Robust Decision Making: Coupling Oil Sands Mine and Waste Disposal Planning

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Abstract

In oil sands mining, providing processable ore and tailings containment at the right time are the main drivers for profitability and sustainability. This paper introduces a Mixed Integer Linear Goal Programming (MILGP) mine planning model to: a) determine the order and time of extraction of ore, dyke material and waste over the mine life that maximizes the operation's net present value; and b) determine dyke material destination that minimizes dyke construction cost. To implement an efficient MILGP model, an initial production schedule was generated and used as an input for the optimization process. This reduced the size and solution time of the oil sands production scheduling and waste management optimization problem leading to the implementation of sustainable mining strategies. This includes an efficiently modeled pushback mining strategy. The model created value and a sustainable operation by generating a practical, smooth and uniform schedule for ore and dyke material. The total NPV generated including dyke construction cost for all pushbacks and destinations is $26,987M for a total mined tonnage of 7377.4Mt.

1. Introduction

Mining is the process of extracting a beneficial naturally occurring resource from the earth (Newman et al., 2010) and historical assessment of mineral resource evaluations has demonstrated the sensitivity of project profitability to decisions based on mine planning. A major aspect of mine planning is the optimization of long-term production scheduling. The aim of long-term production scheduling usually is to determine the time and sequence of extraction and displacement of ore and waste in order to maximize the overall discounted net revenue from a mine within the existing economic, technical and environmental constraints. Long-term production schedules defines the mining and processing plant capacity and expansion potential as well as management investment strategy. In mining projects, deviations from optimal mine plans will result in significant financial losses, future financial liabilities, delayed reclamation and resource sterilization.

Mixed Integer Linear Goal Programming (MILGP) type mathematical models are considered powerful tools in optimizing mine production schedules and there have been some efforts in applying them to mining projects. Though Mathematical Programming Models (MPMs) have been applied in mine production scheduling, very little work has been done in terms of oil sands mine planning, which has a unique scenario when it comes to waste management. Recent mining regulations by Alberta Energy Resources and Conservation Board (Directive 074) (McFadyen, 2008) requires oil sands mining companies to develop integrated mine planning and waste management strategies for their in-pit and external tailings facilities. The MILGP model used for this research incorporates multiple material types with multiple elements for multiple destinations

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in Oil Sands Long Term Production Planning (OSLTPP). This MILGP model however results in a large scale optimization problem which may become intractable or have longer solution times. The objective of this paper is to develop and verify an efficient MILGP model for OSLTPP and waste management that delivers acceptable results in a reasonable time.

Oil sands mining is increasingly becoming challenging as oil sands companies integrate responsible socio-environmental waste management practices into their operations. Together with the limitations in lease areas, it has become necessary to look into effective and efficient waste disposal planning system. This system should be well integrated into the long term mine plan in an optimization framework that creates value and a sustainable operation. In oil sands operations, the pit phase mining occurs simultaneously with the construction of in-pit dykes in the mined out areas of the pit and ex-pit dykes in designated areas outside the pit. These dykes are constructed to hold tailings that are produced during processing of the oil sands ore. The materials used in constructing these dykes come from the oil sands mining operation. The dyke materials are made up of overburden (OB), interburden (IB) and tailings coarse sand (TCS). Any material that does not qualify as ore or dyke material is sent to the waste dump.

In implementing an efficient MILGP model to incorporate waste management into OSLTPP, our targets are to:

- Determine the order and time of extraction of ore, dyke material and waste to be removed from a predefined final pit limit over the mine life that maximizes the Net Present Value (NPV) of the operation.
- Determine the destination of dyke material that minimizes construction cost based on the construction requirements of the various dykes.

Prior to OSLTPP, we assume that the material in the final pit limit is discretized into a three-dimensional array of rectangular or cubical blocks called a block model. Attributes of the material in the block model such as rock types, densities, grades, or economic data are represented numerically (Askari-Nasab et al., 2011; Ben-Awuah and Askari-Nasab, 2011). Fig. 1 shows the schematic representation of the problem definition containing K mining-cuts. Mining-cuts are clusters of blocks within the same level or mining bench that are grouped based on a similarity index defined using the attributes; location, grade, rock type and the shape of mining-cuts that are created on the lower bench. In this research, an agglomerative hierarchical clustering algorithm which seeks to generate clusters with reduced mining-cut extraction precedences compared with other automated methods is used (Tabesh and Askari-Nasab, 2011). Each mining-cut k, is made up of ore o_k, OB dyke material d_k, IB dyke material n_k, and waste w_k. The material in each mining-cut is to be scheduled over T periods depending on the goals and constraints associated with the mining operation. OB dyke material scheduled d_k^T, IB dyke material scheduled n_k^T, and TCS dyke material from the processed ore scheduled, l_k^T, must further be assigned to the dyke construction sites based on construction requirements. For period t_1, the dyke construction material required by site i is $dyke_i$. In addition, the final pit limit block model is divided into pushbacks. The material intersecting a pushback and a bench is known as a mining-panel. Each mining-panel contains a set of mining-cuts and is used to control the mine production operation sequencing.

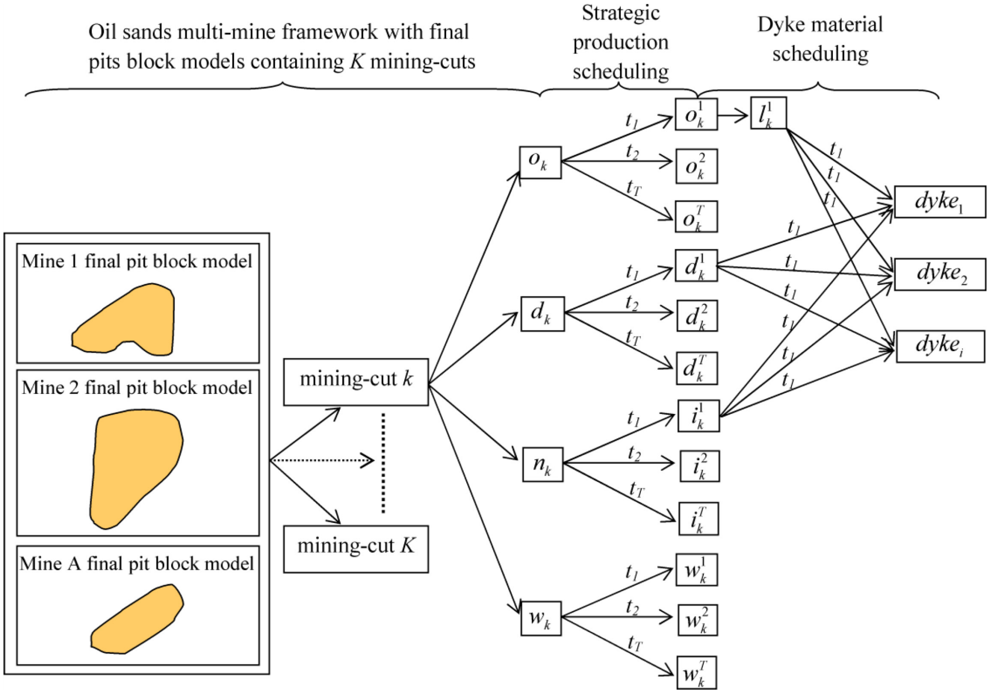

Fig. 1. Schematic representation of the problem definition showing strategic production and dyke material scheduling modified after Ben-Awuah and Askari-Nasab (2011).

The schedules generated for OSLTPP drives the profitability and sustainability of an oil sands mining operation. The strategic production and dyke material schedules control the NPV of the operation and provide the platform for a robust waste management planning strategy. Lack of proper waste management planning can lead to environmental and sustainability issues resulting in major financial liabilities or immediate mine closure by regulatory agencies.

The rest of the paper is organized as follows. Section 2 summarizes the literature on the application of MPMs to the Long Term Production Planning (LTPP) problem. This is followed by a section on the application of MILGP model for OSLTPP and waste management. Section 4 outlines the implementation of an efficient MILGP model for OSLTPP and waste management and a case study presented in section 5. The paper concludes in section 6.

2. Summary of literature review

Applying mathematical programming models (MPMs) like linear programming (LP), mixed integer linear programming (MILP), and goal programming (GP) with exact solution methodologies have proved to be robust. Solving MPMs with exact solution methodologies result in solutions within known limits of optimality. As the solution gets closer to optimality, it results in production schedules that generate higher NPV than those obtained from heuristic optimization methods. This has resulted in extensive research on the application of mathematical programming models (MPMs) like LP and MILP to the long-term production planning (LTPP) problem. The inherent difficulty in applying these models to the LTPP problem is that, they result in large scale optimization problems containing many binary and continuous variables. These are difficult to solve with the current computing software and hardware available and may have lengthy solution times. Though some researchers have made efforts in reducing the solution time associated with solving MPMs, their models were not capable of dealing with large block model sizes or could not generate feasible practical mining strategies (Johnson, 1969; Gershon, 1983; Dagdelen, 1985; Akaike and Dagdelen, 1999; Ramazan, 2001; Caccetta and Hill, 2003; Ramazan and Dimitrakopoulos, 2004). These publications note that the size of the resulting LP and MILP models is a major problem because it contains too many binary and continuous variables.

