Optimizing the Extraction Strategy for Open Pit and Underground Mining Transitions using Mathematical Programming

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ABSTRACT

When an orebody extends from the surface to great depths, the decision to exploit such deposit is an interesting challenge in the mining industry. Simplified decisions are usually made to exploit the deposit by open pit mining followed by underground mining which leads to missed financial and sustainable mining opportunities. Optimized mining strategies that maximize the project value and increase resource extraction ratio include several mining variations. This paper investigates the mining strategy for open pit and/or underground mining options using a Mixed Integer Linear Programming (MILP) optimization framework. The model maximizes the Net Present Value (NPV) of the reserve when exploited using open pit mining, underground mining, or simultaneous open pit and underground mining. The model further features the definition of the 3D location and size of the required crown pillar, and the extraction sequence of the open pit cuts and the underground stopes. The models are implemented on two case studies to investigate the best mining option, and the optimal material extraction strategy that maximizes NPV and increase resource extraction ratio for the mining projects.

1. Introduction

When a mineral deposit extends from the surface to great depth, the portion of the orebody close to the surface is usually preferred to be exploited with open pit (OP) mining options to generate early revenue while the deeper portions are exploited with underground (UG) mining options. In open pit mining, the incremental stripping ratio and overall mining cost with depth makes underground mining profitable beyond a certain depth. This depth has been referred by several authors as transition depth or transition point (Bakhtavar et al., 2009; Opoku and Musingwini, 2013; Roberts et al., 2013; Dagdelen and Traore, 2014; Ordin and Vasil'ev, 2014; De Carli and de Lemos, 2015; King et al., 2016; MacNeil and Dimitrakopoulos, 2017).

During the feasibility study, knowledge of the required mining option(s) in developing the mineral resource at the onset of the mine is a major information that influences several mining and investment decisions. Proper decisions made at the onset of the mine gives much confidence to both mine planners and investors. Knowing the extent or limit of the depth of a typical open pit mining operation is an added advantage to the planning of the mine from geotechnical, resource and financial points of views.

A strategic mathematical framework for determining the mining strategy for a deposit that has the potential to be exploited with several variations of the mining methods has been developed. The mining variations include the exploitation of the deposit completely with (1) open pit mining method, (2) underground mining method, or (3) both open pit and underground mining methods. In the third mining variation, the deposit could be sequentially exploited with an open pit mine before transition into an underground mine or sequentially exploited with an underground mine followed by an open pit mine, or simultaneously exploited with both open pit and underground mining options. In the case where there is the need to exploit the deposit by both open pit and underground mining methods, the location of a required crown pillar is essential for the mining project. The decision to adopt any mining option strategy primarily depends on the economics of the project and the geology of the mining area.

Optimizing the location of a crown pillar is a key factor in the metalliferous mining industry. Finding the most suitable location of the crown pillar in a combined mining method of open pit and underground operations, especially in block caving method, is one of the most interesting problems for mining engineers today (Bakhtavar et al., 2012). The transition from open pit to underground mining involves a complicated geomechanical process.

The paper presents Mixed Integer Linear Programming (MILP) model frameworks for solving the open pit and underground mining transition problems. The model determines an optimized mining strategy for a deposit that is potentially amenable by both open pit and underground mining options. A synthetic copper deposit and a small gold deposit is used as a case study to implement the models for evaluation.

The next section of this research paper covers a summarized literature review on open pit to underground mining transition problem with highlights on research gaps. Section 3 discusses the assumptions and notations used in the proposed MILP models. Section 4 introduces and explains the proposed integrated MILP models for the open pit to underground mining transition complex. Section 5 explains the implementation of the MILP models to the evaluation of two case studied deposits while Section 6 documents the research conclusions and future works.

2. Summary of Literature Review

Strategic open pit and underground mining interface optimization models have been developed based on determining the transition depth between open pit mining and underground mining. These models mainly focus on investigating how an underground mining operation can be exploited after an open pit mine or combined with an existing open pit operation (Ben-Awuah et al., 2016). Acknowledging notable challenges and shortfalls, several researchers have employed techniques, algorithms and/or models to determine the transition depth (Bakhtavar et al., 2009; Opoku and Musingwini, 2013; Roberts et al., 2013; Dagdelen and Traore, 2014; Ordin and Vasil'ev, 2014; De Carli and de Lemos, 2015; King et al., 2016; MacNeil and Dimitrakopoulos, 2017) and the ore block extraction strategy (De Carli and de Lemos, 2015; Ben-Awuah et al., 2016; King et al., 2016; MacNeil and Dimitrakopoulos, 2017).

Kurppa and Erkkilä (1967) assessed the simultaneous mining between open pit and underground (OP-UG) mining during the operations of the Pyhasalmi mine. They indicated that, simultaneous mining was possible due to the geometry of the orebody being worked. Luxford (1997) argued that, cost usually drives the decision to make the transition because as the open pit waste stripping cost keeps increasing with depth, there comes a time when the underground mining cost will be less than the open pit mining cost.

Ben-Awuah et al. (2016) investigated the strategy of mining options for an orebody by using a mathematical programming model – MILP. The research evaluated the financial impacts of applying different mining options separately or concurrently to extract a given orebody. The MILP formulation maximizes the NPV of the reserve when extracted with: (1) open pit mining, (2) underground mining, and (3) concurrent open pit and underground mining. The location of a required crown pillar was not considered in this model.

King et al. (2016) incorporated crown and sill pillar placement into OP-UG transition studies to separate the open pit from the underground mine. In their model, the location of the crown pillar was simulated, and the number of crown pillar locations was selected. Producing several crown pillar locations for a deposit will lead to the difficulty in selecting the optimal crown pillar location that maximizes the value of the mining project.

MacNeil and Dimitrakopoulos (2017) investigated the transition decision at an operating open pit mine that exists within the context of a mining complex comprising five producing pits, four stockpiles and one processing plant. The proposed method improves upon previous developments related to the OP-UG transition problem by incorporating geological uncertainty into the decision-making process while providing a transition depth described in three-dimensions (3D). In their research, MacNeil and Dimitrakopoulos (2017) priori identified the crown pillar envelope for a gold deposit and evaluated four crown pillar locations within this envelope leading to four distinct candidate transition depths.

A mathematical model that solves the transition problem was been developed by Whittle et al. (2018) after modifying the normal pit optimization model based on the maximum graph closure problem (Lerchs and Grossman, 1965). The modifications allow the algorithm to account for the underground mining value of a block, and the requirement for a specified crown pillar thickness and shape above the underground mine. The algorithm of Whittle et al. (2018) is based on what they called the "opportunity cost approach", thus, if a given block is mined by open pit method, its open pit value is gained while its value that would have been obtained by mining blocks using underground mining methods is lost. By applying the opportunity cost approach, the underground value is subtracted from the open pit value for each block before doing pit optimization. This approach compromises the optimal value of the mining project since the optimization process does not fully control the mining sequence with time.

Current formulations for optimizing the open pit to underground transition complexes do not integrate the location and size of a 3D crown pillar into the optimization process and further produce optimal solutions within a known level of confidence. In this research study, we have developed, implemented, and tested a MILP optimization framework capable of evaluating a deposit amenable by either open pit, underground or both open pit and underground mining options. The optimization process of the MILP further decides the best location of the required 3D crown pillar that maximizes the profit of the mining project without human interference on the location of the crown pillar.

