An overview of block caving operation and available methods for production scheduling of block cave mines

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Abstract

Long-term mine scheduling is among one of the optimization problems. Block caving technique is a mass mining method in which gravity is used in combination with internal rock stresses to fracture and break the rock mass into pieces that can be handled by miners. This method is normally applied to large, low-grade ore-bodies because of its low production cost and high capacity.

In this paper, first, production scheduling methods for underground mining are reviewed and then, block caving method is described. Afterwards, available mathematical models of block cave scheduling and their shortcomings are described. Finally, the conclusions and future work are presented.

1. Introduction

Long-term mine scheduling is classified as complex optimization problems. A production schedule must provide a mining sequence that takes into account the physical limitations of the mine and, to the extent possible, meets the demanded quantities of each raw ore-type at each time period throughout the mine life. Mines use the schedules as long-term strategic planning tools to determine when to start mining a production area and as short-term operational guides.

Underground mining is more complex in nature than surface mining (Kuchta et al., 2004). Flexibility of underground mining is less than surface mining due to the geotechnical, equipment and space constraints (Topal, 2008).

Some of the underground mine problems that can be optimized were identified by Alford et al. (2007) and include:

- Primary development,
- Evaluation and selection from alternative stoping methods,
- Sublevel location and spacing,
- Stope envelope,
- Stope sequencing,
- Ore blending,
- Ore transportation,
- Activity scheduling, and
- Ventilation

Scheduling of underground mining operations is primarily characterized by discrete decisions to mine blocks of ore, along with complex sequencing relationships between blocks. Since linear programming (LP) models cannot capture the discrete decisions required for scheduling, mixed integer programming models (MIP) are generally the appropriate mathematical programming model for this purpose.

Williams et al. (1972) planned sublevel stoping operations for an underground copper mine over one year. They used a linear programming approximation model to determine the amount of ore to be mined per month from each stope. Jawed (1993) formulated a linear goal programming model for production planning in an underground room and pillar coal mine. In his formulation, the decision variables determine the amount of ore to be extracted and the objective function minimizes production deviations from target levels, though only for a single period. These two models compromise schedule quality and the length of the time horizon, respectively. Tang et al. (1993) integrated linear programming with simulation to address scheduling decisions, as did Winkler (1998). The linear program handled the continuous variables, which were determination of the amount of ore to be extracted; while simulation model evaluated discrete scheduling decisions. In these two examples, the applications consist of only a single time period. Solving for a single period cannot guarantee optimal solutions because the technique iteratively fixes variable values and optimizes only a portion of the scheduling problem.

Chanda (1990) employed MIP in conjunction with simulation to generate a schedule for producing finger raises in an underground block caving situation.

Trout (1995) was perhaps the first to try integer programming for optimizing underground mine production schedules. He used MIP to schedule the optimal extraction sequence for underground sublevel stoping.

Ovanic (1998) used mixed integer programming of type two special ordered sets to identify a layout of optimal stopes.

Several years later, researchers formulate more manageable MIP models. Carlyle et al. (2001) presented a model that maximized revenue from Stillwater's platinum and palladium mine, which used the sublevel stoping mining method. The problem was focused on strategic mine expansion planning, so that the integer decision variables scheduled the timing of various mining activities: development and drilling, and stope preparation.

Smith et al. (2003) incorporated a variety of features into their lead and zinc underground mine model, including sequencing relationships, capacities, and minimum production requirements. However, they significantly reduced the resolution of the model by aggregating stopes into large blocks.

Topal et al. (2003) generated a long-term production scheduling MIP model for a sub-level caving operation and successfully applied it to Kiruna Mine, one of the largest underground mine in the world. The model determined which section of the ore to mine, and when to start mining them so as to minimize deviation from the planned production quantities, while adhering to the geotechnical and machine availability constraints.

Sarin et al. (2005) scheduled a coal mining operation with the objective of net present value maximization. They expedited the solution time for their model with a Benders' decomposition-based methodology.

Ataee-Pour (2005) critically evaluated some optimization algorithms according to their capabilities, restrictions and application for use in underground mining. Based on his study, only dynamic programming, geostatistical approach and the branch and bound techniques can generate optimal results because these are considered exact algorithms, while all other remaining techniques are heuristics. In fact, optimal results can only be generated by some exact optimization techniques that evaluate the whole model over the life of mine within all the operational constraints and parameters. Heuristic techniques proceed to a solution by a process of trial and error. They do not guarantee optimality; however, they may be used as reasonable starting point for future references (Ataee-Pour, 2004).

