Block-Cave Production Scheduling Using A Multi-Index Clustering Technique

Farshad Nezhadshahmohammad, Firouz Khodayari, and Yashar Pourrahimian
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

Rapid improvement in underground mining technologies causes some disturbs with accumulating huge amounts of the source, destination and extraction time of ore and waste during the life of mine. Long-term planning is one of the most important stages that determines the distribution of cash flows over the life of mine and can significantly impact the feasibility of the project. An optimal production schedule in block caving will not be practical unless the geotechnical constraints are considered. The draw control is one of the main aspects of geotechnical constraints in block-cave operation. Most of the mathematical draw control systems do not have an exact production rate curve to manage draw rates of the drawpoints. In addition, these systems are too complex to provide a practical solution for real block-cave mines. This paper presents a mixed-integer linear programming (MILP) model to optimize the extraction sequence of drawpoints over multiple time horizons of block cave mines with respect to the draw control systems. Also, a multi-index clustering algorithm is presented to reduce the size of the large-scale MILP model to be able to solve the problem in a reasonable time. The results show a significant reduction in the size of the MILP model and CPU time. Application and comparison of the production schedule based on the draw control system with the clustering technique is presented using 2,487 drawpoints to be extracted in 32 years.

1. Introduction

Block and panel caving have become the underground bulk mining methods of choice and expected to continue in the future [1]. Block caving is a complex and large-scale mining method. The application of block caving is for low-grade, caveable, and massive ore-bodies. Block cave mines demand a large capital investment for the development and construction of any production units. Planning of block-caving operations poses complexities in different areas. Generating a production schedule is one of these areas and has involved lots of responsiveness from the researchers during the past decades. Mine planning consists of defining the source, destination and extraction time of ore and waste during the life of mine. Production scheduling of any mining system has an enormous effect on the operation's economics. Nowadays, production scheduling is one of the key components in determining mine viability, because the mining industry faces lower grade and marginal reserves. In block caving projects, deviations from optimal mine plans may result in significant financial losses, future financial liabilities, resource sterilization, unbalanced cave subsidence, the flow of muck, and infrastructure instability.

A major aspect of mine planning is the optimization of long-term production scheduling. The aim of long-term production scheduling is to determine the time and sequence of extraction and displacement of ore and waste in order to maximize the overall discounted net revenue from a mine within the existing economic, technical and environmental constraints [2]. Thus, mining engineers are tasked to design and

A version of this paper has been submitted for publication in International Journal of Mining Science and Technology (IJMST).

optimize the resource recovery to ensure the maximum return on investment whilst still tempered by practical mining and geotechnical constraints [3].

The ore extraction rate and the incorporation of new areas are key operational parameters in block caving. They both control the cave back geometry which is mostly responsible for the induced stress state around productive levels [4]. Rubio [5] pointed out that block caving requires more detailed geotechnical investigations of the ore-body compare to other methods in which conventional drilling and blasting are employed as part of the mine production. Apprehending different operational and geotechnical situations is fundamental to preform and control caving. Geotechnical condition in the form of an exact production rate curve (PRC) controls the draw system. Caveability, in the context of draw control, is primarily concerned with balancing caving rates and production. The draw rate is technically related to the potential seismic activities and geotechnical hazards in caving [6].

Draw control is the bridge which connects production schedule and geomechanic features of the caving operations [7].

A strict planning system depends upon the production control and the ability to implement an effective draw strategy over the life of mine. Introducing an optimum draw control system based on mathematical programming that integrates system's constraints such as economic, environmental, operational, metallurgical, and geological, could result in a successful planning tool to be used by the mining industry. In this paper, a mixed-integer linear programming (MILP) formulation is developed to generate a realistic and practical production schedule. Presented MILP should not over- or under-estimate the value of the operation and have to solve models in a reasonable CPU time for a large-scale block-cave operation. It is intended to help companies to maximize economic outcomes with respect to geotechnical production rate curves. Such a model, which intends to work for real world conditions, must respond to all practical problems which might happen during extraction. This means that numerous constraints must be built into the model; consequently, the size of the MILP model increases substantially. The model must accommodate several decision variables over the life of mine. Solving large-scale problems is a challenge, it could be impossible to find the optimum solution or the solution time is not reasonable. Therefore, it is crucial to reduce the size of the problem using techniques in which the shrunk model can guarantee solution values with minimum deviations from the original model. As a result, this paper outlines an investigation into the application of the hierarchical clustering method for such oversized MILP model. The efficiency of the proposed algorithm is evaluated through a life of mine production scheduling case study with 2,487 drawpoints.

2. Literature review

This section has focused on the mathematical programming application in block-cave mining, and aggregation or clustering techniques for underground mining (notably block-cave projects). Some optimization models exist for production scheduling in block-cave mining, but the literature on draw rates, which is based on the PRC, is relatively new.

Chanda [8] used MIP for combining simulation to production scheduling in a block-caving operation. He concentrated on a short-term planning problem that covers a time horizon of few weeks to few months, applying single step optimization rather than multi-period optimization. This model did consider geometric constraints between drawpoints. Early models did not incorporate the variability and the dynamic behavior of the fundamental models throughout the mine and did not integrate the operational upsets that affect productivity because of lack of draw control planning constraint in models.

Rubio and Diering [9] noted that Chanda has not recognized the fact that the set of constraints is a function of the defined planning horizon. Guest et al. [10] assumed that by following a set of surfaces that conceptually define a draw control strategy, dilution can be minimized and therefore the net present value can be maximized. It should be noted that they stated the importance of the draw strategy on dilution control as part of the production scheduling process.

Rahal et al. [11] reviewed state-of-the-art in production schedule optimization and compared the complications related to caving. They noted that none of the available scheduling methodologies fully address complications associated with caving, in particular ore mixing and frequent loss of drawpoints. Additionally, Rahal [12] used a mixed-integer linear goal programming (MILGP) model in which the model had dual objectives of minimizing the sum of the production and external sources depletion simultaneously. This algorithm assumes that the optimal draw strategy is known. He developed life-of-mine draw profiles for estimated scenarios and showed that by using the results from their integer program, they greatly reduced deviation from ideal drawpoint depletion rates while adhering to a production target. Rahal considered the following constraints in his model: capacity, precedence, material handling, and maximum and minimum levels of draw rates. Rahal emphasized that the major outcomes from his research were a preliminary optimized life-of-mine production plan and the identification of areas where additional work can refine the parameters which were used in the optimization.

Diering [13] presented a non-linear programming optimization method to maximize NPV and minimize the deviation between a current draw profile and the target defined by the mine planner. Diering emphasizes that this algorithm could also be used to link the short-term with the long-term plan. The long-term plan is represented by a set of surfaces used as a target to be achieved based on the current extraction profile when running the short-term plans. Diering used only two boundaries for the draw rate constraint in the mathematical model.

Smoljanovic et al. [14] presented an MILP model to optimize the NPV in a panel cave mine to study the drawpoints' opening sequence. Their emphasis was in the precedence, geometrical, and production constraints. This model was a good starting point for panel or block caving, but the precedence and capacities constraints should be modified and other components need to be included to complete the set of constraints. They did not consider PRC for draw rate constraint.

Parkinson [15] developed three integer programming (IP) models: Basic, Malkin, and 2Cone, for finding the optimal opening sequence in an automated manner. All three models share three basic constraints. The start-once constraint ensures that each drawpoint is opened only once. The global capacity constraint ensures that the number of active drawpoints does not exceed the downstream-processing capacity. The big disadvantage of this research was the lack of draw rate constraint. Parkinson assumed a constant draw rate for the life of the mine.

