Hierarchical Open-Pit Mine Production Scheduling Optimization – Linking the Strategic Plan to Monthly Schedule

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Abstract

A hierarchical open-pit mine production scheduling approach linking the optimal strategic open pit mine plan to the optimal medium-term production schedule is presented. The main objective of this paper is to develop, implement, and verify a mathematical programming framework for the optimal medium-term open-pit mine production scheduling using mixed integer linear programming (MILP). The objective of the medium-term mine production schedule is to minimize operating costs, including waste rehabilitation, processing, mining, haulage, and rehandling costs. To verify the proposed hierarchical model, an iron ore case study is presented. A hierarchical clustering algorithm is used to aggregate blocks into scheduling mining units, referred to as mining-cuts. Then, using an MILP model the optimal life-of-mine production schedule with the objective of maximizing the net present value is generated. Afterwards, an MILP model for medium-term mine production scheduling is proposed and applied. The long-term mine production scheduling model provides a strategic plan of the mining operations – whereas the medium-term mine production scheduling provides an operational scheme for mining while tracking the strategic plan. The results are interpreted and analyzed to verify the applicability of the model in terms of mining practice. The main contributions of the medium-term production scheduling model are: multiple destinations modeling including stockpiles, processing plants, and waste dumps; decision making on selection of routes/ramps to minimize the haulage costs; and modeling stockpiles using mixed integer linear programming.

1. Introduction

Mine production scheduling is defined as a decision making process to determine the extraction sequence of ore and waste and the corresponding destinations for the mined material over time. The goal of long-term mine production scheduling is to determine the mining sequence, which optimizes the company's strategic objective, while honoring the operational limitations over the mine life. There are a number of strategic objectives, which are common in the industry. Usually, the target is to maximize the net present value (NPV) of mining operations within the existing economic, technical, and environmental constraints. However, other objectives such as cost minimization or reserve maximization could also be considered. The production schedule defines the management investment strategy. An optimal plan in mining projects will result in cost reduction, increasing equipment utilization, optimum recovery of marginal ores, steady production rates, and consistent product quality.

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The main objective of this paper is to develop, implement, and verify a mathematical programming framework for the optimal medium-term open-pit mine production scheduling. Also, we present a case study to show the practicality of the model. The proposed mathematical programming model aims at minimizing operating costs, while honoring the long-term strategic open-pit mine plan. The focus of the research has been on developing and testing a correct mathematical programming model that is capable of generating production schedules that have practical merit in real-world mining operations. The medium-term production scheduling optimization is presented in a hierarchical approach linking the medium-term production schedule to the strategic mine plan.

In the hierarchical approach, mine production scheduling is carried out in two sequential steps: long-term and medium-term mine production scheduling. Long-term mine production scheduling considers the mine's operations over the mine-life and specifically aims to increase the profitability – usually net present value (NPV) maximization Chanda (1992). Mathematical modeling is a powerful methodology for long-term mine production scheduling. There are numerous mathematical programming models in the area of long-term open-pit mine production scheduling. Some works have focused on developing mathematical programming models (e.g. MILP) to comprehensively model the production scheduling problem. Some other studies have attempted to develop algorithms to make the large-scale production scheduling problem more tractable in terms of CPU time.

Bley et al. (2010) present three approaches to improve the CPU time to solve an integer programming model: variable elimination through modeling the problem as a knapsack problem, adding inequalities through modeling the problem as the cutting plane with two blocks conflicts, and adding inequalities through modeling the problem as the cutting plane with multiple blocks conflicts. Cullenbine et al. (2011) propose a sliding time window heuristic model for approximately solving the open-pit mine block sequencing. The authors first apply a step called preprocessing to eliminate binary variables that are not necessary for the considered long-term mine planning IP model. Then, the authors apply the recursive sliding time window heuristic which defines and solves the IP model with partitioning the long-term time horizon into three time windows. For other noticeable recent research in the area of open pit long-term production scheduling optimization refer to Askari-Nasab et al. (2010a); Askari-Nasab et al. (2011a); Ben-Awuah and Askari-Nasab (2011); Bienstock and Zuckerberg (2010); Boland et al. (2009); and Chicoisne et al. (2009). For a detailed literature review on applications of mathematical programming in long-term open-pit mine production scheduling see (Newman et al., 2010) and (Osanloo et al., 2008).

Unlike long-term mine production scheduling, medium-term mine production scheduling takes into account certain aspects of mine production such as haulage roads, mining sequence, and equipment investment in more detail (Chanda, 1992). Medium-term mine planning is concerned with determining the removal sequence of ore and waste in mid-range time horizons (e.g. quarters, months, etc.). Medium-term mine production scheduling should satisfy the constraints imposed by the long-term plan. Also, process capacity constraints, inventory restriction constraints, equipment availability constraints, and haulage and ramp constraints must be honored. Unlike long-term mine production scheduling, medium-term mine production scheduling's purpose often is minimization of the operating costs (Kumral and Dowd, 2002). A few studies have been conducted in developing medium-term mine production schedules. Chanda (1992); Fioroni et al. (2008); Fytas et al. (1993); Fytas et al. (1987); Huang et al. (2009); Kumral and Dowd (2002); Rahman and Asad (2010); Sandeman et al. (2010); Souza et al. (2010); Youdi et al. (1992) are some of the works on the medium-term and short-term mine production scheduling available in the literature.

In this paper, a hierarchical open-pit mine production scheduling is proposed to align the optimal medium-term mine production schedule with the optimal long-term strategic plan. The study is based on the assumption that a block model of the orebody using geostatistical methods is available. This block model is the input into the production scheduling stage. The workflow of the study is as follows:

1. Aggregate blocks into practical scheduling units using hierarchical clustering algorithm. These automatically aggregated blocks are referred to as mining-cuts. A hierarchical agglomerative clustering algorithm by Tabesh and Askari-Nasab (2011) is used to create mining-cuts based on a similarity index between blocks. Coordinates, grade, rock-type, and the beneath cluster are the weighted attributes that have been used in calculating similarity matrix among blocks. For more details on the clustering algorithm refer to Askari-Nasab et al. (2010b); Askari-Nasab et al. (2010c); and Tabesh and Askari-Nasab (2011). Block aggregation is required for two reasons: (i) to generate a practical mining schedule that follows a selective mining unit, (ii) to reduce the number of variables, especially binary variables in the MILP formulation to make it computationally tractable. Aggregation is necessary to generate near-optimal realistic mine plans in a reasonable CPU time that could be considered useful in real-world mine planning. Block level resolution mathematical production scheduling models even if could be solved, would generate production schedules that are not practical from equipment access point of view. In the authors' experience the block level resolution mathematical programming models, tend to generate schedules that move the loading equipment with no restrictions. Also, it creates numerous drop-cuts on each bench without considering the extra cost of these drop-cuts. None of these are practical from mining point of view.
2. Generate the optimal long-term multi-period open pit production schedule. This is the strategic yearly production schedule with the objective of net present value (NPV) maximization. The strategic plan honors mining-fleet capacity, shovel limits, grade blending requirements, uniform feed to the processing plant, and most importantly inherently optimizes the cut-off grade requirements. An MILP production scheduler implemented based on the models presented by Askari-Nasab et al. (2010a); Askari-Nasab et al. (2011a); and Ben-Awuah and Askari-Nasab (2011) is used for the strategic mine production scheduling in this study.
3. Generate the optimal medium-term multi-destination open pit production schedule. This is the monthly production schedule with the objective of minimizing the total operating costs. The medium-term production scheduling mathematical programming model is the focus of this paper. It is an extension to the previous work by Eivazy and Askari-Nasab (2012); and Askari-Nasab et al. (2011a). The medium-term MILP model generates an optimal mining sequence while: (i) delivers demanded ore tonnage to processing plants, (ii) satisfies quality requirements of processing plants for each element in each period, (iii) satisfies quantity requirements of each process, and (iv) minimizes deviation from the optimal long-term schedule.

The time horizon of the proposed medium-term model expands one to three years with a monthly resolution. The input data into the long and medium-term production scheduler, such as geological block model, grades, costs, prices, recoveries, and practical mining constraints are based on the best point estimates available at the time of optimization. Any change in the input data requires a re-run of the model with the new input parameters. This is aligned with the mining industry practice of updating yearly, quarterly, monthly, and weekly mine plans as new data becomes available and uncertainty is reduced overtime. In other words, the limitation of this approach is similar to any deterministic model in capturing uncertainty.

The proposed hierarchical mine production scheduling model has the following contributions and improvements over the previous research: 1) considering multiple destinations – stockpiles, processing plants, and waste dumps – to model the real world mining operations, 2) decision making on selection of routes/ramps to minimize the haulage cost, and 3) modeling stockpiles with an MILP model.

The paper is organized as follows: section 2 presents the medium-term mine production scheduling model. In section 3, the proposed model is applied to an iron ore open-pit mine. Results of the application are indicated in terms of mine production schedule and head grade of magnetic weight recovery (MWT) of iron ore. Finally, the conclusions and future research directions are presented in section 4.

2. The proposed model

In this paper, a hierarchical open-pit mine production scheduling model including long-term and medium-term models is proposed. The main objective of the current paper is to find the optimal medium-term mining schedule for an open-pit mine linked to the optimal long-term strategic plan. The open-pit mine considered operates with S stockpiles, P processing plants, and W waste dumps. Waste materials extracted from the mine are sent to the waste dumps. The processes are fed with the ore extracted from the mine and from the buffer stockpiles. The deposit has E elements, one of which is labeled as the major product in accordance with strategic decisions made by the mine's managers. Stockpiles are modeled to:

- Top up processes at the end of a period when the mining rate has been met;
- Buffer the processes from the changes in materials extraction from the mine to avoid future stripping hurdles;
- Blend materials to meet head grade constraints, and
- Store materials above economic cut-off when there is no process available in the pre-stripping stage.