GP is another mathematical programming modeling platform that have been used in solving the LTPP problem. It permits flexible formulation, specification of priorities among goals, and some level of interactions between the decision maker and the optimization process (Zeleny, 1980; Hannan, 1985). This lead to its application to the LTPP problem by Zhang et al. (1993), Chanda and Dagdelen (1995) and Esfandiri et al. (2004). They were however unable to practically implement their models due to the numerous mining production constraints and size of the optimization problem. Recent implementation of MILP models with block clustering techniques were successfully undertaken for an Iron ore deposit (Askari-Nasab et al., 2010; Askari-Nasab et al., 2011). It however lacks the framework for the implementation of an integrated mine planning and waste management system as is the case required for sustainable oil sands mining. Due to the strategy required for sustainable oil sands mining and the regulatory requirements from Directive 074, waste disposal planning is closely related to the mine planning system (McFadyen, 2008; Askari-Nasab and Ben-Awuah, 2011; Ben-Awuah and Askari-Nasab, 2011). Currently, oil sands waste disposal planning is handled as a post-production scheduling optimization activity. Modeling an integrated mine planning system even adds more complexity to the LTPP problem. Ben-Awuah et al. (2012) has implemented a MILGP model for an integrated oil sands production scheduling and waste disposal planning system. The model takes into account multiple material types, elements and destinations, directional mining, waste management and sustainable practical mining strategies. The implementation of the MILGP model resulted in a large-scale optimization problem with lengthy solution times.

This paper presents scheduling models and tests on how to generate MILGP formulations using fewer non-zero decision variables. It also discusses alternative techniques to MILGP pushback mining modeling for efficiency in solving the formulations. The tests show that there are significant differences in the time taken by the various MILGP models generated for the same deposit to maximize NPV and minimize dyke construction cost. An oil sands multi mine data set is used for the case study.

3. MILGP model for OSLTPP and waste management

The OSLTPP and waste management problem is to find the time and sequence of extraction of ore, dyke material and waste mining-cuts to be removed from pre-defined open pit outlines and their respective destinations over the mine life, so that the NPV of the operation is maximized and dyke construction cost is minimized. In general, the MILGP formulation is for multiple material types and destinations as well as pushbacks which ties into the waste management strategy for oil sands operations. The production schedule is subject to a variety of technical, physical and economic goals and constraints which enforce mining extraction sequence, mining and dyke construction capacities and blending requirements. The notations used in the formulation of the OSLTPP and waste management problem have been classified as sets, indices, parameters and decision variables. Some of the details of these notations can be found in the Appendix.

The summary of economic data for each mining-cut known as economic mining-cut value is based on ore parcels within mining-cuts which could be mined selectively. The economic mining-cut value is a function of the value of the mining-cut based on the processing destination and the costs incurred in mining from a designated location and processing, and dyke construction at a specified destination. The cost of dyke construction is also a function of the location of the tailings facility being constructed and the type and quantity of dyke material used. The discounted economic mining-cut value for mining-cut *k* is equal to the discounted revenue obtained by selling the final product contained in mining-cut *k* minus the discounted cost involved in mining mining-cut *k* as waste minus the extra discounted cost of mining OB and IB dyke material, and generating TCS dyke material from mining-cut *k* for a designated dyke construction destination. This can be summarized by Eqs. (1) to (6).

Discounted economic mining-cut value = discounted revenue - discounted costs

$$d_k^{u,t} = v_k^{u,t} - q_k^{a,t} - p_k^{u,t} - m_k^{u,t} - h_k^{u,t} \tag{1}$$

The variables in Eq. (1) can be defined by Eqs. (2) to (6).

$$v_k^{u,t} = \sum_{e=1}^{E} o_k \times g_k^e \times r^{u,e} \times \left(p^{e,t} - cs^{e,t} \right) - \sum_{e=1}^{E} o_k \times cp^{u,e,t} \tag{2}$$

$$q_k^{a,t} = \left(o_k + d_k + n_k + w_k \right) \times cm^{a,t} \tag{3}$$

$$p_k^{u,t} = d_k \times ck^{u,t} \tag{4}$$

$$m_k^{u,t} = n_k \times cb^{u,t} \tag{5}$$

$$h_k^{u,t} = l_k \times ct^{u,t} \tag{6}$$

Where:

$t \in \{1,......,T\}$	index for scheduling periods.
$k \in \{1,.....,K\}$	index for mining-cuts.
$p \in \{1,.....,P\}$	index for mining-panels.
$e \in \{1,.....,E\}$	index for element of interest in each mining-cut.
$j \in \{1,.....,J\}$	index for phases (pushback).
$u \in \{1,.....,U\}$	index for possible destinations for materials.
$a \in \{1,.....,A\}$	index for possible mining locations (pits).
$d_k^{u,t}$	the discounted economic mining-cut value obtained by extracting mining-cut *k* and sending it to destination *u* in period *t*.
$v_k^{u,t}$	the discounted revenue obtained by selling the final products within mining-cut *k* in period *t* if it is sent to destination *u*, minus the extra discounted cost of mining all the material in mining-cut *k* as ore from location *a* and processing at destination *u*.
$q_k^{a,t}$	the discounted cost of mining all the material in mining-cut *k* in period *t* as waste from location *a*.
$q_p^{a,t}$	the discounted cost of mining all the material in mining-panel *p* in period *t* as waste from location *a*. Each mining-panel *p* contains its corresponding set of mining-cuts.
$p_k^{u,t}$	the extra discounted cost of mining all the material in mining-cut *k* in period *t* as overburden dyke material for construction at destination *u*.
$m_k^{u,t}$	the extra discounted cost of mining all the material in mining-cut *k* in period *t* as interburden dyke material for construction at destination *u*.
$h_k^{u,t}$	the extra discounted cost of mining all the material in mining-cut *k* in period *t* as tailings coarse sand dyke material for construction at destination *u*.
o_k	the ore tonnage in mining-cut *k*.

d_k	the overburden dyke material tonnage in mining-cut *k*.
n_k	the interburden dyke material tonnage in mining-cut *k*.
w_k	the waste tonnage in mining-cut *k*.
l_k	the tailings coarse sand dyke material tonnage in mining-cut *k*.
g_k^e	the average grade of element *e* in ore portion of mining-cut *k*.
$r^{u,e}$	the proportion of element *e* recovered (processing recovery) if it is processed at destination *u*.
$p^{e,t}$	the price of element *e* in present value terms per unit of product.
$cs^{e,t}$	the selling cost of element *e* in present value terms per unit of product.
$cp^{u,e,t}$	the extra cost in present value terms per tonne of ore for mining and processing at destination *u*.
$cm^{a,t}$	the cost in present value terms of mining a tonne of waste in period *t* from location *a*.
$ck^{u,t}$	the cost in present value terms per tonne of overburden dyke material for dyke construction at destination *u*.
$cb^{u,t}$	the cost in present value terms per tonne of interburden dyke material for dyke construction at destination *u*.
$ct^{u,t}$	the cost in present value terms per tonne of tailings coarse sand dyke material for dyke construction at destination *u*.

3.1. The MILGP model objective functions

The objective functions of the MILGP model for OSLTPP and waste management can be formulated as: 1) maximizing the NPV of the mining operation, 2) minimizing the dyke construction cost for the waste management plan, and 3) minimizing the deviations from the set goals. We used the concepts presented in Ben-Awuah et al. (2012) as the starting point of our development. The formulation uses continuous decision variables, $y_p^{a,t}$, $x_k^{u,t}$, $z_k^{u,t}$, $c_k^{u,t}$, and $s_k^{u,t}$ to model mining and processing requirements, and OB, IB and TCS dyke material requirements respectively, for all mining locations and processing and dyke construction destinations. Using continuous decision variables allows for fractional extraction of mining-panels and mining-cuts in different periods for different locations and destinations. Continuous deviational variables, $d_1^{-,a,t}$, $d_2^{-,u,t}$, $d_3^{-,u,t}$, $d_4^{-,u,t}$ and $d_5^{-,u,t}$ have been defined to support the goal functions that control mining, processing, OB, IB and TCS dyke material, for all mining locations and processing and dyke construction destinations. The deviational variables provide a continuous range of units (tonnes) that the optimizer can choose from to satisfy the set goals. In the objective function, these deviational variables are minimized. The objective function also contains deviational penalty cost and priority (PP) parameters. The deviational penalty cost parameters, a_1, a_2, a_3, a_4, and a_5, penalizes the NPV for any deviation from the set goals. The priority parameters P_1, P_2, P_3, P_4 and P_5 are used to place emphasis on the goals that are more important. The PP parameters are set up to penalize the NPV if the set goals are not met as well as the most important goal.

