An MILP Model for Long-term Mine Planning in the Presence of Grade Uncertainty

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The complexity of an open pit production scheduling problem is increased by grade uncertainty. A method is presented to calculate the cost of uncertainty in a production schedule based on deviations from the target production. A mixed integer linear programming algorithm is formulated to find the mining sequence of blocks from a predefined pit shell and their respective destinations, with two objectives: (1) to maximize the net present value of the operation and (2) to minimize the cost of uncertainty. An efficient clustering technique reduces the number of variables to make the problem tractable. Also, the parameters that control the importance of uncertainty in the optimization problem are studied. The minimum annual mining capacity in presence of grade uncertainty is assessed. The method is illustrated with an oil sand deposit in northern Alberta.

1. Introduction

Mine planning is an important process in mining engineering that aims to find a feasible block extraction schedule that maximizes net present value (NPV). In the case of open pit mines, Whittle (1989) defines mine planning as: "Specifying the sequence of blocks extraction from the mine to give the highest NPV, subject to variety of production, grade blending and pit slope constraints". Technical, financial and environmental constraints must be considered.

The uncertainty of the ore grade may cause discrepancies between planning expectations and actual production (Vallee, 2000; Osanloo et al., 2008; Koushavand and Askari-Nasab, 2009). Various authors present methodologies to account for grade uncertainty, and demonstrate its impact. Dowd (1994) proposed a risk-based algorithm for surface mine planning. A predefined distribution function is used for some variables such as commodity price, mining costs, processing cost, investment required, grade and tonnages. Different schedules are generated for a number of realizations of the grades. The proposed method leads to multiple schedules reflecting the grade uncertainty.

Ravenscroft (1992) and Koushavand and Askari-Nasab (2009) used simulated orebodies to show the impact of grade uncertainty on production scheduling. They used simulated orebody models one at a time in traditional optimization methods; however, this sequential process does not optimize accounting for uncertainty. Ramazan and Dimitrakopoulos (2004) suggested a mixed integer linear programming (MILP) model to maximize NPV for each realization. Then, the probability of extraction of a block at each period is calculated. These probabilities are used in a second stage of optimization to arrive at one schedule. The uncertainty is not used directly in the optimization process. Godoy and Dimitrakopoulos (2003) and Leite and Dimitrakopoulos (2007) presented a new risk-inclusive long term production plan (LTPP) approach based on simulated

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annealing. A multistage heuristic framework was presented to generate a schedule that minimizes the risk of deviations from production targets. The authors reported a significant improvement in NPV in the presence of uncertainty; however heuristic methods do not guarantee the optimality of the results. Also, these techniques can be difficult to implement, and many parameters may need to be chosen in order to get reasonable results. Dimitrakopoulos and Ramazan (2008) presented a linear integer programming (LIP) model to generate optimal production schedules. Multiple realizations of the block model are considered. This model has a penalty function that is the cost of deviations from the target production and is calculated based on the geological risk discount rate (GRD), which is the discounted unit cost of deviation from target production. They use linear programming to maximize a new function that is NPV less penalty costs. It is not clear how to define the GRD parameter.

The shortcomings of the current mine planning methods include: (1) most of the methods show the effect of uncertainty on the mine plan, but do not suggest a method to minimize the risk of uncertainty, (2) the methods minimize the risk or maximize NPV without using uncertainty explicitly, (3) the methods are not suitable for real-size mining problems, (4) there is no methodology to easily calculate the cost of uncertainty, and (5) none of the presented methods generate an optimum plan in presence of grade uncertainty.

In this paper, a mathematical programming formulism for long term mine planning in presence of grade uncertainty is proposed. The cost of uncertainty is quantified and used in a mixed integer linear programming model. A stockpile is considered in this new model. The cost of uncertainty is needed to determine the optimal trade-off between maximizing the NPV and minimizing the risk of grade uncertainty. The relationship between mining capacity and processing capacity and the cost of uncertainty is shown in this paper.

2. Cost of uncertainty

Typically, the main objective of long-term mine planning is to maximize the NPV of a project subject to technical and other constrains. The goal is to find the sequence of extraction of blocks or mining-cuts. A secondary objective is to account for uncertainty. Recently some authors, such as Dimitrakopoulos and Ramazan (2008), have presented optimization algorithms that aim to maximize NPV and to minimize the negative effects of uncertainty. These methods defer the extraction of more uncertain blocks. In this way the effect of grade uncertainty could be reduced by new information acquired during mining. The key idea is that uncertainty may incur a cost and should be deferred. There are two main costs related to uncertainty:

1. Cost of under production: where the mine may have to react quickly to make up for an unexpected shortfall.
2. Cost of over production: unexpected extra ore available to mine may lead to sub optimal use of resources or a cost for stockpiling.

The cost of under production can be assumed the loss of revenue of tonnage of ore that may not be fed to the processing plant and causes the mine and processing plant to operate sub-optimally. A simple method to calculate the cost of under production is:

$$\text{Cost of under production} = \text{Tonnage of shorfall} \times \left(\text{average revenue per tonne - processing cost per tonne}\right)$$

This equation can be rewritten to calculate the discounted cost of under production for period t:

$$\mathrm{C}_{up}\left(t\right) = T_{up}(t) \times \left(\overline{g}\left(t\right) \times R_p\left(t\right) \times \frac{P\left(t\right)}{(1+i)^t} - \frac{C_p(t)}{(1+i)^t} \right) \quad (1)$$

where $C_{up}(t)$ is the cost of under production, $T_{up}(t)$ is the tonnage of under produced ore, $\bar{g}(t)$ is the average input grade to the mill, $R_p(t)$ is the recovery of processing, $P(t)$ is the commodity price, $C_p(t)$ is the cost of processing in period *t* and i is the interest rate.

This approximation for the cost of under production assumes that under production will lead to a loss of revenue due to the mill running at lower capacity. In practice, it is highly likely that the mine will make up the shortfall somehow; however, there is no doubt that under production will incur a cost. Regarding the cost of over production, there are different components involved. Deferring the extraction of extra ore to the next period entails that the processed ore will have less value due to discounting. The discounting factor also applies to the processing costs. A cost of stockpiling may also be required. The cost of over production could be written as:

$$\text{Cost of over production} = \text{Extra ore tonnage} \times (\text{ lost of value of ore due to processing in next period} + \text{cost of stockpiling and rehandling})$$

The equation below is proposed to calculate the discounted cost of over production:

$$C_{op}(t) = T_{op}(t) \times \left[\underbrace{\bar{g}(t) \times R_p(t) \times \left(\frac{P(t)}{(1+i)^t} - \frac{P(t+1)}{(1+i)^{t+1}} \right)}_{\text{the lost of the value of ore}} + \underbrace{\left(\frac{C_p(t+1)}{(1+i)^{t+1}} - \frac{C_p(t)}{(1+i)^t} \right)}_{\text{the difference of processing costs}} + \underbrace{\frac{C_R(t)}{(1+i)^t}}_{\text{rehandeling cost}} \right] \quad (2)$$

where $T_{op}(t)$ is the tonnage of over produced ore in period *t* that is going to be processed in period *t+1*. $C_R(t)$ is the re-handling cost of stockpile in period *t*. This is an approximation for the cost of over production because the mine may be able to adapt dynamically to the extra ore and divert mining capacity to other locations; nevertheless, there is a cost associated with having more ore available than planned. Therefore it is clear that the cost of over production should be much less than under production in real life. This fact is considered in over production cost calculations. It is assumed that any possible over produced ore that has been transferred to the stockpile will be processed immediately in the beginning of the next period. Therefore, any cost of over production only is related to losing value of ore due to processing of extra ore in the next period and some stockpiling costs.

One should note that for a specific period *t*, a cost of under production and a cost of over production may both be applied. Both $C_{up}(t;l)$ and $C_{op}(t;l)$ are calculated for each realization although one or both will be zero in all cases. The discounted cost of uncertainty in period *t* over all *L* realizations is presented in Eq.(3) :

$$C_u(t) = \frac{1}{L} \sum_{l=1}^{L} \left[C_{up}(t;l) + C_{op}(t;l) \right] \quad (3)$$

The cost of Uncertainty (CoU) is calculated as in Eq.(4) :

$$CoU = \sum_{t=1}^{T-1} C_u(t) \quad (4)$$

CoU gives a single value for the discounted cost of uncertainty over all periods and realizations. It can be used to compare different schedules. The cost of uncertainty is calculated over all periods except the final period; because any ore that is left for the final period will be processed and will not exceed the target production, any shortfall in the final period is not relevant because the project is complete.

