46) and (47).

$$\mathbf{b}1_{vec}^{1\times K} = [1 \quad 0 \quad 0 \quad 0 \quad 0] \tag{42}$$

$$\mathbf{y}1_{vec}^{1\times K} = [0 \quad -1 \quad -1 \quad -1 \quad -1] \tag{43}$$

$$\mathbf{b}1_{mat}^{T\times KT} = \begin{pmatrix} \mathbf{b}1_{vec} & \cdots & \mathbf{0}_5 \\ \vdots & \ddots & \vdots \\ \mathbf{0}_5 & \cdots & \mathbf{b}1_{vec} \end{pmatrix} \tag{44}$$

$$\mathbf{y}1_{mat}^{T\times KT} = \begin{pmatrix} \mathbf{y}1_{vec} & \cdots & \mathbf{0}_5 \\ \vdots & \ddots & \vdots \\ \mathbf{y}1_{vec} & \cdots & \mathbf{y}1_{vec} \end{pmatrix} \tag{45}$$

$$\mathbf{A}_{sy1}^{KT\times KT} = \left(\mathbf{y}1_{mat1}; \quad \mathbf{y}1_{mat2}; \; \cdots\cdots \; ;\mathbf{y}1_{matK}\right) \tag{46}$$

$$\mathbf{A}_{sb1}^{KT\times KT} = \left(\mathbf{b}1_{mat1}; \quad \mathbf{b}1_{mat2}; \; \cdots\cdots \; ;\mathbf{b}1_{matK}\right) \tag{47}$$

Subsequently, we will construct the matrices required to capture Eq. (14). We define $\mathbf{b}2_{vec}$ and $\mathbf{y}2_{vec}$ as $1\times K$ vectors in Eqs. (48) and (49), these vectors are subcomponents used in assembling the matrices required to model Eq. (14). $\mathbf{b}2_{vec}$ and $\mathbf{y}2_{vec}$ vectors are used to assemble matrices $\mathbf{b}2_{mat}$ and $\mathbf{y}2_{mat}$ as presented in Eqs. (50) and (51), for each mining-cut. Afterward, matrices $\mathbf{A}_{sy2}$ and $\mathbf{A}_{sb2}$, which are $KT\times KT$ matrices are constructed for all the mining-cuts in the model, this concatenation is demonstrated by Eqs. (52) and (53).

$$\mathbf{b}2_{vec}^{1\times K} = [-1 \quad 0 \quad 0 \quad 0 \quad 0] \tag{48}$$

$$\mathbf{y}2_{vec}^{1\times K} = [1 \quad 0 \quad 0 \quad 0 \quad 0] \tag{49}$$

$$\mathbf{b}2_{mat}^{T\times KT} = \begin{pmatrix} \mathbf{b}2_{vec} & \cdots & \mathbf{0}_5 \\ \vdots & \ddots & \vdots \\ \mathbf{0}_5 & \cdots & \mathbf{b}2_{vec} \end{pmatrix} \tag{50}$$

$$\mathbf{y}2_{mat}^{T\times KT} = \begin{pmatrix} \mathbf{y}2_{vec} & \cdots & \mathbf{0}_5 \\ \vdots & \ddots & \vdots \\ \mathbf{y}2_{vec} & \cdots & \mathbf{y}2_{vec} \end{pmatrix} \tag{51}$$

$$\mathbf{A}_{sy2}^{KT\times KT} = \left(\mathbf{y}2_{mat1};\quad \mathbf{y}2_{mat2};\ \cdots\cdots\ ;\quad \mathbf{y}2_{matK}\right) \tag{52}$$

$$\mathbf{A}_{sb2}^{KT\times KT} = \left(\mathbf{b}2_{mat1};\quad \mathbf{b}2_{mat2};\ \cdots\cdots\ ;\quad \mathbf{b}2_{matK}\right) \tag{53}$$

Next, we will construct the matrices required to capture Eq. (15). We define $\mathbf{b}3_{vec}$ and $\mathbf{b4}_{vec}$ as $1\times K$ vectors in Eqs. (54) and (55), these vectors are subcomponents used in assembling the matrices required to model Eq. (15). $\mathbf{b}3_{vec}$ and $\mathbf{b4}_{vec}$ vectors are used to assemble matrix $\mathbf{b}3_{mat}$ as presented in Eq. (56), for each mining-cut. Matrix $\mathbf{A}_{sb3}$, which is $K(T-1)\times KT$ matrix is constructed for all the mining-cuts in the model, this concatenation is demonstrated by Eq. (57).

