

# Applications of MILP long-term open pit production scheduler

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## Abstract

*A number of mixed integer linear programming (MILP) formulations have been introduced for production scheduling of open pit mines. One of the main obstacles in using MILP formulations for open pit production scheduling is the size of the problem. The number of integer and continuous decision variables, as well as the number of constraints required to formulate a real-size mine will set up a computationally intractable problem. The main objective of this paper is to present a practical MILP formulation for open pit production scheduling problem. Also, we highlight the achievable economic gains that are possible through production scheduling optimization. We present an application of using mixed integer linear programming formulations for the open pit long-term production scheduling problem. We verify and validate the MILP production scheduler by a comparative case study against Whittle strategic mine planning software. An iron ore deposit with 427 million tonnes of rock and 116 million tonnes of iron ore in the final pit limit at an average grade of 72.9% magnetic weight recovery is studied. The difference between the cumulative discounted cash flow of the MILP schedule and the Whittle Milawa Balanced schedule is \$50.4 million dollars. The considerable difference between the two methods, demonstrates the importance of production scheduling optimization and the necessity for scheduling optimization to turn into a common practice in industry.*

## 1. Introduction

The life-of-mine production schedule defines the strategy of displacement of ore, waste, and overburden over the mine life. The objective of long-term production scheduling is to determine the sequence of extraction and displacement of material in order to maximize or minimize an objective function. Commonly, the goal is to maximize the net present value of mining operation within the existing economic, technical, and environmental constraints. However, other objectives such as cost minimization or reserve maximization could be considered too. Long-term production schedules are the backbone of short-term planning and day to day mining operations. The long-term production schedules determine mine and processing plant capacity and their expansion potential. The production schedule also defines the management investment strategy. Deviations from optimal plans in mega mining projects will result in enormous financial losses, delayed reclamation, and resource sterilization. Current open pit production scheduling methods in the literature and industry are not limited to, but can be divided into two main categories: heuristics and exact algorithms. The main motivation of this paper is to demonstrate that the economic difference between a practical optimal production schedule, generated by exact mathematical methods and production schedules generated by common techniques used in industry is substantial. Optimization practice is becoming more common in well-built mining companies, but still it is a long way until it becomes a common practice in mining industry.

Mixed integer linear programming (MILP) mathematical programming has been used by various researchers to tackle the long-term open-pit scheduling problem. The MILP models theoretically have the capability to model diverse mining constraints such as multiple ore processors, multiple material stockpiles, and blending strategies. However, current MILP formulations developed for open pit production scheduling have two major shortcomings: (i) the inability to generate the global optimal large-scale life-of-mine production schedule within a reasonable timeframe, and (ii) the inability to quantify the geological uncertainty inherent within the problem and as a result, the associated risk with the mine plans. As the first step, we focus on development of a deterministic MILP formulation for large-scale open pit production scheduling problem. We will address the stochastic case in our future research.

The objective of this study is to (i) develop a deterministic MILP formulation for long-term open pit production scheduling problem, (ii) implement and document the details of the MILP numerical models in TOMLAB/CPLEX (Holmström, 2009) environment, (iii) verify and validate the MILP production scheduler by a comparative case study against one of the standard industry tools — Whittle strategic mine planning software (Gemcom Software International, 2008), and (iv) demonstrate the importance of production scheduling optimization to become a common practice industry wide.

In a typical open pit long-term scheduling problem, the number of blocks is in the order of a couple of hundred thousand to millions, and the number of scheduling periods is in the order of twenty periods and more. Evidently, the number of integer and continuous decision variables, and the number of constraints formulating a problem of this size would exceed the capacity of current state of hardware and software. The MILP formulation of open pit production scheduling becomes intractable because of the size of the problem. To overcome the size problem, we aggregate blocks into larger units, we refer to these units as mining-cuts. We present two MILP formulations at two different levels of granularity: (i) processing at block level and mining at mining-cut level; and (ii) processing and mining both at mining-cut level.

The next section of the paper covers the relevant literature to open pit production scheduling problem. Section three presents problem definition, notations of variables, and the mixed integer linear programming formulations of the problem, while the fourth section presents the numerical modeling techniques. The next section represents the verification of the MILP models by a comparative mining case study against the Whittle software (Gemcom Software International, 2008) results. Finally, the last section presents the conclusions and future work followed by the list of references.

## **2. Literature review**

Current production scheduling methods in the literature are not just limited to, but can be divided into two main categories: heuristic and exact algorithms. Some of these algorithms are embedded into available commercial software packages.

Various models based on a combination of artificial intelligence techniques have been developed (Askari-Nasab, 2006; Askari-Nasab, et al., 2009; Denby, et al., 1996; Tolwinski, et al., 1996). Some of the artificial intelligence techniques such as intelligent open pit simulator (Askari-Nasab, 2006) are based on frameworks that theoretically will converge to the optimal solution, given sufficient number of simulation iterations. The main disadvantage of artificial intelligence and heuristic methods however, is that there is no quality measure to solutions provided comparing against the optimum. In addition most of the results are not reproducible.

A variety of operations research approaches including linear programming (LP) and mixed integer linear programming (MILP) have been applied to the mine production scheduling problem. The pioneer work of Johnson (1969) used an LP model, which led to the MIP formulations by Gershon

(1983) for the production scheduling problem. Every orebody is different, but for a typical open pit long-term scheduling problem, the number of blocks is in the order of a couple of hundred thousand to millions, and the number of scheduling periods could vary in the order of twenty and more periods for a life-of-mine production schedule. Evidently, the number of integer and linear decision variables, and the number of constraints formulating a problem of this size would become intractable.

Various models based on mixed integer linear programming mathematical optimisation have been used to solve the long-term open-pit scheduling problem (Boland, et al., 2009; Caccetta, et al., 2003; Dagdelen, et al., 2007; S. Ramazan, et al., 2004). In practice, formulating a real size mine production planning problem by including all the blocks as integer variables will become computationally intractable. Various methods of aggregation have been used to reduce the number of integer variables that are required to formulate the production scheduling problem with MILP techniques. Ramazan and Dimitrakopoulos (2004) illustrated a method to reduce the number of binary integer variables by setting waste blocks as continuous variables instead of integer variables. Ramazan and Dimitrakopoulos (2004) reported a case study on a small single level nickel laterite block model with 2,030 blocks over three periods.

