A review of models and algorithms for strategic mining options optimization

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Abstract

In major mining projects, deviations from optimal mine plans will result in significant financial losses, future financial liabilities, delayed reclamation and resource sterilization. It is important that the strategic mine plan makes optimum use of available resources and provide continuous quality ore to drive sustainable mining and profitability. This requires the development of a well-integrated strategy of mining options for open pit and/or underground mining and their interactions. However, current tools and methodologies used in the mining industry are not adequate in dealing with the complexity of subjecting a deposit to rigorous stochastic mining options optimization with a measure of optimality. Development of innovative technologies, quantification of uncertainty and optimization in strategic mine planning plays a significant role in reducing financial risk and environmental footprints, and promoting sustainable development through improved resource governance and total mine reconciliation. This research reviews existing models and algorithms that evaluate open pit and/or underground mining options optimization. Extensive literature review and gap analysis matrix are used to identify the associated limitations, and opportunities for improvement are outlined.

1. Introduction

Mining is the process of extracting a beneficial naturally occurring resource from the earth crust (Caro et al., 2007; Newman et al., 2010). When an orebody extends from surface to 'great depth', the part of the orebody close to the surface is usually mined with an open pit (OP) to generate early revenue, while the deeper part is subsequently extracted using a cheaper underground (UG) mining alternative (Opoku and Musingwini, 2013) to reduce stripping ratio. As open pit mining deepens, the stripping ratio typically increases, increasing the overall mining cost. As a result, companies often strategically transition between surface and underground to maximize project value and increase resource extraction ratio (Breed, 2016). Fig. 1 is a schematic representation of strategic mining options or the transition problem from open pit to underground mining. The term mining options optimization has been used by researchers and professionals to refer to the initiatives or choices undertaken in the extractive industry to expand, change, defer, abandon and adopt strategies for a mining method(s) and sometimes investment opportunities; based on changing economic, technological or market conditions (Shinobe, 1997; Bakhtavar et al., 2007; Bakhtavar et al., 2008; Bakhtavar et al., 2009; Roberts et al., 2009; Brady and Brown, 2012; Opoku and Musingwini, 2013; Roberts et al., 2013; Ben-Awuah et al., 2016; Marketwired, 2016; Inc., 2017).

Open pit mining usually features a relatively lower mining cost, higher stripping ratio and longer time to access ore (Koushavand et al., 2014; Ben-Awuah et al., 2016) while underground mining on the other hand features a higher mining cost, higher grade and earlier access to ore (Anthony, 2012; Pourrahimian et al., 2013; Terblanche and Bley, 2015; Ben-Awuah et al., 2016). Late stage cut-backs in open pit mining are generally more expensive than earlier stages, but underground mining costs are less likely to rise as much with depth. These late stage cutbacks often have long lead times between waste mining and ore extraction, and the discounting effect of the cash flows must be considered (Earl, 2013).

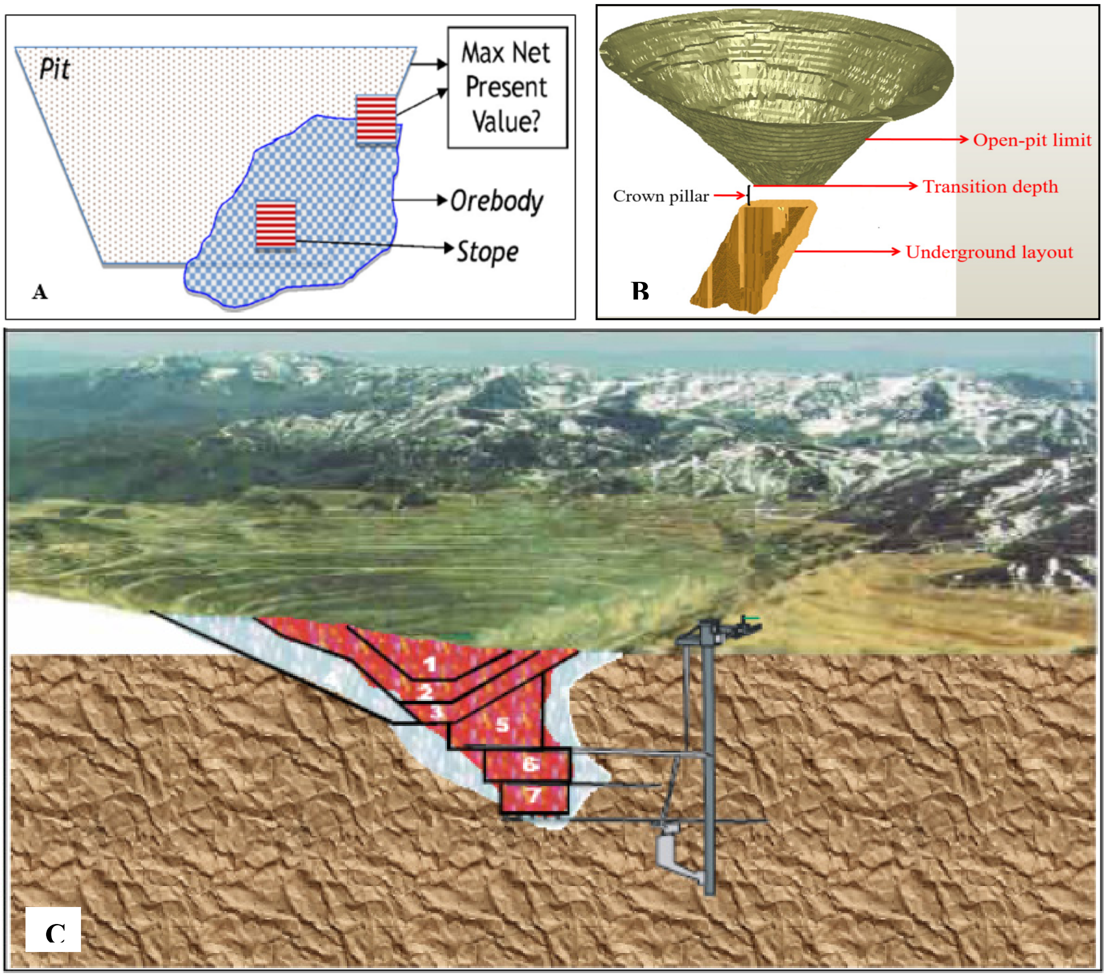

Fig. 1 Schematic representation of strategic mining options or the transition problem from open pit to underground mining [A – Ben-Awuah et al. (2016); B – Oraee and Bakhtavar (2010); C – King (2000)]

The problem of optimizing reserve exploitation depends largely on the mining option used in the extraction. Some mineral deposits have orebodies that extend from near the surface to several meters in depth. Such deposits can be amenable to either open pit mining or underground mining or both, in different variations and forms. This paper reviews relevant literature on algorithms and models for open pit – underground mining options optimization, and further identifies gaps and opportunities that can be explored for further research and implementation in the mining industry.

