Implementation of a Multilevel Mathematical Programming Formulation for Block Cave Production Planning

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Abstract

Production schedules that provide optimal operating strategies while meeting practical, technical, and environmental constraints are an inseparable part of mining operations. Relying only on manual planning methods or computer software based on heuristic algorithms will lead to mine schedules that are not the optimal global solution. Mathematical mine planning models have proven very effective in supporting decisions on sequencing the extraction of material in mines. The objective of this paper is to develop a practical optimization framework for caving operations' production scheduling. To overcome the size problem of mathematical programming models and to generate a robust practical near-optimal schedule, a multi-step method for long-term production scheduling of block caving is presented. A mixed integer linear programming (MILP) formulation is used for each step. The formulations are developed, implemented, and verified in the TOMLAB/CPLEX environment. The production scheduler aims to maximize the net present value of the mining operation while the mine planner has control over defined constraints. Application and comparison of the models for production scheduling using 298 drawpoints over 15 periods are presented.

1. Introduction

The mine production schedule defines the management investment strategy. An optimal plan in mining projects will result in cost reduction, increased equipment utilization, optimum recovery of marginal ores, steady production rates, and consistent product quality. Common strategic objectives in the industry are net present value maximization, cost minimization, and reserve maximization. Relying only on manual planning methods or computer software based on heuristic algorithms will lead to mine schedules that are not the optimal global solution (Pourrahimian et al., 2012a). Therefore, operations research has been used to assist in the decision-making process in the mining industry.

Exact algorithms are guaranteed to find an optimal solution for every instance of an optimization problem. The size of a mathematical formulation for a mine's production scheduling is so great that

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today's commercial software and hardware are not equipped to solve the optimization problem in a reasonable time or to find a solution to the problem at all.

The objective of this paper is to develop, implement, and verify a multi-step framework to address the long-term production scheduling problem of block cave mines. The objective of the theoretical framework is to maximize the net present value (NPV) of the mining operation, while controlling the planning parameters. The planning parameters considered in this study are: (i) mining capacity, (ii) draw rate, (iii) mining precedence, (iv) maximum number of active drawpoints, (v) number of new drawpoints in each period, (vi) continuous mining, and (vii) reserves.

To solve a real size problem, the following general workflow for a block cave operation is proposed:

1. Planner divides the mine into phases based on advancement directions. Fig 1a shows phases and their boundaries for west to east (WE) and east to west (EW) mining directions.
2. The draw columns within each phase are aggregated into practical scheduling units using modified hierarchical clustering algorithms based on an algorithm presented by Tabesh and Askari-Nasab (2011), see. Fig 2b.
3. The optimal life-of-mine multi-period block-cave production schedule is generated at the aggregated drawpoint level (cluster level). Fig 1c shows these aggregated drawpoints. This is the strategic yearly production schedule with the objective of NPV maximization. The strategic plan honors mining capacity and uniform feed to the processing plant (Pourrahimian et al., 2012a).
4. The optimal long-term block-cave production schedule is generated at the drawpoint level. Fig 1d shows the drawpoint level data. At this level, the model deals with draw columns. The output of the cluster level life-of-mine plan (step 3) is used to reduce the number of decision variables in this step. The time horizon for this model can vary as a subset of the life-of-mine to control the size of the MILP to be solved (Pourrahimian et al., 2012a).
5. The optimal schedule is generated at the drawpoint level including slices. Fig 1e shows that at the drawpoint and slice level, the model involves the drawpoints and slices. The output of the drawpoint level (step 4) is used to reduce the number of decision variables at this stage. The time horizon for this detailed 3D model could vary as a subset of the time horizons chosen in the previous step.

In this paper, multi-step MILP formulations are used at three different levels of resolution: (i) aggregated drawpoints (cluster level); (ii) drawpoint level; and (iii) drawpoint and slice level.

A drawpoint aggregation is required to reduce the number of variables, especially binary variables, in the MILP formulation to make it computationally tractable and to generate a practical mining schedule that follows a selective mining unit (Pourrahimian et al., 2012a). Drawpoints are aggregated using an agglomerative hierarchical algorithm that has been modified for the block cave based on the developed algorithm by Tabesh and Askari-Nasab (2011).

The optimization formulation is implemented in the TOMLAB/CPLEX (Holmstrom, 2011) environment. A scheduling case study with real mine data was carried out over 15 periods to verify the MILP models.

The next section of the paper covers relevant literature about the block cave production scheduling problem. Section three defines the problem, methodology and assumptions. Section four explains mixed integer linear programming formulations of the problem, while section five presents problem-solving techniques. Section six presents an example with results and discussion. Finally, the last section presents the conclusions, followed by a reference list.

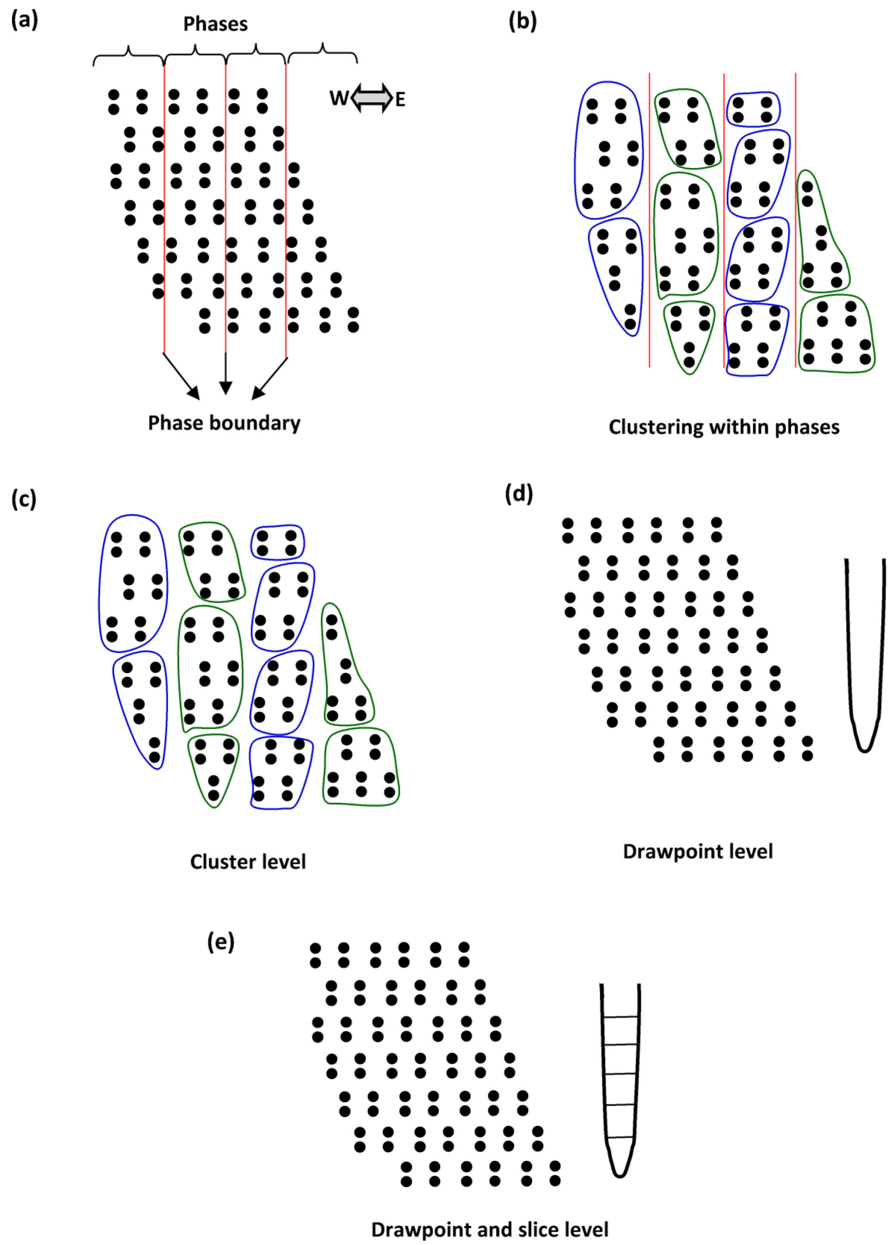


Fig 1. Proposed work flow for block cave production scheduling.

2. Literature review

While there have been some attempts to apply mathematical programming to production scheduling in underground mines, the majority of the scheduling publications to date have been concerned with open pit mining applications. This could be due to the complicated nature of underground mining (Kuchta et al., 2004; Topal, 2008).

Newman et al. (2010) presented a comprehensive review of operations research in mine planning. They summarized authors' attempts to develop methodologies to optimize production scheduling in underground and surface mines using different methods.

Despite the fact that using mathematical programming models with an exact solution methodology to solve the production scheduling optimization problem have proved to be robust, simulation and heuristics methods are generally used to generate a good solution in a reasonable time.

Mathematical programming models are capable of modeling real-world mining problems. Solving these models with exact solution methodologies, results in solutions within known limits of optimality. As the solution gets closer to optimality, it leads to production schedules that generate higher NPV than those obtained from heuristic optimization methods. This has led to extensive research on the application of mathematical programming to long-term production planning problems. Many authors have combined mathematical programming with simulation in an attempt to improve the scheduling process (Song, 1989; Chanda, 1990; Winkler, 1998). Several scheduling methods have been applied to underground mine scheduling. These include alternative scheduling methods, simulation, heuristics, linear programming, mixed integer linear programming, and quadratic programming (Topal et al., 2003; Rahal, 2008; Topal, 2008; Diering, 2012).

The manual draw charts are used to avoid early dilution entry at the beginning of block caving (Rubio, 2006). Over time, different methods have been used to present a good production schedule and optimized outline for block caving. Some of these methods include volumetric algorithms to simulate mixing along the draw column; different draw strategies; and dynamic programming and mathematical programming such as linear programming (LP), mixed integer linear programming (MILP), quadratic programming (QP), and mixed integer linear goal programming (MILGP). Also, different objective functions have been used, such as ones to minimize the fluctuation in the average grade drawn between shifts, to maximize draw control behavior, to maximize net present value (NPV), to maximize mine life, and to minimize the deviation from the ideal draw (Riddle, 1977; Heslop and Laubscher, 1981; Chanda, 1990; Diering, 2000; Guest et al., 2000; Rubio, 2002; Rahal et al., 2003; Diering, 2004; Rubio et al., 2004; Rubio and Diering, 2004; Rahal, 2008; Weintraub et al., 2008; Diering, 2012; Elkington et al., 2012; Pourrahimian et al., 2012a; Pourrahimian et al., 2012b).

