

Determination of the Optimum Undercut Level Elevation in Block Caving

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ABSTRACT

The selection of an elevation for the placement of the undercut level is a decisive initial step in the planning and design of block caving mines. It is key for the success of a caving project to define the undercut elevation that will most likely yield the highest possible Net Present Value (NPV) of the operation, considering the discounting periods from both the vertical draw rate extraction of the ore and the horizontal mining direction advancement.

This paper outlines a mathematical programming framework to determine the best undercut elevation, by formulating a simplified Linear Integer Programming (IP) model that captures the discounting of the profits from the vertical draw rate extraction and the horizontal advance direction that can be applied at the early stages of a caving project. The IP model comprises a simplified block caving scheduling algorithm, that considers "Mining Units" (MU), which are groupings of blocks within each column based on the minimum draw rate condition and the undercut elevation considered. The formulation considers the following constraints: mining capacity, minimum and maximum draw rate, vertical precedence within columns and a horizontal precedence based on a concave mining advancement front. The model is set on an iterative loop over the different possible levels.

The model is tested on a case study, where it provides a tool to evaluate the potential mineable reserves, define the optimal undercut elevation, and a starting schedule for future detailed engineering design.

1. Introduction

One of the initial steps in the long-term planning of Block Caving mines is the selection of an undercut elevation and the definition of a mining footprint, after which a draw point layout is built, and the production schedule is generated (Diering, 2000; Rubio, 2002). The selection of a specific undercut elevation constraints the downstream planning process having a direct influence in the potential NPV of the project and can be thought as an optimization problem on its own.

The current industry standard for the selection of an undercut elevation is the Footprint Finder module in GEMS PCBC software. As explained by (Diering et al., 2008), Footprint Finder evaluates the different levels considering each block within a specific level as a draw point from which the discounted economic value of the column above is calculated, with the capability of applying Laubscher's vertical mixing model (Laubscher, 1994) as well. The level economic value is then

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obtained as the sum of each column within. A method described by (Elkington, 2012) finds the optimal cave outline for a given undercut elevation that maximizes the metal content, by following an approach similar to that of pit outline optimization, using an Integer Programming (IP) method considering operational constraints. This method works at a block level scale and provides an approximation to the potential final cave shape for a given elevation, however it does not involve economic parameters. Another proposed method by (Vargas et al., 2014) considers the caving outline definition as an inversed ultimate pit problem, and for each level calculates the vertically discounted economic value for the caving envelope considering precedence constraints on the inverted block model.

The economic value of the extracted ore is not only subject to vertical discounting on each individual column, as each column is opened at a certain period based on the mining direction and the starting point in the footprint. The starting point and direction of mining are also usually a geotechnical decision, as it is desirable to start extraction in weaker rocks to achieve the hydraulic radius required and steady production rates early in the mine life (Bartlett, 1992; Tukker et al., 2016). Therefore, the horizontal discounting due to the undercut layout sequence has to be considered in order to evaluate and select the best possible undercut elevation. The sequencing and scheduling problem in Block Caving is a complex optimization problem that involves millions of possible combinations for a given set of draw points on a specific undercut level (Khodayari and Pourrahimian, 2015). While the use of mathematical programming techniques can guarantee an optimal solution for Block Caving sequencing and scheduling (Chanda, 1990; Rahal et al., 2008; Pourrahimian et al., 2012; Pourrahimian et al., 2013; Pourrahimian and Askari-Nasab, 2014), it is often at the expense of large and prohibiting processing times.

The following method provides a simplified scheduling Integer Programming (IP) formulation applicable at the block model scale that can be repeated over multiple levels at reasonable processing times, to find the optimal undercut elevation for Block Caving mines considering time discounting over the vertical direction within each column, and the horizontal advancement direction.

