Uncertainty-Based Mine Planning Framework for Oil Sands Production Scheduling and Waste Management

Ahlam Maremi and Eugene Ben-Awuah
Mining Optimization Laboratory (MOL)
Laurentian University, Sudbury, Canada

ABSTRACT

For oil sands mining, the production schedule must be integrated simultaneously with in-pit and ex-pit dyke construction scheduling. The mined ore that exceeds the plant capacity is stockpiled for a limited duration. The topmost layer of the overburden is also stockpiled for land reclamation. Uncertainty is always present in the presence of sparse geological data. Kriged estimates with a variance penalty scheme are used to minimize the financial risk from grade uncertainty associated with the production schedule. An uncertainty-based mathematical programming model is developed based on Mixed Integer Linear Goal Programming (MILGP) for oil sands production scheduling and waste management. The model aims to generate a life of mine plan that maximize the NPV of the project. In addition, the model determines the rehandling strategy for the stockpiled ore and the destination of reclamation and dyke materials to minimize costs. The uncertainty-based model is implemented with two different scenarios. Scenario 1 generates an integrated mine plan with a waste management and stockpiling strategy that maximizes the NPV of the operation and minimizes dyke construction cost. Scenario 2 generates a range of total NPV and estimates the production scheduling risk associated with grade uncertainty.

1. Introduction

In open pit mining, the goal is usually to maximize the Net Present Value (NPV) of the project by providing the plant with ore at full capacity while satisfying physical, operational and economic constraints. The very first and highly important step in the mine planning process is modeling the ore body appropriately. All other activities throughout the mine life starting from evaluating the economic viability of the entire mining operation to undertaking all the processes of mine planning will be based on the ore body model (Hustrulid and Kuchta, 2006). The different phases of the mine planning process include: 1) Block model determination that consists of drilling in different locations and depths of the mine; and obtaining samples of material for grade and density interpolation, dividing the orebody into blocks of equal sizes and assigning estimated tonnage and mineral grades to each block. As a result, the estimated extraction profit or loss for each block in the model needs to be computed. This generates what is referred to as an economic block model; 2) Definition of the Ultimate Pit Limit (UPL) which is the area in which extraction will take place. Before any block can be extracted, all blocks immediately above and at certain angles must be extracted. To determine the UPL, it is necessary to determine the slope angles. This depends on the structural composition of the rocks and the location and depth of each block; 3) Production planning, which involves the decision of which blocks and when and how they should be extracted (Chicoisne, et al., 2012). Extracting mining blocks from an open pit mine in a specific sequence to give the highest NPV is known as open pit mine planning optimization. This is subject to a variety of production, grade blending and pit slope constraints (Whittle, 1989).

The focus of this paper is Long-Term Production Planning (LTPP) optimization with integrated waste management. Oil sands mining of the McMurray formation was studied and used as a case study. The paper seeks to develop an uncertainty-based theoretical framework that maximizes the NPV of an oil sands mining operation and minimizes waste management cost using Mixed Integer Linear Goal Programming (MILGP) model. The model incorporates multiple material types with multiple elements for multiple destinations in oil sands long-term production planning. Although mathematical programming models have been applied in mine production scheduling, "very little work has been carried out in terms of oil sands mine planning, which has a unique scenario when it comes to waste management" (Ben-Awuah and Askari-Nasab, 2013). The proposed model integrates waste management strategy into the production plan as required by recent regulations from Alberta Energy Regulator (AER) Directive 085 (formerly interim directive ID 2001-7) (Alberta Energy Regulator, 2017). Muskeg, the topmost layer of the overburden, is stockpiled and will be used for land reclamation at the end of the mine life. The model also includes a limited duration stockpiling strategy for the mined ore that exceeds the processing capacity. Financial risk is defined as the deviation from expected historical returns within a specific period (Mangram, 2013). The optimal solution to the LTPP problem is affected by uncertainties related to the input parameters. To minimize the financial risk (risk) associated with grade uncertainty, kriging estimates with a variance penalty scheme are used for the production scheduling optimization.

Subsequently, a case study with two different scenarios is examined using the proposed MILGP model. The first scenario, Scenario 1, uses goal functions with a limited duration stockpiling strategy for ore and stockpiling strategy for reclamation material. The second scenario, Scenario 2, evaluates the risk associated with the production schedule based on grade uncertainty and a variance penalty scheme. MATLAB (Mathworks, 2017) is used for coding the mathematical programming formulation and the resulting optimization problem is solved with a large-scale optimization solver IBM/CPLEX (ILOG, 2012). This solver uses a branch and cut algorithm which is a hybrid of branch-and-bound algorithm and cutting plane methods to solve the optimization problem (Horst and Hoang, 1996; Wolsey, 1998).

The next section of this paper gives details of the problem definition. Section 3 covers a summary of the literature review on LTPP optimization problems based on deterministic and uncertainty approaches, clustering and paneling in mine planning, and stockpiling. The process of oil sands mining and waste management is explained in Section 04. Section 5 highlights the concepts of block modeling, variography and kriged estimates with a variance penalty scheme to manage risk. Section 6 presents the theoretical mathematical programming formulation. The implementation of the MILGP model is in Section 7. A case study is presented in Section 8. Finally, Section 9 documents the research conclusions and recommendations.

2. Problem Definition

Oil sands mining operations result in different types of material: ore, interburden (IB), overburden (OB), reclamation material (RM) and waste. Material with a bitumen grade of 7% or more will be classified as ore Directive 082: (Alberta Energy Regulator, 2016). Processing the ore results in a huge amount of tailings. Based on the fines content, the tailings are divided into Tailings Coarse Sands (TCS) and tailings slurry. TCS is used for dyke construction and tailings slurry is deposited in the disposal areas created with dykes. Any ore material that has a bitumen grade less than 7%, known as interburden, will be reclassified based on the fines content. Material with fines content less than 50% will be used for dyke construction; otherwise, it will be sent to the waste dump. Overburden (OB) comprises of the Pleistocene unit and Clearwater formation. OB will be used either for road construction or for dyke construction if it meets the fines requirement. Muskeg, the topmost layer of the overburden, will be extracted, stockpiled and used to reclaim the land at the end of the mine life. Any material that does not meet the requirements of ore, dyke materials or reclamation material is classified as waste and will be sent to the waste dump.

A schematic representation of the problem definition is presented in Fig. 1**.** The final pit block model is divided into pushbacks. The material intersecting a pushback and a bench is known as a mining-panel. Each mining-panel contains a set of mining-cuts and is used to control the mine production operation sequencing. Mining-cuts are clusters of blocks within the same mining bench that are similar in terms of location, grade, rock type and the shape of mining-cuts created on the lower bench. The figure depicts the scheduling of an oil sands ultimate pit block model containing K mining-cuts within P mining-panels. Each mining-cut k, could be made up of one or more of the following materials: ore, O_k, interburden and overburden dyke materials, ib_k and ob_k, muskeg reclamation material, mu_k, and waste, w_k. The material in each mining-cut is to be scheduled over T periods based on the goals and constraints associated with the mining operation. The mined ore extracted from mining-cut k within mining-panel p in period t will be sent to the processing destination a. Any material that exceeds the processing capacity will be sent to the stockpile sp in period t and will be reclaimed in period $t+ts$, where ts is the stockpiling duration limit controlled by the planner to minimize oxidation of the stockpiled material. Oxidized ore reduces processing recovery. The ore extracted in the current period t and the ore that has been sent to the stockpile in period $t-ts$ together will be sent to the processing destination to extract the bitumen. The generated TCS material together with the OB and IB dyke materials will be used for constructing $dyke_i$ at site i. Muskeg material will be sent to the muskeg stockpile area for reclamation. This is referred to as operational material scheduling.

These strategic and operational schedules to be developed are subject to a variety of economic, technical and physical constraints. The constraints control the mining extraction sequence and ore and dyke material blending requirements. The constraints also control mining, processing, muskeg and dyke material goals that specify the quantities of allowable material for the mining operation, processing plant, reclamation works and dyke construction.

Considering uncertain input variables will minimize the difference between the theoretical and actual NPV, and will result in a high degree of confidence for the mining project. Based on this, production risk from grade uncertainty will be minimized using the variance and a penalty scheme. The strategic and operational schedules determine the profitability and sustainability of the project. The schedules control the NPV of the operation and enable a robust waste management planning strategy. Improper waste management planning can lead to environmental issues, resulting in immediate mine closure by regulatory agencies. It is assumed that when a mining-panel is scheduled, all the mining-cuts, blocks or parcels within this mining-panel are extracted uniformly. Also, it is assumed that when modeling the relationship between the mining-panels and mining-cuts, the planner has access to all the mining-cuts within each mining-panel. The stockpiling strategy is considered in the optimization problem for extra ore that exceeds the mill capacity and there are stockpile bins available for each period. It is considered that the exact amount of ore sent to the stockpile in period t will be reclaimed after the stockpiling duration ts controlled by the planner.

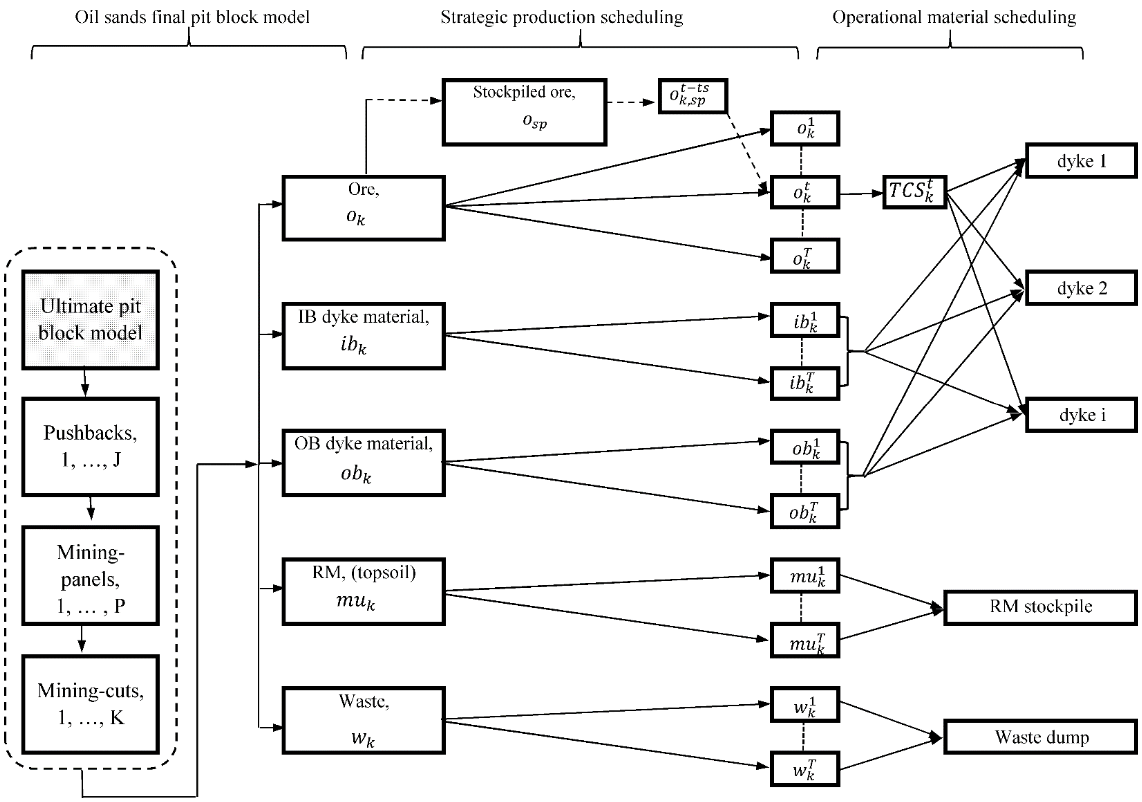

Fig. 1. Schematic representation of the problem definition showing strategic production and operational material scheduling.

3. Summary of Literature Review

Long-term production planning (LTPP) focuses mainly on ore reserves, stripping ratio and major annual investment plans (Newman, et al., 2010). The geologic block model is the backbone of open pit mine design and scheduling processes. Assigning the geological characteristics of each block and their grade can be done using available estimation techniques. Using financial and metallurgical data, the economic value of each block is also calculated (Osanloo, et al., 2008). Since the 1960s, researchers have studied and applied Mathematical Programming Models (MPMs) such as Linear Programming (LP), Integer Programming (IP), and Mixed Integer Linear Programming (MILP). These are commonly used in addition to Dynamic Programming (DP) and Goal Programming (GP) for mine production scheduling (Osanloo, et al., 2008).

Johnson (1969) introduced linear programming as a MPM to the mine planning research area. The author's model was for a long-term multi-destination open pit production planning problem. The results were not optimum and the size of the problem was computationally intractable. Subsequently, the initial LP model was modified by Gershon (1983) and Dagdelen (1985) to a MILP model. The authors considered a set of binary variables to satisfy the precedence of block extraction. The modified models could handle multiple ore processing options and multiple grades. However, their formulations could not ensure feasible solutions for all cases. Also, the number of binary variables makes the model intractable for real-size mine planning projects and difficult to be solved with the current state of hardware and software.

