The Impact of Grade Uncertainty on Optimal Production Scheduling

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Abstract

Mine planning is a technical term in earth science that defines the source, destination and extraction time of ore and waste blocks during the mine life. The result of mine planning is a production schedule that defines the tonnage of ore and waste and the input grade to the processing plant at any given time. For any mining system, the production schedule is a key component in determining mine viability and it has an enormous effect on the economics of mining operations. An optimum production schedule is becoming more critical as the mining industry is forced with more marginal resources with lower grades. In this paper, a mixed integer linear programming method is proposed to maximize the NPV of mine projects using an estimate block model and a stockpile. Although the proposed model deals with stockpiles, it is still a linear model that can be solved using solvers such as CPLEX. A MATLAB code is developed and tested for an oil-sands open pit mine.

1. Introduction

Numerical modeling is the only practical method to quantify the complexity of natural resources. Geostatistical methods that are developed for two decades are widely used for block model generation. A block model is a set of blocks which usually are regular cubic shape, and contains estimated (or simulated) values for variables of interest such as grade of elements. There are various methods to estimate these variables using hard and soft samples. Geostatistical techniques are widely used for this purpose (Deutsch and Journel, 1998; Deutsch et al., 2002; Goovaerts, 1997; Journel and Huijbregts, 1981). Among these techniques the most popular methods is Kriging.

The estimated block model is the main input to the long term mine planning. Usually, in long term mine planning in open pit mine projects, the goal is to maximize the net present value (NPV) of the project during the mine life. It is mostly in yearly resolution. Each year of the production plan is called a period. To achieve this goal, it is required to have a production schedule. The optimal production schedule is the sequence of extraction of ore and waste material from the mine such that all physical constraints are satisfied and the NPV is maximized. In order to solve this optimization problem, the objective function and all the constraints should be constructed at the mathematical form. These constraints fulfill the physical limitations such as: input head grade to the plant, mining and processing capacities, mining precedence, etc.

Askari-Nasab, Hooman (2013), Mining Optimization Laboratory (MOL) – Report Five, © MOL, University of Alberta, Edmonton, Canada, Pages 230, ISBN: 978-1-55195-327-4, pp. 92-101.

Askari-Nasab and Awuah-Offei (2009) reported three general categories of long term mine planning methods that are used in literature : (i) heuristic methods, (ii) artificial intelligence techniques and (iii) operations research methods. The operations research methods are employed in mine production scheduling by several authors. An optimization model is called convex if the solution can be proven to be the global optimum. These methods also called exact methods. Linear programming (LP), integer programming (IP) and mixed integer linear programming (MILP) are commonly used in literature for long term mine planning and they are exact methods.

Johnson (1968) used LP model to maximize NPV of a mine scheduling problem. The main shortcoming of this model is that the precedence of block extraction is not satisfied. This causes some percentage of the overlaying blocks to be suspended in the air. Gershon (1983) modified Johnson's LP model to a general MILP model. A new set of binary variables was considered to satisfy the precedence of block extraction. The main disadvantage of this model is to be intractable for a real size mine planning project. Over the time, several authors tried to make MILP models to be tractable for real size mine planning. Dagdelen and Johnson (1986) decomposed a multi-period problem into smaller single-period problems. Akaike and Dagdelen (1999) presented a 4D-network relaxation method. They used improved Lagrangian relaxation method to solve the model which leads to an optimal solution which is not exact. Caccetta and Hill (2003) presented a customized branch and cut algorithm to speed up the convergence of the MILP model. Some other authors tried to use aggregation methods to reduce the number of variables (Boland et al., 2009; Dimitrakopoulos and Ramazan, 2004; Ramazan, 2007; Ramazan et al., 2005). Askari-Nasab et al. (2010; 2011) presented the objective function of an LP formulation that maximizes the NPV of a mining operation. This model was generalized form a model presented earlier by Caccetta and Hill (2003). Their model is an MILP model which is widely accepted. A MATLAB code has been provided, and TOMLAB (Holmström, 1989-2011) has been employed to solve the proposed optimization problem.

