An Application of Mathematical Programming for Conditional Draw Control Modeling in Block-Cave Mining

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ABSTRACT

Block caving is a complex and large-scale mining method. The application of block caving is for low-grade, caveable, and massive ore-bodies. Among the underground mining methods available, caving methods are favoured because of their low cost and high level of production. Generating a production schedule that will provide optimal operating strategies without geotechnic constraints is not practical in block caving. Some complex theories and mathematical draw control systems have been applied in block-cave mines. However, most did not use an exact production rate curve and depletion conditions among the draw column to manage draw rates of drawpoints in the caved area. Establishing relationships among draw columns to consider depletion rates of other draw columns is complex but essential to provide a reasonable solution for real block-caving mines. This paper presents a mixed-integer linear programming (MILP) model to optimize the extraction sequence of drawpoints over multiple time horizons of block-cave mines with respect to the draw control systems. Four resolutions are formulated in this paper to guarantee practical solutions with respect to draw control managing in mined areas according to the draw rate and conditional draw rate constraints. Dilution and caving are improved indirectly, because the method considers the draw rate strategies. Application and comparison of the four resolutions for production scheduling based on the draw control systems are presented using 325 drawpoints over 15 periods.

1. Introduction

Among the underground mining methods available, caving methods are favoured because of their low cost and high level of production. Block-caving operations generate a much smaller environmental footprint than equivalent open-pit operations because the volume of waste that needs to be moved and handled is much smaller.

Rubio [1] mentioned that block caving is a technique in which gravity is used in conjunction with internal rock stresses to fracture and break the rock mass into pieces that can be handled by miners. Rubio pointed out that block caving requires more detailed geotechnical investigations of the ore-body than do other methods in which conventional drilling and blasting are employed as part of the mine production. Apprehending different operational and geotechnical situations is fundamental to preform and control caving. Geotechnical condition in the form of an exact production rate curve (PRC) controls the draw system. Caveability, in the context of draw control, is primarily concerned with balancing caving rates and production. If draw rates are not controlled, either air gaps or damaging stress concentrations may occur. Stress is important because undercut advance rates must be maintained to prevent stress damage to the production level. Draw must also be maintained across the production level to ensure that local stress

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concentrations and premature dilution entry do not occur [2]. The draw rate is a parameter that is related to the seismic activity and geotechnical hazard potential in caving [3]. Applying draw control to moderate stress distribution in a caved area is one of the most important aspects of draw management. If major problems are to be avoided, production rates have to be tuned to the rate of caving and should not be exceeded. Poor draw control early in the panel's life can, over time, result in compounding difficulties in draw, dilution, low utilization and ground control.

In general, draw control is fundamental to the success or failure of any block cave operations. If draw from the drawpoints is not controlled, many problems and hazards occur: unbalanced cave subsidence as a result of poor ground control over time, decreased recovery and productivity, premature waste ingress and recompaction of broken material in the draw columns, infrastructure instability, fragmentation size distribution, more dilution, ore handling difficulties that can lead to tunnel and ore pass collapse or haulage systems, flow of muck at the drawpoints, and other safety and financial or safety damage to miners. Consequently, a production planning program that does not incorporate the geotechnical properties of the rock mass within the block-cave mining method will not be used for general purposes of production schedules, since such a plan causes many forms of damage. Careful control of the production in long-term planning of a block-cave mine will ensure that the schedule drawing process within the cave moderates unwanted problems and preserves mining economics associated with production targets. Thus, draw control is a basic input and output of a block-cave mine to maximize a company's economic goals by taking into consideration the geotechnical properties of rock mass.

The main cause of over stresses at a cave front is related to the angle of draw, which affects the stress pattern at the cave front. The angle of draw is correlated with the collapses experienced at mines. It is well established that an even draw leads to a more uniform stress distribution on the production level than an isolated draw [4]. Draw control in caving operations has two critical responsibilities: (i) defining a pattern for all drawpoints to deplete by a specific PRC, and (ii) creating an appropriate relationship among all draw columns to adjust their depletion rate by considering adjacent drawpoints according to the caving direction.

This research intended to practice the above mentioned critical responsibilities in planning as an important element of the performance and productivity of a block-caving mine. In this paper some applicable algorithms have been developed to optimize draw control. In order to develop optimization techniques that have the ability to integrate actual planning, more investigation is needed. A strict schedule planning system is needed to maintain production control and implement an effective draw strategy once production commences. Introducing a draw control system based on mathematical programming that integrates constraints from other disciplines like geology, mining and metallurgy will become more acceptable as real business planning tools.

This paper establishes several constraints at different resolutions that can be applied to illustrate a draw control system's effects on mine planning: a draw rate constraint based on PRCs, conditional draw rate constraints, and a maximum number of active periods. These constraints are presented by a set of equations to ensure that the tonnage and grade of material drawn from a drawpoint will be in a practical, specified range. The first resolution seeks to model the exact draw rate constraint. The size and complexity of the mathematical formulation of PRC has forced researchers to consider only minimum and maximum boundaries for draw rate constraint instead of tracking a global model to define how the draw rate is adjusted between these boundaries.

The draw columns should be drawn homogenously across the cave to confirm that the ore-waste interface is maintained between draw columns. In an appropriate planning, difference in draw rate between adjacent drawpoints should not exceed from a specified range. High draw differentials between adjacent drawpoints can result in local problems of uneven mixing, and have the potential to promote packing and convergence in weaker ground [5].

In this paper the conditional draw rate is investigated at three resolutions: (i) draw rates of the considered drawpoint and its adjacent drawpoint along the advancement direction at period t (CDRAT) in which the difference in the draw rate between two drawpoints is controlled by the angle of the draw; (ii) draw rates of the considered drawpoint and its adjacent drawpoints based on the advancement direction at period t (CDRMT) in which difference of the draw rate among multiple drawpoints is controlled by the angle of the draw; and (iii) the difference in extracted tonnage from the considered drawpoint and its adjacent drawpoint along the advancement direction from the beginning of extraction until the end of period t (CDRB). An even draw does not require all of the drawpoints to produce at the same rate. But the drawing process for any drawpoint in a period or among drawpoints in all periods must follow a given rule or range. Minimizing draw variability directly results in a uniform draw and consequently the overall dilution is minimized and the life of the mine is extended.

Conditional draw rate (CDR) establishes a condition such that the extraction of high and low tonnage draw columns in a deterministic distance does not exceed a defined range.

2. Literature review

Most of the mining optimization models that have been developed for block-cave mines consider a special area in planning problems. The literature on draw control systems based on the PRC for block-caving operations is relatively new. Most of the models use simulation in consideration of PRC to evaluate production schedules. PCBC [6] provides a simulation implement to research and assess various production schedules. The literature does not contain clear examples of block-caving production scheduling using a mathematical approach that formulate a draw control system in the block cave according to the PRC. Most of the studies consider only upper and lower boundaries for draw rate in their modeling. Laubscher [7] found that dilution entry is a function of the variation in tonnages drawn from adjacent working drawpoints. Therefore, draw control should limit the relative draw rates between adjacent draw columns. A good draw strategy helps to prevent damaging stress concentration by maintaining an even draw across the panel and it delays dilution entry by prohibiting an isolated draw. It has been shown that rock fragments can travel a substantial distance in the horizontal direction if a differential draw profile is applied [2]. Khodayari and Pourrahimian [8] presented a comprehensive review of operations research in block caving. They summarized several authors' attempts to use different methods to develop methodologies for optimizing production scheduling in block-caving operations. Previous to modern algorithms and computational developments, block-caving scheduling problems, like other underground mining methods in their large size, seemed intractable when formulated in mathematical form, especially in the case of mixed-integer linear programming (MILP) problems. Table 1 summarizes the mathematical models used for block-cave scheduling.

Pourrahimian et al. [9-12] devised other applications of MILP to develop a practical optimization framework for caving production scheduling. They presented a multi-step method for the long-term production scheduling of block caving to overcome the size problem of mathematical programming models and to generate a robust practical near-optimal schedule. Their model aims to maximize the net present value (NPV) of the mining operation at three levels of resolution while the mine planner has control over defined constraints. These levels are: (i) aggregated drawpoints (cluster level); (ii) drawpoint level; and (iii) drawpoint-and-slice level. Pourrahimian attempted to find an optimal schedule for the life of the entire mine, solving simultaneously for all periods by considering all required constraints, but he did not consider geotechnical properties of rock mass through the draw rate constraint. Pourrahimian (2014) mentioned that the formulation tries to extract material from drawpoints with a draw rate within the acceptable range without considering a specific shape. Alonso-Ayuso et al. [13] considered a planning mixed-integer programming (MIP) medium range problem for the El Teniente mine in Chile to maximize NPV by introducing, explicitly, the issue of uncertainty. They presented good work about the stochastic version of the copper extraction planning problem under uncertainty in (volatile) copper prices and used only some operational constraints without taking into account the draw control mechanism.