Ramazan et al. (2005) presented an aggregation method based on fundamental tree concepts to reduce the number of decision variables in the MILP formulation. The fundamental tree algorithm has been used in a case study with 38,457 blocks within the final pit limits. Whittle strategic mine planning software (Gemcom Software International, 2008) has been used to decompose the overall problem into four push-backs. Subsequently, the blocks within the push-backs were aggregated into 5,512 fundamental trees and scheduled over eight periods using the formulation presented in Ramazan and Dimitrakopoulos (2004). Information about the run-time of the MILP models are not presented in Ramazan (2007); also the breakdown of the problem into four push-backs based on the nested pit approach and formulating each push-back as a separate MILP would not generate a global optimum solution to the problem. On the other hand the size of the problem of around thirty thousand blocks over eight periods is more a mid-range planning problem rather than a long-term life-of-mine schedule.

Caccetta and Hill (2003) presented a formulation that only used binary integer variables; they developed and implemented a personalized branch-and-cut (Horst, et al., 1996) method in C++ using CPLEX (ILOG Inc, 2007) to solve the relaxed LP sub-problems. Boland et al. (2009) have demonstrated an iterative disaggregation approach to using a finer spatial resolution for processing decisions to be made based on the small blocks, while allowing the order of extraction decisions to be made at an aggregate level. Boland et al. (2009) reported notable improvements on the convergence time of their algorithm for a model with 96,821 blocks and 125 aggregates over 25 periods. However, combining 96,821 blocks into only 125 aggregates would reduce the freedom of decision variables and the schedule generated could not be considered as an optimal solution in comparison to the case that 96,821 blocks had a decision variable defined for them. Moreover, in Boland et al. (2009) there is no representation of the generated schedules in terms of annual ore and waste production, average grade of ore processed, cross sections, and plan views of the schedules to assess the practicality of the solutions from mining operational point of view.

MineMax (Minemax Pty Ltd, 2009) is a commercially available strategic mine scheduling software, which uses MILP formulation solved by ILOG CPLEX (ILOG Inc, 2007) solver. Given that, MineMax is a commercial software we couldn't find detailed information about the approach and formulation, but our understanding from the evaluation of the demo tutorial version of MineMax is that it initially decomposes the final pit into nested pit shells based on parametric analysis concepts represented by Lerchs and Grossmann (1965). The pit shells define a pit to pit precedence constrained by the minimum and maximum number of benches by which the mining of one specified pit shell is to lag behind the previous one. The other option to define rules for

precedence of extraction is either by proportions mined on each bench or by block precedence based on the overall pit slopes. Then, each pit shell is formulated as a separate MILP model which can contribute to the overall quantity of mining and processing targets within the grade and precedence constraints; this approach results in MILP formulations for each pit shell with smaller size which will converge faster, but it could not be considered a global optimization of the problem since the pit shells are defined by the parametric analysis initially. Another optimization strategy is using sliding windows which are sub-problems tackled on a period by period basis. Other well known proprietary software which tackle the strategic mine production scheduling by MILP techniques are Blasor (Stone, et al., 2007), Prober (Whittle, 2007), and OptiMine (Dagdelen, et al., 2007).

Production scheduling optimization techniques are still not widely used in the mining industry. There is a need to improve the practicality and performance of the current production scheduling optimization tools used in mining industry. Also, to gain more common recognition in industry, there is a need to highlight the considerable achievable economic gains that are possible through production scheduling optimization.

### 3. Mixed integer linear programming model for open pit production scheduling

The basic problem of concern in its simplest form is finding a sequence in which ore and waste blocks should be removed from the predefined open pit outline and their respective destinations over the mine life, so the net present value of the operation is maximized. The production schedule is subject to a variety of physical and economic constraints. The constraints enforce the mining extraction sequence, overall pit slopes, mining, milling, and refining capacities, blending requirements, and minimum mining width. The problem presented here involves scheduling of  $N$  different ore and waste blocks within a predetermined final pit outline over  $T$  different periods of extraction. Blocks within the same mining bench are aggregated into clusters. Aggregation is based on the block attributes such as, location, rock type, and grade distribution. We refer to these clusters of blocks as mining-cuts. Similar to blocks, each mining-cut has coordinates representing the centre of the cut and its location.

As a general assumption for our formulation we define that a parameter  $f$  can take four indices in the format of  $f_{k,n}^{e,t}$ . Where:

$t \in \{1, \dots, T\}$	index for scheduling periods.
$k \in \{1, \dots, K\}$	index for mining-cuts.
$n \in \{1, \dots, N\}$	index for blocks.
$e \in \{1, \dots, E\}$	index for elements of interest in each block.

The objective function of the MILP formulation is to maximize the net present value of the mining operation. Hence, we need to define a clear concept of economic block value based on ore parcels which could be mined selectively. The profit from mining a block depends on the value of that block and the costs incurred in mining and processing the block. The cost of mining a block is a function of its location, which characterizes how deep the block is located relative to the surface and how far it is relative to its final dump. The spatial factor can be applied as a mining cost adjustment factor for each block according to its location to the surface. The discounted profit from block  $n$  is equal to the discounted revenue generated by selling the final product contained in block  $n$  minus all the discounted costs involved in extracting block  $n$ , this is presented by Eqs. (1) and (2).

discounted profit = discounted revenue - discounted costs (1)

$$d_n^t = \underbrace{\left[ \sum_{e=1}^E o_n \times g_n^e \times r^{e,t} \times (p^{e,t} - cs^{e,t}) \right]}_{\text{discounted revenues}} - \underbrace{\left[ \sum_{e=1}^E o_n \times cp^{e,t} \right]}_{\text{discounted costs}} - [(o_n + w_n) \times cm^t] \quad (2)$$

Where

- $d_n^t$  is the discounted profit generated by extracting block  $n$  in period  $t$ ,
- $o_n$  is the ore tonnage in block  $n$ ,
- $w_n$  is the waste tonnage in block  $n$ ,
- $g_n^e$  is the average grade of element  $e$  in ore portion of block  $n$ ,
- $r^{e,t}$  is the processing recovery, which is the proportion of element  $e$  recovered in time period  $t$ ,
- $p^{e,t}$  is the price in present value terms obtainable per unit of product (element  $e$ ),
- $cs^{e,t}$  is the selling cost in present value terms per unit of product (element  $e$ ),
- $cp^{e,t}$  is the extra cost in present value terms per tonne of ore for mining and processing,
- $cm^t$  is the cost in present value terms of mining a tonne of waste in period  $t$ .

For simplification purposes we denote:

$$v_n^t = \left[ \sum_{e=1}^E o_n \times g_n^e \times r^{e,t} \times (p^{e,t} - cs^{e,t}) - \sum_{e=1}^E o_n \times cp^{e,t} \right] \quad (3)$$

$$q_n^t = (o_n + w_n) \times cm^t \quad (4)$$

Where

- $v_n^t$  is the discounted revenue generated by selling the final product within block  $n$  in period  $t$  minus the extra discounted cost of mining all the material in block  $n$  as ore and processing it; and
- $q_n^t$  is the discounted cost of mining all the material in block  $n$  as waste.

