Stochastic Optimization and Deterministic Models in Strategic Mine Planning Problems: An Overview of Solution Methods and Comparisons

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ABSTRACT

Strategic mine planning is a critical aspect of mining, focusing on addressing long-term mining challenges to maximize economic or non-economic value while accounting for operational and financial constraints. There are two primary methods for modeling mining problems: deterministic and stochastic optimization. Deterministic models, or traditional models, are used for problems with fixed input parameters. However, these models often fail to consider the inherent uncertainties in geological, economic, and operational parameters. In real-world situations, it is essential to account for data uncertainty, making the application of stochastic parameters important in mining problems. Stochastic optimization is an ideal method for modeling all constraints and objectives of a mining problem while considering potential data uncertainties. In stochastic optimization, uncertainties are addressed by considering multiple scenarios or probabilistic distributions for the parameters. This study provides an overview of the methods, applications and comparisons of stochastic optimization and deterministic modeling in strategic mine planning. Evaluating the advantages and limitations of these two approaches reveals that stochastic optimization improves problem solutions and enhances the sustainability of mining operations under uncertainty, but it involves more computational complexity.

1. Introduction

Strategic mine planning is a comprehensive problem in the mining industry, involving the design, scheduling, and optimization of mining operations to ensure efficient and cost-effective extraction of minerals and their processing. This process encompasses various steps, from ore body modeling to the final production output. Despite advancements in ore body modeling, a significant gap remains in the integration of these models with downstream mining processes. Current approaches often lack realistic models of these processes, which are crucial for accurate planning and optimization. This gap highlights the need for improved methodologies that can seamlessly incorporate the sequence of inputs and outputs from staged processes, ensuring a more realistic and effective mine planning strategy to maximize financial, environmental and sustainability values [1].

In strategic mine planning there are different methods to model the mining problems, in which the ability to effectively manage and mitigate risk and uncertainty is significant. Initial studies on applying traditional approaches to production planning and scheduling involved methods based on Lagrangian relaxation and the branch-and-cut algorithm. These early methods faced significant limitations, primarily their inability to handle large-scale deposits containing millions of blocks. To address this issue, extensive research has been conducted to reduce the problem size, leading to the

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development of various methodologies and techniques [2]. The main advancements and techniques to solve the mine production planning problems include the fundamental trees methodology, heuristic approaches, dynamic programming and heuristics, and meta-heuristic techniques. However, despite these innovations, a common drawback of these methods is their tendency to produce unrealistic designs by not considering the inherent uncertainties in mining operations. In the next section, each method to solve the traditional model in strategic mine planning is explained in more detail.

After studying traditional models in strategic mine planning studies, researchers recognized the need to address the inherent uncertainties and variability in mining data. To mitigate these uncertainties and enhance the robustness of planning decisions, stochastic optimization techniques were introduced. Stochastic mine planning represents a critical advancement in optimizing mineral resource extraction by incorporating the inherent uncertainties of geological data, market prices, and operational conditions.

Recent advancements in stochastic mine planning have been driven by improvements in computational power and the development of sophisticated algorithms. These innovations enable the processing of large datasets and the simulation of complex models, leading to more accurate predictions. For instance, techniques such as Monte Carlo simulation, geo-statistical simulation, robust optimization and machine learning algorithms are increasingly being employed to enhance the precision and reliability of stochastic models. The mentioned approaches to address stochastic conditions of mine planning problems are investigated in the following sections. Recent studies emphasize the growing importance of stochastic models in improving decision-making and enhancing the economic viability of mining projects [3].

The most stochastic models which are used in mine planning problems consider three primary sources of uncertainty including geological, technical, and economic conditions. The uncertainties in mining impact the reliability and effectiveness of the mine planning model. To achieve more realistic and robust solutions, it is essential to incorporate these uncertainties into the planning process [4]. By addressing the uncertainties associated with geological, technical, and economic factors, the mining industry can develop more effective and sustainable long-term production plans.

2. Review of Literature

In this section, we investigate both stochastic and Traditional (Deterministic) models, exploring the primary approaches used to solve them across various scenarios in mining as well as other general applications. We begin by examining the fundamental principles and methodologies behind these models, highlighting their respective strengths and limitations. Following this, we present some of the most effective and widely used examples, providing detailed case studies to illustrate their practical applications. Finally, we compare these models in terms of their effectiveness, evaluating their performance and suitability for different types of problems. This comprehensive analysis aims to offer valuable insights into the advantages and challenges of each approach, guiding the selection of the most appropriate model for specific situations.

2.1. Traditional (Deterministic) Modeling

Key methods in addressing mine production and schedule planning problems in deterministic conditions encompass the fundamental trees methodology, heuristic approaches, dynamic programming combined with heuristics, and meta-heuristic techniques. These innovations represent significant progress in the field, offering various strategies to enhance the efficiency of mine planning. However, a limitation across these methods is their unrealistic designs in which all data is deterministic and constant. This shortfall arises from their failure to adequately account for the inherent uncertainties present in mining operations, such as variable geological conditions, fluctuating market prices, and unpredictable operational challenges. Addressing these uncertainties is crucial for developing more accurate and feasible mine production plans.

