An Optimization Framework for Cut-and-Fill Mining Production Scheduling using Mixed Integer Linear Programming

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ABSTRACT

Mixed integer linear programming models are recognized to have significant potential for optimizing production scheduling problems. A review of general optimization studies that have been proposed for underground mining shows that previous works lack flexibility, operability and practicality in relation to cut-and-fill mining production scheduling. This paper presents a mixed integer linear programming (MILP) formulation for cut-and-fill mining. The objective function of the model is to maximize the net present value (NPV) of the operation while meeting all mining and processing operational and technical constraints. In addition, stope development and extraction sequencing constraints, and the use of mining and processing tonnage fluctuation, extraction duration and active levels control constraints make the MILP model more practical and expandable. The MILP model is verified and validated with two case studies from an existing mine and the results are compared with the actual mining strategy. The comparison shows a 5% to 7% improved NPV in both case studies resulting from mining higher grades and processing less tonnes thereby generating a better cash flow.

1. Introduction

Mine production scheduling is the determination of the best mining strategy that defines extraction sequence at a given time, and the movement of mineralized material to maximize the overall net present value (NPV) while optimizing the available resource utilization based on identified economics, technical, environmental and geological constraints (Ben-Awuah et al., 2016; Ebbels, 2016). Mine production scheduling is grouped into long-term (usually three to five years or more) and short-term (daily, weekly or monthly plans) production scheduling mainly determined based on various degrees of information provided and the planned working duration. In underground mining, long-term production scheduling specifies the stope extraction sequence and the connected mine developments required to achieve the production targets and maximize the NPV usually from three to five years or more. Short-term production scheduling defines a highly detailed basic operation or guidance for operating over a time frame of days, weeks and months (Alford et al., 2007) to minimize the deviations from the pre-defined production capacities that can maintain a constant plant feed rate (Nehring et al., 2010).

The application of operation research techniques in mining started in the early 1960s (Musingwini, 2016). During the past decades, many researchers have made efforts to use operation research techniques in the mining industry (Osanloo et al., 2008). These works are mainly focused on open pit mining. Although operations research techniques have been extensively used in open pit mining, its use is limited in underground mine planning and scheduling. This is because of the complexities associated with the optimization of development layouts, which are directly related to the underground mining methods used.

This research seeks to develop, implement and verify an optimization framework for an underground cut-and-fill mining production scheduling using mixed integer linear programming (MILP). The objective of the framework is to maximize the net present value (NPV) of an underground mining operation and generate a practical, operational, and long-term cut-and-fill mining schedule.

2. Literature Review

Optimization methods in underground mining were introduced in the 1980s (Lizotte and Elbrond, 1985; Chatterjee and Sridhar, 1986), and a common optimization method applied to underground problems is Linear programming (LP). Jawed and Sinaha (1985), Pendharkar (1997), and Li (2009) used linear programming for optimizing production planning and scheduling of a group of underground mines. However, all these initial models lack generality to some extent and are specific to localized problems, for instance, optimal blending or fleet management problem. Meanwhile, these types of mathematical models do not allow the utilization of integer or binary decision variables that are required to meet precedence needs (Brickey, 2015).

Linear programming is completely dependent on the linear equation in theoretical logic and program implementation. The decision variables are allowed to be fractional, but that is a certain limitation for the practical stopes sequencing problem of the production scheduling of underground mines, which are discrete in nature. Usually, the majority of these activities encountered in mine production management are discrete. Therefore, linear programming sometimes gives results that are not practical.

Mixed integer linear programming (MILP) approaches is a well-suited modeling method developed in the 1990s. Chanda (1990) presented a computerized integer programming model for short-term production scheduling in a copper mine based in Zambia. Trout (1995) developed a general mixed integer programming (MIP) model to solve a 55-stope underground mine scheduling problem with sequencing, production and backfilling constraints. Topal (2003) developed a MIP model and implemented on a small block database for underground sublevel caving operation. These MIP models are unable to handle realistic industrial mining scenarios and solution time may exponentially increase when more integer variables are added. That will make the model very difficult or even impossible to be solved in an actual problem (Ben-Awuah and Askari-Nasab, 2011). More recently, Nehring and Topal (2007) presented a MIP model for a small conceptual sublevel stoping operation. Binary and integer variables were defined to represent four separate mining phases, including preparation, extraction, void and backfilling. To improve on the computational time, a new MIP model was developed by assigning a single binary decision variable for the entire production to reduce the number of variables (Nehring et al., 2010). But, it was necessary to assign earliest and latest possible start times for each production block in advance. Pourrahimian et al. (2012), Pourrahimian (2013), and Pourrahimian and Askari-Nasab (2014) developed, implemented, and verified a theoretical optimization framework based on a MILP model for a long-term production scheduling in underground block-caving with the maximum economic return measured by NPV, and within acceptable technical and operational constraints. However, the introduced optimization framework is suitable only for block-cave mining.

Heuristic approaches have depicted to be effective for large-scale Non-deterministic Polynomial (NP) hard problems, especially in the wider field of mine planning and scheduling. O'Sullivan and Newman (2014) proposed an integer-programming model for underground mine scheduling which aims to maximize metal production subject to resource constraints while considering sequencing of extraction and backfilling. When the determined optimization methods mentioned above are not sufficiently effective, authors have used a heuristic technique to obtain a suboptimal solution within an acceptable time range. Heuristic algorithms generate candidate solutions for the model constraints randomly. Nevertheless, they have their own selecting ways and searching directions to produce better children candidate solutions for the optimization of a model objective function. Although the number of iterations may increase for a larger-scale problem, the consumption increases in computational costs are usually substantially lower than it would be for traditional methods. A terminal problem for these heuristic methods is that there are no mathematical proving methods for the provided solutions to compare to the theoretical optimum. Jie et al. (2016) constructed a MIP model considering underground mining condition of equilibrium and continuity and calculated by an algorithm based on the artificial bee colony model. Similar research has been conducted by Denby and Schofield (1995), Kumral (2004), Yao et al. (2011), and Hu et al. (2013).

Mixed integer programming models are recognized as having significant potential to suit the discrete problem in certain conditions (Ben-Awuah and Askari-Nasab, 2013). The integer variables can represent entities that cannot be divided. However, the downside is that MILP problems are non-convex, they must be solved by some kinds of systematic and possibly exhaustive search. Therefore, the literatures so far report few MILP formulation applications to large-scale underground mine production problems. In addition to the optimization methods and techniques, the current formulations focusing on the implementing optimization procedures in underground mining mostly lack flexibility, operability and practicality. The lack of operational constraints will possibly result in a widely spread mining pattern for various periods. It is sometimes infeasible to incorporate equipment mobility and access requirement into long-term underground production scheduling and ore extraction.