$$\mathbf{b}3_{vec}^{1\times K} = [1 \quad 0 \quad 0 \quad 0 \quad 0] \tag{54}$$

$$\mathbf{b}4_{vec}^{1\times K} = [-1 \quad 0 \quad 0 \quad 0 \quad 0] \tag{55}$$

$$\mathbf{b}3_{mat}^{(T-1)\times KT} = \begin{pmatrix} \mathbf{b3}_{vec} & \mathbf{b}4_{vec} \cdots & \mathbf{0}_5 \\ \vdots \ \ddots & \ddots & \vdots \\ \mathbf{0}_5 \ \ \cdots & \mathbf{b3}_{vec} & \mathbf{b}4_{vec} \end{pmatrix} \tag{56}$$

$$\mathbf{A}_{sb3}^{K(T-1)\times KT} = \left(\mathbf{b}3_{mat1};\quad \mathbf{b}3_{mat2};\ \cdots\cdots\ ;\quad \mathbf{b}3_{matK}\right) \tag{57}$$

Now we can construct the matrix $\mathbf{A}_5$ in Eq. (58), the inequality constraints is represented in Eq.(41), where

- $\mathbf{A}_5$ is a $[2KT+K(T-1)]\times(4K+N)T$ coefficient matrix,
- $\mathbf{b}_5$ is a $[2KT+K(T-1)]\times 1$ zero boundary condition vector,
- $\mathbf{0}_6$ is a $KT\times NT$ zero matrix,
- $\mathbf{0}_7$ is a $KT\times KT$ zero matrix,
- $\mathbf{0}_8$ is a $K(T-1)\times NT$ zero matrix,
- $\mathbf{0}_9$ is a $K(T-1)\times KT$ zero matrix.

$$\mathbf{A}_5^{[2KT+K(T-1)]\times(4K+N)T} = \begin{bmatrix} \mathbf{0}_6 & \mathbf{A}_{sy1} & \mathbf{A}_{sb1} & \mathbf{0}_7 & \mathbf{0}_7 \\ \mathbf{0}_6 & \mathbf{A}_{sy2} & \mathbf{A}_{sb2} & \mathbf{0}_7 & \mathbf{0}_7 \\ \mathbf{0}_8 & \mathbf{0}_9 & \mathbf{A}_{sb3} & \mathbf{0}_9 & \mathbf{0}_9 \end{bmatrix} \tag{58}$$

Maximum number of fractions and reserve constraints

The numerical model for the maximum number of mining fractions and reserve constraints are represented by Eqs. (16) to (19), where $\mathbf{A}_6$ is a $(N+3K)\times(4K+N)T$ coefficient matrix and $\mathbf{b}_6$ is an $(N+3K)\times 1$ boundary condition vector.

$$\mathbf{A}_6.\mathbf{z} \le \mathbf{b}_6 \tag{59}$$

$$\mathbf{A}_6^{(N+3K)\times(4K+N)T} = \begin{bmatrix} \mathbf{A}_{rx} & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_1 & \mathbf{A}_{ry} & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 \\ \mathbf{0}_1 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{A}_u & \mathbf{0}_2 \\ \mathbf{0}_1 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{A}_{uy} \end{bmatrix} \tag{60}$$

$$\mathbf{b}_6^{(N+3K)\times 1} = \left[\ \mathbf{1}_1; \quad \mathbf{1}_2; \quad \mathbf{m}; \quad \mathbf{1}_2 \right] \tag{61}$$

Where

- $\mathbf{1}_1$ is a $N\times 1$ vector with all elements equal to one.
- $\mathbf{1}_2$ is a $K\times 1$ vector with all elements equal to one.
- $\mathbf{A}_{rx}$ is a $N\times NT$ matrix with elements of one for each block in each period, these equality constraints add one constraints per block as defined by Eq. (16).
- $\mathbf{A}_{ry}$ is a $K\times KT$ matrix with elements of one for each mining-cut in each period, these equality constraints add one constraints per mining-cut as defined by Eq. (17).
- $\mathbf{A}_u$ is a $K\times KT$ matrix with elements of one for the periods that a mining-cut is scheduled to be extracted as defined by Eq. (18).
- $\mathbf{m}$ is a $K\times 1$ vector with elements equal to the number of maximum fractions that one mining-cut is allowed to be scheduled as defined by Eq. (18).
- $\mathbf{A}_{uy}$ is a $K\times KT$ matrix.

The equality constraints, in Eqs. (16), (17) and (19) are defined by setting the lower bounds equal to the upper bounds. Finally, we concatenate all the matrices and vectors representing the constraints and bounds into the coefficient matrix, $\mathbf{A}$ with the size of $[(3K+2E+N+4)T+N+2K]\times[(4K+N)T]$ and one boundary condition vector, **b,** with the size of $(4K+N)T\times 1$. The concatenation is represented by Eqs. (62) and (63).