2.1.1. Fundamental Trees Methodology

The Fundamental Trees methodology is a systematic approach used to understand, analyze, and visualize the essential components and relationships within a problem. This method breaks down the problem into its fundamental elements, often represented as nodes in a tree structure, and connects these elements through branches to illustrate their relationships. The key components of Fundamental trees methodology are nodes (root node, leaf node) and branches (connections).

Mixed integer programming (MIP) and linear programming (LP) mathematical optimization models offer the flexibility to incorporate multiple ore processing methods, such as milling and leaching, and various elements during optimization. This adaptability can lead to production schedules with significantly higher net present values (NPV) compared to traditional non-mathematical programming methods. However, MIP formulations for production scheduling often require many binary variables, making them challenging or even impossible to solve for real-world open pit mining operations.

Ramazan [5] introduced the Fundamental Tree Algorithm (FTA) for open pit mine production scheduling, using an LP model to consolidate blocks and reduce binary variables, making MIP models feasible. Also, Ahmad et al.[6] developed random tree and C4.5 decision tree models to predict pillar stability in underground mines, showing high accuracy and reliability. In the general area, Zhao [7] analyzed the Apriori algorithm for association rule mining, emphasizing its efficiency and noise control. Kundu [8] presented a random forest regression methodology for predicting the remaining useful life of spur gears, using a correlation coefficient from residual vibration signals, validated by five experimental datasets and improved by fusing data from multiple accelerometers. Also, deterministic problems can be modeled and solved by a goal programming framework and discrete event simulation for optimizing oil sands mining operations, focusing on production schedules, dyke construction, and tailings management [9].

2.1.2. Heuristic Approaches

Heuristic approaches are problem-solving strategies using practical and efficient methods to generate solutions that may not be perfect but are good enough given the constraints and time available. These methods rely on experience, and intuition rather than precise algorithms. Heuristic approaches are particularly valuable in situations where an exact solution is difficult or impossible to find within a reasonable timeframe. The common Heuristic techniques are Greedy Algorithms, Simulated Annealing, Genetic Algorithms, and Tabu Search.

The mixed integer linear goal programming model is useful to optimize extraction schedules, maximize net present value and minimize dyke construction costs, which can be solved by an agglomerative hierarchical clustering and branch and cut algorithm [12-16].An extraction sequences can be applied for an underground lead and zinc mine, reducing model size with exact and heuristic methods to solve complex problems quickly, tested on Lisheen mine data [10, 13, 17]. An open-pit mining scheduling is able to maximize net present value by hybrid heuristic algorithm[11], improving efficiency and solution quality, and solving previously unsolved instances from the MineLib library. For incorporating a stockpile and a greedy heuristic to enhance computational efficiency, a Simulated Annealing approach in the problem of open-pit mine production scheduling works efficiently [18]. To reduce solution time and memory usage in the sublevel stope layout problems a dynamic programming method including memorization and a greedy heuristic is important[19]. Long-term production scheduling models under grade uncertainty which apply a hybrid model using augmented Lagrangian relaxation and the human mental search algorithm are capable to improve and achieve higher net present value [20]. The mixed-integer programming models can be solved by tabu search algorithm for optimizing autonomous truck trips and speeds to minimize energy consumption, verified in a real-time scheduling system in a coal mine [21].