We present two different formulations for the open pit production scheduling problem. The objective function is to maximize the NPV of the mining operation. We used the concepts presented in Boland et al. (2009) as the starting point of our development. We have developed, implemented, and tested a new MILP formulation taking into account practical shovel movements by controlling the maximum number of extraction periods for a mining-cut.

### 3.1. MILP formulation one - extraction at mining-cut level and processing at block level

In the proposed model, processing is controlled at block level, whereas the extraction is controlled at mining-cut level. The amount of ore processed is controlled by the continuous variable  $x_n^t$ , and the amount of material mined is controlled by the continuous variable  $y_k^t$ . Using continuous decision variables allows fractional extraction of mining-cuts in different periods.  $b_k^t \in \{0,1\}$  is the binary integer variable controlling the precedence of extraction of mining-cuts.  $b_k^t \in \{0,1\}$  is equal to one, if extraction of mining-cut has started by or in period  $t$ ; otherwise it is zero. The objective function is to maximize the net present value of mining operation.

*Objective function:*

$$\max \sum_{t=1}^T \sum_{k=1}^K \left( \sum_{n \in c_k} v_n^t \times x_n^t - \left( \sum_{n \in c_k} q_n^t \right) \times y_k^t \right) \quad (5)$$

Where

- $T$  is the maximum number of scheduling periods, where  $\mathcal{T} = \{1, \dots, T\}$  is the set of all the scheduling time periods in the model,
- $K$  is the total number of mining-cuts to be scheduled, where  $\mathcal{K} = \{1, \dots, K\}$  is the set of all the mining-cuts in the model,
- $c_k$  represents set of blocks within mining-cut  $k$ ,
- $x_n^t \in [0, 1]$  is a continuous decision variable, representing the portion of block  $n$  to be extracted as ore and processed in period  $t$ ,
- $y_k^t \in [0, 1]$  is a continuous decision variable, representing the portion of mining-cut  $k$  to be mined in period  $t$ , fraction of  $y$  characterizes both ore and waste included in the mining-cut.

It should be mentioned that in the objective function given by Eq. (5), mining is controlled at the mining-cut level, whereas the processing is at the higher resolution of block level. The objective function is subject to the following constraints.

*Mining capacity constraints:*

$$\sum_{k=1}^K \left( \sum_{n \in c_k} (o_n + w_n) \right) \times y_k^t \leq mu^t \quad \forall t \in \{1, \dots, T\} \quad (6)$$

$$\sum_{k=1}^K \left( \sum_{n \in c_k} (o_n + w_n) \right) \times y_k^t \geq ml^t \quad \forall t \in \{1, \dots, T\} \quad (7)$$

Where

- $mu^t$  is the upper bound on mining capacity in period  $t$  (tonnes),
- $ml^t$  is the lower bound on mining capacity in period  $t$  (tonnes).

Eq. (6) ensures that the total amount of ore and waste mined in each period is equal to or less than the targeted maximum mining capacity of equipment. The constraints are controlled by the continuous variable  $y_k^t$  at the mining-cut level. Eq. (7) on the other hand, ensures that the minimum amount of material that needs to be mined is achieved; Eq. (7) is useful in achieving a constant stripping ratio over the mine life. A production schedule with an invariable stripping ratio would have significant savings potential by ensuring that fleet size required is matched to targets for material movement. The decision of the proper production rate which leads to the boundaries on mining capacity is an important stage of the production scheduling of open pit mines. Different scenarios of annual ore production rates must be examined and the one with highest NPV and uniform mill feed must be chosen. The mining capacity boundaries are a function of the ore reserve, overall stripping ratio, designed processing capacity, targeted mine-life, and the capital investment available for purchasing equipment. The upper and lower bounds of mining capacity could vary by scheduling periods, allowing the designer to use variable mining capacities throughout the mine life. The shortage of equipment in specific periods could be compensated with contract mining. Eqs. (6) and (7) will generate one constraint per period.

*Processing capacity constraints:*

$$\sum_{n=1}^N o_n \times x_n^t \leq pu^t \quad \forall t \in \{1, \dots, T\} \quad (8)$$

$$\sum_{n=1}^N o_n \times x_n^t \geq pl^t \quad \forall t \in \{1, \dots, T\} \quad (9)$$

Where

- $N$  is the number of blocks in the block model, where  $\mathcal{N} = \{1, \dots, N\}$  is a set of all the blocks in the model,
- $pu^t$  is the upper bound on processing capacity of ore in period  $t$  (tonnes),
- $pl^t$  is the lower bound on processing capacity of ore in period  $t$  (tonnes).

Eqs. (8) and (9) represent inequality constraints controlling the mill feed or processing capacity; These constraints assist the mine planners to achieve an overall mine-to-mill integration by providing a uniform feed throughout the mine-life. Constraints (8) and (9) are at block level, which means the decisions are made based upon the tonnage of ore above the cut-off grade within individual blocks. In practice, the processing capacity constraints must be set within a tight upper and lower bounds to provide a uniform feed to the mill. Based on the shape of the orebody and distribution of ore grades, these constraints might not be honoured under some circumstances, which will lead to an infeasible problem. Pre-stripping could be achieved by setting the upper and lower bounds of processing capacity constraints equal to zero for the desired periods. This approach would enforce the optimizer to only mine waste blocks in the early periods. Eqs. (8) and (9) will generate one constraint per period per processing path.

*Grade blending constraints:*

$$\sum_{n=1}^N g_n^e \times o_n \times x_n^t / \sum_{n=1}^N o_n \times x_n^t \leq gu^{e,t} \quad \forall t \in \{1, \dots, T\}, \quad e \in \{1, \dots, E\} \quad (10)$$

$$\sum_{n=1}^N g_n^e \times o_n \times x_n^t / \sum_{n=1}^N o_n \times x_n^t \geq gl^{e,t} \quad \forall t \in \{1, \dots, T\}, \quad e \in \{1, \dots, E\} \quad (11)$$

Where

- $g_n^e$  is the average grade of element  $e$  in ore portion of block  $n$ , where  $\mathcal{E} = \{1, \dots, E\}$  is the set of all the elements of interest in the model,
- $gu^{e,t}$ , is the upper bound of acceptable average head grade of element  $e$  in period  $t$ ,
- $gl^{e,t}$ , is the lower bound of acceptable average head grade of element  $e$  in period  $t$ .

Production scheduling is concerned with the inherent task of blending the run-of-mine materials before processing. The objective is to mine in such a way that the resulting mix meets the quality specifications of the processing plant. The blending problem becomes more important as the design moves towards mid-range to short-range planning, where the planner is concerned with reducing the grade variability. Constraints (10) and (11) are at block level and there would be one equation per element per scheduling period for both upper and lower bounds.