3. MILP formulation based on grade uncertainty with stockpile

The MILP model described in (Askari-Nasab et al., 2010; Askari-Nasab et al., 2011) will be used.

The profit from mining a block depends on the value of the block and the costs incurred in mining and processing. The cost of mining a block is a function of its location relative to its final destination. The profit from a block in period *t* is equal to the revenue generated by selling the final product contained in block *i* less all the costs involved in extracting the block. The discounted revenue and discounted cost can be written as demonstrated by Eq. (5)and Eq.(6), respectively:

$$v(i;t) = T_o(i) \times \left[g(i) \times R_p(t) \times \frac{P(t)}{(1+i)^t} - \frac{C_p(t)}{(1+i)^t} \right] \tag{5}$$

$$q(i;t) = \left(T_o(i) + T_w(i)\right) \times \frac{C_m(t)}{(1+i)^t} \tag{6}$$

Where *i* is the identification number of the block, $v(i;t)$ and $q(i;t)$ are discounted revenue and discounted cost of extraction from block *i* in period *t*. $T_o(i)$ and $T_w(i)$ are the tonnage of ore and waste for block *i*, and $C_m(t)$ is the mining cost in period *t* per tonne. The revenue of the block will be the sum for each valuable element. In addition the cost of processing contaminants will be deducted from the revenue of that block. The discounted profit of the project is the summation of discounted revenue minus discounted costs over all periods. The objective function is to maximize the total discounted profit including the cost of uncertainty. The first variable, $y(t;i)$, is the portion of the block *i* to be extracted in period *t*, and second variable, $z(t;i)$, is the portion of block *i* to be processed (if it is ore) in period *t*. Two separate variables are defined to control the portion of processing and extraction. Askari-Nasab et al. (2011) explained the reason of defining two separate variables in their model in more details. One reason is that the optimizer has more degrees of freedom with two separate variables to converge faster to higher NPV comparing to the case of using only one variable. In the other words, with two separate variables for extraction and processing, it is possible to generate a schedule that would send some low quality ore blocks on upper benches to waste (or, more likely, a low grade stockpile) to gain access to high quality ore blocks on the lower levels. An estimated block model (krig or e-type) is used to calculate the first part of the objective function. The NPV is calculated from Eq.(7) as:

$$NPV_{es} = \sum_{t=1}^{T} \sum_{i=1}^{N} \left(v(t;i) \times z(t;i) - q(t;i) \times y(t;i) \right) \tag{7}$$

For the second part of the objective function, the simulated realizations are used to measure the cost of uncertainty (*CoU*). The objective function is presented in Eq.(8):

$$Max\{NPV - COU\}$$

$$\Rightarrow Max \sum_{t=1}^{T} \left\{ \underbrace{\sum_{i=1}^{N} \left(v(t;i) \times z(t;i) - q(t;i) \times y(t;i) \right)}_{NPV_{es}(t)} - \underbrace{\frac{1}{L} \sum_{l=1}^{L} \left[C_{up}(t;l) + C_{op}(t;l) \right]}_{C_u(t)} \right\} \tag{8}$$

Subject to: Grade blending constraints:

$$\begin{cases} g_l(t)\times\sum_{i=1}^{N}T_o\left(i\right)\times y(t;i)-\sum_{i=1}^{N}T_o\left(i\right)\times y\left(t;i\right)\times g(i)\le 0 \\ \sum_{i=1}^{N}T_o\left(i\right)\times y\left(t;i\right)\times g(i)-g_u(t)\times\sum_{i=1}^{N}T_o\left(i\right)\times y(t;i)\le 0 \end{cases} \qquad \forall\, t=1,2,\ldots,T \qquad (9)$$

These inequalities ensure that the head grade of estimated block model is within the desired range in each period. $T_o\left(i\right)$ and $g(i)$ are the tonnage of ore and the grade of block *i* both from estimated block model. $g_l(t)$ and $g_u(t)$ are the allowable lower limit and upper limit of the head grade in period *t*. There will be separate constraints for each element of interest and any contaminants in each period. There are two equations (upper bound and lower bound) per element per scheduling period.

Processing capacity constraint:

$$\begin{cases} \sum_{i=1}^{N}T_o(i)\times z(t;i)-p_u(t)\le 0 \\ p_l(t)-\sum_{i=1}^{N}T_o(i)\times z(t;i)\le 0 \end{cases} \qquad \forall\, t=1,2,\ldots,T \qquad (10)$$

Where $p_l(t)$ and $p_u(t)$ are the lower limit and upper limit (target production) for the designed processing plant in period *t*; these inequalities ensure that the total ore processed in each period is within the acceptable range of the processing plant capacity. There are two equations (upper bound and lower) per period per ore type. The same as Eq. (9), estimated block model is used in these constraints too.

Mining Capacity constraint:

$$\begin{cases} \sum_{i=1}^{N}\left(T_o\left(i\right)+T_w\left(i\right)\right)\times z(t,i)-m_u(t)\le 0 \\ m_l(t)-\sum_{i=1}^{N}\left(T_o\left(i\right)+T_w\left(i\right)\right)\times z(t,i)\le 0 \end{cases} \qquad \forall\, t=1,2,\ldots,T \qquad (11)$$

Where $m_l(t)$ and $m_u(t)$ are lower and upper limit for mining capacity in period *t*; these inequalities ensure that the total tonnage of material mined (ore, waste, overburden, and undefined waste) in each period is within the acceptable range of mining equipment capacity in that period. There are two equations (upper bound and lower bound) per period. Estimated block mode is also has been used in these constraints.

The constraints that ensure the portion of extraction should be greater than the portion of the block that is going to be send to processing plant:

$$z(t;i)\le y(t;i) \qquad \forall\, t=1,2,\ldots,T \;,\; i=1,2,\ldots,N \qquad (12)$$

These inequalities ensure that the amount of ore of any block processed in any given period is less than or equal to the amount of rock extracted in the considered time period.

The constrains that specify the block extraction precedence:

$$a(t;i)-\sum_{u=1}^{t}y(u;j)\le 0 \qquad \forall\, t=1,2,\ldots,T \;,\; i=1,2,\ldots,N \;,\; j=1,2,\ldots,C\left(M\right) \qquad (13)$$

$$\sum_{u=1}^{t} y(u;i) - a(t;i) \le 0 \qquad \forall t = 1,2,\ldots,T \;,\; i = 1,2,\ldots,N \tag{14}$$

$$a(t;i) - a(t+1;i) \le 0 \qquad \forall t = 1,2,\ldots,T-1 \;,\; i = 1,2,\ldots,N \tag{15}$$

These equations control the relationship of block extraction precedence by binary integer variables $a(t,i)$, which is equal to one if the extraction of block *i* has started by or in period *t* (otherwise it is zero), and *i*, which is the index for set of the blocks, C(M), that need to be extracted prior to the extraction of block *i*. This model only requires the set of immediate predecessors' blocks on top of each block to model the order of block extraction. This is presented by set C(M) in Eq.(13).

Reserve constraint:

$$\sum_{t=1}^{T} y(t;i) = 1 \qquad \forall\; i = 1,2,\ldots,N \tag{16}$$

All the blocks in the model have to be mined and processed. This is based on the assumption that a fixed final pit limit is used.

Over and under production variables in presence of a stockpile are controlled with these constraints:

$$\begin{cases} \sum_{i=1}^{N} \left\{ -T_o(i;l) \times z(t;i) - \left(T_{op}(t-1;l) + T_{up}(t;l) \right) \right\} + p_u(t) \le 0 \\ \sum_{i=1}^{N} \left\{ T_o(i;l) \times z(t;i) - \left(T_{op}(t-1;l) + T_{op}(t;l) \right) \right\} - p_u(t) \le 0 \end{cases} \qquad \forall t = 1,2,\ldots,T \;,\; l = 1,2,\ldots,L \tag{17}$$

There are two constraints that control the over and under production variables. Where $T_o(i;l)$ is the tonnage of ore in block *i* in realization l.