McIssac (2005) formulated the scheduling of underground mining of a narrow veined polymetallic deposit utilizing MIP. The deposit was divided into eleven zones and scheduled over quarterly time periods. The production schedules were generated for each zone, rather than for the individual stopes within these zones.

Nehring et al.(2007) presented a mixed integer programming formulation for production schedule optimization in underground hard rock mining. The results of his study indicated that the potential benefits of the MIP production scheduling model for the purpose of maximizing NPV were significant. He formulated a new constraint to limit multiple exposures of fill masses for being used in an existing MIP production scheduling model.

2. Block caving method

Underground mining is the planned extraction and transportation of a mineral resource from its underground location to a mill or processing plant on the surface (Alford et al., 2007).

Different methods of extraction of the economic material have been devised depending on the geometry of the ore-body and the geotechnical stability of excavation volumes and the surrounding rock.

This section is focused on block caving method. Block caving is a technique in which gravity is used in conjunction with internal rock stresses to fracture and break the rock mass into pieces that can be handled by miners. Block refers to the mining layout in which the ore-body is divided into large sections of several thousand square meters. Caving of the rock mass is induced by undercutting a block. The rock slice directly beneath the block is fractured by blasting, which destroys its ability to support the overlying rock. Gravity forces on the order of millions of tons act on the block, causing the fractures to spread until the whole block is affected. Continued pressure breaks the rock into the smaller pieces that pass through drawpoints (Hustrulid et al., 2001). The term "block caving" is used for all types of gravity caving methods. There are three major systems of block caving, and the type of production equipment used differentiates them; (1) The first system based on the original block cave system is the grizzly or gravity system and is a full gravity system wherein the ore from the drawpoints flows directly to the transfer raises after sizing at the grizzly and then is gravity loaded into ore cars, (2) The second is the slusher system which uses slusher scrapers for the main production unit, and (3) The last is the rubber-tired system which uses load-haul-dump (LHD) units for the main production unit. The general view of this method is illustrated in Fig 1 . (Hustrulid et al., 2001).

The block caving operation is non-selective, except for the high recovery of ore immediately above the undercut horizon which is virtually certain. Generally, a fairly uniform distribution of values throughout the ore-body is required to assure realization of the maximum ore potential of the deposit (Brady, 2004). The method is applicable to low-grade, massive ore bodies with the following characteristics (Hustrulid et al., 2001):

- Large vertical and horizontal dimensions,
- A rock mass that will break into pieces of manageable size, and
- A surface that is allowed to subside.

The factors to be considered in evaluating the caving potential of an ore-body include the pre-mining state of stress, the frequency of joints and other fractures in the rock medium, the mechanical properties of these features, and the mechanical properties of the rock material. It also appears that the orientations of the natural fractures are important (Brady, 2004).

Based on the undercutting sequence a block cave mine is classified into conventional undercutting, advanced undercutting or pre-undercutting (Barraza et al., 2000).

Conventional undercutting method: It consists of blasting the undercut level once the development and construction of the production level has been finalized.

Advanced undercutting method: It has been introduced to reduce the exposure of the drawpoints to the abutment stress zones induced as a result of the undercutting process. For this method just the production drifts are developed in advance of blasting of the undercut.

Pre-undercutting: It is such that no development or construction takes place on the production level before the undercut has been blasted.

Block caving method needs more detailed geotechnical investigations of the ore-body than other methods where conventional drilling and blasting are employed as part of the production of the mine. This is due to reliance of block caving on natural processes for its success. The main geotechnical parameters affecting the planning of the block cave are presented by Brown (2003) as follows:

- Cavability
- Cave initiation
- Cave propagation
- Fragmentation
- Stress performance surrounding the cave boundary

