# Quantifying the Cost of Grade Uncertainty in Mine Plans

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## Abstract

*Uncertainty is present because of a lack of information. Conditional simulation algorithms are widely used to model grade uncertainty. This uncertainty has some negative effects on the planning process and it is necessary to transfer and measure risk in the long-term production schedule. In this paper, the effect of grade uncertainty in processing and on the economic value of extracted blocks is assessed and a quantitative method is presented to calculate the discounted cost of uncertainty in a production schedule. An oil sand deposit is used to demonstrate the presented methods.*

## 1. Introduction

Grade uncertainty is modeled by generating equal probable realizations using geostatistical conditional simulation techniques. For each block, a local distribution of grade can be generated with simulated values, which show the local uncertainty. Usually, the average grade of a block is used to determine whether a block should be processed or not. The average grade may be the arithmetic mean of all simulated values, or may simply be the estimated grade value of the block using Kriging (Journel and Huijbregts, 1981). The cut-off grade is a critical threshold; any block with a grade above this limit is considered as ore and has an economical value, and any material bellow the cut-off grade is treated as waste with no economic value. Lane (1988) presented the fundamentals of cut-off grade calculation.

There are four different situations that may occur depending on the cut-off grade and local distribution of a block generated by n conditional simulations.

1. All n simulation values are below the cut-off grade (Fig. 1)
2. All n simulation values are above the cut-off grade (Fig. 2)
3. Not all simulation values are above or below the cut-off, but the average grade is below the cut-off grade (Fig. 3)
4. Not all simulation values are above or below the cut-off, but the average grade is above the cut-off grade (Fig. 4)

It is assumed that the number of realizations is sufficient to capture grade uncertainty with a reasonable level of statistical confidence. A synthetic case is assumed to demonstrate all four situations with lognormal distributions for grade of blocks. Four different mean and variances and a cut-grade of 2% have been chosen. Fig. 1 to Fig. 4 show the Probability Density Function (PDF) at left and Cumulative Density Function (CDF) at right for all four situations.

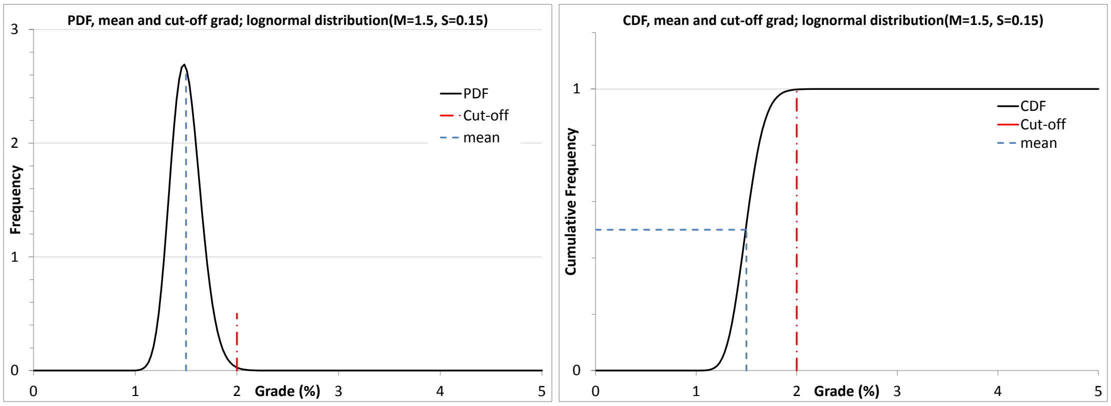


Fig. 1. PDF (left) and CDF (right) for case 1, all n realizations and mean (dashed blue line) are less than cut-off grade (dashed red line).

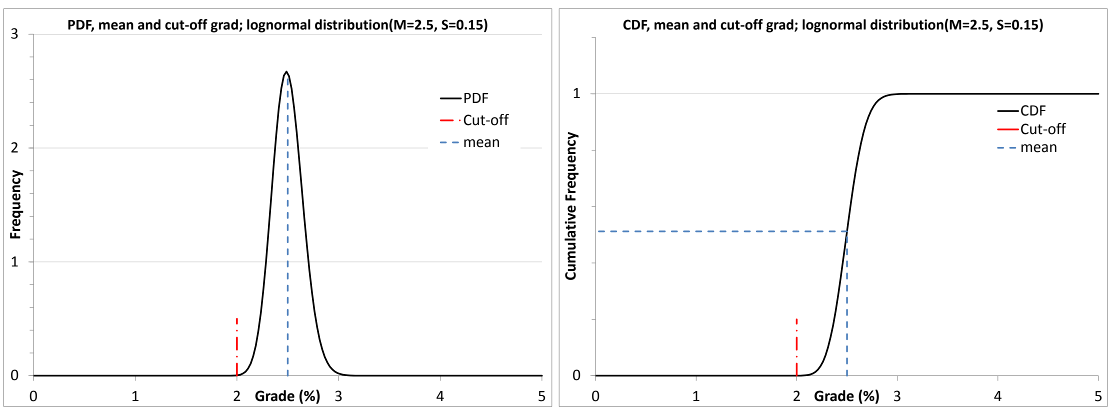


Fig. 2. PDF (left) and CDF (right) for case 2, all n realizations and mean (dashed blue line) are higher than cut-off grade (dashed red line).

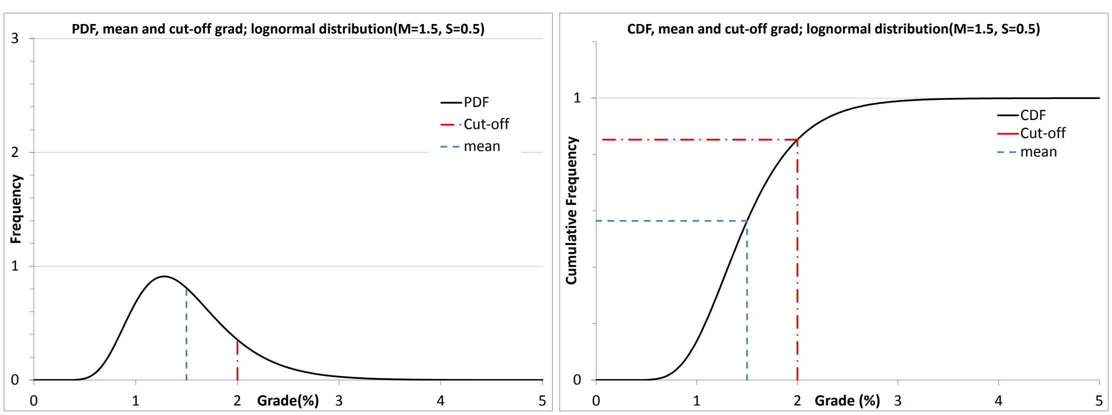


Fig. 3. PDF (left) and CDF (right) for case 3, not all n realizations and mean (dashed blue line) are less than cut-off grade (dashed red line).

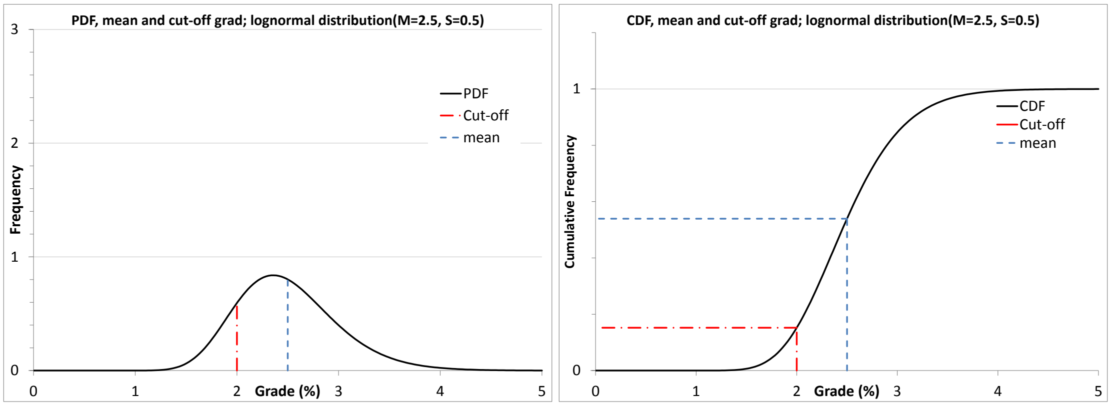


Fig. 4. PDF (left) and CDF (right) for case 3, not all n realizations and mean (dashed blue line) are higher than cut-off grade (dashed red line).

