Underground Mining Stope Layout Optimization and Production Scheduling: A Review of Existing Solvers and Algorithms

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ABSTRACT

Planning of an underground mine poses considerable difficulties in the areas of safety, environment, ground control and production scheduling. To maximize the profitability of these operations, an integrated optimized stope layout and production schedule is required to coordinate underground mining activities such as development, extraction, haulage, dumping, stockpiling and processing. An optimal mine life is detailed in proper scheduling of all available resources and more importantly the mining sequence. The process of preparing such strategic mine plan is an optimization decision-making process which entails a mining sequence that takes into consideration the physical limitations, resource and production constraints, financial performance and uncertainties of the mine, and meets the required quantities of ore type at each period throughout the life of the mine. The current underground stope layout optimization and production scheduling algorithms in literature lack the ability to deal with real-world large-scale complex underground mine planning problems. Most of the algorithms and solvers available do not integrate production scheduling with stope layout optimization. A few that integrate this two important mine planning processes do not have the ability to address: i) the broad range of mining methods (exhaustiveness), ii) practical problem sizes, and iii) stochastic parameters. This paper reviews relevant literature on existing solvers and algorithms for underground stope layout and production scheduling optimization, and outlines their limitations and potential research opportunities.

1. Introduction

Underground mining has aided the extraction of deep occurring hard rock minerals such as gold, copper, silver, and iron for centuries. The extraction of these minerals generally require extensive development work that is time-consuming. Fig. 1 is a section through an underground mine showing some developments. Underground mining has a low level of flexibility in its operations. To maximize the profitability of these operations, an integrated optimized mine plan and schedule are required to coordinate underground mining operations including development, stoping, haulage, dumping, stockpiling and processing. To determine the optimal mine life, proper scheduling of all available resources and a strategic mine plan is required. Preparing such strategic mine plan is an optimization decision-making process that involves a mining sequence that takes into consideration the physical limitations, available resources and production constraints, financial performance, and uncertainties in the mine to meet the required quantities and quality of ore type throughout the mine life. Planning, optimization and production scheduling of underground operations are relevant to ensuring the efficient utilization of resources which in effect reduces production cost.

Planning an underground mine is a complex procedure consisting of five stages (Kuchta et al., 2004): i) determining the geometry and grade (or quality), ii) deciding how to mine the ore by underground mining, iii) designing the mine infrastructure, or layout of the mine to efficiently exploit the ore, iv) planning how to mine and process the ore, and v) decommissioning the mine and restoring the site to an environmentally acceptable state.

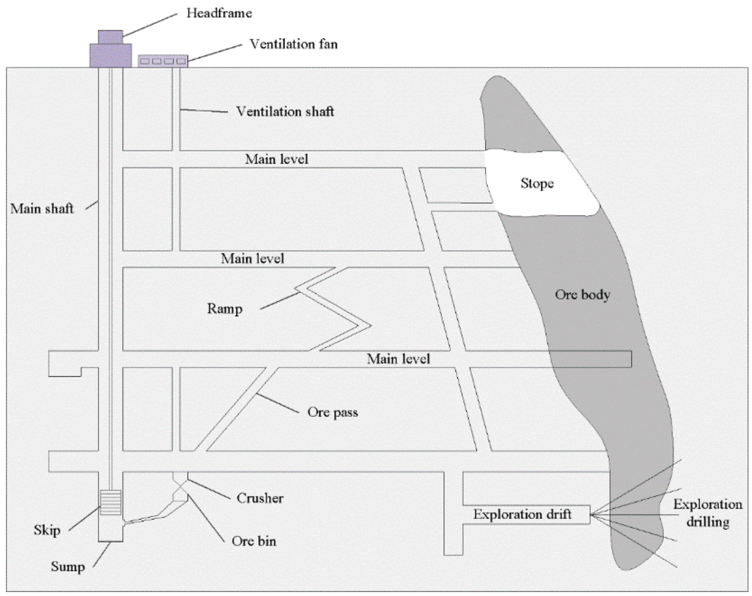

Fig. 1. A cross-section through a typical underground mine modified after Erdogan and Yavuz (2017).

The mining and processing (or production) phase require detailed scheduling. Specifically, a production schedule that must provide a mining sequence that considers the physical limitations of the mine and, meets the demanded quantities of each ore type at each period throughout the life of the mine. Mines, therefore, employ schedules as long-term strategic planning tools to determine when to start mining a production area and as short-term operational guides.

According to Hamrin et al. (2001) and Topal and Sens (2010), a stope is an underground opening from which ore has been excavated. O'Sullivan and Newman (2015) describe stope layout as analogous to the ultimate pit limit in open pit mining, and that it defines the design in underground mining. The optimal stope layout is defined by the size, location and number of stopes within an orebody model. This model is described as consisting of a series of layers for which each is composed of a number of rows referred to as panels, where the panels are made up of a series of rings. Selecting the best combination of available stope layout directly affect the profitability of an underground mining operation (Dimitrakopoulos and Grieco, 2009; Topal and Sens, 2010).

Production scheduling can be defined as the allocation of resources and mined out ore reserves over a period of time with particular sets of constraints. Mine production scheduling is an optimization process which assigns the extraction sequence of mining blocks based on constraints which incorporate, among other things, a method of mining, slope size in other to maximize the Net Present Value (NPV) of a mine (Martinez and Newman, 2011; Gangawat, 2014). Pourrahimian (2013) explains that production scheduling defines the tonnages and grades to be mined throughout the mine-life. Scheduling in any mining system has an enormous effect on the operation's economics. It is becoming essential to generate production schedules that will provide optimal operating strategies while meeting technical and environmental constraints. Improvements in computing power and scheduling algorithms over the past years have allowed planning engineers to develop models to schedule more complex mining systems. Production scheduling problems are usually complex due to the nature and variety mining constraints.

Production scheduling is known to significantly influence the viability of mining operations and scheduling techniques with the capacity to provide optimal results within a reasonable time frame. It presents mining operations with the ability to examine a variety of situations and quickly respond to revised and updated data. This helps to reduce uncertainty and assists in the decision-making process, for future strategies which may result in less volatile operating performance and often higher profitability (Nehring et al., 2010).

Nehring et al. (2012) classified mine production scheduling into two main categories: long-term production scheduling and short-term production scheduling. Long-term production scheduling is done over the life of the mine. In short-term production scheduling, the result generated from the long-term production scheduling is broken down according to task or time period that need to be studied, and different sets of constraints active in short-term are applied to minimize the deviations from the pre-defined capacities.

Long-term underground mine production scheduling problems have been studied by researchers (Jawed, 1993; Nehring et al., 2010; Pourrahimian et al., 2012). These studies have been based on the development of algorithms using operation research methods such as Linear Programming (LP), Mixed Integer Programming (MIP) and Mixed Integer Linear Programming (MILP). The current optimization algorithms in the literature lack the ability to deal with real world large scale and complex, underground mine planning problems. They do not have the ability to address: i) the broad range of mining methods (exhaustiveness) ii) practical problem sizes and iii) stochastic parameters.

This paper reviews relevant literature on existing models and algorithms for underground stope layout and production scheduling optimization for further integration and improvements. The next section of this paper covers summary of the literature review on underground optimization. Section 3 gives details of underground stope layout/limit optimization models and algorithms. Section 4 covers limitations of underground stope layout/limits optimization models and algorithms. Section 5 highlights the underground production scheduling models and algorithms. Section 6 presents the integrated stope layout/limit and production scheduling optimization mathematical programming models and algorithms. In Section 7, stochastic mine planning approaches in underground mining are presented. Research opportunity is presented in section 8. Section 9 documents the rationale behind the proposed PhD research approach and finally, Section 10 presents the summary and conclusions.

2. Summary of Literature Review

Generally, optimization is best described as a process that finds the best or optimal solution for a given problem (Chakraborty, 2010). Mine optimization is an inherent iterative process since all facets of mine design and scheduling are interrelated. It is not possible to change e.g. the level spacing of a mine, without affecting the scheduling, or the stoping efficiencies and number of crews without considering the development requirement (Smit and Lane, 2010). For this reason, changes to design and schedule are usually tested using graphic designs and scheduling applications such as Datamine, Surpac, MineShed, MineSight, NPV Scheduler, Deswik, Maptek Vulcan, Gems and Whittle (Musingwini, 2016; Erdogan et al., 2017).

Optimization in terms of mine planning has also been defined as a descriptive set of techniques that introduce analytic mathematical methods to arrive at an option out of multiple choices; thus bringing it into planning activities. The technique embodies three stages (Shahriar et al., 2007):

- Creation of a mathematical model of the activity;
- Adoption of a criterion; and
- Development of an algorithm.

Underground optimization is considered as a constrained optimization problem, which is aimed at finding the highest profitable stope layout subject to geometric constraints (Bai, 2013). Chakraborty (2010) and Ataee-Pour (2000) further explain that optimization problems are centered around three main factors which are formal procedures of Operations Research (OR):

- An objective function which is either to be maximized or minimized;
- A set of unknowns or variables that affect the objective function; and
- A set of constraints that allow the unknowns to take on certain values but exclude others.

The idea of an optimization problem is therefore to find values of the variables that maximize or minimize the stated or defined objective function while satisfying the stated constraints. The optimal mine layout is greatly influenced by the chosen optimization criteria. The following criteria have been used in mine geometry optimization (Ataee-Pour, 2000):

- Maximization of the total mine economic value;
- Maximization of value per tonne of the saleable product;
- Maximization of the life of the mine provided that the value per tonne does not fall below a certain figure; and
- Maximization of the metal content within the mine.

It is worthy of note that in stope geometry optimization, maximization of the total mine economic value or NPV is frequently used.

Ataee-Pour (2000) asserts that mine geometry or layout is one of the most important stages of mine planning for both surface and underground operations. Outlining the mineable ore assists in determining the amount of reserve, as well as mine life and production scheduling. There is a relationship between mine layout and other mine planning stages such as equipment selection and haulage routes. In effect, mine design and optimization is an interactive process with inconsistency between mine layout definition and production scheduling. However, the optimal layout cannot be determined until the values of the blocks are known or defined. The definition of the block values also depends on factors such as commodity prices and mining costs that in turn depend on when the blocks are to be exploited. Conceptualizing a stope optimizer as a solid box, the input data to the box is the ore block model quantifying the mineral content or economic profit of block volumes on regular grids and the output is the geometry of stope indicating the blocks to be mined (Bai, 2013).

It is a common practice for underground mine plans to be created sequentially, where results from one planning process form the input data for another. The disadvantage to this method is that optimizing an individual mine process, such as stope layouts introduces a likelihood of increasing costs or decreasing revenues associated with other areas such as production scheduling, as harmful decisions must be balanced. This is practical for manual methods; computerized optimization techniques should consider an integrated approach to creating the global mine plan. Considering the interaction and influence that individual underground mine planning processes have on each other during optimization will provide more profitable results than if ignored (Little et al., 2013).