First constraint,

$$\sum_{i=1}^{N} \left\{ -T_o(i;l) \times z(t;i) - \left(T_{op}(t-1;l) + T_{up}(t;l) \right) \right\} + p_u(t) \le 0$$

controls the possible under production of period t for realization l, $T_{up}(t;l)$. If there is any overproduced ore from a previous year in the stockpile, $T_{op}(t-1;l)$, it is transferred to the current year as well. This is the reason that possible under-production is mitigated by over production in the same realization for previous time periods.

The second constraint,

$$\sum_{i=1}^{N} \left\{ T_o(i;l) \times z(t;i) - \left(T_{op}(t-1;l) + T_{op}(t;l) \right) \right\} - p_u(t) \le 0,$$

Also controls the possible over produced ore for realization l in period t, $T_{op}(t;l)$, by adding the possible over production of previous year that has been transferred from the stockpile, $T_{op}(t-1;l)$.

One should note that $T_{op}(t;l)$ and $T_{up}(t;l)$ are tonnages of over production and under production in period t for realization number *l* are decision variables. Both of these variables are present in the objective function (Eq.(8)). $T_{op}(t;l)$ is used to calculate $C_{op}(t;l)$ and $T_{up}(t;l)$ is included in $C_{up}(t;l)$. Because they have negative impact on the objective function, the optimizer tries to assign the lowest positive values to these variables. But there are two main constraints presented in Eq.(17) that enforce these variables to get upper limit values, which for $T_{op}(t;l)$ is the tonnage of over production for realization l in period t and $T_{up}(t;l)$ will be assigned the tonnage of under production in period t for realization *l*.

The concept of stockpile also appears in these two constraints. The decision variable $T_{op}(t-1;l)$ which is the possible over produced ore, remained from the previous period in realization *l* is considered.. Two main factors control these variables. First, the discounting factor that is multiplied by $T_{up}(t;l)$ and $T_{op}(t;l)$ in the objective function, enforces the optimizer to create a schedule with less over and under production in early years. Second, because the tonnage of over production, $T_{op}(t;l)$, is used in the next period, it has less effect on optimization process than $T_{op}(t;l)$. Therefore, tonnage of under production is more penalized in this model.

Upper limit for stockpile:

$$T_{op}(t;l) \le T_{op\_\max}(t) \qquad \forall\, t = 1,2,\ldots,T \ , \ l = 1,2,\ldots,L \tag{18}$$

This equation shows the final constraint that is the upper limit of the stockpile. This value is assigned to $T_{op\_\max}(t)$ at each period and is chosen by the user as an input parameter. The proposed mixed integer linear programming was formulated in MATLAB environment. TOMLAB/CPLEX was used as the mathematical programming solver. The CPLEX solver starts with a relaxed LP model, where the integer variables are relaxed to real variables and a LP model is solved. Then, CPLEX uses the branch and cut algorithm to reach a feasible integer solution. A termination criterion is set explicitly by the user. The difference between the objective function in the current feasible solution and the solution with relaxed variables is called the MILP gap.

There are $3\times(T\times N)+2\times(T\times R)$ decision variables for this model. For example, with a block model of 20,000 blocks and 10 years of mine life and 50 realizations, there are 601,000 variables. Tabesh and Askari-Nasab (2011) tried to solve this problem by clustering the blocks in order to reduce the number of variables. Each cluster of blocks is called a mining-cut. The optimization model may postpone uncertain blocks to the later years; therefore it is important to avoid aggregating high uncertainty blocks with low uncertainty blocks.

4. Discussion

The cost of over-production is less than under production; it is easier to deal with unexpected extra ore by stockpiling or slowing production. Nevertheless, there are still costs for over production: re-handling cost of stockpile, and loss of discounted value of transferred ore to the next period. Figure 1 shows the over and under production discounted costs at different periods. The cost of uncertainty for a unit tonne of under production is higher than the cost of over production. Also, the discounted cost of uncertainty is greater in earlier time periods. The optimizer will postpone the extraction of very high uncertain blocks to the later years – unless the grade value is very high. In the proposed objective function Eq.(8) the two input parameters $C_{up}(t)$ and $C_{op}(t)$ that control the uncertainty part of the optimization problem are the discounted unit cost of deviation from target production. (Dimitrakopoulos and Ramazan, 2008) did not suggest a method to estimate these parameters. They used different discount rates for the cost of deviation from target production than for the calculation of net present value of profit and costs. They defined a new term called the geological risk discount rate (GRD). High GRD rate means that very low uncertain blocks are going to be extracted in early years of production and the generated schedule tends to be conservative. Also, one should note that high GRD value generates lower NPV.

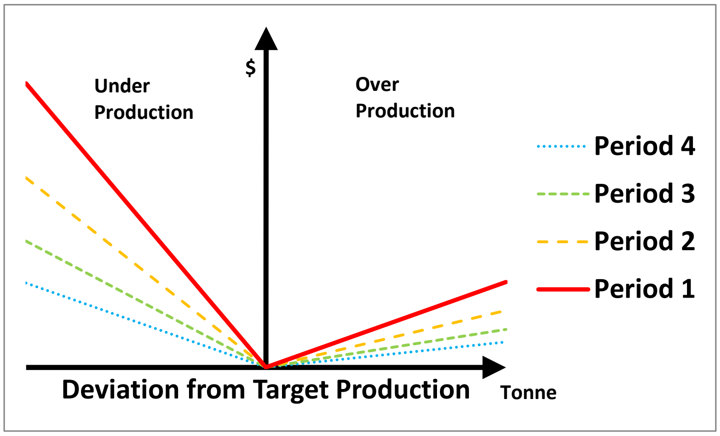

Figure 1. Penalty function for over and under production at different periods based on a discounting factor.

One discount factor is used for all components of the objective function. Two different techniques to calibrate the parameters will be demonstrated. The deterministic method calculates the factors with Eqs.(1) and (2). This method requires the average input grade to the mill for each realization. This cannot be exactly calculated until the optimization finds the solution. An approximate input grade can be used since the optimization appears insensitive to small changes.

A numerical method is the second alternative. A C factor controls the trade-off between NPV of the project and risk of uncertainty. By increasing *C* values, both NPV and cost of uncertainty are reduced, but not with the same rate. A numerical criterion is defined to determine an optimum *C* value. The values of NPV_{es} and CoU calculated by Eqs (4) and **Error! Reference source not found.** (7) are calculated for each C value. The Delta value which is the difference of NPV_{es} and CoU is also calculated as: $Delta = NPV_{es} - CoU$. This Delta value is the amount of money that is lost by generating lower NPV plus the amount of money that is gained by generating lower cost of uncertainty. Therefore, the optimum C value is obtained where the Delta reaches a maximum value. This is shown in Figure 1.

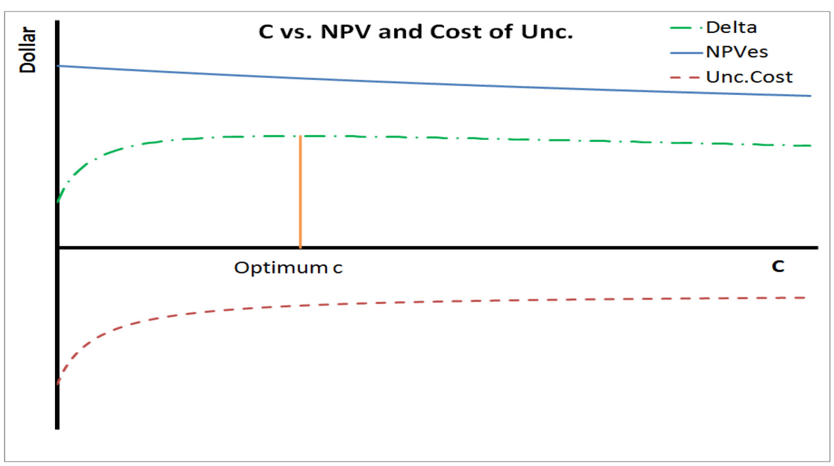

Figure 1. Different C values versus NPV and cost of uncertainty to find the optimum C parameter.

