Incorporating Stockpiling and Cut-off Grade Optimization into Oil Sands Production and Dyke Material Planning using Goal Programming

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ABSTRACT

In achieving maximum benefit in oil sands mining, the time and sequence of removing ore, dyke material and waste from the final pit limit is essential to the long-term production schedule. In-pit waste management strategy requires the simultaneous construction of dykes with the advancement of mining operations. This paper seeks to determine: 1) the time and sequence for removal of ore, dyke material and waste to maximize Net Present Value (NPV); 2) the quantity of dyke material required for dyke construction to minimize construction costs; and 3) the impacts of stockpiling and stockpile reclamation with limited time duration. An Integrated Cut-Off Grade Optimization (ICOGO) model was used to generate an optimum cut-off grade policy and a schedule for mining ore and waste, as well as overburden, interburden and tailings coarse sand dyke materials in long-term production planning. Subsequently, a Mixed Integer Linear Goal Programming (MILGP) model was developed to generate a detailed production schedule for removal of ore, waste and dyke materials from the final pit limit. The cut-off grade profile and schedule generated by the ICOGO model are used as guides to define the grade constraints and production goals required by the MILGP model. The developed models feature stockpiling with limited duration for long-term production scheduling. The models were applied to an oil sands case study to maximize the NPV of the operation. In comparison, whereas the ICOGO model solved the optimization problem faster, the MILGP model results provided detailed mining-cut extraction sequencing for practical mining.

1. Introduction

Effective waste management drives the sustainability and profitability of oil sands mining operations. Currently, more than 80% of the processed oil sands ore are deposited in tailings dams, which are constructed at designated areas outside the final pit limit, or in mined out areas of the active pit (Masliyah, 2010). Generating such large volumes of tailings material during mining has caused several environmental issues. To reduce the environmental footprints for oil sands mining, the regulatory requirements of the Alberta Energy Regulator (AER) Directive 085 require oil sands mining companies to integrate waste management strategies into their long-term production plans (Ellis, 2016b). For this reason, simultaneous in-pit dyke construction and tailings deposition has been introduced as the mine advances; in a way that when a pushback is available, a dyke is constructed to generate a tailings containment area. The material required for dyke construction primarily comes from the mining operation, which includes overburden (OB), interburden (IB) and tailings coarse sand (TCS) dyke material. These materials must meet the fines requirements for

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dyke construction. Material that cannot be classified as ore or dyke material are considered to be waste material (Ben-Awuah and Askari-Nasab, 2011; Ben-Awuah et al., 2012).

Heuristic algorithms and exact solution methods are the two main research areas in optimizing the production scheduling process (Askari-Nasab and Awuah-Offei, 2009). Lane (1964) developed a comprehensive heuristic optimization model to determine the optimum cut-off grade policy and generate the life of mine production schedule in terms of material tonnages. The model does not take into consideration waste management cost as required for integrated oil sands mine and waste disposal planning. This lead to the development of a modified version of Lane's model referred to in this research as the Integrated Cut-Off Grade Optimization (ICOGO) model. The ICOGO model allows for determining the optimum cut-off grade policy in the presence of waste management for dyke construction and stockpiling with limited duration. The developed model considers stockpile re-handling and waste management costs, and generates production schedules for multiple material types. The ICOGO model is fast to implement but does not take into account detailed mining-cut extraction sequencing during optimization.

Subsequently, a theoretical mathematical programming framework based on Mixed Integer Linear Goal Programming (MILGP) model was developed to generate detailed long-term production plans for integrated oil sands mining and waste management. The MILGP model maximizes the NPV and minimizes dyke construction cost, as well as implements stockpiling with limited duration for oil sands mining. Solving long-term production scheduling problems using mathematical programming models with exact solution methods is a preferred approach considering that the extent of optimality of the solution is known. However, mathematical programming frameworks are computationally expensive and this increases exponentially with the problem size (Ben-Awuah and Askari-Nasab, 2011). Heuristic methods are usually computationally cheaper since they follow an iterative process to generate the best results among alternate options, though the optimality of the solution cannot be guaranteed. The cut-off grade profile and schedule generated by the ICOGO model are used as a guide to define the grade constraints and production goals required by the MILGP model. This technique is synonymous to providing a customized initial solution to the MILGP model, thereby decreasing the size of the solution space and facilitating a reduced solution time for the MILGP framework.

The next section of this paper presents a discussion on the problem definition. Section 3 reviews relevant literature related to cut-off grade optimization and open pit production planning algorithms. Section 4 highlights development of the theoretical frameworks for ICOGO and MILGP models and Section 5 discusses the application of the ICOGO and MILGP models with an oil sands case study. The paper concludes in Section 6.

2. Problem definition

Taking waste management into consideration during long-term production scheduling poses challenges related to creating an optimized mining schedule. The integration of the production schedule and waste management strategy increases the size of the optimization problem significantly. Incorporating various material types, elements, and destinations as well as providing an available in-pit area for construction of dykes are a few of the parameters that need to be taken into consideration resulting in a large-scale optimization problem.

An agglomerative hierarchical clustering algorithm developed by Tabesh and Askari-Nasab (2011) is used to create mining-cuts. Blocks within the same level are grouped together based on their attributes such as location, rock type and grade to build up a mining-cut. The intersection of a group of mining-cuts belonging to the same mining bench and a mining-phase (pushback) is referred to as a mining-panel. Each mining-cut within a mining-panel contains: 1) ore material, 2) TCS dyke material (from processed ore), 3) OB and IB dyke material and 4) waste. Figure 1 illustrates the material flow for an oil sands mine containing K mining-cuts and M pushbacks.

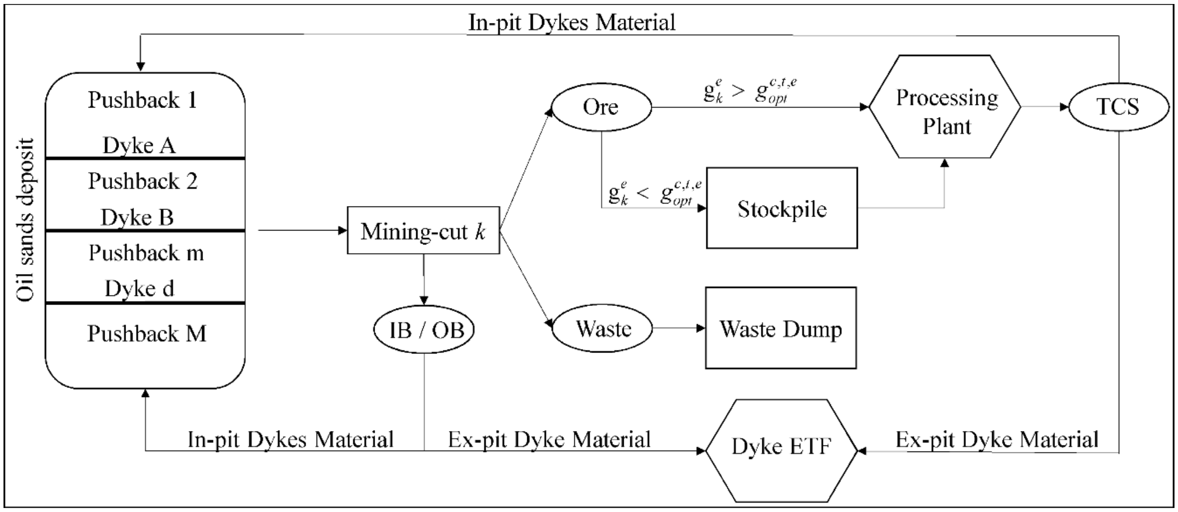

Figure 1: Material flow for oil sands production planning and waste management modified after Ben-Awuah et al. (2012)

Using Ben-Awuah et al. (2012) model as the starting point, this paper focuses on developing an MILGP model to generate a detailed production schedule for different material types and destinations. The Ben-Awuah et al. (2012) model does not provide information on how initial grade boundaries and production targets were defined and they do not consider stockpiling in their model development.

First, a heuristic cut-off grade optimization model was developed considering waste management cost for ex-pit and in-pit dyke construction, and stockpiling with limited duration. The main objective is to develop and implement an Integrated Cut-Off Grade Optimization (ICOGO) model to generate an optimum life of mine cut-off grade profile and production schedule for different material types. The ICOGO model development starts with Lane's (1964) model and includes waste management cost and stockpiling with limited duration as required in oil sands mining.

Subsequently, an MILGP model was developed to use the cut-off grade profile and schedule generated by the ICOGO model as a guide to define the grade constraints and production goals required by the MILGP model. By this approach, the optimization solution space will be limited and the optimum solution found faster. The developed MILGP model features stockpiling with limited duration for long-term production scheduling.

The main objectives of the paper can be classified into four focus areas:

1. Maximize the NPV and minimize dyke construction cost of the operation by determining the time and sequence for removal of ore, dyke material and waste from the final pit limit;
2. Minimize deviations from production goals (grade and tonnage) which are outcomes from the ICOGO model;
3. Evaluate the impact of stockpiling and stockpile with limited duration in oil sands mining;
4. Investigate the impact of the two-step approach on the solution time.

3. Summary of literature review

In 1964, Lane developed a cut-off grade optimization model based on economic factors, grade-tonnage distribution, and operational capacities. The objective function of Lane's model is to maximize the NPV of the operation with respect to capacities of the mining, processing and refinery processes. He considered the concept of opportunity costs in his model to generate a dynamic cut-off grade policy for the life of mine (Lane, 1964). The dynamic nature of Lane's model requires the use of stockpiling. The material between the optimum grade and the lowest cut-off grade can be stockpiled during the mining operation for possible future reclamation (Asad et al., 2016). Lane's model needs the general extraction sequence as an input to the optimization process and generates the production schedule in terms of material tonnages and grades. An extension of Lane's original theory for deposits containing two economic materials was developed by Asad (2005) to incorporate stockpiling into the production scheduling problem. The stockpile acts as an additional pushback when active pit mining is completed. He noted that problems such as leaching, deterioration of material and oxidation, can happen due to long-term stockpile duration. He also demonstrated with a hypothetical case study that his model could increase the NPV of the mining operation.

During the past four decades, many researchers have developed extensions to Lane's model for deposits with single and multiple economic minerals. In order to find the optimum cut-off grade policy and production schedule, Osanloo and Ataei (2003) presented a golden section search method with equivalent grade factor for Lane's model. Genetic algorithm, golden section search, equivalent grade method and iterative grid search have been used by Ataei and Osanloo (2003a; 2003b; 2004) to generate the optimum cut-off grade policy and production plan in complex ore deposits. An application of the grid search technique for deposits with more than two economic minerals was also investigated by Cetin and Dowd (2013).

