Application of Mathematical Programming for Stope Layout and Production Schedule Optimization in Sublevel Stoping

Zeinab Basiri and Yashar Pourrahimian
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

The economics of today's mining industry is such that the major mining companies are increasing the use of massive underground mining methods. Follow to this attraction, mine planning in underground mining and its optimization have been considered more seriously in recent years. Two aspects of mine planning optimization in underground mining are stope layout and production scheduling optimization. Few algorithms have been presented to optimize the stope layout and stope production scheduling for underground mining; however, those are not able to provide an optimum solution. Most of the presented algorithms are heuristic so they cannot guarantee to achieve the optimum solution. The objective of this study is to present the mathematical formulations to find the optimum stopes layout and production scheduling in sublevel stoping method. The stopes layout and production scheduling optimization are applied based on the total blocks in the block model altogether (LOT and SOT methods) and based on the separated levels of block model (LOL and SOL methods) to see the impact of leveling in the stopes layout and production scheduling optimization which has not been clarified in the previous researches. The proposed methodologies maximize the economic value (EV) to determine the stopes layout. The presented production scheduling algorithm maximizes the NPV over the life of mine by using the Binary Integer Programming (BIP) while honoring the constraints such as only one-time mining, stopes adjacency, the connection between mining the stopes and activation of levels, concurrent active levels, and the delay between activation of the levels. The methods have been applied to a block model to check the application of the model in real size problems. Achieved EV by LOT method is 4% higher than LOL method; however, the running time of LOT method is 418 times more than LOL method. Also, from the accessibility of production levels to all the stopes during mining point of view, LOL presented the practical stops layout. The NPV of SOT method is 22% higher than the NPV of SOL method; however, the running time of SOT method is 3.4 times more than SOL method.

1 Introduction

The economics of today's mining industry is such that the major mining companies are increasing the use of massive underground mining methods. They expect that approximately fifty percent of the world's copper production will come from underground mines by 2020. It is a step change for the industry, move from the traditional open-pit mining to the underground mining. Because of attraction to the underground mining, the mine planning in underground mining and its optimization have been considered more seriously in recent years. Compare to the open-pit mine, limited techniques and algorithms are available for underground mining because of the complexity and less flexibility due to geotechnical and operational constraints in underground mining. In fact, to model the problems in underground mining, more constraints and variables are required. The number of variables at mix integer programming (MIP) may exceed hundred thousand in the planning of underground mining (Little et al., 2011). Even, this complexity can be higher in sublevel stoping mining method because of some condition such as the alignment of the extraction level, and non-concurrent production (Copland et al., 2016).

Based on Topal (2008) the optimization of three aspects of the underground mine planning is the center of the attention. These aspects are stope layout, production scheduling and infrastructure. The stope layout optimization determines the dimension and the locations of the stopes. Production scheduling is defined as the sequential order of the preparation, extraction and backfilling of the stopes. The goal of stope layout and production scheduling optimization is profit maximization. In fact, stope layout and production scheduling optimization are tools to maximize the profitability of mining over mine life while the operational and geotechnical constraints are met.

The existing algorithms for underground stope optimization are divided into two sets level-based and field-based. Level-based algorithms of stope optimization implement the optimization on the different levels or panels of the block model; however, field-based stope optimization algorithms are applied on the block model before dividing into levels or panels (Sotoudeh et al., 2017).

Since different mining methods obtain the different geotechnical constraints, it is not reasonable to define a general purpose optimization algorithm suited for all underground mining methods (Bai et al., 2012). As a result, in this study in contrast with the majority of previous researches, the optimization of stopes layout and production scheduling specifically in sublevel stoping method, which is suitable for the wide vein-type steeply dipping deposits, is investigated.

The focus of this study is on two aspects of three aspects of optimization in underground mine planning presented by Topal (2008). Create the mathematical optimization models to find the optimum stopes layout and optimum production scheduling in sublevel stoping are the purposes of this study. Besides, the impact of applying the level-based and field-based algorithms is evaluated. In this research, MATLAB (Math Work Inc., 2017) is employed as a numerical modeling platform and IBM ILOG CPLEX Optimization Studio (12.7.1) as a solver to optimize stopes layout.

2 Literature Review

Sublevel stoping which also is referred to as long-hole stoping or blast-hole stoping is a vertical large-scale underground method. This method is a proper underground mining technique for wide vein-type deposits with stable host rock and competent steeply dipping ore-bodies (Haycocks et al., 1992; Lawrence, 1998). After finishing the development of declines, shafts, raises, orepasses and production levels, a raise slot or a winze is operated into one corner of the stope from one sublevel to the next sublevel, and drawpoints and funnels are provided. After completing the infrastructures and constructing the slot, the drilling, blasting and extraction in the stope can be started. Extracted ore in drawpoints is transported to the crusher or the surface. After extracting the ore within the stope, stope is backfilled by a mixture of mill tailings and cement (Hartman, 1992; Haycocks et al., 1992; Nehring, 2011).

The goal of designing stopes in sublevel stoping is achieving the highest profit by defining the best location, size and number of stopes within an ore-body, while the geotechnical stability concerns are met. The stope size depends on the size and shape of the ore-body (Nehring, 2011). The main consideration with planning and scheduling of sublevel stoping method is geotechnical nature of the ore-body such as faults and principal stress directions. Beside the geotechnical nature of the ore-body, other parameters related to the ore-body including shape, continuity and grade distribution of the ore-body are important parameters in designing and scheduling of sublevel stoping method (Mann, 1998).

Optimization techniques for defining the stope layout back to the more than forty years ago. Riddle (1977) presented the first algorithm, called “Dynamic Programming Algorithm” to find optimum stope layout in block-caving mining method. This method solves the 3D problems by using 2D north-south sections and east-west sections. Although the presented algorithm can optimize the sections, it fails to find the optimum stope in 3D because it does not consider all necessary constraints simultaneously.

Deraisme et al. (1984) used the Downstream Geostatistical Approach to determine optimal stope. This model is a 2D sectional numerical model. Generally, the Downstream Geostatistical Approach is based on a combination of conditional simulation with underground mining simulation to compare selectivity, productivity, and profitability in cut-and-fill and block caving methods.

Cheimanoff et al. (1989) described a heuristic approach with binary-tree division technique, called "Octree Division Approach", to move from geological resources to mineable reserves based on the mining constraints and provides a 3D solution to find optimum stope. In fact, this model is based on the removing the non-desired mining blocks to define minimum stope size. Since this algorithm does not control the amount of the waste in the final mine layout, it cannot guarantee to reach the optimum stopes layout.

Ovanic et al. (1995) developed one-dimensional "Branch and Bound Technique" to optimize outline of the stope based on the optimizing of starting and ending points of mining locations within each row of the blocks. In contrast with previous algorithms, having regular or uniform the shapes blocks is not required in their algorithm.

Alford (1996) described a heuristic model called "Floating Stope Algorithm", which is similar to the "Moving Cone" method in open-pit optimization, to set up the optimal stope boundary. This algorithm applies to use in all underground mining method. The main constraint in this algorithm is the geometry of the stope. Inner and outer envelopes are the model's outputs, and the solution is located between these two envelopes which are defined by users. It means finding the solution relays on the users' experience that can be led to the error. This algorithm is the based algorithm on the Datamine software.

Ataee-pour (2000) and Ataee-pour (2005) presented a heuristic algorithm and called it "Maximum Value Neighborhood" (MVN). He defined the neighborhood concept based on the number of mining blocks equivalent to minimum stope size. The MVN algorithm is applied to all underground mining method. MVN algorithm failed to determine the optimal stope layout. However, it guarantees the optimum value neighborhood for each block.

