Footprint Calculation for Block-Cave Mining under Grade Uncertainty

Saha Malaki and Yashar Pourrahimian
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

Abstract

The initial evaluations of a range of levels for starting the extraction of block-cave mining is an important issue which needs to consider a variety of parameters such as extraction rate, block height, discount rate, block profit, cost of mining and processing and revenue factors. This paper deals with finding the best level to place a block cave extraction level based on the maximum discounted ore profit.

Grade uncertainty has profound impact on the total ore profit of each level and as a result on the placement of extraction level. A set of simulations of the mineral grade is modelled and for each realization the best level for starting extraction is specified.

The maximum NPV is determined using mathematical programming approach (MP) after determining the best extraction level. The model is created in MATLAB and CPLEX as a solver is implemented to solve the optimization problem. The purpose of this paper is to present a methodology to find the best extraction level under grade uncertainty. The extraction level is used to maximize the NPV; given some constraints such as mining capacity, grade of production, extraction rate and precedence.

1. Introduction

Among the underground mining methods available, caving methods are favored because of their low-cost and high-production rates. Production scheduling in block caving, because of its significant impact on the project's value, has been considered a key issue to be improved, so researchers have applied different methods to model production scheduling in block caving.

Grade uncertainty can lead to significant differences between actual production and planning expectations and as a result the NPV of the project (Osanloo et al., 2008; Koushavand and Askari-Nasab, 2009). Various researchers have considered the effects of grade uncertainty in open-pit mines and introduced different methodologies (Dimitrakopoulos, 2011). Dowd (1994) presented a risk-based algorithm for surface mine planning. For different variables such as commodity price, processing cost, mining cost, investment required, grade and tonnages a predefined distribution function is implemented. Several types of schedules are generated for a number of realizations of the grades. The result of this methodology is various schedules that accounts for grade uncertainty. Simulated ore-bodies are utilized by Ravenscroft (1992) and Koushavand and Askari-Nasab (2009) to show the influence of the grade uncertainty on production scheduling. Ramazan and Dimitrakopoulos (2004) used a mixed-integer linear programming

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(MILP) model to maximize NPV for each realization. Then, the probability of extraction of a block at each period is calculated. These probabilities are the input of a second stage of the optimization to have one schedule at the end. Dimitrakopoulos and Ramazan (2008) presented a linear integer programming (LIP) model for generating optimal production schedules. This model considers multiple realizations of the block model and defines a penalty function that is the cost of deviation from the target production. This cost is calculated based on the geological risk discount rate which is the discounted unit cost of deviation from target production. A linear programming is used to maximize a new function that is NPV less penalty costs.

Otherwise, a few numbers of authors have considered geological uncertainty in underground mining. Vargas et al. (2014) developed a tool that took geological uncertainty into account by using a set of conditional simulations of the mineral grades and defining the economic envelope in a massive underground mine. One of the main steps involved in the optimization of underground mines is determining a mining outline and inventory. The open-pit corollary to this is open-pit optimization, which is completed with algorithms such as Lerchs and Grossmann (1965). For optimization of block-caving scheduling, most of researchers have used mathematical programming; Linear Programming (LP), Mixed-Integer Linear Programming (MILP), and Quadratic programming (QP). LP is the simplest one in modelling and solving. Table 1shows some of the applied mathematical methodologies in block-caving production scheduling.

This paper will introduce a method in which the best level for initializing extraction according to the maximum discounted ore profit and grade uncertainty is found. Several realizations are modelled by using geostatistical studies to consider the grade uncertainty. The production schedule is generated for the given advancement direction and in presence of some constraints at the chosen level.

2. Methodology, assumption and notations

A geological block model represents the ore-body. Numerical data are used to represent each block's attributes, such as tonnage, density, grade, rock type, elevation, and profit data. Fig. 1 shows the summary of the methodology.

2.1. Geological Uncertainty

GSLIB (Deutsch and Journel, 1998) is used for geostatistical modeling in this paper. The first step for a geostatistical study is defining different rock types based on the drillhole data. In this study with assuming stationary domain within each rock type, the geostatistical modeling is performed for each of the rock types separately. The following steps are implemented for generating geological model:

1. Declustering
2. Multivariate statistical analysis
3. Determining the principle direction of continuity
4. Transforming data to Gaussian units
5. Calculating the variogram
6. Simulation

First, declustering algorithm is used to adjust the variable distributions for each rock type to decrease the weight of clustered samples (declus program). Then, the correlation of the multivariate data is determined (scatplt program). To determine the principle directions of continuity, global kriging is performed using arbitrary variograms with high range. In the case of rock-type modeling, indicator kriging (ik3d program) and for the rock-property modeling simple kriging (kt3dn program) is used. Afterwards, the data is transformed to Gaussian units; in the case of univariate data, nscore and for multivariate data, ppmt programs are used to removes the correlation between the variables in each rock type (Barnett, 2015).

Table 1. Summary of applied mathematical methodologies in block-caving production scheduling (Khodayari and Pourrahimian, 2014)

Author	Methodology	Model's objective(s)	Features
Song (1989)	MILP	Minimization of total mining cost	**LP:** This method has been used most extensively and it can provide a mathematically provable optimum schedule. But straight LP lacks the flexibility to directly model complex underground operations which require integer decision variables. **MILP:** MILP could be used to provide a series of schedules which are marginally inferior to a provable optimum. Computational ease in solving an integer programming problem is dependent upon the formulation structure. It can provide a mathematically provable optimum schedule. The advantage that MILP has over simulation when used to generate sub-optimal schedules is that the gap between the MILP feasible solution and the relaxed LP solution provides a measure of solution quality. The drawback in using MILP is that it is often difficult to optimize large production systems by the branch-and-bound search method. **QP:** Block caving process is non-linear, so it would not be appropriate to use linear programming for production scheduling in block caving. But solving of this kind of problems could be a challenge because we must change them to LP and then solve them, so we have conversion errors.
Chanda (1990)	Simulation and MIP	Minimization of the deviation in the average production grade between operating shifts	
Guest et al. (2000)	LP	Maximization of NPV	
Rubio (2002)	MIP	Two models (a) maximization of NPV and (b) optimization of the mine life	
Diering (2004)	NLP	Maximizing NPV for M periods and minimization of the deviation between a current draw profile and a defined target	
Rubio and Diering (2004)	LP, IP, QP	Maximization of NPV, optimization of draw profile, and minimization of the gap between long and short term planning	
Rahal et al. (2008)	MILGP	Minimizing deviation from the ideal draw profile while achieving a production target	
Weintraub et al. (2008)	MIP	Maximization of profit	
Smoljanovic et al. (2011)	MILP	Optimization of NPV and mining material handling system	
Parkinson (2012)	IP	Finding an optimal opening sequence in an automated manner	
Epstein et al. (2012)	LP, IP	Maximization of NPV	
Diering (2012)	QP	Objective tonnage (to optimize the shape of the cave)	
Pourrahimian et al. (2013)	MILP	Maximization of NPV	
Alonso-Ayuso et al. (2014)	MILP	Maximization of NPV with considering uncertainty in copper price	
Pourrahimian and Askari-Nasab (2014)	MILP	Maximization of NPV, Determining the BHOD based on the optimization	