3. Assumptions and Notations

The size of the Selected Mining Units (SMUs) of the open pit mining is equal to the stope sizes of the underground mining. In this model, the SMUs are represented by mining block in general or mining-cut for the open pit operation or mining-stope for the underground operation. The location of each mining block or mining-cut or mining-stope is represented by the coordinate of the centroid of the block. It is assumed that a crown pillar is required for the exploitation of the orebody by underground mining. The size of the crown pillar is assumed to be one vertical length of the stope or bench; thus, one bench or level of the block model represents the crown pillar. For the underground mining, ore extraction is achieved by a retreating method of ore extraction. The notation of decision variables, parameters, sets, and constraints are as follows:

3.1. Sets

$K = \{1,\ldots,K\}$	set of all the blocks in the model.
$J = \{1,\ldots,J\}$	set of all the open pit mining-cuts in the model.
$J_s = \{1,\ldots,J_s\}$	set of all the open pit mining-cuts on a level in the model.

$P = \{1,\ldots,P\}$	set of all the underground mining-stopes in the model.
$J_p = \{1,\ldots,J_p\}$	set of all the underground mining-stopes on a particular level in the model.
$C = \{1,\ldots,C\}$	set of all the levels acting as crown pillars (c) in the model.
$O_j(S)$	for each open pit mining-cut, j, there is a set $O_j(S) \subset J$, defining the immediate predecessor mining-cut that must be extracted prior to extracting cut j; where S is the total number of mining-cuts in set $O_j(S)$.
$U_p(S)$	for each underground mining-stope, p, there is a set $U_p(S) \subset P$, defining the immediate predecessor mining-stope that must be extracted prior to extracting stope p; where S is the total number of mining-stopes in set $U_p(S)$.
$C_j(S)$	for each level, there is a set $C_j(S) \subset J$, defining the number of mining-cuts, j, on that level that must be extracted as open pit cut j, or left as unmined level or crown pillar (c); where S is the total number of mining-cuts on a level in set $C_j(S)$.
$C_p(S)$	for each level, there is a set $C_p(S) \subset P$, defining the number of mining-stopes, p, on that level that must be extracted as underground stope p, or left as unmined level or crown pillar (c); where S is the total number of mining-stopes on a level in set $C_p(S)$.

3.2. Indices

A general parameter f can take four indices in the format of $f_{k,l}^{a,t}$. Where:

$t \in \{1,\ldots,T\}$	index for scheduling periods.
$k \in \{1,\ldots,K\}$	index for mining-blocks in the model.
$j \in \{1,\ldots,J\}$	index for open pit mining-cuts in the model.
$p \in \{1,\ldots,P\}$	index for underground mining-stopes in the model.
$c \in \{1,\ldots,C\}$	index for crown pillars in the model.

3.3. Parameter

$v_j^{op,t}$	the open pit (op) discounted revenue generated by selling the final product within mining-cut j in period t minus the discounted extra cost of mining all the mining-cut j as ore and processing it.
$v_p^{ug,t}$	the underground (ug) discounted revenue generated by selling the final product within mining-stope p in period t minus the discounted extra cost of mining all the mining-stope p as ore and processing it.
$q_j^{op,t}$	the open pit (op) discounted cost of mining all the material in mining-cut j in period t as waste.
$q_p^{ug,t}$	the underground (ug) discounted cost of mining all the material in mining-stope p in period t as waste.
g_j	average grade of element in ore portion of mining-cut j.
g_p	average grade of element in ore portion of mining-stope p.
o_k	ore tonnage in mining-block k.

o_j	ore tonnage in mining-cut j.
o_p	ore tonnage in mining-stope p.
w_k	waste tonnage in mining-block k.
w_j	waste tonnage in mining-cut j.
w_p	waste tonnage in mining-stope p.
$g_{lb}^{op,t}$	lower bound on acceptable average head grade of element for the open pit mining (*op*) in period t.
$g_{lb}^{ug,t}$	lower bound on acceptable average head grade of element for the underground mining (*ug*) in period t.
$g_{ub}^{op,t}$	upper bound on acceptable average head grade of element for the open pit mining (*op*) in period t.
$g_{ub}^{ug,t}$	upper bound on acceptable average head grade of element for the underground mining (*ug*) in period t.
$T_{pr,lb}^{op,t}$	lower bound on ore processing capacity from open pit and underground mining in period t.
$T_{pr,lb}^{ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period t.
$T_{pr,ub}^{op,t}$	lower bound on ore processing capacity from open pit and underground mining in period t.
$T_{pr,ub}^{ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period t.
$T_{m,lb}^{op,t}$	lower bound on available open pit mining capacity in period t.
$T_{m,lb}^{ug,t}$	lower bound on available underground mining capacity in period t.
$T_{m,ub}^{op,t}$	upper bound on available open pit mining capacity in period t.
$T_{m,ub}^{ug,t}$	upper bound on available underground mining capacity in period t.
$T_{pr,lb}^{op,ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period t.
$T_{pr,lb}^{op,ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period t.
$T_{pr,ub}^{op,ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period t.
$T_{pr,ub}^{op,ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period t.
r	processing recovery; the proportion of mineral commodity recovered.
sp^t	selling price of mineral commodity in present value terms.

sc^t	selling cost of mineral commodity in present value terms.
pc^t	extra cost in present value terms per tonne of ore for mining and processing in period *t*.
cm_l^t	cost per bench or level l in present value terms of mining a tonne of waste by open pit mining in period *t*.
cm^t	cost in present value terms of mining a tonne of waste by underground mining in period *t*.

3.4. Decision Variables

$x_j^{op,t} \in \{0,1\}$	continuous variable, representing the portion of mining-cut *j* to be extracted as ore and processed in period *t* for open pit.
$x_p^{ug,t} \in \{0,1\}$	continuous variable, representing the portion of mining-stope *p* to be extracted as ore and processed in period *t* for underground mining.
$y_j^{op,t} \in \{0,1\}$	continuous variable, representing the portion of mining-cut *j* to be mined in period *t* for open pit, fraction of *y* characterizes both ore and waste included in the mining-cut.
$y_p^{ug,t} \in \{0,1\}$	continuous variable, representing the portion of mining-stope *p* to be mined in period *t* for underground, fraction of *y* characterizes both ore and waste included in the mining-stope.
$y_c^t \in \{0,1\}$	binary integer variable; equal to one if a level is left as crown pillar (*c*) in period *t,* otherwise zero.
$y_j^t \in \{0,1\}$	binary integer variable; equal to one if a mining-cut *j* or all mining-cuts *Js* on a level is extracted as open pit in period *t,* otherwise zero.
$y_p^t \in \{0,1\}$	binary integer variable; equal to one if a mining-stope *p* or all mining-stopes *Ps* on a level is extracted as underground in period *t,* otherwise zero.
$b_j^t \in \{0,1\}$	binary integer variable controlling the precedence of extraction of mining-cut for the open pit. b_j^t is equal to one if extraction of cut *j* has started by or in period t, otherwise it is zero.
$b_p^t \in \{0,1\}$	binary integer variable controlling the precedence of extraction of mining-stope for the underground. b_p^t is equal to one if extraction of stope *p* has started by or in period t, otherwise it is zero.

4. Integrated MILP Models

Three separate MILP models are formulated to determine the time and sequence of extraction of ore and waste blocks over the life of the mining project. The proposed MILP models interrogate the orebody and determine the best mining option that produces an optimal extraction sequence to maximize the Net Present Value (NPV) of the mining project. The mining options could either be open pit mining or underground mining or simultaneous open pit and underground mining. The NPV of the production schedule is maximized in the presence of technical, geotechnical, geological, and economic constraints to enforce the mining extraction, blending requirements, and mining and processing capacities.

The objective function of the MILP models maximizes the operation of the mining project. The quantity of ore processed is controlled by the continuous decision variables $x_j^{op,t}$ and $x_p^{ug,t}$ for open pit and underground mining respectively. Similarly, the quantity of rock material extracted by open pit and underground mining are respectively controlled by the continuous decision variables $y_j^{op,t}$ and $y_p^{ug,t}$. The continuous variables ensure fractional extraction of the mining-cut or mining-stope in different periods.