Table 1. Mathematical models in block-cave production scheduling optimization

Researcher	Mining capacity	Production grade	Maximum active drawpoints	Precedence	Continuous extraction	New drawpoints	Reserves	Maximum activity life	Draw rate without PRC	Draw rate with PRC	Relative draw rate	Conditional draw rate	Objective function	Model	Publication date
Riddle[1] [15]							*						Profit base	Dynamic	1976
Chanda[2] [16]	*	*	*		*								Min. Grade fluctuation	MIP and simulation	1990
Guest [17]	*	*		*			*		*		*		Max. NPV	MIP	2000
Smith[3] [18]									*				Min. deviation of column heights from ideal	MIP - LP	2001
Rubio [19]			*	*	*	*	*	*	*				Max. NPV and Min. Dilution	MILP	2002
Rahal [20]				*					*		*		Minimization production from ideal draw	MILGP	2003
Rahal[4] [2]	*	*		*	*		*		*		*		Min. production from ideal draw	MILGP	2008
Weintraub[5] [21]	*			*	*								Max. profit	MIP	2008
Queyranne [22]	*	*	*	*	*								Max. NPV	MIP	2008
Smoljanovic [23]	*		*	*	*			*					Max. NPV	MILP	2011
Parkinson [24]	*	*		*	*				*				Max. NPV	IP	2012
Epstein [25]	*	*	*	*	*				*				Max. NPV	MIP	2012
Pourrahimian [12]	*	*	*	*	*	*	*		*				Max. NPV	MILP	2013
Alonso[6] [13]	*	*	*	*	*								Max. NPV	MIP	2014

[1] Riddle presented a dynamic programing model based on ore reserve and profit model.

[2] Chanda's model was in raise level.

[3] Smith presented a review about the importance of a draw control system in a block-cave.

[4] Rahal considered the production rate curve as a goal of programming in the objective function, and a few of the constraints had different descriptions but the same application as the marked constraints.

[5] Weintraub presented a clustering method according to marked parameters.

[6] Alonso studied the risk in the presented research.

All of these studies have concentrated on determining the optimum configuration of a block-caving operation. The main problems associated with the methods presented above can be summarized as follows: some of them did not incorporate, on a routine basis, operational performance to adjust medium and long-term plans because of loss of geotechnical rules in the modeling of actual draw management systems. Constraints must be appropriate with the mining method, objective function, and real geotechnical condition of the rock mass. Maximizing tonnage or mining reserves will not necessarily lead to maximum NPV without aggregation reserve and grade constraint to time dynamic behavior of the fundamental models in linking the mine planning parameters and draw rate curves.

During production, the only control is through the drawpoints. The rate at which material can be drawn from an individual drawpoint depends on several rock mass and design parameters such as equipment size, layout configurations, and stress transfer on the extraction level, haulage infrastructure, and seismic activity [14].

The draw control system is a critical key in the accurate design and operation of block-cave mines. Most of the previous research considers a series of simple definitions of the process of moving and drawing caved material from drawpoints. The researchers have modeled the draw rate constrains regardless of the production rate curves and only by defining lower and upper bounds. Deciding minimum and maximum draw rates of drawpoints is appropriate according to the principles of geotechnical rules in all previous studies. However, during the tire life of any drawpoint, the draw rate varies. The literature review shows that there is no prominent study that models the relationship between the depletion rates of drawpoints in block caving using a mathematical formulation. However, the lack of consideration given to the draw rate curve according to geotechnical rules and new conditional draw rate constraints in the caved area, in addition to the abuse of the objective function and non-optimized scheduling, causes a problematic caving progress. The objective of this study is to develop, implement, and verify a realistic optimization MILP framework for block-cave long-term production scheduling, whereby a mineral is extracted and prepared at a desired market specification, with the maximum economic return measured by NPV, and within acceptable operational constraints with respect to geotechnical rules and draw control management. Figure 1 shows a schematic view of a production rate curve. This profile is usually provided by the geotechnical team to consider many factors such as the engineering and geotechnical properties of the rock mass and safety issues in extraction.

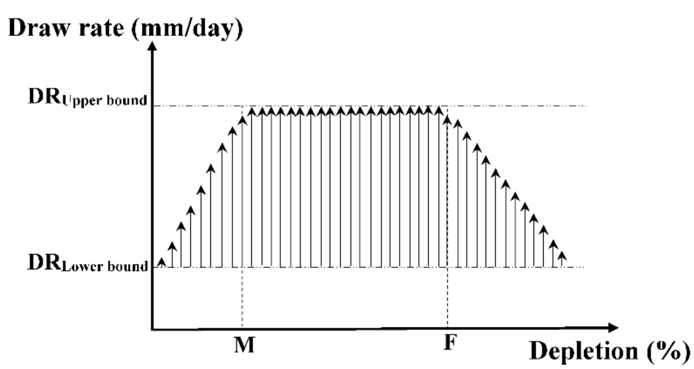

Figure 1. Production rate curve (ramp-up, high-production, and ramp-down)

3. Mathematical Formulation

Developing any mathematical model requires some decision variables, sets, indices, and parameters that correspond to a scheduling program. The following items are introduced according to the current mode:

Indices

$i \in \{1,...,I\}$	Index for drawpoints.
$t \in \{1,...,T\}$	Index for scheduling periods.
l	Index for a drawpoint belonging to set A^i .

Set

A^i	For each drawpoint i, there is a set A^i defining the predecessor drawpoints that must be started prior to the extraction of drawpoint *i*.
R^i	For each drawpoint *i*, there is a set R^i defining the adjacent drawpoints of the drawpoint *i*.

Decision variables

$X_{i,t} \in [0,1]$	Continuous decision variable, representing the portion of draw column *i* to be extracted in period *t*.
$E_{i,t} \in \{0,1\}$	Binary decision variable equal to 1 if drawpoint *i* is active in period *t*; otherwise it is 0.
$S_{i,t} \in \{0,1\}$	Binary decision variable controlling the precedence of extraction of drawpoints. It is equal to 1 if the extraction from drawpoint *i* is started in period *t*; otherwise it is 0.

Parameters

$Value_i$	Economic value of the draw column associated with drawpoint *i*.
$DR_{l,i,t}$	Minimum allowable draw rate of drawpoint *i* in period *t*.
$DR_{u,i,t}$	Maximum allowable draw rate of drawpoint *i* in period *t*.
$N_{Ei,t}$	Maximum allowable number of active drawpoints in period *t*.
$Num_{Nl,i,t}$	Lower limit for the number of new drawpoints, the extraction from which can start in period *t*.
$Num_{Nu,i,t}$	Upper limit for the number of new drawpoints, the extraction from which can start in period *t*.
Ton_i	Total tonnage of material within the draw column associated with drawpoint *i*.
M	Maximum allowable depletion to reach steady region after ramp-up.
F	Maximum allowable depletion to reach ramp-dawn region after steady region.
dsc	Discount rate.
MC_u	Upper limit of mining capacity in period *t*.
MC_l	Lower limit of mining capacity in period *t*.
Act_{life}	Maximum allowable periods that any drawpoints can be active.
$gr_{u,i,t}$	Upper limit of the acceptable average head grade of drawpoint *i* in period *t*.
$gr_{l,i,t}$	Lower limit of the acceptable average head grade of drawpoint *i* in period *t*.
$\widetilde{G}_{i,t}$	Average grade of drawpoint *i*.

DC	Diameter of draw column
□	Angle of draw
X_i , Y_i	Coordinates of drawpoint *i*.
UT	Upper bound of difference in tonnage between drawpoint *i* and *j*
LT	Lower bound of difference in tonnage between drawpoint *i* and *j*
UC	Upper difference in constant coefficients between drawpoint *i* and *j*
LC	Lower difference in constant coefficients between drawpoint *i* and *j*
H_i	Height of draw column *i* after extraction according to angle of draw □

3.1. Objective function

The objective function is to generate a schedule to provide the sequence of drawing from drawpoints over the scheduling periods. It means selecting which drawpoint should be started, activated, and depleted in every period and, finally, closed.

Mine plans are optimized by using many criteria, such as profit, life of mine, mining costs, confidence level, and mineral resources, while attending to constraints related to production rates, plant capacities, and grades. Some companies focus on reducing production costs by maximizing reserves and throughputs in their projects; others try to optimize specific economic indicators such as NPV, the internal rate of return (IRR), the payback period (PB), or the profit investment ratio (PIR) [23, 26, 27].

The maximization of NPV is closely associated with maximizing ore tonnes, as the ore tonnes generate revenue [28]. The NPV approach recognizes the time value of money and represents the sum of the discounted value of future cash flows [29]. Generally in planning, the NPV is maximized because it will yield the maximum profit to the mining companies. Several operations have recognized this strategy as the main driver for the mine planning process [17]. The production rate corresponding to the maximum NPV is the optimum production rate [30].

In this paper, the goal of all models is to maximize the NPV of the block-caving projects over the life of a mine in a time-dynamic manner, while satisfying all draw control system constraints and other operational constraints. The MILP objective function, equation (1), is composed of the economic value of the draw column and a continuous decision variable $X_{i,t}$, which indicates the portion of a draw column which is extracted in each period.

$$Max\sum_{i=1}^{I}\sum_{t=1}^{T}\left[\frac{Value_i}{(1+dsc)^t}\right]X_{i,t} \tag{1}$$

3.2. Constraints

Operational and technical constraints of block-cave mining operations are considered to control the outputs of the optimizations model. Number of decision variables depends on the number of drawpoints and the number of slices in each drawpoint.