Two assumptions are made to simplify the schedule optimization problem that are in line with block caving operational practices. The first assumption is that instead of considering each individual block as a single draw point, a grouping of individual adjacent blocks is performed to represent a draw column or draw zone. Within each draw zone, the individual blocks are then vertically aggregated into representative mining units based on the minimum draw rate required. These mining units are then scheduled based on a concave horizontal mining advancement front, that is desirable to provide a better control of structures and a more secure undercutting (Brown, 2002), and mining capacities constraints to evaluate the economic potential of each undercut level. These assumptions greatly reduce the number of variables and possible combinations while still being representative of the extraction practice in Block Caving operations and provide some useful insight for the planning process at early stages of the project.

2. Methodology

The first is to generate the projected draw column areas on the undercut footprint by grouping the individual block model columns, to generate a more representative scenario of the ore extraction in Block Caving mines where the extraction is carried from draw columns over draw points at the undercut level. The representative columns are selected in order to account for the potential draw point layout geometry configurations, such as the different spacing measures between the draw points and the minimum required hydraulic radius to sustain mining (Malaki et al., 2017), that are selected on a case-by-case basis depending on the properties of the rock mass. Figure 1 show a schematic representation of the aggregation of multiple individual block-model columns into a single representative draw column. This zone is parametrized by two dimensions, a size in X and a size in Y to define a rectangular area of influence to use to the block model information for calculation of

tonnages and metal content within it. The dimensions of this representative column can be adjusted to match the required spacing between draw points, A and B in Figure 1, but is constrained by the dimensions of the underlying block model.

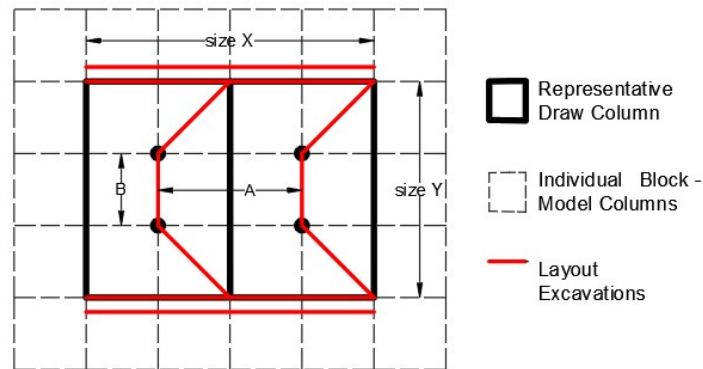


Fig. 1. Schematic of the draw column aggregation from the individual block-model columns.

The grouping of the columns into the representative units for a given undercut is then an optimization problem on its own, and its solution can provide useful insight on the draw point layout design, and a systematic way to evaluate different levels automatically. To optimize the layout, the key parameter considered is to maximize the total metal content of the units. The metal content was selected to not include the economic parameters yet, but still guarantee a priority on the selection of high grade areas. The metal content of each unit is calculated as the summation of the metal contents of each individual block model columns that are part of it. Each column is considered as continuous and the metal summation is carried until a first block of waste is found or a specific maximum column height is reached, to avoid waste material within the cave profile and to consider the operational maximum draw height.

Based on the representative unit dimensions in X and Y, all possible units are generated on a given undercut elevation plan view and the associated total metal content for each representative column is calculated as described above.

The optimization problem is then to find the combination of column units that yield the highest metal content from the deposit. The optimal column units must be adjacent and not include units with individual columns that do not meet a minimum height operational constraint. To achieve this, all column units that do not meet the operational criteria are assigned a zero-metal content value, and the optimization model includes a constraint to avoid overlapping. This initial IP optimization model is described below.