This graph was generated by a synthetic case. The C parameter is shown in the horizontal axis. The vertical axis shows the dollar value of NPV and *CoU*. The bold line shows the NPVs of the project which decreases with higher C values. The dashed line shows the negative values of the cost of uncertainty, which decreases with higher C values. The dashdot line shows the delta value. The shape of this graph and the behaviour of each parameter will be different from one case to another. The reserve constraint shown in Eq.(16) forces the optimizer to extract all the blocks inside the final pit. Therefore even with high *C* values the cost of uncertainty cannot be reduced effectively and consequently, the NPV of the project will not be changed. This explains the flatness of the Delta values after the optimal *C* value. An application of this methodology is to find the optimum mining capacity based on the cost of uncertainty. In traditional design, the mine planer starts with an initial mining capacity and tries to find a reasonable schedule or starts with a schedule and establish the required capacity. The schedule should be uniform at the plant capacity over the mine life.

The objective function presented in Eq. (8) allows the mine planner to consider grade uncertainty. By changing mining capacity, different optimum solutions will be calculated. A stockpile in the optimization process will be used to reduce the cost of uncertainty. Increasing the mining capacity will decrease the cost of uncertainty. An optimum value for mining capacity could be estimated. The optimum mining capacity may be higher than the mining capacity that is found by traditional methods; nevertheless, the optimum mining capacity has the minimum total cost. A synthetic case demonstrates this procedure. Figure 2 shows the relationship between cost of uncertainty and mining and processing capacities. The mining and processing capacities are incremented 10 times each with 5 and 10 unit intervals, respectively. Therefore the optimization process was run for each case with different mining and processing capacities.

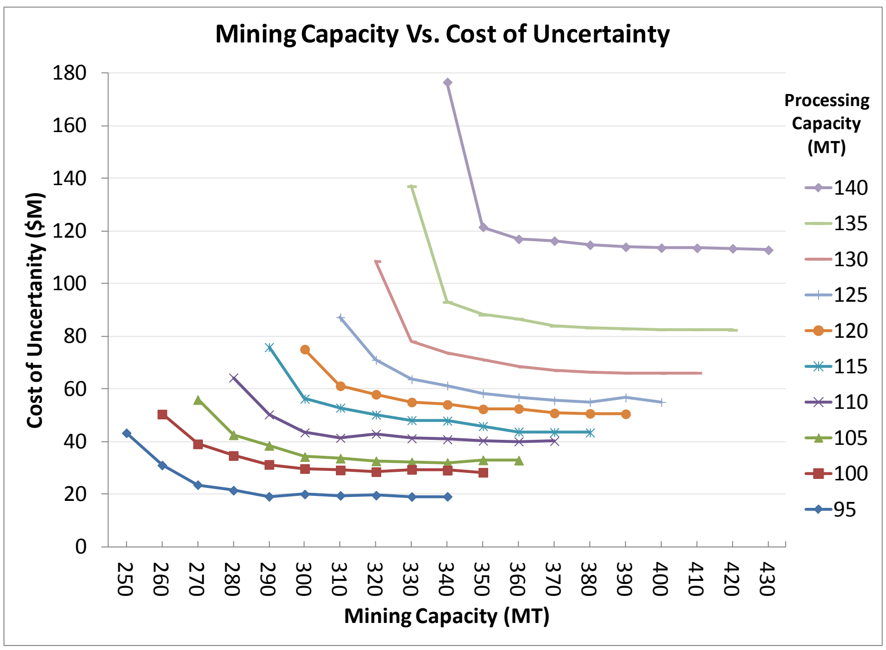

Figure 2. The cost of uncertainty versus different mining and processing capacities in a synthetic case.

The following conclusions can be made from this graph. For a chosen processing capacity (each line), by increasing the mining capacity, the cost of uncertainty decreases until a certain number. After this value, any increment in mining capacity does not affect the cost of uncertainty. A larger processing capacity requires a larger mining capacity to minimize the cost of uncertainty. This can be understood by comparing two processing capacity of 95MT and 140 MT. In these two lines, the mining capacities for minimum cost of uncertainty are 290MT and 350MT, respectively.

5. Case study

A syntactic oil sands deposit is used. Locations of boreholes and a histogram of bitumen grade are presented in Figure 1 and Figure 2. Directional experimental variograms were calculated and fit. The azimuths of the major and minor directions are 50 and 140 degrees. Figure 3 shows the experimental and the fitted variogram models in major (Figure 3a), minor (Figure 3b) and vertical (Figure 3c) directions.

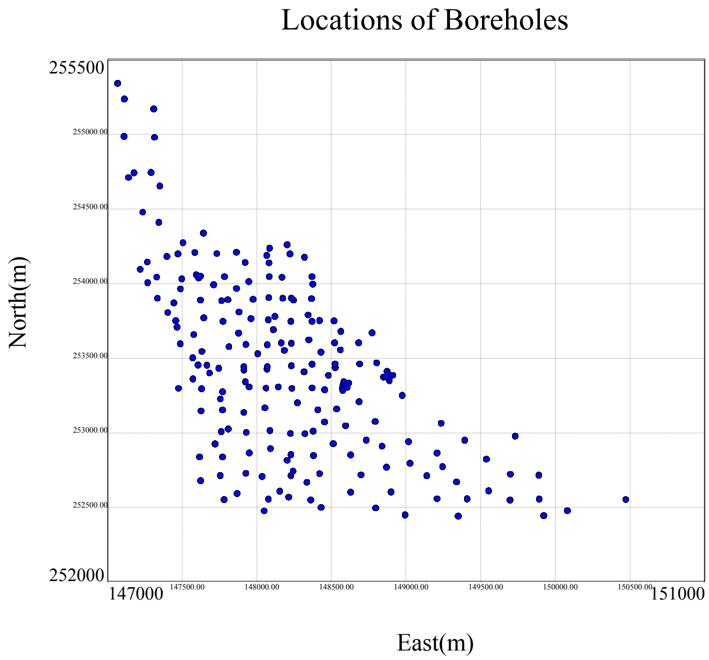

Figure 1: Location map of boreholes of oil sand.

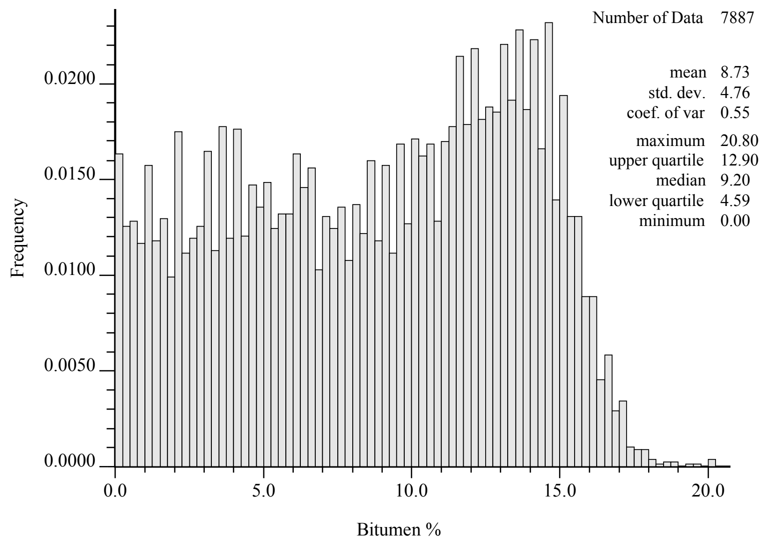

Figure 2: Histogram of bitumen.

Ordinary Kriging (OK) was used to estimate the bitumen grade. Multiple realizations of the bitumen grade were also generated using Sequential Gaussian Simulation (SGS) (Isaaks and Srivastava, 1989) to account for uncertainty in bitumen grade. GSLIB (Deutsch and Journel, 1998) programs were used. Figure 4a, Figure 4b and Figure 4c illustrate the map of bitumen grade from OK, the average of realizations (e-type) and a realization. Kriging is conditionally biased (Isaaks, 2005) and there is no conditional bias of simulation when the simulation results are used correctly (McLennan and Deutsch, 2004). The conditional bias of Kriging can be reduced by tuning estimation parameters but it cannot be eliminated (Isaaks, 2005).