$$\mathbf{A}^{[(3K+2E+N+4)T+N+2K]\times[(4K+N)T]} = \left(\mathbf{A}_1; \quad \mathbf{A}_2; \quad \mathbf{A}_3; \quad \mathbf{A}_4; \quad \mathbf{A}_5; \quad \mathbf{A}_6\right) \tag{62}$$

$$\mathbf{b}^{(4K+N)T\times 1} = \left(\mathbf{b}_1; \quad \mathbf{b}_2; \quad \mathbf{b}_3; \quad \mathbf{b}_4; \quad \mathbf{b}_5; \quad \mathbf{b}_6\right) \tag{63}$$

4. Numerical experiments and mining case study

A case study of scheduling an iron ore deposit was carried out to verify and validate the models. The total number of blocks within the final pit limit is 19,492. We used the fuzzy logic clustering algorithm to aggregate the blocks into 599 mining-cuts. Fig. 2 illustrates aggregating blocks into mining-cuts using fuzzy logic clustering, blocks are spatially grouped together based on rock-type and grade distribution, units in meters. Three types of ore; top magnetite, oxide, and bottom magnetite are classified in the deposit. The block model contains the estimated magnetic weight recovery (MWT%) of iron ore and the contaminants are phosphor (P%) and sulphur (S%). The blocks in the geological model represent a volume of rock equal to $25m \times 25m \times 15m$.

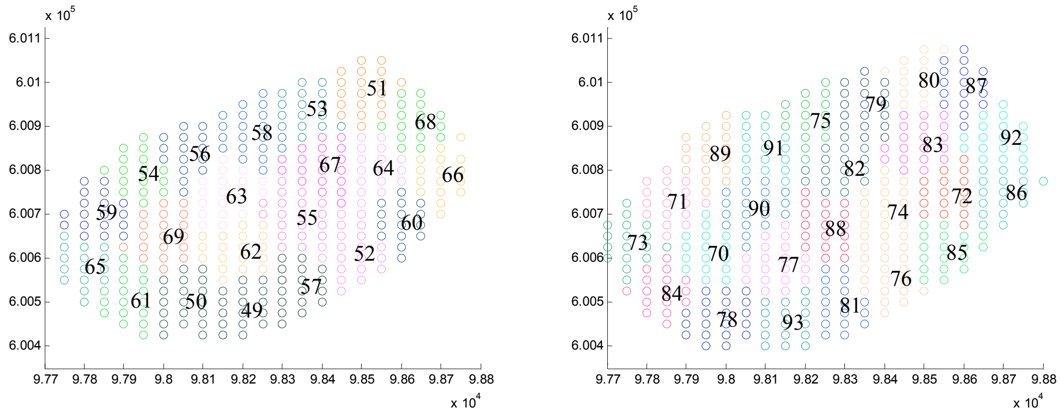

Fig. 2 Aggregating blocks into mining-cuts using fuzzy logic clustering, blocks are spatially grouped together based on rock-type and grade distribution, units in meters.

Table 1 summarizes the information related to the case study. The pit includes 427.33 Mt of rock where 116.29 Mt is ore with an average magnetic weight recovery of grade of 72.9%. Initially a capacity of 30 Mt /year was considered as the upper bound of mining. The objective function aimed to maximize the net present value with a discount rate of 10% per period. TOMLAB/CPLEX was used for implementation and solving the MILP formulation. 19,492 blocks were scheduled over 17 periods this made a coefficient matrix, A of a size $A(63,474 \times 30,549)$ with 510,094 nonzero elements. The CPLEX solver was set to find a solution within 3% gap of the theoretical optimal solution.

Table 2 illustrates the settings for two optimization runs, in the first one we used the default CPLEX settings. The upper and lower bound for MWT, S, and P are defined. A ten Mt/yr processing capacity was set for this case. In the second test we used four years of pre-stripping with setting the upper bound of the processing plant equal to zero. The processing capacity was ramped up gradually to ten Mt/yr by year nine.

Table 1. Final pit and production scheduling information.

Description	Value	Description	Value
Number of blocks	19,492	Minimum mining width (m)	150
Number of mining-cuts	599	Number of periods (years)	17
Total tonnage of rock (Mt)	427.33	A(rows × columns)	63,474×30,549
Total ore tonnage (Mt)	116.29	No. Of nonzero elements in A	510,094
Total tonnage of recovered Fe (Mt)	76.33	Number of decision variables	30,549
Average grade of MWT%	72.9%	Number of integer variables	10,183
Mining capacity (Mt/year)	30		

Table 2. Inputs and numerical results for the data set containing 2598 blocks.

Settings	Processing capacity periods - pu^t / pl^t (Mt)	Grade blending $gu^{e,t}$ / $gl^{e,t}$ (%)	NPV ($M)	Root node gap %	CPU time (S)
1– default	1to 17 - 10/0	$0 \leq S \leq 1.8$ $0 \leq P \leq 0.14$ $55 \leq MWT \leq 85$	2,315.29	2.3	15,574
2- with probing	1 to 4 - 0/0 5 to 6 - 6/5 7 to 8 - 8/7 9 to 17- 10/0	$0 \leq S \leq 1.8$ $0 \leq P \leq 0.14$ $60 \leq MWT \leq 85$	2,141.60	2.02	7,957

As it was expected the net present value in the second case dropped because of the tighter bounds imposed on the model. 114 million tonnes of ore was processed out of 116.29 million tonnes available.