Topal et al. (2010) proposed a heuristic algorithm to find optimum stopes layout in the case of single as well as variable stope sizes in 3D. Their proposed methodology can be used in all underground mining method. Their algorithm works based on two assumptions. Firstly, all stopes have a fixed start-up time, and the production and backfilling time have a linear relation with the stope volume. Secondly, the calculation of NPV is based on the mining of single stope at a given time.

Bai et al. (2012) suggested a new 3D method using "Network Flow Algorithm" to design stopes layout. This model is based on a cylindrical coordinate. They introduced an optimization algorithm that was suitable for sublevel stoping method. Since their algorithm is based on the cylindrical coordinate system with vertical raise, this algorithm is not acceptable in the case of sub-vertical or sub-horizontal deposits that need inclined raise. Furthermore, this approach is based on the small ore-body with single raise parameters, and it is not useful for larger ore-bodies that need many contiguous stopes.

Sandanayake (2014) and Sandanayake et al. (2015a) offered the algorithm to maximizes the economic value regarding the physical and geotechnical constraints. They claim that the algorithm is flexible enough for varying underground mining situations. Sandanayake et al. (2015b) continued their work. The new algorithm considers the fixed and variable stope sizes. They defined an upper bound to limit the number of possible solutions to decrees the solution time.

Nikbin et al. (2018) introduced a Hybrid Algorithm that is a one-dimensional polynomial-time algorithm. This algorithm is a combination of 'Dynamic Programming Algorithm and Greedy algorithm. The algorithm includes three main steps. This algorithm guarantees the optimal solution for a selected row or column of a block model although may fail to reach the optimum solution.

The principal of the work to production scheduling presented by Manchuk (2007) is creating the sequence decisions based on information about the stopes and timing of operations and calculating the probabilities, which dictate the sequence based on this information. He presented the simulated Annealing and a Logic driven algorithm as the optimization techniques. Based on his research, the simulated Annealing technique achieves a better solution in a shorter time.

Nehring (2011) executed the research based on the short and medium term production and activity scheduling for sublevel stoping mining method. His research presented three mathematical optimization models using mixed integer programming to evaluate the relationship between medium and short term scheduling.

Little (2012) presented two approaches to define the stopes layout and production scheduling in sublevel stoping method called integrated approach and isolated approach. In an integrated approach, the optimization of stope layouts and production schedules are done simultaneously. In an isolated approach, the optimum stope layout is defined as the first step, and then the production scheduling is applied.

Copland et al. (2016) proposed a model to maximize the profit from mining the stopes minus the cost of the level development. The model is employed for sublevel stoping underground method. Their primary focus is on using the binary integers programming to help the model in order to decrease the solution time compare to the previously presented production scheduling models.

Generally, the significant limitations of the current stope layout and production scheduling optimization in sublevel stoping are as followed:

- Not any algorithms consider the level-based and field-based optimization with the same data set to be able to compare the results.
- Most of the algorithms are applicable for all mining methods although different mining methods obtain the different geotechnical constraints.
- Only a few numbers of researches are presented the mathematical algorithm with the optimum solution.
- Not all the defined constraints are practical for the case of sublevel stoping method. Also, none of the studies covers all the required constraints for the real situation, for instance, concurrent active levels, and the delay between activation of the levels constraints are ignored in those algorithms.
- Not all the algorithms are able to solve the large-scale problems.

3 Proposed Algorithm Methodology

This study present two methods of stope layout optimization. The first method called LOT method considers total blocks of the block model altogether. The second method called LOL, the block model is divided into the levels, and possible stopes are created for each level independently.

3.1 Layout Optimization Based on Total Blocks (LOT)

The overall process of the proposed algorithm is generated from five main steps (Fig 1). At the first step, based on the blocks' information, economic parameters, and cut-off grade the economic block model is prepared. Block information includes block ID, coordinates or indexes, grade, rock type, the tonnage of each block and economic parameters contains the metal price, cost of selling, mining cost, processing costs, and recovery.

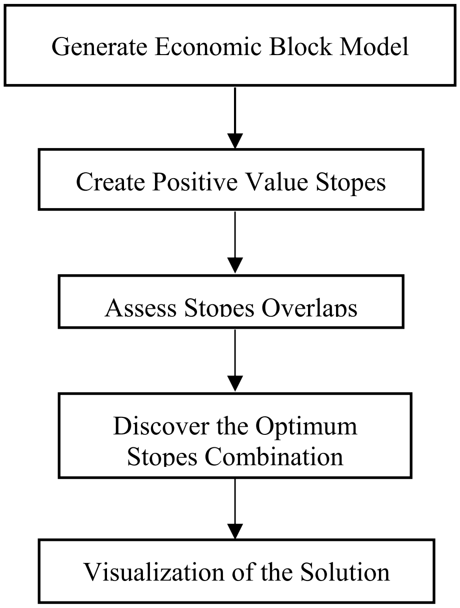

Fig 1. The overall process of the algorithm of layout optimization in LOT method

Next step is creating the positive value which starts by defining the dimension of the stopes. The dimension of the stopes is based on the number of the blocks in three directions X, Y and Z. Then, the stope with this dimension is floated along axes to find all stope possibilities. Then, the economic value of each stope should be calculated, which is the summation of all blocks value in each stope, and stopes with the positive value are separated. Discovering the overlaps between positive possible stopes, which is positive possible stopes with at least one common block is the next step. In reality, the positive stopes combination is not acceptable when there is the overlap between its stopes. After running the optimization model, the best stopes combination with no overlap and the highest economic value is determined. This combination of stopes generates the optimum stopes layout. Finally, the optimum stope layout as the output of the presented algorithm is displayed.

A BIP formulation presents for finding the optimum stopes layout for LOT method. The purpose is to maximize the economic value, while selected stopes in the layout do not have any overlaps. The following assumptions are used in the BIP formulations for optimum stopes layout in LOT method.

- The numerical data such as tonnage, grade, coordinates, and economic value are applied to identify the ore-body attributes in each block.
- In order to create stope, no partial block is considered. In other words, a stope consists of the number of complete blocks.
- The present EV of stopes is considered to find the optimum solution. In fact, the factor of time is not considered.

This problem is formulated as a knapsack problem with conflict graph (see Pferschy et al. (2009)). The required indices, set, decision variable, and parameters to formulate the problem are as follows:

Indices

$s \in \{1,...,S\}$	Index for stopes

Set

Os	Set of all stopes overlaps contains all overlaps for each stope

Decision variable

$x_s \in \{0,1\}$	Binary variable controlling the selection of stope s. It is equal to 1, if the stope s is selected in the stopes combination; otherwise is 0. (Decision variable indicating whether item s is picked in knapsack)

Parameters

EV_s	Economic value of stope s (Utility of items)
Ns	Total number of positive stopes (Capacity of knapsack)
S	Maximum number of positive stopes (Number of items)

Objective function

$$Max\sum_{s=1}^{S} EV_s \times x_s \tag{1}$$

Constraints

$$x_s + x_{s'} \le 1, \qquad \forall (s,s') \in Os \tag{2}$$

$$\sum_{s=1}^{S} x_s \le Ns, \qquad \forall s \in \{1,...,S\} \tag{3}$$

The objective function, equation (1) consists of the stopes EV and a binary decision variable that indicates the selection or not the selection of each stope in the combination. The stope combination with highest EV is the output of this objective function. The stopes overlap constraint, equation ensures that not two stopes, s and s' with overlap can be in the same stope combination. The number of selected stopes constraint, equation (3), presents the maximum allowable number of stopes in the combination that is equal to the total number of positive stopes.