For cases 1 and 3, because the average grade of the blocks is below the cut-off, these cases are considered waste blocks (Fig. 1 and Fig. 3). The average grades of cases 2 and 4 are above the cut-off; hence these blocks are treated and sent to the processing plant (Fig. 2 and Fig. 4). Based on n simulation values, there is a very small risk involved in decisions that are made for cases 1 and 2 (Fig. 1 and Fig. 2). This risk increases in blocks with distributions described in case 3 and 4. In case 3 the block is chosen as waste because average grade is lower than the cut-off grade. But there is 14.7 percent chance of obtaining a higher value than the cut-off grade. This means that there is a 14.7 percent chance that this block should be considered as ore, but it will be treated as waste. Let's assume an extraction schedule that is generated according to the average grade of blocks. In such a case, this block, with probability of 14.7 percent, may generate over-production.

The same situation would occur in case 4 (Fig. 4). This block is classified as ore block because its average grade is higher than the cut-off grade. There is a 15.2 percent chance that the grade of this block is below the cut-off grade. This means that with a block extraction schedule generated based on average grade, this block with a probability of 15.2 percent may be considered as waste. This will cause under production following the designed schedule.

## 2. Grade uncertainty and economic block value

The Economic Block Value (EBV) is presented by Eq. (1).

$$EBV_{\mathrm{n}}^{\mathrm{t}} = \begin{cases} \mathrm{o_n} \times \left(\overline{\mathrm{g}}_{\mathrm{n}} \times \mathrm{p}_r \times \mathrm{price}^{\mathrm{t}} - p_c^t\right) - \left(\mathrm{o_n} + \mathrm{w_n}\right) \times \mathrm{m}_c^t & if\ \overline{\mathrm{g}}_{\mathrm{n}} \geq g_{cut} \\ -\left(\mathrm{o_n} + \mathrm{w_n}\right) \times \mathrm{m}_c^t & if\ \overline{\mathrm{g}}_{\mathrm{n}} < g_{cut} \end{cases} \tag{1}$$

where $\mathrm{o_n}$ is the tonnage of ore, $\mathrm{w_n}$ is tonnage of waste, $\overline{\mathrm{g}}_{\mathrm{n}}$ average grade of block n, $\mathrm{p}_r$ processing recovery, $p_c^t$ processing cost, $\mathrm{m}_c^t$ mining cost, $\mathrm{price}^{\mathrm{t}}$ is the present selling price of final product, $g_{cut}$ is cut-off grade.

EBV is a positive value when the average grade of the block is above the cut-off grade and equal to a negative mining cost for waste blocks (Fig. 5).

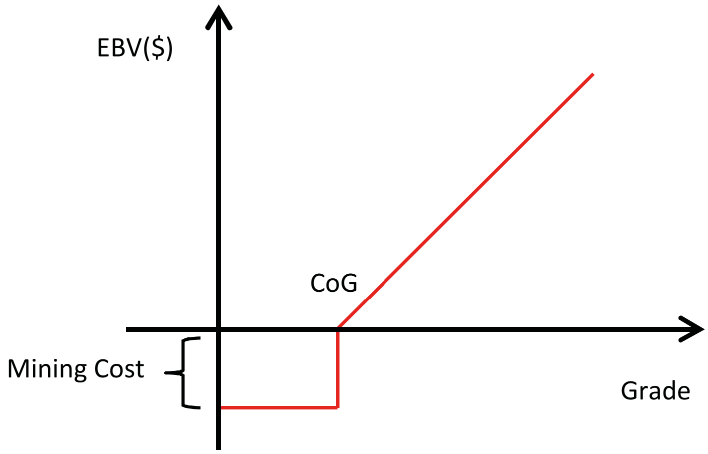

Fig. 5. Cut-off grade and EBV.

To show the effect of grade uncertainty on EBV, a synthetic case is assumed. A block with a log normal distribution of grade with a mean and standard deviation of 2.2% and 0.5% respectively is simulated 10,000 times (Fig. 6). The cut-off grade is assumed to be 2%. Therefore, this block is considered as an ore block because the average grade is above the cut-off grade.

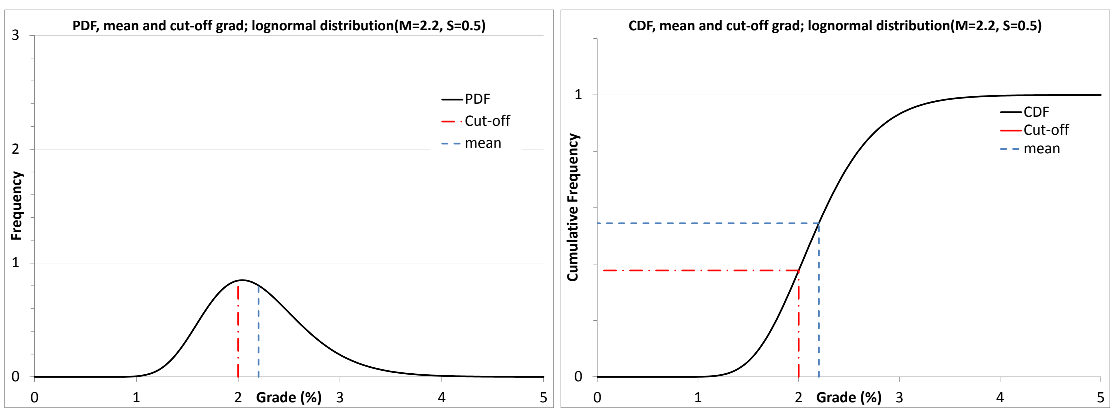

Fig. 6. PDF (left) and CDF (right) for a synthetic case to calculate expected value of EBV.

EBV of this block is calculated for all 10,000 realizations where $p_r = 100\%$, $\text{Price} = 1\$$, $p_c = 0.5\$$ and $m_c = 1.5\$$. The tonnage of this block is assumed to be 1 tonne. Histogram of EBV is shown in Fig. 7. 3744 times (37.44%) over 10,000 generated values, the block is assumed to be waste because the simulated grade is less than the cut-off grade. This is revealed by the trimmed black column in Fig. 7. 62.56% of the times, the simulated grade is above the cut off-grade and the block is assumed to be ore (gray columns at the Fig. 7). The average of EBV is -0.26 $ and less than zero. This means that even for a block with an average grade above the cut-off grade, the average EBV of simulations may be less than zero and it is not economical to be processed.