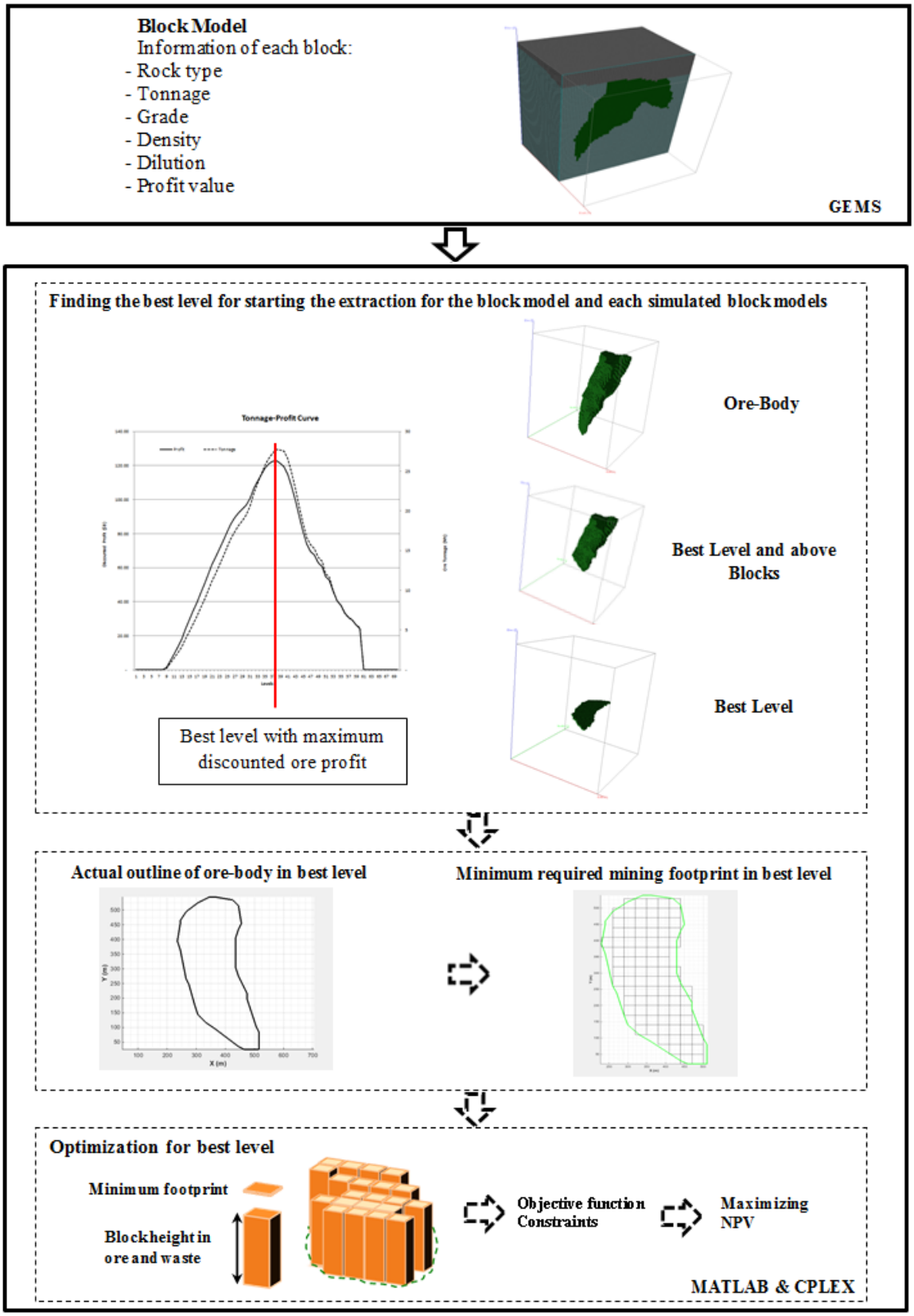

Fig. 1. Schematic representation of the methodology

The experimental variograms are calculated by using the determined directions of continuity in the previous step and a model is fitted to these variograms in different directions. Varcalc, varmodel and varplot programs are utilized for this purpose respectively. In the case of rock-type modeling, indicator variogram and for the rock-property modeling traditional variogram is used.

Rock-type model is generated for the chosen grid definition by using Sequential Indicator Simulation algorithm (SIS). Blocksis program is used for this purpose that generates multiple realizations of the rock-types.

Rock-property model such as grade for each rock-type is generated by using sgsim program which is based on Sequential Gaussian Simulation algorithm (SGS). Then the data is back-transformed to original units by implementing backtr program or ppmt-b program for univariate and multivariate data respectively. In this step histplotsim program can be used for plotting cumulative distribution function (CDF) of each realization and reference distribution in one graph.

Finally, the rock-property model is matched with the rock-type model by using mergemod program for each realization.

At the next stage the best level described in determined for each of the simulated models.

