Emergent Properties of MoFrac-Generated Discrete Fracture Networks

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ABSTRACT

Discrete fracture networks allow for real structural features to be represented when considering a specific volume of rock. Through the construction of valid fracture network models, emergent properties can be identified that can be useful for engineering applications. MoFrac discrete fracture network modelling software can be used to develop realistic models of fracture networks in selected rock masses that are to be blasted. By including statistically derived stochastic fractures into fracture network models, where mapping has not occurred, a complete representation of the structural discontinuities in a specific volume of rock can be achieved. Emergent properties of fracture networks that are useful for characterization have been identified. These properties include in situ block size distributions and heterogeneity of the rock mass character. A method to determine these properties is presented and these characteristics are considered for two DFN models. These models are generated and analyzed using the MoFrac code. For future research, a fragmentation model is proposed that incorporates the identified emergent properties of a discrete fracture network model for blast optimization by means of a rock engineering systems solution.

1. Introduction

A rockmass consists of the rock matrix and the network of discontinuities within it. Rockmass characterization is dependent on both of these qualities. Discrete fracture network (DFN) modelling allows for the discontinuities in a rockmass to be mapped and analyzed. The modelling of discontinuities is independent of the rock matrix characteristics and can be considered separately.

DFN models are generated using software that is designed to create statistically viable fracture networks based on fracture size and orientation distributions. A *deterministic* DFN model is based on mapped data, fractures are generated in the model that honour mapped traces and facets of known fractures. A *stochastic* DFN model is based on probability distributions with fractures seeded randomly within a volume. A hybrid DFN model incorporates both deterministic and stochastic fractures (Lei *et al.*, 2017). A deterministic DFN model is useful when considering individual discontinuities and rockmass failure at known locations whereas a stochastic DFN model is more useful when considering the model as a whole as any particular fracture is likely not located where a discontinuity actually exists. A hybrid DFN model allows for known data to be included in a more complete representation of a rockmass that includes fractures seeded in areas where mapping could not take place. By having stochastic fractures defined using the size and orientation distributions expected to be encountered allows for constraints to be imposed that agree with known data.

A realistic representation of a fracture network is useful for several engineering applications. Hydrogeological modelling, rockmass failure studies, ore reserve analysis and blast optimization are areas of study that are enhanced with knowledge of existing fracture networks.

2. DFN Modelling

The representation of fracture networks in three dimensions is of great interest to engineers but difficult to achieve due to the limitations of mapping. True three dimensional mapping is a destructive process and thus not applicable to most projects. Hybrid DFN models containing both fractures seeded from deterministic traces as well as stochastically derived fractures are useful when considering the DFN model as a whole (Dershowitz and Einstein, 1988). By including deterministically seeded fractures the disaggregate approach of analysis becomes more applicable and thus heterogeneity in a DFN can have meaning other than as derived from the statistical rules used to generate the DFN model.

All DFN models for this study were generated using the Beta v. 0.8 build of MoFrac (MIRARCO, 2018). MoFrac is a DFN modelling tool that generates fractures deterministically from mapped data or stochastically as sampled from size and orientation distributions. Fractures are optionally generated with undulation through tessellation of the fracture mesh to yield a realistic looking DFN model. Priority is given to deterministic fractures as they come first in the modelling process. The software has been verified through validation studies modelling the fracture networks for a tunnel at the SKB's Äspö Hard Rock Laboratory and for the Canadian Shield Dataset further described in Section 2.1 (Junkin, *et al.*, 2017; 2018). These studies validated MoFrac-generated DFN models by considering orientation, intensity and trace maps as Miyoshi *et al.* (2018) describes as necessary for validation.

The goal of this study is to consider a method of measuring the heterogeneity of DFN attributes such as fracture intensity. A voxelization of DFN models allows for a spatial analysis at multiple scales. The subdivision of a DFN into component sections allows for the variability in an attribute such as fracture intensity to be measured. The P_{32} intensity of DFN models will be considered on a deterministic and a stochastic DFN model with subdivisions of 1 (1^3), 8 (2^3), 125 (5^3), 1 000 (10^3), 3375 (15^3) and 15625 (25^3) voxels. Fig. 1 shows an example of voxelization of a simple cube into 1000 component voxels (10^3). P_{32} intensity is a measure of the sum of the surface areas of fractures within a given volume divided by that volume, calculated as shown in Eq. (1).

$$P_{32} = \frac{\sum_{i}^{n} A}{V} \tag{1}$$

The benefit of using P_{32} intensities to characterize a DFN is that this measure is scale independent and there is no bias related to the direction of measurement as occurs with P_{21} values (Dershowitz and Herda, 1992; Alghalandis and Elmo, 2018). By considering fracture intensity by layer and subsections of a DFN model a new method of visualizing DFN models is proposed that honours the location of discrete elements but does not require the elements to be explicitly contained within the model.

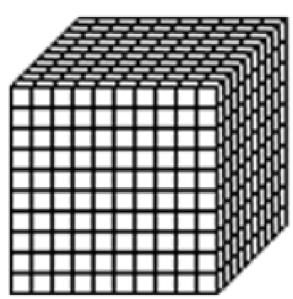

Fig. 1. Example of 10 x 10 x 10 voxelization of a simple cube.

2.1. Canadian Shield dataset

The Canadian Shield dataset developed by R. Mohan Srivastava was used as the input for a deterministic model to be generated. The Canadian Shield dataset was developed in 2002 for the Third Case Study of the Deep Geological Repository technical program of Ontario Power Generation (Gierszewski *et al.*, 2004). The dataset incorporates discontinuities mapped from an aerial photograph of a typical Canadian Shield setting with simulated fracture traces added to have the fracture intensity match expected values for the Canadian Shield (Srivastava, 2002). The dataset covers 200 km^2 of surface area as shown in Fig. 2 and for the purpose of this study a DFN model with a depth of 200 m was used. The total volume of the DFN model being roughly 40 km^3.

This dataset has been used for validation studies on the MoFrac code and the orientations for fracture sets from these studies have been used again here (Junkin *et al.*, 2018). Two major fracture sets are identified with strike angles of 298° and 240° and a third random group identified with strike angle of 8°. These orientations are used with a standard deviation of 10° for the sampling of strike values for the fractures modelled. Dip angle is controlled through a dip undulation scheme with fractures set to dip vertically +/- 10°.

