# Alignment of Optimal Medium-Term Plans with Long-Term Plans using Mixed Integer Linear Programming

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#### **Abstract**

Open pit mine production planning and scheduling is one of the most important steps of any mining project. In this step, how to extract the ore as the valuable asset of mine is defined. Thus, the open pit mine production planning must be concerned well to build a good schedule of extraction. In a hierarchical categorizing, the open pit mine production planning and scheduling is divided into two main groups: long-term, medium-term and short-term planning. The long-term planning provides a strategic plan of the mining project while maximizing the expected profit of the mining project. On the other hand, medium- and short-term mine production scheduling provides operational scheme for extraction while tracking the strategic plan. Primarily, the operational costs are minimized in medium- and short-term mine production scheduling. In this paper, a dataset on an iron ore mine is used to apply long-term and medium-term open pit mine production scheduling. Both models are mixed integer linear programming models for the open pit mine. By applying the long-term and medium-term open pit mine production planning models, it is tried to show the applicability of models in satisfying technical constraints.

#### 1. Introduction

Simply, mining is defined as the process of recovering the valuable materials from the earth crust in the following five steps (Newman et al., 2010).

- Prospecting,
- Exploration,
- Development,
- Exploitation,
- Reclamation.

Prospecting and exploration steps include studies carried out by geologists in order to identify and measure the earth's properties, the value of deposit, mineral concentration and its variability throughout the orebody. The development phase primarily includes detailed engineering design of mine, determination of mining method, and estimating production capacity and invested mining capital. Ore is recovered from the mine in the exploitation step by open pit (surface) and/or underground mining techniques. In other words, mine production is performed in the exploitation stage. Extracted ore is hauled to different destinations such as processing plant, stockpiles, and waste dumps through designed ramps. Reclamation stage consists of restoring the area in which mining operations are done to its natural state to the possible extent (Newman et al., 2010).

Like any business, mining industry seeks the maximum profit. Thus, each of mining stages are planned and carried out in order to guarantee the maximum profit. In each step, data obtained from the previous steps are used for planning of the current stage. Thus, weak planning and performing any phase will create troubles in planning of the next stages which threaten the profitability of the whole mining project. As mentioned, the mine production is done in the exploitation phase. In this stage, the valuable asset of mine, raw ore, is extracted. Fundamentally, mine production planning and scheduling aims to decrease mining costs, maximize the production while considering the quality and operation requirements like mining and processing capacity and truck/shovel allocation (Fioroni et al., 2008). In other words, mine production planning and scheduling optimizes the way to extract the ore reserve as the most valuable asset of any mining operations (Franklin, 1985). Thus, careful planning and scheduling of mine production is a vital part in the chain of mining operations.

The focus of the current study is on open pit mine production planning and scheduling. A mine production schedule has to present a mining sequence scheme that meets physical limitations and aims to produce ore to reply to demand throughout the life of mine (Kuchta et al., 2004). Also, the mine production scheduling can be defined as a decision making process to determine the sequence of extraction of materials (block sequencing) and that how much ore and waste are sent to their corresponding destinations, e.g. processes, stockpiles, and waste dumps, over a time horizon.

Depending on the time horizon of scheduling, the mine production planning and scheduling is classified as long-term, medium-term and short-term planning that each category involves with certain objectives and issues (Chanda, 1992).

The long-term mine production planning mainly focuses on ore reserves, stripping ratio and major yearly investment plans (Fytas et al., 1993). The long-term mine production planning and scheduling is provided while looking at the mine operations in long-range time horizon (e.g. 20 years) of life of mine with specifically aiming to increase the profitability (usually net present value (NPV) maximization) (Chanda, 1992). There are numerous research results available in the area of long term open pit production scheduling using mathematical programming. A number of noticeable recent contributions in this area are carried out by Bienstock and Zuckerberg (2010). Bley et al. (2010), and Askari-Nasab et al. (2010). Mines use long-term schedule as a strategic tool to determine the starting time of mining a production region and as a guideline for medium-term and short-term mine production planning (Kuchta et al., 2004). In fact, the long-term mine production plan presents a general and long-term financial status of mine. However, satisfying blending constraints, head grade constraint, and other technical constraints which are imposed on the long-term planning does not ensure their fulfillment in the medium- and short-term plans (Kumral and Dowd, 2002). In fact, a long-term mine production plan gives a conceptual framework for mine activities while medium-term and short-term mine production plans translate the longterm concept into operational level by presenting effective and doable functions (Franklin, 1985).

