

A mathematical programming formulation for block cave production scheduling

Yashar Pourrahimian and Hooman Askari-Nasab
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

Abstract

Caving is the lowest cost underground mining method provided that the drawpoint spacing, drawpoint size, and ore handling facilities are designed to suit the cave material. Also the drawpoint horizon must be maintained for the life of the draw for a successful operation.

This paper presents a mixed integer programming (MILP) formulation for block cave production scheduling. The objective function of the scheduler is to maximize the net present value (NPV) of operation while meeting technical and operational constraints. In development of the optimization model, in addition to the formulation of the objective function, the set of constraints that define the feasible space of solution is critical to effective mine planning solution.

In this paper, schedule optimization is carried out using six general constraints to model and control production. The constraints define: production target, draw rate, grade blending, maximum number of active drawpoints, precedence of extraction of drawpoints, and draw angle. Application of the model for production scheduling of a synthetic data shows how the constraints regulate production.

1. Introduction

Long-term mine scheduling is one of the optimization problems. A production schedule must provide a mining sequence that takes into account the physical limitations of the mine and, to the extent possible, meets the demanded quantities of each raw ore type at each period throughout the mine life. Mines use the schedules as long-term strategic planning tools to determine when to start mining a production area and as short-term operational guides.

Underground mining is more complex in nature than surface mining (Kuchta et al., 2004). Flexibility of underground mining is less than surface mining due to the geotechnical, equipment and space constraints (Topal, 2008).

In spite of the difficulties associated with the application of mathematical programming to production scheduling in underground mines, authors have attempted to develop methodologies to optimize production schedules.

Williams et al. (1972) planned sublevel stoping operations for an underground copper mine over one year using a linear programming approximation model. Jawed (1993) formulated a linear goal programming model for production planning in an underground room and pillar coal mine. Tang et al. (1993) integrated linear programming with simulation to address scheduling decisions, as did Winkler (1998). Trout (1995) used the MIP method to schedule the optimal extraction sequence for underground sublevel stoping. Ovanic (1998) used mixed integer programming of type two special ordered sets to identify a layout of optimal stopes. Carlyle et al. (2001) presented a model that

maximized revenue from Stillwater's platinum and palladium mine. Topal et al. (2003) generated a long-term production scheduling MIP model for a sub-level caving operation and successfully applied it to Kiruna Mine. Sarin et al. (2005) scheduled a coal mining operation with the objective of net present value maximization. Atae-Pour (2005) critically evaluated some optimization algorithms according to their capabilities, restrictions and application for use in underground mining. McIssac (2005) formulated the scheduling of underground mining of a narrow veined polymetallic deposit utilizing MIP. Nehring et al. (2007) presented a mixed integer programming formulation for production schedule optimization in underground hard rock mining.

Scheduling of underground mining operations is primarily characterized by discrete decisions to mine blocks of ore, along with complex sequencing relationships between blocks. Since linear programming (LP) models cannot capture the discrete decisions required for scheduling, MIPs are generally the appropriate mathematical programming approach to scheduling.

The methods, currently used to compute production schedule in block cave mines, can be classified in two main categories: (a) heuristic methods and (b) exact optimization methods.

Heuristic methods are particularly used to rapidly come to a solution that is hoped to be close to the best possible answer, or optimal solution. These methods are used when there is no known method to find an optimal solution under the given constraints.

The original heuristic methods were the manual draw charts used at the beginning of block caving. These methods evolved through its use at Henderson mine where a way to avoid early dilution entry was described by constraining the draw profile to an angle of draw of 45 degrees (Dewolf, 1981). Heslop et al. (1981) described a volumetric algorithm to simulate the mixing along the draw cone. Carew (1992) described the use of a commercial package called PC-BC to compute production schedules at Cassiar mine. Diering (2000) showed the principles behind the commercial tool PC-BC to compute production schedules, providing several case studies where different draw methods have been applied depending on the ore body geometry and rock mass behavior.

The application of operation research methods to the planning of block cave mines was first described by Riddle (1976). This development intended to compute mining reserves and define the economic extent of the footprint. The final algorithm did not reflect the operational constraints of block caving described above since it worked with the block model directly instead of defining the concept of draw cone as an individual entity of the optimization process.

The first attempt to use mathematical programming in block cave scheduling was made by Chanda (1990) who implemented an algorithm to write daily orders. This algorithm was developed to minimize the variance of the milling feed in a horizon of three days. Guest et al. (2000) made another application of mathematical programming in block cave long term scheduling. In this case, the objective function was explicitly defined to maximize draw control behavior. However, the author stated that the implicit objective was to optimize NPV. There are two problems with this approach. The first one is that maximizing tonnage or mining reserves will not necessarily lead to maximum NPV. The second problem is the fact that draw control is a planning constraint and not an objective function. The objective function in this case would be to maximize tonnage, minimize dilution or maximize mine life.

Rubio (2002) developed a methodology that would enable mine planners to compute production schedules in block cave mining. He proposed a new production processes integration and formulated two main planning concepts as potential goals to optimize as part of the long term planning process, maximization of NPV and maximization of mine life.

Rahal et al. (2003) used a dual objective mixed integer linear programming algorithm to minimize the deviation between the actual state of extraction (height of draw) and a set of surfaces that tend towards a defined draw strategy. This algorithm assumes that the optimal draw strategy is known.

Nevertheless, it is postulated that by minimizing the deviation to the draw target the disturbances produced by uneven draw can be mitigated.

Diering (2004) presented a non-linear optimization method to minimize the deviation between a current draw profile and the target defined by the mine planner. He emphasized that this algorithm could also be used to link the short with the long-term plan. The long-term plan is represented by a set of surfaces that are used as a target to be achieved based on the current extraction profile when running the short-term plans. Rubio et al. (2004) presented an integer programming algorithm and an iterative algorithm to optimize long-term schedules in block caving integrating the fluctuation of metal prices in time.

We critically review the MILP formulations of the block cave production scheduling problem. We model the problem considering different possible directions of extraction and different draw angles. We divide the major decision variables into two categories, continuous variables representing the portion of a drawpoint that is going to be extracted in each period and binary integer variables controlling the order of extraction of drawpoints and the number of active drawpoints in each period. The MILP formulation also ensures that the angle of draw along the advance direction is 45 degree. We have implemented the optimization formulation in TOMLAB/CPLEX (Holmstrom, 1989-2009) environment. A scheduling case study with synthetic data is carried out over fifteen periods to verify the MILP model.

The next section of the paper covers the assumptions, problem definition, and the notations of variables. Section 3 presents mixed integer linear programming formulation of the problem, while section 4 presents the numerical modeling techniques. Section 5 presents an example, conclusions and future work followed by the list of references in the next section.

2. Assumptions, problem definition, and notation

We assume that a geological block model represents the orebody, which is a three-dimensional array of rectangular or cubical blocks used to model orebodies and other sub-surface structures. Numerical data are used to represent a single attribute of the orebody such as densities, grades, elevations, or economic data. The draw cones are created based on the block model. We assume that all draw cones have the same height. Average grade of all blocks inside each draw cone is presented as grade of that draw cone. Each draw cone is representative by two drawpoints (see Fig. 1). The coordinates of the center of each draw cone defines the spatial location of each drawbell. There is no selective mining and the whole material in the draw cone has to be extracted. Fig. 1 shows the flow chart from initial block model to the created draw cones.

The problem is finding a sequence of drawpoints extraction over the mine life, provided the NPV is maximized. The material in each draw cone is to be scheduled over T periods depending on the goals and constraints associated with the mining operation. To solve the problem, three decision variables are employed for each draw cone, one continuous decision variable and two binary integer variables. Continuous decision variable indicates the portion of extraction from each draw cone in each period and two binary integer variables control the number of active drawpoints and precedence of extraction. This formulation is implemented for eight advance directions to maximize the NPV. Fig 2. illustrates a schematic plan view of these directions.

According to the advance direction for each drawpoint, d , there is a set $C_d(K)$ which defining the predecessor drawpoints that must be started prior to extraction of drawpoint d , where K is the total number of drawpoints in set $C_d(K)$. Based on the search direction, eight different predecessor data sets can be defined for each drawpoint. Fig. 3. illustrates these data sets.