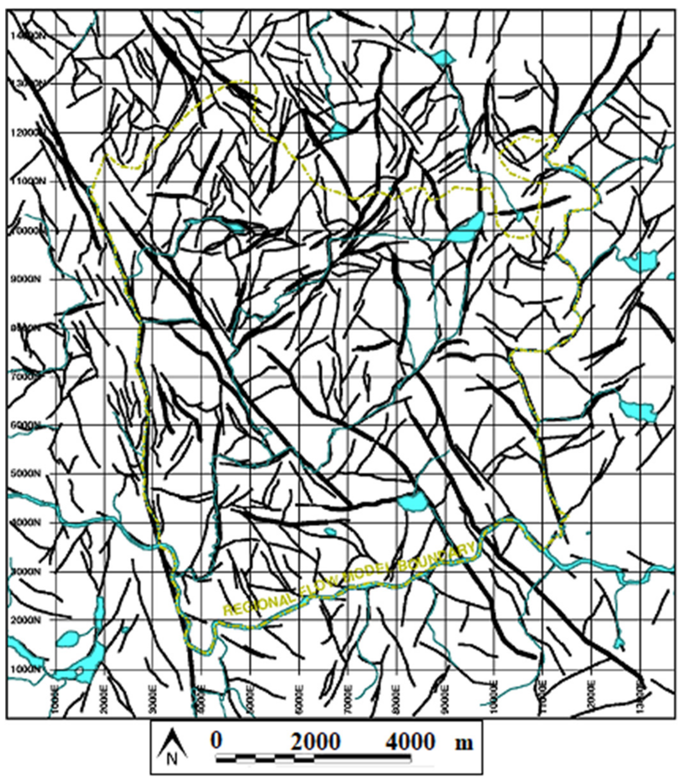

Fig. 2. Integrated Canadian Shield dataset shown with surficial water features as seen in original aerial photograph (Srivastava, 2002).

2.2. Stochastic DFN models

A stochastic DFN model was also designed in order to consider the proposed characterization technique. This model was designed with the eventual goal of blast optimization. The model was set up with 25 rectangular regions in a 5 x 5 pattern as viewed from surface. These rectangular columns are 10 x 10 x 50 units, in combination they form a 50 x 50 x 50 cube. Fig. 3 shows the defined regions in the stochastic DFN model.

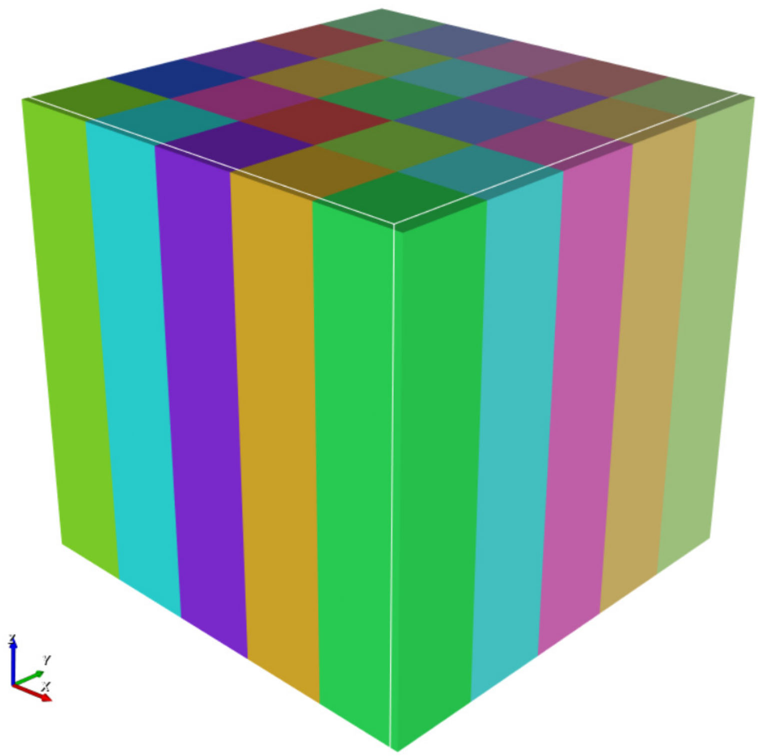

Fig. 3 Location of defined regions in stochastic DFN models. Unique colours represent individual regions and simulated deterministic traces shown in black.

Three fracture groups are defined for the model with orientation distributions as given in Table 1. Stochastic fractures also have size defined through a Cumulative Area Distribution (CAD). The input CAD values define a straight line on a log-log plot of fracture size and the number of fractures greater than the threshold divided by the model volume. These values are shown in Fig. 4 along with the distribution of actual fracture surface areas as measured from a resulting DFN model. The resultant orientation distributions for each group are shown in Wulff stereonets in Fig. 5 with values calculated using Dips v. 6.017 (Rocscience, 2016) included in Table 1 for comparison with input values.

Stochastic fractures are seeded in regions individually allowing for variability in orientation and intensity between regions if desired. Size distributions define that stochastic fractures will have surface areas less than 100 m^2. The size of each region is 10 x 10 x 50 m^3 thus all stochastic fractures will be non-persistent through the model and their region of origin. Fractures are defined not to terminate against other fractures nor to terminate at a regional boundary. There is an expected boundary effect in terms of intensity as regions in the center of the DFN model. This is because regions will have fractures propagate into them from neighbouring regions and thus final fracture intensity will be related to the number of neighbouring regions.

Ten realizations of the stochastic DFN model were generated for analysis. The model was also designed with an external region 1 unit thick surrounding the 25 stochastic regions. This external region allows for the inclusion of deterministic traces. By including deterministic traces, studies to determine the minimum size of fracture that needs to be included in mapping can be determined. The size should reflect the minimum sized feature that is required to have a representative distribution of fracture intensity and blocks. A second set of smaller, non-persistent fractures can be included into the model in order to achieve the predicted overall intensity.

Table 1. Comparison of input orientation distributions with realized orientations for a single stochastic DFN model.

Fracture group	Input dip (°)	Standard deviation of dip (°)	Input dip direction (°)	Standard deviation of dip direction (°)	Resultant dip (°)	Resultant dip direction (°)	Fisher K statistic
1	90	15	90	15	88.54	89.6	15.8
2	90	15	180	15	89.98	179.14	16.59
3	0	5	125	15	0.44	141.74	158.973

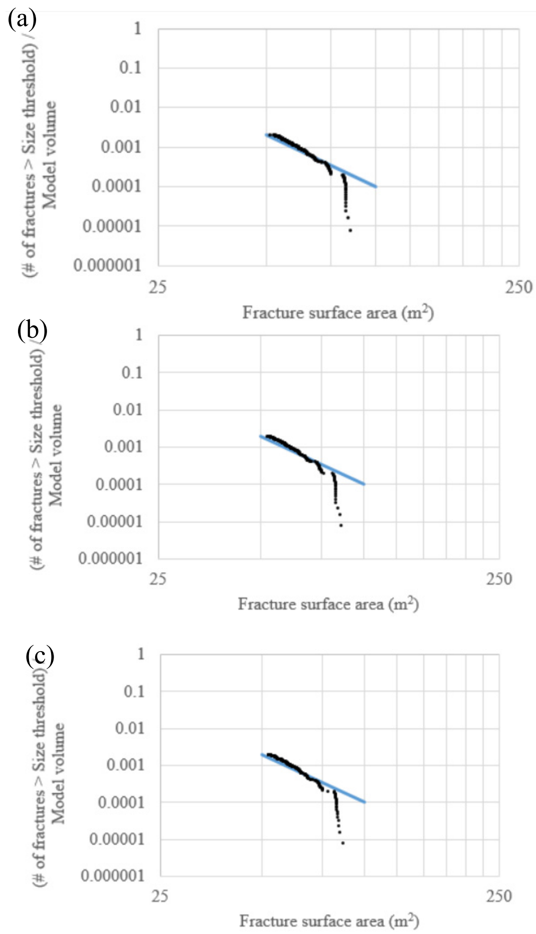

Fig. 4. Input CAD parameters shown as straight line and data points for realized fractures in (a) group 1, (b) group 2, and (c) group 3.