Unlike the long-term mine planning, in the medium-term mine production planning some aspects of mine production such as haulage roads, mining sequence, and equipment investment are involved with more detail (Chanda, 1992). The medium- and short-term mine production planning is concerned with determination of sequence of ore and waste removal in medium-term periods (e.g. month, week, etc.). The medium- and short-term mine production scheduling should match with constraints applied by long-term plan, plant capacity, inventory restriction, equipment availability, haul and road. In short-term mine production scheduling, usually more detailed data on ore grade distribution are available (Muge et al., 1992). Unlike long-term mine production planning which tries to maximize profit, often medium-term mine production planning's purposes include minimizing the operational cost (Kumral and Dowd, 2002). The medium- and short-term mine production plan must obey the long-term plan to guarantee the expected profitability of mining project (Chanda, 1992). Thus, ensuring the profitability in the medium-term and short-term

planning, some operational costs involved in the mining process are going to be taken into account. Then, minimizing the cost is the main objective of medium-term and short-term plans.

The rest of the paper is organized as follows: in section 2, objectives of the paper are explained. Section 3 indicates the theoretical framework of the research. Different steps of research including both long-term and medium-term open pit mine production scheduling models are briefly described. In section 4, a dataset regarding an iron ore open pit mine is used to apply long-term and medium-term open pit mine production scheduling formulations. The long-term mine production planning is performed for 21 years. For medium-term mine production scheduling, the corresponding MILP model is applied for two years 10 and 16, separately. The results of implementation of mine production scheduling are elaborated and discussed. Finally, conclusion and future research directions are stated in section 5.

### 2. Objectives of research

The main objective of the current research is to apply long-term and medium-term open pit mine production scheduling to an iron ore mine case study. Also, the present study aims to provide both long-term and medium-term mine production schedules. The schedules and different plan views and issues of mining are plotted to show the results of mine production planning.

#### 3. Theoretical framework

The general framework of the present study is as follows:

- Applying a long-term open pit mine production scheduling model to a dataset related to an iron ore open pit mine;
- Selecting a few long-term periods (years) for applying medium-term mine production planning;
- Applying a medium-term open pit mine production scheduling in the selected years;
- Interpreting the results of mine production planning models in form of plan view and resulted schedules.

The long-term open pit mine production scheduling model is the formulation # 2 developed by Askari-Nasab and Awuah-Offei (2009). The long-term model as a mixed integer linear program (MILP) maximizes the discounted cash flow of mining project in the life of mine subject to a number of constraints as follows:

- Mining capacity constraints;
- Processing capacity constraints;
- Head grade constraints;
- Precedence or slope constraints:
- Complete extraction of whole pit.

On the other hand, the medium-term open pit mine production scheduling model of this research is the formulation developed by Eivazy and Askari-Nasab (2010). The medium-term open pit mine production scheduling model considers a number of stockpiles, processes, and waste dumps that are named as the destinations of extracted material. Generally, the medium-term open pit mine production scheduling model decides on followings:

- Sequence of block extraction including amount of extraction of each block in each period through the time horizon,
- Amount of total material that must be extracted from the mine in each period,
- Destinations that the mined materials are sent to,

Amount of ore reclaimed from the stockpiles to the processes in each period.

Briefly, the assumptions of the medium-term open pit mine production scheduling model are as follows:

- Each destination can receive material with a specific rock-type or a combination of rock-types,
- Each element has an acceptable grade range at different processes and stockpiles. Generally, stockpiles are separated by rock-type and grade range of ore.
- There are numerous routes to send extracted materials from the mine to different destinations. Mined material of each block could be hauled to corresponding destination through some routes.
- Material within each stockpile is homogeneous and the ore reclaimed from each stockpile has a certain grade equivalent to the average grade of stockpiled material.

The medium-term open pit mine production scheduling formulation minimizes total operational costs including total processing cost, total waste rehabilitation cost, total rehandling cost, and total haulage cost. The minimization of total costs is performed under a number of constraints as follows:

- Complete extraction of all blocks within the considered year of long-term plan,
- Assigning one destination for the extracted ore and waste in each period,
- Minimum extraction of blocks in each period,
- Mining capacity constraints,
- Processing capacity constraints,
- Stockpile capacity constraints,
- Head grade constraints,
- Vertical precedence in extraction of blocks (slope constraint),
- Route selection constraints,