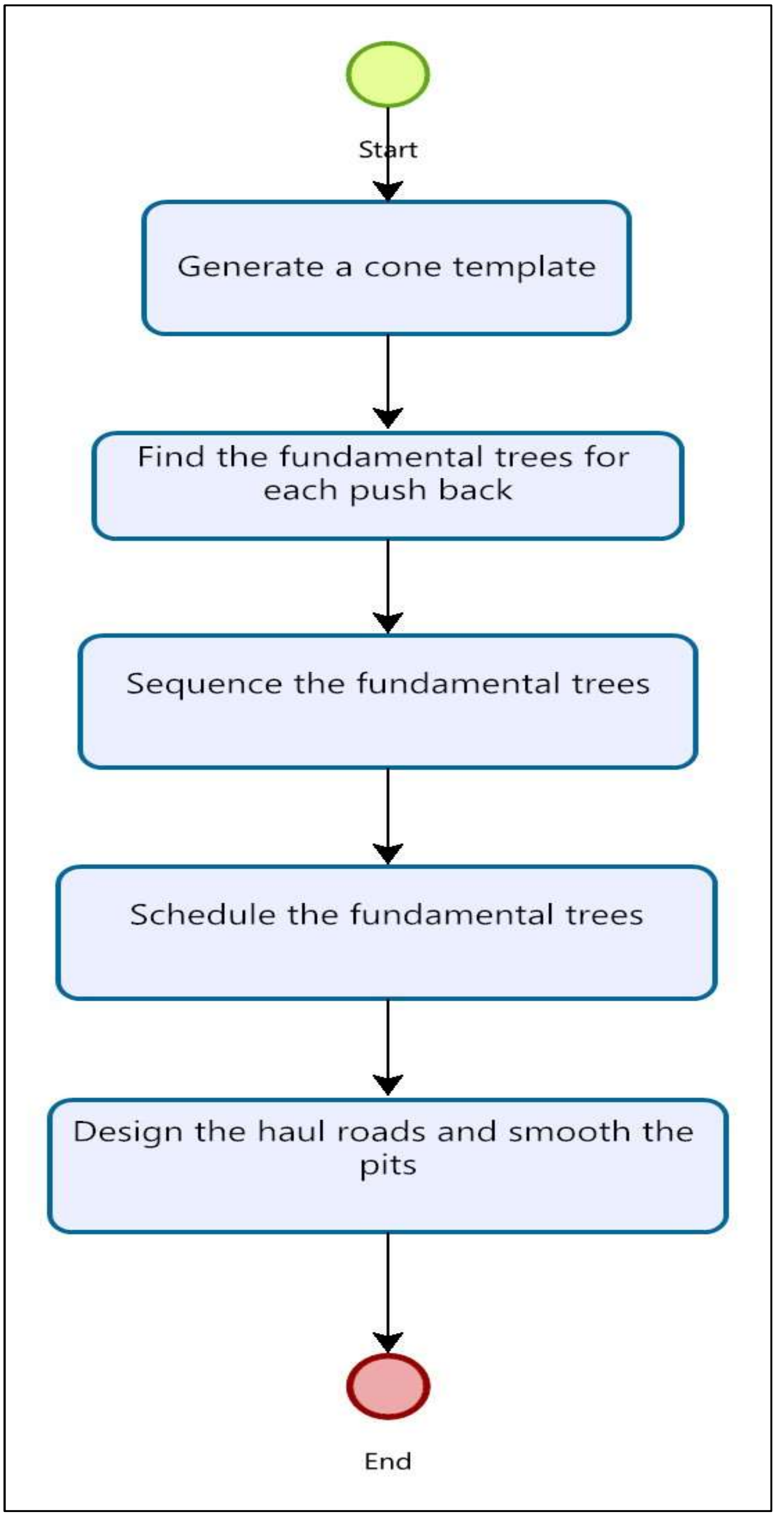

Figure 1, Mine planning based on the fundamental tree algorithm [5].

2.1.3. Dynamic Programming

Dynamic programming (DP) is a powerful algorithmic technique to solve complex problems by breaking them down into smaller sub-problems. The key idea behind dynamic programming is to use previously computed results to efficiently solve new sub-problems to avoid redundant work.

A dynamic stochastic programming approach was studied by Rimele [22] for open pit mine planning with geological and commodity price uncertainty. Their optimization method addressed both geological and market uncertainties, providing a life-of-mine production schedule that is scenario-independent regarding geological uncertainty but scenario-dependent concerning commodity price non-anticipatively. This allows the model to adapt to commodity price changes within reasonable limits for metal production targets and feasible mine designs. The method integrates a two-stage stochastic integer program with a stochastic dynamic programming algorithm. They used a case study to illustrate how the proposed method identifies policies that correlate price variations with metal production targets and consider both geological and commodity price uncertainties.

2.1.4. Meta-heuristic techniques

Meta-heuristic algorithms are high-level problem-solving strategies that guide underlying heuristics to find, generate, or select a heuristic that may provide a sufficiently good solution to an optimization problem. These algorithms are particularly useful for solving complex and large-scale problems where traditional methods are impractical due to the size and complexity of the search space. The most commonly used meta-heuristic algorithms in mining area in the recent years are Genetic Algorithms [23, 24], Simulated Annealing [25, 26], Particle Swarm Optimization [27-29], Ant Colony Optimization, Tabu Search [30, 31], and Differential Evolution [32]. These meta-heuristic algorithms are used in different mining fields and other areas to solve problems that are difficult to tackle using traditional optimization methods.

Optimization and decision-making in engineering are crucial today, with data processing demands escalating due to massive data volumes. Addressing these demands, heuristic and meta-heuristic algorithms play a significant role, especially when combined into hyper-heuristic solutions that optimize time and space complexities. The study by Chandra [33] explores nature-inspired computing, meta-heuristic models, hybrid models, and hyper-heuristics, proposing a hyper-heuristic approach derived from meta-heuristic algorithms for general problem domains. They categorize heuristic algorithms into population-based, evolutionary-based, swarm-based, agent-based, search-based, and hybrid-based, with agent-based multi-objective approaches proving efficient for global optimal solutions. It reviews various optimization algorithms, emphasizing the transition from meta-heuristics to hyper-heuristics and demonstrating performance improvements with fine-tuned algorithms. The paper highlights the evolution and application of these algorithms, advocating for multi-level heuristics to tackle real-world challenges effectively.

2.2. Stochastic Optimization

To consider uncertainty of data in mine planning models, and designing stochastic models, we can apply several methodologies, where the main methods are stochastic integer programming [34] and conditional stochastic [35]. After applying uncertainty in the mathematical models, the complexity level of the model is increased. Therefore, recent studies in stochastic mine planning have solved their proposed models by advanced methodologies reducing complexity and improving solutions. These methods are explained in the following.

2.2.1. Monte Carlo Simulation

Since presenting one method for each problem can exclude valid solutions and results, there is not any single and definitive Monte Carlo method. However, many simulations generally follow this pattern.

1. Model the system using one or more probability density functions (PDFs).

2. Repeatedly make samples from these PDFs.

3. Compute and analyze the statistics of interest.

Harrison [36] indicated the main considerations required to be defined in modeling Monte Carlo simulation including desired output, purpose of outputs, accuracy/precision, modeling specificity, input definition, and process modeling.