One of the main goals of long-term mine planning is to integrate internal and external mine planning factors that affect the performance of the mine operation. At the same time, long-term planning is responsible for coordinating strategic goals and operational activities [31]. Getting the best solution requires using an operations research technique to limit the objective function by some constraints. These constraints appear in several different forms: geotechnical, grades, period, advancement direction, and priority of productive units, productivity, production rates, and many others that depend on the mining method. The construction of the optimization problems has required rational studies of which mining constraints are applicable in each case [32]. The constraints considered in the integer program are generally quality and quantity requirements. Knowledge, experiments of mine planners, and corresponding planning horizons have a critical role in the process of assigning constraints to the optimization problems. The following constraints are part of the problem in deriving the formulation:

3.2.1. Mining capacity

Equation (2) and (3) ensure that the total tonnage of material extracted from drawpoints in each period is within the acceptable range that allows flexibility for potential operational variations.

$$\sum_{i=1}^{I}(Ton_i)\times X_{i,t} \leq MC_u \tag{2}$$

$$\sum_{i=1}^{I}(Ton_i)\times X_{i,t} \geq MC_b \tag{3}$$

3.2.2. Production grade

Equations (4) and (5) force the mining system to achieve the desired grade. The average grade of the element of interest has to be within the acceptable range and between certain values.

$$\sum_{i=1}^{I}Ton_i \times X_{i,t} \times (gr_{l,i,t} - \widetilde{gr_{i,t}}) \leq 0 \tag{4}$$

$$\sum_{i=1}^{I}Ton_i \times X_{i,t} \times (\widetilde{gr_{i,t}} - gr_{u,i,t}) \leq 0 \tag{5}$$

3.2.3. Maximum number of active drawpoints

According to equations (6), (7), and (8) in each period, the number of active drawpoints must not exceed the allowable number and has to be constrained according to the size of the ore-body and the available infrastructure and equipment. A large number of active drawpoints might lead to serious operational problems.

$$E_{i,t} - \frac{Ton_i}{DR_{l,i,t}} \times X_{i,t} \leq 0 \tag{6}$$

$$X_{i,t} - E_{i,t} \leq 0 \tag{7}$$

$$\sum_{i=1}^{I}E_{i,t} \leq Num_{E_{i,t}} \tag{8}$$

3.2.4. Precedence of drawpoints

The precedence between drawpoints is controlled in a horizontal direction. Controlling the order of extraction of drawpoints through an advancement direction is the goal of the precedence constraint. According to the advancement direction, for each drawpoint *i* there is a set A^i which defines the predecessor drawpoints among adjacent drawpoints that must be started before drawpoint *i* is extracted. Equation (9) controls the precedence of extraction.

$$S_{i,t} - \sum_{j=1}^{t}S_{l,j} \leq 0 \tag{9}$$

3.2.5. Continuous extraction

Equations (10) and (11) force the mining system to extract material from drawpoints continuously after opening until closing. Equation (12) is only used for period 1.

$$\sum_{t=1}^{T} S_{i,t} = 1 \tag{10}$$

$$E_{i,t} - E_{i,(t-1)} - S_{i,t} \leq 0 \tag{11}$$

$$E_{i,1} - S_{i,1} = 0 \tag{12}$$

3.2.6. Number of new drawpoints

Based on the footprint geometry, the geotechnical behavior of the rock mass, and the existing infrastructure of the mine, the maximum feasible number of new drawpoints to be opened at any given time within the scheduled horizon must be defined on the basis of equations (13) and (14).

$$Num_{Nl,i,t} \leq \sum_{i=1}^{I} S_{i,t} \leq Num_{Nu,i,t} \tag{13}$$

$$\sum_{i=1}^{I} S_{i,1} \leq Num_{Ei,1} \tag{14}$$

3.2.7. Reserves

Equation (15) ensures that the sum of the fractions of the draw column that are extracted over the scheduling periods in maximum value is one, which means there is selective mining. For that reason, all the material in the draw column may not be extracted.

$$\sum_{t=1}^{T} X_{i,t} \leq 1 \tag{15}$$

3.2.8. Maximum activity life

The activity period of the drawpoint, in the context of draw control, is mainly concerned with the assessment of draw rates to adjust extraction tonnage of any drawpoint and prevent any recompaction or dilution in the activity life of drawpoints. Maximum activity life constraint causes the maximum activity life of any drawpoint to be limited to a deterministic value, so the draw rate of the drawpoint must be large enough to maximize the NPV and small enough to prevent over-dilution. Equation (16) indirectly affects the draw rate by controlling the number of activity periods of any drawpoint. So if the activity period has a large value, then the draw rate can have smaller values, and increasing the drawpoint activity life increases the probability of recompaction and dilution.

$$\sum_{t=1}^{T} E_{i,t} \leq Act_{life} \tag{16}$$

3.2.9. Draw rate constraint

The development of an overlap and disjunctive (OD) system for regulating drawpoint production begins by breaking the production profile into a number of regions, each of which has a binary indicator variable. When the binary variable for each level takes a value of zero the draw constraint for that level is relaxed;

otherwise that region binds production. The OD system determines which region is active. In the first step of the proposed OD system, the line equation for each region is written. For this purpose, as shown in Figure 2, the general equation for the ramp-up region is written based on the depletion and the draw rate (see equation (17)).

$$(DR_{i,(t+m)} - DR_{l,i,t}) = \frac{(DR_{u,i,t} - DR_{l,i,t})}{M - X_t} \times (X_{i,(t+m)} - X_{i,t}) \tag{17}$$

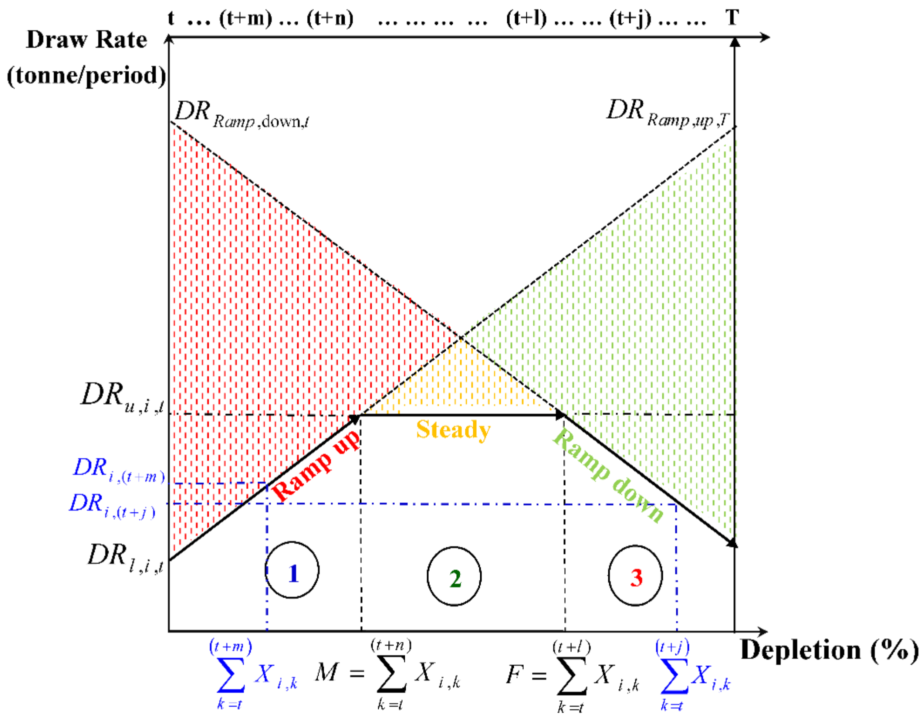

Figure 2. Ramp-up-steady-ramp-down situation

In the simple form it can be written that (see equation (18))

$$X_{i,(t+m)} = \sum_{t=1}^{t+m} X_{i,t} \tag{18}$$

$$DR_{i,(t+m)} = X_{i,(t+m)} \times Ton_i \tag{19}$$

Equation (20) shows the mathematical structure for the area under the ramp-up region.

$$X_{i,(t+m)} \times Ton_i - \left(\frac{(DR_{Ramp,up,T} - DR_{l,i,t})}{1 - \frac{DR_{l,i,t}}{Ton_i}}\right) \times \sum_{t=1}^{t+m} X_{i,(t+m)} \le DR_{l,i,t} - \left(\frac{(DR_{Ramp,up,T} - DR_{l,i,t})}{1 - \frac{DR_{l,i,t}}{Ton_i}}\right) \times \frac{DR_{l,i,t}}{Ton_i} \tag{20}$$

With a similar process, the equations for the steady and ramp-down regions can be written as follows. Equations (21) and (22) are related to the steady and ramp-down regions respectively:

$$X_{Steady,T} \times Ton_i \le DR_{u,i,t} \tag{21}$$

$$X_{i,(t+m)} \times Ton_i + \left(\frac{(DR_{Ramp,down,T} - DR_{l,i,t})}{1-\dfrac{DR_{l,i,t}}{Ton_i}}\right) \times \sum_{t=1}^{t+m} X_{i,(t+m)} \le DR_{Ramp,down,T} + \frac{(DR_{Ramp,down,T} - DR_{l,i,t})}{1-\dfrac{DR_{l,i,t}}{Ton_i}} \times \frac{DR_{l,i,t}}{Ton_i} \tag{22}$$

Based on equations (20), (21), and (22), the problem can be formulated for the required PRC. In each period that drawpoint depletion is started, the draw rate must be equal to the minimum acceptable draw rate. Equation (23) forces models to start depletion with a minimum acceptable draw rate.

$$DR_{l,i,t} \times S_{i,t} - Ton_i \times X_{i,t} \le 0 \tag{23}$$

3.2.10. Conditional draw rate constraint (CDR)

The presented conditional draw rate constraints in this paper seek to control undesirable problems among the draw columns according to special characters and geotechnical rules of any caving mine. The conditional draw rate (CDR) constraint is a fundamental part of schedule planning to ensure that an even draw is maintained across the cave.

