Production scheduling with minimum mining width constraints using mathematical programming

Yashar Pourrahimian and Hooman Askari-Nasab
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

Abstract

The successful implementation of branch and cut algorithms for combinatorial optimization problems in mathematical optimizers has reduced the gap between theory and practice in optimization of large-scale industrial problems. Mixed integer linear programming (MILP) methods are used for optimizing production planning in open pit mines with an objective function of maximizing the net present value. Mine production schedules generated by MILP formulations occasionally create a scattered block extraction order that cannot be implemented in practice. In this paper, two alternative MILP production scheduling formulations are presented with minimum mining width integrated into the models as linear constraints. The proposed MILP formulations are implemented and tested in a TOMLAB/CPLEX optimization solver. The results show that the new formulations prevent scattering of the excavation sequence in a given scheduling period and have an acceptable computing time.

1. Introduction

Open pit mine production scheduling can be defined as specifying the sequence in which "blocks" should be removed from the mine in order to maximize the total discounted profit from the mine subject to a variety of physical and economic constraints. Typically, the constraints relate to the mining extraction sequence; mining, milling and refining capacities; mill head grades; and various operational requirements such as minimum pit bottom width. Various methods have been used for optimization of mining problems (Ramazan et al., 2003; Ramazan et al., 2004a; Ramazan et al., 2004b; Caccetta, 2007). Some examples include: linear programming and mixed integer linear programming (Caccetta et al., 2003; Ramazan et al., 2003; Ramazan et al., 2004a; Ramazan et al., 2004b), dynamic programming (Tolwinsky et al., 1996), graph and network theory (Fan et al., 2003), simulation (Dimitrakopoulos, 1998) and artificial intelligence (Denby et al., 1994; Denby et al., 1996; Tolwinsky et al., 1996; Askari-Nasab et al., 2008; Askari-Nasab et al., 2009). Among these optimization techniques, mixed integer linear programming is recognized as having significant potential to optimize production plans in large open pit mines with the objective of maximizing the total net present value.

In this paper, we will present two mixed integer linear programming (MILP) production scheduling formulations with minimum mining width integrated into the models as linear constraints.

2. MILP production scheduling formulations

2.1. Parameters

The parameters used in the mine production scheduling are listed in Table 1.

Table 1. MILP production scheduling parameters

Parameter	Description
BEV_n^t	block economic value of block n in period t
T	maximum number of scheduling periods
N	number of blocks to be scheduled
K	number of mining cuts to be scheduled
i	interest rate
X_n^t	binary integer variable controlling the sequence of extraction of blocks, equal to 1 if block n is to be mined in period t, otherwise 0
gu^t	upper bound on acceptable average head grade of ore send to the mill in period t
gl^t	lower bound on acceptable average head grade of ore send to the mill in period t
g_n	average ore grade of block n
g_k	average ore grade in mining cut k
Ot_n	ore tonnage in block n
Ot_k	ore tonnage in mining cut k
W_n	tonnage of waste material in block n
W_k	tonnage of waste material in mining cut k
$(PC_{\max})^t$	upper bound on ore processing capacity in period t
$(PC_{\min})^t$	lower bound on ore processing capacity in period t
$(MC_{\max})^t$	upper bound on ore processing capacity in period t
$(MC_{\min})^t$	lower bound on ore processing capacity in period t
Wb	working block
m	number of the blocks forced to be mined with working block (8 or 24)
v_k^t	discounted revenue generated by selling the final product within mining cut k in period t minus the extra discounted cost of mining all the material in mining cut k as ore and processing it
s_k^t	continuous variable, representing portion of mining cut c_k to be extracted as ore and processed in period t
q_n^t	discounted cost of mining all the material in block n as waste
y_k^t	continuous variable, representing portion of mining cut c_k to be mined in period t
b_k^t	binary integer variable controlling precedence of extraction of mining cuts