*Ore processed and material mined constraints:*

$$x_n^t \leq y_k^t \quad \forall n \in \{1, \dots, N\}, \quad n \in c_k, \quad t \in \{1, \dots, T\} \quad (12)$$

Where  $x_n^t$  is the portion of block  $n$  to be extracted as ore and processed in period  $t$ , and  $y_k^t$  is representing the portion of mining-cut  $k$  to be mined in period  $t$ , where fraction of  $y$  characterizes both ore and waste included in the mining-cut. Eq. (12) demonstrates inequalities that ensure the amount of ore in any block which is processed in any given period is less than or equal to the amount of rock extracted from the mining-cut  $k$  in any given scheduling period. A very important assumption in the formulation is that each mining-cut is extracted homogeneously; this means that  $y_k^t$  illustrates the fraction of mining-cut  $k$  to be extracted in time period  $t$  and all the blocks within the cut  $n \in c_k$  are extracted with the same proportion of  $y_k^t$ . Eq. (12) generates one equation per block per period.

*Precedence of mining-cuts extraction and slope constraints*

$$b_k^t - \sum_{i=1}^t y_s^i \leq 0 \quad \forall k \in \{1, \dots, K\}, \quad t \in \{1, \dots, T\}, \quad s \in H(S) \quad (13)$$

$$\sum_{i=1}^t y_k^i - b_k^t \leq 0 \quad \forall k \in \{1, \dots, K\}, \quad t \in \{1, \dots, T\} \quad (14)$$

$$b_k^t - b_k^{t+1} \leq 0 \quad \forall k \in \{1, \dots, K\}, \quad t \in \{1, \dots, T-1\} \quad (15)$$

Where

- $b_k^t \in \{0,1\}$  is a binary integer decision variable controlling the precedence of extraction of mining-cuts.  $b_k^t$  is equal to one if extraction of mining-cut  $k$  has started by or in period  $t$ , otherwise it is zero,
- $H_k(S)$  is a set  $H_k(S) \subset \mathcal{K}$  for each mining-cut  $k$ , defining the immediate predecessor cuts that must be extracted prior to extracting mining-cut  $k$ , where  $S$  is the total number of cuts in set  $H_k(S)$ .

For each mining-cut  $k$ , Eqs. (13) to (15) check the set of immediate predecessor cuts that must be extracted prior to mining-cut  $k$ . This precedence relationship ensures that all the blocks above the current mining-cut are extracted prior to extraction of mining-cut. As it could be deduced from Eq. (15), the formulation is based on the temporal sequence of extraction rather than checking for all the periods. Eqs. (13) to (15), represent one equation per mining-cut per period.

For each block  $n$  there is a set  $C_n(L) \subset \mathcal{N}$ , which includes all the blocks that must be extracted prior to mining block  $n$  to ensure that block  $n$  is exposed for mining with the desired overall pit slopes, where  $L$  is the total number of blocks in set  $C_n(L)$ . We use a directed graph to model the precedence of extraction between blocks. We define a directed graph  $G_b(\mathcal{N}, \mathcal{A})$  by the set of vertices,  $\mathcal{N}$  (blocks); connected by ordered pairs of elements called arcs,  $\mathcal{A}$ . More detailed information about directed graphs could be reviewed in (Siek, et al., 2002).

During the clustering of blocks into mining-cuts, another directed graph at mining-cut level is constructed capturing the precedence relationship of mining-cuts. This directed graph is denoted by  $G_c(\mathcal{K}, \mathcal{B})$  where  $\mathcal{B} = \{1, \dots, B\}$  is the set of all edges in the mining-cuts precedence directed graph. The directed graph  $G_c(\mathcal{K}, \mathcal{B})$  is constructed in a way that while satisfying the order of extraction at mining-cut level, it would also satisfy the relationships defined by the graph  $G_b(\mathcal{N}, \mathcal{A})$  at block level. This approach of defining two directed graphs at mining-cut and block level enables us to model variable pit slopes with small acceptable slope errors in the different regions of the open pit. In other words, mining is controlled at the mining-cut level, while the slopes are modelled at the block level.



### 3.2. Alternative MILP formulation

Eqs. (5) to (15) represent the MILP formulation for long-term open pit production scheduling. The proposed formulation requires  $(2 \times K + N) \times T$  number of decision variables, where  $K \times T$  of these variables are binary integers. One of the major obstacles in using the MILP formulations for mine production scheduling is the sheer size of the problem. The number of blocks,  $N$ , in the model is usually between tens of thousands to millions. Moreover, the main physical constraint in open pit mining is the block extraction precedence relationship modelled by binary integer variables. The most common problem in the MILP formulation is size of the branch and cut tree. The tree becomes so large that insufficient memory remains to solve an LP sub-problem. The number of binary integer variables in the formulation determines the size of the branch and cut tree. As a general strategy in our formulations we aimed at reducing the number of binary integer variables. We have reduced the number of binary integer variables to  $K \times T$ , where to some extent we have control over the number of mining-cuts  $K$  created during the clustering algorithm.

We investigated the effect of using continuous decision variables ( $x'_n$  and  $y'_k$ ), which leads to fractional block extraction on the quality and practicality of the generated schedules. There is a possibility that block  $n \in c_k$  get extracted over multiple periods. Our computational experiments on different models using the formulation presented by Eqs. (5) to (15) shows that block fractions are usually scheduled over consecutive periods and in the worst case examined, some blocks were extracted over three periods. However, this could not be extended as a general rule. We should also emphasize again that blocks are uniformly extracted as part of mining-cuts. The total tonnage of ore processed in the MILP formulation presented by Eqs. (5) to (15) is related to how mining and processing capacities are set in accordance with the ore reserve total tonnage. There is the possibility that quantities of ore above the cut-off grade would not get processed due to the processing capacity limitations. It is feasible to overcome the abovementioned problems by adding reserve and maximum number of fractions constraints to the MILP formulation presented by Eqs. (5) to (15).

*Maximum number of fractions and reserve constraints*

$$\sum_{t=1}^T x'_n = 1 \quad \forall n \in \{1, \dots, N\} \quad (16)$$

$$\sum_{t=1}^T y'_k = 1 \quad \forall k \in \{1, \dots, K\} \quad (17)$$

$$\sum_{t=1}^T u'_k \leq m \quad \forall k \in \{1, \dots, K\} \quad (18)$$

$$\sum_{t=1}^T u'_k \times y'_k = 1 \quad \forall k \in \{1, \dots, K\} \quad (19)$$

Where

- $u'_k \in \{0, 1\}$  is a binary integer decision variable equal to one if mining-cut  $k$  is scheduled to be extracted in period  $t$ , otherwise zero,
- $m$  is an integer number representing the maximum number of fractions that mining-cuts are allowed to be extracted over and the previously defined terms apply.