The problem of open pit production scheduling can be described as specifying the sequence in which mining blocks should be removed to maximize the NPV of the deposit, with respect to physical and economic constraints. The main constraint for production scheduling is the block extraction sequencing (Whittle, 1989). Some optimization frameworks for mine production planning such as Lane's model define production levels in terms of tonnage and grade. This concept can reduce complex computations; however, it ignores the detailed block extraction sequencing, which is the most important part of mine production planning (Gershon, 1983).

Linear Programming (LP) and Mixed-Integer Linear Programming (MILP) models are some of the most robust techniques used for solving mine production scheduling problems since the 1960s. These models can consider thousands of decision variables and constraints. LP and MILP problems are solved using exact optimization methods which provide a single solution within the set optimality tolerance. The LP and MILP models are generated as a system of equations which makes them easy to use for multiple projects, requiring only minor changes to be made to them. On the other hand, like other Mathematical Programming Models (MPMs), LP and MILP models are computationally costly, which can be difficult to handle for large problems with thousands of variables and equations (Huttagosol and Cameron, 1992).

Manula (1965), Johnson (1969) and Meyer (1969) were among the firsts to initiate development of LP and MILP models in mine planning optimization. One of the main obstacles that these authors encountered was solving large integer programming problems. Despite the models' remarkable success, LP and MILP have not become the preferred method for mine planning due to computational difficulties (Gershon, 1983). One of the most critical aspects of the production scheduling process is to determine a feasible mining sequence. Therefore, it is vital to follow the block extraction precedence relationships in the optimization process to ensure the long-term plan is practically feasible (Gershon, 1983).

During the past three decades, many authors have made efforts to overcome the problem of solving large scale optimization problems in a timely manner. The lagrangian relaxation algorithm is one of the methods that was adopted by Dagdelen (1985) and Dagdelen and Johnson (1986). Another method is the branch-and-cut algorithm which was used by Caccetta and Hill (2003) to solve large scale optimization problems. Binary variables are the main reason which makes solving the optimization problem difficult. One technique for solving the large-scale problem is to reduce the size of the problem prior to optimization. Ramazan and Dimitrakopoulos (2004) reduced the number of binary variables to solve the optimization problem faster. In order to reduce the number of binary variables even more, Ramazan et al. (2005) and Ramazan (2007) used an aggregation method and solved the problem with a fundamental tree algorithm. However, using their method may eliminate the global optimum solution due to the method of reduction of the problem size. Askari-Nasab et al. (2010) applied MILP formulations to an open pit iron ore mine production schedule and compared their results to an industry strategic mine planning software, Whittle (GEOVIA Whittle, 2013). In order to reduce the size of the optimization problem, they aggregated the mining blocks into mining-cuts using a clustering algorithm and claimed that the generated NPV of the MILP model was 2.6% higher than the NPV generated by Whittle Milawa Balanced algorithm (GEOVIA Whittle, 2013). They conclude that MILP formulations based on processing and extraction at mining-cut level, are models that maximize NPV and suitable for long-term planning with efficient computation time.

Another MPM used for Long-Term Production Planning (LTPP) problems is Goal Programming (GP). The benefit of using GP over other mathematical programming models is being able to prioritize one goal over another. Zhang et al. (1993) used GP for LTPP of a mining operation with a single ore type process. They verified their model by applying it to an open pit coal mine. Chanda and Dagdelen (1995) and Esfandiari et al. (2004) also applied GP to the LTPP problem; however, they mentioned that the application of GP is impractical due to the size of the problem and large number of constraints. Research shows that there is a greater advantage using MILP and GP together. Industries such as manufacturing and operations management are taking advantage of the application of MILGP models (Selen and Hott, 1986; Liang and Lawrence, 2007; Sen and Nandi, 2012).

Ben-Awuah and Askari-Nasab (2011) formulated, implemented and tested a theoretical MILGP framework for oil sands production scheduling and waste management. Their model could handle multiple material types and elements in LTPP, and maximize the NPV of the operation. Ben-Awuah et al. (2012) completed their work by considering multiple destinations for dyke material, including in-pit and external tailings facilities for waste management. They used MILGP because the formulation structure allows the optimizer to achieve a set of goals, whilst some goals can be traded-off against others based on their priority. In addition, hard constraints that could result in infeasible solutions can be changed to soft constraints. Implementation of their model resulted in maximum NPV while creating timely tailings storage areas. It should be mentioned that the main limitation with their model is the absence of stockpile management and long runtime (Ben-Awuah et al., 2012). In order to reduce the solution running time, Ben-Awuah and Askari-Nasab (2013) used a pre-processing approach to reduce the number of non-zero and integer decision variables. Results from their case studies showed a reduction in the solution time by more than 99% (Ben-Awuah and Askari-Nasab, 2013).

Over the past decades, researchers have improved the cut-off grade optimization framework introduced by Lane in 1964 by incorporating different parameters into the cut-off grade calculation. This research seeks to incorporate waste management cost and stockpiling simultaneously into an integrated cut-off grade optimization model for oil sands mining. Furthermore, the research explores the incorporation of a limited duration stockpiling strategy in a two-step mathematical programming framework for integrated mine planning and waste management as required in oil sands mining. The practical implementation of the cut-off grade optimization and mathematical programming models are discussed.

4. Theoretical framework

4.1. Block Modelling

For scheduling optimization of an open pit mine, the orebody is discretized as a block model comprising of three-dimensional arrays of cubical blocks. The number of blocks in the block model is related to the size of the deposit. The geology of the deposit and the preferred size of mining equipment can be used to identify the dimensions of the blocks in the block model. Characteristics of the blocks including rock type, density, grade and economic data can be expressed numerically (Askari-Nasab et al., 2011). The blocks in the block model consist of smaller units called parcels, which contain information on rock-type, tonnage and element content. The ore tonnage and the block grade can be used to estimate the quantity of minerals in a block. The spatial location of each block within the block model is determined by the coordinates of its center. However, the shape and location of the parcels within each block are not specified (Askari-Nasab and Awuah-Offei, 2009). The ultimate pit limit (UPL) can be generated using the block model as input to Whittle strategic mine planning software (GEOVIA Whittle, 2013) which is developed based on the Lerchs and Grossmann (LG) algorithm (Lerchs and Grossmann, 1965).

4.2. Break-Even Cut-Off Grade Optimization

Cut-off grade optimization was used to maximize the NPV of oil sands mining operations with respect to limited processing capacity. The ICOGO model incorporates dyke construction costs into cut-off grade optimization and generates the optimum cut-off grade profile and production schedule for ore, dyke material, and waste. The results from the ICOGO model were used as a guide to define grade boundaries and production targets in the MILGP model.

The oil sands ore has grade dependent recovery factor. According to Directive 082 (Ellis, 2016a), the processing recovery factor for oil sands ore can be calculated based on average weight percent bitumen content of the as-mined ore. Figure 2 shows the profile for oil sands ore processing recovery factor.

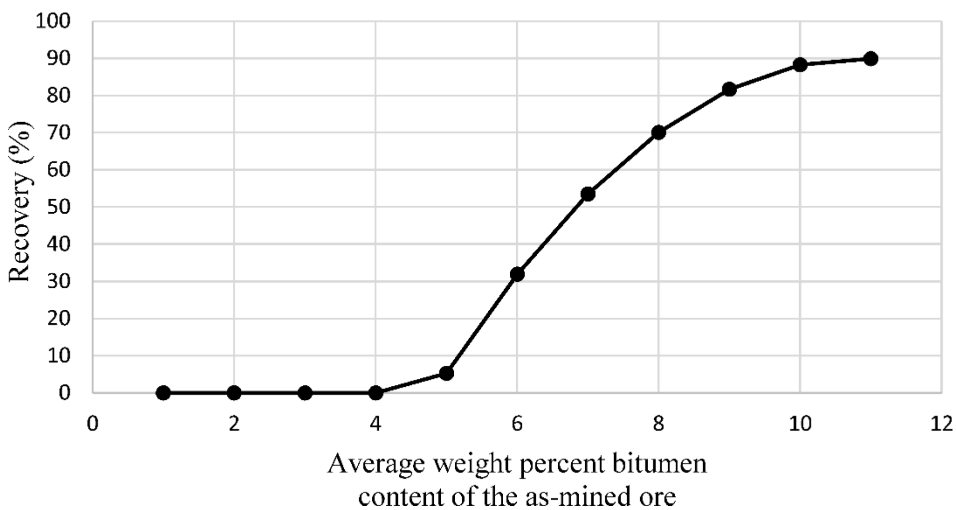

Figure 2: Profile for oil sands ore processing recovery factor

Based on the grade-recovery relationship, the break-even cut-off grade for oil sands mining was determined with Whittle (GEOVIA Whittle, 2013). In this paper, the break-even cut-off grade is referred to by the term 'lowest acceptable grade'. Whittle generated a lowest acceptable grade of 6% for the oil sands ore. From Figure 2, material with bitumen grade less than 6% have less than 31% recovery and are therefore not economical to process. During stockpiling, the ore material oxidizes and the processing recovery begins to deteriorate. For the purpose of this research, it was assumed that the processing recovery deteriorates by 1% for each year stockpiled. In addition during the cut-off grade optimization calculation, a weighted average recovery factor was used which is more representative of the mineralization.

4.3. The Integrated Cut-Off Grade Optimization (ICOGO) Model

Considering the lowest acceptable bitumen cut-off grade of 6%, the material in the final pit limit was classified as ore, dyke material, and waste. Extending Lane's (1964) model, the ratio of the amount of dyke material should be related to the total amount of ore and waste to allow the incorporation of dyke construction costs into the cut-off grade optimization process. Equation (1) shows the ratio of the TCS dyke material to the total amount of ore, and Equations (2) and (3) show the ratio of OB and IB dyke material to the total amount of waste, respectively. The variables used in developing the equations have been defined in the Appendix.

$$R_{TCS} = \frac{Total\ amount\ of\ TCS\ dyke\ material}{Total\ amount\ of\ ore} \tag{1}$$

$$R_{OB} = \frac{Total\ amount\ of\ OB\ dyke\ material}{Total\ amount\ of\ waste} \tag{2}$$

$$R_{IB} = \frac{Total\ amount\ of\ IB\ dyke\ material}{Total\ amount\ of\ waste} \tag{3}$$

Using the calculated ratios for dyke material, the profit expression for oil sands mining and waste management operations can be determined by Equation (4). The mine life and the amount of product can be calculated by Equations (5) and (6) respectively, for a processing limited mining operation.