2.2. Placement of extraction level

After importing the block model into the MATLAB (Math Works Inc, 2014), the ore tonnage and discounted profit for each level are calculated. Discounted profit of each ore block (Diering et al., 2008) and the total discounted profit of each level are calculated through Eqs. (1) and (2).

$$Dis\ \mathrm{P}_{blL} = \frac{\mathrm{P}}{(1+i)^{d/ER}} \tag{1}$$

$$Dis\ \mathrm{P}_{L} = \sum_{bl=1}^{n} Dis\ \mathrm{P}_{blL} \tag{2}$$

Where $Dis\ \mathrm{P}_{blL}$ is the discounted profit of ore block bl in level L; $Dis\ \mathrm{P}_{L}$ is the total discounted profit of level L; P is the profit of ore block bl and its above ore blocks; i is the discount rate; d is the distance between the center points of ore block bl in level L and its above blocks; ER is the extraction rate per period. The profit of each block is calculated by the following equations:

$$TR = G \times Ton \times R \times (P - SC) \tag{3}$$

$$TC = Ton \times (MC + PC) \tag{4}$$

$$P = TR - TC \tag{5}$$

Where TR is the total revenue; R is the processing plant recovery; P is the price per tonne of the product; SC is the selling cost per tonne of material; G is the element grade; TC is the total cost; PC is the processing plant cost and MC is the cost of mining per tonne of material.

Then the tonnage-profit curve is plotted and the level with the highest profit is selected for starting the extraction.

2.3. Making decision

Two methods are introduced to select the best level among all the realizations:

Method I)

If the numbers of realizations are n, there are n block models with separate grade numbers for each of them. In this method just one block model is considered instead of n block models in which the average grade of all the block models for each cell is calculated. At the next step the best level of

extraction is found for the created block model according to section 2.2. Fig. 2 shows the summary of the method.

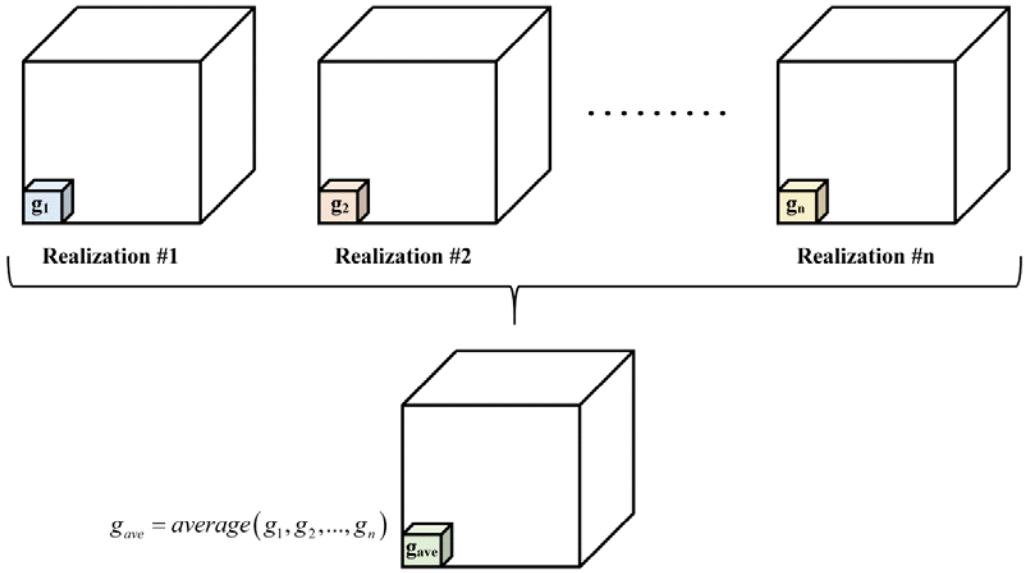

Fig. 2. Schematic representation of method I

Method II)

The best level for each realization is found according to section 2.2. The next step is calculating the probability of each selected levels based on the number of repetition. The level with the probability higher than p , is defined by planner, will be chosen for starting the extraction.

2.4. Production Scheduling

The actual outline of the ore-body for the best elevation is determined. Then the inside of the ore-body outline is divided into rectangles based on the given information about the minimum mining footprint. The minimum mining footprint (plan view) represents the minimum sized shape that will induce and sustain caving. This is similar to the hydraulic radius in a caving operation. It could be a rectangle, a circle, or any other shape. Finally the NPV through the objective function and subjected to a set of constraints is maximized.

The notation of sets, indices and decision variables are as follows:

2.4.1. Notation

Indices

$t \in \{1,....,T\}$	Index for scheduling periods
$bl \in \{1,...,BL\}$	Index for big blocks

Set

S^{bl}	For each block, bl , there is a set S^{bl} defining the predecessor blocks that must be started prior to extraction of block bl .

Decision Variables

$B_{bl,t} \in \{0,1\}$	Binary variable controlling the precedence of the extraction of blocks. It is equal to one if the extraction of block bl has started by or in period t; otherwise it is zero.

$x_{bl,t} \in [0,1]$	Continuous variable, representing the portion of block bl to be extracted in period t.

Parameters

$MCL\,(Mt)$	Lower bound of mining capacity
$MCU\,(Mt)$	Upper bound of mining capacity
g_{bl}	The average grade of considered element in the ore portion of block bl
$GL\,(\%)$	Lower bound of the acceptable average head grade of considered element
$GU\,(\%)$	Upper bound of the acceptable average head grade of considered element
$ExtU\,(Mt)$	Maximum possible extraction rate from each big block
$i\,(\%)$	Discount rate
$R\,(\%)$	Processing plant recovery
$P\,(\$/tonne)$	Price per ton of the product
$MC(\$/tonne)$	Cost of mining per ton of material
$PC(\$/tonne)$	Cost of processing per ton of material
$SC(\$/tonne)$	Selling Cost per ton of material
L	An arbitrary big number
T	Maximum number of scheduling periods
BL	The number of big blocks in the model

2.4.2. Objective function and constraints

The objective function of the MILP formulation is to maximize the net present value (NPV) of the mining operation which depends on the value of the big blocks (based on distances between drawpoints and footprint size, the blocks are placed into bigger blocks.). The objective function, Eq. (6). is composed of the block profit value, discount rate, and a continuous decision variable that indicates the portion of a block, which is extracted in each period. The most profitable blocks will be chosen to be part of the production in order to maximize the NPV.

$$Max \quad \sum_{t=1}^{T}\sum_{bl=1}^{BL}\frac{\mathrm{Pr}ofit_{bl} \times x_{bl,t}}{(1+i)^t} \tag{6}$$

Where $\mathrm{Pr}ofit_{bl}$ is the profit value of the block bl which is equal to the summation of the blocks profit within the big blocks; and i the discount rate.