Each destination can receive material with a specific rock-type or a combination of rock-types. Also, each element has an acceptable grade range at different processes and stockpiles. Generally, stockpiles are separated by rock-type and grade range of ore. In addition, it is assumed that stockpiles are homogeneous and the ore reclaimed from each stockpile has a certain grade equivalent to the average grade of stockpile material. Regarding the haulage of materials, there are numerous routes to send extracted materials from the mine to different destinations. Mined material of each block could be hauled to corresponding destinations through some routes. A number of ramps have also been designed in each bench for haulage of extracted material of blocks in the bench.

2.1. Medium-term mine production scheduling

Data required for the proposed formulation of medium-term open-pit mine production scheduling are as follows:

- Specifications of orebody and deposit such as rock and grade variability, rock tonnage, and mineral tonnage in the form of a block model;
- Capacity of processes and mining equipment;
- Long-term mine production schedule;
- Specifications of stockpiles and processes such as acceptable grade values; processing plant capacities, and storage capacity of stockpiles;
- Dumping capacity of each waste dump;
- Coordinates of the exit of the mine, processing plants, stockpiles, concentrators, ramps, pit limits and their technical specifications.

Also, the proposed MILP model makes a decision about the following issues:

1. The extraction sequence of blocks including amount of extraction of each block in each period through the time horizon (K periods with a span of t) in accordance with the long-term mine plan. Therefore, in the medium-term mine production scheduling, the optimal sequence of extraction of these blocks has to be determined;

2. The total amount of material that must be extracted from the mine in each period, which indicates the total amount of ore and waste materials that are mined and sent to different destinations in each period;
3. The destinations of mined materials. Here, the destinations for the extracted material of each block in each period are determined. This decision making is performed mainly regarding the rock-type of material, demand of processes, haulage cost of sending material to different possible destinations, grade of elements, and type of material (ore or waste); and
4. The amount of ore reclaimed from the stockpiles and sent to the processes in each period.

The proposed MILP model including objective function and constraints (Equations (1)-(26) and Equation (30)) are included in the Appendix. However, definition of parameters/decision variables and explanation of the proposed MILP formulation are mentioned as follows:

2.1.1. Parameters

1- t : period of scheduling ($t=1,\ldots,K$)
2- e : element e ($e=1,..,E$)
3- p : process p ($p=1\ldots P$)
4- s : stockpile s ($s=1\ldots S$)
5- w : waste dump w ($w=1\ldots W$)
6- r : ramp r
7- $J(n)$: set of blocks that must be extracted during the medium-term time horizon (set in long-term planning)
8- N : number of blocks in $J(n)$
9- ME^t : minimum fraction of each block that could be extracted in period t if it is going to be extracted
10- O_n : tonnage of mineralized zone in block n
11- R_n : rock tonnage in block n
12- g_n^e : grade of element e in block n
13- $VPB(n)$: set of vertical precedent blocks of block n
14- $N_{VPB(n)}$: number of blocks in set $VPB(n)$
15- $R(n)$: set of corresponding ramps for haulage of extracted materials of block n
16- $P(n)$: set of processes that can receive ore from block n
17- $SP(n)$: set of stockpiles that can receive ore from block n
18- $W(n)$: set of waste dumps that can receive waste material from block n
19- MC^t : unit mining cost in period t

20- WC_w^t : unit waste rehabilitation cost in period t for waste dump w,(this cost includes the unit cost of waste rehabilitation and unit cost of haulage of waste to waste dump w.)

21- PC_p^t : Unit processing cost in period t for process p, (this cost includes the unit cost of processing by process and unit cost of haulage of ore to process p.)

22- $RH_s^{t,p}$: Unit rehandling cost for stockpile s, sends ore to process p in period t, (this cost includes the unit cost of haulage of ore from stockpile s to process p.)

23- H^t : Unit haulage cost in period t, (this cost includes the unit cost of haulage of material inside pit to the pit exit.)

24- MU^t : maximum acceptable tonnage that could be mined based on maximum available mining capacity in period t

25- ML^t : minimum acceptable tonnage that could be mined based on minimum available mining capacity in period t

26- PU_p^t : maximum acceptable ore tonnage that process p can process based on maximum process p capacity in period t

27- PL_p^t : process p minimum ore acceptable tonnage in period t

28- $gu_p^{t,e}$: upper bound on acceptable grade of element e for process p in period t

29- $gl_p^{t,e}$: lower bound on acceptable grade of element e for process p in period t

30- $OGSP_s^e$: grade of element e in ore which is sent from stockpile s to processes

31- $UGSP_s^e$: upper bound of grade of element e in stockpile s

32- $LGSP_s^e$: lower bound of grade of element e in stockpile s

33- CSP_s : maximum storage capacity of stockpile s

34- IMC_s^e : initial metal content of element e in stockpile s

35- dE_n^r : distance of block n to exit by the route passing from ramp r

36- $P(SP(s))$: set of processes that can receive ore from stockpile s (destination of stockpile s)
$SP(P(p))$: set of stockpiles that can send ore to process p (source of process p)

2.1.2. Decision variables

1- u_n^t : fraction of block n extracted in period t $(n \in J(n))$

2- $uw_n^{t,w}$: fraction of block n extracted in period t and sent to waste dump w $(w \in W(n))$

3- $us_n^{t,s}$: fraction of block n extracted in period t and sent to stockpile s $(s \in SP(n))$

4- $up_n^{t,p}$: fraction of block n extracted in period t and sent to process p $(p \in P(n))$

5- b_n^t : binary variable, if block n is extracted in period t it gets value of one otherwise zero

6- $x_n^{t,r}$: amount of extracted material of block n in period t which is sent to exit by ramp r

7- $b_n^{t,r}$: binary variable, if ramp *r* is selected for haulage of extracted material from block *n* in period *t*, it gets value of one, otherwise zero

8- $y_s^{t,p}$: ore tonnage sent from stockpile *s* to process *p* in period *t* ($s \in SP(n)$ is the set of stockpiles that send ore to process *p)*

9- I_s^t : inventory of stockpile *s* at the end of period *t*

2.1.3. Elaboration of the MILP formulation

Equation (1) presents the objective function. The objective function includes the following cost terms:

1. Total processing cost is the overall cost of processing the ore throughout *K* periods.
2. Total waste rehabilitation cost is the summation of costs for rehabilitating waste materials in the waste dumps during *K* periods.
3. Total rehandling cost is the total haulage cost of ore that is sent to processes from stockpiles in *K* periods.
4. Total haulage cost is the overall haulage cost of sending materials extracted inside the mine to the exit of the open-pit.

The objective function is minimized under some constraints that are reflected in Equations (2) to (26). Equation (2) shows the constraints regarding the complete extraction of all blocks; all blocks must be mined completely during *K* periods. As mentioned, from the long-term plan, a set of blocks that are going to be extracted over a long-term period, e.g. year, are determined. Equation (2) enforces obeying the long-term plan related to staying in the pit space in the given year of the mine life. An extracted portion of each block is sent to different destinations; this is presented by Equation (3). To ensure the entire extraction of mineralized zone, Equation (4) is imposed. According to Equation (4), all the mineralized material of each block are going to be extracted completely through *K* periods. In other words, after *K* periods, all the mineral portions of each block will be extracted completely. Equations (2) and (4) lead to complete mining of waste portions of each block. Equation (2) ensures complete mining of each block, while Equation (4) enforces complete extraction of mineralized material of each block.

Equation (5) defines the relationship between binary variable b_n^t and the continuous variable u_n^t. Based on Equation (5), only if a portion of block n is going to be mined, $u_n^t > 0$ the corresponding binary variable b_n^t is enforced to get a value of one. Equation (6) guarantees minimum extraction of blocks in each period. According to Equation (6), if a portion of a block is going to be mined, this portion cannot be less than a certain value. Equation (6) is imposed to avoid extraction of very small fraction of blocks in each period.

Equations (7) and (8) indicate limitations on capacity of mining equipment and processes, respectively. These constraints prevent the mine production in the medium-term plan to deviate from the long-term target production. In fact, these constraints enforce the medium /short-term mine schedule to completely obey the long-term plan in a given year of LOM. Equation (7) presents that the total rock tonnage mined in each period must be in a certain acceptable range, according to the available mining equipment capacity. Equation (8) forces that the total tonnage of ore sent to each process in each period must be in an acceptable range based on the process's capacity. Equation (9) guarantees the vertical precedence in extraction of blocks (slope constraint). Equation (10) imposes the head grade constraint of each process in each period. Each process can work with ore of a certain grade for each element. Thus, the average grade of elements in the ore sent to each process should be in a certain acceptable grade range specified for the process and each element. The numerator of Equation (10) represents the total amount of metal content of element e available in the ore sent to process p from stockpiles and the mine in period t. The denominator stands for ore tonnage sent to process p from stockpiles and the mine in period t. Equation (11) shows the balancing equation for inventory of stockpiles thorough K periods. Inventory of stockpiles at the end of period t is the summation of total received material in period t and the current inventory minus total ore tonnage reclaimed in period t. $\forall n{:}s \in SP(n)$ represents the set of blocks that can send ore to stockpile s. Equation (12) stresses that total ore tonnage sent out from each stockpile to the processes in each period should be less than the inventory of that stockpile at the beginning of that period. The main assumption hidden in Equation (12) is that stockpiles can send materials to processes only at the beginning of each period. Equation (13) constrains the grade value of element e in the ore sent from the mine to stockpiles to be in a certain acceptable range corresponding to each stockpile and element. The numerator and denominator of Equation (13) represent the total metal content tonnage of element e carried from the mine to stockpile p in period t and the total tonnage of ore sent from the mine to stockpile p in period t, respectively.