Most of the methods presented above do not incorporate factors such as technical uncertainty and geotechnical constraints. On the other hand, because of the large number of integer and continuous decision variables, and the number of constraints needed to formulate a typical block cave long-term scheduling problem, this problem cannot be solved in a reasonable time using current versions of hardware and software. In addition, it should be mentioned that production scheduling optimization techniques are still not widely used in the mining industry, particularly for underground mining. There is a need to show how optimization methods create a practical production schedule. In the available block cave scheduling software, the mining sequence is controlled manually and, consequently, cannot yield an optimum solution for the problem.

3. Problem definition, methodology and assumptions

Scheduling a block cave mine is a matter of finding the goal that best represents the strategic planning vision subject to several mine design, geomechanical, operational, and environmental constraints. The production schedule is subject to a variety of physical and economic constraints. The constraints enforce the production target, draw rate, mining precedence, maximum number of active drawpoints, number of new drawpoints in each period, continuous mining, and complete extraction of reserves. The production schedule defines (1) the amount of the material to extract from each drawpoint in every period of production to achieve a given planning objective and (2) the number of new drawpoints to construct in every time period and their sequence to support a given production target.

Several assumptions are used in the proposed MILP formulations. The orebody is represented by a geological block model. The draw columns, which are vertical, are created based on the block model. The total column is divided into slices, which match the vertical spacing of the geological block model. Numerical data are used to represent orebody attributes in each slice, such as tonnage, density, grade of elements, elevation, percentage of dilution, and economic data. It is assumed that the physical layout of the production level is offset herringbone (Brown, 2003). The source model is assumed to be static with time. There is no selective mining, meaning that after applying the best height of draw (BHOD), all the material in the draw column must be extracted. A range for some of the constraints is used such as average grade and tonnage of production, development rate, and draw rate. A precedence role for clusters, drawpoints, and slices is used to control the horizontal and vertical mining advancement direction.

In order to create a slice file, all stages before scheduling are done using Gemcom GEMS and PCBC (GemcomSoftwareInternational, 2011). The slices file and the height that produces the best economic value for each draw column are imported to our developed software. Then, slices within the same draw column are grouped based on the BHOD. The total tonnage and draw column economic value are summation of these values for slices within the considered draw column. Afterwards, a block cave mine's production schedule can be optimized using the multi-step formulations presented in this paper. At cluster level and drawpoint level formulations, the precedence between clusters or drawpoints is controlled in a horizontal direction, but at drawpoint and slice level formulation, the precedence between drawpoints and slices is controlled in horizontal and vertical directions, respectively.

Clustering algorithms are usually performed by defining a measure of similarity or dissimilarity between the objects. To create the clusters, the draw columns are grouped into clusters based on similarities between their physical location, average grade, and tonnage. Similar to drawpoints, each cluster has coordinates representing the center of the cluster and its coordinates. The portion scheduled to be extracted from each cluster is assumed to be taken from all the drawpoints, based on the ratio of each draw column's tonnage in the cluster.

To solve the problem by a MILP formulation, both discrete decisions and continuous variables are used. To solve the problem at the cluster or drawpoint level, one continuous decision variable and two binary integer variables are employed per cluster or per drawpoint. When the problem is solved at the cluster level, the continuous decision variable indicates the portion of extraction from each cluster in each period. Two binary integer variables control the number of active clusters, precedence of extraction between clusters, the opening and closing time of each cluster, the extraction rate from each cluster, and the number of new clusters that need to be constructed in each period. These variables perform similar roles for drawpoints when the problem is solved at the drawpoint level (Pourrahimian et al., 2012a). To solve the problem at the drawpoint and slice level, one continuous decision variable and one binary integer variable for slices and two binary integer variables for drawpoints are employed. The continuous decision variable indicates the portion of extraction from each slice in each period. The role of binary integer variables is similar to the former formulations with one difference: here, the precedence of extraction between slices must also be considered, and one of them controls this case.

For cluster level and drawpoint level formulations, binary variables are defined to identify at what period a given drawpoint is started and active. For the drawpoint and slice level formulation, instead of using binary variables to define explicitly at what period a drawpoint is started or is active, the binary variables specify whether a drawpoint is started by a certain period. Caccetta and Hill (2003) employed this approach in their formulation of the open pit mine scheduling problem. Instances using the "*by period*" formulations tend to be solved faster than those the *"at period"* formulations, because the resultant branch and bound tree is more balanced.

At the cluster level, the MILP formulation is implemented for different advancement directions that can be four cardinal and four inter-cardinal directions. Then, for the other levels, the problem is solved for the direction with the best-obtained NPV from the cluster level. The results of each level are used as the input into the next level of resolution to eliminate unnecessary variables from the model based on the obtained earliest starting period.

3.1. Clustering

In the MILP formulation, the size of the branch and cut tree becomes so large that insufficient memory remains to solve the LP sub-problems. The size of the branch and cut tree can actually be affected by the specific approach one takes in performing the branching and by the structure of each problem. So, there is no way to determine the size of the tree before solving the problem (Pourrahimian et al., 2012a).

Attempts have been made to overcome the curse of dimensionality, which causes difficulties for long-term production scheduling combinatorial optimization in open pit and underground mines (Weintraub et al., 2008; Askari-Nasab et al., 2011; Tabesh and Askari-Nasab, 2011). As a general strategy in the formulation presented here, the number of binary integer variables was reduced. For this purpose, the algorithm by Tabesh and Askari-Nasab (2011) was modified for application in block cave mining.

The algorithm aggregates draw columns into selective mining units based on a similarity index that is defined based on the draw columns' tonnage, average grade, and physical location. The planner has to divide the mine into phases based on advancement directions (see Fig 1a). Clustering is done within each phase. This means two drawpoints that are located within two different phases cannot be within the same cluster. The general procedure of the algorithm is as follows:

(1) Define the maximum number of required clusters and the maximum number of allowed draw columns within each cluster.
(2) Each draw column is considered as a cluster, so that if there are D draw columns, then there are D clusters. The similarities between clusters are the same as the similarities between the objects they contain.
(3) Similarity values are calculated.
(4) The most similar pair of clusters is merged into a single cluster.
(5) Similarity between the new cluster and the rest of the clusters is calculated.
(6) Steps 2 and 3 are repeated until the maximum number of clusters is reached or there is no pair of clusters to merge because of the maximum number of allowed draw columns within each cluster.

Similarity value between draw column i and j is defined as in Eq.(1)

$$S_{ij} = \frac{1}{D_{ij}^{W_D} \times G_{ij}^{W_G} \times T_{ij}^{W_T}} \qquad (1)$$

Where Dij represents the normalized distance value between draw columns i and j, Gij represents the normalized grade difference between draw columns i and j, and Tij represents the normalized tonnage difference between draw columns i and j. W_D, W_G, and W_T are weighting factor for distance, grade, and tonnage, respectively.

A mine with D draw columns has a D by D distance matrix. The matrix is normalized by dividing all of its elements by the maximum value. The calculated normalized distance factor is then powered to W_D. The same approach is taken for grade and tonnage differences. To avoid infinite numbers in similarity indices, the grade and tonnage differences values are considered equal to a very small number wherever these values for two draw columns are the same. The mine planner defines the weights.

3.2. Mining precedence

First, according to the advancement direction, the precedence between clusters is defined. Then the precedence between drawpoints of the mine is defined based on the obtained precedence for clusters and the precedence between drawpoints within each cluster.

According to the advancement direction, for each cluster cl there is a set S^{cl} which defines the predecessor clusters among adjacent clusters that must be started before cluster cl is extracted. Based on the search direction, eight different predecessor data sets can be defined for each cluster. The set S^{cl} is created in three steps. In step one, the boundary drawpoints of the considered cluster are determined. These boundary drawpoints are located behind an imaginary line perpendicular to the desired advancement direction at the cluster's center point. In step two, all clusters that have at least one adjacent drawpoint with the boundary drawpoints are determined. Afterwards, clusters whose center point is behind the center point of the considered cluster are defined as members of set S^{cl} in the considered advancement direction out of selected clusters in the previous step (Pourrahimian et al., 2012a).

Fig 2a illustrates the above-mentioned method. For advancement from south to north, the boundary drawpoints for cluster 25 are d17, d24, d31, and d37. Only drawpoints d24, d31, and d37 have adjacent drawpoints (d30, d36, and d43) behind an imaginary line perpendicular to the SN direction of advancement. Drawpoints d30 and d36 belong to cluster 23 and drawpoint d43 belongs to cluster 26, so these two clusters can be a member of set S^{25}. Since the center point of cluster 23 is behind the center point of cluster 25 in the considered direction, it is only selected as a member of set S^{25}.

According to the advancement direction, for each drawpoint d there is a set S^{d}, which defines the predecessor drawpoints that must be started before drawpoint d is extracted. Based on the search direction, eight different predecessor data sets can be defined for each drawpoint. The members of set S^{d} in each direction are determined based on clusters and drawpoints within clusters. Each drawpoint d belongs to a cluster that has its predecessor clusters. Therefore, members of the predecessor clusters are members of set S^{d}. The other members of set S^{d} are defined according to the advancement direction and from among adjacent drawpoints located in the same cluster as drawpoint d.

For this purpose, a line perpendicular to the desired advancement direction at the location of the considered drawpoint is imagined. All located adjacent drawpoints behind the imaginary line that are in the same cluster as drawpoint d are defined as members of set S^{d} in the considered advancement direction. For instance, Fig 2b shows that drawpoint d30 belongs to cluster CL32. So, all drawpoints within clusters CL20 and CL15 belong to set S^{30}. On the other hand, among adjacent drawpoints for drawpoint d30 within cluster CL32, the extraction of drawpoints d37, d43, and d44 has to be started prior to drawpoint d30. Therefore, these drawpoints also belong to set S^{30}.

4. Mixed integer linear programming model for block cave production scheduling

4.1. Cluster level and drawpoint level

The mixed integer linear programming models for block cave production scheduling optimization are explained in this section. The models for the cluster and drawpoint levels are described briefly, but a more detailed explanation can be found in Pourrahimian et al. (2012a).

