Short-term Production Scheduling in Open-pit Mines with Shovel Allocations over Continuous Time Frames

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ABSTRACT

Short term mine production scheduling is essential to attain the desired capacity utilization of equipment, meet blending targets and adhere to the strategic schedules. Although, several optimization models exist in the literature, size and complexity remains a problem which poses a limitation to solve and generate the schedules at smaller resolutions for larger time frames. This paper presents a short term production scheduling model which allocates shovels over continuous time frames, thus controlling the model size from growing with the resolution. This approach also models the mine operating environment in a more representative way to create practical and achievable schedules. A verification study of the model is presented in this paper using an iron ore mine case study. A comparison of solutions in the pareto optimal space also justifies the goal optimization as an efficient approach to attain best outcomes from among multiple conflicting objectives of the problem.

1. Introduction

Mining is the backbone of world economy. The origin of most of the products that we use in our regular life can be traced to mining of its constituent ingredients. The market price and demand makes an operation profitable and hence viable, however, the dynamic price fluctuations in the market may pose a threat to the entire investment. A detailed planning is thus carried out before making the investment to realize a profitable operation. Through a long term production planning, it is desired to maximize the profit in the early years of the production so that breakeven can be achieved early and maximum NPV can be attained. Although, strategic planning process is significant, it is at the mine operation level that the actual profit can be realized. Efficient utilization of equipment and human resources coupled with the realization of long term planned schedules and adjusting to the existing market demand, bears a significant potential to realize or even exceed projected returns. Short term production scheduling and planning should therefore be considered an important step in the mine planning process. Attaining even marginal improvements at the operation level using efficient short term plans can lead to major gains at the strategic level.

Short term mine production scheduling has started to gain attention of the researchers in recent years and very few researchers have addressed the problem so far. Bjørndal et al. (2012) attributes the size and complexity of the problem for the lack of research in this area. These types of problems belong to NP-hard class as proved by Souza et al. (2010). A comprehensive review of the existing models and solution techniques is presented by Blom et al. (2018). The review (Blom*, et al.*, 2018) presents a vast

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difference in various existing models, in terms of mine operation features/activities considered, varying level of details and the objectives of the problem. Eivazy and Askari-Nasab (2012) propose a MILP model for multi-destination short term production scheduling model to minimize mining, processing, haulage, re-handling and rehabilitation costs, incorporating ramps, horizontal directional mining and buffer and blending stockpiles. Gholamnejad (2008) proposes a binary integer programming model to solve the problem at the block resolution, emphasizing on the accessibility of blocks for mining. L'Heureux et. al. (2013), Liu et. al.(2013), Kozan and Liu (2016; 2014) propose multi-stage MIP models to incorporate drilling, blasting and extraction operations. Topal and Ramazan (2010) provide a MIP model for scheduling a fixed fleet of mining trucks in a mine operation to minimize the maintenance costs. Due to the vastness of open pit mine operations, there exists numerous objectives and elements of the operations that need to be accounted for at the short term planning time resolution. For the short term production scheduling purposes, this paper assumes four major objectives as to maximize the production, meet tonnage and grade requirements of the processing operations and minimize the shovel movements (Upadhyay and Askari-Nasab, 2017, 2018).

Short term mine production scheduling problem requires to determine optimal sequence of faces to be drilled, blasted and mined by the available equipment resources for the strategically scheduled faces in the given time horizon and the amount of material to be transported from the faces to their respective destinations to achieve production and grade blend requirements of the processing plants over daily or weekly time frames. A detailed short term production scheduling model is required to account for equipment resources and corresponding drilling/charging/production and movement capabilities between faces; throughput rates of the processing operations; precedence among faces for drilling, blasting and excavation operation; production between faces and various dump destinations; and discrete haulage capacities based on the available haul truck resources and road network architecture of the mine. L'Heureux et. al. (2013) also incorporates the selection of blocks for each blast in their model. Mousavi et. al.(2016a) incorporates drop cuts in the predetermined periods and maximum vertical distance between two blocks extracted in a period to consider bench access design (haul roads).

Although, a detailed short term planning model is desired, it poses a limitation on solvability. Number of faces considered in the optimization time frame is one major reason for the size and complexity of the problem. The definition of faces is debatable in the existing literature where an excavator resource can be allocated. Block aggregation is a common technique to group similar blocks together and considered as a face. L'Heureux et. al.(2013) defines a face as an aggregate of 1 to 4 adjacent blocks having similar characteristics and considers a single block as a face while solving the model. Kozan and Liu (2016) considers a set of several same-grade block units on the same bench in the same pit as a face. Mousavi et al. (2016a, 2016b) solves the problem at the block resolution.

Uncertainty in mine operations is another problem that leads to deviations between planned and reality at this resolution. Dimitrakopoulos and Jewbali (2013) proposes a joint stochastic optimization approach to link the variability in short term with long term production schedules. Matamoros & Dimitrakopoulos (2016) proposes a stochastic integer programming formulation to simultaneously optimize fleet and production schedule accounting for uncertainty in grade and fleet parameteres and availability. Simulation and optimization approaches have also been used to capture the uncertainties of the mine operations to generate short term production schedules (Fioroni*, et al.*, 2008; Upadhyay and Askari-Nasab, 2018).

One important contribution of this paper is the use of continuous time variables to control shovel allocations and movements while capturing the production and quality of material produced in discrete periods. It serves two purposes. First, as shovel allocation and movement variables are independent of the periods, model size does not increase drastically with increasing resolution. We can solve a problem at larger or smaller resolutions without having big impacts on the size of the model. The second purpose is related to practicality. It is essential at this planning resolution that precedence are accurately modeled and that the production plans are practical. Allocating shovels within discrete periods (L'Heureux*, et al.*, 2013; Mousavi*, et al.*, 2016a, 2016b) does not model the exact time production finished or started at a face. Thus a shovel, in such models, can be allocated to a face in the beginning of a period even if the precedence requirements are met towards the end of the period as shown in Fig. 1. Such a solution would not be practical and the planned production could not be achieved in reality, especially when the resolution of the model is larger. Kozan and Liu (2016) propose a framework strictly for equipment allocations at the various operational stages using continuous time variables, however, they do not capture production and quality at discrete time resolutions, rendering the model not usable for blending and other objectives which are measured over discrete time frames.