Indices

$p \in \{1, \dots, P\}$ Index for all possible column units

$b \in \{1, \dots, B\}$ Index for all possible ore blocks in a given undercut level

Sets

O^b Set containing all column units that overlap with block b , with number of elements $N(O^b)$

Decision Variables

$x_p \in \{0, 1\}$ Binary variable controlling the decision to include unit p in the layout

$y_b \in \{0,1\}$ Binary variable used to ensure that all ore blocks in the undercut are covered with no gaps in the layout

Parameters

M_p Metal content of column unit p

Objective Function

$$\text{Max} \sum_1^P M_p \times x_p \quad (1)$$

Constraints

$$\sum_1^{N(O^b)} x_p \leq y_b \quad \forall b \in \{1, \dots, B\} \quad (2)$$

$$y_b = 1 \quad \forall b \in \{1, \dots, B\} \quad (3)$$

Equation (1) represents the objective function to maximize the metal content of the selected units, with the consideration that units with any individual column that does not meet the minimum column height (continuous ore from the undercut) are assigned a 0 value. Equation (2) and Equation (3) represent the overlapping constraint to assure that only one column unit from all those that share at least a single block is selected, while also guaranteeing that all ore blocks in the undercut level are covered to avoid gaps in the layout.

After this initial step a layout is generated for any given undercut, that simplifies the number of variables involved for the next scheduling optimization step while still being representative of the operating conditions in block caving operations.

The second assumption mentioned above is the grouping of block slices along the vertical section of each of the column units based on the minimum draw rate. The decision to group the blocks on these mining units is based on the condition that on each period at least the minimum tons, or inches, must be drawn continuously from each unit. The tonnage of each mining unit is the summation of the tonnage of the individual blocks within it, and the grade of the mining unit is the tonnage-weighted average grade of the individual blocks. As observed in Figure 1 this minimum draw rate would apply to each pair of draw points on the same excavation and extracting ore from the same draw zone. While resolution is lost for the scheduling along the vertical section of each column unit due to the grouping, it further reduces the number of variables and possible combinations to achieve an optimal solution for the scheduling problem under reasonable processing times.

The simplified IP model then is formulated to maximize the discounted profit, or NPV, of the extraction of each mining units over several periods. The model is constrained by the total mining capacity, the maximum draw rate from each column unit, the vertical precedence within each column and the horizontal precedence across the layout based on a concave advancement front.

Indices

$u \in \{1, \dots, U\}$ Index for all mining units

$t \in \{1, \dots, T\}$ Index for all periods

$c \in \{1, \dots, C\}$ Index for all column units

Sets

C^c	Set containing the mining units that are within column unit c . Each set has a total number of elements C^c .
$V^{u,c}$	Single element set containing the mining unit directly below unit u of column c .
$H^{u,c}$	Set containing the column unit that must be opened previously to the extraction of unit u of column c based on a horizontal concave mining direction. Each set has a total number of elements H_u .

Decision Variables

$z_{u,c}^t \in \{0,1\}$ Binary variable controlling the decision to extract mining unit u of column c .

Parameters

$P_{u,c}^t$ Discounted profit for the extraction of unit u of column c .

\bar{M} Maximum mining capacity (tons/period)

\underline{M} Minimum mining capacity (tons/period)

T_u Tonnage of mining unit u .

\overline{DR} Maximum draw rate from each column unit (tons/period)

\underline{DR} Minimum draw rate from each column unit (tons/period)

Objective Function

$$\text{Max} \sum_{t=1}^T \sum_{c=1}^P P_{u,c}^t \times z_{u,c}^t \quad (4)$$

Constraints

$$\underline{M} \leq \sum_{c=1}^P T_u \times z_{u,c}^t \leq \overline{M} \quad \forall c \in \{1, \dots, C\} \quad \forall t \in \{1, \dots, T\} \quad (5)$$

$$\sum_{c=1}^C z_{u,c}^t \leq 1 \quad \forall u \in \{1, \dots, U\} \quad \forall c \in \{1, \dots, C\} \quad (6)$$

$$z_{u,c}^t \leq z_{V^{u,c}}^t + z_{V^{u,c}}^{t-1} \quad \forall u \in \{2, \dots, U\} \quad \forall c \in \{1, \dots, C\} \quad \forall t \in \{2, \dots, T\} \quad (7)$$