There should be a close relationship between the depletion of a given production drawpoint and its adjacent drawpoints in all activated area. If the production difference between adjacent drawpoints is too much, material from a draw column with higher tonnage moves to the draw column with lower tonnage. The draw strategy of each drawpoint should not only be connected to the PRC, but should take into account the amount of the extracted material from the adjacent drawpoints during the periods in which the drawpoint is active. Therefore, to optimize draw control in block-cave production scheduling, two fundamental factors must be considered: (i) the draw rate of the drawpoint, and (ii) the conditional draw rate between the drawpoint and its adjacent drawpoints. These two factors result in a uniform extraction across the cave. The CDR constraint includes two components: upper and lower difference tonnage coefficients (UT and LT), and upper and lower difference constant coefficients (UC and LC).

- *Difference tonnage coefficients (DTC)*

The UT and LT difference tonnage coefficients of CDR are defined based on the angle of the draw in the advancement direction. The angle of the draw controls the tonnage of extraction from adjacent drawpoints. The angle of the draw can be steep or shallow and is defined by the operation's production rate, stress distribution in the rock mass, strength of the host rock, cave stability, engineering judgment, and practical strategies from other projects. It is important to maintain the determined angle of the draw during the mine life and to prevent sudden changes of the angle of the draw between periods. A significant variation in the depletion rate among the draw columns in the advancement direction leads to an increased amount of chaotic movement within the broken rock. Stress is then redistributed throughout the mining area as a function of the depletion rate in all drawpoints. Also, dilution which is the result of mixing of fragmented material along the draw column and uneven draw among the draw columns seriously increases. Whenever the angle of draw is tightened, the adjacent drawpoints are required to have a closer production rate (Figure 3). The UT and LT of drawpoint *i* and *j* are calculated by equation (24).

$$DTC = \frac{\pi}{4} \times DC^2 \times \rho_j \times \tan\beta \times \sqrt{(\mathrm{X_i}-\mathrm{X_j})^2 + (\mathrm{Y_i}-\mathrm{Y}_j)^2} \quad \begin{cases} \text{For } UT & \beta = \beta_{\max} \\ \text{For } LT & \beta = \beta_{\min} \end{cases} \tag{24}$$

The aim of the CDRAT constraint is to control the total difference in depletion rates of adjacent drawpoints along the advancement direction in period *t*. If the adjacent drawpoints are still mining at low waste contents, it may be necessary to continue to extract ore from the high waste drawpoints to maintain the correct shape to maximize the extraction from the more economic drawpoints [33]. The difference in draw tonnage between adjacent drawpoints is restricted by the CDRAT constraint. This constraint ensures that

(i) the dilution is controlled and (ii) over- or under-depletion does not occur. This prevents damage to the extraction level and other mine structures that can result from induced stresses.

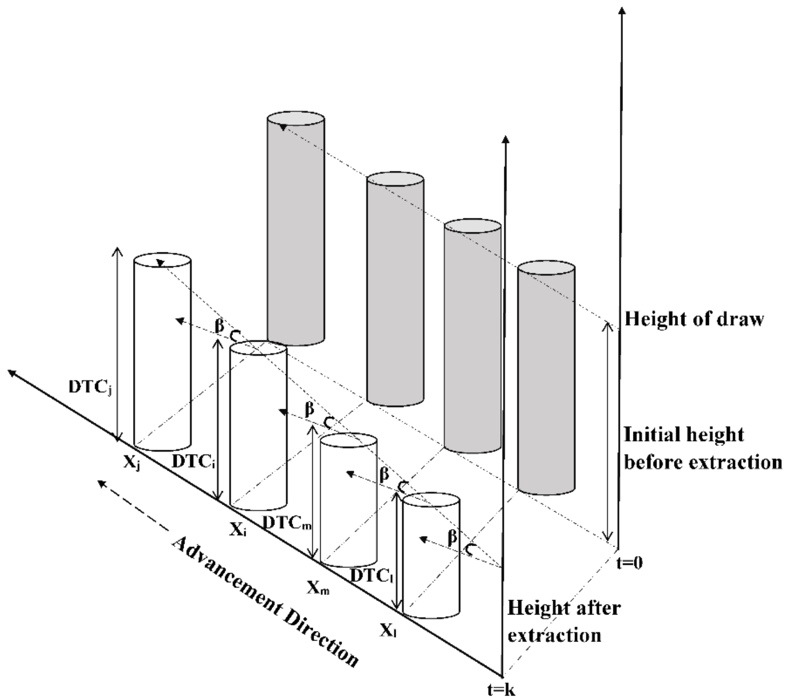

Figure 3. Influence of angle of draw on adjacent drawpoints

Equation (25) shows the mathematical structure of the CDRAT constraint:

$$LT_j \times DR_{i,t} + MinDR_{j,t} \times (1-E_{i,t}) \leq DR_{j,t} \leq UT_j \times DR_{i,t} + MaxDR_{j,t} \times (1-E_{i,t}) \tag{25}$$

The draw rate is a function of the draw column's tonnage and the portion of extraction in each period (see equation (26))

$$LT_j \times X_{i,t} \times Ton_i + MinDR_{j,t} \times (1-E_{i,t}) \leq X_{j,t} \times Ton_j \leq UT_j \times X_{i,t} \times Ton_i + MaxDR_{j,t} \times (1-E_{i,t}) \tag{26}$$

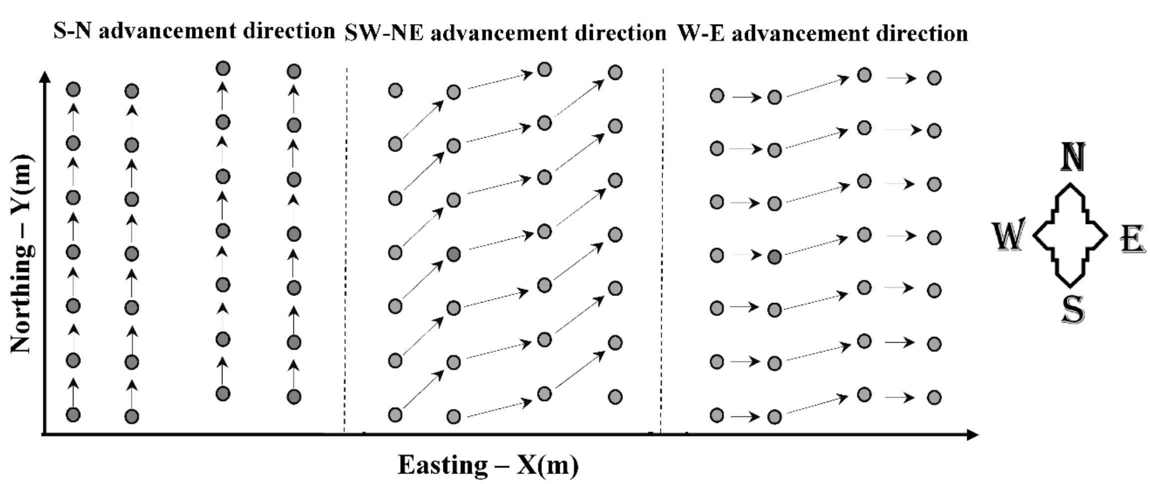

Figure 4. One adjacent drawpoint for any drawpoint in different advancement directions

- *The conditional draw rate of the a drawpoint and its adjacent drawpoints based on the advancement direction at period t (CDRMT)*

This section investigates the effect of the adjacent drawpoints on the depletion rate from a drawpoint in the advancement direction.

Increasing the distance from the considered drawpoint reduces the influence of adjacent drawpoints. It is necessary to define a radius to determine the number and location of adjacent drawpoints. This radius is defined in a way that all drawpoints available in this area have a direct effect on the considered drawpoint. Figure 5 shows the search radius and adjacent drawpoints of the considered drawpoint in different advancement directions.