Epstein et al. [16] presented and solved a MIP model with an objective function of maximizing the NPV that was successfully used in Chilean copper mines by Codelco for both underground and open-pit extraction. Their model uses drawpoint as the exploitation unit for underground operations. The drawpoint is represented as a column composed of a discrete number of blocks. In the model, the sequence is an input for a constraint in the model. Other constraints that appear are production capacity, maximum extraction rate per drawpoint, maximum allowed horizontal extraction per sector planted in terms of areas, not shapes, regularity in heights, and interaction with neighborhoods (considering interactive draw).

Alonso-Ayuso et al. [17] considered a MIP medium range planning problem for the El Teniente mine in Chile to maximize NPV by introducing the uncertainty problem. They presented a stochastic version of copper extraction planning problem under uncertainty in the (volatile) copper prices and used only some operational constraint. But they did not consider the draw control mechanism.

Khodayari and Pourrahimian [18] presented a comprehensive review of application of operations research in block-cave production scheduling. They summarized researchers' efforts of using mathematical programming for developing methodologies to optimize production schedules in block-caving operations. It was stated that more works need to be done on including geotechnical aspects of the operations, the uncertainty associated with material flow, and price uncertainties in the production scheduling. In addition, clustering methods was suggested as a solution for reducing the size of the problem in block-cave mining.

Rubio and Fuentes [4] described a simulation methodology to compute production schedule reliability and derive into the risk-return space such that mine planners can provide a portfolio of planning scenarios to decision makers. The construction of scenarios needs to be efficiently managed in order to cover the whole space of technical options feasible for a mine operation. Maximum tonnage that can be drawn from drawpoints based on the overall drawing strategy assumed as the objective function. They used a simple linear mathematical formulation for draw rate constraint. Khodayari and Pourrahimian [3] applied mixed-integer quadratic programming (MIQP) in the long-term scheduling of block-cave mines. Their aim was to minimize deviations of the production schedule from targets while operational constraints are satisfied.

Some of the methods which were presented above, did not incorporate, operational performance to adjust medium- and long-term plans because of loss of geotechnical rules in the modeling of actual draw management systems. Including the PRC constraint in the mathematical model can cause complexity and as a result a significant increase in the size of the problem.

Cluster analysis organizes data by abstracting underlying structure either as a grouping of individuals or as a hierarchy of groups. The representation can then be investigated to determine whether the data grouping is according to preconceived ideas or, if not, to suggest new experiments [19]. The hierarchical algorithm is a statistical method used to build clusters gradually based on the measured characteristics. It starts with each case in a separate cluster and then combines the cluster sequentially, reducing the number of clusters at each step, until one cluster is left. When there are N cases, the algorithm consists of N–1 clustering steps or fusions. In simple terms, a sequence of partitions of N samples into C clusters is considered. The first of these is a partition into N clusters, each containing exactly one sample. The next is a partition into N–1 clusters, and then into N–2, and so on [20].

In brief, clustering is defined as the process of grouping similar entities together so that maximum intra-cluster similarity and inter-cluster dissimilarity are achieved [20-22]. Clustering can be categorized into two major groups: hierarchical and partitional clustering.

Epstein et al. [23] used aggregation for underground block-sequencing operations and embedded it into an optimization-based heuristic. Weintraub et al. [24] considered a priori and a posteriori aggregation clustering methods based on a K-means algorithm to reduce the size of the MIP model which they had developed for El Teniente, a large Chilean block-caving mine. In establishing a method for measuring the dissimilarity between clusters, they considered a number of characteristics: tonnage, the copper grade, the molybdenum grade, and the rate of extraction. Each characteristic had a different importance, so, a set of weights associated with the characteristics was defined. Weintraub and his colleagues also identified a number of constraints that need to be satisfied: (1) each cluster can be extracted only once; (2) the defined sequence of extraction must be satisfied; (3) the allowable extraction rate and capacity of extraction must not be exceeded; and (4) flows and logical relationships between variables must be conserved. The objective function was maximizing the profit. The planning process was considered for a 25-year horizon. The authors noted that their aggregation procedure faced difficulties in defining aggregations and weights. Because each characteristic has different levels of importance, a set of weights associated with the characteristics was defined. This was done by mine planning experts. Draw rate as one of the important parameters was not established as a characteristic to measure the dissimilarity between clusters.

Newman and Kuchta [30] formulated an MIP model to schedule an iron-ore production over multiple time periods. To overcome the size of problem, they designed a heuristic approach based on solving a smaller, more tractable model. In this approach, they aggregated the time periods and then solved the original model using information gained from the aggregated model. They calculated an upper bound based on the difference between the original optimum solution and the restricted model. The non-aggregated models in Newman and Kuchta consist of 500 binary variables, while aggregated models consist of 260 binary variables [25].

Pourrahimian et al. [26, 27] applied an MILP model to develop a practical optimization framework for cave mining production scheduling. They presented a multi-step method for long-term production scheduling of block caving. To overcome the problem about the size of mathematical programming models and to generate a robust practical near-optimal schedule, they used a hierarchical clustering method. Their model aims to maximize the NPV of the mining operation at three different levels of resolution: (i) aggregated drawpoints (cluster level); (ii) drawpoint level; and (iii) drawpoint-and-slice level. Their model extracts material from drawpoints within an acceptable rate, however, it does not consider the geotechnical properties of the rock mass through the draw rate constraint.

One of the shortcomings of these methods is their dependency on the definition of similarity and their high sensitivity to the weights used in determining similarity. The proposed clustering algorithm in this paper is based on a hierarchical approach and is specifically developed to be used in solving block-cave mine production scheduling problem.

3. Clustering algorithm

Clustering is defined as the process of grouping similar objects together in a way that maximum intra-cluster similarity and inter-cluster dissimilarity are achieved. Hierarchical clustering procedures are among the best known statistical methods of clustering.

Clustering is applied in planning programs due to its low computational effort requirement and the reasonable quality of the solutions that generates [6]. After applying clustering in the MILP model, the percentage error in the value of the objective function must be negligible and the reduction of solving time must be significant when comparing the original model with the new clustered model. Determining the similarity index for grouping objects in clusters is the main key in clustering algorithms. However, the similarity of constituent items is not always the sole factor in determining the groups. One can name situations in which generated clusters have to satisfy some constraints, such as mutually exclusive and inclusive objects, minimum and maximum cluster sizes, and constraints on the cluster shapes. Although it is possible to form a mathematical model for finding the optimum clustering scheme and add different constraints to the formulations, the clustering problem has been proven to be an NP-Hard problem [4, 5].

Hierarchical clustering can be divided into two distinct classes; agglomerative and divisive. Agglomerative (bottom up, clumping) procedures start with n singleton clusters and form the sequence by successively merging clusters. Divisive (top down, splitting) procedures start with all of the samples in one cluster and form the sequence by successively splitting clusters. The computation required to go from one level to another is simpler for the agglomerative procedures than the divisive one [20]. Aggregation techniques are highly dependent on the structure of the problem, and, in general, are tailored specifically for a class of problems or even for a specific instance of a problem [25]. Clustering reduces the number of variables, especially binary variables in the MILP formulation, to make the formulation computationally tractable.