The objective function is subject to the following constraints:

Mining capacity:

$$MCL_t \le \sum_{bl=1}^{BL} Ton_{bl} \times x_{bl,t} \le MCU_t\,, \quad \forall t \in \{1,....,T\} \tag{7}$$

These constraints ensure that the total tonnage of material extracted from big blocks in each period is within the acceptable range. The constraints are controlled by the continuous variable $x_{bl,t}$.

Grade Blending:

$$GL_t \le \frac{\sum_{bl=1}^{BL} g_{bl} \times Ton_{bl} \times x_{\mathrm{bl},t}}{\sum_{bl=1}^{BL} Ton_{bl} \times x_{\mathrm{bl},t}} \le GU_t, \quad \forall t \in \{1,...,T\} \tag{8}$$

These constraints control the production's average grade.

Block Extraction Rate:

$$Ton_{bl} \times x_{bl,t} \le ExtU_t, \quad \forall bl \in \{1,...,\mathrm{BL}\}, \mathrm{t} \in \{1,...,\mathrm{T}\} \tag{9}$$

These constraints ensure that the extraction rate from each big block is less than or equal to the given maximum value.

Binary Constraints:

$$B_{bl,t} \le L \times x_{bl,t}, \quad \forall bl \in \{1,...,BL\}, \ t \in \{1,...,T\} \tag{10}$$

$$\mathrm{x}_{bl,t} \le B_{bl,t}, \quad \forall bl \in \{1,...,BL\}, \ t \in \{1,...,T\} \tag{11}$$

$$B_{bl,t} - B_{bl,t+1} \le 0, \quad \forall bl \in \{1,...,BL\}, \ t \in \{1,...,T\} \tag{12}$$

Where $B_{bl,t}$ is a Binary variable controlling the precedence of the extraction of blocks. It is equal to one if the extraction of block bl has started by or in period t ; otherwise it is zero and L is an arbitrary big number. In these constraints every block has two variables (continuous and binary) in which constraints (10) and (11) ensure if the extraction of one block is started its binary variable should be one and otherwise it should be zero. Also Eq. (12) controls the fact that if the extraction of one block in period t has been started, so for the next period the extraction of that block is started. The results of these constraints is been used for the precedence constraint that the maximum number of active blocks is needed.

Precedence Constraints:

$$n \times Bl_{bl,t} \le \sum_{k=1}^{n} B_{k,t}, \quad k \in S^{bl}, \quad \forall bl \in \{1,...,BL\}, \ t \in \{1,...,T\} \tag{13}$$

These constraints ensure that all the predecessor blocks of a given block bl have been started prior to the extraction of this block.

For applying this constraint at first the adjacent blocks of each block is determined then by considering the extraction direction and determining the perpendicular line to it which crosses the center point of a given block, the adjacent blocks under this line have been specified which are the blocks that should be extracted before that given block. The formulas that have been used in finding the predecessor blocks of each block are described in the following equations:

$$Y_{new} - y_{bl} = -m(X_{new} - x_{bl}) \tag{14}$$

$$D = (x_{adj} - x_{bl})(Y_{new} - y_{bl}) - (y_{adj} - y_{bl})(X_{new} - x_{bl}) \tag{15}$$

Where $-m$ is the slope of the perpendicular line to the mining direction; y_{bl} and x_{bl} are the coordinates of each big block in the extraction level; X_{new} is an arbitrary coordinate and as a result

Y_{new} is calculated by Eq. (14). (perpendicular line crosses both the center point of each big block and a point with coordinates of (X_{new}, Y_{new})). In Eq. (15). x_{adj} and y_{adj} are the coordinates of the adjacent blocks of each big block. By calculating D, the blocks with $D < 0$ are below the perpendicular line and considered as the predecessors of a given big block.

Reserve Constraints:

$$\sum_{t=1}^{T} x_{bl,t} = 1, \quad \forall bl \in \{1,...,BL\} \tag{16}$$

In this formulation, all material inside of the big blocks should be extracted which is controlled by Eq. (16).

3. Case study

3.1. Grade Uncertainty

Geostatistical study based on the drillholes data and according to what mentioned in section 2.2. is performed. The data is belonged to cupper grade, so it is a univariate data and multivariate statistical analysis to find the correlation between the variables is not needed. The initial inspection of drillholes locations demonstrated equally-spaced drillholes and as a result declustering algorithm is not implemented.

The next step is grid definition for simulation. Distance between the grid nodes in each direction, the number of grid nodes in each direction and the coordinates of first grid node are important parameters for defining a grid. By considering all of these parameters the size of the grid is chosen according to

Table 2.

Table 2. Grid Definition for Geostatistical Study

Direction	Number of nodes	Center coordinates of first node	Grid Spacing
Easting	45	145	10
Northing	60	25	10
Elevation	70	395	10

The modeling has two parts: rock-type modeling and rock-property modeling. There are two types of blocks in this model: ore blocks (18%) and waste blocks (82%). Rock-property modeling should be implemented for both rock-types (ore and waste) separately.

3.1.1. Rock-type Modeling

The principal directions of continuity are found using indicator kriging based on two categories; 0 (waste) and 1 (ore). The azimuths of major and minor directions were chosen to be 0 and 90 degrees (Fig. 3) that are used for variogram calculation at the next step.

Afterwards indicator variograms are calculated and a theoretical variogram model is fitted with three structures, nugget effect of 0 and sill of 0.14 (Fig. 4).

At the next step, 20 realizations for rock-types are generated using Sequential Indicator Simulation (SIS) algorithm. Fig. 5 illustrates the results for the first realization.