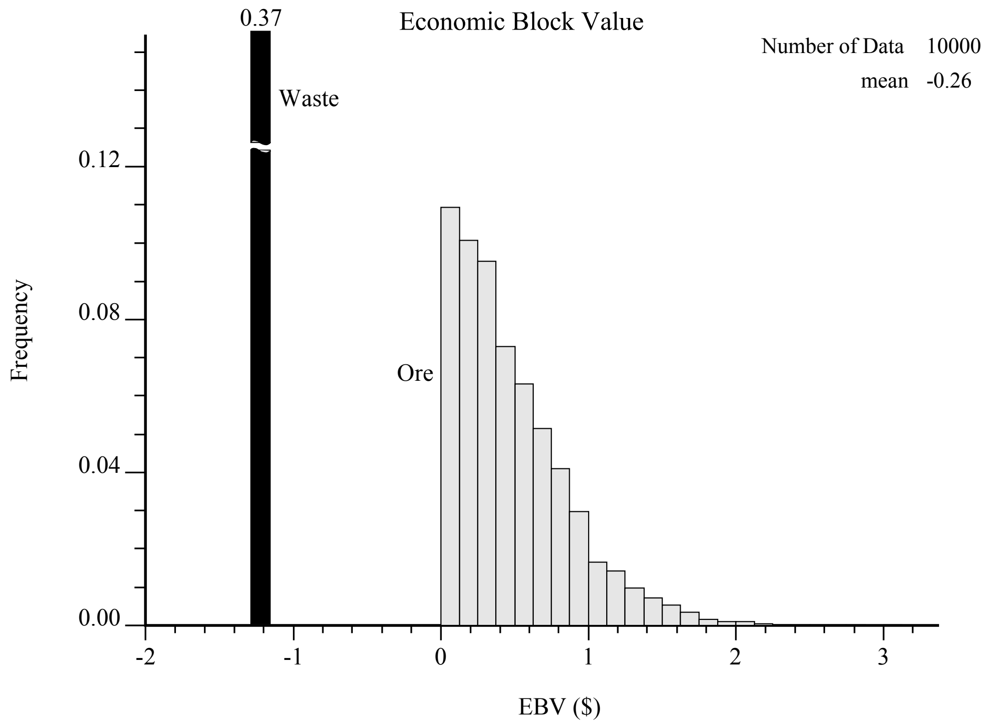


Fig. 7. Histogram of EBV for a block with lognormal distribution, mean=2.2 and Std.dev=0.5.

## 3. Cost of uncertainty

The main objective of long-term mine planning is usually to maximize the net present economic value of a project subject to technical and other (e.g. environmental) constrains. Such an objective is usually modeled using optimization techniques. The goal is usually to find the sequence of extraction of blocks or mining-cuts to reach the maximum achievable net present value of the project. The input data used in production scheduling optimization, such as geological block model, grades, costs, prices, recoveries, and practical mining constraints are usually based on the best point estimates available at the time of optimization. Traditional mine planning techniques do not consider grade uncertainty and only one estimate of the block grade is used.

As noted in the previous section, grade uncertainty might cause shortfalls from and surpluses over the designed target production at the processing plant. This is because decisions are made only based on an estimated grade value for a block. Different estimation techniques, such as different Kriging methods or the E-type mean method, are used to determine the average grade of a block. If the average grade of a block is less than the cut-off grade, then the block is classified as waste and vice versa. Therefore, uncertainty in the grade can result in misclassification; a block that has been classified as waste can be an ore block and vice versa. This misclassification is not considered in traditional mine planning methods, which use only a smooth block model, in most cases, an average grade estimation such as Kriging or E-type is used.

The secondary objective seen increasingly in the literature is to take uncertainty into account. The fact that the input variables into the optimization model are uncertain, affects the optimization process. Recently some authors, such as Dimitrakopoulos and Ramazan (2008), have presented optimization algorithms which aim to maximize expected value of the target function, which is net present value (NPV), and to minimize the negative effect of uncertainty, which is called risk. These methods try to maximize NPV and minimize the risk of grade uncertainty by deferring the extraction of more uncertain blocks into the future so that the effect of grade uncertainty is reduced by gathering new information during mine life. The main idea is that uncertainty somehow costs money and should be deferred. The challenging question is how to quantify of the cost of uncertainty. The cost of uncertainty has two main reasons:

1. For any shortfalls that may happen at processing plant, there is a missing profit. This profit could be achieved by a feeding plant with full capacity.
2. On the other hand, assume that the processing plant is working at full capacity and there is enough ore to feed the plant for a while. At this moment, trucks and shovels are working to remove waste blocks as planned. Because of grade uncertainty and misclassification, a block that has been classified as waste is extracted and has a grade above cut-off grade. In real life, this block would not be extracted and the schedule would be changed or in most cases, there would be a stockpile where this extra ore would be sent to be used in the future. Not having a stockpile is very unlikely in real life. We may, however, assume a hypothetical case in which, there are no stockpiles to store extra ore and in which the block is being extracted to follow exactly the same schedule. In such a case, this ore block will be sent to the waste dump and its revenue will be lost. This lost profit is also part of the cost of uncertainty.

To quantify these two costs, a cost of not meeting the target production is presented in Eq. (2)

$$C_t = \left|P_t - \text{Target}_t\right| \times \left(\overline{g}_t \times P_r \times \text{Price - } P_c\right) \tag{2}$$

Where $C_t$ is the cost on not meeting the target production in period t, $P_i$ is the input ore to the mill at period t, $\text{Target}_t$ is target production for period t, $\overline{g}_t$ average grade of input ore at period t, $P_r$ processing recovery, Price is the selling price of final product and $P_c$ is the processing cost. By averaging $C_t$ over all realization and discounting it over all the planning periods, the discounted cost of uncertainty is defined by Eq. (3)

$$C_u = \frac{1}{L}\sum_{t=1}^{T-1}\sum_{l=1}^{L}\left(\frac{C_t^l}{(1+i)^t}\right) \tag{3}$$

Where L is the number of realizations, T is the number of periods and $i$ is the discounting rate. The cost of uncertainty is calculated over all period except final period. Because any ore that is left for final period will be processed and will not exceed the target production, any shortfall in the final period does not occur because of grade uncertainty.

## 4. Case study

An oil sands deposit in Fort McMurray, Alberta, Canada was used for this case study. The location of boreholes and the histogram of data are presented at Fig. 8 and Fig. 9 respectively.

GSLIB (Deutsch and Journel, 1998) programs were used in this case study. Directional experimental variograms were calculated and fit using `gamv` and `vmodel` programs. The azimuths of major and minor directions were 50 and 140 degrees. Fig. 10 shows the experimental and the fitted variogram models in major, minor and vertical directions.

`KT3d` was employed to estimate the bitumen grade (with no normal score transform) at each block location using Ordinary Kriging (OK). Multiple realizations of the bitumen grade were generated using Sequential Gaussian Simulation (SGS) (Isaaks and Srivastava, 1989) at a very high resolution three-dimensional grid in the point scale. To determine the average grade of a block, simple arithmetic averaging was done between all simulated points inside the block. This step is called up-scaling, and a program called `blkavg` was used at this stage. The E-type mean was calculated using the `postsim` program. Fig. 11 illustrates the map of the bitumen grade for the Kriged and the E-type methods, and of realization no. 26 at block scale. It is well-known that

Kriging is conditionally biased (Isaaks, 2005) and that, on the other hand, "there is no conditional bias of simulation when the simulation results are used correctly" (McLennan and Deutsch, 2004). The conditional biasness of Kriging can be reduced by tuning estimation parameters but it cannot be eliminated (Isaaks, 2005). The Grade-Tonnage curve is a good tool for use in checking the impact of Kriging biasness.

Fig. 12 shows the grade tonnage curve of simulation realization (dashed lines), Kriging (bold solid line) and E-type (bold dashed line). Although an effort was made to minimize the conditional biasness of Kriging, there are still differences between the Kriging and simulation results because SGS uses Simple Kriging(SK). Also, E-type is slightly different than Kriging; theoretically, the E-type model is identical with the SK result in Gaussian space (Journel and Huijbregts, 1981).

Histogram and variogram reproduction were checked using `gamsim` and `histpltsim` respectively. Fig. 13 shows the histogram reproduction, and Fig. 14 shows the variogram reproduction in major and minor horizontal and vertical directions. Generally, SGS should reproduce the histogram and variograms of the original data if it is carefully implemented. In this case, the reproduction of the histogram and variograms was acceptable.