Due to the significant nature of optimization in the designing and planning of mines, researchers have carried out several works. However, little has been done in the formulation, optimization of design and the long-term production scheduling problems for underground mines. In addition, limited algorithms are available for the optimization and scheduling of underground mine production. It is worth noting that until the last few years, comprehensive computer-based designing, planning, and scheduling tools for underground mining did not exist hence process was manual, tedious and inefficient. Mining software developers were able to tackle underground mine design, planning and production problems after their success in applying computer techniques that have been embedded in commercial software packages to open pit mining. This can be attributed to the large number of underground mining methods, lack of versatility and complexity of underground mining economic block modeling (Ataee-Pour, 2000; Newman et al., 2010).

Little et al., (2013) reported that software tools for strategic underground mine planning and scheduling are lacking and that the demand for tools will increase as deeper deposits amenable to underground mining continue to grow in value. A number of optimization techniques have been developed for stope layout design and progress continues to be made in the area of production scheduling, both of these mine planning areas are amenable to improve in terms of guaranteeing optimal results.

3. Underground Stope Layout/Limit Optimization Models and Algorithms

A stope is an underground opening from which ore has been or is to be extracted. Selection of the best combination of available stope boundary will directly affect the profitability of the operation (Topal and Sens, 2010).

From 1965-2000, out of 62 publications that contributed to mine geometry optimization, only 10 papers focused on underground mining related. Generally, algorithms for open pit limit optimization are numerous, amounting to approximately two and a half times more than that for underground methods. True optimum solution for the open pit limit optimization is often guaranteed and, additionally, several computer packages are available to the industry. However, only few algorithms have been developed for optimization of ultimate stope layout or boundaries in underground mining (Ataee-Pour, 2005). Nikbin and Ataee-Pour (2016) reported that the first algorithms were presented about five decades ago for optimization of mining limits. Most of the algorithms presented are applied to optimization of open pit limits. Few algorithms have been presented for underground cases, most of which have been tailored for a specific mining method or based on heuristic techniques. These heuristic algorithms do not guarantee the true optimum. There are, however; some rigorous algorithms, which provide a true optimum, but fail to provide a 3D solution. Each of the algorithms developed for ultimate underground layout has only considered few constraints. Consequently, the Floating Stope of Datamine, Maximum Value Neighborhood (MVN), can be run on 3D block models, but they are heuristic and cannot find the true optimum layout. Although Branch and Bound technique and Dynamic programming also called Optimum Limit Integrated Probable Stope (OLIPS) are rigorous, they have been presented for 1 and 2-dimension problems only. Underground mine optimization has attracted more attention in the last 15 years. Less effort has been committed to the field of underground mining and only a few algorithms are available for economic optimization of underground stope boundaries (Sotoudeh et al., 2017).

The available algorithms for underground stope boundaries or layout limit optimization are illustrated in Fig. 2 (Ataee-Pour, 2000; Ataee-Pour, 2005; Grieco and Dimitrakopoulos, 2007; Shahriar et al., 2007; Bai, 2013; Little et al., 2013; Sandanayake, 2015; Erdogan et al., 2017; Sotoudeh et al., 2017; Nhleko et al., 2018): These optimization techniques are discussed in the subsequent sessions.

3.1. Stope boundary/layout limit algorithms based on heuristic logic

Porumbel (2012) explains that a practical approach to solving many intractable problems of large size is by using heuristic search algorithms or simply, heuristics. They are generally used to explore only a small part of a very vast search space. Heuristic algorithms use reasonable resources and are able to produce acceptable solutions, but without any theoretical guarantee. They have the ability to produce competitive results not only on well-known Nondeterministic Polynomial (NP) complete problems, but on any computational problem for which exact or rigorous algorithms require prohibitive time.

Cheimanoff et al. (1989) developed the Octree Division Approach. The technique evolved from the prototype production scheduling tool BONANZA which forms part of GEOCAD package used for the development of geological resources to mining reserves. The features of CAD and artificial intelligence modules are used to develop a rule-based system to generate the shape of the mineable ore while imposing the underground mining constraints. The program uses the octree space division algorithm to remove undesired mining blocks based on the minimum size.

BONANZA is designed in three steps, the first step is gathering data, including borehole data or geological underground sampling, results from geostatistical analysis or interpreted contours supplied by the geologist and intuition from geoscientist of the mining engineer such as the shapes of geological objects. These data are used to build a geometrical model. The second step is transforming the modelled geological resources into mineable reserves. In this step different production factors are simulated to build possible workable volumes, using the available geological resources. In the final step the geological resources are economically evaluated to determine workable reserves and mining sequence. The mine layout is determined by using a rule-based programming method. The two main constraints are geometric and economic ones (Topal and Sens, 2010). According to Sandanayake (2015) the algorithm consequently divides the reserves into sub-volumes for further economic evaluations. These sub-volumes are either stored or removed from the model, if their mineral content or dimensions violates the constraints set by the algorithm.

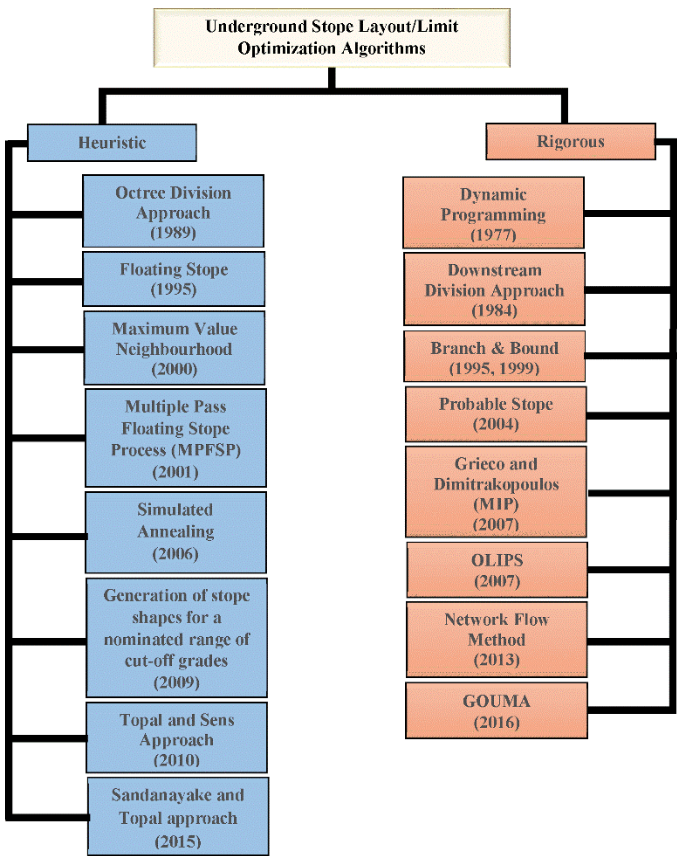

Fig. 2. Taxonomy of underground stope layout/limit optimization algorithms and models.

Alford (1995) explains that the mineable volumes, well-adapted to these constraints are determined through two main modules: Objective Manipulator and Shape Generator. The first phase "Objective Manipulator" gathers mineralized veins into convex blocks distinguishing "large" veins that justify a mineable block by themselves, merges those "close enough" into a single block, and separates those "too far from one another" into two separate blocks. A second phase, "Shape Generator", progressively subdivides a boundary volume in an octree until the smallest subdivision matches the smallest unit. The Octree division algorithm generates a 3D feasible solution for the stope geometry, but it leads to more waste being added to the final mine layout because the algorithm does not analyze the sub-volumes jointly. It includes individual sub-volumes that consider the minimum allowable dimensions in the final layout but not the amount of waste in these sub-volumes. As such, the algorithm does not guarantee the optimality of stope layouts hence, heuristic.

The Floating Stope algorithm was developed by Alford (1995) as an optimization tool used by Datamine to define optimal limit for mineable ore or stope envelope which can be economically extracted by underground stoping methods and was implemented on a fixed economic block model of an orebody (Shahriar et al., 2007). This tool is analogous to the Moving Cone method of open pit limit optimization. The main constraint in this algorithm is the geometry of the stope, which is translated into a minimum stope dimension in three orthogonal directions. The term floating stope is derived from the technique of floating a stope shape of the minimum stope dimensions around any block to locate the stope position of the highest stope grade. Different optimization objectives offered by the floating stope algorithm include the maximization of ore tons, the grade, and accumulated (dollar) value and minimization of the cutoff wastes. The problem is then to determine if any block above the specified cutoff grade can be included into a stope. The Floating Stope technique and its open pit analogue have different constraints. The Moving Cone technique examines cones independently. In fact, the mutual support between blocks is ignored in this method. As a result, two different cones may individually be uneconomical but when they are jointly considered the cones may be profitable. The Floating Stope approach has the opposite limitation. Two overlapping best stopes considered individually may be economical due to a number of high grade blocks. However, when they are considered together, the combination of the stopes may be uneconomical. Sandanayaka (2015) illustrates the issue using a hypothetical 2D scenario. The two stopes measure 5×5 blocks, i.e., five blocks along the x axis and five blocks along the y axis, overlap and share six mining blocks. The two overlapping stopes are designed as stope 1 and stope 2 and the economic values of their shared mining blocks are indicated.

The total economic value of the combination of stopes differs from the actual economic value. Therefore, the algorithm does not guarantee an optimal solution to the problem. The most significant advantage of this algorithm is its simplicity. A new version of the algorithm, the Multiple Pass Floating Stope Process (MPFSP) employs multiple-optimization process. It provides a 3D evaluation over the optimization and sensitivity analysis for mineable reserves and stope geometry while considering partial blocks in the final layout. Moreover, the algorithm benefits from generality. It is not specialized for a certain mining method. Finally, a commercial software package has been developed for the Floating Stope algorithm and it is has been reported by Shahrair et al. (2007) to give true optimality among the list of stope limit optimizers. However, it is a heuristic approach and lacks the rigorous mathematical support (Ataee-Pour, 2000).

The Maximum Value Neighbourhood (MVN) algorithm is another heuristic approach that was developed and implemented by Ataee-Pour (2000). He developed a Stope Limit Optimizer (SLO) on a Fortran 90 based program with Winteracter software developer to implement MVN algorithm. He proposed this algorithm in 1997 to optimize stope boundaries using a 3D dimensional fixed economic block model to locate the best neighborhood of a block which guarantees the mine geometry constraints (Shahriar et al., 2007). According to Ataee-Pour (2005), the neighborhoods are restricted by the mine geometry constraints. The neighborhood concept is based on the number of mining blocks equivalent to the minimum stope size (stope/block ratio). Since several neighborhoods are available for each block, the one that provides the maximum net value is located for inclusion in the final stope. Sandanayake (2015) explains with an example using a hypothetical row of six mining blocks. The six mining blocks are labelled a-f in alphabetical order for easy reference. This algorithm can also be applied to any underground mining method although it does not guarantee the true optimum stope layout (Shahriar et al., 2007).

The Multiple Pass Floating Stope Process (MPFSP) was developed by Cawrse (2001) to extend and improve the functionality of Datamine software. The process of the algorithm relies on the same principals as the floating stope algorithm. The MPFSP process is divided into three steps: input parameter definition, file generation and file management. Input parameters (multiple), such as the head grade, cutoff grade and maximum waste, are defined by the user. During the file generation step, economic stope envelopes are created for each set of parameters. Finally, the data files or the statistical files generated in the process are converted into a Microsoft Excel compatible (CSV) format. The generated envelopes based on input parameters can provide extra information during the mine design process helping the designers to improve the efficiency of their mine design. Although the method can assist in stope boundary selection and design, it does not generate optimum stope layouts (Topal and Sens, 2010; Sandanayake, 2015).