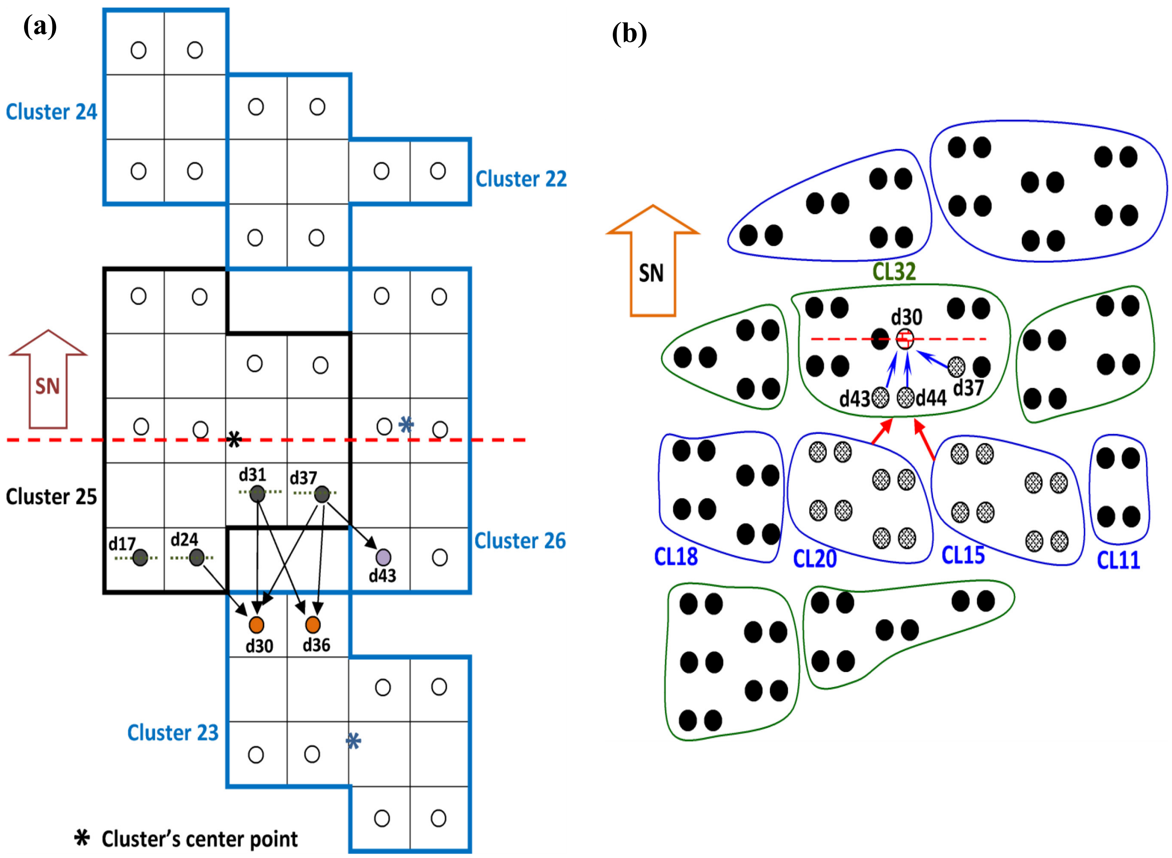


Fig 2. Determination method of members for set S^{cl} and S^{d} in different directions.

The notations used in the formulations have been classified as sets, indices, subscripts, superscripts, parameters, and decision variables. Continuous decision variables $U_{d,t} \in [0,1]$ and $U_{cl,t} \in [0,1]$ are used to determine the portion of the draw column associated with drawpoint d to be extracted in period t and the portion of the cluster cl to be extracted in period t, respectively. Since constraints such as precedence, maximum number of active drawpoints/clusters, number of new drawpoints/clusters, and continuous extraction cannot only be satisfied using these variables, another set of binary variables is defined.

$A_{d,t} \in \{0,1\}$ is a binary integer decision variable equal to one if drawpoint d is active in period t, otherwise it is zero. $Z_{d,t} \in \{0,1\}$ is a binary integer variable controlling the precedence of the extraction of drawpoints. It controls the sequence of opening. It is equal to one if the extraction from drawpoint d is started in period t, otherwise it is zero. $A_{cl,t} \in \{0,1\}$ and $Z_{cl,t} \in \{0,1\}$ play the same role for clusters. S^{d} is a set defining the predecessor drawpoints that must be started prior to extraction of drawpoint d. Based on the advancement direction, extraction from drawpoints belonging to the relevant set S^{d} must be started prior to extraction of drawpoint d. S^{cl} is a set defining the predecessor clusters that must be started before cluster cl is extracted. Table 1 summarizes these formulations. The objective function of the MILP formulations is to maximize the net present value of the mining operation. The objective functions are composed of the draw column economic value (DEV)/cluster economic value (CLEV), discount rate, and a continuous decision variable.

Table 1. MILP formulations for cluster level and drawpoint level (Pourrahimian et al., 2012a)

Cluster Level	Drawpoint Level
Maximize $\sum_{t=1}^{T}\sum_{cl=1}^{N_{cl}}\left[\frac{CLEV_{cl}}{(1+i)^{t}}\right]U_{cl,t}$	Maximize $\sum_{t=1}^{T}\sum_{d=1}^{N_{d}}\left[\frac{DEV_{d}}{(1+i)^{t}}\right]U_{d,t}$
$\underline{M_t} \le \sum_{cl=1}^{N_{cl}} U_{cl,t} \times (Ton_{cl}) \le \overline{M_t}$ $\forall t \in \{1,...,T\}, \quad cl \in \{1,...,N_{cl}\}$	$\underline{M_t} \le \sum_{d=1}^{N_{d}} U_{d,t} \times (Ton_{d}) \le \overline{M_t}$ $\forall t \in \{1,...,T\}, \quad d \in \{1,...,N_{d}\}$
$A_{cl,t} \le L.U_{cl,t}, \; U_{cl,t} \le A_{cl,t}, \; \sum_{cl=1}^{N_{cl}} A_{cl,t} \le N_{Acl,t}$ $\forall t \in \{1,...,T\}, \quad cl \in \{1,...,N_{cl}\}$	$A_{d,t} \le L.U_{d,t}, \; U_{d,t} \le A_{d,t}, \; \sum_{d=1}^{N_{d}} A_{d,t} \le N_{Ad,t}$ $\forall t \in \{1,...,T\}, \quad d \in \{1,...,N_{d}\}$
$L = \left(\frac{\max\{Ton_{cl}\}}{\min\{\text{number of drawpoints within a cluster}\} \times \text{minimum draw rate}}\right)$	$L = \frac{\max\{Ton_{d}\}}{\text{minimum draw rate}}$
$Z_{cl,t} - \sum_{j=1}^{t} U_{k,j} \le 1-\varepsilon \; \forall cl \in \{1,...,N_{cl}\}, \quad t \in \{1,...,T\}, \quad k \in S^{cl}$ $\varepsilon = \left(\frac{\min\{\text{number of drawpoints within a cluster}\} \times \text{minimum draw rate}}{\max\{Ton_{cl}\}}\right)$	$Z_{d,t} - \sum_{j=1}^{t} U_{k,j} \le 1-\left(\frac{\text{minimum draw rate}}{\max\{Ton_{d}\}}\right)$ $\forall d \in \{1,...,N_{d}\}, \quad t \in \{1,...,T\}, \quad k \in S^{d}$
$\sum_{t=1}^{T} Z_{cl,t} = 1, \; \forall cl \in \{1,...,N_{cl}\}, \quad t \in \{1,...,T\}$	$\sum_{t=1}^{T} Z_{d,t} = 1, \; \forall d \in \{1,...,N_{d}\}, \quad t \in \{1,...,T\}$
$A_{cl,t} - A_{cl,(t-1)} \le Z_{cl,t}, \; \forall cl \in \{1,...,N_{cl}\}, \quad t \in \{2,...,T\}$	$A_{d,t} - A_{d,(t-1)} \le Z_{d,t}, \; \forall d \in \{1,...,N_{d}\}, \quad t \in \{2,...,T\}$
$A_{cl,1} - Z_{cl,1} \le 0.5, \; \forall cl \in \{1,...,N_{cl}\}$	$A_{d,1} - Z_{d,1} \le 0.5, \; \forall d \in \{1,...,N_{d}\}$
$A_{cl,t} \times (NDP_{cl} \times \underline{DR}_{k,t}) \le U_{cl,t} \times (Ton_{cl}) \le (NDP_{cl} \times \overline{DR}_{k,t})$ $\forall cl \in \{1,...,N_{cl}\}, \quad t \in \{1,...,T\}, \quad k \in S^{cl}$	$A_{d,t} \times \underline{DR}_{d,t} \le U_{d,t} \times (Ton_{d}) \le \overline{DR}_{d,t}$ $\forall t \in \{1,...,T\}, \quad d \in \{1,...,N_{d}\}$
$\underline{N_{Ncl,t}} \le \sum_{cl=1}^{N_{cl}} Z_{cl,t} \le \overline{N_{Ncl,t}}$ $\forall t \in \{2,...,T\}, \quad cl \in \{1,...,N_{cl}\}$	$\underline{N_{Nd,t}} \le \sum_{d=1}^{N_{d}} Z_{d,t} \le \overline{N_{Nd,t}}$ $\forall t \in \{2,...,T\}, \quad d \in \{1,...,N_{d}\}$
$\sum_{cl=1}^{N_{cl}} Z_{cl,1} \le N_{Acl,1}, \; \forall cl \in \{1,...,N_{cl}\}$	$\sum_{d=1}^{N_{d}} Z_{d,1} \le N_{Ad,1}, \; \forall d \in \{1,...,N_{d}\}$
$\sum_{t=1}^{T} U_{cl,t} = 1, \; \forall t \in \{1,...,T\}, \quad cl \in \{1,...,N_{cl}\}$	$\sum_{t=1}^{T} U_{d,t} = 1, \; \forall t \in \{1,...,T\}, \quad d \in \{1,...,N_{d}\}$

4.2. Drawpoint and slice level

This model is an extension of the formulated model by Pourrahimian et al. (2012b). The notation of sets, indices, and decision variables are as follows:

Sets:

S^d	For each drawpoint, d, there is a set S^d defining the predecessor drawpoints that must be started prior to extraction of drawpoint d.
S^{ds}	For each drawpoint, d, there is a set S^{ds} defining the slices in the draw column associated with drawpoint d.
S^s	For each slice, s, there is a set S^s defining the predecessor slice that must be extracted prior to the extraction of slice s.
S^{dls}	For each drawpoint, d, there is a set S^{dls} defining the lowest slice within the draw column associated with drawpoint d.