The ultimate pit limit design was carried out based on the Syncrude's costs in CAN$/bbl of sweet blend for the third quarter of 2008 (Jaremko 2009). The price of oil was assumed to be US$45, with an exchange rate of 1.25:1 equal to CAN $56.25/bbl SSB in the same time period. We assumed that every two tonnes of oil sands with an average grade of 10% mass would produce one barrel of sweet blend, which is approximately 200 kg. We also assumed a density of 2.16 tonne/m$^3$ of oil sands, and a density of 2.1 tonne/m$^3$ of waste material, including clay and sand.

Table 1 shows the pit design and production scheduling input parameters. The mining cost of of $4.6/tonne and processing cost of $0.5025/tonne was applied. Thirty-three pit shells were generated using 49 fixed revenue factors ranging from 0.1 to 2.5, based on the Kriged block model. The number of pit shells was reduced to 14 after applying the minimum mining width of 150 meters for the final pit and the intermediate pits. Table 2 summarizes the information related to the final pit limit based on Kriged block model at 6% bitumen cut-off grade. The minimum slope error, the average slope error and the maximum slope error respectively are: 0.0 degrees, 0.2 degrees, and 0.4 degrees. The final pit limit was designed for the E-type model and all fifty simulation realizations with the exact same input variables.

The final pit, based on the Kriging block model was used at this stage. There are 14607 blocks inside the final pit. Using the MATLAB (MathWorks Inc., 2007) c-mean clustering function, 1834 mining cuts were generated by aggregating blocks at the same level with similar grades. The Kriging block model was used at LP optimization(Askari-Nasab and Awuah-Offeri, 2009). Two years of pre-stripping were assumed to provide enough operating space and ore availability. No stockpile was defined and the target production was set to 36 million tonnes of ore per year with a mining capacity of 135 million tonnes per year. The interest rate is 10%. The mine life was 10 years. There were 653.61 million tonnes of material inside the final pit, 282.44 million tonnes of which were ore. The strip ratio was 1.31; in addition, there were 37.4 million tonnes of ore with an average grade less than the cut-off grade. These were assumed as waste blocks.

The mixed integer programming was solved using TOMLAB CIPLEX (Holmström, 1989-2009) with a gap of 1%. Fig. 15 shows the schedule generated by MILP and using Kriging block model. Gray and yellow bars present removed waste and ore materials respectively. The plan view and two cross sections of blocks and their extraction periods are shown in Fig. 16. To capture the effect of grade uncertainty, the Kriged value of blocks were replaced by simulated values and the same extraction schedule was followed. Any blocks that had less simulated grade than cut-off grade were sent to waste dump even if they were considered to be ore using the Kriging block model and vice-versa. Following the same schedule, some realizations are generated over-produced ore, This revenue was counted in the NPV. This NPV is incorrect, because is it impossible to process over-produced ore where there is no stockpile. Therefore the revenue of these surplus ores must be removed. For this reason, there are two versions of results. One is the row results, surplus ore is not removed and revenue is counted. In the second version the removed over-produced ore is removed and called "Cleaned Version". From Fig. 17 to Fig. 20, row results are shown on the left and the cleaned version is at right. Fig. 17 to Fig. 19 show the effect of grade uncertainty on the generated plan in terms of cumulative NPV, head grade and feed respectively. The bold black line is Kriging, the bold dashed blue line is Etype and the dashed red lines are 50 conditional simulation results. Fig. 17 demonstrates that the NPV of the cleaned realizations is less than of the row version. For example in the row version there is a realization that generates more NPV than the Kriging block model, but in the cleaned one there is not any realization to exceed the NPV of Kriging. Fig. 20 illustrates the box plot of input ore into the plant calculated with using simulation realizations. Yellow bars show the deviation from target production. As it is clear from this graph and Fig. 19, surplus ore production and under-production are probable in periods 2 and 4 respectively, while shortfall may only happen in period 3.

Table 3 shows the summary statistics for the generated schedule for the two versions. The NPV of Kriging is $2461 million. The expected NPV is calculated from all 50 realizations. In the first table, the statistics show the row results and in the second table, the results are for removed surplus ores. The average NPV for row and cleaned version respectively are 2335.4 and 2317.5, and are less than the Kriged NPV. It is reasonable to have a lower expected NPV, because the whole MILP algorithm was solved to maximize NPV of the Kriging and the solution is optimized for the Kriged block model. Table 4 shows the summary of statistics for cumulative discounted case flow at different periods. In the first two periods, the cumulative discounted case flow is negative because of pre-striping at these periods and because there is no extraction of ore.

The discounted cost of uncertainty was calculated based row results. Therefore the discounted cost of uncertainty based on periods 3 to 9 was $178.9 Million.

For this case study, 861 of 1834 cuts were considered to be ore based on comparison of the Kriging average grade to the cut-off grade. 84 mining cuts of 861 ore cuts had less expected EBV than the minimum acceptable EBV, which is calculated based on cut-off grade. Therefore, if the average EBV is the criterion for choosing whether a block will be processed or not, then not all of these 84 mining cuts should be considered as ore cuts. This shows the effect of grade uncertainty on misclassification even with an average grade above the cut-off grade. The difference summation of EBVs of these 86 mining cuts is $64 million.

## 5. Conclusions

In this paper the effect of grade uncertainty on four synthetic cases was illustrated. Four possible situations may occur, depending upon on the estimated mean and a cut-off grade. Two cases may cause shortfalls or extra ore production in the generated schedule using only an estimated value. This is because of grade uncertainty.

It has also been shown that the cut-off grade and one average value for a block are not good criteria to choose a block as ore or waste. There are some situations in which the average grade of a block is above cut-off grade, but the expected dollar value of block is less than minimum threshold. Therefore the average EBV of a block over realizations may be a better criterion for classifying a block as either ore or waste.

The cost of not meeting the target production and cost of uncertainty were presented. Using all simulation realizations, the cost of uncertainty can be calculated. It is a good criterion for use in comparing two different schedules. The Cost of Uncertainty is an average dollar value that, by following a schedule over all realizations, is imposed on the plant because of probability of not meeting the target production.

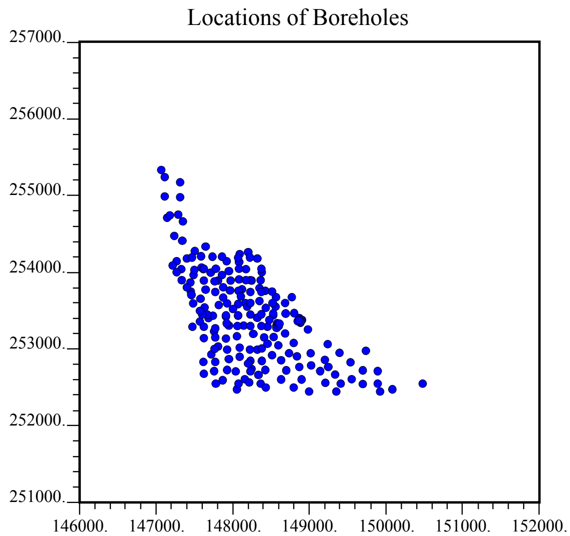


Fig. 8. Location map of boreholes.

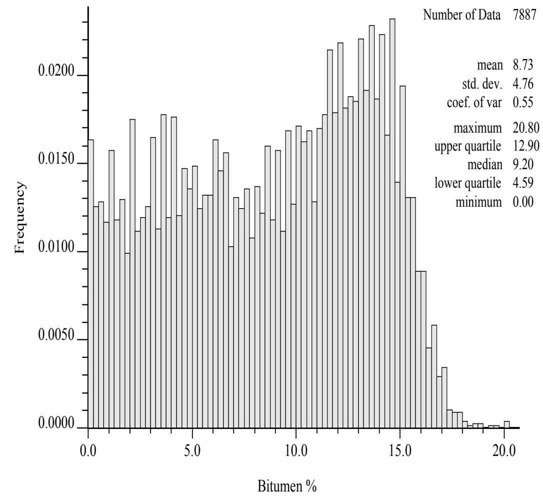


Fig. 9. Histogram of Bitumen.