The Simulated Annealing (SA) approach is an algorithm that was developed by Manchuk (2006) and Manchuk and Deutsch (2008) for stope geometry and sequencing optimization. In this method, a stope is parameterized as a geometric object consisting of a set of vertices and edges that formed a triangulated mesh. This notably facilitated the manipulation of stope geometric constraints in optimization. The rationale behind the optimization is to randomly adjust the shape of stopes, respecting the geometric constraints, in order to find the shell enclosing maximum profits. The algorithm offers a general 3D solution to engineers, integrating full geometry constraints regardless of the mining methods selected. One of the limitations of the simulated annealing process is that it can be long. This is seen especially when the number of vertices is large to construct a complex geometry. Another is that the time of convergence to optimality can be unrealistic. Therefore, in practice, SA is regarded as a heuristic for which the quality of the approximation to the real optimal solution is difficult to assess (Bai, 2013).

Alford and Hall (2009) developed a method for automated stope design. In this method, the stope optimization is run at a sequence of cut-off grades to generate a series of 'nested stopes' which is a similar concept in the pit optimization where 'nested pits' are generated. The method is able to define the best set of extraction levels and stope heights. However, it does not take into account mining cost as a function of the size and the shape of the stop. It specifies a fixed cutoff grade in the design process, which does not allow an optimum solution (Erdogan et al., 2017).

Topal and Sens (2010) heuristic approach is a methodology that consists of three basic elements: the block converter, stope optimizer and visualizer (Topal and Sens, 2010). During initialization, the given mining block model is converted to a regularized block model, i.e., a block model that constitutes mining blocks with consistent sizes. Given their dimensions (height, length and width) stopes are generated from the regularized block model. Finally, a heuristic stope optimization algorithm is implemented in Matlab software based on the economic value of stopes and the results are visualized with respect to different user-defined parameters. The advantage is that it locates stope boundaries using different stope sizes and selection strategies in 3D. To its disadvantage, the algorithm selects the stopes in descending order of the stope economic values while removing the overlapping stopes. To clarify, this process discards the possibility of multiple stope combinations that can be derived from a given stope set. Among these multiple combinations, there may be combinations with higher total economic values (Topal and Sens, 2010; Sandanayake, 2015).

Sandanayake (2015) also proposed and developed a new 3D heuristic algorithm for the stope layout optimization. It finds a unique solution that maximizes the economic value of the stope layout under physical and geotechnical constraints. The suggested algorithm recommends a unique solution for stope layout and generates non-overlapping stopes. Moreover, it includes variable stope sizes with or without pillars and satisfies the mining and geotechnical constraints. However, in order to find the optimal solution, the algorithm needs to evaluate all the possible unique combinations that require significant computational power in large-scale applications. This limits the algorithm to find the optimal solutions in large-scale data-sets (Erdogan et al., 2017).

3.2. Stope boundary/layout limit algorithms based on rigorous logic

Rigorous algorithms can be described as the algorithms based on a mathematical model and hence they guarantee an optimum solution. Such algorithms are robust, possess objectivity, they are tractable, provide model solutions and facilitate sensitivity. Furthermore, optimal solutions provided by refined rigorous algorithms can often be reached more rapidly with basic meta-heuristics (Martinich, 2008; Porumbel, 2012; Nhleko et al., 2018).

Dynamic Programming (DP) algorithm was proposed by Riddle (1977) to undertake optimization of stope layout of block caving mining method. It was developed as an extension of 3D dynamic programming method for ultimate open pit optimization that had been developed by Johnson and Sharp in 1971. The algorithm was developed in Fortran and implemented on hypothetical economic block models. The algorithm is a multi-section 2D solution for 3D problems. This simply implies that the approach is provided as an optimum stope in 2D section but fails to determine the actual optimal stope in 3D (Ataee-Pour, 2005; Shahriar et al., 2007; Sotoudeh et al., 2017). In running the algorithm, it assumes that there is no footwall in the optimization process and that the maximum profit is achieved irrespective of the footwall design. It then defines a footwall in operational regions and examines the profitability of all feasible solutions for mining and non-mining regions. In the subsequent steps, the process divides the defined footwall region into two sub-regions if the profit is greater than or equal to an assumed profit. However, the process terminates when there are no more profitable footwalls to examine, or no further feasible footwalls can be introduced into the operational region (Sandanayake, 2015). The algorithm is a rigorous mathematical solution to the problem. This method is known to be limited to block caving mines and for that matter unable to function in optimizing the layout of other underground stopping methods (Ataee-Pour, 2005; Shahriar et al., 2007).

The downstream geostatistical approach also known as the mathematical morphology approach was proposed by Deraisme et al. (1984). The introduction of this approach was to determine the outline of the mineable orebody shape in underground mines. This approach was constructed for cut-and-fill and sublevel stoping methods and was based on a 2D sectional block models of the deposit. Mathematical morphology was used to transform the image of ore blocks above cut-off grade to another image satisfying the stope geometry constrains (Erdogan et al., 2017). The main disadvantage of the algorithm is its complexity. The traditional algorithm has been developed for specific mine (project) and tailored for two specific mining methods (cut and fill and sublevel stoping methods). The algorithm benefits from a mathematical support and is expected to provide the real optimum solution at least in 2D (Erdogan and Yavuz, 2017). However, reported by the developers that the economically optimized images do not necessarily respect the geometrical constraints. Further modifications of the images are required to make them meet the constraints and this makes the results non-optimum (Ataee-Pour, 2005). The approach takes into account grade uncertainty in the optimization process. It controls only the stope geometry so it does not consider the economic profit related. This limits the optimality of results in 3D (Erdogan et al., 2017).

Branch and Bound Technique is a mathematical programming technique for economic optimization of stope boundary. It was developed by Ovanic and Young (1995). An optimal economic stope boundary was developed by optimizing the starting and ending locations for mining within each row of blocks. To determine these locations, two piecewise linear, cumulative functions known as Type-Two Special Ordered Sets (SOS2) were used for each row (Ataee-Pour, 2005; Shahriar et al., 2007). SOS2 technique, referred to a separate programming, is used to optimize piecewise linear functions. It is defined as an ordered set of special variables for which the solution allows at most two variables to be non-zero, they must be adjacent. The first function sums block values along the row for inclusion within the stope boundary, while the second function sums block values for exclusion. It is an apparent advantage that the branch and bound algorithm removes some restrictions from mine layout design and, in particular, stope geometry optimization. There are no restrictions for blocks to be treated as a whole, but rather partial blocks can be included in the optimal stope. Moreover, blocks are not limited to be regular or uniform in shape. Whatever their shape or size it does not influence the optimization since the block cumulative function is used in modeling. In other words, branch and bound method allows regular or orthogonal block geometry so blocks can be formed following the geologic variation and discontinuities. However, it optimizes the stope boundary in one dimension and neglects the wall slope constrains hence lacks a 3D implementation (Shahriar et al., 2007; Sandanayake, 2015; Erdogan et al., 2017; Sotoudeh et al., 2017).

Ovanic and Young (1999) introduced a Mixed Integer Programming (MIP) model to optimize stope boundaries. The model works by locating the optimal starting and ending points for mining within a row (mining panel) in a block model and in effect, it establishes the optimum stope boundaries. In order to determine the optimum starting and ending location of each panel, two piece-wise linear cumulative functions are calculated. The major advantage of this technique compared to the others is that the block geometry is not required to be regular or orthogonal. A disadvantage, however, is that the algorithm optimizes the stope boundary along the row of blocks in only 1D. Therefore, examples and results for using the algorithm in 3D do not exist. Another disadvantage is that the algorithm also only allows the user to optimize the design of extraction rows only, as the locations of these rows are determined earlier in the design process the algorithm only partially optimizes the mine design (Topal and Sens, 2010).

The Probable Stope (PS) algorithm was established by Jalali and Ataee-Pour (2004) based on Riddle's Dynamic Programming (DP) algorithm to optimize the stope limits of the mining methods which are feasible for vein deposits. The main difference between PS technique and the others is that, the algorithm is implemented on a particular economic block model including the constraints of stope dimensions. In effect, the most significant constraints within the objective function are eliminated. It has resulted in the algorithm being simple in concept, easy to program and reaches a solution quickly (Shahriar et al., 2007).

A probabilistic algorithm based on MIP was developed by Grieco and Dimitrakopoulos in 2007 and the application of the method at Kidd Creek Mine, Canada has been reported (Grieco and Dimitrakopoulos, 2007). In this approach, the ore body is initially divided into layers. Each layer is then subdivided into a number of panels and each panel is further subdivided into a series of rings. Each ring is assigned to a binary variable of the mixed integer model. The objective function of the algorithm maximizes the metal content at a given time. The minimum and maximum mining rings, and the sizes of the pillars that are to be left unmined between two primary stopes, are limited by the constraints of the model. Although the proposed method is the first stope design methodology that has acknowledged uncertainty and greatly assists stope design by considering geological uncertainty, it has potential drawbacks. Because the methodology is based on rings that have been predefined in terms of location and size to determine the most profitable stopes, it finds optimum stopes layout based on those rings. This approach does not allow accurate examination of the orebody over smaller areas and examination of stopes in different locations. In addition, as each ring is represented as a binary variable within the MIP model, it encourages a very long solution time as the number of rings increases within the model (Grieco and Dimitrakopoulos, 2007; Dimitrakopoulos and Grieco, 2009; Topal and Sens, 2010; Sandanayake, 2015).

Optimum Limit Integrated Probable Stope (OLIPS) is an algorithm developed in 2007 based on the Dynamic Programming method by Jalali et al. (2007). The algorithm complies with all technical and geometric constraints, provides mathematical proof and optimality. OLIPS algorithm has two major steps. In the first step, a conventional economic model of mining panel is constructed and in the second step, the probable stope economic model and integrated probable stope economic model are derived from a conventional model. Based on OLIPS algorithm, a computer program named Stope Boundary Optimizer (SBO) was developed and validated by 2D hypothetical models (Sotoudeh et al., 2017).

Network Flow method is an optimization algorithm that was introduced by Bai (2013) to optimize stope design based on graph theory and specifically applicable for the sublevel stoping method. The algorithm is based on a cylindrical coordinate system, which is defined around a specified vertical raise (initial) (Bai, 2013; Sandanayake, 2015). In the process, the graph is constructed using vertical arcs for the footwall and hanging wall slope constraints, with horizontal arcs for the stope width constraint. Selecting a particular mining block for a stope should aim to maximize the stope value subject to two additional constraints: the maximum distance of a block from the raise and the horizontal width required to bring the most distant mining block to the raise. The stope profit is optimized as a function of location and height of the raise and consequently, the best location and height for raises are identified. Finally, the performance of the algorithm is compared with the floating stope algorithm and obtaining better solution values. However, the algorithm is limited to relatively small sub-vertical deposits mined by the sublevel stoping method (Sandanayake, 2015; Erdogan et al., 2017).