Indices:

$t \in \{1,....,T\}$	Index for scheduling periods.
$d \in \{1,...,D\}$	Index for drawpoints.
$s \in \{1,...,S\}$	Index for slices.
$e \in \{1,...,E\}$	Index for elements of interest in each slice.
m	Index for a slice belonging to one of the sets S^s or S^{dls}
j	Index for a drawpoint belonging to one of the sets S^d or S^{ds}

Decision variables:

$X_{st} \in [0,1]$	Continuous variable, representing the portion of slice s to be extracted in period t
$E_{dt} \in \{0,1\}$	Binary integer variable controlling the starting period of drawpoints and the precedence of extraction of drawpoints. E_{dt} is equal to one if the extraction of drawpoint d has started by or in period t; otherwise it is zero.
$C_{dt} \in \{0,1\}$	Binary integer variable controlling the closing period of drawpoints. C_{dt} is equal to one if the extraction of drawpoint d has finished by or in period t; otherwise it is zero.
$B_{st} \in \{0,1\}$	Binary integer variable controlling the precedence of the extraction of slices. It is equal to one if the extraction of slice s has started by or in period t; otherwise it is zero.

4.2.1. Objective function:

The objective function of the drawpoint and slice model is to maximize the net present value of extracting material from the drawpoints. The profit from mining a drawpoint depends on the value of the slices and the costs incurred in mining. The objective function, Eq.(2), is composed of the slice economic value (SEV), discount rate, and a continuous decision variable that indicates the portion of a slice, which is extracted in each period. The most profitable slices will be chosen to be part of the production in order to maximize the NPV.

$$\text{Maximize} \quad \sum_{t=1}^{T}\sum_{s=1}^{S}\left(\frac{SEV_s}{(1+i)^t}\right)\times X_{st} \tag{2}$$

Where

- T is the maximum number of scheduling periods,
- S is the maximum number of drawpoints in the model,
- SEV_s is the economic value of the slice s,
- i is the discount rate,

The objective function is subject to the following constraints:

4.2.2. Mining capacity:

$$\underline{M_t} \le \sum_{s=1}^{S}(Ton_s)\times X_{st} \le \overline{M_t} \qquad \forall t \in \{1,...,T\} \tag{3}$$

Where

- $\overline{M_t}$ is the upper limit of mining capacity in period t,
- $\underline{M_t}$ is the lower limit of mining capacity in period t,
- Ton_s is the total tonnage of material within slice s,

These constraints force a mining rate between the desired and maximum mining capacity available. In other words, they ensure that the total tonnage of material extracted from slices in each period is within the acceptable range that allows flexibility for potential operational variations. The constraints are controlled by the continuous variable X_{st}. There is one constraint per period.

4.2.3. Grade blending:

$$\underline{G_{et}} \le \frac{\sum_{s=1}^{S} G_{es}\times(Ton_s)\times X_{st}}{\sum_{s=1}^{S}(Ton_s)\times X_{st}} \le \overline{G_{et}} \qquad \forall t \in \{1,...,T\}, \quad e \in \{1,...,E\} \tag{4}$$

Where

- G_{es} is the average grade of element e in the ore portion of slice s.
- $\overline{G_{et}}$ is the upper limit of the acceptable average head grade of element e in period t,
- $\underline{G_{et}}$ is the lower limit of the acceptable average head grade of element e in period t,

These constraints force the mining system to achieve the desired grade. The average grade of the element of interest has to be greater than or equal to a certain value, $\underline{G_{et}}$, and less than or equal to a certain value, $\overline{G_{et}}$, for each period t.

4.2.4. Maximum number of active drawpoints and continuous extraction from drawpoint:

$$X_{st} - E_{dt} \le 0 \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,D\}, \quad s \in S^{dls} \tag{5}$$

$$E_{dt} - E_{d(t+1)} \le 0 \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,D\} \tag{6}$$

$$E_{dt} - C_{dt} \le L \times \sum X_{mt} \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,D\}, \ m \in S^{ds}$$
$$L > \left(\frac{\max\{Ton_d\}}{\text{minimum draw rate}} \right) \tag{7}$$

$$C_{dt} - C_{d(t+1)} \le 0 \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,D\} \tag{8}$$

$$\sum_{d=1}^{D} (E_{dt} - C_{dt}) \le N_{at} \qquad \forall t \in \{1,...,T\} \tag{9}$$

Where

- N_{at} is the maximum number of active drawpoints in period t,
- L is the big enough number which is determined based on the minimum acceptable draw rate.

Fig 3 shows three different situations of a drawpoint during the mine life. In each period, the number of active drawpoints must not exceed the allowable number. This constraint controls the maximum number of active drawpoints at any given period of the schedule. Each draw column is divided into slices. The lowest slice controls the starting period of extraction from each drawpoint. This means that the extraction from the draw column associated with drawpoint d is started by the extraction from the relevant lowest slice. When the extraction of the last portion of a slice is finished in period t, extraction of the slice above can start in the period t or $t+1$. In other words, the extraction of a slice can start if the slice below is totally extracted. If the extraction of a slice is not started after finishing the extraction of the slice below in period t or $t+1$, the relevant drawpoint must be closed.

The concept is applied using Eqs. (5), (6), (7), and (8). Eq. (7) ensures that when drawpoint d is open, at least a portion of one of the slices within the draw column associated with drawpoint d is extracted; otherwise the drawpoint must be closed. This means extraction must be continuous; otherwise the drawpoint will be closed. Eqs. (6) and (8) ensure that when variables E_{dt} and C_{dt} change to one, they remain one until the end of the mine life. This recognizes the periods when the drawpoint is active. Fig 4 shows the relationship between opening time, closing time, and active time. Eq. (9) controls the maximum number of active drawpoints in each period. N_{at} should be given as an input to the algorithm.

Mining precedence:

$$E_{jt} - E_{dt} \le 0 \qquad \forall d \in \{1,...,D\}, \quad t \in \{1,...,T\}, \ j \in S^{d} \tag{10}$$

$$B_{st} - \sum_{i=1}^{t} X_{mi} \le 0 \qquad \forall s \in \{1,...,S\}, \ t \in \{1,...,T\}, \quad m \in S^{s} \tag{11}$$

$$\sum_{i=1}^{t} X_{si} - B_{st} \le 0 \qquad \forall s \in \{1,...,S\}, \quad t \in \{1,...,T\} \tag{12}$$

$$B_{st} - B_{s(t+1)} \le 0 \qquad \forall s \in \{1,...,S\}, \quad t \in \{1,...,T\} \tag{13}$$

$$\frac{\sum X_{mt}}{Ns_d} \leq E_{dt} - C_{dt} \qquad \forall d \in \{1,...,D\}, \quad t \in \{1,...,T\}, m \in S^{ds} \tag{14}$$

Where

- Ns_d is the number of slices within the draw column associated with drawpoint d,

Mining precedence must be controlled in horizontal and vertical directions. The extraction precedence of drawpoints controls the horizontal mining precedence, and the extraction precedence of slices controls the vertical mining precedence. Eq. (10) ensures that all drawpoints belonging to the relevant set, S^d, are started prior to the extraction of drawpoint d. This set is defined based on the selected mining advancement direction. This set can be empty, which means the considered drawpoint can be extracted in any time period in the schedule. Eq. (10) also ensures that only the set of the immediate predecessor drawpoints needs to start prior to starting the drawpoint under consideration.

Extraction of the slice, s, can be started if the slice below it has been extracted totally. For each slice within the draw column except the lowest, there is a set S^s defining the predecessor slice that must be extracted prior to the extraction of slice s. The extraction precedence of the slice within each draw column is controlled by Eqs. (11), (12) and (13). Eqs. (11) and (12) ensure that the extraction of the slice belonging to the relevant set, S^s, has been finished prior to the extraction of slice s. Eq. (14) ensures that slice s is extracted when the relevant drawpoint is active.

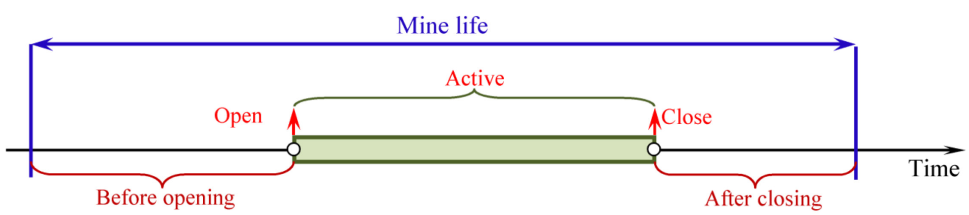


Fig 3. Changes of drawpoint situation during the mine life (Pourrahimian and Askari-Nasab, 2011)

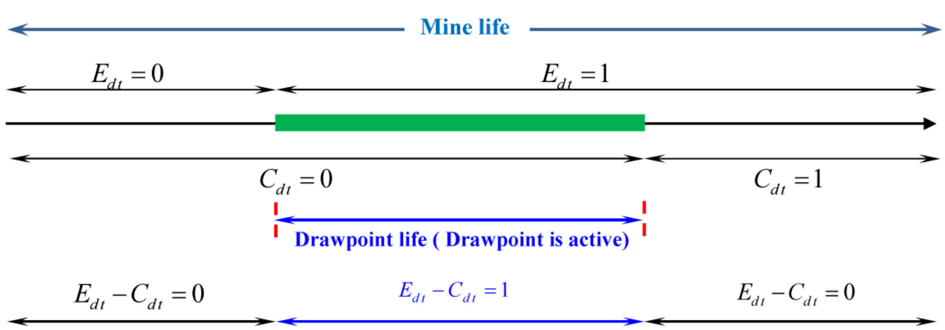


Fig 4. Values and relationships between binary integer variables based on different situations of the drawpoint during the mine life (Pourrahimian and Askari-Nasab, 2011).

4.2.5. Number of new drawpoints:

$$\underline{N_{nt}} \leq \sum_{d=1}^{D} E_{dt} - \sum_{d=1}^{D} E_{d(t-1)} \leq \overline{N_{nt}} \qquad \forall t \in \{2,...,T\} \tag{15}$$

$$\sum_{d=1}^{D} E_{d1} \leq N_{a1} \tag{16}$$

Where

- $\overline{N_{nt}}$ is the upper limit of number of new drawpoints that must be opened in period t,
- $\underline{N_{nt}}$ is the lower limit of number of new drawpoints that must be opened in period t,

This constraint defines the maximum feasible number of drawpoints to be opened at any given time within the scheduled horizon. This constraint is usually based on the footprint geometry, the geotechnical behavior of the rock mass, and the existing infrastructure of the mine, which will typically define available mining faces.

