Stochastic Optimization of Block Cave Production Scheduling with Material Flow Uncertainty

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ABSTRACT

Block-cave mining has become more popular in the last few years, because of its lower operating costs and less waste removal requirements, the trend is expected to continue. Production scheduling, as one of the most important steps in any mining project, can be complicated for block caving mining because of the material flow and its uncertainties. The uncertainties should be considered within the production schedule, otherwise, the production schedule could be far from the real operations. This research uses stochastic optimization for production scheduling in block-cave mining. The proposed model maximizes the net present value of the mining project while minimizing the production grade deviations from a target grade. A number of scenarios are considered to capture the material flow uncertainties. Testing the model for a real case block-cave mining operation shows that the proposed model can take the material flow uncertainties into the production schedule in order to achieve more reliable plans; the optimum production schedule is accomplished based on different scenarios which can happen in the real operations. The model also calculates the optimum height of draw as part of the optimization.

1. Introduction

Any planning and financial analysis in a mining project depends on production schedule in which the amount of ore and waste removal in each period of time is determined. An optimum realistic production schedule can significantly improve the overall practicality and profitability of the project. Block-cave mining operation is involved with uncertainties which cannot be ignored in the production scheduling; while the caving is occurring, the flow of material (which happens because of the gravity) can be unpredictable. This will result in grade and tonnage uncertainties in the production during the life of mine. Numerical methods are useful tools to model the material flow. With stochastic optimization, it is possible to capture the uncertainty of material flow within the production schedule.

Production schedule in a block-cave mining operation can be investigated from different levels of resolutions: cluster level, drawpoint level, or slice level (Pourrahimian, et al., 2013). In this research, the slices are the smallest production units. The output of the production schedule at this level would be the periods in which each of the slices within a draw column is extracted and sent to the processing plant. These decisions are made based on the defined goal(s) in the objective function while considering the limitations of the operations as the constraints of the model. The proposed production scheduling model is a stochastic optimization model in which the net present value of the project is maximized during the life of the mine while the deviations from a target production grade are minimized. Different scenarios of grades for the slice model are generated to analyze the uncertainty of the production grade which exists because of the material flow during production.

2. Block caving

These days, most surface mines work in a higher stripping ratio than in the past. In the following conditions, a surface mine can be less attractive to operate and underground mining is used instead: (i) too much waste has to be removed in order to access the ore (high stripping ratios), (ii) waste storage space is limited, (iii) pit walls fail, or (iv) environmental considerations could be more important than exploitation profits (Newman, et al., 2010). Among underground methods, block-cave mining, because of its high production rate and low operation cost, could be considered an appropriate alternative. Projections show that 25 percent of global copper production will come from underground mines by 2020. Mining companies are looking for an underground method with a high rate of production, similar to that of open-pit mining. Therefore, there is an increased interest in using block-cave mining to access deep and low-grade ore bodies. A schematic view of block cave mining is shown in Fig 1.

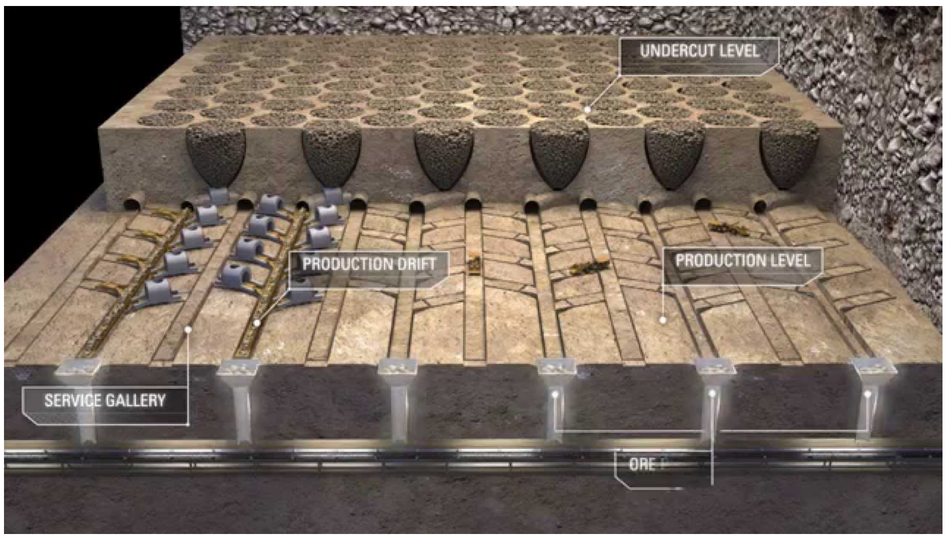

Fig 1. Block cave mining, LHD method (Caterpillar, 2015))

3. Literature review

There is a significant amount of research on production scheduling in mining operations, mostly in open-pit mining (Newman, et al., 2010). In block-cave mining, the production schedule determines which drawpoints to be opened/closed in each period, which slices to be extracted, what the best direction for mining development is, and what is the production rate and the production grade during the life of mine. A detailed literature review of production scheduling in block-cave mining can be found in (Khodayari & Pourrahimian, 2015).

Production scheduling in block-cave mining is more complicated to be optimized, mainly because of the material flow and its uncertainties. Researchers have also been trying to model the flow of material and how it can impact the production in cave mining for almost three decades. Numerical models (Alford (1978); R. L. Castro, et al. (2009); Edward Pierce (2010)), pilot tests (Raúl L. Castro, et al. (2014);Jin, et al. (2017)), and full scale experiments (Power (2004); Brunton, et al. (2016); Garcés, et al. (2016)) have been used to study the flow of the material. Pilot models have many limitations and in most cases cannot describe the behavior of the flow. Full-scale methods are usually expensive to use. Numerical models can be more efficient and less expensive if they are properly modeled. Gibson (2014) tried to use Pascal cone to understand the probabilities of blocks moving down as the production occurs in caving operations. Although his model was dependent on the cell size and the probabilities, it was shown that stochastic models can be used to present the behavior of material flow.

This research proposes a stochastic optimization model in which, the uncertainty of the material flow which results in production grade uncertainty, is brought to the production scheduling optimization. This optimum production schedule will not only maximize the NPV of the project but also minimizes the deviation of the production grade from a target grade in the considered scenarios.