If the maximum processing rate is the limiting operational constraint, to calculate the optimum cut-off grade, Equations (5) and (6) should be substituted in Equation (4) to get Equation (7). Taking the derivative of Equation (7) with respect to grade and setting it to zero will result in the optimum cut-off grade. In Equation (8), the amount of material to be mined is independent of the grade; which makes *dqm/dg = 0*. Therefore, to make Equation (8) equal to zero, Equation (9) which is the first part of Equation (8) should be equal to zero. Solving Equation (9) for grade generates the optimum cut-off grade (Equation (10)). Details of an iterative algorithm used in generating the optimum cut-off grade profile and production schedule for oil sands mining considering waste management for dyke construction, and stockpiling with limited duration for a processing limited operation is presented in Seyed Hosseini (2017). Figure 3 illustrates the flow diagram of the iterative algorithm for implementing the ICOGO model.

Profit = Revenue - Processing Cost - Mining Cost - TCS Cost - OB Cost - IB Cost - Annual Fixed Cost

$$pr = (sp - sc)qr - pc.qp - mc.qm - tc.R_{TCS}.qp - bc.R_{OB}.(qm - qp) - ic.R_{IB}.(qm - qp) - FT \tag{4}$$

$$T_p = \frac{qp}{QP} \tag{5}$$

$$qr = g_{avg}.r_{avg}.qp \tag{6}$$

$$pr = \left((sp - sc).g_{avg}.r_{avg} - pc - tc.R_{TCS} + bc.R_{OB} + ic.R_{IB} - \frac{F}{QP} \right).qp - \left(mc + bc.R_{OB} + ic.R_{IB} \right).qm \tag{7}$$

$$\frac{dpr}{dg}=\left(\left(sp-sc\right).g_{avg}.r_{avg}-pc-tc.R_{TCS}+bc.R_{OB}+ic.R_{IB}-\frac{F}{QP}\right).\frac{dqp}{dg}-\left(mc+bc.R_{OB}+ic.R_{IB}\right).\frac{dqm}{dg}=0 \quad (8)$$

$$\left(\left(sp-sc\right).g_{avg}.r_{avg}-pc-tc.R_{TCS}+bc.R_{OB}+ic.R_{IB}-\frac{F}{QP}\right)=0 \quad (9)$$

$$g_p=\frac{pc+tc.R_{TCS}-bc.R_{OB}-ic.R_{IB}+\dfrac{F}{QP}}{\left(sp-sc\right).r_{avg}} \quad (10)$$

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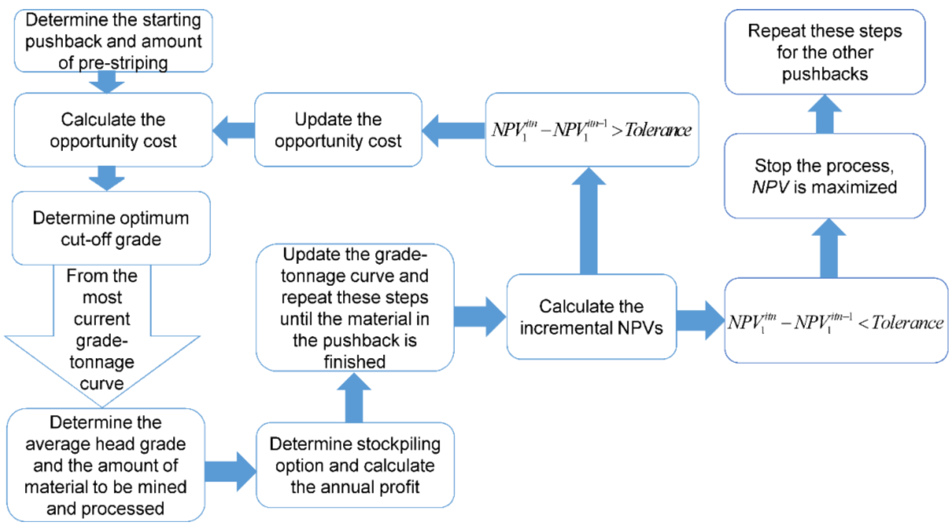

Figure 3: Flow diagram of iterative algorithm for implementing ICOGO model (Seyed Hosseini, 2017)

4.4. The Mixed Integer Linear Goal Programming (MILGP) Model

4.4.1. Block Clustering

A substantial challenge in finding the long-term optimal production schedule is a lack of adequate computer memory space during optimization calculations due to the exponential growth of the problem size with an increase in the number of blocks. The integer decision variables used in constructing the block mining precedence constraints require large computational resources during optimization. Employing clustering and paneling approaches reduce the optimization problem size and ensure minimum mining width is practical for the large mining equipment used in oil sands mining.

In this research, an agglomerative hierarchical clustering algorithm developed by Tabesh and Askari-Nasab (2011) is used in aggregating mining blocks into mining-cuts for solving the mine production scheduling problem. The clustering algorithm is customized for mine production planning problems. Using this algorithm, ore data is summarized as well as the total quantity of elements contained in the mining-cuts from the mining blocks. Also, the separation of lithology is maintained. Mining-cuts are made up of blocks within the same level and are grouped based on their attributes; location, rock type, and grade. Mining-panels are made up of mining-cuts and are used to control the mine extraction sequence. A mining-panel is the intersection of the material in a mining phase/pushback and a mining bench (Ben-Awuah and Askari-Nasab, 2013). Figure 4 shows the relation between blocks, mining-cuts and mining-panels on a level.

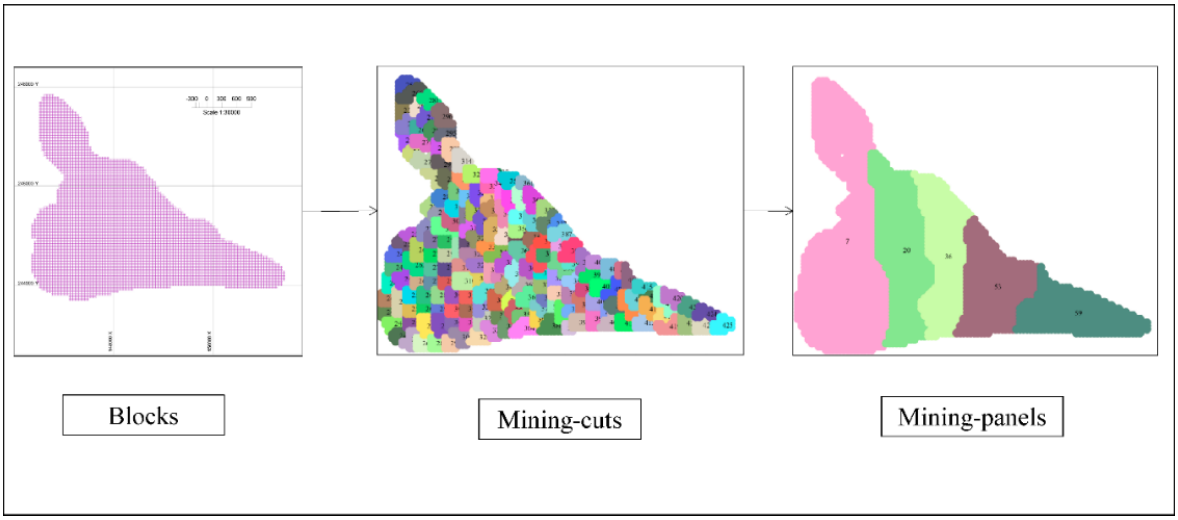

Figure 4: Relation between blocks, mining-cuts and mining-panels on a level

4.4.2. Economic Mining-cut Value

Each mining-cut has an economic value based on mining blocks which can be mined selectively within the mining-cut. The total discounted cost involved in excavating each mining-cut are: the base discounted mining costs for excavating mining-cut *K* as waste; the extra discounted costs of processing the ore parcels contained in the mining-cut *K* at the designated processing destination; the extra discounted costs of excavating OB, IB and the generated TCS dyke material from mining-cut *K* for dyke construction at a designated destination; and the discounted annual fixed cost. The discounted profit generated from extracting each mining-cut can be defined based on the total discounted revenue generated from selling the final product within each mining-cut minus the total discounted cost involved in extracting the mining-cut. Mining-panels are made up of mining-cuts that belong to the same pushback and mining bench. The sum of the discounted economic mining-cuts values within each mining-panel determines the discounted economic mining-panel value.

Equation (11) shows the discounted economic mining-cut value for mining-cut *K* that is sent from the mine to the plant. Equation (12) shows the discounted economic mining-cut value for mining-cut *K* that is sent from the stockpile to the plant.

$$d_k^{d,t} = v_k^{c,t} - c_k^{l,t} - eo_k^{d,t} - ei_k^{d,t} - et_k^{d,t} \qquad (11)$$

$$d_{k,s}^{d,t} = sv_{k,s}^{c,t} - c_k^{l,t} - eo_k^{d,t} - ei_k^{d,t} - et_k^{d,t} \qquad (12)$$

Equations (13) to (18) define the parameters in Equations (11) and (12). Equation (13) defines the discounted revenue generated from selling the final product within each mining-cut *K* minus the discounted cost of processing, minus the discounted annual fixed cost. Equation (14) defines the discounted revenue generated from selling the final product within each mining-cut *K* processed from the stockpile, minus the discounted cost of processing, minus the extra discounted cost of re-handling the stockpile material, minus the discounted annual fixed cost. Equation (15) defines the base discounted mining cost for extracting mining-cut *K* as waste. Equations (16) to (18) show the extra discounted cost of mining OB, IB and TCS dyke material respectively, from mining-cut *K* to the appropriate dyke construction destinations.

$$v_k^{c,t} = o_k g_k^e r_{avg}^{c,e} (sp^{e,t} - sc^{e,t}) - o_k pc^{c,e,t} - o_k \left(\frac{F^t}{PT^{c,t}} \right) \quad (13)$$

$$sv_{k,s}^{c,t} = o_k g_k^e r_{avg,s}^{c,e} (sp^{e,t} - sc^{e,t}) - o_k pc^{c,e,t} - o_k kc_s^{c,e,t} - o_k \left(\frac{F^t}{PT^{c,t}} \right) \quad (14)$$

$$c_k^{l,t} = (o_k + od_k + id_k + w_k) mc^{l,t} \quad (15)$$

$$eo_k^{d,t} = od_k bc^{d,t} \quad (16)$$

$$ei_k^{d,t} = id_k ic^{d,t} \quad (17)$$

$$et_k^{d,t} = td_k tc^{d,t} \quad (18)$$

Using these equations, the economic mining-cut value of material from mine to plant or from stockpile to plant can be evaluated.