The drawpoint opening is controlled by the variable E_{dt}, which takes a value of one from the opening period to the end of the mine life. From period two to the end of the mine life, the difference between the summation of opened drawpoints until and including period t, and the summation of opened drawpoints until and including previous period $t-1$, indicates the number of new drawpoints. Eq. (15) ensures that the number of new drawpoints that are open in each period except period one is within the acceptable range. Eq. (16) ensures that in period one, the number of new drawpoints is equal to the number of active drawpoints.

4.2.6. Reserves:

$$\sum_{t=1}^{T} X_{st} = 1 \qquad \forall s \in \{1,...,S\} \tag{17}$$

Eq. (17) ensures that the fractions of draw columns that are extracted over the scheduling periods are going to sum up to one, which means there is no selective mining for the slices, and thereby all the material in the draw column must be extracted.

4.2.7. Draw rate:

$$\left(E_{dt} - C_{dt}\right).\underline{DR_{dt}} \le \sum \left(Ton_m\right).X_{mt} \le \overline{DR_{dt}} \qquad \forall d \in \{1,...,D\}, t \in \{1,...,T\}, m \in S^{ds} \tag{18}$$

Where

- $\overline{DR_{dt}}$ is the maximum possible draw rate of drawpoint d in period t,
- $\underline{DR_{dt}}$ is the minimum possible draw rate of drawpoint d in period t,

This constraint controls the maximum and minimum rate of draw and is a function of fragmentation and caveability. This rate should be fast enough to avoid compaction and slow enough to avoid air gaps. The fragmentation process usually determines the maximum limit to the draw rate since time is required to achieve good fragmentation. However, sometimes the maximum rate may be dictated by the LHD productivity. Inequalities in Eq. (18) ensure that the draw rate from each drawpoint is within the desired range in each period. Eq. (18) imposes upper and lower bounds for the draw rate. When drawpoint d is not active, the variable $\left(E_{dt} - C_{dt}\right)$ is equal to zero and this relaxes the lower bound of the equation. A boundary limit for the draw rate is assumed. Another way to control the draw rate is using a production rate curve, in which production depends on the tonnes mined from a drawpoint. In this formulation, the production curve is managed the relationship between portions of extraction from each drawpoint in different periods.

5. Solving the optimization problem

At the cluster level and drawpoint level formulations, binary variables are used to identify at what period a given drawpoint is started and is active, while in the drawpoint and slice level formulation, they are employed to specify whether a drawpoint is started by a certain period.

The proposed MILP models have been developed, implemented, and tested in the TOMLAB/CPLEX environment (Holmstrom, 2011). TOMLAB/CPLEX integrates the solver package CPLEX with the MATLAB environment (MathWorksInc, 2011). The algorithms are also coded in MATLAB. Exact solution methods are an approach commonly used to solve MILP problem. The branch and bound algorithm is the most common exact solution method used to solve MILP problems. Using a branch and bound algorithm to solve MILP problem formulations guarantees an optimal solution if the algorithm is run to completion. However, this may require excessively long computational time; instead the algorithm is often terminated and the gap is reported between the best integer objective and the objective of the best node remaining.

Variable reduction techniques are used to improve the solution time. These techniques endeavor to limit the search space by eliminating certain variables or by a priori setting the values of other variables. Clustering brings two advantages to the problem. First, it reduces the number of variables, especially binary variables in the MILP formulation to make it computationally tractable. The second advantage of clustering lies in generating a practical mining schedule. After solving the problem at the cluster level, the earliest period that each cluster can be reached and the cluster life if all the constraints are satisfied are known. The drawpoints of each cluster are known, so the earliest start time and the cluster life allow elimination from the drawpoint level model of any variables that would mine each drawpoint before its earliest start time. Two years flexibility for the earliest start time is assumed at the drawpoint level. For this purpose, if at the cluster level, the extraction of a drawpoint is started in period t with the cluster life of n, at the drawpoint level any variables that would mine this drawpoint before period $t-2$ and after $(t+n)+2$ are eliminated. The results of the drawpoint level are used at drawpoint and slice level to eliminate the unnecessary variables. The same concept is used to eliminate the variables related to the drawpoints. Some of the variables related to the slices are eliminated based on earliest extraction time of each slice as well. According to the maximum allowed draw rate, the earliest extraction time for each slice is defined. This number is added to the starting period of the drawpoint. For example, the draw column associated with drawpoint d50 has 42 slices. Based on the maximum allowable draw rate, this draw column is divided into eight slice groups, which are numbered from bottom to top. This means if extraction from drawpoint d50 is started in period t , only the first grouped slices can be extracted in period t; the earliest starting time for the second grouped slices is $t+1$ and so on. This concept allows elimination from the drawpoint and cluster level any variables that would mine each slice before its earliest start time.

6. Results and discussion

The performance of the proposed models was analyzed based on NPV, mining production, and practicality of the generated schedules. The models were tested on a Dell Precision T7500 computer at 2.7 GHz, with 24GB of RAM. The goal is to maximize the NPV at a discount rate of 12%, while assuring that all constraints are satisfied during the mine life. Two different methods were applied on a real data set. In the first one, the presented multi-step algorithm was applied and the results of each level were used to reduce the number of variables in the next level. In the second method, the results of the previous level were not used to reduce the number of variables at the drawpoint level and the drawpoint and slice level. In other words, these two steps were solved independently.

A real data set containing 298 drawpoints with the slice height of 10 meters is considered. The minimum and maximum numbers of slices within draw columns are 29 and 36, respectively. The initial slice file contains 9790 slices, of which 4251 are eliminated after applying the BHOD. The BHOD is limited to not less than 50 m. After applying this assumption, minimum and maximum heights of the draw column are 50 m and 290 m, respectively. The models are verified by numerical experiments on a real data set containing 298 drawpoints and 5539 slices over 15 periods in four different advancement directions. Fig 5 illustrates a plan view of the drawpoint configuration based on the relevant coordinates. The total tonnage of material to extract is almost 37 Mt. The tonnage from individual drawpoints varies between 28 and 233 kt. A capacity of 2.5 Mt/yr was considered as the upper bound on the mining capacity for all formulations. The maximum number of active clusters and drawpoints in each period is set to 15 and 135, respectively. The maximum number of new clusters that can be opened in each period at the cluster level is equal to 5. The maximum number of new drawpoints that can be opened in each period at the drawpoint level and drawpoint and slice level is 50. The lower bound and upper bound of the draw rate for drawpoints are set to 10 and 50 kt/yr for all the models.

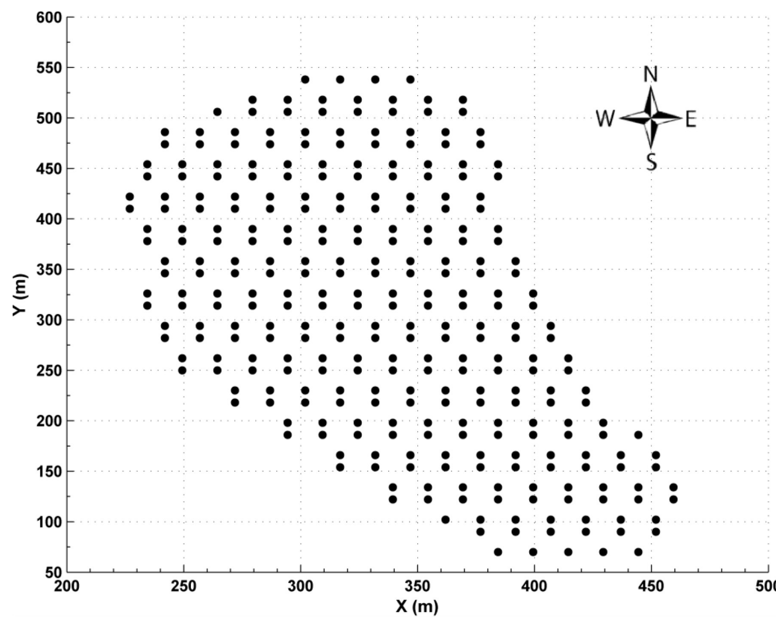
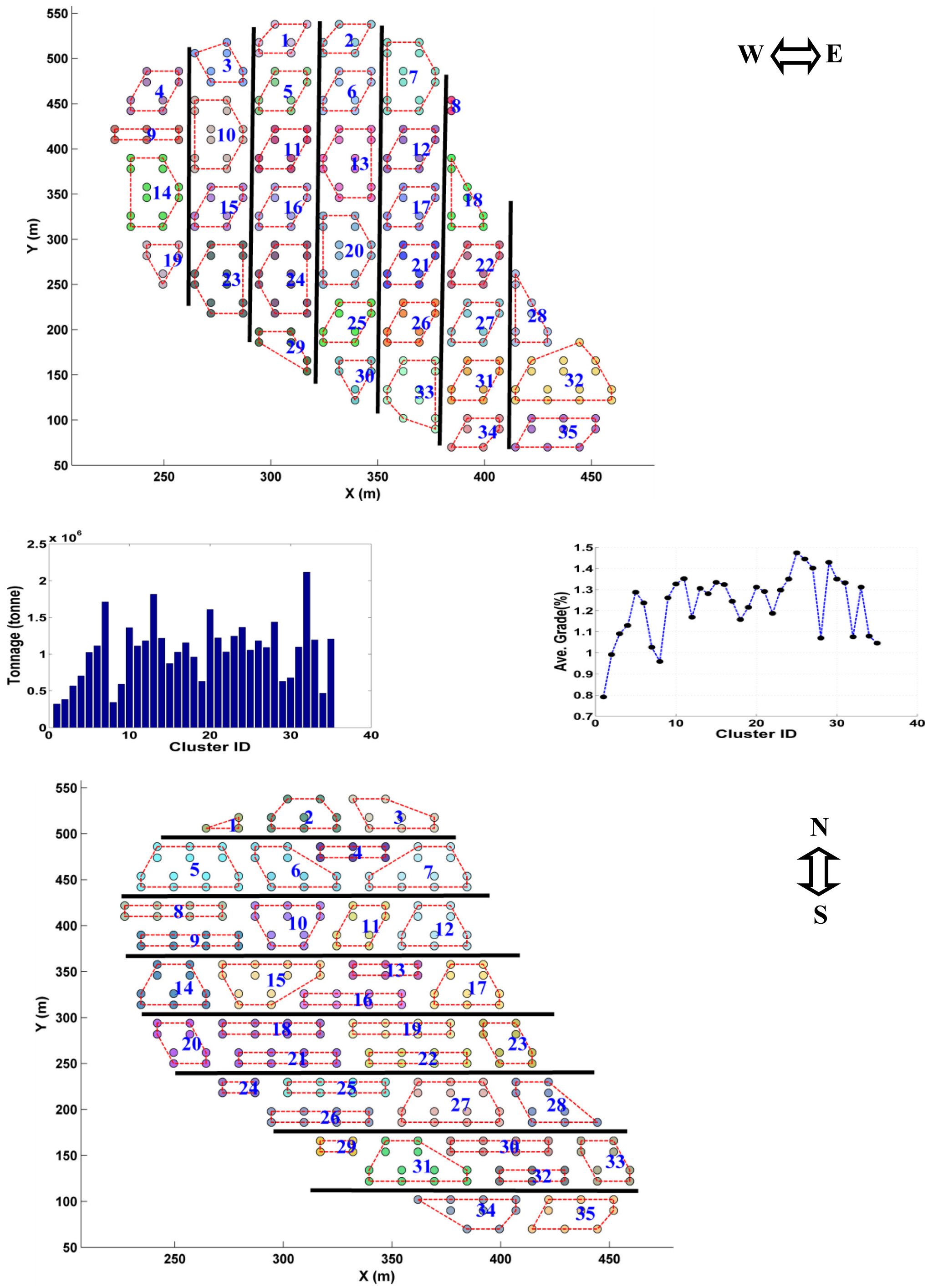
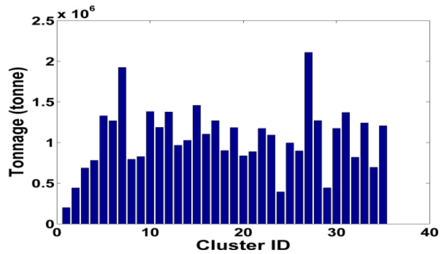