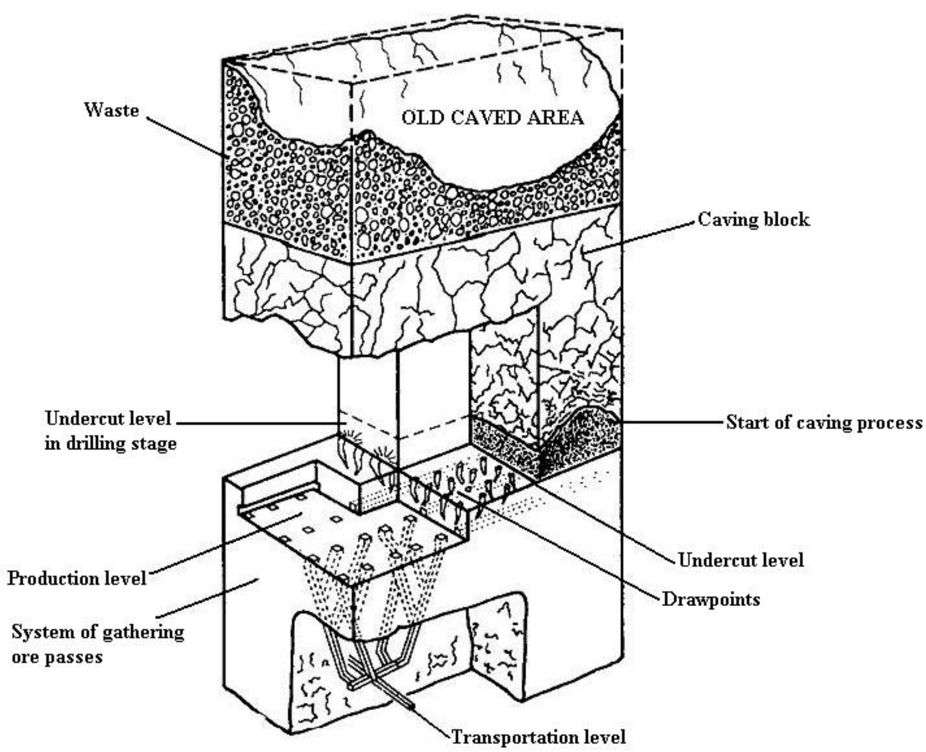
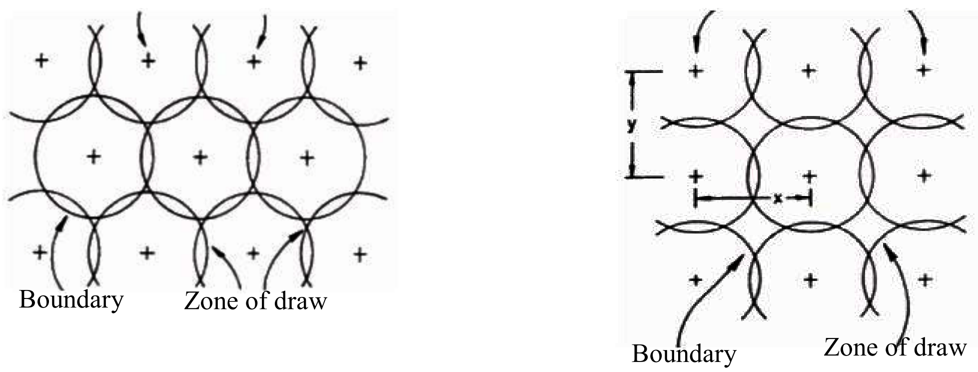


Fig 1. Schematic view of block caving operation (Brady, 2004)

Determining the cavability of an ore body is the first task to be undertaken. For a good caving action generally the ore body should have fractures in three orientations (Julin, 1992). To investigate the cavability of the ore body, drill cores are obtained throughout the ore body using exploration openings and then rock quality designation (RQD) analysis is done. The RQD value helps to identify the caving characteristics of the rock mass.

The draw control and drawpoint spacing are the second applications of caving mechanics after determining ore cavability.

Fig 2 illustrates plan view of drawpoints arrangement (Richardson, 1981).



a) Suitable hexagonal spacing

b) Suitable square spacing

Fig 2. Plan view of drawpoints configuration

3. Block cave production scheduling

The production rates of a mining system are defined by production planning. In underground mining, the extraction and rates of production should be scheduled during the life of the mine from feasibility through to the final production phase. In the case of a block cave mine, the production schedule mainly defines the amount of tonnage to be mined from the drawpoints in every period of the plan to achieve a given planning objective. The mine plan also defines the number of new drawpoints that need to be constructed and their sequence to support a given production target.

The planning parameters that consider computing a production schedule of a block cave mine are as follows (Rubio, 2006):

- *Development rate:* this defines the maximum feasible number of drawpoints to be opened at any given time within the scheduled horizon.
- *Drawpoint construction sequence:* this defines the order in which the drawpoints will be constructed. It is usually defined as a function of the undercut sequence.
- *Maximum opened production area at any given time:* this is an operational constraint which depends on infrastructure and equipment availability as well as on ventilation resources. A large number of active drawpoints might lead into serious operational problems such as excessive haulage distance and problems related to the movement of equipment within the active drawpoints.
- *Draw rate:* this parameter limits the production yield of a drawpoint at any given time within the production schedule. The draw rate is a function of the fragmentation and the cavability model. It should be fast enough to avoid compaction and slow enough to avoid air gaps.