4.4.3. MILGP Objective Function

In order to maximize the NPV of the mine operation, the MILGP model objective function should contain all of the following parameters: determining the time and sequence for removal of ore, dyke and waste material from the UPL; minimizing the dyke construction cost; and minimizing deviations from production goals which are inputs from the ICOGO model. Here, the model presented by Ben-Awuah et al. (2012) was used as a starting point. The MILGP model uses two sets of decision variables: binary integer decision variables to control precedence relation of mining-panels extraction; and continuous decision variables to control the mining, processing, stockpiling, OB, IB and TCS dyke material production requirements. In addition, continuous deviational variables have been defined to control the mining, processing, OB, IB and TCS dyke material production goals, and processing plant head grade goals. These variables provide an option for the user to set a continuous range of units for the optimization process to achieve the targeted goals with acceptable deviations. To prioritize goals and set precedence for achieving one goal over another, priority parameters were defined. The main goal that the user wants to achieve can have the highest priority parameter, ensuring that the optimization process will achieve that goal. For any deviation from the targeted goals, a penalty cost that reduces the NPV can be charged. Prioritized penalty parameters were defined to control deviations from the targeted goals as shown in Equation (21).

Based on the regulatory requirements (Ellis, 2016a), oil sands mining companies cannot leave behind any ore material containing more than 7% bitumen. Based on the initial cut-off grade analysis conducted with Whittle (GEOVIA Whittle, 2013), materials containing more than 6% bitumen have economic potential. During production, it is assumed that all material sent to the stockpile will be reclaimed for processing after a specified stockpiling duration (kd). Stockpiling oil sands ore for longer periods results in oxidation that causes challenges during the bitumen extraction process. To add stockpiling to the MILGP model, a new set of decision variable, $x_{k,s}^{c,t}$, was introduced. Tabesh et al. (2015) modeled a stockpiling decision variable in an MILP model. In their model, stockpiling bins with known grade ranges for each period are considered to avoid a non-linear problem when adding stockpiling to the production scheduling problem. In the MILGP model developed in this research, it was assumed that for every period there are stockpile bins available where material can be sent, and after the stockpiling duration, this material can be reclaimed in entirety with known grades.

The maximization of NPV and minimization of dyke construction costs are determined using Equations (19) and (20). In these equations, continuous decision variables

$y_p^{l,t}$, $x_k^{c,t}$, $x_{k,s}^{c,t}$, $u_k^{d,t}$, $n_k^{d,t}$ and $z_k^{d,t}$ are controlling the mining, processing, stockpiling, OB, IB and TCS dyke material production, respectively. Equation (21) shows the minimization of the deviation variables from the set targets. For mining, processing, OB, IB and TCS dyke material production goals, we define negative deviational variables which are $gd_1^{-,l,t}$, $gd_2^{-,c,t}$, $gd_3^{-,d,t}$, $gd_4^{-,d,t}$ and $gd_5^{-,d,t}$, respectively. However, for average processing plant head grade goal we define negative $\left(gd_6^{-,c,t}\right)$ and positive $\left(gd_6^{+,c,t}\right)$ deviational variables.

$$Max\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{s=1}^{S}\sum_{c=1}^{C}\sum_{t=1}^{T}\left(\sum_{\substack{k\in C_p\\ p\in C_m}}\left(v_k^{c,t}x_k^{c,t}+sv_{k,s}^{c,t}x_{k,s}^{c,t-kd}-c_p^{l,t}y_p^{l,t}\right)\right) \quad (19)$$

$$Min\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{d=1}^{D}\sum_{t=1}^{T}\left(\sum_{\substack{k\in C_p\\ p\in C_m}}\left(eo_k^{d,t}u_k^{d,t}+ei_k^{d,t}n_k^{d,t}+et_k^{d,t}z_k^{d,t}\right)\right) \quad (20)$$

$$Min\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{d=1}^{D}\sum_{t=1}^{T}\left[\sum_{\substack{k\in C_p\\ p\in C_m}}\left(\begin{array}{l}PP_1gd_1^{-,l,t}+PP_2gd_2^{-,c,t}+PP_3gd_3^{-,d,t}+PP_4gd_4^{-,d,t}+PP_5gd_5^{-,d,t}\\ +PP_6gd_6^{-,c,t}+PP_7gd_6^{+,c,t}\end{array}\right)\right] \quad (21)$$

To formulate a single objective function for the MILGP model, Equations (19) to (21) are combined to generate Equation (22).

$$Max\sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{d=1}^{D}\sum_{s=1}^{S}\sum_{c=1}^{C}\sum_{t=1}^{T}\left[\sum_{\substack{k\in C_p\\ p\in C_m}}\left[\begin{array}{l}\left(v_k^{c,t}x_k^{c,t}+sv_{k,s}^{c,t}x_{k,s}^{c,t-kd}-c_p^{l,t}y_p^{l,t}\right)-\left(eo_k^{d,t}u_k^{d,t}+ei_k^{d,t}n_k^{d,t}+et_k^{d,t}z_k^{d,t}\right)-\\ \left(\begin{array}{l}PP_1gd_1^{-,l,t}+PP_2gd_2^{-,c,t}+PP_3gd_3^{-,d,t}+PP_4gd_4^{-,d,t}+PP_5gd_5^{-,d,t}+\\ PP_6gd_6^{-,c,t}+PP_7gd_7^{+,c,t}\end{array}\right)\end{array}\right]\right] \quad (22)$$

4.4.4. MILGP Goal Functions

The MILGP model uses goal functions to accomplish the long-term production targets generated by the ICOGO model. The goal functions for production targets in tonnages are defined by Equations (23) to (27) for mining, processing, OB, IB and TCS dyke material. The average head grade goal function, Equation (28), is defined in terms of grade unit (%mass).

$$\sum_{m=1}^{M}\left(\sum_{p\in C_m}\left(o_p+od_p+id_p+w_p\right)y_p^{l,t}\right)+gd_1^{-,l,t}=MT^{l,t} \quad (23)$$

$$\left[\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}}\left(o_kx_k^{c,t}\right)\right)+\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}}\left(o_kx_{k,s}^{c,t-kd}\right)\right)\right]+gd_2^{-,c,t}=PT^{c,t} \quad (24)$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}}\left(od_ku_k^{d,t}\right)\right)+gd_3^{-,d,t}=OT^{d,t} \quad (25)$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}}\left(id_k n_k^{d,t}\right)\right)+gd_4^{-,d,t}=IT^{d,t} \tag{26}$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}}\left(td_k z_k^{d,t}\right)\right)+gd_5^{-,d,t}=TT^{d,t} \tag{27}$$

$$\frac{\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k g_k^e x_k^{c,t}\right)+\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k g_k^e x_{k,s}^{c,t-kd}\right)}{\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k x_k^{c,t}\right)+\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k x_{k,s}^{c,t-kd}\right)}+gd_6^{-,c,t}-gd_7^{+,c,t}=HT^{c,t} \tag{28}$$

Equation (23) controls the total quantity of material to be mined in each period and $gd_1^{-,l,t}$ allows the acceptable deviation from the mining target defined by the user. Equation (24) determines the total quantity of ore sent to the processing destination in each period from the mine and the stockpile. The quantity of material that was sent to the stockpile in period *t-kd,* is equal to the quantity of material sent to the processing destination from the stockpile in period *t*. In this equation, $gd_2^{-,c,t}$ controls the acceptable deviation from the set processing target. Equations (25) to (27) are the dyke material goal functions. Using these equations, the dyke material production target was set for different dyke construction destinations, which provides a practical schedule for dyke construction. Equation (28) controls the average head grade of the material being sent to the processing destination from the mine and stockpile. The acceptable negative and positive deviations from the set targets are controlled by $gd_6^{-,c,t}$ and $gd_6^{+,c,t}$, respectively. Equation (28) has a non-linear format. The numerator of the first part of the equation is equal to the quantity of element content in each production period and the denominator is equal to the quantity of material processed in each period. Dividing these two, will generate the grade of the material processed. In order to convert Equation (28) to a linear format, the head grade target and deviational variables are multiplied by the processing target to generate element content target. This requires that the set periodic processing targets must be achieved to ensure Equation (28) accurately monitors the head grade in each period.

In general, these goals are defined with guidance from the production schedule generated by the ICOGO model.

4.4.5. MILGP Cut-Off Grade Constraints

The optimum cut-off grade profile was generated by the ICOGO model. Here, the cut-off grade values for each period are used to control the grade of the material that can be sent to the processing destination in each period. From Equation (29), if the grade of the mining-cut *K* is less than the optimum cut-off grade in period *t*, then mining-cut *K* cannot be sent to the processing destination in period *t*. Equation (30) controls the grade of material that can be sent to the stockpile in each period. Based on this equation, if the grade of mining-cut *K* is higher than the optimum cut-off grade of period *t* or is less than the minimum acceptable grade, mining-cut *K* cannot be sent to the stockpile in period *t*.

$$x_k^{c,t}\le 0 \qquad \forall\ g_k^e < g_{opt}^{c,t,e} \tag{29}$$

$$x_{k,s}^{c,t} \leq 0 \qquad \forall\ g_k^e \geq g_{opt}^{c,t,e}\ \ or\ \ g_k^e < g_l \tag{30}$$

4.4.6. MILGP Fines Blending Constraints

To control the quality of material sent to dyke construction destinations, materials should meet the fines requirements. The ore material sent to the processing destination should have the quality required at the processing destination in terms of bitumen and fines. Inequality Equations (31) and (32) ensure that the ore material sent to the processing destination is between the minimum and maximum fines requirements. Equations (33) and (34) verify the same requirements for the ore material that has been sent to the stockpile, since they will be processed in subsequent years. Based on the dyke construction requirements, IB dyke material should have the required fines content. Inequality Equations (35) and (36) guarantee that the IB dyke material sent to the different dyke construction destinations have between the minimum and maximum fines requirements.

$$\underline{fi}^{c,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k x_k^{c,t}\right)-\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k fi_k^e x_k^{c,t}\right)\leq 0 \tag{31}$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k fi_k^e x_k^{c,t}\right)-\overline{fi}^{c,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k x_k^{c,t}\right)\leq 0 \tag{32}$$

$$\underline{fi}^{c,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k x_{k,s}^{c,t}\right)-\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k fi_k^e x_{k,s}^{c,t}\right)\leq 0 \tag{33}$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k fi_k^e x_{k,s}^{c,t}\right)-\overline{fi}^{c,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k x_{k,s}^{c,t}\right)\leq 0 \tag{34}$$

$$\underline{fi}^{d,t,\mathrm{id}}\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} id_k n_k^{d,t}\right)-\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} id_k fi_k^{id} n_k^{d,t}\right)\leq 0 \tag{35}$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} id_k fi_k^{id} n_k^{d,t}\right)-\overline{fi}^{d,t,\mathrm{id}}\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} id_k n_k^{d,t}\right)\leq 0 \tag{36}$$

4.4.7. MILGP Mining-Panels Extraction Precedence Constraints

Mining-panels have been used to reduce the number of integer variables and to help solve the optimization problems in a more efficient manner. Mining-panels also provide good minimum mining width for the large cable shovels and trucks used in oil sands mining.