In this study, a multi-step approach is developed to generate aggregates with respect to the direction of mining and control the shape and size of the generated aggregates. The methodology can determine the best order of extraction of material from the clusters by maximizing the net present value (NPV) of the operation over the life of mine. In the clustering method it has been assumed that the scheduled portion to be extracted from each cluster is taken from all of the drawpoints, based on the ratio of each drawpoint's tonnage in the cluster. The maximum draw rate ($MaxDR_i$) of drawpoint *i* is calculated from the PRC by considering depletion percentage (M). Equation (1) represents the maximum draw rate of drawpoint response to the PRC:

$$MaxDR_i = (M - \frac{MinDR_i}{Ton_i}) \times Ton_i \quad (1)$$

Where $MinDR_i$ is the minimum draw rate of drawpoint i which is a given value for mines according to the geotechnical properties of rock mass and Ton_i is the overall tonnage of draw column associated with drawpoint i.

Bartlet [28] recommends starting the caving where the weak rock is located so that the hydraulic ratio can be reached earlier in the life of the mine and the time required to recover the investment is shortened.

Another method, which consists of starting where the high-grade ore is located, leads to early payback of the investment or higher NPV [29]. The direction of undercut developing into the principal stress direction will influence the magnitude of abutment stresses. Therefore, to reduce clamping stresses in the cave back, the undercuts are usually extracted in the direction of the maximum principal stress [30].

Considering the direction of advancement in forming the clusters is a key strategy when dealing with economical or geotechnical problems. It is essential to develop a direction factor to be included in the similarity index and account for the advancement direction. For this purpose, the method by Tabesh and Askari-Nasab was modified for its application in block-cave mining [31]. According to the Tabesh and Askari-Nasab method, after determining the advancement direction, the planner should define two points at the starting and ending point in the direction of advancement. Afterward, the direction factor can be calculated using equation (2).

$$N_i = sign\left((N_i^1)^2 - (N_i^2)^2\right) \times \sqrt{\left|(N_i^1)^2 - (N_i^2)^2\right|} \tag{2}$$

Where N_i^1 and N_i^2 are the distance from drawpoint i to start and end points respectively. The sign function returns +1 if the value is positive and -1 if the value is negative.

3.1. Multi-similarity index

Aggregation procedure needs a similarity measure or similarity index that quantifies the similarity between two objects. Various properties can be taken into account when defining similarities between draw columns. In this section, a multi-similarity index clustering algorithm is developed for draw column aggregation. The multi-similarity indices are used to remove the similarity index dependency for the weight factors that are defined by the planner in the existing models.

Increasing the number of properties engaged in similarity calculations increases the complexity of the index, in terms of not representing a unique physical attribute. Multi-similarity indexes aggregate the draw columns into clusters based on center-by-center distance, grade distribution, maximum draw rate according to the PRC, and advancement direction. The general procedure of proposed algorithm is as follows:

1. Define the number of required similarity indexes according to the mining operation.
2. Define a search radius.
3. Each draw column is considered as a cluster. The similarities between clusters are the same as the similarities between the objects they contain in each index.
4. Define the maximum number of required clusters and the maximum number of allowed draw columns within each cluster for each index.
5. Similarity values are calculated for the considered similarity index.
6. The most similar pair of clusters is merged into a single cluster.
7. The similarity between the new clusters and the rest of the clusters is calculated. Steps 3 to 6 are repeated until the maximum number of clusters is reached or there is no pair of clusters to merge because the maximum number of allowed draw columns has been reached.
8. For the next similarity index, define an intra-cluster adjacency matrix for draw columns that are located within two different clusters.
9. Repeat steps 3 to 7 for the similarity index defined in step 8.

For the first step, the similarity index is calculated based on the distance and the most similar pair of clusters is merged into a single cluster (equation (3)):

$$SI_1 = \frac{1}{D_{ij} \times N_{ij\text{-}SI_1}} \times A_{ij} \tag{3}$$

Where SI_1 is the similarity index of step 1 (distance), $\frac{1}{D_{ij} \times N_{ij\text{-}SI_1}}$ is the similarity value between draw columns i and j, D_{ij} is the normalized distance value between the center line of draw columns i and j, $N_{ij\text{-}SI_1}$ is the normalized Euclidean distance between values N_i and N_j for SI_1, and A_{ij} is the adjacency factor between draw columns i and j. If the distance is less than the defined search radius, A_{ij} is 1; otherwise, it is 0. The similarity between the new clusters and the rest of clusters is calculated. After calculating the similarity, the mentioned steps are repeated until the maximum number of clusters is reached or there is no pair of clusters to be merged, because the maximum number of allowed draw columns has been reached.

For the second step, similarity based on the maximum allowable draw rate, an intra-cluster adjacency matrix for draw columns that are located within two different clusters is required (equation (4)):

$$SI_2 = SI_1 \times \frac{1}{MaxDR_{ij} \times N_{ij\text{-}SI_2}} \times ISA_{ij} \tag{4}$$

Where $SI_1 \times \frac{1}{MaxDR_{ij} \times N_{ij-SI_2}}$ is the similarity value between draw columns i and j, and ISA_{ij} is the second-inter-cluster adjacency factor between draw columns i and j. If draw columns i and j are in the same cluster as they were in the first step, ISA_{ij} is 1; otherwise, it is 0.

In the third step, similarity is calculated based on the grade of draw columns (equation (5)):

$$SI_3 = SI_1 \times \frac{1}{Grade_{ij} \times N_{ij\text{-}SI_3}} \times ITA_{ij} \tag{5}$$

Where $SI_1 \times \frac{1}{Grade_{ij} \times N_{ij-SI_3}}$ is the similarity value between draw columns i and j, $Grade_{ij}$ is the normalized grade difference between draw columns i and j, and ITA_{ij} is the third-intra-cluster adjacency factor between draw columns i and j. If draw columns i and j are in the same cluster as they were in the second step, ITA_{ij} is 1; otherwise, it is 0. This algorithm controls practical cave advancement.

4. MILP model

Developing any model requires some decision variables, sets, indices, and parameters that correspond to a scheduling program. Indices are for drawpoints and periods. The model identifies two types of variables: (i) continuous and (ii) binary. $U_{cl,t} \in [0,1]$ is the portion of cluster cl to be extracted in period t. The binary variable indicates the time when the cluster is opened, active, or in any working states during a specified time period. $A_{cl,t} \in \{0,1\}$ *is* equal to 1 if cluster cl is active in period t; otherwise it is 0. $Z_{cl,t} \in \{0,1\}$ *is* controlling the precedence of extraction of clusters. It is equal to 1 if extraction from cluster cl is started in period t; otherwise, it is 0. $cl \in \{0,CL\}$ is an index for clusters, $t \in \{0,T\}$ is an index for scheduling periods, and l is an index for a cluster belonging to set S_{cl}. For each cluster, cl, there is a set S_{cl} defining the predecessor clusters that must be started prior to extraction of cluster cl.