Figure 2 illustrates a brief introduction to Monte Carlo Simulation. The real-world problem represents the initial problem that needs to be solved. It is the starting point of the simulation process. By analyzing the data related to the real-world problem and developing a conceptual model, the conceptual model can be presented to capture the essential features of the problem. The conceptual model is a theoretical framework to outline the system's behavior. Then, with applying computer programming and implementing, the conceptual model is translated into a computer model through programming. It includes coding the model, implementing algorithms, and setting up simulations. Also, in the phase of simulation, the computer model is executed to perform simulations. This involves running multiple iterations to explore different outcomes based on varying inputs and random variables.

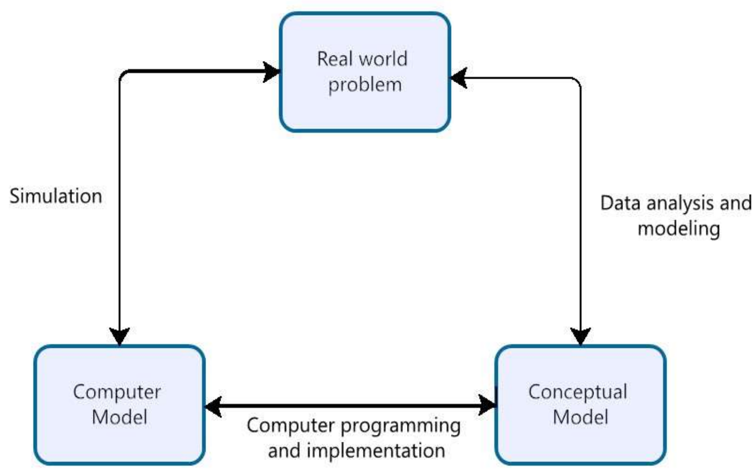

Figure 2. Framework of computer simulation for Monte Carlo simulation [37].

2.2.2. Geostatistical Simulation

Geostatistical simulation methods are valuable tools for generating multiple equally probable realizations of a spatial phenomenon, allowing for the quantification of uncertainty in the generated patterns. Before selecting a geostatistical simulation method for a specific problem, it is crucial to determine the nature of the variables involved. According to Emery [38] and Silva [31], variables can be categorized into three main types. Continuous variables typically represent physical properties such as percentage of tree cover. Categorical variables which are defined by a fixed number of states or categories, such as soil type. Also, objects have varying shapes, locations, and orientations, such as buildings, trees, or water bodies.

Geostatistical methods have become increasingly practical to predict spatial attributes and model the uncertainty of predictions, which are crucial for mineral resource estimation and ore reserve evaluation [39]. One of the key geostatistical interpolation methods is the linear robust estimator of Kriging [40] which has the drawbacks of smoothing effect with highly skewed data. In contrast, Gaussian simulation [41] provides more precise results for many continuous variables by transforming them to a Gaussian distribution.

2.2.3. Robust Optimization

Robust optimization (RO) does not rely on known probability distributions for uncertain data. Instead, it assumes that uncertain data exists within a specified uncertainty set. Basic RO models enforce strict constraints, meaning that no constraint violations are allowed for any possible realization of the data within the uncertainty set. RO is widely used because it is computationally efficient for many types of uncertainty sets and problems [42-44].

Robust optimization is a widely used method for handling optimization under uncertainty. The main idea is to indicate an uncertainty set containing possible values of the uncertain parameters and then optimize for the worst-case scenario within this set. When the uncertainty sets are well-chosen, robust models typically result in manageable optimization problems that perform better than other methods. However, if the sets are poorly chosen, robust models can become too conservative or difficult to solve. Therefore, selecting an appropriate uncertainty set is essential. In the recent studies, several theoretically and experimentally validated methods for constructing uncertainty sets are presented [43].

As illustrated in Figure 3, the optimizer generates solutions for the problem, which operates within a specific environment. The problem uses these solutions, and its performance is assessed using evaluation methods. This performance evaluation is shown back for the optimizer to inform the quality of the solutions. During operation, the system encounters uncertainties from both internal and external sources [44].