The grade-tonnage curve is a useful tool to check the impact of the smoothness of the kriged estimates. Figure 5 shows the grade-tonnage curve of realizations (dashed lines), OK (bold solid line) and etype (bold dashed line). The systematic conditional bias of OK was minimized but still there are differences between OK and the simulation results. The Etype model is not used directly.

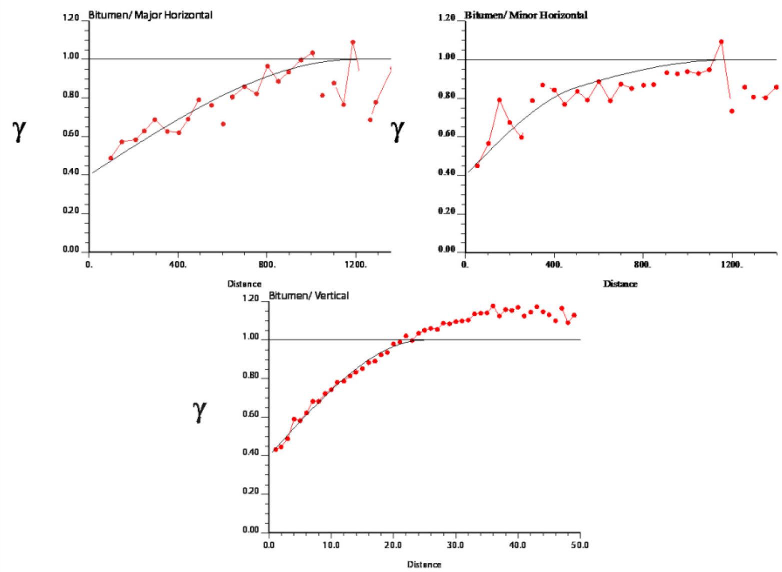

Figure 3: Experimental directional variograms (dots) and the fitted variogram models (solid lines), distance units in meters.

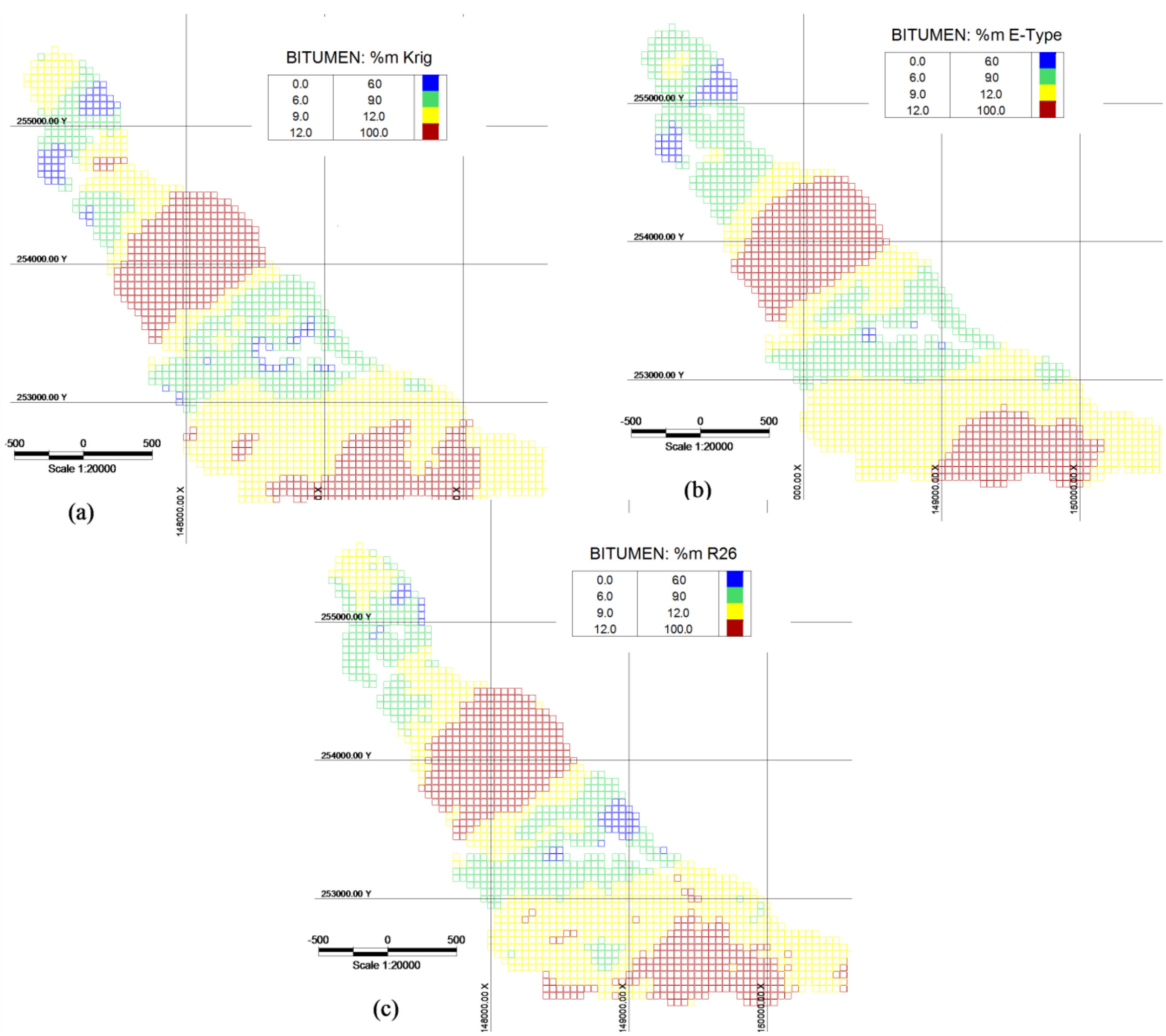

Figure 4: Plan view at 260m; (a) Kriged model, (b) E-type model, (c) realization 26.

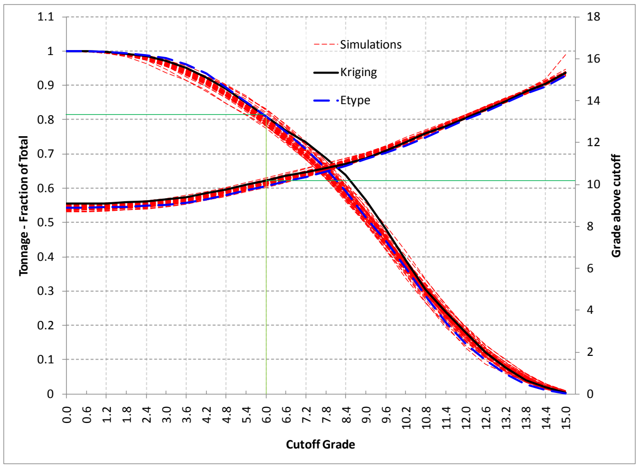

Figure 5: Grade tonnage curve of simulation realizations, kriged, and Etype block models.

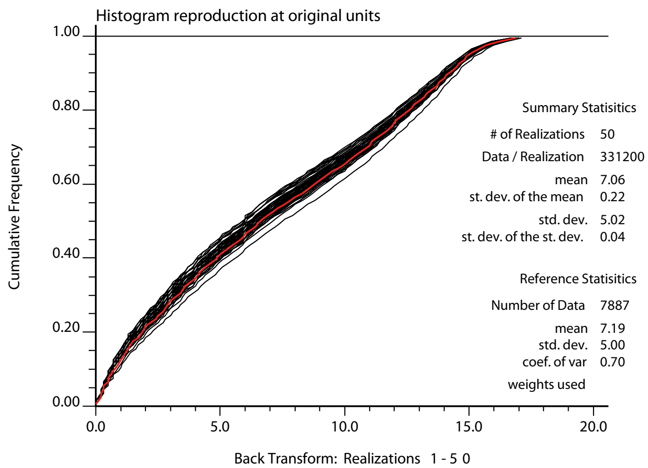

Figure 6: Histogram reproduction of simulation realizations (dashed lines) and histogram of original data. (bold line)