3. Emergent Properties of DFN Models

An emergent property of a system can be defined as a property resulting from the collective effects of the interaction of the component parts of a system. For DFN modelling, emergent properties are only observable when considering the DFN as a whole and cannot be determined by simply examining the input values. For the purpose of DFN modelling with MoFrac several input variables are required. Fracture orientation distributions are required for each defined fracture group, size distributions are also required which when considered with respect to the model size can define fracture persistence. Fracture intensity is also required as an input, MoFrac accepts fracture intensities and size distributions as two points and a slope on a Cumulative Length Distribution (CLD) or CAD curve.

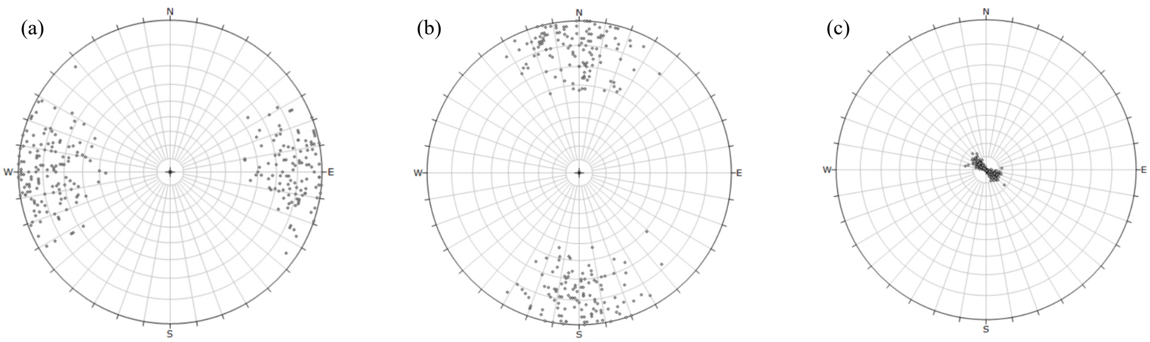

Fig. 5. Stereonets showing fracture orientation distributions in a single DFN realization for fractures in (a) group 1, (b) group 2, and (c) group 3.

Following the generation of a DFN model based on the required inputs several measures of emergent properties can be considered. The *in situ* Block size Distribution (ISBD) of a rock mass has been identified as vital for optimization of processes such as rock blasting and rock slope angle determination (Mavrouli, *et al.*, 2015). Fracture intensity is also important to these two areas of optimization and also is significant for hydraulic modelling of a rockmass (Lei, *et al.*, 2017; Appleyard, *et al.*, 2018) and ore reserve estimation (Mooney and Boisvert, 2016).

This study identifies a simple method of querying blocks within a DFN model for fracture intensity (P_{32}) at different scales. This method allows for a characterization of the variability of intensity within a DFN model as well as for the measurement of the changes in the coefficient of variation (CV %) when considering different sample block sizes. The data collected also allows for an estimation of ISBD within the DFN model.

Where DFN models consisting of non-persistent fractures have defined boundaries where fractures will not be seeded a boundary effect is to be expected. By considering the fracture intensity in the layer of exterior blocks compared to the interior blocks for different block sizes an estimation of the boundary effect based on fracture size and orientation is possible, again using the data collected to characterize fracture intensity.

In summary four emergent properties will be explored using the method of querying sample blocks of a DFN model for fracture intensity. The four emergent properties under study are:

- Measuring heterogeneity of fracture intensity;
- Measuring the coefficient of variation for different inspection block sizes;
- Estimation of *in situ* block size distribution;
- Estimation of the area of influence of any boundary effect.

These properties of a DFN model are considered as well as regular validation of DFN models. For stochastic fractures, these include honouring fracture orientation and size distributions with respect to inputs; for deterministic fractures, validation is based on location and orientation errors with respect to input data of mapped fractures.

There is ongoing work considering the mapping of rock bridges that occur within a rock mass. A rock bridge can be defined as the competent sections of rock between discontinuities. Alghalandis and Elmo (2018) demonstrate a means of quantifying the degree of rock bridging in a rock mass in an analogous way to determining fracture intensity as presented by Dershowitz and Herda (1992). For example a P_{32} intensity quantifies the sum surface area of fractures within a defined volume and a RB_{32} value considers the surface area of rock bridges within a volume of rock. The method outlined in this study is based on calculations of P_{32} intensity in sample blocks, it should be noted that a new set of emergent properties could evolve from similar analysis of RB_{32} values in the same sample blocks. A relation between fracture intensity and rock bridge formation could also arise through this type of analysis.

3.1. Fracture intensity

Fracture intensity in a DFN model is often described as a P_{10} (number of fractures per linear unit), P_{21} (sum length of fractures per aerial unit), or P_{32} (sum area of fractures per volumetric unit) value (Dershowitz and Herda, 1992). Fracture intensity can also be described using CLD and CAD curves that separate size and intensity as separate factors. Fractures can be seeded in DFN models according to either type of input intensity (Ryan, *et al.*, 2000). Input intensities can be easily verified against resulting DFN models, as in Fig. 4, or by calculating the appropriate intensity for comparison to input values.

The use of inspection blocks to analyze the DFN infers that using the P_{32} intensity measure represents information representative of the entire cube. Having a single value allows for a consideration of the change in intensity between sampled blocks in the DFN model. The degree of spatial variability of fracture intensities is a characteristic that is useful for optimization of blast design. By having fracture intensity data for all component cubes of the DFN it is possible to view the DFN in new ways. The area of influence for a blast hole can be considered independently, layers of a DFN can be assessed for connectivity of high intensity blocks, and boundary effects of the DFN can be quantified.

The mean intensity is a useful measure to characterize a sampled volume as a whole and the standard deviation gives a measure of variance. The coefficient of variation (CV%) is defined as the ratio of the standard deviation to the mean and is useful in normalizing data for comparison, for example, DFN models of different scales. By sampling a DFN several times with inspection cubes of different volumes the change in the CV% is considered as a characteristic and emergent property of the DFN models. This characteristic parameter shows potential to be useful in optimizing the size of blast holes during blast design, namely by limiting the area of influence to areas of low variance (CV%<1).

3.2. In situ block size distribution (ISBD)

The advancements in the development of methods to determine *in situ* block size distributions in undisturbed rock has been outlined by Elmouttie and Poropat (2012). Methods have become more sophisticated over time but four limitations were identified in their review:

- The ability to account for non-persistent fractures;
- The ability to account for more than three fracture groups;
- The use of fully detailed DFN models in Monte Carlo simulations;
- Simplifications and approximations required (ie. convex blocks).