In order to control the mining-panels extraction precedence, a set of binary integer variables, $\left(b_p^t \in [0,1]\right)$ are used. If the extraction of mining-panels p has started in or by period t, b_p^t is equal to one, otherwise, it is zero. Equation (37) ensures that all the immediate preceding mining-panels above mining-panel p are extracted before mining-panel p can be extracted. $F_p(L)$ is the set containing all the immediate predecessor mining-panels above mining-panel p. Equation (38) ensures that all the immediate preceding mining-panels in the horizontal mining direction of mining-panel *p* are extracted before mining-panel *p* can be extracted. $R_p(Z)$ is the set containing all the immediate preceding mining-panels in the horizontal mining direction, preceding mining-panel *p*. Equation (39) ensures that before mining-panel *p* can be extracted, all the immediate predecessor mining-panels in a mining phase, are extracted. $C_m(H)$ is the set containing all the immediate preceding mining-panels to mining-panel *p* in a mining phase. Equation (40) ensures that if mining-panel *p* has not been extracted in previous periods, then the extraction of mining-panel *p* can start. Equation (41) ensures that if mining-panel *p* extraction starts in period *t*, then mining-panel *p* will be available for extraction in subsequent periods.

$$b_p^t - \sum_{v=1}^{l}\sum_{i=1}^{t} y_w^{v,i} \le 0 \qquad w \in F_p(L) \tag{37}$$

$$b_p^t - \sum_{v=1}^{l}\sum_{i=1}^{t} y_j^{v,i} \le 0 \qquad j \in R_p(Z) \tag{38}$$

$$b_p^t - \sum_{v=1}^{l}\sum_{i=1}^{t} y_g^{v,i} \le 0 \qquad g \in C_m(H) \tag{39}$$

$$\sum_{v=1}^{l}\sum_{i=1}^{t} y_p^{v,i} - b_p^t \le 0 \tag{40}$$

$$b_p^t - b_p^{t+1} \le 0 \tag{41}$$

4.4.8. MILGP Variables Control Constraints

In the MILGP model, the decision variables logics are controlled by applying the variables control constraints, ensuring the requirements of each variable are met. Inequality Equation (42) ensures that the material mined as ore and dyke material from mining-cuts belonging to mining-panel *p* in period *t* are less or equal to the total material mined from mining-panel *p* in period *t* from any mining location. Equation (43) is a reserve constraint that ensures that the total available ore in each mining phase will be mined. This facilitates in-pit tailings deposition once a phase is completely extracted. Inequality Equations (44) to (47) ensure that the summation of the portions of the mining-panels and mining-cuts scheduled for different destinations in different periods are less than or equal to one. Since the TCS dyke material is produced from processed ore, Equation (48) ensures that the fraction of TCS scheduled in each period is less than or equal to the fraction of ore processed in that period.

$$\sum_{d=1}^{D}\sum_{s=1}^{S}\sum_{c=1}^{C}\sum_{\substack{k\in C_p\\ p\in C_m}} \left(o_k x_k^{c,t} + o_k x_{k,s}^{c,t} + od_k u_k^{d,t} + id_k n_k^{d,t} \right) \le \sum_{l=1}^{L}\sum_{p\in C_m} \left[y_p^{l,t}\left(o_p + od_p + id_p + w_p \right) \right] \tag{42}$$

$$\sum_{s=1}^{S}\sum_{c=1}^{C}\sum_{t=1}^{T} x_k^{c,t} + x_{k,s}^{c,t} = 1 \tag{43}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} y_p^{d,t} \le 1 \tag{44}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} u_k^{d,t} \le 1 \tag{45}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} n_k^{d,t} \le 1 \tag{46}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} z_k^{d,t} \le 1 \tag{47}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} z_k^{d,t} \le \sum_{c=1}^{C}\sum_{t=1}^{T} x_k^{c,t} + \sum_{s=1}^{S}\sum_{t=1}^{T} x_k^{s,t-kd} \qquad t-kd>0 \tag{48}$$

4.4.9. MILGP Non-Negativity Constraints

Equation (49) ensures that the decision variables for mining, processing, stockpiling, OB, IB and TCS dyke material cannot be negative. To support the goal functions, Equation (50) also ensures that the deviational variables cannot be negative.

$$y_p^{l,t},\ x_k^{c,t},\ x_{k,s}^{c,t},\ u_k^{d,t},\ n_k^{d,t},\ z_k^{d,t} \ge 0 \tag{49}$$

$$gd_1^{-,l,t},\ gd_2^{-,c,t},\ gd_3^{-,d,t},\ gd_4^{-,d,t},\ gd_5^{-,d,t},\ gd_6^{-,c,t},\ gd_7^{+,c,t} \ge 0 \tag{50}$$

5. Application to an oil sands deposit

The ICOGO model and the MILGP framework discussed in Section 4 are applied to an oil sands case study. Different stockpiling scenarios were investigated to assess the impact of the stockpiling duration on the mining operation. The final pit limit was considered to have three main pushbacks for phased mining. The horizontal mining precedence is defined based on these three main pushbacks. Figure 5 shows the three pushbacks within the final pit limit and the bitumen grade distribution on level 302.5 m.

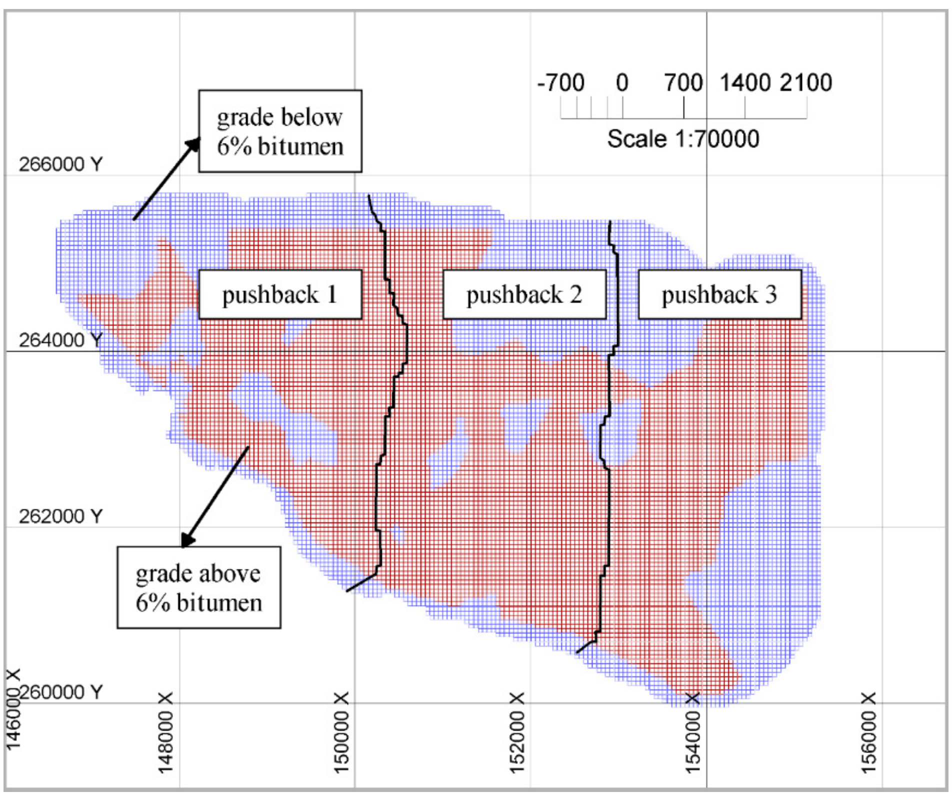

Figure 5: Pushbacks and bitumen grade distribution in the case study area on level 302.5 m

Table 1 contains information about the oil sands deposit for the case study and Table 2 shows the economic data extracted and compiled from Ben-Awuah (2013) and Burt et al. (2012). Since all the input parameters are considered to be deterministic, a discount rate of 15% is used to factor in the risks associated with the development of oil sands resources.

Table 1: Oil sands deposit pushbacks and final pit characteristics

Description	Value			
	Pushback 1	Pushback 2	Pushback 3	Final pit (Total)
Total tonnage of material (Mt)	1989.3	2294.6	2246.3	6530.2
Total ore tonnage (Mt)	695.4	875.8	728.4	2299.6
Total TCS dyke material tonnage (Mt)	476.1	582.4	573.5	1632.0
Total OB dyke material tonnage (Mt)	600.7	676.8	595.2	1872.7
Total IB dyke material tonnage (Mt)	448.3	579.3	600.7	1628.4
Total waste tonnage (Mt)	244.8	162.6	321.9	729.3
Number of blocks	26,334	30,129	28,706	85,169
Number of mining-cuts	754	858	814	2,426
Number of mining-panels	45	41	39	125
Number of mining benches	9	9	9	9

Table 2: Economic parameters (Ben-Awuah, (2013); Burt et al., (2012))

Parameter	Value
Mining cost ($/tonne)	2.5
Processing cost ($/tonne)	5.03
Stockpiling cost ($/tonne)	0.5
TCS dyke material cost ($/tonne)	0.92
OB dyke material cost ($/tonne)	0.95
IB dyke material cost ($/tonne)	0.95
Selling price ($/bitumen %mass)	4.5
Annual fixed cost (M$/year)	1,590
Discount rate (%)	15

The ICOGO model was coded in Matlab (Mathworks, 2015) and implemented on the oil sands deposit. The model was implemented based on two stockpiling management scenarios: Scenario 1a) reclaiming stockpile simultaneously with the mining operation after one year duration and Scenario 2a) reclaiming stockpile simultaneously with the mining operation after two years duration. The optimization problem was solved in 2.2 seconds for Scenario 1a and 1.9 seconds for Scenario 2a. Figures 6 and 7 show the production schedule and average head grade for Scenario 2a. Details of the implementation of the ICOGO model for the case study have been documented in Seyed Hosseini (2017). The cut-off grade profile, average head grade and production schedule generated by the ICOGO model are used as a guide for setting up the MILGP mine planning model requirements.