$$\sum_{c=1}^{C_c} T_{u,c} \times z_{u,c}^t \leq \overline{DR} \quad \forall u \in \{1, \dots, U\} \quad \forall c \in \{1, \dots, C\} \quad \forall t \in \{1, \dots, T\} \quad (8)$$

$$H_u \times z_{1,c}^t \leq \sum_{c=1}^T \sum_{c=1}^{H_u} z_{1,c}^t \quad \forall u \in \{1, \dots, U\} \quad \forall c \in \{1, \dots, C\} \quad \forall t \in \{1, \dots, T\} \quad (9)$$

Equation (4) shows the objective function of the optimization model, that maximizes the total NPV of the operation by considering the discounted profit from the extraction of each block. The profit is calculated as the difference between the revenues and the costs associated to mining each block and is then discounted based on the period of extraction. Equation (5) corresponds to the mining capacity constraint, and forces that the total ore tonnage extracted from all columns over each period is within the minimum and maximum established ranges. Equation (6) guarantees that no mining unit is extracted more than once. Equation (7) establishes the vertical precedence within mining units of the same column, and guarantees that to extract a unit, it's directly below unit must be extracted on the

same or previous period to assure a continuous draw of the column. Equation (8) forces that the total ore tonnage extracted from each column unit on each period is less or equal than the maximum draw rate. Since the mining units are built based on the minimum draw rate volume there is no need to include a minimum draw rate constraint. Finally, Equation (9) forces the horizontal precedence constraint. The horizontal precedence constraint is based on a concave mining advancement front and defined by three angles, the azimuth of the main advancement direction a , and the two diagonal angles $a1$ and $a2$, as shown in Figure 2. The constraints guarantee that a column unit can initiate extraction only after all the directly preceding columns, based on the mentioned direction, have started extraction.

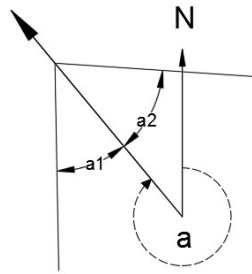


Fig. 2. Concave advancement front for the establishment of the horizontal precedence constraint.

The optimization framework then corresponds that for a given undercut elevation, the column unit layout that maximizes the metal content is found first, then the mining units are built based on the optimized layout and the minimum draw rate to be scheduled considering the vertical and horizontal profit discounting, and operating constraints as well. The process is then carried over the different possible levels of the deposit to select the level that yields the highest NPV. The model then can be used to individually evaluate multiple scenarios, such as different mining directions or operational parameters, on any specific undercut level of interest.

3. Case Study

The optimization model was tested on a copper deposit case study. A block model of the deposit is built with blocks of $10\text{ m} \times 10\text{ m} \times 10\text{ m}$, as shown in Figure 3. The deposit has a total of 58.5 Mtons of ore with a weighted average grade of 1.46% copper.

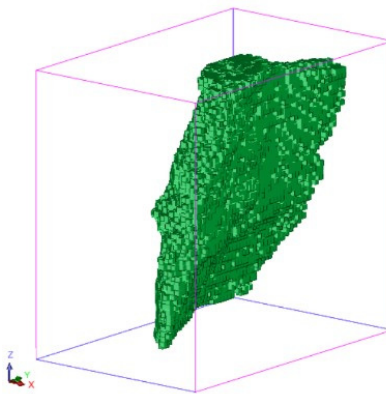


Fig. 3. Isometric view of the block model for the case study.

The IP optimization framework is solved using a MATLAB environment (MathWorksInc, 2018) and the IBM ILOG CPLEX engine (IBM, 2015) with a 5% gap (the feasible integer solution found is proven to be within the five percent of the optimal), on an Intel Core i7 machine with 3.40 GHz and 16 GB of RAM.