When setting up these parameters, the planner needs to monitor how continuous mining proceeds period by period and the uniformity of tonnages mined per period; as well as the corresponding NPV generated, to keep track of how parameter changes affect these key performance indicators. In some scenarios, the limit for setting the PP parameters depends on the extent to which the planner wants to trade off NPV to meet the set goals. A higher PP parameter may enforce a goal to be met whilst reducing the NPV of the operation. A case showing this trend was analyzed in Ben-Awuah et al. (2012). In general, the magnitude of the PP parameters should be calibrated based on the objectives of management. More weight should be assigned to a goal that has a higher priority for the management.

The three objective functions of the MILGP model for OSLTPP and waste management are represented by Eqs. (7) to (9) respectively.

$$Max\sum_{a=1}^{A}\sum_{j=1}^{J}\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{\substack{k\in B_p\\ p\in B_j}}\left(v_k^{u,t}\times x_k^{u,t}-q_p^{a,t}\times y_p^{a,t}\right)\right) \tag{7}$$

$$Min\sum_{a=1}^{A}\sum_{j=1}^{J}\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{\substack{k\in B_p\\ p\in B_j}}\left(p_k^{u,t}\times z_k^{u,t}+m_k^{u,t}\times c_k^{u,t}+h_k^{u,t}\times s_k^{u,t}\right)\right) \tag{8}$$

$$Min\sum_{a=1}^{A}\sum_{j=1}^{J}\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{\substack{k\in B_p\\ p\in B_j}}\begin{bmatrix}P_1\left(a_1d_1^{-,a,t}\right)+P_2\left(a_2d_2^{-,u,t}\right)+P_3\left(a_3d_3^{-,u,t}\right)+\\ P_4\left(a_4d_4^{-,u,t}\right)+P_5\left(a_5d_5^{-,u,t}\right)\end{bmatrix}\right) \tag{9}$$

Eqs. (7) to (9) can be combined as a single objective function formulated as in Eq. (10).

$$Max\sum_{a=1}^{A}\sum_{j=1}^{J}\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{\substack{k\in B_p\\ p\in B_j}}\begin{bmatrix}\left(v_k^{u,t}\times x_k^{u,t}-q_p^{a,t}\times y_p^{a,t}\right)-\left(p_k^{u,t}\times z_k^{u,t}+m_k^{u,t}\times c_k^{u,t}+h_k^{u,t}\times s_k^{u,t}\right)-\\ \begin{pmatrix}P_1\left(a_1d_1^{-,a,t}\right)+P_2\left(a_2d_2^{-,u,t}\right)+P_3\left(a_3d_3^{-,u,t}\right)+\\ P_4\left(a_4d_4^{-,u,t}\right)+P_5\left(a_5d_5^{-,u,t}\right)\end{pmatrix}\end{bmatrix}\right) \tag{10}$$

3.2. The MILGP model goal functions

In the proposed model, the goals to be achieved are the mining and processing targets, and OB, IB and TCS dyke materials targets in tonnes for all mining locations, and processing and dyke construction destinations. These goal functions are represented by Eqs. (11) to (15) respectively.

$$\sum_{j=1}^{J}\left(\sum_{p\in B_j}\left(o_p+d_p+n_p+w_p\right)\times y_p^{a,t}\right)+d_1^{-,a,t}=T_m^{a,t} \tag{11}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in B_p}\left(o_k\times x_k^{u,t}\right)\right)+d_2^{-,u,t}=T_{pr}^{u,t} \tag{12}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in B_p}\left(d_k\times z_k^{u,t}\right)\right)+d_3^{-,u,t}=T_d^{u,t} \tag{13}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in B_p}\left(n_k\times c_k^{u,t}\right)\right)+d_4^{-,u,t}=T_n^{u,t} \tag{14}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in B_p}\left(l_k \times s_k^{u,t}\right)\right) + d_5^{-,u,t} = T_l^{u,t} \tag{15}$$

Eq. (11) represents the mining goal function which ensures the total amount of ore, dyke material and waste mined in each period from all mining locations equals the total available equipment capacity with a defined acceptable deviation. This goal is controlled by the continuous variable $y_p^{a,t}$. With the deviational variable $d_1^{-,a,t}$, Eq. (11) will be used in achieving a uniform stripping ratio over the mine life. A production schedule with a constant stripping ratio ensures that the mining equipment fleet size required is matched to material movement targets. To establish a proper production rate, among other things, multiple scenarios of yearly ore production rates must be investigated and the one with a uniform mill feed and the highest NPV considered. A variable mining goal that allows the mine planner to use different mining capacities throughout the mine life can be implemented with this model. This allows for consideration of future expansion projects either by owner or contract mining which in most cases increases profitability considerably. The set mining goal is a function of the ore reserve, targeted mine-life, designed processing capacity, overall stripping ratio, and the available capital for mining fleet acquisition.

Eq. (12) represents the processing goal function which controls the mill feed. This goal helps the mine planner to provide a uniform feed throughout the mine life resulting in an effectively integrated mine-to-mill operation. In practice, the processing goal must be set with minimal periodic deviations to ensure maximum utilization of the mill. Depending on the ore grade distribution of the orebody, the processing goal may not be achieved in some periods. In such cases, pre-stripping could be explored to provide a uniform mill feed. This amounts to forcing the optimizer to mine waste in the early periods, or mining more ore than needed when available and feeding the mill with the stockpiled ore when required.

Eqs. (13), (14) and (15) represent the dyke material goal functions which control dyke construction scheduling. These goals enable the mine planner to schedule the required dyke materials for all dyke construction destinations. The TCS dyke material generated from the processing plant is directly dependent on the mill feed. The schedules generated from the MILGP model give the planner good control over dyke material and provide a robust platform for effective dyke construction planning and tailings storage management. Movement of dyke material and dyke construction scheduling can be well integrated with the mining fleet management plan. Thus, timely tailings containment areas can be created for the storage of fluid fine tailings. In oil sands mining, being able to efficiently plan the waste management strategy results in a more profitable and sustainable operation.

3.3. The MILGP model grade blending constraints

The MILGP model grade blending constraints control the grade of ore bitumen, ore fines and interburden fines in the mined material for all processing and dyke construction destinations. These constraints are formulated by Eqs. (16) to (21).

$$\sum_{p=1}^{P}\sum_{k\in B_p} g_k^e \times o_k \times x_k^{u,t} - \overline{g}^{u,t,e}\sum_{p=1}^{P}\sum_{k\in B_p} o_k \times x_k^{u,t} \le 0 \tag{16}$$

$$\sum_{p=1}^{P}\sum_{k\in B_p} g_k^e \times o_k \times x_k^{u,t} - \underline{g}^{u,t,e}\sum_{p=1}^{P}\sum_{k\in B_p} o_k \times x_k^{u,t} \ge 0 \tag{17}$$

$$\sum_{p=1}^{P}\sum_{k\in B_p} f_k^e \times o_k \times x_k^{u,t} - \overline{f}^{u,t,e}\sum_{p=1}^{P}\sum_{k\in B_p} o_k \times x_k^{u,t} \le 0 \tag{18}$$

$$\sum_{p=1}^{P}\sum_{k\in B_p} f_k^{e}\times o_k\times x_k^{u,t} - \underline{f}^{u,t,e}\sum_{p=1}^{P}\sum_{k\in B_p} o_k\times x_k^{u,t}\geq 0 \tag{19}$$

$$\sum_{p=1}^{P}\sum_{k\in B_p} f_k^{d}\times n_k\times c_k^{u,t} - \overline{f}^{u,t,d}\sum_{p=1}^{P}\sum_{k\in B_p} n_k\times c_k^{u,t}\leq 0 \tag{20}$$

$$\sum_{p=1}^{P}\sum_{k\in B_p} f_k^{d}\times n_k\times c_k^{u,t} - \underline{f}^{u,t,d}\sum_{p=1}^{P}\sum_{k\in B_p} n_k\times c_k^{u,t}\geq 0 \tag{21}$$

Eqs. (16) to (19) represents inequality constraints which monitors the mill feed quality. They specify the limiting grade requirements for ore bitumen and ore fines for processing. The objective of blending in production scheduling is to mine in a way that the run-of-mine materials meet the quality and quantity specification of the processing plant and dyke construction destinations. As more detailed planning is done in the short term, the planner is more concerned with reducing the mill head grade variability and hence blending of mill feed material becomes critical. The mill head grade is a function of the ore grade distribution, processing plant design and mine cash flow requirements.

Eqs. (20) and (21) represents inequality constraints that controls the IB dyke material quality. They specify the limiting grade requirements for IB dyke material fines for dyke construction. The designs for dykes at different destinations come with specific dyke material requirements. Among other things, the dyke material quality defines the integrity of the tailings containment facilities constructed. Since in oil sands mining it is required by law to store large volumes of tailings with less environmental footprints, waste management directly impacts profitability and sustainability (McFadyen, 2008).