Ramazan and Dimitrakopoulos (2004a) developed MILP formulations to reduce the number of binary variables and solution times. They set certain variables as binary and others as continuous. Their model resulted in partial mining of blocks that have the same ore value affecting the NPV generated. Askari-Nasab, et al. (2011) developed MILP models that use block clustering techniques. The models use a combination of continuous and binary integer variables and were applied to a large-scale problem. The authors stated that they successfully implemented the models for some basic large-scale production scheduling problems. A Dynamic Programming (DP) model that maximizes the NPV, subject to production and processing constraints was presented by Osanloo, et al. (2008). This model considers both the time value of money and block sequencing to determine the UPL. However, it cannot be applied to large-scale problems and there is no guarantee that mining and processing constraints will be satisfied. In general, applying MPMs to the LTPP result in large scale optimization problems with many integer and continuous variables which are difficult to solve with the available software and hardware and might need lengthy solution time. The efforts that have been made in reducing the solution time were inefficient for large-scale problems or could not generate integrated practical mining strategies.

One of the deterministic approaches used to solve long-term production planning and scheduling problems is Goal Programming (GP). It is a popular deterministic approach for solving multiple objective optimization problems. The main idea of GP is that the optimizer provides results for the objective very close to the required goals, regardless of whether the goals are achievable or not. GP minimizes the deviations between the target values of the objectives and the satisfying solution (Orumie and Ebong, 2014). Ben-Awuah and Askari-Nasab (2013) formulated the oil sands LTPP and waste disposal planning problem using a combination of MILP and GP formulations. The hybrid termed as Mixed Integer Linear Goal Programming (MILGP) has an objective function, goal functions and constraints. These goals are prioritized according to the impact of a deviation from their targets on the entire mining operation. The authors stated that using MILGP is appropriate for their framework because, based on the importance of the goals, the MILGP structure will allow the planner to achieve some goals while others are traded off. According to the authors, solutions with known optimality limits are generated when using exact solution methods for LTPP problems. For the resulting production schedule, a higher NPV is achieved as the solution gets closer to optimality.

In LTPP, the size of the problem grows exponentially as the number of blocks increases resulting in insufficient computer memory during optimization. Researchers have tried to classify the large amount of data into relatively few classes of similar entities (cluster) by maximizing both intra-cluster similarity and inter-cluster dissimilarity. This classification is known as aggregation or clustering. Clustering will minimize the number of integer decision variables as well as maintaining the minimum mining width for large mining equipment (Askari-Nasab and Awuah-Offei, 2009). Clustering algorithms can be categorized into hierarchical, partitional or overlapping clustering (Tabesh and Askari-Nasab, 2011). In mine planning, only hierarchical and partitional clustering can be used because all blocks must belong to a single cluster. Tabesh and Askari-Nasab (2011) developed a new clustering approach more suitable to the mining industry. Paneling is another technique that has been introduced in production scheduling to maintain practical mining widths and reduce the size of the optimization problem. The intersections of pushbacks and mining benches generate mining panels (Ben-Awuah and Askari-Nasab, 2013). Each mining-panel contains a set of mining-cuts and is used to control the mine production operation sequencing. For this paper, hierarchical clustering algorithm developed by Tabesh and Askari-Nasab (2011) was used.

In the implementation of most LP and MILP models, the material flow post-extraction is not considered (Moreno, et al., 2017). “In particular, the use of stockpiling to manage processing plant capacity, and the interplay of material flows from the mine to a stockpile, the mine to a processing plant, and a stockpile to a plant, have not been treated as an integrated part of mine extraction sequence optimization” (Moreno, et al., 2017). Stockpiling can be used in mine operations for many reasons such as the blending of material, storage of overproduced ore or low-grade ore for future processing, and storage of waste material for reclamation purposes. Asad (2005) cautioned that long-term stockpiling could result in problems such as leaching, deterioration of material and oxidation, which might result in poor recovery in the treatment process. For oil sands mining, the stockpiled material must also be processed within a limited duration due to oxidation that affects efficiency of the processing recovery process. For this paper, the stockpiling duration is limited to a maximum of two years to ensure there is no significant effect on the ore recovery.

The geologic block model is the main input data for optimizing LTPP problems. Some random input variables such as grades, rock types, costs, prices, and recoveries might affect the optimal solution. In deterministic approaches, the best-estimated values of these random variables available at the time are used for the optimization run. A rerun is required when new data become available. Uncertainty can be reduced only by getting more data over time. Most researches have focused on minimizing the negative impacts of grade, geological, and market uncertainties on production schedules. The uncertainties involved in mine planning can be classified as: 1) orebody model and in-situ grade uncertainty and material type distribution; 2) technical mining specification uncertainty, for example, mining and processing capacities and slope consideration; and 3) financial uncertainties including prices and costs (Dimitrakopoulos, 1998). Results have shown that significant differences might exist between actual production and theoretical expectations due to geological and economic uncertainties (the most important sources of risk in mining operations), especially in the first years of production. A loss of $1.4 billion to the Canadian mining industry in early 1991 has been reported due to geological and economic uncertainties (Sabour and Dimitrakopoulos, 2011). A survey of mining operations in the early production years show that 60% of mines had 70% less production than designed capacity (Osanloo, et al., 2008). As hardware, software, and solution techniques evolve, more accurate models are expected (Osanloo, et al., 2008).

Gholamnejad and Osanloo (2007) presented an uncertainty-based model for production scheduling. They considered grade uncertainty in which each block has a probability distribution function obtained using geostatistical simulation. However, they do not give details or examples. Koushavand, et al. (2009) presented two different methodologies based on grade uncertainty for open pit mine production schedules. They evaluated the output parameters such as NPV, ore tonnage, head grade, stripping ratio, amount of final production and annual target production. They stated that there is significant uncertainty in the long-term production schedules. Also, the long-term schedule based on one particular simulated ore body model is not optimal for other simulated geological models. Dimitrakopoulos and Ramazan (2008) presented a stochastic integer programming model to generate the optimal production schedule using multiple realizations as input. They used a penalty scheme for the deviation from the target production. The function is calculated from a Geological Discount Rate (GDR). They use linear programming to maximize NPV minus penalty costs. They stated that the production schedule is the optimum solution. However, they did not show how to define the GDR parameter. Sabour and Dimitrakopoulos (2011) built their model based on metal price and exchange rate uncertainties. They developed a system for mine planning and scheduling that instantly reacts to new information. They claimed that their results showed a significant difference and improved the design-ranking process.

Solving MPMs with deterministic approaches generate solutions within known limits of optimality. Though, they result in large-scale optimization problems that are difficult to solve with the current computing software and hardware and may have lengthy solution times. In order to decrease the size of the optimization problem, clustering is used in mine planning. It generates practical schedules, the formulations are faster to solve and it is easy to implement. Stockpiling is introduced for material blending, storage of overproduced ore or low-grade ore for future processing, and storage of waste material for reclamation purposes. Geological and economic uncertainties are also used in mine planning to minimize the differences between actual production and theoretical expectations that will result in a high degree of confidence in the NPV of the mining project. The aim of this research is to propose a MILGP optimization framework that integrates oil sands production planning and waste management for multiple material types, elements and destinations with limited duration stockpiling. The research also aims to introduce an uncertainty-based MILGP model that evaluates production schedule financial risk associated with grade uncertainty.

4. Oil Sands Mining and Waste Management

Oil sands mining is one of the most rapidly developing industries in North America. This industry started in the 1960s with surface mining operations that used Clark Hot Water Extraction (CHWE) to extract bitumen from the McMurray formation (Masliyah, 2010 ; Sanford, 1983). A truck-shovel system is used to extract the oil sands from the Athabasca Wabiskaw-McMurray deposit, which is located in the northeastern part of the province of Alberta. It is the largest deposit in the world and mostly located near the surface. The Pleistocene unit is the topmost layer of the formation. It contains muskeg, also known as bog peat, which is comprised mainly of organic matter. The Clearwater formation overlying the McMurray formation is comprised of marine clay, fine sand and siltstone. Both the Pleistocene and Clearwater formation are known as Overburden (OB). The McMurray formation contains bitumen, the element of interest. It is informally subdivided into Upper, Middle and Lower formations based on the environment of sediments deposition. Devonian carbonates mark the end of the oil sands deposit (Masliyah, 2010). Fig. 2 shows a sketch of the vertical soil profile for an oil sands formation.

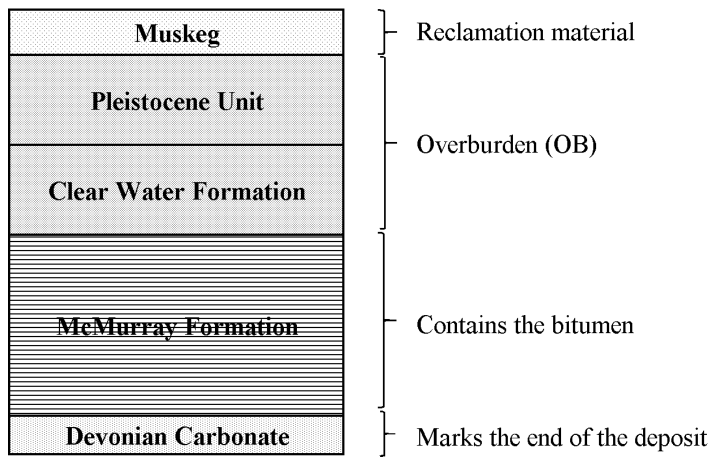

Fig. 2. Schematic view of the soil profile for an oil sands formation.

During oil sands mining, huge amounts of bituminous sands are sent to the processing plant, which results in a huge amount of a mixture of water, fine materials, sands and residual bitumen known as tailings (Masliyah, 2010). There are three significant aspects in dealing with oil sands tailings, the most unwanted by-product of oil sands processing. First, the greenhouse gas emissions resulting from the CHWE process (Devenny, 2009). Second, the environmental challenges due to the toxicity of the tailings resulting in the contamination of the fresh water table by polluted tailings' water leaks. Third, space limitations increases the need for in-pit tailings containment, and storage space since more mining processes lead to additional volume of tailings slurry (Devenny, 2009).

Presently, plans for tailings deposition and mine reclamation are prepared after the optimization of the long-term mine production plans (Ben-Awuah and Askari-Nasab, 2013). Directive 085 (Alberta Energy Regulator, 2017) issued by the Alberta Energy Regulator (AER) requires oil sands operators to periodically publish their waste disposal and tailings plans publically (McFadyen, 2008; Ben-Awuah and Askari-Nasab, 2013).

It should be mentioned that according to Directive 082, any ore with a bitumen grade of 7% or more should not be left behind (Alberta Energy Regulator, 2016). Air flotation technique is used to separate bitumen from the fines results in tailings. Using the hydro-cyclone, tailings is classified into Tailings Coarse Sands (TCS) and Fine Sands (Kalantari, et al., 2013). The ore with a bitumen grade of less than 7%, known as interburden (IB), will be used together with Overburden (OB) and (TCS) for dyke construction if they meet the dyke material requirements. It is important that the sequence of extracting the ore and the supply of material for dyke construction be integrated to guarantee a uniform material supply to the plant and for dyke construction throughout the mine life (Fauquier and al., 2009). Fig. 3 shows a conceptual mining model, which includes an oil sands deposit area to be mined and simultaneously used as an in-pit tailings storage facility. As mining advances in the specified direction, the in-pit tailings dyke footprints are released for dyke construction.

Waste management is a significant part of oil sands mining operations. It requires special geotechnical considerations and tailings management techniques that may lead to economic liabilities and delayed reclamation if not well managed (Boratynec, 2003; Ben-Awuah and Askari-Nasab, 2013; Azam and Scott, 2005). Additional documentation on oil sands solid waste and tailings management can be found in Ben-Awuah, et al., (2012); Badiozamani and Askari-Nasab, (2014); Badiozamani and Askari-Nasab, (2016).

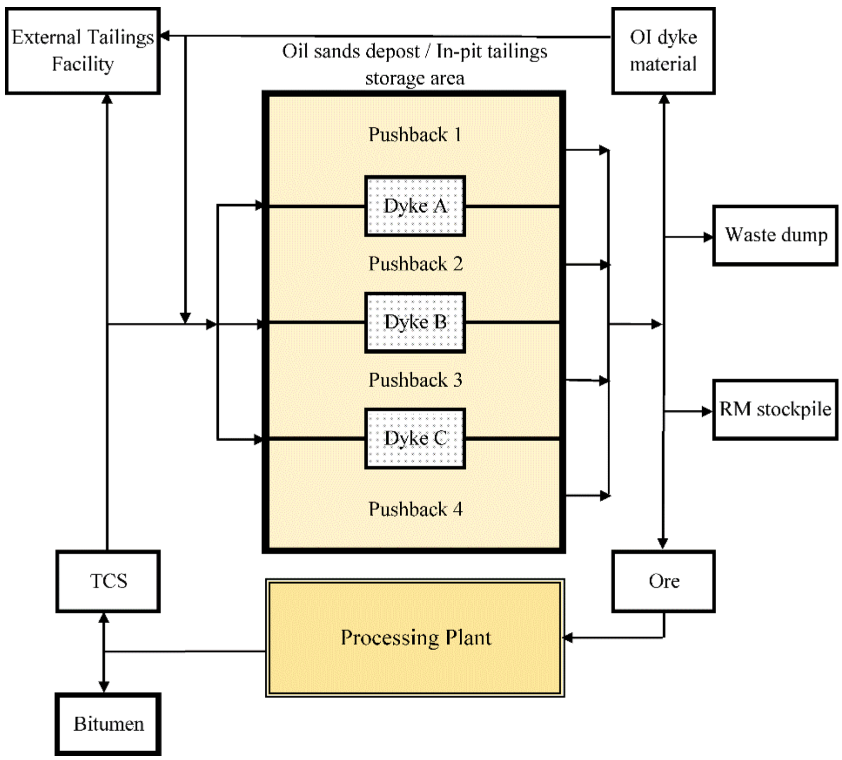

Fig. 3. Conceptual model for mining and waste management strategy modified after Ben-Awuah and Askari-Nasab (2013).