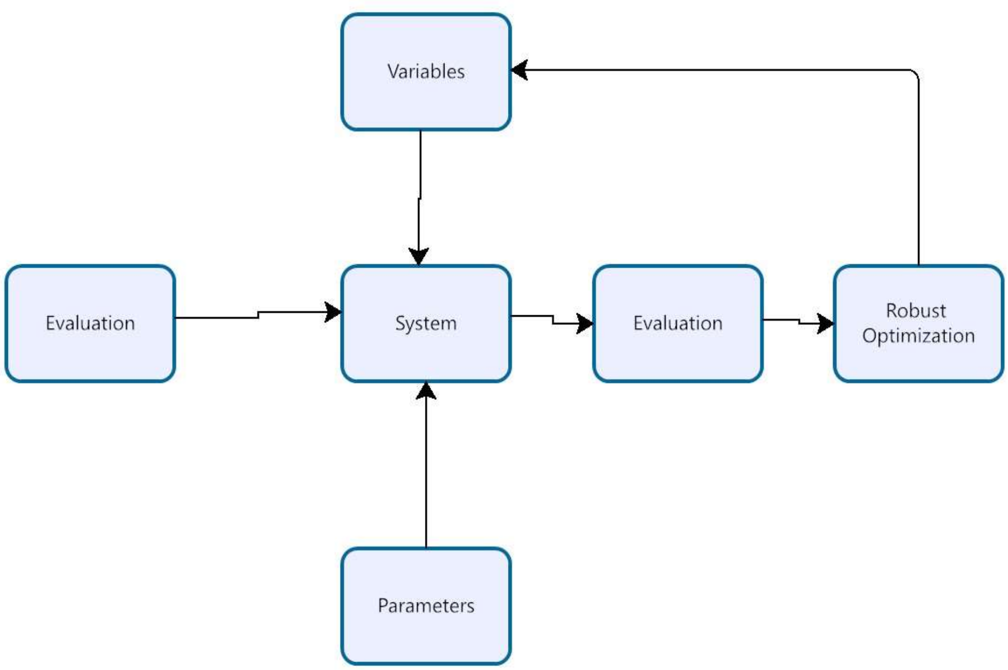

Figure 3. Sources of dynamics and uncertainties [44].

2.2.4. Machine Learning Algorithm

In the era of the Fourth Industrial Revolution (Industry 4.0), vast amounts of data are generated from IoT, cybersecurity, mobile devices, business transactions, social media, and healthcare. Effective analysis of this data for developing intelligent, automated applications requires knowledge of artificial intelligence (AI) and machine learning (ML). Sarker [45] provided an extensive overview of machine learning algorithms including supervised, unsupervised, semi-supervised, reinforcement learning, and deep learning, and their applications in domains such as cybersecurity, smart cities, healthcare, e-commerce, and agriculture. Their study defines the scope by considering the nature of real-world data and the capabilities of various learning techniques, enhancing the intelligence of data-driven applications. It discusses the applicability of ML-based solutions, potential research directions, and challenges for intelligent data analysis. The study emphasizes the importance of quality data and algorithm performance for successful ML models and highlights how ML methods can address real-world issues. By addressing these challenges, the paper opens new avenues for research and application development, serving as a valuable reference for academia, industry professionals, and decision-makers. Haul trucks in open-pit mines consume significant amounts of diesel fuel, resulting in high operating costs and substantial greenhouse gas emissions. Improving fuel efficiency is vital for cost reduction and environmental sustainability. Alamdari [46] modeled diesel fuel consumption in an iron ore mine using machine learning techniques. Payload was identified as the most significant factor, impacting fuel consumption by 47.9%. The study emphasizes the need for accurate, data-driven models to reduce costs and emissions in mining operations.

Hazrathosseini [47]explored intelligent Fleet Management Systems (FMSs) in surface mining, using SWOT analysis to identify strengths, weaknesses, opportunities, and threats, concluding that benefits outweigh drawbacks. Bnouachir proposed a distributed FMS architecture for real-time vehicle control using AI and IoT, enhancing agility and interoperability. Zhang [48] developed a reinforcement learning approach for vehicle dispatching, improving productivity and learning efficiency in dynamic environments. Huo used Q-learning to enhance fleet productivity and reduce GHG emissions, showing significant improvements in haulage efficiency. Carvalho[49] presented an adaptive truck dispatching policy using deep Q-learning, improving adherence to operational plans in mining complexes. Mastui introduced a deep reinforcement learning algorithm for real-time dispatching of autonomous haulage trucks, increasing efficiency and reducing fuel consumption. Noriega[50] developed a DRL-based dispatching system for open-pit mining, achieving robust policy learning and maintaining ore feed quality in a case study.

2.2.5. Sources of Uncertainty in Stochastic Optimization

Most stochastic models used in mine planning problems account for three primary sources of uncertainty: geological conditions, which involve the variability in mineral deposits and rock formations; technical conditions, which pertain to the efficiency and reliability of mining equipment and processes; and economic conditions, which encompass fluctuations in market demand, commodity prices, and operational costs.

1. Geological Uncertainty: Geological uncertainty refers to the variability in the spatial distribution of mineral resources within the earth. This uncertainty arises due to limited data from exploration drill holes, variability in orebody geometry, mineral grade distribution, and unknown geological features.

Brika [51] presented an innovative method for optimizing long-term production scheduling in open pit mines, incorporating multiple processing streams and investment decisions under geological uncertainty. Their approach begins with solving the linear relaxation using an extended Bienstock-Zuckerberg algorithm tailored for stochastic optimization. This is followed by a topological sorting-based rounding heuristic and a parallel multi-neighborhood Tabu search. The model, applied to a multi-product open pit mine, allows investments in shovels, trucks, or crushers to enhance capacity. By incorporating capital expenditure options and geological scenarios, the aim is to maximize NPV of the mining complex from various customers and the spot market. The research enhances previous models by integrating investment options and using a stochastic Bienstock-Zuckerberg decomposition method with heuristics. A case study on an iron open pit mine demonstrated the model's effectiveness, achieving an optimality gap of 1.5% within 12–22 minutes and showing significant NPV improvement through early investment in additional crushing capacity.

Jelvez [11] introduced a multi-stage methodology for long-term production planning in open-pit mines, addressing ore grade uncertainty. The approach integrates geological uncertainty into defining the final pit, optimizing pushbacks, and scheduling production, using new mathematical optimization models and conditional simulation. The main contributions of their research include a model optimizing final pit value while minimizing conditional value at risk (CVaR), a pushback optimization model selecting nested pits with similar total tonnages, and a mixed-integer program for scheduling bench phases to minimize production target deviations. They revealed that the final pit limit stage contributes 31.4%, pushback optimization 12.6%, and production scheduling 56% to the overall value. This multi-stage approach helps determine the relative impact of each planning stage, guiding efforts to maximize value and minimize uncertainty costs. Results show that incorporating uncertainty reduces the risk of failing to meet production goals compared to a deterministic approach.