Equations (14) and (15) guarantee that after reclaiming ore from one stockpile, the average grade of elements in a stockpile remains in its acceptable grade range. Figure 1 indicates the procedure of reclaiming ore from a typical stockpile s in period t. Suppose $N_s^{e,t-1}$ stands for the metal content of element e in stockpile s at the beginning of period t (or at the end of period t-1). Since ore is sent with an average grade $OGSP_s^e$, the remaining metal content for element e is calculated by Equation (27). Also, the remaining ore tonnage available in stockpile s is calculated by Equation (28). As the average grade of element e in stockpile s should be between its predefined lower and upper bounds, then, Equation (29) is created. After re-organizing Equation (29), the two equations, Equations (14) and (15), are obtained. It should be mentioned that the metal content of element e in stockpile s in period t-1 is calculated by Equation (30).

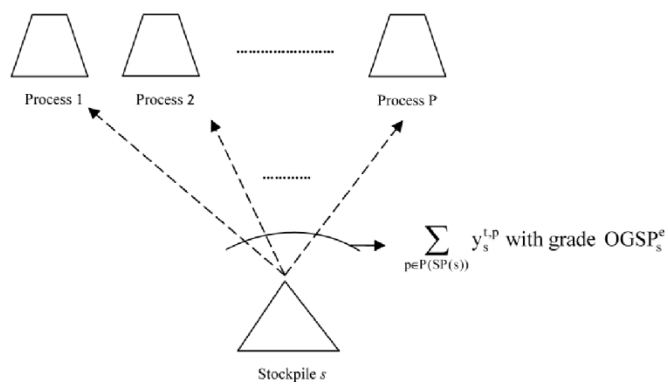

Figure 1. Procedure of feeding processes by stockpiles.

In Equation (30) the metal content of element e in period t-1 for stockpile s is calculated by the summation of all received metal tonnages in the previous periods and the initial metal content of element e, minus the total metal tonnages that the stockpile has sent to the processes by period t-1.

Equations (16) to (18) indicate that total rock tonnage mined from each block in period t is hauled to destinations by just one of the possible ramps that the block can access. Here, the decision making is on the selection of one possible ramp for haulage of extracted material of each block to destinations. Equation (16) stresses that total hauled material of block n in period t to different ramps equals to the total extracted material from that block in that period. Equations (17) and (18) indicate the extracted tonnage can be sent only to one of the possible ramps. Equations (19) to (26) represent the sign constraints regarding decision variables. Among these constraints, Equation (25) shows the limitation of storage for stockpiles.

3. Experimental results and discussion

This section is concerned with the verification of the proposed hierarchical model. The dataset of an iron ore open-pit mine located in southern Iran, is used to apply the proposed model. The main element of interest in the deposit is Iron (Fe). The processing plant uses magnetic separators; therefore, the main criterion in selecting ore to be sent to the concentrator is the magnetic weight recovery (percent MWT) of iron ore. The contaminants are phosphor and sulfur, considered as secondary elements to be controlled. The open-pit has 20 benches. For this pit, only one exit has been designed. Extracted materials are hauled to this unique exit through different routes/ramps and then sent to final destinations. Also, 34 ramp access points have been designed for this mine – on average, two ramp accesses for each bench. Table 1 presents general specifications of the case study. For this problem, two stockpiles, two processes, and one waste dump are considered. Stockpile 1 can only send ore to process 1 while stockpile 2 can only send ore to process 2. In addition, waste dump 1 has unlimited capacity. Figure 2 illustrates a schematic view of the location of processing plants, stockpiles, and waste dump relative to the pit exit.

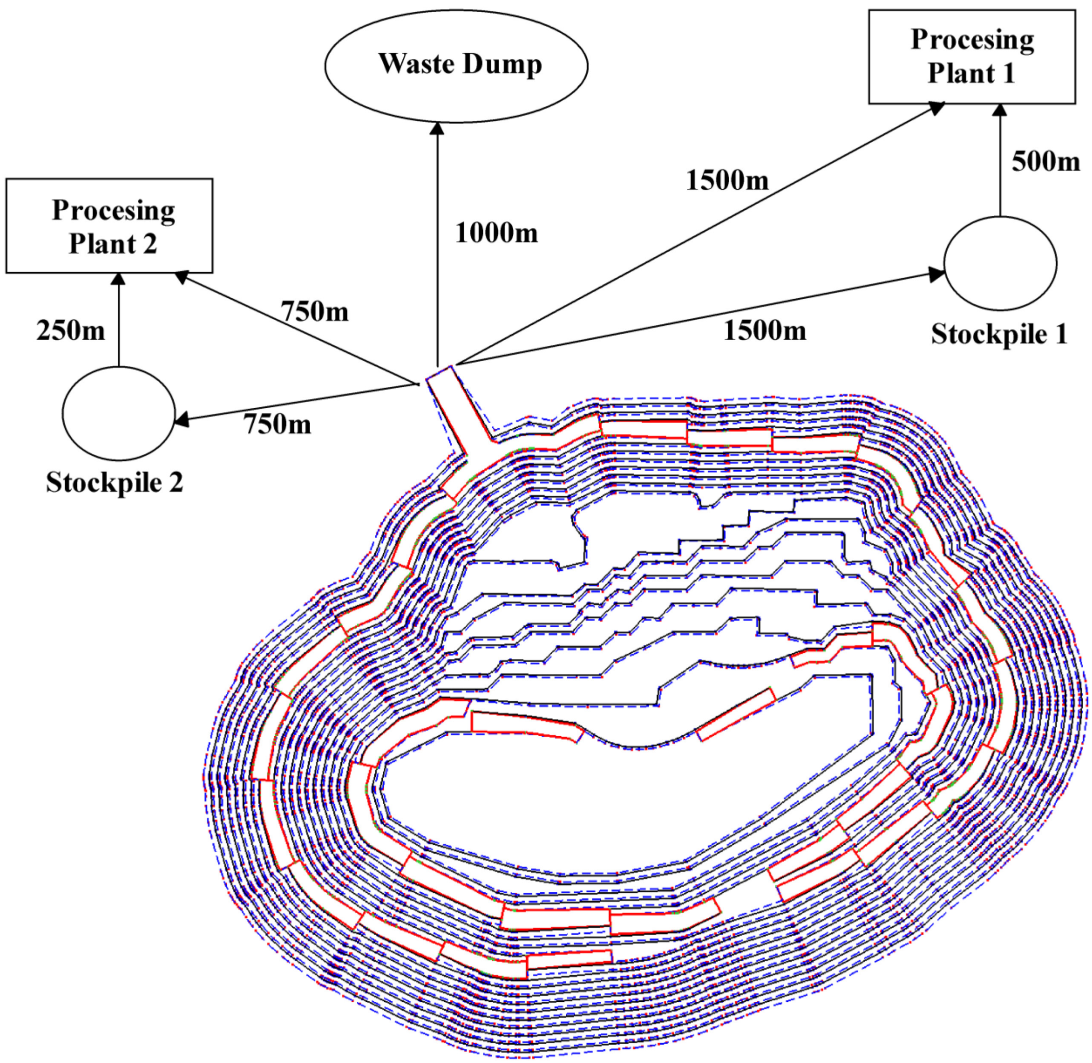

Figure 2. Schematic view of the location of processing plants and stockpiles in relevance to the pit exit point.

Table 1. Specifications of the iron ore open-pit mine.

Case	Value
Range of elevation	1470 to 1740 m
Z-value of the first bench (bottom of pit)	1470 m
Block size	25 ×25×15 m
Bench height	15 m
Number of benches	20
Number of blocks	21378
Number of designed ramps	34
Element of interest	MWT
Contaminant elements	Sulfur and phosphor
Total rock tonnage	463 million tonnes

The long-term mine production scheduling using the MILP model presented by Askari-Nasab et al. (2011b) is run for 17 scenarios. These scenarios are distinguished by the average number of blocks per mining-cut. Block aggregation is carried out to generate practical mining schedules that follow a selective mining unit. Also, aggregation reduces the number of variables, especially binary variables in the MILP formulation to make it computationally tractable. Real size open-pit mine planning problems at the block model resolution will lead to mathematical formulations with tens of millions of variables – a situation which is not tractable using current available hardware and software. The clustering algorithms aggregate blocks into selective mining units based on rock types, grades, and distances between blocks. The aggregated blocks are referred to as mining-cuts, which are consequently used in the aforementioned mathematical formulation. The algorithm used to aggregate blocks into mining-cuts in this study is hierarchical clustering technique by Tabesh and Askari-Nasab (2011).

Table 2 shows the results of long-term mine production scheduling for 17 scenarios. Also, Figure 3 illustrates the difference in NPV for the scenarios. All scenarios are solved with TOMLAB/CPLEX (Holmström, 2011) on on a 2.79 GHz two quad core and 24.0 GB RAM Intel ® Xeon CPU computer. Also, all scenarios are solved with 19 year time horizon, 1% gap, 27 million tonnes mining capacity between year 1 and 9 and 25 million tonnes mining capacity for year 10 and after. Also, process 1 and process 2 have 4 million tonnes of processing capacity per year.

The production schedule of all the scenarios are imported into mine planning software and the plan views and cross sections are examined in terms of practicality of the schedules generated. It has been decided that the schedule generated by using mining-cuts with 30 blocks per cut is the most practical schedule. This practicality is measured by the minimum mining-width required for the equipment to practically mine a face. Using mining-cuts with average of 30 blocks per cut, generates an NPV of $2010.03 million, which is 4.66% less than the highest NPV generated with mining-cuts with 20 blocks per cut. Figure 4 illustrates the long-term schedule over 19 years of mine-life. Figure 5 to Figure 7 show the MWT%, S%, and P% head grade for process 1 and process 2 over the mine-life starting from year 6. Also, five years of pre-stripping is enforced to make sure a uniform feed is provided to the processing plants in the later years while the mine is in production.