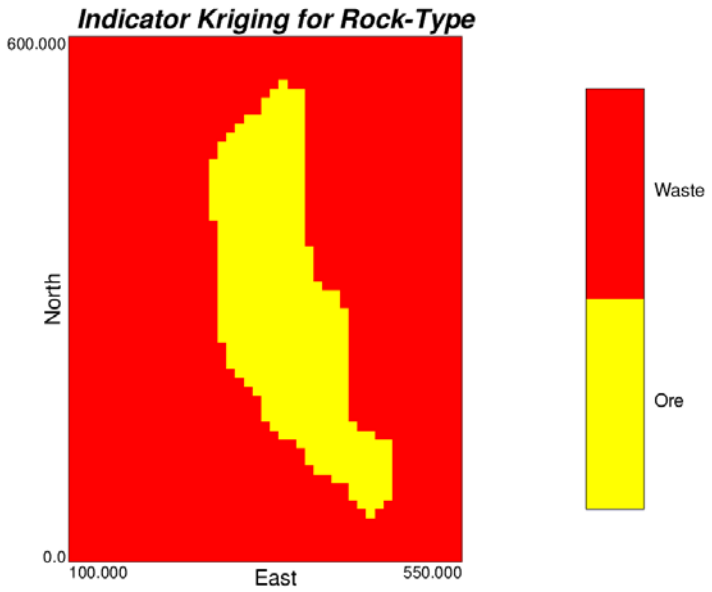

Fig. 3. Plan view of maximum direction of continuity for rock-types at elevation 40

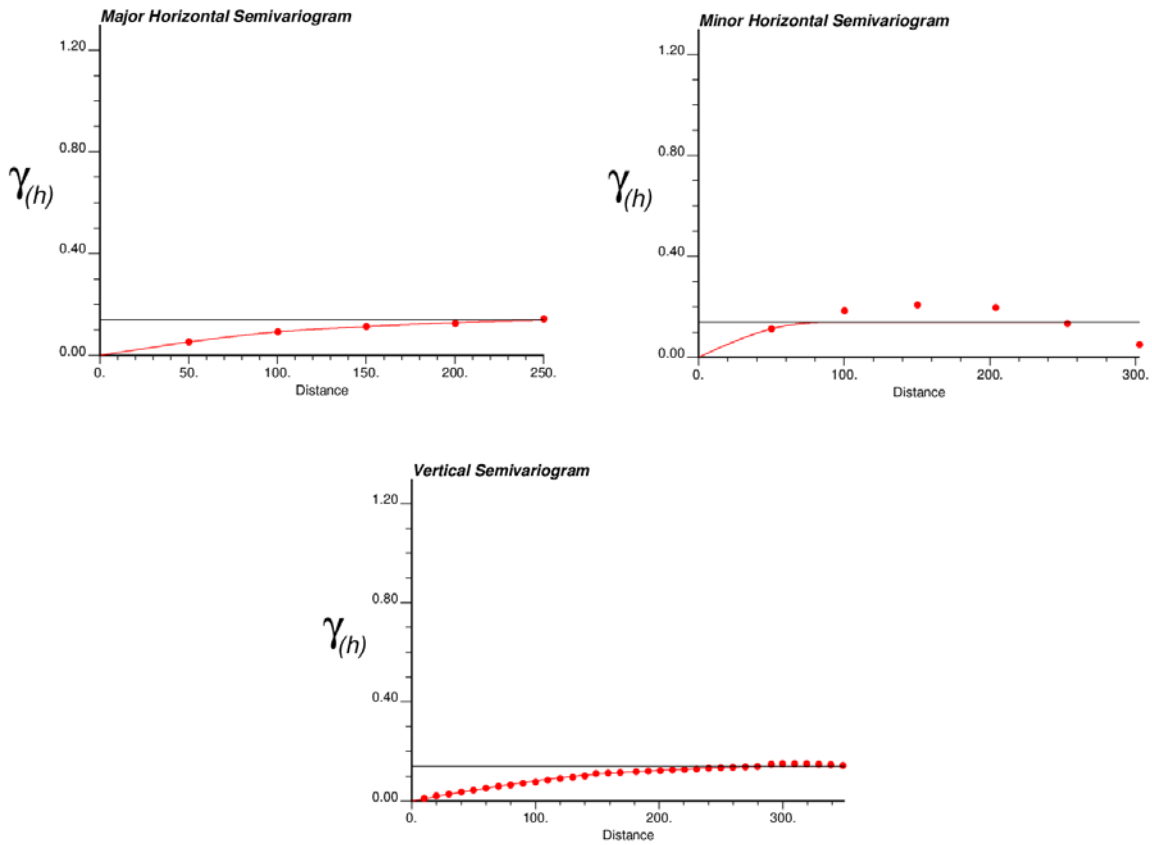

Fig. 4. Experimental directional variograms (dots) and the fitted variogram models (solid lines) for rock-type, distance units in meter

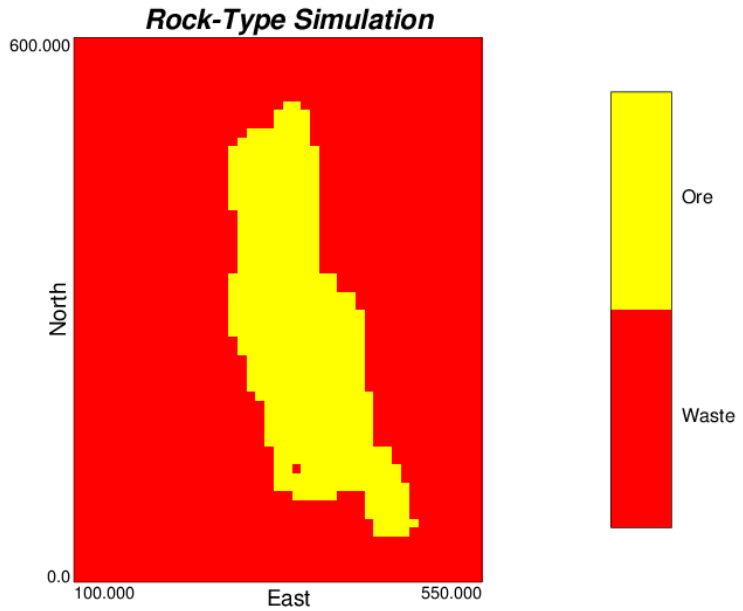

Fig. 5. Plan view of rock-type simulation for first realization at elevation 40

3.1.2. Rock-Property Modeling

As in this case all the cupper grades related to waste blocks were zero, rock-property modeling is unnecessary to be performed for the waste and just zero values equal to grid-size are considered for waste modeling.

