Production Scheduling with Horizontal Mixing Simulation in Block-Cave Mining

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ABSTRACT

High production rates and low operating costs highlight block caving as one of the favorable underground mining methods. However, the uncertainties involved in the material flow make it complicated to optimize the production schedule for such operations. In this paper, a stochastic mixed integer linear optimization model is proposed in order to capture horizontal mixing that occurs among the draw columns within the production scheduling optimization. The goal is to not only consider the material above each drawpoint for extraction from the same drawpoint, as traditional production scheduling does, but also to capture the horizontal movements among the adjacent draw columns. In this approach, different scenarios are generated to simulate the horizontal mixing among adjacent slices within a neighborhood radius. The best height of draw for draw columns is also calculated as part of the optimization. The model is tested for a block-cave mine with 640 drawpoints to feed a processing plant for 15 years. The resulting NPV is 473M$ while the deviations from the targets in all scenarios during the life of the mine are minimized. Using the proposed model will result in more reliable mine plans as it takes the horizontal mixing into account in addition to achieving the production goals. Using different penalties for grade deviations shows that the model is a flexible tool in which the mine planners can achieve their goals based on their priorities.

1. Introduction

Gravity is the main driver in block-cave mining: an undercut is developed beneath the orebody, the rock fractures because of the created empty space, the caved material is extracted using the designed drawpoints, and finally, the ore is transported to the surface for processing (Fig 1). For scheduling purposes, the resulting block model from the resource estimation is used to create a slice model in which each slice is an aggregation of several blocks (Fig 2). Likewise the blocks, each slice is represented by its grade, density, volume, and a decision variable (for the mathematical modeling). Production scheduling for this type of operation determines the mining direction in the layout as well as the amount of material to extract from each of the drawpoints in each period.

Similar to open pit mining, production scheduling has significant impacts on the feasibility of the project as it directly controls the cash flow. However, in block caving, because of the uncertainties involved in the flow of the caved material, the material movement influences the production (Fig 3). As a result, the production scheduling optimization is more complicated, and an optimum mine plan without consideration of the material flow can be impractical.

In this paper, a stochastic model is presented that aims to capture some of the uncertainties involved in material flow to achieve production schedules that can be applied in real operations. To do this, the Height of Interaction Zone (HIZ) is used to consider the material flow within the optimization model (Fig 3). The slices that fall into the HIZ are directly extracted from their associated drawpoint which means no horizontal mixing occurs. Different scenarios are generated to simulate the potential horizontal movements of the material within an adjacency radius from each slice. The aim is to achieve production targets and maximize the net present value (NPV) of the project as well as consider the horizontal mixing between draw columns as the caved rock is extracted from the

This paper, in its entirety, has been submitted for publication in the international Journal of Mining Science

drawpoints. Integer and continuous decision variables in the stochastic mixed-integer optimization model (SMIOM) represent slices, drawpoints, and deviations. In addition, the best height of draw (BHOD) is calculated as part of the optimization, which means that this part of optimization is not separated from the production scheduling.

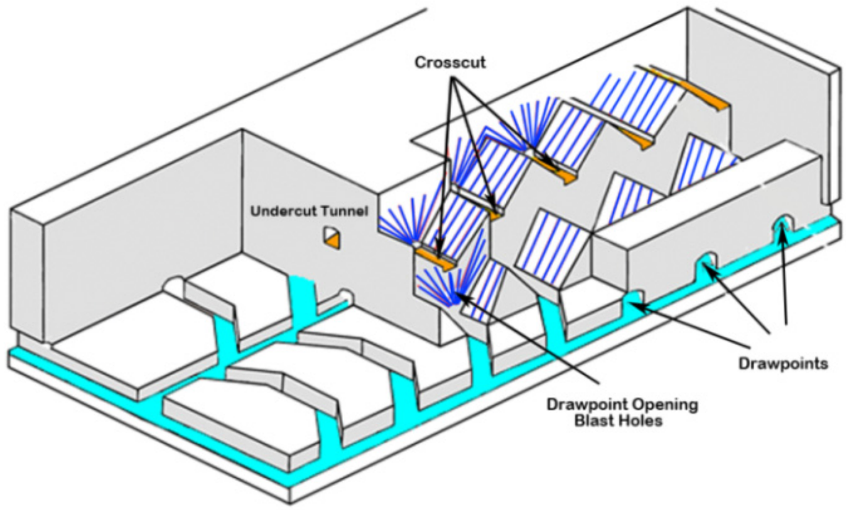

Fig 1. Block-cave mining

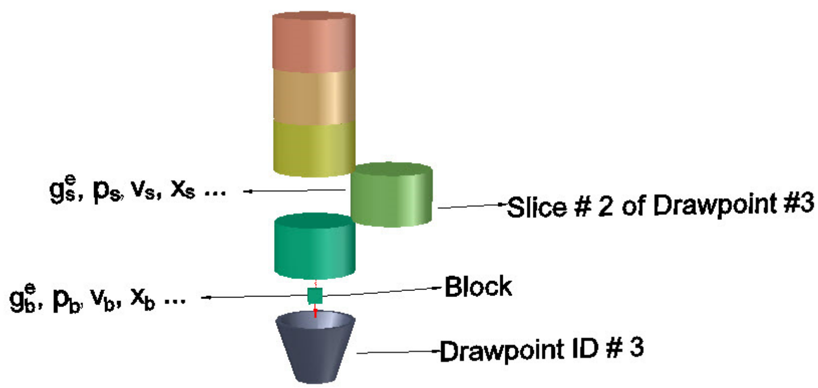

Fig 2. Slice model

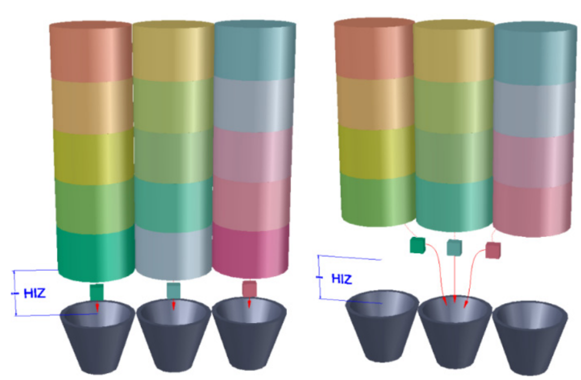

Fig 3. Horizontal mixing and its impact on production: below HIZ (left) and above HIZ (right)

2. Literature review

Since the proposed model includes horizontal mixing, literature on both mine planning optimization and material flow is presented in the following sections.