A comparative study on a gold mining complex proposed by Morales [52], evaluating two stochastic mine plans: one using conventional wireframes for geological domains and the other using high-order simulations. The study found that simulated geological domains produced a 20% higher expected net present value (NPV) and a 40% wider risk profile due to reduced waste handling costs and a lower environmental footprint. Key contributions include a methodology for simulating geological domains using high-order spatial statistics, validated up to the fourth-order statistics, and the integration of these simulations into a stochastic optimization framework to produce mine schedules. The study highlights the importance of incorporating geological uncertainty in mine planning, leading to different extraction sequences, ultimate pit limits, and practical adjustments in mine design, such as ramps and waste dump layouts. The 20% increase in expected NPV results from reduced waste handling rather than increased metal recovery, emphasizing significant financial and environmental benefits.

Quantitative modeling of geological heterogeneity is crucial for effective resource management in mining, particularly when data is limited to exploration drill holes, introducing uncertainty in the empirical cumulative distribution function (CDF). Erten [53] addressed this challenge by employing a multivariate spatial bootstrap procedure to quantify parameter uncertainty and incorporating it into geo-statistical simulations. Using data from the Lisheen lead and zinc mine, they compared three scenarios: (1) using all available data, (2) a representative subset without parameter uncertainty, and (3) the same subset with parameter uncertainty. The results showed that including parameter uncertainty provides a more realistic assessment of resource risk.

In the area of underground mine design, Noriega [54] introduced a novel approach using deep reinforcement learning (DRL) to address geological and mineral grade uncertainties, which are often overlooked by traditional methods, resulting in suboptimal decisions. The proposed framework employs the proximal policy optimization (PPO) algorithm and integrates multiple numerical realizations of a mineral resource to enhance financial performance. In the gold mine case study, the DRL approach achieved an 8.3% higher expected profit and extracted 3.4% more gold compared to standard industry methods. The primary benefit of this approach lies in its ability to incorporate geological uncertainty into mine planning, leading to improved decision-making.

2. Technical Uncertainty: Technical uncertainty encompasses the variability associated with the mining process, equipment performance, and operational conditions. Factors contributing to technical uncertainty include equipment reliability, mining method efficiency, processing plant performance, and variations in ore recovery rates.

Based on [55] study, uranium in situ recovery faces challenges in predicting fluid flow and geochemical reactions due to geological uncertainties. Traditional simulations are computationally intensive, making uncertainty propagation impractical. Therefore, they proposed a Scenario Reduction Method for 100 realizations of a production block to select a subset of geo-statistical simulations and approximate a larger set. This method, validated against full simulations, maintains accuracy in predicting median uranium production and confidence bounds while significantly reducing computational time. The results offer an effective way to quantify production uncertainty in uranium in situ recovery mining and reduce computational time.

Upadhyay [56; 57] developed a simulation optimization framework for mining operations, combining discrete event simulation with goal programming to account for uncertainties and enhance short-term production planning. This tool allows proactive decision-making through scenario analysis and was verified using an iron ore mine case study. Tabesh [58] presented four agglomerative hierarchical clustering algorithms incorporating geostatistical realizations to handle geological uncertainties, resulting in more robust and minable aggregates. In a separate study, Tabesh [59] used discrete event simulation to model iron ore processing plants, focusing on output quality and the impact of input uncertainties. Moradi-Afrapoli [60] created an integrated simulation and optimization framework for truck dispatching in surface mines, improving production by 11% compared to current models. Moradi-Pirbalouti [61] examined different mathematical models for mine planning under deterministic and uncertain conditions, comparing approaches to manage data uncertainty. Jelvez [62] proposed new optimization models and conditional simulation to address geological uncertainty, improving solution quality in a real case study and helping decision-makers identify risks.

3. Economic Conditions: Economic uncertainty results in fluctuations and in market conditions, commodity prices, and economic factors that affect the financial aspects of mining operations. Economic uncertainty arises from market demand, changes in commodity prices, exchange/inflation rate fluctuations, and broader economic trends.

Kizilkale [63] introduced a dynamic programming framework to optimize production rates across multiple metal mines under financial uncertainty. Each mine's production rate is treated as a stochastic optimization problem, with solutions provided via dynamic programming and Markovian feedback control based on current price data. The hierarchical approach separates individual mine scheduling from global extraction rate targets, addressing metal content uncertainty locally and financial uncertainty globally. Using a distributed policy iteration method, each mine's extraction rate is optimized in parallel until a unique equilibrium is achieved. Numerical results indicate that this global optimization method outperforms individual mine optimizations, enhancing financial performance and robustly managing mining rates under uncertainty.

3. Discussion and Results

In the following there is some analysis about the studies on stochastic optimization and traditional modeling in mine planning problems.

Table 1. The summary of publications on Stochastic Optimization in mine planning.