2. Summary of Literature Review

Based on the geometry and orientation of the orebody, open pit mining becomes more favorable than underground mining or vice versa or both. There is a depth within the mineralized zone where comparisons are made between ore extraction using surface mining methods or underground mining methods or both. This depth is broadly referred to as the transition depth or transition interface or cut-over point. According to Opoku and Musingwini (2013), the transition point is often determined anytime from project pre-feasibility stage to several years after commencement of the mining operation. Bakhtavar et al. (2009) commented that, the most sensitive problem for a deposit that has the potential to be mined by a combined method of open pit and underground is the determination of the optimal transition depth from open-pit to underground or vice versa.

Current strategic open pit and underground mining interface optimization models have been developed mainly based on determining the transition point or depth between open pit mining and underground mining. These models mainly focus on investigating how an underground mining operation can be exploited after an open pit mine or combined with an existing open pit operation. (Ben-Awuah et al., 2016). Acknowledging notable challenges and shortfalls, several researchers have employed techniques, algorithms and/or models to determine the transition depth (Bakhtavar et al., 2009; Opoku and Musingwini, 2013; Roberts et al., 2013; Dagdelen and Traore, 2014; Ordin and Vasil'ev, 2014; De Carli and de Lemos, 2015; King et al., 2016; MacNeil and Dimitrakopoulos, 2017) and the strategy for extracting these ore blocks (De Carli and de Lemos, 2015; Ben-Awuah et al., 2016; King et al., 2016; MacNeil and Dimitrakopoulos, 2017). According to Roberts et al. (2013), optimization of an open pit mine in conjunction with an existing high production underground operation is more complex. This challenge is faced by a growing number of operations throughout Australia and around the world and has not been fully addressed in the literature. Ordin and Vasil'ev (2014), King et al. (2016) and MacNeil and Dimitrakopoulos (2017) have developed mathematical programming models (MPMs) to handle the optimization of an open pit mine in conjunction with an underground mine for a specific orebody.

Available robust, risk-based and practical models and techniques to directly optimize the open pit-underground mining interface and interactions with integrated waste management are currently limited. A stochastic model that comprehensively and simultaneously schedules an optimized open pit mine, determines the transition interface and further schedules an optimized underground mine for an orebody will add significant value to the mining industry. Fig. 2 is a schematic representation of some research studies on the optimization of mining options in literature.

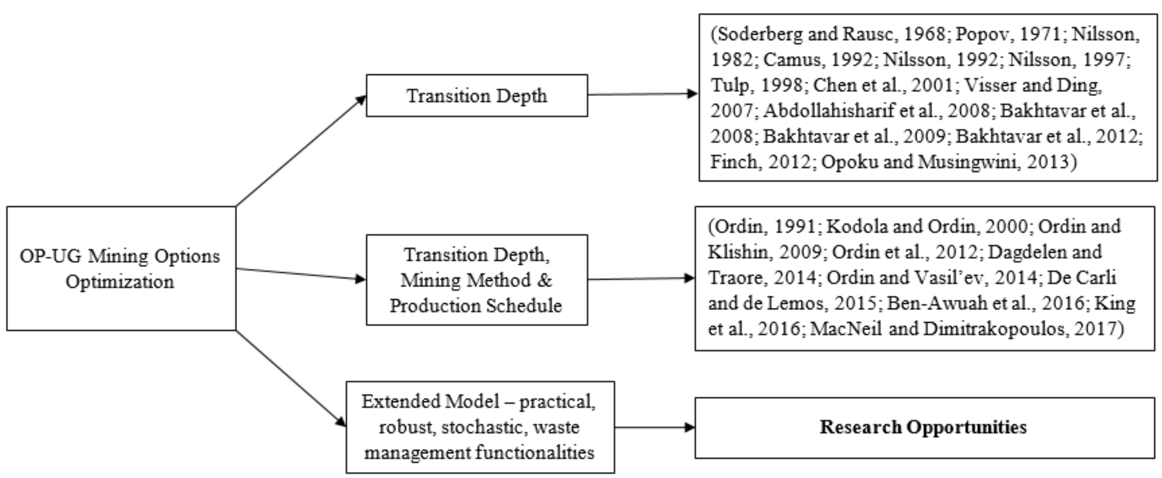

Fig. 2 Schematic representation of some research studies on the optimization of mining options

3. Open pit – underground (OP-UG) mining options

Historical assessment of mineral resource evaluations has demonstrated the sensitivity of project profitability to decisions based on mine planning (Ben-Awuah et al., 2016). Two main kinds of mining method exist when mining options are being considered: sequential and parallel mining. In sequential mining, the economic mineralization is continuous over depths that could be economically extracted by open pit and underground methods. The open pit and underground operations are competing for the same resource. In parallel mining, there is an opportunity to exploit a distinct independent deposit by both open-pit and underground operations simultaneously (Finch, 2012).

Mining options scenarios have been broadly grouped into three: (1) open pit mine to underground mine or underground mine after open pit mine, (2) concurrent open pit mine and underground mine, and (3) underground mine to open pit mine or underground mine after open pit mine. From the mining options above, the transitioning is the main challenge to the mine planner. From a practical point of view, planning for the transition requires a long lead-time as the implications on the ultimate pit and the underground design can be significant. This means that determination of the cut-over point and strategy should be thoroughly examined prior to the commencement of construction (Opoku and Musingwini, 2013). The transition problem is the determination of the optimal transition point with the aim of maximizing the project's value and resource utilization.

The decision to adopt any particularly mining option surely depends mainly on the economics of the project. Earl (2013) explained that, it is important to undertake rigorous analysis and model the transition zone over the widest range of conditions possible. Finch (2012) identified three typical approaches to determining the cut-over point. These are (1) biggest economic pit, (2) incremental undiscounted cashflow, and (3) automated scenario analysis. According to Earl (2013), it is worth exploring all options to reduce unit costs and minimize risk, thus, canceling that big cutback and changing to underground mining may provide one such avenue.

For the biggest economic pit approach, the open pit to underground cut-over is determined by focusing on the economic size of the open pit. Consideration of underground mining is secondary, and it is based on the remaining resource outside this pit. The biggest economic pit is the simplest and most common approach. It can be determined using any one of the several commercially available pit optimization software packages. The pit will terminate at the point that the marginal cost of waste stripping outweighs the marginal revenue generated by additional ore processing. (Finch, 2012).

For the incremental undiscounted cashflow approach, the marginal profit derived from the pit associated with depth decreases with depth. Given that underground mining profits are less dependent on depth, there will likely be a shallower point where the marginal profit of the underground exceeds that of the open pit. Using this method, the cut-over point is the depth at which the marginal profit from the open-pit is equal to the marginal profit from the underground. This is usually shallower than the largest economic pit method. This method can also be undertaken using commercially available software (Finch, 2012).

For the automated scenario analysis approach, the method, unlike the incremental undiscounted cashflow, accounts for discounting. As the underground mine operates at a higher cut-off grade than the open pit, it will deliver a higher grade and normally higher cashflow for the same throughput. Therefore, there is likely a discounted cashflow benefit from generating this cashflow early which may elevate the optimal transition point. The only way to test this is to complete schedules (which include open pit and underground mining) and derive an NPV for each potential transition point. Depending on the complexities of the mine, deriving a new schedule for each transition point can be very time consuming, and to test a reasonable number of transition points in a reasonable time, automated optimizing scheduling software should be used. The software should be able to handle both underground and open pit mine scheduling simultaneously, and should be able to develop an optimized schedule for each transition point. In this way, each schedule generated reflects the best possible schedule for a given transition point. Using this kind of software, a suite of transition points can be evaluated, and their results compared so that the point that offers the highest value can be chosen (Finch, 2012).

