A Mathematical Programming Model for Optimal Truck-Shovel Allocation

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Abstract

The fundamental objective of any mine plan is to maximize the mine profit by extracting ore at the lowest possible cost over the mine life. Since the costs associated with the operation of trucks and shovels as resources are significant, the optimum allocation and dispatching of these resources is an essential issue. This paper presents a Mixed Integer Linear Programming (MILP) model to determine the optimum number of shovels and trucks required to meet the short-term plan's goals. Also, the model takes into account the allocation of trucks and shovels to mining faces. This model minimizes operational costs, while attempting to meet the production demand and consider technological constraints.

1. Introduction and literature review

Mine planning consists of two the planning level and the operational level. The fundamental objective of any mine plan is to maximize the mine profit by extracting ore at the lowest possible cost over the mine life. Geological, operational, technological and financial requirements constrain this objective. Mining equipment is one of the most expensive necessities of a mine. At the operational level, the goal is to use the trucks and shovels efficiently, minimizing the resources required which results in reduction in hauling, operating and maintenance costs, while meeting production targets.

In this paper resource allocation refers to the allocation of trucks and shovels to mining faces over a shift. Shovels are used to extract the ore and trucks are used to haul the ore to various destinations for further processing. Since the costs associated with the operation of trucks and shovels are significant, the allocation of these resources is an essential issue. Many mining companies try to allocate the trucks and shovels to produce an optimal schedule in a way that the operating costs are minimized and the utilization of resources is maximized through the planning horizon. Increasing the efficiency of the trucks and shovels results in savings.

Allocation of resources in the mining context is a complex and important process. The main factor that makes the allocation problem complex is the uncertainties in the operation of trucks and shovels such as truck cycle time, load tonnage, and truck and shovel reliabilities. These factors affect the production of a mine. Ignoring such uncertainties in the operation of a mine could result in deviations from the optimal plans. Any deviation from the production plan because of operational uncertainties increases the overall cost. One of the solutions to prevent the risk of not meeting the production demand is to provide extra trucks and shovels which again imposes extra costs to the system.

In many of the open pit mining systems, dispatching is considered a two level process. The first level is to allocate the shovels and the trucks at the beginning of the period and the second level is to implement the solution for real time operations. Most of the studies develop a mathematical programming model to solve the allocation problem. They usually aim to minimize the overall operating costs or maximize the profit, while meeting the target production. Other studies apply heuristic rules or stochastic approaches to solve the allocation and dispatching problems.

Li (1990) proposed a new dispatching methodology called intertruck-time deviation to keep truck flows as close to the optimum as possible. This methodology is based on material transportation rather than operational costs and can be used in real time open pit mining operations. Czaplicki (1999) proposed a procedure based on the queuing theory to assess the optimum number of operating trucks and reserve trucks in a mine. Two types of truck-and-shovel systems are considered: (1) one shovel and a certain number of trucks and (2) a certain number of shovels and trucks. This study considers many important technical and stochastic properties of the system.

A mixed integer programming (MIP) model was proposed by Topal and Ramazan (2010) to produce an optimum schedule for a fixed fleet of trucks over a year. This model minimizes the maintenance costs, while trying to achieve the target production. Yuriy and Vayenas (2008) developed a reliability assessment model based on genetic algorithm to evaluate and generate the time between truck failures. The output of the model is used as an input to a discrete event simulation model to analyze the impact of failures on production. Two different simulation software are used to compare the merits.

Temeng et al. (1997) developed a transportation algorithm for real time dispatching system. This algorithm evaluates the criteria called cumulative production ratio and minimizes the deviation of this criteria from the mean for each shovel route. Yan et al. (2008) and Yan and Lai (2007) developed an integrated mixed integer model to study the production scheduling and truck dispatching problems in the same framework. The methodology was applied to a ready mixed concrete (RMC) case in Taiwan. Fioroni et al. (2008) presented a two stage method; firstly a mathematical programming model is used to allocate the shovels and the trucks; secondly simulation is used to assess the results in real time operations.

Muduli and Yegulalp (1996) modeled the dispatching system as a closed queuing network considering different classes of trucks with various attributes. Mean value analysis (MVA) is used to evaluate performance measures. Erselebi and Bascetin (2009) presented a two stage procedure to optimize the truck and shovel system. In the first stage a model based on the closed queuing network theory is used to determine the optimal number of operating trucks. At the second stage a linear programming (LP) model is used to specify the dispatching sequence of trucks to shovels.

This paper presents a mixed integer linear programming (MILP) model to determine the number of shovels and trucks and solve the allocation problem to the mining faces available at any given shift. The model minimizes operational costs, while trying to meet the production demand and considers technological constraints. The next section introduces the problem and defines the assumptions considered in this study. Section 3 presents the mathematical formulation of the problem. The last section includes the conclusion and future work.