2.1. Production scheduling optimization in block caving

Mine planning optimization for both open pit and underground mining has been a focus of many studies. For block-cave mining, researchers have mostly used mixed-integer linear programming (MILP); also, there are some models with application of Linear programming (LP) and quadratic programming (QP) for optimizing the production scheduling problem (Khodayari & Pourrahimian, 2015b). A summarized review of the existing research in this area is discussed in the following.

Song (1989) was one of the first researchers who used mathematical programming for block-cave operations. He used MILP for sequence optimization at the Tong Kuang Yu mine in China. Guest et al. (2000) formulated a long-term production scheduling model for a diamond mine in South Africa using LP and MILP to maximize the NPV of the project. Rubio (2002) had two strategic goals in his model: to maximize the NPV and to achieve an optimum mine life for the project. Rahal et al. (2003) developed an MILP goal program to minimize deviation from ideal draw coupled with accomplishing a production target. In their model, material-mixing impact on the long-term plan was assumed insignificant. Rahal (2008) used MILP to control monthly production and cave shape, he tried to integrate geotechnical constraints in the production schedule by draw rate constraint; it was shown that strict relative draw rates (RDR) result in more uniform cave shape but less production rates. Using an MILP model by Rahal et al. (2008) to optimize the production schedule of Northparkes E48 mine showed that mathematical programming models could generate optimum results while being practical. A mixed-integer programming model (MIP) was developed by Weintraub et al. (2008) to determine the times in which each block is extracted within the draw columns. The production schedule was based on a five-year plan. In their model, they used aggregation methods to reduce the size of the model and as a result the solution time. Smoljanovic et al. (2011) adapted a model from open pit mining to find the optimum sequence of drawpoints in block-cave layout. Testing the model for a case study with 322 drawpoints, they proved that an optimum sequence could improve the objective function up to 50%.

Three integer programming models were developed by Parkinson (2012) to optimize the sequence of extraction. Using the models for two datasets, she concluded that the proposed models could help to find the opening sequence but it was not easy to find the best one among different models. Pourrahimian et al. (2012) developed MILP models to maximize the NPV of the block caving project during the life of the mine. They also proposed three formulations to optimize the production schedule in three levels of resolution: cluster level, drawpoint level and drawpoint-and-slice level (Pourrahimian et al., 2013). The goal was to provide the mine planner with different alternatives based on their expected solution time and level of resolution. Alonso-Ayuso et al. (2014) tried to capture the uncertainty of copper price to maximize the NPV of a block-cave mining project in Chile using an MIP model. Their results showed the advantage of stochastic models in mining projects. Khodayari and Pourrahimian (2014) proposed a model in which the best height of draw (BHOD) is determined as part of the sequence optimization in block caving. In other words, the mining reserve was an output of the optimization, not an input.

Nezhadshahmohammad et al. (2017) proposed an MILP model to optimize the mine plan with consideration of draw rate curve as a geotechnical constraint. Nezhadshahmohammad et al. (2017) used a clustering model to optimize a long-term production schedule for block caving operations. They also used a draw control system within the neighborhood while maximizing the NPV of the project (Nezhadshahmohammad et al., 2017). Malaki et al. (2017) applied sequential Gaussian simulation and MILP to find the best level of extraction in block caving while considering the grade estimation uncertainties. An LP model was developed by Diering (2004) to determine the tonnages to extract from each drawpoint; his goal was to maximize the NPV. He also proposed a QP model to control the shape of the cave by minimizing the deviations between the production and target tonnages (Diering, 2012). The interaction between neighboring drawpoints was not considered in his model. Khodayari and Pourrahimian (2016) proposed a quadratic programming model for production scheduling, the goal was to achieve a uniform extraction profile in order to reduce the potential dilutions in the production.

Among the above-mentioned models, the following were the most common constraints: allowable output per shift, number of working drawpoints per shift, grade limitations, tonnage limitations, draw rate, draw ratio, draw life, and mining precedence. In addition to what presented here, the authors of this paper have published a detailed literature review on application of mathematical programming in block-caving in (Khodayari & Pourrahimian, 2015b).

2.2. Material flow

The literature on material flow can be divided into three categories: numerical models, pilot tests, and full-scale experiments. Castro et al. (2009), as part of the development of FlowSim software, used numerical methods to simulate the gravity flow of caved rock. They calibrated the proposed model with real data from two mining operations in Chile. Another numerical model was developed by Pierce (2010), he used distinct element method to identify different zones of movement (or isolations) in the caved material. A stochastic approach was proposed by Gibson (2014) to study the material flow in block-cave operations. He used Pascal cone theory to assess the probabilities of material that move within their neighborhood when empty space is created because of extraction from drawpoints. He found it useful to use stochastic models instead of Finite Element methods although it needs a certain amount of information about the rock mass. The cell size was one of the main criteria in the Gibosn's model, and any changes can remarkably affect the results.

Castro et al. (2014) designed a test setup to study the flow mechanisms of cohesionless material if extracting from a single drawpoint. It was shown that the fragmentation size, the diameter of the opening, and the vertical load (the amount of material above) are the factors that affect the material flow. Jin et al. (2017) studied the shape of the ellipsoid draw by some pilot tests; they proposed a new methodology to predict the extraction and movement zone shapes and their change trends. Brunton et al. (2016) performed a full-scale experiment in Ridgeway Deeps and Cadia East block caving mines in Australia. They used markers to quantify and assess the development and shape of the extraction zone and find the mechanisms that control the flow behavior. It was shown that marker experiments can provide a good understanding about the development of the extraction zone both during undercutting and production stages. Garcés et al. (2016) applied smart marker technology in block 2 of Esmeralda mine, El Teniente to study the gravity flow. The main objectives were to check the interaction between extraction zones, estimate the mining recovery, evaluate the flow characteristics of caved rock, and develop a model for flow. Results showed that the extraction zone mainly depends on the fragmentation condition and the mine plan. It was also observed that the horizontal mixing is higher in the case of a non-concurrent draw.

In summary, a significant number of research exist in production scheduling optimization and material flow; however, both concepts have been investigated separately, and there is a gap of optimizing the mine plan with the material flow consideration. An optimized mine plan will provide a guide towards a successful operation, and a material flow simulation model can give us a valuable understanding of the caving process; however, these two will not be helpful enough if they are not studied simultaneously. The goal of this research is to consider the material flow uncertainties as part of the production scheduling optimization; in other words, to achieve an optimum production schedule in which not only the technical constraints are satisfied but also the horizontal mixing is modeled within the optimization.

3. Problem statement and formulation

In this section, the formulations of the proposed model are presented. The model includes technical constraints that are common among block caving operations; other project-specific constraints can be added to the model as it is needed.