Generally these limitations were found to result in over estimation of fragmentation, thus an over estimation of fracture intensity. Rogers *et al.* (2007) documents the first attempts to derive an ISBD from a fully defined DFN model. It was noted that using this approach blocks can be formed from the interaction of an unlimited number of fractures. The difficulties in determining block size distributions when fractures are smaller relative to intensity (less persistent) was also highlighted in the work, showing that smaller fractures can often intersect a block but do not cross it. This leads to concave blocks to be modelled. The significance between the occurrence of convex and concave blocks would affect different optimization problems in different ways.

The method of estimating *in situ* block size distribution suggested by Elmouttie and Porpat (2012) address the limitations as listed above. A DFN model that consists of polygonal approximations is used to calculate the ISBD thus allowing for non-persistent joints and blocks formed from multiple joints to exist within a DFN model that is analyzed. The fractures included in the models of Elmouttie and Poropat are planar, there is no undulation of fractures as there is with MoFrac-generated fractures. The inclusion of undulating fractures in a MoFrac DFN further complicates the potential geometry of block shapes that can be encountered. It is accepted that a three dimensional DFN model represents the possibilities of block formation in a rock mass to a greater degree of accuracy than is possible from analysis of two dimensional slices taken from a DFN and analyzed for closed polygons (Miyoshi, *et al.*, 2018). A new method of using data obtained when considering fracture intensity is proposed that directly queries the DFN model for a variety of block sizes for the presence of any fractures. This allows for block sizes to be determined from any number of fracture groups with any degree of persistence as they are measured directly from the DFN model. From this data an estimate of ISBD can be obtained as demonstrated in Section 6.4.

3.3. Rock bridges

Rock bridges are defined as the parts of intact rock surrounded by fractures (Alghalandis and Elmo, 2018). A definition that is very similar to what one would consider an *in situ* block. Whereas block size distributions are significant to fragmentation and the magnitude of potential slope failure, due to the quantity of rock that may fail, rock bridges are important in that they can define a route that existing fractures will coalesce during rock failure. Both of these factors are integral to a subject such as blast design as the block size will determine the amount of explosives to be used and the knowledge of rock bridges could be used to determine blasthole location that will optimize the use of the blast energy in the area that is to be fragmented. Aghlandis and Elmo propose the use of a measurement analogous to fracture intensity measures. In two dimensions an RB_{21} measurement gives the minimum linear length of intact rock between fracture traces needed to contact two boundaries of a DFN model.

The RB_{21} value could be useful as an additional emergent property of DFN models useful for design optimization. An RB_{32} measurement can also be made in three dimensions that could be calculated using the same approach of voxelation as proposed in this paper. The degree to which rock bridge intensities change with sample size would be a useful parameter to determine how energy will flow through a rock mass during failure or blasting.

3.4. Orientation and fracture size distributions

Fracture group orientation and size distributions are required to model fractures with MoFrac. Being the main inputs to the DFN recipe it is important to verify that the inputs assigned have in fact been realized. Figs. 4 and 5 visually show the process of verifying orientation and size distributions. This is the main method used to verify a stochastic DFN model compared to measuring the degree to which traces and assigned orientations are honoured in a deterministic DFN. There are additional tests and comparisons that can be accomplished once a DFN has been generated that would qualify as an emergent property. For example in Table 1 the input orientations for the stochastic model were given as a mean and standard deviation for strike and dip separately. An alternative method is to give the strike and dip along with the Fisher k statistic which is a measurement of clustering when visualized on a stereonet. A larger Fisher k value is associated with a tighter cluster of orientations. The stereonets show that fracture group three is much more clustered than groups one and two. This is due to the lower standard deviation assigned to the dip value for the third group. This clustering is verified by calculating the Fisher k value as shown in Table 1. In a similar way the fracture intensities for the stochastic models are defined using a CAD curve, once a DFN model has been generated it is then possible to consider P_{32} values as a verification and comparison. There is a need for future work to consider the relation between both types of intensity and orientation measurements specifically related to MoFrac-generated DFN models.

4. DFN Characterization Technique

DFN models have shown to demonstrate opportunities in modelling heterogeneity in a rock mass when compared to equivalent medium techniques. DFN models that represent actual rock masses are generally considered to be a combination of fractures seeded from mapped fractures with stochastic fractures derived from statistics associated with what has been mapped (Elmouttie and Poropat, 2012). Stochastic fracture size distributions are generally shown to be power law when the effect of sampling bias is minimized and thus power law distributions are used for determining fracture size through use of a Cumulative Area Distribution (CAD) (Tonon and Chen, 2007).

A DFN characterization is proposed that allows for a DFN model to be analyzed after discretization through voxelization as shown in Fig. 2. The P_{32} values of component voxels is determined and mapped by layer using a heat map designed using Excel Visual Basic for Applications (VBA). The goal of this method is to create a heat map that characterizes the DFN model implicitly where fracture location can be identified by increases in fracture intensity, the second goal is to identify the distribution of intensities throughout the DFN establishing a measure of heterogeneity of intensity within the DFN models generated. The characterization of these regions will be verified by using simulated boreholes in the DFN models used to measure P_{10} values that will show the same trends as predicted using the analysis of individual voxels.

5. Analysis of Canadian Shield DFN Model

Ten realizations of the Canadian Shield deterministic DFN were generated to be analyzed at different scales of voxelization. All five models were verified using a metric for the location error of the fracture traces. Results were similar to those found in previous validation studies using the dataset (Junkin *et al.*, 2018). Longitudinal Root Square Mean Error (LRSME) values comparing the input traces to traces from an inspection slice on the surface averaged 4.09 m with a standard deviation of 11.91 m. The method followed that of previous validation studies (Junkin *et al.*, 2017; 2018) Variability in the strike to dip ratio and Trace Area Constraint (TAC) factor resulted in randomness to the geometry of fractures thus yielding a variety of different realizations for analysis. The location error remained constant meaning that fracture traces were honoured to the same degree despite the geometry of individual fractures changing. One realization of the Canadian Shield DFN is shown in Fig. 6, the model extends to 200 m depth with a surface area of 200 km^2. The trace map and an inspection plan on surface from a single realization are shown superimposed on each other in Fig. 7 to visualize the degree of fit between fractures and input traces.

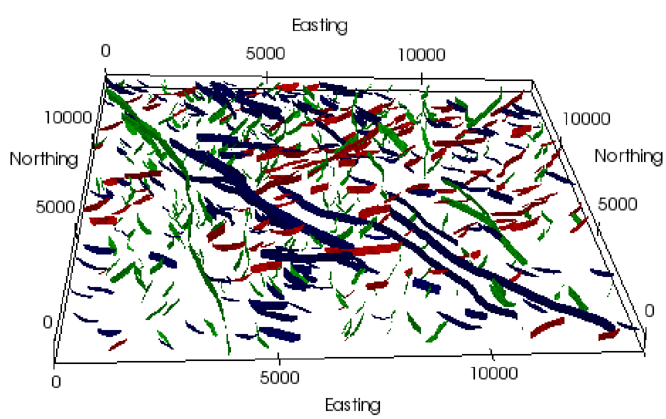

Fig. 6. Single DFN realization of the Canadian Shield dataset, coloured by fracture group.