Global Optimization for Underground Mining Area (GOUMA) algorithm was presented as a new comprehensive algorithm that proves optimality in 2016 (Sotoudeh et al., 2017). The algorithm has two important characteristics, which makes it very useful. Firstly, it is executed on the Variable Value Economic model (VVEM). Secondly, constraints and technical limitations of all conventional underground mining methods used for tabular deposits are applied in this algorithm. Therefore, the main difference between this computer program and other programs and software tools is the type of algorithm used. For easy use of this algorithm on large□scale problems, a computer program called GOUMA□CP was written in C++ programming language (Jalali et al., 2016).

3.3. Comparative implementation and performance of existing models and algorithms

Intrinsically, underground mine planning is more complex because there are numerous permutations of the direction of mining, such as advance or retreat mining, depending on the mining method chosen. These contribute to the reasons why there is a lack of extensive optimization algorithms and commercial software packages for underground mining, and why most of the work on optimization in underground mine planning is largely academic (Musingwini, 2016).

One of the most effective ways to identifying the strengths and weaknesses of existing models and algorithms for further research is by assessing their comparative implementation and performance. A number of software packages and optimization tools that have the algorithms embedded in them have been developed to enable the implementation of the algorithms. As a result, most of the existing stope layout/limit models and algorithms have been implemented and their results assessed (Bai et al., 2013; Musingwini, 2016; Erdogan and Yavuz, 2017).

The Floating stope algorithm (heuristic) and its extension, the Multiple pass floating stope have been implemented in some commercial software packages, such as Datamine, Vulcan stope optimizer by Maptek Snowden's Stopesizer mining software package, used internally by Snowden Consultants to produce a single mining outline for a selected cut-off grade and the Anglo Platinum Optimization Tool (APMOT) (Smit and Lane, 2010; Bai, 2013; Musingwini, 2016). In Bai (2013), a new algorithm, the Network flow algorithm to optimize stope design for sublevel mining method was proposed and implemented by a comparative work with the Floating stope technique. In the work, the floating stope technique and the network flow technique were applied on three synthetic cases and one real case study. In the three synthetic cases, the profits obtained with respect the Network flow were larger than with the inner and outer envelopes obtained with the Floating stope algorithm. On the real case study based on a metal deposit located in Canada (name and location withheld), the Network flow approach provided an equivalent profit to the inner envelope and more profit than the outer envelope (+53%). It is worth noting also that neither of the floating stope envelopes met the slope angle constraints, as these constraints are simply not considered in the Floating stope algorithm.

Nikbin and Ataee-pour (2016) described a new Integer Programming (IP) model for optimization of underground stope layout. Their model is run on a specific block model, which is called secondary block model. The objective function of this model has been presented based on maximizing the profit. Almost all of the important and critical constraints in this scope have been considered and even some of them such as the minimum width of rib pillars are taken into account for the first time. The value of stopes found by the new model is the same as that obtained from OLIPS algorithm by Jalali et al., (2007). Results show that the new model provided more stope value compared to using Dynamic Programming of Riddle (1977). Also, the wide range of application and mathematical formulation of this model are its advantage. However, it is a 2D model and it is recommended that a 3D model is presented in the future researches.

Nelis et al. (2016) proposed a new algorithm for the optimal stope design problem. The algorithm is based on the methodology developed by Bai et al. (2013) where a cylindrical coordinate system is used to define geomechanical restrictions and to find the optimal stope around an initial raise. The proposed algorithm extends this work based on an Integer Programming (IP) formulation incorporating a new set of constraint, directed to solve geomechanical issues present on the original methodology. The results of a test of the new formulation on two synthetic and one real deposits were reported. An economic, geomechanical and feasibility analysis was performed and results compared with Bai's approach. Results indicated that the new approach allows the determination of optimal stope designs, fulfilling geomechanical requirements. It also generates feasible stopes in disseminated orebodies, avoiding unstable and irregular geometries. The generated designs comply with technical requirements such as the stope drilling pattern. The model can be solved in reasonable times for real life cases, and it can be extended to allow further regularization of stope design or faster resolution times with heuristics.

In the work by Erdogan et al. (2017), the capabilities and limitations of four heuristic algorithms are evaluated and compared to each other and then, compared with the stope optimization results from an existing underground mine. The algorithms and the corresponding commercial software packages selected were Floating Stope (Datamine), Maximum Value Neighbourhood (MVN) (Minesight), and two special applications that were developed by Sens and Topal and Sandanayake and Topal. Mineable Shape Optimizer (MSO), an optimization tool introduced by DATAMINE software it was also included in the study to compare with other algorithms. For the success of the comparison, a section of an economic block model that represents a gold deposit located in South Australia and a relatively deep, extending from 150 m below the surface to 600 m was used. The research aimed to find optimal solutions and maximize the value of the operation based on the defined constraints and rules. Results reflected that in terms of net profit values, MSO tool produced the highest followed by Sandanayake and Topal approach, MVN approach, the real mine case, Sens and Topal approach and the Floating Stope approach respectively. However, none of them can give a true optimum solution; they however provide only approximate solutions in 3D.

Sotoudeh et al. (2017) reviewed some existing underground stope layout algorithms for optimization and developed a computer program called Stope Layout Optimizer 3D (SLO3D) with C sharp user interface to implement a heuristic algorithm for optimization of underground stope boundaries on data from an actual copper mine located in southeast Iran. The developed optimizer provides an interactive environment to define and edit important parameters related to the stope layout optimization, including block model parameters, stope geometry, cutoff grade and economic factors. Finally, an example was presented to demonstrate the implementation of the algorithm with different stope limits and selection type strategies. At the end of the implementation, the SLO3D optimizer generated an optimized non-overlapping layout with 29 stopes

Nikbin et al. (2018) introduced a new hybrid algorithm that is a combination of dynamic programming and greedy algorithm. Although this proposed algorithm fails to provide a true optimum solution, it generates better solutions than some existing algorithms. The new proposed algorithm and three existing algorithms were used to find the optimal stope boundaries on a real case ore body. The results demonstrate that the proposed algorithm improved the profit by 117.78%, 16.86% and 0.42% compared to Floating Stope, Maximum Value Neighborhood (MVN), and Greedy algorithm solutions, respectively, on a real case study at a reasonable CPU time.

4. Limitations of Existing Underground Stope Layout/Limit Models and Algorithms

A variety of optimization techniques have been developed to generate optimal stope layouts. These techniques use the term optimization to describe their approach, but all fail to truly generate optimal results in 3D (Little et al., 2013). The complexity of underground stope optimization problem generally necessitates solution by heuristic approaches, i.e. search algorithms. However, the few algorithms that have been developed for optimizing underground mining stope envelopes fail to guarantee an optimum solution in 3D space. For real 3D stope definition, earlier approaches employed include: mathematical morphology tools, floating stope technique, maximum value neighborhood method, octree division, and network flow approach. These approaches share two major drawbacks: they are heuristic approaches which do not guarantee true optimality in stope layout and unable to directly incorporate specific geotechnical constraints, such as load carrying capacity of rock mass, wall stability of stopes etc., which influence not only the stope boundary but also production scheduling. Some of these methods may incorporate an additional step to test the stability of the current solution in order to generate stable designs, but they cannot address the geotechnical restrictions directly. Although the Simulated Annealing-based algorithm, had some mining constraints incorporated, it is very slow and the convergence to a global optimum is impractical. In addition, the restriction of disturbance to movements respecting all constraints on slopes in 3D setting can seriously halter the capacity of the algorithm to finding a good solution. To this effect, the mining engineer has to adjust the stope solution in other to obtain a feasible stope (Manchuk, 2005; Bai, 2013; Bai et al., 2014; Musingwini, 2016; Erdogan et al., 2017; Nhleko et al., 2018).

In the case of rigorous algorithm, works on stope optimization relied mostly on strong simplifications of the initial problem. For example, the 3D problem was simplified by considering optimization along only 1D or 2D. The dynamic programming method, and branch and bound techniques which find optimality were developed in this manner. Although the simplifications decrease the complexity of the optimization, it precludes incorporating realistic geotechnical constraints into the optimization (Ataee-Pour, 2000; Ramazan et al., 2005; Bai et al., 2013; Musingwini, 2016; Nelis et al., 2016).

Another limitation identified in almost all the optimization algorithms was grade uncertainty and the contributions of individual ore types to stope value. Consideration of grade uncertainty should be evaluated in the stope optimization procedure. For example, in Bai et al., (2014) the true grade values of the deposit used were assumed known everywhere or obtained using a conditionally unbiased estimator for simplicity sake and so the effect of the uncertainty on grades with regard to the stope design is not considered in this study. With the exception of downstream geostatistical approach or mathematical morphology and the probabilistic optimization algorithm based on MIP developed by Grieco and Dimitrakopoulos that considers grade uncertainty and geological uncertainty respectively in the optimization process, the others do not incorporate uncertainty (Grieco and Dimitrakopoulos, 2007; Dimitrakopoulos and Grieco, 2009; Topal and Sens, 2010; Sandanayake, 2015; Erdogan et al., 2017; Erdogan and Yavuz, 2017).

Again, a limitation identified in the algorithms is the problem of mining methods to which they can be applied. The existing algorithms cannot be applied to all the mining methods. Algorithms like Network flow can only be applied to Sublevel stoping method. The existing algorithms therefore require some improvements in terms of design criteria in order to take into account different mining or stoping methods, which could affect the economics of the mining operation (Erdogan et al., 2017).

Lastly, Musingwini (2016), Erdogan et al. (2017) and Erdogan and Yavuz (2017) assert that, the underground optimization techniques should consider an integrated approach for stope boundary optimization, development and production schedule optimization. This approach will provide more realistic and profitable results than individual evaluations of the stope boundary optimization methods.

5. Underground Production Schedule Optimization Models and Algorithms

Production scheduling is said to specify the mining sequence for economic stopes and the associated mine development required to bring the specified stopes into production. The decision to optimize centres on variables that represent the time at which to mine each stope in order to maximize NPV subject to operational constraints such as mining infrastructure, production capacity constraints, milling capacity constraints, grade and geometallurgical constraints, and rules on precedence relationships between specified stopes (Musingwini, 2016). The complexity of underground mining has resulted in the delay of software for underground operations and hence a lot of the scheduling concepts and algorithms produced for surface mining operations have eventually found their way into underground mining. The stoping methods employed in underground mining are branded by complex decision combinations, conflicting goals and interaction between production constraints (Kuchta et al., 2004; Pourrahimian, 2013).

As chronicled by Askari-Nasab et al. (2010); Askari-Nasab et al. (2011); Pourrahimian et al. (2012); and Pourrahimian (2013), production scheduling optimization models and algorithms available in literature are not just limited to, but can be divided into two main areas: heuristic and exact algorithms. However, the main disadvantage of heuristic methods is that there is no quality measure to solutions provided compared to the optimum. In addition, most of the results are not reproducible. Generally, heuristic methods are employed when there is no available approach to establishing an optimal solution under given constraints though they are widely known to generate good solutions within reasonable times. Currently, underground mine scheduling practice has inclined toward the use of simulation and heuristic software to determine feasibility rather than optimal draw schedules. Notwithstanding the flaws associated with heuristics such as frequently required intervention and lack of ways to improve optimality, simulations and heuristics are capable of handling non-linear relationships as part of the scheduling procedure. It was concluded by Gershon (1987) that schedules generated with heuristic techniques should only be considered as useful guide. Other methods such as queuing theory, network analysis, and dynamic programming have also been used to schedule production and/or material transport (Pourrahimian, 2013).