Author	Year	Method	Outcome
Gilani [2]	2020	Particle Swarm Optimization (Geological Uncertainty)	High performance on a large-scale open pit mine; higher NPV with reduced solution time.
Rimele [64]	2018	Stochastic Integer Optimization Programming	Integrated disposal of mining waste and tailings directly inside the pit into life-of-mine planning, saving costs and environmental impact.
Fattahi [65]	2021	Multi-Stage Stochastic Optimization	Designed a sustainable mining supply chain with renewable energy, showed the impact of improving environmental and social objectives on total cost.
Dimitrakopoulos [66]	2022	Simultaneous Stochastic Optimization	Highlighted managing uncertainty and technical risks; increased metal production, operational capability, and net present value.
Morales [52]	2024	High-Order Simulation Of Geological Domains	Presented a 20% higher NPV and up to 40% wider risk profiles.
Nelis [67]	2022	Optimization Model For Scheduling And Mining Cuts	Captured profit and met mining, processing, and operational constraints.
Joshi [68]	2022	Integrated Parametric Graph Closure and Branch-and-Cut Algorithm	Reduction in computational time with reasonable optimality gap.
Guo [69]	2021	Artificial Neural Network Approach	Estimating mining capital cost with ANN model; demonstrated superior performance
Levimson [70]	2023	Reinforcement Learning with Mathematical Programming	Increase in annual cash flow
Whittle [71]	2018	Global Optimization Principles	Discussed global optimization principles and their application in various industries
Kumral [72]	2010	Robust Stochastic Optimization	Improved resilience and efficiency in a case study of production scheduling.
Carpentier [73]	2016	Stochastic Integer Programming	Designed a SIP model for underground mine production scheduling; showed higher project value and better production risk control.
Dimitrakopoulos [74]	2011	Stochastic Simulation And Optimization	A risk-based framework for strategic mine planning; increased production schedule value by 25%.
Montiel [75]	2015	Simulated Annealing	Optimized mining complexes achieved a 5% increase in NPV and reduced risk.
Montiel [76]	2017	Heuristic Approach	Significant improvement in expected NPV and capacity management.
Lin [44]	2024	Mixed Integer Programming (MIP)	Sustainable production scheduling with reduced GHG emissions by 47%; superior performance in NPV maximization.
Carvalho [77]	2023	Actor-Critic Reinforcement Learning	Production planning and fleet management with 47% increase in cash flow.
Faria, et al. [78]	2022	Integrated Stochastic Optimization	Stope design and long-term scheduling with 11% higher NPV and shorter mine life.
Yaakoubi [79]	2022	Learn-to-Perturb (L2P) Hyper-Heuristic	Reduced iterations and computational time with enhanced efficiency in mining complexes.

Minniakhmetov [80]	2021	Data-Driven High-Order Simulation	Practical utility demonstrated in copper deposits.
Rendu [81]	2014	Cut-off Grade Estimation	Enhanced understanding of cut-off grade estimation; crucial for profitability and socio-economic impacts.
Saliba [82]	2019	Stochastic Optimization	Improved operational flexibility and profitability for gold mining operation.
Sepúlveda [83]	2020	Metaheuristic and Simulation Tools	Incorporated uncertainties in mine planning; recommended inclusion of environmental and social variables.

Table 1 shows that most of the research has been conducted in recent years (2018-2024), indicating an increasing focus on optimization methods in mining. Particularly, there is a notable increase in publications from 2020 onwards. Earlier studies (e.g., from 2010 and 2011) laid the foundation for robust and stochastic optimization, which evolved into more sophisticated and integrated approaches seen in recent years.

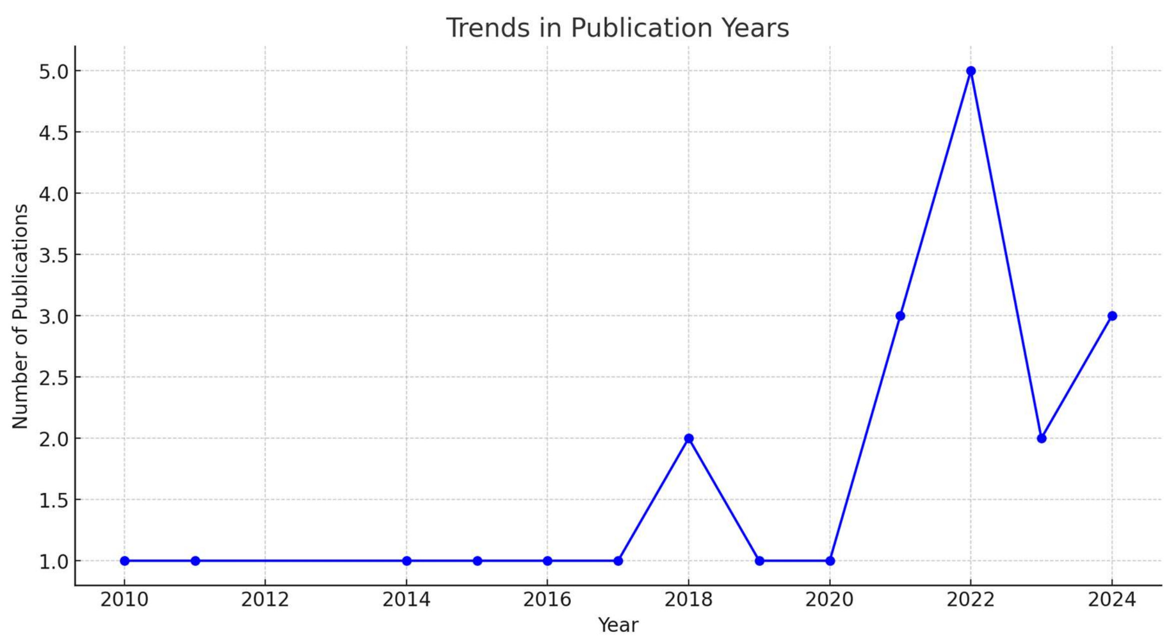

Figure 4. Trends in publications on stochastic optimization in mine planning problems.

