MILP formulation for open pit scheduling with multiple materials destinations

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Abstract

A mixed integer linear programming formulation for open pit production scheduling with multiple material destinations is presented.

1. Notation

We will present an MILP formulation for the open pit production scheduling problem with multiple materials destinations. The notation of decision variables, parameters, sets, and constraints are as follows:

1.1. Sets

$\mathcal{K} = \{1,...,K\}$	set of all the mining-cuts in the model.
$\mathcal{P} = \{1,...,P\}$	set of all the phases (push-backs) in the model.
$\mathcal{D} = \{1,...,D\}$	set of all the possible destinations for materials in the model.
$\mathcal{A}_k = \{1,..., A_k\}$	set of all the directed arcs in the mining-cuts' precedence directed graph denoted by G_k ($\mathcal{K}, \mathcal{A}_k$).
$C_k(L)$	for each mining-cut k, there is a set $C_k(L) \subset \mathcal{K}$ defining the immediate predecessor mining-cuts that must be extracted prior to extracting mining-cut k. Where L is an integer number presenting the total number of blocks in the set C_k.
$B_p(M)$	for each phase p there is a set $B_p(M) \subset \mathcal{K}$ defining the mining-cuts constructing phase p. Phases are constructed to be sets of mining-cuts partitioning $\mathcal{K}$. where M is an integer number denoting the total number of blocks in the set B_p.
$H_P(R)$	for each phase p, there is a set $H_p(R) \subset \mathcal{K}$ defining the mining-cuts within the immediate predecessor pit phases (push-backs) that must be extracted prior to extracting phase p. Where R is an integer number presenting the total number of blocks in the set H_p.

1.2. Indices

A general parameter can take the following indices in the format of $f_{k,p}^{d,t,e}$. Where:

$k \in \{1,...,K\}$	index for mining-cuts.
$p \in \{1,...,P\}$	index for phases.
$d \in \{1,...,D\}$	index for possible destinations for materials.
$t \in \{1,....,T\}$	index for scheduling periods.
$e \in \{1,...,E\}$	index for elements of interest in each block.

1.3. Parameters

$u_k^{d,t}$	the discounted dollar value generated by extracting mining-cut k and sending it to destination d in period t.
$v_k^{d,t}$	the discounted revenue generated by selling the final products within mining-cut k in period t, if it is sent to destination d, minus the extra discounted cost of mining all the material in mining-cut k as ore and processing at destination d.
$q_k^{d,t}$	the discounted cost of mining the material in mining-cut k in period t as waste and sending it to destination d.
g_k^e	average grade of element e in ore portion of mining-cut k.
$\overline{g}^{d,t,e}$	upper bound on acceptable average head grade of element e, in period t, at processing destination d.
$\underline{g}^{d,t,e}$	lower bound on acceptable average head grade of element e, in period t, at processing destination d.
o_k	ore tonnage in mining-cut k.
w_k	waste tonnage in mining-cut k.
$\overline{p}^{d,t}$	upper bound on processing capacity of ore in period t at destination d (tonnes).
$\underline{p}^{d,t}$	lower bound on processing capacity of ore in period t at destination d (tonnes).
$\overline{m}^t$	upper bound on mining capacity in period t (tonnes).
$\underline{m}^t$	lower bound on mining capacity in period t (tonnes).
$r^{d,e}$	processing recovery, is the proportion of element e recovered if it is processed at destination d.
$s^{t,e}$	price in present value terms obtainable per unit of product (element e).
$cs^{t,e}$	selling cost in present value terms per unit of product (element e).
$cp^{d,t,e}$	extra cost in present value terms per tonne of ore for mining and processing at destination d.
$cm^{d,t}$	cost in present value terms of mining a tonne of waste in period t sending it to destination d.

h_k	bench number corresponding to mining-cut k, benches are numbered from the top of the pit towards the bottom accordingly.
$\overline{h}_p$	maximum acceptable lead between phase p and $p+1$. Where the lead is the number of benches by which the mining of a specified phase must be ahead of the next one.
$\underline{h}_p$	minimum acceptable lead required between phase p and $p+1$.
M	the total number of mining-cuts in the set $B_p(M)$.
R	the total number of mining-cuts in the set $H_p(R)$.
L	the total number of mining-cuts in the set $C_k(L)$.

1.4. Decision Variables

$x_k^{d,t} \in [0,1]$	continuous variable, representing the portion of mining-cut k sent to processing destination d, in period t.
$y_k^{d,t} \in [0,1]$	continuous variable, representing the portion of mining-cut k mined in period t , and sent to destination d.
$b_k^t \in \{0,1\}$	binary integer variable controlling the precedence of extraction of mining-cuts. b_k^t is equal to one if extraction of mining-cut k has started by or in period t, otherwise it is zero.
$z_p^t \in \{0,1\}$	binary integer variable controlling the precedence of mining phases. z_p^t is equal to one if extraction of phase p has started by or in period t, otherwise it is zero.