Fig. 7. Comparison of input traces to an inspection plane of a single realization of the Canadian Shield DFN model cut on the same plane as input traces.

5.1. Coefficient of variation (CV %) plot

In order to characterize heterogeneity of fracture intensity at multiple scales the coefficient of variation (CV %) is considered for each size of inspection cube. This is a useful measure of fracture intensity variation in a DFN as multiple scales are considered and the trends in changes in CV% dependent on sample size can be easily visualized. It is proposed that the curve resulting from plotting CV% against block size is characteristic to a representative DFN model. This method can be used to verify that multiple realizations of the same DFN are demonstrating similar characteristics over the scales used for sampling. A CV% value is considered low variance when below 1, meaning the standard deviation is less than the mean. A high variance occurs when CV% is greater than 1. Fig. 8 shows the plot of the CV% vs block size for the Canadian Shield model as well as the stochastic model which will be discussed in Section 6. Based on the Canadian Shield curve it can be seen that when considering the whole model high variance occurs when a block size of approximately 200 000 m^2 is used. These curves will be dependent both on the DFN and the geometry of the sample blocks.

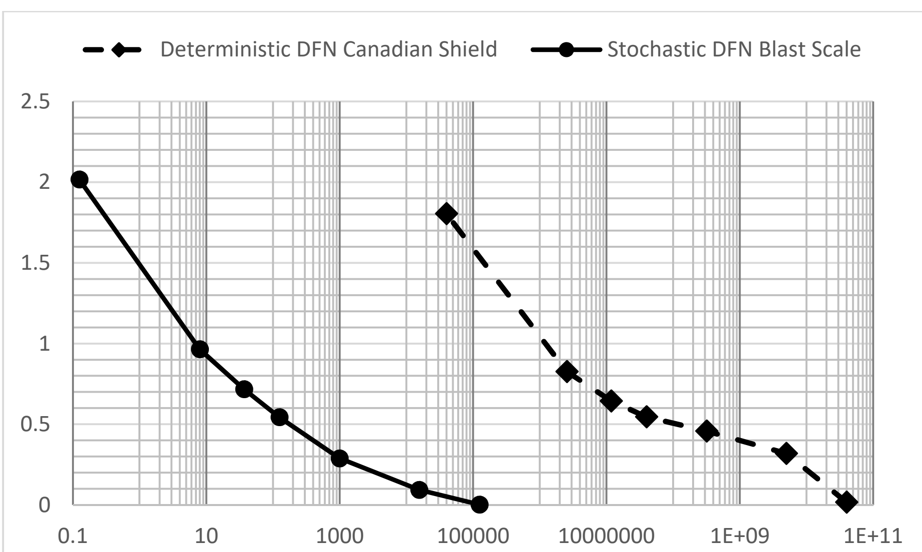

Fig. 8. Coefficient of variation plotted against inspection block size for both the Canadian Shield and stochastic DFN models.

5.2. *In situ* block size estimation

A benefit of calculating the P_{32} fracture intensity for a variety of sizes of blocks within a DFN and to calculating the intensity for all blocks is that an estimate of the ISBD can be achieved by counting null blocks, blocks that have a fracture intensity of 0. For this case ISBD curves were achieved by considering the number of null blocks. The largest block encompasses the entire DFN and thus a fracture intensity of 0 will not occur if any fractures are present. The largest null cubes are encountered when using a voxelization of 10 x 10 x 10 for analysis. This equates to a block size of 200 000 m^3. As progressively more null blocks are found and higher resolutions the volume of blocks accounted for by larger blocks are subtracted from the new sum volume of null blocks. This ensures that null blocks are not counted twice when creating this plot. It can be seen that the resulting distribution follows the power law as is expected with a block size distribution. As there is no association of null blocks with neighbouring null blocks it is expected that this estimate will overestimate small blocks and underestimate large blocks. This will result in an overestimation of fracturing in the model but still provides an indication as to the actual ISBD for the DFN and could be incorporated into optimization models. The ISBD estimate associated with a single DFN realization of the Canadian Shield DFN models is presented in Fig. 9.

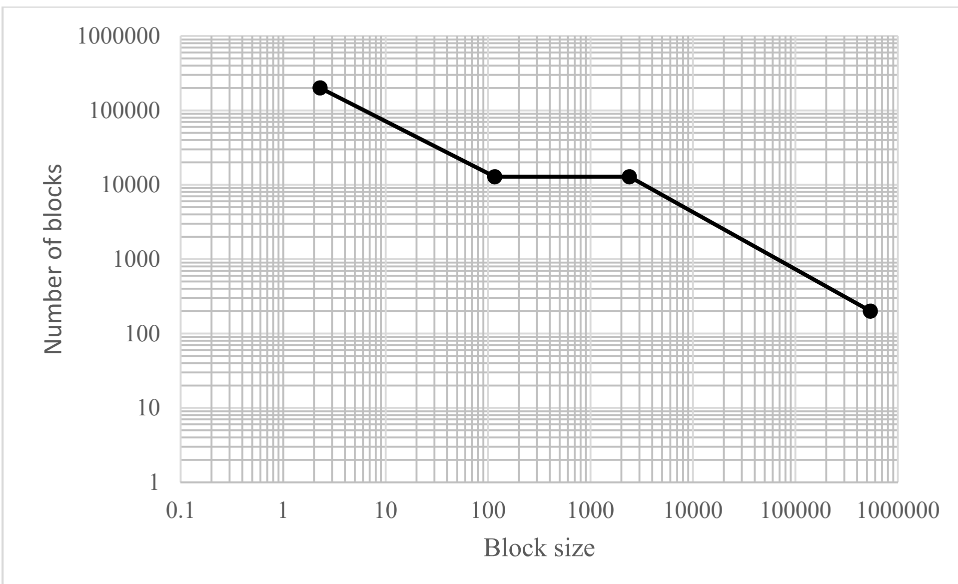

Fig. 9. Estimated *In situ* block size distribution from counting null cubes for a single realization of the Canadian Shield DFN model as measured over ten DFN realizations.

6. Analysis of Stochastic DFN Models

Ten realizations of the stochastic DFN model were generated for analysis. By including both a deterministic and stochastic DFN in this study the methods of analysis presented are shown to be highly versatile, it is also inferred that these methods will be applicable to analysis of hybrid models and will be integral for determining the scale of geological mapping that should be included in DFN models used for engineering design. Fig. 10 shows two views of a single realization of the stochastic model, one view showing the fractures coloured by group and another showing the fractures coloured by region. Although all regions are defined as having the same intensity and the result is that each region is composed of 30 fractures it can be seen that they differ greatly when sampled individually. A boundary effect is expected on exterior blocks that will result in a slightly lower intensity as no fractures are able to leak into these regions from the exterior of the model which would not be representative of the natural case. The orientations and intensities of these models were validated as shown in Fig. 4 and Fig. 5.