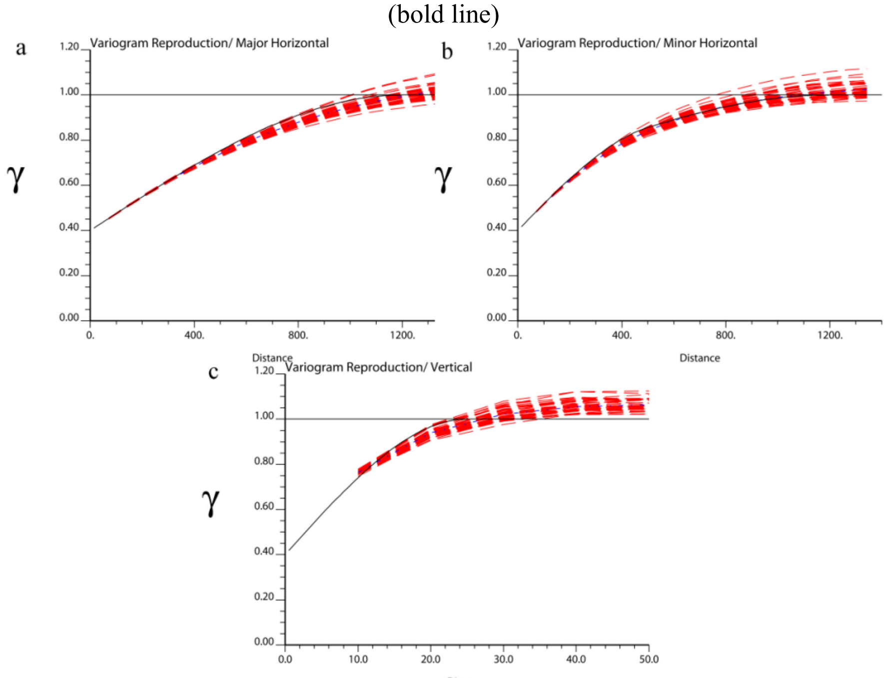

Figure 7: Variogram reproduction of simulation realizations (red dash lines) and reference variogram model (black line).

The sizes of the blocks used in mine planning are a function of the selective mining unit (SMU). The original grid was up-scaled (arithmetic average) to get the right block scale values.

Table 1 summarizes the costs used in the pit limit design. A mining cost of $4.6 per tonne of oil sands was used; processing cost is assumed $0.5025 per tonne; mining and processing recovery factors are taken 88 percent and 95 percent. Also, an interest rate of 10% is chosen.

Table 1: Summary of costs and the interest rate used in pit limit design.

Description	Value
Mining Costs (CAN $/tonne)	4.6
Upgrading Costs (CAN $/tonne)	0.5
Interest Rate	10%

Table 2. shows the pit design and production scheduling input parameters. 33 pitshells were generated using 49 fixed revenue factors ranging between 0.1 to 2.5, based on the OK block model. No pitshell is generated for revenue factors between 0.1 to 0.30 because there is not any feasible LG pit with those revenue factors.

Table 2: Final pit limit and mine planning input parameters.

Description	Value	Description	Value
Cutoff grade (%mass bitumen)	6	Processing limit (M tonne/year)	71
Mining recovery fraction	0.88	Mining limit (M tonne/year)	135
Processing recovery factor	0.95	Overall slope (degrees)	20
Minimum mining width (m)	150	Pre-stripping (years)	1

Also, for some revenue factors same pit shell is generated. The number of pitshells is reduced to 14 after applying the minimum mining width of 150 meters for the final pit and the intermediate pits. Table 3 summarizes the information related to the final pit limit based on OK block model at 6 percent bitumen cut-off grade. The target overall slope of the final pit was 20 degrees, where the minimum slope error, the average slope error and the maximum slope error respectively are: 0.0 degrees, 0.2 degrees, 0.4 degrees.

Table 3: Material in the final pit using the kriged block model.

Description	Value
Total tonnage of material (M tonne)	653.6
Tonnage of ore (M tonne)	282.4
Tonnage of material below cutoff (M tonne)	37.4
Tonnage of waste (M tonne)	371.2
Bitumen recovered (M tonne)	23.3
Stripping ratio (waste:ore)	1.31

The Lerchs-Grossman algorithm (1965) was used to find the final pit using the OK block model. Whittle (Gemcom Software International, 1998-2008) was used for this purpose. There were 14607 blocks with 653.61 million tonnes of material and 282.44 million tonnes of ore inside the final pit. The strip ratio was 1.31. Using the MATLAB (MathWorks Inc., 2007) c-mean clustering function, all 14607 blocks inside the final pit were aggregated to 2000 mining cuts. First the number of mining cuts for each level was calculated and then for each level Fuzzy C-mean clustering technique was used to aggregate based on similar grades. Two years of pre-stripping were assumed to provide enough operating space and ore availability. The target production was set to 36 million tonnes of ore per year with a mining capacity of 135 million tonnes per year. The mine life was 10 years.

The OK block model was used in the first part to maximize NPV and the realizations were used in the second part to minimize the cost of uncertainty. The stockpile is only used for uncertainty where over produced ore is sent to the stockpile and used in the next period. In this case study a \$0.4 per tonne penalty value was considered for under-production $C_{up}(t)$ and no upper limit for the stockpile has been considered. The cost of over-production was calculated by the difference in the discounted value of $C_{up}(t)$ at each period related to the previous period. The gap of 0.05 percent was used in CPLEX optimizer.

Figure 8 shows the resulting schedules. The feed to the plant is uniform over the 7 years of production. The plan view and two cross sections of blocks and their extraction periods are shown in Figure 9. Clustering causes the generated schedule to be smooth however this schedule may not be practical in real life. To solve this problem fewer number of cluster should be used or directional mining is suggested. To assess the effect of uncertainty, the generated schedule was followed by each realization. Figure 10 shows the cumulative cash flow for Kriging, e-type and realizations.

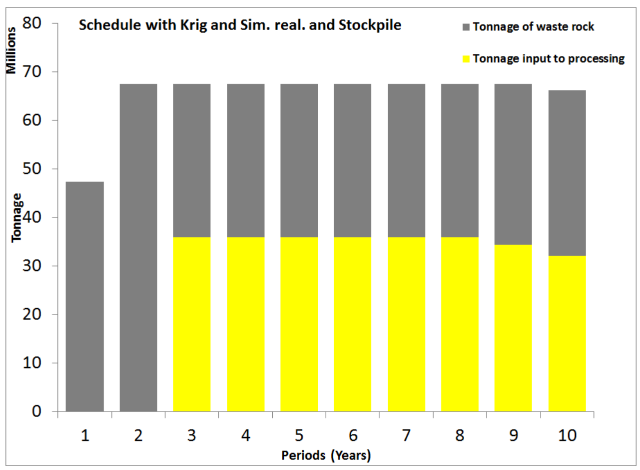

Figure 8. Schedule generated using OK block model and realizations with stockpile.

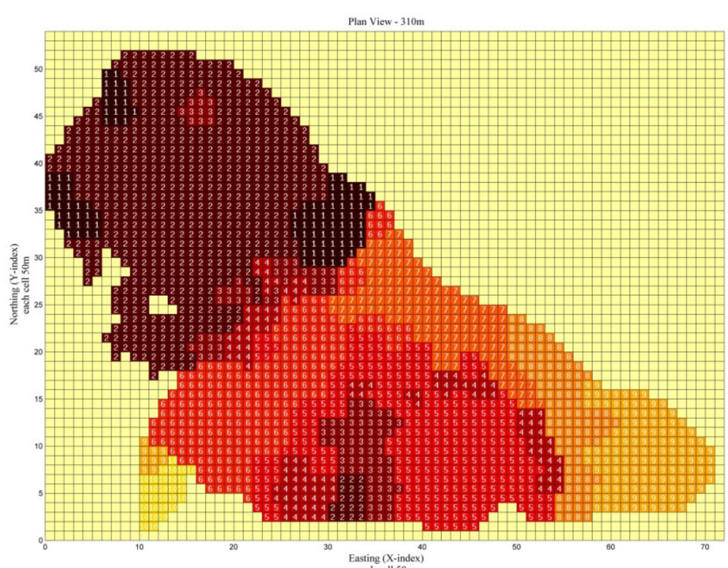

Figure 9. Plan view, cross section looking north and east for the generated schedule.