Within the past few decades many researchers have employed the use of Operations Research (OR) techniques in the mining industry. The application of OR techniques to optimization in the mining industry started to emerge in the early 1960s. Since then, optimization techniques have been applied to solve widely different mine planning problems. One of such OR techniques is mathematical programming. Mathematical programming techniques have been employed for many open pit optimization and scheduling and has therefore been investigated for similar applications in the underground mine environment. Mathematical models are capable of providing mathematically provable optimum schedules. To this effect, mathematical programming models that have been used for underground mine planning production scheduling, (short and long term) problems include: Linear Programming (LP), Integer Programming (IP), Mixed Integer Programming (MIP), Mixed Integer Linear Programming (MILP), Goal Programming (GP), and Quadratic Programming (QP) approaches (Alford et al., 2006; Nehring et al., 2010; Newman et al., 2010; Pourrahimian, 2013; Musingwini, 2016).

The application of LP in underground mine planning was first reported by Williams et al. (1973). LP was used to produce a schedule for the Nchangua Consolidated Copper Mines, limited which used to undertake production scheduling manually based on the experience of the mine planner for three underground ore sources. The mine planning system had already employed a heuristic solution but had no favorable results. The results from the LP schedule for one out of the three mines showed reduction in deviations from set targets subject to hoisting capacity, tramming capacity, profile constraints and manpower requirements. The application of LP on the other two shafts was less effective due to their much smaller size and the complicated mining sequencing resulting from the folded nature of the orebody.

Another application of LP was reported by Jawed (1985). In a case study on a group of coal mines, LP was employed to optimize production planning and scheduling. The application of LP resulted in the minimization of deviation from prescribed targets subject to operational constraints, manpower requirements, extraction capacity, plant capacity and lower bounds on the quality of extraction.

In 1997, Pendharkar (1997) developed and used a fuzzy LP model which incorporated fuzzy measures of quality to evaluate different production alternatives in the context of the coal industry. The authors applied the developed model on a hypothetical problem. The goals of the formulated model were to determine production outputs by coal mines, the satisfactory quality level of the coal to be delivered to the market and profitability. Results from the research indicated that the model had potential for solving decision making problems (production schedule problems) in the coal mining industry and could solve both simple to very complex problems that were dynamic in nature. They further proposed that the model could be extended to consider Non-Linear objective function since Non-Linear Programming has an advantage over LP because it considers certain factors like economies of scale and economies of scope which are ignored by LP.

An application of mathematical programming (Integer Programming/Mixed Integer Programming) in short-term and long-term production scheduling was reported by Guest et al. (2000) in block caving. the objective function in the work was defined as, to maximize draw-control behavior. In his conclusion, he iterated the benefits and advantages of using LP/MIP options and added that in comparison with spreadsheet manual systems, LP and MIP results had improved by 20%. However, Pourrahimian (2013) attests that the objective was to optimize Net Present Value (NPV). He further identifies two major problems associated with the approach: maximization of tonnage or minimization of reserves will not necessarily lead to maximization of NPV and draw control is a planning constraint rather than an objective function. He concluded that the objective function will therefore be to maximize tonnage, minimize dilution, or maximize mine life.

The application of integer programming to optimize underground mine production scheduling was first attempted by Trout (1995). The author uses MIP to develop an optimal production schedule with maximization of Net Present Value (NPV) as an objection function. The work was carried out on a sublevel stoping underground copper ore operation situated at Mt Isa, Australia. The objective function was subject to stope sequencing, stope extraction and backfilling quantities, equipment capacities and production grade as constraints. Due to an interruption from lack of memory, the proof of optimality could not be realized. Solution after 1.6 hours indicated a 25% improvement over the NPV generated by current operational policies. The model was not implemented at the mine due to additional improvements. Nevertheless, the study indicated the relative merit of employing MIP techniques over manual techniques for scheduling stopes (Little et al., 2008; Pourrahimian, 2013).

Carlyle and Evans (2001) developed a large MIP model which factors several constraints such as planned mine layout, projected ore quality, and projected costs for basic mining activities. The model presented maximized discounted ore revenue when applied to only a part of the sublevel stoping platinum and palladium mine in Stillwater, Montana. The system obtained a near optimal schedule of various planning scenarios (Pourrahimian, 2013).

An application of mathematical programming in block caving was also presented by Rubio (2002). The author presented a methodology and subsequently, an application that would enable mine planners to compute production schedules in block caving. The research demonstrated an integration and formulation of two main planning concepts as potential goals to optimize the long-term planning process, thereby maximizing NPV and mine life.

Smith et al. (2003) constructed a large-scale, time-dynamic, life-of-complex mixed integer program production schedule to optimize copper and zinc cash flow based on a detailed life of project production-scheduling at a copper and zinc underground mine at Mount Isa, Australia. The objective was to maximize NPV subject to operational constraints such as ore availability, mill capacity, mine infrastructure production capacity, grade limits, continuous production rules, and precedence relationship between production blocks. However, the authors were unable to solve all instances of their problem in a reasonable amount of time (Newman et al., 2010).

A mathematical programming model to determine the levels of extracted ore from several different underground copper mines to maximize profit was presented by Epstein et al. (2003) at El Teniente, the largest underground copper mine in the world. In the formulated MIP model, the geological area of interest is strategically outlined by profitable extraction points, resulting from vertical aggregation of blocks. Mined material is sent through a network of alternative technologies and infrastructure investments. This capacitated network contains hundreds of thousands of variables and constraints. A rounding heuristic, developed and implemented by the authors gave a resulting solution indicating an improvement of more than 5% on the current operations of the mine (Newman et al., 2010).

Rahal et al. (2003) also described a Mixed Integer Goal Program (MIGP) model with dual objectives of minimizing the deviation from the ideal draw while achieving a production target in block caving. The developed algorithm assumes that the optimal draw strategy is known. In addition, a life-of-mine draw profiles for notional scenarios were developed and showed that by using the results from their integer program, they greatly reduced deviation from ideal draw point depletion rates while adhering to a production target.

Rubio and Diering (2004) in their work expand the analysis to include cost considerations based on previous work by Rubio (2002) by using his operational constraints. They described the application of mathematical programming to formulate optimization problems in block-cave production planning by formulating two main planning strategies: maximization of NPV and maximization of mine life (Newman et al., 2010; Pourrahimian, 2013).

Sarin and West-Hansen (2005) developed a MIP model for generating mine production schedules with constraint set consisting primarily of enforcing precedence, smoothing quality, and production levels and limiting the quantity of sections simultaneously mined. The authors in their presentation maximize NPV for an underground coal mine that consists of sections mined with longwall, room-and pillar, and retreat mining. The model is based on the definition of the mine layout as a precedence network, with the nodes representing mining sections. Binary variables track whether a section is scheduled to start being mined by a given set of equipment at a given time, whereas continuous variables track the quality and production volume of the material extracted. They tailored and developed a general solution methodology based on the Benders' decomposition model to successfully solve the original problem in mining (Newman et al., 2010).

McIssa (2005) also employed MIP to generate a long-term mine plan for a polymetallic narrow vein mine. The objective function was to maximize the cash flow of the operation. His model determined the sequence of development and stoping activities of eleven separate sectors based on three month intervals for a four-year period set at a production rate of 500 tonnes per day. Production schedules were generated for each zone rather than individual stopes. Failure on the part of the formulated model to schedule individual stopes but rather just specify zones, however, provides the mine with only a very general indication of where production should take place. Nehring and Topal (2007) reported that if the schedule model was applied to weekly or monthly periods, this general specification may not actually assist in generating optimal results. Though the model was solved in 30 minutes with 1,200 variables, it is not stated how many of these variables are integer variables and there is no mention of a previously generated manual production schedule with which to compare results.

Nehring (2006) further advances on the work of Trout (1995) by formulating a new constraint to limit multiple fill mass exposures without breaking other operational constraints. The model contained a total of 315 integer variables and 774 binary variables. His research also demonstrated the benefits of using MIP for generating production schedules over a common manual method of selecting production from the next highest available cash flow stope. It is reported by Nehring and

Topal (2007) that based on a conceptual nine stope operation, results showed an improvement in NPV.

Newman and Kuchta (2007) formulated a multi-period mixed integer program model for both short and long-term production scheduling of iron ore production at Kiruna, Sweden. They describe how they modify the model to consider several different levels of time resolution in the short versus long-term, and provided guidance for increasing model tractability. They then demonstrate numerically the increase in schedule quality and model tractability as a result of these modifications aggregating the periods and then solved the original model using information gained from the aggregated model (Pourrahimian, 2013).

Similarly, Nehring and Topal (2007) presented a new constraint formulation for limiting multiple exposures of fill masses for a small conceptual sublevel stoping operation. Comparative results of a schedule generated by the MIP production scheduling model and a manually generated model on a nine stope example was presented. Results from the work indicated that the potential benefits of the MIP production scheduling model for the purpose of maximizing NPV are significant. Finally, the new constraint was implemented in conjunction with other constraints without violating or limiting a mixed integer programming production schedule's capacity to optimize production from a series of stopes.

Little et al. (2008) and Nehring et al. (2010) in their research presented a classical MIP model that generates optimal production schedules for sublevel stoping operations. They further proposed a new formulation that significantly reduces the number of integer variables, specifically binary variables, and solution times without altering results while maintaining all constraints. The proposed model proved to have a better ability to provide all sublevel stope mining operations with the capacity to quickly produce new schedules in response to updated data and examined a variety of situations. Their work was based on two theories relating to natural sequence and natural commencement. However, they did not recognize stope grades over smaller volumes within each stope hence assumed an average grade for the entire stope. Therefore, in situations where a particular metal quantity must be satisfied, this requirement could violate the constraint where stope extraction occurs over more than one-time period. Also, timing constraints were not formulated to limit stope production to the extent of development. Generally, not all stopes are available over all time periods for production. This is particularly relevant if a great deal of development is required to access a stope because development costs could outweigh the benefits of mining that specified stope in that particular time period. Other researchers like Bley and Terblanche (2012) and King (2016) also worked to reduce the solution times while maximizing the mine's discounted value.

Pourrahimian et al. (2012) presented two Mixed Integer Linear Programming (MILP) formulations for long-term production scheduling of block caving mining operations. They first solved the problem at the draw point level and then aggregated the draw points into larger units referred to as clusters. Their formulations were developed, implemented and verified in the TOMLAB/CPLEX environment. The objective of the production scheduler is to maximize the Net Present Value (NPV) of the mining operation. The models also provided the mine planner the flexibility to have control over the development rate, vertical mining rate, lateral mining rate, mining capacity, maximum number of active draw point and advancement direction.

As an improvement of the previous work by Pourrahimian et al. (2012), Pourrahimian (2013) in his study developed, implemented, and verified a theoretical optimization framework based on a mixed integer linear programming (MILP) model for block cave long-term production scheduling to generate near-optimal life-of-mine production schedules for block cave mining operations. He introduced three MILP formulations for three levels of problem resolution; cluster level, draw point level, and draw point-and-slice level. These formulations could be applied in two ways; i) as a single-step method in which each of the formulations is used independently; ii) as a multi-step method in which the solution of each step is used to reduce the number of variables in the next level and consequently to generate a practical block cave schedule in a reasonable amount of CPU runtime for large-scale problems. His model however was based on deterministic data and hence did consider attribute uncertainties, stochastic variables, ore dilution and other mine planning parameters.