In recent researches on the problems of mining options, the automated scenario analysis approach has been employed in several modified ways by Bakhtavar et al. (2012), Opoku and Musingwini (2013), Ordin and Vasil'ev (2014), King et al. (2016) and MacNeil and Dimitrakopoulos (2017). Table 1 shows a matrix comparison of notable research on the OP-UG transition problem for the last decade. Discussions to these modifications have been reviewed in later sections.

Table 1: Notable research on OP-UG transition problem for the past decade

	Name of Author(s)	MacNeil and Dimitrakopoulos	King et al.	Ben-Awuah et al.	De Carli and de Lemos	Ordin and Vasil'ev	Dagdelen and Traore	Roberts et al.	Opoku and Musingwini	Bakhtavar et al.	Bakhtavar et al.
	Year	2017	2016	2016	2015	2014	2014	2013	2013	2012	2009
	Research Focus	Optimal Transition Depth - Instances	Optimal Transition Depth - Instances	Assessment of Transition Problem	Optimal Transition Depth - Instances	Optimal Transition Depth - Dynamic	Optimal Transition Depth	Assessment of Transition Problem	Optimal Transition Depth - Dynamic	Optimal Transition Depth	Optimal Transition Depth
Approach Used	Algorithm										Heuristic
Approach Used	Model	Stochastic Integer Model	MILP	MILP		Dynamic Programming - lag, trend, nonlinear	MILP - OptiMine® used to optimise the transition problem	MILP - OP & UG		(0-1) Integer Programming	
Approach Used	Software Application			Evaluator	Studio 3 and NPV Scheduler		OptiMine, Whittle, Studio 5D and EPS	Open pit Optimization - Blasor; Open Pit Schedule - COMET; UG Optimization - Evaluator	Open pit Optimization - Whittle; Open Pit Schedule - XPAC; UG Optimization - Datamine's MRO; Excel analysis		
Outputs / Performance / Transition Indicators	NPV	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Outputs / Performance / Transition Indicators	IRR			Yes	Yes						
Outputs / Performance / Transition Indicators	Mining Recovery				Yes						
Outputs / Performance / Transition Indicators	Avg. ROM Grade							Yes	Yes		
Outputs / Performance / Transition Indicators	Gold Price to Cost Ratio								Yes		
Outputs / Performance / Transition Indicators	Production Rate					Yes		Yes	Yes		
Outputs / Performance / Transition Indicators	Stripping Ratio				Yes	Yes			Yes		
Outputs / Performance / Transition Indicators	Mine Life				Yes	Yes	Yes	Yes			
Outputs / Performance / Transition Indicators	Revenue					Yes					
Outputs / Performance / Transition Indicators	Mining Cost							Yes			
Outputs / Performance / Transition Indicators	Production Schedule	OP & UG		OP, UG & OP-UG		Standalone for OP & UG	Standalone for OP & UG				
	Commodity	Gold		Gold-silver-copper	Gold	Coal mining - kimberlite pipe	Gold	Iron	Gold - 4 different deposits	Hypothetical	
	Notable Remarks	20 simulated grades used. Compared to a deterministic model - 9% better NPV							Deterministic not good		One level assumed as the crown pillar

3.1. Optimization models and algorithms for OP-UG mining options

Identifying the transition depth or interface in the transitional problem and the ore extraction strategy of any mining option study is very strategic. According to Ben-Awuah et al. (2016), the aim of long-term production scheduling is to determine the strategy, thus, the time and sequence of extraction and displacement of ore and waste, that maximizes the overall discounted net present revenue from a mine within the existing economic, technical and environmental constraints. Long-term production schedules define the mining and processing plant capacity, and expansion potential as well as management investment strategy. The problem of optimizing reserve exploitation depends largely on the mining option used in the extraction (Ben-Awuah et al., 2016). Significant value can be generated by rigorously investigating these mining options using optimization tools to arrive at the appropriate strategic plan that maximizes the overall NPV of the deposit (Epstein et al., 2012; Roberts et al., 2013; Ben-Awuah et al., 2016).

Kurppa and Erkkilä (1967) assessed the simultaneous mining between open pit and underground mining during the OP-UG mining at Pyhasalmi mine. They indicated that, the simultaneous mining was possible due to the geometry of the orebody being worked on. Luxford (1997) argued that cost usually drives the decision to make the transition because as the open pit waste stripping cost keeps rising with depth, there comes a time when the underground mining cost will be less than the open pit mining cost. Stacey and Terbrugge (2000) considered the OP-UG transition for an economically designed pit, which they argued, should have slopes that are close to their stability limits so that there is little scope for extending the open pit to greater depths, other than with a pushback. Finch (2012) dwelled much on the transition point evaluation so that the point which offers higher values can be chosen. He noted that, the transition point depends largely on transition indicators which are uncertain over time.

Roberts et al. (2013) followed three-step processes to determine the material that can be mined from underground ahead of the open pit advancement: (1) open pit optimization to determine an optimal open pit only sequence and schedule, (2) underground with open pit optimization to determine the discounted value of each block being mined from the open pit using the schedule generated from the (1). The discounted value was deducted from the potential underground mining value and the underground is optimized on the net objective, and (3) combined open pit and underground optimization is determined by integrating the open pit and underground sequences into an overall project schedule.

When optimizing a schedule for a single mining operation the value ranking of a block for mining is determined by variables such as the net value per ton, net mill return or net smelter return. However, Roberts et al. (2013) used opportunity cost represented by open pit mining to rank the variables, thus, if the discounted value of a block is greater when mined by open pit then it should not be extracted from underground. Therefore, to rank blocks correctly for an optimized underground with open pit strategy, a measure of 'incremental value' (IV) was developed. When considering a single block, the discounted value that the block adds to the overall operation, its IV will be equal to the discounted value when extracted by the underground mine, minus the discounted value of the block if it were to be extracted by the open pit mine (Roberts et al., 2013).

The problem on optimization of the depth for transition from open pit to underground mining and the design capacity of the open pit and underground mine based on the condition of maximum NPV while accounting for the lag factor has been solved using the optimality principle of dynamic programming by Ordin and Vasil'ev (2014). The mine design capacity optimization procedure based on lag modeling has been developed, tested by design institutions such as Giprougol, Kuzbassgiproshakht and Yakutniproalmaz, and used in some projects on open pit and underground mine planning (Ordin, 1991; Kodola and Ordin, 2000; Ordin and Klishin, 2009; Ordin et al., 2012). Ordin and Vasil'ev (2014) used lag models to account for influence of time lags on future profit and allow estimating economic benefit of freezing of investment in the mine construction period

Dagdelen and Traore (2014) used an iterative approach by evaluating a set of selected transition depths through optimizing the life of mine production schedules of both the open pit and underground mines using mixed linear integer programming (MILP) techniques, a mathematical programming model. The authors begin by using Geovia's Whittle software to generate a series of pits which provide an ultimate pit contour. The locations of the ultimate pit and crown pillar provide a basis for the underground mine design. Optimized life of mine production schedule is then created to determine yearly cash flow and resulting NPV. This procedure is repeated for progressively deeper transition depths until the NPV observed in the current iteration is less than what was seen for a previously considered transition depth, at which point the authors conclude that the previously considered depth, with a higher NPV, is optimal (Dagdelen and Traore, 2014). The MILP model categorically factored the different rock types (3) and ore stockpiling (3 – oxide, transition and fresh) in the model.