4. Modeling

The proposed model maximizes the NPV of the mining project during the life of mine while trying to minimize the deviations of production grade from a defined target grade. To be able to capture the uncertainty of production in block-cave mining, the model is a stochastic optimization in which different scenarios are considered. The formulation of the objective function was inspired by a stochastic optimization model which was used by MacNeil and Dimitrakopoulos (2017) for determining the optimal depth of transition from open pit to underground mining. The scenarios are defined based on the grade distribution in the mine reserve. Each scenario represents one circumstance that can happen during the production based on the flow of the material. Fig 2 shows the flow of the material and how it can impact the production.

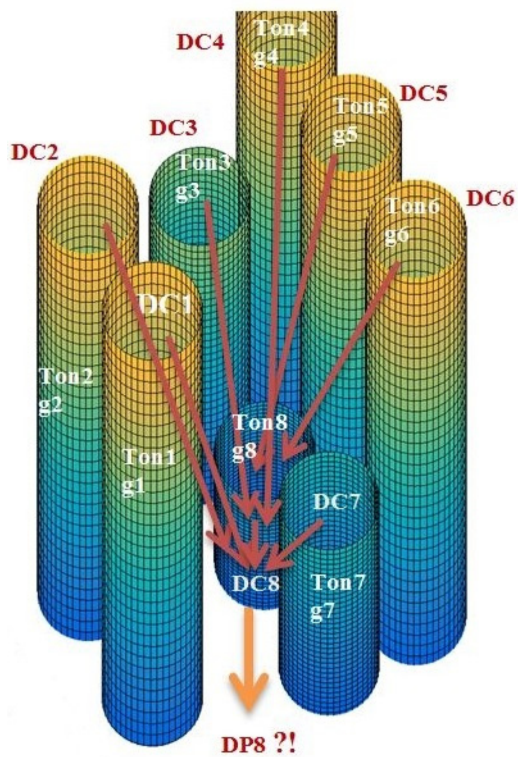

Fig 2. Flow of the material and its impact on the production grade

It can be seen that while extracting from a drawpoint, the material can move not only from the column above (DC8) but also from the columns in its neighborhood (DC1… DC7) into the intended drawpoint. This kind of movements of material during the caving is the main source of the uncertainties in the operations. A production schedule would be more realistic if the uncertainties are captured; this research's aim is to optimize the production schedule while capturing the material flow uncertainties. As it was mentioned, the decision units for the production schedule are the slices; the slice model is built based on the estimated block model, the column above each drawpoint is divided into slices. In this section, the mathematical programming model is presented in details.

4.1. Notation

- **Indices**

$t \in \{1,...,T\}$ Index for scheduling periods

$sl \in \{1,...,Sl\}$	Index for individual slices
$dp \in \{1,...,Dp\}$	Index for individual drawpoints
$s \in \{1,...,S\}$	Index for individual scenarios
$a \in \{1,...,A\}$	Index for the adjacent drawpoints

- **Variables**

$X_{sl}^{t} \in [0,1]$	Binary decision variable that determines if slice *sl* is extracted in period *t* [$X_{sl}^{t} = 1$] or not [$X_{sl}^{t} = 0$]
$Y_{dp}^{t} \in \{0,1\}$	Binary decision variable which determines whether drawpoint *dp* in period *t* is active [$DpAct_{dp}^{t} = 1$] or not [$DpAct_{dp}^{t} = 0$]
$Z_{dp}^{t} \in \{0,1\}$	Binary decision variable which determines whether drawpoint *dp* till period *t* (periods 1, 2,..,t) has started its extraction [$DpStart_{dp}^{t} = 1$] or not [$DpStart_{dp}^{t} = 0$]
$d_{us}^{t} \in \{0,\infty\}$	Excessive amount from the target grade (the metal content)
$d_{ls}^{t} \in \{0,\infty\}$	Deficient amount from the target grade (the metal content)

- **Model Parameters**

g_{sl}	Copper (Cu) grade of slice *sl*
g_{E}	Expected copper grade based on all scenarios
ton_{sl}	Ore tonnage of slice *sl*
t_{c}	Current period
sl_{dp}	Number of slices associated with drawpoint *dp*

- **Input Parameters**

$TarGrade$	The target grade of production which is defined based on the production goals and processing plant's requirements
$M_{\min}$	Minimum mining capacity based on the capacity of plant and mining equipment
$M_{\max}$	Maximum mining capacity based on the capacity of plant and mining equipment
$ActMin$	Minimum number of active drawpoints in each period
$ActMax$	Maximum number of active drawpoints in each period
M	An arbitrary big number
MinDrawLife	Minimum drawpoint life
MaxDrawLife	Maximum drawpoint life
DR_{Min}	Minimum draw rate

DR_{Max}	Maximum draw rate
$IntRate$	Discount rate
$RampUp$	Ramp up
$ScenNum$	Number of scenarios
$DP_{dp}^{t_c}$	Drawpoint depletion percentage which is the portion of draw column *dp* which has been extracted from drawpoint *dp* till period t_c
Price	Copper price ($/tonne)
Cost	Operating cost ($/tonne)
C_u	Cost (penalty) for excessive amount ($)
C_l	Cost (penalty) for deficient amount ($)

4.2. Objective function

The objective function is defined as follows:

$$Maximize \ \sum_{t=1}^{T}\sum_{sl=1}^{Sl} E\{(NPV_{sl}^{t})\}X_{sl}^{t} - \sum_{t=1}^{T}\sum_{s=1}^{S}\{Grade\ deviations\}_{s}^{t}$$
$$= \sum_{t=1}^{T}\sum_{sl=1}^{Sl}\frac{(Price\times Rec\times(g_{Esl}\times ton_{sl}/100))-(Cost\times ton_{sl})}{(1+IntRate)^{t}} * X_{sl}^{t}$$
$$-\sum_{t=1}^{T}\sum_{s=1}^{S}\frac{1}{S}\left(\frac{d_{ls}^{t}\times c_{l}+d_{us}^{t}\times c_{u}}{(1+IntRate)^{t}}\right) \quad (1)$$

The first part of the objective function maximizes the NPV of the project during the life of mine, which is the optimum sequence of extraction of the slices in the mine reserve. The second part minimizes the deviations of the production grade from the target grade by allocating a penalty to the deviations that might happen in different scenarios.