O'sullivan and Newman (2014) employed mathematical optimization technique to determine a production schedule of extraction and subsequent filling of resultant voids for a complex underground mine in Ireland. The goal of the mining operation was to maximize metal production over the life of the mine, subject to constraints on maximum monthly extraction and backfilling quantities, maximum and minimum monthly metal production, and sequencing between extraction and backfilling operations. The authors solved the integer programming model with a heuristic to produce a schedule that adds value to the mining operation by: shifting metal production forward, reducing waste mining and backfilling delays, avoiding expensive mill-halting drops in ore production, and enabling smoother workforce management. The formulated module produced near-optimal production schedules for the mine that is more consistent with managerial goals than the mine's previous manual scheduling method was. Although both the manual and the IP schedules added value to the mining operation. It is reported by Brickey (2015) that the authors utilized aggregation techniques to reduce variables and developed methods for assigning precedence to the various activities; however, the disadvantage of aggregation is that it is limited as a result of the complicated precedence structures.

Brickey (2015) presented a generalized, mathematical formulation that results in a large-scale integer optimization model. His model maximizes NPV and determines the optimal or near-optimal sequence of activities related to the development, extraction and backfilling of an underground mine. Defined constraints in his work include physical precedence and resource capacities. He used data from an existing underground mine. Interestingly, the formulated model had the ability to serve other underground mines with similarly structured data and provides the ability to customize constraints. He included a constraint that treats available mine ventilation as a consumable resource.

Terblanche and Bley (2015) proposed a MIP model which is in relation to similar works by Little et al. (2008); Nehring et al. (2010) and Newman and Kuchta (2007). The rationale behind the approach is that the number of variables in the formulation was reduced by introducing a lower time period resolution while maintaining enough information to enable the optimization model to boost profits through selective mining. In addition, a generic formulation of the mine scheduling optimization problem cast within a resource production framework is introduced, with the purpose of simplifying notation used in formulating underground mine scheduling optimization problems. The presented MIP model is applicable to underground selective mining but not limited to a specific mineral where the ore grade is highly variable and by selecting out only high-grade areas to mine a reduction in cost can be achieved, thus improving profitability.

Ben-Awuah et al. (2016) developed, implemented and verified a MILP formulation and methodology with the objective to maximize the Net Present Value (NPV) of an orebody using different mining options with complex production requirements. The mining options considered were: i) open pit mining; ii) underground mining; and iii) concurrent open pit and underground mining. The MILP optimization framework proved to be robust in providing a global optimization solution when assessing the different mining options. In addition, the framework could also be extended to determine the change-over point between an open pit mining operation and an underground mining operation. The different mining options were evaluated based on the assumption of a high preproduction capital investment with low operating cost. The authors concluded that implementing open pit mining generated a higher NPV than underground mining but considering the investment required for these mining options, underground mining generated a better return on investment than open pit mining. For concurrent open pit and underground mining scenario, the optimizer preferred extracting blocks using open pit mining. Although the underground mine could access ore sooner, the mining cost differential for open pit mining was more than compensated for by the discounting benefits associated with earlier underground mining. They further recommended that an additional study could be done to investigate the mining options including their pre-production capital expenditure requirements.

6. Mathematical Programming Models and Algorithms for Integrated Underground Stope Layout/Limit and Production Schedule Optimization

Available literature as discussed under Sections 3 and 4 of this paper indicates that there has been several optimization techniques that have been developed for stope layouts and progress is being made in the area of production scheduling. However, both areas are still open to improvement.

It is reported by Little et al. (2011) that stope layout methods struggle to produce true optimal results whereas scheduling techniques are hindered by excessive solution times. The available techniques aim to provide local optimal solutions that do not consider simultaneously how other designs or planning areas will affect the optimality of results. Clearly, to date, no method has been proposed, or given consideration, to expanding the scope of underground mine optimization to integrating key areas of long term underground mine design and planning such as to merge stope layouts and production scheduling into one technique. Chadwick (2009) states that, optimization has the ability to add significant value to a mining operation but only if the whole process is considered. However, if concentration is based solely on one part of the process there is a possible risk of transferring cost or reducing revenue elsewhere. Simultaneous optimization would allow for the interaction and influence of one area on another to be assessed and mine plans appropriately improved, in effect, providing more profitable mine plans than currently possible (Little et al., 2011).

Poniewierski et al. (2003) in his investigation into the relationship between stope layouts, production schedule and cut-off grade optimization acknowledges and establishes the complex interaction that exist between the three aforementioned areas. His work describes aspects of the detailed investigation into estimating a practical static cutoff grade for the two main orebodies in Mount Isa Mine's Enterprise Mine, a deep underground copper mine. The approach described by Poniewierski et al. (2003) shows the first step towards achieving a practical application of Lane's theory on optimum cutoff grade in selective underground mining. He iterates in his work that, the key to determining an optimum cutoff grade is the ability to rapidly perform complex mine layout designs combined with rapid output of multiple potential schedules. For the over 200 schedules that were generated for 13 stope layouts, he examined each case using mining, concentrating and smelting net cash flows and the NPV. For each orebody, a cutoff grade was selected based on the cutoff grade which achieved the maximum NPV while allowing for an appropriate production rate. However, due to the limitations of the commercially available software package, the process did not consider all possible designs and schedules together. The process in effect illustrated the connection and complex interaction between stope design and production scheduling and established the need to take into consideration stope design and production scheduling simultaneously if globally optimal results are to be achieved in underground mine planning. However, work completed by Smith and O'Rourke (2005) concludes that using cutoff grades to drive mine design and planning is unnecessary as the optimization process will mine the portion of the deposit which is most economical within the limits of the operation.

As an attempt to address the issue of obtaining globally optimal results by simultaneous optimization of stope layout and production scheduling, Little et al. (2011) and Little (2012) proposed a model that simultaneously optimizes stope layouts and production schedules using integer programming technique mixed integer programming for sublevel stoping operations. They emphasized that MIP techniques are used because it has been shown that they can successfully handle multi-constrained problems while providing a multi-period production schedule that satisfies a required objective. The authors successfully applied the MIP model to 64 443 blocks over a 23-year mine life, which demonstrated the model's ability to produce optimal long term mine plans for a sublevel stoping operation with a \$22.7m maximized NPV. The formulated model works by selecting the best stope sizes, identifying the best stope locations and generating a corresponding optimal long term production schedule simultaneously for a given block model. In solving the optimization problem, the model did not necessarily select the most profitable stope to be produced, it rather resulted to producing different stopes in different configurations, while identifying the best timing for preparation, extraction and backfilling for each stope, which eventually maximizes the mine's NPV. As part of the future works and recommendations, the authors reiterate the need to revise the model formulation to reduce the solution time and memory requirements due to increases in problem sizes. This will make the model more applicable because the reduction in the solution time will make the model more acceptable for practicable mining purposes. They emphasized the need to employing stochastic techniques to geological and economic uncertainties, such as grade and metal prices as these will increase the reliability and success of the mine plan than using deterministic assumptions only. They conclude that poor consideration of these uncertainties recurrently becomes the main reason why many mine sites consistently do not achieve their forecasted performance.

To compare the strength of an integrated stope layout and production schedule optimization with the isolated optimization, Little et al. (2013) developed an Integer Programming (IP) model that allows for either integrated or isolated optimization. He applied both approaches separately to a block model of a gold deposit. Results demonstrate the formulated model's ability to produce optimal long-term sublevel stoping mine plans and the benefits of using an integrated approach. Though the optimization of a part of a mine plan generally tend to yield better results than that of the manual planning methods, the integrated optimization approach provides better operational and financial advantages owing to its ability to assess the interaction and influence between planning areas. It is realized that the isolated approach has a narrow, outlook because its initial focus is solely on the selection of stope layouts without consideration for the impacts on production scheduling. Invariably, this limits the scope in which production scheduling optimization can work due to the various constraints specified, and this ultimately harms the NPV because costs are either transferred or revenues reduced elsewhere in the mine plan. However, the integrated approach, rather takes a holistic look at the operation. This approach predicts and manages issues between the mine planning areas and can accommodate all constraints. As an advantage over the isolated approach, it ensures generating a more considerable and comprehensive mine plan, hence achieving a better NPV. The integrated approach further allows for risks and opportunities to be identified more readily, hence risks can be reduced or opportunities capitalized on more easily in the planning process. Musingwini (2016) highlights the fact that this approach of integrated optimization means a more realistic and sound comparative evaluation of projects within reasonable time frames, facilitating better informed operational and financial decisions is to be expected. The model however did not incorporate development optimization. It only has one development consideration; the location of extraction levels this does not assess if other development requirements, such as shaft or decline construction time or their placement underground, can satisfy the proposed production schedule, or if the proposed stope layouts and schedule works well with development requirements both technically and financially. It is worthy to note that optimizing development along with stope layouts and the production schedule will provide a globally optimal strategic mine plan.

Gangawat (2014) developed an integer programming formulation and solved using CPLEX solver for a single stope. His work is divided into two parts; optimal designing of stopes for open stoping method based on constraints of stope extraction angle and stope height and production scheduling of the generated stopes employing heuristic approaches. The algorithm was solved using part of the zinc mine of India with data containing 4,992 blocks. After successfully solving and eliminating the first stope and respective crown pillar data from the data set, the algorithm was solved again for the second and subsequently a third stope from the remaining data set. In all, a total of 3 stopes were designed. After stope design, production scheduling of the stope blocks was carried out heuristically satisfying extracting angle, mining and processing constraints. However, heuristic approaches are known to give solutions which are just close to the optimal but not an optimal solution though they have the ability to significantly reduce computational time. A flaw in his work, however, was that consideration was not given to the cost of development in the objective function. Considering it could bring the generated NPV close to the real time solution.

7. Stochastic Mine Planning Approaches in Underground Mining

There are two approaches of conducting mine planning, namely; deterministic and stochastic approaches. Deterministic mine planning uses historical data collected from previous mine production activities and assumes that this trend will continue in future processes. Conversely, stochastic mine planning can be described as a premeditated course of action based on data collected from historical mine production figures and also incorporating uncertainty into the modelling processes. Deterministic approaches are known to ignore variations; hence stochastic mine planning is promoted because it incorporates variations considering the fact that mine production activities are erratic in nature and not static. Stochastic mine planning is a complex scheduling process due to its ability to incorporate uncertainties such as geological, technical and economic, inherent in mining operations (Magagula, 2016).

The application of stochastic mine planning in production scheduling has been reported by various researchers in open pit mining taking into consideration grade uncertainty and associated risk (Dimitrakopoulos et al., 2002; Dimitrakopoulos and Ramazan, 2004; Leite and Dimitrakopoulos, 2007), orebody uncertainty, in situ grade and geological uncertainty (Ramazan and Dimitrakopoulos, 2004; Dimitrakopoulos et al., 2007; Dimitrakopoulos and Ramazan, 2008; Godoy and Dimitrakopoulos, 2011; Silva et al., 2015), geological and market uncertainty (Sabour and Dimitrakopoulos, 2011) and NPV (Ramazan and Dimitrakopoulos, 2012) to mention a few.