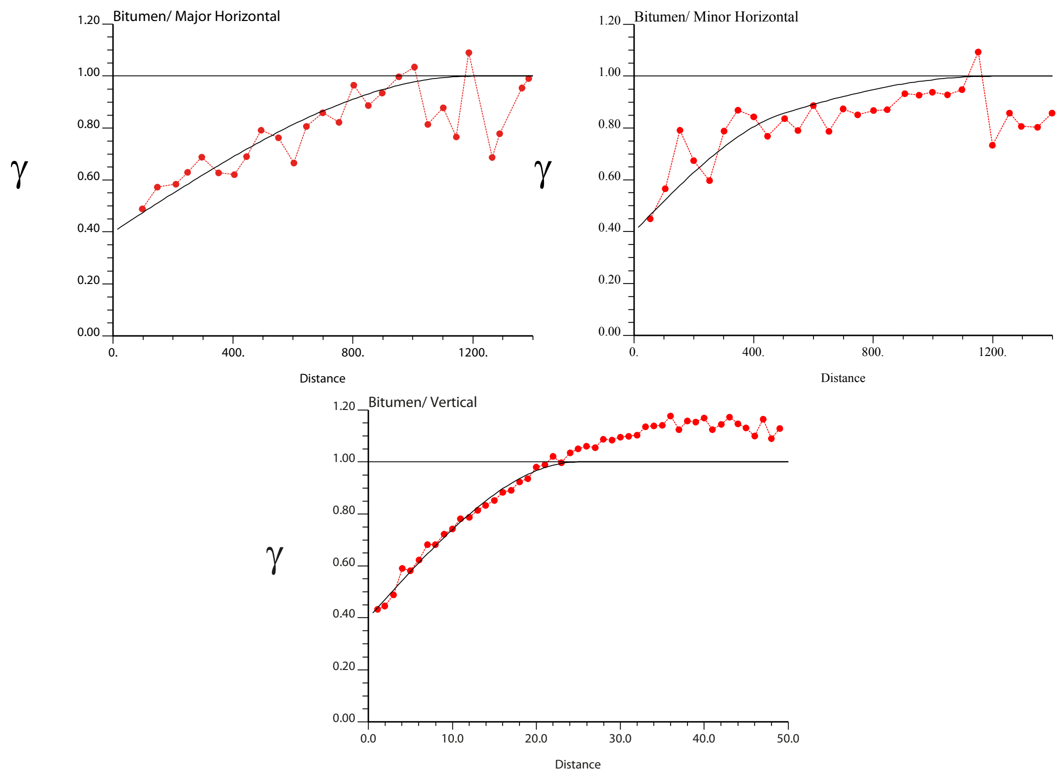


Fig. 10. Experimental directional variograms (dots) and the fitted variogram models (solid lines)

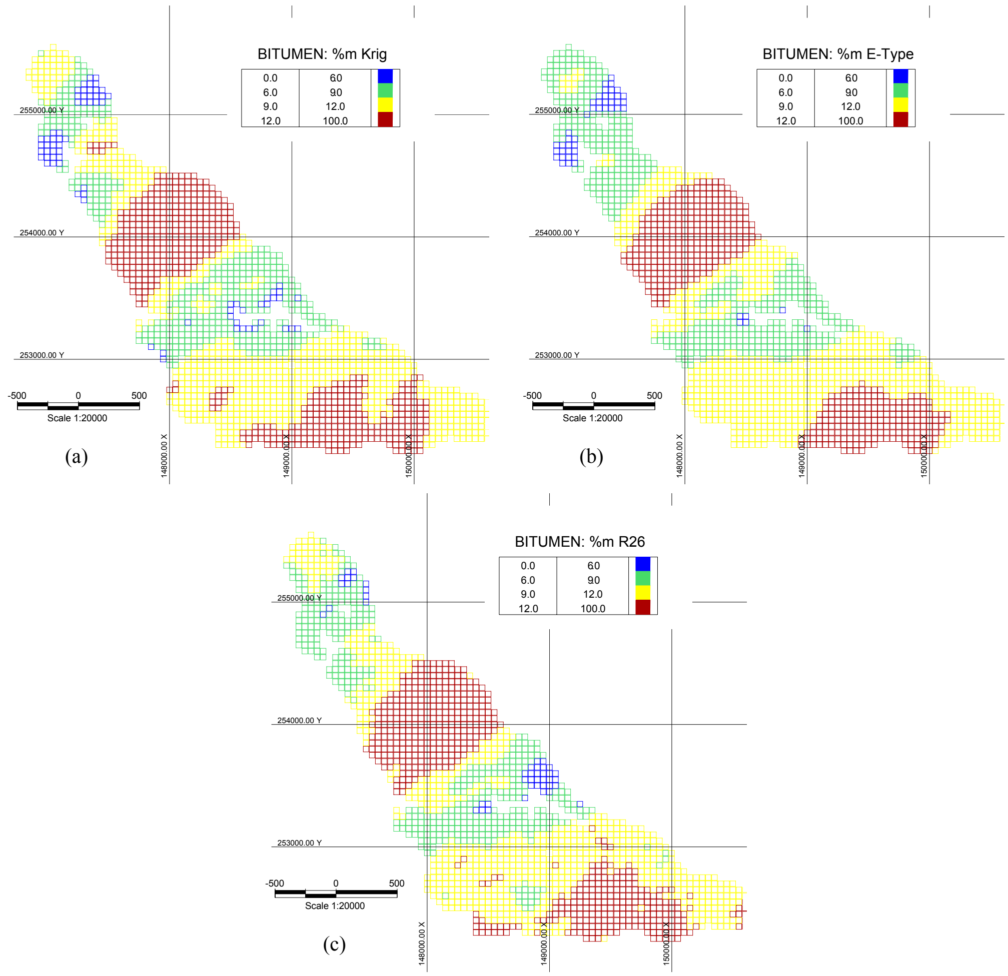


Fig. 11. Plan view at 260m; (a) Kriged model, (b) E-type model, (c) realization 26.

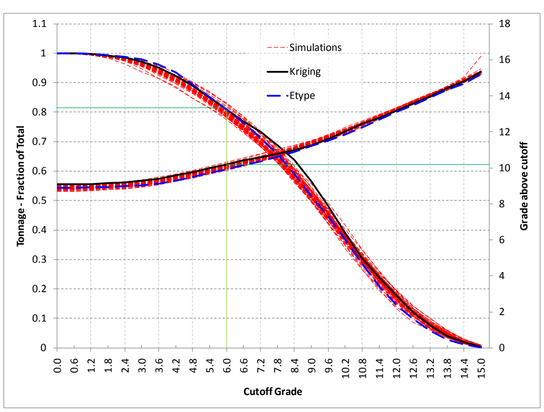


Fig. 12. Grade tonnage curve of simulation realizations, kriged, and Etype block models

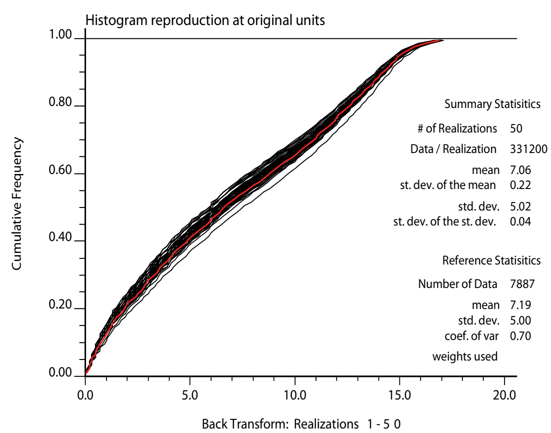


Fig. 13. Histogram reproduction of simulation realizations (dashed lines) and histogram of original data (bold line)