Fig 5. Plan view of 298 drawpoints configuration

To aggregate the draw columns, the user defines the advancement lines for each direction. Afterwards, clustering is done between lines for each direction. Fig 6 shows defined advancement lines and clustered drawpoints for WE/EW and NS/SN directions. The tonnage and average grade of each cluster are also shown. The total tonnage of material is calculated for each cluster based on the tonnage of draw columns within the cluster. The maximum number of clusters is set to 35. The weight factors of the tonnage, average grade, and distance between the draw columns are 0.1, 0.1, and 4. The maximum number of draw columns in each cluster cannot be more than 15.

Table 2 compares the number of decision variables for three different models. The problem was solved in four cardinal directions. For the cluster level, an EPGAP of 1% was set for optimization of all defined directions. This means the gap between the best integer objective and the feasible integer solution must be equal to or less than 1%. The results show that the maximum NPV is gained in the west to east direction. A comparison between the difference in percent from the maximum NPV shows that the difference in percent for east to west and north to south is more than the defined EPGAP. As a result, these two directions do not have the potential to be considered as mining directions. The difference in percent between the NPVs of west to east and south to north directions is less than the defined EPGAP, so the south to north direction can also be considered as a mining direction. Table 3 shows a summary of the details for each direction at the cluster level . Fig 7 compares the maximum number of active clusters, number of new clusters that need to be constructed in each period, and the cash flow between the west to east and south to north directions.





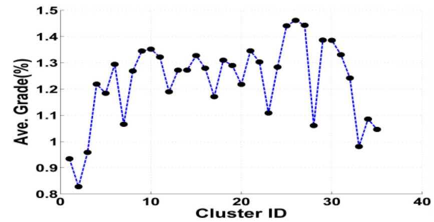


Fig 6. Clustered draw points and their tonnage and grade

Table 2. Number of variables for three different levels

Member description	Number of members	Variable		
		Total	Continuous	Binary
Cluster	35	1575	525	1050
Drawpoint	298	13410	4470	8940
Drawpoint and slice	298 / 5539	175110	83085	92025

Table 3. Numerical results for cluster level formulation

Direction	CPU time 4 CPUs @ 2.7 GHz	EPGAP (%)	Optimality GAP (%)	NPV ($M)	Difference from Max. (%)
west to east (WE)	00:00:2	1	0.36	48.25	0
east to west (EW)	01:16:56	1	1.00	46.87	2.9
north to south (NS)	00:28:15	1	0.98	46.02	4.6
south to north (SN)	00:00:3	1	0.99	47.93	0.66

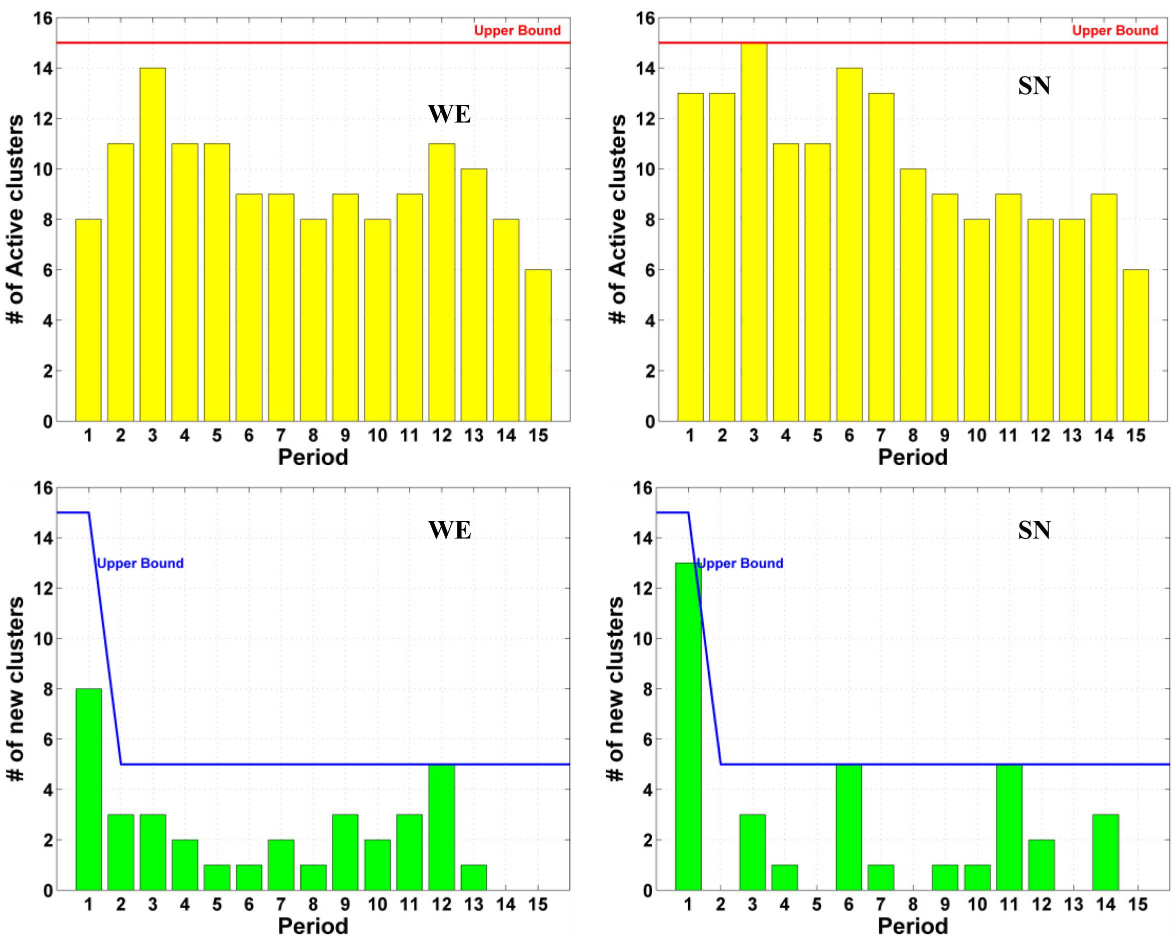
The formulation wants to maximize the NPV so it tries to keep mining capacity at the upper bound. This results in the same yearly production for both directions. But during the early years of the south to north direction, the number of active drawpoints is higher. This means more clusters and consequently more drawpoints are required. It is obvious that the number of new clusters that are opened in each period for the west to east direction is more uniform than the number opened in the south to north direction.

The cash flow for the first four years in the west to east direction is more than that in the south to north direction. However, the number of active clusters in the south to north direction is more than that in the west to east direction. Based on the information presented, the west to east direction is selected as the preferred mining direction and the problem for the two next levels is solved only for this direction.

To solve the problem at the drawpoint level, the obtained starting periods from the cluster level are used to eliminate variables. Fig 8 shows the starting period of drawpoints based on the cluster level solution. Two years flexibility is assumed for the earliest start time at the drawpoint level. The total variable number of 13410 is reduced to 8883, of which the 1095-variable from continuous variables and the 3432-variable from binary variables. The problem was solved in the west to east direction at the drawpoint level formulation. For this level, an EPGAP of 2% was set for optimization. The obtained NPV from the drawpoint level is $48.45M, with the optimality gap of 1.45%. Fig 9 shows the results obtained for the drawpoint level. Based on the drawpoint level formulation, there are always fewer than 120 drawpoints active based on the drawpoint level formulation. However, the maximum number of allowable active drawpoints is 135. The trend is for the number of active drawpoints to increase until period 11, after which it gradually decreases.

To solve the problem at the drawpoint and slice level in the west to east direction, an EPGAP of 5% was set for optimization. The lower bound and upper bound of the average grade were set to 0.7% and 1.6% in this model. For this level also, two years flexibility for the earliest start period of each drawpoint was considered in comparison with the drawpoint level.

Consequently, the total variable of 175110 is reduced to 125613, of which the 23712-variable from continuous variables and the 25785-variable from binary variables are eliminated. The resulting NPV of the drawpoint and slice level is $50.43M, with the optimality gap of 4.8%. The reason for the higher NPV for this level is the resolution of the level. In other words, when the problem is solved in the drawpoint and slice level, the method deals with the slices. The economic value of each slice is taken into account. Therefore, the model tries to mine slices with higher economic values and grades, earlier than other slices.