All of the proposed methods by these authors do not consider any stockpiles. The destination of the blocks is either the processing plant or the waste dump. However, in practice, there is usually a stockpile to store the ore material. The ore tonnage that cannot be processed immediately due to the processing capacity and other limitations, are sent to the stockpile to be used and processed at later periods. There are few attempts in literature to consider stockpiles in the optimization model. The reason is that two different decision variables are required to model the stockpile: the tonnage of ore reclaimed from stockpile and the average grade of stockpile. In order to maximize NPV in presence of a stockpile, these two variables are multiplied and therefore the optimization problem turns into a nonlinear problem which is more challenging to solve with integer variables. The model may not be convex and it may be very time consuming to solve. The new approach presented in this paper considers the stockpile and the model is still a mixed integer linear optimization model.

2. Methodology

The proposed method is based on two main assumptions:

1. The average grade of the stockpile is assumed to be a pre-defined input that is specified by user. This can be considered as a severe assumption. However, due to the blending material in the stockpile, it can be assumed that the average grade of the stockpile is very close to the average input head grade of the mill. First, by assuming some numbers for each period the project is ran and the results are rechecked. If the assumed numbers are way off from the actual average grade at each period, new numbers can be replaced and the optimization is reran. Therefore, the average grade of the stockpile at each period can be estimated with a good precision by a recursive scheme. However, the sensitivity analysis showed that the

generated schedule is not too sensitive to the stockpile grade at each period which is defined by user in the proposed method.

2. The destination of each block is determined by the cut-off grade. If the average grade of a block is higher than operational cut-off grade, the block is sent to either the processing plant or the stockpile; otherwise it is sent to the waste dump. Between the plant and the stockpile, the priority is to feed the plant with full capacity. Any extra ore is sent to the stockpile to be used at later years. Let's assume a block that is extracted at period t and sent to the stockpile to be processed at period t+1. To calculate NPV without defining any extra variables, it is assumed that this block is extracted and processed at period t .The value of ore that is lost due to discounting factor is assumed as a cost and it is deducted from the NPV.

Therefore, the objective function that should be maximized is calculated using the discounted economic block value at the period that blocks are extracted (not processing) minus the loss of the value of ore tonnage that is transferred from the stockpile to the processing plant at each period (due to the discounted factor). The re-handling cost of material also can be considered as well. The general form of this model is presented in Eq. (1)

$$\text{Max} \quad NPV = \sum_{t=1}^{T} \left\{ \sum_{i=1}^{N} \left[DEBV(t;i) \times y(t;i) \right] - T_{Stockpile}(t) \times c_{op,RH}(t) \right\} \tag{1}$$

Where

- T is the number of periods or the mine life
- N is the total number of blocks.
- $y(t;i)$ is the decision variable and indicates the portion of block *i* that should be extracted at period t.
- DEBV is the discounted economic block value which is calculated by discounting the EBV (economic block value) of block *i* at period *t* by a given interest rate (IR) as shown in Eq. (2)

$$DEBV(i;t) = \frac{EBV(i)}{(1+IR)^t} \tag{2}$$

EBV of a block is also calculated from the revenue of a block minus the mining cost and it is calculated by Eq.(2).

$$\begin{aligned} EBV(i) &= V(i) - Q(i) \\ &= T_o(i) \times v(i) - \left[T_o(i) + T_w(i) \right] \times q(i) \end{aligned} \tag{3}$$