Table 2: Results of long-term mine scheduling for 17 cluster sizes.

Scenario Number	Averrage # of blocks in clusters	No. of clusters	CPU time (Second)	NPV (Million Dollars)	Differnce in NPV from worst case
1	1	21378	3683.83	2079.44	34.98%
2	2	10695	1109.00	2079.16	34.96%
3	5	4347	144.89	2078.87	34.94%
4	8	2804	95.87	2080.99	35.08%
5	12	2013	91.75	2081.62	35.12%
6	16	1561	25.47	2080.68	35.06%
7	20	1318	16.53	2081.81	35.13%
8	24	1030	20.17	2060.28	33.73%
9	30	874	15.75	2010.03	30.47%
10	35	778	21.43	1921.90	24.75%
11	40	690	7.20	1891.58	22.78%
12	45	614	14.53	1838.88	19.36%
13	50	510	2.86	1706.28	10.75%
14	60	446	3.81	1660.66	7.79%
15	70	394	2.66	1589.73	3.19%
16	80	349	2.42	1595.16	3.54%
17	90	321	3.02	1540.61	0.00%

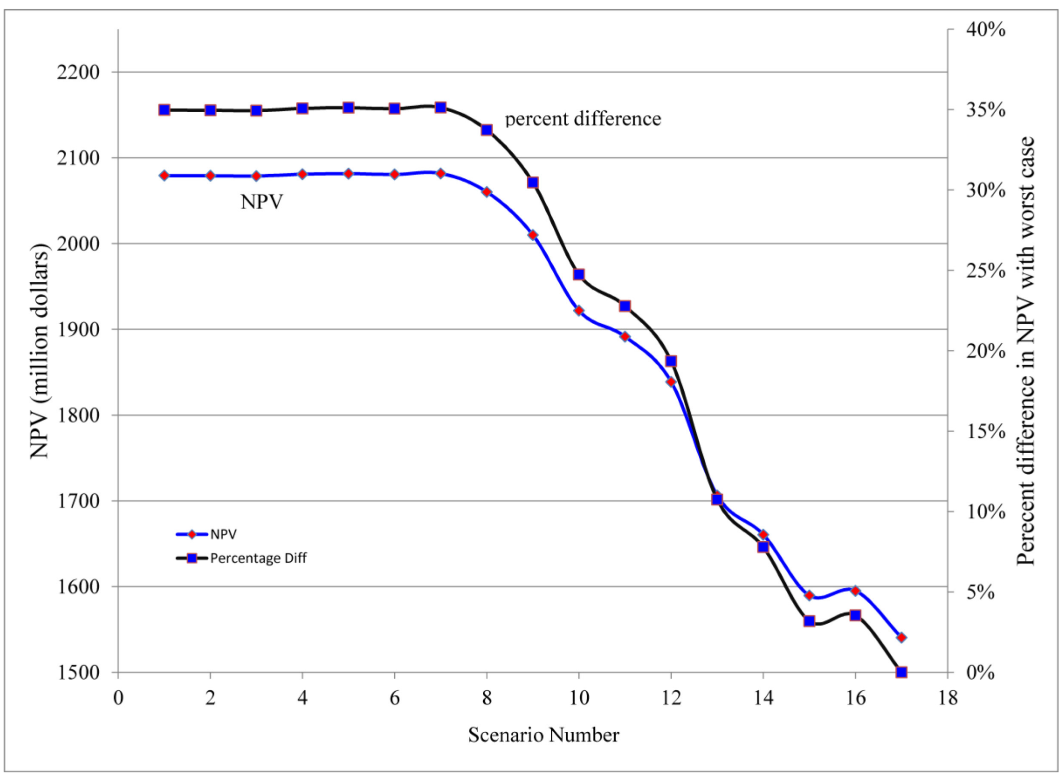

Figure 3. Comparison of difference in NPV for different mining-cut resolution.

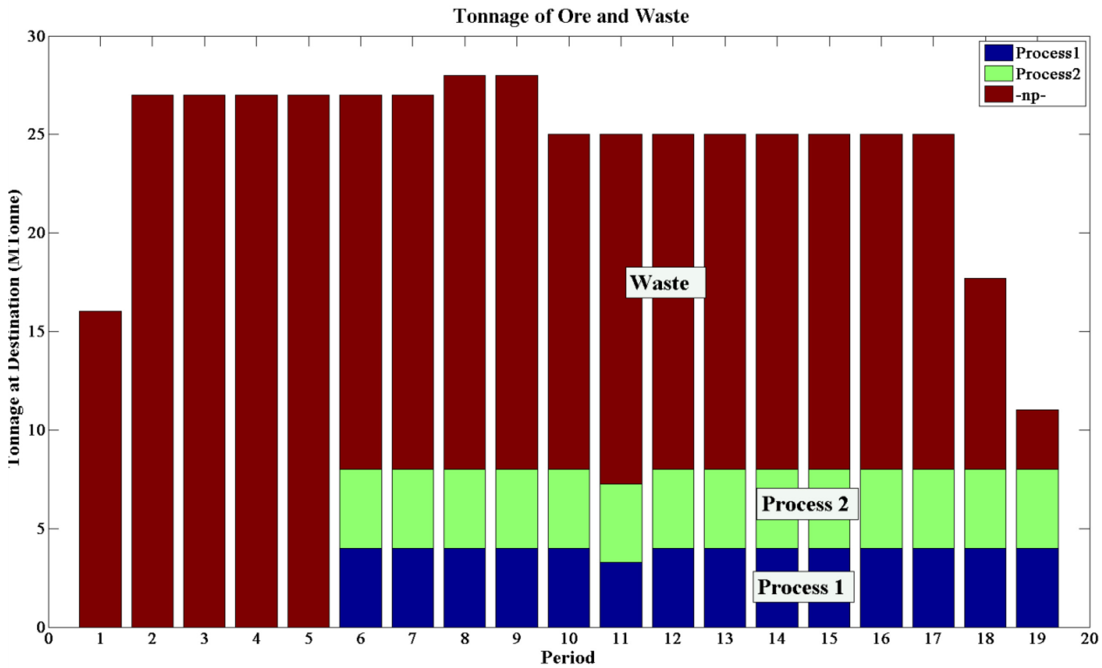

Figure 4. Production schedule generated for 30 blocks per mining-cut.

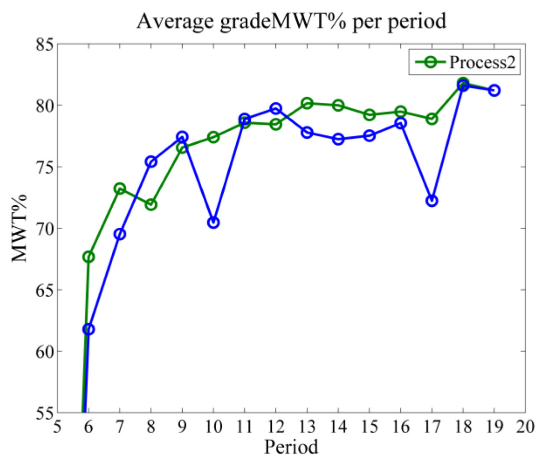

Figure 5. MWT head grade - process 1 and process 2.

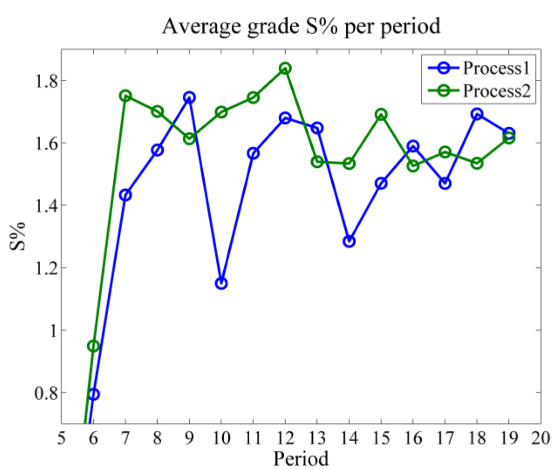

Figure 6. Sulphur head grade - process 1 and process 2.

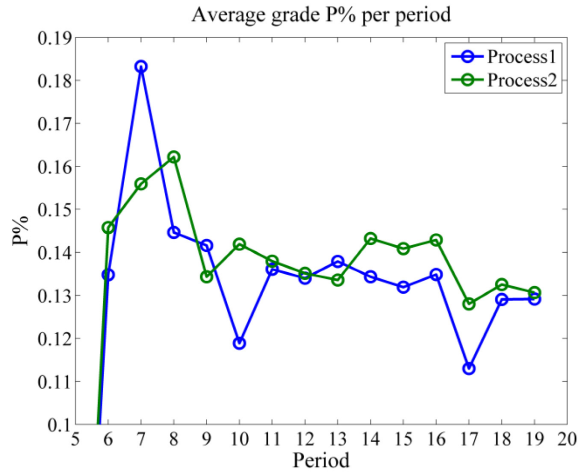

Figure 7. Phosphor head grade - process 1 and process 2.

To apply the proposed MILP model over a monthly plan, the generated long-term schedule for year 10 is considered. Blocks that are going to be extracted within year 10 are the input into the medium-term MILP model. Based on the long-term schedule, 1797 blocks have to be extracted in year 10. Total rock tonnage and ore tonnage of these blocks are 25 and 8 million tonnes, respectively. To apply the MILP model presented in the Appendix for medium-term, five scenarios are considered based on the number of blocks per mining-cuts generated. The scenarios 1 to 5 are called Cuts (5-10), Cuts (10-15), Cuts (20-25), Cuts (30-35), and Cuts (40-50). The first and second numbers inside the parenthesis represent the average number of blocks per mining-cut and the maximum number of allowable blocks per mining-cut. All five scenarios are solved to 1% gap, 2.2 million tonnes mining capacity per month, 12 month time horizon, and constraints on stockpiles and processing plants are presented in Table 3 and Table 4 as follows:

Table 3. Processing plants constraints and parameters.