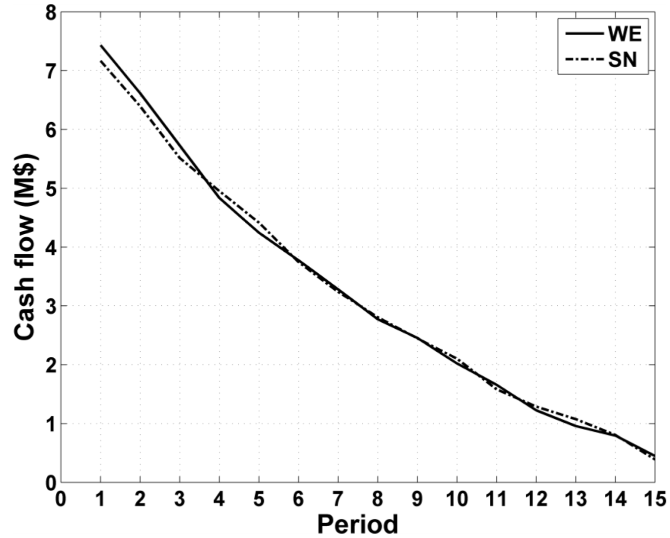


Fig 7. Comparison between the obtained results for two directions: west to east (left side) and south to north (right side)

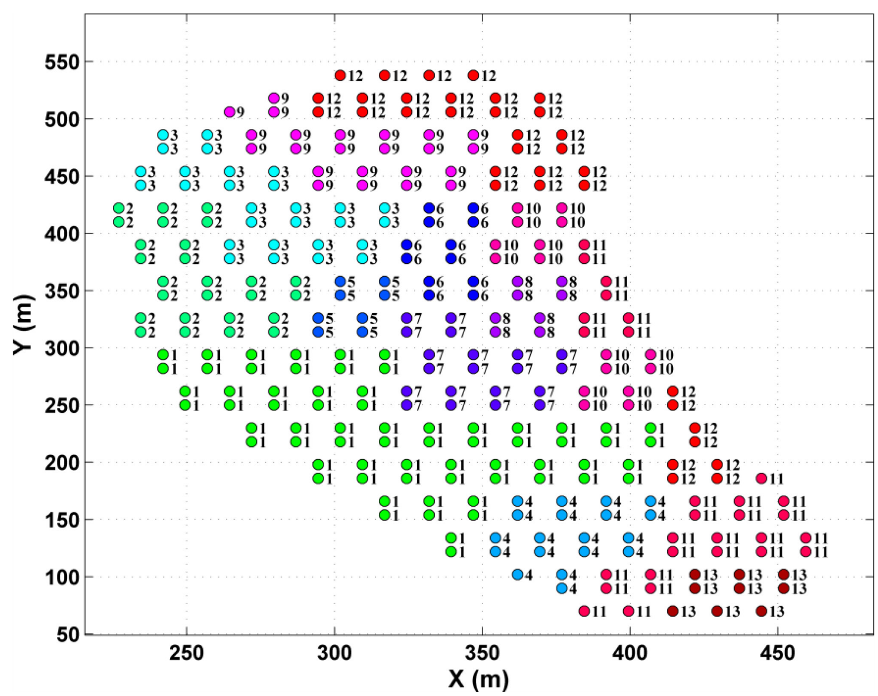
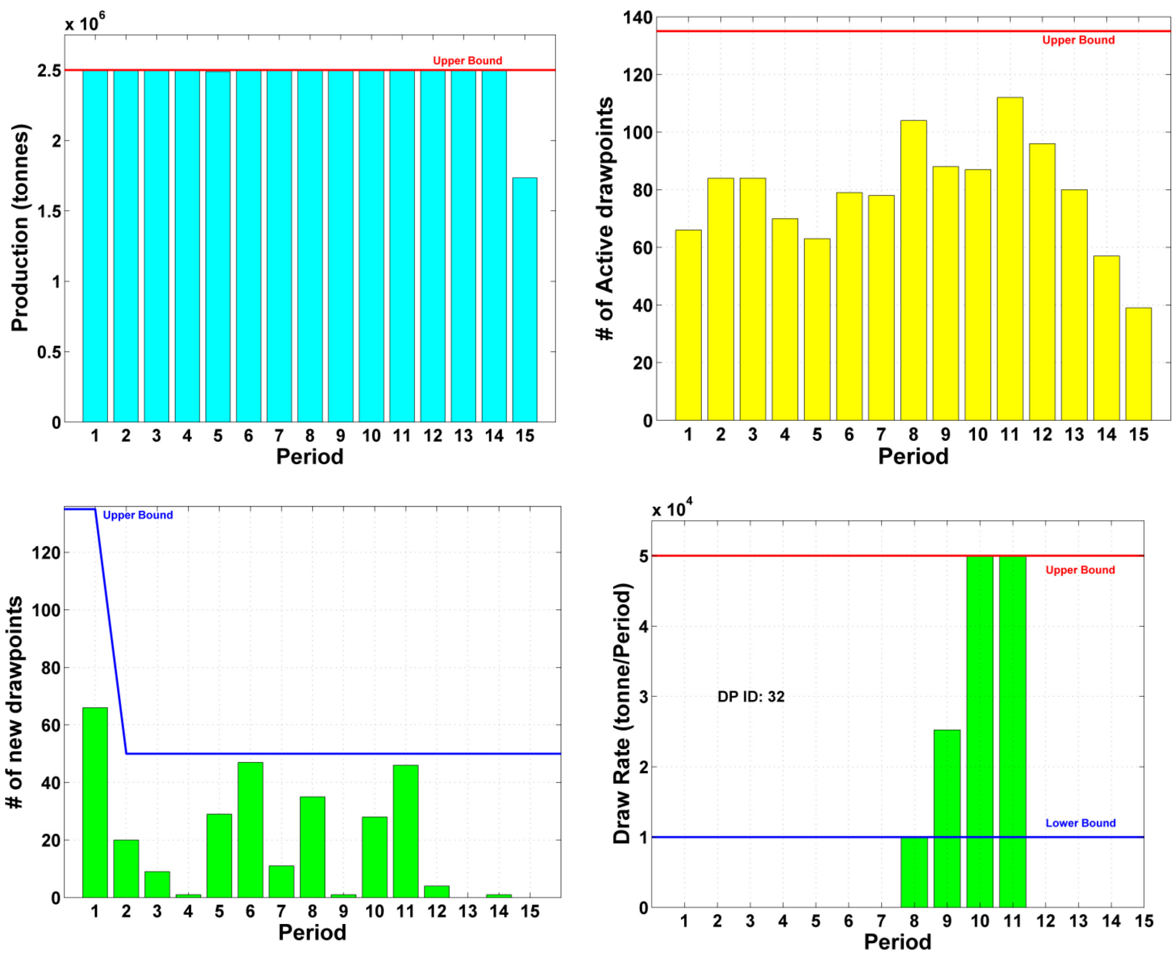


Fig 8. Starting period of drawpoints based on cluster level solution for west to east direction.

At the drawpoint level, a weighted average is calculated for the drawpoint economic value based on the slices within the related draw column. A portion of that is extracted during the optimization. The same explanation can be used for the cluster level. Cash flow over the mine life and the NPV for the three levels is shown in Fig 10. It is obvious that the cash flow during the first three years of the mine life for drawpoint and slice level formulation is greater than it is for other levels. Fig 11 shows that all defined constraints have been satisfied. The number of active drawpoints is increased gradually until period 5. Afterwards, during the next seven years, the mine works with maximum allowable active drawpoints. From period 12 to 15, this number gradually goes down.

It can be seen that the model tries to mine the slices with a higher grade earlier so that the average grade of production has a descending trend. During the last periods, because of more dilution, the average grade of production is less than in previous years. Fig 12shows how to extract the material from the draw column associated with drawpoint d32. Fig 12a shows the extraction periods and depletion tonnages from this drawpoint during its life. Fig 12b shows the percentage extracted from each slice located within the draw column associated with drawpoint d32. In this Fig, the vertical axis represents the ID number of slices located within the considered draw column. The numbers in front of each slice indicate the percentage extracted from that slice in the related period. It is obvious that there is a continuous extraction order between slices, and the defined precedence between slices of a draw column is observed. For instance, 21.8% of slice 362 is extracted in period 6, so extraction from slice 363 is not started until the rest of the material is extracted from slice 362 in period 7.

To better understand the effect of the multi-step approach on a real case data, the problem was solved at the drawpoint level and the drawpoint and slice level without using the mentioned approach. Table 4 summarizes the results. Although there is no significant difference between NPVs at the drawpoint level, the execution time for the multi-step method is 124 times faster than the independent single method. Furthermore, the multi-step method permits solution of the problem with a smaller EPGAP in a reasonable time. At the drawpoint and slice level, the independent single method was still running after five days without any feasible solution.



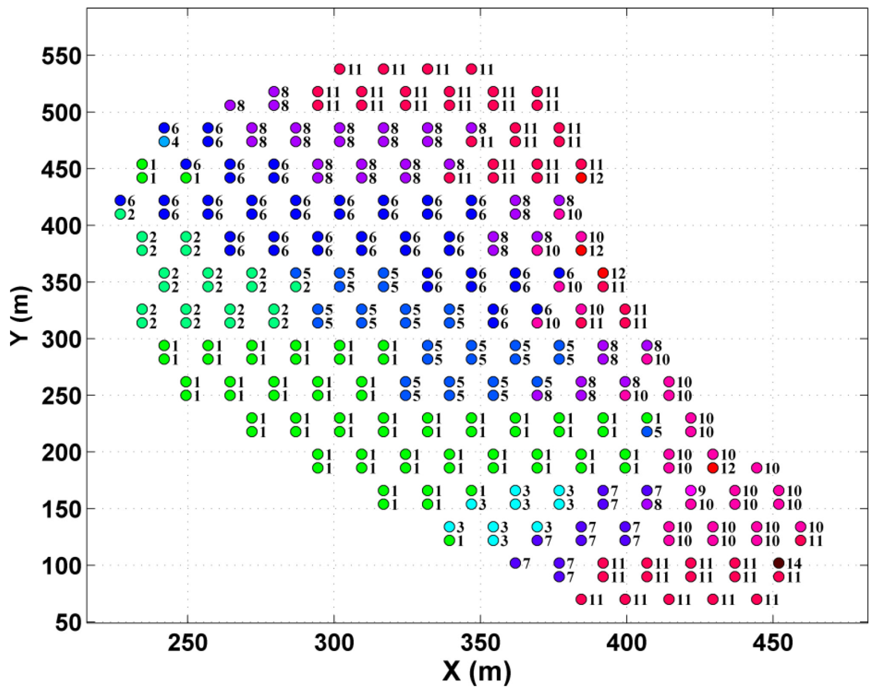


Fig 9. Satisfied constraints and starting period of drawpoints based on drawpoint level solution for west to east direction