Equality constraints presented by Eq. (16) ensures that all the ore within the predefined pit limits or the targeted push-back is processed during the optimization. Eq. (16) adds one constraint per block. Eq. (17) ensures that all the material within the predefined pit outline is to be mined; this adds one

constraint per mining-cut. Eq. (18) and (19) guarantee that the maximum number of fractions of mining-cuts in the solution for  $y_k^t$  is not going to exceed the integer number  $m$ . For large-scale models with many scheduling periods  $m$  is set equal to two or three. This would ensure that the generated schedule is practical from the equipment movement point of view. Eq. (19) is a set of non-linear constraints, which introduces a mixed integer non-linear programming (MINLP) problem. MINLP problems are very difficult, if not possible, to solve. We linearize Eq. (19) by introducing a new continuous variable,  $a_k^t$ , to replace the product  $a_k^t = u_k^t \times y_k^t$ . Eq. (19) is replaced by  $\sum_{t=1}^T a_k^t = 1$  and linear constraints represented by Eqs. (20) to (23) are added to force  $a_k^t$  to take the value of  $u_k^t \times y_k^t$ .

$$a_k^t \leq u_k^t \quad \forall k \in \{1, \dots, K\}, \quad t \in \{1, \dots, T\} \quad (20)$$

$$a_k^t \leq y_k^t \quad \forall k \in \{1, \dots, K\}, \quad t \in \{1, \dots, T\} \quad (21)$$

$$a_k^t \geq y_k^t - (1 - u_k^t) \quad \forall k \in \{1, \dots, K\}, \quad t \in \{1, \dots, T\} \quad (22)$$

$$a_k^t \geq 0 \quad \forall k \in \{1, \dots, K\}, \quad t \in \{1, \dots, T\} \quad (23)$$

#### 4. Results and discussion

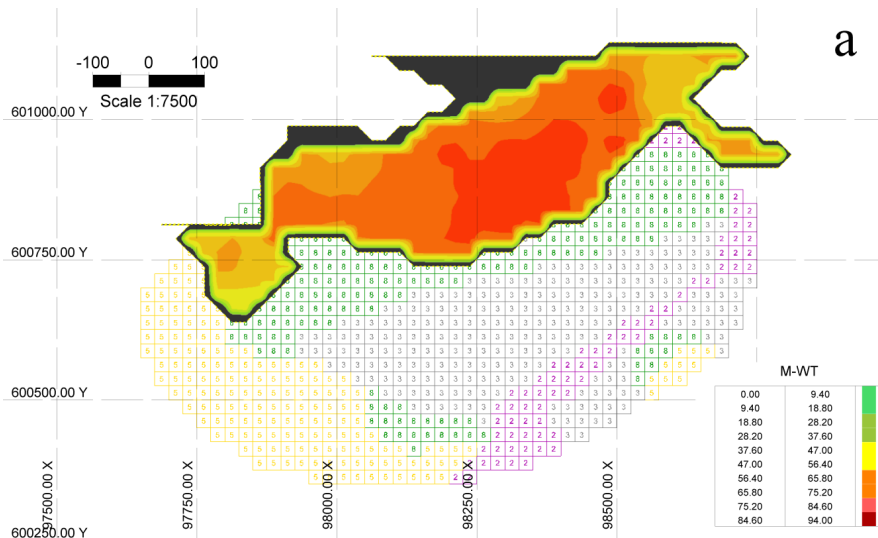
A production scheduling case study is carried out to verify and validate the MILP models. An iron ore deposit is considered with three types of ore classified as top magnetite, oxide, and bottom magnetite. The block model contains the estimated magnetic weight recovery (MWT%) of iron ore. The contaminants are phosphor (P%) and sulphur (S%). Blocks in the geological model represent a volume of rock equal to  $25m \times 25m \times 15m$ . We compare our model against Whittle strategic mine planning software (Gemcom Software International, 2008). Whittle is one of the tools extensively used in industry for open pit optimization and long-term production scheduling. The input parameters and the mining strategies in Whittle and MILP scheduler are scrutinized carefully to make sure an unbiased comparative study is undertaken. The goal is to maximize the NPV at a discount rate of 10%, while assuring a constant uniform feed to the processing plant through a 21-year mine-life. We aimed at generating a practical schedule taking into account the minimum operational room required, the number of active benches in each period, the number of benches added to the pit in each period, uniformity of processing plant feed, and variability of the stripping ratio. The pit includes 427.33 Mt of rock where, 116.29 Mt is iron ore with an average magnetic weight recovery grade of 72.9%. Initially a capacity of 26 Mt/yr was considered as the upper bound on mining capacity, subsequently it was reduced to 25 Mt/yr from year 6 to the end of the mine life. We use Milawa Balanced scheduling algorithm in Whittle, with minimum lead of 3 and maximum lead of 6 benches. The maximum number of active benches is set to 6. In the pit limit optimization process a minimum mining width of 100 meters is used for intermediate pits. Examination of the orebody and cross sections of the open pit reveals that (Figure 2a) due to the shape of the deposit providing a uniform feed to the processing plant is a challenging task. Five years of pre-stripping is considered to ensure the deposit is exposed for open pit mining with adequate operating room in the future. Table 1 summarizes the information related to the comparative case study.

Table 1. Final pit and production scheduling information.

Description	Value	Description	Value
Number of mining-cuts	895	Minimum mining width (m)	100
Total tonnage of rock (Mt)	427.33	Number of periods (years)	21
Total ore tonnage (Mt)	116.29	Maximum number of active benches	6
Total tonnage of recovered Fe (Mt)	76.33	MILP sulphur grade constraint (%)	$0 \leq S \leq 1.9$
Average grade of MWT%	72.9%	MILP phosphor grade constraint (%)	$0 \leq P \leq 0.2$
Mining capacity (Mt/year) (years 1 to 5)	26	MILP MWT grade constraint (%)	$60 \leq \text{MWT} \leq 80$
(years 6 to 21)	25	Minimum lead between benches	3
Processing (Mt/year) (years 1 to 5)	0	Maximum lead between benches	6
(years 6 to 21)	8	Total number of blocks in the pit	19,492

In the MILP model, the mining and processing are both set at mining-cut level. The total number of blocks within the final pit limit is 19,492. We use fuzzy logic clustering to aggregate blocks into 895 mining-cuts to reduce the number of variables required in the MILP model. The clustering algorithm, aggregates blocks based on three main criteria: location, rock type, and grade distribution. Blocks that have a higher order of similarity are aggregated into mining-cuts. Figure 1a illustrates the plan view of bench at 1575m elevation; the magnetic weight recovery and rock-types are shown in Figure 1a. Magnetic weight recovery (MWT%), rock-type code, and location of each block is used to aggregate the blocks into mining-cuts, representing a selective mining unit. Figure 1b illustrates the result of the clustering at the 1575m bench. The total tonnage (quantity) of ore, waste, iron ore (MWT), phosphor, and sulphur are calculated for each mining-cut. The aggregated mining-cut model is the input into the MILP scheduler. Aggregation of blocks into mining-cuts, also impose the MILP scheduler to generate a mining schedule at the mining selective unit (SMU). This is very important in generating a practical production schedule from mining operation point of view. Grade blending constraints are set for magnetic weight recovery, sulphur, and phosphor as described in Table 1.