4.3. Constraints

Operational and technical constraints of block-cave mining operations are considered to control the outputs of the optimizations model. Number of decision variables depends on the number of drawpoints and the number of slices in each drawpoint.

- *Logical constraints*

There are two sets of binary decision variables in the proposed model, which will be required for defining different constraints. Logical constraints connect the continuous decision variables to the binary ones; each set contains two inequality equations.

$$\text{Set 1: } Y_{dp}^{t}\in\{0,1\},\begin{Bmatrix} dp\in Dp \\ t\in T\end{Bmatrix}$$

$$\forall t\in T\ \&\ dp\in Dp\rightarrow \quad Y_{dp}^{t} - M*\sum_{sl=1}^{sl_{dp}} X_{sl}^{t}\le 0 \quad (2)$$

$$\forall t\in T\ \&\ dp\in Dp\rightarrow \quad \sum_{sl=1}^{sl_{dp}} X_{sl}^{t} - M*Y_{dp}^{t}\le 0 \quad (3)$$

$$\text{Set 2: } Z_{dp}^{t} \in \{0,1\}, \begin{Bmatrix} dp \in Dp \\ t \in T \end{Bmatrix}$$

$$\forall\, dp \in Dp \rightarrow DP_{dp}^{t_c} = \sum_{t=1}^{t_c} Y_{dp}^{t} \tag{4}$$

$$\forall t \in T \;\&\; dp \in Dp \rightarrow \quad DP_{dp}^{t_c} - M * Z_{dp}^{t} \le 0 \tag{5}$$

$$\forall t \in T \;\&\; dp \in Dp \rightarrow \quad Z_{dp}^{t} - M * DP_{dp}^{t_c} \le 0 \tag{6}$$

- *Mining Capacity*

Mining capacity is limited based on the production goals and the availability of equipment.

$$\forall t \in T \rightarrow \quad M_{\min} \le \sum_{sl=1}^{Sl} ton_{sl} \times X_{sl}^{t} \le M_{\max} \tag{7}$$

- *Production grade*

This constraint ensures that the production grade is as close as possible to the target grade in different scenarios. The deviations from the target grade for all scenarios in different periods of production during the life of mine are considered for this constraint.

$$\forall t \in T \,\&\, \forall s \in S \rightarrow \quad \sum_{sl=1}^{Sl} (g_{sl} - G_{tar}) \times ton_{sl} \times X_{sl}^{t} + d_l - d_u = 0 \tag{8}$$

- *Reserve*

This constraint makes sure that not more than the mining resources can be extracted, the output of the model would be the mining reserve.

$$\forall sl \in Sl \rightarrow \quad \sum_{t=1}^{T} X_{sl}^{t} \le 1 \tag{9}$$

- *Active drawpoints*

A limited number of drawpoints can be in operation at each period of time; the mining layout, equipment availability, and geotechnical parameters can define this constraint.

$$\forall t \in T \rightarrow \quad ActMin \le \sum_{dp=1}^{Dp} Y_{dp}^{t} \le ActMax \tag{10}$$

- *Mining precedence (horizontal)*

The precedence is defined based on the mining direction in the layout. Production from each drawpoint can be started only if the drawpoints in its neighborhood which are located ahead (based on the mining direction) are already in production. Equation (11) presents this constraint.

$$\forall dp \in Dp \;\&\; t \in T \rightarrow \quad A * Z_{dp}^{t} \le \sum_{a=1}^{A} Z_{a}^{t} \tag{11}$$

Where *A* is the number of drawpoints in the neighborhood of drawpoint *dp* which are located ahead (based on the defined direction) and *Z* is the second set of binary variables.

- *Mining precedence (vertical)*

The sequence of extraction between the slices within the draw columns during the life of the mine is defined by this constraint.

$$\forall dp \in Dp \ \ \& \forall sl \in Sl \ \ \& \ \ t \in T \rightarrow \qquad X_{sl}^{t} \leq \sum_{t=1}^{t_c} X_{sl-1}^{t} \tag{12}$$

This equation ensures that in each period of *t*, slice *sl* in the draw column associated with drawpoint *dp,* is extracted only if slice *sl-1* beneath it is already extracted in the periods before or at the same period t_c.

- *Continuous mining*

This constraint guarantees a continuous production for each of the drawpoints during the life of mine. In other words, if a drawpoint is opened, it is active in consecutive years (with the minimum draw rate of DR_{Min}) till it is closed.

$$\forall dp \in Dp \,\&\, t \in T \rightarrow \qquad Y_{dp}^{t} \leq DpAct_{dp}^{t-1} + (1 - Z_{dp}^{t}) \tag{13}$$

- *Draw rate*

The total production of each drawpoint in each period of *t* is limited to a minimum and maximum amount of draw rate.

$$\forall dp \in Dp \,\&\, t \in T \rightarrow \qquad DR_{Min} \times Y_{dp}^{t} \leq \sum_{sl=1}^{sl_{dp}} ton_{sl} \times X_{sl}^{t} \leq DR_{Max} \tag{14}$$

- *Draw life*

Drawpoints are allowed to be in operation during a certain period of time which is called draw life. The draw life is limited to the minimum and maximum years of operations by the following equation:

$$\forall dp \in Dp \rightarrow \qquad MinDrawLife \leq \sum_{t=1}^{T} Y_{dp}^{t} \leq MaxDrawLife \tag{15}$$

5. Solving the optimization problem

The proposed stochastic model has been developed in MATLAB (TheMathWorksInc., 2017), and solved in the IBM ILOG CPLEX environment("IBM ILOG CPLEX Optimization Studio," 2017) CPLEX uses branch-and-cut search for solving the problem to achieve a solution within the defined mip gap (or the closest lower gap). The case study in this research was solved by gap of 3% (a feasible integer solution proved to be within three percent of the optimal).