- *Draw ratio:* this defines a temporary relationship in tonnage between one drawpoint and its neighbor. In fact this parameter controls the dilution entry point and the damage of the production level due to induced stress.

Period constraints: these force the mining system to achieve the desired production target usually keeping it within a range that allows flexibility for potential operational variations.

Fig 3 illustrates the planning parameters of a block cave mine. There are some fundamental models for determination of planning parameters. The modeling is normally used to estimate parameters such as stress distribution at the front cave to decide upon the mining sequence and stress re-distribution on the cave back to estimate ultimate fragmentation. These models include:

- Geomechanical model
- Fragmentation model
- Geological model
- Reconciliation model

The geomechanical model affects the following aspects of the design and planning of a block cave mine:

- Drawpoint sequence would be affected by the structural pattern. Usually, the undercut sequence is oriented perpendicular to the major structures to produce blocks than can enhance the cavability of the rock mass (Rojas et al., 2000).
- Abutment stress at the cave front would be a function of the pre-mining stresses and the angle of draw. This affects the stability of the excavation located on the undercut, production level and haulage level immediately below the front of the caving boundary (McKinnon et al., 1999). The angle of draw is commonly measured in a vertical cross section perpendicular to the mining sequence displaying the height of draw (HOD) of the drawpoints.

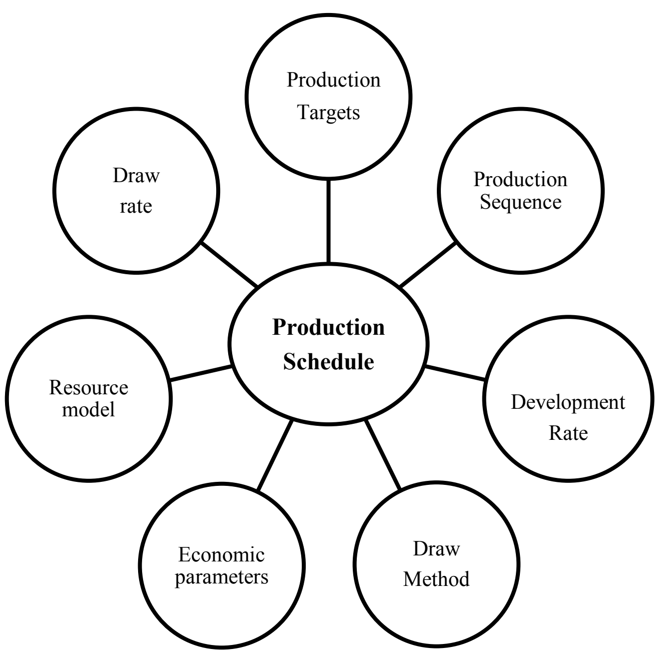


Fig 3. Planning parameters of block cave mine

- Seismicity is the response of the rock mass to the stresses developed at the cave back as the cave propagates to surface and also the response of the rock mass surrounding the excavations exposed to the abutment stress such as undercut, production, ventilation and haulage drifts and rib tunnels (Glazer et al., 2004).
- Induced stresses due to uneven draw. By performing uneven draw high stresses are transferred to the zones of low draw due to the compaction of the broken rock overlying the production level (Febrian et al., 2004).

The fragmentation model affects several aspects of the planning of a block cave mine, the most important aspects are as follows:

- Dilution entry point which is the result of mixing of fragmented material along the draw column (Heslop et al., 1981).
- Drawpoint spacing is the result of the draw column diameter which is believed to be a function of the ultimate fragmentation of the draw column (Kvapil, 1965).
- Drawpoint secondary breakage activity is the result of the frequency of oversize boulders, typically larger than $2m^3$ that cannot be handled by the LHD.
- Oversize and hang up frequency which severely affects the productivity of the mining system (Barraza et al., 2000; Moss et al., 2004).
- Drawpoint yield is the maximum productivity of a drawpoint in the free flow state. As the drawpoint matures, the fragmentation becomes finer because of secondary fragmentation. Therefore, the void ratio decreases as the drawpoint matures leading to an increase in LHD bucket capability, consequently achieving higher drawpoint productivity (Esterhuizen et al., 2004).

The geological model links data relating to structure, lithology and mineralogy with the ultimate metallurgical recovery.

The reconciliation model captures the production performance of the mine. If this model is available, it is used to feedback key performance indicators to the fundamental models to calibrate their behavior. This model is also used to check the validity of different assumptions made regarding to a production schedule. Thus this model will be used as a guide to frame the production planning of the mine based on historical performance (Rubio, 2006). This model affects the following aspects of the design and planning of a block cave mine:

- Draw rate is adjusted based on the historical production performance of drawpoints located in a given rock mass domain.
- Development rate is adjusted depending on the rock mass stress regime in which the construction will take place.
- Draw strategy is compared against the historical performance of the mine.