The scheduling of the open pit is carried out using Whittle and the MILP scheduler under the same input data and similar mining strategies. Figure 1c shows bench 1575m scheduled with the MILP scheduler, the periods of extraction are labelled on each block. Figure 1d illustrates bench 1575m scheduled with Whittle Milawa Balanced strategy. Figure 2a to 2c demonstrates cross section 98500m looking east. Figure 2a illustrates the orebody, MWT grade distribution, and rock-types. Figure 2b shows the MILP production schedule with periods labelled and Figure 2c demonstrates Whittle Milawa Balanced production schedule with periods numbered accordingly.



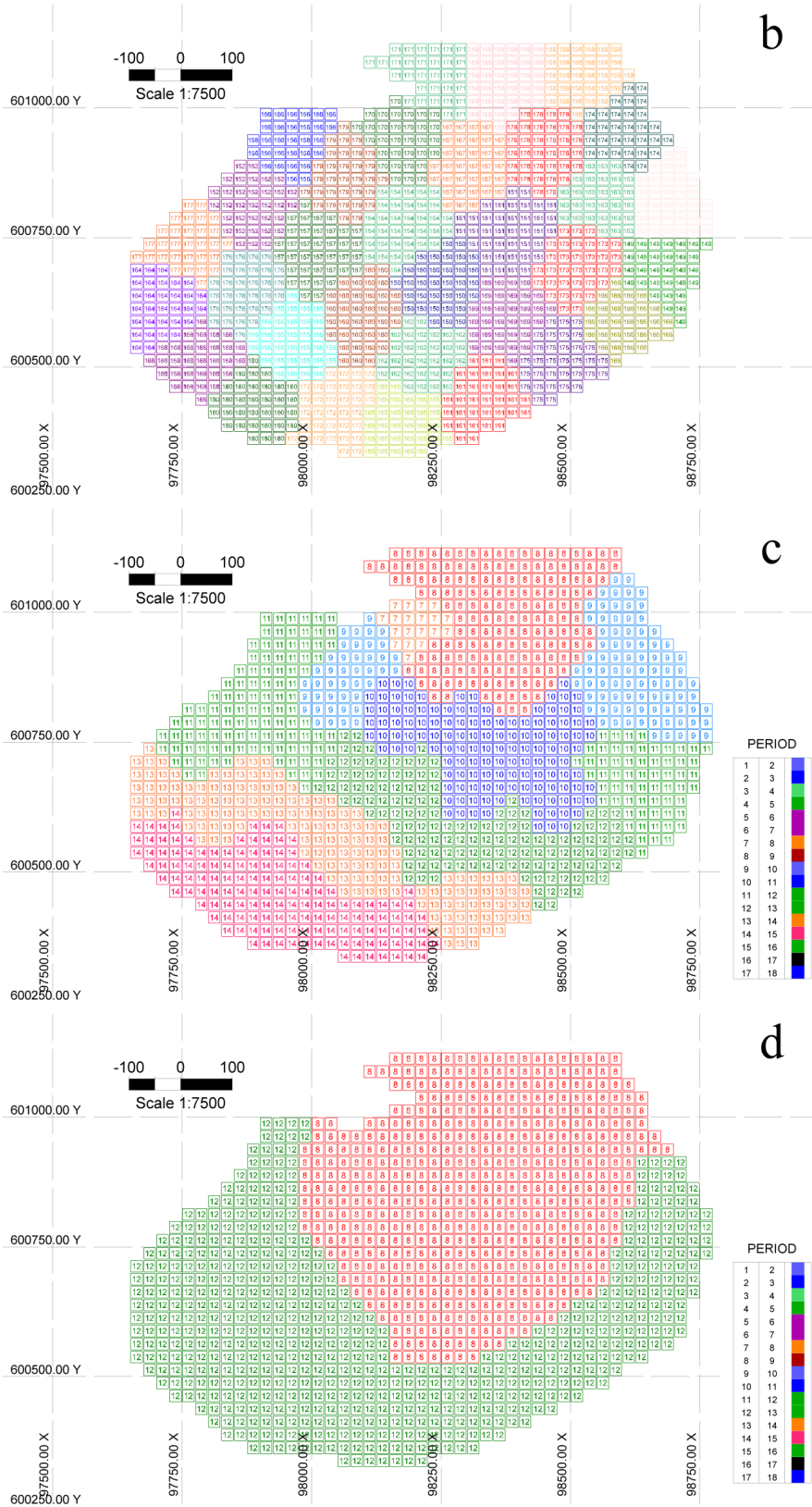


Figure 1. Bench plan view at elevation 1575m. (a) orebody outline and grade distribution with rock-types labelled; (b) clustering results aggregating blocks into mining-cuts; (c) MILP production schedule; (d) Whittle production schedule.