3.1. Notation

- ***Indices***

$t \in \{1,...,T\}$	Index for scheduling periods
$s \in \{1,...,S\}$	Index for individual slices
$d \in \{1,...,D\}$	Index for individual drawpoints
$n \in \{1,...,N\}$	Index for individual scenarios
$a \in \{1,...,A\}$	Index for the adjacent drawpoints

- ***Variables***

$x_{s,t} \in \{0,1\}$	Binary decision variable that determines if slice *s* is extracted in period *t* ($x_{s,t}=1$) or not ($x_{s,t}=0$)
$y_{d,t} \in \{0,1\}$	Binary decision variable which determines whether drawpoint *d* in period *t* is active ($y_{d,t}=1$) or not ($y_{d,t}=0$)
$z_{d,t} \in \{0,1\}$	Binary decision variable which determines whether drawpoint *d* at period *t* has started its extraction ($z_{d,t}=1$) or not ($z_{d,t}=0$)
$dev_{n,t}^{uo} \in [0,\infty)$	Excessive amount from the target production ore in scenario n at period *t*
$dev_{n,t}^{lo} \in [0,\infty)$	Deficient amount from the target production ore in scenario n at period *t*
$dev_{n,t}^{ug} \in [0,\infty)$	Excessive amount from the target production grade (the metal content) in scenario *n* at period *t*
$dev_{n,t}^{lg} \in [0,\infty)$	Deficient amount from the target production grade (the metal content) in scenario *n* at period *t*

- ***Model Parameters***

g_s^e	Grade of element *e for* slice *s*
Eg_s^e	Expected grade of element *e* for slice *s* based on all scenarios
ρ_s	Density of slice *s* (t/m^3)
$E\rho_s$	Expected density of slice *s* (t/m^3) based on all scenarios
V_s	Volume of slice *s* (m^3)
t'	Current period
N_{Sd}	Number of slices associated with drawpoint *d*
d_p	Drawpoint that its extraction should be started before drawpoint d based on the defined mining direction of extraction
s_p	Slice which is located below the slice *s* and must be extracted before *s*

- ***Input Parameters***

$\hat{g}_t^e$	The target grade of production for element *e* in period *t,* which is defined based on the production goals and processing requirements
$o_{s,t}$	The ore production resulting from extraction slice *s* in period *t*
$\hat{o}_t$	The target ore production in period *t* which is defined based on the production goals and processing requirements
$\min Act_t$	Minimum number of active drawpoints in period *t*
$\max Act_t$	Maximum number of active drawpoints in period *t*

M	A big number, this is chosen based on the maximum number of slices in draw columns
DL_{min}	Minimum drawpoint life
DL_{max}	Maximum drawpoint life
DR_{min}	Minimum draw rate
DR_{max}	Maximum draw rate
i	Discount rate
$Ramp-up$	Ramp-up time
$ScenNum$	Number of scenarios
$DP_{d,t'}$	Drawpoint depletion percentage: the portion of draw column associated with drawpoint d that has been extracted from $t=1$ to t'
p^e	Price of element e ($/tonne)
c	Operating costs ($/tonne)
c^{lo}	Cost (penalty) for deficient amount of ore production from the target ($)
c^{uo}	Cost (penalty) for excessive ore production from the target ($)
c^{lg}	Cost (penalty) for deficient production grade from the target ($)
c^{ug}	Cost (penalty) for excessive production grade from the target ($)
r^e	Recovery of the processing plant for element e (%)
R	Adjacency radius

3.2. Preliminaries

The scenarios are generated based on the neighborhood concept within the caved material. For each slice, a horizontal neighborhood is defined in a radius (adjacency radius) from the center of the slice; all of the slices that fall into that neighborhood (the distance from their center to the center of slice s is less than or equal to R) are called adjacent slices for that slice (Fig 4). Therefore, a population (P_s) is created for slice s that includes slice s and all of its adjacent slices. In the next step, random samples are generated from the population to simulate different scenarios for each slice in terms of horizontal movement while extracting from each drawpoint during the operations.

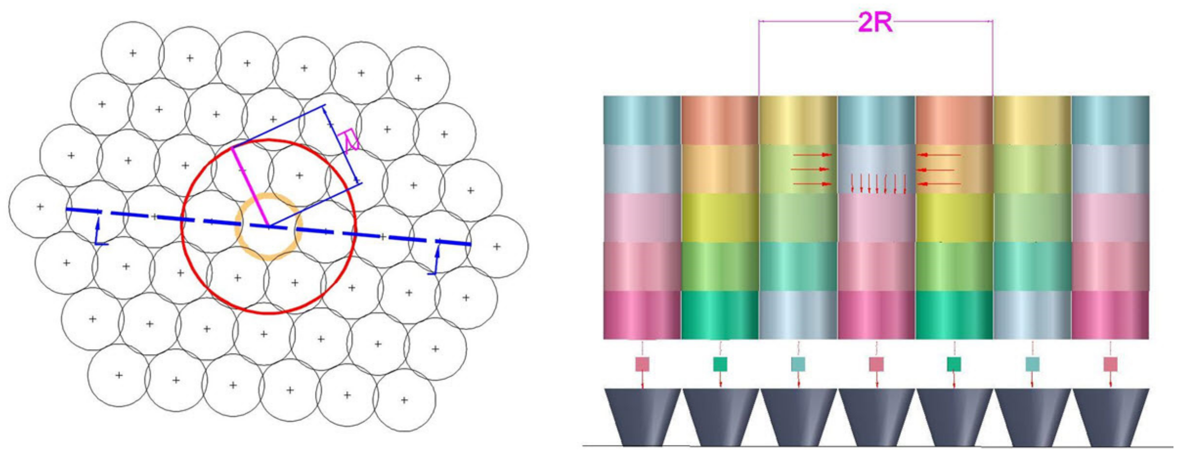

Fig 4. Adjacency concept: plot view of adjacent drawpoints (left Fig) and adjacent slices (right Fig); in the plan view, the black circles represent the drawpoints with the cross sign as their center point, the red circle is the adjacent neighborhood for the orange-colored drawpoint

In this research, only the horizontal movements are considered as the neighborhood is defined in two dimensions (2-D). The ore tonnage ($P_{s,o}$) and grade ($P_{s,g}$) populations for each slice are defined based on the adjacent slices, and then the scenarios are generated by sampling from those populations. The following equations present the process:

$$P_{s,o}=\{o_1,...,o_A\} \qquad s\in\{1,...,S\} \tag{1}$$

$$o_{s,n}=\text{sample}(P_{s,o}) \qquad \forall n\in\{1,...,N\},\ s\in\{1,...,S\} \tag{2}$$

$$P_{s,g}=\{g_1,...,g_A\} \qquad s\in\{1,...,S\} \tag{3}$$

$$g_{s,n}=\text{sample}(P_{s,g}) \qquad \forall n\in\{1,...,N\},\ s\in\{1,...,S\} \tag{4}$$

As an example, for slice *s* with an original grade of 0.51% and considering the closest 19 slices that are located in its adjacency (based on the adjacency radius), the population is defined as a set of the original grades of all adjacent slices and slice s itself (Table 1). In the next step, the scenarios, in this case 15 scenarios, are generated (Table 1).