4. Mathematical programming in block cave production scheduling

Mathematical programming contains all the tools needed to formulate the block cave scheduling in a comprehensive manner. The block cave scheduling should always pursue a goal, which can be represented by an objective function. The actual constraints in the production schedule also can be represented explicitly as constraints to the optimization.

The methods, currently used to compute production schedule in block cave mining, can be classified in two main categories: (a) heuristic methods and (b) exact optimization methods.

Heuristic methods are particularly used to rapidly come to a solution that is hoped to be close to the best possible answer, or optimal solution. These methods are used when there is no known method to find the optimal solution under the given constraints.

The original heuristic methods were the manual draw charts used at the early days of block caving. These methods evolved to be used at Henderson mine where a way to avoid early dilution entry was described by constraining the draw profile to an angle of draw of 45 degrees (Dewolf, 1981). Heslop et al. (1981) described a volumetric algorithm to simulate the mixing along the draw column. Carew (1992) described the use of a commercial package called PC-BC (Diering, 2000) to compute production schedules at Cassiar mine. Diering (2000) showed the principles behind the commercial tool PC-BC to compute production schedules, providing several case studies where different draw methods have been applied depending on the ore body geometry and rock mass behavior.

The application of operations research methods to the planning of a block cave mines was first described by Riddle (1976). This development was intended to compute mining reserves and define the economic extent of the footprint. The final algorithm did not reflect the operational constraints of block caving described above since it worked with the block model directly instead of defining the concept of draw column as an individual entity of the optimization process.

The first attempt to use mathematical programming in block cave scheduling was made by Chanda (1990) who implemented an algorithm to write daily orders. This algorithm was developed to minimize the variance of the milling feed in a horizon of three days. Guest et al. (2000) made another application of mathematical programming in block cave long term scheduling. The objective function was explicitly defined to maximize draw control behavior. However, the author stated that the implicit objective was to optimize NPV. Two problems potentially can arise with this approach are: 1) that maximizing tonnage or mining reserves will not necessarily lead to maximum NPV, 2) that draw control is a planning constraint and not an objective function. The objective function in this case would be to maximize tonnage, minimize dilution or maximize mine life.

Rahal et al. (2003) used a dual objective mixed integer linear programming algorithm to minimize the deviation between the actual state of extraction (height of draw) and a set of surfaces that tend towards a defined draw strategy. This algorithm assumes that the optimal draw strategy is known. Nevertheless, it is postulated that by minimizing the deviation to the draw target the disturbances produced by uneven draw can be mitigated.

Diering (2004) presented a non-linear optimization method to minimize the deviation between a current draw profile and the target defined by the mine planner. Diering emphasized that this algorithm could also be used to link the short with the long-term plan. The long-term plan is represented by a set of surfaces that are used as a target to be achieved based on the current extraction profile when running the short-term plans. Rubio et al.(2004a) presented an integer programming algorithm and an iterative algorithm to optimize long-term schedules in block caving integrating the fluctuation of metal prices in time.

5. Uncertainty in Block cave production scheduling

Uncertainty in block cave production scheduling is due to the lack of formal link between the fundamental models and the planning parameters. Summers (2000) described the main source of uncertainty in block cave mining.

The treatment of uncertainty in production planning as generally being discussed by several authors such as Samis et al.(1998). Singh et al.(1991) and Kajner et al. (1992) have also looked at the flexibility needed in mineral resource industry as a function of the level of uncertainty. Commonly, simulation of the mining system has been the main tool used to assess the amount of flexibility needed in a mine design. The main problem with this approach is that often simulation models do not integrate the fundamental models such as stress distribution, cavability and gravity flow.

Flexibility or the ability to deal with changes and upsets has often been proposed as a response to uncertainty in mine planning. Real options have been used to estimate the value of flexibility (Trigeorigs, 1990).

There are several methods developed to quantify the impact of uncertainty on the financial valuation of the mine. Often Monte Carlo simulations have been used to quantify the risk related to metal price uncertainty. The existing methods concentrate mainly in uncertainty derived from metal prices and grades. Rubio (2006) applied block cave mine infrastructure reliability to production planning.

6. Mathematical formulation for production scheduling of block cave mines

6.1. Chanda (1990)

Chanda (1990) presented a computerized model for short-term production scheduling in a typical block caving mine with a stratiform ore-body. The model combined two separate operation research techniques; mixed integer programming and simulation.