To assess the mining option to employed, De Carli and de Lemos (2015) used the premise suggested by Bakhtavar et al. (2008) that the overall stripping ratio (OSR) in the deposit must be smaller than the allowable stripping ratio (ASR) for the feasibility of the open pit mining method to underground mining method. The authors used Studio 3 to determine the ultimate optimal pit limit and the NPV Scheduler was used to generate the production schedule of the ultimate pit. By integrating the use of the mining software, the depth of transition was analyzed using cyclical calculation of the Cut of Grade of the ore blocks.

Ben-Awuah et al. (2016) investigate the strategy of mining options for an orebody using a mathematical programming model – mixed integer linear programming (MILP) optimization framework. The purpose of the framework and methodology is to evaluate the financial impacts of applying different mining options separately or concurrently to extract a given orebody. The MILP formulation maximizes the NPV of the reserve when extracted with (1) open pit mining, (2) underground mining, and (3) concurrent open pit and underground mining. According to Ben-Awuah et al. (2016), the production schedule for a combined open pit and underground mining scenario requires that both mining options compete for the same reserve during optimization. This model did not consider the capital expenditure of the projects, equipment requirement and stochasticity of certain parameters of the problem.

King et al. (2016) incorporated crown pillar placement that separates the open pit from the underground mine, and of the sill pillars, i.e., levels left in situ that can grant earlier access to stopes by creating a false bottom in their OP-UG transition studies. King et al. (2016) developed a mathematical programming model based on an ad-hoc branch-and-bound approach that incorporates decomposition methods for solving precedence constrained production scheduling problem (PCPSP) linear programming relaxations, and that includes rounding heuristics. In their model, they first presented a surface extraction formulation, followed by an underground formulation, and conclude with a preliminary transition formulation which is essentially a combination of the two. The solution strategy of King et al. (2016) are (i) exhaustively searching possible crown and sill pillar placement options using an ad-hoc branch-and-bound strategy and solving the resulting LP relaxations, (ii) using a rounding heuristic to convert the LP relaxation solutions with favorable objective function values into integer solutions, and (iii) using integer solutions to eliminate a number of possible crown and sill pillar placement options to reduce the amount of computation required in (ii).

MacNeil and Dimitrakopoulos (2017) investigated the transition decision at a currently operating open pit mine that exists within the context of a mining complex that is comprised of five producing open pits, four stockpiles and one processing plant. In this paper, the financial viability of a set of candidate transition depths was evaluated to identify the most profitable transition depth. To generate an accurate projection of the yearly cash flows that each candidate transition depth can generate, a yearly life of mine extraction schedule was produced for both the OP and UG components of the mine. A two-stage mathematical programming model (MPM) for production scheduling similar to the work developed by Ramazan and Dimitrakopoulos (2005; 2013) was presented. The proposed method improves upon previous developments related to the OP-UG transition problem by simultaneously incorporating geological uncertainty into the long-term decision-making while providing a transition depth described in three-dimensions that can be implemented and understood by those who operate the mine.

In MacNeil and Dimitrakopoulos (2017) work, a stochastic integer programming formulation used to produce a long-term production schedule for each of the pre-selected candidate transition depths was presented. In addition to a unique transition year, each candidate transition depth corresponds to a unique ultimate open pit limit, crown pillar location and underground orebody domain, all of which are described in the three-dimensional space. An optimization solution outlining a long-term schedule that maximizes NPV is produced separately for the OP and UG operations at each of the candidate transition depths considered.

3.2. Optimization models and algorithms for OP-UG transition interface

Bakhtavar et al. (2009) have reviewed several models and algorithms for determining the transition depth. Bakhtavar et al. (2009) discussed some models and algorithms used in determining the transition depth. The first method for determining transition depth from open-pit to underground was the allowable stripping ratio, which is a relation between the exploitation cost of 1 ton of ore in underground (and in open pit) and the removal cost of waste in relation to 1 ton of ore extracting by open pit (Soderberg and Rausc, 1968; Popov, 1971). In 1982, an algorithm by Nilsson based on cash flow and Net Present Value (NPV) was presented (Nilsson, 1982). However, to consider the transition depth as a critical issue with respect to deposits with combinational extraction, the previous algorithm (1982) was again represented and reviewed (Nilsson, 1992).

In 1992, Camus introduced another algorithm for the transition depth. This algorithm was presented based on the block models and NPV values of blocks for open pit and underground exploitation. The approach basically consists of running the open pit algorithm considering an alternative cost due to underground exploitation (Camus, 1992). In 1997, Nilsson underlined discount rate as the most sensitive parameter in the process of handling the transition depth problem (Nilsson, 1997). In 1998, Whittle programming (4-x) which has been developed to assist in the interfacing of open-pit and underground mining methods was argued and studied. Due to the applied method in the programming, management can make decisions based on quantified operational scenarios of the open pit to underground transition (Tulp, 1998). In 2001 and 2003, an approach with allowable stripping ratio method was developed, and a mathematical form for the objective was introduced. Volumes of ore and waste within the open pit limit were assumed as a function of constant (ultimate open pit) depth (Chen et al., 2001; Chen et al., 2003).

To determine the optimal transition depth from open pit to underground mining, a software based on a heuristic algorithm was prepared by Visser and Ding in 2007 (Visser and Ding, 2007). In the same year Bakhtavar, Shahriar and Oraee introduced a simple heuristic method on the foundation of economical block models with open pit and underground block values. The main process in the algorithm is a comparison between total values of open pit and underground Bakhtavar et al. (2007). A heuristic model was established upon a two-dimensional block model with the values of open pit and underground presented (Bakhtavar et al., 2008).

In 2009, Bakhtavar, Shahriar and Oraee developed their model by modifying Nilsson's algorithm (Bakhtavar et al., 2009). According to Bakhtavar et al. (2009), until 2009, only some of the represented methods can solve but not carefully, problems on researches and studies in this nature. He further added that, the few methods (algorithms) have some disadvantages and deficiencies in finding the transition depth optimally.

The model of Bakhtavar et al. (2009) generates two different optimized mines – open pit mine and underground mine. Each mining method is employed on the same level of mining blocks in series. The NPVs of the optimized mines for each level block are compared and if the NPV of the open pit mine is larger than that of the optimized underground mine, the algorithm transcends by adding the next series of level block to the previously optimized blocks and the NPV of the open pit and underground mines are compared again. The evaluation from the first level to the last level is followed so that a certain level is assigned as an optimal transition depth (level) to establish the crown pillar. The remaining levels below the optimal transition level or crown pillar are emphasized and attended to extract but only utilize the underground stoping method (Bakhtavar et al., 2009). The major problem with this approach is the use of one level as the crown pillar without major consideration to the geotechnical parameters of the intercepting rock formation.