Equation **Error! Reference source not found.** controls the CDRMT constraint.

$$\begin{aligned} &LT_j \times X_{i,t} \times Ton_i + MinDR_{j,t} \times (1-E_{i,t}) \le X_{j,t} \times Ton_j \le UT_j \times X_{i,t} \times Ton_i + MaxDR_{j,t} \times (1-E_{i,t}) \\ &\mathrm{j}: 1,2,3,... \in R^i \end{aligned} \quad (27)$$

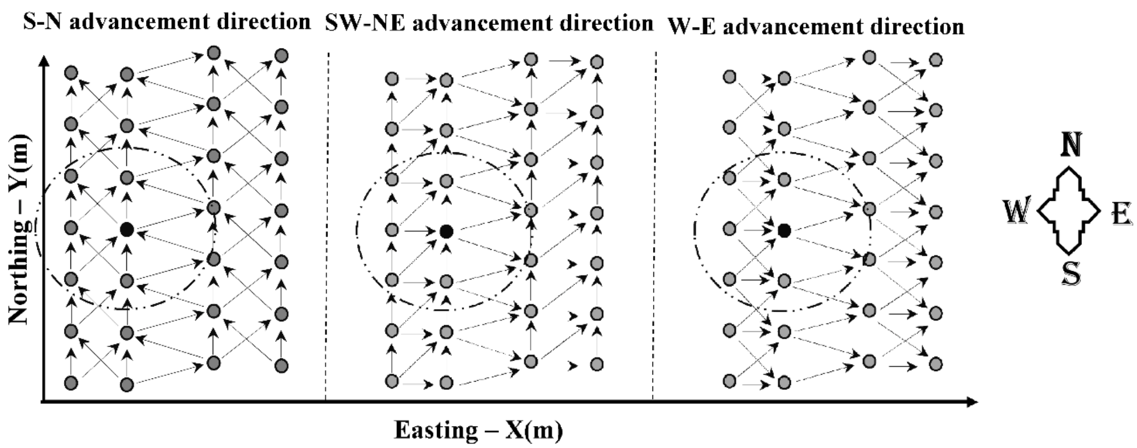

Figure 5. Multi-neighboring of any drawpoints in different advancement directions

- *Conditional draw rate constraint by period t (CDRB)*

Draw column height is the height of ore above the drawpoint with an acceptable height of dilution that can be drawn from the drawpoint [12]. The main parameter to simulate the mixing process is the height of the interaction zone. The greater the height of interaction, the sooner the dilution will appear in the drawpoint. The CDRB reduces the total difference of depleted tonnage between adjacent drawpoints (*i* and *j*) by period *t*. The total difference in tonnage between adjacent drawpoints (*i* and *j*) before the extraction has a constant value (CV). After starting the depletion from drawpoints *i* and *j* the remaining material in the draw columns can vary and have any tonnage. So the drawn surface can be uneven if the difference in depleted tonnage cannot be controlled in drawpoints *i* and *j*. Equation (27) defines the condition that seeks to limit the total difference in any adjacent drawpoints' depletion tonnage to a given value such as CV (see Figure 6). However, the CDRB controls the summation of the total difference in depleted tonnage from drawpoints *i* and *j* in each period and additionally, from the starting period to the considered period. It is important to note that, the primary objective of the conditional draw rate constraint in period *t* is to regulate block-cave production to conform to the complete drawn program.

$$\left[Ton_i - Ton_j + Ton_j(1-S_{j,t})\right] \times LC \le \sum_{t=1}^{T} DR_{i,t} - \sum_{t=1}^{T} DR_{j,t} \le \left[Ton_i - Ton_j + Ton_j(1-S_{j,t})\right] \times UC \quad (27)$$

Equation (28) shows the CDRB constraint based on extracted tonnage by period *t*:

$$\left[Ton_i - Ton_j \times S_{j,t})\right] \times LC \leq (Ton_i \times \sum_{t=1}^{T} X_{i,t} - Ton_j \times \sum_{t=1}^{T} X_{j,t}) \leq \left[Ton_i - Ton_j \times S_{j,t})\right] \times UC \tag{28}$$

LC and UC are the minimum and maximum values of CV. Rules of thumb, feedback from the rock mechanics and mine stability experts, and practical strategies from previous extractions can define CV.

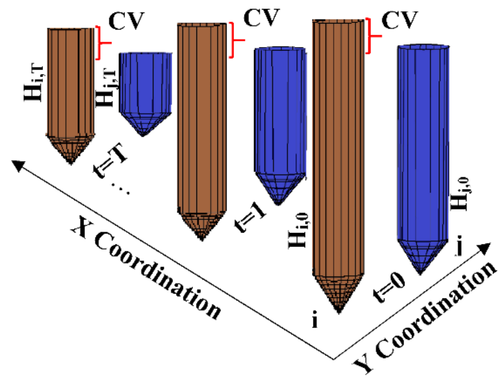

Figure 6. Draw column height in all periods

4. Solving the optimization problem

In this paper, the presented MILP models were developed in MATLAB [34] and solved in the IBM ILOG CPLEX [35] environment. CPLEX uses a branch-and-bound algorithm to solve the MILP model, assuring an optimal solution if the algorithm is run to completion. A gap tolerance (EPGAP) of 9% is used as an optimization termination criterion. This is an absolute tolerance between the gap of the best integer objective and the objective of the remained best node.

5. Case study

The production schedule of 325 draw columns over 15 periods for each draw rate strategy is investigated. Figure 7 shows the drawpoints' herringbone layout. The total tonnage of material is 38.71 (Mt) with an average density of 2.7 (t/m^3) and an average grade of 0.41% Cu. Figure 8 illustrate the tonnage and grade distribution of the draw columns. The searching radius in the CDRMT constraint is 20 m. UC and LC in the CDRB constraint are 0.25 and 3, respectively. The performance of the proposed MILP models was analyzed based on maximizing the NPV at a discount rate of 12%. The draw control system, by enrolling an exact production rate curve and conditional draw rate constraints, seeks to optimize and present practical block-cave planning.

The models were tested on a Dell Precision T7600 computer with Intel(R) Xeon(R) at 2.3 GHz, with 32 64 GB of RAM. In all models, as part of their implementation, the maximum depletion of the draw column from ramp up to steady (M) is assumed to be 30%, the maximum depletion of the draw column from steady to the ramp-down region is assumed to be 90%. The other scheduling parameters have been summarized in Table 2. The models are verified in the south to north (SN) advancement direction. This advancement direction was selected using the methodology developed by Khodayari and Pourrahimian [36]. Khodayari showed that the cumulative economic value of all drawpoints can be used to determine the best advancement direction to maximize the NPV. Figure 9 shows the results of the model for 325 drawpoints. As shown, the best direction is SN.

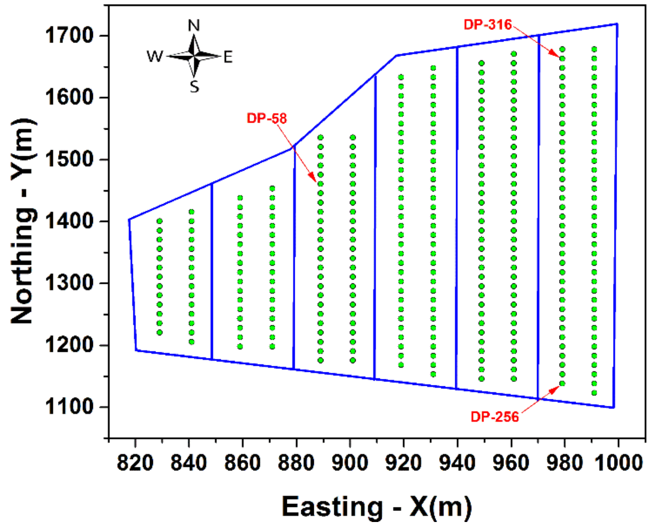

Figure 7. Drawpoints' herringbone layout

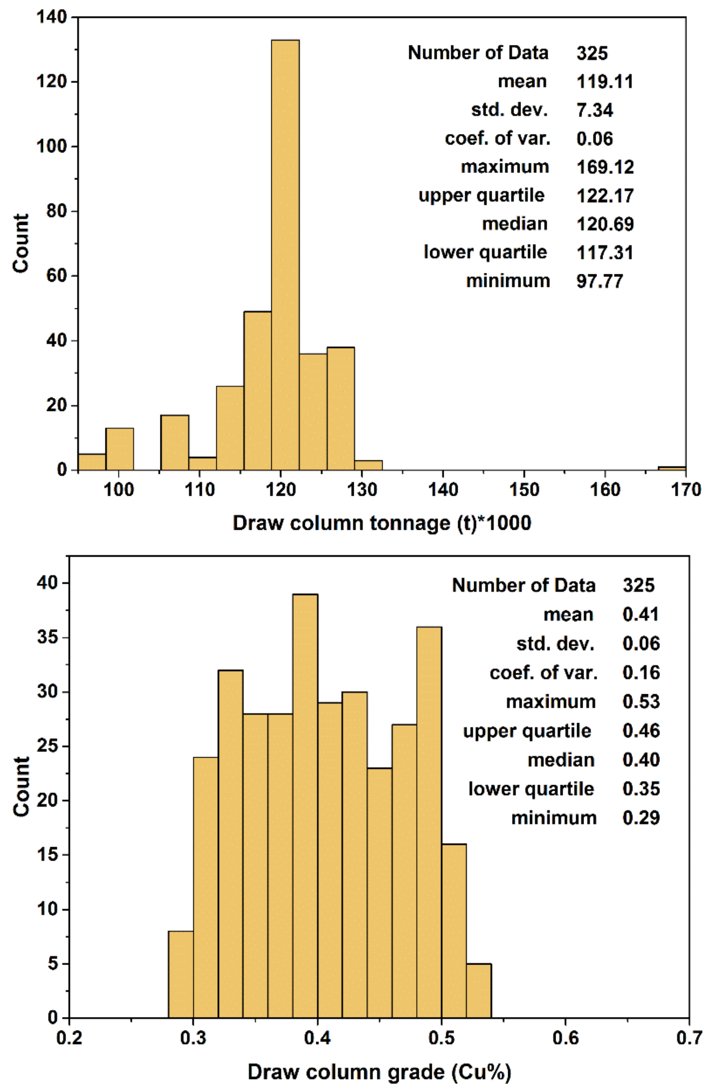

Figure 8. Grade and tonnage distribution for 325 draw columns

The problem was solved in the SN direction for four resolutions: draw rate (DR), CDRAT, CDRMT, and CDRB. Table 3 `shows the results of the all models. The DR model is considered a basic model because it has all of the other models' details. The obtained NPV from the DR is $451.07M with the optimality gap of 9%.