Underground mine production scheduling has developed over the last two decades but proven to be an intractable and a challenging task. There is currently no suitable underground mine production scheduling optimization model available to the application of cut-and-fill mining. Hence, based on the critical review of the optimization of mine production planning and scheduling studies that have been proposed for underground hard rock mining, we present a MILP model for cut-and-fill mining that seeks to maximize the project NPV and provide a feasible, practical and flexible ore extraction and development scheduling.

3. Assumptions, Definitions and Notations

Cut-and-fill mining is applied to steeply dipping orebodies, good to moderate geological strength and stability, and a comparatively high-grade mineralization. It provides better selectivity and is preferred to orebodies with irregular shape, discontinuities and scattered mineralization (Hamrin et al., 2001).

Cut-and-fill mining method extracts the ore in horizontal slices, starting from a bottom undercut, advancing upward. The ore is drilled, blasted, loaded and removed from a stope, which is then backfilled with tailings from the dressing plant combined with cement, or waste rock obtained from any part of the underground working. The backfilling body serves both to support stope walls, and as a working platform for the equipment when the next slice is mined.

There are horizontal development levels to work mines from the main shaft at regular intervals along the orebody. In a plan view, the orebody is broken-up into a series of stopes which are generally square or rectangular in shape. A crosscut is a horizontal opening driven from the shaft and near right angles to the orebody, which is connecting a level with the main shaft. A horizontal drift is a necessary access for mine equipment transporting broken materials from the stopes. Fig. 1 shows a schematic diagram of stopes layout.

Generally, some mining exploration developments, such as the main shaft, auxiliary shaft, main return airways and dewatering systems are selected and designed considering future development. These are often completed at one time as a permanent facility for subsequent development and operation. However, these infrastructure projects are not within the scope of this research.

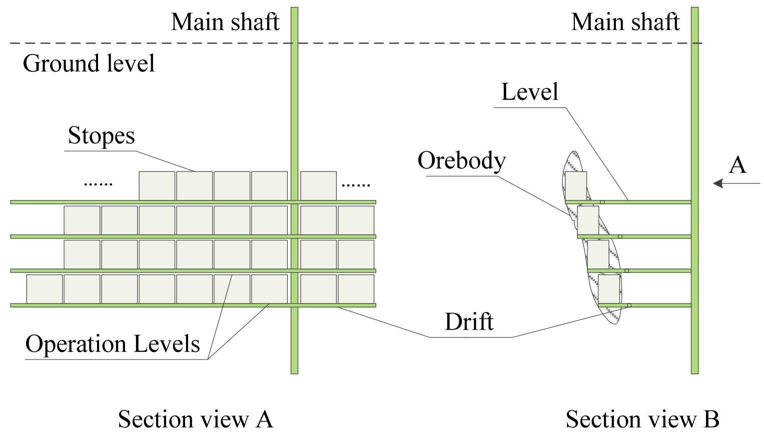

Fig. 1. A schematic diagram of stopes layout.

This study is focused on the optimization of mining production phases for cut-and-fill underground mining. Fig. 2 shows the stope production phases modeled in three separate phases including advancement, extraction, and backfilling. In cut-and-fill mining, all work is done inside the stope, therefore, advancement, including necessary crosscuts, transport drifts, and proper ventilation rises, must be provided to the worker. After these activities, stope extraction activities such as drilling, blasting and mucking, can begin. Finally, every mined-out stope is backfilled for ground control purpose and to provide a surface to work from for future stopes. The advancement, extraction and backfilling are repeated until mining has progressed along the entire strike of the stope.

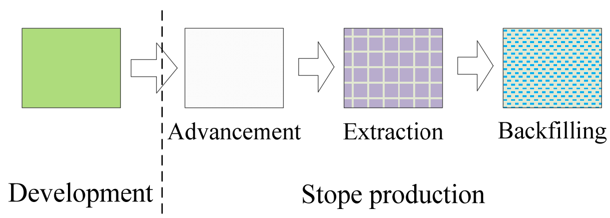

Fig. 2. Stope production phases.

3.1. Notations

The following notations, sets, parameters, and definitions are used to construct the objective function and constraints for formulation of the mathematical mixed integer linear programming model.

t	schedule time periods: t = 1, 2, …, T, where T is the schedule duration.
n	stope identification: n = 1, 2, …, N, where N is the total number of stopes.
l	level identification: l = 1, 2, …, L, where L is the total number of levels.

3.2. Decision variables

$a_n^t \in \{0,1\}$	binary integer variable; equal to one if the advancement for the stope n is to be scheduled in time period t, otherwise zero.
$e_n^t \in \{0,1\}$	binary integer variable; equal to one if the extraction for the stope n is to be scheduled in time period t, otherwise zero.
$b_n^t \in \{0,1\}$	binary integer variable; equal to one if the backfilling for the stope n is to be scheduled in time period t, otherwise zero.

$x_n^t \in [0,1]$ — continuous variable, representing the portion of the stope n is to be extracted in time period t. This variable is linked to the stope extraction variable e and only provides a non-zero value when e is non-zero.

3.3. Sets

$A(J)$ — for each stope n, there is a set $A(J)$ defining the adjacent stopes that cannot be mined simultaneously with the extraction or backfilling of the stope n; where J is the total number of stopes in set $A(J)$.

$D(I)$ — for each stope n, there is a set $D(I)$ defining the development that must be completed prior to the advancement and extraction of the current stope n. where I is the total number of advancement activities in set $D(I)$.

$L(M)$ — For each level l, there is a set $L(M)$ defining all the stopes in this level, where M is the total number of stopes in set $L(M)$.

3.4. Parameters

R_n^t — discounted revenue, generated by selling the final product.

i — discounted rate.

o_n — ore tonnage in stope n.

g_n — average grade of mineral in ore portion of stope n.

r^t — processing recovery, the proportion of mineral recovered in period t.

p^t — selling price in present value terms obtainable per unit of product.

CA_n^t — cost in present value terms of advancement for stope n in period t.

ca_n^t — variable cost in present value terms of advancement in period t.

ad_n^t — advancement tonnage of stope n in period t.

CE_n^t — cost in present value terms of extraction in period t.

ce_n^t — variable cost in present value terms of extraction in period t.

w_n — waste tonnage in stope n.

CF_n^t — cost in present value terms of backfilling in period t.

cf_n^t — variable cost in present value terms of backfilling in period t.

f_n^t — backfilling amount of stope n in period t.