Here we analyze the summary of stochastic optimization studies based on year, method, and findings.

- Stochastic Optimization: Most papers reflect the importance of handling geological and operational uncertainties in mining.
- Many studies focus on maximizing the Net Present Value (NPV) of mining operations, showing that this is a critical measure of success. Studies such as those by Gilani et al., Morales et al., Joshi et al., Levimson et al., and Lin et al. report significant improvements in NPV.
- A significant portion of the findings emphasize managing risk and uncertainty. For instance, the work by Dimitrakopoulos [66] and Faria [78] show that integrating uncertainty into planning leads to better risk control and higher project values.
- Several studies, such as those by Rimélé [64] and Lin [44] , highlight cost savings and reduced environmental impacts as key outcomes, demonstrating the dual focus on economic and environmental sustainability.

- The integration of advanced technologies such as AI and reinforcement learning ([69; 77] is a notable trend, indicating a shift towards leveraging modern computational tools for optimization.
- The increasing number of publications in recent years reflects growing interest and advancements in mining optimization techniques.

Top and commonly used methods in 2010-2024 on the studies on stochastic optimization in mine planning are as follows respectively. While Meta-heuristics, Simulation, and Mixed-integer programming remains dominant, there is significant exploration of other innovative methods such as data-driven simulation and heuristic algorithms.

1. Meta-heuristic algorithms
2. Stochastic simulation and optimization
3. Mixed integer programming (MIP)
4. Data-driven high-order simulation method
5. Heuristic approach

Table 2. Summary of publications on Traditional Modeling in mine planning.

Author	Year	Method	Finding
Ramazan [5]	2005	Fundamental Tree Algorithm (FTA) Based on LP Model	Consolidates blocks reducing binary variables for production scheduling in large open pit mines, making MIP models feasible.
Sullivan [10]	2015	Integer Programming	Improving computational efficiency, and practical applicability for mine scheduling.
Jelvez [11]	2020	Hybrid Heuristic Algorithm	Providing feasible and near-optimal solutions for scheduling problems, improving efficiency and solution quality.
Desale [84]	2015	Heuristic and Meta-Heuristic Algorithms	Provide robust near-optimal solutions efficiently, invaluable for NP-complete challenges.
Denardo [85]	2012	Dynamic Programming	Shares features among sequential decision processes, identifying it as a branch of applied mathematics.
Souza [86]	2022	Dynamic Programming Review	DP may not be extensively explored for scheduling, but advances come from DP's theoretical findings.
Chandra [33]	2022	Hyper-Heuristic Approach Derived From Meta-Heuristic Algorithms	Explores nature-inspired computing models and hyper-heuristics, advocating for multi-level heuristics for real-world challenges.
Tolouei [87]	2021	Hybrid of Lagrangian Relaxation (LR) with Meta-Heuristic Methods	Hybrid LR-meta-heuristic approach outperforms others in NPV, and computational time, suggesting potential of LR-BA method.

It appears that research methodologies in the summary of deterministic modeling studies are different. This might indicate a diverse range of methods used across the papers. The most used methodologies are as follows.

1. Integer programming
2. Heuristic and meta-heuristic algorithms
3. Hybrid heuristic algorithm
4. Fundamental Tree Algorithm (FTA) based on LP model
5. Dynamic programming

We observe that the most common method in traditional modeling is integer programming. This method is favored for its efficiency and straightforward modeling process. Additionally, heuristic and meta-heuristic methods are widely used in many studies. These methods are particularly useful for tackling large-scale problems due to their ability to provide robust and near-optimal solutions.

4. Conclusion

This study highlights the critical role of stochastic optimization in strategic mine planning, especially in addressing the inherent uncertainties of geological, economic, and operational parameters. Traditional deterministic models, while useful in scenarios with fixed input parameters, often fall short in capturing the dynamic nature of real-world mining conditions. The integration of stochastic methods, such as Monte Carlo simulation, geostatistical simulation, and robust optimization, offers a significant improvement in the reliability and effectiveness of mine planning by considering multiple scenarios and probabilistic distributions. While stochastic optimization presents a more complex computational challenge, its ability to produce more realistic and robust solutions makes it an invaluable tool in strategic mine planning.

Recent advancements in computational power and algorithm development have enabled more precise and reliable stochastic models, enhancing the decision-making process and the economic viability of mining projects. Techniques such as machine learning and reinforcement learning are becoming increasingly prevalent, further refining the accuracy of predictions and operational strategies.

Furthermore, the emphasis on sustainability and environmental considerations within stochastic mine planning is a significant development. By incorporating environmental impact assessments and sustainability metrics, these models ensure that resource extraction is conducted responsibly, aligning with regulatory requirements and the growing demand for sustainable mining practices.

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