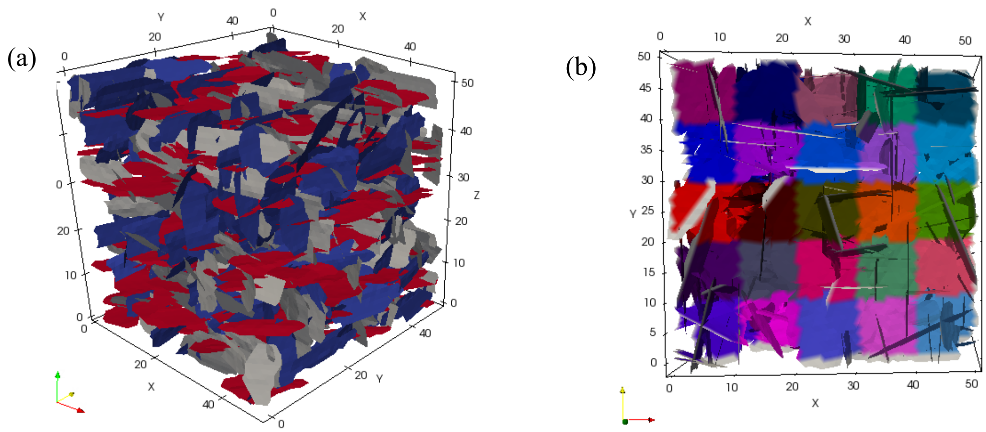

Fig. 10. A single realization of the stochastic model including deterministic fractures (a) coloured by fracture group and (b) coloured by region.

6.1. Fracture intensity mapping

By considering the intensities of all cubes in a DFN model it is possible to generate maps showing changes in intensity throughout a DFN. As an example Fig. 11 shows data from ten layers of a single realization of a stochastic model. From top down, left to right, layers 1 to 10 are shown as a trace map, a fracture intensity heat map and a histogram showing the distribution of intensities specific to the layer. The boundary effects can be seen by comparing the slices from the top and bottom of the model to the interior slices. It can be seen visually that there is a larger number of low intensity blue squares for these layers. This method of visualization can be applied both to layers in a DFN or defined volumes. This allows for the area of influence of a blast hole to be isolated and considered alone. By changing the amount of explosive used the area of influence is changed and thus can be optimized to incorporate only the desired range of fracture intensity in the area of influence for a single blast hole.

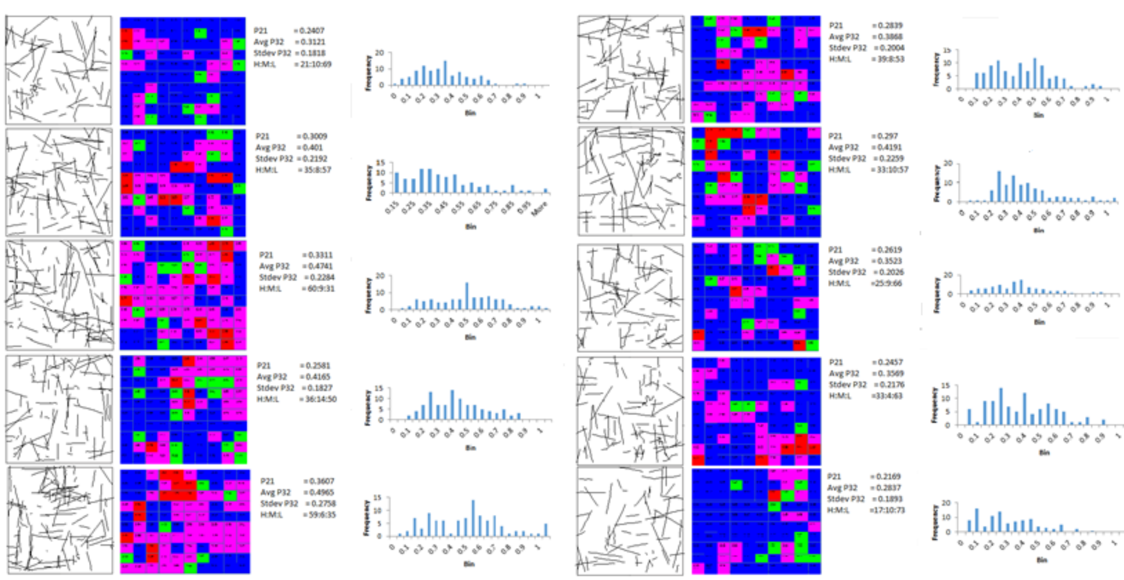

Fig. 11. Comparison of fracture intensities graphed on a heat map with histogram showing intensity distribution by layer of a purely stochastic DFN model (green-low intensity, green-medium intensity, pink-high intensity, red-very high intensity).

The mapping of fracture intensity also allows for a visual validation of the method of visualization. By superimposing fracture traces on the heat map generated a shown in Fig. 12 the location of high fracture intensities can be seen to match the high intensity squares (pink and red). This shows that if the location of specific fractures is not required but it is still desired to consider a disaggregate representation of a rock mass that a three dimensional heat map can be used. This would require significantly less memory for visualization and for incorporating DFN data into other engineering computer models.

6.2. Boundary effects

As fractures can propagate from one region to another depending on their seed point within a region a boundary effect is expected that will result in lower intensities on exterior blocks. This is due to the fact that no fractures are seeded out of bounds that could leak into a region that is within the DFN. For example a block in the center of a DFN would have 6 neighbouring blocks but a block on the corner of a DFN would only have 3 neighbouring blocks. It would be expected that the corner blocks would have half as many fractures propagate into them as would blocks in the center of the DFN.

The P_{32} fracture intensity is plotted by layer in Fig. 13 with the standard deviation of intensity for each layer. This graph shows that there is an observable decrease in intensity for the top and bottom layers. The standard deviation for these layers does fall into the range of intensities of the interior layers. This demonstrates that although the overall intensity is lower that there will still be some areas of high intensity on the exterior of the DFN models as can be seen by comparing layers in Fig. 11.

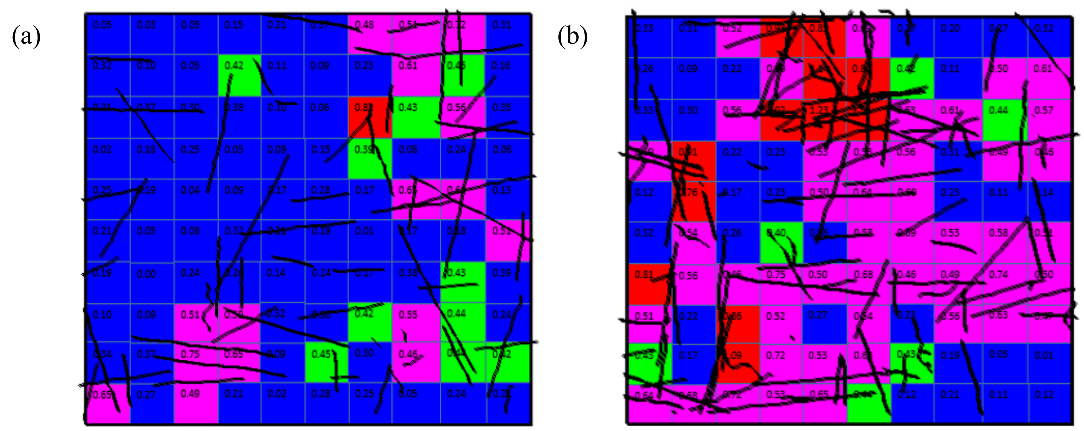

Fig. 12. Close up of heat map for layer 5 and layer 10 of a purely stochastic DFN model with superimposed fracture traces from the midpoint of the layer (green-low intensity, green-medium intensity, pink-high intensity, red-very high intensity).