As shown in Fig 3, this model assumes that in each draw column, the material that is located in the interaction zone is extracted from the same drawpoint (the drawpoint associated with that draw column); in other words, horizontal mixing will not occur for the slices that are located within the HIZ. HIZ can be calculated based on the curve that was presented by Laubscher (1994).

Table 1. An example of creating a population within a slice's neighborhood and Generating 15 scenarios

Population ID	Grade (%)	Scenarios	Grade (%)
1	0.51	1	0.45
2	0.71	2	0.51
3	0.61	3	0.64
4	0.71	4	0.51
5	0.6	5	0.71
6	0.64	6	0.44
7	0.56	7	0.47
8	0.41	8	0.56
9	0.49	9	0.43
10	0.41	10	0.51
11	0.46	11	0.49
12	0.44	12	0.41
13	0.45	13	0.49
14	0.47	14	0.49
15	0.45	15	0.71
16	0.49		
17	0.44		
18	0.49		
19	0.44		
20	0.43		

3.3. Objective function

The objective function is defined as follows:

$$Maximize\ \sum_{t=1}^{T}\sum_{s=1}^{S}E\{(NPV_{s,t})\}\,x_{s,t}-\sum_{t=1}^{T}\sum_{n=1}^{N}\{Ore\ deviations\}_{n,t}-\sum_{t=1}^{T}\sum_{n=1}^{N}\{Grade\ deviations\}_{n,t}$$

$$=\sum_{t=1}^{T}\sum_{s=1}^{S}\left(\frac{\left(p^{e}\times r^{e}\times Eg_{s}^{e}-c\right)\times E\rho_{s}\times V_{s}}{(1+i)^{t}}\right)x_{s,t}$$
$$-\sum_{t=1}^{T}\sum_{n=1}^{N}\frac{1}{N}\left(\frac{c^{lo}\times dev_{n,t}^{lo}+c^{uo}\times dev_{n,t}^{uo}}{(1+i)^{t}}\right)-\sum_{t=1}^{T}\sum_{n=1}^{N}\frac{1}{N}\left(\frac{c^{lg}\times dev_{n,t}^{lg}+c^{ug}\times dev_{n,t}^{ug}}{(1+i)^{t}}\right) \quad (5)$$

The first part of the equation (5) maximizes the NPV of the project; this is calculated based on the smallest units of production, which are the slices. Decision variable $x_{s,t}$ is a member of the first set of decision variables and represents slice s in period t; it is 1 if slice s is extracted in period t or zero if it is not. The second and third parts minimize the deviations of the production from target tonnages and grades, respectively (during the life of the mine). The expected values of tonnage and grade are considered for calculation of NPV and both the revenue and deviations are discounted by i. Four penalties control the deviations in the objective function: two for excessive and deficient amounts of tonnage (c^{lo}, c^{uo}) and two for excessive and deficient amounts of grade (c^{lg}, c^{ug}) from the targets.

3.4. Constraints

Two types of constraints, logical and technical constraints, are defined and discussed in this section.

- *Logical constraints*

Logical constraints connect the decision variables. The first set of decision variables, $x_{s,t}$, is associated with the slices and the second set, $y_{d,t}$, is associated with the drawpoints. The following equations show the logic:

$$y_{d,t}-M\times\sum_{s=1}^{N_{Sd}}x_{s,t}\leq 0 \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \quad (6)$$

$$\sum_{s=1}^{N_{Sd}}x_{s,t}-M\times y_{d,t}\leq 0 \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \quad (7)$$

The third set of decision variables is used to define the continuous mining and precedence of extraction constraints. This set is defined using the draw percentage concept. $z_{d,t}$ represents drawpoint d in period t, which is 0 if draw percentage of drawpoint d at period t is zero and 1 if not; equations (8) to (10) connect the second and third set of decision variables.

$$DP_{d,t'}=\sum_{t=1}^{t'}y_{d,t} \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \quad (8)$$

$$DP_{d,t'}\leq M\times z_{d,t} \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \quad (9)$$

$$z_{d,t}\leq M\times DP_{d,t'} \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \quad (10)$$

where t' is the current period and the constraints have to be satisfied for all periods ($t\in\{1,...,T\}$). Technical constraints model the practical restrictions of the operations and are as follows:

- *Production targets*

Production tonnage is limited by the mining equipment, requirements of the processing plant, the market demand, and the goals of the management team. This constraint ensures that the extractions in different scenarios are as close as possible to the production targets during the life of the mine ($\hat{o}_t$).

$$\left\{\sum_{s=1}^{S}\rho_{s,n}\times V_{s}\times x_{s,t}\right\}-\hat{o}_{t}+dev_{n}^{lo}-dev_{n}^{uo}=0 \qquad \forall n\in\{1,...,N\},\ t\in\{1,...,T\} \quad (11)$$

- *Grade targets*

Production grade is limited by the requirements of the processing plant and it is difficult to predict in block-cave mining because of the material flow. This constraint ensures that the production grade of element e is as close as possible to the target grade in different scenarios during the life of the mine ($\hat{g}_t^e$).

$$\sum_{s=1}^{S}(g_{s,n}-\hat{g}_t^e)\times\rho_{s,n}\times V_s\times x_{s,t}+dev_n^{lg}-dev_n^{ug}=0 \qquad \forall n\in\{1,...,N\},\ t\in\{1,...,T\} \qquad (12)$$

- *Reserve*

This is a control constraint that ensures the model does not extract more than the existing mining resource. It can also force the model to extract the whole resource (in case of equality) if maximum ore extraction is the goal.

$$\sum_{t=1}^{T}x_{s,t}\le 1 \qquad \forall s\in\{1,...,S\} \qquad (13)$$

- *Active drawpoints*

Because of operational considerations, only a certain number of drawpoints can be extracted (active) at the same time.

$$\min Act_t\le\sum_{d=1}^{D}y_{d,t}\le\max Act_t \qquad t\in\{1,...,T\} \qquad (14)$$

- *Mining direction*

The decision of the starting point and the mining direction within the designed layout of block-cave drives the development and operational priorities during the life of the mine. The mining direction, which is the horizontal sequence of extraction among drawpoints, can be chosen based on economic or geotechnical criteria. If there are no geotechnical limitations that dictate the direction, the economic value of the draw columns is the main driver of the starting point and the direction. Khodayari and Pourrahimian (2015a) proposed a methodology to find the optimum mining direction based on the economic value of the draw columns. This methodology is used in this paper to find the best starting point and the optimum mining direction, and then the sequence of extractions between the drawpoints in the given layout is defined. Equation (15) ensures that the sequence of extraction is achieved in the production schedule.

$$z_{d,t}\le y_{d_p,t} \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \qquad (15)$$

- *Slice sequence (vertical)*

This constraint controls the sequence of extraction between the slices in each of the draw columns during the life of mine.

$$x_{s,t}\le\sum_{t=1}^{t}x_{s_p,t} \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \qquad (16)$$

This equation ensures that in each period of t, slice s (in the draw column associated with drawpoint d) is extracted only if slice s_p, which is located beneath it, is already extracted in the periods before or at the same period t.