- $V(i)$ and $Q(i)$ are discounted revenue and discounted cost of extraction of block *i*.
- $v(i)$ and $q(i)$ are discounted revenue and discounted cost per tonne of extraction of block *i*.
- $T_o(i)$ and $T_w(i)$ are tonnage of ore and waste of block *i*.
- $T_{Stockpile}(t)$ is the tonnage of ore at the stockpile in period *t*.
- $c_{op,RH}(t)$ is the discounted cost of re-handling material from stockpile and loss of ore value due to transfer to the stockpile. When an ore tonnage is extracted and transferred to the stockpile to be used at the later years, ore tonnage loses the value by the discount factor for each year. This is considered as a cost and is implemented in $c_{op,RH}(t)$. It can be calculated by Eq. (4) as below:

$$c_{op,RH}\left(t\right)=\left[\frac{v_s\left(t\right)}{\left(1+IR\right)^t}-\frac{v_s\left(t+1\right)}{\left(1+IR\right)^{t+1}}\right]+\frac{c_{\text{rehandling}}}{\left(1+IR\right)^t} \tag{4}$$

- $v_s\left(t\right)$ is the value of ore per tonne in the stockpile at period t. This value is calculated by assuming that the average grade of the stockpile at period *t* is predefined by user and is calculated by Eq. (5)

$$v_s\left(t\right)=P\times R_p\times \overline{g}_s\left(t\right)-C_p \tag{5}$$

Where

- P is the price of commodity.
- R_p is the processing recovery.
- $\overline{g}_s\left(t\right)$ is the average grade of the stockpile at period *t*.
- C_p is the cost of processing.

The optimization model presented in Eq.(1) is subject to the following constraints:

- Grade blending: By adding this constraint to the optimization problem, the input head grade to the mill is forced to be in a user defined range. There are two constraints (upper and lower) for each element that is processed in each period.
- Mining capacity: There are two (upper and lower) mining limits at each period.
- Mining precedence: These constraints enforce the production schedule to be feasible. This means that if a block is planned to be extracted in period t, it is necessary that all the blocks above has been already extracted or they are going to be extracted in that period. By using binary variables, a set of constraints are constructed such that a block is not extracted until all the blocks located directly above are extracted.
- Upper limit for tonnage of stockpile: These constraints are required to limit the tonnage of ore in the stockpile. There is one constraint per period.
- The following constraints are required to calculate tonnage of ore at the stockpile:

$$\sum_{i=1}^{N}\left[T_o\left(i\right)\times y\left(t;i\right)\right]+T_{Stockpile}(t-1)-T_{Stockpile}(t)\le P_u\left(t\right)\qquad \forall\, t=1,2,\ldots,T\ ,\ l=1,2,\ldots,L \tag{6}$$

- $P_u\left(t\right)$ is the capacity of the processing plant. It is important to note that the tonnage of stockpile at period zero is zero and $T_{Stockpile}(t)$ shows the amount of ore in the stockpile at the end of period t.

The proposed optimization model is mixed integer linear programming (MILP) problem. There are different software packages to handle MILP problems. However, an easy way to start with is MATLAB (MathWorks Inc., 2011) and the optimization toolbar called TOMLAB/CPLEX (Holmström, 1989-2011); both are available through Mining Optimization Laboratory (MOL) at University of Alberta. TOMLAB/CPLEX is a commercial toolbox for MATLAB environment and has so many powerful optimization engines and algorithms that can be used for different purposes such as CPLEX(ILOG Inc, 2007). CPLEX has been developed since 1988 and it is a very powerful solver for large scale mixed integer linear and quadratic optimization problems.

A computer with 8 CPUs and 20Gbyte of ram has been used for all codes. The operation system was Microsoft Windows 7 Professional. The detailed processor information is shown below: Intel(R) Core(TM) i7 CPU 930 @ 2.80GHz, 2801 Mhz, 4 Core(s), 8 Logical Processor(s). All the methods are solved by TOMLAB/CPLEX solver.