The constraints of the proposed MILP models control the mining, processing, plant head grade, geotechnical slope, crown pillar, and the surface and underground mining precedence. Acceptable upper and lower targets have been defined for the mining, processing, and plant head grade inequality constraints.

The mining capacity is a function of the ore reserve, targeted mine-life, designed processing capacity and available capital for mining fleet acquisition for the mining operation. The processing capacity constraints define the ore production schedule in each period and further ensures the run-of-mine (ROM) material satisfies the quantity specification of the processing plant. Based on the cut-off grades or head grade of the plant, limiting grade requirements for appropriate ore blending from each mining option are defined within a lower and upper grade targets for the mining operation.

4.1. Modeling the economic block value

The economic block values are defined based on the SMU, thus, mining-cut and mining-stope for the open pit and underground mining respectively. The value of the block is a function of the recovered quantity of mineral present in the block, the discounted revenue of selling the commodity, and the discounted costs of mining, processing and selling costs.

The discounted revenues generated by selling the final product within block k being extracted in period t by open pit mining $v_j^{op,t}$ and underground mining $v_p^{ug,t}$ are respectively shown in Eqs. (1) and (2). Similarly, the discounted costs of mining all the material within block k being extracted in period t by open pit mining $q_j^{op,t}$ and underground mining $q_p^{ug,t}$ are respectively shown in Eqs. (3) and (4).

$$v_j^{op,t} = \sum_{j=1}^{J} o_j \times g_j \times r \times (sp - sc) - \left(\sum_{j=1}^{J} o_j \times pc \right) \tag{1}$$

$$v_p^{ug,t} = \sum_{p=1}^{P} o_p \times g_p \times r \times (sp - sc) - \left(\sum_{p=1}^{P} o_p \times pc \right) \tag{2}$$

$$q_j^{op,t} = \sum_{j=1}^{J} (o_j + w_j) \times \sum_{l=1}^{L} cm_l^t \tag{3}$$

$$q_p^{ug,t} = \sum_{p=1}^{P} (o_p + w_p) \times cm^t \tag{4}$$

4.2. Model 01 – open pit mining option

Objective function:

$$Max \left[\sum_{t=1}^{T} \sum_{j=1}^{J} \left(v_j^{op,t} \times x_j^{op,t} - q_j^{op,t} \times y_j^{op,t} \right) \right] \tag{5}$$

Subject to:

$$T_{m,lb}^{op,t} \le \sum_{j=1}^{J}\left[\left(o_j + w_j\right)\times y_j^{op,t}\right] \le T_{m,ub}^{op,t} \qquad \forall t \in \{1,\dots,T\}, j \in \{1,\dots,J\}; \tag{6}$$

$$T_{pr,lb}^{op,t} \le \sum_{j=1}^{J}\left(o_j \times x_j^{op,t}\right) \le T_{pr,ub}^{op,t} \qquad \forall t \in \{1,\dots,T\}, j \in \{1,\dots,J\}, p \in \{1,\dots,P\}; \tag{7}$$

$$g_{lb}^{op,t} \le \left[\sum_{j=1}^{J}\left(\frac{g_j \times o_j}{o_j}\right)\right]\times x_j^{op,t} \le g_{ub}^{op,t} \qquad \forall t \in \{t,\dots,T\}, j \in \{1,\dots,J\}; \tag{8}$$

$$x_j^{op,t} - y_j^{op,t} \le 0 \qquad \forall t\in\{1,\dots,T\}, j\in\{1,\dots,J\}; \tag{9}$$

$$b_j^t - \sum_{t=1}^{T} y_s^{op,t} \le 0 \qquad \forall t \in \{1,\dots,T\}, s \in O_j(S), j\in\{1,\dots,J\}; \tag{10}$$

$$\sum_{t=1}^{T} y_j^{op,t} - b_j^t \le 0 \qquad \forall t \in \{1,\dots,T\}, j\in\{1,\dots,J\}; \tag{11}$$

$$b_j^t - b_j^{t+1} \le 0 \qquad \forall t \in \{1,\dots,T\text{-}1\}, j \in \{1,\dots,J\}; \tag{12}$$

$$\left(\sum_{j=1}^{Js} b_{j,s}^t \times \frac{1}{Js}\right) - y_j^t \le 0 \qquad \forall t \in \{1,\dots,T\}, j\in\{1,\dots,Js\}, s\in C_j(S), j\in\{1,\dots,J\}; \tag{13}$$

$$\sum_{t=1}^{T} y_j^{op,t} \le 1 \qquad \forall t \in \{1,\dots,T\}, j\in\{1,\dots,J\}; \tag{14}$$

The formulation of Model 01 follows after Askari-Nasab et al. (2011). The objective function of Model 01 shown in Eq. (5) maximizes the operation of the open pit mining option. The quantity of ore processed, and rock material mined are controlled by the continuous decision variables $x_j^{op,t}$ and $y_j^{op,t}$ respectively.

Eq. (6) defines the mining capacity constraints for the open pit mining. This inequality ensures that the total tonnage of rock material mined in each period is within the acceptable lower and upper limits of the total available equipment capacity for the open pit mining operation.

Eq. (7) is the processing capacity constraint that controls the quantity of mill feed for the open pit mining option. The processing constraints are controlled by the continuous decision variables $x_j^{op,t}$. This inequality ensures that uniform ore is fed to the processing plant throughout the mine life of the open pit mining operation within acceptable lower and upper targets of ore for the processing plant in each period.

Eq. (8) represents the grade blending constraints for the open pit mining option. This inequality is controlled by the continuous variable $x_j^{op,t}$ and further ensures quality ore is delivered to the processing plant in each period. Eq. (9) defines the relationship between the ore and mining-cut tonnages controlling the mining and processing decisions. Thus, the continuous variable $x_j^{op,t}$ is always smaller than or equal to the continuous variable $y_j^{op,t}$.

Eqs. (10), (11) and (12) control the downward precedence relationship of mining-cut extraction and the appropriate geotechnical mining slope for the open pit mining option. For the open pit mining, nine overlying mining-cuts are mined before the underlying mining-cut is mined.

Eq. (13) ensures that a level y_j^t is categorized for open pit mining when a block on that level is extracted. Eq. (14) defines the reserve constraints and ensures that each block is extracted once in the life of the open pit mine.

4.3. Model 02 – underground mining option

Objective function:

$$Max\left[\sum_{t=1}^{T}\sum_{p=1}^{P}\left(v_p^{ug,t}\times x_p^{ug,t}-q_p^{ug,t}\times y_p^{ug,t}\right)\right] \tag{15}$$