3.2 Layout Optimization Based on Levels (LOL)

Fig 2 shows the seven main steps of the process. First step, preparing the block model is same as LOT method. Next Step is defining the possible levels which are any elevation in Z direction that can be the elevation as the base for creating the stopes. The process of creating positive value stopes and assessing stopes overlap in LOL method is same as creating possible positive stopes in LOT method. The only difference is in the LOL method the process should be repeated for every single possible level. As a result, stopes with positive economic value and their overlaps in all possible levels are generated. Next step is determining the best stopes combination with the highest economic value and without stopes overlap in each possible level. The algorithm needs to be run for all possible levels separately. At the next step, the presented algorithm compares the economic value of possible levels, which is the value of best stopes combination and finds the best set of levels with the highest value among all possible sets. It is obvious that the difference between elevations of levels in each set should be more than the dimension of stope in the Z direction. Based on the optimum levels set, the optimum stope layout generates. In fact, optimum stope layout is the combination of selected stopes in selected levels. The last step is displaying the solution.

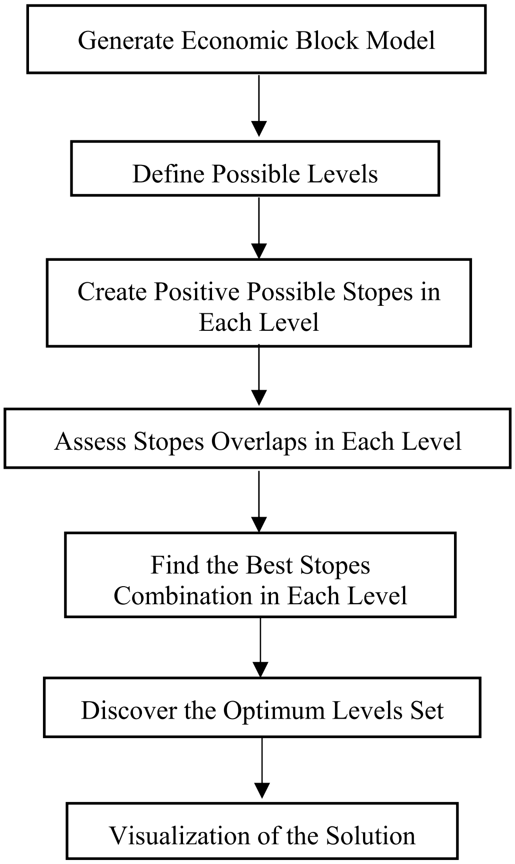

Fig 2. The overall process of the algorithm of layout optimization in LOL method

The formulation of finding the optimum stopes in each level in LOL method is same as LOT method. Nevertheless, the problem is not solved only once. The problem should be formulated for every single level, and the optimum stopes should be found at all levels. To find the optimum levels in LOL method, the knapsack problem with conflict graph is applied as well. Beside the mentioned assumption for LOT method, the following assumptions are applied in the BIP formulations for finding the optimum stope layout in LOL method.

- In order to create a level, no partial stope is considered. In fact, levels must create only at top or bottom of the stopes, not middle.
- A level covers of all possible stopes with the same base elevation, not some of those stopes.

In the LOL method, the objective function of the BIP formulation is to maximize the EV of stopes in each level, and the EV of levels compose the objective function. The required indices, set, decision variable, and parameters to formulate the problem are as followed.

Indices

$l \in \{1,...,L\}$	Index for levels

Set

O_l	Set of all levels overlaps which contain all overlaps for each levels

Decision variable

$x_l \in \{0,1\}$	Binary variable controlling the selection of level l. It is equal to 1, if the levels l are selected in the levels set; otherwise is 0. (Decision variable indicating whether item l is picked in knapsack)

Parameters

EV_l	Economic value of level l (Utility of items)
Nl	Total number of levels (Capacity of knapsack)
L	Maximum number of levels (Number of items)

Objective function

$$Max\sum_{l=1}^{L} EV_l \times x_l \tag{4}$$

Constraints

$$x_l + x_{l'} \le 1, \qquad \forall l \,\&\, l' \in Ol \tag{5}$$

$$\sum_{l=1}^{L} x_l \le Nl, \qquad \forall l \in \{1,...,L\} \tag{6}$$

The objective function, equation (4) consists of the levels EV of levels and a binary decision that indicates the selection or not the selection of each level in the optimum level set. The optimum level set with the highest EV is the output of this objective function. The levels overlap constraint, equation (5), ensures that two levels l and l' with overlap cannot be in the same level set. The number of selected levels, equation (6), presents the maximum allowable number of levels in a set that is equal to the total number of levels.

After defining the optimum stopes layout by LOL method and discovering the best stopes to mine, finding the optimum mining sequence of those stopes during the mine life is a next goal, which is called production scheduling optimization. In this research, two methods are applied to produce the production scheduling, SOT and SOL. SOT considers total selected stopes as a one set and employs the optimization process on those stopes and SOL method applies the optimization process for each selected level separately.

3.3 Production Scheduling Optimization Based on Total Stopes (SOT)

The overall process of the proposed algorithm to implement the production scheduling includes seven steps (Fig 3). At the first step based on the tonnage of stopes in optimum stopes layout, the life of mine is defined. By using Long's rule, the production rate can be calculated and then based on the production rate and tonnage of material, the life of mine is determined. Next step is determining the discounted economic value of selected stops to generate the objective function.

In order to decrease the chance of geotechnical failures, adjacent stopes are not allowed to be mined concurrently during any periods. In SOT method, adjacent stopes are defined as the stopes with the shared boundaries in any coordinate planes. Also, in order to determine practical production scheduling, mining the stopes from different levels should follow the reasonable sequence.

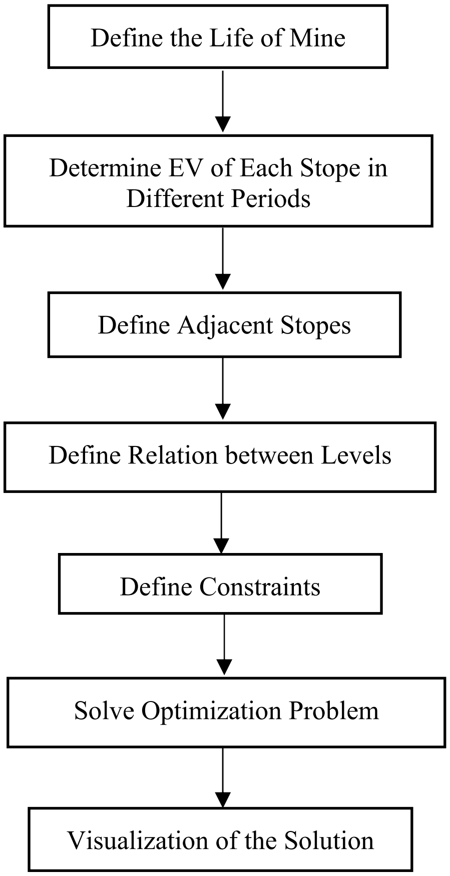

Fig 3. The overall process of the algorithm of production scheduling optimization in SOT method

In addition, the reasonable delay between activation of subsequent levels needs to be defined. Also, the maximum number of active levels at the same time should be defined. Defining other constraints such as mining capacity, grade blending and desirable direction of mining between levels are necessary. After creating the objective function and all the constraints, the optimum stope production schedule with the highest NPV is generated. Finally, the last step is displaying the solution.