3. Case Study

An oil sand data set has been used. The GSLIB (Deutsch and Journel, 1998) is employed for comprehensive geostatistical study. Ordinary Kriging (OK) is used to generate the estimation block model. The 3D LG (Lerchs and Grossmann, 1965) method has been used to determine the optimum final pit limit. All the blocks outside this range has been eliminated from the model. A MATLAB function called 'fcm', Fuzzy c-means clustering, has been used to aggregate the blocks with similar grade at each level. This reduces the number of variables and therefore the optimization can be solved much faster without compromising the optimality of the solution. Also by using mining cuts, the generated schedule is more applicable in the real industrial size project. The generated schedule in block scale can be very scatter which means that it requires a lot of transportation of the excavators at a single period. This is not a desirable situation at real size problems.

Table 1 shows the parameters that are used for this case study. There are 14612 blocks inside the final pit limit and they are aggregated into 1834 mining cuts. The total tonnage of rock inside the final pit is 653.6 million tonnes. The 6 mass percent (%m) of cut-off grade is applied to the blocks. Therefore the total tonnage of ore inside the pit is 282.5 million tonnes with average grade of 10.31 %m. The average grade of the stockpile is assumed to be 11.0 m%. Therefore, the value of one tonne ore at period zero is 25.4$ ($v_s\left(t=0\right)=25.4$). One should note that $c_{op,RH}\left(t\right)$ at the final period is higher because it is assumed that any ore that is left at the end of final period will not be processed. Therefore, the higher cost of over production at final period prevents any leftover ore not be processed at the end of the mine life. The other assumption is that 5 million tonnes of ore should exist in the stockpile at any period except last two periods (9 and 10). This ore is used to feed the plant in any case that an unexpected shortfall occurs in the input ore from mine to the plant due to failures at mine operations or due to weather conditions.

Table 1: Summary of costs and the interest rate used in pit limit design.

Description	Value
Mining Costs (CAN $/tonne)	4.6
Upgrading Costs (CAN $/tonne)	0.5
Interest Rate	10%
Overall slope (degrees)	20
Cutoff grade (%mass bitumen)	6
Mining recovery factor	0.88
Processing recovery factor	0.95
Processing capacity (M tonne/year)	36
Mining capacity (M tonne/year)	67.5
Stockpile capacity (M tonne)	36
Rehandling cost (CAN $/tonne)	0
Pre-stripping (years)	2
Mine Life	10
Selling price ($/tonne)	281.25

The proposed model has been solved by TOMLAB/CPLEX solver in MATLAB environment. The Relative MIP gap (EPGAP) tolerance is 0.005 or 0.5%. It means that the answer which is reported here is within 0.5 percent of theoretical optimal solution. The CPU time and real runtime for this model respectively are 141,449 and 22,839 seconds. The CPU time is sum of all seconds that any of CPUs was busy with solving the problem. The real time is the elapsed real runtime that CPLEX takes to solve the optimization problem and terminates based on the EPGAP parameter. Table 3 shows the summary of the generated schedule that satisfies the input constraints. The first two periods is the pre-striping years and any ore tonnage that is extracted in these two periods is sent to the stockpile. The annual processing tonnages at all periods are less than the processing capacity which is 36 million tonnes. The annual mining tonnage also is less than or equal to 67.5 million tonnes. The stockpile capacity is also satisfied since at all periods the total tonnage of stockpile is less than the stockpile capacity and in all periods except last two periods the tonnage of ore in stockpile is more than 5 million tonnes. The average grade of ore tonnage is 11.32 m% which is very close to the input assumption which was 11 m%. Therefore, there is no need to readjust and rerun the whole model. Finally, the cumulated discounted cash flow at each period is shown in the final column of this table. The NPV of this production schedule is 2442.63 Million dollars.

Table 2: Dollar value of one tonne of ore in the stockpile at different periods ($v_s(t)$) and the loss of ore value due to the deferring the processing of one tonne of ore to the next period ($c_{op,RH}(t)$).