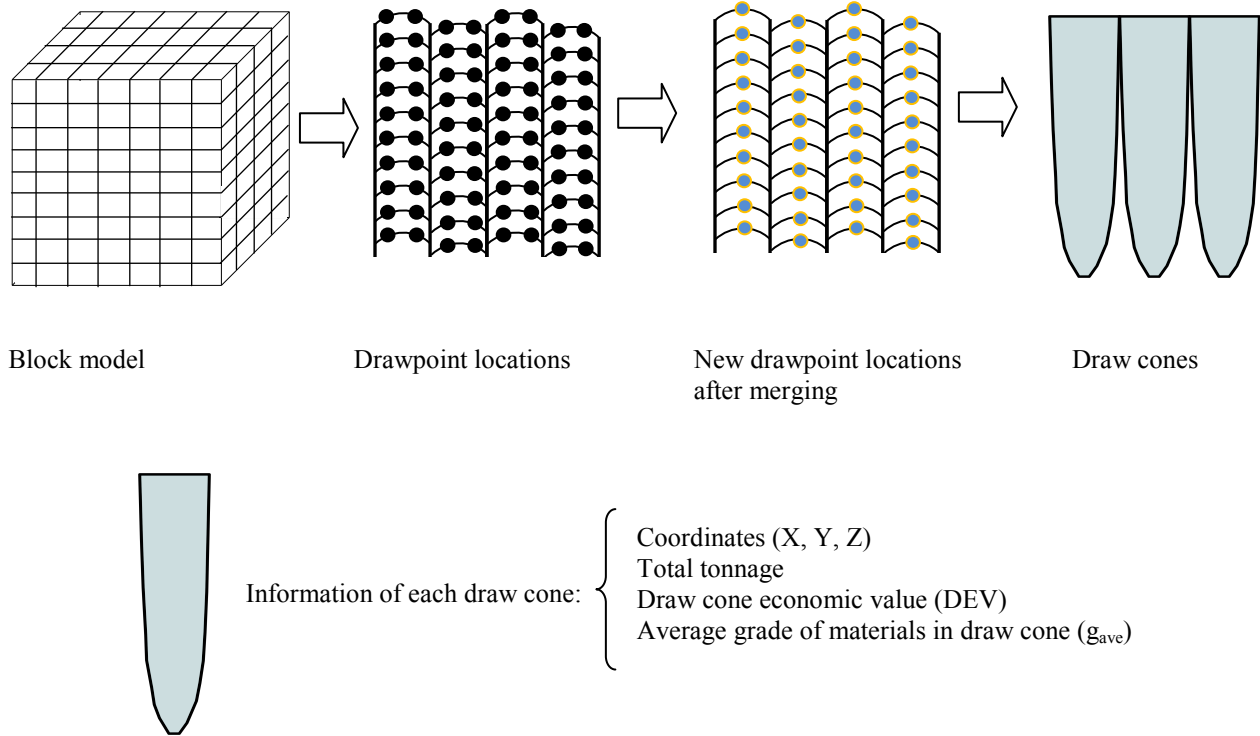


Fig. 1. Overall flowchart from initial block model to draw cones

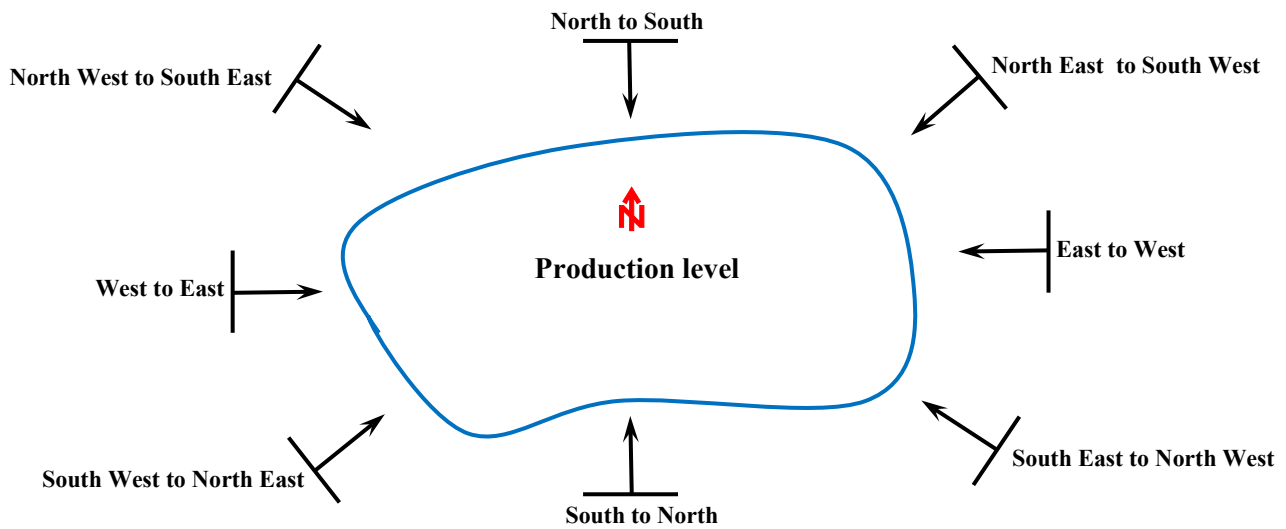
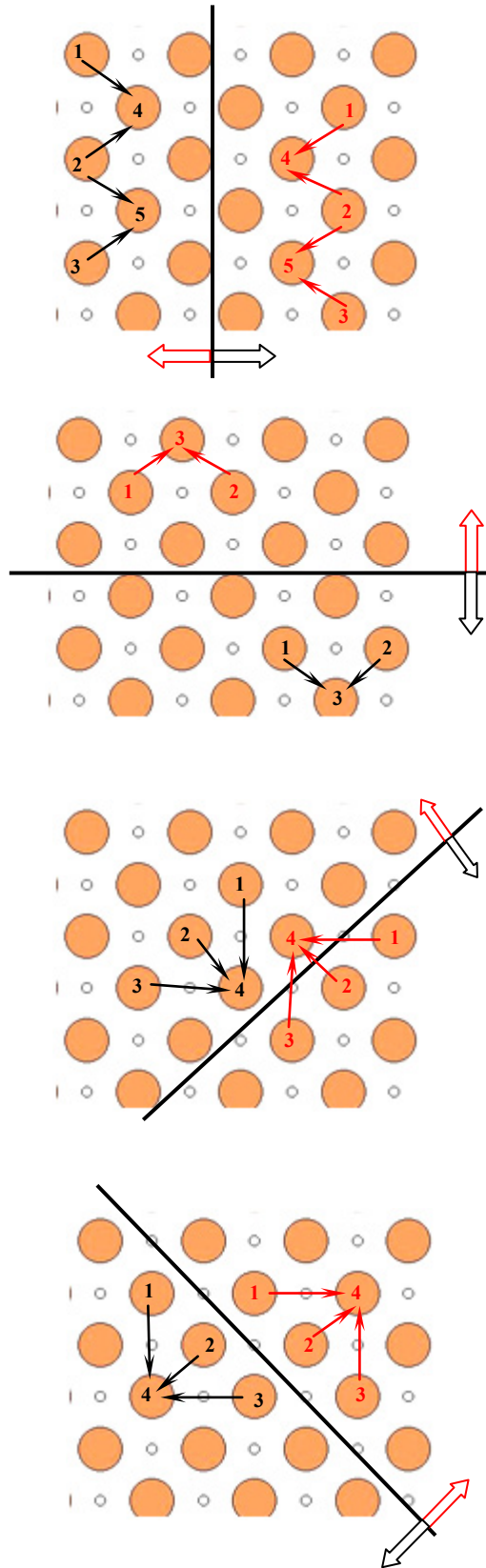


Fig. 2. Alternative cave advance directions



W to E or E to W

For each of the above directions, there is a set $C_d(K)$, which includes all drawpoints that must be started prior to drawpoint d . Set $C_4(K)$ belongs to drawpoint 4 and it has two members, drawpoints 1 and 2.

$$C_4(2) = \{1, 2\} \quad \text{and} \quad C_5(2) = \{2, 3\}$$

N to S or S to N

For each of the above directions, there is a set $C_d(K)$, which includes all drawpoints that must be started prior to drawpoint d . Set $C_3(K)$ belongs to drawpoint 3 and it has two members, drawpoints 1 and 2.

$$C_3(2) = \{1, 2\}$$

SE to NW or NW to SE

For each of the above directions, there is a set $C_d(K)$, which includes all drawpoints that must be started prior to drawpoint d . Set $C_4(K)$ belongs to drawpoint 4 and it has three members, drawpoints 1, 2 and 3.

$$C_4(3) = \{1, 2, 3\}$$

SW to NE or NE to SW

For each of the above directions, there is a set $C_d(K)$, which includes all drawpoints that must be started prior to drawpoint d . Set $C_4(K)$ belongs to drawpoint 4 and it has three members, drawpoints 1, 2 and 3.

$$C_4(K) = \{1, 2, 3\}$$

Fig. 3. Constructing the predecessor data set

2.1. NOTATION

The notation of decision variables, parameters, sets, and constraints are as follows:

2.1.1 Sets

$C_d(K)$ For each drawpoint, d , there is a set $C_d(K)$ defining the predecessor drawpoints that must be started prior to extraction of drawpoint d , where K is the total number of drawpoints in set $C_d(K)$.