The BIP formulation to optimize the production scheduling based on SOT method is presented. The purpose is to maximize NPV while considering mining capacity, grade blending, only one-time mining, stope adjacency, the connection between mining the stopes and activation of levels, concurrent active levels and delay between activation of the levels conditions. The following assumptions are used in the BIP formulations for production scheduling optimization in SOT method.

- No partial stope is considered in production scheduling. In other words, a stope must thoroughly be mined in a period or not be mined at all.
- When the model selects a stope in a period, it includes all the required preparation, extraction and backfilling of that stope during that period. In this research, the combination of these three functions called mining.
- The market fluctuations during mine life are not considered.
- There is no material mixing between stopes and within a stope during mine operation.

The required indices, sets, decision variables, and parameters to formulate the problem are as followed. Since the stopes and levels in this section are the output of stope layout optimization by LOL method and are shown by * ($s*$ and $l*$).

Indices

$s^* \in \{1,...,S^*\}$	Index for selected stopes as the output of LOL and input of SOT
$l^* \in \{1,...,L^*\}$	Index for selected levels as the output of LOL and input of SOT
$t \in \{1,...,T\}$	Index for scheduling period in SOT method

Sets

A_{s^*}	Set of all stopes adjacency which contains all adjacency for every stopes
$B_{l^*}^{s^*}$	Set of the stopes s^* in level l^*

Decision variables

$x_{s^*}^{t} \in \{0,1\}$	Binary variable controlling mining of stope s^* in period t. It is equal to 1, if the stope s* is mined in period t; otherwise is 0.
$y_{l^*}^{t} \in \{0,1\}$	Binary variable controlling the activation of level l^* in period t. It is equal to 1, if the level l^* is active in period t; otherwise is 0.

Parameters

EV_{s^*}	Economic value of stope s^* in SOT method
S^*	Maximum number of stopes
T	Maximum number of scheduling periods (Mine life)
L^*	Maximum number of levels l^*
Ton_{s^*}	Tonnage of stope s^*
Cu_t	Upper bound of mining capacity in period t
Cl_t	Lower bound of mining capacity in period t
G_{s^*}	Average grade of stope s^*
Gu_t	Upper bound of the acceptable average grade in period t
Gl_t	Lower bound of the acceptable average grade in period t
$N_{l^*}^{s^*}$	Number of selected stopes s^* in level l^*
Mcl	Maximum number of concurrent active levels
D	The required delay between activation of levels
R_{t}	Metal processing recovery in period t

Objective function

$$Max\sum_{t=1}^{T}\sum_{s*=1}^{S*}\frac{EV_{s*}}{(1+\mathrm{i})^{\mathrm{t}}}\times x_{s*}^{t} \tag{7}$$

Constraints

$$Cl_t \le \sum_{s*=1}^{S*} Ton_{s*}\times x_{s*}^{t} \le Cu_t, \qquad \forall t\in\{1,...,T\} \tag{8}$$

$$Gl_t \le \sum_{s*=1}^{S*} Ton_{s*}\times x_{s*}^{t}\times G_{s^*}\times \mathrm{R}_{\mathrm{t}} \le Gu_t, \qquad \forall t\in\{1,...,T\} \tag{9}$$

$$\sum_{t=1}^{T} x_{s*}^{t} = 1, \qquad \forall s*\in\{1,...,S*\} \tag{10}$$

$$x_{s*}^{t} + x_{s*'}^{t} \le 1, \qquad \forall s*\,\&\,s*'\in As* \tag{11}$$

$$\sum_{s*\in B_{l*}^{s*}} x_{s*}^{t} \le N_{l}^{s*}\times y_{l*}^{t}, \qquad \forall t\in\{1,...,T\}, l*\in\{1,...,L*\} \tag{12}$$

$$\sum_{l*}^{L*} y_{l*}^{t} \le Mcl, \qquad \forall t\in\{1,...,T\} \tag{13}$$

$$\mathrm{y}_{l*}^{t} \le \sum_{t'=1}^{t-D} y_{l*+1}^{t'}, \qquad \forall t\in\{1,...,T\}, l*\in\{1,...,L*\} \tag{14}$$

The objective function, equation (7) consists of the EV of selected stope, discount rate, and a binary decision that indicates mining or not mining of each stope in each period. The stope with the highest EV is chosen to be part of the production in order to maximize the NPV. The required constraints are presented by equations (8) to (14). Equation (8) represents the mining capacity. The binary variable x_{s*}^{t} controls this constraint. One constraint should be defined for each period. Equation (9) ensures that the production's average grade is in the acceptable range. The binary variable x_{s*}^{t} controls this constraint. One constraint per period is required. Equation (10) shows every single stope s* must be mined only once. The binary variable x_{s*}^{t} controls this constraint. One constraint should be defined for each stope. Equation (11) is related to the adjacent stopes. This constraint is controlled by the binary variable x_{s*}^{t}. This constraint indicates stopes $s*$ and $s*'$ must not be mined at the same period, if $s*$ and $s*'$ belong to A_{s*}. Equation (12) presents the connection between mining the stopes and activation of levels. This constraint is controlled by binary variables x_{s*}^{t} and y_{l*}^{t}. x_{s*}^{t} controls stopes that belong to level $l*$ and y_{l*}^{t} controls the levels. One constraint should be defined for each level in each period. Equation (13) is defined for concurrent active levels. This constraint is controlled by the binary variable y_{l*}^{t}. One constraint should be defined for each period. Equation (14) keeps the desired delay between the activation of levels. The binary variable y_{l*}^{t} controls this constraint. One constraint should be defined for each level in each period.

3.4 Production Scheduling Optimization Based on Levels (SOL)

As Fig 4 indicates, the production scheduling optimization in SOL method includes seven steps.

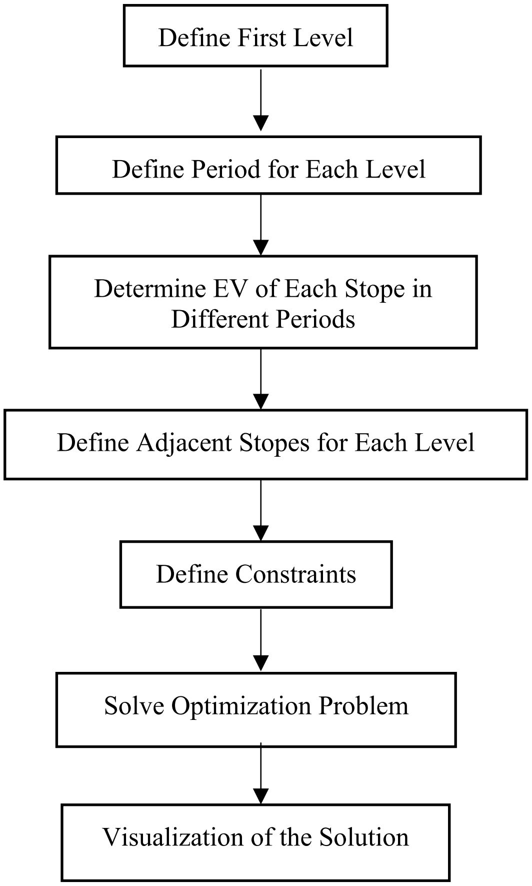

Fig 4. The overall process of the algorithm of production scheduling in SOL method

Since starting production scheduling optimization in a level occurs after finishing schedule optimization at the previous level, selecting the first level is critical. In order to choose the first level, a desired direction of mining must be known. Based on the information of the selected levels and selected stopes by LOL method, the required mining period is defined for each level. In SOL method since all stopes are in the same level with the same Z coordinate, adjacent stopes are defined as the stopes with the shared boundaries in X-Y coordinate planes. Mining capacity, grade blending and precedence between levels also are considered as the constraints. After considering all conditions, the optimum stope production scheduling with the highest NPV is created for each level separately. The last step is displaying the result of optimum stope production scheduling in SOL method. Comparison of the results of SOT and SOL methods is the last step in production scheduling optimization.