Conceptual developments in open pit mining have gradually led to the application of stochastic mine planning in underground mine planning notwithstanding the intricate nature of underground mine planning. The uncertainty related to orebody is a critical aspect affecting the forecasted performance of designs and is linked to the failing of meeting production targets and project financial expectations in mine planning (Grieco and Dimitrakopoulos, 2007).

Grieco and Dimatrakopoulos (2007) developed and explored a new probabilistic mixed integer programming model to optimise stope designs, including size, location and number of stopes under consideration of grade uncertainty and predefined levels of acceptable risk. The model was applied on data from Kidd Creek Mine, Ontario, Canada to demonstrate its practicality. The application exhibited aspects including risk quantification for contained ore tonnes, grade and economic potential. The authors reported that unlike any conventional stope optimization approach, the stope designs generated based on the concept of acceptable risk gives the mine planner control over the final stope layout and its potential future performance while considering grade uncertainty. The application of the proposed approach is based on the ability to stochastically simulate equally probable representations of the deposit.

Martinez (2009) gives a classic presentation on why accounting for uncertainty and risk leads to an improvement in decision making so far as mine evaluation is concerned. He introduced a new mine evaluation framework; the Integrated Valuation/Optimization Framework (IVOF). He presented this as an alternative tool for mine project evaluation where uncertainty and risk are incorporated in the evaluation process. His paper highlights two main objectives; i) it shows what problems can arise when single estimated values are substituted for a distribution of values when evaluating a mine project in the face of uncertainty and ii) it shows how the ability to deal with uncertainty and risk in mine project evaluation can have a significant impact on the owners' and stakeholders' investment decision-making. The author asserts that the complexity of mine projects makes it a business that requires constant assessment of risk due the fact that the value of a mine project is typically influenced by many underlying economic and physical uncertainties, such as metal prices, metal grades, costs, schedules, quantities and environmental issues, among others, which are not known with absolute certainty. He identifies the main sources of uncertainty that arise at the commencement of a mine project evaluation to be uncertainty in orebody modelling, uncertainty in metal prices and costs, and uncertainty and risk in mine planning and design.

Dimitrakopoulos and Grieco (2009) adopted risk-based concepts developed in open pit mining to the underground stoping environment and shows examples employing data from Kidd Creek Mine. The example illustrates how conventional technologies cannot quantify risk since they are unable to foresee a significant upside potential and/or downside risk for the conventionally produced designs. The work quantified risk is in terms of the uncertainty a conventional stope design has in expected: contained ore tones, grade and economic potential. They further outlined a new probabilistic mathematical formulation that optimizes the size, location and number of stopes in the presence of grade uncertainty with an additional constraint introduced as the minimum acceptable risk allowed in a design. The model was applied to demonstrate the advantages of a user-defined level of acceptable risk. It is worth mentioning, that the authors recommended that further developments of the work could be to include: i) the formulation of a stope optimization formulation that replaces the probability of grades above cutoff with the direct use of all available simulated orebodies, integrating more geological information; ii) consider sequencing and thus accommodate risk management and/or geological risk discounting as part of the stope design process; and iii) extend to integrate geotechnical uncertainties starting from over-breaking and under-breaking.

Dimitrakopoulos (2011) proposed a risk-based Stochastic Integer Programming (SIP) optimization model which incorporates uncertainties from both the geological and economic factors while minimizing cost. The model integrates two elements: stochastic simulation and stochastic optimization. These elements provide an extended mathematical framework that allows modelling and direct integration of orebody uncertainty to mine design, production planning, and valuation of mining projects and operations. This stochastic framework increases the value of production schedules by 25%. Case studies also show that stochastic optimal designs i) can be about 15% larger in terms of total tonnage when compared to the conventional design, while ii) adding about 10% of NPV comparing to the traditional scheduling using a determined averaging orebody model designs. Results suggest a potential new contribution to the sustainable utilization of natural resources. However, Magagula (2016) reports that the author indicated the difficulties surrounding creating such planning process.

To integrate ore/metal uncertainty into the optimization of mine production scheduling, a Stochastic Integer Programming (SIP) formulation is proposed and tested at a copper deposit by Leite and Dimitrakopoulos (2014). The stochastic solution maximizes the economic value of a project and minimizes deviations from production targets in the presence of ore/metal uncertainty. Unlike the conventional approach, the SIP model accounts and manages risk in ore supply, leading to a mine production schedule with a 29% higher NPV than the schedule obtained from the conventional, industry-standard optimization approach, thus contributing to improving the management and sustainable utilization of mineral resources.

Macneil (2015) uses stochastic mine planning methods to identify the optimal open pit to underground mining transition depth by identifying a series of candidate scenarios where it is feasible to make an OP-UG transition. The author evaluated the economic viability of each member of the set of candidate transition depths by producing uncertainty-based life-of-mine production plans that were used to outline expected yearly cash flows. The benefits of using stochastic mine planning to provide well-informed long-term strategic decision-making criteria are observed. Results of the application of the stochastic approach produced operational schedules with an increase NPV compared to the corresponding deterministic framework. An application of the proposed method at Geita gold mine, a large gold mine in Eastern Africa indicate that future ore production is forecasted to fall well below the mill's capacity, and to supplement this deficiency a transition from open pit to underground mining is being considered. Interestingly, results of the analysis from the proposed stochastic framework reflects that the most profitable decision favored continuing production through solely open pit mining for the foreseeable future. Valuable insights towards the risk associated with the proposed mine design are gained through stochastic risk analysis.

As the need to incorporate multiple components of the mining value chain increased a number of methods were developed over the past decades. Efforts have been made by these new methods to incorporate more decisions and flexibility to the mining optimization of a mining complex, however, they either ignore uncertainties associated with the mining project or consider decisions taken before optimization (Montiel et al., 2016). A method that optimizes mining complexes comprised of multiple open-pits, underground operations and processing destinations was presented by Montiel et al. (2016). The proposed method simultaneously optimizes mining, blending, processing and transportation decision variables while accounting for geological uncertainty. The method employs a simulated annealing algorithm at different decision levels in order to generate a stochastic-based extraction sequence and processing policies. An application based on a case study shows the methods ability to generate a higher NPV while facing a reduced amount of risk when compared to traditional optimization methods.

Malaki (2016) employs the application of grade uncertainty in block cave mining. The author presents a methodology to find the best extraction level and the optimum sequence of extraction for that level under grade uncertainty. The work uses stochastic sequential simulation to address this problem by modelling a set of simulated realizations of the average mineral grade. MILP model was formulated to obtain the maximum NPV given some constraints such as mining capacity, production grade, extraction rate and precedence. Finally, risks associated with grade uncertainty are investigated and analyzed, considerably helping the decision makers in better understanding of various cases and conditions. The author concludes among other things that more uncertain attributes other than grade should be added to the optimization problem. Hence, the MILP model should be extended to take stochastic variables into account during optimization.

A new SIP model incorporating geological uncertainty to optimize long-term scheduling of an underground project extension was introduced by Carpentier et al. (2016). In order to represent a deposit integrating the uncertainty, they stochastically generated a set of simulations and put them into the optimization model. The results show that the schedule generated has a higher expected value when considering and managing grade risk. They also demonstrate the benefits of risk control, allowed by the approach.

MacNeil and Dimitrakopoulos (2017) provide an approach to determine an optimal depth at which a mine should transition from open pit to underground mining, based on managing technical risk. The proposed approach is tested on a gold deposit. This work aims to approve on previous attempts to solve this problem by jointly considering geological uncertainty and describing the optimal transition depth effectively in 3D. Results show the benefits of managing geological uncertainty in long-term strategic decision-making frameworks. The stochastic result produces a 9% increase in NPV over a similar deterministic formulation. The risk-managing stochastic framework also produces operational schedules that reduce a mining project ‘s susceptibility to geological risk. The authors direct future studies to aim at improving the method by considering more aspects of financial uncertainty such as inflation and mining costs. Table 1 presents the summary of current stope layout/limit optimization algorithms and their limitations.

Table 1. Limitations of stope layout/limit optimization models and algorithms.

Classification	Algorithm/Author (s)	Mining method	Dimensional shape	Optimality	Stochastic Consideration	Production Scheduling
Heuristic	Octree Division (1989)	All	3D	No	No	No
	Floating Stope (1995)	All	3D	No	No	No
	Maximum Value Neighbourhood (MVN) (2000)	All	3D	No	No	No
	Multiple Pass Floating Stope Process (2001)	All	3D	No	No	No
	Simulated Annealing (2006)	All	3D	No	No	No
	Generation of Stope shapes for nominated range of Cut-off grades (2009)	Not indicated	Not indicated	No	No	No
	Topal and Sens (2010)	Not indicated	3D	No	No	No
	Sandanayake and Topal (2015)	Sublevel stoping	3D	No	No	No
Rigorous	Dynamic Programming (1977)	Block caving	2D	No	No	No
	Downstream Division (1984)	Cut and fill; Sublevel stoping	2D	No	Yes (Grade uncertainty)	No
	Branch and Bound (MIP) (1995, 1999)	All	1D	Yes	No	No
	Probable Stope (2004)	All	2D	No	No	No
	Grieco and Dimitrakopoulos (MIP) (2007)	Blasthole/Longhole stoping with cemented backfill	Not indicated	No	Yes (Geological uncertainty)	No
	Network Flow (2013)	Sublevel stoping	3D	No	No	No
	OLIPS (2007)	All	2D	Yes	No	No
	GOUMA (2015)	All	2D	Yes	No	No

8. Research Opportunities

It is evident from literature that underground stope layout/limit and production scheduling optimization remains a vital area in the underground mining industry and opportunity for research into improving and/or obtaining global optimality is still open. Most of the research carried out in the area has been towards developing algorithms and models separately for stope layout/limit optimization and for production scheduling optimization. According to Erdogan et al. (2017), Sotoudeh et al. (2017), and Nhleko et al. (2018), various algorithms and models have been developed to solve the problem of underground stope layout/limit optimization. The authors further explain that majority (70%) of these algorithms have gradually managed to define stope boundary in 3D, this used to be the problem with earlier developed algorithms and models due to the fact that mining is a 3D problem. Almost, about 70% of these algorithms are heuristic based and hence true optimality is not guaranteed with these algorithms even in 3D. A majority of the algorithms and models are found to be applicable to all mining methods except the Dynamic Programming by Riddle (1977) and Downstream Geostatistical Algorithm by Deraisme et al. (1984). However, Nhleko et al. (2018) concludes that all these algorithms are based on deterministic orebody models and, therefore, fail to consider the existence of uncertainty in ore deposits. Consequently, there is a need for further research in the field of stope boundary optimization.

Erdogan and Yavuz (2017) mentioned that there is a clear need for improved algorithms and software that guarantee optimal solutions and that the underground optimization techniques should be considered as a whole and techniques should be developed that covers all three areas of optimization: stope boundary optimization, development and production schedule optimization in the future. Currently, the only exception can be mentioned in the case of Little (2012) and Gangawat (2014) who considered a holistic look at simultaneously optimizing stope layout and production scheduling.

In the report by Musingwini (2016), it is mentioned that the idea behind Little's algorithm was to integrate the two and optimize them simultaneously. He highlighted on the fact that the algorithm was developed using the Integer Programming (IP) technique and that previously, most underground optimization techniques focused on optimizing stope layouts and production schedules separately.