Based on the fact that a boundary effect is expected and the fact that it is demonstrated through visual inspection and quantitatively when comparing fracture intensities, it was deemed necessary to further investigate this phenomenon. Fig. 14 shows a curve that considers the difference between the mean fracture intensity for exterior and interior blocks in a DFN model based on block size. It is proposed that the extent of the boundary effect can be determined through this analysis. A peak is noted in the difference in intensities for a block size of 20 m^3. This equates to a boundary effect of approximately 2.75 m, or one quarter of a 10 x 10 m^2 region with fractures from 50 to 100 m^2 being seeded.

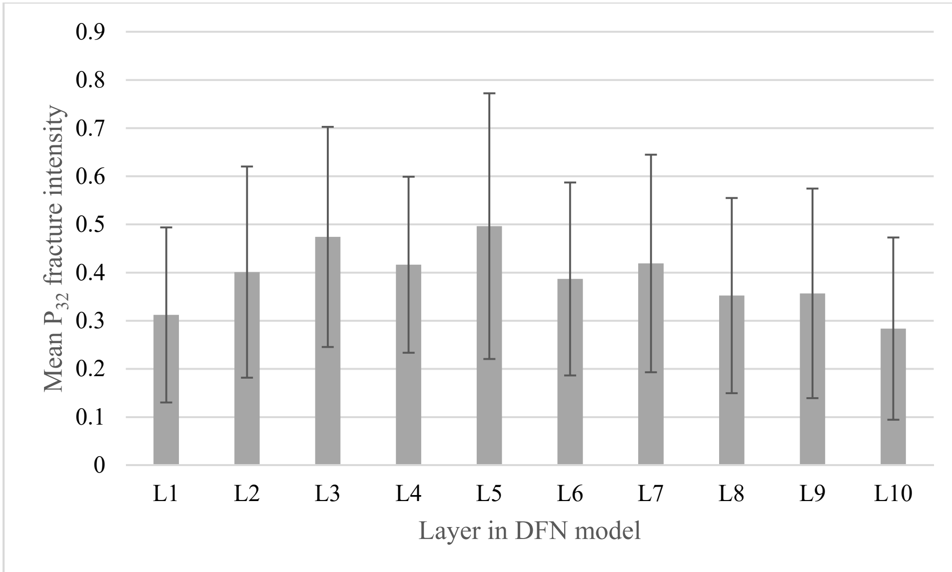

Fig. 13. Variability in P_{32} in a single realization of a stochastic DFN model by layer (5 m thick, numbered from bottom to top), error bars show standard deviation.

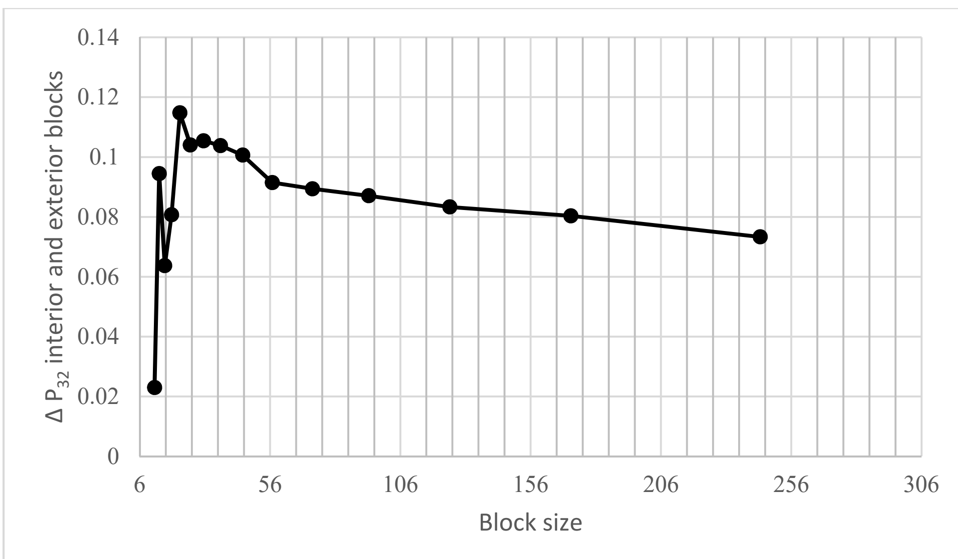

Fig. 14. Difference between interior and exterior P_{32} intensities vs. sample block size

6.3. Fracture intensity distributions and CV% plot

In order to view the distributions of fracture intensities that result from stochastic seeding Fig. 15 was created. By considering how the distribution of fracture intensities changes with sample size and estimation can be made as to what size of block should be used for sampling and what distribution would be expected. This allows for a prediction of fracture intensity if there is a limited number of sampled cubes. For the case of Fig. 20 only 10 000 blocks were sampled for the 100 x 100 x 100 voxelation. The distribution has appeared to break down at this point with peaks at P_{32} of 2, 4, and potentially 6. These peaks need to be further investigated to determine if they are representative of the entire DFN and if so to determine the cause.

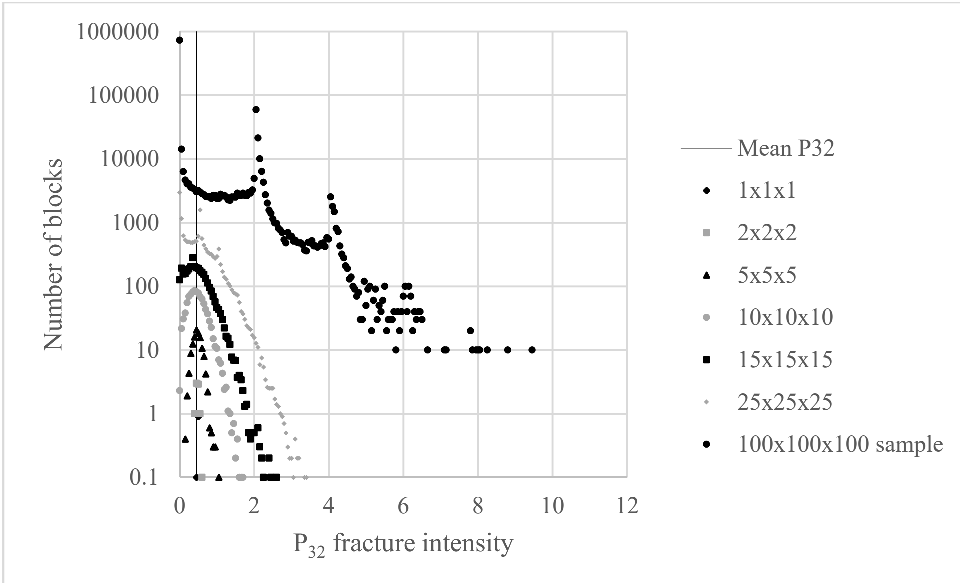

Fig. 15. Fracture intensity distributions with respect to voxelation.