Fig. 3 illustrates the plan view of bench 1567m, the orebody is outlined with the final pit and the waste blocks are represented by their rock type color profile. Fig. 4 illustrates the generated schedule on bench 1567m with the order of extraction color profiled. Fig. 5 illustrates the schedule at the block level on bench 1567m. Fig. 6 to 9 represent the schedule on cross section 98300m looking east and cross section 600840 looking north.

Fig. 10 and 11 show the yearly schedule of the ore and waste production along with the average grade of MWT, S, and P through out the mine life.

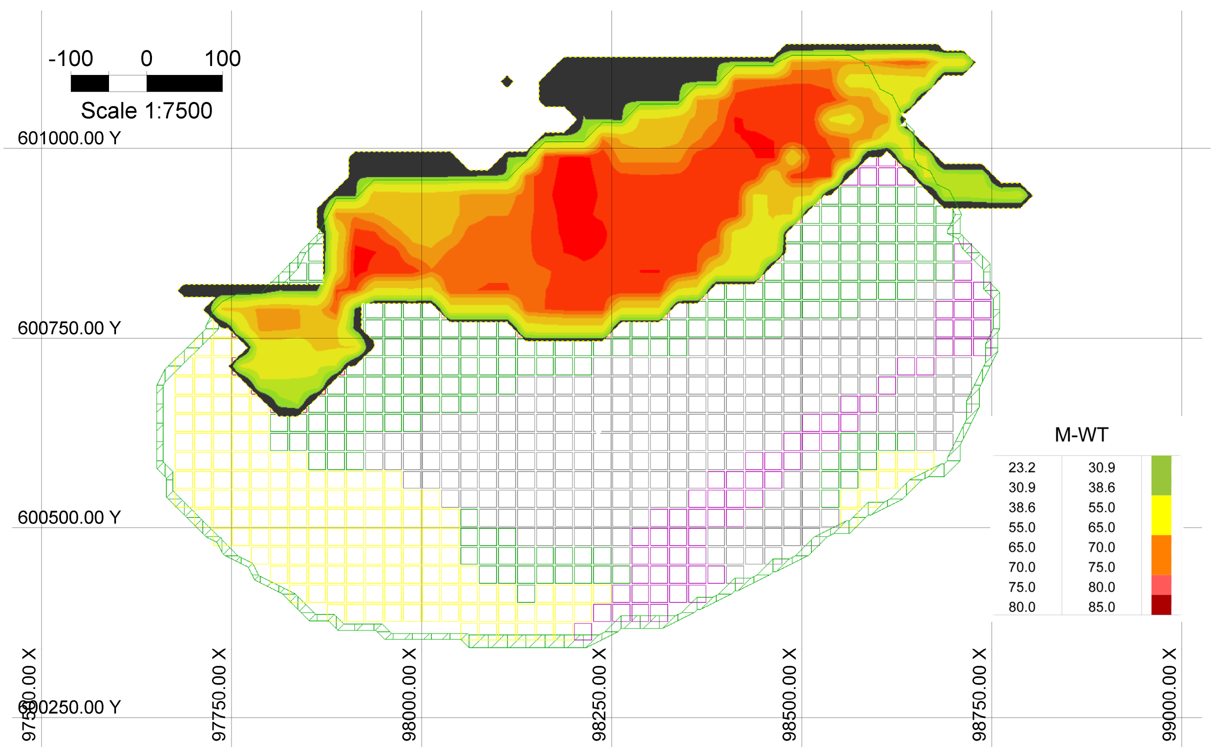

Fig. 3. Plan view of bench 1567m, the orebody is outlined, with the MWT grade; the green line shows the outline of final pit and the waste blocks are represented by their rock type color profile.