- *Continuous mining*

In block-cave mining, when a drawpoint is opened, its extraction has to be continued until the end of its life. Any discontinuation can cause compaction resulting in the secondary blasting or the loss of remaining ore in the draw column. In other words, if a drawpoint is opened, it is active in consecutive years with at least a minimum draw rate of DR_{min} utill it is closed.

$$y_{d,t} \le y_{d,t-1} + (1 - z_{d,t}) \qquad \forall d \in \{1,...,D\}, t \in \{1,...,T\} \tag{17}$$

- *Draw rate*

The draw rate, which is the total extraction from each drawpoint in each period of *t*, is limited to a minimum and maximum amount of material. The geomechanical parameters, the geometry of drawpoints and the ore passes, the mining equipment, and the production targets will dictate this constraint. This constraint keeps the draw rate of each drawpoint within a defined range (DR_{min} to $DR_{\max}$); however, it is possible to consider a production rate curve.

$$DR_{min} \times y_{d,t} \le \sum_{s=1}^{s_d} \rho_s \times V_s \times x_{s,t} \le DR_{max} \qquad \forall d \in \{1,...,D\}, t \in \{1,...,T\} \tag{18}$$

- *Draw life*

Drawpoints can be in production during a certain time which is called draw life. Draw life is dictated by geomechanical, operational, and economic parameters. This constraint ensures that each drawpoint is active during certain periods.

$$DL_{\min} \le \sum_{t=1}^{T} y_{d,t} \le DL_{\max} \qquad \forall d \in \{1,...,D\} \tag{19}$$

3.5. Solving the optimization model

MATLAB R2017a (2017) was used to build the model based on the mentioned objective function and the constraints. Then CPLEX IBM12.7.1 is used to solve the model. CPLEX uses branch-and-cut algorithm to solve the MILP problem. In this algorithm, a search tree consisting of nodes is created; the nodes represent LP subproblems to be solved and analyzed further. Nodes are processed until either no more active nodes are available or some limit has been reached. Creation of two nodes from a parent is called a branch. A cut is a constraint added to the model in order to reduce the size of the solution domain (IBM, 2017).

4. Numerical results

The proposed model was applied on a block-cave mining project with a production layout of 640 drawpoints (Fig 5). According to the designed layout, each draw column above a drawpoint is divided into a number of slices, and the total number of slices is 5,260. For the first set of decision variables, one decision variable has to be assigned to each slice in each period. Fig 6 and Fig 7 show tonnage and grade of the slice model, respectively. The mine life is 15 years with a starting production of 3 million tonnes and a ramp-up period of 3 years to reach the full production of 7 million tonnes per year as the target. The "datasample" function in MATLAB is used to generate 15 mixing scenarios. The sequence of extraction between the drawpoints was defined using the distribution of the economic value of the ore in the layout. In this case study, the materials with higher economic value are located on the northwest of the layout (the red area in Fig 8). The model was set in a way to start the extraction from the area with higher economic value, and then propagation of the cave occurs as the caved block is expanded through the rest of the mining layout (lateral extension). Also, a V-shape mining advancement was considered to have a concave face for the undercut level which provides better control of major structures and more secure undercut. Therefore, the extraction starts from location A (x=1962, y=1434) and moves towards southeast and northwest by two V-shapes at the same time (Fig 8). The penalties for deviations are set based on the target grades and tonnages. For the current caving operations, based on the targets, two different sets of penalties (case A and B) were tested to study the impact of the penalties on the results (Table 2). More details of the input parameters are presented in Table 3.