Process	Lower Grade (%)			Upper Grade (%)			Capacity (Million Tonnes)	
	MWT	Sulfur	Phosphor	MWT	Sulfur	Phosphor	Minimum	Maximum
Process 1	73	0	0	78	2	0.3	0.32	0.4
Process 2	78	0	0	82	2	0.3	0.32	0.4

Table 4. Stockpile constraints and parameters.

Stockpile	Lower Grade (%)			Upper Grade (%)			Output Grade (%)			Initial Inventory (Million Tonnes)
	MWT	Sulfur	Phosphor	MWT	Sulfur	Phosphor	MWT	Sulfur	Phosphor	
Stockpile 1	71	0	0	74	3	0.3	72.5	1.5	0.15	0.0
Stockpile 2	75	0	0	77	2	0.35	76	1	0.17	0.0

Table 5 summarizes information about the five scenarios of medium-term production schedule at different mining-cut resolution. All scenarios had an objective function value (operating cost) within 1% of $137 million dollars.

Table 5. Results of medium-term plan scenarios with different cut numbers

Scenario	No. of mining-cuts	No. of variables	No. of integer variables	No. of constraints	CPU time (second)
Cuts (5-10)	330	40992	10572	29700	30.9
Cuts (10-15)	183	22692	5832	16722	26.5
Cuts (20-25)	101	12540	3216	9442	12.2
Cuts (30-35)	75	9324	2388	7146	8.2
Cuts (40-50)	53	6588	1680	5194	4.7

In each scenario, around 2.2 million tonnes of material is mined every month. In all scenarios, processing plants are running with full capacity with ore delivered from the mine and stockpiles in each month. The resulting schedules were imported into a mine planning design software to verify practicality from the mining point of view. Since the costs associated with these scenarios vary within 1% of each other, the practicality of the schedule was used as the main measure of selecting the mining-cut size for medium-term planning. The scenario with 20 blocks per cut was found to be the most practical production schedule.

Figure 8 shows the trend of the head grade of MWT in processing plant 1 and 2 for five scenarios. Figure 8 illustrates how head grade of process 1 is controlled between 73-78% MWT and the head grade of process 2 is controlled between 78-82% MWT. Evidently, the MWT head grade is controlled while generating an optimal monthly schedule minimizing the operating costs. Figure 8 shows that the MWT grade fed to process 1 mainly changes around the 78% defined upper bound for MWT grade. Also, the head grade of MWT in process 2 changes around the 78% defined lower bound.

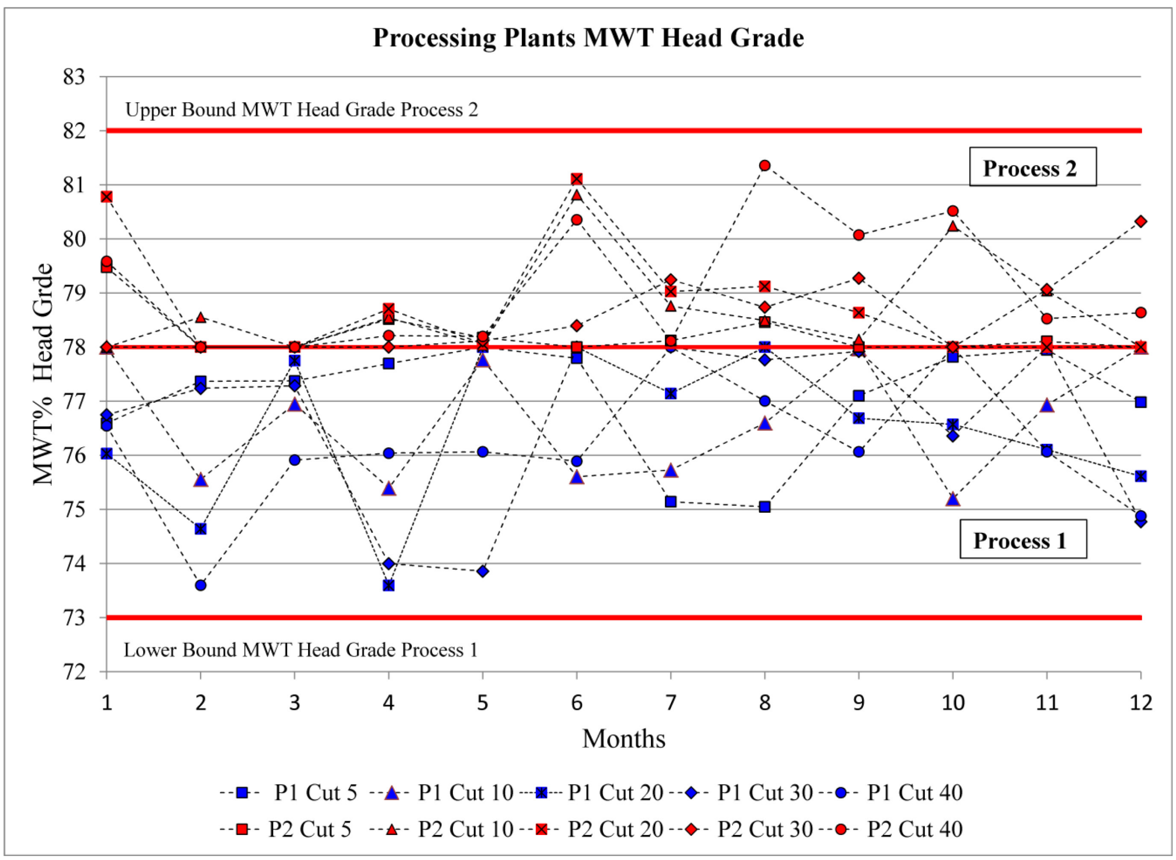

Figure 8. Head grade of processing plants 1 and 2 for the five mining-cut scenarios of the medium-term plan. Process 1 head grade is controled between 73-78% MWT and Process 2 head grade is controled between 78-82% MWT.

Figure 9 shows the plots of MWT grade of five scenarios in stockpile 1 and 2. The acceptable range of material sent to stockpile 1 varies between 71-74% MWT. Whereas, the grade range for stockpile 2 is between 75-77% MWT. In the proposed MILP model, the stockpiles are assumed to be homogenous. Therefore, to generate valid production schedules, material with a limited grade range must be sent to the stockpiles. The main assumption for reclamation of material is that the reclaimed grade of material is equal to the average grade of the upper bound and lower bound of the stockpile in each period. The model tracks the total metal content as well, so the total tonnage of iron ore sent and reclaimed from the stockpile is balanced. Figure 9 illustrates the average grade present in each stockpile in each month. Also, Figure 9 illustrates the grade reclaimed from the stockpiles by the model in each period. Material in stockpile 1 is reclaimed with an average grade of 72.5% MWT and stockpile 2 is reclaimed with an average grade of 76% MWT. According to Figure 9, the actual MWT grade of stockpiles changes around the assumed output grade. For example, in Figure 9, the MWT grade of stockpile 1 for scenario 3 is near to its defined output grade of MWT. For the same stockpile, MWT grade for the second scenario changes within less than 1% of the output grade. The scenarios that are not plotted did not have any ore in the stockpile.

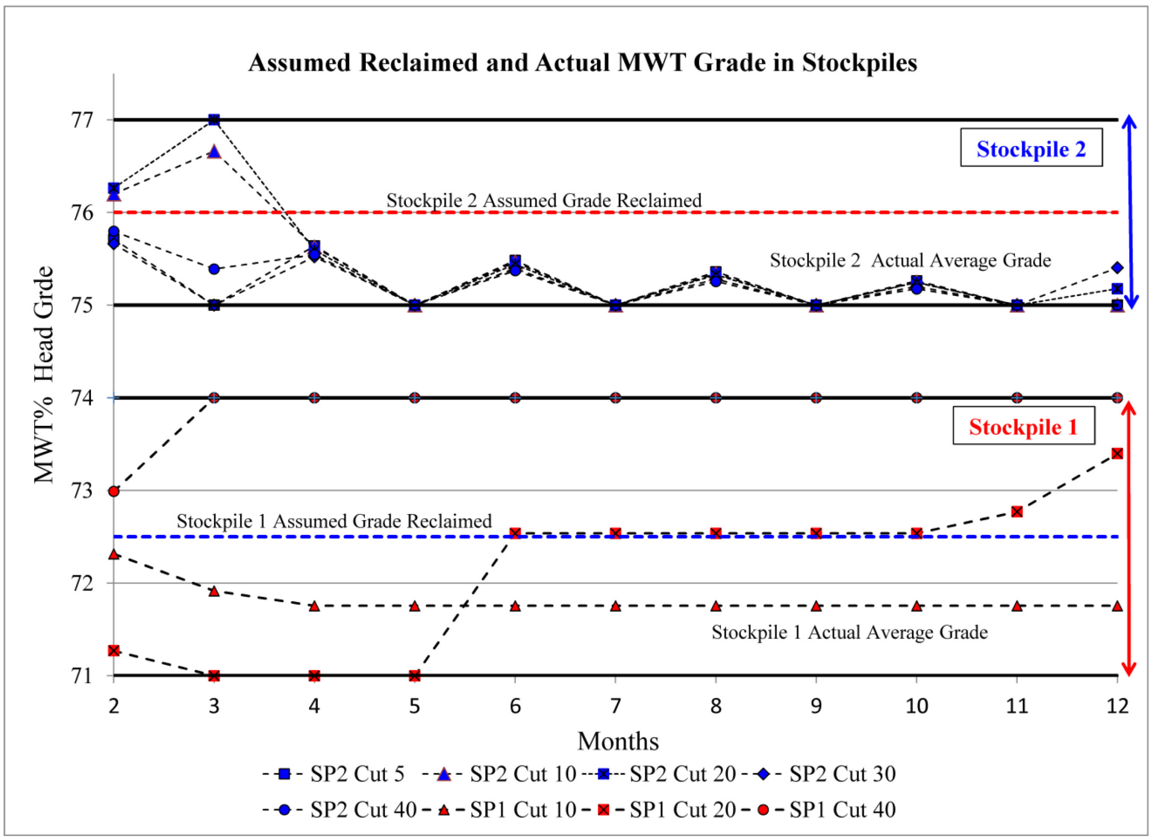

Figure 9. Assumed and actual grades reclaimed from stockpiles.