2.1.2 Indices

A general parameter f can take three indices in the format of $f_{d,k}^t$. Where:

$t \in \{1, \dots, T\}$ Index for scheduling periods.
 $d \in \{1, \dots, D\}$ Index for drawpoints.
 $k \in \{1, \dots, K\}$ Index for drawpoints in the set $C_d(K)$.

2.1.3 Parameters

DEV The drawpoint economic value.
 i The discount rate.
 N^t Maximum number of active drawpoints in period t .
 Q_d Total tonnage of material in draw cone associated with drawpoint d .
 g_d Average grade of material in draw cone associated with drawpoint d .
 $\overline{g^t}$ Upper bound on acceptable average head grade in period t .
 $\underline{g^t}$ Lower bound on acceptable average head grade in period t .
 \overline{R}_d^t Upper bound on draw rate from drawpoint d in period t .
 \underline{R}_d^t Lower bound on draw rate from drawpoint d in period t .
 \overline{M}^t Upper bound on feasible production target (tonne) from the mine in period t .
 \underline{M}^t Lower bound on feasible production target (tonne) from the mine in period t .
 L_{dk} The shortest distance between drawpoint d and drawpoint k from set $C_d(K)$
 A_d Cross section area between undercut level and top of extraction.
 γ_d Density of material in drawpoint d .
 f Extraction height factor. It is calculated by $f = \frac{Q_d}{A_d \cdot \gamma_d}$

2.1.4 Decision Variables

$u_d^t \in [0,1]$ Continuous variable, representing the portion of draw cone d to be extracted in period t .

- $a_d^t \in \{0,1\}$ Binary integer variable controlling the active drawpoints. a_d^t is equal to one if drawpoint d is active in period t , otherwise it is zero.
- $z_d^t \in \{0,1\}$ Binary integer variable controlling the precedence of extraction of drawpoints. z_d^t is equal to one if extraction of drawpoint d has started by period t , otherwise it is zero.

3. Mathematical formulation for NPV maximization

3.1. Objective function

The objective function is composed of the draw cone economic value, discount rate and a continuous decision variable that indicates the portion of a draw cone which is extracted in each period. The DEV value is defined based on economic block values. The profit from mining a block that is located inside the cone depends on the value of the block and the costs incurred in mining and processing. The cost of mining a block is a function of its location, which characterizes how deep the block is located relative to the surface and how far it is located relative to its final dump.

The objective function is to maximize the NPV over all periods, $1 \leq t \leq T$. The objective function is expressed by Eq.(1),

$$\text{Maximize} \quad \sum_{t=1}^T \sum_{d=1}^D \left[\frac{DEV_d}{(1+i)^t} \right] u_d^t \quad (1)$$

3.2. Definition of constraints

3.2.1 Draw rate

These inequalities, Eq.(2), ensure that the draw rate from drawpoint d in period t is within the acceptable tonnage range.

$$\underline{R}_d^t \leq u_d^t \times Q_d \leq \overline{R}_d^t \quad \forall t \in \{1, \dots, T\}, \quad d \in \{1, \dots, D\} \quad (2)$$

3.2.2 Target production

These inequalities, Eq.(3), ensure that the total tonnage of material extracted from drawpoints in each period is within the acceptable range of production targets for the mine.

$$\underline{M}^t \leq \sum_{d=1}^D u_d^t \times Q_d \leq \overline{M}^t \quad \forall t \in \{1, \dots, T\} \quad (3)$$

3.2.3 Grade blending

These inequalities, Eq.(4), ensure that the average grade of production is within the desired range in each period.

$$\underline{g}^t \leq \frac{\sum_{d=1}^D u_d^t \times Q_d \times g_d^t}{\sum_{d=1}^D u_d^t \times Q_d} \leq \overline{g}^t \quad \forall t \in \{1, \dots, T\} \quad (4)$$

Eq.(4) is rearranged as Eq.(4a) and Eq.(4b).

$$\text{Upper bound: } \sum_{d=1}^D Q_d \times (\bar{g}_d^t - g^t) \times u_d^t \leq 0 \quad (4a)$$

$$\text{Lower bound: } \sum_{d=1}^D Q_d \times (g^t - \bar{g}_d^t) \times u_d^t \leq 0 \quad (4b)$$

3.2.4 Maximum number of active drawpoints in each period

These constraints control the maximum number of active draw points at any given period of the schedule. N^t should be given as an input to the algorithm. u_d^t is a continuous variable and can get a value between zero and one. When it is greater than zero, it means a portion of draw cone d is extracted in the relevant period and the related drawpoint is active during this time. In order to control the number of active drawpoints, a binary decision variable, a_d^t , is employed. According to Eq.(5), when a portion of draw cone d is extracted in period t ($u_d^t > 0$), the relevant binary variable to drawpoint d , must be equal to 1, ($a_d^t = 1$), because of the activity of drawpoint d in period t . Eq. (6) ensures that summation of active drawpoints do not exceed the maximum number allowed in each period.

$$u_d^t - a_d^t \leq 0 \quad \forall t \in \{1, \dots, T\}, \quad d \in \{1, \dots, D\} \quad (5)$$

$$\sum_{d=1}^D a_d^t \leq N^t \quad \forall t \in \{1, \dots, T\} \quad (6)$$

3.2.5 Precedence of drawpoints

These constraints control the extraction precedence of drawpoints. Eq. (7) ensures that drawpoints belonging to the relevant set, $C_d(K)$, have been started prior to extraction of drawpoint d . This set is defined based on the selected mining direction (see Fig. 3). This set can be empty, which means the considered drawpoint can be extracted in any time period in the schedule. To control the precedence of extraction, a binary decision variable, z_d^t , is employed. If extraction of drawpoint d is started in period t , the variable z_d^t has to be 1. If summation of extracted portions of each draw cone from the respective set is greater than zero, then, extraction from the considered drawpoint can be started. Parameter ε in the Eq. (7) is a very small positive real number defined based on the maximum portion that can be extracted and it must be greater than zero and less than one. If this parameter is assumed zero, it means drawpoint d can be started before the drawpoints of set $C_d(K)$, and this leads to incorrect solution. Eqs. (7), (8) and (9) must be satisfied simultaneously to complete the precedence constraint. Eq. (8) ensures that if extraction of drawpoint d is started in period t , that drawpoint has not been extracted before. Eq. (9) ensures that extraction of drawpoint d is started by period t .

$$z_d^t - \sum_{j=1}^t u_k^j \leq 1 - \varepsilon \quad \forall d \in \{1, \dots, D\}, \quad t \in \{1, \dots, T\}, \quad k \in C_d(K) \quad (7)$$

$$\sum_{j=1}^t u_d^j - z_d^t \leq 0 \quad \forall d \in \{1, \dots, D\}, \quad t \in \{1, \dots, T\} \quad (8)$$

$$z_d^t - z_d^{t+1} \leq 0 \quad \forall d \in \{1, \dots, D\}, \quad t \in \{1, \dots, T-1\} \quad (9)$$

3.2.6 Draw angle

A uniform draw rate during the mine life will affect three aspects of design and planning of a block cave mine; (1) less dilution at the drawpoints (Laubscher, 1994), (2) reduced stresses (Barlett et al., 2000), and (3) finer fragmentation at the drawpoints.

One of the greatest dangers of block caving is the potential for excessive dilution that could significantly reduce the grade of the ore or result in a significant loss of ore. There are various methods of draw control. One that has been used successfully at the Climax and Henderson mines is to limit the draw in the drawpoints against uncaved ore to a small percentage of the ore column. The draw is increased by equal percentage increments in each line of drawpoints as they progress away from the cave line. The principle is to maintain an ore-to-waste contact of 45 to 50° (Julin, 1992). The angle of draw affects the stress pattern at the cave front and causes over stress at this area. Fig. 4 illustrates this angle. Rubio (2006) analyzed the effect of angle of draw in a mine using historical collapses experienced at that mine. He realized that collapse of the cave front in that mine occurred under three conditions: shallow angle of draw, steep angle of draw, and sudden changes of the angle of draw between periods. Generally, range of angle of draw is site dependent and it is a function of insitu stresses, rock mass strength, and undercutting method. To control draw angle, Eq. (10) is employed. It is assumed that draw angle should be 45°. This means difference between the remaining tonnage of draw cone, d , and the remaining tonnage of the relevant draw cones from set $C_d(K)$ at any given period of time should be kept in a specific range. In other words, the draw rate from set of predecessor drawpoints must be controlled in a way that the angle between the horizontal line and a line which connects the top of extraction point of drawpoints (A and B) be around 45 degrees along the advance direction (see Fig. 5). For this purpose, the extracted tonnage is converted to height using height of extraction factor (f). This factor can be calculated using Eq. (11). It should be noted, upper and lower bound of this equation is set based on the geomechanical condition of the study area.