5. Block Modeling, Variography and Kriging

For this paper, a block model was created using Ordinary Kriging (OK) as the best linear unbiased estimator (Isaaks and Srivastava, 1989). For oil sands resource modeling, it has been found that OK with a large number of search data results in a low observed mean squared error (Deutsch, et al., 2014). This makes OK a preferred resource estimation technique. With Geovia GEMS (Gemcom Software International, 2015a), the block model was used to define the orebody for reserve estimation and mine planning. The creation of interpolation profiles was required to determine the spatial correlation between the observations. A semi-variogram was used for this purpose. Variogram models are used in kriging estimation procedures, which can be used further to construct search parameters for interpolation techniques. To create a variogram model, an experimental variogram was calculated for the oil sands McMurray Formation being used as case study. Omnidirectional variograms for bitumen grades were prepared to identify the sill while vertical variograms were used to identify the nugget effect. Primary variogram maps were calculated to determine the orientation of the major axis in the presence of anisotropy. The directional horizontal variogram (major axis) was calculated and modeled. The secondary variogram maps (semi-major axis) were calculated and modeled. The variograms were extracted along the ellipse axis. The major, semi-major and minor axes were selected for final modeling. Search ellipse profiles and semi-variogram profiles were updated from the semi-variogram models and used for interpolation within the ore rock types for bitumen and fines grades.

Ordinary Kriging estimates are calculated for each block in the block model. Using the concept of Block Kriging (Gringarten and V. Deutsch, 2001), these kriging estimates are used together with the sill, the nugget effect and the range to calculate the Ordinary Kriging grade and variance for each mining-cut and mining-panel in the final pit limit. The kriged estimates together with a variance penalty scheme are used for mine planning to estimate the financial risk (risk) associated with grade uncertainty as explained in Section 5.2.

5.1. Kriged estimates and a variance penalty scheme

Ordinary Kriging (OK) estimates the expected value of a spatial variable (the grade in this case) and the surrounding uncertainty at a given location (Isaaks and Srivastava, 1989). It is the best linear unbiased estimator since it tries to have the mean residuals equal to zeros and aims to minimize the variance of the errors. OK uses statistical distance instead of geometric distance, which controls redundancy of estimation samples. As mentioned in Section5, the spatial continuity for bitumen and fines grades was modeled with semi-variograms, so the pattern and anisotropy are incorporated into the estimation using OK. The general equation for anisotropic spherical variogram is given by Eq. (1). The variogram models for bitumen and fines consists of two nested spherical models Table 1.

$$\gamma(h) = \begin{cases} C_0 & \text{if } h = 0.0 \\ C_0 + C \times \left(1.5\left(\frac{h}{r}\right) - 0.5\left(\frac{h}{r}\right)^3\right) & \text{if } 0 < h < r \\ C_0 + C & \text{if } h \geq r \end{cases} \tag{1}$$

Table 1. Variogram model for bitumen and fines grades.

Element	Nugget Effect, C_0	Spherical 1		Spherical 2	
		Sill, C	Range, r	Sill, C	Range, r
Bitumen	0.026	0.673	46.34	0.292	46.61
Fines	0.000	0.600	29.15	0.400	44.13

Using the semi-variograms, the kriged grade and kriged variance are determined using OK. The kriged variance is robust to most errors likely to be made in the semi-variogram model selection for spherical models. However, the nugget effect should be carefully selected to avoid understating the kriged variance (Brooker, 1986).

The kriged block estimates (bitumen grades), sill, nugget effect and the range are used to determine the mining-cut and mining-panel kriged grades and kriged variances following the concept of Block Kriging. The estimated kriged variances for all mining-panels are used to calculate the frequency and probability of occurrence of the kriged variance. The latter is used to calculate the expected variance of the mining-panels.

5.2. The effect of grade uncertainty

Grade uncertainty affects the metal content in the material sent to the plant which will subsequently affects the NPV of a project. This uncertainty exists because of our ignorance or lack of knowledge. It is not an inherent feature of the deposit (Isaaks and Srivastava, 1989).

For this study, the effect of grade uncertainty on the mine plan is investigated through the grade variance. A more robust method is to minimize the grade-variance effect while the expected value of NPV is maximized. It is called the mean-variance method. The grade uncertainty can affect the Economic Block Values (EBV) that will affect the production plan due to the variation in ore tonnages sent to the mill. Consequently, this will affect the NPV of the project. This approach works based on the mean-variance method referred to as the Modern Portfolio Theory (MPT) for risk-based portfolio optimization (Markowitz, 1952). MPT requires calculating the expected value and expected variance of a portfolio's return considering a weighted average combination of the assets' return (Fig. 4). MPT is a quadratic optimization problem that maximizes the expected return and minimizes the standard deviation. The decision variables are the weights or the portions of each asset's contribution towards the objective function.

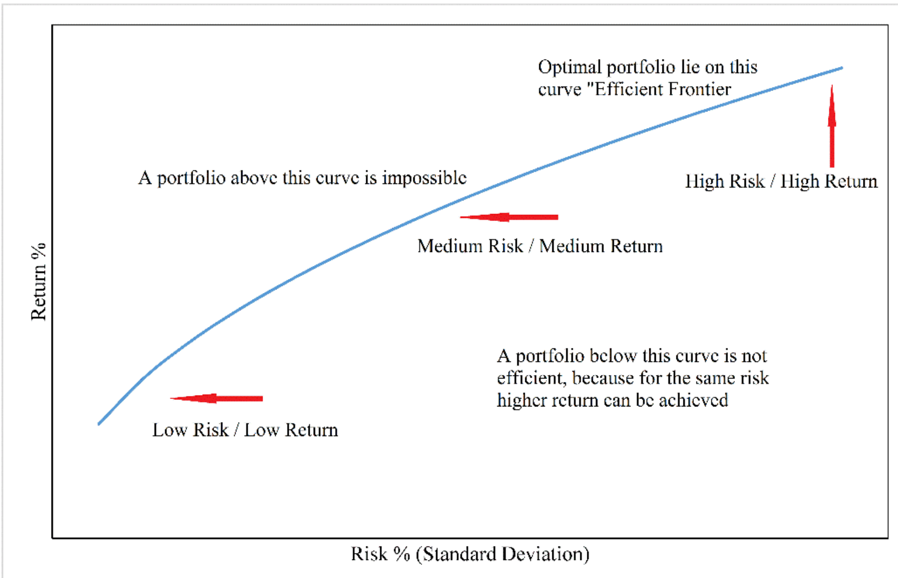

Fig. 4. Return versus standard deviation, modified after dos Santos and Brandi (2017).

In this paper, the mean-variance method is adapted and applied to production planning optimization in the presence of grade uncertainty using an uncertainty-based MILGP model to reduce project risk. The kriged mean and kriged variance for each mining block, mining-cut and mining-panel are calculated as well as the EBVs. The main concept of the mean-variance method deployed for long-term mine planning is to find the low variance mining blocks, mining-cuts or mining-panels to be extracted earlier, such that the average project NPV is maximized and the mining of high-variance blocks, cuts or panels is deferred to later years.

In the uncertainty-based MILGP model, the kriged variance for mining-panels together with a penalty scheme is used to minimize the risk associated with the production schedule. Applying the penalty postpones mining of high variance mining-panels to later years. Fig. 5 shows the kriged variance for mining-panels within the UPL for a case study. This paper investigates two options in applying the variance penalty for mining-panels. The first option is to apply the penalty for high variance mining-panels only. In this case, the expected variance of the deposit is the threshold above which the variance of a mining-panel is classified as high. The expected variance is used to determine whether a mining-panel should be penalized or not. Mining-panels with kriged variance greater than the expected variance will be penalized. The second option is to apply the penalty for all mining-panels. The optimizer will give preference to the mining of low variance mining-panels earlier in the mine life to reduce the risk associated with achieving the production schedule. The term "grade uncertainty cost" is used in this study to represent a pseudo cost for each mining-panel calculated as a product of a penalty value and the mining-panel kriged variance. Subsequently, the grade uncertainty cost is used to enforce the mining of low variance mining-panels early in the mine life to reduce project risk. The "overall uncertainty cost" is therefore a quantitative parameter which estimates the difference in NPV of the production schedule from the MILGP model and uncertainty-

based MILGP model due to grade uncertainty. The grade uncertainty cost is calculated from Eq. (2)
.

$$\text{Grade uncertainty cost } \left(PN_u\right) = \text{Penalty cost} \times \text{mining-panel kriged variance} \tag{2}$$

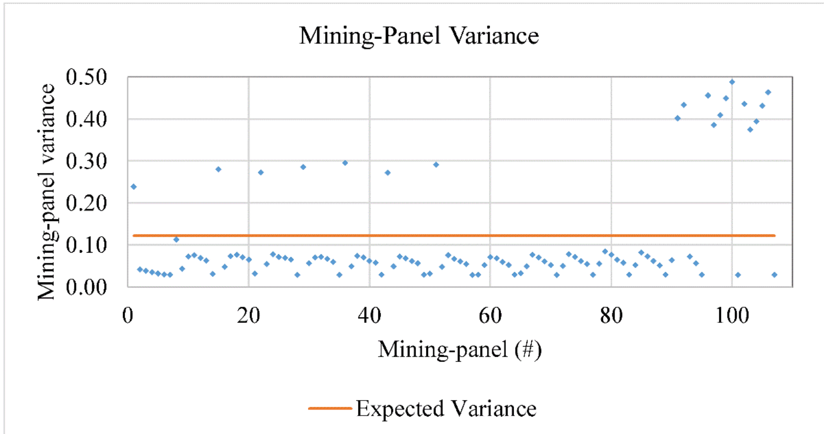

Fig. 5. Kriged variance for the mining-panels within the ultimate pit limit.

5.3. Penalty cost determination

The penalty cost is controlled by the planner. It starts from zero and increases gradually until the production schedule starts to change and consequently the NPV. As the penalty applied increases, the optimizer changes the production schedule, looking for low-variance mining-panels to be mined earlier to minimize the risk associated with the production schedule. At some point, increasing the penalty cost has no effect on the production schedule and subsequently the NPV (Fig. 6). Low penalty means high NPV and high risk, and vice versa. Increasing the penalty will minimize the risk that results in high degree of confidence in the mining project.

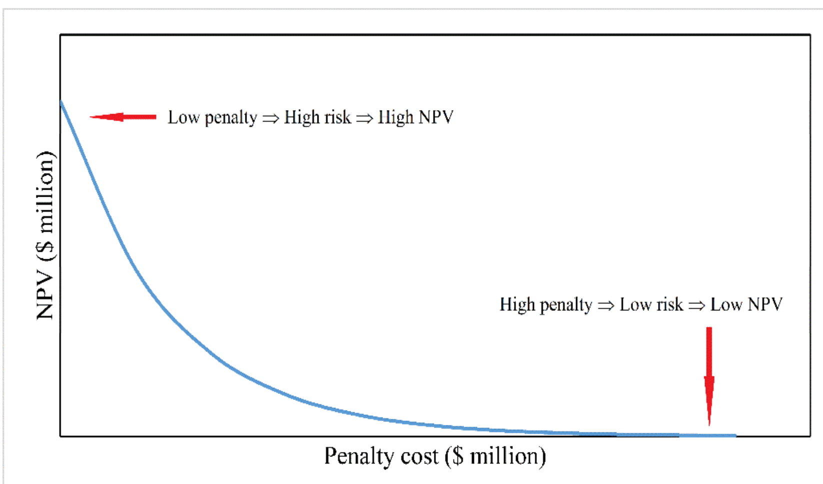

Fig. 6. Net present value versus penalty cost.

6. MILGP Theoretical Model Formulation

The strategic production schedule considers the time and sequence of extracting the ore, muskeg, overburden, interburden and waste blocks, as well as their destinations from a predefined UPL. The proposed MILGP model is capable of considering multiple mining locations, multiple pushbacks and different types of materials. The stockpiled ore can be reclaimed after open pit mining is completed or simultaneously during active open pit mining with a pre-determined reclamation duration.

However, long-term stockpiling could result in problems such as leaching, the deterioration of material and oxidation, which can affect the processing recovery efficiency. For oil sands mining, to avoid the risk of oxidation, the ore will be reclaimed in a pre-determined period controlled by the planner. The proposed oil sands production scheduling model integrates waste management through dyke construction and stockpiling for a limited duration. Stockpiling is for the mined ore that exceeds the plant capacity in any given year. The MILGP model is subject to economic, technical and physical constraints that control the mining operation. The notations used in the formulation of the oil sands long-term production planning and waste management framework have been classified as indices, sets, parameters and decision variables. The details of these notations can be found in the list of nomenclature in Appendix.