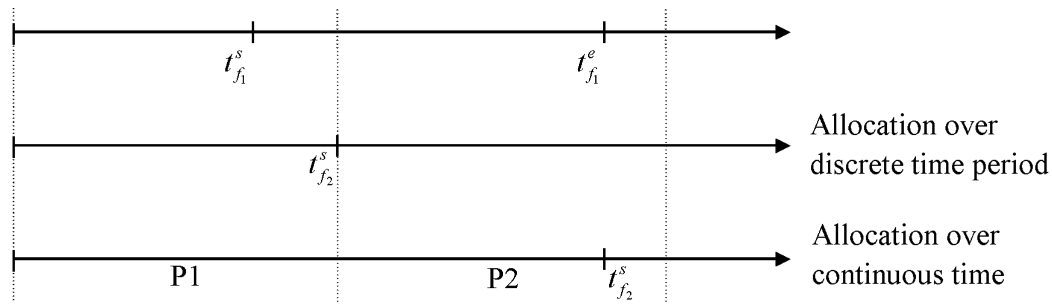

Fig. 1. Comparison between allocations over discrete and continuous time frames for the mining start time at a face f_2 based on the finish time of its precedence face f_1.

Another drawback observed with the models which allocate shovels within discrete time periods with single possible assignment in a period, is an induced, but not desired, characteristic to the model that tries not to assign a shovel to a face based on the remaining tonnage at the face. If the remaining tonnage is small compared to capacity or minimum production constraints at the model resolution, a different assignment will be preferred by the model and the optimality of the solution would be questionable. Shovel movements can be allowed within periods for such models (Upadhyay and Askari-Nasab, 2018), but it becomes hard to capture the movements across periods.

A mixed integer programming (MIP) model is presented in this paper to formulate the short term production scheduling problem by providing shovel allocations over continuous time frames, and discrete production between faces and destinations to achieve production and grade blend objectives. Model verification and a comparison of solution times is presented with an iron ore mine case study. The model is solved following a goal optimization approach due to the conflicting nature of model objectives. A comparison of solution for each objective within the pareto optimal space is also presented to justify the goal programming approach and obtain best solution based on the decision makers (DMs) preference.

2. Model characteristics and assumptions

In this paper a face is an aggregate of multiple blocks together based on their similarity (Tabesh and Askari-Nasab, 2013) to reduce the size of the problem and have practical solutions for an open pit mining framework. The characteristics of a face are the average values of the blocks constituting the face. Shovels are allocated to the faces to mine and send the material to various destinations based on their grade and tonnage requirements and capacity constraints in each period.

The model presented in this paper assumes that a shovel will move to a new face only after the current face is completely mined out. The validity of this assumption is dictated mainly by the size of the faces. Using large faces poses a problem on the validity of using the average characteristics of the constituting blocks and neglecting possible and may be significant variations. A very small face, on the other hand, loses its purpose to reduce the problem size and practically minable dimension requirements. This assumption is different from some existing models where shovel may move from and to a face multiple times and that a face may be mined intermittently, and not continuously, over multiple periods. In the model presented, decreasing the size increases the number of faces and allows for more possible shovel movements if desired. Thus possible movements can be controlled by the size of the faces in this model.

3. Optimization model

3.1. Goals

Indices, variables and parameters used in the model are described in Table 2, 3 and 4 respectively in the Appendix section. This model considers four main operational objectives as goals of the short term production scheduling model: (1) maximize production by minimizing the negative deviation in production by shovels compared to their capacities, (2) minimize the deviation in ore tonnage received at processing plants compared to their capacities, (3) minimize the deviation in grades delivered to ore destinations compared to desired grades and (4) minimize the movement times of shovels (Upadhyay and Askari-Nasab, 2017).

$$\Psi_1 = \sum_p \sum_s x^-_{s,p} \tag{1}$$

$$\Psi_2 = \sum_{d^c} \sum_p (\delta^-_{d^c,p} + \delta^+_{d^c,p}) \tag{2}$$

$$\Psi_3 = \sum_p \sum_{d^o} \sum_k (g^-_{k,d^o,p} + g^+_{k,d^o,p}) \tag{3}$$

$$\Psi_4 = \sum_s \sum_{f_1} \sum_{f_2} \Gamma_{s,f_1,f_2} \times a_{s,f_1,f_2} \tag{4}$$

3.2. Objective function

The model optimizes these objectives as a goal programming function by carefully choosing priority weights for each objective as per the requirements of the decision maker (Romero, 2004). The objectives are first normalized in the pareto optimal space and then combined using weighted sum approach to form the objective function (Grodzevich and Romanko, 2006; Tamiz*, et al.*, 1998) as given in Eq. (5).

$$\Psi = W_1 \times \overline{\Psi}_1 + W_2 \times \overline{\Psi}_2 + W_3 \times \overline{\Psi}_3 + W_4 \times \overline{\Psi}_4 \tag{5}$$

Where:

$$\overline{\Psi}_i = \left(\Psi_i - Utopia_i\right) / \left(Nadir_i - Utopia_i\right) \tag{6}$$

3.3. Constraints

$$\sum_{f_1} \sum_s a_{s,f_1,f_2} = 1 \qquad \forall f_2 \neq f^0 \tag{7}$$

$$\sum_{f_1} \sum_s a_{s,f_1,f_2} = \overline{S} \qquad f_2 = f^0 \tag{8}$$

$$\sum_{f_1} a_{s,f_1,f} - \sum_{f_2} a_{s,f,f_2} = 0 \qquad \forall s, f \tag{9}$$

$$t^s_{f_1} - t^s_{f_2} + \sum_s \left(\Pi_{s,f_1} + \Gamma_{s,f_1,f_2}\right) \times a_{s,f_1,f_2} + B \times \sum_s a_{s,f_1,f_2} \le B \qquad \forall f_1 \,\&\, \forall f_2 \neq f^0 \tag{10}$$

$$t^s_{f_1} + \sum_s \left(\Pi_{s,f_1} + \Gamma_{s,f_1,f_2}\right) \times a_{s,f_1,f_2} + B \times \sum_s a_{s,f_1,f_2} \le 2 \times B \qquad \forall f_1 \,\&\, f_2 = f^0 \tag{11}$$

$$t_f^s + \sum_s \sum_{f_1} \frac{O_f}{X_s} \times a_{s,f,f_1} \le t_f^e \qquad \forall f \qquad (12)$$

$$t_{f_1}^e + \sum_s \Gamma_{s,f_1,f_2} \times a_{s,f_1,f_2} \le t_{f_2}^s + (1 - \sum_s a_{s,f_1,f_2}) \times B \qquad \forall f_1, \forall f_2 \ne f^0 \qquad (13)$$