3.4. The MILGP model variables control constraints

In the proposed model, the variables control constraints monitor the logics of the variables that define mining, processing, dyke materials and goal deviations to ensure they are within acceptable ranges. These variables control constraints are represented by Eqs. (22) to (29).

$$\sum_{u=1}^{U}\sum_{k\in B_p}\left(o_k x_k^{u,t} + d_k z_k^{u,t} + n_k c_k^{u,t}\right)\leq \sum_{a=1}^{A}\sum_{p\in B_j}\left(y_p^{a,t}\left(o_p + d_p + n_p + w_p\right)\right) \tag{22}$$

$$\sum_{u=1}^{U} s_k^{u,t}\leq \sum_{u=1}^{U} x_k^{u,t} \tag{23}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} x_k^{u,t}\leq 1 \tag{24}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} y_p^{u,t}\leq 1 \tag{25}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} z_k^{u,t}\leq 1 \tag{26}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} c_k^{u,t}\leq 1 \tag{27}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} s_k^{u,t}\leq 1 \tag{28}$$

$$d_1^{-,a,t},\ d_2^{-,u,t},\ d_3^{-,u,t},\ d_4^{-,u,t},\ d_5^{-,u,t} \geq 0 \tag{29}$$

Eq. (22) outlines inequalities that ensure that the total material mined from mining-panel *p* in any given scheduling period from any mining location exceeds or is equal to the sum of the ore and OB and IB dyke material mined from the mining-cuts belonging to that mining-panel. It is assumed that when a mining-panel is scheduled, all the mining-cuts, blocks or parcels within the mining-panel are extracted uniformly. Eq. (23) states that the fraction of TCS dyke material produced in each period should be less or equal to the fraction of ore mined for all destinations. This constraint manages the direct relationship between ore and TCS dyke material. TCS dyke material is only generated when ore is processed for bitumen extraction. Eqs. (24) to (28) ensure that the total fractions of mining-panel *p* and mining-cut *k* mined and TCS dyke material produced and sent to all destinations in all periods is less or equal to one. This keeps track of the different portions of mining-panels and mining-cuts that are scheduled for various destinations. Eq. (29) defines the non-negativity of the deviational variables defined to support the goal functions.

3.5. The MILGP model mining-panels extraction precedence constraints

The mining-panels extraction precedence in the MILGP model are defined by Eqs. (30) to (34). Binary integer decision variable, $b_p^t \in [0,1]$ is used to control precedence of mining-panels extraction. b_p^t is equal to one if the extraction of mining-panel *p* has started by or in period *t*, otherwise it is zero. These equations together implement the vertical and horizontal mining-panel extraction sequence.

$$b_p^t - \sum_{a=1}^{a}\sum_{i=1}^{t} y_s^{a,i} \leq 0 \qquad s \in C_p(L) \tag{30}$$

$$b_p^t - \sum_{a=1}^{a}\sum_{i=1}^{t} y_r^{a,i} \leq 0 \qquad r \in M_p(Z) \tag{31}$$

$$b_p^t - \sum_{a=1}^{a}\sum_{i=1}^{t} y_h^{a,i} \leq 0 \qquad h \in B_j(H) \tag{32}$$

$$\sum_{a=1}^{a}\sum_{i=1}^{t} y_p^{a,i} - b_p^t \leq 0 \tag{33}$$

$$b_p^t - b_p^{t+1} \leq 0 \tag{34}$$

For each mining-panel *p*, Eqs. (30) to (34) check the set of immediate predecessor mining-panels that must be mined prior to mining mining-panel *p* for all periods and from all locations. This precedence relationship ensures that: 1) all the immediate predecessor mining-panels above the current mining-panel *p* are extracted prior to extraction of mining-panel *p*; represented by the set $C_p(L)$, 2) all the immediate predecessor mining-panels preceding the current mining-panel *p* in the horizontal mining direction are extracted prior to extraction of mining-panel *p*; represented by the set $M_p(Z)$, and 3) all the mining-panels within the immediate predecessor mining phase that precedes the current mining phase, *j* are extracted prior to extraction of mining-panel *p* in the current mining phase; represented by the set $B_j(H)$.

Specifically, Eqs. (30) to (32) ensures that all the immediate predecessor mining-panels which are members of $C_p(L)$, $M_p(Z)$, and $B_j(H)$ are mined prior to mining mining-panel *p*. Eq. (33) checks that extraction of mining-panel *p* can start only when the mining-panel has not been extracted before. Eq. (34) monitors that once the extraction of a mining-panel starts in a period, this mining-panel is available for extraction during the subsequent periods. These equations work together to ensure mining proceeds in the specified horizontal mining direction as the mine goes deeper.

Implementing the mining operation sequencing at mining-panel level helps reduce the number of binary variables to be solved for during optimization. It also ensures practical mining sequencing with reduced number of required drop-cuts ensuring efficient equipment utilization.

4. Implementation of an efficient MILGP model for OSLTPP and waste management

We have progressively developed an efficient and robust MILGP model for solving the OSLTPP and waste management problem which involves multiple destinations, material types, mining locations and pushbacks (Askari-Nasab and Ben-Awuah, 2011; Ben-Awuah and Askari-Nasab, 2011). This leads to a large scale optimization problem with numerous decision variables and constraints that takes large memory overheads and time to solve. Thus, resulting in a sophisticated production scheduling problem which calls for improved numerical modeling and optimization techniques to deliver acceptable results in a timely manner. We have further developed techniques to reduce the number of non-zero decision variables and pushback mining constraints in the production scheduling problem. We also implemented a practical mine production sequencing with mining-panels which results in reduced number of binary variables to be solved for during optimization. The formulated MILGP production scheduling problems are solved using Tomlab/CPLEX (Holmström, 2009).

4.1. MILGP implementation with fewer non-zero decision variables

The main set-back in solving large scale MILGP problems is the size of the branch and cut tree. During optimization, the size of the branch and cut tree becomes so large that insufficient memory remains to solve an LP sub-problem. The size of the branch and cut tree depends on the number of decision variables in the formulation. The general strategy in formulating the MILGP for OSLTPP and waste management is therefore to reduce the number of decision variables in the production scheduling problem, thereby reducing the solution time significantly. This is implemented using an initial production schedule generated based on a practical oil sands directional mining strategy and the annual mining capacity.

The general form of the MILGP formulation can be represented by Equation (35) as:

$$\min_{r} f(r) = c^{T}.r \tag{35}$$

subject to: goals and constraints of the MILGP model

The objective function for the OSLTPP problem as stated by Equation (10) maximizes the NPV and minimizes the dyke construction cost. The objective function coefficient vector, *c*, is a column vector containing the discounted revenue and cost values for all mining-panels and mining-cuts in all periods and for all destinations. This is shown by Equation (36). The objective function decision variables vector, *r*, is a column vector containing mining-cut or mining-panel precedence, ore, mining, overburden, interburden and tailings coarse sand production and deviational variables. This is shown by Equation (37). The notations used have been defined in the Appendix. The decision variables vector, *r* is therefore made up of $\sum_{dv=1}^{DV} KTU + \sum_{dn=1}^{DN} TU$ non-zero decision variables to be solved for in the MILGP model during optimization. This vector ensures that each mining-cut or mining-panel is available for production scheduling during the entire mine life. As shown in section 4.1.1, by having an initial production schedule, the number of non-zero decision variables in *r* can be reduced, thereby reducing the size of the production scheduling problem.

$$c^{\left(\sum_{dv=1}^{DV} KTU+\sum_{dn=1}^{DN} TU\right)\times 1}=\left[0;v;q;p;m;h;a_1;a_2;a_3;a_4;a_5\right] \tag{36}$$

$$r^{\left(\sum_{dv=1}^{DV} KTU+\sum_{dn=1}^{DN} TU\right)\times 1}=\left[b;x;y;z;c;s;d_1;d_2;d_3;d_4;d_5\right] \tag{37}$$

where $dv\in\{1,...,DV\}$ are the decision variables in the objective function; $dn\in\{1,...,DN\}$ are the deviational variables in the objective function; K is the number of mining-cuts or mining-panels involved depending on the coefficient or variable being set up.

4.1.1. Generating and applying an initial production schedule

This technique is based on a practical directional oil sands mining and the continuous depletion of material from a given mining and processing capacity. An initial production schedule can be generated using i) a fast heuristic production scheduling algorithm like Whittle's Fixed Lead algorithm (Gemcom Software International, 2008) or ii) a moving production bin calculated estimate. Before optimization with the MILGP model, Whittle can be used to generate a production schedule and then some periodic tolerance applied to the schedule and used as an initial production schedule. Similarly, a moving production bin can be initiated at one end of the deposit and with the annual mining and processing targets and mining direction, a schedule can be generated. Applying a periodic tolerance, an initial schedule can be generated for the MILGP model as well.