The BIP formulation for optimizing the production scheduling for SOL method is presented. The purpose is to maximize NPV of mining stopes in each level separately while considering mining capacity, grade blending, only one-time mining, stope adjacency. In addition to assumptions in the SOT method, the following assumptions are used in the BIP formulations for production scheduling optimization in SOL method.

- Only based on the proportion of material tonnage in each level out of total material tonnage, the proportion of mine life is assigned to each level and other factors are not considered.

- Production scheduling is done at each level separately, and the connection between levels is not considered.
- Mining capacity and required average grade in each period in SOL method are assumed same as SOT method other factors are not considered.

The required indices, sets, decision variables, and parameters to formulate the problem are as follows.

Indices

$s_{l*}^{*} \in \{1,...,S_{l*}^{*}\}$	Index for selected stopes $s*$ in selected level $l*$ as the output of LOL method
$t_{l*} \in \{1,...,T_{l*}\}$	Index for scheduling period in level $l*$ in SOL method

Set

A_{l*}^{s*}	Set of all stopes adjacency which contains all adjacency for every stopes

Decision variable

$x_{s*,l*}^{t_{l*}} \in \{0,1\}$	Binary variable controlling mining of stope $s*$ of level $l*$ in period t_{l*} in SOL method. It is equal to 1, if the stope s*is mined in period t; otherwise is 0.

Parameters

EV_{l*}^{s*}	Economic value of stope s* of level l* in SOL method
S_{l*}^{*}	Maximum number of stopes in level $l*$
T_{l*}	Maximum number of scheduling periods for level $l*$
Ton_{l*}^{s*}	Tonnage of stope $s*$ in level $l*$
$Cu_{l*}^{t_{l*}}$	Upper bound of mining capacity in level $l*$ in period t_{l*}
$Cl_{l*}^{t_{l*}}$	Lower bound of mining capacity in level $l*$ in period t_{l*}
G_{l*}^{s*}	Average grade of stope $s*$ in level $l*$
$Gu_{l*}^{t_{l*}}$	Upper bound of the acceptable average grade in level $l*$ in period t_{l*}
$Gl_{l*}^{t_{l*}}$	Lower bound of the acceptable average grade in level $l*$ in period t_{l*}
R_{l*}^{t}	Metal processing recovery in period t_{l*}

Objective function

$$Max\sum_{t_{l*}=1}^{T_{l*}}\sum_{s_{l*}^{*}=1}^{S_{l*}^{*}}\frac{EV_{l*}^{s*}}{(1+i)^{t_{l*}}}\times x_{s*,l*}^{t_{l*}} \tag{15}$$

Constraints

$$Cl_{l*}^{t_{l*}} \le \sum_{s_{l*}^{*}=1}^{S_{l*}^{*}} Ton_{l*}^{s*} \times x_{s*,l*}^{t_{l*}} \le Cu_{l*}^{t_{l*}}, \qquad \forall t_{l*} \in \{1,...,T_{l*}\} \tag{16}$$

$$Gl_{l*}^{t_{l*}} \le \sum_{s_{l*}^{s*}=1}^{S_{l*}^{*}} Ton_{l*}^{s*} \times x_{s*,1*}^{t_{l*}} \times G_{l*}^{s*} \times R_{l*}^{t_{l*}} \le Gu_{l*}^{t_{l*}}, \qquad \forall t_{l*} \in \{1,...,T_{l*}\} \tag{17}$$

$$\sum_{t_{l*}=1}^{T_{l*}} x_{s*,1*}^{t_{l*}} = 1, \qquad \forall s_{l*}^{*} \in \{1,...,\mathrm{S}_{l*}^{*}\} \tag{18}$$

$$x_{s*,l*}^{t_{l*}} + x_{s'*,l*}^{t_{l*}} \le 1, \qquad \forall s_{l*}^{*} \;\&\; s_{l*}^{*'} \in A_{l*}^{s*} \tag{19}$$

The objective function, equation (15) consists of the economic value of selected stopes in selected levels, discount rate, and a binary decision that indicates mining or not mining of each stope of each level in each period. The constraints are presented by equations (16) to (19). Equation (16) represents the mining capacity. This constraint is controlled by the binary variable $x_{s*,l*}^{t_{l*}}$. One constraint should be defined for each period in a level. Equation (17) ensures that the production's average grade in each level is in the acceptable range. This constraint is controlled by the binary variable $x_{s*,l*}^{t_{l*}}$. One constraint per period in a level is required. Equation (18) indicates every single stope s_{l*}^{*} in each level must be mined only once. The binary variable $x_{s*,l*}^{t_{l*}}$ controls this constraint. One constraint should be defined for each stope in a level. Equation (19) is related to the adjacent stopes. This constraint is controlled by the binary variable $x_{s*,l*}^{t_{l*}}$. This constraint indicates stopes s_{l*}^{*} and $s_{l*}^{'*}$ must not be mined at the same period, if s_{l*}^{*} and $s_{l*}^{'*}$ belong to A_{l*}^{s*}.

In this research, MATLAB (Math Work Inc., 2017) is employed as a numerical modeling platform and IBM ILOG CPLEX Optimization Studio (12.7.1) as a solver to optimize stopes layout. MATLAB is a high-level language to analyze data, develop algorithms and models. CPLEX solver is a tool to solve a large-scale mixed-integer linear and quadratic programming (Pourrahimian, 2013).

4 Implementation of the Algorithm

In this section, the mathematical formulations will be implemented on a drillhole data set in order to demonstrate how the presented methodologies work. The block model belongs to a silver (AG) deposit. Table 1 indicates the block model information. The total tonnage of ore-body in the block model is 37.9 (Mt). Fig 5 shows different range of grade of silver. Also, Fig 6 indicates the grade distribution of ore-body.

Table 1. Block model information

Number of blocks	48,362
Blocks size (m3)	10×10×10
Blocks tonnage (tonne)	2700
Blocks grade (g/tonne)	0 - 1580
X Coordinate (X index)	1-60
Y Coordinate (Y index)	1-13
Z Coordinate (Z index)	1-62