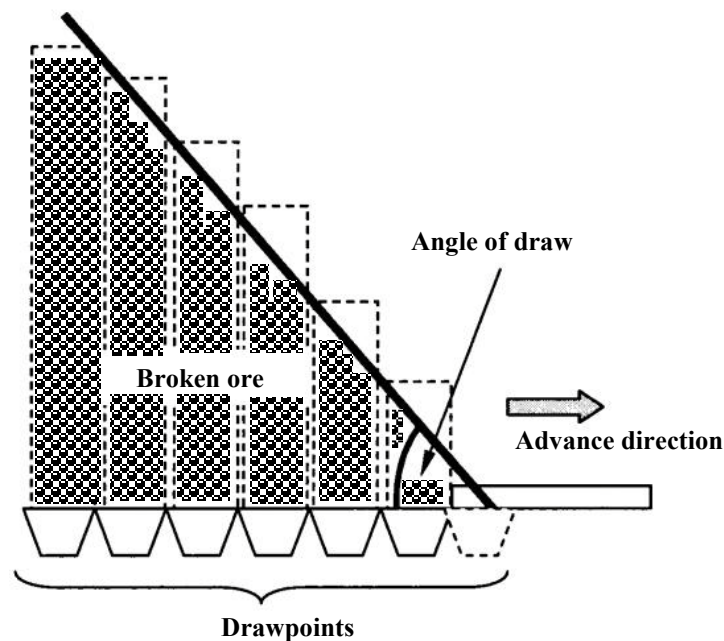


Fig. 4. Angle of draw along the advance direction

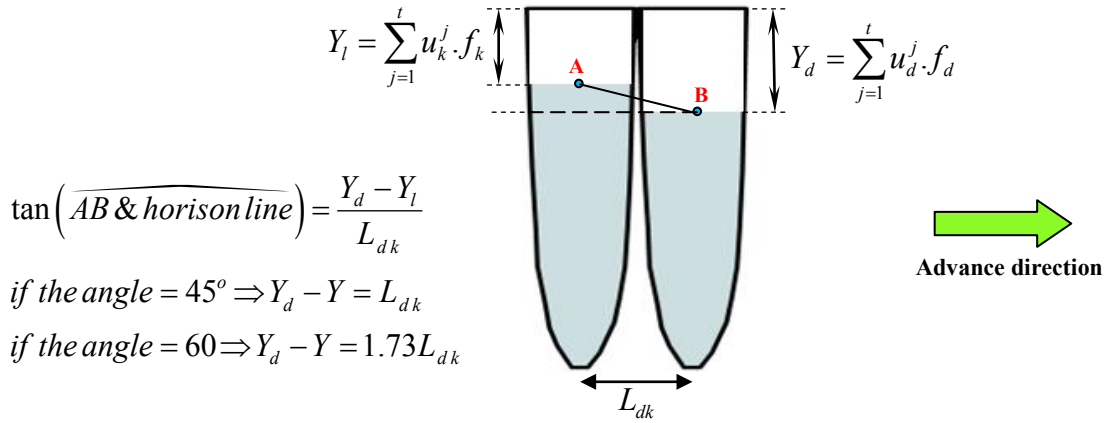


Fig. 5. Draw angle control section

$$\sum_{j=1}^t u_d^j \cdot f_d - \sum_{j=1}^t u_k^j \cdot f_k \leq L_{dk} \quad \forall d \in \{1, \dots, D\}, \quad t \in \{1, \dots, T\}, \quad k \in C_d(K) \quad (10)$$

$$f_i = \frac{Q_i}{A_i \times \gamma_i} \quad \forall i \in \{1, \dots, D\} \text{ or } i \in C_d(K) \quad (11)$$

3.2.7 Reserve constraint

This constraint, Eq.(12), ensures that the fractions of draw cone that are extracted over the scheduling periods are going to sum up to one, which means all the estimated draw cone tonnage is going to be scheduled.

$$\sum_{t=1}^T u_d^t = 1 \quad \forall t \in \{1, \dots, T\}, \quad d \in \{1, \dots, D\} \quad (12)$$

4. Numerical modeling

In most linear optimization problems, the variables of the objective function are continuous in the mathematical sense, with no gaps between real values. To solve such linear programming problems, ILOG CPLEX implements optimizers based on the simplex algorithms (Winston, 1995) (both primal and dual simplex) as well as primal-dual logarithmic barrier algorithms.

Branch and cut is a method of combinatorial optimization for solving integer linear programs. The method is a hybrid of branch and bound and cutting plane methods (Horst et al., 1996). Refer to Wolsey (1998) for a detailed explanation of the branch and cut algorithm, including cutting planes. In this study we used TOMLAB/CPLEX version 11.2 (Holmström, 1989-2009) as the MILP solver. TOMLAB/CPLEX efficiently integrates the solver package CPLEX (ILOG Inc, 2007) with MATLAB environment (MathWorks Inc., 2007).

Table 1 shows the number of decision variables and the number of binary integer variables required for the proposed MILP formulations as a function of number of drawpoints, ND, and number of scheduling periods, T.

Table 1. Number of decision variables in the presented formulation

Number of continuous variables	Number of binary integer variables
$ND \times T$	$2 \times ND \times T$

5. Illustrative example

The presented model has been implemented and tested in TOMLAB/CPLEX environment. It was verified on a synthetic data set containing 70 drawpoints. Radius of each draw cone is 15 meters. Fig. 6 shows the plan view of the center of merged drawpoints based on relevant coordinates. Average grade of drawpoints is varying between 0.65 and 0.8 percent. The tonnage of material between under cut and top of extraction for all drawpoints is same and equivalent to 26.5 MT. The top of extraction is a flat surface. To calculate the height of each drawpoint, it is assumed that (1) tonnage of each drawpoint is located between undercut level and top of extraction, (2) density of material is equal to 2.5 t/m^3 , and (3) radius of each draw cone is equal to 15 m.

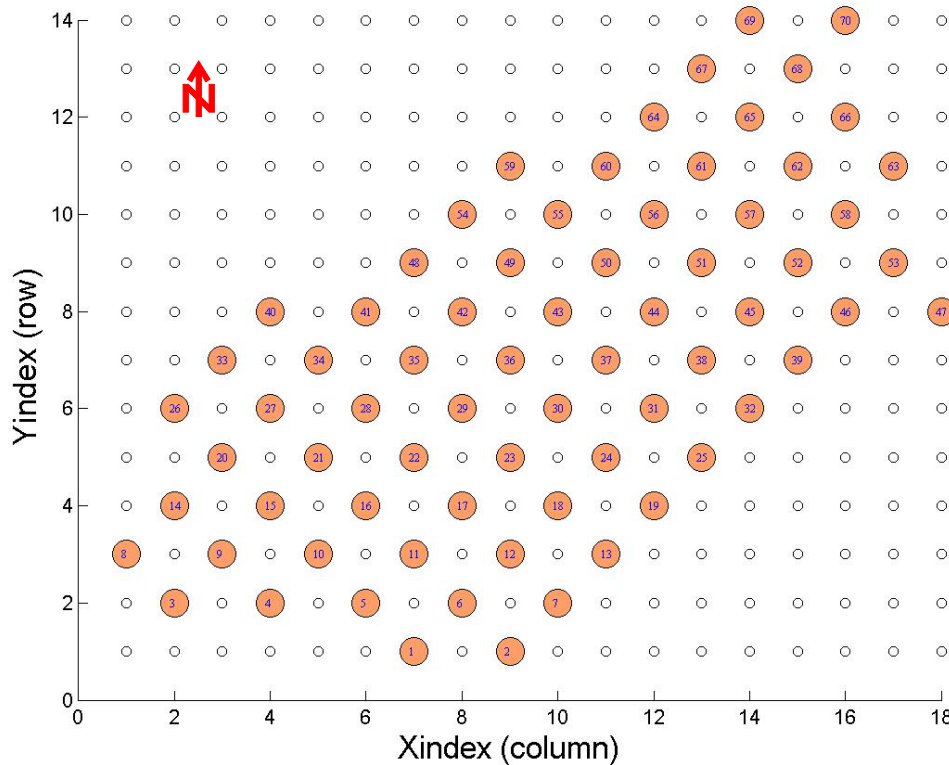


Fig. 6. Plan view of center of draws

6. Results and discussion

To find the optimum production schedule, the formulation was implemented along all the advance directions presented in section 2 over 15 periods scheduling horizon.