For ore modeling, the principal directions of continuity are extracted by doing simple kriging with the help of arbitrary variograms. As it can be seen in Fig. 6 the azimuth of 90° (major) and zero (minor) in horizontal direction are selected for the following variogram calculation.

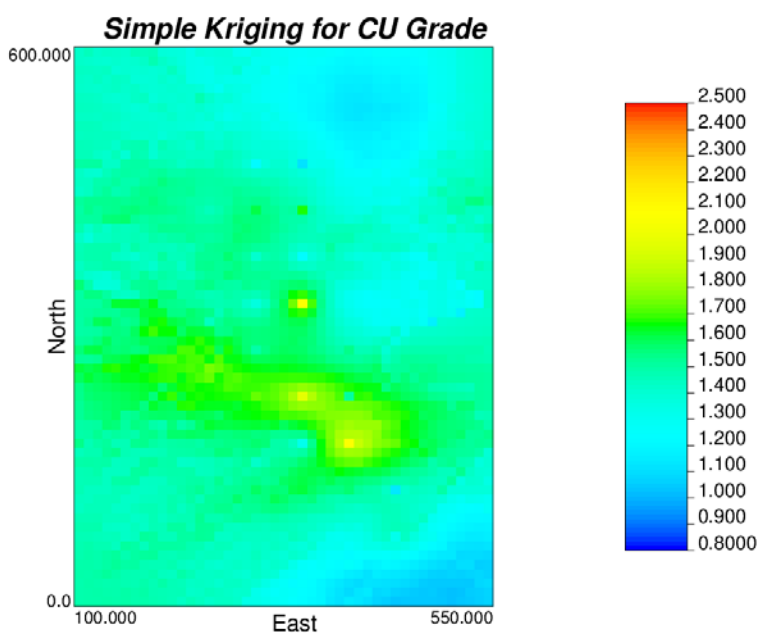

Fig. 6. Plan view of maximum direction of continuity for cupper grade at elevation 40

At the next step the cupper grades should be transformed to Gaussian space. Then traditional variogram calculation and modeling with three structure and nugget effect of 0.1 are done for the cupper grade and the results are demonstrated in the Fig. 7.

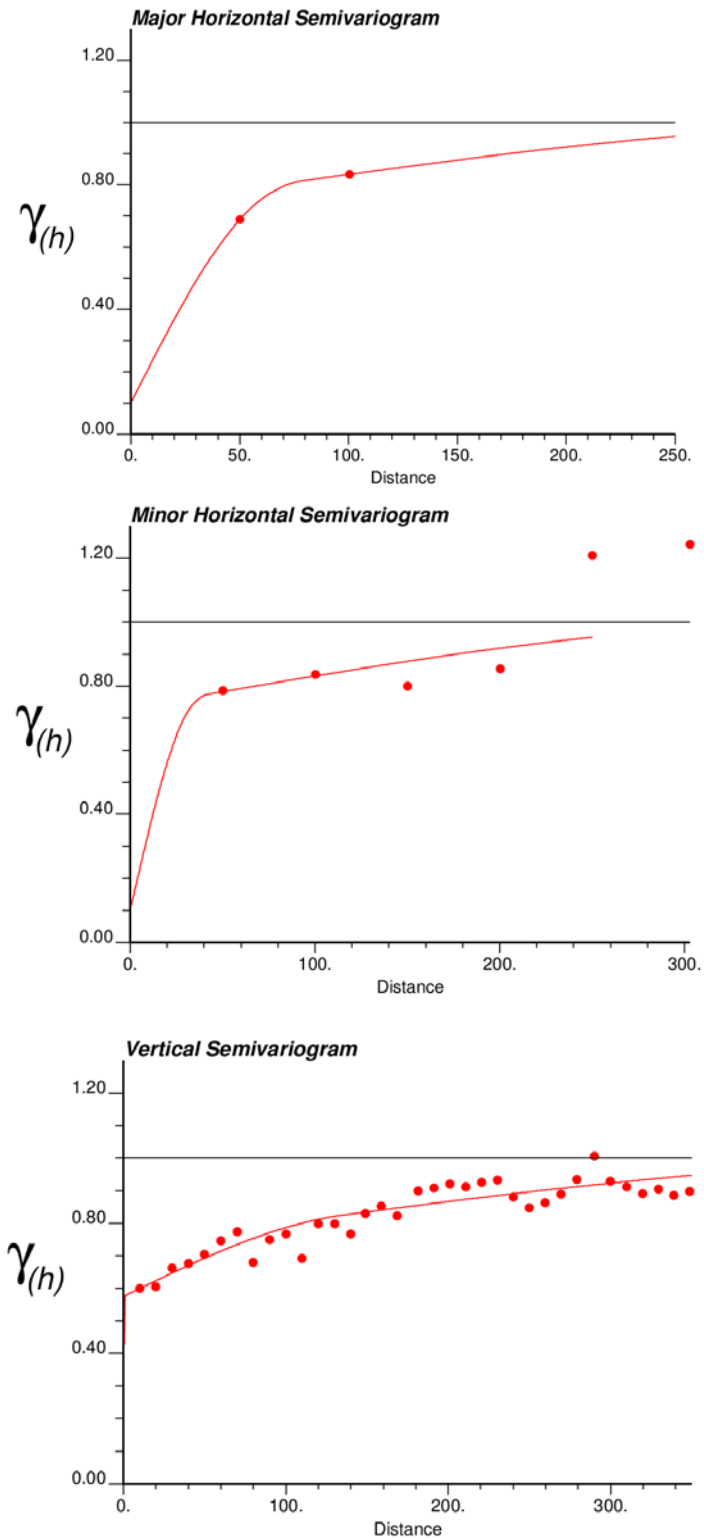

Fig. 7. Experimental directional variograms (dots) and the fitted variogram models (solid lines) for Cu grade of ore blocks, distance units in meter

At the next step, 20 realizations for cupper grade are generated using Sequential Gaussian Simulation (SGS) algorithms. The SGS needs a back-transformation to original units that the results are demonstrated in Fig. 8.