CP_n^t — cost in present value terms of processing in period t.

cp_n^t — variable cost in present value terms of processing in period t.

al^t	lower bound on advancement capacity in period *t*.
au^t	upper bound on advancement capacity in period *t*.
h	hoist rate, representing the portion of material to be hoisted in period *t*.
hl^t	lower bound on hoisting capacity in period *t* (tonnes).
hu^t	upper bound on hoisting capacity in period *t* (tonnes).
pl^t	lower bound on processing capacity of ore in period *t* (tonnes).
pu^t	upper bound on processing capacity of ore in period *t* (tonnes).
gl^t	lower bound on acceptable average grade of mineral in period *t*.
gu^t	upper bound on acceptable average grade of mineral in period *t*.
D_m	deviation for mining tonnage fluctuation constraints.
D_P	deviation for processing tonnage fluctuation constraints.
l_l^t	binary integer variable; when it is equal to one, it means there are activities in progress in this level in period *t*.
N_{ed}	the maximum length of the extraction duration of a stope.
N_{al}	the maximum number of the active levels in period *t*.

4. Mathematical Formulation

4.1. Objective function

The objective function of the MILP formulation seeks to maximize the NPV of the mining operation. The profit from mining a stope depends on the value of that stope and the costs incurred in advancement, extraction, backfilling and processing. Hence, the discounted profit from mining a stope can be represented by the discounted revenue minus the discounted costs. The objective function is expressed by Eq. (1), composed of the revenue, discounted rate, costs of advancement, extraction, backfilling and processing, two binary decision variables indicating the time of advancement and backfilling to be scheduled, and a continues decision variable indicating the portion of a stope which is extracted in each period. The component equations of the discounted revenue and costs are shown in Eqs. (2) to (6).

$$Max \quad \sum_{t=1}^{T}\sum_{n=1}^{N}\left[\frac{\left(R_n^t - CE_n^t - CP_n^t\right)\times x_n^t - CA_n^t \times a_n^t - CF_n^t \times b_n^t}{\left(1+i\right)^t}\right] \tag{1}$$

$$R_n^t = o_n \times g_n \times r^t \times p^t \tag{2}$$

$$CA_n^t = ca_n^t \times ad_n^t \tag{3}$$

$$CE_n^t = ce_n^t \times \left(o_n + w_n\right) \tag{4}$$

$$CF_n^t = cf_n^t \times f_n^t \tag{5}$$

$$CP_n^t = cp_n^t \times o_n \tag{6}$$

4.2. Productivity constraints

The constraints can generally be classified into three categories, namely productivity constraints, sequencing and geotechnical constraints and operational constraints. Advancement capacity, grade blending, and hoisting capacity are considered as productivity constraints and a vital part in the development of an optimal production schedule. Sequencing and geotechnical constraints are required to satisfy geotechnical and technical conditions. Operational constraints include the mining and processing tonnage fluctuation constraints, extraction duration constraints and active levels control constraints. They are expressed in mathematical form as follows.

4.2.1. Advancement capacity constraints

Eq. (7) ensures that the total amount of advancement in each period cannot exceed the predefined and acceptable range of advancement capacity of the equipment in that period. al^t and au^t are the lower and upper bounds for the advancement capacity respectively.

$$al^t \le \sum_{n=1}^{N} (ad_n^t \times a_n^t) \le au^t \qquad \forall t \in \{1, ..., T\} \tag{7}$$

4.2.2. Hoisting capacity constraints

Eq. (8) ensures that the total tonnage of material hoisted in each period cannot exceed the predefined and acceptable range of hoist capacity in that period. The inequality accounts for the total rock material from the stopes and advancement that must be hoisted from the mine. hl^t and hu^t are the lower and upper bounds for the hoisting capacity respectively.

$$hl^t \le h \times \sum_{n=1}^{N} \left[\left(o_n + w_n \right) \times x_n^t + ad_n^t \times a_n^t \right] \le hu^t \quad \forall t \in \{1, ..., T\} \tag{8}$$

4.2.3. Grade blending constraints

Eq. (9) ensures that the average grade of the ore or production is within the desired range in each period. It enforces the production scheduling to achieve the desired quality of material for the processing plant. gl^t and gu^t are the lower and upper bounds for the desired grade range.

$$gl^t \le \frac{\sum_{n=1}^{N} g_n \times o_n \times x_n^t}{\sum_{n=1}^{N} o_n \times x_n^t} \le gu^t \qquad \forall t \in \{1, ..., T\} \tag{9}$$

4.3. Sequencing and geotechnical constraints

4.3.1. Advancement sequencing constraints

If a stope is scheduled to be mined in a period, advancement must be ready in or before that period. Thus, all the advancements prior to the current advancement necessary to exploit a stope should be completed first. To meet these requirements, we use a set *D(I)* representing the development that must be completed prior to the advancement of the current stope *n*. The same with a_n^t, a_i^t.is a binary integer variable representing the development in *D(I)* to be scheduled or not in time period *t*. Eq. (10) guarantees the reasonable advancement sequence.

$$a_n^t \le \sum_{k=1}^{t} a_i^k \qquad i \in D\left(I\right), \forall n \in \{1, ..., N\}, \forall t \in \{1, ..., T\} \tag{10}$$

4.3.2. Stope status and sequencing constraints

Sequencing constraints were required to satisfy geotechnical and technical conditions. These constraints define where to start the advancement, extraction and backfilling for a particular level and how to progress specifically. Eq. (11) ensures that an individual stope could only be in one extraction or backfilling phase during any specific period. These two activities cannot be in progress at the same time for a particular stope. Eqs. (12) to (13) ensure that production mining phases progress sequentially through all mining phases including advancement, extraction and backfilling. Eq. (14) ensures that each stope is in either the extraction or backfilling phase immediately after a non-zero stope extraction period. The stope status constraints and sequencing constraints together function to ensure discrete mining activities progress sequentially, through all phases for all the periods. On the other hand, these constraints limit the exposure time of the void area, that potentially leads to excessive geotechnical stresses, increasing the stability to a certain extent.

$$e_n^t + b_n^t \leq 1 \qquad \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{11}$$

$$e_n^t - \sum_{k=1}^{t} a_n^k \leq 0 \qquad \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{12}$$

$$b_n^t - \sum_{k=1}^{t} x_n^k \leq 0 \qquad \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{13}$$

$$e_n^t \leq e_n^{t+1} + b_n^{t+1} \qquad \forall n \in \{1,...,N\}, \forall t \in \{1,...,T-1\} \tag{14}$$

4.3.3. Non-adjacent stope production constraints

Non-adjacent stop production constraint is a kind of geotechnical constraints, which focuses on limiting the size of unsupported void areas. Fig. 3 is a vertical view of nine adjacent stopes showing two kinds of spatial constraints required to ensure geotechnical stability of the mine. When a stope is being extracted, the adjacent stopes cannot be mined. For instance, as shown in Fig. 3 (1), if the central stope *S5* is in the extraction state, all kinds of activities are forbidden in the near adjacent stopes (*S2, S4, S6, S8*). Similarly, when a stope is being backfilled, the adjacent stopes cannot be exploited. Eqs. (15) and (16) ensure that all forms of simultaneous adjacent stope production must be avoided to prevent excessively large unsupported voids.