The first set of constraints model the shovel allocations and corresponding movement between faces. These constraints are formulated similar to the modeling of a multiple travelling salesman problem or vehicle routing problem. Given a set of faces and a starting dummy face where all shovels are initially located, it is required to visit each face exactly ones by the shovels and return to the starting dummy face. The distance between all the faces and the dummy face is zero along with its tonnage. Constraint (7) restricts only one possible movement to a face, i.e. only one shovel can move to a face from only one of the available faces. Constraint (8) models the return of all shovels to the dummy face. Constraint (9) is the flow balancing equation which states that if a shovel has moved to a face, it must move out from that face. The sub-tour elimination constraint is modeled using a continuous time variable in equations (10) and (11), which captures the start time of the allocated shovel at each face. Constraint (10) dictates that start time of the face f_2 must be greater than or equal to the summation of start time and mining time of face f_1, and movement time of the corresponding shovel from face f_1 to f_2. If there is no movement between the faces, the difference between the start times will always be lesser than the maximum time B. When there is a movement to the dummy face, Constraint (11) models the same equation (10) considering the start of the dummy face at the maximum time B, preserving its actual start time variable as zero. Constraint (12) ensures that finish time of a face is always greater than or equal to the start time plus the minimum mining time of the face. Coupled with constraint (12), constraint (13) restricts start time of a face to be greater than the finish time of its predecessor face plus the movement time of the corresponding shovel between them.

$$\left(1 - m_{f,p}^s\right) \times B + m_{f,p}^s \times p \times T \ge t_f^s \qquad \forall f, p \qquad (14)$$

$$\left(1 - m_{f,p}^e\right) \times B + m_{f,p}^e \times p \times T \ge t_f^e \qquad \forall f, p \qquad (15)$$

$$\left(1 - m_{f,p}^s\right) \times p \times T \le t_f^s \qquad \forall f, p \qquad (16)$$

$$\left(1 - m_{f,p}^e\right) \times p \times T \le t_f^e \qquad \forall f, p \qquad (17)$$

$$m_{f,p}^s \ge m_{f,p-1}^s \qquad \forall f \,\&\, \forall p \ne 1 \qquad (18)$$

$$m_{f,p}^e \ge m_{f,p-1}^e \qquad \forall f \,\&\, \forall p \ne 1 \qquad (19)$$

$$\sum_s \sum_d x_{s,f,d,p} \le m_{f,p}^s \qquad \forall f, p \qquad (20)$$

$$m_{f,p}^e \le \sum_s \sum_d x_{s,f,d,p} \qquad \forall f, p \qquad (21)$$

$$m_{f,p}^e \le m_{f,p}^s \qquad \forall f, p \qquad (22)$$

$$m_{f_2,p}^s \le \sum_s \sum_d x_{s,f_1,d,p} + 1 - \sum_s a_{s,f_1,f_2} \qquad \forall f_1, f_2, p \qquad (23)$$

$$\sum_d \left(x_{s,f,d,p} - x_{s,f,d,p-1}\right) \times O_f \le \begin{pmatrix} (m_{f,p}^s - m_{f,p-1}^s) \times p \times T + \\ \left(1 - (m_{f,p}^s - m_{f,p-1}^s)\right) \times B - t_f^s \end{pmatrix} \times X_s \qquad \forall s, f, p \qquad (24)$$

$$\sum_{d}\left(x_{s,f,d,p}-x_{s,f,d,p-1}\right)\times O_f \le \left(t_f^e-(m_{f,p}^e-m_{f,p-1}^e)\times(p-1)\times T\right)\times X_s \qquad \forall s,f,p \tag{25}$$

$$t_{f'}^e \le t_f^s \qquad \forall f\ \&\ f'\in F_f \tag{26}$$

As shovel movement variable is not indexed over periods, constraints (14) to (25) are used to relate the continuous time variable with discrete periods from which production can start from a face. Constraint (14) and (15) coupled with constraints (16) and (17) restricts the mining start and mining end variables to be true for the periods corresponding to mining start and end times respectively. Constraints (18) and (19) make sure mining start and end variables remains true after they first became true. Constraint (20) states that no production is possible from a face until mining start variable becomes true, i.e. until a shovel has reached to the face. Constraint (21) on the other hand restricts the mining end variable to become true until the face is completely mined. Constraint (22) simply states that mining end variable can be true only if mining start variable is true. Constraint (23) ensures zero production from a face until its predecessor, from where shovel has moved, is completely mined. Constraint (24) determines the maximum production possible from the face since the shovel arrived at the face in that period. It states that if mining start variable became true in a period, i.e. a shovel arrived to the face, the maximum production possible from the face in that period will be limited by the remaining time. Similarly constraint (25)ensures that mining end time of a face is accurately captured based on the amount of material mined in that period. Precedence requirements are modeled using constraint (26) which ensures that a shovel cannot start mining a face before its precedence faces are finished.