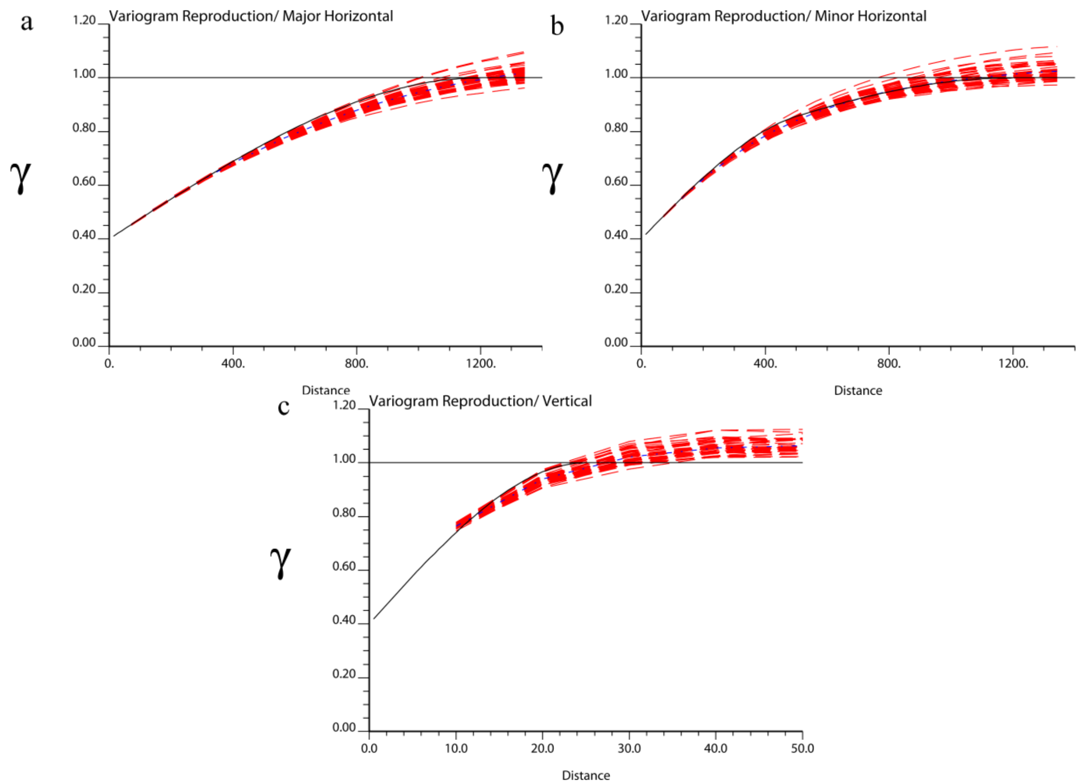


Fig. 14. Variogram reproduction of simulation realizations (red dash lines) and reference variogram model (black line).