Periods	0	1	2	3	4	5	6	7	8	9	10
$v_s(t)$	25.4	23.09	20.99	19.08	17.35	15.77	14.34	13.03	11.85	10.77	9.79
$c_{op,RH}(t)$	N/A	2.10	1.91	1.73	1.58	1.43	1.30	1.18	1.08	0.98	9.79

Table 3: Summary of production schedule in each period for OK block model.

Period	Input Ore MT	Waste MT	Mined MT	SR	Grade M%	Stockpile MT	Stockpile grade M%	CDCF MD
1	0.00	44.31	50.56	Inf.	0.00	6.25	10.59	-211.45
2	0.00	38.84	67.50	Inf.	0.00	34.91	11.32	-468.06
3	36.00	35.87	67.50	1.13	11.55	30.54	11.32	119.90
4	36.00	57.04	67.50	5.45	11.03	5.00	11.32	619.96
5	36.00	31.50	67.50	0.88	10.05	5.00	11.32	1016.05
6	36.00	31.50	67.50	0.88	9.79	5.00	11.32	1362.28
7	36.00	31.50	67.50	0.88	9.53	5.00	11.32	1663.82
8	36.00	31.50	67.50	0.88	9.96	5.00	11.32	1957.66
9	36.00	35.30	67.50	1.10	10.65	1.20	11.32	2252.79
10	30.44	33.81	63.05	1.16	9.81	0.00	0.00	2442.63
Total	282.5	371.2	653.6	1.31	10.31	0.00	0.00	2442.63

Figure 1 shows the production schedule at each period. The tonnages of mined ore, waste and the ore tonnage that is sent to the stockpile and mill from stockpile are shown in the column bars with different colors. The tonnage of ore in the stockpile also is indicated by green line. The plant is fed at full capacity at all periods except final period. The shortfall from target production at this period is due to the less ore tonnage left for final period. This schedule is quite reasonable for a real size industrial problem.

Figure 2 shows the discounted cash flow (red line) and cumulative cash flow (dashed line). The breakeven for this project without considering the capital costs is at end of period two.

Figure 3 shows the extraction periods of the blocks in the plan view at 290m and two cross sections at 252750m looking north and 148250m looking east. There is no gap effect at this production plan and it looks reasonably smooth. All the extraction precedences are also satisfied in this model.

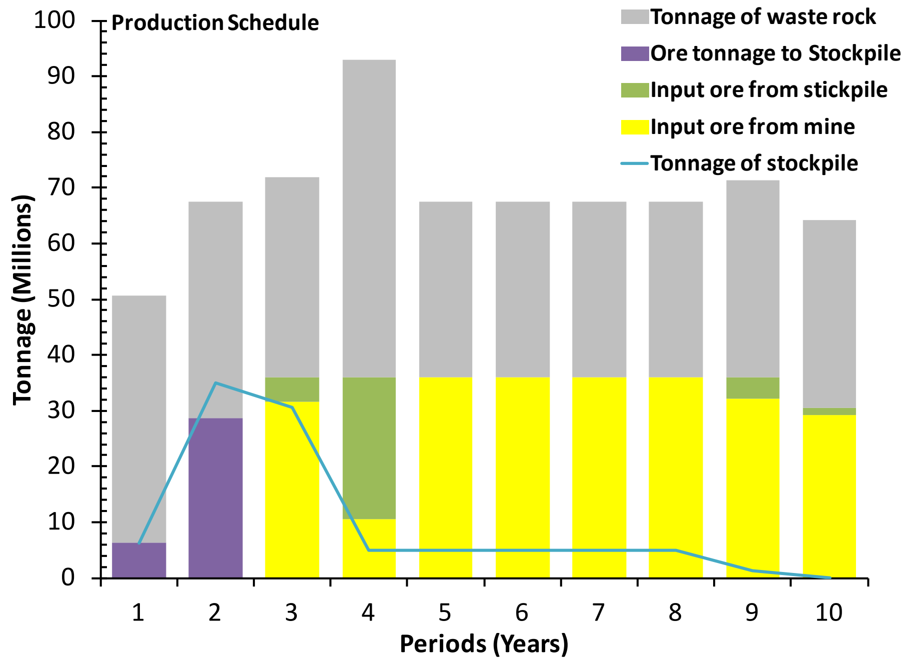

Figure 1: Production schedule using OK block model with a stockpile.