$$\sum_{d} x_{s,f_2,d,p}-\sum_{f_1} a_{s,f_1,f_2}\le 0 \qquad \forall s,f_2,p \tag{27}$$

$$x_{s,f,d,p}\le x_{s,f,d,p+1} \qquad \forall s,f,d,p \tag{28}$$

$$\sum_{d}\sum_{f}(x_{s,f,d,p}-x_{s,f,d,p-1})\times O_f \le X_s^+ \qquad \forall s,p \tag{29}$$

$$\sum_{d}\sum_{f}(x_{s,f,d,p}-x_{s,f,d,p-1})\times O_f \big/ X_s^+ + x_{s,p}^- = 1 \qquad \forall s,p \tag{30}$$

$$\sum_{s}\sum_{d^c} x_{s,f,d^c,p}\le Q_f \qquad \forall f,p \tag{31}$$

$$\sum_{s}\sum_{d^w} x_{s,f,d^w,p}\le (1-Q_f) \qquad \forall f,p \tag{32}$$

$$\sum_{s}\sum_{f}(x_{s,f,d^c,p}-x_{s,f,d^c,p-1})\times O_f \big/ \left(Z_{d^c}\times T\right)+\delta_{d^c,p}^- - \delta_{d^c,p}^+ = 1 \qquad \forall d^c,\forall p \tag{33}$$

$$\delta_{d^c,p}^- \le \Lambda_{d^c}^- \big/ Z_{d^c} \qquad \forall d^c,p \tag{34}$$

$$\delta_{d^c,p}^+ \le \Lambda_{d^c}^+ \big/ Z_{d^c} \qquad \forall d^c,p \tag{35}$$

$$\begin{aligned}&\sum_{s}\sum_{f}(x_{s,f,d^c,p}-x_{s,f,d^c,p-1})\times O_f\times \overline{G}_{f,k}+g_{k,d^c,p}^- - g_{k,d^c,p}^+ = \\ &\sum_{s}\sum_{f}(x_{s,f,d^c,p}-x_{s,f,d^c,p-1})\times O_f\times G_{k,d^c}\end{aligned} \qquad \forall k,d^c,p \tag{36}$$

Constraint (27) allows no production by a shovel from a face, if there is no movement of the shovel to the face. Constraint (28) insures that production from a face by a period is always greater than previous period. Constraint (29) is the capacity constraint restricting total production in a period to the maximum production capacity of shovels. Negative deviation in production compared to the maximum shovel capacities are modeled by constraint (30). Constraints (31) and (32) restrict ore production to be sent to crushers and waste production to the waste dumps. Negative or positive deviation in tonnage received at crushers compared to desired rate is modeled using constraint (33). Constraint (34) and (35) provides lower and upper limits on the deviation in tonnage received at crushers. Grade deviations are captured using constraint (36) as the negative or positive deviation in tonnage of received metal content as compared to desired metal content based on the received tonnage of ore at the crushers.

4. Model Implementation

The model is verified by implementing it over an iron ore open pit mine case study with 5 shovels (2 ore and 3 waste mining shovels). There exists two crushers (C1 and C2) and a waste dump. The crushers are required to be fed at-least at half of its desired capacity with a MWT grade of 65% and75%. The model is implemented with IBM ILOG CPLEX solver version12.7.1 through Matlab, on a Windows 7 environment with Intel(R) Core(TM) i7-3770 processor of 3.40 GHz and 16 GB RAM.

Table 1. Comparison of models, sizes, solution time and relative MIP gap

Problem	Faces	Periods	Variables	Binary variables	Constraints	Solution time - Goal (s)	Relative MIP gap (%)
P1	24	4	4963	3325	7628	12	0
P2	24	20	12099	4125	33980	185	0
P3	43	8	15924	10384	33252	36000	0.31%
P4	62	3	23251	20223	27189	-	-

Four set of problems (P1-P4) are formulated with increasing time frame and resolution to study the impact on model size and solution time which is presented in Table 1. It shows that number of binary variables does not grow drastically with increase in the resolution, which is not the case if shovel allocation variables are indexed over periods.

To assist the model runtime, initial faces for shovel assignments are fixed in this implementation. Moreover, based on the prior knowledge that ore shovels should work only on ore faces and waste shovels on the waste faces, and no movement is possible from a face to any of its precedence faces, corresponding variables are fixed prior to solving the model. Also, based strictly on the precedence requirements and shovel capacities, a minimum possible start time of all the faces is determined to fix the mining start and mining end binary variables of the faces for the relevant periods to be zero.

Table 1 shows the run time and MIP gap of the goal function obtained by running the model for each problem with a stopping criteria of 36000 s or 10 h. P1 and P2 could be easily solved to optimality within a reasonable time, however a parito optimal space could not be determined for P4 within the stopping criteria. For problem P3, all individual objectives could be solved to optimality within 45 minutes combined, however goal function could not converge to optimality within the stopping criteria after achieving a 0.36% relative MIP gap within half an hour.

Solutions for problems P1 and P2 are compared in Fig. 2 to 4 for the major objectives considered in the optimization. Fig. 2 and 3 show ore and waste productions sent to two crushers (C1 and C2) and the waste dump over four periods in P1 and 20 periods in P2 respectively. As P2 correspond to the same time frame as P1 with a smaller resolution, one period in P1 is equivalent to 5 periods in P2. The capacity utilization values are presented with respect to desired feed. It can be observed that the two solutions are not the same entirely. The reason for the difference is the grade blend objective function, which evaluates grade as an average over a larger time resolution in P1, but a smaller and practical resolution in P2.