The dimensions considered for the column unit layout optimization of each undercut sections are 20m in the X direction and 30m in the Y directions, to mimic some common spacing measures of 10m between draw points on the same drawbell and 20m between different draw point excavations.

The operational and economic parameters considered are shown in Table 1.

Table 1. Operational and economical parameters for the application of the optimization framework.

Parameter	Value	Unit	Description
T	15	Year	Number of periods
\overline{M}	1200	kton/period	Maximum mining capacity per period
\underline{M}	500	kton/period	Minimum mining capacity per period
\overline{DR}	150	kton/column/period	Maximum draw rate per column unit per period
\underline{DR}	60	kton/column/period	Minimum draw rate per column unit per period
\overline{H}	300	meters	Maximum column height
\underline{H}	100	meters	Minimum column height
Recovery	85	%	Metal process recovery
Price	6000	\$/ton	Selling price of the metal
Cost	26.1	\$/ton	Extraction cost (Mining + Processing)
Discount Rate	10	%	Discount rate
a	290	degree	Main horizontal mining direction
a1	45	degree	Concave face angle
a2	45	degree	Concave face angle

The minimum draw rate from which the mining units are built on this test model was 60 kton/column/period, which considering the dimensions of the column units used is roughly equivalent to drawing about 5 inches per day from each column unit. This value is in line with some reported operational parameters. The optimization model is run for an interval of levels between 30 (elevation 705 meters) and 45 (elevation 555 meters) from the block model z index. Figure 4 shows the NPV and metal content obtained by the optimization model from each level.

The optimization model yielded level 38, at an elevation of 625 m, as the optimal undercut elevation for the case study, with a NPV of 587.85 MUSD and a total of 318 kton of Cu. This level is selected by analyzing not only the total blocks above each undercut but considering operating constraints and a mining advancement direction, with the mining reserves and NPV obtained subjected to operational constraints. The undercut layout consists of 64 column units, which are representative of 128 draw points. Overall, the computing time for each level was about 30 – 60 minutes depending on the number of ore blocks in each undercut plan section. The undercut ore areas varied between 38,000 m^2 to 57,000 m^2 . On larger undercuts from massive deposits, the processing times can be expected to be longer, however an optimal solution can still be reached in a reasonable time.

Figure 5 shows the optimal layout for level 38. This layout maximizes the metal content of the deposit based on the described aggregation of individual columns into the column units to represent the

extraction practice in block caving, considering operational constraints. This layout also gives some insight on the potential drawpoint layout for detailed engineering studies at a later stage of the project.

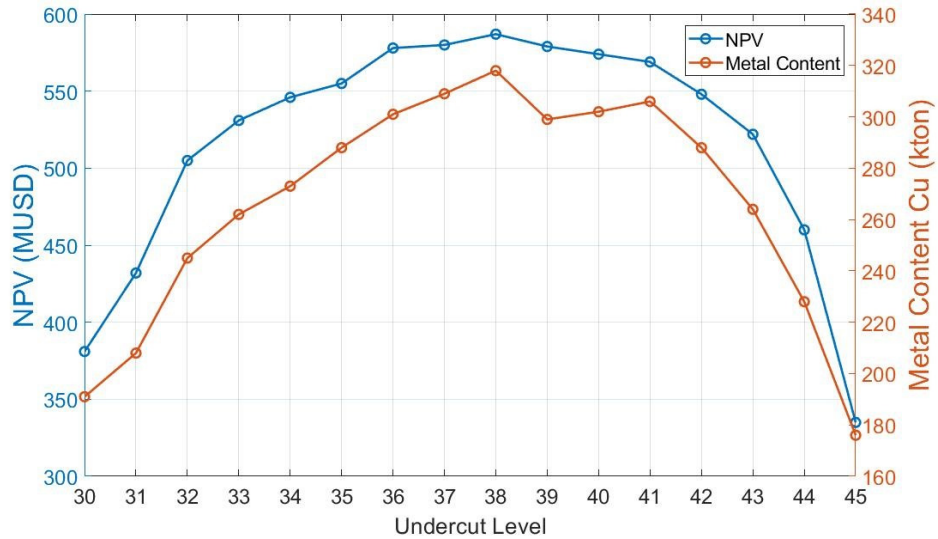


Fig. 4. NPV and metal content obtained from the optimization model for each level. Level 38 appears as the optimal undercut level.