Subject to:

$$T_{m,lb}^{ug,t}\le\sum_{p=1}^{P}\left[\left(o_p+w_p\right)\times y_p^{ug,t}\right]\le T_{m,ub}^{ug,t} \quad \forall t\in\{1,\ldots,T\},\ p\in\{1,\ldots,P\}; \tag{16}$$

$$T_{pr,lb}^{ug,t}\le\sum_{p=1}^{P}\left(o_p\times x_p^{ug,t}\right)\le T_{pr,ub}^{ug,t} \quad \forall t\in\{1,\ldots,T\},\ p\in\{1,\ldots,P\}; \tag{17}$$

$$g_{lb}^{ug,t}\le\left[\sum_{p=1}^{P}\left(\frac{g_p\times o_p}{o_p}\right)\right]\times x_p^{ug,t}\le g_{ub}^{ug,t} \quad \forall t\in\{1,\ldots,T\}; \tag{18}$$

$$x_p^{ug,t}-y_p^{ug,t}\le 0 \quad \forall t\in\{1,\ldots,T\},\ k\in B_p,\ p\in\{1,\ldots,P\}; \tag{19}$$

$$b_p^t-\sum_{t=1}^{T}y_s^{ug,t}\le 0 \quad \forall t\in\{1,\ldots,T\},\ s\in U_p(S_{1,2,3}),\ p\in\{1,\ldots,P\}; \tag{20}$$

$$\sum_{t=1}^{T}y_p^{ug,t}-b_p^t\le 0 \quad \forall t\in\{1,\ldots,T\},\ p\in\{1,\ldots,P\}; \tag{21}$$

$$b_p^t-b_p^{t+1}\le 0 \quad \forall t\in\{1,\ldots,T\text{-}1\},\ p\in\{1,\ldots,P\}; \tag{22}$$

$$\sum_{t=1}^{T}x_p^{ug,t}\le 1 \quad \forall t\in\{1,\ldots,T\},\ p\in\{1,\ldots,P\}; \tag{23}$$

$$y_c^t+y_p^t\le 1 \quad \forall t\in\{1,\ldots,T\},\ p\in\{1,\ldots,P\},\ c\in\{1,\ldots,C\}; \tag{24}$$

$$\left(\sum_{p=1}^{Ps}b_{p,s}^t\times\frac{1}{Ps}\right)-y_p^t\le 0 \quad \forall t\in\{1,\ldots,T\},\ p\in\{1,\ldots,Ps\},\ s\in C_p(S),\ p\in\{1,\ldots,P\}; \tag{25}$$

$$y_c^t-y_c^{t+1}\le 0 \quad \forall t\in\{1,\ldots,T\},\ c\in\{1,\ldots,C\}; \tag{26}$$

$$\sum_{c=1}^{C}y_c^t=1 \quad t\in\{1,\ldots,T\},\ c\in\{1,\ldots,C\}; \tag{27}$$

The objective function of Model 02 shown in Eq. (15) maximizes the operation of the underground mining option. The quantity of ore processed, and rock material mined are controlled by the continuous decision variables $x_p^{ug,t}$ and $y_p^{ug,t}$ respectively.

Eq. (16) defines the mining capacity constraints for the underground mining. This inequality ensures that the total tonnage of rock material mined in each period is within the acceptable lower and upper limits of the total available equipment capacity for the underground mining operation.

Eq. (17) is the processing capacity constraint that controls the quantity of mill feed for the underground mining option. The processing constraints are controlled by the continuous decision variables $x_p^{ug,t}$. This inequality ensures that uniform ore is fed to the processing plant throughout the mine life of the underground mining operation within acceptable lower and upper targets of ore for the processing plant in each period.

Eq. (18) represents the grade blending constraints for the underground mining option. This inequality is controlled by the continuous variables $x_p^{ug,t}$ and further ensures quality ore is delivered to the processing plant in each period. Eq. (19) defines the relationship between the ore and mining-stope tonnages controlling the mining and processing decisions. Thus, the continuous variable $x_p^{ug,t}$ is always smaller than or equal to the continuous variable $y_p^{ug,t}$.

Eqs. (20), (21) and (22) control the lateral precedence relationship of mining-stope extraction on each level for the underground mining option. For the underground mining, ore extraction sequence is implemented in a retreating manner towards the main entrance of the mine for each underground level.

Eq. (23) defines the reserve constraints and the ore selective nature of underground mining. This inequality ensures that each stope is extracted once in the life of the underground mine.

Eqs. (24), (25), (26) and (27) define the location and size of the required crown pillar for the underground mining option. Eq. (24) ensures that, a level is either considered for underground mining or left as crown pillar or unmined level. Eq. (25) ensures that a level belongs to an underground mining option when a block or more on that level is extracted by underground mining. Eq. (26) ensures that a level acting as the crown pillar stays the same throughout the life of the mining operation while Eq. (27) ensures that one level always acts as the crown pillar throughout the life of the underground mine.

4.4. Model 03 – simultaneous open pit and underground mining option

Objective function:

$$Max\left[\sum_{t=1}^{T}\left(\sum_{j=1}^{J}\left(v_j^{op,t}\times x_j^{op,t}-q_j^{op,t}\times y_j^{op,t}\right)+\sum_{p=1}^{P}\left(v_p^{ug,t}\times x_p^{ug,t}-q_p^{ug,t}\times y_p^{ug,t}\right)\right)\right] \quad (28)$$

Subject to:

$$T_{m,lb}^{op,t}\le\sum_{j=1}^{J}\left[\left(o_j+w_j\right)\times y_j^{op,t}\right]\le T_{m,ub}^{op,t} \quad \forall t\in\{1,\dots,T\}, j\in\{1,\dots,J\}; \quad (29)$$

$$T_{m,lb}^{ug,t}\le\sum_{p=1}^{P}\left[\left(o_p+w_p\right)\times y_p^{ug,t}\right]\le T_{m,ub}^{ug,t} \quad \forall t\in\{1,\dots,T\},\ p\in\{1,\dots,P\}; \quad (30)$$

$$\left[o_j\times b_j^t\right]+\left[o_p\times b_p^t\right]\le o_k \quad \forall t\in\{1,\dots,T\}, j\in\{1,\dots,J\}, p\in\{1,\dots,P\}, k\in\{1,\dots,K\}; \quad (31)$$

$$T_{pr,lb}^{op,ug,t}\le\sum_{j=1}^{J}\left(o_j\times x_j^{op,t}\right)+\sum_{p=1}^{P}\left(o_p\times x_p^{ug,t}\right)\le T_{pr,ub}^{op,ug,t} \quad \forall t\in\{1,\dots,T\}, j\in\{1,\dots,J\}, p\in\{1,\dots,P\}; \quad (32)$$

$$T_{pr,lb}^{ug,t}\le\sum_{p=1}^{P}\left(o_p\times x_p^{ug,t}\right)\le T_{pr,ub}^{ug,t} \quad \forall t\in\{1,\dots,T\}, p\in\{1,\dots,P\}; \quad (33)$$

$$g_{lb}^{op,t}\le\left[\sum_{j=1}^{J}\left(\frac{g_j\times o_j}{o_j}\right)\right]\times x_j^{op,t}\le g_{ub}^{op,t} \quad \forall t\in\{t,\dots,T\}, j\in\{1,\dots,J\}; \quad (34)$$

$$g_{lb}^{ug,t} \leq \left[\sum_{p=1}^{P} \left(\frac{g_p \times o_p}{o_p} \right) \right] \times x_p^{ug,t} \leq g_{ub}^{ug,t} \qquad \forall t \in \{1,\ldots,T\}; \tag{35}$$

$$x_j^{op,t} - y_j^{op,t} \leq 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{36}$$

$$b_j^t - \sum_{t=1}^{T} y_s^{op,t} \leq 0 \qquad \forall t \in \{1,\ldots,T\}, s \in O_j(S), j \in \{1,\ldots,J\}; \tag{37}$$

$$\sum_{t=1}^{T} y_j^{op,t} - b_j^t \leq 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{38}$$

$$b_j^t - b_j^{t+1} \leq 0 \qquad \forall t \in \{1,\ldots,T\text{-}1\}, j \in \{1,\ldots,J\}; \tag{39}$$

$$\sum_{t=1}^{T} y_j^{op,t} \leq 1 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{40}$$

$$x_p^{ug,t} - y_p^{ug,t} \leq 0 \qquad \forall t \in \{1,\ldots,T\}, k \in B_p, p \in \{1,\ldots,P\}; \tag{41}$$

$$b_p^t - \sum_{t=1}^{T} y_s^{ug,t} \leq 0 \qquad \forall t \in \{1,\ldots,T\}, s \in U_p(S_{1,2,3}), p \in \{1,\ldots,P\}; \tag{42}$$