Table 2 shows the numerical results of the test of the MILP model with the data set containing 70 drawpoints over 15 periods of extraction. To solve the problem we set same boundaries for each advance direction (see Table 3). Table 3 shows input data and obtained results for each advance direction. The NPV for different directions is illustrated in Fig. 7. It can be seen that difference

between the highest and the lowest NPVs is more than \$6.5 million. The maximum NPV is obtained in South East to North West direction. Fig. 8 shows that all constraints have been satisfied for this direction. Fig. 9 illustrates drawpoints schedule. In each period, the red color indicates that extraction of drawpoint is started in that period while the yellow color indicates that extraction of that drawpoint has been started in the previous periods and it is extracted in this period too. The blue star sign indicates that extraction of that drawpoint has been finished. It can be seen that extraction of all drawpoints is started before period 13. Recovered profiles for all drawpoints in different periods are shown in Fig. 10, which show the location of top point of each draw cone at the end of each period.

Table 2. Numerical results for the synthetic data set containing 70 drawpoints

Direction	CPU time (s)	Size of matrix A (row×col)
West to East	62.37	8865 × 3150
East to West	12.47	8865 × 3150
North to South	66.67	8865 × 3150
South to North	29.28	8865 × 3150
South East to North West	4.94	10245 × 3150
North West to South East	86.95	10245 × 3150
South West to North East	56.79	10425 × 3150
North East to South West	89.75	10425 × 3150
Necessary time to solve this problem	409.22 = 6.82 (min)	

Table 3. Inputs and NPV for the data set containing 70 drawpoints

Direction	N^t	$\underline{g}^t / \overline{g}^t$ (%)	$\underline{M}^t / \overline{M}^t$ (MT)	$\underline{DR} / \overline{DR}$ (T/day)	NPV (\$M)
West to East	25	0.65 / 0.8	0 / 2	0 / 4.16	145.7
East to West	25	0.65 / 0.8	0 / 2	0 / 4.16	147.39
North to South	25	0.65 / 0.8	0 / 2	0 / 4.16	141.62(min)
South to North	25	0.65 / 0.8	0 / 2	0 / 4.16	148.04
South East to North West	25	0.65 / 0.8	0 / 2	0 / 4.16	148.24 (max)
North West to South East	25	0.65 / 0.8	0 / 2	0 / 4.16	146.27
South West to North East	25	0.65 / 0.8	0 / 2	0 / 4.16	146.99
North East to South West	25	0.65 / 0.8	0 / 2	0 / 4.16	146.51

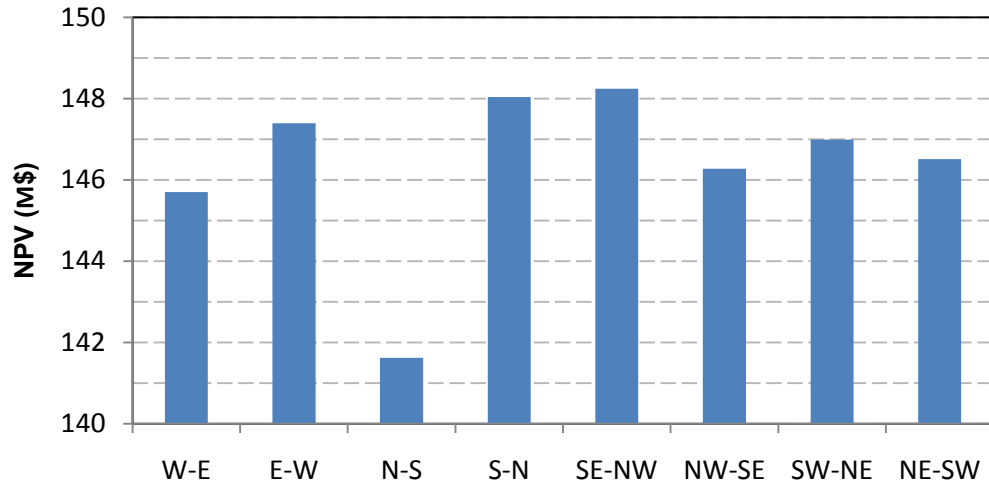


Fig. 7. Amount of NPV for different directions over 15 years scheduling horizon

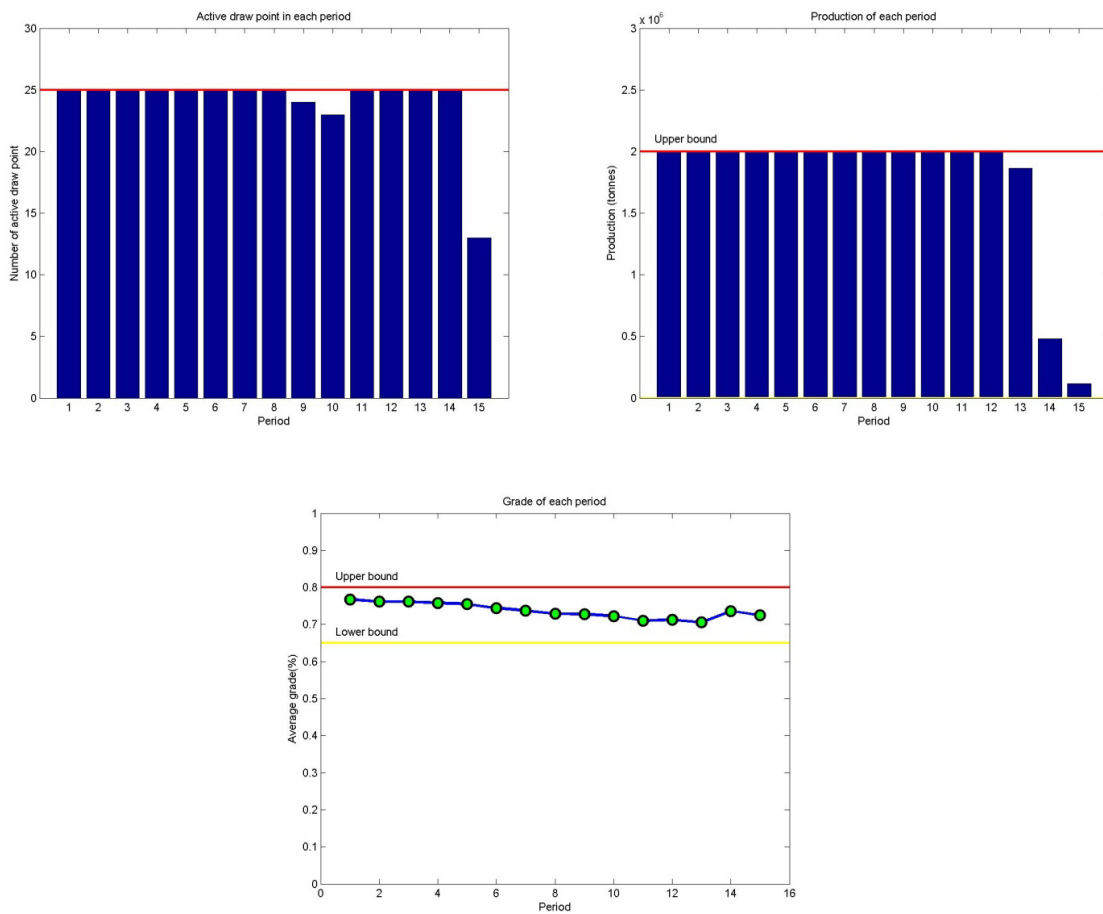


Fig. 8. Number of active drawpoints, tonnage of production and average grade for each period