6.1. Modeling of economic mining-cut value

Based on the value of the mining-cut and the costs incurred during mining and processing operations, the discounted economic value of each mining-cut can be calculated after applying a discount rate to calculate its present value since the mining-cuts are extracted in different periods. The discounted economic mining-cut value can be calculated using the following formula:

Discounted Economic Mining-Cut Value = Discounted Revenue – Discounted Costs

For a mining-cut, if there are valuable elements, its discounted economic value if it is sent from the mine to the processing plant ($dm_k^{d,t}$) or from the stockpile to the processing plant ($ds_{k,sp}^{d,t}$) is given by Eqs. (3) and (4), respectively.

$$dm_k^{d,t} = rm_k^{a,e,t} - dw_k^{l,t} - dmu_k^{d,t} - dob_k^{d,t} - dib_k^{d,t} - dt_k^{d,t} \tag{3}$$

$$ds_{k,sp}^{d,t} = rs_k^{a,e,t} - dw_k^{l,t-ts} - dmu_k^{d,t-ts} - dob_k^{d,t-ts} - dib_k^{d,t-ts} - dt_k^{d,t-ts} \tag{4}$$

Eqs. (5) to (11) define the parameters used in Eqs. (3) and (4). Eq. (5) defines the discounted revenue generated from selling the final product within each mining-cut k minus the discounted processing cost; for mining-cuts sent directly from the mine to the processing plant. Eq. (6) defines the discounted revenue generated from selling the final product within each mining-cut k minus the discounted processing cost, minus the extra discounted re-handling cost; for mining-cuts sent from the mine through the stockpile to the processing plant. Eq. (7) defines the discounted cost for extracting mining-cut k as waste. Eq. (8) shows the extra discounted cost of re-handling reclamation material (Muskeg unit). Eqs. (9) to (11) show the extra discounted cost of mining the OB , IB and TCS dyke materials from mining-cut k , respectively.

$$rm_k^{a,e,t} = \sum_{e=1}^{E} o_k \times g_k^e \times rp_{avg}^{a,e} \times \left(p^{e,t} - sc^{e,t} \right) - \sum_{e=1}^{E} o_k \times pc^{a,e,t} \tag{5}$$

$$rs_k^{a,e,t} = \sum_{e=1}^{E} o_k \times g_k^e \times rp_{avg,sp}^{a,e} \times \left(p^{e,t} - sc^{e,t} \right) - \sum_{e=1}^{E} o_k \times pc^{a,e,t} - \sum_{e=1}^{E} o_k \times pc_{sp}^{a,e,t} \tag{6}$$

$$dw_k^{l,t} = (o_k + mu_k + ob_k + ib_k + w_k) \times mc^{l,t} \tag{7}$$

$$dmu_k^{d,t} = mu_k \times muc^{d,t} \tag{8}$$

$$dob_k^{d,t} = ob_k \times obc^{d,t} \tag{9}$$

$$dib_k^{d,t} = ib_k \times ibc^{d,t} \tag{10}$$

$$dt_k^{d,t} = t_k \times tc^{d,t} \tag{11}$$

6.2. Uncertainty-based MILGP objective function

The MILGP model objective function for oil sands long-term production planning and waste management is developed based on deterministic and risk-based approaches. The objective functions in general aim to:

- Maximize the NPV of the project;
- Minimize financial risk (risk) associated with the production schedule;
- Minimize reclamation material mining cost;
- Minimize dyke material mining cost;
- Minimize deviations from the production goals.

To develop the models, the concepts presented in Ben-Awuah and Askari-Nasab, (2013) are used as the starting point. The formulation uses continuous decision variables, $y_p^{l,t}$, $x_k^{a,t}$, $c_{k,sp}^{a,t}$, $v_k^{d,t}$, $z_k^{d,t}$, $u_k^{d,t}$, and $q_k^{d,t}$ to model mining, processing from mine, processing from stockpile, RM, and OB, IB and TCS dyke material requirements, respectively; for all mining locations, and processing and dyke construction destinations. Continuous decision variables are used to allow for fractional extraction of mining-panels and mining-cuts in different periods for different locations and destinations. The model features the parameter grade uncertainty cost, PN_u, which is a pseudo cost used to enforce preferential mining of low variance mining-panels. This is used to estimate risk associated with the generated production schedule. Continuous deviational variables, $dv_1^{-,l,t}$, $dv_2^{-,a,t}$, $dv_3^{-,d,t}$, $dv_4^{-,d,t}$, $dv_5^{-,d,t}$, and $dv_6^{-,d,t}$ are defined to support the goal functions that control mining, processing, RM, OB, IB and TCS dyke materials, for all mining locations and processing, reclamation and dyke construction destinations. They provide a continuous range of units (tonnes) that can be determined by the optimizer to satisfy the set goals. In the objective function, these deviational variables are minimized. There are also deviational penalty cost and priority parameters in the objective function used to model the focus of mine management in the presence of multiple conflicting goals. The deviational penalty cost parameters PN_1, PN_2 , PN_3, PN_4, PN_5, and PN_6 penalize the NPV for any deviation from the set goals. The priority parameters, P_1 , P_2, P_3, P_4, P_5, and P_6 are used to place emphasis on the most important goals. In general, the deviational penalty cost and priority parameters are set up to penalize the NPV if the set goals and the most important goals are not met. When setting up these parameters, the planner has to monitor how continuous mining proceed period by period, the uniformity of tonnages mined per period, and the corresponding NPV generated, to keep track of how parameter changes affect these key performance indicators. More weight should be assigned to a goal that has a higher priority for mine management.

The uncertainty-based objective function presented in Eq. (12), aims to maximize the NPV of the project, minimize grade uncertainty cost, minimize reclamation and dyke material mining cost, and minimize deviations from the production goals. The production schedule risk is estimated through the grade uncertainty cost calculated by applying a mining-panel grade-variance penalty scheme to generate varying production schedules with different NPVs.

$$Max \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{d=1}^{D}\sum_{sp=1}^{SP}\sum_{a=1}^{A}\sum_{e=1}^{E}\sum_{t=1}^{T}\left(\sum_{\substack{k\in MK_p\\ p\in Mp_j}}\left[\begin{array}{l}\left(rm_k^{a,e,t}\times x_k^{a,t}+rs_{k,sp}^{a,e,t}\times c_{k,sp}^{a,t}-(dw_p^{l,t}+PN_u)\times y_p^{l,t}\right)-\\ \left(dmu_k^{d,t}\times v_k^{d,t}\right)-\left(dob_k^{d,t}\times z_k^{d,t}+dib_k^{d,t}\times u_k^{d,t}+dt_k^{d,t}\times q_k^{d,t}\right)\\ -P_1\left(PN_1\times dv_1^{-,l,t}\right)-P_2\left(PN_2\times dv_2^{-,a,t}\right)-P_3\left(PN_3\times dv_3^{-,d,t}\right)\\ -P_4\left(PN_4\times dv_4^{-,d,t}\right)-P_5\left(PN_5\times dv_5^{-,d,t}\right)-P_6\left(PN_6\times dv_6^{-,d,t}\right)\end{array}\right]\right) \quad (12)$$

6.3. MILGP model goal functions

There are tonnage targets for all mining locations and processing destinations as well as tonnage targets for muskeg as reclamation material and dyke material for dyke construction destinations. Eqs. (13) to (18) represent all required goal functions. Eq. (13) defines the mining goal function that controls the total amount of material mined from mining-panel p within pushback j in each period. The negative allowable deviation from the set mining goal is controlled by the planner using the $dv_1^{-,l,t}$ decision variable. Eq. (14) defines the processing goal function that controls the total amount of ore from mining-cut k within mining-panel p sent from both the mine and stockpile in each period to the processing destination. It should be mentioned that the amount of ore sent to the processing destination from the stockpile in period t and the amount of ore sent to the stockpile from the mine in period $t-ts$ must be equal. The negative allowable deviation from the set processing goal is controlled by the planner using the $dv_2^{-,a,t}$ decision variable. Eq. (15) defines the RM tonnage goals that control the total amount of muskeg material to be mined from mining-cut k within mining-panel p in each period. The negative allowable deviation from the set RM goal is controlled by the planner using the $dv_3^{-,d,t}$ decision variable. OB, IB and TCS dyke material goal functions control the dyke material production targets for different dyke construction destinations. These are defined by Eqs. (16) to (18), respectively. These functions provide a feasible schedule for dyke construction. The negative allowable deviation from the set OB, IB and TCS dyke material goals are controlled by the planner using $dv_4^{-,d,t}$, $dv_5^{-,d,t}$ and $dv_6^{-,d,t}$ decision variables.

$$\sum_{j=1}^{J}\left(\sum_{p\in MP_j}\left(o_p+mu_p+ob_p+ib_p+w_p\right)\times y_p^{l,t}\right)+dv_1^{-,l,t}=Mg^{l,t} \tag{13}$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}}\left(o_k\times x_k^{a,t}\right)+\sum_{sp=1}^{SP}\sum_{\substack{k\in MK_p\\ p\in MP_j}}\left(o_k\times c_{k,sp}^{a,t}\right)\right)+dv_2^{-,a,t}=Pg^{a,t} \tag{14}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(mu_k\times v_k^{d,t}\right)\right)+dv_3^{-,d,t}=MUg^{d,t} \tag{15}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(ob_k\times z_k^{d,t}\right)\right)+dv_4^{-,d,t}=OBg^{d,t} \tag{16}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(ib_k\times u_k^{d,t}\right)\right)+dv_5^{-,d,t}=IBg^{d,t} \tag{17}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(cs_k\times q_k^{d,t}\right)\right)+dv_6^{-,d,t}=CSg^{d,t} \tag{18}$$

6.4. MILGP model constraints

6.4.1. Stockpiling capacity constraints

Constraints that control ore stockpile tonnages are presented in Eqs. (19) and (20). These equations control the amount of ore sent from mining-cut k to stockpile sp in period t. Material sent to the stockpile in period t are reclaimed in period $t+ts$, where ts is the stockpiling duration controlled by the planner. The planner also controls the upper and lower capacity limits for stockpile bins.

$$\sum_{sp=1}^{SP}\sum_{k=1}^{K} o_k \times s_k^{sp,t} \leq \overline{os}^{sp,t} \tag{19}$$

$$\sum_{sp=1}^{SP}\sum_{k=1}^{K} o_k \times s_k^{sp,t} \geq \underline{os}^{sp,t} \tag{20}$$

6.4.2. Bitumen and fines grade blending constraints

The MILGP bitumen and fines grade blending constraints ensure that the quality requirements of the processing plant, stockpile and dyke construction destinations are achieved. These constraints are formulated using Eqs. (21) to (30). Ore bitumen grade blending constraints ensure the extracted ore from mining-cut k within mining-panel p sent to either processing destination a or to stockpile sp in period t meets the grade quality requirements. Ore bitumen grade blending constraints are formulated using Eqs. (21) to (24). Eqs. (21) and (22) represent inequality constraints that control the limiting ore bitumen grade sent from the mine and stockpile to the processing plant. Eqs. (23) and (24) represent inequality constraints that control the limiting ore bitumen grade sent from the mine to the stockpile.

$$\sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) - \overline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) \leq 0 \tag{21}$$

$$\underline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) - \sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) \leq 0 \tag{22}$$

$$\sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times s_k^{a,t} - \overline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times s_k^{a,t} \leq 0 \tag{23}$$

$$\underline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times s_k^{a,t} - \sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times s_k^{a,t} \leq 0 \tag{24}$$

Ore fines grade blending constraints ensure the extracted ore from mining-cut k within mining-panel p sent to either processing destination a or to stockpile sp in period t meets the fines requirements. Interburden fines grade blending constraints also ensure that the interburden fines for dyke construction are within the upper and lower limits required. Fines grade blending constraints are formulated using Eqs. (25) to (30). Eqs. (25) and (26) represent inequality constraints used to control the limiting grade of ore fines sent from the mine and stockpile to the processing plant. Eqs. (27) and (28) represent inequality constraints used to control the limiting grade of ore fines sent from the mine to the stockpile. Eqs. (29) and (30) represent inequality constraints used to control the limiting grade of interburden dyke material fines sent from the mine to dyke construction destinations.