$$\sum_{t=1}^{T} y_p^{ug,t} - b_p^t \leq 0 \qquad \forall t \in \{1,\ldots,T\}, p \in \{1,\ldots,P\}; \tag{43}$$

$$b_p^t - b_p^{t+1} \leq 0 \qquad \forall t \in \{1,\ldots,T\text{-}1\}, p \in \{1,\ldots,P\}; \tag{44}$$

$$\sum_{t=1}^{T} x_p^{ug,t} \leq 1 \qquad \forall t \in \{1,\ldots,T\}, p \in \{1,\ldots,P\}; \tag{45}$$

$$y_j^t + y_c^t + y_p^t \leq 1 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}, p \in \{1,\ldots,P\}, c \in \{1,\ldots,C\}; \tag{46}$$

$$\left(\sum_{j=1}^{Js} b_{j,s}^t \times \frac{1}{Js} \right) - y_j^t \leq 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,Js\}, s \in C_j(S), j \in \{1,\ldots,J\}; \tag{47}$$

$$\left(\sum_{p=1}^{Ps} b_{p,s}^t \times \frac{1}{Ps} \right) - y_p^t \leq 0 \qquad \forall t \in \{1,\ldots,T\}, p \in \{1,\ldots,Ps\}, s \in C_p(S), p \in \{1,\ldots,P\}; \tag{48}$$

$$y_c^t - y_{j-1}^t \leq 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}, c \in \{1,\ldots,C\}; \tag{49}$$

$$y_j^t - y_{j-1}^t \leq 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{50}$$

$$y_{c-1}^t - y_p^t \leq 0 \qquad \forall t \in \{1,\ldots,T\}, p \in \{1,\ldots,P\}, c \in \{1,\ldots,C\}; \tag{51}$$

$$y_c^t - y_c^{t+1} \leq 0 \qquad \forall t \in \{1,\ldots,T\}, c \in \{1,\ldots,C\}; \tag{52}$$

$$\sum_{c=1}^{C} y_c^t = 1 \qquad t \in \{1,\ldots,T\}, c \in \{1,\ldots,C\}; \tag{53}$$

The objective function of Model 3 shown in Eq. (28) maximizes the simultaneous open pit and underground mining operation. The quantity of ore processed is controlled by the continuous decision variables $x_j^{op,t}$ and $x_p^{ug,t}$ for open pit and underground mining respectively. Similarly, the quantity of rock material extracted by both open pit and underground mining options are respectively controlled by the continuous decision variables $y_j^{op,t}$ and $y_p^{ug,t}$. The continuous variables ensure fractional extraction of the mining-cut or mining-stope in different periods.

Eqs. (29) and (30) are the mining capacity constraints respectively for the open pit and underground mining operations. The open pit and underground mining are respectively controlled by the continuous decision variables $y_j^{op,t}$ and $y_p^{ug,t}$. These inequalities ensure the total tonnage of rock material mined in each period is within the acceptable lower and upper limits of the total available equipment capacity for the mining operations.

Eq. (31) represents the interaction of the open pit and underground mining options. The inequality ensures that the mining block is extracted by only one mining option or left us unmined level or bench.

Eqs. (32) and (33) are the processing capacity constraints that control the quantity of the mill feed for each mining option. Eq. (32) represents the contribution of ore production from both open pit and underground mining option to the processing plant. The constraints are controlled by the continuous decision variables $x_j^{op,t}$ and $x_p^{ug,t}$ for open pit and underground mining respectively. These inequalities ensure that uniform ore is fed to the processing plant throughout the mine life of the mining operation within acceptable lower and upper targets of ore for the processing plant in each period.

Eqs. (34) and (35) represent the grade blending constraints for both open pit and underground mining respectively. These inequalities are also respectively controlled by the continuous variables $x_j^{op,t}$ and $x_p^{ug,t}$ for open pit and underground mining and ensure quality ore is delivered to the processing plant in each period. The ore production schedule in each period ensures the run-of-mine (ROM) material satisfies the ore quality specification of the processing plant. Based on the cut-off grades or head grade of the plant, limiting grade requirements for appropriate ore blending from each mining option are defined within a lower and upper grade targets for the mining operation.

Eq. (36) defines the relationship between the ore and mining-cut tonnages controlling the mining and processing decisions. Thus, the continuous variable $x_j^{op,t}$ is always smaller than or equal to the continuous variable $y_j^{op,t}$.

Eqs. (37), (38) and (39) control the downward precedence relationship of mining-cut extraction and the appropriate geotechnical mining slope for the open pit mining option. For the open pit mining, nine overlying mining-cuts are mined before the underlying mining-cut is mined.

Eq. (40) defines the reserve constraints and ensures that a block is extracted once in the life of the open pit mine. Eq. (41) defines the relationship between the ore and mining-cut tonnages controlling the underground mining and processing decisions. Thus, the continuous variable $x_p^{ug,t}$ is always smaller than or equal to the continuous variable $y_p^{ug,t}$.

Eqs. (42), (43) and (44) control the lateral precedence relationship of mining-stope extraction on each level for the underground mining option. For the underground mining, ore extraction sequence is implemented in a retreating manner towards the main entrance of the mine for each underground level.

Eq. (45) defines the reserve constraints and the ore selective nature of underground mining. This inequality ensures that each stope $x_p^{ug,t}$ on a level y_p^t is extracted once in the life of the underground mine.

Eqs. (46), (47), (48), (49), (50), (51), (52) and (53) define the location and size of the required crown pillar for the commencement of the underground mining option. Eq. (46) ensures that, each level or bench is either considered for open pit mining or underground mining or left as crown pillar or unmined level within the block model. Eq. (47) ensures that, a level belongs to an open pit mining option when one block or more on that level is extracted by open pit mining while Eq. (48) ensures that a level belongs to an underground mining option when one block or more on that level is extracted by underground mining. Eq. (49) ensures that a level can be the crown pillar when the levels above are being considered for open pit mining option. Thus, crown pillar immediately follows an open pit mining option.

Equation (50) ensures that when a level is considered for open pit mining, the immediate level below could be considered for open pit mining option. Equation (51) ensures that the crown pillar is always above the level being considered for underground mining. Equation (52) ensures that a level acting as the crown pillar stays the same throughout the life of the mining operation while Equation (53) ensures that one level always acts as the crown pillar.

4.5. Non-negativity constraints

Eq. (54) ensures that the decision variables for open pit and underground mining and processing, crown pillar, open pit mining benches, and underground mining levels are non-negative. The inequality further defines the binary variables controlling the geotechnical and extraction sequence in the open pit and underground mining are integers.

$$x_j^{op,t},\ y_j^{op,t},\ x_p^{ug,t},\ y_p^{ug,t} \geq 0 \text{ and } b_j^t,\ b_p^t,\ y_j^t,\ y_p^t,\ y_c^t \text{ are integers} \tag{54}$$

5. Computational Implementation of the MILP Models

The formulated models were implemented on two experimental case studies. IBM ILOG CPLEX Optimization Studio V12.6.3 (ILOG, 2015) was integrated within MATLAB 2018a V9.4.0.813654 (Mathworks, 2018) environment to define the modelled framework and solve the optimization problem at a MILP gap tolerance of 5%. The CPLEX solver stops at an integer feasible solution when the gap tolerance of optimality is 5%. The model was tested on an Intel(R) Core™ i7-7700HQ CPU Dell computer @ 2.80GHz, with 32 GB RAM.

The block model of the deposit is prepared by dividing it into two halves with an operating ore drive on each level. Crosscuts development extend from this ore drive to the ends of the minefields through each SMU, acting as stope drives. Fig. 1 is a representation of the block model showing the main underground operating developments and the retreating ore extraction sequence on a typical level.