Let us consider an oil sands deposit containing 980 mining-cuts in 2 pushbacks which is to be mined from west to east over 12 periods for the processing plant and 4 dyke construction destinations; as shown in Fig. 2. The production scheduling and waste disposal planning strategy to be used here is based on a practical directional oil sands mining similar to the conceptual mining model implemented by Askari-Nasab and Ben-Awuah (2011). This includes complete extraction of pushback 1 before the mining of pushback 2 to ensure that pushback 1 can be used for tailings disposal planning. From Fig. 2, based on the mining and processing goals and direction of mining we can estimate that mining-cut 6 may be mined in say period 4. Assuming we apply a periodic tolerance of 3, then in the initial schedule for the MILGP model, mining-cut 6 can be said to be extractable over periods 1 to 7; whilst the rest of the periods are set to zeros. Conventionally, mining-cut 6 will have been modeled to be extractable over the entire 12 years mine life. With this technique, the number of non-zero decision variables, r to be solved for in the MILGP model during production scheduling will reduce from 176,568 to 94,472. This reduces the size of the production scheduling problem significantly.

Theoretically, this variable reduction technique decreases the solution space for the optimization problem. Thus during optimization, some of the branches in the branch and cut tree are eliminated, ensuring that the solution for the practical production scheduling problem is reached faster. It is important to note that, reducing the solution space unreasonably can cause one to miss the desired practical production scheduling solution.

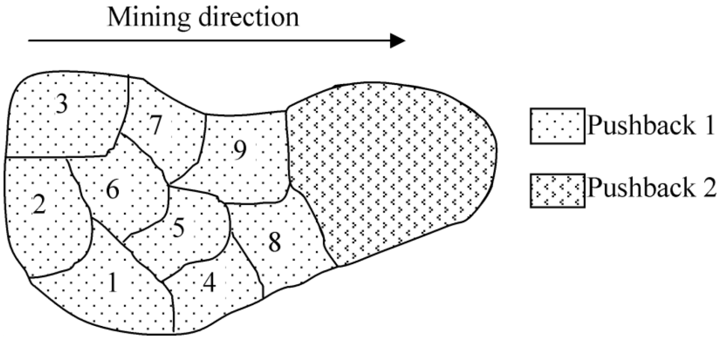

Fig. 2. Schematic representation of an oil sands deposit showing mining-cuts and pushbacks.

4.2. MILGP multi mine implementation with fewer pushback mining precedence constraints

In OSLTPP and waste management, it is important to have a pushback mining precedence strategy that ties into the waste disposal plan. This requires the development of a well-integrated strategy of directional and pushback mining, and tailings dyke construction for in-pit and ex-pit tailings storage management. This includes the complete extraction of one pushback before the mining of the next pushback in the direction of mining, thus enabling the release of the dyke footprints of the recently mined pushback for dyke construction to start and then subsequently tailings deposition. Details of this integrated mine planning and waste management strategy has been well documented by Askari-Nasab and Ben-Awuah (2011). Multiple mines final pits are modeled as pushbacks and the MILGP model applied appropriately.

To implement the complete extraction of pushbacks during optimization, pushback mining precedence constraints must be developed and implemented whilst ensuring that the optimization problem is still feasible within a reasonable time. This requires an efficient modeling of the pushback mining precedence constraints to reduce the number of variables being added to the problem. The strategy used by the MILGP model has been tied into the vertical and horizontal extraction precedence constraints of the mining-panels. Three cases and strategies have been identified and are illustrated in Fig. 3.

The first case in Fig. 3 (i) and (ii) assumes that pushbacks in the final pit being used as an input for the MILGP model has flat topography and bottom. This means that with the west to east mining direction, mining will proceed in pushback 1 until it reaches the bottom of the pit where the list of bounding mining-panels in set A becomes the last set of mining-panels for complete extraction of pushback 1 prior to pushback 2. Set B also contains the list of bounding mining-panels at the top of pushback 2 where mining starts. Set A therefore becomes the preceding mining-panels set to set B. The pushback mining precedence constraints here involves identifying the list of bounding mining-panels that belongs to set A and B and applying the mining-panels extraction precedence constraints in Eqs. (32) to (34).

The second case in Fig. 3 (iii) is when the final pit has undulating topography and bottom which is almost always the case. Here, we look for the set C which is made up of the bounding mining-panels at the bottom of pushback 1 mined last. The set D also contains the list of bounding mining-panels at the top of pushback 2 which must be mined first when mining of pushback 2 starts. This approach becomes necessary because the mining-panels at the bottom of pushback 1 and top of pushback 2 belong to different mining benches therefore the vertical and horizontal mining-panels extraction precedence constraints are not able to tie the mining of these mining-panels together. Set C becomes the preceding mining-panels set to set D. Similarly, the mining-panels extraction precedence constraints in Eqs. (32) to (34) can then be applied to implement the complete extraction of pushback 1 prior to pushback 2.

The third case is when you have a similar situation in Fig. 3 (iii). The strategy here is by adding air mining-panels to the final pit both at the top and bottom, converting it from case 2 to case 1. The case 1 strategy can then be applied to implement the pushback mining precedence constraints.

The strategy used in the second case was implemented in the case study in this paper.

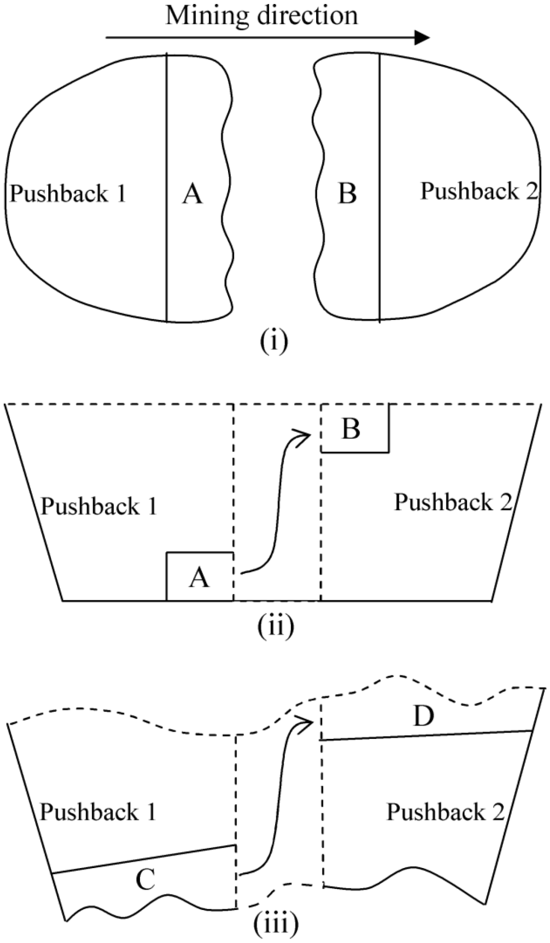

Fig. 3. Developing pushback mining precedence constraints: (i) Plan view and (ii) Cross sectional view; of final pit with flat topography and bottom showing pushbacks 1and 2 and the sets of bounding mining-panels (iii) Cross sectional view of final pit with undulating topography and bottom showing pushbacks 1 and 2 and the sets of bounding mining-panels.

5. Case study: implementation of the MILGP model

The MILGP model was coded in Matlab (Mathworks Inc., 2009) and implemented on an oil sands deposit which has 2 final pits covering an area of about 3900 ha. The mineralized zone of this deposit occurs in the McMurray formations. The deposit is to be scheduled for 16 periods equivalent to 16 years for the processing plant and dyke construction destinations. The performance of the proposed MILGP model was analyzed based on NPV, mining production goals, smoothness and practicality of the generated schedules, the availability of tailings containment areas at the required time and the computational time required for convergence. The model was implemented on a Quad-Core Dell Precision T7500 computer at 2.8 GHz, with 24GB of RAM. Table 1 provides information about the orebody model within the ultimate pits limits used in the case study.

The area to be mined are divided into 4 pushbacks in consultation with tailings dam engineers based on required tailings cell capacities and the timelines required in making the cell areas available for tailings containment. These 4 pushbacks are further divided into 20 intermediate pushbacks to enable the creation of practical mining-panels to be used in controlling the mining operation. These intermediate pushbacks are created using an approximately equal distribution of tonnages to be mined across the deposit. A hierarchical clustering algorithm is used in clustering blocks within each intermediate pushback into mining-cuts (Tabesh and Askari-Nasab, 2011).

Clustering blocks into mining-cuts ensures the MILGP scheduler generates a mining schedule at a selective mining unit that is practical from mining operation perspective. In solving the MILGP model with CPLEX, the absolute tolerance on the gap between the best integer objective and the objective of the best node remaining in the branch and cut algorithm, referred to as EPGAP, was set at 5% for the optimization of the mining project. The mining targets, processing plant feed, dyke construction requirements, bitumen grade and fines percent need to be controlled within acceptable ranges. These requirements have been summarized in Table 2. Mining will proceed from pushback 1 to 4 with complete extraction of each pushback prior to the next. In addition to the processing plant, dyke material requirements for 4 dyke construction destinations will be scheduled. Details of the waste management strategy implemented here has been well documented by Askari-Nasab and Ben-Awuah (2011).

Table 1: Oil sands deposit characteristics within the ultimate pit limits to be scheduled for 16 periods.