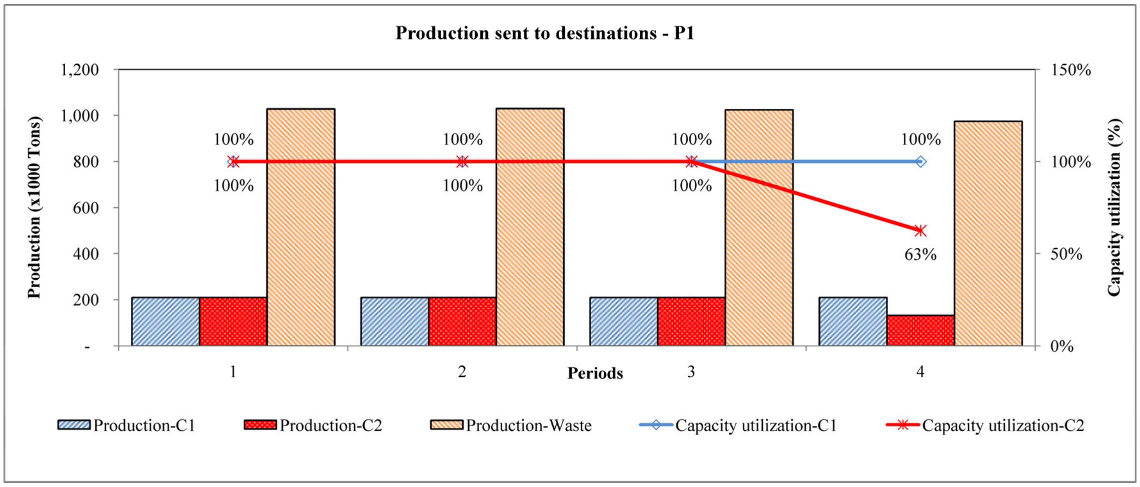

Fig. 2. Production and capacity utilizations for problem P1

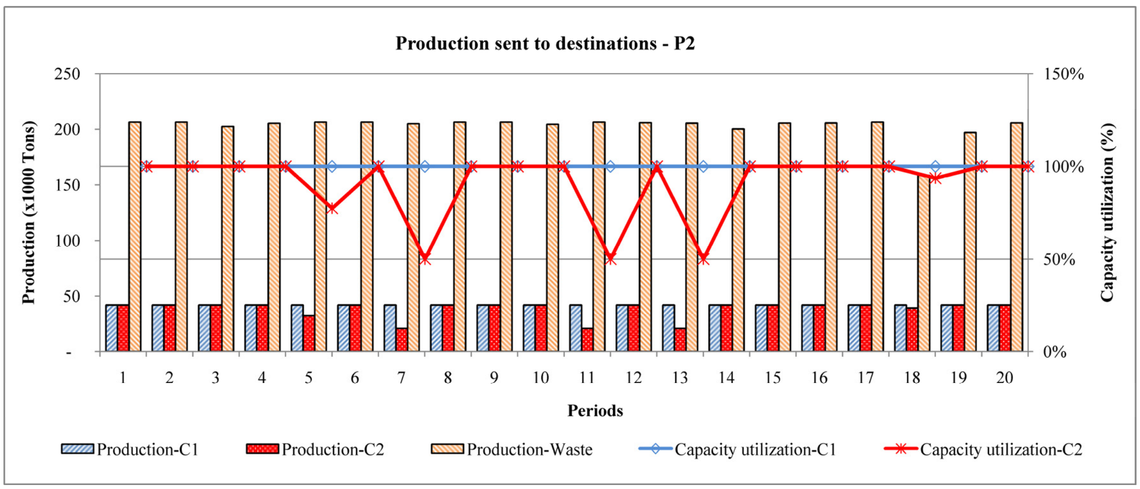

Fig. 3. Production and capacity utilizations for problem P2

Fig. 2 and 3 show that crusher 1 operates at its desired capacity in both the solutions for P1 and P2. However capacity utilization of crusher 2 drops to 63% of desired capacity in the last period for P1. In comparison, crusher utilization for P2 drops to the minimum limit at three occasions. The reason for this behavior can be attributed to the grade blend objective which sends less material to crusher 2 whenever desired grade blend cannot be achieved as observed from Fig. 4. A comparison of grade blend delivered over both resolutions (P1 and P2) is shown in Fig. 4, which categorically suggests the practicality of solution for P2 in comparison to P1, by showing the variations in grade blend deliverable at a higher resolution.

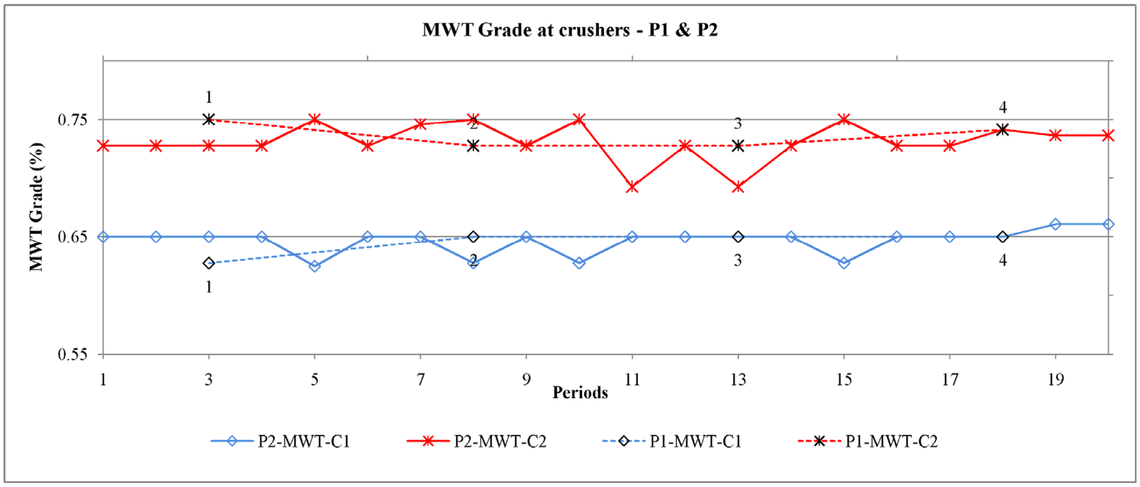

Fig. 4. MWT Grade delivered to crushers for problems P1(4 periods) and P2 (20 periods)

Similarly, solutions for problem P3 are presented in Fig. 5 and 6. The first four periods in this problem correspond to P1, however the results are observed to be different. The obvious reason is the inclusion of more available faces in the problem, which provides more options to achieve the objectives spread over more periods. The production and capacity utilizations of the crushers show promising results, however the grade blend objective for crusher 2 performed poorly.