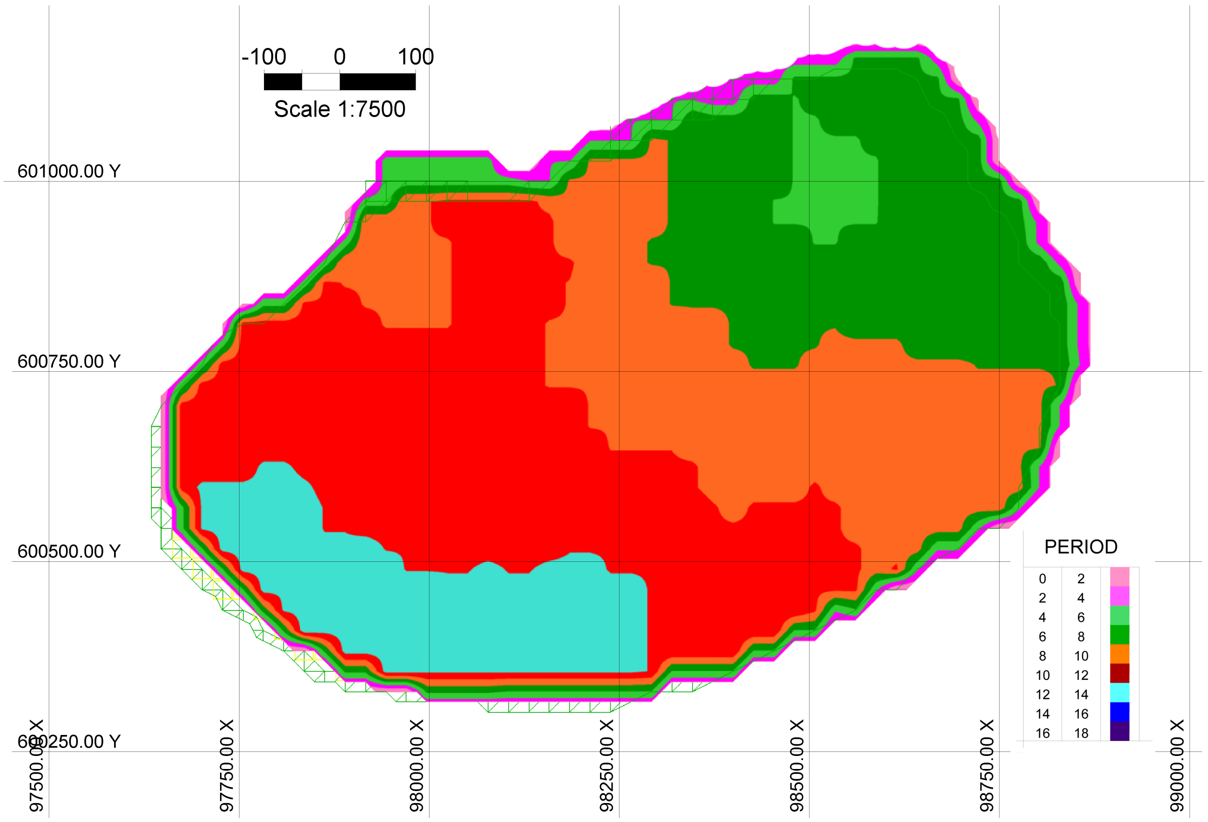

Fig. 4. Plan view of bench 1567m schedule, the extraction periods are colored sequentially.

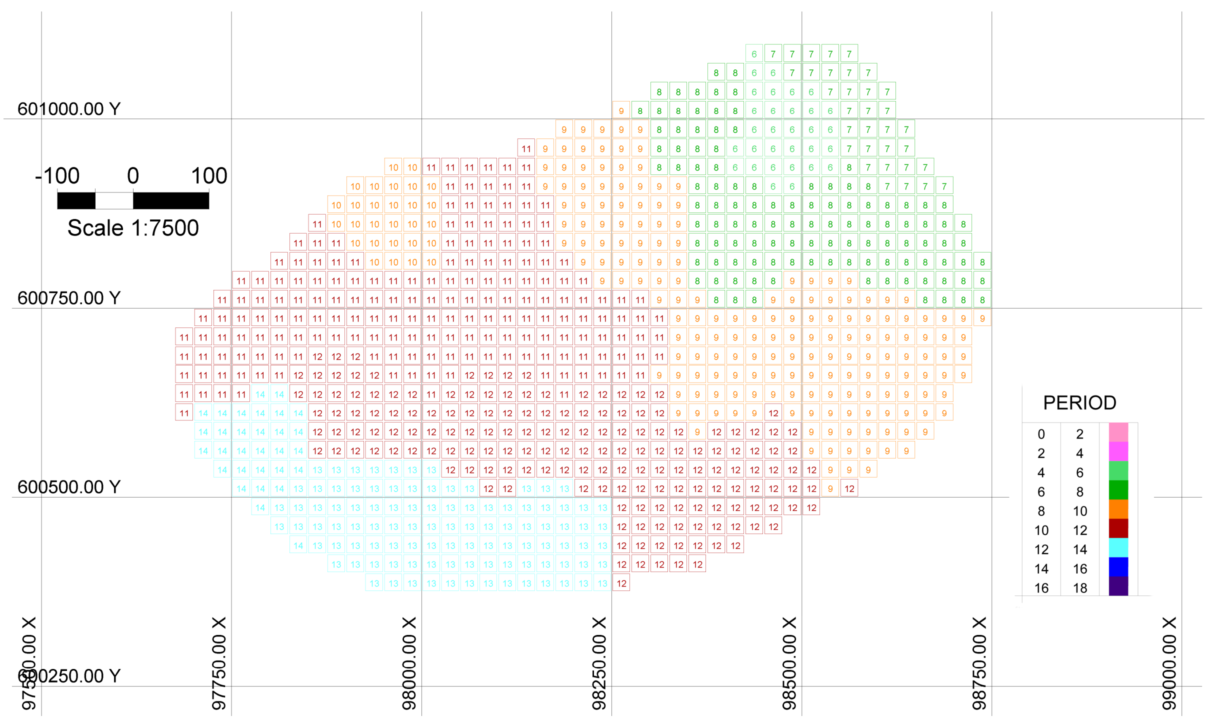

Fig. 5. Plan view of bench 1567m schedule, the extraction periods are defined at the block level, units in meters.