Characteristic	Value			
	Pit 1	Pit 2		
	Pushback 1	Pushback 2	Pushback 3	Pushback 4
Tonnage of rock (Mt)	1,244.8	2,165.9	2027.9	2068.7
Ore tonnage (Mt)	394.8	673.0	693.1	549.7
OB dyke material tonnage (Mt)	406.4	667.5	633.9	564.9
IB dyke material tonnage (Mt)	204.4	589.5	597.8	686.0
TCS dyke material tonnage (Mt)	298.8	468.0	454.0	428.0
Waste tonnage (Mt)	239.2	235.9	103.1	268.1
Average ore bitumen grade (wt%)	10.25	10.22	10.46	9.92
Average ore fines (wt%)	11.39	16.27	20.48	11.00
Average IB dyke material fines (wt%)	17.30	25.73	26.81	11.31
Number of blocks	16,985	28,700	26,667	26,393
Number of mining-cuts	380	630	579	564
Number of mining-panels	43	44	40	39
Block dimensions (m)	50 x 50 x 15			
Number of benches	9			

Table 2: Mining and processing goals, OB, IB and TCS dyke material goals, ore and IB dyke material grade requirements for the MILGP model.

Production scheduling parameter	$T_m^{a,t}$ (Mt)	$T_{pr}^{u,t}$ (Mt)	$T_d^{u,t}$ (Mt)	$T_n^{u,t}$ (Mt)	$T_l^{u,t}$ (Mt)	$\overline{g}^{u,t,e} / \underline{g}^{u,t,e}$ (wt%)	$\overline{f}^{u,t,e} / \underline{f}^{u,t,e}$ (wt%)	$\overline{f}^{u,t,d} / \underline{f}^{u,t,d}$ (wt%)
Value	470	145	36	33	26	16 / 7	30 / 0	50 / 0

5.1. Analysis

Run 2 was chosen for analysis due to its significantly reduced solution time. After optimization, the overall NPV generated including the dyke construction cost for all pushbacks and destinations is \$26,987M and the total dyke construction cost is \$3,821M at a 4.98% EPGAP. The scenario implemented here focuses on a practically integrated OSLTPP and waste management strategy that generates value and sustainability. This includes mining in a specified direction and making completely extracted pushbacks available for in-pit dyke construction and subsequently tailings management. This reduces the environmental footprints of the external tailings facility by commissioning in-pit tailings facilities when the active pushback is completely mined. The mining direction was decided on during the initial production schedule run using the Fixed Lead heuristic algorithm in Whittle (Mathworks Inc., 2009). The mining direction with the best NPV was selected for the MILGP model. The mining sequence at level 320m and 305m for all pushbacks with a west-east mining direction can be seen in Fig. 4 and Fig. 5. Fig. 4 and Fig. 5 also show the complete extraction of each pushback prior to mining the next, to support tailings management. The mining sequence shows a progressive continuous mining in the specified direction to ensure least mobility and increased utilization of loading equipment. This is very important in the case of oil sands mining where large cable shovels are used. The size of the mining-cuts and mining-panels also enables good equipment maneuverability and supports multiple material loading points. It enables mining to proceed with a reduced number of required drop-cuts.

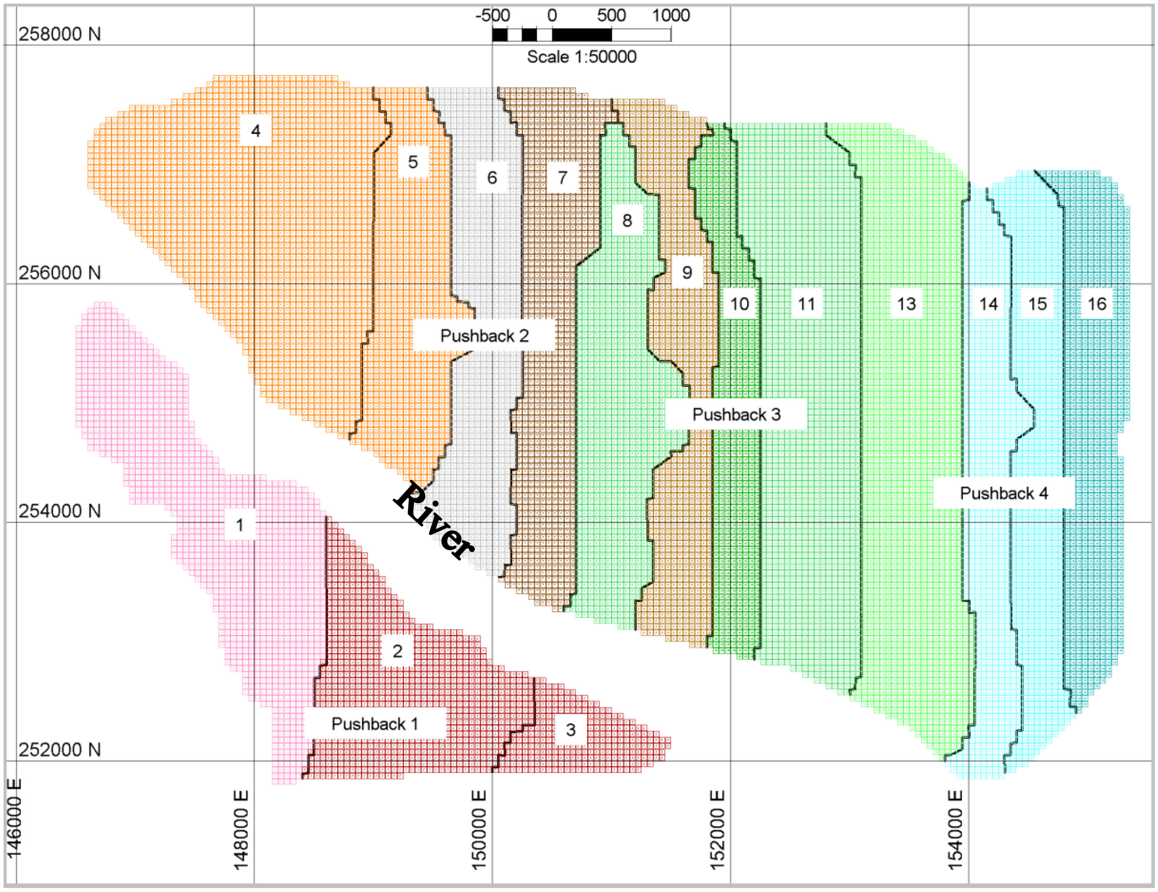

Fig. 4. Mining sequence at level 320m for all pushbacks with a west-east mining direction.

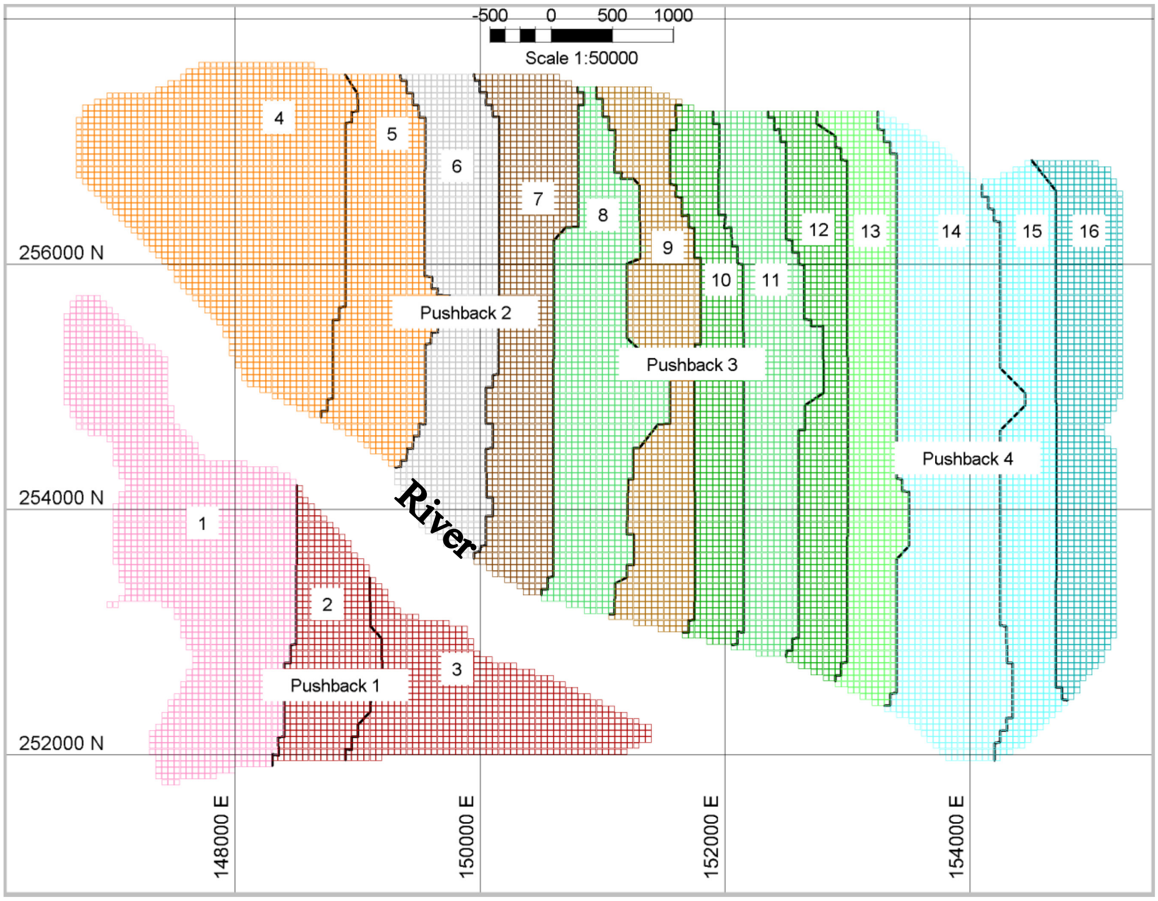

Fig. 5. Mining sequence at level 305m for all pushbacks with a west-east mining direction.