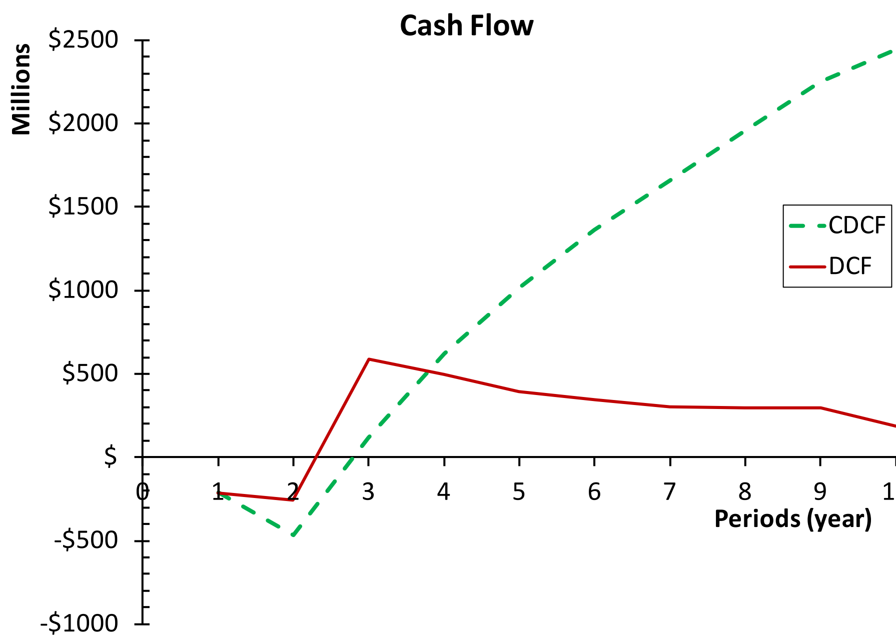

Figure 2: Discounted cash flow (bold line) and cumulative discounted cash flow (dash line).