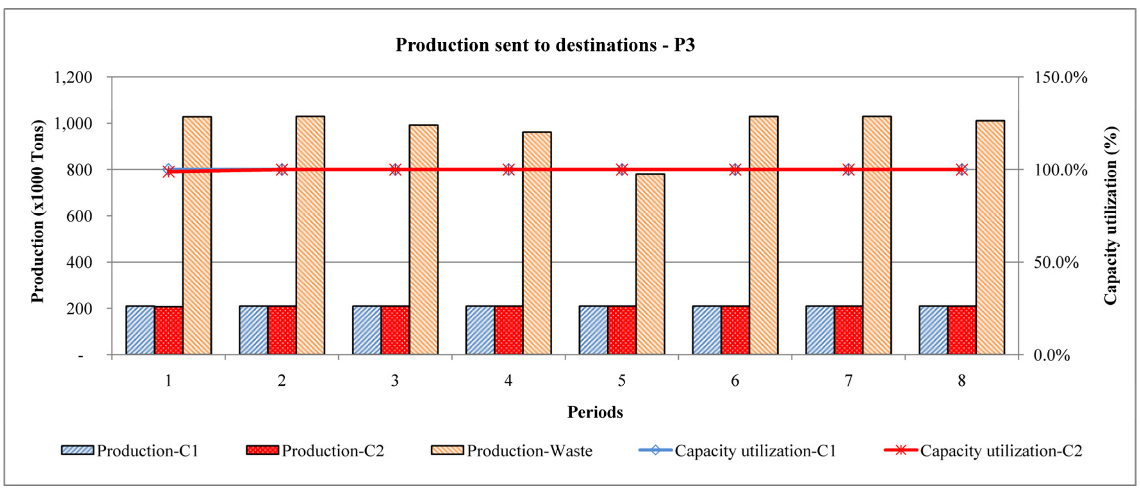

Fig. 5. Production and capacity utilizations for problem P3

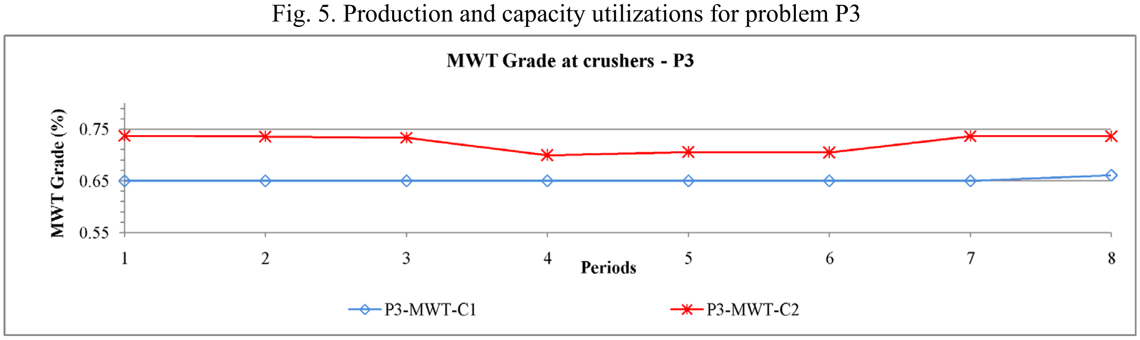

Fig. 6. MWT grade delivered to crushers for problem P3

Although goal function provides the desired solution based on decision makers (DMs) preference to each objective, it is imperative to understand and analyze the best solution for individual objectives as well. Such an analysis supports the DMs judgment of preference weights used in the goal function. A similar analysis is presented for problem P3 through Fig. 7, 8, 9 and 10. The solutions are recorded during the determination of parito optimal space which involved optimizing each objective separately.

Best possible production scenario is presented in Fig. 7, which falls short by 3.5% of the maximum production capacity of the mine. The lesser material handling capacity of the crushers in comparison to production capacity of the ore shovels could be one major reason for the short fall, coupled with lost production in movement and idling due to precedence requirements. Moreover, goal function schedule's 0.8% less production in comparison to best production case scenario to compensate for other objectives. It can be observed from Fig. 8 that by having a 0.8% production loss, a much better grade blend is achieved by the goal function (Fig. 6). Also, crushers are overfed in best production case scenario and made to operate at their peak capacities, more than desired, in comparison to schedule provided by goal function. Similarly, the best grade blend scenario can provide an improvement in the grade blend delivered to crushers (see Fig. 9), however, it cannot feed the plants at their desired capacity (see Fig. 10).

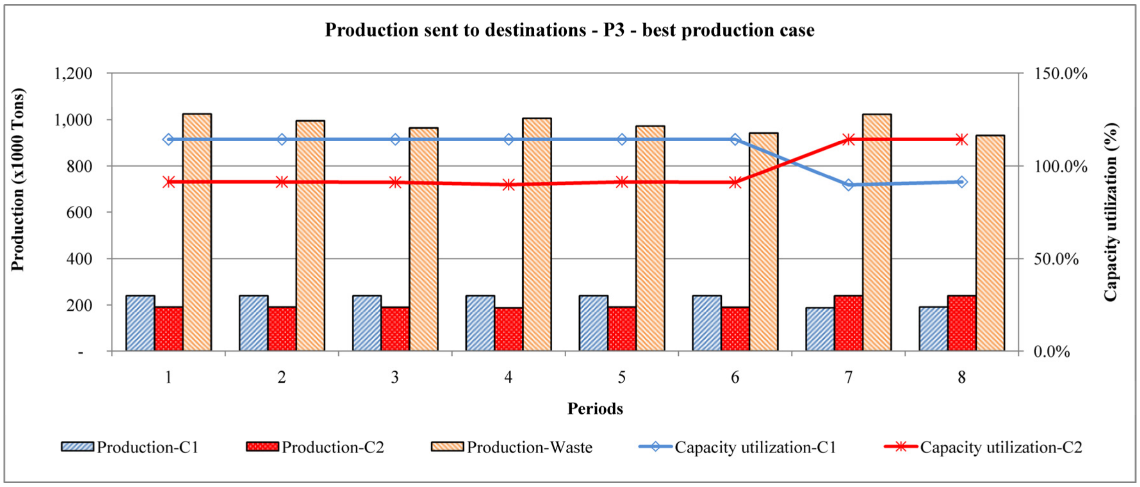

Fig. 7. Production and capacity utilizations for problem P3 - best production case