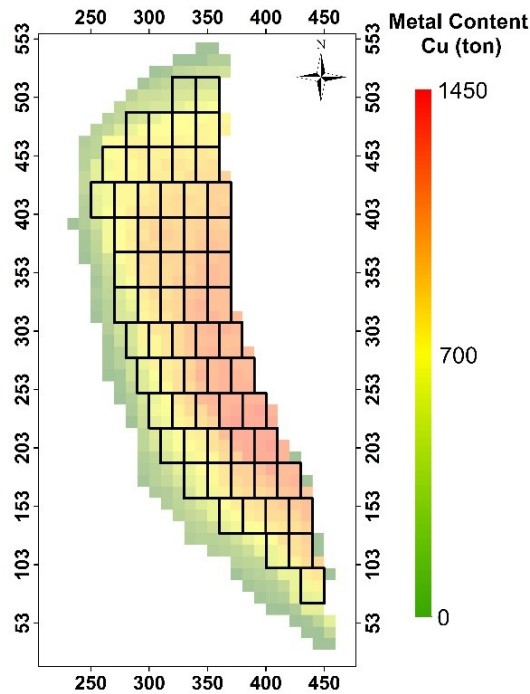


Fig. 5. Draw layout for the optimal undercut level 38, showing the metal content.

Figure 6 shows the extraction opening sequence of the column units in the optimal undercut level. This sequence achieves the maximum NPV considering the horizontal concave mining direction. At the moment of the development of this model, the starting point is selected to be the farthest column unit based on the direction advancement.

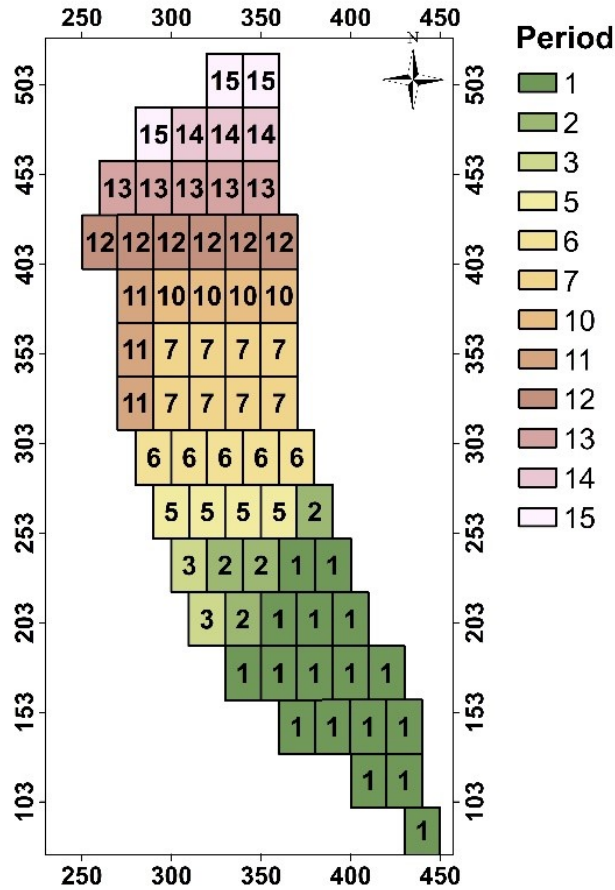


Fig. 6. Sequence of column opening to apply the horizontal discounting on the NPV at undercut level 38.