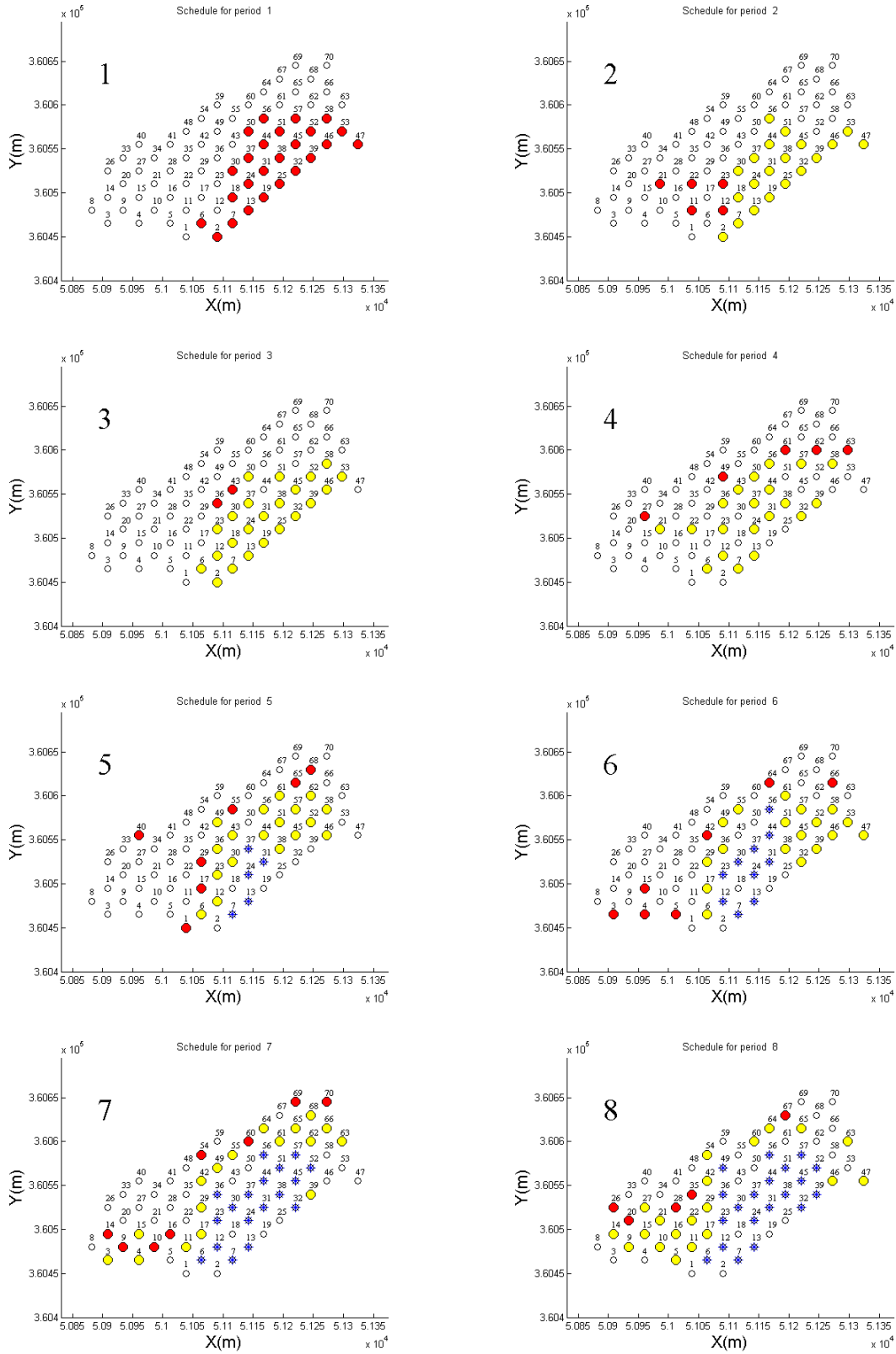


Fig. 9. Sequence of extraction

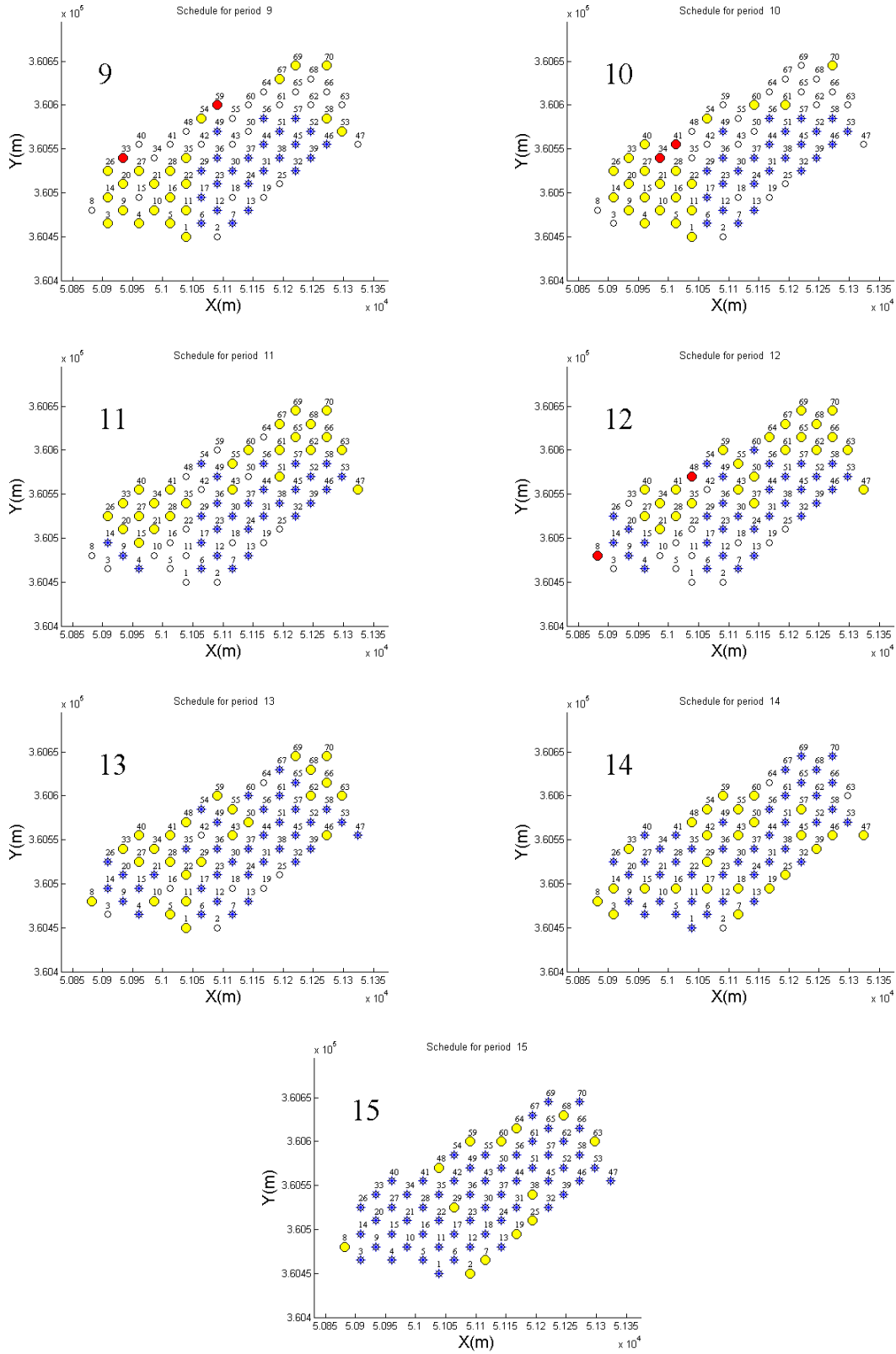


Fig. 9. Continued

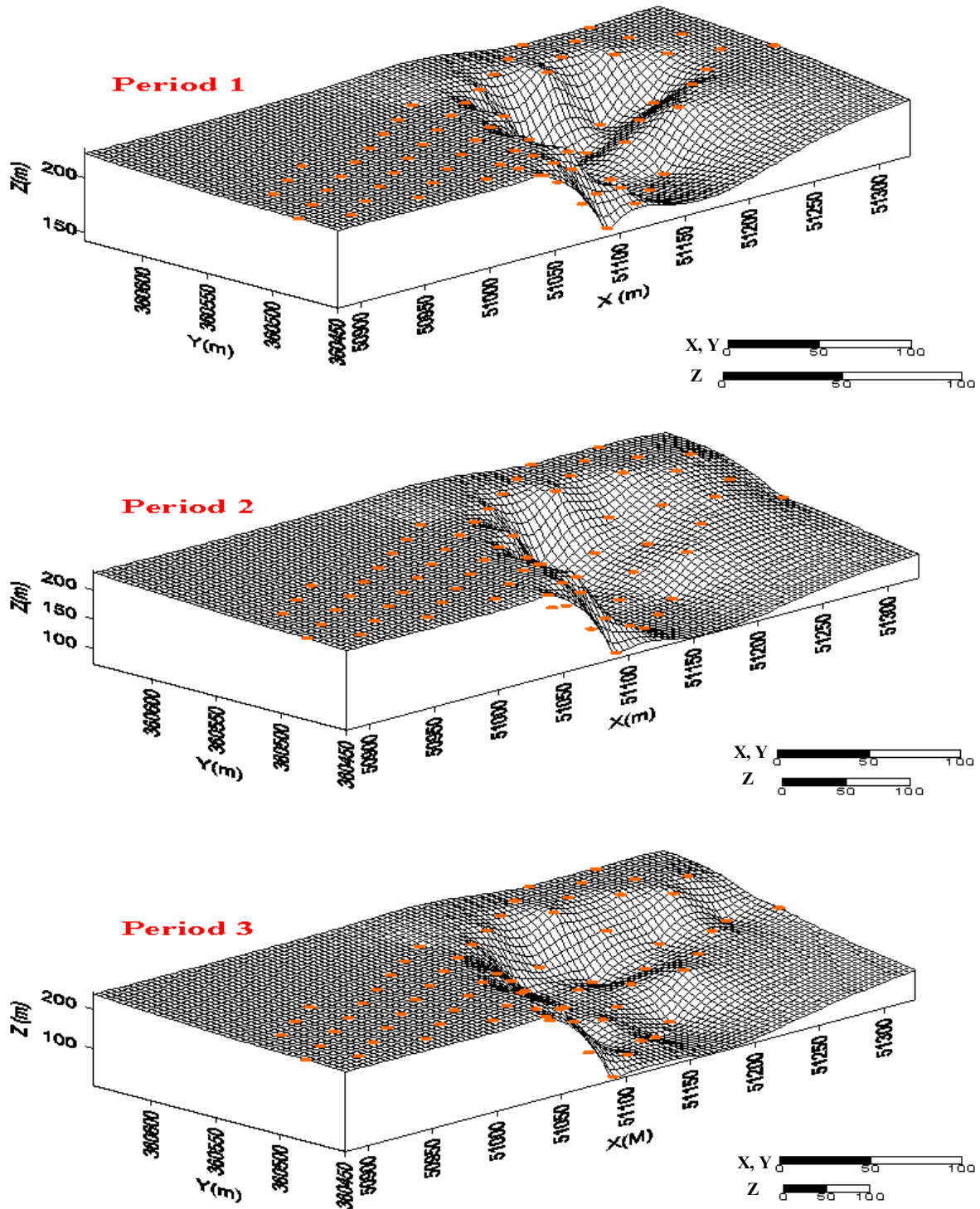


Fig. 10. Recovered drawpoints profile showing the end of the relevant period

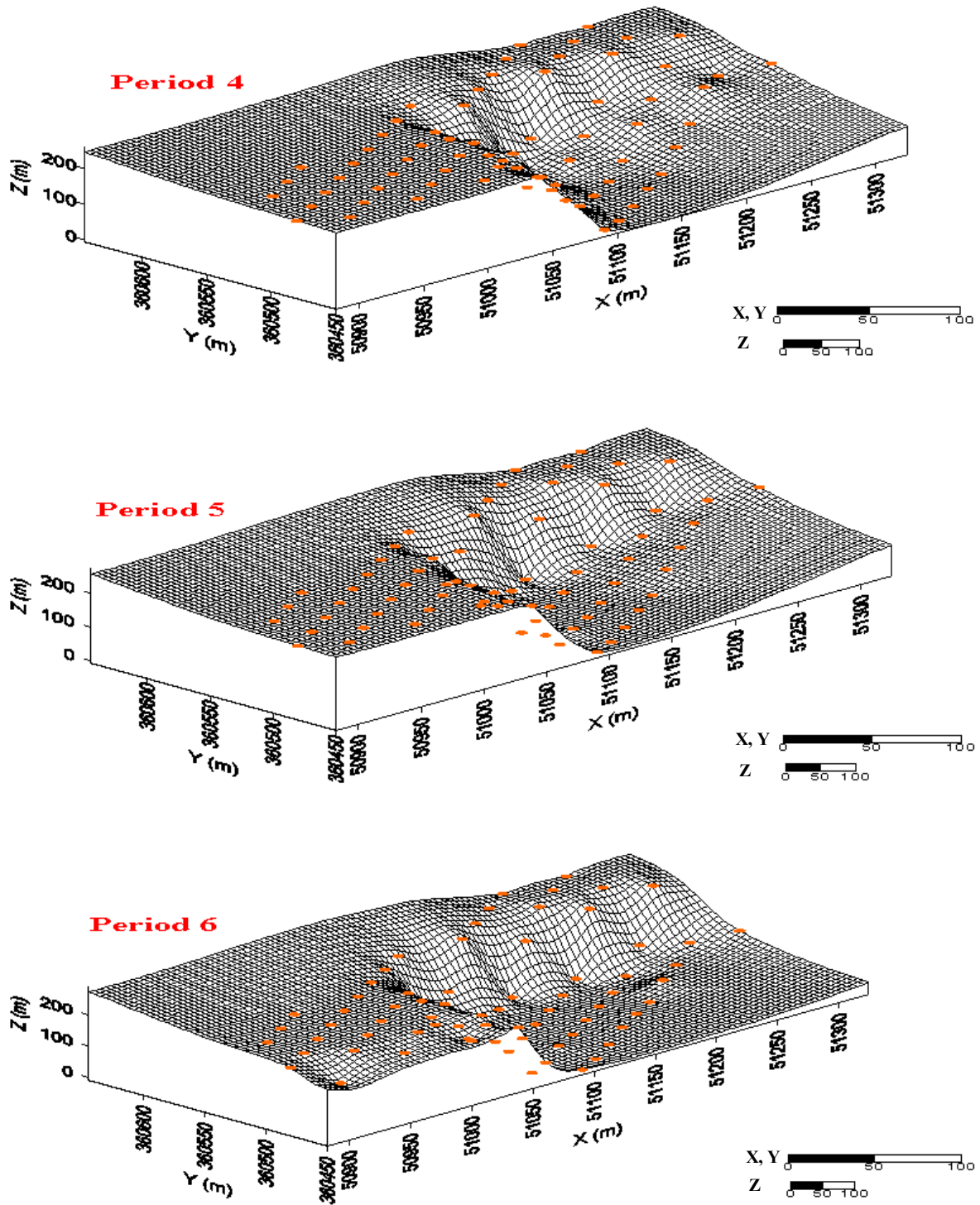


Fig. 10. Continued

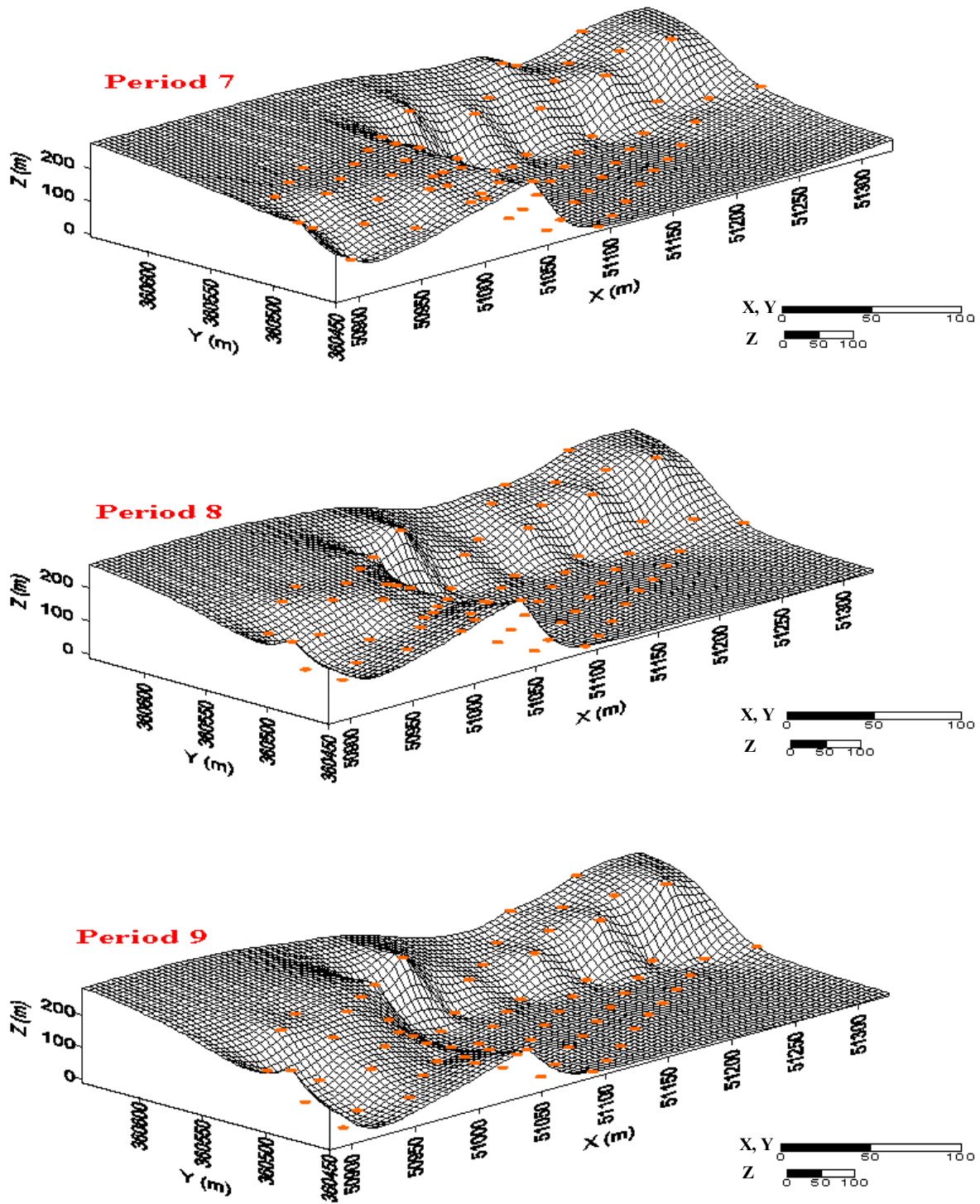


Fig. 10. Continued

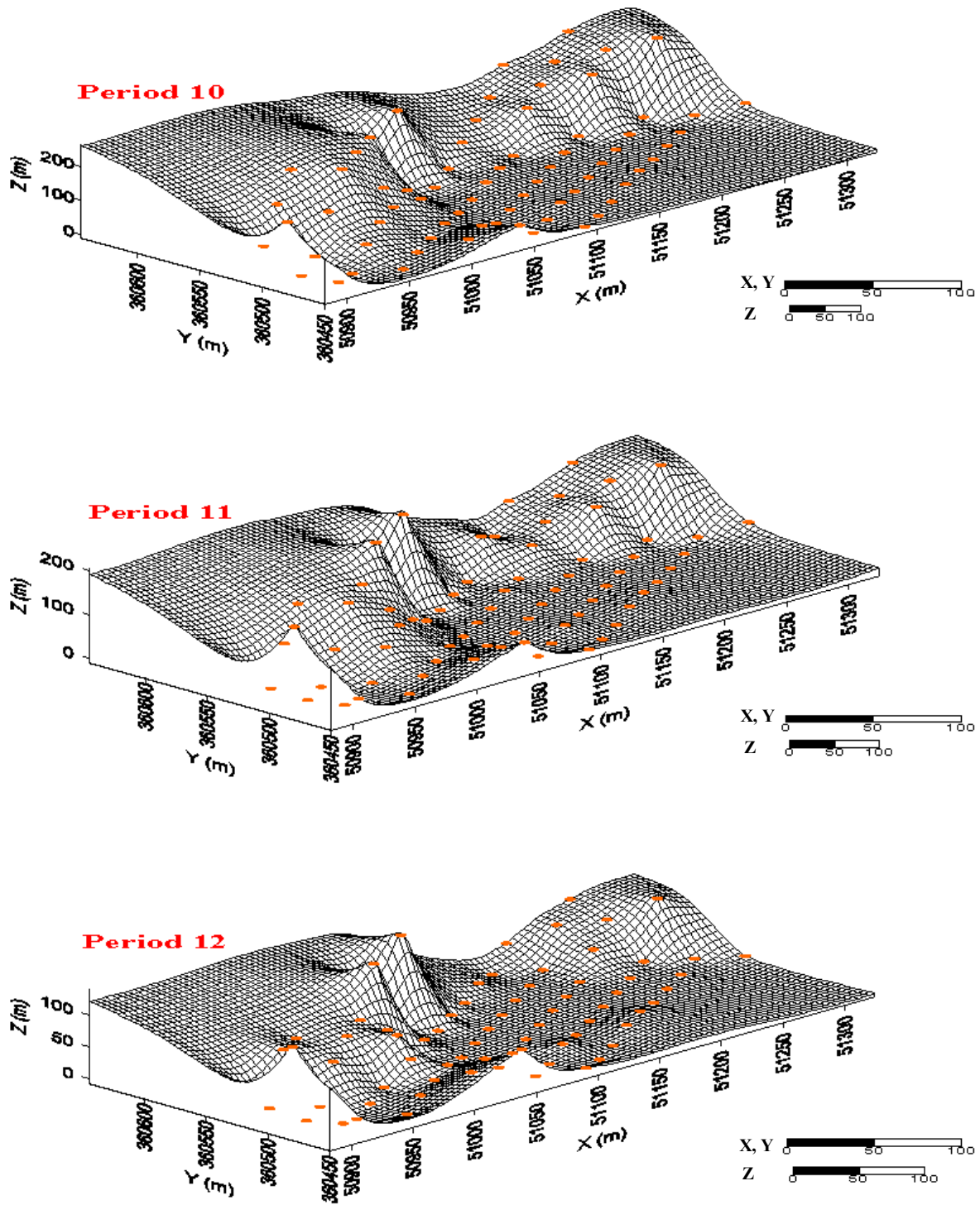


Fig. 10. Continued