Fig. 6 shows uniform mining and processing schedules that ensures efficient utilization of mining fleet and processing plant capacity throughout the mine life. The schedule provides the quality and quantity of dyke material needed to build the dykes of the external tailings facility and in-pit tailings cells in a timely manner and at a minimum cost. Pre-stripping of pushback 1 and 2 starts in the first and fourth years, resulting in less ore being mined. Subsequently, uniform ore feed is provided at the required processing plant capacity throughout the mine life. The dyke material mined is sent to the scheduled dyke construction destination. Table 3 shows the total material mined, ore, OB and IB dyke material tonnage mined and TCS dyke material tonnage generated from the processing plant. The schedules give the planner good control over dyke material and provides a robust platform for effective dyke construction planning and tailings storage management.

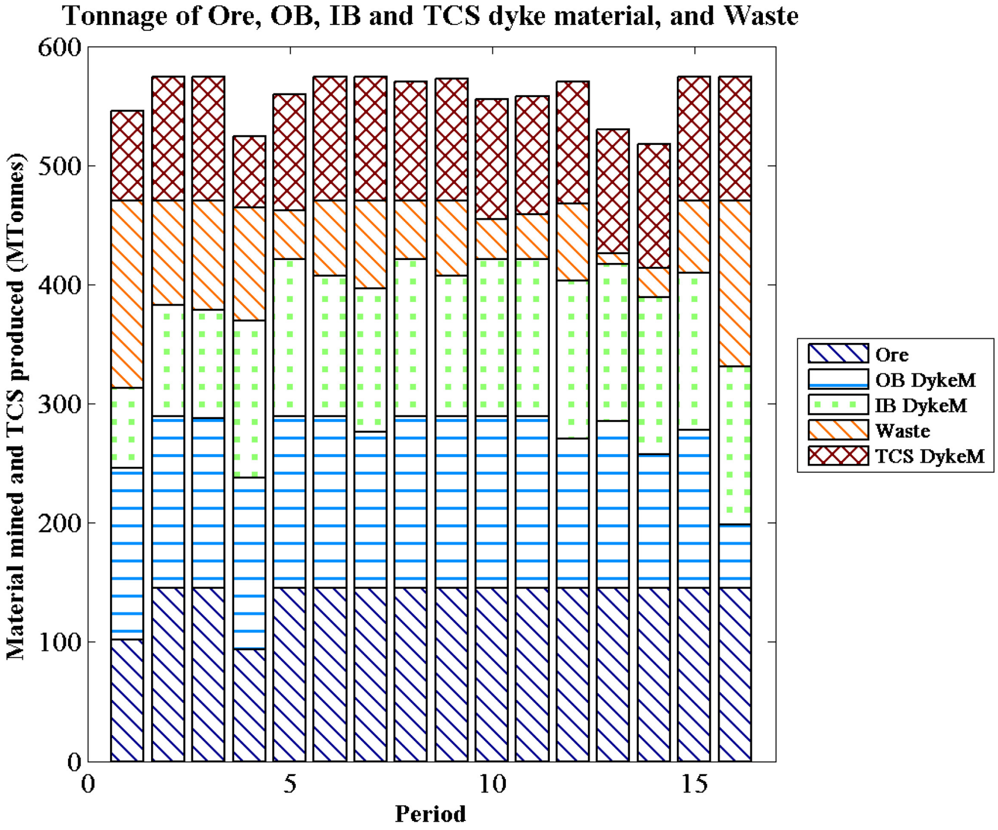

Fig. 6. Schedules for ore, OB, IB, and TCS dyke material, and waste tonnages.

Table 3: Summary of production scheduling results.

Production scheduling results	Tonnage of rock (Mt)	Ore tonnage (Mt)	OB dyke material tonnage (Mt)	IB dyke material tonnage (Mt)	TCS dyke material tonnage (Mt)	Average IB dyke material fines (w%)	
						Min	Max
Value	7377.4	2225.8	2135.4	1927.1	1570.3	9.0	50.0

The ore and dyke material quality is obtained by blending the run-of-mine material. The targeted processing plant head grade and IB dyke material grade that was set were successfully achieved in all periods and for all destinations. We targeted to reduce the periodic grade variability by setting tighter lower and upper grade bounds. The periodic grades in each pushback can be varied depending on the processing plant or dyke construction requirements whilst ensuring a feasible solution is obtained. Fig. 7 and Fig. 8 show the average ore bitumen grades and ore fines percent over the mine life. The IB dyke material fines percent range obtained for all destinations has been summarized in Table 3.

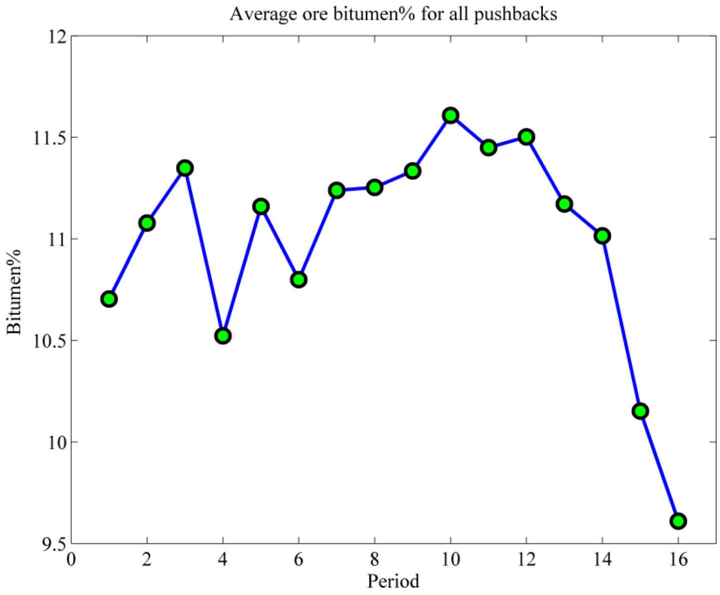

Fig. 7. Average ore bitumen grades in all periods.

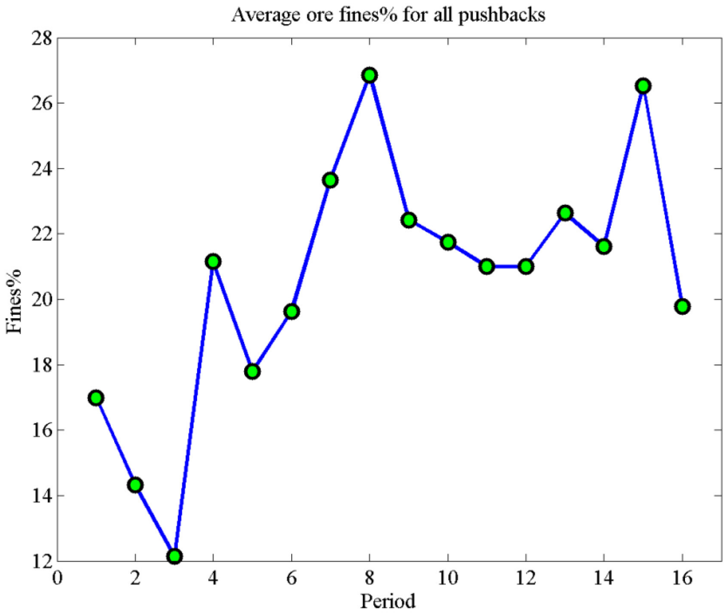

Fig. 8. Average ore fines percent in all periods.