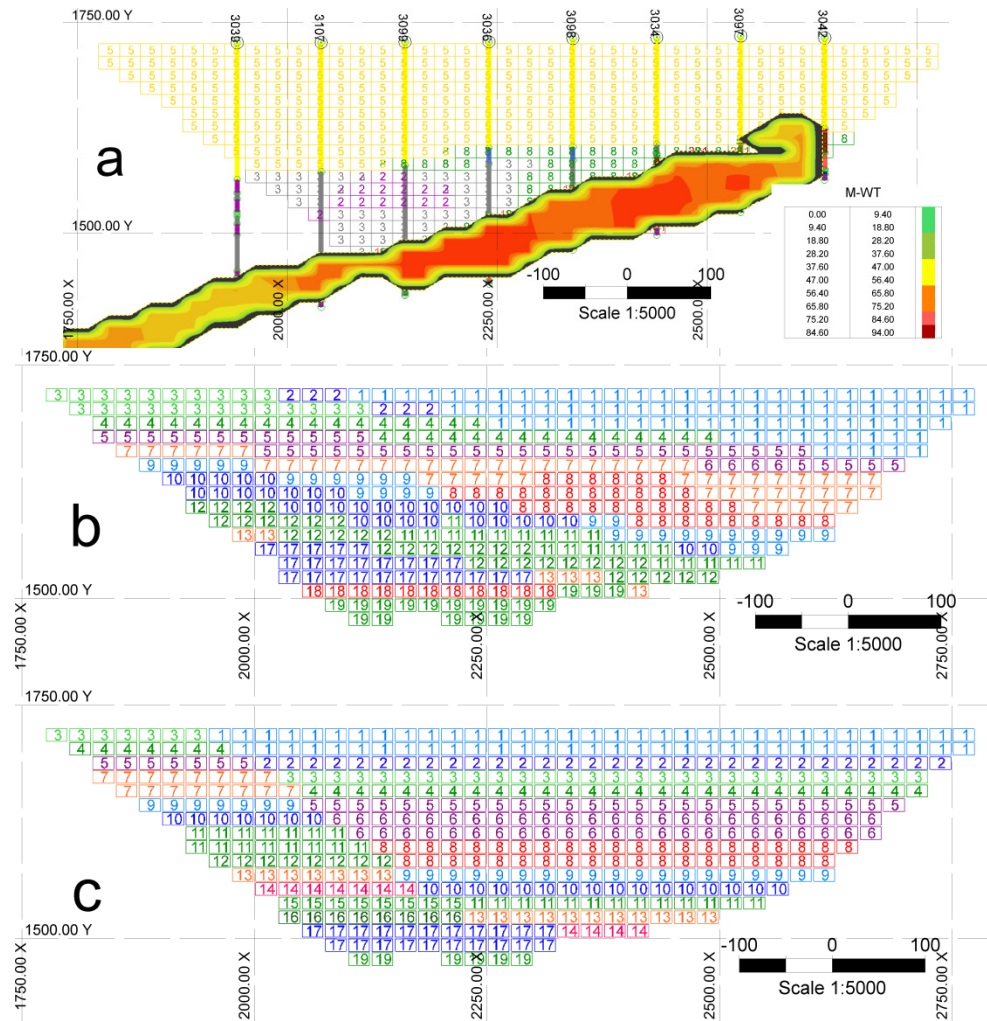


Figure 2. Cross section 98500m looking east ; (a) orebody, grade distribution, and rock-types; (b) MILP production schedule with periods labelled; (c) Whittle production schedule with periods labelled.

Figure 3a illustrates the yearly ore and waste production generated by MILP scheduler and Whittle software. There are 5 years of pre-stripping in both cases. A processing maximum capacity of 8Mt/yr is set from year 6 to 21. We have tried different scenarios and options with Whittle Milawa Balanced strategy and Whittle Milawa NPV strategy to generate a reasonable schedule with the least amount of deviation from target production. Figure 3b illustrates that there is a shortfall of ore feed in years 6 and 7; this shortfall does not occur in the MILP schedule (Figure 3a), since a minimum processing requirement of 6Mt/yr is set for years 6 to 9. The lower bound on processing requirements will ensure that ore is going to be delivered to the processing plant in the schedule.

Figure 4a and 4b illustrate the sulphur and phosphor average grade at the processing plant. The contaminant grade constraints for sulphur and phosphor as set up in Table 1 are met in both schedules. Figure 5 illustrates the head grade of magnetic weight recovery of iron ore. Comparison of Whittle and MILP schedule head grades in Figure 5 illustrates a higher average head grade by the MILP schedule. This is especially notable in the starting years of processing (years 6 to 9), which has a higher impact on the NPV of the operation. The higher average head-grade and less deviation from the 8 Mt/yr target production, translates into higher cash flows in the early years by the MILP schedule. Figure 6 illustrates the annual cash flow generated by MILP scheduler and

Whittle software. It is evident that the cash flows generated in years 6 and 7 by the MILP scheduler are substantially greater than the Whittle results. In Whittle the mine planner does not have control over the lower boundary constraints for mining and processing. Figure 7 illustrates the cumulative discounted cash flow of the MILP and Whittle schedule; it also demonstrates the difference between the cumulative discounted cash flow at the end of each period.

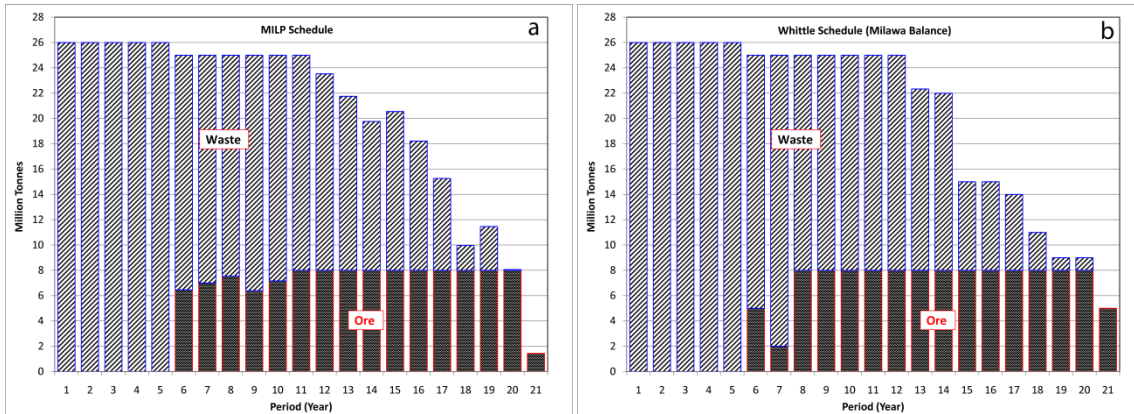


Figure 3. Yearly production schedule of ore and waste; (a) MILP schedule; (b) Whittle schedule.

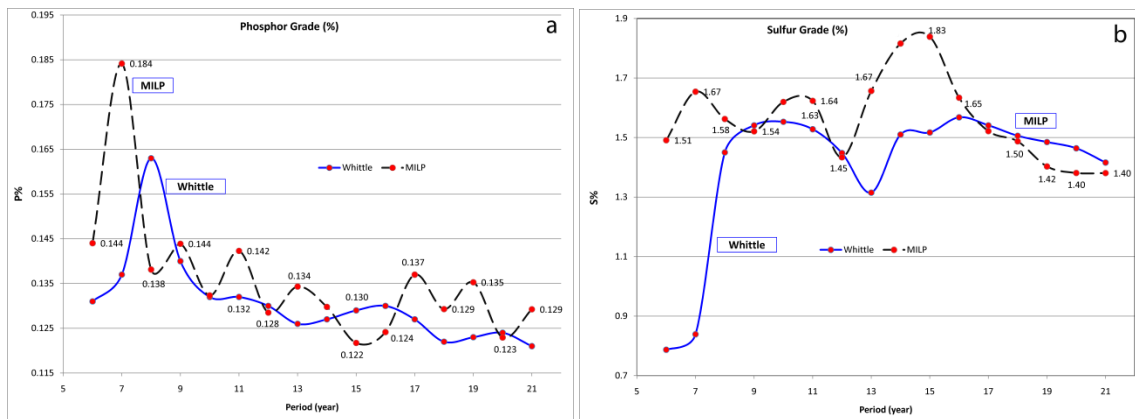


Figure 4. Contaminants grade over the mine life for MILP and Whittle schedule; (a) phosphor grade in percent; (b) sulphur grade in percent.