Mixed-integer linear programming (MILP) formulation presented in this paper maximizes the NPV subject to various operational and geotechnical constraints. The model is developed in MATLAB [32] and solved using IBM/CPLEX [33]. The mathematical equations are presented as follows:

$$Maximize \sum_{cl=1}^{CL} \sum_{t=1}^{T} \left[\frac{\mathrm{Re}_{cl}}{(1+i)^t} \right] \times U_{cl,t} \tag{6}$$

$$M_l \leq \sum_{cl=1}^{CL} (Ton_{cl}) \times U_{cl,t} \leq M_u \tag{7}$$

$$\sum_{cl=1}^{CL} (Ton_{cl} \times (G_{l,cl,t} - \widetilde{G}_{cl,t}) \times U_{cl,t} \leq 0 \tag{8}$$

$$\sum_{cl=1}^{CL} (Ton_{cl} \times (\widetilde{G}_{cl,t} - G_{u,cl,t}) \times U_{cl,t} \leq 0 \tag{9}$$

$$A_{cl,t} \leq L.U_{cl,t} \tag{10}$$

$$U_{cl,t} \leq A_{cl,t} \tag{11}$$

$$\sum_{cl=1}^{CL} A_{cl,t} \leq N_{Acl,t} \tag{12}$$

$$Z_{\mathrm{cl},t} - \sum_{j=1}^{t} Z_{l,j} \leq 0 \tag{13}$$

$$\sum_{t=1}^{T} Z_{\mathrm{cl},t} = 1 \tag{14}$$

$$A_{\mathrm{cl},t} - A_{\mathrm{cl},(t-1)} \leq Z_{\mathrm{cl},t} \tag{15}$$

$$A_{\mathrm{cl},1} = Z_{\mathrm{cl},1} \tag{16}$$

$$N_{Nl,\mathrm{cl},t} \leq \sum_{\mathrm{cl}=1}^{CL} Z_{\mathrm{cl},t} \leq N_{Nu,\mathrm{cl},t} \tag{17}$$

$$\sum_{\mathrm{cl}=1}^{CL} Z_{\mathrm{cl},1} \leq N_{A\,\mathrm{cl},1} \tag{18}$$

$$\sum_{t=1}^{T} U_{\mathrm{cl},t} \leq 1 \tag{19}$$

$$\sum_{t=1}^{T} A_{\mathrm{cl},t} \leq Max_{Life-Activity} \tag{20}$$

$$U_{\mathrm{cl},(t+m)} \times Ton_{\mathrm{cl}} - \left(\frac{(DR_{Ramp,\mathrm{up},T} - DR_{l,\mathrm{cl},t})}{1 - \frac{DR_{l,\mathrm{cl},t}}{Ton_{\mathrm{cl}}}} \right) \times \sum_{t=1}^{t+m} U_{\mathrm{cl},(t+m)} \leq DR_{l,\mathrm{cl},t} - \left(\frac{(DR_{Ramp,\mathrm{up},T} - DR_{l,\mathrm{cl},t})}{1 - \frac{DR_{l,\mathrm{cl},t}}{Ton_{\mathrm{cl}}}} \right) \times \frac{DR_{l,\mathrm{cl},t}}{Ton_{\mathrm{cl}}} \tag{21}$$

$$U_{Steady,T} \times Ton_{\mathrm{cl}} \leq DR_{u,\mathrm{cl},t} \tag{22}$$

$$U_{\mathrm{cl},(t+m)} \times Ton_{\mathrm{cl}} + \left(\frac{(DR_{Ramp,down,T} - DR_{l,\mathrm{cl},t})}{1 - \frac{DR_{l,\mathrm{cl},t}}{Ton_{\mathrm{cl}}}} \right) \times \sum_{t=1}^{t+m} U_{\mathrm{cl},(t+m)} \le DR_{Ramp,down,T} + \left(\frac{(DR_{Ramp,down,T} - DR_{l,\mathrm{cl},t})}{1 - \frac{DR_{l,\mathrm{cl},t}}{Ton_{\mathrm{cl}}}} \right) \times \frac{DR_{l,\mathrm{cl},t}}{Ton_{\mathrm{cl}}} \tag{23}$$

$$DR_{l,\mathrm{cl},t} \times Z_{\mathrm{cl},t} - Ton_{\mathrm{cl}} \times U_{\mathrm{cl},t} \le 0 \tag{24}$$

The objective function, equation (6), is composed of the economic value of the cluster, the continuous decision variable, and the discount rate.

Knowledge, experiments of mine planners, and corresponding planning horizons have a critical role in the process of assigning constraints to the optimization problems. These constraints appear in several different forms: geotechnical, grades, period, advancement direction, and priority of the clusters, productivity, production rates, and many others that depend on the mining method.

Equation (7) ensures that the total tonnage of material extracted from active clusters in each period is within an acceptable range that allows flexibility for potential operational variations. Where Ton_{cl} is the total tonnage of material within the cluster *cl*, M_u and M_l are the upper and lower limits of mining capacity in period *t*, respectively.

Equations (8) and (9) force the mining system to achieve the desired production grade. The average grade of the element of interest has to be within the acceptable range. Where $\overline{G}_{cl,t}$ is the average grade of the cluster *cl*, and $G_{u,cl,t}$ and $G_{l,cl,t}$ represent the upper and lower limits of the acceptable average head grade of cluster *cl* in period *t*.

According to Equations (10), (11) , and (12) the number of active clusters must not exceed the allowable number in each period and has to be constrained according to the size of the ore-body and the available infrastructure and equipment. Where $N_{Acl,t}$ is the maximum allowable number of active clusters in period *t*. The precedence between clusters is controlled in a horizontal direction. Controlling the order of extraction of clusters in an advancement direction is the goal of the precedence constraint. According to the advancement direction, for each cluster *cl* there is a set S^{cl} which defines the predecessor clusters among adjacent clusters that must be started before cluster *cl* is extracted. Equation (13) controls the precedence of extraction. In equation (13), *l* is the index for a cluster belonging to set S^{cl}, $Z_{l,t}$ ensures that all clusters belonging to set S^{cl} are started before or in period *t*, if cluster *cl* is started in period *t*.

Equations (14) and (15) force the mining system to extract material from each cluster continuously after opening until closing. Equation (16) is only used for period 1. Equations (17) and (18) control the number of new clusters to be opened at any given time within the scheduled horizon. This parameter is determined based on the footprint geometry, the geotechnical behavior of the rock mass, and the existing infrastructure of the mine. $N_{Nl,cl,t}$ and $N_{Nu,cl,t}$ are the lower and upper limits for the number of new clusters, the extraction from which can start in period *t*. The number of new clusters that should be opened in period 1 is equal to the number of active clusters. Equation (19) ensures that the model cannot extract more than the available material in each cluster.

Maximum activity life, controls the number of periods in which the cluster is active. This constraint ensures that the extraction rate from the cluster is large enough to maximize the NPV and small enough to prevent over-dilution. Equation (20) indirectly affects the draw rate by controlling the number of activity periods of any cluster. Greater activity life results in higher recompaction and dilution. $Max_{Life-Activity}$ is the maximum number of allowable periods in which any cluster can be active.

Draw control in a caving operation means optimizing the extraction from drawpoints in order to achieve the production goals while minimizing the dilution. In addition, the draw rate controls the distribution of the induced stresses in the caving environment [34].

Production rates are specified as tonnes per square meter per day per drawpoint. The changes in draw rate are normally classified as a drawpoint opening to ramp up production, steady state production, and ramp down to closure. Such pattern named as ramp up-steady-ramp down (USD) (see Fig 1). The USD model is best because the low draw rate in the early years of the life of drawpoints is caused by the geotechnical characteristics of the surrounding environment aligned with the caved material, and in fact, increasing stress caused by draw and caving process relaxes with a relatively soft tendency. Therefore, appropriate management of stress relaxation can be done. The main goal of draw rate formulation is to find the draw rate of each drawpoint in each period during the optimization of scheduling based on the defined objective function, constraint, and PRC.