One of the main concerns in mine planning is the risk of building an initial design based on a set of economic assumptions that are uncertain in the future. To deal with this uncertainty, a sensitivity analysis is carried to understand the behavior of a decision or design over multiple scenarios. The model proposed can be easily set up to evaluate multiple scenarios for any required economical or technical parameter.

As an example, a sensitivity analysis on the price of the metal is presented. The price of the metal is a decisive factor on the profitability over the life of a mine, but it is also one of the most uncertain parameters as it is dictated by global market conditions.

For the model presented, the response of the NPV and, most importantly, the most profitable level is evaluated over different price scenarios. The selection of an undercut elevation is not a flexible decision, once development has started it not possible or very costly to start over. Therefore, it is important to evaluate multiple economic conditions to make a decision. There is also geotechnical uncertainty, however the model takes as input parameters a starting point and mining direction as well as draw rates, all of which are assumed to be defined considering the proper geotechnical environment. Table 2 shows the results for the sensitivity analysis on different prices. Two lower cases and two upper cases were considered at 10% intervals each. The model was set up to run on the same technical parameters as specified previously.

The optimum level remains at level 38, at an elevation of 625 m, for both the upper cases and up to a -10% variation in the price. That means that starting the operation with the undercut placed at that level will maximize the profitability of the mine even over optimistic scenarios, and up to 10% metal price value reductions. The optimum level does change when a -20% reduction in the copper price is considered, and moves up to level 39, at an elevation of 635 m.

Table 2. Sensitivity analysis for different price scenarios.

	Lower Cases		Base Case	Upper Cases	
Variation	-20%	-10%	-	+10%	+20%
Price (\$/ton)	4800	5400	6000	6600	7200
Optimum Level	39	38	38	38	38
NPV(M\$)	327.34	430.95	587.85	712.32	824.61

On a pessimistic scenario, with the price set at 4800 \$/ton, a better NPV while scheduling based on the mining direction is found at a different level. However, setting the undercut at level 38 seems as a robust decision, as it maximizes the NPV over multiple price scenarios and would likely be the most profitable decision for this case study.

4. Conclusions

The optimization framework achieves an optimal solution in under 20 minutes for each level and gives insight on the possible alternatives for footprint layout design and development sequence. The fast processing times allow for an iterative process to find the most profitable undercut elevation considering operational constraints and guaranteeing an optimal solution via an IP model. While the assumptions used to simplify the problem cause a loss of resolution for the long-term scheduling of the caving mine, the final goal of this model is not to find an optimal schedule but to include the mining direction discounting on the NPV evaluation for the selection of an undercut elevation.

Although it would be expected for the computing time to increase for massive deposits with a larger footprint area, the model could still perform on reasonable times as on this particular case at a footprint of 57,000 m^2 the optimal solution was reached after 20 minutes of processing, and the iterating process over multiple levels is a computational parallelisable process.

The model could be still improved by considering dilution mixing models such as Laubscher's (non-linear mixing models would require another type of mathematical formulation), considering grade uncertainty via multiple geostatistical simulations of the deposit, and considering specific starting points for the extraction sequence. Multiple metals can also be considered when building the objective function, that is when calculating the economic block values. Considering multiple economic metals, common in many porphyry deposits suitable to block caving, would not add to the computing times.

The mining direction and starting point in the sequence still play a major role in the NPV of a caving project. The model takes as an input a mining direction, usually defined considering the geotechnical conditions of the deposit and evaluates the profitability of setting the undercut level at different elevations considering the economic discounting from this direction. Once an elevation or a small number of undercut levels have been found to yield the highest NPV for the deposit, the model could be applied to evaluate different mining directions and scenarios.

The initial layout optimization step is also of interest as it could be improved to account for the different geometrical constraints of the common draw point layout schemes, and possible underground infrastructure placement, in order to develop a tool to optimize the draw point locations in the caving footprint.

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