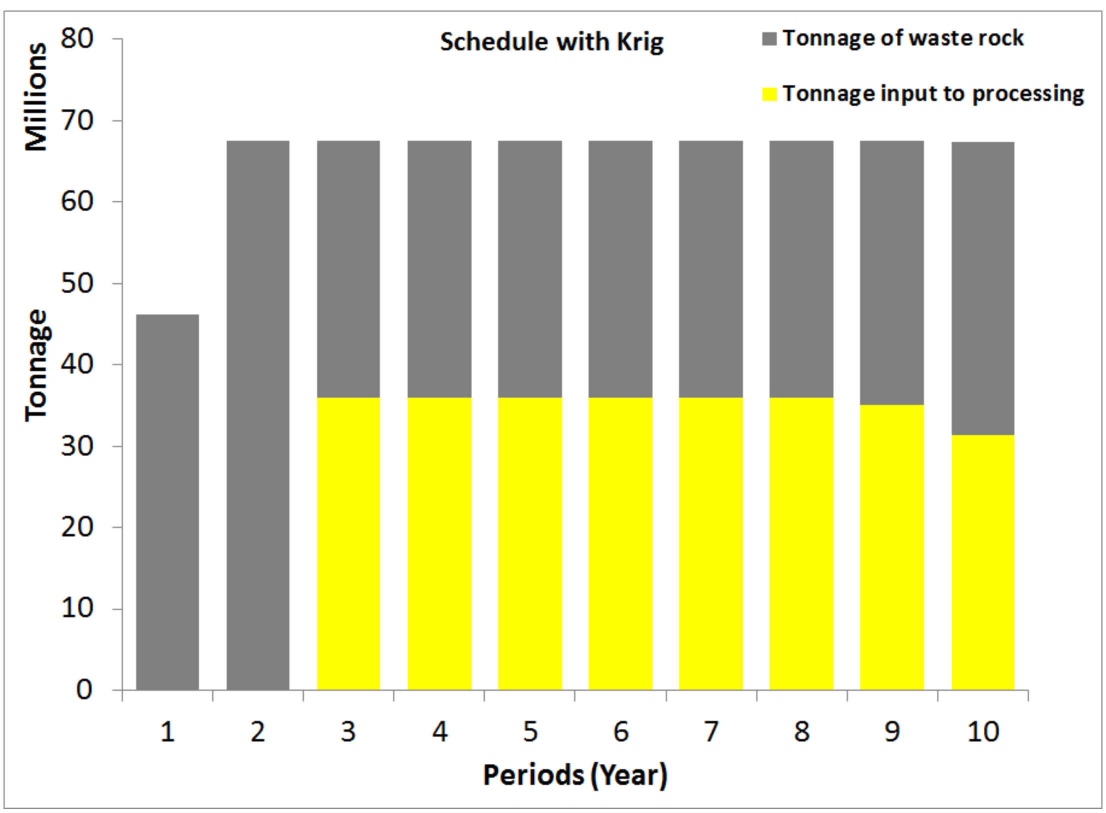


Fig. 15. Schedual generated using krig model

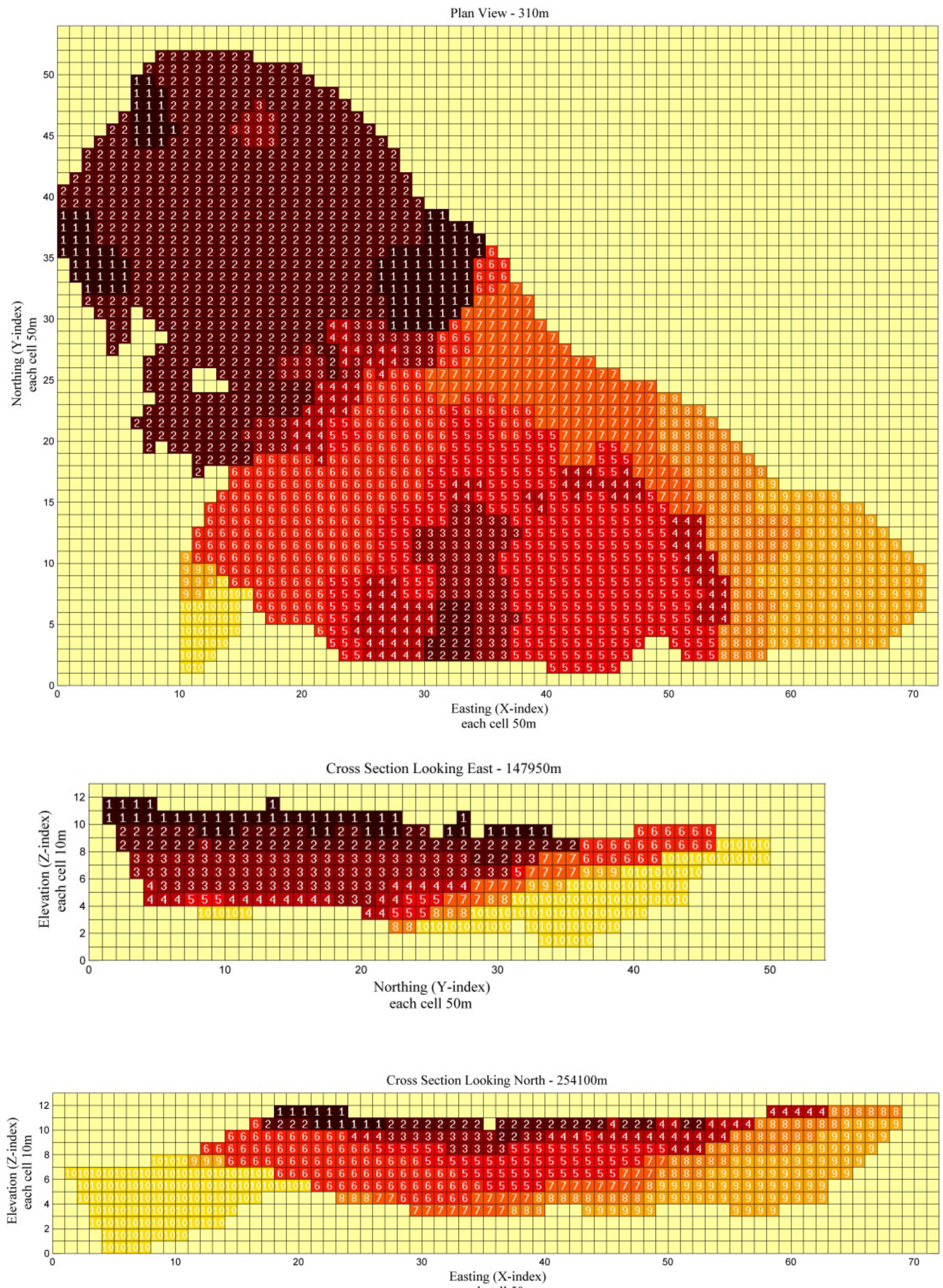


Fig. 16. Plan view, cross section looking east and north for generated schedual

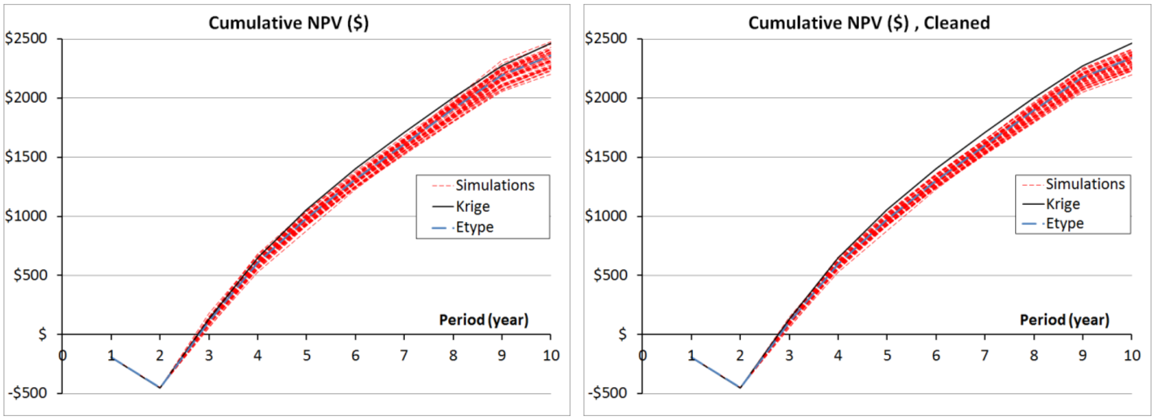


Fig. 17. Cumulated NPV over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

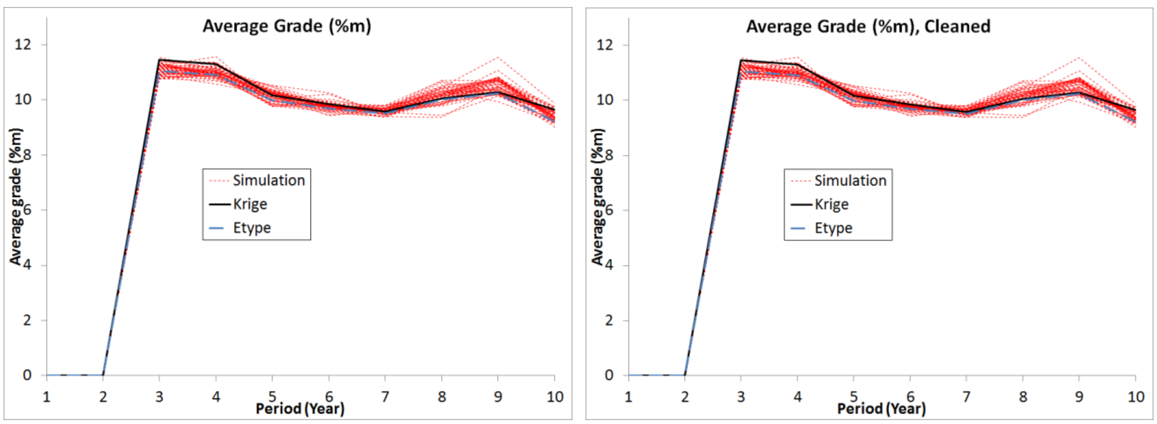


Fig. 18. Input head grade to the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

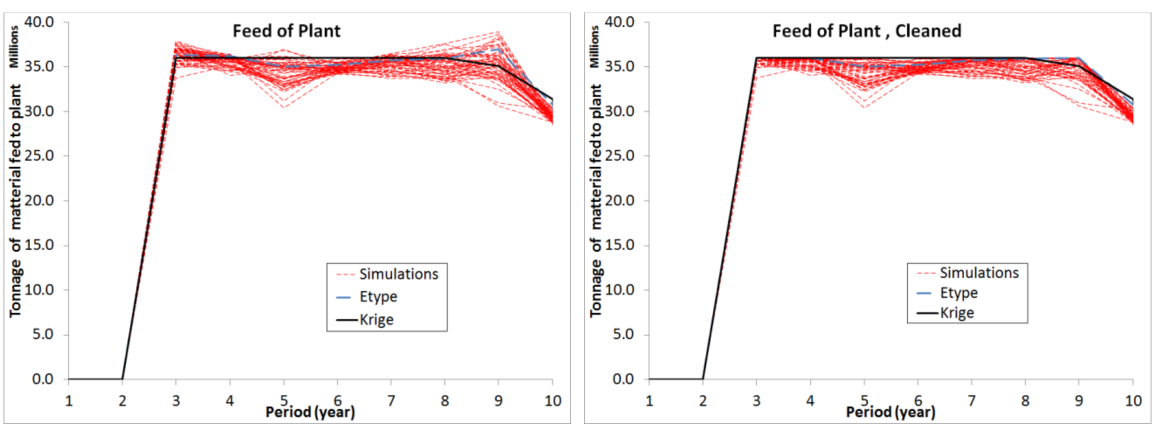


Fig. 19. Feed of the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

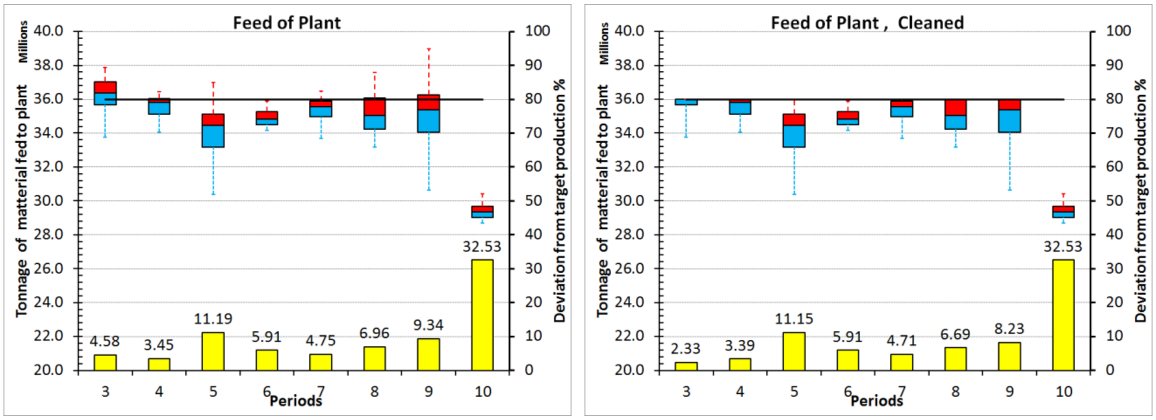


Fig. 20. Boxplot and deviation from target production (yellow bars), calculated using simualtion values, surplus ore not removed at left and cleaned version at right