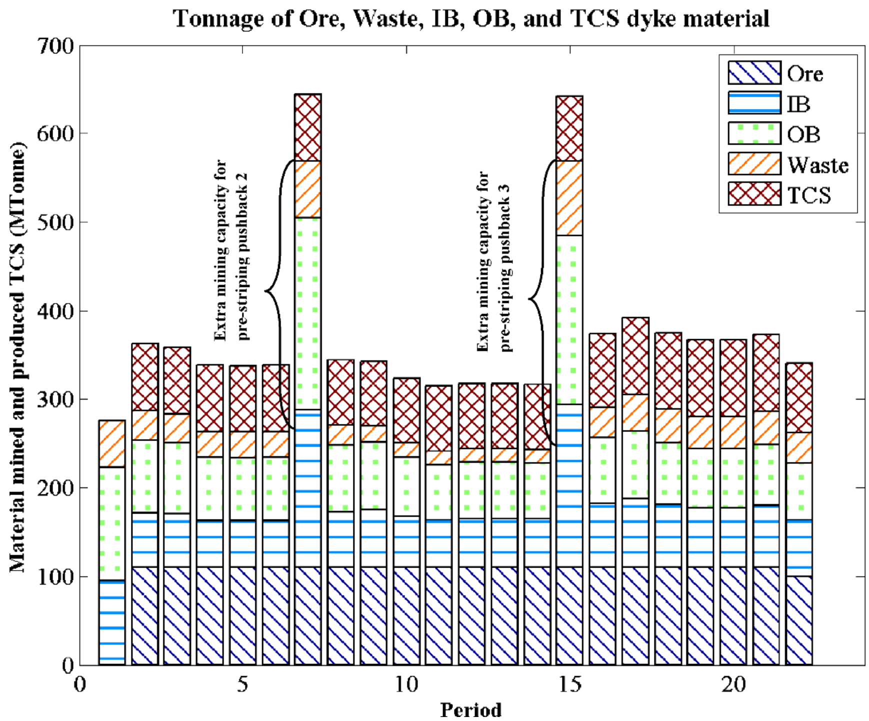

Figure 6: Schedule for material mined, processed and produced TCS for Scenario 2a

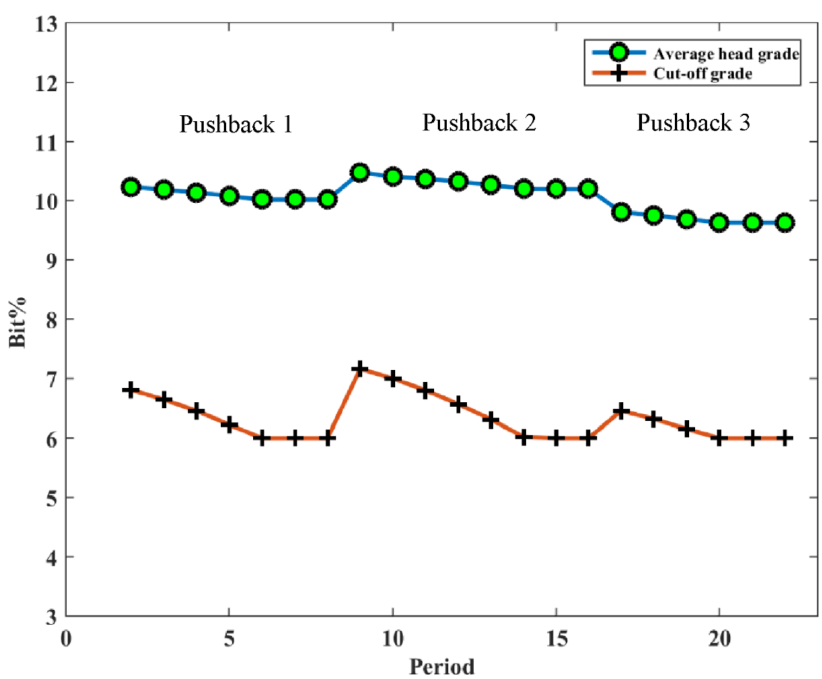

Figure 7: Average head grade and cut-off grade profile for Scenario 2a

The MILGP model was implemented in Matlab (Mathworks, 2015) and IBM CPLEX (IBM ILOG, 2012) was used as the optimization solver. IBM CPLEX (IBM ILOG, 2012) uses the branch-and-cut algorithm which is a hybrid of branch-and-bound algorithm and cutting plane methods to solve the optimization problem. The termination criterion, which is known as the gap tolerance (EPGAP), needs to be set by the user. EPGAP sets a relative tolerance on the gap between the best integer objective and the objective of the best node remaining in the branch-and-cut algorithm. CPLEX will terminate the optimization process when a feasible integer solution within the set EPGAP has been reached. An EPGAP of 10% was set as the termination criterion for the optimization process. The MILGP model was implemented on a Lenovo P510 computer with Intel Xeon (E5) at 3.6 GHz with 64 GB RAM. The optimization problem was solved in 108.2 hours for the scenario with one year stockpile duration (Scenario 1b) and 58.1 hours for the scenario with two years stockpile duration (Scenario 2b).

5.1. Discussion of Results: Scenario 2b

One of the main advantages of the MILGP model is the ability to setup production goals with allowable deviational variables which ensures that a feasible solution can be achieved each time whereas an infeasible solution will have been reported for other mathematical programming frameworks. The prioritized penalty parameters provide options for planners to ensure some goals are closely achieved over others. Different goals were defined based on mine management requirements. In the case study, one of the main objectives is to get a uniform ore production rate for the processing plant.

Figure 8 shows the schedule for material mined, processed, and OB, IB and TCS dyke material for different dyke construction destinations for Scenario 2b. The MILGP model generated an NPV of $6,014.9 M for the life of mine including the waste management cost for Scenario 2b. A summary of the numerical results from the MILGP model can be seen in Table 3.

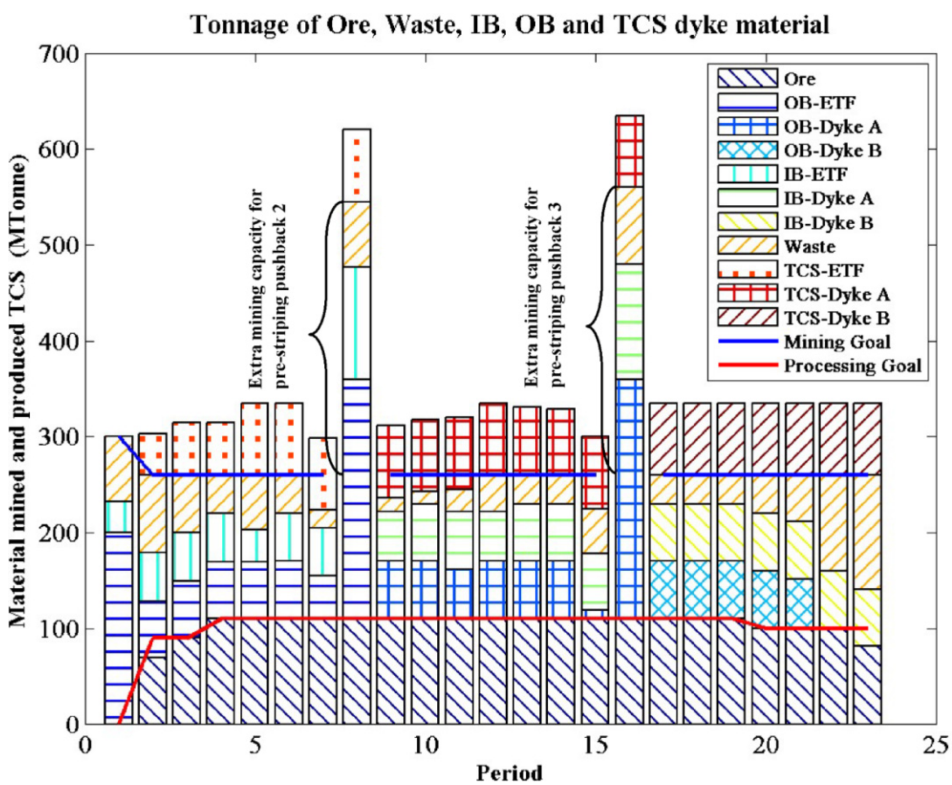

Figure 8: Schedule for material mined, processed and produced TCS for Scenario 2b

Table 3: Numerical results from the MILGP model

Stockpile management scenario	Total material mined (Mt)	Total material Processed (Mt)	Material processed from stockpile (Mt)	Total dyke Material (Mt)	NPV (M$)
Scenario 1b: Reclamation after 1 year	6,501.0	2,299.6	43.8	4,666.2	5,950.4
Scenario 2b: Reclamation after 2 years	6,478.2	2,299.6	42.7	4,641.8	6,014.9

From the production schedule, pre-stripping was enforced in year one and the preferred processing target in year two had to be adjusted. This was due to the location of ore material in the pit and mining precedence of the mining-cuts. The ICOGO model does not take into consideration the actual mining precedence of mining-cuts and hence generated a production target which was unachievable in year two. In the remaining years, all processing targets were achieved until the last year when the ore material was finished. The ore production target starts with 90 Mt and then ramps up to a maximum capacity of 110 Mt. In the last four years, the target is reduced to 100 Mt. Figure 9 shows the stockpile material scheduled for processing in Scenario 2b. The stockpile schedule shows that all material sent to stockpile in any year is fully reclaimed after two years duration. In the MILGP model, the average bitumen head grade is calculated based on the mining-cuts that are processed directly from the mine and reclaimed from the stockpile in each period, taking into consideration the practical mining-cut extraction sequencing. Figure 10 shows the average bitumen head grade target for each period and the scheduled average bitumen head grade with the MILGP model for Scenario 2b. It can be seen that the average bitumen head grade generated by the MILGP model fluctuates around the target provided by the ICOGO model for the most part. Figure 11 illustrates the mining sequence on level 302.5 m for Scenario 2b.