6. Case study

The proposed model was tested on a block-cave mining operation with 102 drawpoints. It was a copper-gold deposit with the total ore of 22.5 million tonnes and the weighted average grade of 0.85% copper. The draw column heights vary from 320 to 351 meters. Fig 3 and Fig 4 show the drawpoints layout (2D) and a conceptual view of the draw columns (3D). Each draw column consists of slices with the height of 10 meters (33 to 36 slices for each draw column). In total, the model was built on 3,470 slices. The mine life of 10 years with the maximum production of 2 million tonnes of ore per year was considered. The details of the input parameters for the case study are presented in Table 1.

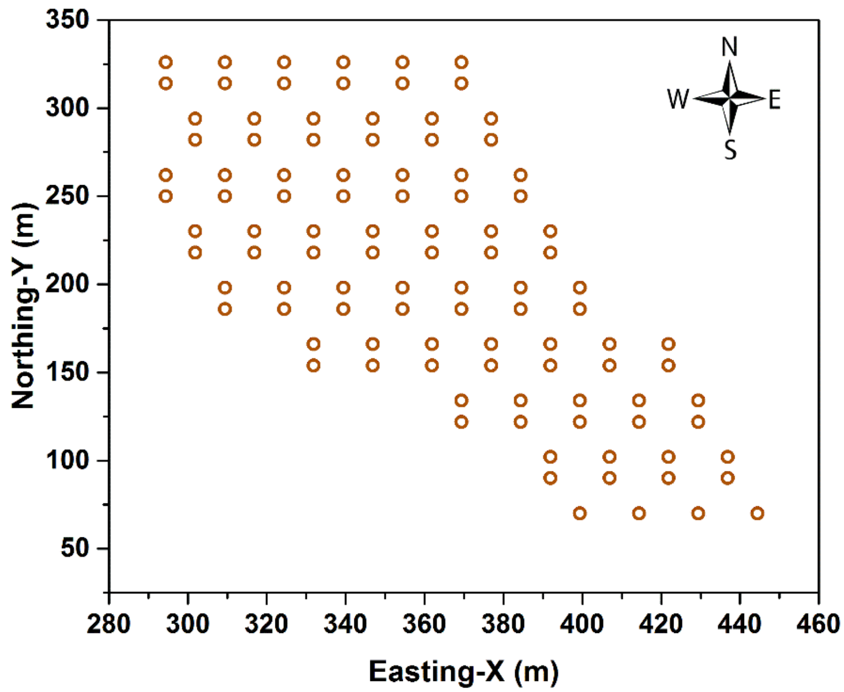

Fig 3. Drawpoints layout (circles represent drawpoints)

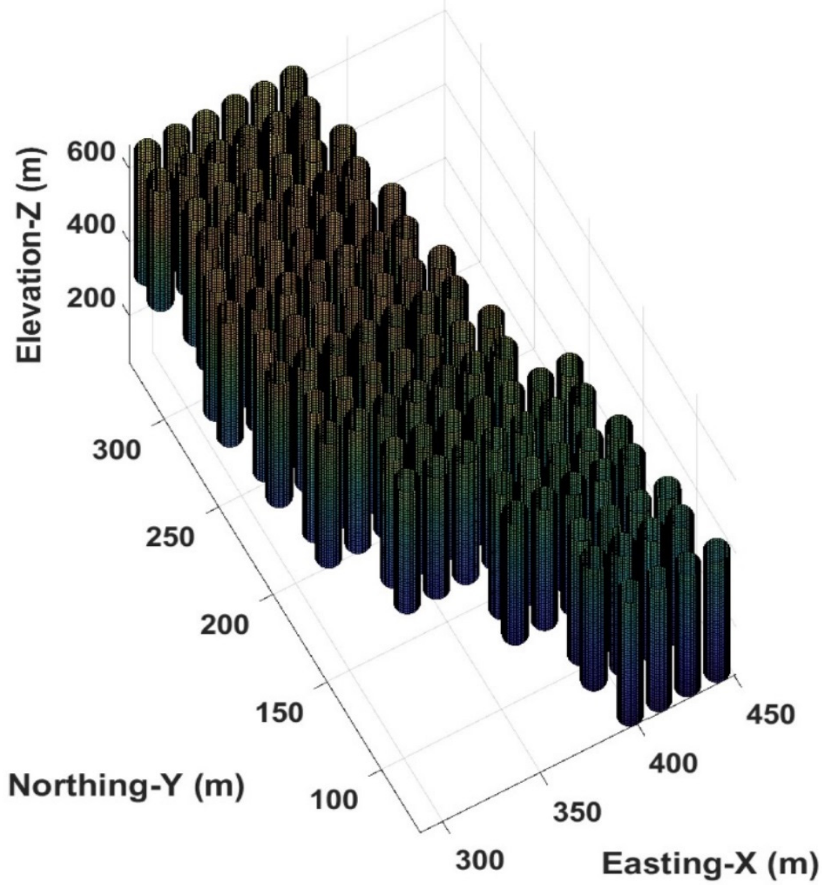

Fig 4. A conceptual view of the draw columns

Table 1. Scheduling parameters for the case study

Parameter	Value	Unit	Description
T	10	Year	Number of periods (life of the mine)
G_{min}	0.5	%	Minimum production average grade for Cu per each period
G_{max}	1.6	%	Maximum production average grade for Cu per each period
G_{Tar}	1.3	%	Target production grade (Cu)
M_{min}	0	Mt	Minimum mining capacity per period
M_{st}	0.5	Mt	Mining capacity at the first year of production
M_{max}	2	Mt	Maximum mining capacity per period
Ramp-up	3	Year	The time period in which the production is increased from starting amount to maximum
ActMin	0	-	Minimum number of active drawpoints per period
ActMax	70	-	Maximum number of active drawpoints per period
MIPgap	5	%	Relative tolerance on the gap between the best integer objective and the objective of the best node remaining
Radius	8.2	m	The average radius of the drawpoints
Density	2.7	t/m^3	The average density of the material
M	100	-	An arbitrary big number
MinDrawLife	0	Year	Minimum life of drawpoints
MaxDrawLife	4	Year	Maximum life of drawpoints
DR_{Min}	13,000	Tonne/year	Minimum draw rate
DR_{Max}	75,000	Tonne/year	Maximum draw rate
Recovery	85	%	Recovery of the processing plant
Price	5,000	\$/tonne	Copper price per tonne of copper
Cost	15	\$/tonne	Operating cost per tonne of ore (Mining+Processing)
IntRate	10	%	Discount rate
S	50	-	Number of scenarios