In the previous steps the decision was made for the right size of the mining-cuts in medium-term planning. The average of 20 blocks per mining-cut generated a practical monthly mining schedule. Figure 10 illustrates the plan view of bench 1590m in year 10 – mining-cuts generated based on average 20 blocks per mining-cut. In the next step, three scenarios based on specific mining direction are assessed. The proposed medium-term MILP model with 20 blocks per mining-cut is applied to three scenarios as follows:

1. Scenario 1: implementing the proposed MILP without imposing any horizontal mining direction. Figure 11 shows the plan view of bench 1590m in year 10 – monthly schedule generated with no horizontal extraction precedence among mining-cuts. The numbers on the plan view represent the month of extraction of blocks.
2. Scenario 2: implementing the proposed MILP with imposing North-West to South-East (NW-SE) horizontal directional extraction. In this scenario, precedence of extraction of materials in each bench is performed from the North-West corner of bench and advances to South-East corner. Figure 12 illustrates the plan view of bench 1590m in year 10 – monthly schedule generated with NW-SE horizontal extraction precedence among mining-cuts.
3. Scenario 3: applying the proposed MILP with imposing South-East to North-West (SE-NW) horizontal extraction. This scenario enforces the precedence of extraction of materials in each bench from the South-East corner and advancing to North-West corner. Figure 13 shows the plan view of bench 1590m in year 10 – monthly schedule generated with SE-NW horizontal extraction precedence among mining-cuts.

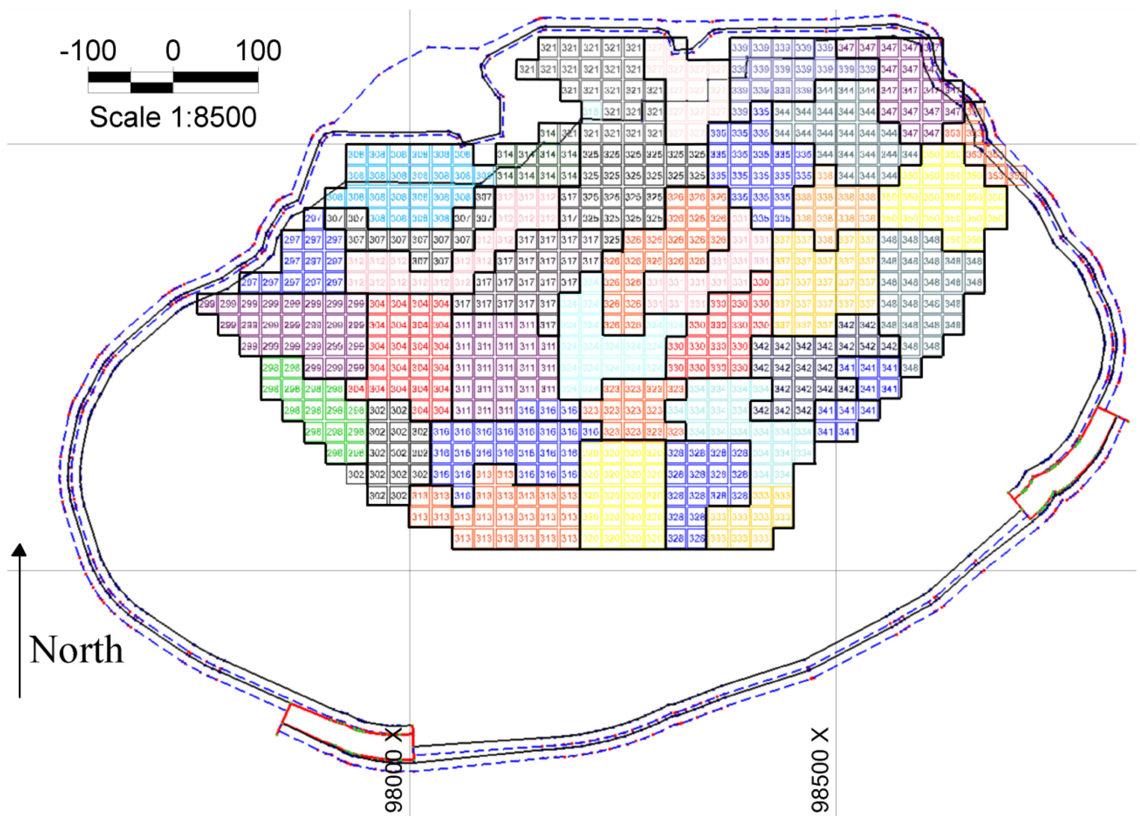

Figure 10. Plan view of bench 1590m in year 10 – mining-cuts generated based on average 20 blocks per mining-cut – units in meters.

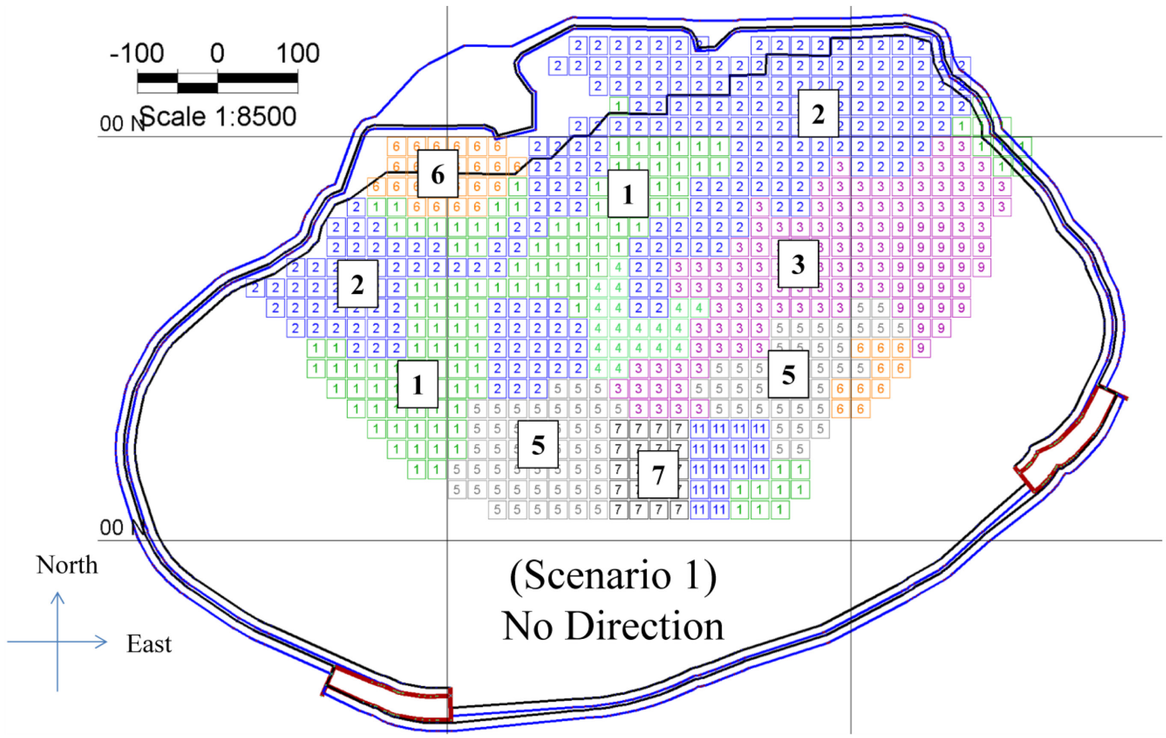

Figure 11. Plan view of bench 1590m in year 10 – monthly schedule generated with no horizontal extraction precedence among mining-cuts – units in meters – numbers on the plan represent the month of extraction.