Table 2. Production scheduling parameters

Parameters		Value
Number of periods		15
Maximum number of activity year of drawpoints		4
Draw rate (kt/year)	Min	11
	Max	40
	M (%)	30
	F (%)	90
Angle of draw (°)	β max for UT	15
	β min for LT	5
Number of new drawpoints per year	Max	50
	Min	3
Average grade of production (%)	Max	0.6
	Min	0.4
Number of maximum active drawpoints per year		90

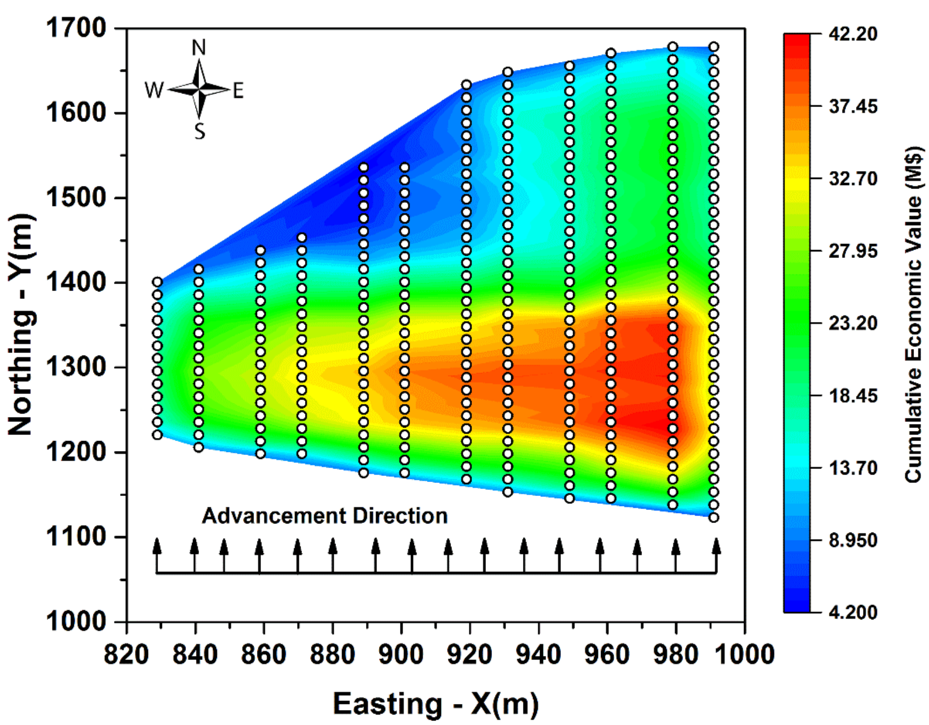

Figure 9. Mining advancement direction

Table 3. Numerical results for all models

Resolution	CPU time	Extraction (Mt)	Extraction (%)	NPV (M$)	Constraint number	Variable number	
						Con.	Bin.
DR	02:08:30	30.56	79	451.07	53650	4875	9750
CDRAT	11:20:43	30.97	80	436.66	53650	4875	9750
CDRMT	10:42:40	28.91	75	424.37	53650	4875	9750
CDRB	07:07:42	32.51	84	445.07	58525	4875	9750

The maximum obtained NPV from all four models belongs to DR model. The total running time in this resolution is 02:08:30. This time, according to the complex structure of the DR model, is desirable. The total extracted tonnage from all drawpoints in this resolution is 30.56Mt. This means that about 79% of all material is extracted in this model. The value of the NPV for the CDRMT resolution is lower than other resolutions and the tonnage of the material that can be depleted from the active drawpoints in this model is less than that of other models. The difference between the obtained NPV of the DR and CDRMT models is less than 6.3%. The minimum percentage of extracted material belongs to CDRMT model. The difference in extracted materials in both models is less than 5.07%. Thus, CDRMT model can simultaneously provide economic and geotechnical goals. Figure 10 shows the cash flow for different resolutions.

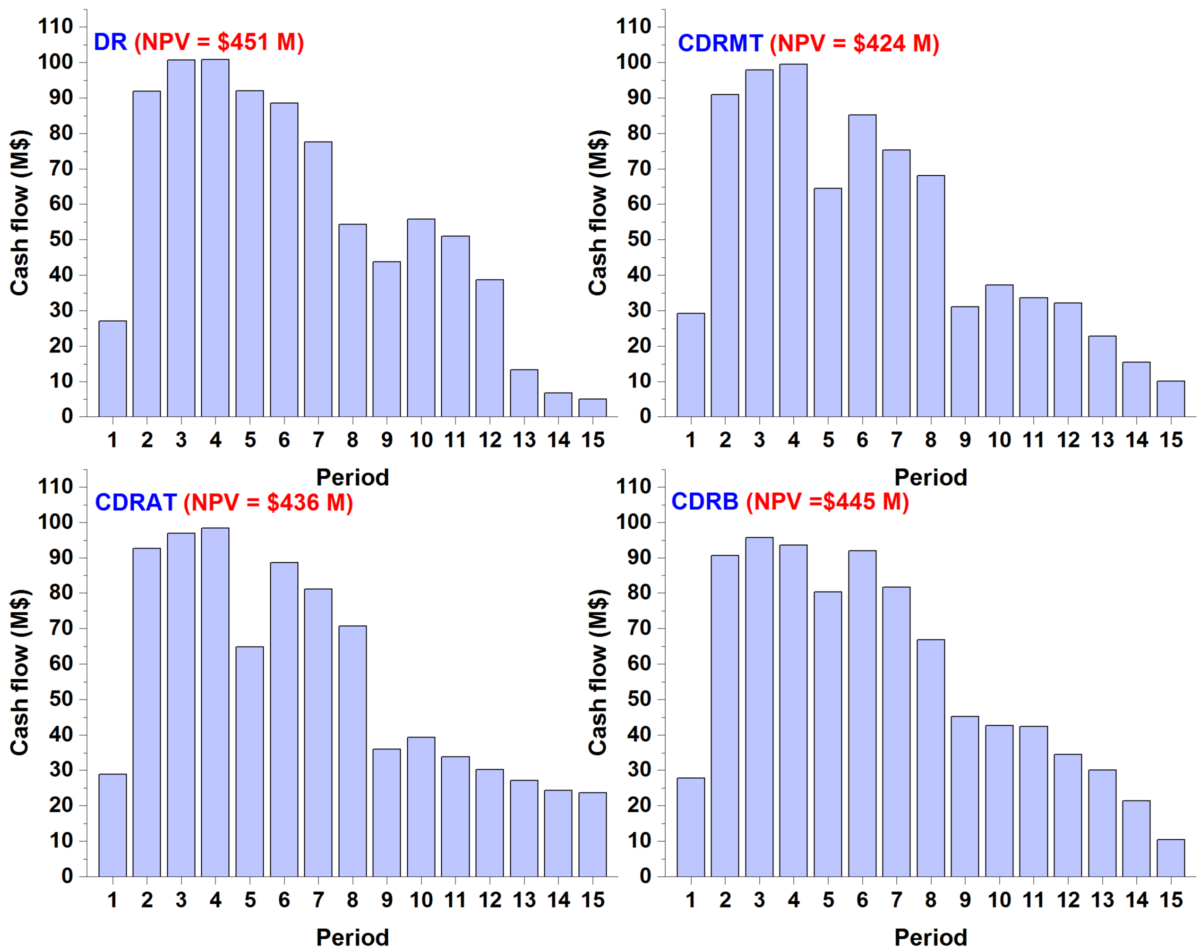

Figure 10. Comparison of cash flow for all resolutions

Figure 11 shows the production tonnage and the average grade of production in each period of all resolutions. In period 1 because of the minimum draw rate and total allowable active drawpoints, production is less than the maximum mining capacity. In the CDRAT model, after period 1, the tolerance of production tonnage is less than other models. Therefore, the CDRAT model seeks to evenly extract materials from drawpoints during the life of the mine. After period 11, the production tonnage decreases gradually in all of the models. The average grade of production increases gradually from periods 1 to 5 and after period 5 it shows a descending trend until the end of the mine life for all resolutions. During the last periods, because of reaching the top of the draw column and increasing the probability of dilution, the average grade of production is less than early years. It should also be noted that dilution is reduced because of the controlling feature of the production rate curve in the draw control system.