2. Problem definition

In the mining industry, trucks and shovels are used as resources to extract ore and haul it to various destinations for further processing or dumping as waste. Shovels are used to extract the material and load them to the trucks. Trucks operate continuously to haul the material to other locations and to feed the processing plant.

The number and the type of trucks and shovels are important elements in optimal designing of open pit hauling systems. The truck-and-shovel allocation problem involves determining the number and size of trucks and shovels, and the matching between them. Availability, useful economic life, spare parts availability, maintenance, and operating costs are factors affecting the type of trucks and shovels to be chosen for hauling.

The following assumptions are the basis of the truck-shovel allocation mathematical model. This paper studies a mine consisting of different mining faces. A number of shovels and trucks of different types are available. Each type of truck has a specific size and hauls a different volume of material. Due to the failures and predicted maintenance the number of available trucks of each type and available shovels may vary for each period.

At the beginning of each period the decision is made about assigning trucks and shovels to the mining faces which are ready to be extracted. The type of the material of each mining face specifies each truck's destination. If the material type is ore, assigned trucks go to the mill and if it is waste, they go to the waste dump. The number of trips of each type of truck to different destinations is another variable to be decided in the model. This assignment must be in a manner that the loading and haulage costs are minimized.

The grade of different minerals and metals directly affects the mining costs. The grades of materials in the ore faces are considered in the model. Shovels and trucks are allocated to the mining faces in order to meet the blending constraints at the mill or stockpiles. Any deviation from the target production at the mill results in extra system costs as a penalty. Other assumptions of the problem are as follow:

- Specific types of trucks can work with specific types of shovels;
- The number of available trucks of each type is known at the beginning of the period;
- The number of available shovels is known at the beginning of the period;
- Maximum and minimum production capacity of shovels and load capacity of trucks are known;
- Only one shovel operates at each mining face at a time;
- Each shovel can operate at only one mining face at a time;
- The time horizon for the model is an 8-hour shift.

3. Mathematical formulation

In this section the MILP model of the problem is presented.

3.1. Sets

$I = \text{set of mining faces}$

$J = \text{set of shovels}$

$K = \text{set of types of trucks}$

3.2. Indices

$i \in I =$ Index for mining faces

$j \in J =$ Index for shovels

$k \in K =$ Index for types of trucks

3.3. Parameters

$$MAT_i = \begin{cases} 1 & \text{if current material type of mining face } i \text{ is ore} \\ 0 & \text{otherwise} \end{cases}$$

ORE_i = remaining ore tonnage at mining face i (ton)

$WASTE_i$ = remaining waste tonnage at mining face i (ton)

$$AVL_i^{face} = \begin{cases} 1 & \text{if mining face } i \text{ is available} \\ 0 & \text{otherwise} \end{cases}$$

$$AVL_j^{shovel} = \begin{cases} 1 & \text{if shovel } j \text{ is available} \\ 0 & \text{otherwise} \end{cases}$$

$SHCAP_j^{\max}$ = maximum production capacity of shovel j (ton/hour)

$SHCAP_j^{\min}$ = minimum production capacity of shovel j (ton/hour)

NUM_k = number of available trucks of type k

$$COMP_{jk} = \begin{cases} 1 & \text{if truck } k \text{ is compatible with shovel } j \\ 0 & \text{otherwise} \end{cases}$$

CT_{ik}^{ore} = cycle time of truck type k transferring ore from mining face i to the mill (second)

CT_{ik}^{waste} = cycle time of truck type k transferring waste from mining face i to the waste dump (second)

CAP_k^{ore} = capacity of truck type k transferring ore (ton)

CAP_k^{waste} = capacity of truck type k transferring waste (ton)

GR_{il} = grade of variable l at mining face i (%)

UB_l = upper bound of grade blending for variable l (%)

LB_l = lower bound of grade blending for variable l (%)

$PMAX$ = maximum processing capacity of the mill (ton)

$PMIN$ = minimum processing capacity of the mill (ton)

MC_{ij} = moving cost of shovel j from its current location to minign face i (%)

TRC_{ik}^{ore} = trip cost of truck type k from mining face i to the mill ($)

TRC_{ik}^{waste} = trip cost of truck type k from mining face i to the waste dump ($)

DC = cost of deviation from target production ($/ton)

T = planning time duration (hour)

3.4. Decision variables

$$a_{ij} = \begin{cases} 1 & \text{if shovel } j \text{ is assigned to mining face } i \\ 0 & \text{otherwise} \end{cases}$$

n_{ik}^{ore} = number of trips of truck type k from mining face i to the mill

n_{ik}^{waste} = number of trips of truck type k from mining face i to the waste dump

x_i = extracted tonnage from mining face i (ton)

3.5. Objective Function

$$\text{MIN } Z = \sum_{i\in I}\sum_{j\in J} MC_{ij}\cdot a_{ij} + \sum_{i\in I}\sum_{k\in K}(TRC_{ik}^{ore}\cdot n_{ik}^{ore} + TRC_{ik}^{waste}\cdot n_{ik}^{waste}) + DC\cdot(PMAX - \sum_{i\in I} MAT_i \cdot x_i) \quad (1)$$