#### 4. Case study and discussion of results

Here, a dataset regarding an iron ore open pit mine is used to apply the long-term and medium-term mine production scheduling models. The mixed integer linear program developed for the long-term mine production scheduling, is applied for 21 years. In addition, to apply medium-term mine production scheduling model, two year 10 and 16 are selected. Figs. 1 and 2 show the 3D and 2D views of whole pit, respectively. Totally, 33 ramps have been designed for the iron ore mine. The ramps can be seen in Fig. 2. As there are large number of blocks, solving the problem in this size could take much time. Thus, blocks are aggregated by fuzzy C-means method into 1217 cuts. Fig. 3 to Fig. 13 show the resulting long-term schedule in terms of plan view plots for levels with specific z-values. These specific levels are selected in which their blocks are extracted in years 10 and 16. The numbers inside the plan view plots shown in Fig. 3 to Fig. 13 indicate the year that the maximum portion of corresponding block is extracted.

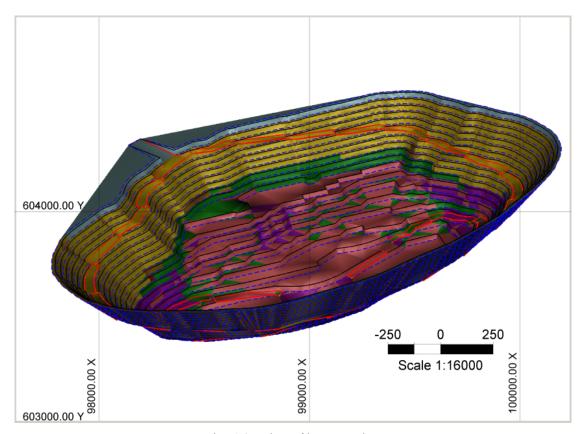


Fig. 1 3D view of iron ore mine

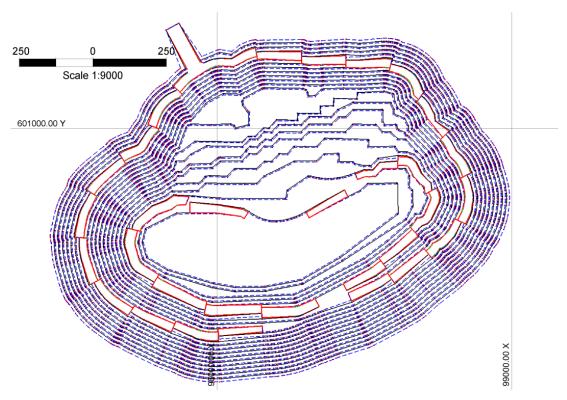


Fig. 2 2D view of pit with designed ramps

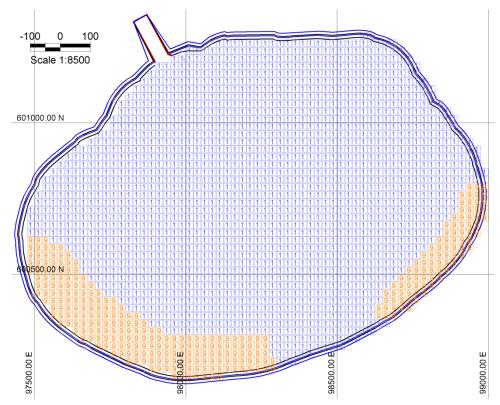


Fig. 3 Plan view plot of long-term schedule-Z=1725 m.

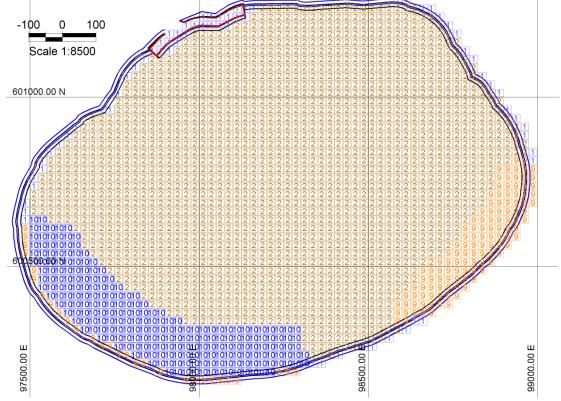


Fig. 4 Plan view plot of long-term schedule-Z=1710m.