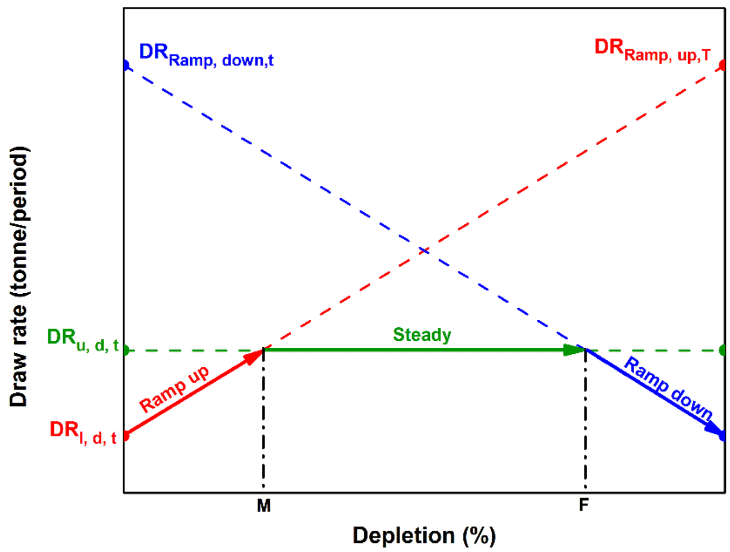


Fig 1. PRC for the draw rate control system

It should be noted that the PRC of each cluster is determined based on the number of drawpoints and the PRC of the drawpoints within the cluster. The problem can be formulated for the given PRC Based on the equations (21), (22) and (23). $DR_{u,cl,t}$ and $DR_{l,cl,t}$ are the maximum and minimum possible draw rate of cluster cl in period t, M is the maximum allowable depletion to reach $DR_{u,cl,t}$, F is maximum allowable depletion to reach ramp-dawn region after steady production, U_t is the percentage of depletion at the first period extraction of the cluster Draw control in a caving operations means optimizing the extraction from drawpoints in order to achieve the production goals while minimizing the dilution. In addition, the draw rate controls the distribution of the induced stresses in the caving environment [34]. $U_{d,(t+m)}$ is the percentage of total depletion after m period of extraction, $DR_{Ramp.up,t}$ is the draw rate with full depletion for ramp up states, and $DR_{Ramp,down,t}$ is the draw rate with full depletion for ramp down states. Equation (21) shows the mathematical structure for the area under the ramp up region. Also, equations (22) and (23) are related to the steady and ramp down regions respectively.

5. Illustrative Example

The production schedule of 2,487 draw points according to defined PRC and cluster approach is investigated in this section. The total tonnage of material is 803.91 (Mt) with an average density of 2.2 (t/m^3) and an average grade of 0.36%Cu. Fig 2 illustrates the tonnage and grade distribution of the draw columns. The performance of the proposed MILP models was analyzed based on maximizing the NPV at a discount rate of 12%. The model was tested using a Dell Precision T7600 computer with Intel(R) Xeon(R) at 2.3 GHz, with 64 GB of RAM. The maximum depletion of the drawpoints from the ramp up to steady (M) and steady to the ramp down (F) were 40% and 85%, respectively. The draw control system, by enrolling an exact production rate curve seeks to optimize and present a practical block-cave production schedule. A gap tolerance (EPGAP) of 5% was used as an optimization termination criterion. The other scheduling parameters have been summarized in **Error! Reference source not found.**.

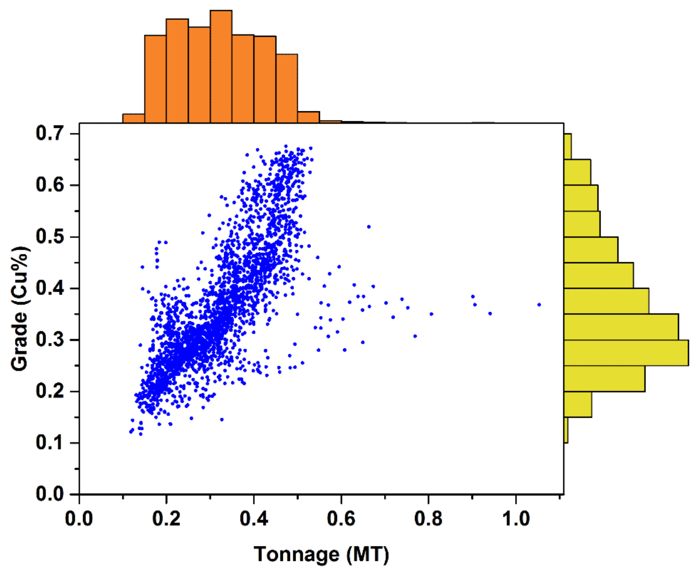
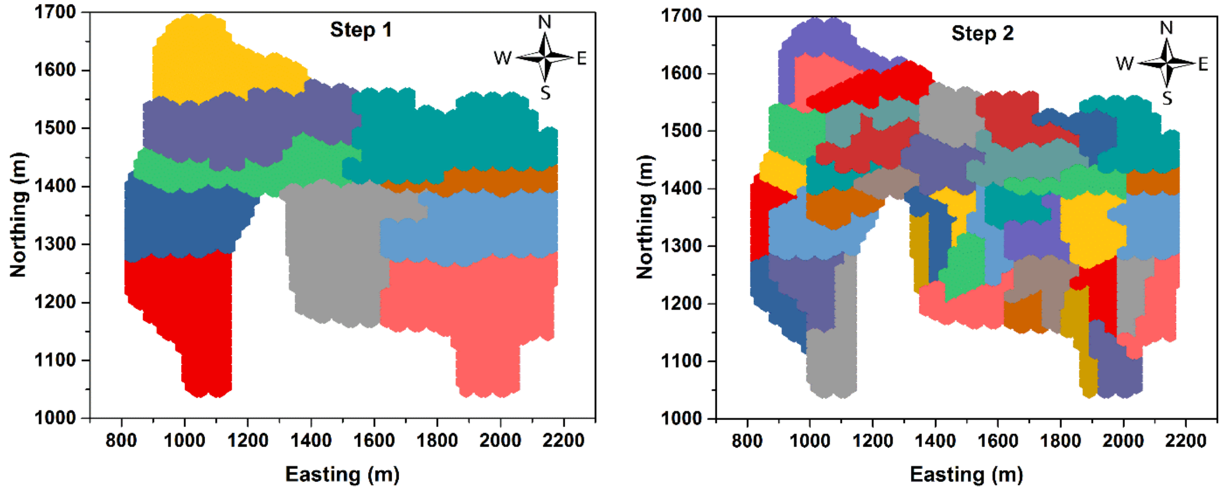


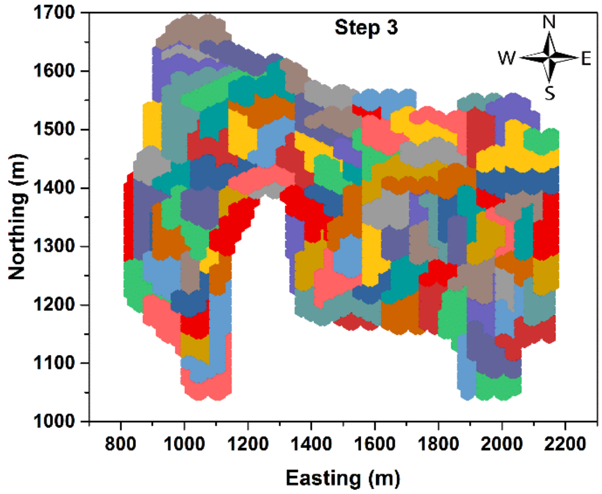
Fig 2. Tonnage and grade distributions of draw columns

Table 1. Production scheduling parameters

Parameters	Value	
Maximum activity (periods)	5	
Mining capacity (Mt)	15 – 27.5	
Draw rate of draw columns (kt/period)	30 – 100	
No. of new clusters per period	0 – 11	
Production grade (%Cu)	0.3 – 0.6	
No. of maximum active clusters per period	25	
Max. number of clusters	First step	10
	Second step	40
	Third step	109
Max. number of draw columns	First step	350
	Second step	80
	Third step	25
Adjacency radius (m)		22

Fig 3 shows the proposed clustering method for 2,487 drawpoints. The clustering was done in three steps. These steps are based on (i) the distance between drawpoints in the advancement direction, (ii) draw rate of drawpoints, and (iii) grade of draw columns. The advancement direction was determined based on the method presented by Khodayari and Pourrahimian [35]. The advancement direction is from south to north. The maximum number of clusters which was defined in the first, second, and third step were 10, 40, and 109 respectively.