3.6. Constraints

$$\sum_{j\in J} a_{ij} \le AVL_i^{face} \qquad \forall i\in I \quad (2)$$

$$\sum_{i\in I} a_{ij} \le AVL_j^{shovel} \qquad \forall j\in J \quad (3)$$

$$CT_{ik}^{ore}\cdot n_{ik}^{ore} \le 3600\cdot T\cdot NUM_k \cdot MAT_i \qquad \forall i\in I, k\in K \quad (4)$$

$$CT_{ik}^{waste}\cdot n_{ik}^{waste} \le 3600\cdot T\cdot NUM_k \cdot (1-MAT_i) \qquad \forall i\in I, k\in K \quad (5)$$

$$n_{ik}^{ore} \le \sum_{j\in J} a_{ij}\cdot COMP_{jk} \qquad \forall i\in I, k\in K \quad (6)$$

$$n_{ik}^{waste} \le \sum_{j\in J} a_{ij}\cdot COMP_{jk} \qquad \forall i\in I, k\in K \quad (7)$$

$$\sum_{i\in I} n_{ik}^{ore}\cdot CT_{ik}^{ore} + \sum_{i\in I} n_{ik}^{waste}\cdot CT_{ik}^{waste} \le 3600\cdot T\cdot NUM_k \qquad \forall k\in K \quad (8)$$

$$x_i \le \sum_{j\in J} T\cdot SHCAP_j^{\max}.a_{ij} \qquad \forall i\in I \quad (9)$$

$$x_i \ge \sum_{j\in J} T\cdot SHCAP_j^{\min}.a_{ij} \qquad \forall i\in I \quad (10)$$

$$\sum_{i\in i} x_i\cdot MAT_i \le PMAX \quad (11)$$

$$\sum_{i\in i} x_i\cdot MAT_i \ge PMIN \quad (12)$$

$$x_i\cdot MAT_i \le ORE_i \qquad \forall i\in I \quad (13)$$

$$x_i\cdot\left(1-MAT_i\right) \le WASTE_i \qquad \forall i\in I \quad (14)$$

$$x_i = \sum_{k\in K} CAP_{ik}^{ore}\cdot n_{ik}^{ore} + \sum_{k\in K} CAP_{ik}^{waste}\cdot n_{ik}^{waste} \qquad \forall i\in I \quad (15)$$

$$\sum_{i\in I} GR_{il}\cdot x_i \le \sum_{i\in I} UB_l\cdot x_i \qquad \forall l\in L \quad (16)$$

$$\sum_{i\in I} GR_{il}\cdot x_i \ge \sum_{i\in I} LB_l\cdot x_i \qquad \forall l\in L \quad (17)$$

$$a_{ij} \in \{0,1\} \quad \forall i \in I, j \in J \tag{18}$$

$$n_{ik}^{ore}, n_{ik}^{waste} \in Z \quad \forall i \in I, k \in K \tag{19}$$

$$x_i \geq 0 \quad \forall i \in I \tag{20}$$

Objective function seeks to minimize the operational costs associated with the mine. The first term in Eq. (1) is the total cost of moving shovels to new faces, the second term is the total transportation cost of trucks moving to the waste dump or to the mill, and the last term is the cost of negative deviation from the production target at the mill. Constraint Eq. (2) indicates that at each available mining face only one shovel can operate, and if a face is not available, no shovel should be assigned to that face. Constraint Eq. (3) assures that each available shovel can operate at only one face. Eq. (4)limits the number of trips for a fleet of trucks travelling from each mining face to the mill. Eq. (5)restricts the number of trips for a fleet of trucks travelling from each mining face to the waste dump. Constraints Eq. (6) and Eq. (7) guarantee that a truck could travel to a mining face only if a shovel is assigned to that face and the shovel is compatible with that truck type. Equation Eq. (8) denotes that the total number of trips of each truck type travelling to the mill or the waste dump is less than the maximum possible trips of that truck type. Constraints Eq. (9) and Eq. (10) ensure that the production of each mining face is between minimum and maximum possible production of the shovel assigned to that face. Eq. (11) and Eq. (12) aim to meet the limits of processing capacity of the mill. Constraints Eq. (13) and Eq. (14) force the production of each mining face to be smaller than the maximum amount of available material. Eq. (15) defines the production of each mining face based on the number of trips of each fleet of trucks. Eq. (16) and Eq. (17) ensure that the grade blending at the mill is between specified upper and lower limits. Eqs. (18), (19), and (20) define types of decision variables.

4. Conclusions and future work

This paper presented a mixed integer programming model to determine the number and the optimum allocation of trucks and shovels in an open pit mine. The model is proposed for the problem under certain assumptions. The next stage in this study is to code, solve, and verify the model using optimization tools. The approach should be applied to a real case to validate the model and to study the efficiency of the model and the solution algorithm.

5. References

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