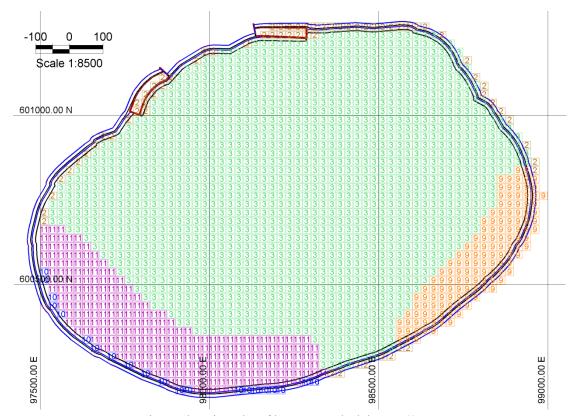


Fig. 5 Plan view plot of long-term schedule-Z=1695m.

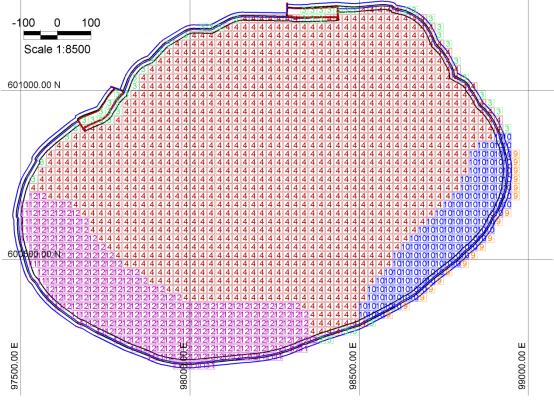


Fig. 6 Plan view plot of long-term schedule-Z=1680m.

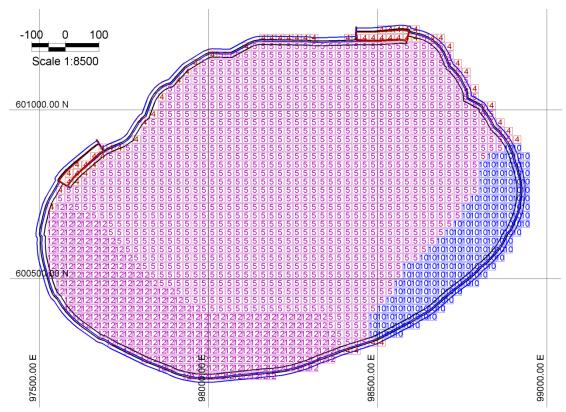


Fig. 7 Plan view plot of long-term schedule-Z=1665m.

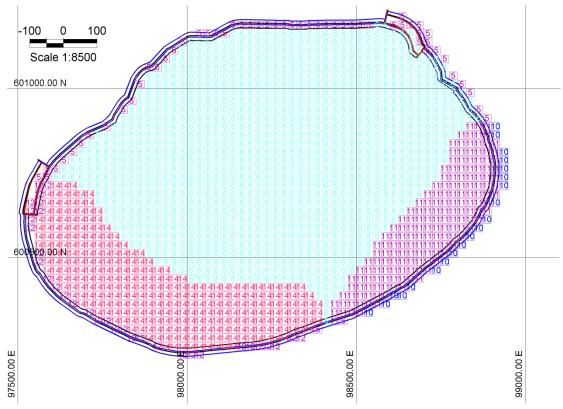


Fig. 8 Plan view plot of long-term schedule-Z=1650m.

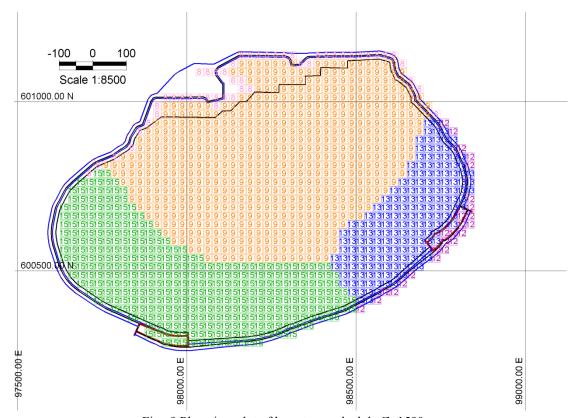


Fig. 9 Plan view plot of long-term schedule-Z=1590m.

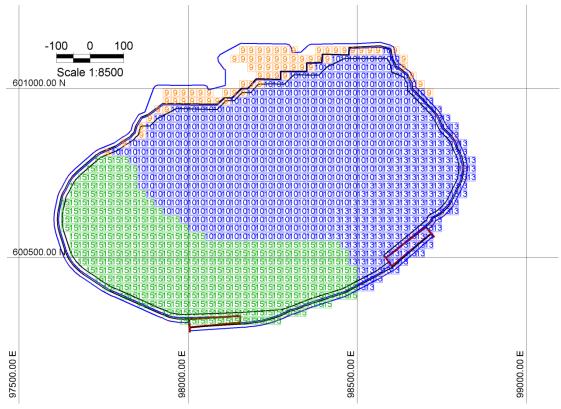


Fig. 10 Plan view plot of long-term schedule-Z=1575m.