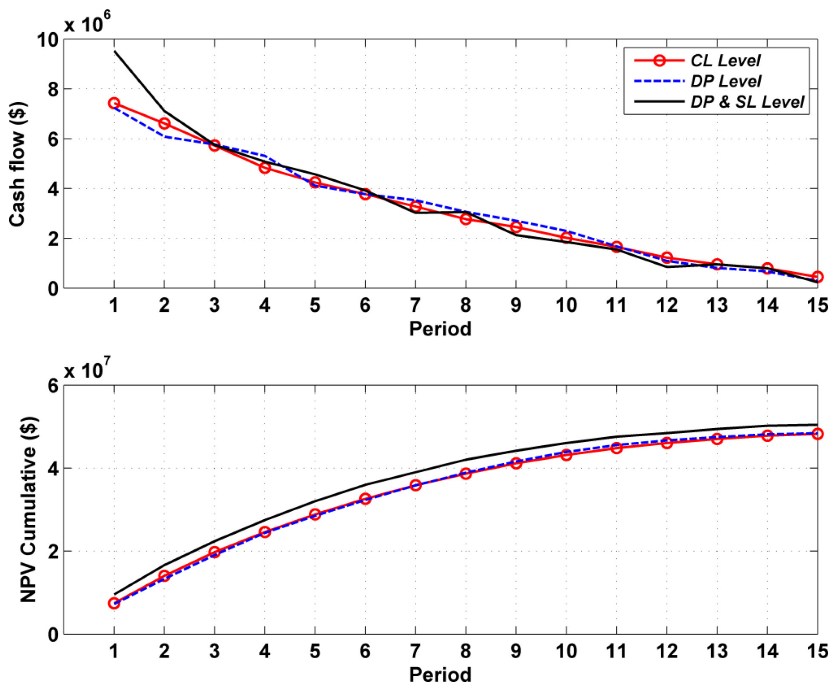


Fig 10. Comparison of cash flow and NPV for different formulation levels in west to east direction.

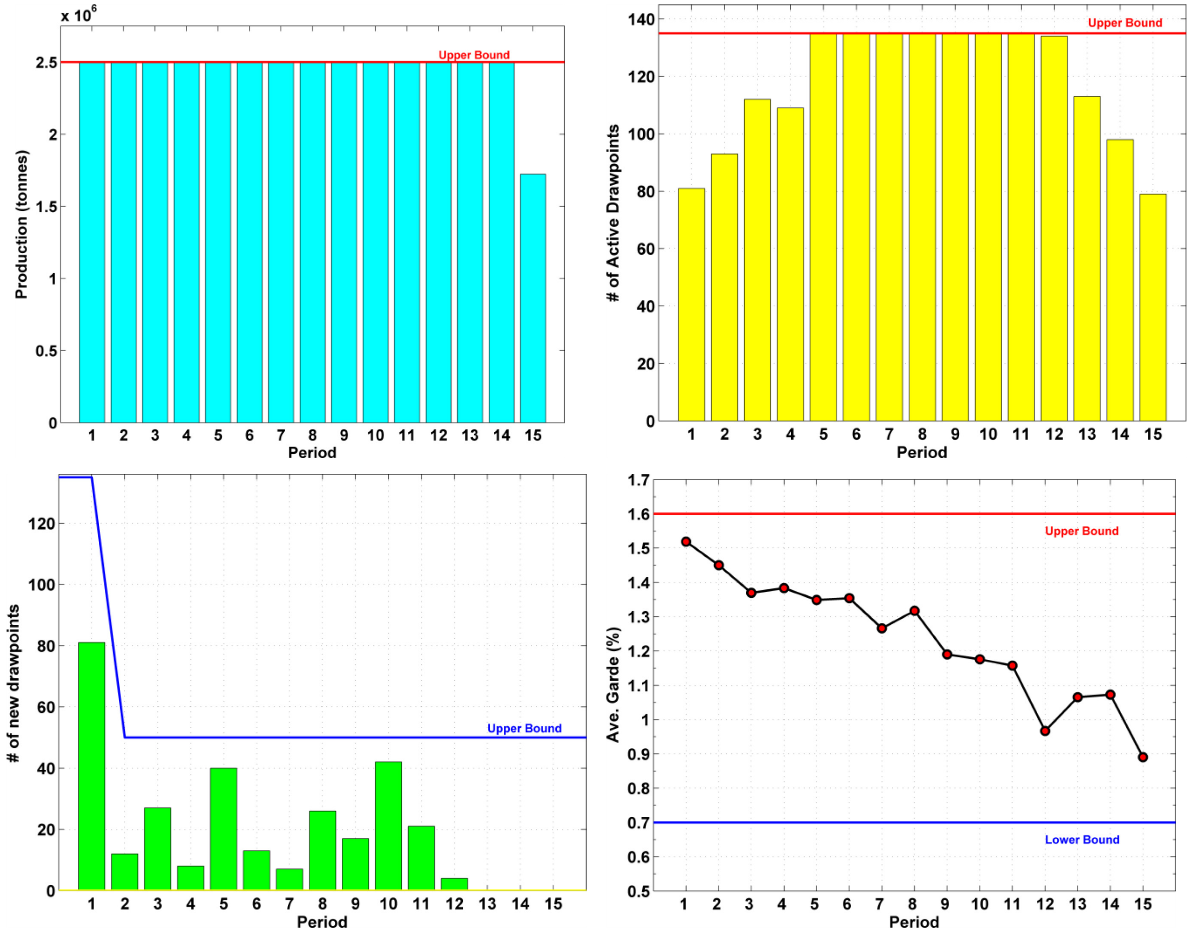


Fig 11. Satisfied constraint for drawpoint and slice level formulation in west to east direction

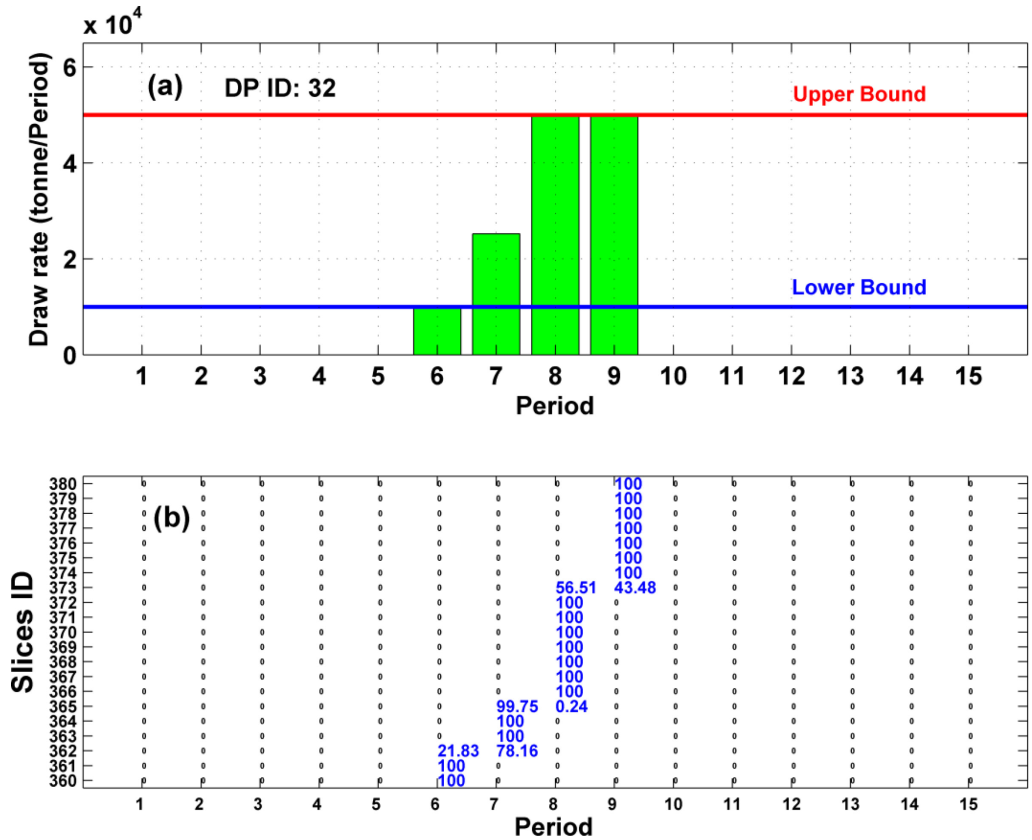


Fig 12. How to extract from drawpoint 32: (a) draw rate, and (b) percentage extraction of from each slice within draw column associated with drawpoint 32.

Table 4. Results of MILP formulations for multi-step approach and independent single step.

Direction	Level of formulation	Dependent multi-step			Independent single step		
		CPU time 4 CPUs 2.7 GHz	NPV ($M)	Optimality GAP (%)	CPU time 4 CPUs 2.7 GHz	NPV ($M)	Optimality GAP (%)
west to east	Drawpoint	00:22:45	48.45	1.45	46:54:31	48.27	4.97
	DP & Slice	25:59:27	50.43	4.79	I stopped running after five days.		

This comparison shows the effectiveness of the proposed method. A higher number of drawpoints increases the execution time of the problem exponentially and reduces the probability of finding a near-optimal solution. Fortunately, the method presented here can solve large-size problems.

7. Conclusions and future work

This paper presents a multi-step approach for block cave production scheduling optimization. MILP formulations for block cave production schedule were developed, implemented, and tested in the TOMLAB/CPLEX (Holmstrom, 2011) environment. The schedule of each step gives the mine planner good control over the number of new drawpoints that need to be constructed in each period to support the mine production, the number of active drawpoints in each period, and the average grade of production. The mine planner also has the flexibility of choosing the best direction for mining to maximize the NPV.

The problem was solved at three different levels of resolution. The result of each level is used to reduce the number of continuous and binary variables in the next level. All formulations maximize the NPV subject to several constraints such as the vertical mining rate, lateral mining rate and mining capacity, and the maximum number of active drawpoints or clusters.

The resulting NPV value for the drawpoint and slice level was greater than that for other levels. At the drawpoint and slice level, the method deals with the slices' economic value. The model extracts slices with a higher economic value earlier than other slices. But at the drawpoint level, the method deals with a draw column whose economic value is a weighted average of slices within the related draw column.

Future research will focus on modifying the approach for handling multiple-lift and multiple-mine scenarios. In addition, other efficient mathematical formulation techniques will be explored in an attempt to will reduce the execution time for large-scale block cave production scheduling.

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