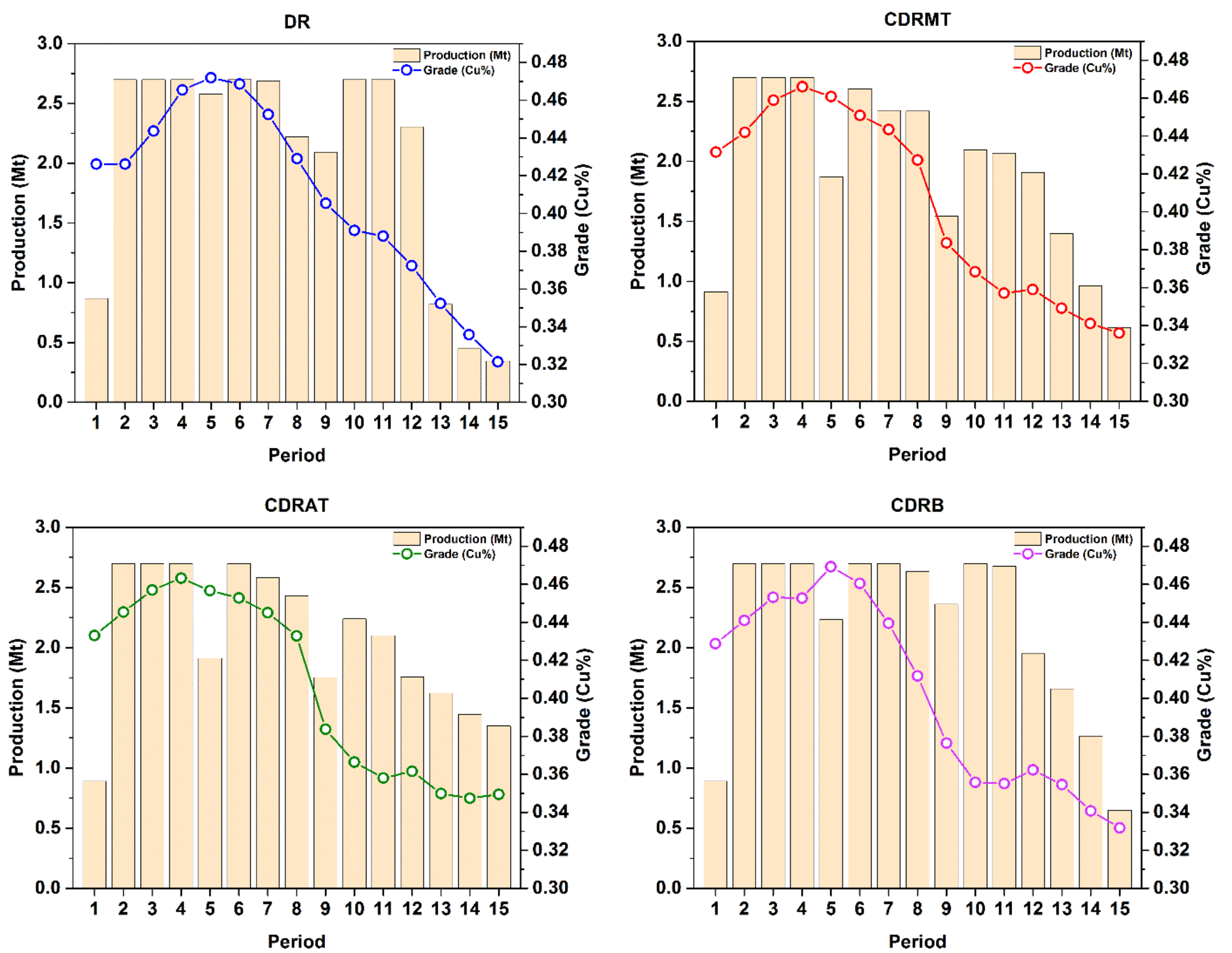

Figure 11. Production tonnage during the mine life for all resolutions

Figure 12 shows the maximum number of active drawpoints in each period for all models. The maximum number of active drawpoints in all periods is less than 90. The number of active drawpoints in the CDRAT, CDRMT, and CDRB models in the last periods is more than the number in the DR model. In the CDRAT model, the difference between the number of active drawpoints in the first and last periods is less than that in other models. The conditional draw rate models seek to deplete draw columns evenly by keeping the same number of active drawpoints in different periods.

According to the defined PRC, drawpoints cannot be depleted arbitrarily. On the other hand, the material with a lower economic value and grade can remain in the number of drawpoints because of the defined objective function and the constraints. Figure 13 illustrates the draw rate changes for three different drawpoints in all models. It is clear that the defined PRC for the selected drawpoints is satisfied in all models. Extraction from drawpoint 256 (DP256) in the DR model is started in period 1 with the minimum acceptable draw rate (11 kt). Then it increases gradually to reach the maximum acceptable draw rate (40 kt) in period 3. The tonnage of extraction from drawpoints varies based on the drawpoints' economic values. The objective function maximizes the NPV and the tonnage of extraction from each draw column is the result of optimization.

The results show that all the defined constraints have been satisfied in all models. The draw rate amount for each drawpoint and starting and finishing periods are obtained as a result of optimization. The models extract the material from each draw column based on the defined draw rate model while maximizing the NPV of the operation.

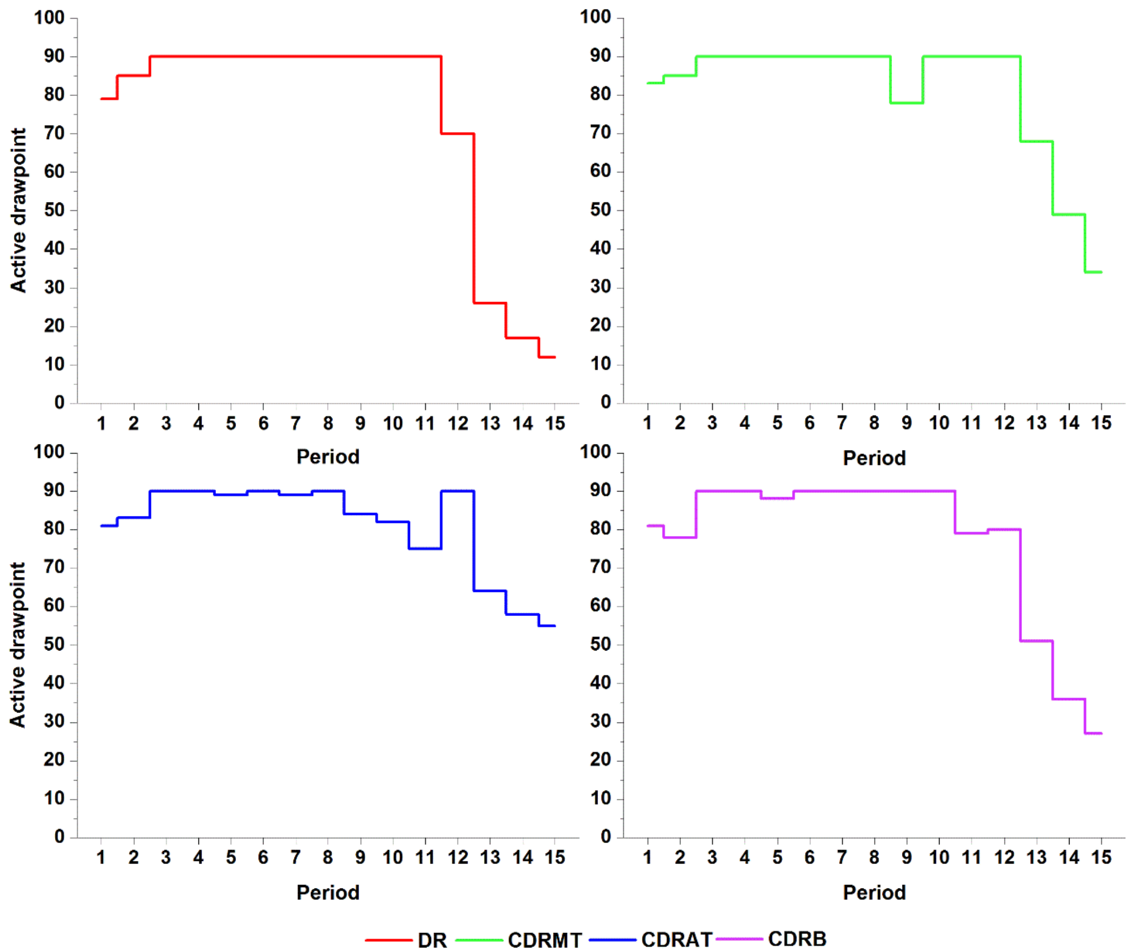

Figure 12. Number of active drawpoints in each period during the mine life

According to Figure 13, the drawpoints that started to deplete in the first periods are less influenced by CDR. The DP256 is one of the southern drawpoints which is started in period 1 in all resolutions. The DP58 has different start periods because it is almost at the end of the advancement direction. Different resolutions have different effects on the DP58. The maximum draw rate in all models is 40000 (tonne/year). The DR and CDRB models can reach the maximum draw rate of 40000 (tonne/year). In the CDRMT and CDRAT, the draw rates are influenced by the angle of draw. Therefore, the depletion rate, according to determined angles and the advancement direction, has a reduction trend. By reducing the draw rate, the CDRMT and CDRAT seek to increase the activity periods of drawpoints. In the CDRAT and CDRMT, all drawpoints are active in four periods but in other models the maximum activity life is less than four periods.

Figure 14 shows the height of the draw columns at the end of different periods for all resolutions. The defined advancement direction has also been satisfied. The height of the drawpoints represents the surface displacement at the end of each period. One of the advantages of the draw control system is that it controls surface displacement by using the draw rate in all drawpoints during the life of the mine. Whenever geotechnical constraints have tightened conditions, the overall displacement in the surface is controlled better. Figure 14**Error! Reference source not found.** shows that the extraction rate from the draw column in the CDRAT and CDRMT models follows an even pattern. Finally, the dilution in the last periods in the marginal draw column reduces because the CDR models restrict more depletion of material in those draw columns. Surface displacement is in the expected shape according to allowable depletions in all CDR resolutions compared to the DR model.