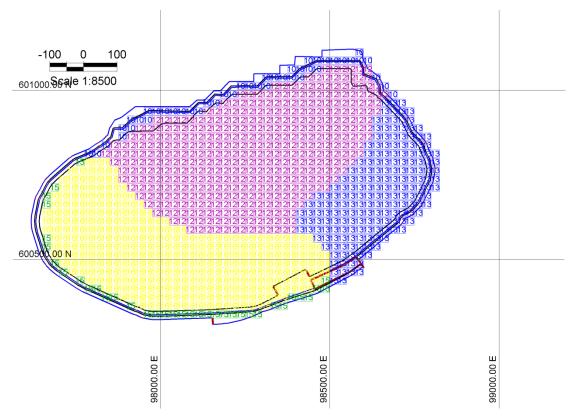


Fig. 11 Plan view plot of long-term schedule-Z=1560m.

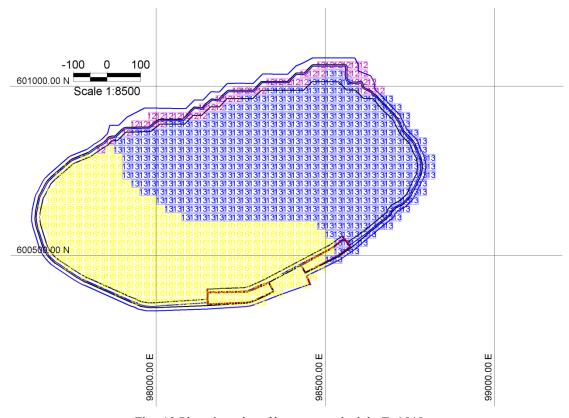


Fig. 12 Plan view plot of long-term schedule-Z=1545m.

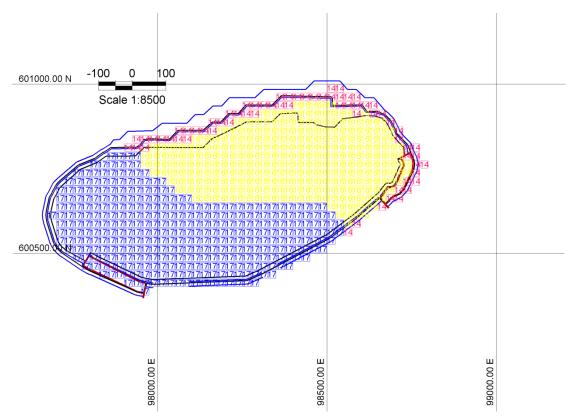


Fig. 13 Plan view plot of long-term schedule-Z=1515m.

Table 1 and 2 indicate the general information about the domain of medium-term scheduling in years 10 and 16, respectively. In addition, Table 3 shows the values of unit costs (waste rehabilitation, processing, mining, rehandling, and haulage) considered in the medium-term scheduling model. In Table 3, SP and P indicate stockpile and process, respectively.

Table 1 General specifications of the domain considered in year 10

Number of periods	12 months
Total number of blocks	2464
Total number of cuts	142
Block size	$25 \times 25 \times 15 \text{ (m}^3\text{)}$
Total rock tonnage	25 million tonnes
Total mineral tonnage	8 million tonnes

Table 2 General specifications of domain considered in year 16

Number of periods	12 months
Total number of blocks	1296
Total number of cuts	72
Block size	25×25×15 (m <sup>3</sup> )
Total rock tonnage	25 million tonnes
Total mineral tonnage	8 million tonnes

Unit Processing Cost (\$.tonne <sup>-1</sup> )		Co	Unit Rehandling Cost (\$.tonne <sup>-1</sup> )		te ation onne <sup>-1</sup> )	Unit Mining Cost (\$.tonne <sup>-1</sup> )	Unit Haulage Cost (\$.tonne <sup>-1</sup> .m <sup>-1</sup> )
P1	P2	SP1to p1	SP2 to p2	W1	W2	1	0.001
5.5	5 75	0.5	0.25	1.75	2		

Table 3 Values of different unit costs

Tables 4 and 5 show process 1 and 2's features that are involved in the medium-term mine production scheduling model in years 10 and 16, respectively. These features include the values of lower and upper bound on ore tonnage and grade fed into them. In addition, Table 5 and 6 indicate different specifications of stockpiles such as the values of lower and upper bound on ore tonnage and grade fed into them and the values of their output grade in years 10 and 16, respectively.