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} o_k \times fn_k^e \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) - \overline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) \leq 0 \tag{25}$$

$$\underline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) - \sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} o_k \times fn_k^e \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) \leq 0 \tag{26}$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} o_k \times fn_k^{e} \times s_{k,sp}^{a,t}\right) - \overline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MPj}} o_k \times s_{k,sp}^{a,t}\right) \le 0 \tag{27}$$

$$\underline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MPj}} o_k \times s_{k,sp}^{a,t}\right) - \sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} o_k \times fn_k^{e} \times s_{k,sp}^{a,t}\right) \le 0 \tag{28}$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} ib_k \times fn_k^{ib} \times u_k^{d,t}\right) - \overline{fn}^{d,t,ib}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MPj}} ib_k \times u_k^{d,t}\right) \le 0 \tag{29}$$

$$\underline{fn}^{d,t,ib}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MPj}} ib_k \times u_k^{d,t}\right) - \sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} ib_k \times fn_k^{ib} \times u_k^{d,t}\right) \le 0 \tag{30}$$

6.4.3. Mining-panels extraction precedence constraints

Five precedence constraints presented in Eqs. (31) to (35) are used to define the precedence extraction sequence for each mining panel p based on its spatial location. These equations use the binary integer decision variable b_p. This variable is equal to one if the extraction of mining-panel p has started by or in period *t*; otherwise, it is zero. Specifically:

- Eq. (31) defines the vertical mining precedence. Prior to the extraction of a specific mining-panel, all the mining-panels above it must be extracted so that the mining-panel is accessible. The set $IP_p\left(Z''\right)$ represents the set of immediate mining-panels that are above mining-panel p.
- Eq. (32) defines the horizontal mining precedence. Prior to the extraction of a specific mining-panel, all the mining-panels in a specified horizontal mining direction on a level must be extracted. The set $IH_p\left(Z'\right)$ represents the set of immediate mining-panels in the specified horizontal mining direction.
- Eq. (33) defines the pushback mining precedence. Eq. (33) checks all the mining-panels within the immediate predecessor pushback that must be extracted prior to the extraction of mining-panels in pushback *j*. The set $MP_j\left(H''\right)$ represents the set of mining panels in the predecessor pushback.
- Eq. (34) ensures that mining-panel p can only be extracted if it has not been extracted before.
- Eq. (35) ensures that once the extraction of a mining-panel starts in period t, this mining-panel is available for extraction during the subsequent periods.

$$b_p^t - \sum_{c=1}^{c}\sum_{m=1}^{t} y_{u_1}^{c,m} \le 0,\ \ u_1 \in IP_p\left(Z''\right) \tag{31}$$

$$b_p^t - \sum_{c=1}^{c}\sum_{m=1}^{t} y_{u_2}^{c,m} \le 0,\ \ u_2 \in IH_p\left(Z'\right) \tag{32}$$

$$b_p^t - \sum_{c=1}^{c}\sum_{m=1}^{t} y_{u_3}^{c,m} \le 0,\ \ u_3 \in MP_j\left(H''\right) \tag{33}$$

$$\sum_{c=1}^{c}\sum_{m=1}^{t} y_p^{c,m} - b_p^t \le 0 \tag{34}$$

$$b_p^t - b_p^{t+1} \le 0 \tag{35}$$

6.4.4. Decision variables' control constraints

In the MILGP model, all decision variables used to control mining, processing, stockpiling, reclamation material, dyke materials and goal deviations are continuous variables. Inequality Eq. (36) makes sure that all the material mined as ore (sent to either the processing destination a or the stockpile sp), and all reclamation and dyke materials extracted from the mining-cuts belonging to mining-panel p in period t are less than or equal to the total material mined from mining-panel p in period t from any mining location.

Eq. (37) ensures that the total fractions of ore mined from a mining-cut (sent to either the processing destination a or the stockpile sp) is less than or equal to one. Eq. (38) ensures that the fraction of ore extracted from mining-cut k and sent to the stockpile sp in period $t-ts$ must be equal to the fraction of ore reclaimed from the stockpile sp and sent to the processing plant a in period t; where ts is the stockpiling duration.

Eq. (39) ensures that the fractions of TCS dyke material produced from processed ore is less than or equal to the fractions of ore sent from the mine and stockpile to the processing plant in each period. Eq. (40) ensures that the fractions of mining-panel p extracted and sent to different destinations in different periods is less than or equal to one. Eq. (41) ensures that the total fractions of reclamation material extracted from the mine and sent to its destinations in different periods is less than or equal to one. Eqs. (42) to (44) ensure that the total fractions of dyke materials extracted from the mine (OB and IB) or generated from the processing plant (TCS) and sent to all destinations in different periods is less than or equal to one.

$$\left[\begin{array}{l} \sum_{d=1}^{D}\sum_{sp=1}^{SP}\sum_{a=1}^{A}\sum_{\substack{k\in MK_p \\ p\in MP_j}} \left(o_k \times x_k^{a,t} + o_k \times s_{k,sp}^{a,t} + mu_k \times v_k^{d,t} + ob_k \times z_k^{d,t} + ib_k \times u_k^{d,t}\right) \\ \le \sum_{l=1}^{L}\sum_{p\in MP_p}\left[y_p^{l,t}\left(o_p + mu_p + ob_p + ib_p + w_p\right)\right] \end{array}\right] \tag{36}$$

$$\sum_{a=1}^{A}\sum_{sp=1}^{SP}\sum_{t-1}^{T}\left(x_k^{a,t} + s_{k,sp}^{a,t}\right) \le 1 \tag{37}$$

$$\sum_{a=1}^{A}\sum_{sp=1}^{SP}\sum_{t=1}^{T}\left(c_{k,sp}^{a,t} - s_{k,sp}^{a,t-ts}\right) = 0, \quad t-ts \ge 0 \tag{38}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} q_k^{d,t} \le \sum_{a=1}^{A}\sum_{t=1}^{T} x_k^{a,t} + \sum_{sp=1}^{SP}\sum_{t=1}^{T} c_{k,sp}^{a,t} \tag{39}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} y_p^{d,t} \le 1 \tag{40}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} v_p^{d,t} \le 1 \tag{41}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} z_p^{d,t} \le 1 \tag{42}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} u_p^{d,t} \leq 1 \tag{43}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} q_p^{d,t} \leq 1 \tag{44}$$

6.4.5. Non-negativity constraints

Eq. (45) ensures that the decision variables for mining, processing, stockpiling (ore sent and reclaimed), RM, OB, IB and TCS dyke material are non-negative. Eq. (46) ensures that the deviational decision variables that support the goal functions are non-negative as well.

$$\mathrm{y}_p^{l,t},\ x_k^{a,t},\ \mathrm{s}_{k,sp}^{d,t},\ \mathrm{c}_{k,sp}^{a,t},\ v_k^{d,t},\ z_k^{d,t},\ u_k^{d,t},\ q_k^{d,t} \quad \geq 0 \tag{45}$$

$$dv_1^{-,l,t},\ dv_2^{-,a,t},\ dv_3^{-,d,t},\ dv_4^{-,d,t},\ dv_5^{-,d,t},\ dv_6^{-,a,t} \quad \geq 0 \tag{46}$$

7. Implementation of the MILGP Framework for an Oil Sands Deposit

A kriged block model is developed using Geovia GEMS Software (Gemcom Software International, 2015a). The ultimate pit limit is generated using Geovia Whittle Software (Gemcom Software International, 2015b) which is based on 3D LG algorithm (Lerchs and Grossmann, 1965). The optimized pit shell from Whittle is used to design the final pit in GEMS software (Gemcom Software International, 2015a). The mining blocks within the pit limit are used as input data for the MILGP model. An agglomerative hierarchical clustering algorithm is used in clustering blocks within each intermediate pushback into mining-cuts (Tabesh and Askari-Nasab, 2011). The intersection between benches and intermediate pushbacks are used in creating mining-panels. MATLAB (Mathworks, 2017) application is used as the programming platform to define the MILGP framework and IBM/CPLEX (ILOG, 2012) solver which uses a branch and cut optimization algorithm is employed to solve the MILGP problem. This section documents the application and results from the developed MILGP model for an oil sands dataset.

Two implementation scenarios highlighting different aspects of the developed MILGP model are outlined in Fig. 7. These scenarios are designed to highlight features of the MILGP model including: 1) integrating waste management, reclamation and limited duration stockpiling strategy into oil sands mine planning (Scenario 1); and 2) estimating the effect of grade uncertainty on the generated NPVs of the project (Scenario 2).

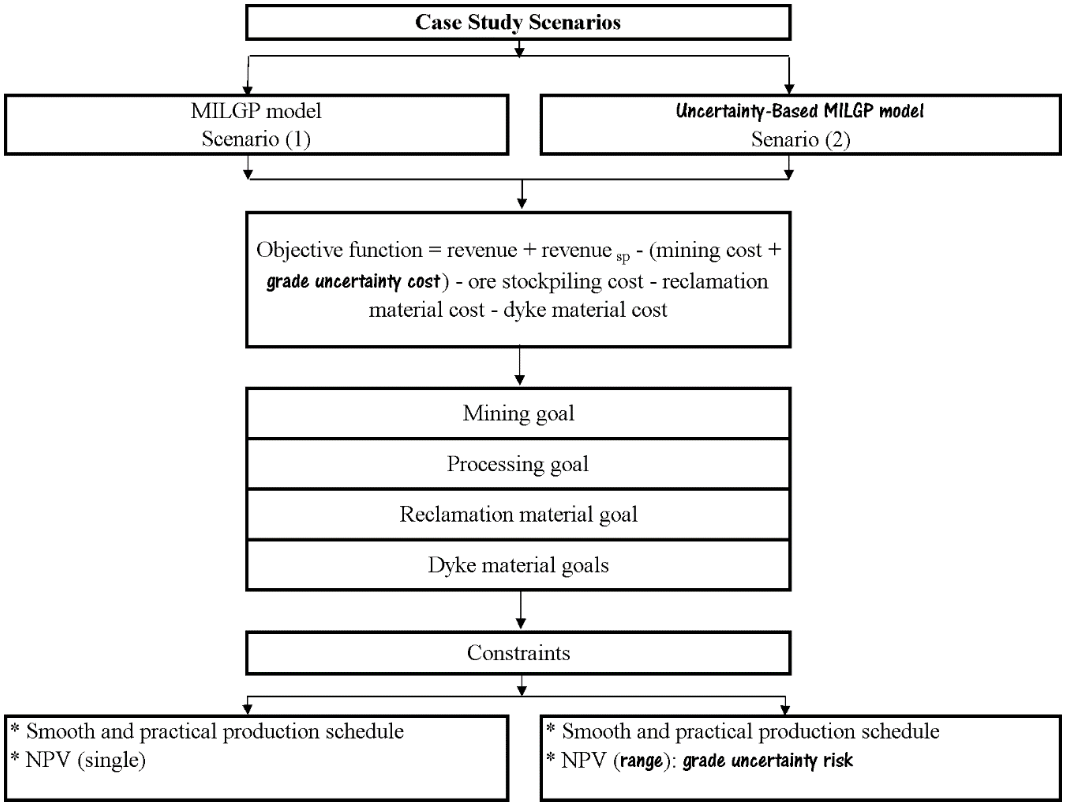

Fig. 7. Case study scenarios.

7.1. MILGP model: Scenario 1

The developed MILGP model for Scenario 1 features goal functions for mining, processing, dyke material, reclamation material and a stockpiling strategy for ore. The ore that exceeds the processing capacity in each period will be stockpiled for a limited stockpiling duration controlled by the mine planner. The revenue generated from processing the stockpiled ore is calculated. The grade uncertainty cost is set to zero. This MILGP model generates a smooth and practical production schedule, and an NPV with known limits of optimality. To model the mining, ore processed from mine, ore processed from stockpile, ore sent to stockpile, RM, OB, IB and TCS dyke material, the following continuous decision variables $\mathrm{y}_p^{l,t}$, $x_k^{a,t}$, $\mathrm{c}_{k,sp}^{a,t}$, $\mathrm{s}_{k,sp}^{d,t}$, $v_k^{d,t}$, $z_k^{d,t}$, $u_k^{d,t}$ and $q_k^{d,t}$ are used respectively in the proposed MILGP model, for all mining locations l, processing destination a and other possible destinations d. To control the precedence of mining-panel extraction, the binary integer decision variable b_k^t is used.

For Scenario 1, continuous deviational variables $dv_1^{-,l,t}$, $dv_2^{-,a,t}$, $dv_3^{-,d,t}$, $dv_4^{-,d,t}$, $dv_5^{-,d,t}$, and $dv_6^{-,d,t}$ are used to support the goal functions that control mining, processing, reclamation material, OB, IB and TCS dyke materials for all locations and destinations. The deviational variables make a continuous range of tonnes available to the optimizer to choose from to satisfy the set goals. These deviational variables are minimized in the objective function. Deviational penalty cost and priority parameters are included in the objective function. Deviational penalty cost parameters PN_1, PN_2, PN_3, PN_4, PN_5 and PN_6 are used to penalize the NPV for any deviation from the set goals. The priority parameters P_1, P_2, P_3, P_4, P_5 and P_6 are used to place emphasis on the most important goals. These parameters also penalize the NPV if the most important set goal is not met. While setting up the deviational variables, penalty and priority cost parameters, the smoothness of the mining schedule from one period to the next and the uniformity of tonnages mined per period needs to be monitored together with the corresponding NPV generated. Setting up the priority and penalty cost depends on whether the planner needs to trade off NPV to meet some set goals consistent with mine management's objective.