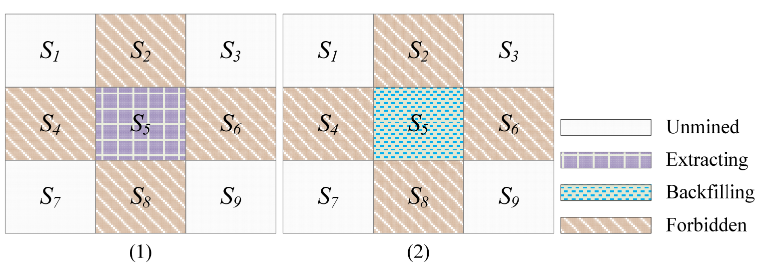

Fig. 3. Schematic representation of activities around the stopes.

$$e_n^t + e_j^t \leq 1 \qquad j \in A(J), \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{15}$$

$$b_n^t + e_j^t \leq 1 \qquad j \in A(J), \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{16}$$

4.4. Operational constraints

4.4.1. Mining and processing tonnage fluctuation constraints

In previous works, the upper and lower bounds (mining and processing target/capacity constraints) are usually used to ensure the total tonnage of material or ore mined in each period is within the acceptable range of production targets or mining equipment capacity. Unfortunately, this kind of constraints cannot control the production fluctuations with time and have no motivation to make full use of the mining capacity to enforce higher ore production in the early periods of the mining operation. Tonnage fluctuation constraints on mining and processing shown in Eqs. (17) and (18) respectively are used instead of production target/capacity constraints to ensure that the total rock tonnage and ore tonnage mined increase steadily and stable during the production periods for an improved cash flow of the mining operation. D_m and D_p are function of the maximum mining and processing capacity respectively.

$$\sum_{t=1}^{T}\left(\sum_{n=1}^{N}\left[\left(o_n+w_n\right)\times(x_n^{t+1}\text{-}x_n^t)\right]\right)\leq D_m \tag{17}$$

$$\sum_{t=1}^{T}\left(\sum_{n=1}^{N}\left[o_n\times(x_n^{t+1}\text{-}x_n^t)\right]\right)\leq D_P \tag{18}$$

4.4.2. Extraction duration constraints

Eq. (19) ensures that the extraction duration of a stope cannot exceed the predefined periods. The inequality helps the model to provide an implementable schedule that better reflects the actual mining environment to control the maximum length of the extraction duration. According to the variety of applicable scenarios, the length of the extraction duration can customize depending on inter alia, stope size, production rate and mining strategies. Therefore, it provides added flexibility and functionality to the formulation.

$$\sum_{t=1}^{T}e_n^t\leq N_{ed}\qquad\forall n\in\{1,...,N\} \tag{19}$$

4.4.3. Active levels control constraints

Eqs. (20) to (22) control the maximum number of simultaneous active levels at each period in the schedule. For the inequalities (20) and (21), when an advancement activity is active or a portion of a stope is being extracted in period t in this level, the relevant binary variable l_l must be equal to one. N_{al} is given as an input to the algorithm. In each period, the number of simultaneous active levels must not exceed the allowable number N_{al}, shown in Eq. (22). A large number of active levels might lead to serious operational problems. Therefore, a customized maximum number of simultaneous active levels allow mine planners to adjust constraints that adapt to different application scenarios depending on available infrastructure, production rates, equipment capacities, and the stability of the surrounding rocks. Accordingly, the active levels control can also be considered as a kind of geotechnical constraints. It takes into account the equipment mobility and access required for the underground mine production scheduling optimization framework. Overall, the active levels control constraints help the optimization model to generate a more operational mining pattern, indirectly leading to a practical maximum NPV.

$$\sum_{k=1}^{t}\left(\frac{1}{M}\sum_{m=1}^{M}a_m^k-l_l^k\right)\leq 0\qquad m\in L\left(M\right),\forall l\in\{1,...,L\},\forall t\in\{1,...,T\} \tag{20}$$

$$\sum_{k=1}^{t}\left(\frac{1}{M}\sum_{m=1}^{M}x_m^k-l_l^k\right)\leq 0\qquad m\in L\left(M\right),\forall l\in\{1,...,L\},\forall t\in\{1,...,T\} \tag{21}$$

$$\sum_{l=1}^{L}l_l^t\leq N_{al}\qquad\forall t\in\{1,...,T\} \tag{22}$$

4.4.4. Variables constraints

In the proposed model, the variables constraints control the logics and interrelationships of the binary and continuous variables that define advancement, extraction and backfilling to ensure all of them are within desired ranges. Eq. (23) links the continuous extraction variable *x* to the binary extraction variable *e*, and ensures to only provide a non-zero value *x* when *e* is non-zero. Eqs. (24) and (25) ensure that all stopes are going to be advanced and backfilled once in the mine life. Eq. (26) ensures that the total fractions of stopes that are extracted over the scheduling periods are going to be sum up to one, which means all the underground stopes are going to be scheduled. Some other variables defining the deviation for mining and processing tonnages are not written here, they should be non-negative.

$$x_n^t - e_n^t \le 0 \qquad \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{23}$$

$$\sum_{t=1}^{T} a_n^t = 1 \qquad \forall n \in \{1,...,N\} \tag{24}$$

$$\sum_{t=1}^{T} b_n^t = 1 \qquad \forall n \in \{1,...,N\} \tag{25}$$

$$\sum_{t=1}^{T} x_n^t = 1 \qquad \forall n \in \{1,...,N\} \tag{26}$$

5. Case Study

5.1. Case study 1 - Forty stopes

For the purpose of evaluating and demonstrating the potential benefits of optimized production scheduling, the proposed model is implemented on a small underground cut-and-fill gold mining operation containing forty stopes. Fig. 4 shows the layout of stopes in the mine. The size of the stope is 50 m in length, 40 m in height, and the width is the same as orebody's width ranging from 10 to 30 m. The total mineral resource in forty stopes is 4.3 Mt with an average Au grade of 2.73 g/t. The goal is to obtain an ore production scheduling over eight periods. A discount factor of 8 percent per annum is applied.