Table 4 Specifications of processes for medium-term planning in year 10

Process	Lower	Grac 6)	le	Uppei	· Grade	Capacity (Million Tonnes)		
	MWT	S	P	MWT	S	P	Min	Max
Process 1	73	0	0	83	2	0.18	0.31	0.35
Process 2	60	0	0	73	1.8	0.2	0.32	0.35

Table 5 Specifications of processes for medium-term planning in year 16

Process	Lower	Grac 6)	le	Uppei	· Grade	Capacity (Million Tonnes)		
	MWT	S	P	MWT	S	P	Min	Max
Process 1	73	0	0	83	2	0.18	0.35	0.35
Process 2	60	0	0	73	1.8	0.2	0.33	0.35

Table 6 Specifications of stockpiles for medium-term planning in year 10

Stockpile	Lower Grade (%)		Upper Grade (%)			Outpu	t Grad	e (%)	Initial Inventory	
	MWT	S	P	MWT	S	P	MWT	S	P	(Million Tonnes)
Stockpile 1	75	0	0	85	2	0.16	80	1.5	0.15	0.1
Stockpile 2	64	0	0	78	2.5	0.18	69	1	0.17	0

Table 7 Specifications of stockpiles for medium-term planning in year 16

Stockpile		Lower Grade (%)			Upper Grade (%)			t Grad	le (%)	Initial Inventory	
	MWT	S	P	MWT	S	P	MWT	S	P	(Million Tonnes)	
Stockpile 1	75	0	0	80	2	0.16	80	1.5	0.15	0.2	
Stockpile 2	64	0	0	75	2.5	0.18	69	1	0.17	0.1	

Figs. 14 to 21 show the plan view plots of the resulting medium-term schedule in year 10. In addition, Figs. 22 to 25 show the plan view plots of medium-term schedule for year 16. The numbers inside these figures represents the month that the maximum portion of the block is extracted within year 10. For example, in Fig. 14, value of 1 indicates that green blocks are at most extracted in the first month of year 10. Also, numbers 2 in Fig. 22 present that blue blocks are at most extracted in the second month of year 16. Fig. 26 and Fig. 27 show the medium-term mine production schedules in years 10 and 16, respectively. These figures present following information about the production:

- Amount of rock tonnage which is extracted in each month of the considered year
- Amount of waste tonnage which is fed into waste dump in each month of the considered year
- Amount of ore tonnage which is fed into processes in each month of the considered year
- Amount of ore tonnage which is fed into stockpiles in each month of the considered year

As well as above information, the optimal values of operational costs resulted from medium-term scheduling are as follows:

- 137 million dollars for year 10
- 157 million dollars for year 16



Fig. 14 Plan view plot of medium-term schedule-Year 10 (Z=1725m)

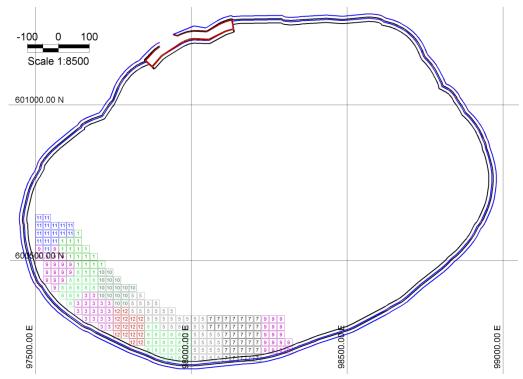


Fig. 15 Plan view plot of medium-term schedule-Year 10 (Z=1710m)

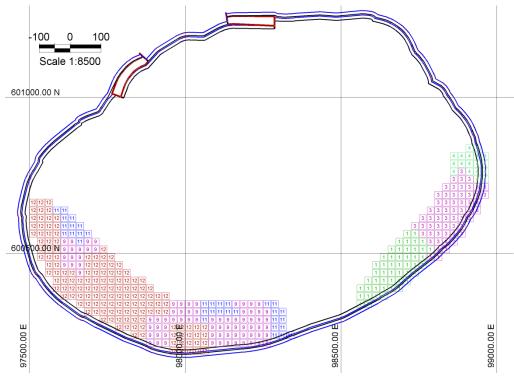


Fig. 16 Plan view plot of medium-term schedule-Year 10 (Z=1695m).