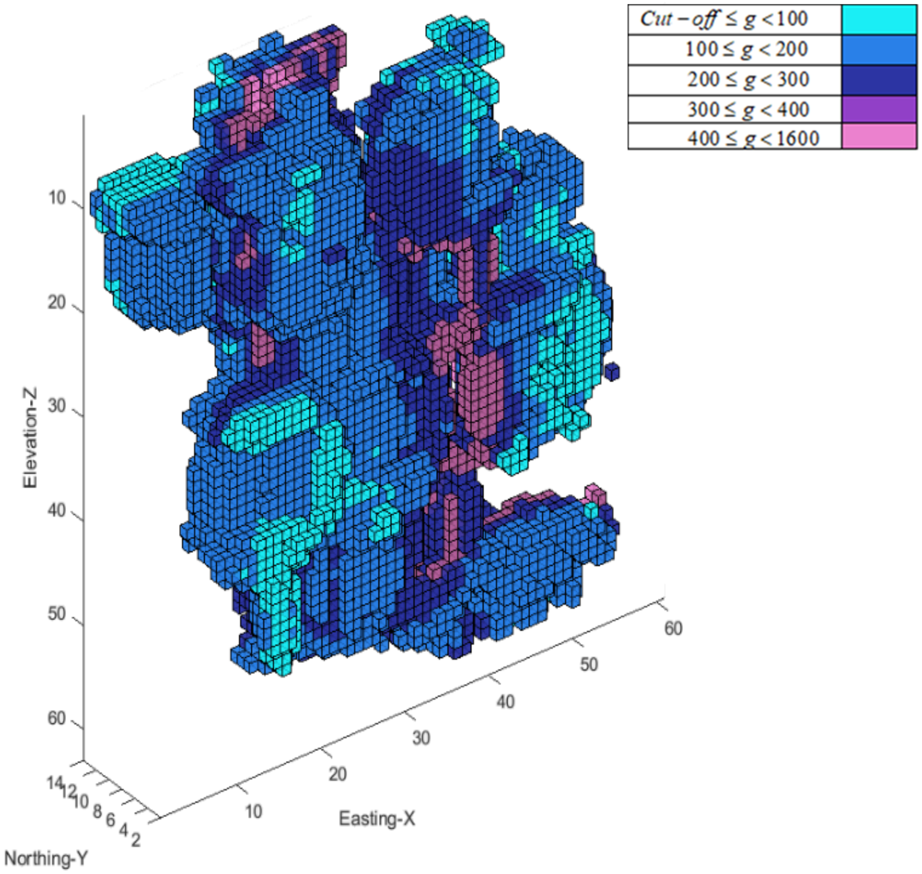

Fig 5. Grade of Ag (g/ton) in the ore blocks

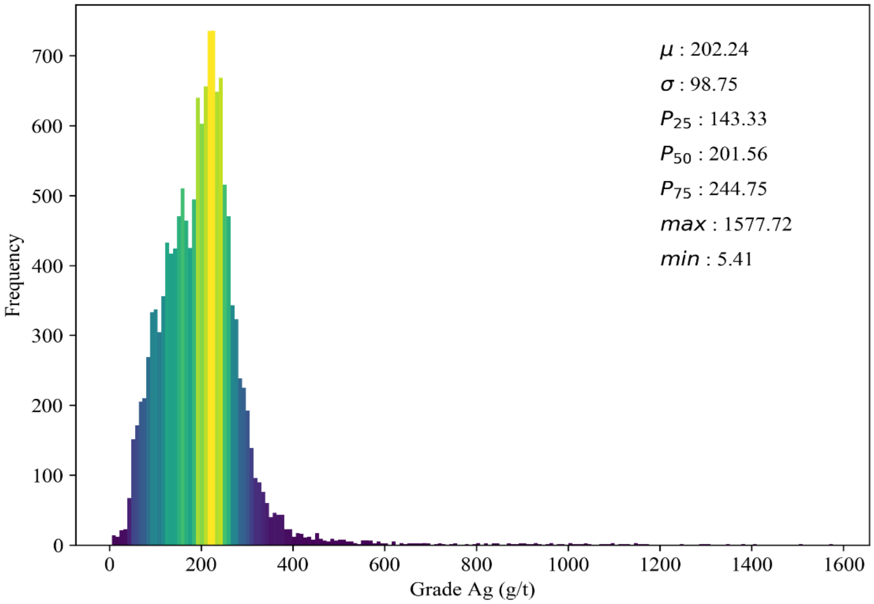

Fig 6. Grade distribution of the ore-body

4.1 Layout Optimization Based on Total Blocks (LOT)

The economic value (EV) of the blocks was calculated based on the parameters listed in Table 2. After calculating the cut-off grade, waste or ore blocks were determined and the BEV was calculated. The calculated cut-off grade for this block model was 66.7 (g/t), and the block values were between $ -0.64M and $ +2,20M.

Table 2. Economic parameters

Metal price ($/g)	0.6
Cost of mining ($/t)	24
Cost of processing ($/t)	12
Recovery	90%

According to (Hartman, 1992; Haycocks et al., 1992), stope width at least must be 6 meters, and the range for length and height of the stopes is between 45 and 120 meters. In order to have a practical condition, the stope dimension 40×40×120 cubic meters (4×4×12 blocks) was chosen in this study. However, there is the possibility of changing this dimension and scanning the impacts of changing. Creating the stopes and separating the positive ones were the following steps in this section. In LOT method, 29,070 initial stopes were generated which 13,403 of those stopes had positive economic value. By running the LOT algorithm and removing all overlaps, the combination of 85 stopes was discovered. In fact, this combination of stopes was the optimum stope layout. The EV of stopes in this layout was $ 2,341.1M. Also, the total solution time was 62(hr):02(min):07(sec). Fig 7 illustrates the stopes in the result of the LOT method. The selected stopes in the optimum stopes layout are demonstrated with the different colors.

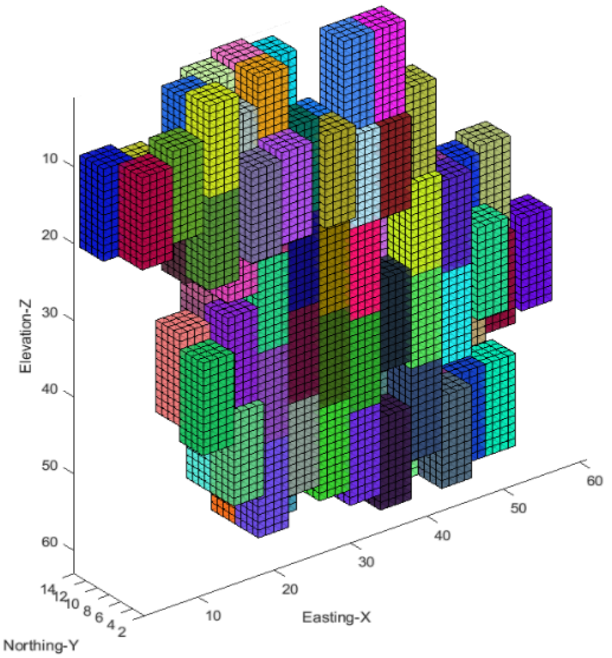

Fig 7. Optimum stopes layout based on the LOT method

4.2 Layout Optimization Based on Levels (LOL)

In LOL method, the primary block model was divided into a number of possible levels that was the base elevation for creating the stope. In the case study, the number of blocks in direction Z is 62 and dimension of stope in the Z direction is 12, so number of possible levels is equal to 51. For all 51 possible levels, the process of creating stopes was applied, and initial and positive stopes were defined in each level. For instance, in the first level, 597 initial stopes were determined which 165 of those have positive EV. In total, 30, 447 initial stopes and 13,403 positive stopes were generated in LOL method. A set of possible levels 2 (Z=13 of block model), 14 (Z=25 of block model), 26 (Z=37 of block model), 38 (Z=49 of block model) and 50 (Z=61 of block model) was determined as the optimum set with the highest EV. Table 3 demonstrates a number of initial stopes, positive stopes and the selected stopes in each selected level. In addition, the EV of the selected levels are indicated in this table. Generally, 5 levels including 93 stopes and total value of $ 2,252.2M were the result of LOL method. The LOL method reached the answer in 00:08(min):54(sec). Fig 8 indicates the stopes layout based the result of the LOL method.