Each linear programming model includes an objective function and constraints. The open pit mine production schedule can be defined as specifying the sequence in which blocks should be removed from the mine to maximize the total discounted economic value, or the net present value (NPV) from the mine, subject to a variety of physical and economic constraints. The first model is a modification of the approach used by Ramazan et al.(2004b); in this model, equipment access and mobility have been added. The second model (Askari-Nasab et al., 2009) is developed based on a combination of concepts from Caccetta et al. (2003) and Boland et al. (2009). In this model, mining and processing are both at mining-cut level. The blocks are clustered prior to schedule optimization and the ore processing and mining are controlled by two continuous variables. Blocks within the mining bench are grouped into clusters based on their attributes, spatial location, rock type, and grade distribution. Similar to blocks, each mining cut has coordinates representing the centre of the cut and its spatial location.

2.2. Model I

Objective function of model I is:

$$\text{Maximize} \quad \sum_{t=1}^{T}\sum_{n=1}^{N}\frac{BEV_n^t}{(1+i)^t}\times X_n^t \tag{1}$$

This objective function is subject to the following constraints:

$$\sum_{n=1}^{N}(g_n - gu^t)\times Ot_n \times X_n^t \le 0 \qquad \forall\, t\in\{1,...,T\} \tag{2}$$

$$\sum_{n=1}^{N}(g_n - gl^t)\times Ot_n \times X_n^t \ge 0 \qquad \forall\, t\in\{1,...,T\} \tag{3}$$

$$(PC_{\min})^t \le \sum_{n=1}^{N}(Ot_n \times X_n^t) \le (PC_{\max})^t \qquad \forall\, t\in\{1,...,T\} \tag{4}$$

$$(MC_{\min})^t \le \sum_{n=1}^{N}(Ot_n + W_n)\times X_n^t \le (MC_{\max})^t \qquad \forall\, t\in\{1,...,T\} \tag{5}$$

$$\sum_{t=1}^{T}X_n^t = 1 \qquad \forall\, n\in\{1,...,N\} \tag{6}$$

$$0.4\,(Int(m))\,X_{Wb}^t - \sum_{n=1}^{m} X_n^t \le 0 \qquad \forall\, t\in\{1,...,T\},\quad Wb\in\{1,...,N\} \tag{7}$$

Eqs. (2) and (3) are grade blending constraints; these inequalities ensure that the average grade of the material sent to the mill is within the desired range in each period. Eq. (4) imposes processing capacity constraints; these inequalities satisfy that the total tonnage of ore processed is within the acceptable range of the processing plant capacity. Eq. (5) applies mining capacity constraints; these inequalities satisfy that the total amount of material (waste, ore and overburden) mined is within the acceptable mining equipment capacity in each period. Eq. (6) is a reserve constraint; this constraint is applied to each block such that that all the blocks in the model considered have to be mined once. Eq. (7) is equipment access and mobility constraints; these constraints ensure that there is sufficient access for equipment for mining a given block and they prevent spreading of scheduling pattern over each period. Eq. (7) also minimizes equipment movement in a given period. In order to consider equipment access to each block, the optimization model should enforce extraction of a working block with a number of surrounding blocks in the same extraction period. To perform this, we define a concentric window around a working block (Fig. 1). The number of blocks in a window can be 8 or 24. The optimization model should force extraction of a working block and blocks numbered 1 to 8 or 1 to 24 in the same scheduling period, with at least *m* blocks within this window. Experience shows that it is better to use 40 percent of blocks in given window, either 8 or 24. Using a higher percentage than 40 percent would tighten up the constraints and most of the time the MILP model will not result in a feasible solution.