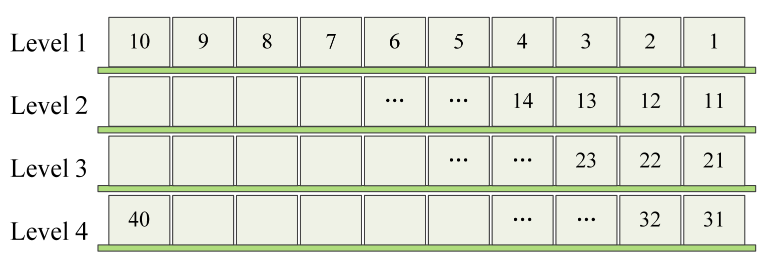

Fig. 4. Forty stopes layout.

Each stope is available for scheduling from the first year period onwards. Advancement and backfilling for each stope is expected to take one time period and extraction of each stope is expected to take no more than a predefined time period length (e.g., three time periods). Therefore, a total of three to five time periods are required for full stope production from the commencement of advancement to the completion of backfilling. The non-adjacent extracting and backfilling production and sequencing constraints are applied to ensure that mining activities progress sequentially through all stope production phases and maintain stability. The technical and economic parameters used for testing the models are shown in Table 1.

Table 1. The technical and economic parameters for the case study 1.

No.	Description	Value
1	Mining cost ($/t)	33.97
2	Processing cost ($/t)	9.82
3	Backfilling cost ($/t)	6.59
4	Advancement cost ($/t)	32.12
5	Price of gold ($/g)	45.19
6	Discount rate (%)	8.00
7	Mining recovery (%)	92.47
8	Mining dilution (%)	4.50
9	Mining loss (%)	8.00
10	Maximum grade (g/t)	2.90
11	Minimum grade (g/t)	2.60
12	Maximum mining capacity (Mt/Period)	0.55
13	Maximum processing capacity (Mt/Period)	0.55
14	Maximum advancement capacity (Mt/Period)	0.07
15	Maximum hoisting capacity (Mt/Period)	0.62
16	Maximum extraction duration (Periods)	2
17	Maximum active levels	3

Note: Period = two months.

The models were formulated in Matlab (2017a) (MATLAB and Release, 2016) and CPLEX environment (IBM., 2015). All stopes are extracted while meeting all mining operational constraints. Solution times for the model during all the testes are less than 10 seconds.

Results are shown in Fig. 5 to Fig. 8. It can be seen that the ore production increases to the maximum capacity in the second period and kept stable during all the production periods before the end of schedule periods. The mining and processing fluctuation constraints help to avoid peaks and dips often found in other conventional target/capacity constraints production schedules. The average ore grade can reasonably feed the processing. All the stope production phases including advancement, extraction and backfilling execute sequentially. Advancement is indicated by the color blue. It is followed by several periods of extraction (green) then backfilling (yellow). The stopes extraction duration and the number of simultaneous active levels during all the schedule periods fall within the boundary of the related constraints. In comparing the actual mining strategy of the underground gold mining operation in 2016 to the MILP production schedule, it is apparent that the MILP model optimized schedule improved NPV by $ 9.61 M (approximately 7.19% in this case).

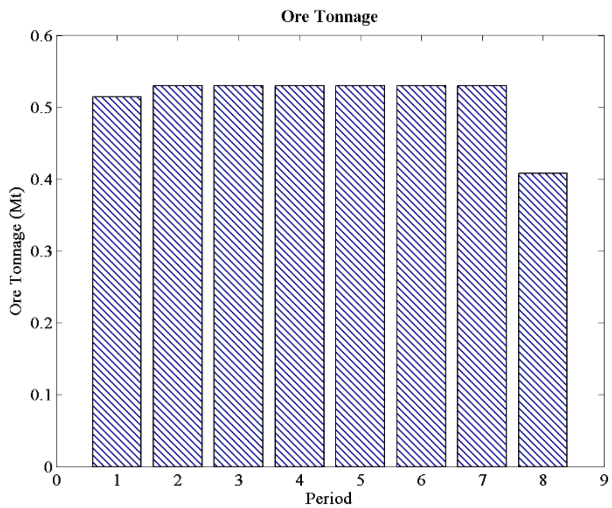

Fig. 5. Ore tonnage over 8 periods.

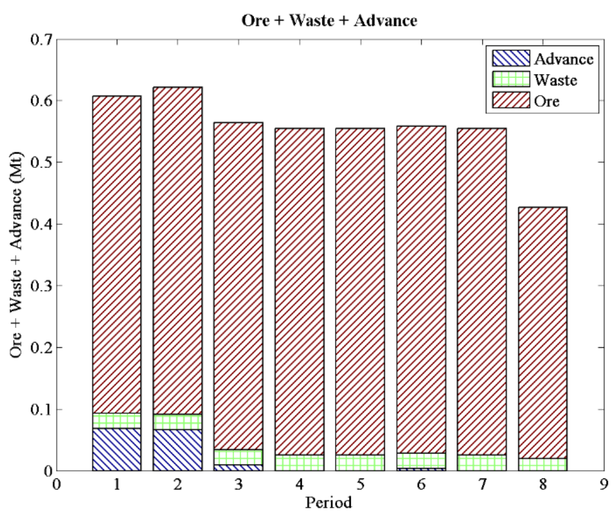

Fig. 6. Total tonnage over 8 periods.

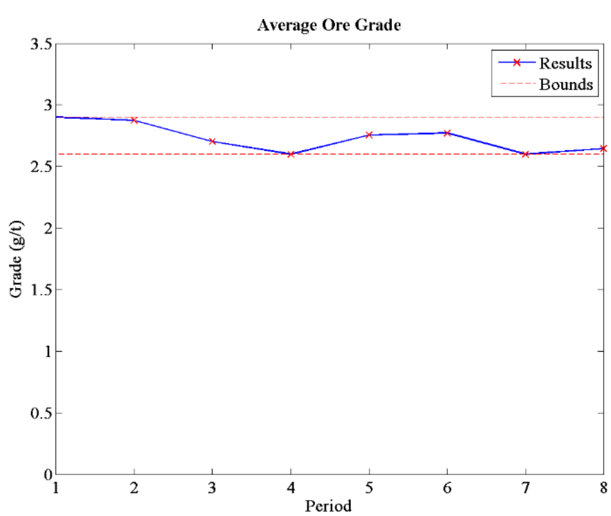

Fig. 7. Average ore grade over 8 periods.