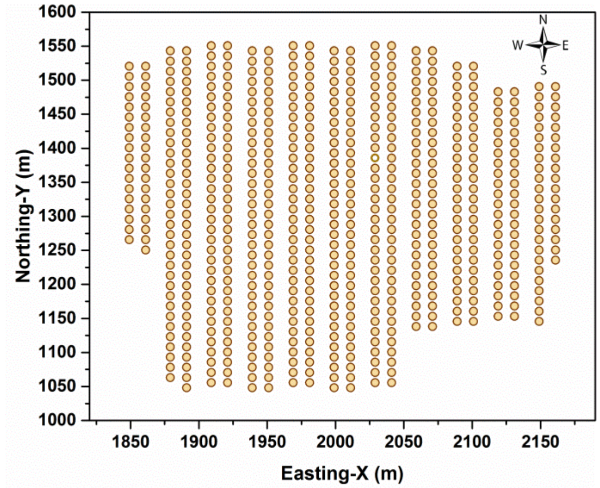

Fig 5. Layout of the drawpoints

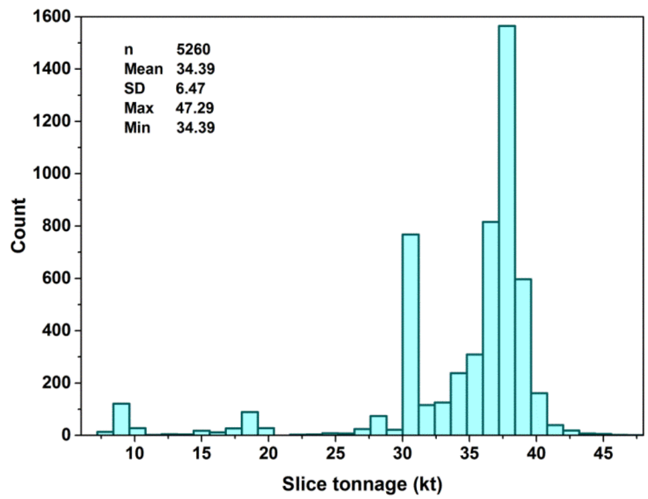

Fig 6. Histogram of the tonnage for the slice model

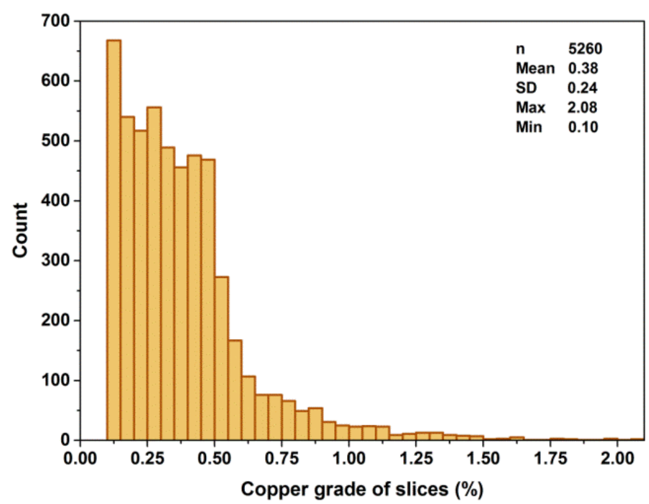

Fig 7. Histogram of the copper grade for the slice model

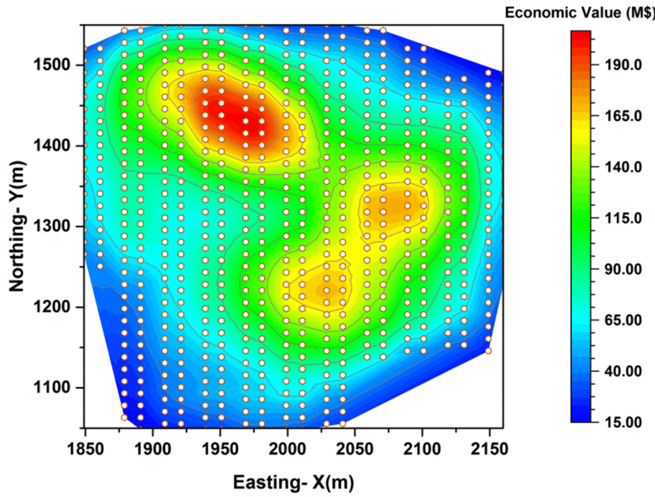

Fig 8. Distribution of economic value of ore in the mining layout

Table 2. Applying two sets of penalties for the case study

Case/Penalties	Deviations from grade ($)	Deviations from tonnage ($)
A	10	10
B	50	10

Table 3. Scheduling parameters for the case study

Parameter	Value	Unit	Description
T	15	Year	Production schedule timeline (the life of the mine)
$\hat{g}_t^{cu}$	0.52	%	Target production grade for copper (Cu) $\forall t \in \{1,...,T\}$
$\hat{o}_1$	3	Mt	The ore target at the first year of production
$\hat{o}_2$	5	Mt	The ore target at the second year of production
$\hat{o}_t$	7	Mt	The ore target $\forall t \in \{3,...,T\}$
Ramp-up time	3	Year	The time in which the production is increased from starting amount to the full capacity
minAct	70	-	Minimum number of active drawpoints per period
maxAct	200	-	Maximum number of active drawpoints per period
MIPgap	5	%	Relative tolerance on the gap between the best integer objective and the objective of the best node remaining
DL_{min}	0	Year	Minimum life of drawpoints
DL_{max}	6	Year	Maximum life of drawpoints
DR_{min}	30,000	Tonne/year	Minimum draw rate
DR_{max}	50,000	Tonne/year	Maximum draw rate
p^{cu}	5,000	$/tonne	Copper price
c	15	$/tonne	Operating costs per tonne of ore (Mining+Processing)
i	10	%	Discount rate
r^{cu}	85	%	Recovery of the processing plant for copper (cu)
N	15	-	Number of scenarios
R	50	meter	Adjacency radius
HIZ	76	meter	Height of Interaction Zone

The case study was solved with CPLEX 12.7.1 (Academic license) in a MATLAB environment on a computer with two Intel Xeon CPU E5-2630 version 0 @ 2.3 GHz processors and 64 GB RAM. The results show that both production tonnages and grades are as close as possible to the defined targets with the NPV of 473 M$ for the project (considering the first part of the objective function). The productions for the 15 generated scenarios follow the target production line with minor deviations in some periods (Fig 9).

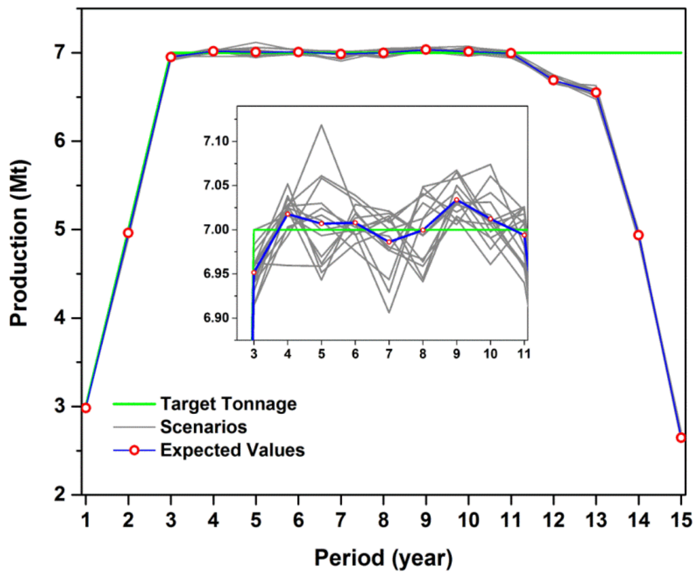

Fig 9. Production during the life of mine (case A)

However, compared to tonnage deviations, the resulting production grade shows visible deviations in some periods (Fig 10); this is because of the grade distribution in the orebody and the defined mining direction. In other words, the extraction starts from high-grade parts of the orebody, which has high economic values, and then moves to the low-grade area. Therefore, the production grade is higher than the target in the first few years and lower in the last few years of production. The model is defined in a way that the penalties control the deviations; a reduction in the deviations is expected by increasing the penalties although the NPV might decrease.

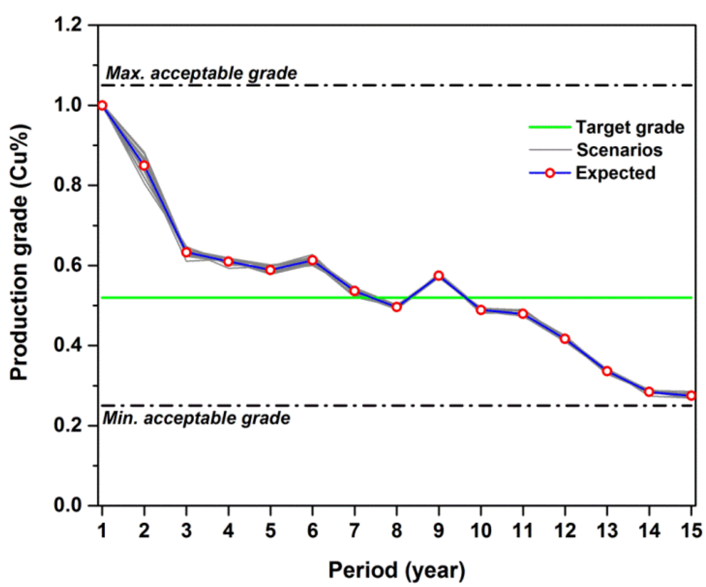

Fig 10. Production grade compared to the target grade (case A)

The model was run for the same case study and input parameters but with higher penalties for grade deviations. The results show a decrease in deviations of production grade compared to the target grade among all scenarios during the life of mine (Fig 11). On the other hand, the deviations of production tonnages from the targets have increased because, in this case, the weight of grade in the objective function is higher than tonnage (Fig 12). In addition, as expected, the NPV of the project decreased to 450 M$. As a result, the model is flexible and the mining engineer, or management team, can make the decision based on their priorities: less deviation from the target grades with less NPV or accepting some deviations for achieving a higher NPV.