5.2. Comparison

In implementing the efficient MILGP model with fewer non-zero decision variables, two optimization scenarios were executed to assess our model. Table 4 shows a summary of the results of the scenarios with different number of decision variables remaining before and after applying an initial schedule with a periodic tolerance. The results show less than 1% change in NPV and more than 99% change in solution time due to differences in solution space. Run 1 have a lower NPV due to the increase in dyke material tonnage and the associated waste material mined. This resulted in a higher tonnage mined in run 1. The results also show run 2 terminating at a branch closer to the optimal solution than run 1 as shown by the EPGAP. The ore tonnages sent to the processing plant in the two scenarios were the same. However there is a significant decrease in the CPU time as the number of decision variables are reduced using the initial schedule with a periodic tolerance. After applying a periodic tolerance of 2, the number of decision variables in run 1 reduced from 453,360 to 121,884 whilst the CPU time reduced from 243.79 to 0.84 hours which represents over 99% decrease in solution time for run 2. This technique can be used to overcome the long CPU time associated with solving mathematical models like the MILGP model thus bringing its daily use to the front due to its advantages. For a chosen application, the periodic tolerance required to be applied to an initial schedule from a heuristic could be established and used appropriately each time.

In general, it should be noted that the solution time for MILGP models do not depend only on the number of decision variables, but also on the tightness of the model which includes the data set used, the objective function and the constraints. The data used determine the coefficients in the objective function, and coefficients and bounds of the goals and constraints which have major impact on the solution time of an MILGP model.

Table 4: Summary of results before and after applying an initial schedule with a periodic tolerance

Run #	Periodic tolerance (yrs)	No. of decision variables	NPV (M$)	Tonnage mined (Mt)	Ore tonnage (Mt)	Dyke material tonnage (Mt)	EPGAP (%)	CPU time (hrs)
1	-	453360	26,791	7504.6	2225.8	5654.2	5.00	243.79
2	2	121884	26,987	7377.4	2225.8	5632.8	4.98	0.84

6. Conclusions

We have developed, implemented and verified a MILGP formulation which takes into account practical shovel movements by selecting mining-panels and mining-cuts that are comparable to the selective mining units of oil sands mining operations. Different techniques have been presented for implementing an efficient MILGP model that serves as a guide for optimization of OSLTPP and waste management. The model created value and a sustainable operation by generating a practical, smooth and uniform schedule for ore and dyke material. The schedule gives the planner good control over dyke material and provides a robust platform for effective dyke construction and waste disposal planning. The schedule ensures that the major factors affecting oil sands profitability and sustainability are taken care of within an optimization framework by maximizing NPV whilst creating timely tailings storage areas.

It has been shown that using an initial schedule with a periodic tolerance results in reduced number of decision variables to be solved for in the optimization problem. This variable reduction technique reduced the CPU time by over 99%, changing the long CPU times associated with solving mathematical models like the MILGP. In addition to its advantages, the reduced solution time will make the use of such mathematical models more appealing in solving mine planning problems. For a chosen application, the periodic tolerance required to be applied to an initial schedule from a heuristic could be established and used appropriately each time.

The total NPV generated including dyke construction cost for all pushbacks and destinations is $26,987M. The average bitumen grade for the scheduled ore was 11.0%. The average ore and IB dyke material fines percent ranges between 12.1 and 26.9, and 9 and 50, respectively. The total material mined was 7377.4Mt, which includes: 2225.8Mt of ore; 2135.4Mt of OB dyke material and 1927.1Mt of IB dyke material whilst 1570.3Mt of TCS dyke material was generated from the processing plant.

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8. Appendix

1.1. Notations

1.1.1 Sets

$\boldsymbol{K} = \{1, \ldots\ldots, K\}$	set of all the mining-cuts in the model.
$P = \{1, \ldots\ldots, P\}$	set of all the mining-panels in the model.
$\boldsymbol{J} = \{1, \ldots\ldots, J\}$	set of all the phases (push-backs) in the model.
$\boldsymbol{U} = \{1, \ldots\ldots, U\}$	set of all the possible destinations for materials in the model.
$A = \{1, \ldots\ldots, A\}$	set of all the possible mining locations (pits) in the model.
$B_p(V)$	for each mining-panel *p*, there is a set $B_p(V) \subset K$ defining the mining-cuts that belongs to the mining-panel *p*, where *V* is the total number of mining-cuts in the set $B_p(V)$.
$C_p(L)$	for each mining-panel *p*, there is a set $C_p(L) \subset P$ defining the immediate predecessor mining-panels above mining-panel *p* that must be extracted

	prior to extraction of mining-panel *p*, where L is the total number of mining-panels in the set $C_p(L)$.
$M_p(Z)$	for each mining-panel *p*, there is a set $M_p(Z) \subset P$ defining the immediate predecessor mining-panels in a specified horizontal mining direction that must be extracted prior to extraction of mining-panel *p* at the specified level, where Z is the total number of mining-panels in the set $M_p(Z)$.
$B_j(H)$	for each phase *j*, there is a set $B_j(H) \subset P$ defining the mining-panels within the immediate predecessor pit phases (push-backs) that must be extracted prior to extracting phase *j*, where H is an integer number representing the total number of mining-panels in the set $B_j(H)$.

1.1.2 Parameters

$\underline{g}^{u,t,e}$	the lower bound on the required average head grade of element *e* in period *t* at processing destination *u.*
$\overline{g}^{u,t,e}$	the upper bound on the required average head grade of element *e* in period *t* at processing destination *u*.
f_k^e	the average percent of fines in ore portion of mining-cut *k*.
$\underline{f}^{u,t,e}$	the lower bound on the required average fines percent of ore in period *t* at processing destination *u.*
$\overline{f}^{u,t,e}$	the upper bound on the required average fines percent of ore in period *t* at processing destination *u*.
f_k^d	the average percent of fines in interburden dyke material portion of mining-cut *k*.
$\underline{f}^{u,t,d}$	the lower bound on the required average fines percent of interburden dyke material in period *t* at dyke construction destination *u.*
$\overline{f}^{u,t,d}$	the upper bound on the required average fines percent of interburden dyke material in period *t* at dyke construction destination *u*.
o_p	the ore tonnage in mining-panel *p*.
d_p	the overburden dyke material tonnage in mining-panel *p*.
n_p	the interburden dyke material tonnage in mining-panel *p*.
w_p	the waste tonnage in mining-panel *p.*
$T_m^{a,t}$	the mining goal (tonnes) in period *t* at location a.
$d_1^{-,a,t}$	the negative deviation from the mining goal (tonnes) in period *t* at location a.
$T_{pr}^{u,t}$	the processing goal in period *t* at destination *u* (tonnes).
$d_2^{-,u,t}$	the negative deviation from the processing goal in period *t* at destination *u* (tonnes).

$T_d^{u,t}$	the overburden dyke material goal in period *t* at destination *u* (tonnes).
$d_3^{-,u,t}$	the negative deviation from the overburden dyke material goal in period *t* at destination *u* (tonnes).
$T_n^{u,t}$	the interburden dyke material goal in period *t* at destination *u* (tonnes).
$d_4^{-,u,t}$	the negative deviation from the interburden dyke material goal in period *t* at destination *u* (tonnes).
$T_l^{u,t}$	the tailings coarse sand dyke material goal in period *t* at destination *u* (tonnes).
$d_5^{-,u,t}$	the negative deviation from the tailings coarse sand dyke material goal in period *t* at destination *u* (tonnes).
P_1	the priority level associated with minimizing the deviations from the mining goal.
P_2	the priority level associated with minimizing the deviations from the processing goal.
P_3	the priority level associated with minimizing the deviations from the overburden dyke material goal.
P_4	the priority level associated with minimizing the deviations from the interburden dyke material goal.
P_5	the priority level associated with minimizing the deviations from the tailings coarse sand dyke material goal.
a_1	the penalty paid per tonne in deviating from the mining goal.
a_2	the penalty paid per tonne in deviating from the processing goal.
a_3	the penalty paid per tonne in deviating from the overburden dyke material goal.
a_4	the penalty paid per tonne in deviating from the interburden dyke material goal.
a_5	the penalty paid per tonne in deviating from the tailings coarse sand dyke material goal.

1.1.3 Decision variables

$x_k^{u,t} \in [0,1]$	a continuous variable representing the ore portion of mining-cut *k* to be extracted and processed at destination *u* in period *t*.
$z_k^{u,t} \in [0,1]$	a continuous variable representing the overburden dyke material portion of mining-cut *k* to be extracted and used for dyke construction at destination *u* in period *t*.
$c_k^{u,t} \in [0,1]$	a continuous variable representing the interburden dyke material portion of mining-cut *k* to be extracted and used for dyke construction at destination *u* in period *t*.
$s_k^{u,t} \in [0,1]$	a continuous variable representing the tailings coarse sand dyke material portion of mining-cut *k* to be extracted and used for dyke construction at destination *u* in period *t*.

$y_p^{a,t} \in [0,1]$	a continuous variable representing the portion of mining-panel p to be mined in period t from location a, which includes both ore, overburden and interburden dyke material and waste.
$b_p^t \in [0,1]$	a binary integer variable controlling the precedence of extraction of mining-panels. b_p^t is equal to one if the extraction of mining-panel p has started by or in period t, otherwise it is zero.

9. Appendix

MATLAB and TOMLAB/CPLEX documentation for oil sands production scheduling