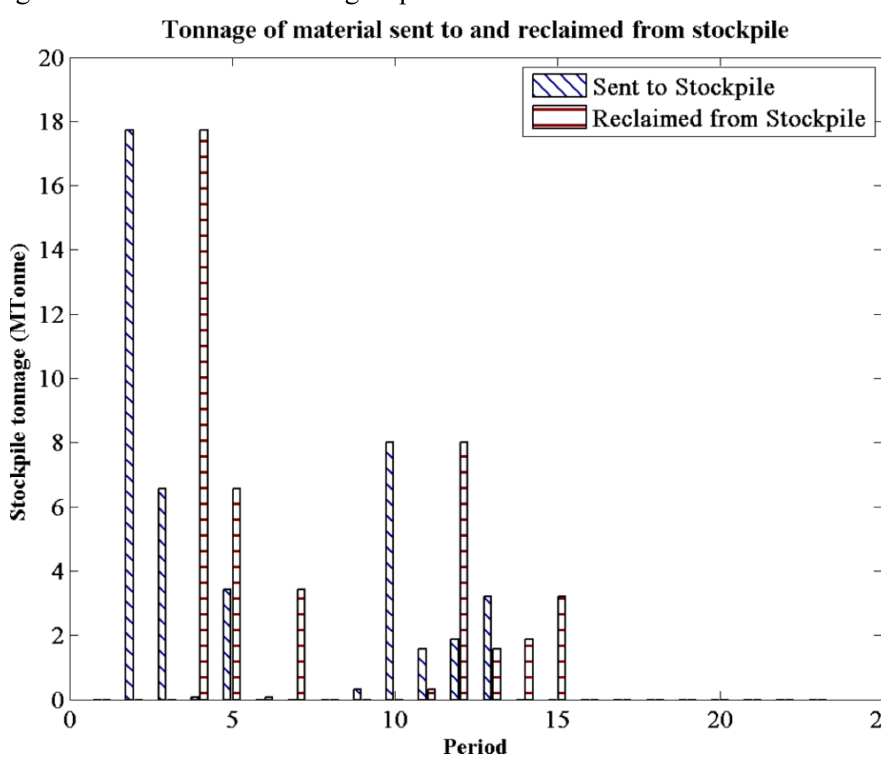

Figure 9: Schedule for material stockpiled and reclaimed after two years in Scenario 2b

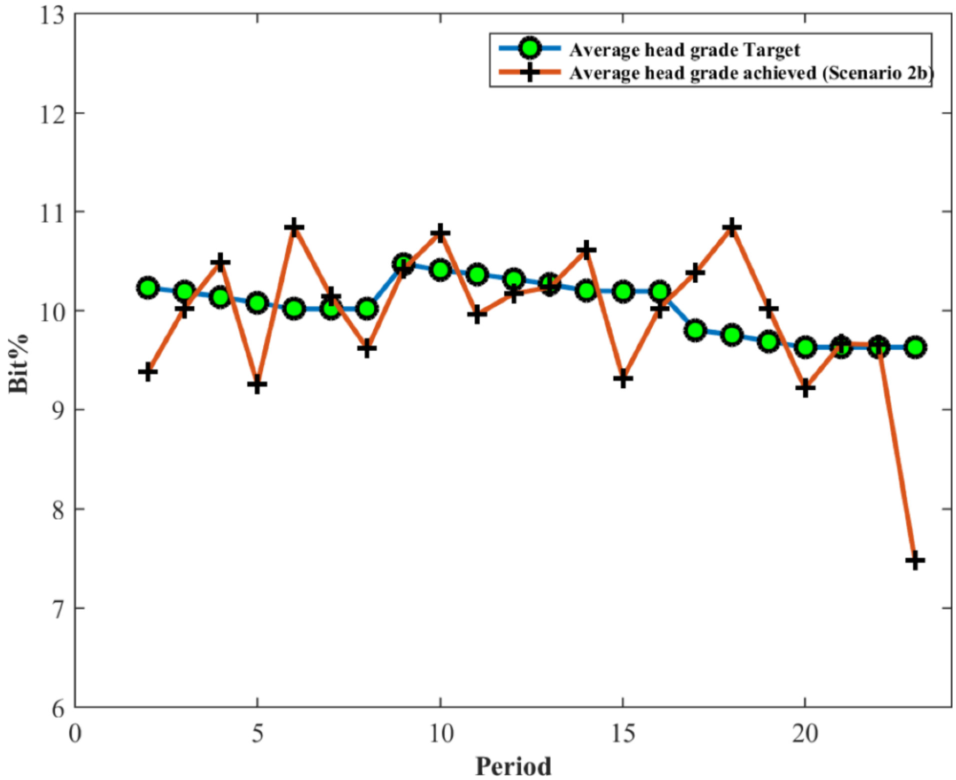

Figure 10: Average bitumen head grade for Scenario 2b

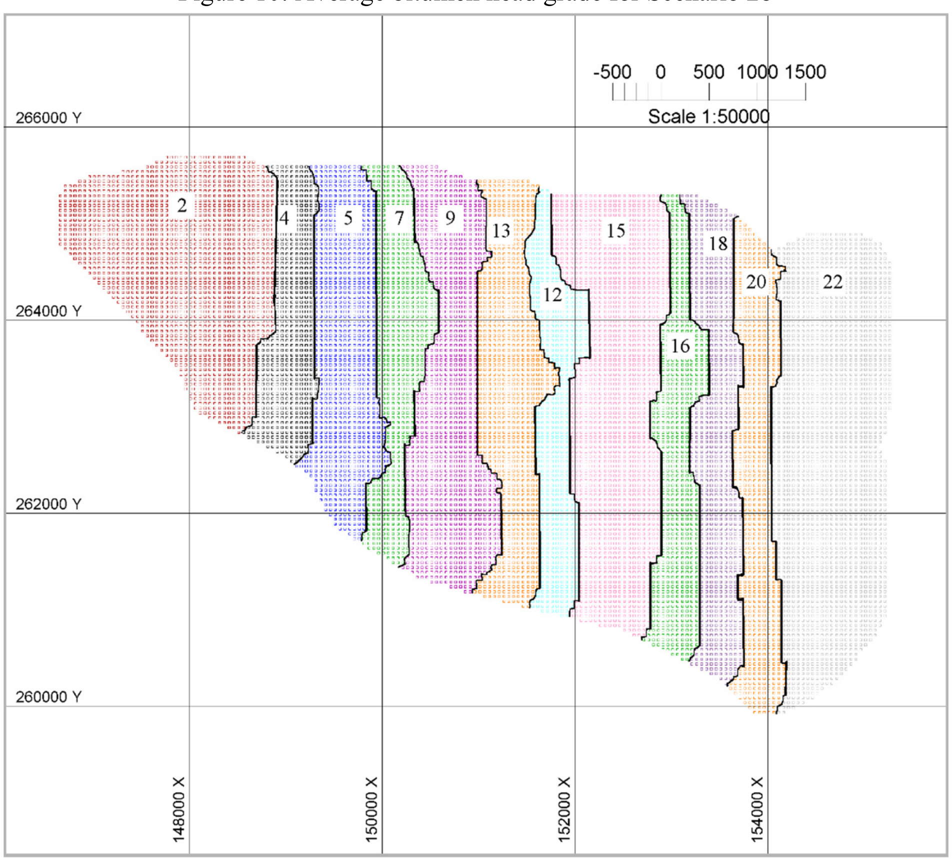

Figure 11: Mining sequence on level 302.5 m for Scenario 2b

5.2. Comparative Experiment

In the previous experiments (Scenarios 1b and 2b), the production schedule and average head grade generated by the ICOGO model were used as inputs to setup targets for the MILGP goal functions. In order to evaluate the impact of the average head grade target provided by the ICOGO model, another optimization experiment (Scenario 3) was conducted by replacing the average head grade goal function (Equation (28)) with Equation (51). Equation (51) specifies the average head grade limiting bounds for ore processing.

$$\underline{g}^{c,t,e} \leq \frac{\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k g_k^e x_k^{c,t}\right) + \sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k g_k^e x_{k,s}^{c,t-kd}\right)}{\sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k x_k^{c,t}\right) + \sum_{p=1}^{P}\left(\sum_{\substack{k\in C_p\\ p\in C_m}} o_k x_{k,s}^{c,t-kd}\right)} \leq \overline{g}^{c,t,e} \tag{51}$$

These experiments were run to 10% EPGAP and were designed to highlight the impact of the two-step approach on the solution time. For the case study investigated, the new optimization problem (Scenario 3) was solved in 87.2 hours compared to 58.1 hours for Scenario 2b. This comparative analysis showed that using the initial results provided by the ICOGO model to setup a goal function limits the solution space thereby causing the near-optimal solution to be reached faster. The two-step approach improved the solution time by about 33%. Table 4 shows a summary of the results from the comparative experiment.

Table 4: Summary results for comparative experiment

Experiment	Number of constraints	Number of continuous variables	Number of binary variables	Solution time (hrs)	NPV (M$)
Scenario 2b	152,764	845,710	8,372	58.1	6,014.9
Scenario 3	152,787	845,664	8,372	87.2	5,975.3

6. Conclusion

Long-term production scheduling optimization is one of the important aspects of mine planning. In achieving the maximum benefit from a mining operation, the long-term production schedule should detail the time and sequence of removing ore and waste material from the final pit limit. Improving the efficiency of production scheduling optimization tools' performance in the mining industry is a high priority task since the economic gains are considerably high.

In the case of oil sands mining, the waste management strategy drives the sustainability and profitability of the mining operation. It makes it necessary to consider the waste management cost and its constraints in the cut-off grade optimization process for integrated long-term production scheduling. In this research, the ICOGO model which is based on a fast heuristic optimization framework was developed. The ICOGO model determines the optimum cut-off grade policy taking into consideration stockpiling with limited duration, and waste management costs for dyke construction. Subsequently, the cut-off grade profile, average head grade and production schedule generated by the ICOGO model were used as guides in setting up the inputs for the MILGP model.

The MILGP model generates a more practical and detailed schedule for extracting ore, waste and dyke material from the final pit limit. The MILGP model provides simultaneous stockpile reclamation with the specified stockpiling duration taking into consideration processing recovery changes resulting from oxidation of stockpiled ore. The MILGP model provides a framework consistent for sustainable oil sands mining with respect to regulatory requirements. Table 3 presents a summary of the MILGP model results for Scenarios 1b and 2b. The NPV generated by Scenario 2b was 1% higher than Scenario 1b. This increase in NPV is due to different dyke material tonnages scheduled and the flexibility in stockpiling and reclamation in Scenario 2b allowing the optimizer to send higher grades for processing in early years to generate more profit.