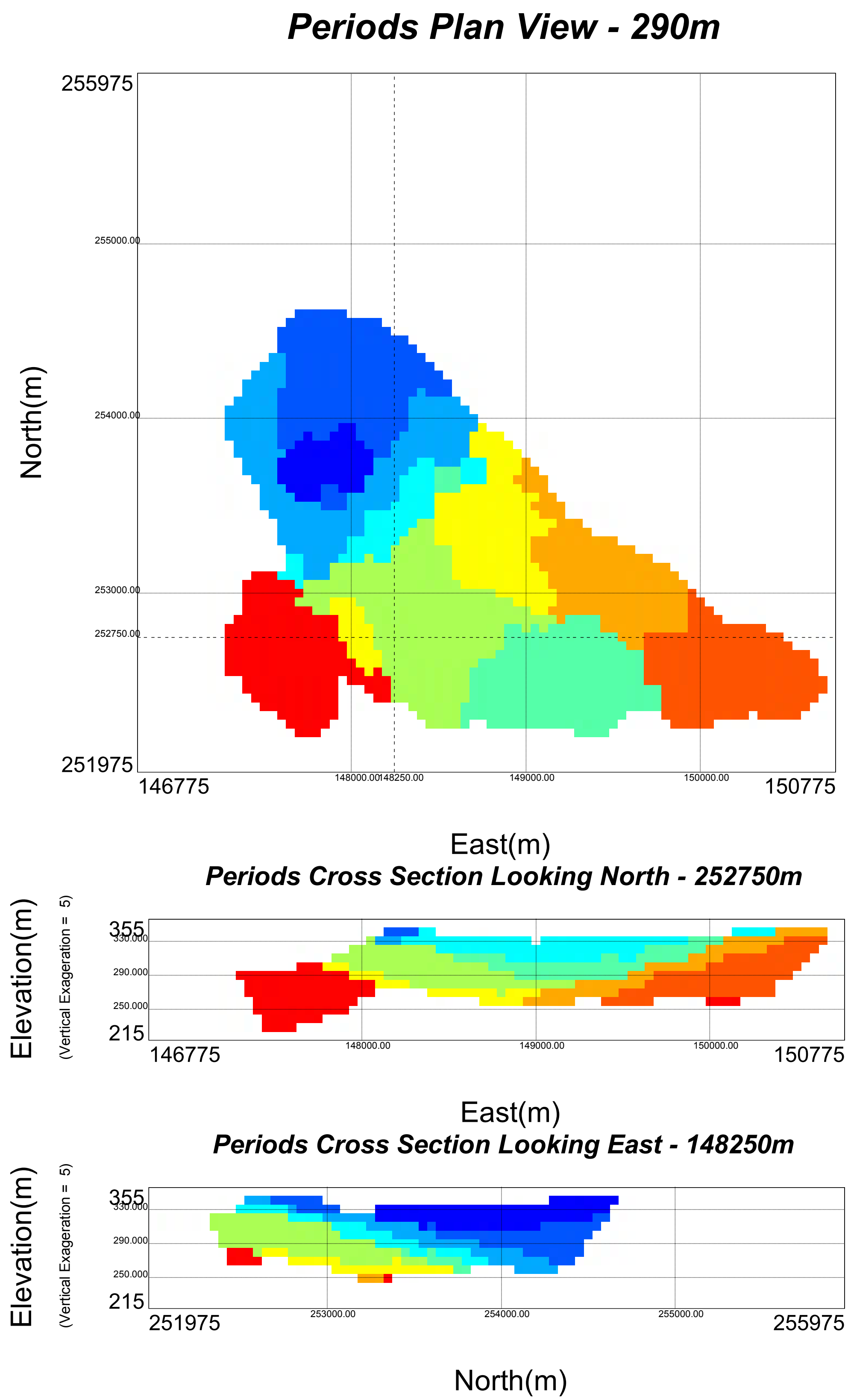

Figure 3: Plan view and cross section of the periods of the extractions.

4. Conclusion

Considering the stockpile in the mine plan production optimization is a very difficult task since it leads to nonlinear optimization problems. However, the optimization model that is presented here is kept linear by assuming the average ore grade in the stockpile as a user defined value. The generated schedule is checked to validate the input parameters. A recursive scheme can be followed in order to determine the average ore grade at the stockpile. Since the model is a mixed integer programming model, it converges to the optimum solution much faster than nonlinear models. The other advantage of this model is that the optimality of the solution is always guaranteed because linear models always have global optimum solution and it can be determined that how close is the answer to the theoretical solution. The clustering method is required to reduce size of the problem and prevent generating the production schedule with gap effects and scatter extraction schedule. The provided source codes for the proposed model are in MATLAB environment. TOMLAB/CPLEX solver is an industrial scale optimization engine that is capable of handling very large size problems. Also, the parallel computation is implemented inside the CPLEX which improves the speed of the algorithm very significantly. Therefore, a large production scheduling problems can be easily handled with this model given source codes.

On the other hand, the proposed method is limited to the single stockpile. The proposed method only provides the time and portion of the extraction and does not give destination of the blocks. The decision between ore and waste blocks are made by the cut-off grade. If a block is waste, it will be sent to the waste dump and if it is ore it either will be sent to the plant or stockpile. Therefore, in order to determine destination of each ore block, another linear optimization model needs to be solved which is called the short term production planning problem.

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