According to Bakhtavar et al. (2012), the most significant problem in the transition problem was the determination of the optimal transition depth from open pit to underground (OP-UG) mining. In 2012, Bakhtavar et al. developed a heuristic model based on block economic values of open pit and underground methods together with the Net Present Value (NPV) attained through mining (Bakhtavar et al., 2012). The NPV of the open pit operation is compared to the NPV of the underground operations for the similar levels. According to Bakhtavar et al. (2012), the model can optimally solve the transition problem based on technical and economic considerations, but did not consider the social effects, requirement of the working force in relation to the open pit mining lifetime, and equipment considerations after open pit mining. Bakhtavar et al. (2012) did not also consider the uncertainties in the geological and geotechnical characteristics of the orebody.

Finch (2012), discussed the important of determining the range of possible transition points within the largest economic pit. Schedules and cash flows developed for each option can be compared to find the best alternative. The main disadvantage highlighted by Finch is that, the effort of generating open pit and underground schedules for all likely transition points at all likely processing rates (and by inference cut-off grades) can be costly and time consuming. This often means that consequently, the problem is not thoroughly explored and therefore the result may be sub-optimal. By applying modern automated optimization software solutions to this problem, the effort can be significantly reduced and the likelihood of developing an optimal result in a palatable time frame at an acceptable cost is dramatically increased (Finch, 2012).

In 2013, Opoku and Musingwini suggested in their studies that, the OP-UG transition should not be based on the transition depth but more appropriately on other dynamic transition indicators (Opoku and Musingwini, 2013). According to Opoku and Musingwini (2013), Musendu (1995) suggested and discussed about the transition indicators as essential for determining the optimum transition level at which a change from OP-UG mining should occur. Opoku and Musingwini (2013) made the transition depth dynamic and reviewed the OP-UG transition decision problem from a stochastic perspective. To capture the dynamic nature of the decision problem, the transition length (T_L) should be T_{Lt}, where t is the point in time at which the parameters are obtained or estimated (Opoku and Musingwini, 2013).

3.2.1. Geomechanics of the crown pillar

A crown pillar is a horizontal part of an orebody between the first stope of an underground mine and the surface of the earth or an open pit or open excavation. A crown pillar is often provided to prevent water entering from the open pit floor into the stope, as well as to reduce surface subsidence and caving. Finding the most suitable crown pillar in a combined mining method using open pit and underground operations, especially block caving, is one of the most interesting and useful problems for mining engineers today (Bakhtavar et al., 2012). The transition from open-pit to underground mining is a complicated geomechanical process.

In the open pit to underground transition, the problems of displacement, deformation and stability of open pit rocks should be properly addressed; otherwise they will directly affect the production, safety and environment of underground mining (Ma et al., 2012). Very large and thick pillars cause the loss of the reserves whilst undersized pillars may cause failure and instability in the mine (Tavakoli, 1994). According to Ma et al. (2012), most of the geometrical and mechanical analyses, analytical analysis, numerical and physical simulations used in the past to study the ground movement and deformation of open pit (Singh and Singh, 1991; Singh and Singh, 1993; Pariseau et al., 1997; Sun et al., 2000; Wang et al., 2000; Bye and Bell, 2001; Liu et al., 2004; Rose and Hungr, 2007; He et al., 2008), lack the long-term monitoring of ground movement and deformation of open-pit after transition from open-pit to underground mining.

The optimization of crown pillar dimension is very important for the metalliferous mining industry. Prediction of optimum thickness of crown pillar is complex, generally based on practical experience with input from numerical analysis and various empirical techniques (Tavakoli, 1994). Numerous parameters affect the stability of a crown pillar (Brady and Brown, 2012). These parameters according to Brady and Brown (2012) are grouped broadly into geological and mining. The geological parameters include the dip of orebody; rock types, hangingwall, footwall and orebody; strength and deformation characteristics of hangingwall, footwall and orebody, as defined by rock mass classification; geometry of multiple ore zones (if applicable); virgin stress conditions and properties of contact zones between ore and country rock while the mining parameters include the geometry of crown pillar and surrounding stopes; support methods (including backfilling); mining sequence and stress redistribution caused by mining.

3.2.2. Placement of crown pillar in OP-UG transition

Crown pillar placement invariably defines the transition interface or zone of the open pit to underground transition problem. Appropriately defining the suitable location of this crown pillar is the beginning to the strategic long-term planning of the mining option optimization, hence, the fundamental burden of most researchers on the mining options optimization problem. According to Bakhtavar et al. (2012), leaving a pillar with adequate thickness will minimize detrimental interference between the open pit and underground mining operations, while maximizing ore recovery. Bakhtavar et al. (2012) assumed that at most one uniform crown pillar with constant height being a multiple of the row (level) height. In their work, the number of required rows to act as the crown pillar was considered with reference to the selected underground stoping method, economic aspects, and geotechnical concerns, and mathematically modelled through two set of constraints.

In their work, MacNeil and Dimitrakopoulos (2017) priori identified the crown pillar envelope for the gold deposit and evaluated four crown pillar locations within this envelope leading to four distinct candidate transition depth. MacNeil and Dimitrakopoulos (2017) further added that, the size of the crown pillar remains the same, although the location changes.

According to King et al. (2016), industry practice places the crown pillar based on: (1) largest economically viable open pit mine, or (2) the extraction method that results in the largest undiscounted profit for each three-dimensional discretization of the orebody and surrounding rock. They determined the sill pillar placement, i.e., locations in which material is left in-situ to allow for a change in mining direction, which adds a layer of complexity. King et al. (2016) used an ad-hoc branch-and-bound strategy to exhaustively search the possible crown and sill pillar placement options before solving the resulting LP relaxations. A rounding heuristic was used to convert the LP relaxation solutions with favorable objective function values into integer solutions. The integer solutions were used to eliminate several possible crown and sill pillar placement options to reduce the amount of computation required in the previous step. Due to the involvement of geology, King et al. (2016), incorporated bound dominance to heavily prune their ad-hoc branch-and-bound tree. They further mentioned that, only 40 of the over 3500 crown and sill pillar placement options have an LP relaxation objective function value greater than the best-known IP objective function value.

The stability of this proposed crown pillar or transition interface is key to strategic decision on the mining options optimization for any potential orebody that could be extracted with either open pit or underground or both mining methods. The placement of the crown pillar or transition interface significantly affects the NPV (Opoku and Musingwini, 2013; Roberts et al., 2013; Ordin and Vasil'ev, 2014; King et al., 2016; MacNeil and Dimitrakopoulos, 2017).

Nowadays, researching methods of influence made by transition from open pit to underground mining are mainly math and physics model research, sliding failure mechanism analogy and engineering analogy. According to Bo-lin et al. (2014), numerical simulation is currently one of the most effective means of studying the stability of the crown pillar. The FLAC software was used by Bo-lin et al. (2014) to study the influence of underground room-and-pillar mining to an open pit slope stability in Changba lead-zinc mine, China by using the equivalent rock parameter identification method to determine the slope rock mass parameters and the safety factor of the slope based on median approximation and strength reduction method.