7.2. Uncertainty-based MILGP model: Scenario 2

The uncertainty-based MILGP model for Scenario 2 features the developments for the MILGP model in Scenario 1. In addition, the uncertainty-based MILGP model is implemented with a mining-panel grade variance penalty scheme through the grade uncertainty cost to estimate the risk associated with the production schedule. This is implemented using two different risk-based techniques: 1) applying the grade uncertainty cost for high variance mining-panels, and 2) applying the grade uncertainty cost for all mining panels. In applying either technique, different production schedules are generated as the pseudo penalty cost is varied resulting in a range of NPVs. An estimated overall risk value associated with the production schedule can then be calculated. This uncertainty-based MILGP model generates a smooth and practical production schedule, an NPV with known limits of optimality, is easy to setup with more flexibility for the optimizer and provides a quantified risk value associated with the production schedule.

8. Case Study

For this case study, no pushbacks prior to the UPL were considered. However, to create mining-panels, the ultimate pit was divided into four pseudo pushbacks. Blocks in each mining-panel were clustered into mining-cuts using hierarchical clustering algorithm (Tabesh and Askari-Nasab, 2011). Two implementation scenarios as outlined in Section 7 are investigated with the oil sands data. The deposit is to be scheduled for 8 years for the processing plant, reclamation and dyke construction destinations. Summarized information on the oil sands deposit final pit design is presented in Table 2. Table 3 shows the economic parameters and operational capacities for production scheduling. The economic data are extracted and compiled based on (Ben-Awuah and Askari-Nasab, 2013). Table 4 shows the upper and lower bounds of material quality requirements for ore and interburden dyke material. The model is implemented on a Lenovo Think Pad computer with i5 Core at 2.2 GHz, and 8.0 GB of RAM.

8.1. MILGP model: Scenario 1

In Scenario 1 for this case study, the mining operation is limited by mining and processing capacities using the operational capacities in Table 2 and material quality requirements in Table 3. The main target is to achieve a smooth processing rate throughout the mine life and generate a uniform production schedule that generates the highest NPV. The overall NPV generated is $ 18,108.0 M and the results of the production schedule are shown in Table 5 and **Error! Reference source not found.** and **Error! Reference source not found.**.

Table 2. Oil sands deposit final pit design characteristics.

Description	Value
Total tonnage of material (Mt)	5,735.60
Total ore tonnage (Mt)	1,906.90
Total TCS dyke material tonnage (Mt)	1,351.60
Total OB dyke material tonnage (Mt)	1,515.50
Total IB dyke material tonnage (Mt)	1,925.50
Total RM tonnage (Mt)	218.74
Total waste tonnage (Mt)	168.96
Number of blocks	79,095
Number of mining-cuts	4,494
Number of mining-panels	107
Number of benches	8

Table 3. Economic parameters and operational capacities.

Parameter (unit)	Value	Parameter (unit)	Value
Mining cost ($/tonne)	4.60	Processing capacity (Mt/year)	82.59
Processing cost ($/tonne)	5.03	RM capacity (MT/year)	220.00
Ore re-handling cost ($/tonne)	0.50	OB capacity (MT/year)	230.00
Selling price ($/bitumen %mass)	4.50	IB capacity (MT/year)	200.00
TCS dyke material cost ($/tonne)	0.92	TCS capacity (MT/year)	75.00
OB dyke material cost ($/tonne)	1.38	Mining recovery fraction (%)	100.00
IB dyke material cost ($/tonne)	1.38	Processing recovery (%)	90.00
RM extra mining cost ($/tonne)	0.50	Discount rate (%)	10.00
Mining capacity (Mt/year)	239.76		

Table 4. Material quality requirements.

Parameter	Value
Upper bound of ore bitumen grade (wt %)	16.0
Lower bound of ore bitumen grade (wt %)	7.0
Upper bound of ore fines percent (wt %)	30.0
Lower bound of ore fines percent (wt %)	0.0
Upper bound of IB dyke material fines percent (wt %)	50.0
Lower bound of IB dyke material fines percent (wt %)	0.0

Table 5. Production schedule using goal functions with a 2-year ore stockpiling duration from the MILGP model.

Period	Average bitumen grade (wt %)	Material mined (Mt)	Material processed (Mt)
1	8.33	239.76	13.49
2	10.33	239.76	62.43
3	9.96	239.76	67.08
4	10.32	239.76	58.24
5	10.44	239.76	82.59
6	10.51	239.76	82.59
7	10.16	239.76	82.59
8	10.74	239.76	82.59
9	10.80	239.76	82.59
10	11.13	239.76	82.59
11	10.94	239.76	82.59
12	10.44	239.76	82.59
13	9.58	239.76	82.59
14	9.60	239.76	82.59
15	10.64	239.76	82.59
16	11.28	239.76	82.59
17	10.67	221.39	82.59
18	10.58	209.76	82.59
19	10.84	209.76	82.59
20	10.38	209.76	82.59
21	10.65	209.76	82.59
22	9.79	209.76	82.59
23	10.23	209.76	82.59
24	8.89	209.76	82.59
25	9.37	209.76	53.89

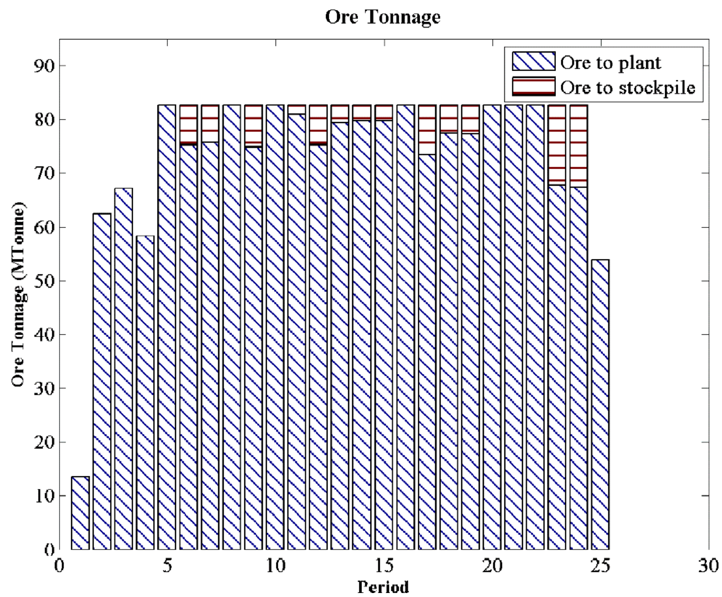

Fig. 8. Processing schedule using goal functions with a 2-year ore stockpiling duration from the MILGP model.

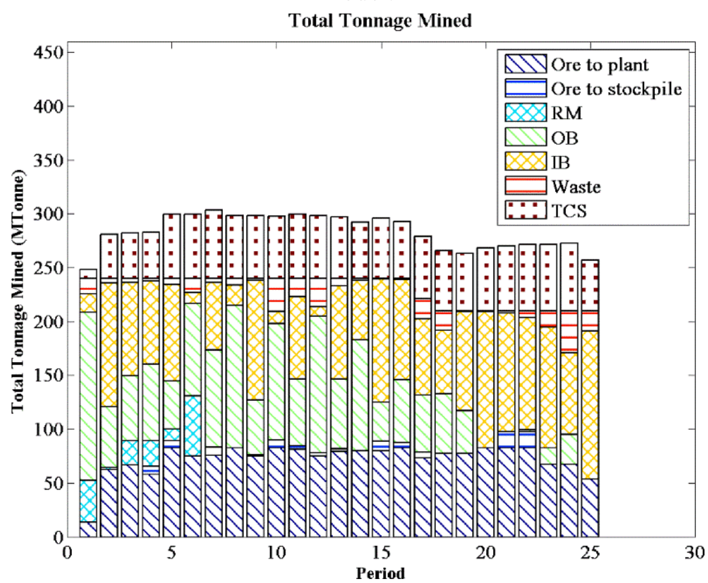

Fig. 9. Production schedule using goal functions with a 2-year ore stockpiling duration from the MILGP model.

8.2. Uncertainty-based MILGP model: Scenario 2

In Scenario 2 for this case study, the mining operation is limited by mining and processing capacities. To investigate in detail the features of the Uncertainty-based MILGP model, the scenario was setup with goal functions and no stockpiling, reclamation material or dyke materials are considered. First, this scenario is run with no penalty applied for mining-panels (Scenario 2A). Two different risk-based techniques are examined: 1) applying penalty cost for only high-variance mining-panels (Scenario 2B) and; 2) applying penalty cost for all mining panels variance (Scenario 2C). The main focus is to achieve a smooth processing rate throughout the mine life and generate a range of NPVs corresponding to the varying penalty cost values. As the penalty cost values increases, the grade uncertainty cost increases, which forces the optimizer to preferentially mine lower variance mining-panels thereby affecting the NPV. The relationship between NPV and penalty cost can be seen in Fig. 10 with selected data points. The selected results of the production schedule using both techniques are shown in Table 6. Fig. 11 shows the production schedule using Scenario 2A which is similar to 2B and 2C. It can be noticed that the extraction sequence is postponed to later years for some of the high variance mining-panels such as mining-panels number 15, 43, 51, 91, 96, 98, 100 and 103. At the same time, the extraction sequence for some low variance mining-panels are extracted earlier such as mining-panel number 12, 35, 38, 45, 64, 65, 70, 83 and 93. In addition, there is an exceptional effect of penalty cost on the low variance mining-panels such as mining-panels 26, 44, 52, 55 and 87 as the will give access to lower variance mining-panels. It is important to note that, the primary objective function of the uncertainty-based MILGP model is to maximize NPV and hence the optimizer looks for high grade mining-cuts in addition to low mining-panels variance.

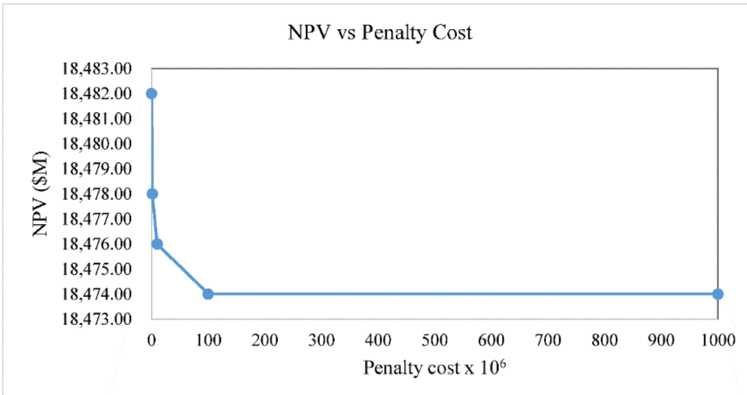

Fig. 10. Relationship between net present value and penalty cost.

Table 6. Production schedule for selected mining-panels using goal functions from the uncertainty-based MILGP model.

Mining-Panel #	**Mining-Panel Variance**	**Extraction Sequence (year)**		
		Scenario 2A	Scenario 2B	Scenario 2C
12	0.0691	4	3	3
15	0.2809	3	4	4
26	0.0697	5	6	6
35	0.0291	10	10	8
38	0.0743	7	6	6
43	0.2721	6	7	7
44	0.0487	6	7	7
45	0.0724	8	7	7
51	0.2911	7	8	8
52	0.0485	7	8	8
55	0.0613	10	11	11
64	0.0291	21	17	17
65	0.0328	25	22	22
70	0.0526	18	17	17
83	0.0291	22	21	21
87	0.0617	18	19	19
91	0.4013	17	18	18
93	0.0723	22	19	20
96	0.4564	19	20	20
98	0.4090	20	23	22
100	0.4883	23	24	23
103	0.3747	23	24	24

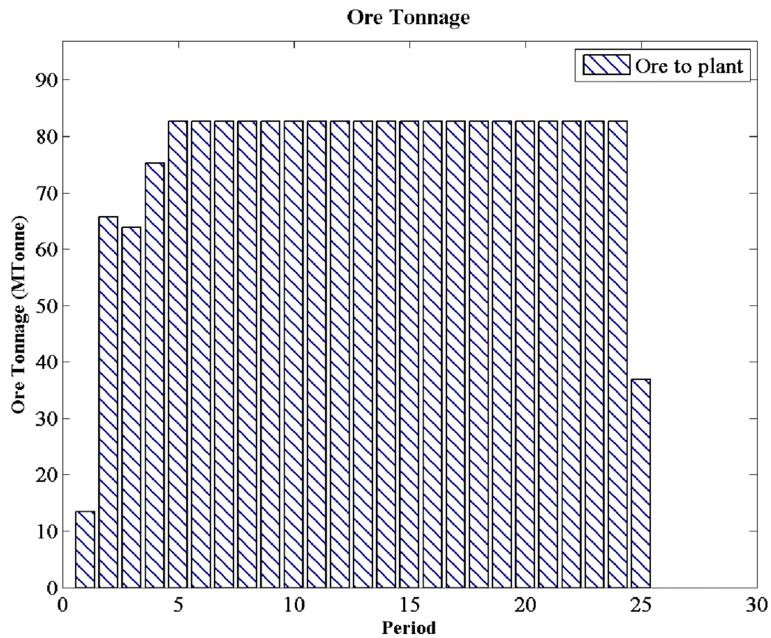

Fig. 11. Processing schedule using goal functions from the uncertainty-based MILGP model (Scenario 2A).