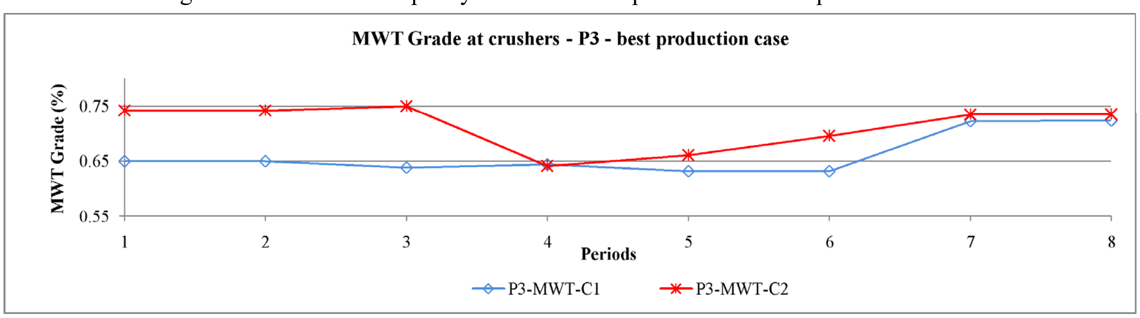

Fig. 8. MWT grade delivered to crushers for problem P3 - best production case

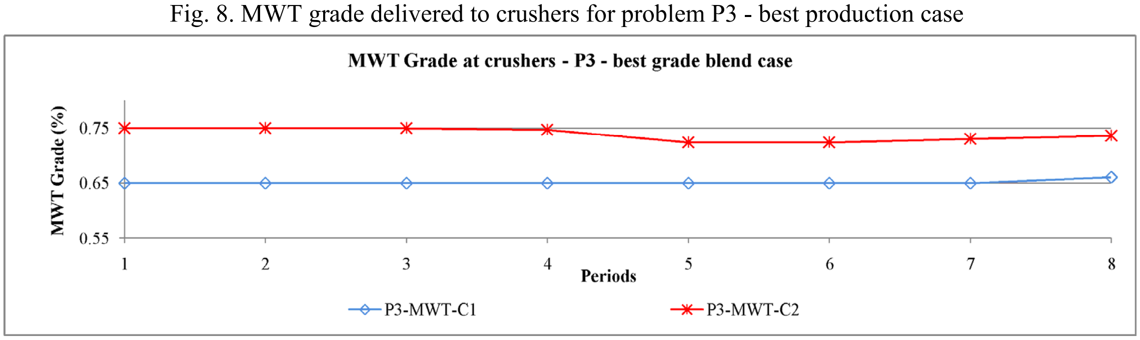

Fig. 9. MWT grade delivered to crushers for problem P3 - best grade blend case

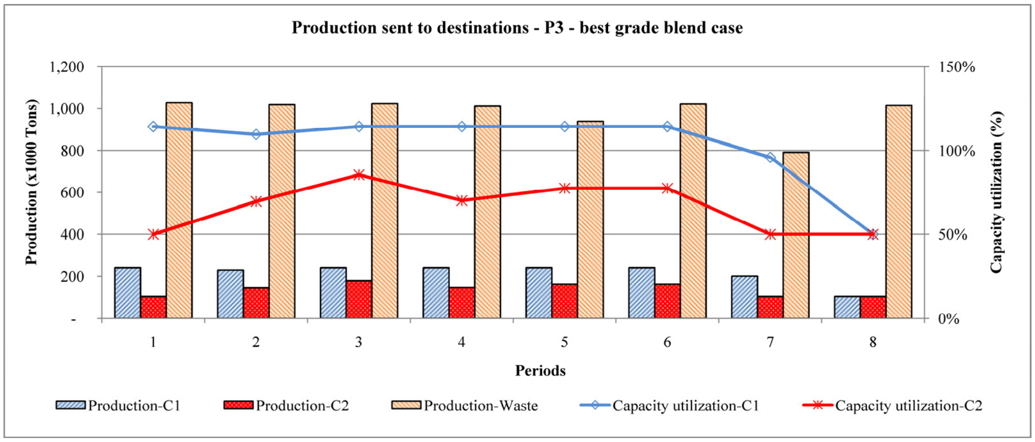

Fig. 10. Production and capacity utilizations for P3 - best grade blend case

To summarize and justify the goal function, production values obtained by the optimization of each objective is compared against the best production scenario in Fig. 11. A similar comparison for crusher feed objective is presented in Fig. 12. Negative values in Fig. 12 correspond to overfeeding the crushers above desired feed. It can be seen that production deviation can be as large as 5.4% when only grade blend is optimized, whereas goal function could reduce it to 0.8% (Fig. 11) while maintaining a satisfactory grade blend as shown in Fig. 6. Crusher feed is also compromised by only 0.7% in the optimal goal solution in comparison to best crusher scenario as shown in Fig. 12.

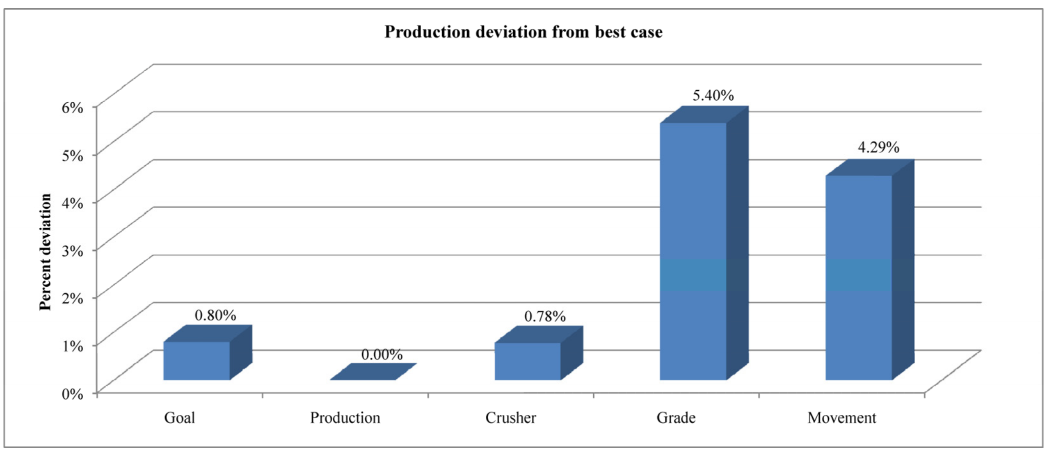

Fig. 11. Production deviation from best production scenario for the optimal solution of each objective function