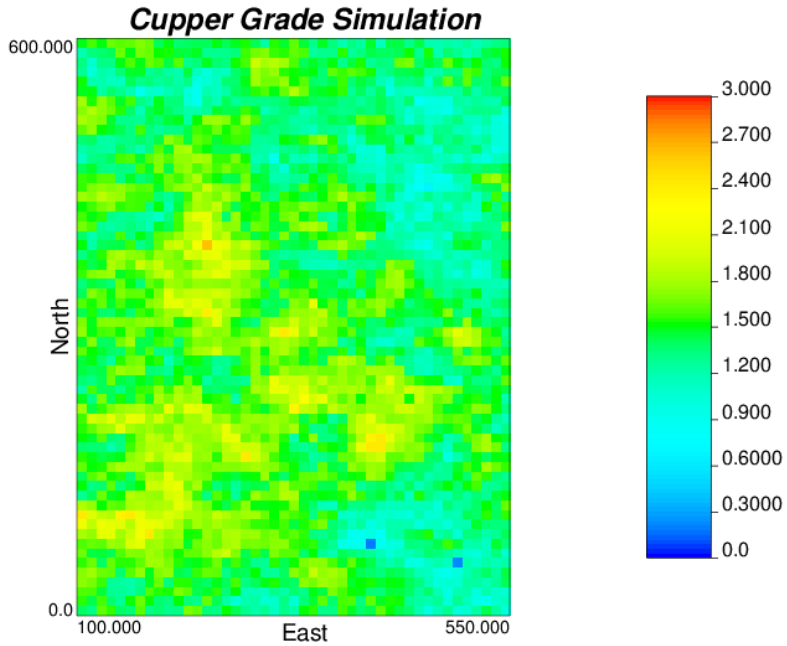

Fig. 8. Plan view of Cu grade simulation for first realization at elevation 40

3.1.3. Merging rock-type and rock-property models

The next step is merging rock-type model with rock-property model. The plan view of final simulation for first realization is shown in Fig. 9.

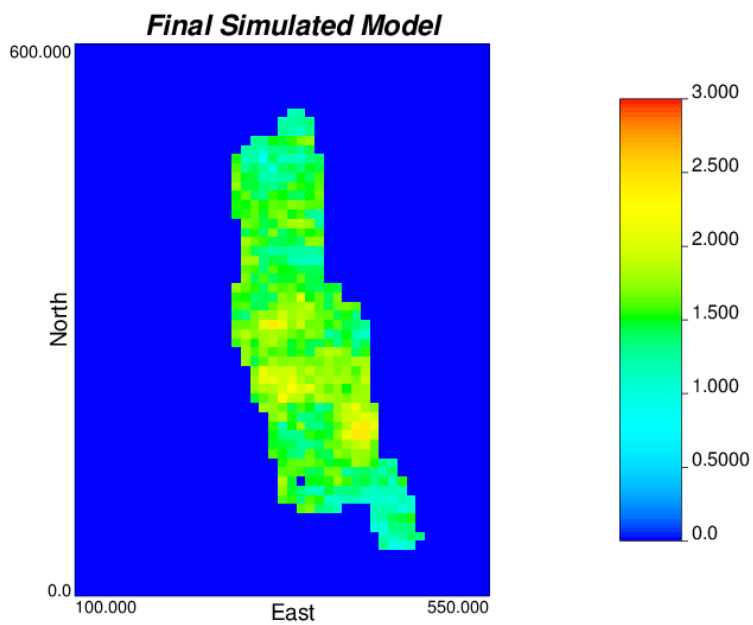

Fig. 9. Final simulation at elevation 40 and realization 1

Fig. 10 shows the variogram reproduction of rock-property (ore) simulation (left) and rock-type simulation (right) in three major, minor and vertical directions. Since the variogram is reproduced quite reasonably, the generated realizations are considered representative of the grade uncertainty.