Although the level of detail of the production schedule generated by the ICOGO model is not similar to the MILGP model, it provides an initial production schedule for the life of mine planning. The results from the ICOGO model can be used as a guide for detailed long-, medium-, and short-term mine planning with any production scheduling optimization framework. It should be mentioned that the main advantage of the ICOGO model over the MILGP model is the significantly less solution time. For the case study investigated, the ICOGO model was solved in less than 3 seconds whereas the MILGP model required 58.1 hours to solve on a Lenovo P510 computer with Intel Xeon (E5) at 3.6 GHz with 64 GB of RAM. In conclusion, whereas the ICOGO model solved the optimization problem faster, the MILGP model results provide detailed mining-cut extraction sequencing for practical mining.

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8. Appendix

Indices and Sets

$c \in C$, $\mathrm{C} = \{1,2,......,\mathrm{C}\}$ index and set for all the possible processing destinations.

$d \in D$, $\mathrm{D} = \{1,2,......,\mathrm{D}\}$ index and set for all the possible destinations for materials.

$e \in \mathrm{E}$, $\mathrm{E} = \{1,2,......,\mathrm{E}\}$ index and set for all the elements of interest in each mining-cut.

$k \in \mathrm{K}$, $\mathrm{K} = \{1,2,......,\mathrm{K}\}$ index and set for mining-cuts.

$l \in \mathrm{L}$, $\mathrm{L} = \{1,2,......,\mathrm{L}\}$ index and set for all the possible mining location.

$m \in \mathrm{M}$, $\mathrm{M} = \{1,2,......,\mathrm{M}\}$ index and set for all the phases (pushbacks).

$p \in \mathrm{P}$, $\mathrm{P} = \{1,2,......,\mathrm{P}\}$ index and set for mining-panels.

$t \in \mathrm{T}$, $\mathrm{T} = \{1,2,......,\mathrm{T}\}$ index and set for all the scheduling periods.

$s \in S$, $\mathrm{S} = \{1,2,......,\mathrm{S}\}$ index and set for all stockpiles.

$C_p(V)$ for each mining-panel p, there is a set $C_p(V) \subset \mathrm{K}$ defining the mining-cuts that belongs to the mining panel p, where V is the total number of mining-cuts in the set $C_p(V)$.

$C_m(H)$ for each phase m, there is a set $C_m(H) \subset \mathrm{P}$ defining the mining-panels within the immediate predecessor pit phases (pushbacks) that must be extracted prior to extracting phase m, where H is an integer number representing the total number of mining panels in the set $C_m(H)$.

$F_p(L)$ for each mining-panel p, there is a set $F_p(L) \subset \mathrm{P}$ defining the immediate predecessor mining-panels above mining-panel p that must be extracted prior to extraction of mining-panel p, where L is the total number of mining-panels in the set $F_p(L)$.

$R_p(Z)$ for each mining-panel p, there is a set $R_p(Z) \subset \mathrm{P}$ defining the immediate predecessor mining-panels in a specified horizontal mining direction that must be extracted prior to extraction of mining-panel p at the specified level, where Z is the total number of mining-panels in the set $R_p(Z)$.

Decision variables

$b_p^t \in [0,1]$**,** a binary integer variable controlling the precedence of extraction of mining-panels. If the extraction of mining-panel p has started by or in period t, b_p^t is equal to one, otherwise it is zero.

$gd_1^{-,l,t}$**,** the amount of negative deviation from the mining target (tonnes) at location l in period t.

$gd_2^{-,c,t}$**,** the amount of negative deviation from the processing target (tonnes) at processing destination c in period t.

$gd_3^{-,d,t}, gd_4^{-,d,t}, gd_5^{-,d,t}$**,** the amount of negative deviation from the overburden, interburden and tailings coarse sand dyke material target (tonnes) at destination d in period t, respectively.

$gd_6^{-,c,t}$, the amount of negative deviation from average head grade target (%mass) at processing destination c in period t.

$gd_6^{+,c,t}$, the amount of positive deviation from average head grade target (%mass) at processing destination c in period t.

$n_k^{d,t}, u_k^{d,t}, z_k^{d,t} \in [0,1]$, a continuous variable representing the interburden, overburden and tailings coarse sand dyke material portion of mining-cut k to be extracted and used for dyke construction at destination d in period t, respectively.

$x_k^{c,t} \in [0,1]$, a continuous variable representing the ore portion of mining-cut k to be extracted and processed at destination c in period t.

$x_{k,s}^{c,t} \in [0,1]$, a continuous variable representing the ore portion of mining-cut k to be extracted and sent to stockpile s in period $t-kd$ and reclaimed to be processed at destination c in period t.

$y_p^{l,t} \in [0,1]$, a continuous variable representing the ore portion of mining-panel p to be mined in period t from location l, which includes ore, overburden and interburden dyke material and waste from the associated mining-cuts.

Parameters

bc, the cost per tonne of overburden dyke material for dyke construction.

$bc^{d,t}$, the cost in present value terms per tonne of overburden dyke material for dyke construction at destination d.

$c_k^{l,t}, c_p^{l,t}$, the discounted cost of mining all the material in mining-cut k and mining-panel p as waste from location l in period t, respectively.

$d_k^{d,t}$, the discounted economic mining-cut value obtained by extracting mining-cut k and sending it to destination d in period t.

$d_{k,s}^{d,t}$, the discounted economic mining-cut value obtained by extracting mining-cut k and sending it to stockpile s and reclaiming it to destination d in period t.

$ei_k^{d,t}, eo_k^{d,t}, et_k^{d,t}$, the extra discounted cost of mining all the material in mining-cut k as interburden, overburden and tailings coarse sand dyke material for construction at destination d in period t, respectively.

F, the annual fixed cost.

F^t, the discounted annual fixed cost in period t.

fi_k^e, the average percent of fines in ore portion of mining-cut k.

$\underline{fi}^{c,t,e}, \overline{fi}^{c,t,e}$, the lower and upper bound on the required average fines percent of ore at processing destination c in period t.

fi_k^{id}, the average percent of fines in interburden dyke material portion of mining-cut k.

$\underline{fi}^{d,t,id}, \overline{fi}^{d,t,id}$, the lower and upper bound on the required average fines percent of interburden dyke material at dyke construction destination d in period t.

g_{avg_n}, the average head grade.

g_k^e, the average head grade of element e in ore portion of mining-cut k.

$g_{opt}^{c,t,e}$, the cut-off grade of element e at processing destination c in period t.

g_p, the processing limited cut-off grade.

$\underline{g}^{c,t,e}, \overline{g}^{c,t,e}$, the lower and upper bound on the required average bitumen head grade of ore at processing destination c in period t

$HT^{c,t}$, the average head grade target at processing destination c in period t.

i, the discount rate.

ic, the cost per tonne of interburden dyke material for dyke construction.

$ic^{d,t}$, the cost in present value terms per tonne of interburden dyke material for dyke construction at destination d.

id_k, id_p, the interburden dyke material tonnage in mining-cut k and in mining-panel p, respectively.

$IT^{d,t}$, the interburden dyke material target (tonnes) at destination d in period t.

$kc_s^{c,e,t}$, the extra cost in present value terms per tonne of ore for re-handling from stockpile s and processing at processing destination c in period t.

kd, the duration of stockpiling the material.

kt_n, the amount of material (tonnes) sent to the stockpile in each period.

mc, the cost of mining a tonne of waste.

$mc^{l,t}$, the cost in present value terms of mining a tonne of waste in period t from location l.

$MT^{l,t}$, the mining target (tonnes) at location l in period t.

o_k, o_p, the ore tonnage in mining-cut k and in mining-panel p, respectively.

od_k, od_p, the overburden dyke material tonnage in mining-cut k and in mining-panel p, respectively.

$OT^{d,t}$, the overburden dyke material target (tonnes) at destination d in period t.

pc, the extra cost per tonne of ore for mining and processing.

$pc^{c,e,t}$, the extra cost in present value terms per tonne of ore for mining and processing at processing destination c in period t.

$PP_1, PP_2, PP_3, PP_4, PP_5$, the prioritize penalty parameter associated with the deviation from the mining target, processing target, overburden, interburden and tailing coarse sand dyke material target, respectively.

PP_6, PP_7, the prioritize penalty parameter associated with the deviation from the average head grade target.

pr_n, the annual profit.

$PT^{c,t}$, the processing target (tonnes) at processing destination c in period t.

qm, the amount of material to be mined (tonnes)

QP, the maximum processing capacity in terms of tonnes per year

qp, the amount of material to be processed (tonnes)

qr, the amount of material to be refined (tonnes)

r_{avg}, the weighted average processing recovery factor.

$r_{avg}^{c,e}$, the proportion of element e recovered if it is processed at processing destination c (weighted average processing recovery).

$r_{avg,s}^{c,e}$, the proportion of element e recovered if it is reclaimed from stockpile s and processed at processing destination c (weighted average processing recovery).

R_{IB}, the ratio of the total amount of interburden dyke material over the total amount of waste material.

R_{OB}, the ratio of the total amount of overburden dyke material over the total amount of waste material.

R_{TCS}, the ratio of the total amount of tailings coarse sand dyke material over the total amount of ore material.

sp, the selling price per unit of product.

$sp^{e,t}$, the selling price of element e in present value terms per unit of product.

sc, the refinery and selling cost per unit of product.

$sc^{e,t}$, the refinery and selling cost of element e in present value terms per unit of product.

$sv_{k,s}^{c,t}$, the discounted revenue obtained by selling the final products within mining-cut k if it is sent to processing destination c in period t from stockpile s, minus the extra discounted cost of mining all the material in mining-cut k as ore from location l and processing at processing destination c; minus the extra discounted cost of re-handling for stockpile material; and minus the discounted annual fixed cost.

tc, the cost per tonne of tailings coarse sand dyke material for dyke construction.

$tc^{d,t}$, the cost in present value terms per tonne of tailings coarse sand dyke material for dyke construction at destination d.

td_k, the tailings coarse sand dyke material tonnage in mining-cut k.

$TT^{d,t}$, the tailings coarse sand dyke material dyke material target (tonnes) at destination d in period t.

$v_k^{c,t}$, the discounted revenue obtained by selling the final products within mining-cut k if it is sent to processing destination c in period t, minus the extra discounted cost of mining all the material in mining-cut k as ore from location l and processing at processing destination c; and minus the discounted annual fixed cost

w_k, w_p, the waste tonnage in mining-cut k and the waste tonnage in mining-panel p, respectively.