8.3. Discussion of the results

The performance of the MILGP models in Scenarios 1 is analyzed based on: NPV, mining and processing production targets and smoothness and practicality of the generated schedules. Scenario 1 generates smooth schedules for mining, processing, reclamation and dyke materials. The total material mined is 5,735.60 Mt. This is made up of 1,906.9 Mt of ore, 218.74 Mt of RM, 1,515.50 Mt and 1,925.50 Mt of OB and IB dyke materials, whilst 1,351.60 Mt of TCS dyke material is generated. The model generated a uniform production schedule for OB, IB and TCS dyke material over the 25 periods. This ensures the effective utilization of the mining fleet and processing plant throughout the mine life.

The uncertainty-based MILGP model, Scenario 2, is developed based on grade uncertainty using the mining-panels grade variance. The first run (Scenario 2A) no penalty is applied and the NPV obtained is $ 18,482.00 M. For (Scenario 2B), the optimizer penalizes mining-panels with grade variance higher than the expected grade variance, calculated as 0.1219. The penalty cost applied starts from zero (no grade uncertainty cost) and increases gradually until the production schedule starts to change by postponing the mining of high variance mining-panels to later years. As the penalty cost increases, the NPV of the project decreases due to the changes in the production schedule. At some point, increasing the penalty cost has no effect on the production schedule and subsequently the NPV. The NPV obtained ranges from $ 18,482.00 M to $ 18,474.00 M. Thus, the potential risk associated with the production schedule can be estimated as a loss of $ 8.00 M in the most pessimistic case. For (Scenario 2C), the optimizer penalizes all mining-panels to give preference to low variance mining panels to be mined earlier. The NPV obtained ranges from $ 18,482.00 M to $M 18,475.00 M. Thus, the potential risk associated with the production schedule can be estimated as a loss of $ 7.00 M in the most pessimistic case. Among other things, by taking into consideration the NPV range and the production schedule risk, investors can make more pragmatic choices when managing their mining investment risk profile. Table 7 shows the production schedule and grade profile over 8 periods for Scenarios 2A, 2B, and 2C. Fig. 12 shows the grade profile comparisons when no penalty applied and when it is applied for the two risk-based techniques.

Table 7. Production schedule and grade profile for the two risk-based techniques.

Period	Scenario 2A			Scenario 2B			Scenario 2C		
	Mining Schedule (Mt)	Processing Schedule (Mt)	Grade Profile (%)	Mining Schedule (Mt)	Processing Schedule (Mt)	Grade Profile (%)	Mining Schedule (Mt)	Processing Schedule (Mt)	Grade Profile (%)
1	239.76	13.49	8.33	13.49	13.49	8.33	239.76	13.49	8.33
2	239.76	65.71	10.37	65.71	65.71	10.37	239.76	65.71	10.37
3	239.76	63.80	9.89	63.80	63.80	9.89	239.76	63.80	9.89
4	239.76	75.24	10.41	75.24	75.24	10.41	239.76	75.24	10.41
5	239.76	82.59	10.37	82.59	82.59	10.37	239.76	82.59	10.37
6	239.76	82.59	10.34	82.59	82.59	10.34	239.76	82.59	10.34
7	239.76	82.59	10.44	82.59	82.59	10.44	239.76	82.59	10.44
8	239.76	82.59	10.77	82.59	82.59	10.77	239.76	82.59	10.77
9	239.76	82.59	10.74	82.59	82.59	10.74	239.76	82.59	10.74
10	239.76	82.59	10.86	82.59	82.59	10.89	239.76	82.59	10.86
11	239.76	82.59	11.11	82.59	82.59	11.07	239.76	82.59	11.11
12	239.76	82.59	10.43	82.59	82.59	10.43	239.76	82.59	10.43
13	239.76	82.59	9.64	82.59	82.59	9.66	239.76	82.59	9.66
14	239.76	82.59	10.22	82.59	82.59	10.16	239.76	82.59	10.21
15	239.76	82.59	10.86	82.59	82.59	10.23	239.76	82.59	10.85
16	239.76	82.59	10.17	82.59	82.59	10.63	239.76	82.59	10.19
17	209.76	82.59	9.88	82.59	82.59	10.07	209.76	82.59	9.87
18	211.52	82.59	10.70	82.59	82.59	10.79	209.76	82.59	11.07
19	209.76	82.59	11.15	82.59	82.59	11.08	209.76	82.59	10.77
20	209.76	82.59	10.93	82.59	82.59	10.92	209.76	82.59	10.93
21	209.76	82.59	10.67	82.59	82.59	10.64	209.76	82.59	10.66
22	209.76	82.59	10.03	82.59	82.59	10.06	209.76	82.59	10.05
23	209.76	82.59	9.98	82.59	82.59	9.93	210.92	82.59	9.93
24	209.76	82.59	9.25	82.59	82.59	9.29	209.76	82.59	9.29
25	209.76	36.88	8.93	36.88	36.88	8.93	209.76	36.88	8.93

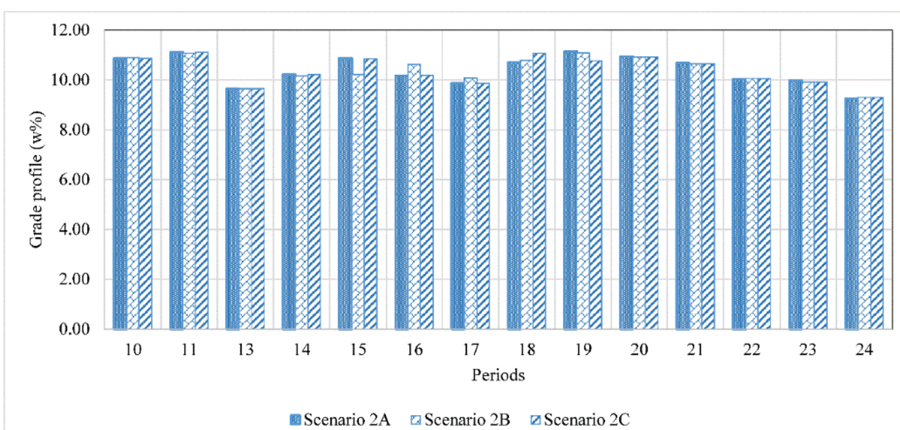

Fig. 12. Grade profile comparison.

9. Conclusions

The proposed uncertainty-based MILGP model for oil sands long-term production planning involves the interactions of their three main subcomponents: the objective function, the goal functions and the constraints in an optimization framework to achieve the research objectives. The model generates a strategic production schedule for the ore, reclamation material and dyke materials using two different scenarios. In Scenario 1, using goal functions for mining and processing with uncertainty penalty cost set to zero, the MILGP model illustrates how production scheduling with limited duration stockpiling strategy for ore can be effectively integrated with waste disposal planning and reclamation material stockpiling in oil sands mining. Based on dyke construction requirements, schedules are generated to provide the required dyke materials to support engineered dyke construction that will help in reducing environmental impacts. This schedule gives the planner good control over dyke materials and provide a solid platform for effective dyke construction and waste management planning.

The effect of grade uncertainty on the NPV of a mining project is investigated using the uncertainty-based MILGP model (Scenarios 2A, 2B, and 2C). The research is based on the concept of mean-variance analysis, which is a process of weighing risk (variance) against expected NPV. The main goal in Scenario 2 is to feed the plant with ore that has less potential grade variation especially in the early years of mine life. By deferring ore with highly uncertain grades to later years, the risk associated with generating the estimated mineral content will be reduced; as well as the potential uncertainty cost associated with the production schedule. The cost of uncertainty is a good indicator of the level of risk associated with generating the expected NPV from the mine plan.

The future work for this research will focus on integrating the optimization of pushback size for waste management in long-term production planning and scheduling.

10. References

[1] Directive 082-Operating Criteria: Resource Recovery Requirements for Oil Sands Mine and Processing Plant Operations, (2016).

[2] Directive 085-Fluid Tailings Management for Oil Sands Mining Projects, (2017).

[3] Asad, M. W. A. (2005). Cutoff grade optimization algorithm with stockpiling option for open pit mining operations of two economic minerals. *International Journal of Surface Mining, Reclamation and Environment, 19*(3), 176-187.

[4] Askari-Nasab, H., and Awuah-Offei, K. (2009). *Mixed integer linear programming formulations for open pit production scheduling*. Edmonton: University of Alberta, Mining Optimization Laboratory Report One.

[5] Askari-Nasab, H., Pourrahimian, Y., Ben-Awuah, E., and Kalantari, S. (2011). Mixed integer linear programming formulations for open pit production scheduling. *Journal of Mining Science, 47*(3), 338-359.

[6] Azam, S., and Scott, J. D. (2005). *Revisiting the ternary diagram for tailings characterization and management*. Edmonton: The Oil Sands Tailings Research Facility.

[7] Badiozamani, M. M., and Askari-Nasab, H. (2014). Integration of reclamation and tailings management in oil sands surface mine planning. *Environmental Modelling & Software, 51*, 45-58.

[8] Badiozamani, M. M., and Askari-Nasab, H. (2016). Integrated mine and tailings planning: a mixed integer linear programming model. *International Journal of Mining, Reclamation and Environment, 30*(4), 319-346.

[9] Ben-Awuah, E., and Askari-Nasab, H. (2013). Incorporating waste management into oil sands long term production planning. *Transactions of the Institution of Mining and Metallurgy, Section A, 122*(1), 33-45.

[10] Ben-Awuah, E., Askari-Nasab, H., and Awuah-Offei, K. (2012). Production scheduling and waste disposal planning for oil sands mining using goal programming. *Journal of Environmental Informatics, 20*(1), 20-33.

[11] Boratynec, D. J. (2003). *Fundamentals of rapid dewatering of composite tailings.* Unpublished MSc, University of Alberta, Edmonton.

[12] Brooker, P. I. (1986). A Parametric Study of Robustness of Kriging Variance as a Function of Range and Relative Nugget Effect for a Spherical Semivariogram. *Mathematical Geology, 18*(5), 12.

[13] Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno, E., and Rubio, E. (2012). A new algorithm for the open-pit mine production scheduling problem. *Operations Research, 60*(3), 517-528.

[14] Dagdelen, K. (1985). *Optimum multi-period open pit mine production scheduling by Lagrangian parameterization.* Unpublished PhD, University of Colorado, Colorado.

[15] Deutsch, J.L., Szymanski, J., and Deutsch, C.V. (2014). Checks and measures of performance for kriging estimates. *Journal of the Southern African Institute of Mining and Metallurgy, 114*, 223-223.

[16] Devenny, D. W. (2009). *Overview of oil sands tailings*. Calgary: Alberta Energy Research Institute.

[17] Dimitrakopoulos, R. (1998). Conditional simulation algorithms for modelling orebody uncertainty in open pit optimisation. *International Journal of Mining, Reclamation and Environment, 12*, 173-179.

[18] Dimitrakopoulos, R., and Ramazan, S. (2008). Stochastic integer programming for optimising long term production schedules of open pit mines: Methods, application and value of stochastic solutions. *Institute of Materials, Minerals and Mining, 117*(4), 155-160.

[19] dos Santos, Silvio Francisco, and Brandi, Humberto Siqueira. (2017). Selecting portfolios for composite indexes: application of Modern Portfolio Theory to competitiveness. *Clean Technologies and Environmental Policy, 19*(10), 2443-2453.

[20] Fauquier, R., and al., et. (2009). *In-pit dyke construction planning*. Ft. McMurray: Shell Upstream Americas.

[21] GemcomSoftwareInternational. (2015a). Gemcom GEMS (Version 6.7.1). Vancouver: Gemcom Software International Inc.

[22] GemcomSoftwareInternational. (2015b). Whittle strategic mine planning software (Version 4.5.5). Vancouver: Gemcom Software International Inc.

[23] Gershon, M. E. (1983). Optimal mine production scheduling: evaluation of large scale mathematical programming approaches. *International Journal of Mining Engineering, 1*(4), 315-329.

[24] Gholamnejad, J., and Osanloo, M. (2007). Using chance constrained binary integer programming in optimising long term production scheduling for open pit mine design. *Institute of Materials, Minerals and Mining, 116*(2), 58-66.

[25] Gringarten, E., and V. Deutsch, C. (2001). *Teacher's Aide Variogram Interpretation and Modeling* (Vol. 33).

[26] Horst, R., and Hoang, T. (1996). *Global optimization: deterministic approaches* (3rd ed.). New York: Springer.

[27] Hustrulid, W. A., and Kuchta, M. (2006). *Open pit mine planning and design* (2nd ed.): Taylor and Francis/Balkema.

[28] ILOG, IBM. (2012). CPLEX reference manual and software (Version 12.5). Pullman, WA, USA: ILOG S.A. and ILOG Inc.

[29] Isaaks, Edward H., and Srivastava, R. Mohan. (1989). *An Introduction to Applied Geostatistics*. New York: Oxford University Press, Inc.,.

[30] Johnson, T. B. (1969). *Optimum open-pit mine production scheduling.* Paper presented at the 8th International Symposium on the Application of Computers and Operations Research in the Mineral Industry, Utah.

[31] Kalantari, Samira, Ben-Awuah, Eugene, and Askari-Nasab, Hooman. (2013). Towards an integrated oil sands mine plan and composite tailings plan. *International Journal of Mining, Reclamation and Environment, 27*(2), 103-126.