In this case study, different scenarios were defined by generating random numbers in MATLAB; a linear function was defined based on the original grades of the slices to produce different scenarios. The model was built in MATLAB (R2017a) and solved using IBM/CPLEX (Version 12.7.1.0). Also, the model was solved as a deterministic model in which there was no penalty for deviation from defined target grade. The production grade in different scenarios (stochastic model) and deterministic model is shown in Fig 5. It can be observed that in all scenarios the production grade is as close as possible to the defined target grade during the life of mine. The deterministic model tries to maximize the NPV and the higher-grade ore is extracted at the first years of production and then the lower grades at the latter years. Ore production is regulated by the mining capacity constraint and the ramp-up and ramp-down are almost achieved in both stochastic and deterministic models (Fig 6 & Fig 7).

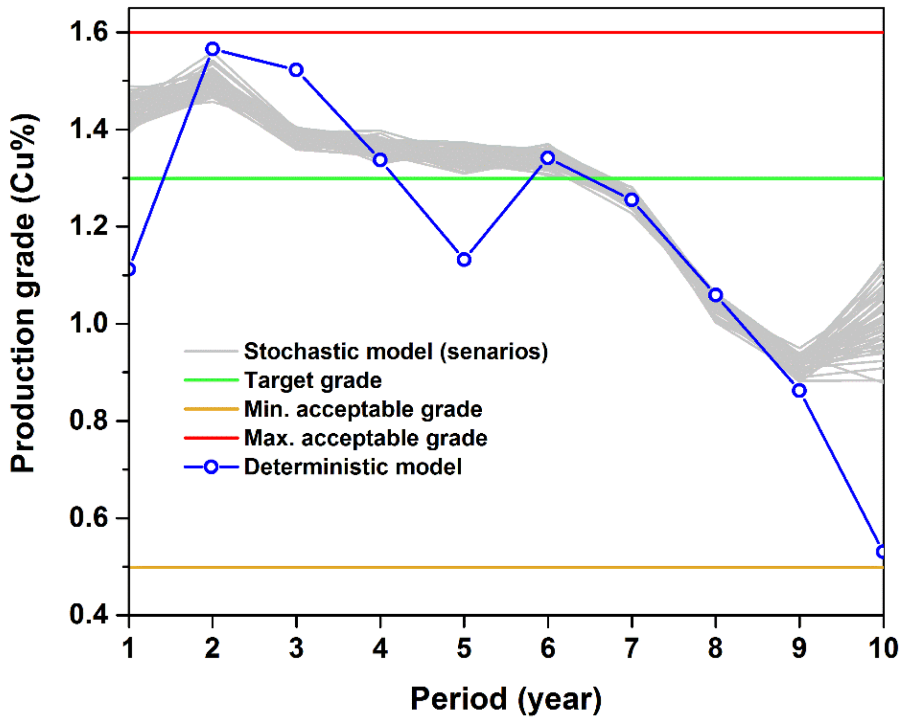

Fig 5. Average production grade based on the stochastic and deterministic models

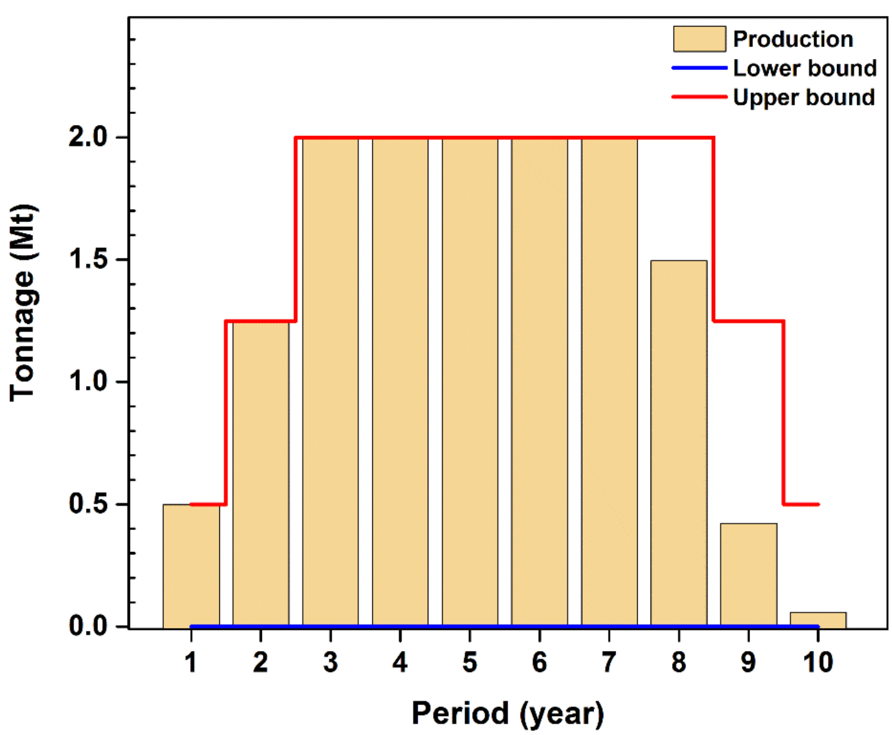

Fig 6. Ore production during the life of mine (stochastic model)