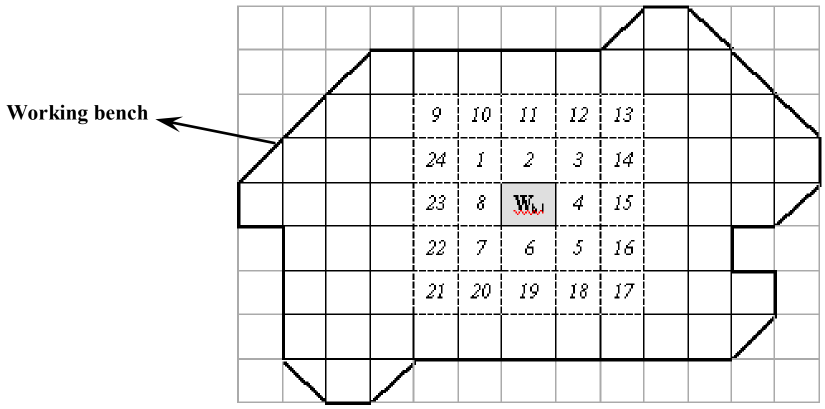


Fig. 1. Block configuration around a working block in a working bench

2.3. Model II

Objective function of model II is:

$$\text{Maximize} \quad \sum_{t=1}^{T}\sum_{n=1}^{N}(v_n^t \times s_n^t - q_n^t \times y_n^t) \tag{8}$$

This objective function is subject to the following constraints:

$$gl^t \le \sum_{k=1}^{K} g_k \times Ot_k \times s_k^t \Big/ \sum_{k=1}^{K} Ot_k \times s_k^t \le gu^t \qquad \forall t \in \{1,...,T\} \tag{9}$$

$$(PC_{\min})^t \le \sum_{k=1}^{K} Ot_k \times s_k^t \le (PC_{\max})^t \qquad \forall t \in \{1,...,T\} \tag{10}$$

$$(MC_{\min})^t \le \sum_{k=1}^{K} (Ot_k + W_k) \times y_k^t \le (MC_{\max})^t \qquad \forall t \in \{1,...,T\} \tag{11}$$

$$s_k^t \le y_k^t \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{12}$$

$$\sum_{i=1}^{t} y_k^i - b_k^t \le 0 \qquad \forall t \in \{1,...,T\}, \quad \forall k \in \{1,...,K\} \tag{13}$$

$$b_k^t - b_k^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\}, \quad \forall k \in \{1,...,K\} \tag{14}$$

Eqs. (9) to (11) control grade blending, processing capacity, and mining capacity constraints at the mining-cut level with fractional extraction from mining cuts. Eq. (12) ensures that the amount of ore extracted and processed from any mining cut in any given period is going to be less than or equal to the amount of rock extracted from that mining cut. Eqs. (13) and (14) check the set of the immediate predecessor cut that must be extracted prior to extracting mining cut, *k*.

3. Application of models to production scheduling in an iron mine

A TOMLAB/CPLEX environment (Holmstrom, 1989-2009) is used to develop and test the two models. The models are then used to schedule a bench of an iron ore mine. The block model includes estimated values for percentage of sulphur, phosphor and iron ore. The main mineral considered for profit is iron ore. The bench is divided into 415 blocks with 25m×50m×15m dimension. The bench contains 11.2 million tonnes of ore, with an average grade of about 72% magnetic weight recovery (MWT) of iron ore. A production schedule for the bench is developed to maximize the total discounted economic value at a 10% discount rate. Model I and model II generated M$531.64 and M$531.21 total discounted economic value for this bench over five periods of extractions respectively. To meet the physical mining constraints we have used a mining capacity upper bound of five million tonnes per period, whereas the processing capacity is 2.5 million tonnes per period. In model II, the bench block model is divided into 30 mining cuts. Fig. 2 shows the extraction sequences of this bench for two models. Model I has created scattered block extraction order, while the schedule generated by model II is smooth and feasible to implement in practice. The yearly tonnage of ore processed, waste mined, and the total tonnage of material mined in each period of production is compared in Fig 3. Fig. 4 and Fig. 5 show average iron ore grade and cash flow per period, respectively. There are not significant differences between results from the two models, but model I has many binary integer variables and the CPU processing time is almost thirty thousand times more, comparing to model II.