Step	# of clusters	Clustering based on
1	10	Distance
2	40	Draw rate
3	109	Grade

Fig 3. Application of the multi-index clustering method for 2,478 draw columns

The problem was modeled both with and without clustering approach. The total number of constraints in the model without clustering was 610,548. The numbers of continuous and binary variables were 79,296 and 158,592 respectively. This model did not reach a solution after being ran for 15 days. On the other hand, the model with the clusters was built on 30,516 constraints, and the number of continuous and binary variables was 3,488 and 6,976 respectively. Using the multi-similarity index aggregation system resulted in 95% reduction in the number of binary variables. The cluster model was solved in 37(hr):48(min):11(sec). 761.87Mt ore was extracted during 32 years of production with the NPV of $304.6 B.

Fig 4 shows the cash flow, the production tonnage, and average grade of production in each period for the cluster model. The ramp up and down for the total production is achieved in the resulted production plan.

Fig 5 shows the grade distribution of the clusters. Fig 6 illustrates the starting period of the clusters in the cluster model during the life of the mine. The start period of drawpoints shows the advancement direction of caving has been achieved within in the optimization. To follow the defined sequence of extraction, the high-grade clusters are extracted during periods 15 to 21 (Fig 6). As a result, the grade of production increases during that period of time (Fig 4). Because of the application of the production rate curve for draw control, it is expected to have less dilution during the life of mine.

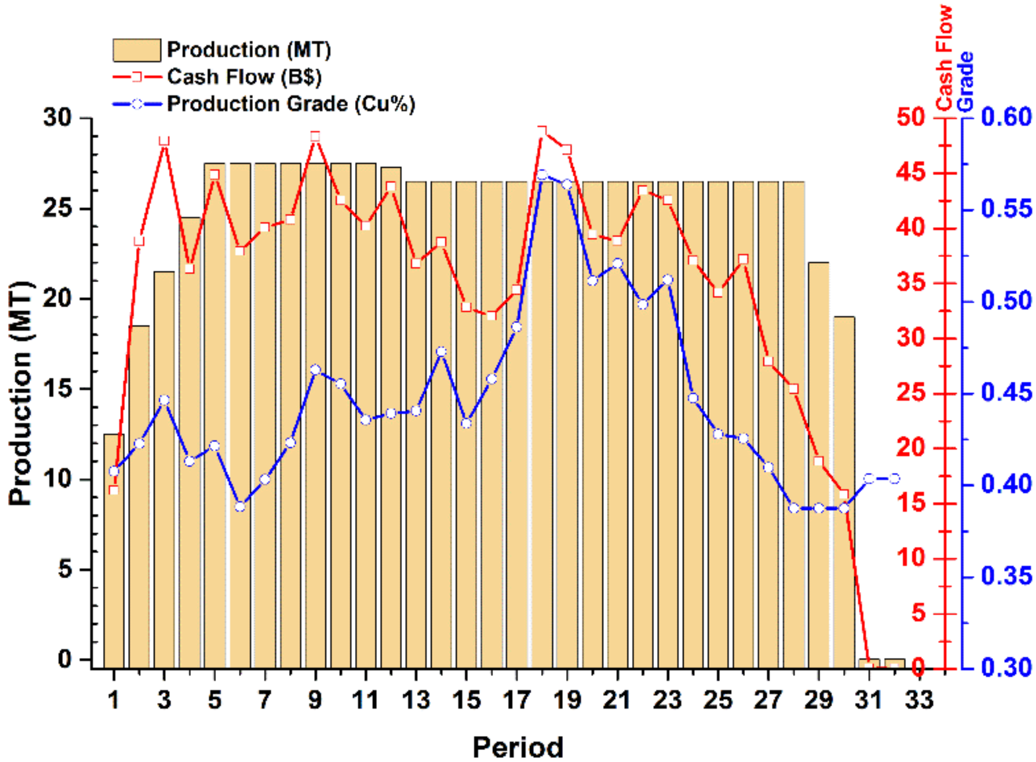


Fig 4. Cash flow, yearly production and average grade of production in the cluster model