Table 1. Final pit limit and mine planning input parameters.

| Description | Value | Description | Value |
|---|---|---|---|
| Mining Cost ($/tonne) | 4.6 | Processing Cost ($/tonne) | 0.5025 |
| Cutoff grade (%mass bitumen) | 6 | Processing limit (M tonne/year) | 36 |
| Mining recovery fraction | 0.88 | Mining limit (M tonne/year) | 67.5 |
| Processing recovery factor | 0.95 | Overall slope (degrees) | 20 |
| Minimum mining width (m) | 150 | Pre-stripping (years) | 2 |

Table 2. Material in the final pit using the kriged block model.

| Description | Value |
|---|---|
| Total tonnage of material (M tonne) | 653.61 |
| Tonnage of ore (M tonne) | 282.44 |
| Tonnage of material below cutoff (M tonne) | 37.4 |
| Tonnage of waste (M tonne) | 371.166 |
| Bitumen recovered (M tonne) | 23.29 |
| Stripping ratio (waste:ore) | 1.31 |

Table 3. Summary statistic of realization simulations when generated schedule with kriging is followed, surplus ore not removed at top and cleaned version at bottom

| Row Version | Ore(MT) | STRO | Input Bitumen (MT) | Average % | NPV (M$) |
|---|---|---|---|---|---|
| Mean | 276.2 | 1.4 | 28.3 | 10.2 | 2335.4 |
| Std. dev | 3.9 | 0.0 | 0.5 | 0.1 | 64.2 |
| Min | 269.2 | 1.3 | 27.3 | 10.0 | 2201.9 |
| Quartile 1 | 273.3 | 1.3 | 27.9 | 10.2 | 2285.1 |
| Median | 276.4 | 1.4 | 28.3 | 10.2 | 2332.2 |
| Quartile 2 | 278.7 | 1.4 | 28.6 | 10.3 | 2385.8 |
| Max | 287.3 | 1.4 | 29.4 | 10.5 | 2473.8 |
| Krig | 282.4 | 1.3 | 29.1 | 10.3 | 2461.0 |
| Etype | 282.2 | 1.3 | 28.5 | 10.1 | 2360.8 |

| Cleaned Version | Ore(MT) | STRO | Input Bitumen (MT) | Average % | NPV (M$) |
|---|---|---|---|---|---|
| Mean | 275.0 | 1.4 | 28.1 | 10.2 | 2317.5 |
| Std. dev | 3.1 | 0.0 | 0.4 | 0.1 | 56.0 |
| Min | 268.0 | 1.3 | 27.2 | 10.0 | 2195.8 |
| Quartile 1 | 272.7 | 1.4 | 27.8 | 10.2 | 2267.7 |
| Median | 275.2 | 1.4 | 28.2 | 10.2 | 2322.7 |
| Quartile 2 | 277.0 | 1.4 | 28.4 | 10.3 | 2363.0 |
| Max | 280.5 | 1.4 | 28.8 | 10.5 | 2415.6 |
| Krig | 282.4 | 1.3 | 29.1 | 10.3 | 2461.0 |
| Etype | 280.7 | 1.3 | 28.3 | 10.1 | 2341.5 |

Table 4. Summery statistics of Cumulative NPV at each period, surplus ore not removed at top and cleaned version at bottom.

| Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Mean | -193.2 | -449.8 | 112.4 | 603.3 | 975.7 | 1,303.9 | 1,600.6 | 1,893.0 | 2,175.6 | 2,335.4 |
| Std. dev | 0.0 | 0.0 | 24.6 | 32.7 | 36.1 | 38.9 | 42.6 | 51.7 | 62.9 | 64.2 |
| Min | -193.2 | -449.8 | 58.8 | 525.9 | 879.5 | 1,228.8 | 1,522.8 | 1,794.7 | 2,055.0 | 2,201.9 |
| Quartile 1 | -193.2 | -449.8 | 95.4 | 578.6 | 947.2 | 1,280.9 | 1,565.7 | 1,849.1 | 2,124.9 | 2,285.1 |
| Median | -193.2 | -449.8 | 110.6 | 604.1 | 975.3 | 1,297.5 | 1,602.6 | 1,904.7 | 2,169.6 | 2,332.2 |
| Quartile 2 | -193.2 | -449.8 | 129.9 | 620.1 | 1,002.2 | 1,337.7 | 1,638.8 | 1,935.7 | 2,230.3 | 2,385.8 |
| Max | -193.2 | -449.8 | 180.6 | 679.3 | 1,049.9 | 1,378.2 | 1,678.9 | 1,987.6 | 2,319.8 | 2,473.8 |
| Krig | -193.2 | -449.8 | 132.0 | 650.6 | 1,054.0 | 1,403.1 | 1,707.8 | 2,005.4 | 2,274.7 | 2,461.0 |
| Etype | -193.2 | -449.8 | 110.5 | 606.3 | 982.7 | 1,313.2 | 1,611.0 | 1,905.4 | 2,194.3 | 2,360.8 |

| Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Mean | -193.2 | -449.8 | 102.2 | 592.2 | 964.0 | 1,292.2 | 1,588.4 | 1,879.1 | 2,157.7 | 2,317.5 |
| Std. dev | 0.0 | 0.0 | 17.8 | 27.4 | 32.1 | 35.0 | 38.7 | 46.4 | 54.8 | 56.0 |
| Min | -193.2 | -449.8 | 58.8 | 525.8 | 879.5 | 1,228.8 | 1,520.7 | 1,792.7 | 2,048.9 | 2,195.8 |
| Quartile 1 | -193.2 | -449.8 | 89.4 | 574.5 | 941.0 | 1,266.4 | 1,554.6 | 1,840.6 | 2,112.6 | 2,267.7 |
| Median | -193.2 | -449.8 | 102.1 | 593.6 | 962.3 | 1,290.7 | 1,585.8 | 1,889.9 | 2,161.6 | 2,322.7 |
| Quartile 2 | -193.2 | -449.8 | 116.8 | 608.6 | 984.1 | 1,320.6 | 1,618.2 | 1,917.1 | 2,202.5 | 2,363.0 |
| Max | -193.2 | -449.8 | 144.5 | 654.0 | 1,030.6 | 1,360.8 | 1,655.3 | 1,964.5 | 2,255.1 | 2,415.6 |
| Krig | -193.2 | -449.8 | 132.0 | 650.6 | 1,054.0 | 1,403.1 | 1,707.8 | 2,005.4 | 2,274.6 | 2,461.0 |
| Etype | -193.2 | -449.8 | 103.4 | 596.2 | 972.5 | 1,303.0 | 1,600.8 | 1,895.3 | 2,175.0 | 2,341.5 |

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