[32] Koushavand, B., Askari-Nasab, H., and Deutsch, C.V. (2009). Transfer geological uncertainty through mine planning. *MPES. MPES-International Symposium of Mine Planning/Equipment Selection*.

[33] Mangram, M. (2013). *A Simplified Perspective of the Markowitz Portfolio Theory* (Vol. 7).

[34] Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance, 7*(1), 77-91.

[35] Masliyah, J. (2010). *Fundamentals of oilsands extraction*. Edmonton: University of Alberta.

[36] Mathworks, Inc. (2017). MATLAB Software (Version 9.2 (R2017a)) [Software]: Mathworks Inc.

[37] McFadyen, D. (2008). *Directive 074*. Calgary: Energy Resources Conservation Board.

[38] Moreno, E., Rezakhah, M., Newman, A., and Ferreira, F. (2017). Linear models for stockpiling in open-pit mine production scheduling problems. *European Journal of Operational Research, 260*(1), 212-221.

[39] Newman, Alexandra M., Rubio, Enrique, Caro, Rodrigo, Weintraub, Andres, and Eurek, Kelly. (2010). A review of operations research in mine planning. *Interfaces, 40*(3), 222-245.

[40] Orumie, U. C., and Ebong, D. (2014). A Glorious Literature on Linear Goal Programming Algorithms *American Journal of Operations Research, 4*, 59-71.

[41] Osanloo, M., Rashidinejad, F., and Rezai, B. (2008). Incorporating Environmental Issues into Optimum CutOff Grades Modeling at Porphyry Copper Deposits. *Resources Policy, 33*(4), 222-229.

[42] Ramazan, S., and Dimitrakopoulos, R. (2004a). *Recent applications of operations research and efficient MIP formulations in open pit mining.* Paper presented at the SME Annual Meeting, Cincinnati, Ohio.

[43] Sabour, S. A., and Dimitrakopoulos, R. (2011). Incorporating Geological and Market Uncertainties and Operational Flexibility into Open Pit Mine Design. *Journal of Mining Science, 47*(2).

[44] Sanford, E. C. (1983). Processibility of athabasca oil sand: Interrelationship between oil sand fine solids, process aids, mechanical energy and oil sand age after mining. *The Canadian Journal of Chemical Engineering, 61*(4), 554-567.

[45] Tabesh, M., and Askari-Nasab, H. (2011). Two stage clustering algorithm for block aggregation in open pit mines. *Transactions of the Institution of Mining and Metallurgy, Section A, 120*(3), 158-169.

[46] Whittle, J. (1989). *The facts and fallacies of open pit design*. North Balwyn, Victoria, Australia: Whittle Programming Pty Ltd.

[47] Wolsey, L. A. (1998). *Integer programming*. New York: J. Wiley.

11. Appendix

Indices

$a \in A, A = \{1.....,A\}$	index and set for all possible processing destination in the model.
$d \in D, D = \{1,.....,D\}$	index and set for all possible destinations for materials in the model.
$e \in E, E\{1,....,E\}$	index and set for all elements of interest in each mining-cut.
$j \in J, J = \{1,......,J\}$	index and set for all pushbacks in the model.
$k \in K, K = \{1,.....,K\}$	index and set for all mining-cuts in the model.
$l \in L, L = \{1,......,L\}$	index and set for all possible mining locations (pits) in the model.
$p \in P, P = \{1,......,P\}$	index and set for all mining-panels in the model.
$sp \in SP, SP = \{1,....,SP\}$	index and set for all possible stockpiles in the model.
$t \in T, T = \{1,....,T\}$	index and set for all the scheduling periods, years.
$ts \in TS, TS = \{1,....,TS\}$	index and set for all possible stockpiling durations, years.

Sets

$IH_p(Z')$	For each mining-panel p, there is a set $IH_p(Z') \subset P$ defining the immediate predecessor mining-panels in a specified horizontal mining direction that must be extracted prior to extraction of mining-panel p at the specified level, where Z' is the total number of mining-panels in the set $IH_p(Z')$.
$IP_p(Z'')$	For each mining-panel p, there is a set $IP_p(Z'') \subset P$ defining the immediate predecessor mining-panels above mining-panel p that must be extracted prior to extraction of mining-panel p, where Z'' is the total number of mining-panels in the set $IP_p(Z'')$.
$MK_p(H')$	For each mining-panel p, there is a set $MK_p(H') \subset K$ defining the mining-cuts that belongs to the mining-panel p, where H' is the total number of mining-cuts in the set $MK_p(H')$.
$MP_j(H'')$	For each phase j, there is a set $MP_j(H'') \subset P$ defining the mining-panels within the immediate predecessor pit phases (pushbacks) that must be extracted prior to extracting phase j, where H'' is an integer number representing the total number of mining-panels in the set $MP_j(H'')$.

Decision Variables

$b_p^t \in [0,1]$	Binary integer variable controlling the precedence of extraction of mining-panels. b_p^t is equal to one if the extraction of mining-panel *p* has started by or in period *t*, otherwise it is zero.
$c_k^{a,t+ts} \in [0,1]$	Continuous variable representing the ore portion of mining-cut *k* to be reclaimed and processed at destination *a* in period *t+ts*.
$dv_1^{-,l,t}$	Negative deviation from the mining goal (tonnes) in period *t* at location *l*.
$dv_2^{-,a,t}$	Negative deviation from the processing goal (tonnes) in period *t* at processing destination *a* (tonnes).
$dv_3^{-,d,t}$	Negative deviation from the reclamation material goal (tonnes) in period *t* at destination *d* (tonnes).
$dv_4^{-,d,t}$	Negative deviation from the overburden dyke material goal (tonnes) in period *t* at destination *d* (tonnes).
$dv_5^{-,d,t}$	Negative deviation from the interburden dyke material goal (tonnes) in period t at destination d (tonnes).
$dv_6^{-,d,t}$	Negative deviation from the tailings coarse sand dyke material goal in (tonnes) period *t* at destination *d* (tonnes).
$q_k^{d,t} \in [0,1]$	Continuous variable representing the tailings coarse sands dyke material portion of mining-cut *k* to be extracted and used for dyke construction at destination *d* in period *t*.
$s_k^{sp,t} \in [0,1]$	Continuous variable representing the ore portion of mining-cut *k* to be extracted and sent to the stockpile *sp* in period *t*.
$u_k^{d,t} \in [0,1]$	Continuous variable representing the interburden dyke material portion of mining-cut *k* to be extracted and used for dyke construction at destination *d* in period *t*.
$v_k^{d,t} \in [0,1]$	Continuous variable representing the muskeg reclamation material portion of mining-cut *k* to be extracted and stockpiled at destination *d* in period *t*.
$x_k^{a,t} \in [0,1]$	Continuous variable representing the ore portion of mining-cut *k* to be extracted and processed at destination *d* in period *t*.
$y_p^{l,t} \in [0,1]$	Continuous variable representing the portion of mining-panel *p* to be mined in period *t* from location *l*, which includes ore, overburden and interburden dyke material, muskeg reclamation material and waste from the associated mining-cuts.
$z_k^{d,t} \in [0,1]$	Continuous variable representing the overburden dyke material portion of mining-cut *k* to be extracted and used for dyke construction at destination *d* in period *t*.

Parameters

$CSg^{d,t}$ Tailings coarse sand dyke material goal in period t at destination d (tonnes).

cs_k Tailings coarse sand dyke material tonnage in mining-cut k.

$dib_k^{d,t}$ Extra discounted cost of mining all the material in mining-cut k in period t as interburden dyke material for dyke construction at destination d.

$dm_k^{d,t}$ Discounted economic mining-cut value obtained by extracting mining-cut k and sending it to destination d in period t.

$dmu_k^{d,t}$ Extra discounted cost of mining all the material in mining-cut k in period t as muskeg reclamation material at destination d.

$dob_k^{d,t}$ Extra discounted cost of mining all the material in mining-cut k in period t as overburden dyke material for dyke construction at destination d.

$ds_{sp,k}^{d,t}$ Discounted economic mining-cut value obtained by extracting mining-cut k and sending it to stockpile sp and reclaiming it to destination d in period t.

$dt_k^{d,t}$ Extra discounted cost of mining all the material in mining-cut k in period t as tailings coarse sand dyke material for dyke construction at destination d.

$dw_k^{l,t}$ Discounted cost of mining all the material in mining-cut k in period t as waste from location l.

fn_k^e The average percent of fines in ore portion of mining-cut k.

$\overline{fn}^{a,t,e}$ Upper bound on the required average fines percent of ore in period t at processing destination a.

$\underline{fn}^{a,t,e}$ Lower bound on the required average fines percent of ore in period t at processing destination a.

fn_k^{ib} The average percent of fines in interburden dyke material portion of mining-cut k.

$\overline{fn}^{d,t,ib}$ Upper bound on the required average fines percent of interburden dyke material in period t at dyke construction destination d.

$\underline{fn}^{d,t,ib}$ Lower bound on the required average fines percent of interburden dyke material in period t at dyke construction destination d.

g_k^e The required average head grade of element e in ore portion of mining-cut k.

$\underline{g}^{a,t,e}$ Lower bound on the required average head grade of element e in period t at processing destination a.

$\overline{g}^{a,t,e}$ Upper bound on the required average head grade of element e in period t at processing destination a.

i	The discount rate.
$IBg^{d,t}$	Interburden dyke material goal in period t at destination d (tonnes).
$ibc^{d,t}$	Cost in present value terms per tonne of interburden dyke material for dyke construction at destination d in period t.
ib_k	Interburden dyke material tonnage in mining-cut k.
ib_p	Interburden dyke material tonnage in mining-panel p.
$Mg^{l,t}$	Mining goal (tonnes) in period t at location l.
$mc^{l,t}$	Cost in present value terms of mining a tonne of waste in period t from location l.
$MUg^{d,t}$	Muskeg, reclamation material goal (tonnes) in period t at destination d.
mu_k	Reclamation material tonnage in mining-cut k.
mu_p	Reclamation material tonnage in mining-panel p.
$muc^{d,t}$	Cost in present value terms per tonne of reclamation material at destination d.
o_k	Ore tonnage in mining-cut k.
o_p	Ore tonnage in mining-panel p.
$OBg^{d,t}$	Overburden dyke material goal in period t at destination d (tonnes).
$obc^{d,t}$	Cost in present value terms per tonne of overburden dyke material for dyke construction at destination d in period t.
ob_k	Overburden dyke material tonnage in mining-cut k.
ob_p	Overburden dyke material tonnage in mining-panel p.
$\overline{os}^{sp,t}$	The upper bound of ore tonnage sent to stockpile sp from mining-cut k in period t that exceeds the processing capacity.
$\underline{os}^{sp,t}$	The lower bound of ore tonnage sent to stockpile sp from mining-cut k in period t that exceeds the processing capacity.
$p^{e,t}$	The selling price of element e in present value terms per unit of product.
P_1	Priority level associated with minimizing the deviations from the mining goal.

P_2	Priority level associated with minimizing the deviations from the processing goal.
P_3	Priority level associated with minimizing the deviations from the reclamation material goal.
P_4	Priority level associated with minimizing the deviations from the overburden dyke material goal.
P_5	Priority level associated with minimizing the deviations from the interburden dyke material goal.
P_6	Priority level associated with minimizing the deviations from the tailings coarse sand dyke material goal.
$pc^{a,e,t}$	Extra cost in present value terms per tonne of ore for mining and processing at processing destination *a* in period *t*.
$pc_{sp}^{a,e,t}$	Extra cost in present value terms per tonne of ore for stockpiling at stockpile *sp* and processing at destination *a* in period *t*.
$Pg^{d,t}$	Processing goal in period *t* at destination *d* (tonnes).
PN_1	Penalty paid per tonne in deviating from the mining goal.
PN_2	Penalty paid per tonne in deviating from the processing goal.
PN_3	Penalty paid per tonne in deviating from the reclamation material goal.
PN_4	Penalty paid per tonne in deviating from the overburden dyke material goal.
PN_5	Penalty paid per tonne in deviating from the interburden dyke material goal.
PN_6	Penalty paid per tonne in deviating from the tailings coarse sand dyke material goal.
PN_u	Grade uncertainty cost, a pseudo cost for each mining-panel calculated as a product of a penalty value and the mining-panel kriged variance
$rp_{avg}^{a,e}$	Proportion of element *e* recovered (processing recovery) if it is sent from the mine to processing destination *a*.
$rp_{avg,sp}^{a,e}$	Proportion of element *e* recovered (processing recovery) if it is sent from the stockpile to processing destination *a*.
$rm_k^{a,e,t}$	Discounted revenue obtained by selling the final products within mining-cut *k* in period *t* if it is sent to processing destination *a*, minus the extra discounted cost of mining all the material in mining-cut *k* as ore from location *l* and processing at destination *d*.

$rs_k^{a,e,t}$ Discounted revenue obtained by selling the final products within mining-cut k from stockpile sp in period t if it is sent to destination a in period t, minus the extra discounted cost of processing and re-handling

$sc^{e,t}$ Selling cost of element e in present value terms per unit of product.

$tc^{d,t}$ Cost in present value terms per tonne of tailings coarse sand dyke material for dyke construction at destination d in period t.

w_k Waste tonnage in mining-cut k.

w_p Waste tonnage in mining-panel p.