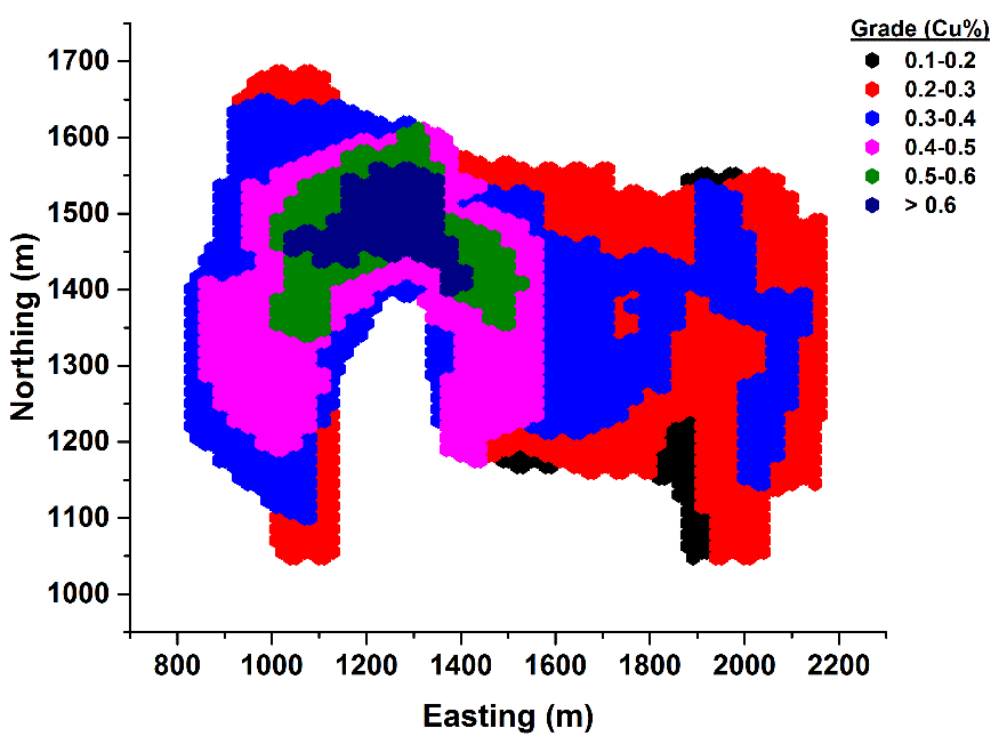


Fig 5. Grade distribution in the cluster model

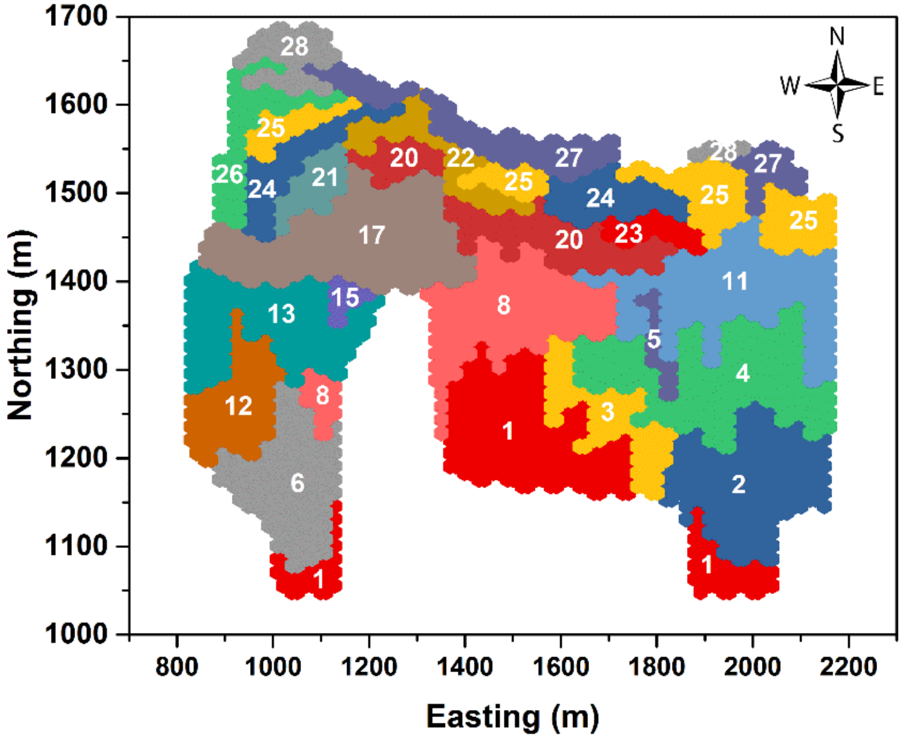


Fig 6. Starting period of the different areas in the mine over the life of mine

Fig 7 shows the maximum number of active clusters and number of new clusters which had to be opened in each period. The number of active clusters in period one is equal to the number of new clusters which opened. From periods 12 to 15, this number gradually reduces. The number of new clusters opened in period one could be equal to the maximum allowable number of active clusters to reach the required production in this period. There was no need to open new clusters in periods 7, 9, 10, 14, 16, 18, 19, 20, 30, 31, and 32.

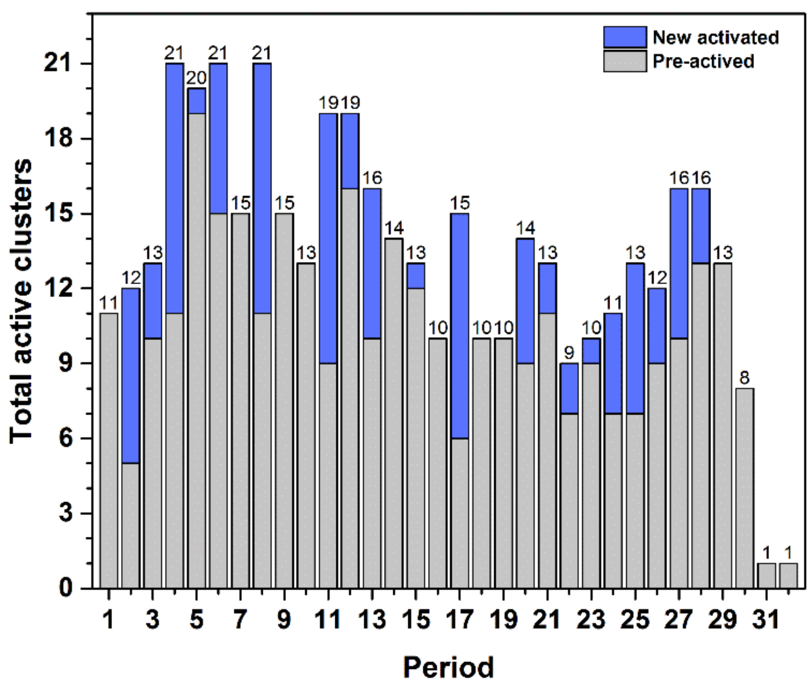


Fig 7. Number of active and new clusters in the model

Fig 8 presents the draw rate changes for three different clusters in the cluster model. According to the defined PRC, clusters cannot be depleted arbitrarily. On the other hand, the material with a lower economic value and grade can remain in the clusters because of the defined objective function and the constraints. The minimum and maximum draw rate of each drawpoint are 30,000 (kt/period) and 100,000 (kt/period), respectively. The draw rate of each cluster varies based on the number of draw columns in the cluster. In Fig 8, it can be seen that extraction from cluster 29 is started in period 25 with the minimum acceptable draw rate, then it gradually increases to reach the maximum acceptable draw rate in period 27, the steady state with the maximum draw rate continues for two periods, and finally the draw rate reduces till it is closed. Cluster 29 contains 30 drawpoints, so the minimum and maximum draw rates for this cluster are 900 and 3000 (kt/period), respectively.

The results show that all the defined constraints have been satisfied. The draw rates for each cluster and starting and finishing periods are the outputs of the optimization. The model extracts the material from each cluster based on the defined draw rate model while maximizing the NPV of the operation.

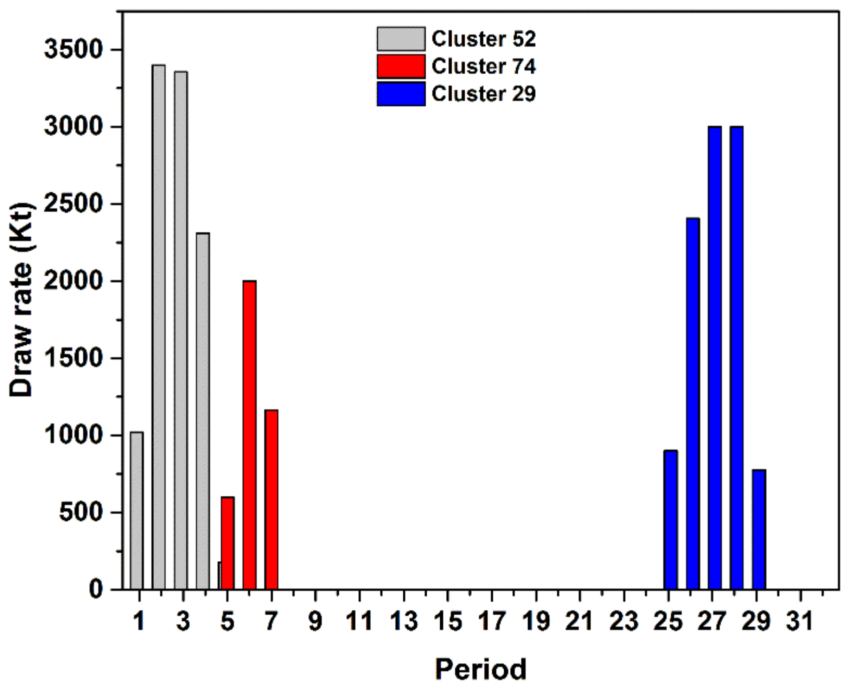


Fig 8. Draw rate of clusters 29, 52, and 74

6. Conclusion

This paper presented a multi-similarity index aggregation approach for the oversize MILP models. The proposed model maximizes the NPV subject to operational and geotechnical constraints. The MILP formulation for a block-cave production schedule was developed, implemented, and tested in IBM/CPLEX environment. A practical draw control system was considered in the optimization model based on PRC in order to manage draw rates of block-cave operations. Because of the size of the problem, the model without clustering could not solved in a reasonable time, therefore, the cluster model was implemented. The clustering techniques resulted in 95% of the reduction in the number of binary variables which made it possible to solve the same problem in an acceptable processing time. This solution time would enable the mine planner to analyze different scenarios during the feasibility studies. The presented aggregation approach eliminates dependency to the weight factor in clustering technique, as a result, human errors will not affect the optimality of the production schedule. Using the presented clustering approach and the MILP formulation, production schedule of large-scale block cave operations can be optimized during the life of mine.