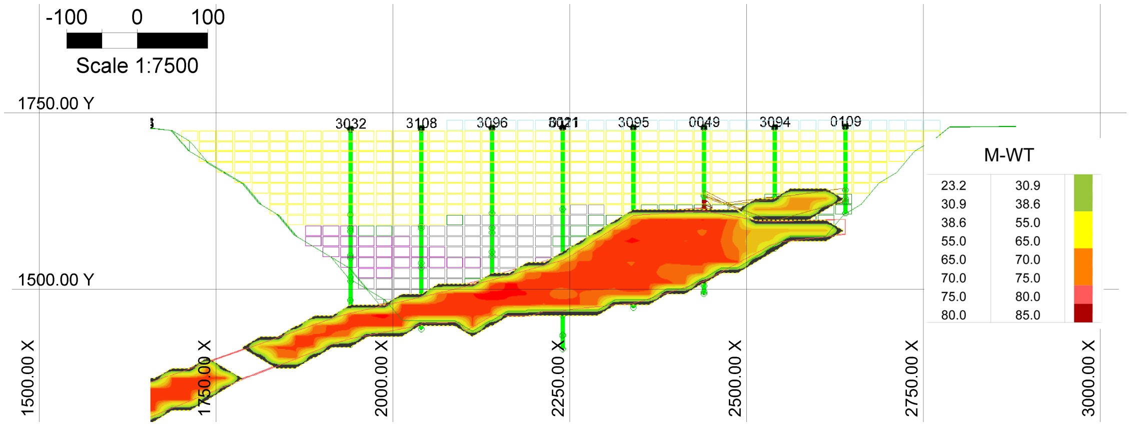

Fig. 6. Cross section 98300m looking east, the orebody is outlined, with the MWT grade, the waste blocks are represented by their rock type color, units in meters.

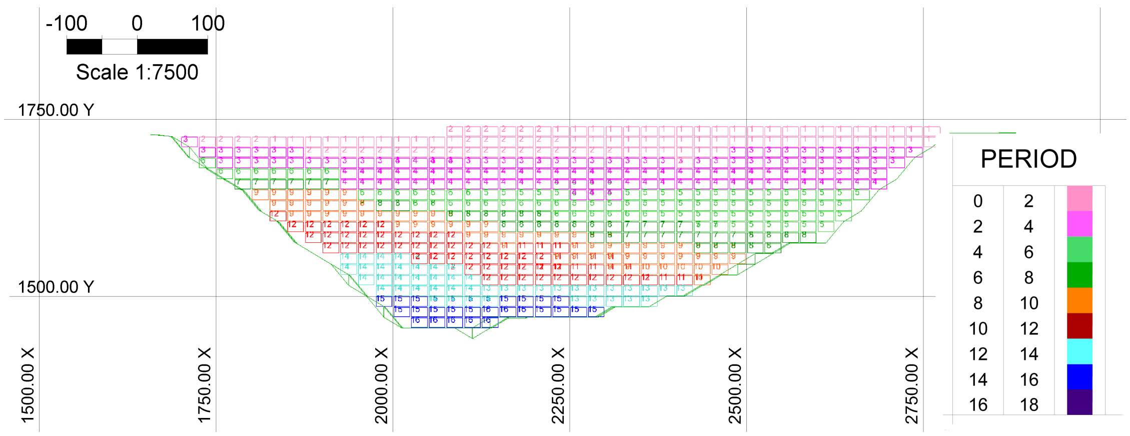

Fig. 7. Schedule of cross section 98300m looking east, units in meters.