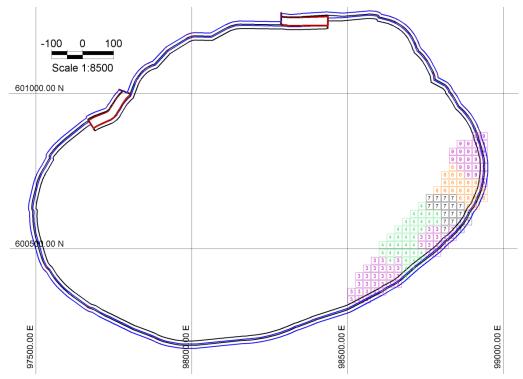


Fig. 17 Plan view plot of medium-term schedule-Year 10 (Z=1680m)

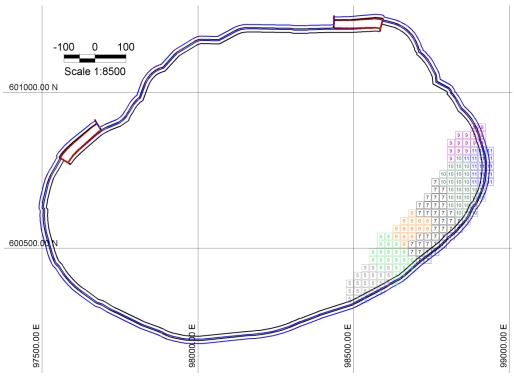


Fig. 18 Plan view plot of medium-term schedule-Year 10 (Z=1665m)

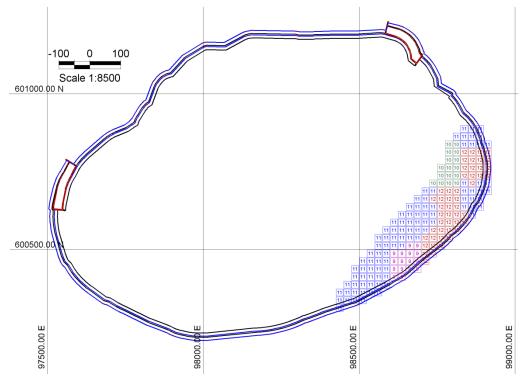


Fig. 19 Plan view plot of medium-term schedule-year 10 (Z=1650m)

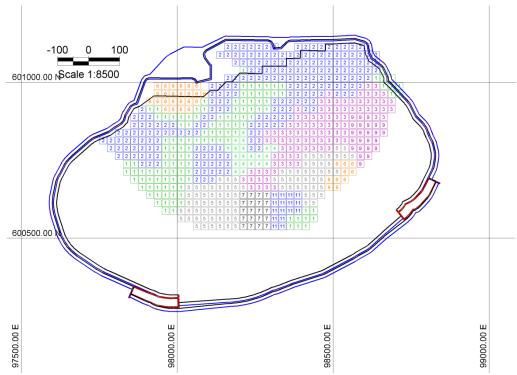


Fig. 20 Plan view plot of medium-term schedule-Year 10 (Z=1590m)

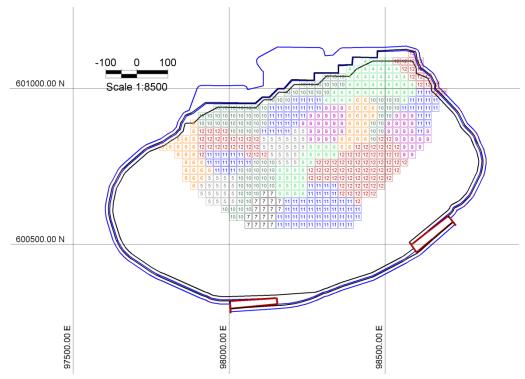


Fig. 21 Plan view plot of medium-term schedule-Year 10 (Z=1575m)

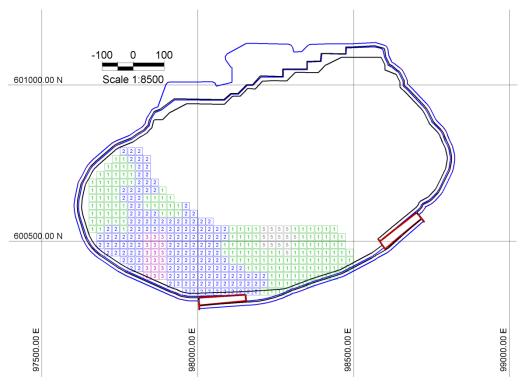


Fig. 22 Plan view plot of medium-term schedule-year 16 (Z=1575m)