The flaw in the attempt by Little (2012) was the inability of the IP model formulation to reduce the solution time and memory requirements due to increase in problem sizes. This would have made the model more applicable because the reduction in the solution time will make the model more acceptable for practicable mining purposes. The model did not also employ stochastic techniques to geological and economic uncertainties, such as grade and metal prices that would help increase the reliability and success of mine plans. The consideration of these uncertainties will help mine sites to achieve their forecasted performance. Finally, the model was developed and tested for sublevel stoping mining method. No mention was made of its applicability on other underground mining methods.

Similarly, the failure on the part of Gangawat (2014) after using an IP formulation to undertake stope design was the use of a heuristic approach to undertake production scheduling of the stope blocks satisfying extracting angle, mining and processing constraints. The flaw in his work was that heuristics are known to give solutions that are just close to the optimal but not an optimal solution though they have the ability to significantly reduce computational time. In addition, he gave no consideration to the cost of development in the objective function which could have brought the generated NPV close to the real time solution. Again, the author did not make mention of the applicability of the model to other underground mining methods in addition to open stoping.

Recently, numerous researchers have explored and advanced industrial knowledge on the use of Mixed Integer Programming (MIP)/Mixed Integer Linear Programming (MILP) for the purpose of generating optimal mine production schedules from defined stope boundaries (Trout, 1995; Kuchta et al., 2004; McIsaac, 2005; Nehring and Topal, 2007; Pourrahimian, 2013). The account by these authors clearly demonstrate the strength of mathematical programming techniques when applied in the area of production schedule and stope sequence optimization compared to manual production scheduling. Scheduling underground mining operations is primarily characterized by discrete decisions to mine blocks of ore, along with complex sequencing relationships between blocks. Since LP models cannot capture the discrete decisions required for scheduling, MIPs are generally the appropriate mathematical programming approach to scheduling. The advantage of using these methods for production scheduling is that they can provide a mathematically provable optimum schedule. These methods are able to approximate some non-linear systems, though not as flexible as simulation.

In the MILP method, any feasible schedule produced has an associated gap that provides a measure of how far the feasible schedule is from its linear relaxation. This gap is based on the difference between the best node and best integer feasible value found within the branch-and-bound search tree. In a typical mine production scheduling problem, the number of integer and continuous decision variables and the number of constraints determine the complexity and solution time. The model decision variables need to assume integer variables to suit the discrete problems in certain conditions. When this restriction as well as the binary restriction is added to the problem, i.e., IP and MIP, the mine production scheduling problems are nicely addressed. Mixed integer programming models are recognized as having significant potential to optimize production scheduling for underground mines. The integer variables can represent entities that cannot be divided. And the binary variables can be used to represent Yes/No decisions such as to extract or not to extract the ore in a given time period (Topal, 2008; Pourrahimian et al., 2012; Little et al., 2013; Ben-Awuah et al., 2016)

It can be deduced from Musingwini (2016) that in order to meaningfully interpret and communicate results for decision-making it is always important to understand the optimization processes in mine planning. This implies: i) developing more robust mine planning through stochastic optimization by considering probability underground excavation design of stopes and development in order to improve confidence in the placement and sizing of excavations; ii) integrating stochastic optimization within the four broad areas in underground mine planning i.e. development layout, sizing stope envelopes, production scheduling, and equipment selection and utilization so that these can be executed simultaneously and enable planning of just-in-time development. Finally, the integrated optimization should guarantee true optimality in 3D space and incorporate uncertainty, thus making a case for integrated 3D stochastic optimization.

9. Rationale behind PhD and Proposed Research Approach

Mine planning should be geared towards producing an optimized plan. An optimized mine plan is expected to be sufficiently robust to ensure that actual outcomes are close or equal to planned targets, provided that variances arising from poor performance are minimal. However, due to the geological, technical, and economic uncertainties inherent in mining operations, this does not always happen in practice; hence the gulf that mine plans tend to be based on deterministic frameworks, while actual mining operations are stochastic in nature. This observation explains the emerging paradigm shift towards stochastic mine planning. The failure to have actual outcomes close to or the same as planned targets is widely acknowledged in the mining industry due to the way the industry models its systems (Magagula, 2016; Musingwini, 2016).

Literature review has shown that a holistic look at underground mine optimization problem in the areas of stope layout/limit and production scheduling and integrating stochastic considerations is imperative. Few attempts have proved unsuccessful. This therefore calls for the need to formulate robust and tactical models capable of solving the problem of global optimality by maximizing NPV while satisfying all available constraints and not affecting the function of other constraints. To solve such a task will require employing Operations Research (OR) techniques capable of solving such complex real-world modern optimization problems. One of such techniques to employ would be an optimization model; a mathematical formulation that is solved using an exact algorithm to produce a single optimal solution through a process of either maximization or minimization. To this effect, Mixed Integer Linear Programming (MILP) with its advantages in the formulation and solving of such complex real-world optimization problems should be employed.

Long-term underground mine production scheduling problems have been studied by researchers including Jawed (1993), Nehring et. al. (2010) and Pourrahimian et al. (2012). These studies have been based on the development of algorithms using operation research methods such as Linear Programming (LP), Mixed integer Programming (MIP) and Mixed Integer Linear Programming (MILP). The current optimization algorithms in the literature lack the ability to deal with real□world large□scale complex underground mine planning problems. They do not have the ability to address i) the broad range of mining methods (exhaustiveness) ii) practical problem sizes, and iii) stochastic parameters.

The research objective will be to investigate the application of algorithms or mathematical programming optimization for strategic underground mine planning in the case of both selective and bulk mining methods. Subsequently, the research will develop efficient algorithms to reduce the size and complexity of the real□world large□scale problems. Finally, the research will investigate techniques to incorporate stochastic variables such as grade and price into the optimization framework. The research will result in optimization models and methodology that can be used to generate a comprehensive stope designs and life of mine plan for a given orebody. The optimization framework will consider the economical mineable sections of the deposit and optimal sequence of extraction to maximize the Net Present Value (NPV). To validate the optimization framework, the research project will be applied to case studies of different orebodies that will be provided during the project. The research will therefore focus on the following tasks:

- Investigate the application of algorithms or mathematical programming optimization for strategic underground mine planning;
- Develop efficient algorithms to reduce the size and complexity of the real□world large□scale problems;
- Investigate techniques to incorporate stochastic variables including grade and price into the optimization framework; and
- Formulate optimization model and methodology that can be used to generate stope designs and a comprehensive life of mine plan for a given orebody.

Appropriately scheduling the long-term production of a typical underground mine is very critical to the overall success of the mine. However, the major problem in long-term production scheduling for underground ore bodies generally relate to the substantial number of variables which makes it too complex to solve (Nehring and Topal, 2007). The numerous constraints and uncertainties especially with the geomechanics of an underground mine makes long-term underground mine production scheduling very challenging. A long-term plan must solve three main problems: i) Investment: determine the selection and timing of investments; ii) Extraction: determine the production in the mine; and iii) Processing: determine the operation of the plants. Solving each one of these problems, even separately, is a complex task. However, the need for a strategic underground production schedule approach cannot be dismissed. For instance, the real value of an investment is appreciated only once its coherence is verified with respect to the extraction and processing decisions (Epstein et al., 2012).

The shortfalls of the available models in literature with underground mine production scheduling include: 1) considered constraints for the models are not exhaustive; 2) models are mostly applicable to only one underground mining method; 3) they cannot deal with practical and complex problem sizes; 4) they cannot handle stochastic variables such as grade and price, and 5) limited total schedule quality in terms of tractability (ease to apply), realism (applying to deployment settings) and generality (applying to many protocols). These main factors would often make real-world underground production scheduling problems very difficult to solve by models in existing literature. In most cases, either the existing models cannot deliver optimal solutions in a reasonable time frame or they do not have the framework to overcome these limitations. Experimental based solutions have therefore been extensively relied upon in managing these challenges in the real-world underground mine settings. These have often affected the overall NPV, equipment and labor utilization, increased ore losses and dilutions, and increased mining costs for the operation. Therefore, finding an optimization framework that is devoid of the above-mentioned shortfalls would be of great benefit in the mining environment. Fig. 3 is a schematic of the proposed research protocol.

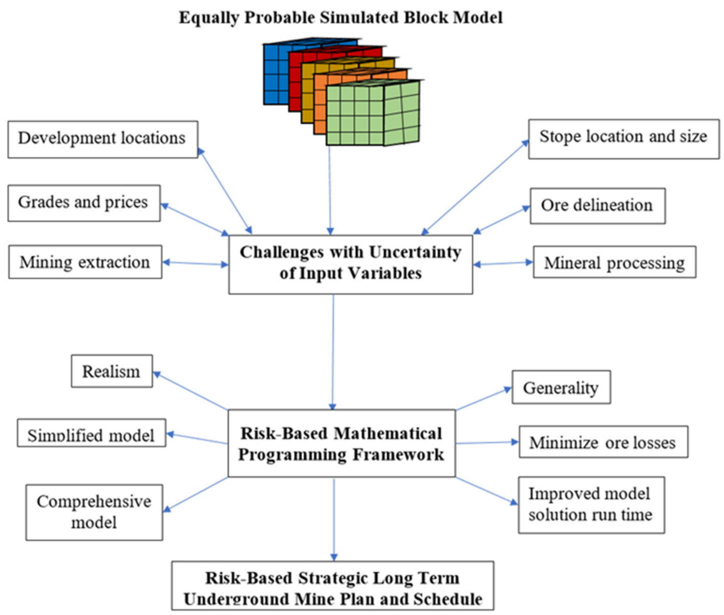

Fig. 3. Schematic diagram of the proposed research protocol.

10. Summary and Conclusions

Discussions so far indicate that underground mine planning optimization work completed to date has focused on solving an individual mine planning area with little consideration for its flow-on effects to other planning areas. It is seen and generally accepted that the optimized solution for the first problem (stope layout/limit) forms the input for the following problem, and so on until a mine plan has been completed. Consequently, the acknowledgement of the importance of an integrated approach towards optimization is growing. Recent studies have shown that decisions made in individual planning areas naturally influence the overall outcome, and focusing on local sub-objectives can be counterproductive to the overall objective (Little et al., 2013).

Current literature shows that optimization in underground mine planning still remains largely fertile for new developments because the direction of mining in underground mines has numerous permutations, depending on the mining method used, making the underground mining optimization problem intrinsically more complex to solve. This complexity explains the isolated piecemeal developments for solving parts of the overall optimization problem in underground mine planning in four key interdependent areas, namely: development layout, sizing stope envelopes, production scheduling, and equipment selection and deployment. Many opportunities therefore, exist for the development of integrated 3D stochastic optimization models for underground mine planning. The failure to obtain actual outcomes close to our targets is widely acknowledged in the mining industry. This is due to the way the industry models its systems (Musingwini, 2016; Nhleko et al., 2018). Sabour and Poulin (2010) and Musingwini (2016) proposed that mining companies ought to develop adaptable mine plans to manage the impact of uncertainties such as commodity price volatility, thus implicitly advocating stochastic mine planning. In effect, mine plans can incorporate uncertainty such as price volatility if a shift is made away from deterministic to stochastic optimization.

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