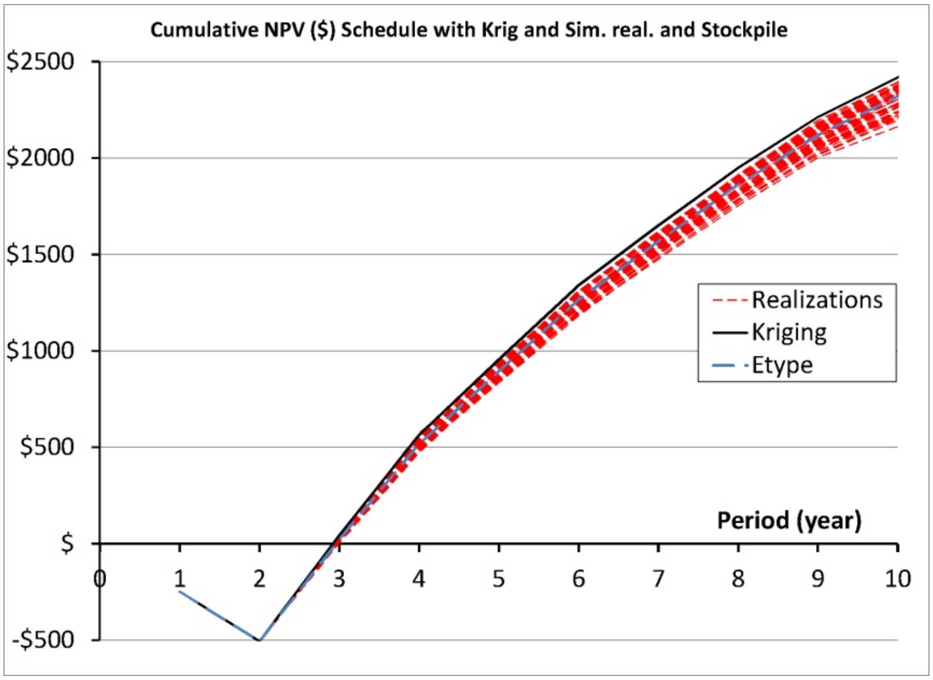

Figure 10. Comulative NPV over periods for Kriging (back line), etype (dashed blue line) and realizations (dash red line).

Figure 11 illustrates input tonnage to the mill. Figure 12 shows the box plot of realizations and deviations from target production. The average input grade to the mill in each period is presented in Figure 13.

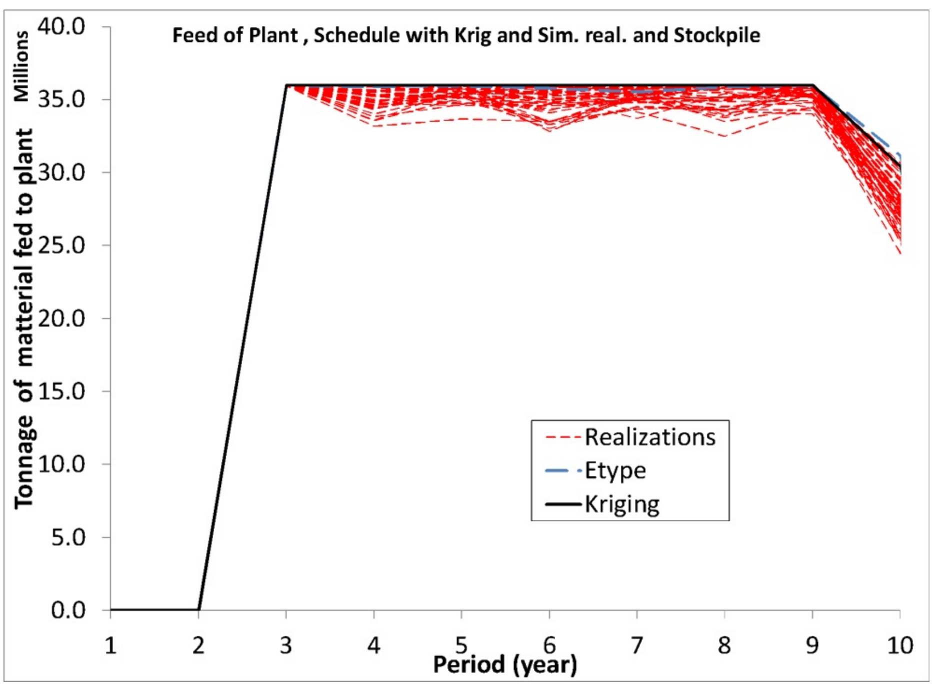

Figure 11. Feed of the plant over periods for kriging (back line), etype (dashed blue line) and realizations (dash red line).

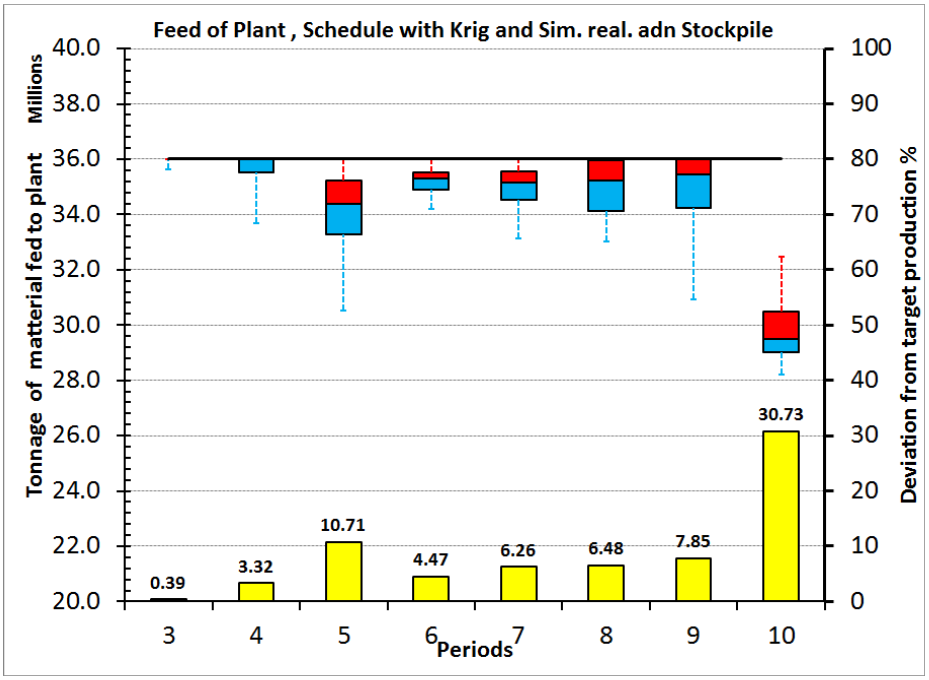

Figure 12. Boxplot and deviation from target production (yellow bars), calculated using simualtion values.

Table 4 summarizes the statistics following the generated schedule. Because the optimization of NPV has been applied using the OK block model, the NPV of the Kriging is higher than average NPV calculated from realizations ($2,323 million versus $2,454 million). Figure 14 shows the tonnage of ore at the stockpile in each period for every realization (dash lines) and the solid line shows the average amount of ore in the stockpile for each period.

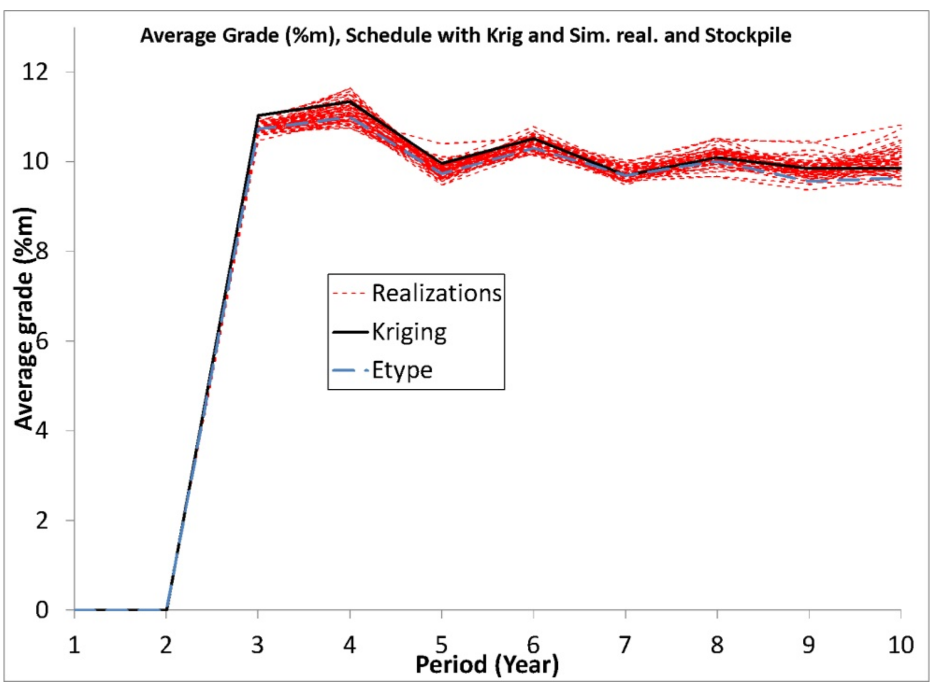

Figure 13. Input head grade to the plant over periods for kriging (back line), etype (dashed blue line) and realizations (dash red line).