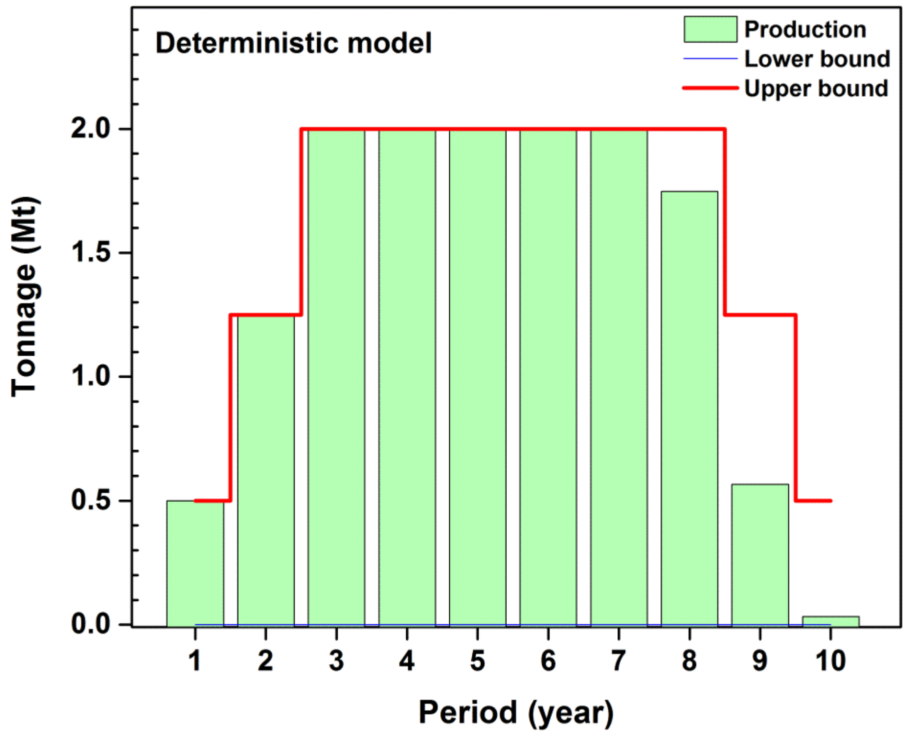

Fig 7. Ore production during the life of mine (deterministic model)

Horizontal precedence, which is the sequence of extraction between drawpoints, was achieved for both of models based on the defined v-shaped precedence (Fig 8).

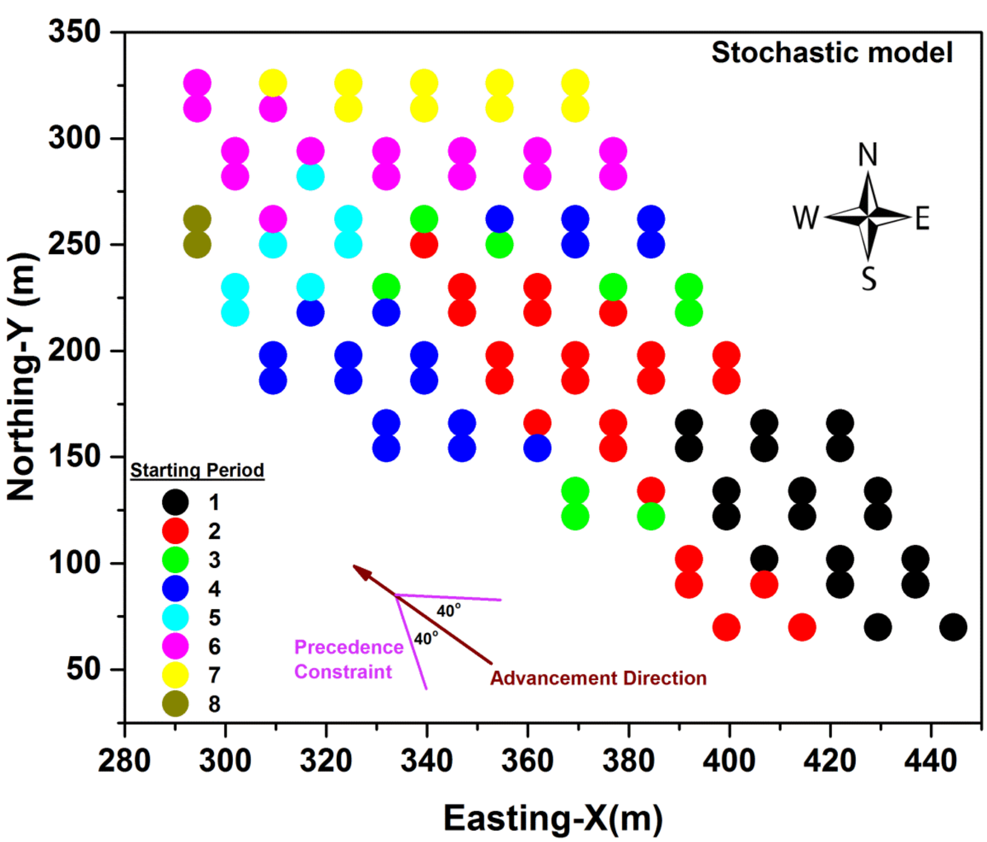

Fig 8. Sequence of extraction for drawpoints based on the stochastic model (2D precedence)