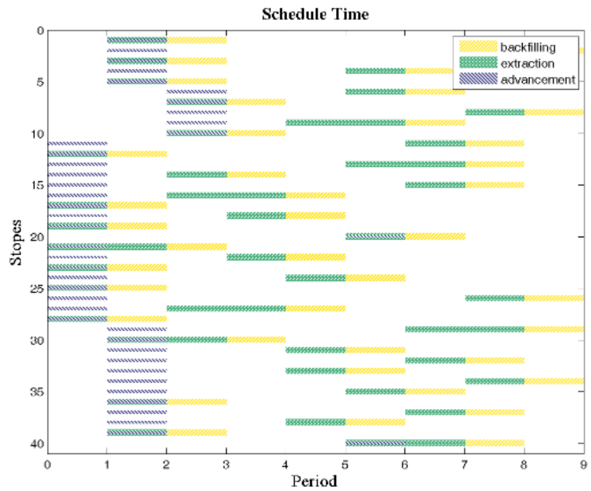

Fig. 8. Stopes mining schedule.

5.2. Case study 2

The case study 2 is a large-scale underground gold mining operation using cut-and-fill mining method. Annual production is approximate 3.6 million tons of ore, and 8000 kg of gold. The case study 2 is carried out using the actual stopes data as well as production parameters in order to facilitate the comparison research with actual mining strategy. The case study area contained 104 stopes, distributed in 8 levels. The size of the stope is 50 m in length, 40 m in height, and the width varies according to the width of the orebody ranging from 10 m to 30 m. the total tonnage of material in this area is almost 11 Mt with an average density of 2.78 t/m^3. Almost all the rock mass is ore with an average grade of 2.35 g/t. The technical and economic parameters used for evaluating the models are shown in Table 2.

The objective function of the model is to maximize the NPV at a discount rate of 0.08, while all the constraints are satisfied during a three years mining operation. The MILP model was solved by using the Matlab (2017a) (MATLAB and Release, 2016) programming and CPLEX solver (IBM., 2015) on a machine with Intel(R) Xeon(R) CPU E5-1650 v4 processor and 64 Gigabytes of RAM. The performance of the proposed MILP model is analyzed on the basis of the NPV, stability as well as practicality of the generated schedules. Comparisons between the actual mining strategy and the optimized schedule are presented herein.

The results of the optimized MILP model are shown in Fig. 9 to Fig. 12. Production schedule is produced per six monthlies instead of yearly in this case study. The tonnage and average grade of ore within each period are shown in Fig. 9, Fig. 10 and Fig. 11. It is obvious that the model can keep the mining capacity at the upper bound; moreover, it avoids production fluctuations benefiting from the mining and processing fluctuation constraints so as to provide a more stable ore and metal supply for the plant.

Fig. 12 illustrates the stope production phases is sequential in specific order of advancement, extraction and backfilling, in the meantime, extraction phrases are all within two time periods. The maximum number of the simultaneous active levels is limited to four. When considering control of active levels, the optimized scheduling production is more concentrated and requires significantly less equipment investment and movement.

The graphs shown in Fig. 13, Fig. 14 and Fig. 15 show the cumulative metal, rock production, and cash flow comparison between optimized results and actual mining strategy over six time periods. The MILP optimized schedule improved NPV by $ 17.15M, about 5.29 percent on the actual mining strategy. Apparently, the results show a significant improvement in metal production and NPV while less total tonnages mined. This reflects that, the MILP model has the motivation to extract high-grade stopes earlier to generate better cash flow.

Table 2. The technical and economic parameters for case study 2.

No.	Description	Value
1	Mining cost ($/t)	33.97
2	Processing cost ($/t)	9.82
3	Backfilling cost ($/t)	6.59
4	Advancement cost ($/t)	32.12
5	Price of gold ($/g)	45.19
6	Discount rate (%)	8.00
7	Mining recovery (%)	92.47
8	Mining dilution (%)	4.50
9	Mining loss (%)	8.00
10	Maximum grade (g/t)	2.50
11	Minimum grade (g/t)	2.10
12	Maximum mining capacity (Mt/Period)	1.80
13	Maximum processing capacity (Mt/Period)	1.80
14	Maximum advancement capacity (Mt/Period)	0.15
15	Maximum hoisting capacity (Mt/Period)	1.95
16	Maximum extraction duration (Periods)	2
17	Maximum active levels	4

Note: Period = six months.

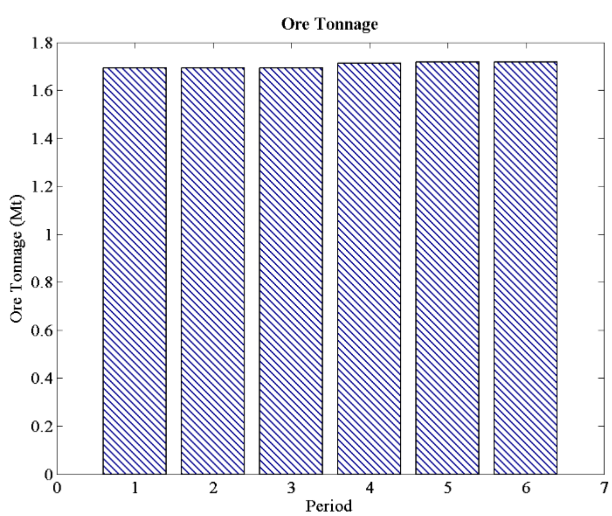

Fig. 9. Ore tonnage over 6 periods.

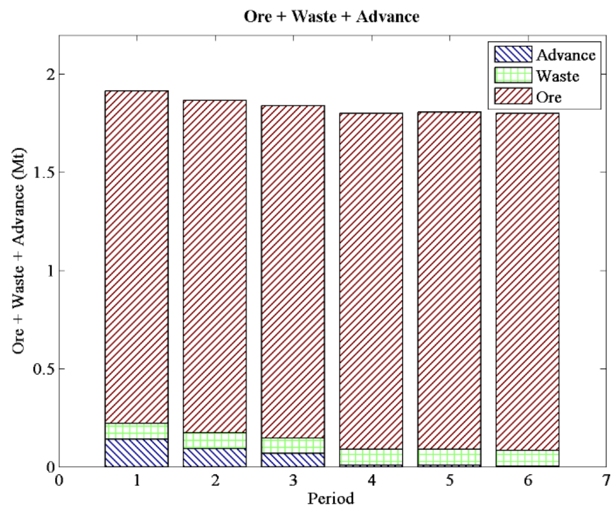

Fig. 10. Total tonnage over 6 periods.