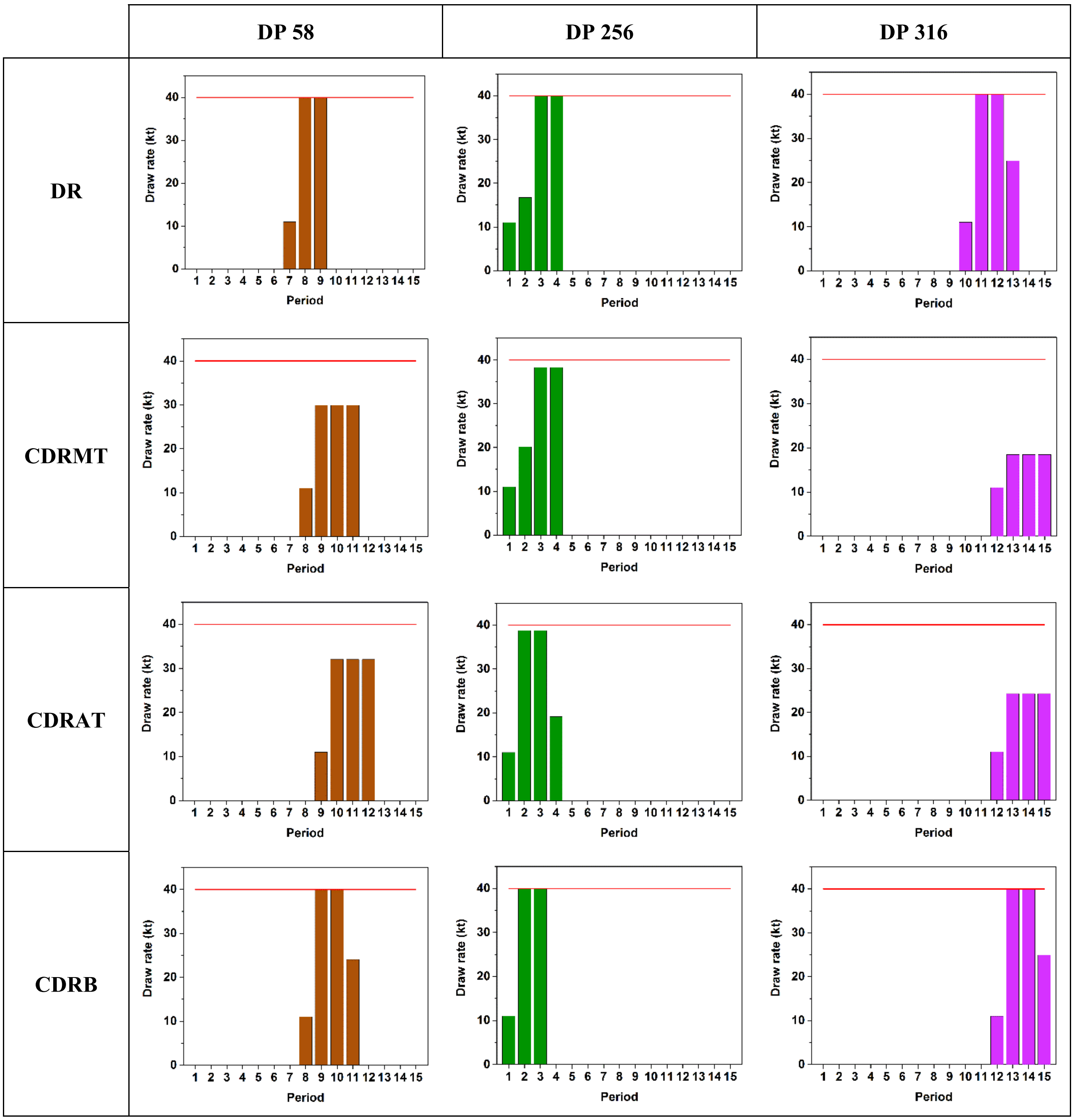

Figure 13. Obtained draw rate as result of optimization for different drawpoints in all models

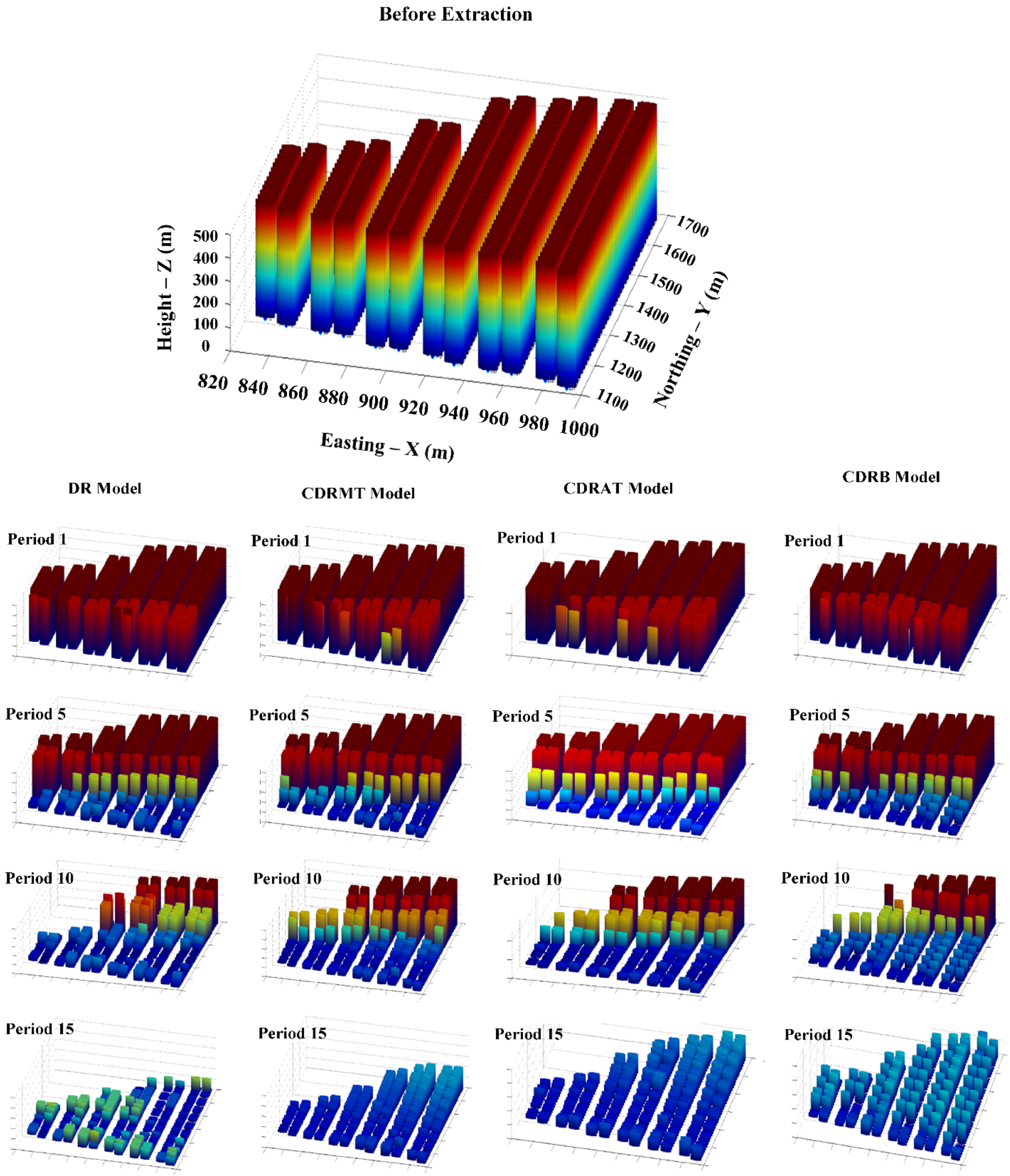

Figure 14. Height of drawpoints at the end of periods 1, 5, 10, and 15 for all resolutions

6. Conclusion

This paper presented a practical draw control system based on some conditional approaches to manage draw rates of block-cave operations. The presented model maximizes the NPV subject to all operational and geotechnical constraints. The MILP formulation for a block-cave production schedule was developed, implemented, and tested in the CPLEX/IBM environment. To manage drawpoint production, a draw rate constraint based on the PRC was established. Three different resolutions' responses to conditional constraints were added to the exact management system of PRC to control the depletion rate among all drawpoints in the caved area.

The draw control system was classified in four alternative resolutions to be modelled and practiced by all mines according to their draw requirements. Among these four resolutions, the depletion rate from the draw column in CDRAT and CDRMT models follows an even extraction pattern. The CDR models by controlling depletion rate among drawpoints, number of active drawpoints, and production grade during the mine life in compared to the DR model seek to deplete broken rock from the draw columns evenly. The CDRAT model depletes more evenly than all of the other models. Although the DR model reduces unfavorable displacement and dilution in block-cave mining, the CDR models work better and reduce unfavorable displacement more than the DR model. The CDR constraints increase the size of the model and solving time but the obtained NPVs are acceptable and the generated schedule is practical and realistic.

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