2. Economic block value modeling

The objective function of the MILP formulation is to maximize the net present value of the mining operation. Hence, we need to define a clear concept of economic value based on the amount of ore within mining-cuts, which can be mined selectively. The profit from mining depends on the value of the mining-cut based on its processing destination and the costs incurred in mining and processing it. The cost of mining a cut is a function of its location, which characterizes how deep the mining-cut is located relative to the surface and how far it is relative to its final dump. The spatial factor can be applied as a mining cost adjustment factor for each mining-cut according to its location to the surface. The discounted profit from mining-cut k is equal to the discounted revenue generated by selling the final product contained in mining-cut k minus all the discounted costs involved in extracting mining-cut k, this is presented by Eq. (1).

$$u_k^{d,t} = \underbrace{[\sum_{e=1}^{E} o_k \times g_k^e \times r^{d,e} \times (s^{t,e} - cs^{t,e})}_{discounted\ revenues} - \underbrace{\sum_{e=1}^{E} o_k \times cp^{d,t,e}] - [(o_k + w_k) \times cm^{d,t}]}_{discounted\ costs} \qquad \forall d \in \{1,...,D\} \qquad (1)$$

For simplification purposes we denote:

$$v_k^{d,t} = \sum_{e=1}^{E} o_k \times g_k^e \times r^{d,e} \times (s^{t,e} - cs^{t,e}) - \sum_{e=1}^{E} o_k \times cp^{d,t,e} \qquad \forall d \in \{1,...,D\}, \quad k \in \{1,...,K\} \qquad (2)$$

$$q_k^{d,t} = (o_k + w_k) \times cm^{d,t} \qquad \forall d \in \{1,...,D\}, \quad k \in \{1,...,K\} \tag{3}$$

3. Model

Objective function:

$$\max \sum_{d=1}^{D} \sum_{t=1}^{T} \sum_{p=1}^{P} \left(\sum_{k \in B_p} (v_k^{d,t} x_k^{d,t} - q_k^{d,t} y_k^{d,t}) \right) \tag{4}$$

Subject to:

$$\underline{g}^{d,t,e} \le \sum_{p=1}^{P} \left((\sum_{k \in B_p} g_k^e o_k \Big/ \sum_{k \in B_p} o_k) x_k^{d,t} \right) \le \overline{g}^{d,t,e} \quad \forall t \in \{1,...,T\}, \quad d \in \{1,...,D\}, \quad e \in \{1,...,E\} \tag{5}$$

$$\underline{p}^{d,t} \le \sum_{p=1}^{P} \left(\sum_{k \in B_p} o_k x_k^{d,t} \right) \le \overline{p}^{d,t} \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,D\} \tag{6}$$

$$\underline{m}^{t} \le \sum_{p=1}^{P} \left(\sum_{k \in B_p} (o_k + w_k) y_k^{d,t} \right) \le \overline{m}^{t} \qquad \forall t \in \{1,...,T\}, \quad d \in \{1,...,D\} \tag{7}$$

$$\sum_{d=1}^{D} x_k^{d,t} \le \sum_{d=1}^{D} y_k^{d,t} \qquad \forall k \in \{1,...,K\}, \quad t \in \{1,...,T\} \tag{8}$$

$$b_k^t - \sum_{d=1}^{D} \sum_{i=1}^{t} y_s^{d,i} \le 0 \qquad \forall k \in \{1,...,K\}, \quad t \in \{1,...,T\}, \quad s \in C_k(L) \tag{9}$$

$$\sum_{d=1}^{D} \sum_{i=1}^{t} y_k^{d,i} - b_k^t \le 0 \qquad \forall k \in \{1,...,K\}, \quad t \in \{1,...,T\} \tag{10}$$

$$b_k^t - b_k^{t+1} \le 0 \qquad \forall k \in \{1,...,K\}, \quad t \in \{1,...,T-1\} \tag{11}$$

$$\underline{h}_p \ge h_l b_l^t - h_j b_j^t \ge \overline{h}_p \qquad \forall p \in \{1,...,P\}, \quad t \in \{1,...,T\}, \quad l \in B_p, \quad j \in B_{p+1} \tag{12}$$

$$R.z_p^t - \sum_{r=1}^{R} \sum_{d=1}^{D} \sum_{i=1}^{t} y_r^{d,i} \le 0 \qquad \forall p \in \{1,...,P\}, \quad t \in \{1,...,T\}, \quad r \in H_p(R) \tag{13}$$

$$\sum_{m=1}^{M} \sum_{d=1}^{D} \sum_{i=1}^{t} y_m^{d,i} - M.z_p^t \le 0 \qquad \forall p \in \{1,...,P\}, \quad t \in \{1,...,T\}, \quad m \in B_p(M) \tag{14}$$

$$z_p^t - z_p^{t+1} \le 0 \qquad \forall p \in \{1,...,P\}, \quad t \in \{1,...,T-1\} \tag{15}$$