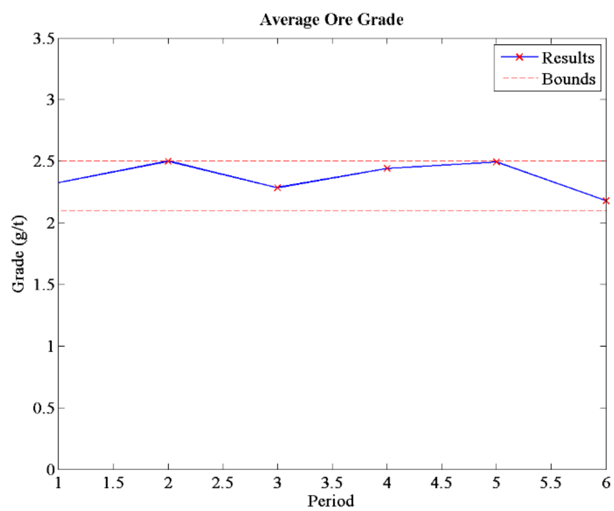

Fig. 11. Average ore grade over 6 periods.

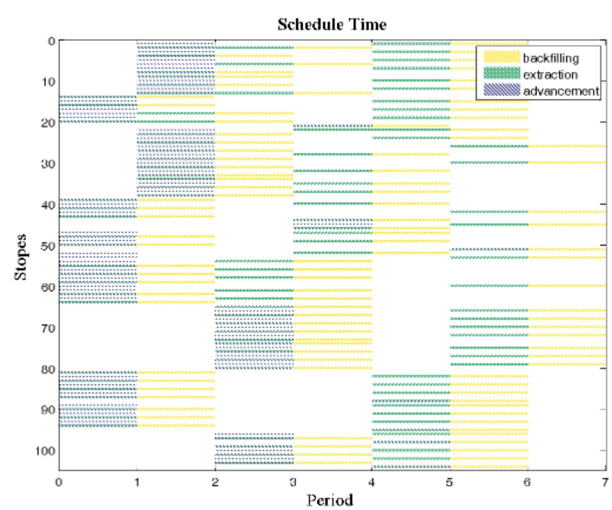

Fig. 12. Stopes mining schedule.

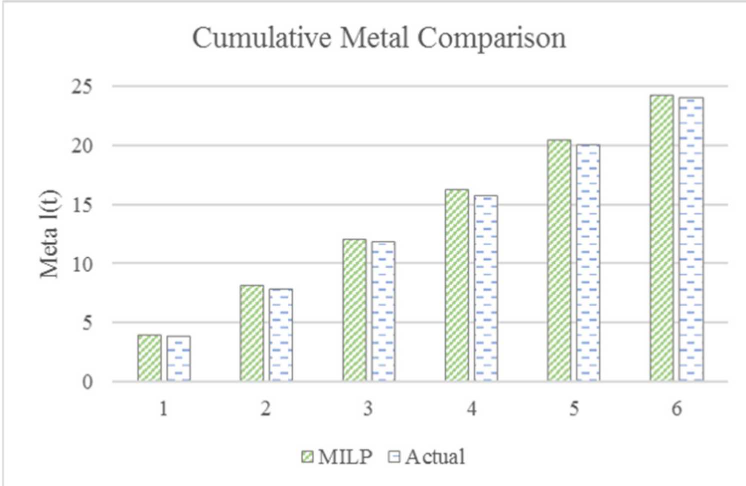

Fig. 13. Cumulative metal comparison.

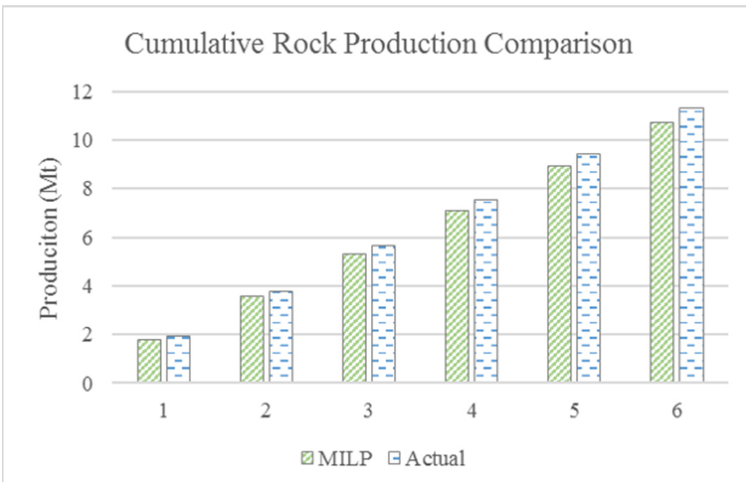

Fig. 14. Cumulative rock production comparison.

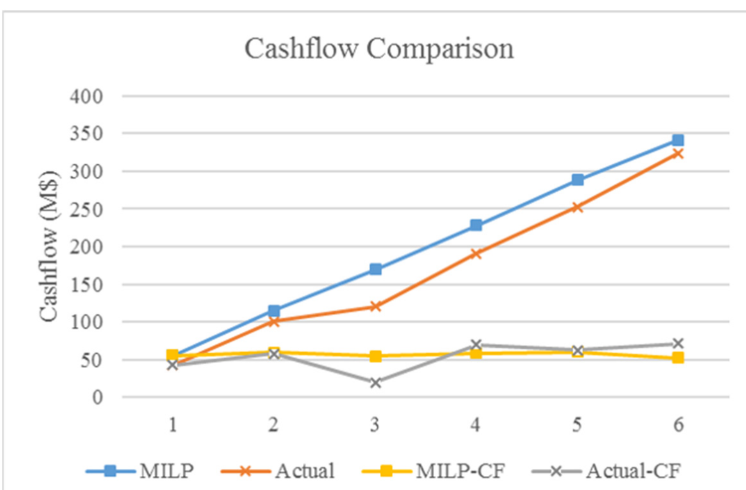

Fig. 15. Cumulative cash flow comparison.

6. Conclusions

This research presented an optimization framework for cut-and-fill mine production scheduling using mixed integer linear programming. The objective function of the model is to maximize the NPV of the operation while meeting all mining resource and operational constraints.