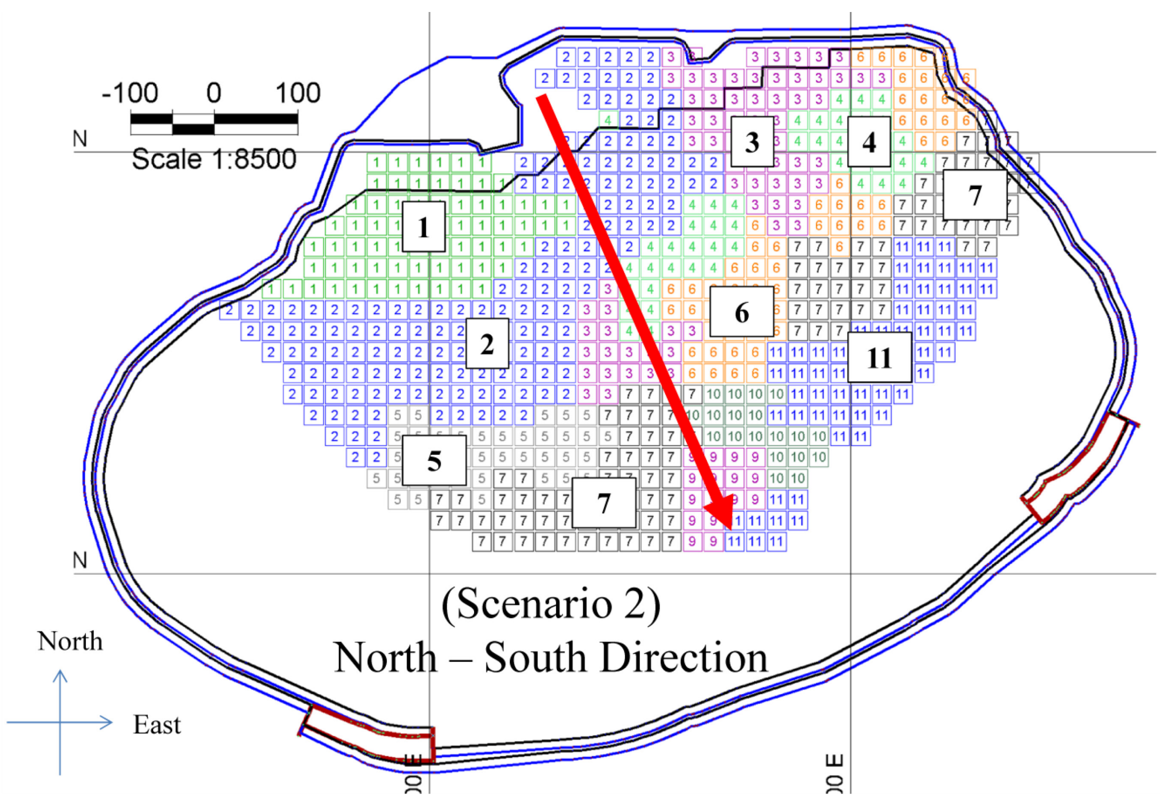

Figure 12. Plan view of bench 1590m in year 10 – monthly schedule generated with NW-SE horizontal extraction precedence among mining-cuts – units in meters – numbers on the plan represent the month of extraction.

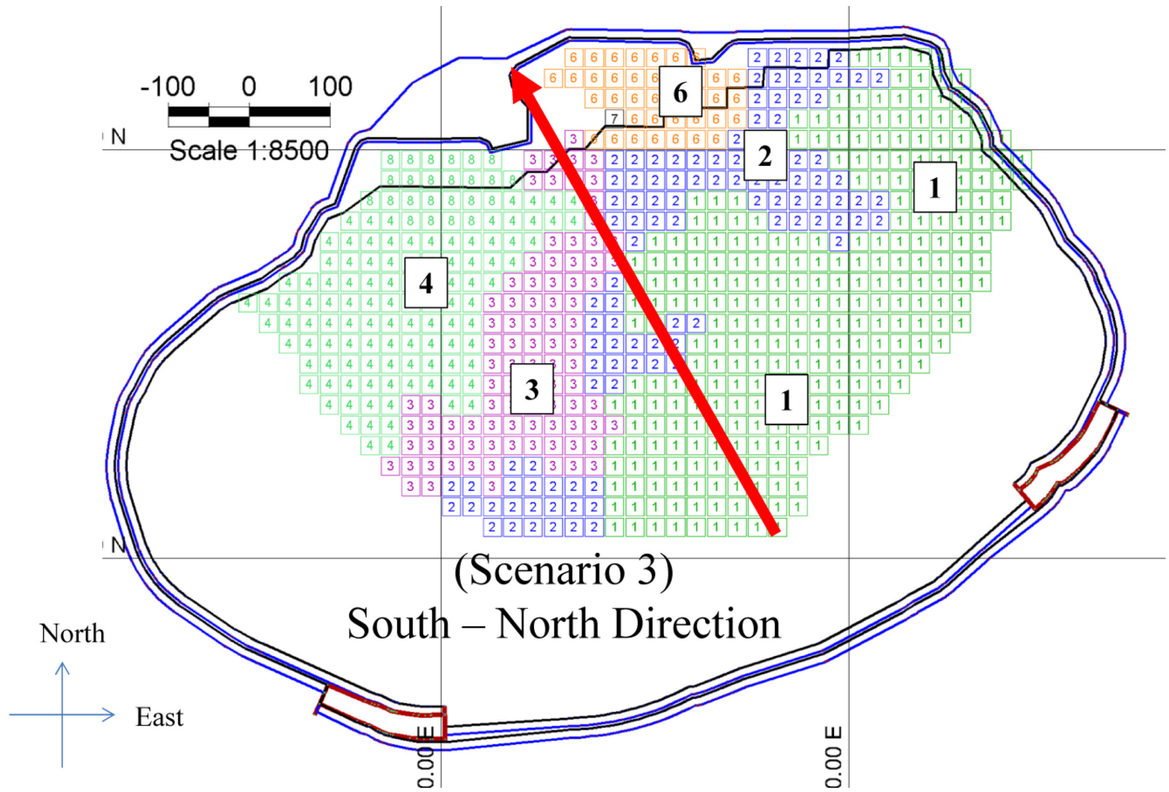

Figure 13. Plan view of bench 1590m in year 10 – monthly schedule generated with SE-NW horizontal extraction precedence among mining-cuts – units in meters – numbers on the plan represent the month of extraction.

The operating costs of the three scenarios are: \$136.9M – no mining direction, \$137.2M NW-SE mining direction, and \$137.3M SE-NW mining direction. Obviously, the first scenario should have the least total cost because the other scenarios contain an extra set of constraints – preferred horizontal mining direction – which makes the feasible solution space smaller. The difference between worst and best case from operating cost point of view is 0.3%, which for a monthly plan over a period of a year is negligible. The decision is made based on checking the minimum mining-width generated by each scenario to make sure a smooth progression of mining faces over the one year planning horizon is carried out. Scenario 3 with SE-NW mining direction is chosen as the best scenario. Figure 14 illustrates the production schedule of three scenarios based on mining direction precedence.

4. Conclusions and future work

Mine production planning and scheduling is a challenging problem in the mining industry. Since production planning and scheduling generate plans for extraction of ore as the most valuable asset of the mine, it is an important tool for the profitability of mining operations. In this paper, a hierarchical open-pit mine production scheduling model has been proposed. The first step of the proposed model includes aggregating blocks into mining-cuts based on the hierarchical clustering algorithm proposed by Tabesh and Askari-Nasab (2011). The second step, is the long-term mine production scheduling optimization using the mixed integer linear programming model proposed by Askari-Nasab et al. (2011b). This model generates the optimal strategic mine plan over the mine-life with the objective of net present value maximization. In the last step, the long-term strategic plan is broken down into an optimal monthly plan using the medium-term mathematical programming model presented in this paper.

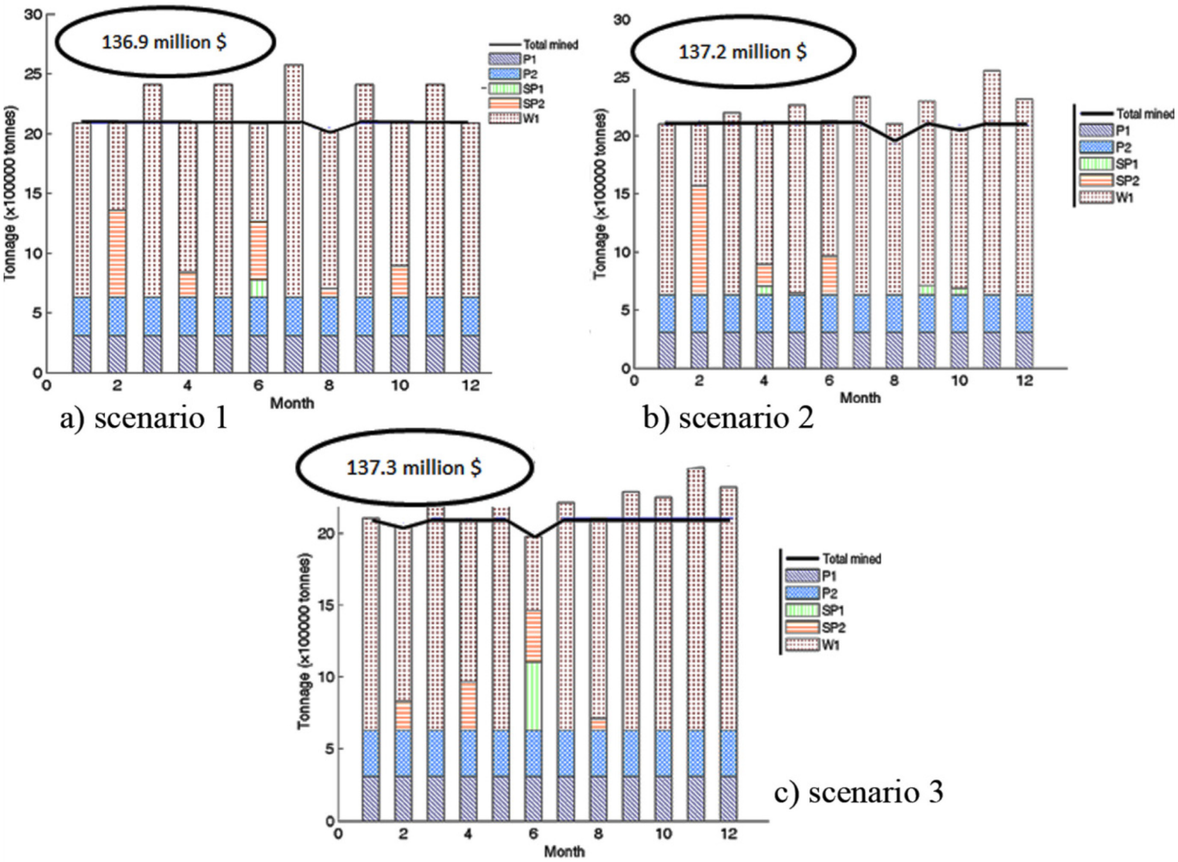

Figure 14. Production schedule of three scenarios based on mining direction precedence.