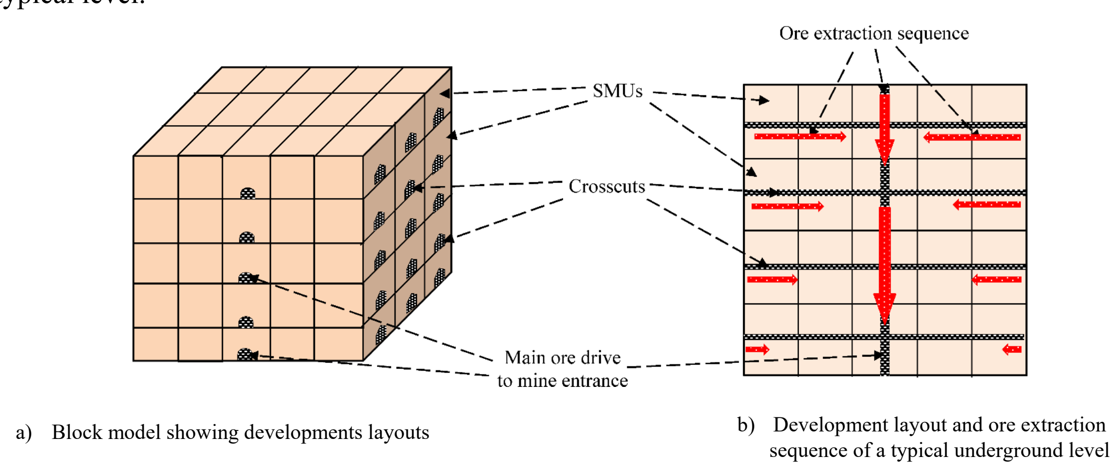

Fig. 1. Isometric view of the block model and development layout of an underground level.

In the open pit mining, the ore is exploited from the top to the bottom with a 45° slope to ensure a geotechnical control of the mine. However, in underground mining, the ore is extracted from the ends of the minefield in a retreating manner towards the main entrance of the mine through the crosscuts and the ore drives.

5.1. Case study 1 – synthetic dataset for copper deposit

The strategic Mixed Integer Linear Programming (MILP) models were implemented and tested on a synthetic copper deposit. The copper deposit consists of 500 mining blocks representing the Selective Mining Units (SMUs). The deposit is represented in a geological block model, in a 3D array of cubical blocks containing 500 SMUs of sizes 250 x 200 x 15. The orebody in the block model is irregularly shaped with a total mineral resource of 69.32 Mt of ore with an average Cu grade of 0.7664 g/t. Table 1 is a description of the copper deposit used to implement the models.

Table 1. Statistical description of the gold deposit.

No.	Description	Value
1	Total material (Mt)	675.00
2	Total mineralized material (Mt)	69.32
3	Minimum value of Cu (g/t)	0.7190
4	Maximum value of Cu (g/t)	0.8160
5	Average value of Cu (g/t)	0.7664
6	Variance	0.0006
7	Standard deviation	0.0251

5.1.1. Economic and mining data for case study 1

Taylor's mine life rule was used to estimate the mine life between 15.0 to 22.5 years. A mine life of 20 years was used to assess three mining options. Based on the quantity of mineralized material present in the deposit, the yearly mining and processing capacities were determined. An incremental bench cost of $ 2.0 per 7.5 m was used as the open pit mining variable cost as the pit transcends downwards. The economic, mining and processing data used for evaluating the gold deposit is shown in Table 2.

5.1.2. Results and discussions for case study 1

The Net Present Value (NPV) of the three mining options were evaluated and the best mining option selected for the deposit. The results of the integrated MILP models for the open pit mining option, simultaneous open pit and underground mining option with a crown pillar and underground mining option with a crown pillar on case study 2 are shown in

Table 3. The copper deposit is best exploited with simultaneous open pit and underground mining option for an operational mine life of 20 years since the generated NPV of $ 102.02 billion is greater than that determined for the other mining options.

From the resource depletion point of view, the investor, mine operator and government regulations have broader understanding to the resource exploitation from the implementation of each mining option. Although simultaneous open pit and underground mining option (OP-UG) yields the maximum discounted revenue, 86.7% of the resource will be depleted as compared to the other mining options. The remaining 13.3% of the available mineral resource per the OP-UG mining option remains in the crown pillar as unmined.

The yearly ore production schedule for the open pit mining (OP), underground mining, and simultaneous open pit and underground mining with a crown pillar option for case study 1 are respectively shown in Fig. 2, Fig. 3 and Fig. 4. The ore production schedules show that the ore target is attained in the initial years of the mine life before dropping to a stable schedule for the OP mining option (Fig. 2) but fluctuates in the case of the UG (Fig. 3) and OP-UG (Fig. 4) throughout the mine life. The ore fluctuating scenarios are usually impractical and must be controlled to ensure a steady cashflow of the mining project. A practical ore production schedule demonstrates a gradual rise in production tonnages at the beginning of the mine life and a gradual fall in the later years of the life of the mine. The practicality of the ore production is an integral part of the improvement phase of these models.

Table 2. Economic, mining and processing data for evaluating the copper deposit.

No.	Description	Value
1	Open pit mining cost ($/t)	8.0
2	Underground mining cost ($/t)	300.0
3	Processing cost ($/t)	15.0
4	Selling cost ($/lb)	1.5
5	Selling price of copper ($/lb)	3.5
6	Discount rate (%)	10.0
7	Mining recovery (%)	100.0
8	Mining dilution	0.0
9	Max open pit (OP) processing capacity (Mt/year)	4.6
10	Min open pit (OP) processing capacity (Mt/year)	3.1
11	Max open pit (OP) mining capacity (Mt/year)	45.0
12	Min open pit (OP) mining capacity (Mt/year)	30.0
13	Max underground (UG) processing capacity (Mt/year)	3.5
14	Min underground (UG) processing capacity (Mt/year)	1.2
15	Max underground (UG) mining capacity (Mt/year)	45.0
16	Min underground (UG) mining capacity (Mt/year)	0.0
17	Max open pit & underground (OP-UG) processing capacity (Mt/year)	6.0
18	Min open pit & underground (OP-UG) processing capacity (Mt/year)	1.0
19	Max open pit & underground (OP-UG) mining capacity (Mt/year)	45.0
20	Min open pit & underground (OP-UG) mining capacity (Mt/year)	0.0
21	Incremental bench cost ($/7.5 m)	2.0
22	Average mine life (years)	20.0

Table 3. Computed results of the integrated MILP model for case study 1.

No.	Description	Discounted Revenue ($)	Ore Tonnage (Mt)	Rock Tonnage (Mt)	Resource Depletion (%)
1	Open pit mining ($ bn)	99.59	69.32	600.00	100.0
2	Underground mining ($ bn)	43.99	29.09	37.80	42.0
3	Open pit and underground mining (Simultaneous) ($ bn)	102.02	60.10	78.30	86.7

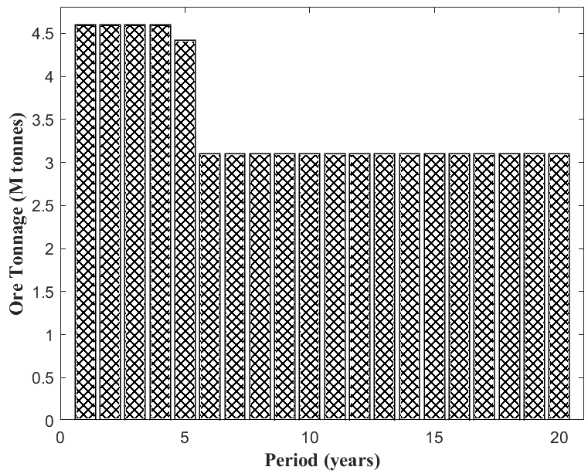

Fig. 2. Yearly ore production schedule for the open pit mining option in case study 1.