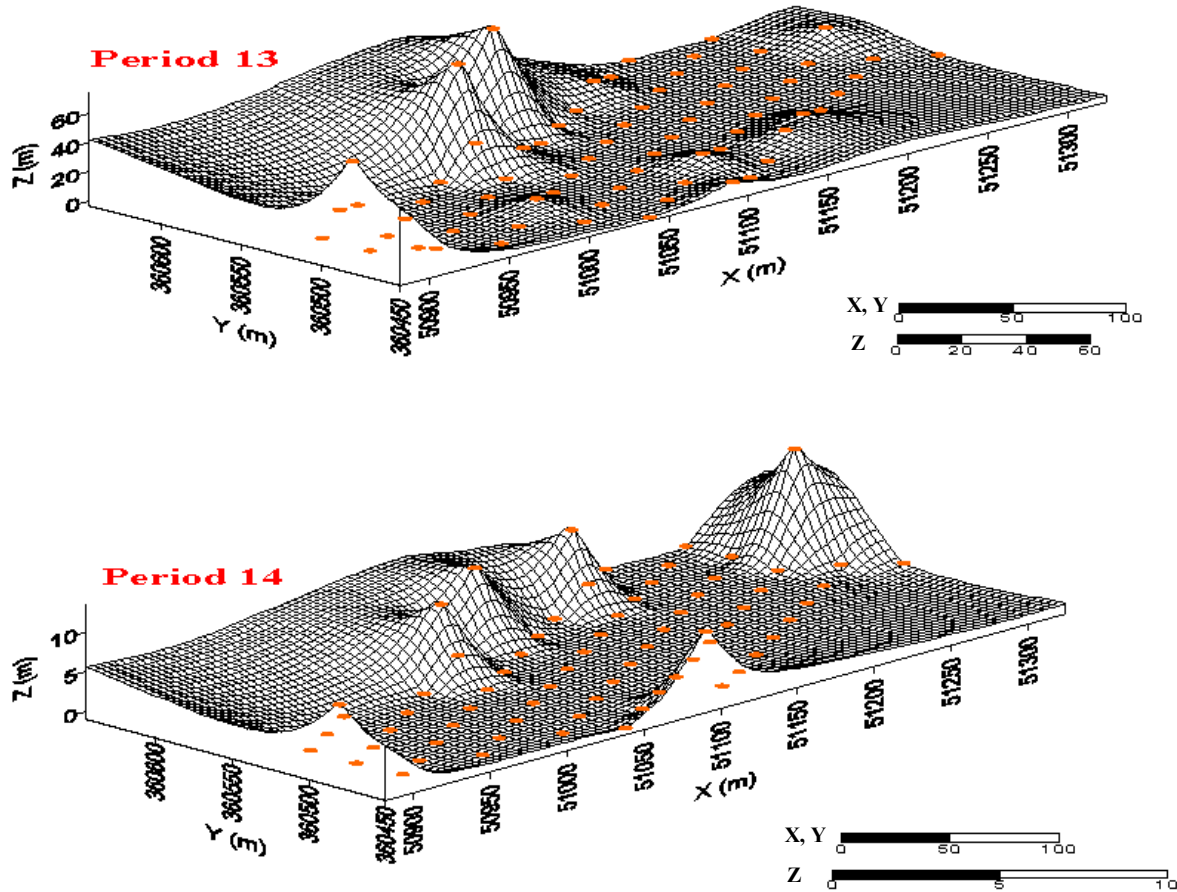


Fig. 10. Continued

7. Conclusion and future work

The paper presented an MILP formulation for block cave mines production scheduling. The formulation maximizes the NPV subject to several constraints such as draw rate, production target, grade blending, drawpoints precedence, and draw angle. We have implemented the optimization formulation in TOMLAB/CPLEX (Holmstrom, 1989-2009) environment. Further focused research is underway to add and test geomechanical constraints on the real data. There were several parameters such as grade that had been assumed to be constant; we will try to introduce probabilistic behavior of these parameters into the process of block cave mine planning.

8. References

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9. Appendix

[MATLAB and TOMLAB/CPLEX documentation for block cave mines scheduling](#)