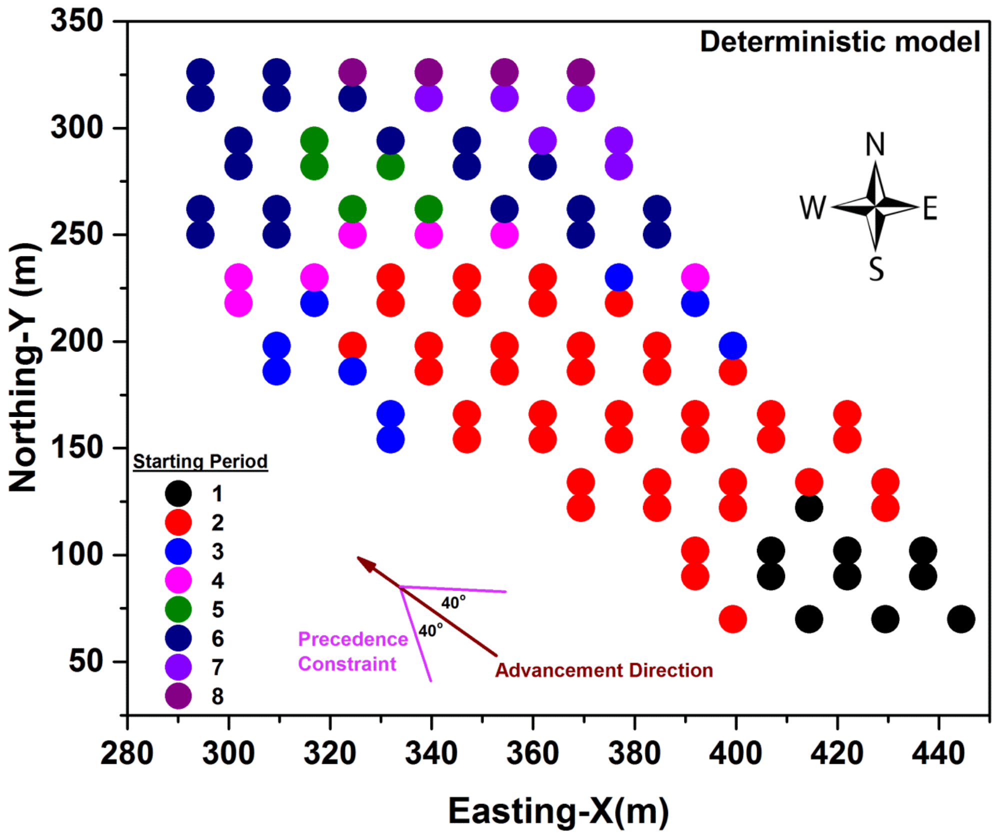

Fig 9. Sequence of extraction for drawpoints based on the deterministic model (2D precedence)

Vertical precedence determines the sequence of extraction between slices in each of draw columns. Fig 10 shows the sequence of extraction in draw column 75. It can be seen that extraction from this drawpoint starts from year 5 and ends at year 8; the sequence of extraction is well maintained and the production is continuous which means both the vertical precedence and continuous mining constraints were satisfied. The original height of draw column 75 is 330.1 meters with the total ore of 212,397 tonnes which contains 34 slices. Based on the optimization results (Fig 10), the optimum height of draw (best height of draw) was 260 meters with the optimum draw tonnage of 168,650; this means that 26 out of 34 slices are extracted during the life of mine.

Number of active drawpoints and number of new drawpoints which are opened in each year for both models are presented in Fig 12 and Fig 13. Comparing the new drawpoints to be opened in each year for two models, the stochastic model does not suggest big changes from one year to another while the deterministic model shows such a pattern. In other words, the results of the stochastic model is more practical than the deterministic model.

A brief comparison among the original ore resource model, the results of the deterministic model, and the results of the stochastic model is presented in Table 2. For this case study, the mining reserve and the NPV of the project for both models are almost the same (-2% in ore reserve and 0.7% for NPV). The stochastic model takes longer to solve, it is because of the number of decision variables and also the number of constraints. The stochastic model has more decision variables because of the deviation variables and more constraints for of the scenarios.

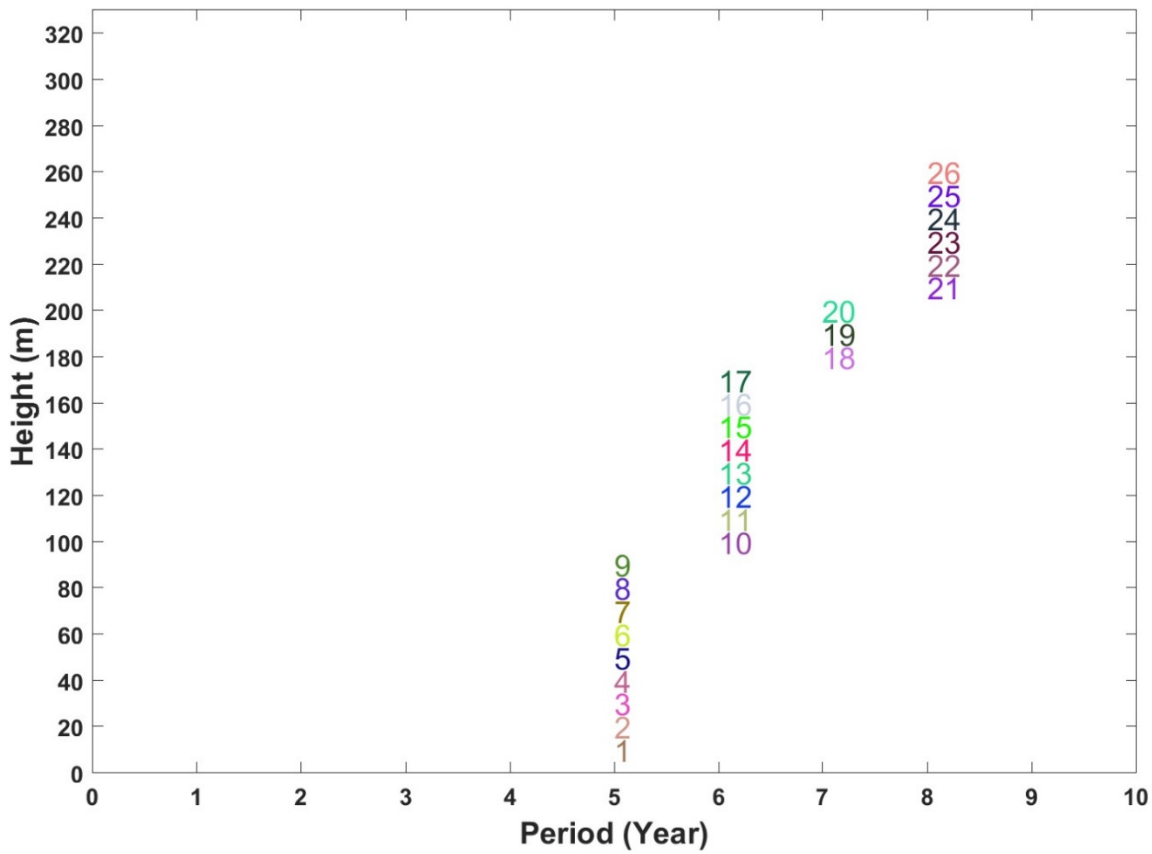

Fig 10. Sequence of extraction for slices in draw column associated with drawpoint 75 (numbers represent ID of slices within the draw column)