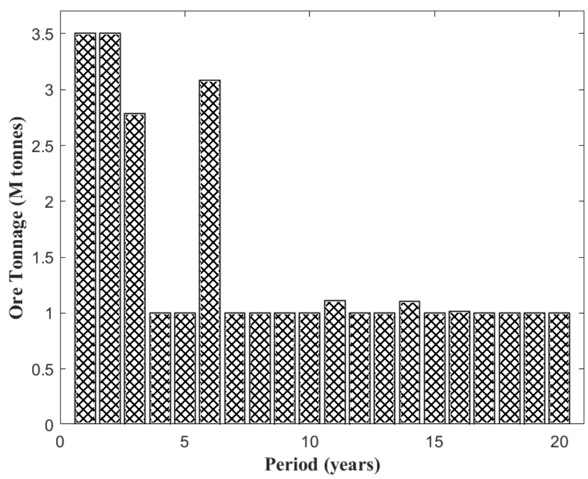

Fig. 3. Yearly ore production schedule for the underground mining option in case study 1.

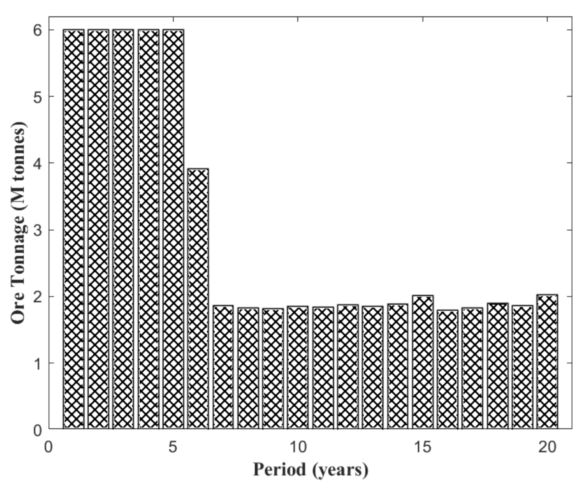

Fig. 4. Yearly ore production schedule for the simultaneous open pit and underground mining option in case study 1.

In the case of the simultaneous open pit and underground mining option (OP-UG) with a required crown pillar, Model 3; when the open pit mining cost is $ 8.0 per tonne and the underground mining cost is $ 300 per tonne, level 2 acts as the crown pillar and the NPV of the mining project is $ 102.02 billion. A representation of the location and size of the crown pillar for exploiting the copper deposit is shown in Fig. 5.

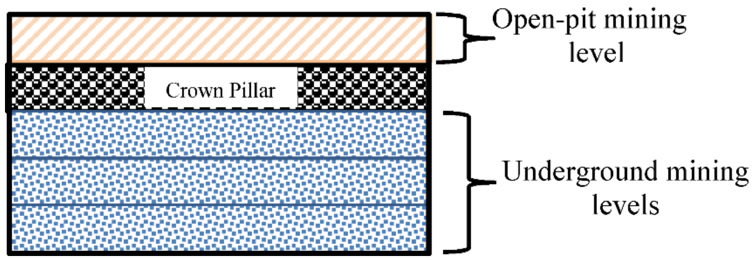

Fig. 5. Representation of the location of the crown pillar for the gold deposit in case study 1.

5.2. Case study 2 – dataset on gold deposit

The strategic Mixed Integer Linear Programming (MILP) models were implemented and tested on the gold deposit. The gold deposit consists of 420 mining blocks representing the Selective Mining Units (SMUs), referred to as mining-cuts in open pit mining and stopes in underground mining. The gold deposit is represented in a geological block model, which is a 3D array of cubical blocks containing 6,720-unit blocks of sizes 15 x 15 x 5. These unit blocks were re-blocked into the 420 SMUs of sizes 30 x 30 x 20 using GEOVIA GEMS 6.8 (Dassault, 2018). The orebody in the block model is irregularly shaped with a total mineral resource of 25.7 Mt of ore with an average Au grade of 0.7689 g/t. Table 4 is a description of the gold deposit used to implement the models.

Table 4. Statistical description of the gold deposit.

No.	Description	Value
1	Total material (Mt)	860.24
2	Total mineralized material (Mt)	25.70
3	Minimum value of Au (g/t)	0.7187
4	Maximum value of Au (g/t)	0.8765
5	Average value of Au (g/t)	0.7659
6	Variance	0.0006
7	Standard deviation	0.0252

5.2.1. Economic and mining data for case study 2

Taylor's mine life rule was used to estimate the mine life between 11.7 to 17.6 years. A mine life of 16 years was used to assess three mining options. Based on the quantity of mineralized material present in the deposit, the yearly mining and processing capacities were determined. With knowledge of a similar gold mining company in Canada (Centerra Gold Inc. and Premier Gold Mines Limited, 2016), an incremental bench cost of $ 2.0 per 10 m was used as the open pit mining variable cost as the pit transcends downwards. The economic, mining and processing data used for evaluating the gold deposit is shown in Table 5.

Table 5. Economic, mining and processing data for evaluating the gold deposit.

No.	Parameter	Values
1	Open pit mining cost ($/t)	5.51
2	Underground mining cost ($/t)	275.5
3	Processing cost ($/t)	8.93
4	Selling cost ($/oz)	48.0
5	Selling price of gold ($/oz)	1500
6	Discount rate (%)	10.0
7	Mining recovery (%)	95.0
8	Mining dilution	0.0
9	Max open pit (OP) processing capacity (Mt/year)	2.0
10	Min open pit (OP) processing capacity (Mt/year)	1.5
11	Max open pit (OP) mining capacity (Mt/year)	55.0
12	Min open pit (OP) mining capacity (Mt/year)	10.0
13	Max underground (UG) processing capacity (Mt/year)	1.40
14	Min underground (UG) processing capacity (Mt/year)	0.80
15	Max open pit & underground (OP-UG) processing capacity (Mt/year)	2.20
16	Min open pit & underground (OP-UG) processing capacity (Mt/year)	0.80
17	Max underground (UG) mining capacity (Mt/year)	50
18	Min underground (UG) mining capacity (Mt/year)	0.0
19	Incremental bench cost ($/10 m)	2.0
20	Average mine life (years)	16

5.2.2. Results and discussions for case study 2

The Net Present Value (NPV) of the three mining options were evaluated and the best mining option selected for the gold deposit. The results of the integrated MILP models for the open pit mining option, simultaneous open pit and underground mining option with a crown pillar and underground mining option with a crown pillar on case study 2 are shown in Table 6. The gold deposit is best exploited with open pit mining option for an operational mine life of 16 years since the generated NPV of $ 460.46 billion is greater than that determined for the other mining options.

Table 6. Computed results of the integrated MILP model for case study 2.

No.	Description	Discounted Revenue ($)	Ore Tonnage (Mt)	Rock Tonnage (Mt)	Resource Depletion (%)
1	Open pit mining ($ bn)	460.46	25.70	740.68	100.0
2	Underground mining ($ bn)	347.62	22.23	392.05	86.5
3	Open pit and underground mining (Simultaneous) ($ bn)	453.67	24.53	481.82	95.4

From the resource depletion point of view, the investor, mine operator and government regulations have broader understanding to the resource exploitation from the implementation of each mining option. The open pit mining option with the highest NPV depletes all the available mineral resource while the OP-UG mining option depletes 95.4% of the available mineral resource. The remaining 4.6% of the available mineral resource per the OP-UG mining option remains in the crown pillar as unmined.