The model was based on a given layout of the mining block, which was considered fixed during the planning period. The model could be used in the selection of finger raises to put on draw in each drift in a block and to simulate in a shift.

In comparison with manual scheduling, the computerized model was faster and generated near-optimal schedules. In the mixed integer programming formulation of the problem 0/1 variables are introduced to represent decisions on whether to draw or not to draw from a particular finger raise during a shift. Fig 4 shows the block layout represented in the mathematical model. It should be noted that this formulation does not incorporate economic parameters such as costs or revenue. In this model, variables are as follows:

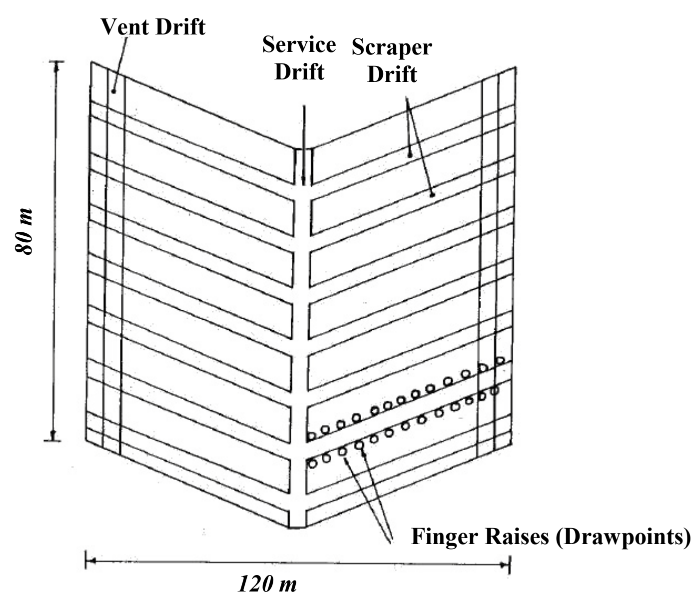


Fig 4. Block layout represented mathematical model

$$d_{it} = \begin{cases} 1 & \text{if raise } i \text{ is drawn in shift } t, \\ 0 & \text{otherwise,} \end{cases}$$

$$y_{it} = \begin{cases} 1 & \text{if raise } i \text{ is on draw in shift } t, \\ 0 & \text{otherwise,} \end{cases}$$

$X_{it} =$ Tonnage of ore from raise i in shift t, (this is a continuous variable)

$T_t =$ Tonnage of blended ore produced in shift t, (this is a continuous variable)

$M_i =$ The maximum allowable output per shift from raise i based on production control considerations.

$Q_t =$ The grade of ore from raise i

$P_t =$ The required or call grade in shift t

6.1.1 Objective function

Chanda (1990) used an objective function, which would reduce as much as possible the fluctuation between shifts in the average grade drawn:

$$Minimize \sum_{i,t} \left| X_{it} \times Q_{it} - X_{it-1} \times Q_{it-1} \right| \tag{1}$$

6.1.2 Constraints

The constraints considered in the integer program are mainly quality and quantity requirements. Other constraints such as profile constraints or slushing capacity are handled by the simulation part of the model.

$$X_{it} - M_i d_{it} \leq 0 \tag{2}$$

This constraint implies that if raise i is not drawn in shift t, there can be no output from it.

$$d_{it} \leq N \qquad \text{for all } t \tag{3}$$

This constraint allows no more than N out of the whole raises to be on draw in a shift.

$$d_{it} - y_{it} \leq 0 \qquad \text{for all } i,t \tag{4}$$

According to this constraint if raise i is barricaded in shift t, it cannot be drawn in that shift.

$$y_{it+1} - y_{it} \leq 0 \qquad \text{for all } i,t \tag{5}$$

This constraint forces a raise to be declared exhausted.

$$\sum_i Q_i \times X_{it} - P_t T_t = 0 \qquad \text{for all } t \tag{6}$$

$$\sum_i X_{it} - T_t = 0 \qquad \text{for all } t \tag{7}$$

This constraint ensures that the tonnage of blended ore in each shift equals the combined tonnage of the constituents.

6.2. Rubio(2002)

Rubio (2002), proposed a methodology to compute production schedules in block caving. He used two objective functions as the main goal of mine planning. The first one was the maximization of NPV by explicitly defining the mathematical expression of NPV. The second proposed objective function was to minimize dilution entry by controlling the profile of the caving back. The second algorithm indirectly represented the maximization of the life of the mine. In this formulation there is no relation constraining the total tonnage drawn from a drawpoint. Thus, mining reserves are not included in the formulation, since it is believed that these reserves should be an output of the optimization rather than an input. This formulation is a mixed integer problem, since it contains integer and real variables. The objective function is also non-linear and the interior point algorithm has been used to solve the problem.