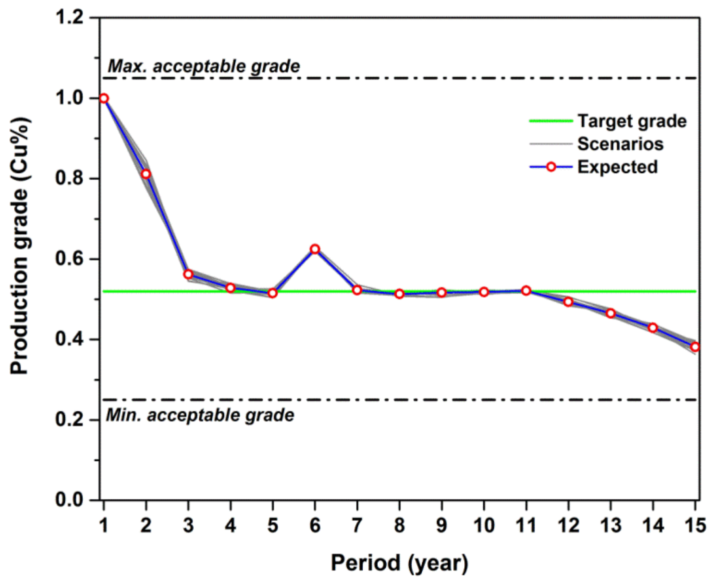

Fig 11. Production grade compared to the target grade (case B)

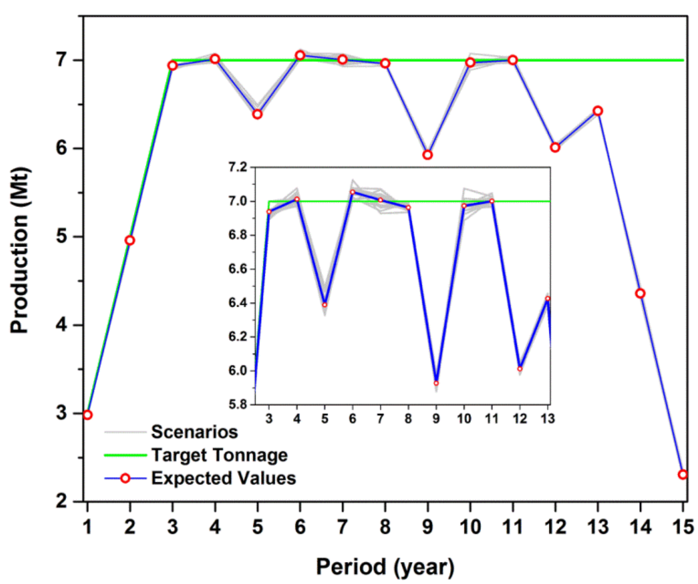

Fig 12. Production during the life of mine (case B)

The period in which extraction from each drawpoint is started (starting period) shows the resulting sequence of extraction in the mining layout (Fig 13). Comparing the defined mining direction in Fig 8 and the resulting sequence of extraction shows that the production schedule follows the desired mining direction in addition to satisfying the number of active drawpoints constraint (Fig 14). The

BHOD and as a result the mining reserve is calculated as one of the outputs of the production schedule. Fig 15 shows the BHOD among draw columns compared to their initial heights. Additionally, the mine reserve can be determined according to the BHODs as an output of optimization.

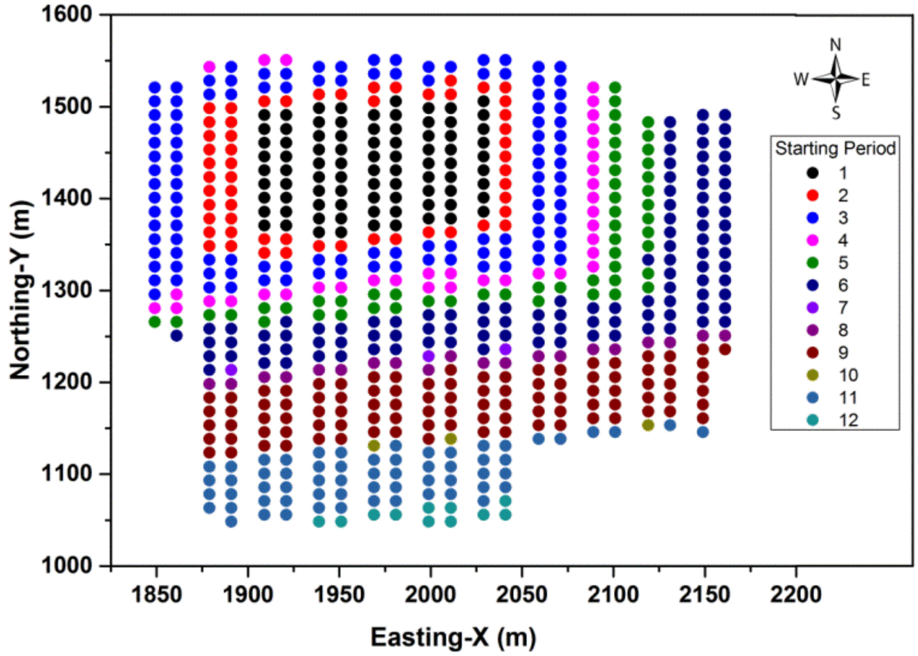

Fig 13. Sequence of extraction for drawpoints based on the defined mining direction