Through the generation of this figure it is possible to view the fracture intensity distribution, estimation of ISBD (y axis values) and the CV% by considering the deviation of the distribution from normal. A CV% curve was plotted for all ten realizations of the stochastic DFN model and is included in Fig. 8. When compared directly to the Canadian Shield curve it can be seen that both the location and shape of the curve has changed. It is proposed that this shape is characteristic of a DFN. The shift in the curve to the left means that there is a lower degree of variance at smaller scales than the Canadian Shield which is likely a function of fracture size and orientation with respect to the experimental volume.

6.4. *In situ* block size estimation

Using the method of ISBD estimation presented in Section 5.2 a block size distribution for a realization of the stochastic was created and is presented in Fig. 21. This curves again is indicative of a power law relation as is expected. The largest blocks are predicted to be about 125 m^3 or 5 x 5 x 5 m^3. By using progressively smaller sample blocks it is possible to define blocks using this method to a very small size given the computing power. The smallest block size shown in Fig. 21 is a result of a 22 x 22 x 22 m^3 inspection of the DFN.

7. Discussion

A new method of analysis is presented in this study that applies directly to MoFrac-generated DFN models. MoFrac DFN models contain non-persistent, undulating fractures and thus estimates of fracture intensity and block size distribution must be derived from generated models. DFN models are both generated and analyzed using the MoFrac code.

By dividing the DFN into component blocks at different scales and measuring fracture intensity it is possible to derive several tools for analysis. Three of these tools are:

- The CV% plot
- Intensity distribution analysis
- ISBD estimation

The values used to determine values that enable these three tools are all visible in Fig. 16. Further to these three tools a new method of visualizing MoFrac fractures is presented that replaces actual fractures with inspection cubes that shows fracture intensity in a sense analogous to a heat map. This visualization allows for understanding of any boundary effect with MoFrac that was characterized in further detail in this study. It would be necessary to calculate the boundary effect for any new DFN until relation between fracture intensity and orientation in a MoFrac DFN can be used to generate a function that predicts block size. The use of CV% is a measure of fracture intensity variation over different scales and is useful to determine if fractures of the correct size are included in DFN models for engineering design and also of use to optimize such designs. AN estimation of ISBD has been presented that utilizes the data collected to obtain the CV% plot. Although this method serves as an estimate only there is indication that with refinement the method can be used to measure actual block sizes within a DFN by querying null blocks for neighbouring null blocks and summing the volume of connected blocks. This would mean that only one analysis would be required with a block size equivalent to the smallest block volume desired for consideration.

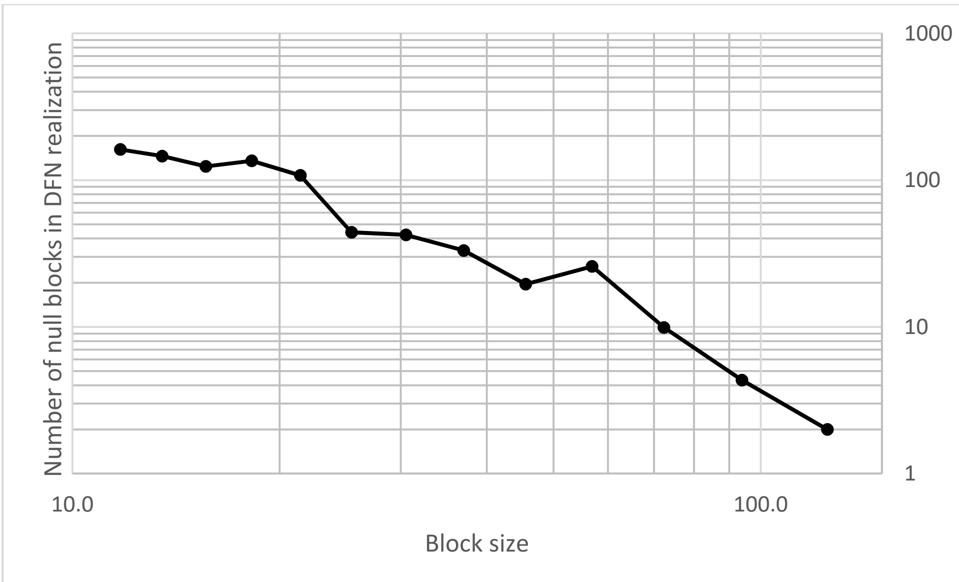

Fig. 16. Estimated *In situ* block size distribution from counting null cubes for a single realization of a stochastic DFN model.

7.1. Applications to Rock Engineering Systems solutions

Rock Engineering Systems (RES) solutions to civil engineering problems were developed by Hudson (1992) as part of research to develop a new methodology for civil engineering design that considers the interaction of a variety of parameters that are identified to be pertinent to a specific project (Hudson , 1994). This approach to developing designs for civil projects has been applied to blasting by a variety of researchers. This application was initiated by Lu in 1997 which was followed up with papers to characterize the blastability of a rock mass using a RES approach (Lu and Latham, 1999).

A rock engineering systems approach involves identifying the parameters of interest to a specific problem and formulating an interaction matrix that describes the relationship between each parameter. This approach is a total systems approach that develops an analytic model defining the environment for a particular system that is to be analyzed (Hudson, 1992). Interaction matrices developed for determining blastability have been previously developed (Lu, 1997; Lu and Latham, 1999). These interaction matrices involve the parameters listed below:

- Strength
- Resistance to fracturing
- Sturdiness of rock
- Elasticity of rock
- Resistance to dynamic loading

- Hardness of rock
- Deformability
- Resistance to breaking
- *In-situ* block sizes
- Fragility of the rock mass
- Integrity of the rock mass
- Discontinuity plane's strength

It is suggested that this model can be used with modifications to incorporate emergent variables derived from MoFrac-generated models. Parameters that could be included in a modified interaction matrix would include the coefficient of variability for a DFN, mean fracture intensity for the area of influence of a blast hole, and *in situ* block size distributions as calculated by MoFrac. Together with the ability to visualize rock surfaces and determine exact burdens and fracture locations and orientations through laser scanning, a new blast optimization technique using the emergent properties in calculating optimized hole locations and powder factors can be developed. Improvements will also be available for the visualization of the blast and for exact calculations of rock volumes (Jian, *et al*., 2018). It is also suggested to develop an interaction matrix that is friendly to the skill set of the blaster and not the rock engineer. Often it is the blaster that makes decisions about blast design based on previous blasting results at the same site. To have a method to verify changes in a blast plan or to develop entirely new blast plan strategies, such as irregular patterns that utilize information available to the blaster would prove to be very useful under a variety of conditions where blast results are unpredictable.

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