Wang and Zheng (2010) built up a v-SVR model reflecting the non-linear regularity between underground mining and open slope deformation based on the v-SVR to forecast deformation. Sun et al. (2000) discussed slope rock mass sliding mechanism by analysis method; Nan et al. (2010) used Ansys program to analyze slope stability in Shirengou iron mine and proposed relative safety measures; Shi et al. (2011) used FLAC software to analyze the character of deformation failure about surrounding rocks in complex condition in transition from open-pit to underground mining of Tonglv Mountain NO.1 ore-body. Obviously, numerical simulation is one of the most effective means to analyze slope stability and safety predication (Bo-lin et al., 2014).

3.3. Constraints for OP-UG mining options

According to industry professionals, the production schedule is subject to a variety of technical, physical and economic constraints which enforce the mining extraction sequence, blending requirements, and mining and processing capacities. The transition indicators used; net present value (NPV), stripping ratio, and commodity price, are mostly dynamic over time. Factors that impact the ideal transition from surface to underground operations includes (1) cut-off grades, waste stripping, stockpile generation and stockpile reclamation in surface operations; (2) access to higher grades, dilution, proportion of resource extracted (due sterilization associated with mining method), production costs and capabilities, capital requirement, etc. in underground operations; and (3) tailings capacity, closure cost implications, etc. in combined surface and underground operations. The main constraints applied in the optimization studies of the OP-UG transition have been identified to include the total mining capacity, mining capacity in each ventilation district, total processing capacity, total lateral development capacity, lateral development capacity in each ventilation district, reserve constraints (i.e. to ensure that production activities were completed to a maximum of 100% and did not exceed reserve), and sequencing constraints.

According to Opoku and Musingwini (2013), the transition indicators that Musendu (1995) considered were mining recovery (higher recoveries favor OP over UG), price and grade (higher price and higher grade favor OP over UG), cost (higher OP costs favor UG mining), cost of stripping waste (the higher this cost the earlier the transition), production rate (higher rate favours OP over UG), and underground dilution (does not favor UG as it reduces run-of-mine (ROM) grade. Shinobe (1997) developed a software that enables the mine operator to determine the optimum time of conversion based on discounted cash flow (DCF) techniques and O'Hara and Subo1eski's cost estimation equations. This software assumed that the existence of underground reserves has been confined and that their extraction is technically feasible.

Luxford (1997) highlighted some critical issues to be considered when planning to make the transition. These include cost, workforce recruitment, orebody geometry, production rate, and geomechanics. Hayes (1997) emphasized the importance of economic considerations in OP-UG transition. Hayes indicated that, the following factors affect the decisions on OP-UG: management competence, geological and geotechnical characteristics of the orebody, stripping ratio, and productivity and capital cost of the underground option.

Finch (2012) identified the following as important transition indicators to be considered when making the OP-UG transition decision: feed grade, stripping ratio, commodity price, production rate, and mining cost (open pit and underground mining costs). A set of the constraint considered by Ordin and Vasil'ev (2014) in the optimization studies of the OP-UG transition includes parameters of geotechnologies. These were used to define the depth of transition from open pit to underground mine, mine design capacity, ultimate stripping ratio (the ultimate stripping ratio is only used to find the breakeven point of an open pit mine relative to its depth), and the rate of discounting of expected money flows.

Deterministic approaches fail to account for the uncertain nature of the transition indicators used for the decision-making as well as the geological uncertainties, hence, the failure to address the dynamic nature of the problem (Opoku and Musingwini, 2013). According to Opoku and Musingwini (2013), transition indicators used to inform the OP-UG transition decision are not clearly defined as they vary from company to company, orebody to orebody, and commodity to commodity. However, the commonly used quantitative indicators which address most of the issues identified and applied to gold mines are the margin (as a ratio of gold price to cost) which was chosen to avoid conflicting, views that might arise if price and cost are considered in isolation, average ROM grade, stripping ratio of the open pit mining, NPV of either the open pit alone, underground alone or the combined method; and processed ounces as a proxy for production rate. To account for supplementary and qualitative information on the diversity of issues and differences in ore bodies that must be considered concurrently with the key quantitative transition indicators, Opoku and Musingwini (2013) developed a checklist based on the geology, operational, and geotechnical to guide the transition decision.

3.4. Implementation of models and algorithms for OP-UG mining options

Most of the existing models and algorithms have been implemented and their results assessed. Some of these models have been incorporated into a software. According to Ordin and Vasil'ev (2014), the problem solution to mining options optimizations generally uses the Lerchs–Grossman algorithm, Seymour algorithm, floating cone technique, dynamic programming, neural network, theory of graphs, network flows, etc. Based on these methods, programs of Surpac Vision, NPV Sheduler, Four-X, MineShed, integrated 3D CAD systems of Datamine, Vulcan, MineScape, MineSight, Gemcom and others are widely used (Achireko, 1998; Kaputin, 2004; Kaputin, 2008).

The use of the MIGP formulation for an orebody model usually results in a large scale optimization problem (Askari-Nasab et al., 2011). According to Askari-Nasab et al. (2011), one of the optimization solvers capable in handling such problems is the ILOG CPLEX (2007). This optimizer was developed based on branch and cut algorithm and makes the solving of MIGP models possible for large-scale problems.

To implement their approach in dealing with the transition problem, Opoku and Musingwini (2013) used mining specific software including Datamoine®, Isatis®, Whittle®, XPAC®, and Mineable Reserve Optimizer (MRO®). Isatia software was used to generate the simulated models. MRO was used to determine the mineable stopes for the underground option, and the XPAC was used to schedule the output of the optimization to obtain realistic mining schedule.

In Roberts et al. (2013) work, a standalone open pit is optimized using a combination of the mathematical programming models of the Blasor pit optimization tool and the COMET cut-off grade and schedule optimizer. Blasor uses a mixed integer programming (MIP) formulation to produce an optimized pushback sequence while COMET uses a dynamic programming approach to cut-off and schedule optimization based on a given set of pushback designs. The Blasor output (in the form of an optimum mining sequence) is exported to a software tool called COMET for optimization of the mining schedule. Output from COMET was used to code the optimum open pit schedule into the resource model. For the underground mine, the process of creating mining outlines from the IV0 resource model is conducted using Snowden's Stopesizor software (Myers et al., 2007). Stopesizor modifies a geological block model to identify the optimum mining outline for a range of cut-off values (usually grade based). The mining schedule optimization is conducted using Snowden's Evaluator software package.

In their work, De Carli and de Lemos (2015) used Studio 3 and NPV Scheduler software to assist in the search of the required results in most of the steps of their model in solving the transition problem. King et al. (2016) used the OMP and AMPL/CPLEX solvers in their research.

3.5. Performance evaluation of the models and algorithms for OP-UG mining options

According to Fiscor (2010), the Palabora Mine transited from open pit mine to underground mine in 1996 when the mine announced to proceed with the development of an underground block cave mine with a production rate of 30,000 Mt/day. Fiscor (2010) explains that, Palabora Mining's engineering design work set a precedent for converting from open-pit to underground design. After careful studies, the Palabora Mine transited from open pit to underground mine with a transition zone (crown pillar) of 400 m below the 800 m deep pit. A slope failure occurred at the Northing wall of the open pit after the transition in 2003 (Brummer et al., 2006). Evaluating the performance of models and algorithms are essential to the strength and weakness of such models and algorithms that could lead the way for further studies in the subject area.