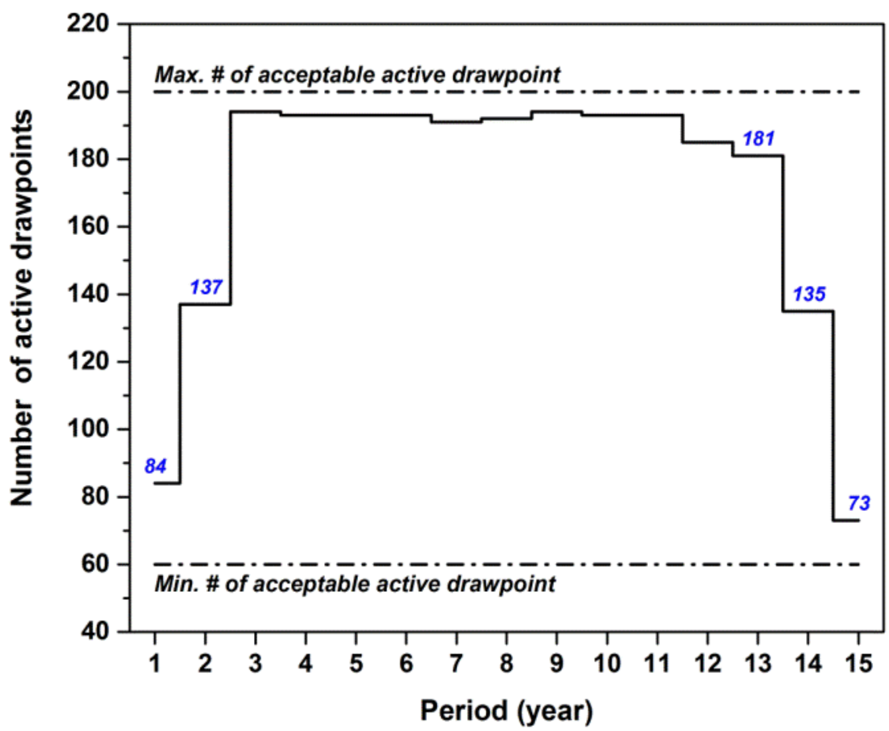

Fig 14. Active drawpoints during the life of the mine

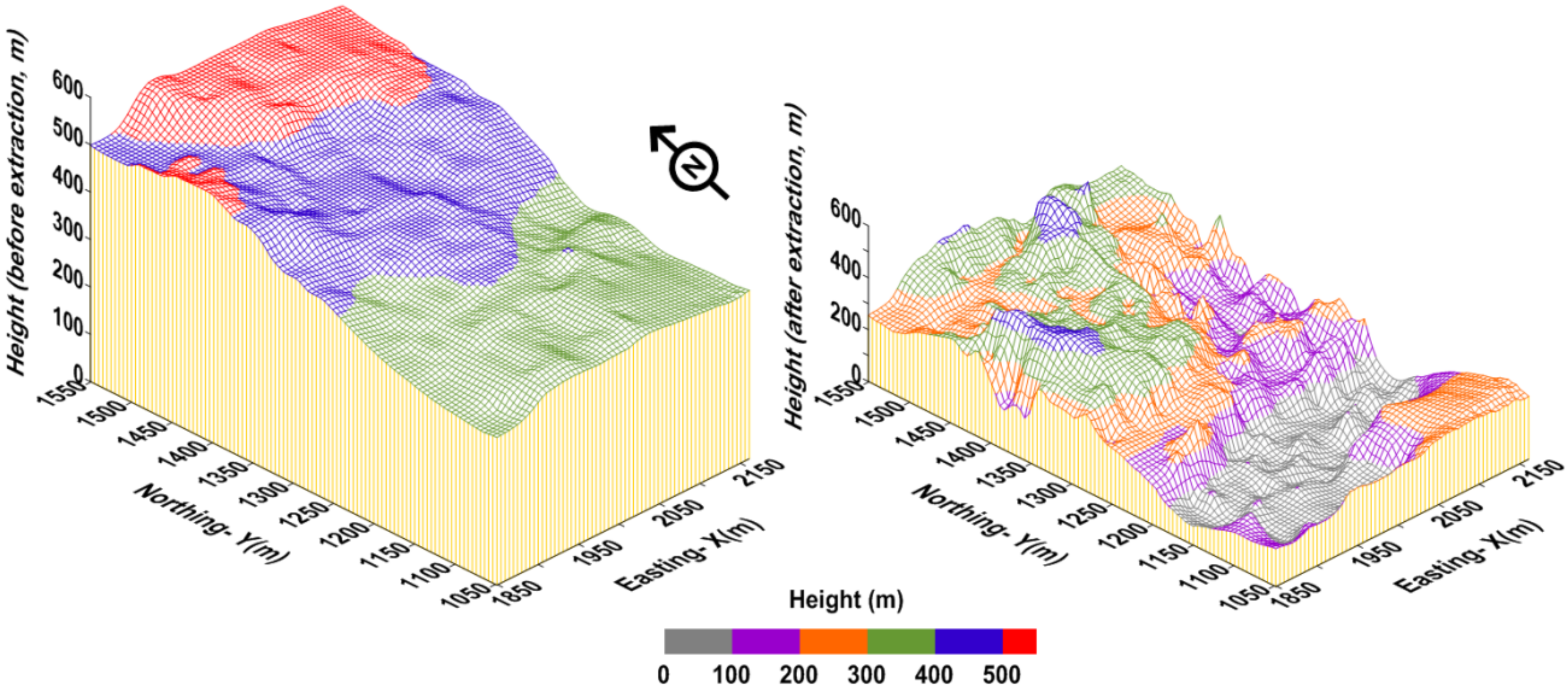

Fig 15. Height of the material after extraction compare to initial height of the draw columns

5. Discussion and future research direction

Despite its significant impact on production scheduling, material flow and its uncertainties have not been part of production scheduling models. The existing uncertainties can be one of the main reasons for gaps between the production goals and the actual operations resulting in an inefficient mine to mill system. In this paper, using stochastic optimization, an MILP model was proposed to capture the horizontal mixing that occurs in material flow when extracting from drawpoints in block-cave mining. Adjacent slices located within the same neighborhood were used to generate scenarios in order to simulate the horizontal movements of the material. To address the processing plant's objectives, in addition to technical and operational constraints, the model was built based on two production targets: tonnage and grade. For the presented case study, the resulting tonnages show only minor deviations from the targets in all scenarios. There are visible deviations between the target grade and the resulting production grade which is mainly because of the defined mining direction (extracting high grade material in the first years of the mine life). The BHOD calculation was also brought into the optimization, which means that the mine reserve is the output of the production scheduling and not an input; this takes us one step closer towards overall optimization. The model was run for two different sets of penalties for grade deviations and proved that increasing the penalties will provide us with closer results to the targets but lower NPVs. As a result, mine planners can use the proposed stochastic model as a flexible tool; they can define the penalties for deviations from their specified targets based on their priorities: higher penalties will result in less deviation from the targets but lower NPVs, and vice versa. Our future work will be developing models that can capture vertical mixing in addition to horizontal mixing.

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