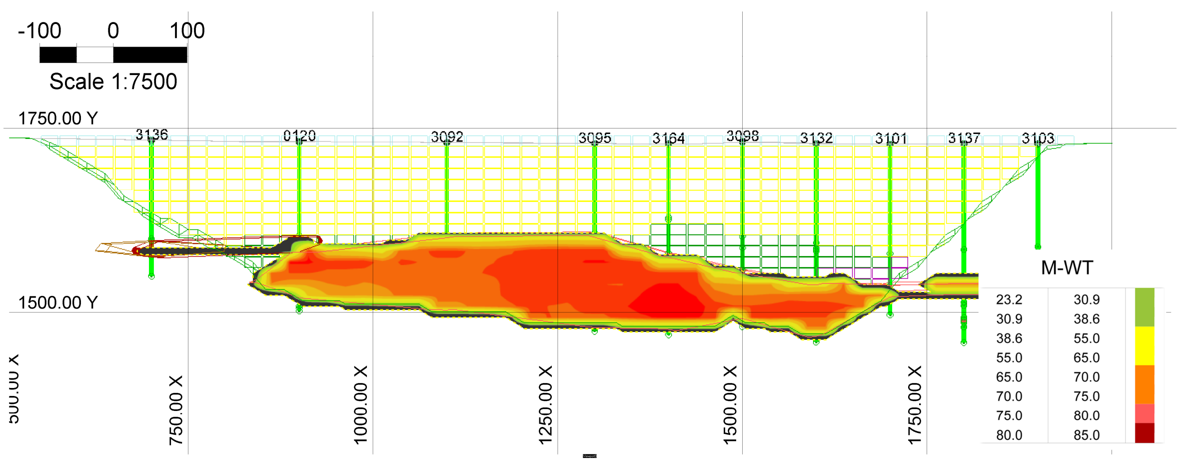

Fig. 8. Cross section 600840m looking north, the orebody is outlined, with the MWT grade, the green line shows the outline of final pit and the waste blocks are represented by their rock type, units in meters.

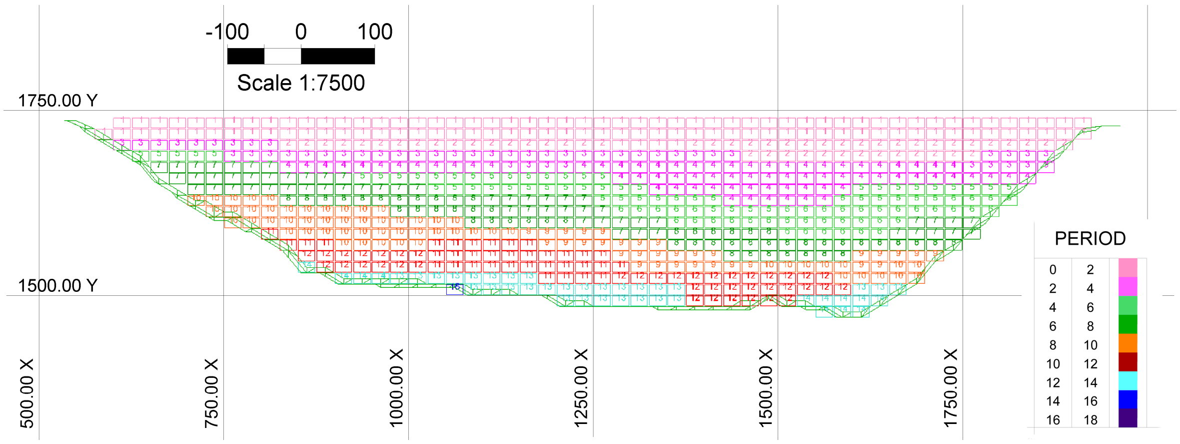

Fig. 9. Schedule of cross section 600840m looking north, extraction periods are color profiled, units in meters.

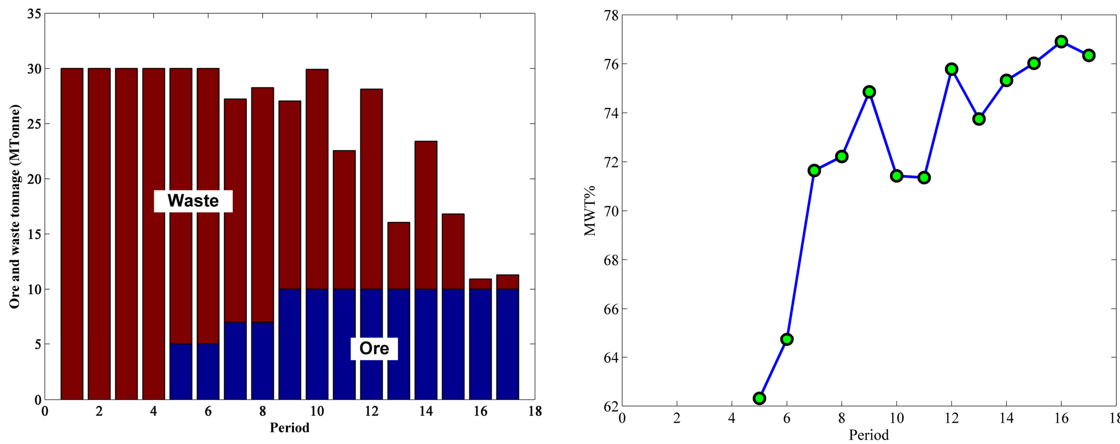

Fig. 10. Ore and waste tonnage schedule (left-a), Average grade MWT% per period (right-b), period in years.