Assumptions which are used in his formulation are as follows:

- There is just one mine in production at the time ignoring possible blending between mines.
- The undercut sequence is known.
- The size of the layout should be fixed, and defined as part of the previous planning steps.

In this model, variables are as follows:

$P_t =$ The profit to be earned in period *t*.

$d_{it} =$ Tonnage to be drawn from drawpoint *i* in period *t*.

$a_{it} = \begin{cases} 1 & \text{if drawpoint } i \text{ is drawn in period } t. \\ 0 & \text{otherwise,} \end{cases}$

$S_{it} = \begin{cases} 1 & \text{if drawpoint } i \text{ is opened in period } t \\ 0 & \text{otherwise} \end{cases}$

$C_{it} = \begin{cases} 1 & \text{if drawpoint } i \text{ is closed in period } t \\ 0 & \text{otherwise} \end{cases}$

$G_{kit} =$ The diluted grade of element *k* from drawpoint *i* in period *t*.

$RF_{kt} =$ The revenue factor of element *k* in period *t*.

$MR_k =$ The metallurgical recovery of element *k*.

$MC_{it} =$ The mining cost of drawpoint *i* in period *t*.

$PC_{it} =$ The processing cost of drawpoint *i* in period *t*.

$\delta =$ Discount rate of one period

$v_t =$ The total number of new drawpoints to open in period *t*.

$DV =$ The development cost of opening a new drawpoint.

$TU_{it} =$ The maximum draw rate of drawpoint *i* in period *t*.

$TL_{it} =$ The minimum draw rate of drawpoint *i* in period *t*.

$TTU_t =$ The maximum feasible production targets for the mine.

$TTL_i =$ The minimum feasible production targets for the mine.

6.2.3 Objective function

To maximize NPV, the objective function is composed of the tonnage to be drawn from the drawpoints and a binary variable that indicates whether a specific drawpoint has a production call in a particular period of the schedule or not.

The profit to be earned per period of the mine schedule is computed as follows

$$P_t = \sum_{i=1}^{l} d_{it} a_{it} \left[\sum_{k=1}^{K} \left(G_{kit} RF_{kt} MR_k \right) \right] - MC_{it} - PC_{it} \tag{8}$$

The objective function is to maximize the NPV over all periods $1 \le t \le T$

$$MAX \left[\sum_{t=1}^{T} \frac{P_t - v_t\, DV_t}{(1+\delta)^t} \right] \tag{9}$$

The objective function is non-linear since the relation between tonnage and grade along a draw column is non-linear. A draw column tonnage (V_{it}) and grade (G_{kit}) are coupled by a non-linear function given by the initial block model and the mixing process.

6.2.4 Constraints

1. Development rate

$$\sum_{i=1}^{l} S_{it} = v_t \qquad \forall t, 1 \le t \le T \tag{10}$$

$$0 \le v_t \le New_t \qquad \forall t, 1 \le t \le T \tag{11}$$

Note that v_t is an integer variable and New_t is an upper bound for the problem.

2. Undercut sequence

$$\sum_{k=1}^{t} S_{ik} \ge S_{i+lt} \tag{12}$$

This constraint guarantees that drawpoints are opened in sequence.

$$\sum_{t=1}^{T} S_{it} \le 1 \tag{13}$$

This constraint guarantees that every drawpoint is opened just once.

3. Drawpoint status

To avoid the closure of the drawpoint before it is opened the following constraint needs to be formulated

$$\sum_{k=1}^{t} S_{ik} \ge \sum_{k=1}^{t-1} C_{ik} \tag{14}$$

Also to avoid that drawpoint is opened twice the following needs to the set of constraints

$$\sum_{t=1}^{T} C_{it} \le 1 \tag{15}$$

To link a_{it} with C_{it}, the below constraint should be used

$$a_{it} = \sum_{k=1}^{t} S_{ik} \left(1 - \sum_{k=1}^{t} C_{ik}\right) \tag{16}$$

If drawpoint *i* has not been opened $a_{it} = 0$ because $\sum_{k=1}^{t} S_{ik} = 0$, thus the drawpoint *i* will not be part of the schedule in period t.

If $\sum_{k=1}^{t} S_{ik} = 1$, then a_{it} can either be 1 or 0 depending on the need to use drawpoint *i* in period t of the schedule.

If the drawpoint *i* is not drawn in period t, then this drawpoint is closed automatically ($C_{it} = 1$).