4. Conclusion

Two mixed integer linear programming (MILP) models were presented. Model I only consists of binary integer decision variables. This model generates a production schedule at block level resolution. In model II, extraction, processing, and the order of block extraction are controlled at the mining-cut level. Model II reduces the size and computational time of the problem. The models were compared using block model data from a bench of an iron ore mine. Although, model I generates a higher total discounted economic value than model II, the run time for model I is 30,000 times more than model II. The results show that model II generates a practical mining schedule that includes enough space for equipment to maneuver and it prevents scattering of the excavation sequence in a given scheduling period.

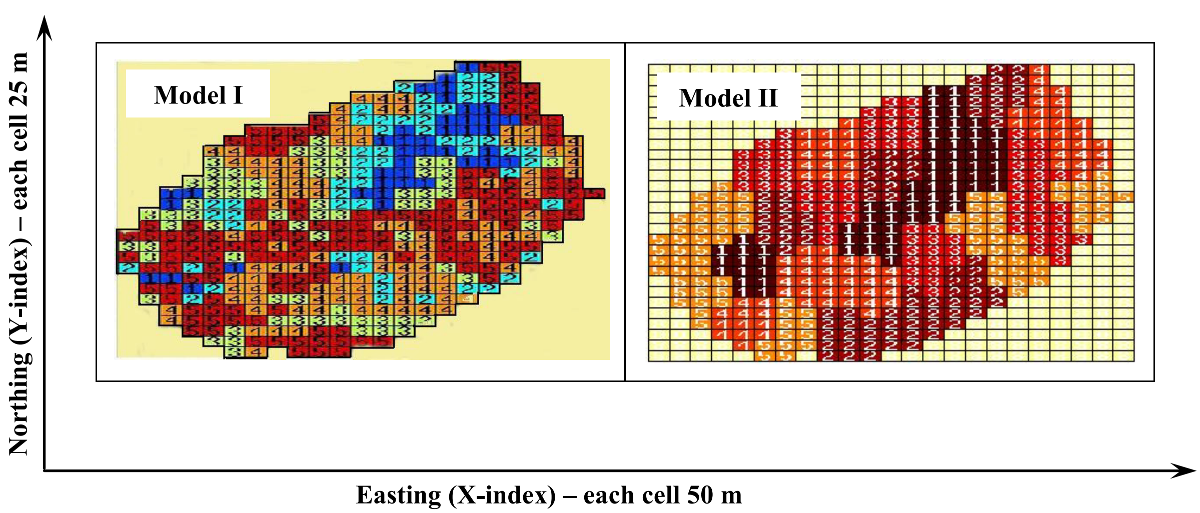
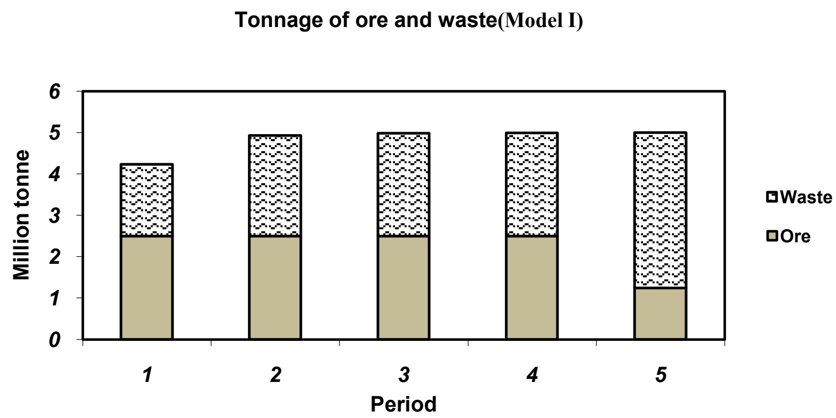