The proposed MILP model considers different destinations, e.g. processes, stockpiles, and waste dumps. The objective function is to minimize the total cost of mining and haulage, processing, rehandling, and waste rehabilitation. This function is subject to different mining operational constraints such as head grade, precedence among mining-cuts, mining capacity, and processing capacity. The proposed medium-term model has the following significant contributions:

- Multiple destinations modeling: the proposed formulation takes into account multiple destinations, stockpiles, processes, and waste dumps to model the real world medium-term mining operations more realistically.
- Route/ramp selection: the proposed mathematical model decides on the selection of the routes/ramps that incur the minimum haulage costs, while meeting all the desired constraints imposed by the mine planner.
- Modeling stockpiles using mathematical programming: The introduction of stockpiles into production scheduling mathematical models creates non-linear formulations, which practically can't be solved. A linear mathematical model is presented in this paper that realistically models homogenous stockpiles.

To verify the hierarchical mine production scheduling model, a dataset of an open-pit iron ore mine has been used. First, the long-term mine production schedule is generated. Then, the MILP model for the medium-term scheduling model is applied for selected year 10. All blocks to be extracted in year 10 are scheduled in 12 months by applying the MILP model. Due to the large number of blocks, they are aggregated into a number of mining-cuts. Five scenarios have been considered to aggregate the blocks at different resolutions. The MILP model has been solved by using TOMLAB/CPLEX for five scenarios. Results of applications have been shown and analyzed in terms of mine production schedules with different mining directions. Also, the trend of head grade and the grade in stockpiles have been assessed.

To improve the proposed hierarchical mine production scheduling model, the following future research directions are recommended:

- Simulation study of the developed schedules: discrete event simulation could capture the uncertainties in equipment availability, utilization, break-down times, and unscheduled maintenance. Also, the variability in cycle times due to seasonal and different road and ramp conditions could be taken into account. Considering these uncertainties using simulation and examining the schedules can show the strengths and weaknesses of the hierarchical model in detail.

- Extension of the MILP formulation for pits with multiple exit locations: in this paper, the proposed model only considered one exit point for sending the extracted material to different destinations. Adjusting the MILP model with multiple exit spots makes the model more realistic while complicating the model, especially in the selection of routes/ramps.

5. Appendix -MILP formulation

$$\left[\begin{array}{l} Minimize\ Total\ Cost = \underbrace{\sum_{t=1}^{K}\sum_{n=1}^{N} R_n \cdot u_n^t \cdot MC^t}_{total\ mining\ cost} + \underbrace{\sum_{t=1}^{K}\sum_{n=1}^{N}\sum_{w\in W(n)} uw_n^{t,w} \cdot R_n \cdot WC_w^t}_{total\ waste\ rehabilitation\ cost} + \\ \underbrace{\sum_{t=1}^{K}\sum_{n=1}^{N}\sum_{p\in P(n)} up_n^{t,p} \cdot R_n \cdot PC_p^t + \sum_{t=1}^{K}\sum_{p=1}^{P}\sum_{s\in SP(P(p))} y_s^{t,p} \cdot PC_p^t}_{total\ processing\ cost} + \\ \underbrace{\sum_{t=1}^{K}\sum_{s=1}^{S}\sum_{p\in P(SP(s))} y_s^{t,p} \cdot RH_s^{t,p}}_{total\ rehandling\ cost} + \underbrace{\sum_{t=1}^{K}\sum_{n=1}^{N}\sum_{r\in R(n)} x_n^{t,r} \cdot dE_n^r \cdot H^t}_{total\ haulage\ cost} \end{array}\right] \tag{1}$$

Subject to:

$$\sum_{t=1}^{K} u_n^t = 1,\ \forall n = 1...N \tag{2}$$

$$\sum_{p\in P(n)} up_n^{t,p} + \sum_{s\in SP(n)} us_n^{t,s} + \sum_{w\in W(n)} uw_n^{t,w} = u_n^t,\ \forall n = 1...N, \forall t = 1...K \tag{3}$$

$$R_n \cdot \sum_{t=1}^{K}\Big(\sum_{p\in P(n)} up_n^{t,p} + \sum_{s\in SP(n)} us_n^{t,s}\Big) = O_n,\ \forall n = 1...N \tag{4}$$

$$u_n^t \le b_n^t,\ \forall n = 1...N, \forall t = 1...K \tag{5}$$

$$ME^t \cdot b_n^t \le u_n^t,\ \forall n = 1...N, \forall t = 1...K \tag{6}$$

$$ML^t \le \sum_{n=1}^{N} R_n \cdot u_n^t \le MU^t\,,\ \forall t = 1...K \tag{7}$$

$$PL_p^t \le \sum_{\forall n:\, p \in P(n)} R_n \cdot up_n^{t,p} + \sum_{s \in SP(P(p))} y_s^{t,p} \le PU_p^t,\ \forall t = 1...K, \forall p = 1...P \tag{8}$$

$$N_{VPB(n)} \cdot b_n^t \le \sum_{\tau=1}^{t} \sum_{i \in VPB(n)} u_i^\tau,\ \forall n = 1...N, \forall t = 1...K \tag{9}$$

$$gl_p^{t,e} \le \frac{\sum_{s \in SP(P(p))} y_s^{t,p} \cdot OGSP_s^e + \sum_{\forall n:\, p \in P(n)} up_n^{t,p} \cdot R_n \cdot g_n^e}{\sum_{s \in SP(P(p))} y_s^{t,p} + \sum_{\forall n:\, p \in P(n)} up_n^{t,p} \cdot R_n} \le gu_p^{t,e}, \tag{10}$$
$$\forall t = 1...K, \forall p = 1...P, \forall e = 1...E$$

$$\sum_{\forall n:\, s \in SP(n)} R_n \cdot us_n^{t,s} - \sum_{p \in P(SP(s))} y_s^{t,p} + I_s^{t-1} = I_s^t,\ \forall t = 1...K, \forall s = 1...S \tag{11}$$

$$\sum_{p \in P(SP(s))} y_s^{t,p} \le I_s^{t-1},\ \forall t = 1...K, \forall s = 1...S \tag{12}$$

$$LGSP_s^e \le \frac{\sum_{\forall n:\, s \in SP(n)} us_n^{t,s} \cdot R_n \cdot g_n^e}{\sum_{\forall n:\, s \in SP(n)} us_n^{t,s} \cdot R_n} \le UGSP_s^e, \tag{13}$$
$$\forall t = 1...K, \forall s = 1...S, \forall e = 1...E$$

$$\sum_{p \in P(SP(s))} y_s^{t,p} \le \frac{N_s^{e,t-1} - LGSP_s^e \cdot I_s^{t-1}}{OGSP_s^e - LGSP_s^e}, \tag{14}$$
$$\forall t = 1...K, \forall s = 1...S, \forall e = 1...E$$

$$\sum_{p \in P(SP(s))} y_s^{t,p} \le \frac{-N_s^{e,t-1} + UGSP_s^e \cdot I_s^{t-1}}{-OGSP_s^e + UGSP_s^e}, \tag{15}$$
$$\forall t = 1...K, \forall s = 1...S, \forall e = 1...E$$

$$\sum_{r \in R(n)} x_n^{t,r} = R_n \cdot u_n^t,\ \forall n = 1...N, \forall t = 1...K \tag{16}$$

$$x_n^{t,r} \le M.b_n^{t,r},\ \forall n = 1...N, \forall t = 1...K, \forall r = 1...R(n) \tag{17}$$
$$(M\ is\ a\ large\ number)$$

$$\sum_{r \in R(n)} b_n^{t,r} = b_n^t,\ \forall n = 1...N, \forall t = 1...K \tag{18}$$

$$0 \le u_n^t \le 1, \forall n = 1...N, \forall t = 1...K \tag{19}$$

$$0 \le uw_n^{t,w} \le 1, \forall n = 1...N, \forall t = 1...K, \forall w \in W(n) \tag{20}$$

$$0 \le up_n^{t,p} \le 1, \forall n = 1...N, \forall t = 1...K, \forall p \in P(n) \tag{21}$$

$$0 \le us_n^{t,s} \le 1, \forall n = 1...N, \forall t = 1...K, \forall s \in SP(n) \tag{22}$$

$$0 \le x_n^{t,r}, \forall n = 1...N, \forall t = 1...K, \forall r \in R(n) \tag{23}$$

$$0 \le y_s^{t,p}, \forall t = 1...K, \forall s = 1...S, \forall p \in P(SP(s)) \tag{24}$$

$$0 \le I_s^t \le CSP_s \quad \forall t = 1...K, \forall s = 1...S \tag{25}$$

$$b_n^t \text{ and } b_n^{t,r} = 0/1, \forall n = 1...N, \forall t = 1...K, \forall r \in R(n) \tag{26}$$

$$N_s^{e,t-1} - OGSP_s^e \cdot \sum_{p \in P(SP(s))} y_s^{t,p}, \quad \forall e = 1...E \tag{27}$$

$$I_s^{t-1} - \sum_{p \in P(SP(s))} y_s^{t,p} \tag{28}$$

$$LGSP_s^e \le \frac{N_s^{e,t-1} - OGSP_s^e \cdot \sum_{p \in P(SP(s))} y_s^{t,p}}{I_s^{t-1} - \sum_{p \in P(SP(s))} y_s^{t,p}} \le UGSP_s^e, \ \forall e = 1...E \tag{29}$$

$$N_s^{e,t-1} = \sum_{\tau=1}^{t-1} \sum_{\forall n: s \in SP(n)} us_n^{\tau,s} \cdot R_n \cdot g_n^e + IMC_s^e - \sum_{\tau=1}^{t-1} \sum_{p \in P(SP(s))} y_s^{\tau,p} \cdot OGSP_s^e, \quad \forall e = 1...E, \forall t = 1...K, \forall s = 1...S \tag{30}$$

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