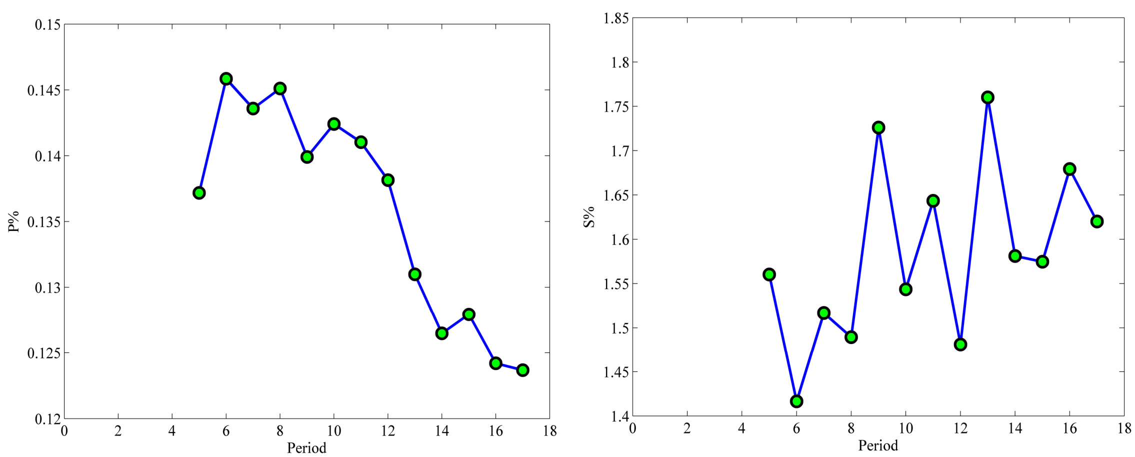

Fig. 11. Average grade P% per period (left-a), average grade S% per period (right-b), periods in years.

5. Conclusions

The applications of the MILP model developed in this study showed that it has the capability of generating production schedules within a close gap to the theoretical optimal net present values for mining operations. The balance between the number of mining-cuts and the total number of blocks in the model is very important. Research is underway to test the developed models on large-scale open pit problems with up to two hundred thousand blocks within the final pit over thirty years of mine-life. In the future we will focus on reformulating the problem with reduced number of integer variables and complexity and tackling the geological uncertainty within the mine planning domain.

6. References

[1] Bixby, R. E., (1987-2009), "ILOG CPLEX", ver. 11.0, Sunnyvale, CA , USA: ILOG, Inc.

[2] Boland, N., Dumitrescu, I., Froyland, G., and Gleixner, A. M., (2009), "LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity", *Computers and Operations Research*, Vol. 36, 4, pp. 1064-89.

[3] Boland, N., Fricke, C., and Froyland, G., (2007), "A strengthened formulation for the open pit mine production scheduling problem", Optimization Online, © the Mathematical Programming Society and by the Optimization Technology Center, Retrieved Jan. 9, 2009 from: http://www.optimization-online.org/DB_HTML/2007/03/1624.html

[4] Caccetta, L. and Hill, S. P., (2003), "An application of branch and cut to open pit mine scheduling", *Journal of Global Optimization*, Vol. 27, November, pp. 349 - 365.

[5] Dagdelen, K. and Kawahata, K., (2007), "Oppurtunities in Multi-Mine Planning through Large Scale Mixed Integer Linear Programming Optimization", *in Proceedings of 33rd International Symposium on Computer Application in the Minerals Industry (APCOM)*, © GECAMIN LTDA, Santiago, Chile, pp. 337-342.

[6] Holmström, K., (1989-2009), "TOMLAB /CPLEX - v11.2", ver. Pullman, WA, USA: Tomlab Optimization.

[7] Horst, R. and Hoang, T., (1996), "Global optimization : deterministic approaches", © Springer, Berlin ; New York, 3rd ed, Pages xviii, 727 p.

[8] ILOG Inc, (2007), "ILOG CPLEX 11.0 User's Manual September", ver. 11.0: ILOG S.A. and ILOG, Inc.

[9] MathWorks Inc., (2007), "MATLAB 7.4 (R2007a) Software", MathWorks, Inc.

[10] Ramazan, S. and Dimitrakopoulos, R., (2004), "Traditional and new MIP models for production scheduling with in-situ grade variability", *International Journal of Surface Mining, Reclamation & Environment*, Vol. 18, 2, pp. 85-98.

[11] Winston, W. L., (1995), "Introduction to mathematical programming : applications and algorithms", © Duxbury Press, Belmont, Ca., 2nd ed, Pages xv, 818, 39 p.

[12] Wolsey, L. A., (1998), "Integer programming", © J. Wiley, New York, Pages xviii, 264 p.