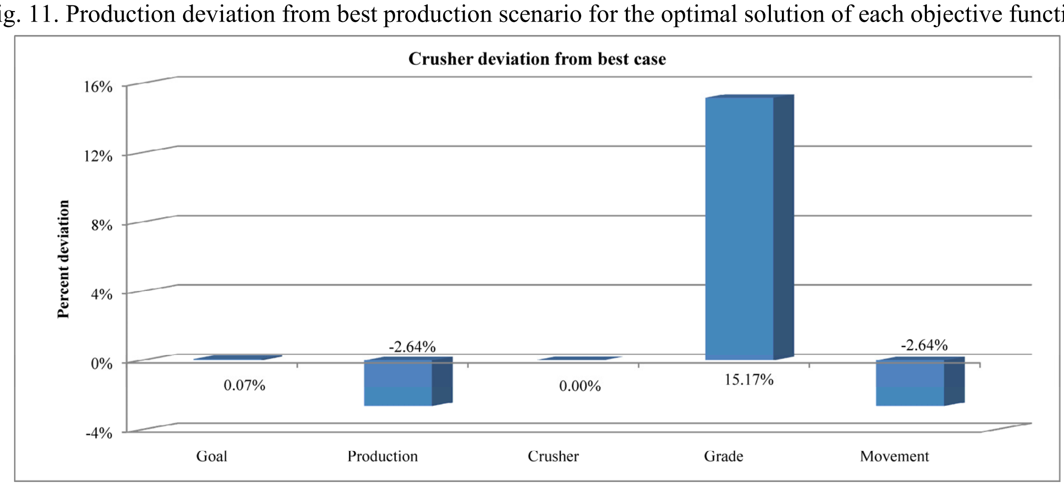

Fig. 12. Crusher feed deviation from best crusher feed scenario for the optimal solution of each objective function

Due to the complexity of the mine operations and multiple objectives to be taken care of at the short term scheduling stage, it is imperative to optimize all the objectives simultaneously to have a reasonable solution. Optimizing the goal function in the pareto optimal space is also essential to obtain non-dominated solution for each objective based on DMs preferential weights as shown in Fig. 11 and 12.

5. Conclusions and recommendations

This paper presented a model for short term production scheduling of open pit mines. Through a comparison in modeling approach and the selection of variables, the presented model can be observed to be superior in the context of modeling the problem correctly. The use of continuous time variables for shovel allocations and capturing productions and grades over discrete periods models the system precisely. The modeling approach also shows its capability to control the size of the problem at increasing resolutions which is necessary for practical solutions especially for grade blend deliverability. An implementation of the model on a verification case study shows its strength in solving the short term production scheduling model at the size of problem P3 in a reasonable time, and capturing the practical deliverance of the objectives of the model.

A comparison of all objective function values in the pareto optimal space is also presented to compare the final solution against the best possible solution for each objective considered. The comparison presented can be used to justify the DMs preference weight of each objective and can be subsequently adjusted as desired. However, in the case presented, the solution obtained fairly justifies the weights used.

Run time for larger problems still remains a major hurdle for this model. Selection of blast areas, drilling and blasting processes will also be included in the model as a future work for more practical production schedules.

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7. Appendix

Table 2**.** Indices for variables, parameters and sets.

Indices	Description
s	Index for set of shovels (s = 1, … $\overline{S}$)
f	Index for set of faces (f = 1, … $\overline{F}$)

k	Index for set of material types (k = 1, … $\overline{K}$)
d	Index for set of destinations (crushers, waste dumps)
d^c	Index for set of crushers/processing plants (d^c = 1, … $\overline{C}$)
d^w	Index for waste dumps (d^w = 1, … $\overline{W}$)
p	Index for periods (p=1,… $\overline{P}$)

Table 3. Variables considered in the model

Variables	Description
a_{s,f_1,f_2}	Movement variable if shovel s moved from face f_1 to face f_2 (binary)
$m^s_{f,p}$	Mining start variable if any shovel reached to face f by period p (binary)
$m^e_{f,p}$	Mining end variable if any shovel finished a face f by period p (binary)
t^s_f	Continuous time variable at which production start from face f
t^e_f	Continuous time variable at which production end at face f
$x_{s,f,d,p}$	Fraction of tonnage at face f sent by shovel s, to destination d by period p
$x^-_{s,p}$	Fraction of maximum capacity of shovel to model negative deviation in production by shovel s compared to its capacity in period p
$\delta^-_{d^c,p}, \delta^+_{d^c,p}$	Negative and positive deviations in production received at crusher destinations d^c in period p, as a fraction of processing plant capacities
$g^-_{k,d^c,p}, g^+_{k,d^c,p}$	Negative and positive deviations in tonnage equivalent of grade of material type k compared to desired grade at crusher destinations d^c in period p

Table 4. Parameters of systems considered.

Parameters	Description
T	Time horizon of a period (hr)
B	Time horizon in which all faces can be mined out completely (hr)
X_s	Shovel hourly production capacity (tonne/hr)
X^+_s	Maximum possible shovel production in decision time frame 'T' (tonne)
Γ_{s,f_1,f_2}	Movement time of shovel s from face f_1 to face f_2 (hr)
$\Pi_{s,f}$	Mining duration of face f by shovel s at maximum production capacity (hr)
$\overline{\Pi}_f$	Minimum mining duration possible for face f (hr)
Fi_s	Face where shovel is initially located (start of the optimization time frame)
f^0	Dummy face, which has zero tonnage and distance to all the faces
F_f	Set of precedence faces to be mined before starting face f
z_{d^c}	Maximum capacity of the crushers/processing plants (tonne/hr)

$\Lambda_{d^c}^{+}$	Maximum positive deviation in tonnage accepted at crushers/processing plants (tonne/hr)
$\Lambda_{d^c}^{-}$	Maximum negative deviation in tonnage accepted at crushers/processing plants (tonne/hr)
G_{k,d^c}	Desired grade of material types at the crushers
$\overline{G}_{f,k}$	Grade of material type k at face f
O_f	Tonnage available at face f at the beginning of optimization (tonne)
Q_f	1 if material at face is ore, 0 if it is waste (binary parameter)
M_s^{ore}	0 if shovel s is locked to an ore face, 1 for waste face, 2 otherwise.
W_i	Normalized weights of individual goals (i = 1, 2, 3) based on priority