Fig. 2. Extraction sequence for the two models



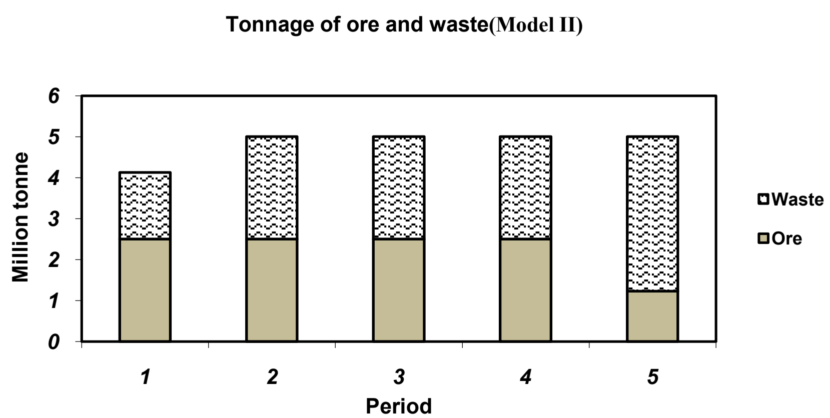


Fig 3. Tonnage of ore and waste per period

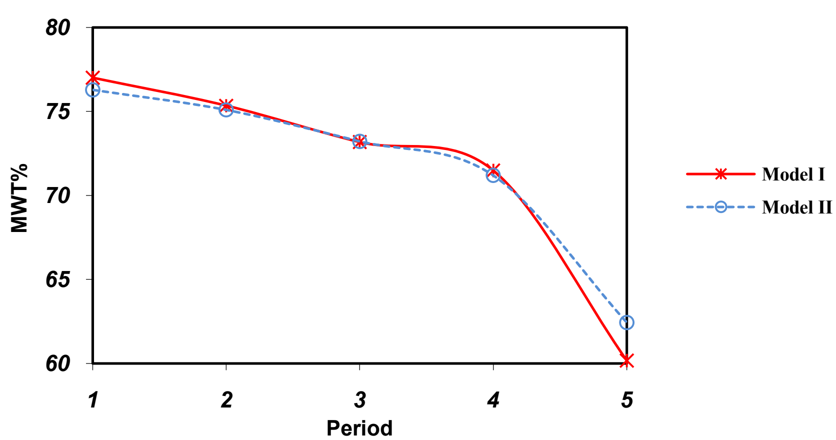


Fig. 4. Average iron ore (MWT%) grade per period

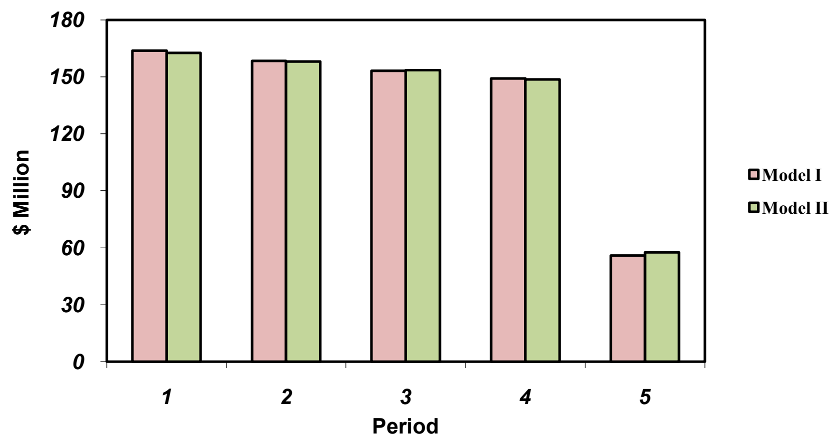


Fig. 5. Comparison between amount of cash flow

5. References

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6. Appendix

MATLAB and TOMLAB/CPLEX code and documentation for Model I & Model II