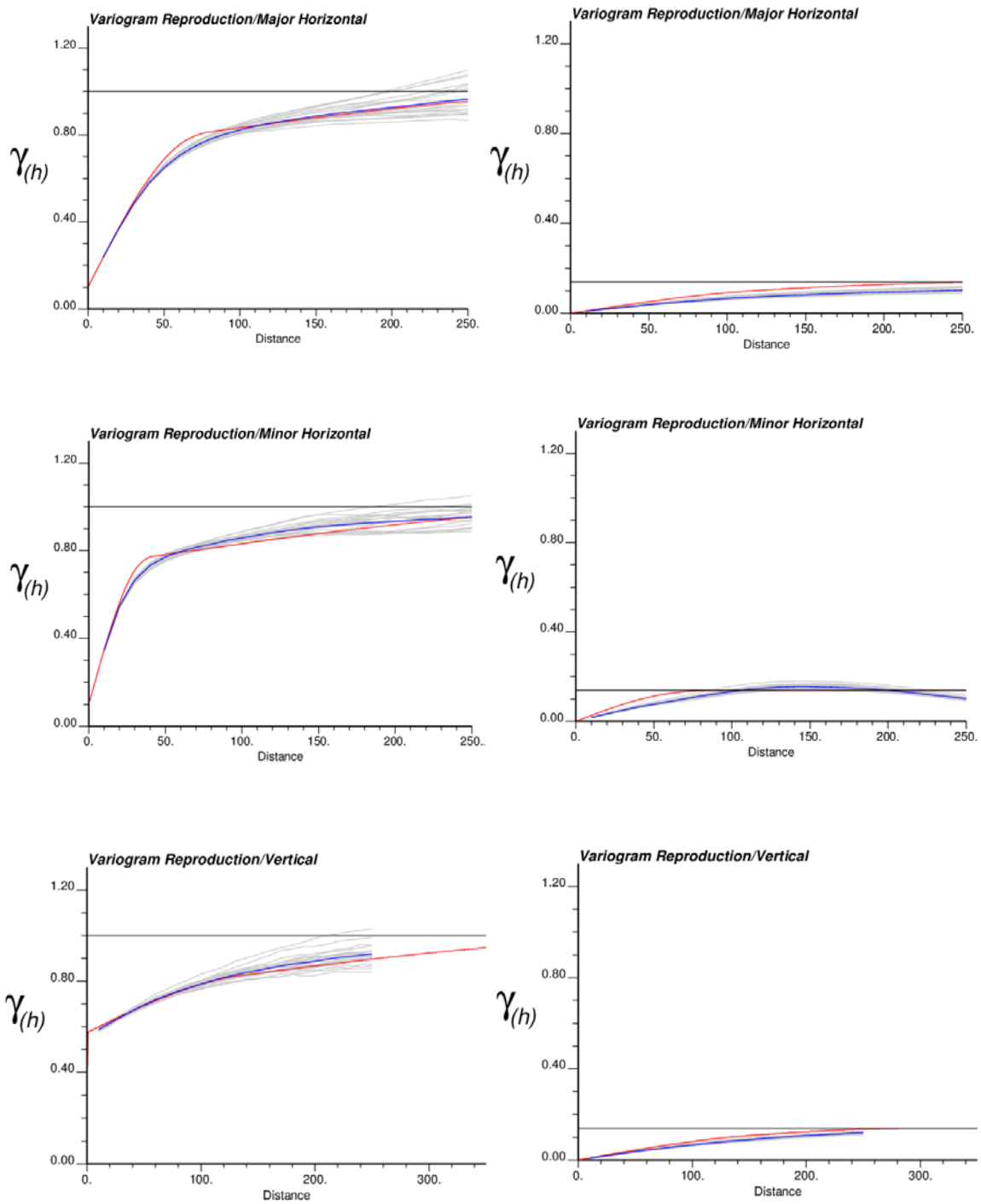

Fig. 10. Variogram reproduction at Gaussian units of cupper grade (left) and rock-type (right) realizations (gray lines), the reference variogram model (red line) and the average variogram from realizations (blue line) in three directions.

3.2. Placement of extraction level

For each block model, at the first step the discounted profit and tonnage of the ore blocks above each ore block in each level is calculated and the profit-tonnage curve is plotted which leads to selecting the best level for starting extraction based on the maximum profit (Fig. 11).

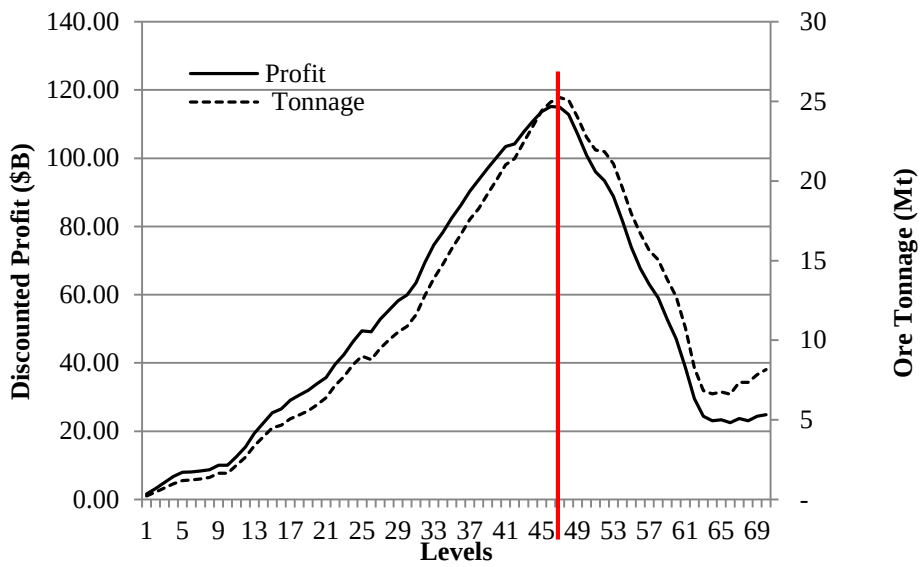

Fig. 11. Tonnage-Profit curve against the levels for one of the simulated block models

This method has been implemented for all the realizations (Table 3). The average grade of all realizations is 1.4794 that is quite near the average grade of original data which is 1.4759.

Table 3. Best level for a number of realizations

Realization	Average grade	Best level	Ore Tonnage (Mt)	Ore discounted profit ($B)
1	1.4784	48	28.884	123.907
2	1.4873	46	24.108	109.964
3	1.4845	46	24.964	115.114
4	1.4648	49	25.015	110.174
5	1.4649	47	25.300	112.752
6	1.4629	48	26.910	116.964
7	1.4607	45	25.188	109.274
8	1.4986	47	24.284	108.451
9	1.4901	46	23.892	108.673
10	1.4831	46	25.452	113.599
11	1.4779	47	25.930	116.430
12	1.4822	46	25.130	110.634
13	1.4921	49	26.630	116.611
14	1.4817	47	24.012	109.150
15	1.4747	47	22.920	101.623
16	1.4798	46	25.636	117.686
17	1.4942	46	26.642	120.052
18	1.4872	46	23.746	110.302
19	1.4767	47	24.276	107.577
20	1.4664	47	26.210	114.670

3.3. Making Decision

Method II is implemented to select the best level. The probability of each level from Table 3 is calculated. Level 46 with the highest probability is selected for the placement of extraction level and for production scheduling at the next step.

3.4. Production Scheduling

The proposed mathematical model will be used in further studies to generate the production schedule for the level 46 to maximize the NPV.

4. Conclusion

Geological uncertainty has been utilized recently in open-pit mining, but less studied in underground mining especially in block caving in which revision in production plan is not easy after caving (Vargas et al., 2014). This methodology is able to find the best extraction horizon placement under grade uncertainty. Also it will be able to implement mathematical programming through defining an optimization model in MATLAB and running it by CPLEX in the future studies.

The results related to the profit of each level have been compared with each other thorough the profit-tonnage curve and as a consequence, the best level is recommended for initializing the extraction from all the realizations. Then by optimizing the proposed model maximum NPV will be obtained for the selected level.

5. References

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