Table 3. Information of selected levels

Level ID	Possible level ID	Initial stopes (#)	Positive stopes (#)	Selected stopes (#)	Value (M$)
Level 1	2	597	202	12	263.6
Level 2	14	597	283	20	434.8
Level 3	26	597	265	21	599.6
Level 4	38	597	294	22	630.1
Level 5	50	597	232	18	324.1

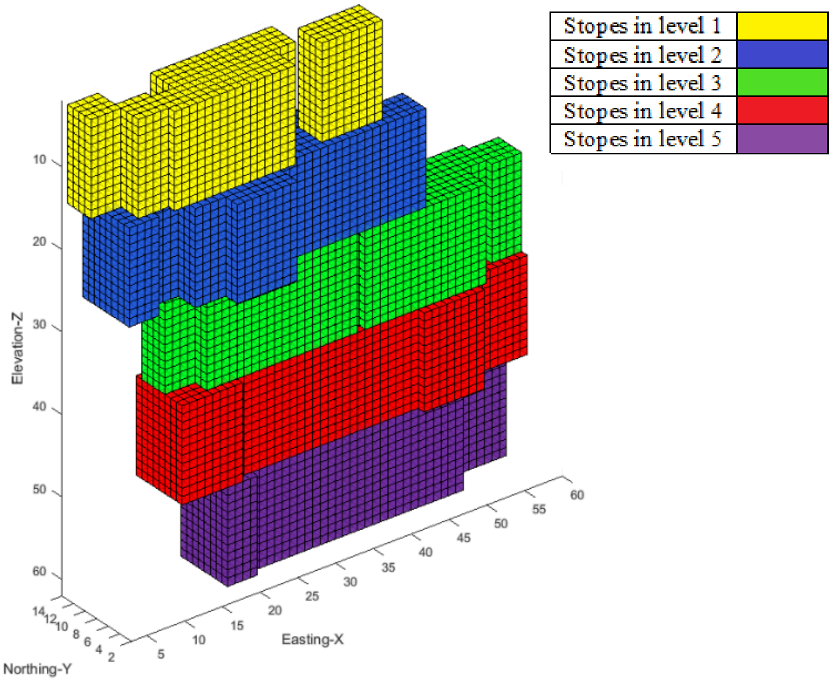

Fig 8. Optimum stopes layout based on the LOL method

Number of selected stopes in the optimum layout defined by LOL method, is eight stopes more than LOT method and total mined tonnage in LOL method is 8.5% higher than LOT method. The achieved value by LOT method is 4% more than that value for LOL method. However, there is a world of difference between the running times the methods. As in LOT method all the stopes are evaluated at one set, some steps of the algorithm including assessing stopes overlaps, creating the overlap constraint and running optimization model are time-consuming processes. Nevertheless, in LOL method the stopes divided to the smaller sets, levels, so the processing time drops significantly. As indicated in Fig 8, in LOL method, selected stopes in the optimum stopes layout were in the same levels. As a result, this method achieved to the practical solution. However, in

LOT method Fig 7, selected stopes in the optimum stopes layout were in different elevations so it is not possible to design production levels to have access to all the stopes. Therefore, the achieved solution by LOT method was not practical.

4.3 Production Scheduling Optimization Based on Total Stopes (SOT)

The total tonnage of 93 stopes was 48.21Mton. The life of mine was 22 years. A discount rate of 10% was used in this case study for production scheduling. Table 4 provides other required parameters for production scheduling based on the SOT method.

The achieved NPV by running SOT method was $ 1,288.1M. Running time for SOT method was 00:46(min):14(sec) and the optimality gap is zero percent. Fig 9 illustrates the sequence of mining the stopes based on SOT method. Different colors show the different periods.

Table 4. Production scheduling parameters in SOT method

Life of mine (year)	22
Discount rate (%)	10
Minimum mining capacity (Mt)	1.55
Maximum mining capacity (Mt)	3.1
Minimum average grade (g/t)	55
Maximum average grade (g/t)	220
Delay between activation (period)	2
Maximum number of active levels at a period	3
Direction of mining the levels	Upward

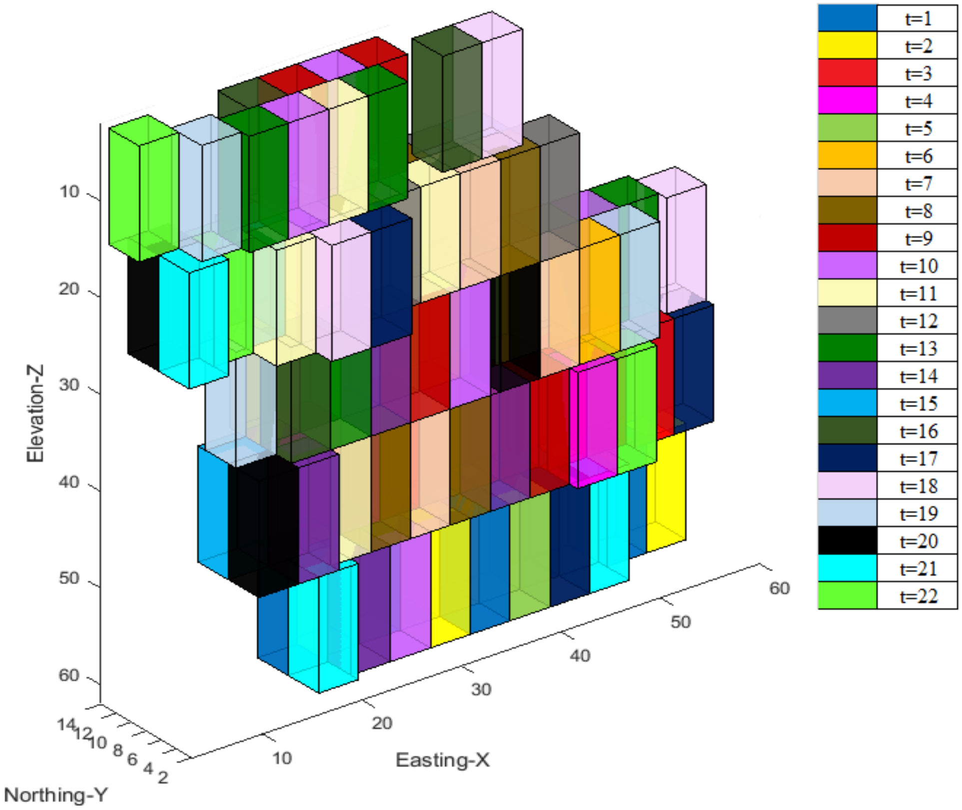

Fig 9. Optimum production scheduling based on the SOT method

4.4 Production Scheduling Optimization Based on Levels (SOL)

22 years was considered as life of the mine. By distributing the mine life to levels based on the total tonnage of stopes in each level, mining period for each level was determined. Table 5 shows the number of stopes in each level, total tonnage of each level and the assigned period for each level.

Table 6 provides the required parameters for production scheduling based on the SOL method. The total obtained NPV and the running time for the SOL method with the optimality gap of zero percent were $ 1,052.5 M and 00:13(min):34(second), respectively. Table 7 indicates the NPV achieved by applying the formulation at each level. According to this table, Level 4 has the highest NPN, and Level 1 has the lowest NPV. Since Level 5 contains less number of stopes than Level 4, Level 4 reaches to the highest NPV. Fig 10 displays the solution in levels 1, 2, 3, 4 and 5 respectively. Different colors show the different mining periods.