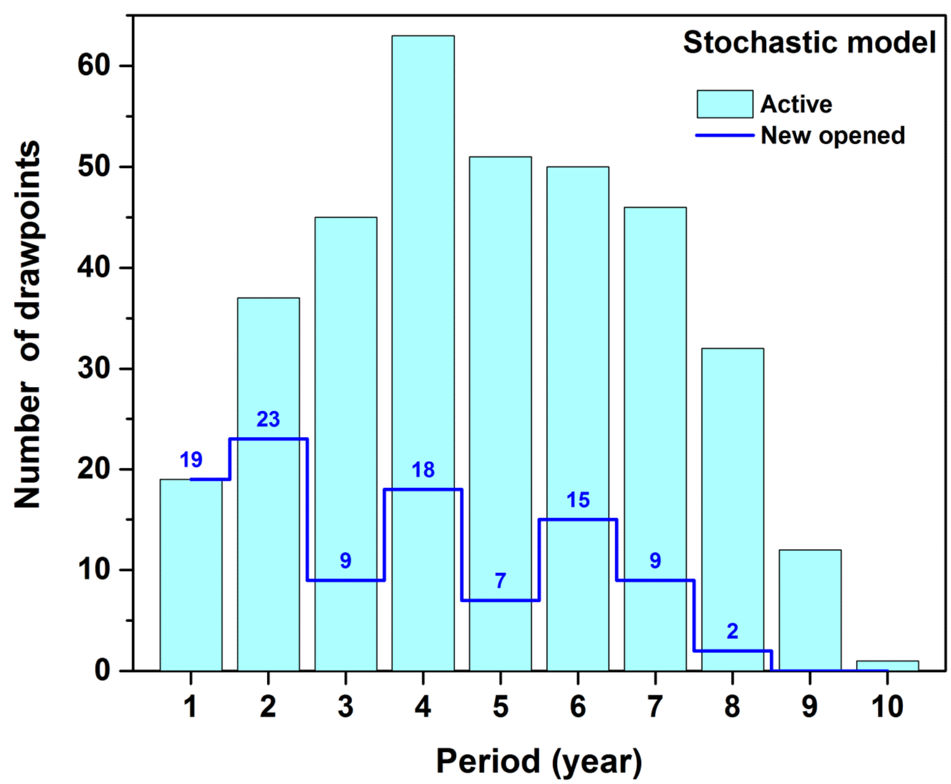

Fig 11. Active and new opened drawpoints for the stochastic model

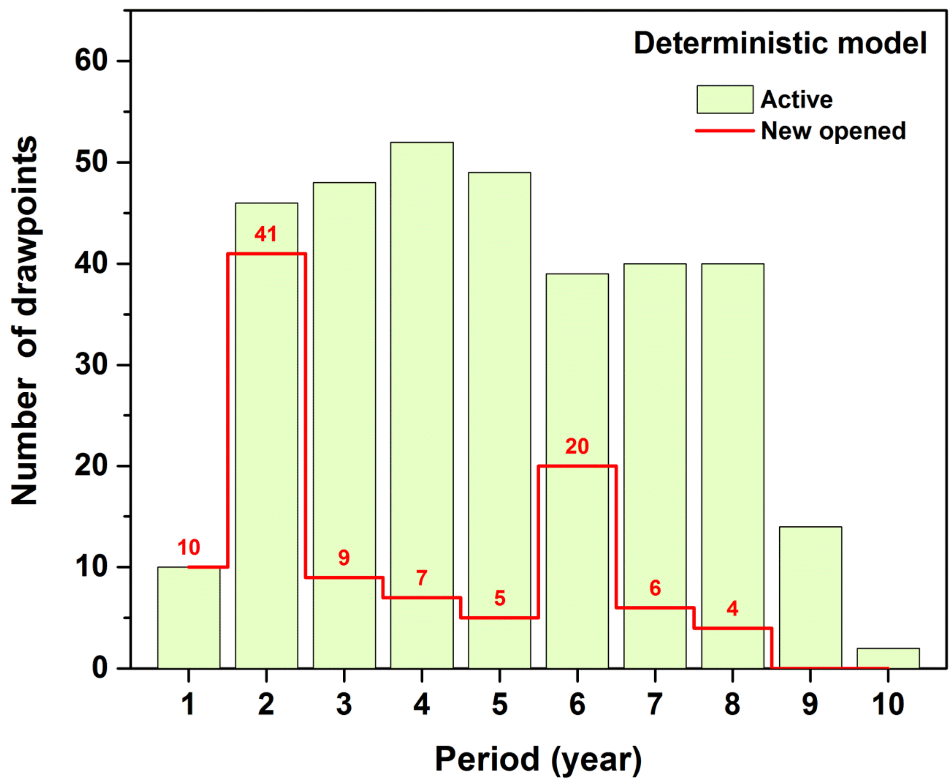

Fig 12. Active and new opened drawpoints for the deterministic model

Table 2. Comparing the original model with the deterministic and stochastic models results

Comparison item	Original Model (Mine Resource)	Mine Reserve	
		Deterministic Model	Stochastic Model
Ore tonnage (Mt)	22.5	14	13.7
Number of slices	3,470	2,160	2,106
Average weighted grade (%)	0.85	1.28	1.3
Height of draw in individual draw columns	320-351	30-320	30-310
Number of slices in individual draw columns	33-36	3-32	3-31
NPV (M$)	-	357.1	359.7
Solution time (Seconds)	-	5,292	55,601

7. Discussion

Production scheduling for block-cave mining operations could be hard because of the uncertainties which are involved in the operations. A stochastic optimization model was proposed in this research in order to maximize the NPV of the project while minimizing the production grade deviations from the project's targets. Results show that stochastic models can be so effective in production scheduling for block-cave mining: the production goals are achieved, the constraints of the mining project are satisfied, the uncertainty of the material flow is captured, optimum height of draw (best height of draw) is calculated as part of optimization, as well as the net present value of the project is maximized. Unlike deterministic models which do not consider the uncertainty of the material flow, stochastic models can maximize the profitability of the project while minimizing the unexpected events. The future work will be on consideration of both grade and tonnage deviations in the production schedule, and also generating more realistic scenarios.

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