The yearly ore production schedule for the open pit mining (OP), underground mining (UG), and simultaneous open pit and underground mining (OP-UG) with a crown pillar option are respectively shown in Fig. 6, Fig. 7 and Fig. 8. The ore production schedule for the OP and OP-UG (Fig. 6 and Fig. 8) show a quick rise in the ore production at the initial years of the mine life but falls through the middle sections of the mine life towards the end. This indicates the readily available ore characterized by open pit mining option as opposed to underground mining option shown in Fig. 7. The cashflow is however steady for the UG mining option (Fig. 7) compared to the OP (Fig. 6) and OP-UG (Fig. 7) mining options. The huge drop in the ore production at the initial years compared to the later years of the mine life needs to be controlled to reflect practical mining operations.

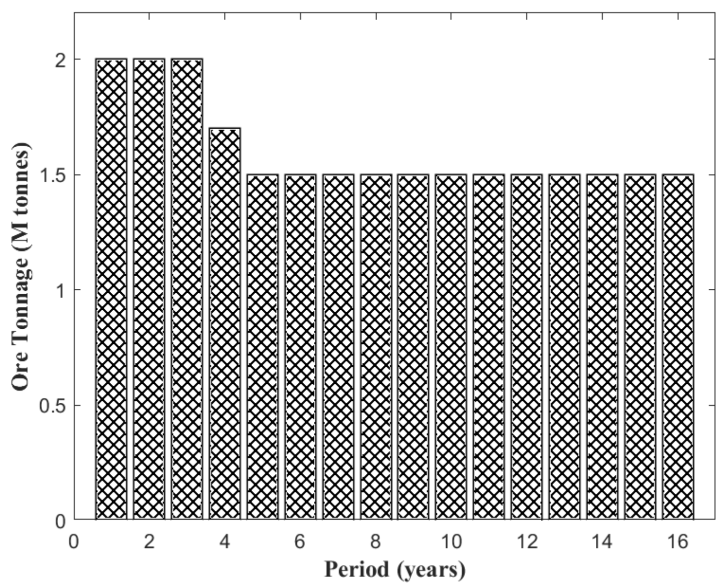

Fig. 6. Yearly ore production schedule for the open pit mining option in case study 2.

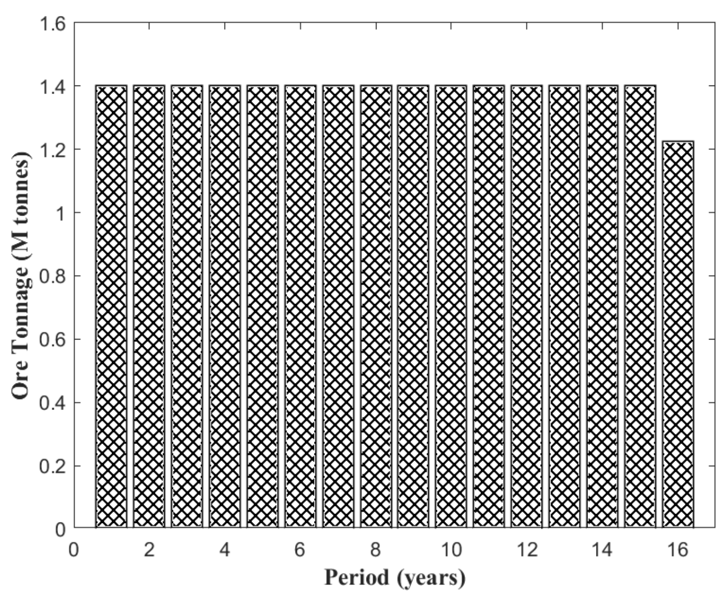

Fig. 7. Yearly ore production schedule for the underground mining option in case study 2.

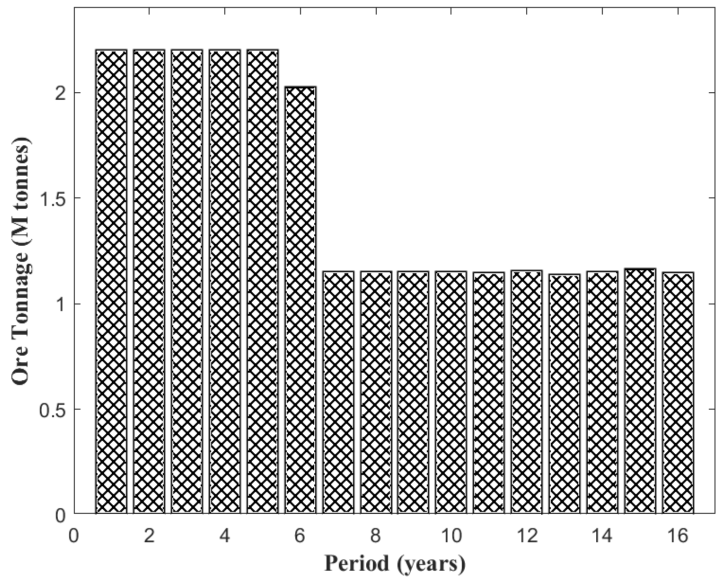

Fig. 8. Yearly ore production schedule for the simultaneous open pit and underground mining option in case study 2.

For the simultaneous open pit and underground mining option (OP-UG) with a required crown pillar, Model 3; when the open pit mining cost is \$ 5.51 per tonne and the underground mining cost is \$ 275.5 per tonne, level 3 acts as the crown pillar and the NPV of the mining project is \$ 453.67 billion. A representation of the location and size of the required crown pillar for the exploitation of the deposit is shown in Fig. 9.

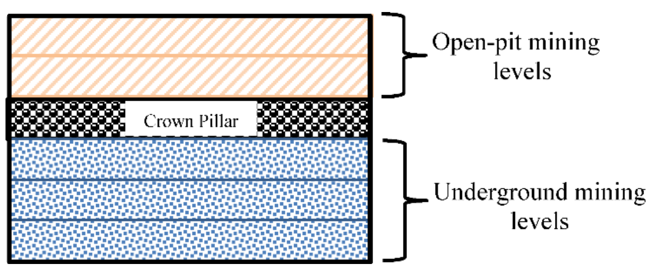

Fig. 9. Representation of the location of the crown pillar for the gold deposit in case study 2.

6. Conclusions and Future Works

In this paper, three Mixed Integer Linear Programming (MILP) models have been developed, implemented, and tested on two datasets. The MILP models produce a deterministic open pit production scheduling (Model 1), underground production scheduling with a required crown pillar (Model 2) and simultaneous open pit and underground mining production scheduling with a required crown pillar (Model 3).

For Models 2 and 3, the block model is organized by determining the location of the underground operating developments and the extraction sequence. The models are applicable to any preferred direction of mineral extraction sequence. All the models require that the cost of open pit is defined per depth (m) of ore extraction, thus, the cost of open pit mining increases with depth. This concept allows the optimization to decide when to stop the open pit mining, introduce a crown pillar and initiates underground mining.

Two novel models have been developed to interrogate a deposit and decide whether the deposit should be extracted by open pit mining or underground mining with a crown pillar or simultaneous open pit and underground mining with a crown pillar. For the underground mining and simultaneous open pit and underground mining options, the location of the 3D crown pillar is decided by the optimization process and not by human interferences.

Further research work is focused on controlling the respective gradual rise and fall in achieving ore production targets at the initial and final years of the mine life to ensure a steady cashflow of the mining project. Similarly, it is necessary to integrate prescheduled underground developments to Models 2 and 3 as required for exploiting the ore to ensure the discounted costs and revenues depicts the practicalities of the mining industry. The models will further be extended from its current deterministic approach to a stochastic mathematical programming domain to address the geological uncertainty of the deposit and the risk associated with the mining project.

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