Table 5. The assigned period for each level

Level ID	Number of Stope	Average grade of the level (g/t)	Tonnage (Mt)	Period
Level 1	12	138.3	6.2	3
Level 2	20	137.8	10.4	5
Level 3	21	163	10.9	5
Level 4	22	162.9	11.4	5
Level 5	18	120.7	9.3	4
Total	93	146.2	48.2	22

Table 6. Production scheduling parameters in SOL method

Life of mine (year)	22
Discount rate (%)	10
Minimum mining capacity (Mt)	1.55
Maximum mining capacity (Mt)	3.1
Minimum average grade (g/t)	55
Maximum average grade (g/t)	220
Direction of mining the levels	Upward

Table 7. Maximum achieved value of each level in SOL method

Level ID	NPV (M$)
Level 1	39.7
Level 2	102.2
Level 3	230.9
Level 4	388.7
Level 5	291.0
NPV	1,052.5

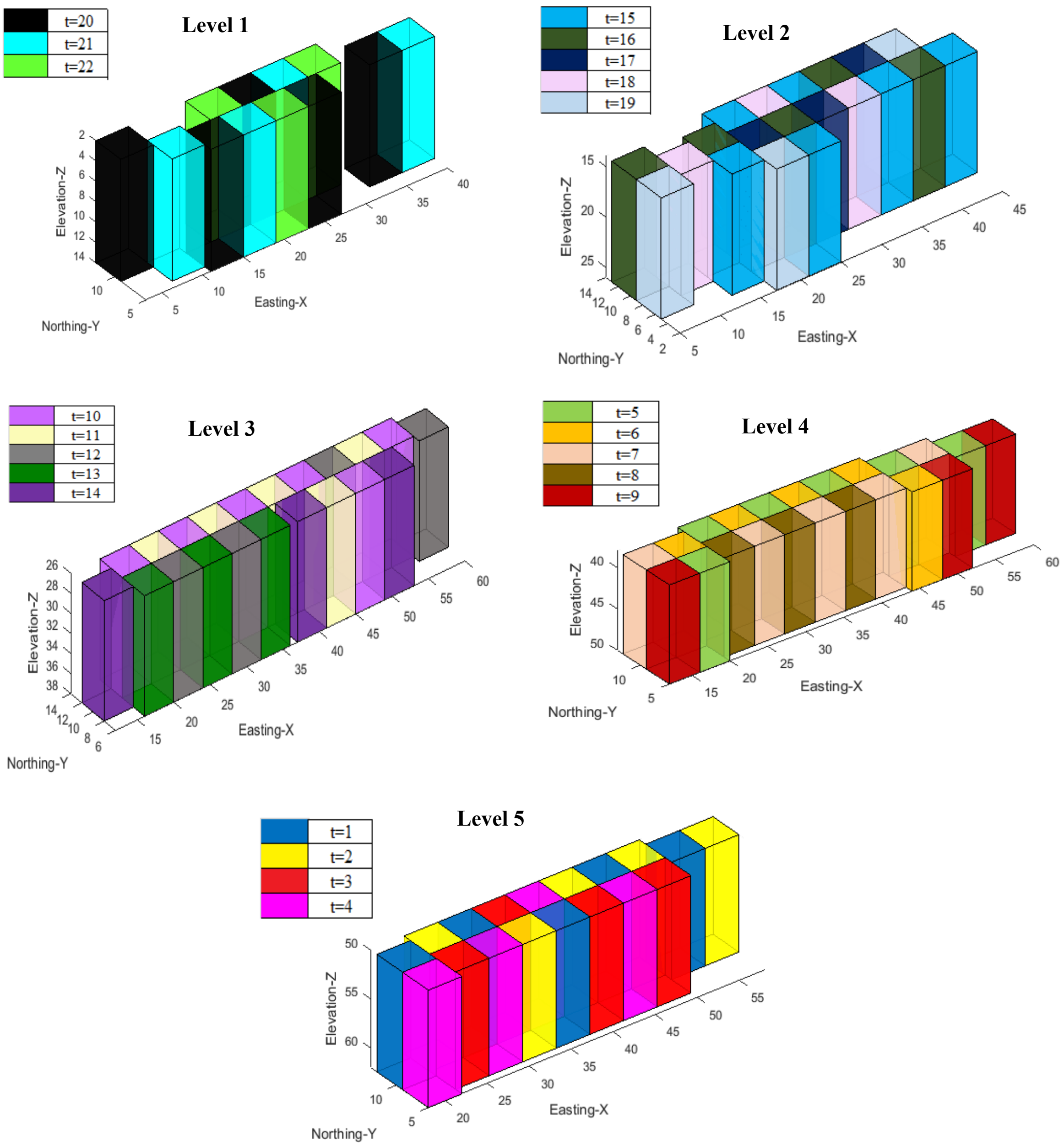

Fig 10. Optimum production scheduling in all levels based on the SOL method

The achieved NPV by SOT method is 22% more than what is got by SOL method. However, the running time of SOT method is 241% more than SOL method. Since in SOL method, stopes are divided to the few sets, levels, and then the optimization algorithm is applied for each set separately, the running time of SOT method is higher than the running time of SOL method.

5 Summary and Conclusion

The presented methods are able to find the optimum stopes layout in sublevel stoping based on the total blocks and based on the levels while considering the constraints. Also, the proposed methodologies are verified regarding both feasibility and optimality on a case study and the optimality gap for all four methods are zero percent. According to the results, for stope layout optimization, it is reasonable to use the leveling to save the time although the achieved economic value is a little bit lower. Also, from a practicality point of view, LOL reaches the better solution.

However, for production scheduling, the model does not show a specific advantage of using leveling method. Fig 11 indicates the comparisons between achieved NPVs and running times. One of the assumptions in this study was to create stope, no partial block and to create a level, no partial stope were considered and considering those for future studies is recommended. Also, this study presented the deterministic model, so it was not able to capture the uncertainty. It is good to consider this matter in future.

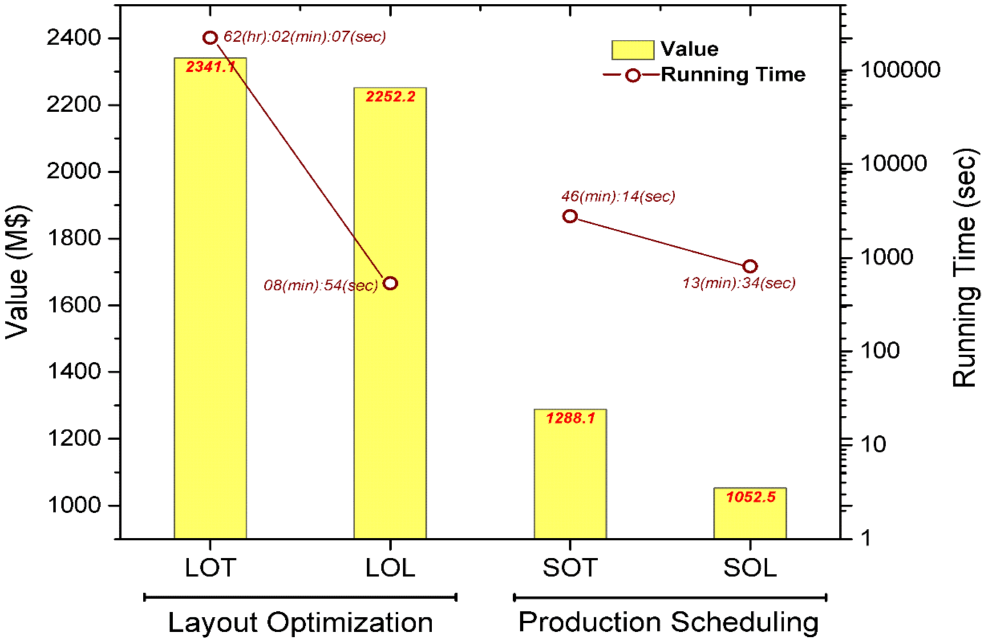

Fig 11. Summary of results of all presented method

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