In addition to the mining and processing productivity constraints, and stope development and extraction sequencing constraints, the use of mining and processing tonnage fluctuation constraints ensure full utilization of mining equipment and further avoid excessive fluctuations during the production periods. Since the extraction duration and simultaneous active levels in each time period are controlled in the optimization, the production schedule obtained from the MILP model is more concentrated and requires significantly less equipment movement. All these operational constraints make the new MILP model more flexible and adjustable to different solutions and processes in relation to cut-and-fill mining production scheduling, as well as other underground mining variations.

The optimization formulation was implemented in a Matlab/CPLEX environment. Apart from the first and the last periods in two case studies, the main periods all reached the processing capacity at the upper bound (0.55 Mt/Period in case study 1 and 1.8 Mt/Period in case study 2). The average grade is reasonably within the desired range (2.6g/t to 2.9 g/t in case study 1 and 2.1g/t to 2.5 g/t in case study 2) in each period. The results compared with the actual mining strategy demonstrate the potential benefits of production schedule optimization for cut-and-fill mining using MILP. The comparison shows a 5% to 7% improved NPV resulting from mining higher grades and processing less tonnes thereby generating a better cash flow.

7. References

[1] Alford, C., Brazil, M., and Lee, D. (2007). Optimisation in underground mining. *Handbook of operations research in natural resources*, 561-577.

[2] Ben-Awuah, E. and Askari-Nasab, H. (2011). Oil sands mine planning and waste management using mixed integer goal programming. *International Journal of Mining, Reclamation and Environment*, *25* (3), 226-247.

[3] Ben-Awuah, E. and Askari-Nasab, H. (2013). Incorporating waste management into oil sands long term production planning. *Mining Technology*, *122* (1), 33-45.

[4] Ben-Awuah, E., Richter, O., Elkington, T., and Pourrahimian, Y. (2016). Strategic mining options optimization: Open pit mining, underground mining or both. *International Journal of Mining Science and Technology*, *26* (6), 1065-1071.

[5] Brickey, A. J. (2015). *Underground production scheduling optimization with ventilation constraints.* Colorado School of Mines.

[6] Chanda, E. (1990). An application of integer programming and simulation to production planning for a stratiform ore body. *Mining Science and Technology*, *11* (2), 165-172.

[7] Chatterjee, P. and Sridhar, C. (1986). Computer-aided mine design and planning for underground mines. *CIM BULLETIN*, *79* (893), 55-60.

[8] Denby, B. and Schofield, D. (1995). *The use of genetic algorithms in underground mine scheduling*. in Proceedings of Proceedings of the 25th international symposium application of computers mineral industries, pp. 389-394.

[9] Ebbels, A. M. (2016). Optimising underground mining strategies. *Australasian Institute of Mining and Metallurgy Publication Series*, p 60-65.

[10] Hamrin, H., Hustrulid, W., and Bullock, R. (2001). Underground mining methods and applications. *Underground mining methods: Engineering fundamentals and international case studies*, 3-14.

[11] Hu, N. L., Li, Y., Li, G. Q., and Yao, X. L. (2013). Optimization of open-pit-mining operational planning by using a particle swarm algorithm. *Journal of University of Science & Technology Beijing*, *35* (4), 537-543.

[12] IBM. (2015). IBM ILOG CPLEX optimization studio CPLEX user's manual, Version 12 Release 6. IBM Corp Armonk, NY.

[13] Jawed, M. and Sinha, D. (1985). System approach to production planning and scheduling with special reference to coal mines-A case study. *Journal of Mines, Metals and Fuels, February*, 37-41.

[14] Jie, H., Nai-Lian, H. U., Guo-Qing, L. I., Zhao-Yang, M. A., Di, L. I., and Jian, G. (2016). Dynamic optimization of production plans for multi-metal underground mines. *Chinese Journal of Engineering*.

[15] Kumral, M. (2004). Genetic algorithms for optimization of a mine system under uncertainty. *Production Planning & Control*, *15* (1), 34-41.

[16] Li, S. (2009). Application of Linear Programming Approach in Mining Schedule Planning. *Metal Mine*.

[17] Lizotte, Y. and Elbrond, J. (1985). Optimal layout of underground mining levels. *CIM BULLETIN*, *78* (873), 41-48.

[18] MATLAB and Release, S. T. (2016). The MathWorks, Inc. United States Natick, Massachusetts.

[19] Musingwini, C. (2016). Optimization in underground mine planning-developments and opportunities. *Journal of the Southern African Institute of Mining and Metallurgy*, *116* (9), 809-820.

[20] Nehring, M. and Topal, E. (2007). Production schedule optimisation in underground hard rock mining using mixed integer programming. *Proceedings-Project Evaluation. The Australasian Institute of Mining and Metallurgy, Melbourne*, 169-175.

[21] Nehring, M., Topal, E., and Knights, P. (2010). Dynamic short term production scheduling and machine allocation in underground mining using mathematical programming. *Mining Technology*, *119* (4), 212-220.

[22] Nehring, M., Topal, E., and Little, J. (2010). A new mathematical programming model for production schedule optimization in underground mining operations. *Journal of the Southern African Institute of Mining and Metallurgy*, *110* (8), 437-446.

[23] O'Sullivan, D. and Newman, A. (2014). Extraction and backfill scheduling in a complex underground mine. *Interfaces*, *44* (2), 204-221.

[24] Osanloo, M., Gholamnejad, J., and Karimi, B. (2008). Long-term open pit mine production planning: a review of models and algorithms. *International Journal of Mining, Reclamation and Environment*, *22* (1), 3-35.

[25] Pendharkar, P. C. (1997). Fuzzy linear programming model for production planning in coal mines. *Computers and Operations Research*, *24* (12), 1141-1149.

[26] Pourrahimian, Y. (2013). *Mathematical programming for sequence optimization in block cave mining.* University of Alberta (Canada).

[27] Pourrahimian, Y. and Askari-Nasab, H. (2014). An application of mathematical programming to determine the best height of draw in block-cave sequence optimisation. *Mining Technology*, *123* (3), 162-172.

[28] Pourrahimian, Y., Askari–Nasab, H., and Tannant, D. (2012). Mixed–Integer Linear Programming formulation for block–cave sequence optimisation. *International Journal of Mining and Mineral Engineering*, *4* (1), 26-49.

[29] Topal, E. (2003). Advanced underground mine scheduling using mixed integer programming.

[30] Trout, L. (1995). *Underground mine production scheduling using mixed integer programming*. in Proceedings of 25th International APCOM Symposium, Melbourne, Australia, pp. 395-400.

[31] Yao, X. L., Hu, N. L., Zhou, L. H., and Li, Y. (2011). Ore blending of underground mines based on an immune clone selection optimization algorithm. *Journal of University of Science & Technology Beijing*, *33* (5), 526-531.