Table 4. Summary statistics of realizations when Kriging schedule is followed.

LP With Krig & Sim. Realizations With Stockpile	Ore (MT)	STRO	Input Bitumen Millions Tonnes	Average %	NPV Millions Dollars
Mean	275.92	1.37	28.24	10.24	2322.60
Std. dev	3.61	0.03	0.44	0.09	59.42
Min	269.29	1.29	27.22	10.02	2191.25
Quartile 1	272.97	1.35	27.84	10.19	2269.99
Median	276.37	1.36	28.23	10.24	2318.70
Quartile 2	278.39	1.39	28.56	10.29	2372.09
Max	284.47	1.43	29.10	10.52	2430.31
Krig	282.44	1.31	29.11	10.31	2453.85
Etype	282.07	1.32	28.48	10.10	2349.99

The LP optimization (Askari-Nasab and Awuah-Offeri, 2009) that uses only one block model is used to compare the performance of the presented method. As a sensitivity analysis, the Kriging block model was used. Figure 15 and Figure 16 show the tonnage of ore to the mill at different periods and realizations. The proposed method that uses the realizations reduces the deviation from target production particularly in periods 3, 4 and 5.

6. Conclusions

The cost of uncertainty is accounted for in mine planning. A mixed integer linear programming model considering stockpiles was presented to generate long term production schedules. The NPV was maximized based on an estimated block model resulted from Kriging. The second objective is to minimize cost of uncertainty. The idea is to defer extraction of highly uncertain blocks to later years when more information is provided by new infill drillholes. This is controlled by the cost of over and under production. Two methodologies to calculate the values of these parameters were shown: a deterministic method that needs average input grade to the processing plant for each realization and the numerical method that needs to run the optimization for different parameters to get the optimum value.

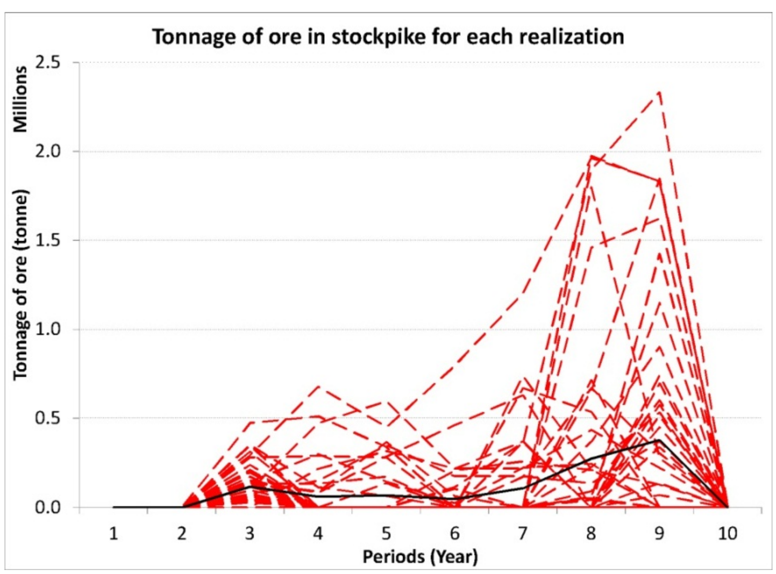

Figure 14. Tonnage of ore in the stockpile for each realization (dashed lines) and the average tonnage (solid line)

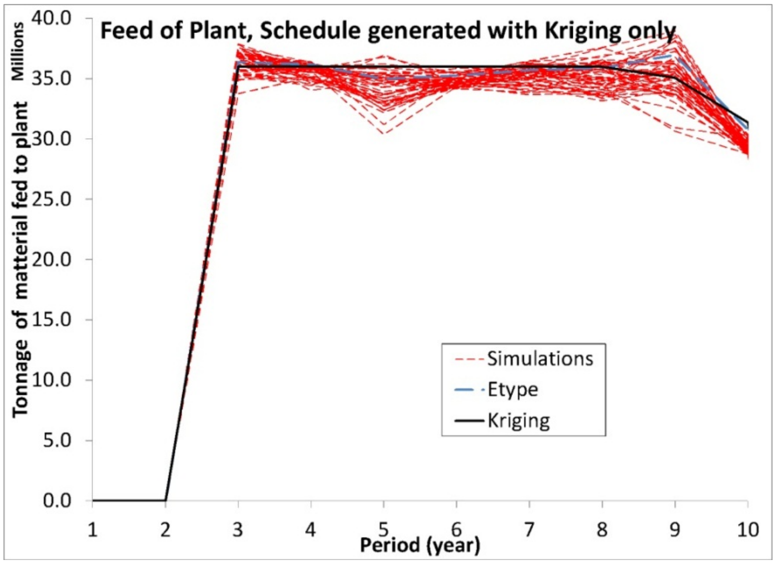

Figure 15. Feed of the plant over periods for kriging (back line), etype (dashed blue line) and realizations (dash red line). Only Kriging block model has been used and no realizations.

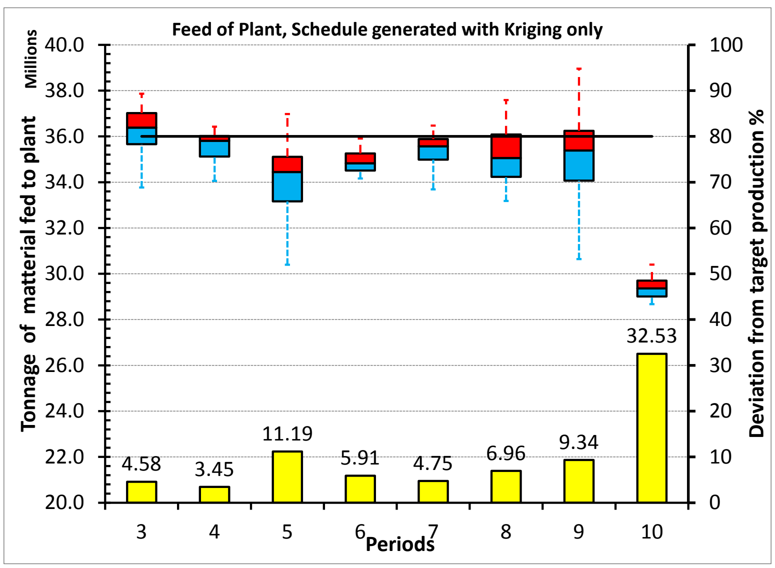

Figure 16. Boxplot and deviation from target production (yellow bars), calculated using simualtion values. Only Kriging block model has been used and no realizations.

The optimum mining capacity can be estimated considering the grade uncertainty. A new methodology was presented to show the impact of different mining and processing capacities on the cost of uncertainty and the generated schedule.

There is no shortfall in the generated schedule for the Kriging block model. The probability of deviations from target production in each period is calculated using a set of simulated realizations. The generated schedules are more robust because the probability of not meeting the target production is lower in the early years of production. The size of the optimization problem with the original block model is too large for current commercial solvers such as CPLEX. A clustering technique was used to aggregate similar blocks into groups called mining-cuts. This reduces the number of variables and smoothen the generated schedule.

Future work is to use pushbacks to further reduce the size of the problem. Also directional mining can be implemented by adding some new constraints.

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