In their research, Askari-Nasab et al. (2011) compare the performances of the proposed models based on Net Present Value (NPV) generated, practical mining production constraints, size of the mathematical programming formulations, the number of integer variables required in formulation, and the computational time required for convergence.

Opoku and Musingwini (2013) analyzed the results from their model using normal distributions and their associated and cumulative probability distributions to predict the values of transition indicators at different probability levels. According to Opoku and Musingwini (2013), the transition indicators at a probability of 95% for the four case study mines favored only one mine to transit from OP-UG (combined mining) under the current techno-economic conditions. Opoku and Musingwini (2013) based their analyses on the transition indicators on suggestions from Wright (2012), that, a stripping ratio of 4–17 is considered as a good indicator for the UG option, NPV should be positive and the margin (gold price to cash cost) of 2 is also acceptable since the industry value for 2011 was 1.58.

According to Roberts et al. (2013), to reduce the optimization problem to a manageable size for efficient computation, stope blocks are accumulated into a series of groups or 'bins'. It is assumed that within each group, the contained blocks are to be depleted at the same rate. To produce the best possible approximation of a block by block optimization, each group contains blocks with comparable properties. The blocks are grouped according to ventilation district, IV0 outline, and the year in which the blocks were planned to be extracted by the open pit. In addition to being grouped by ventilation district, blocks are also grouped by their planned open pit extraction year. The extraction year is the governing factor which determines at what time the IV0 value of a block becomes negative.

Ben-Awuah et al. (2016) assessed the performance of their proposed model based on the NPV and smoothness of the generated schedules. The MILP model was setup for open pit and open stope mining to compete for the same material during optimization subject to each method's respective mining and economic parameters. Similarly, Askari-Nasab et al. (2011) assessed the performance of their proposed model based on the NPV, mining production goals and smoothness and practicality of the generated schedules. They tested their model on a Dell Precision T3500 computer at 2.4 GHz, with 3 GB of RAM.

4. Limitations with current models and algorithm for OP-UG mining options

Two key challenges to the mining options optimization problem are the exhaustive consideration of stochasticity of the contributing variables to the models and/or algorithms, and geotechnical considerations in defining the transition interface or zone. Jakubec (2001) and McCracken (2001) stressed on the need to integrate geotechnical models in the strategic long-term mine plans at the prefeasibility stage similar to how geologic models are incorporated. Incorporating geological and financial uncertainties in the mining options optimization models and/or algorithms will result in robust mining projects. Corporate capital budgeting and cost of capital estimation are among the most important decisions made particularly in relation to the impact they may have on the business (Wooldridge et al., 2001). The main challenges of mining variables to the business environment include organizational differences in cost reporting structures, global assumptions, risk appetite and strategic global outlook (Gabryk et al., 2012).

Some of the limitations with current models and algorithms for OP-UG mining options optimization include one or more of the following:

a) optimality assessment,

b) consideration of geotechnics for transition zone,

c) consideration of stochastic variables, and

d) comprehensiveness and efficiency of models.

4.1 Optimality assessment

Optimality assessment of the model is a real challenge to current models and algorithms for OP-UG mining options. The optimality of the problem is therefore compromised with time and cost and the level or gap of optimality is always uncertain. Some of these represented models could solve the transition problems but not carefully, usually giving producing near optimal solutions (Bakhtavar et al., 2012; Finch, 2012). Bakhtavar et al. (2009) noted that, few methods (algorithms) have some disadvantages and deficiencies in finding the transition depth optimally. According to Askari-Nasab et al. (2011), the main disadvantage of heuristic algorithms are that the solution may be far from optimal and in mega mining projects, this is equivalent to huge financial losses. Finch (2012) also highlighted that, the effort of generating open pit and underground schedules for all likely transition points at all likely processing rates (and by inference cut-off grades) can be costly and time consuming, thus, the main disadvantage of current models and algorithms used to solve the transition problem. This main disadvantage implies that, the transition problem is not thoroughly explored and therefore the result may consequently be sub-optimal. By applying modern automated optimization software solutions to this problem, the effort can be significantly reduced and the likelihood of developing an optimal result in a palatable time frame at an acceptable cost is dramatically increased (Finch, 2012).

4.2 Consideration of geotechnics for transition zone

Roberts et al. (2013) acknowledged that, geotechnical constraints were not considered in their work. To verify the impact of geotechnical constraints on the optimal solution, Roberts et al. (2013) recommended that, such constraints need to be incorporated in subsequent studies. The location of the crown pillar, which defines the interface of the open pit to underground transition was priori selected and treated as deterministic. The selection approach to the location of the crown pillar was not known and, according to Opoku and Musingwini (2013), fails to account for uncertainties. MacNeil and Dimitrakopoulos (2017) recommended in their work that, the impact of the size of OP and UG mines on the dimensions of the crown pillar should be investigated.

4.3 Consideration of stochastic variables

Currently, most mine operators schedule the open pit and underground operations independently and then merge the two. However according to King et al. (2016), this approach creates a myopic solution. King et al. (2016) confined the discussions of their approach to open stoping and its associated sequencing options. No stochastic variables like grade and price uncertainty were employed in their model; thus, their model was deterministic. King et al. (2016) further acknowledged that, their methodology in handling the transition problem require additional work to address the accuracy, applicability and optimality gap.

Although MacNeil and Dimitrakopoulos (2017) incorporated grade uncertainty in their work, they further identified some important notable geological uncertainties such as material types, metal and pertinent rock properties and their impact on the strategic long term planning of a mining project. MacNeil and Dimitrakopoulos (2017) further recommended that, future studies should aim to improve on their method by considering more aspects of financial uncertainty such as inflation and mining costs.

4.4 Comprehensiveness and efficiency of models

Shinobe's software based on mathematical programming model (1997) assumed that the existence of underground reserves has been confined and that their extraction is technically feasible. This is a challenge to this model. He later recommended that the results of the program should be viewed only as a preliminary level indication of the economics of underground conversion. No final decision to proceed with the conversion should be taken, solely based on the program's output (Shinobe, 1997). Stacey and Terbrugge (2000) suggested that the transition problem was known but the lack of a model to guide the transition remained an issue. They further noted that the planning and implementation period for transition from OP-UG could take as long as 20 years and so should commence at an early stage.

According to Ordin and Vasil'ev (2014), majority of the existing researches on transition problem lack some constraints and solving of a more general problem – joint optimization of the depth for transition from open pit to underground mining and the design capacities of the open pit and underground mine. Ordin and Vasil'ev (2014) generated curves of NPV and depth of transition from open pit to underground mining for Botuobinskaya pipe deposit. From the generated model, it was interesting to note that comparatively, at the optimum mining depth between 250 m to 400 m, the total NPV of combined open pit and underground mining was higher than the NPV for the open pit mine and the NPV for the underground mine for the same mining depth. In their work, Ben-Awuah et al. (2016) did not consider uncertainty in their model formulation and further recommended that an additional study is undertaken to investigate the mining options including pre-production capital expenditures (CAPEX).

Re-handling, mixing and degradation costs were omitted from the formulation of the model to ensure easy exposition and to enable the use of a special solution strategy. According to King et al. (2016), the re-handling cost proves to be insignificant when incorporated into the transition model. King et al. (2016) observed some fluctuations in both the open pit and underground production, which is undesirable from an operational standpoint, and would require smoothening to create an operationally feasible schedule. However, they added that, these fluctuations are not uncommon in a strategic plan.