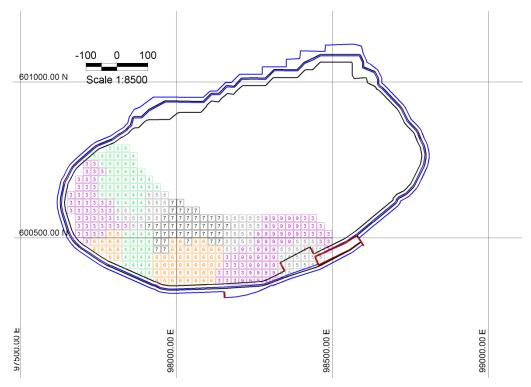


Fig. 23 Plan view plot of medium-term schedule-Year 16 (Z=1560m)

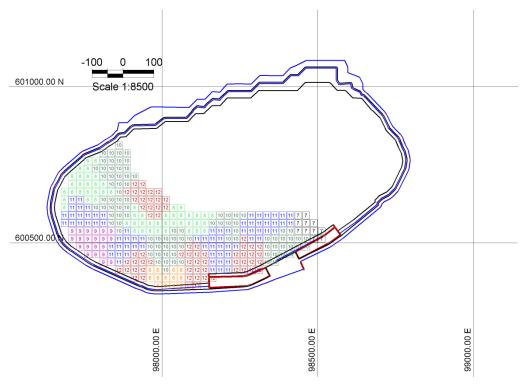


Fig. 24 Plan view plot of medium-term schedule-Year 16 (Z=1545m)

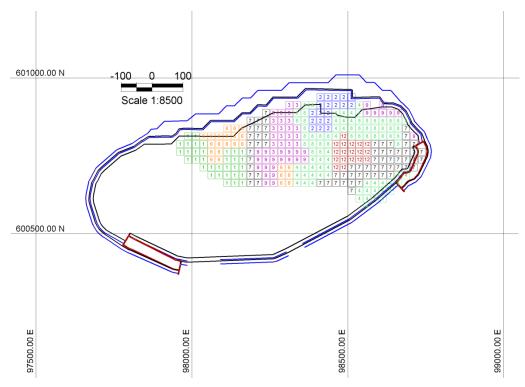


Fig. 25 Plan view plot of medium-term schedule-Year 16 (Z=1515m)

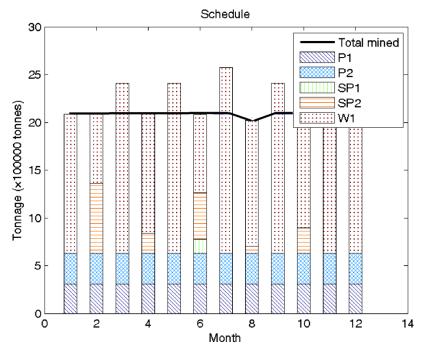


Fig. 26 Medium-term mine production schedule-Year 10

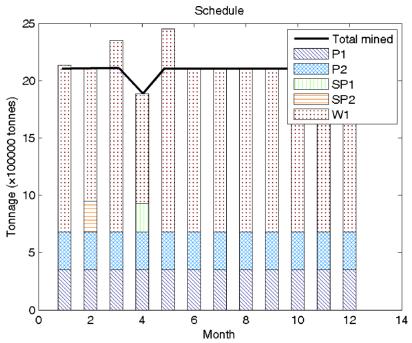


Fig. 27 Medium-term mine production schedule-Year 16

#### 5. Conclusions

In this paper, an open pit mine production scheduling model including long-term and medium-term was applied to a dataset on an iron ore mine. Both of the production schedule models were mixed integer linear programming. The long-term scheduling model tried to maximize the discounted cash flow in the life of mine while the medium-term production plan minimized the total operational costs. Since there were many blocks, solving the models in this large size could take so long time. Thus, blocks were aggregated into a number of cuts and then solved by TOMLAB/CPLEX. After applying the long-term mine production scheduling model, two years 10 and 16 were selected to apply the medium-term mine production scheduling. For each year, the medium-term mine production scheduling were presented in terms of plan view plots of schedule and mine production schedule plots. The plan view plots of schedule showed the plan view plots of the considered domain of planning in which the month that each bock is extracted was shown. The medium-term mine production schedule indicated that in each month of the considered year, what amount of material is extracted and sent to different destinations (stockpiles, processes, and waste dump).

#### 6. References

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## 7. Appendix

MATLAB and CPLEX Code Documentation