7. References

[1] G.P. Chitombo, *Cave mining: 16 years after Laubscher's 1994 paper 'Cave mining - state of the art'*, Mining Technology 119 (2010), pp. 132-141.

[2] E. Ben-Awuah, O. Richter, T. Arlington, and Y. Pourrahimian, *Strategic mine options optimization: open pit mining, underground mining or both*, International Journal of Mining Science and Technology 26 (2016), pp. 1065-1071.

[3] F. Khodayari, and Y. Pourrahimian, *Quadratic programming application in block-cave mining*, 1st International Conference on underground Mining (Umining 2016), Santiago, Chile, (2016), pp. 427-438.

[4] E.Rubio, and M. Fuentes, *A portfolio Approach for Mine Planning under Uncertainty*, 1st International Conference on underground Mining (Umining 2016), Santiago, Chile, (2016), pp. 496-407.

[5] E. Rubio, *Block cave mine infrastructure reliability applied to production planning* 2006.

[6] J. Díaz, Y. Y Sepúlveda, and P. P Lledó, *Geomechanical ground control in block/panel caving,* Caving Santiago, Chile, 2014.

[7] M.L. Smith, and D. Rahal, *Draw Control Optimisation in the Context of Production Scheduling*, 17th International Mining Congress and Exhibition of Turkey- IMCET2001 (2001), pp. 831-838.

[8] E.C.K. Chanda, *An application of integer programming and simulation to production planning for a stratiform ore body*, Mining Science and Technology 11 (1990), pp. 165-172.

[9] *block cave production planning using operation research tool*, Gemcom.

[10] A.R. Guest, G.J. Van Hout, and A. Von Johannides, *An Application of Linear Programming for Block Cave Draw Control,* MassMin, Brisbane, Australia, 2000.

[11] D. Rahal, M. Smith, G. Van Hout, and A. Von Johannides, *The use of mixed integer linear programming for long-term scheduling in block caving mines*, Application of Computers and Operations Research ill the Minerals Industries, South African Institute of Mining and Metallurgy (2003), pp. 123-132.

[12] D. Rahal, *Draw Control in Block Caving Using Mixed Integer Linear Programming*. PhD The University of Queensland, 2008.

[13] T. Diering, *Combining Long Term Scheduling and Daily Draw Control for Block Cave Mines,* Massmin, Santiago, Chile, 2004.

[14] M. Smoljanovic, E. Rubio, and N. Morales, *Panel Caving Scheduling Under Precedence Constraints Considering Mining System,* 35th APCOM Symposium, Wollongong, NSW, Australia, 2011.

[15] A. Parkinson, *Essays on Sequence Optimization in Block Cave Mining and Inventory Policies with Two Delivery Sizes*. PhD, The University Of British Columbia, 2012.

[16] R. Epstein, M. Goic, A. Weintraub, J. Catalán, P. Santibáñez, R. Urrutia, R. Cancino, S. Gaete, A. Aguayo, and F. Caro, *Optimizing Long-Term Production Plans in Underground and Open-Pit Copper Mines*, Operations Research 60 (2012), pp. 4-17.

[17] A. Alonso-Ayuso, F. Carvallo, L.F. Escudero, M. Guignard, J. Pi, R. Puranmalka, and A. Weintraub, *Medium range optimization of copper extraction planning under uncertainty in future copper prices*, European Journal of Operational Research 233 (2014), pp. 711-726.

[18] F. Khodayari, and Y. Pourrahimian, *Mathematical programming applications in block-caving scheduling: a review of models and algorithms*, International Journal of Mining and Mineral Engineering (IJMME) 6 (2015), pp. 234-257.

[19] P. Andritsos, *Data Clustering Techniques*, University of Toronto, Department of Computer Science, 2002.

[20] D.C. Panigrahi, and H.B. Sahu, *Application of hierarchical clustering for classification of coal seams with respect to their proneness to spontaneous heating*, Mining Technology 103 (2004), pp. 97-106.

[21] M. M. Tabesh, and H. Askari-Nasab, *Two-stage clustering algorithm for block aggregation in open pit mines*, Mining Technology 120 (2011), pp. 158-169.

[22] Y. Pourrahimian, *Mathematical programming for sequence optimization in block cave mining*. NR92565 Ph.D., University of Alberta (Canada), 2013.

[23] R. Epstein, S. Gaete, F. Caro, A. Weintraub, P. Santibañez, and J. Catalan, *Optimizing long term planning for underground copper mines,* 5th International conference Proceedings of Copper 2003-Cobre 2003, , Santiago, 2003.

[24] A. Weintraub, M. Pereira, and X. Schultz, *A Priori and A Posteriori Aggregation Procedures to Reduce Model Size in MIP Mine Planning Models*, Electronic Notes in Discrete Mathematics 30 (2008), pp. 297–302.

[25] A.M. Newman, and M. Kuchta, *Using aggregation to optimize long-term production planning at an underground mine*, European Journal of Operational Research 176 (2007), pp. 1205-1218.

[26] Y. Pourrahimian, H. Askari-Nasab, and D. Tannant, *A multi-step approach for block-cave production scheduling optimization*, International Journal of Mining Science and Technology 23 (2013), pp. 739-750.
[27] Y. Pourrahimian, and H. Hooman Askari-Nasab, *An application of mathematical programming to determine the best height of draw in block-cave sequence optimisation*, Mining Technology (Trans. Inst. Min. Metall. A) 123 (2014), pp. 162-172.
[28] P.J. Bartlett, *The Design and Operation of a Mechanized Cave at Premier Diamond Mine,* Massmin, Johannesburg, SAIMM, 1992.
[29] E. Rubio, *Long term planning of block caving operations using mathematical programming tools*. Master of Applied Science, The University of British Columbia, 2002.
[30] D.H. Laubscher, *Block caving manual (prepared for International Caving Study)*, JKMRC and Itasca Consulting Group, Inc: Brisbane, 2000.
[31] M. Tabesh, and H. Askari-Nasab, *Automatic Creation of Mining Polygons Using Hierarchical Clustering Techniques*, Mining Science 49 (2013), pp. 426-440.
[32] *MathWorksInc. MATLAB (R2016a) (2016) 'Ver. 9.0.0.341360, MathWorks, Inc.'.*
[33] *IBM ILOG,ILOG CPLEX, ILOG, Inc.*, Sunnyvale, CA, USA 2015.
[34] C.A. Preece, and B. Liebenberg, *Cave management at Finsch Mine*, The Journal of The Southern African Institute of Mining and Metallurgy 107 (2007), pp. 775-781.
[35] F. Khodayari, and Y. Pourrahimian, *Determination of development precedence for drawpoints in block-cave mining,* 5th International Symposium Mineral Resources and Mine Development (AIMS 2015), Aachen, Germany, 2015.