In their work, MacNeil and Dimitrakopoulos (2017) incorporated the constraints of mining, processing, metal content and precedence relationships in their model. According to the constraints identified in the works of Opoku and Musingwini (2013) affecting the transition problem, those constraints considered by MacNeil and Dimitrakopoulos (2017) are not exhaustive. Capital investment required to ramp up UG mining was not considered in the application presented for the gold deposit case study (MacNeil and Dimitrakopoulos, 2017).

5. Summary and conclusions

The problem of optimizing reserve exploitation depends largely on the mining option used in the extraction. Some mineral deposits have orebodies that extend from near the surface to several meters in depth. Such deposits can be amenable to both open pit mining and/or underground mining. Current strategic open pit and underground mining interface optimization models have been developed mainly based on determining the transition point or depth between open pit mining and underground mining. An algorithm or model that comprehensively and simultaneously determine an optimized open pit mine, determine the transition interface and further determine an optimized underground mine for any orebody with the potential to be exploited by both surface or underground mining methods or both will add significant value to the mining industry. A matrix showing the various approaches adopted by researchers in tackling the OP-UG transition problem in the last decade has been developed. Challenges and performance evaluation of notable research on mining options strategy have been discussed and opportunities for further research identified. A research approach for further studies on the strategic mining options problem has been outlined.

Notable limitations of existing models and algorithms for the OP-UG mining option have been identified to include one or more of the following: a) optimality assessment of the models and/or algorithms, b) models and/or algorithms did not assess the geotechnical condition of the transition zone, c) consideration of the stochastic variables are not exhaustive, and d) the models and/or algorithms are not comprehensive and efficient.

Although the main sources of uncertainties in mining options study have been found to include financial, technical and geological, research on strategic mining options have handled these uncertainties independently. The incorporation of these geological uncertainties in current strategic mining options have been applied in different forms, including, grade and tonnage uncertainties, probability indices, and the use of algorithms to further define these uncertainties. The incorporation of financial uncertainties together with geological uncertainties are limited in current research on mining option studies. As uncertainties cannot be eliminated in the mining options problem, the best strategy is to quantify uncertainty, reduce this uncertainty as far as investment allows, and finally manage the associated risk during the scheduling procedure.

Over the years, several algorithms (heuristics and meta heuristics) and models (deterministic and stochastic) have been developed to handle the open pit – underground mining options. In the last decade, however, different variations of mathematical programming models have been used by researchers to solve the numerous challenges of the OP-UG mining options problem. The main variations of mathematical programming models are either deterministic (linear programming, integer and mixed-integer programming) or stochastic (dynamic programming, stochastic programming) or combination of both. The authors' conclude by proposing further research into the application of an integrated stochastic mathematical programming model for the mining options optimization problem.

Mathematical programming models are known to be robust and their solutions have a measure of optimality. Some benefits of mathematical programming models include:

a) Robust – mathematical programming models have a well-defined structure that describes the thought process of the modeler in terms of the decision variables (objective functions), and the decision environment (constraints).

b) Objectivity – mathematical programming models are more objective since all assumptions and criteria are clearly specified. Although these models may reflect the experience and biases of the modelers, these biases can be identified by outside observers.

c) Tractability – mathematical programming models allow large and complex problems to be solved in their reduced formed by employing the significant interrelationships among the variables constituting the problem. Thus, they provide a relatively simple and compact approximation of complex decision-making problems.

d) Model solution – mathematical programming models make problems amenable to mathematical and computer solution techniques.

e) Facilitates sensitivity or parametric analysis - mathematical programming models make it relatively easy to find the optimal solution for a specific model and scenario.

f) Optimality measure – mathematical programming models can determine the level of optimality and/or certainty of the solutions to the problem. An optimality gap could be defined to ascertain the optimality level of the solution to the problem.

6. Research opportunities

Quantification of uncertainty and optimization in strategic mine planning plays a significant role in reducing financial risk and environmental footprints, and promoting sustainable development through improved resource governance and total mine reconciliation. The strength of mathematical programming models as opposed to heuristic and metaheuristic techniques will be explored to solve the open pit – underground (OP-UG) mining option problems. Fig. 3 is a schematic representation of the proposed approach to the strategic OP-UG mining options optimization.

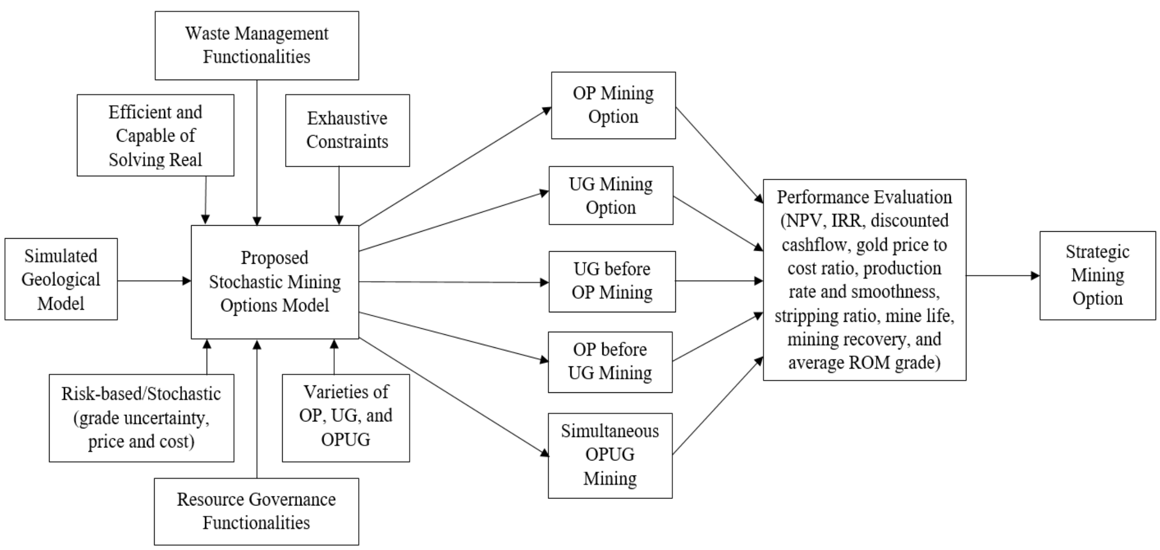

Fig. 3 Schematic representation of the research approach for the strategic OP-UG mining option studies.

Suggested recommendations in addressing the research gap in current models and algorithms will be incorporated in a mathematical programming model with the following characteristics:

a) Robust - when implemented results in:
   i. Separate mining strategy for OP or UG
   ii. Sequential mining strategy for OP to UG or UG to OP
   iii. Simultaneous mining strategy for OP and UG

b) Risk-based/stochastic
   i. Grade uncertainty
   ii. Price and cost

c) Practicality/tractability/generality
   i. Easy to apply
   ii. More varieties of UG and OP mining methods/strategies
   iii. Exhaustive constraints
   iv. Real problem sizes/efficiency
   v. Practical solution run time with known optimality
d) Integrated waste management
   i. Synergy in waste disposal planning
   ii. Improved resource governance and total mine reconciliation

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