4. Maximum opened production area

$$\sum_{i=1}^{I} a_{it} \le A_t \tag{17}$$

This constraint controls the maximum open area at any given period of the schedule. A_t should be given as an input to the algorithm.

5. Draw rate

$$d_{it} \le TU_{it} a_{it} \tag{18}$$

$$d_{it} \ge TL_{it} a_{it} \tag{19}$$

This constraint can also be written as follows

$$TL_{it} = \frac{TU_{it}}{dcf_t} \tag{20}$$

dcf_t represents the desired draw control factor to be used in period t of the schedule. The draw control factor is an indicator of how even the draw is performed among active drawpoints at a given horizon of time. The following formula has been traditionally used to compute the draw control factor

$$dcf_t = \frac{\left[\dfrac{\sum_{k=1}^{K}\left(d_i - d_k^i\right)^2}{K}\right]^{0.5}}{\dfrac{1}{K+1}\sum_{k=1}^{K+1} d_k} \tag{21}$$

Where d_i is the tonnage drawn from drawpoint *i* in a period of time, d_k^i is the tonnage drawn from drawpoint k, a neighbor of drawpoint *i* in the same period of time and K is the number of neighbors of drawpoint *i*.

6. Period constraints

$$\sum_{i=1}^{L} d_{it} \le TTU_i \tag{22}$$

$$\sum_{i=1}^{L} d_{it} \ge TTL_i \tag{23}$$

6.3. Diering (2004)

With production scheduling for block cave operations we want to predict or schedule the best tonnages to extract from a number of drawpoints for various periods of time (Diering, 2004). The time period can be very short, such as a day or it can extend over the life of the mine. In this model variables are as follows:

$N =$ Number of drawpoints

$M =$ Number of time periods

$i =$ The i^{th} drawpoint

$j =$ The j^{th} time period

$t_{ij} =$ Tons for drawpoint i for time period j

$T_i =$ Total tonnage allowed for drawpoint i

$T_j =$ Total tonnage allowed for time period j

$c_{ij} =$ PRC tonnage limit for drawpoint i for period j
(PRC: Production Rate Curve)

$b_{ij} =$ Minimum tonnage allowed for drawpoint i in period j. ($b_{ij} \ge 0$)

$e_{ij} =$ Allowable difference in tonnage between drawpoint and its neighbors

$m_{ij} =$ Mean tonnage of neighbors

$r_{ij} =$ Revenue from drawpoint i in period j (adjusted for mining costs)

$d_j =$ Discount fraction in period j

$g_{ij} =$ Grade of drawpoint i in time period j

6.3.5 Objective function

Diering considered the sample problem of maximizing NPV for M periods. In this function, the grades are variable and are function of t_{ij} and t_{ij} for previous period. Therefore, the objective is actually non-linear. The function is as follows:

6.3.6 Constraints

1. PRC limits per drawpoint

$$t_{ij} \le c_{ij} \tag{24}$$

2. Lower limit per drawpoint

$$t_{ij} \ge b_{ij} \tag{25}$$

3. Total tonnage limits per drawpoint

$$\sum t_{ij} \leq T_i \quad (26)$$

4. Tonnage limits for each period

$$\sum t_{ij} = T_i \quad (27)$$

According to the above equations, the program will simply take the maximum allowable tons from the highest-grade drawpoints as soon as possible. Therefore, extra constraints, which limit how tonnages are related to neighboring drawpoints, should be added.

$$t_{ij} \leq m_{ij} + e_{ij} \quad (28)$$

$$t_{ij} \geq m_{ij} - e_{ij} \quad (29)$$

7. Conclusions

Much of the logic around the scheduling of large open pits applies to scheduling of block caves. In both cases, geotechnical constraints are very important. In both cases, the potential to add value to the overall project through careful scheduling is significant.

The methods currently used to compute production schedule in a block cave mine can be classified in two main categories: heuristic methods and exact optimization methods. Some of the main problems associated with the methods are as follows:

- They do not incorporate the variability and the dynamic behavior of the fundamental models (see section 3) throughout the ore-body.
- They do not have a rational way to link the mine planning parameters with the fundamental models which discussed in section 3.
- They do not integrate the operational upsets that affect productivity.
- They do not incorporate, in a routine basis, operational performance to adjust the medium and the long-term plans.

There are parameters in the production schedule methodology that are assumed to be constant, mainly because there are currently no planning tools to introduce the probabilistic behavior of these parameters into the process of planning a block cave mine. These parameters include draw rates, grade, undercut sequence, development rate and air gaps.

8. References

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