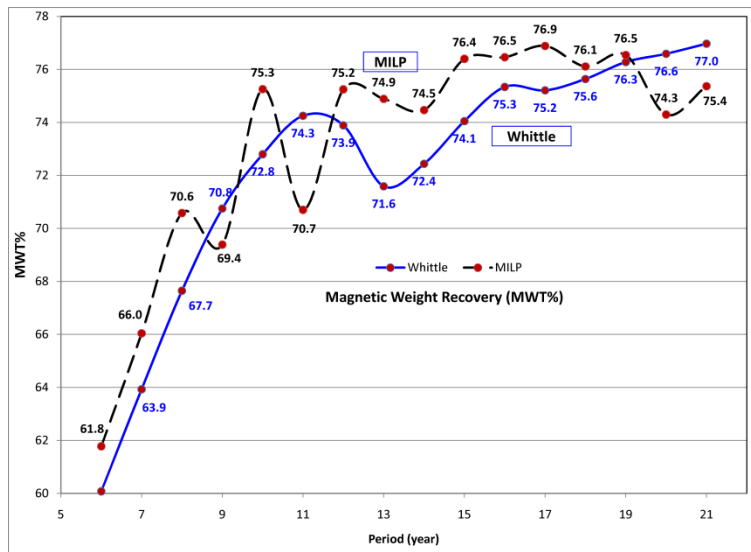


Figure 5. Magnetic weight recovery head grade for the MILP and Whittle schedule.



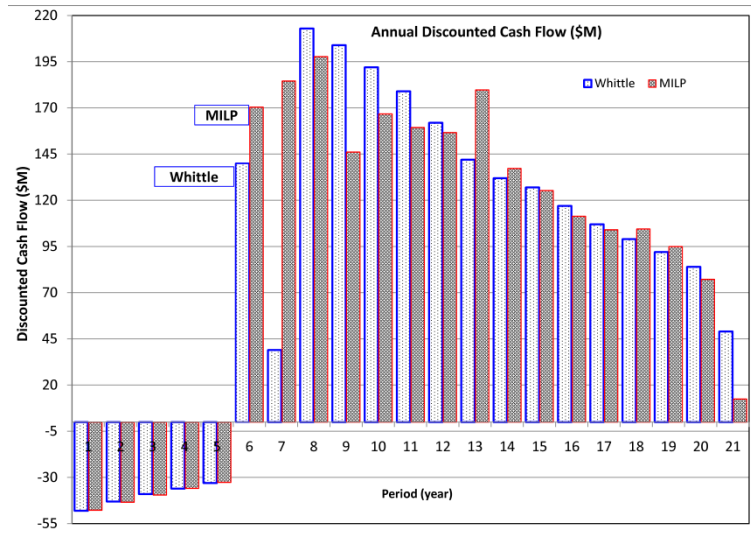


Figure 6. Annual cash flow of the MILP and Whittle schedule in million dollars.

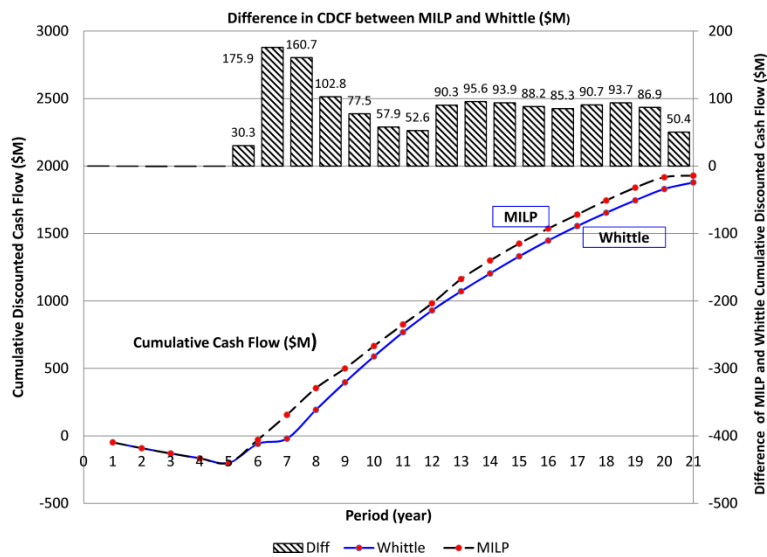


Figure 7. Cumulative discounted cash flow of the MILP and Whittle schedule- the difference between the cumulative discounted cash flows are illustrated on top.

The discounted cash flow of the MILP scheduler is \$1929.4 million dollars; whereas the discounted cash flow generated by Whittle Milawa Balanced is \$1879.0 million dollars. The discounted cash flow is compared over the same mine-life of 21 years and at a discount rate of 10%. We also kept all the input scheduling variables and strategies with both models the same. The difference between the cumulative discounted cash flow of the MILP scheduler and the Whittle Milawa Balanced results is \$50.4 million dollars. This is a substantial amount considering the relatively small size of the open pit. We emphasize again that our goal was to compare the results under the same mining strategies, such as minimum mining width, minimum and maximum lead number, and maximum active benches in each period. We examined generating the schedule with Whittle Milawa NPV under the exact same input parameters. The cumulative discounted cash flow by Whittle Milawa NPV is equal to \$1896.3 million dollars over similar mine-life. The generated schedule had a very variable stripping ratio through the mine life which makes it almost impossible to implement in practice.

## 5. Conclusions and future work

This paper investigated the development of a mixed integer linear programming (MILP) formulation for open pit production scheduling optimization. We developed, implemented, and tested practical MILP models for open pit production scheduling in TOMLAB/CPLEX (Holmström, 2009) environment. The MILP formulation of open pit production scheduling becomes intractable because of the size of the problem. To reduce the number of continuous and binary variables in the model, we aggregated blocks into larger units, referred to as mining-cuts using clustering algorithms.

The main objective of this paper was to highlight the considerable achievable economic gains that are possible through production scheduling optimization. Also, we aimed at improving the practicality and performance of the MILP production scheduling formulations. We verified and validated the MILP production scheduler by a comparative case study against one of the standard industry tools — Whittle strategic mine planning software (Gemcom Software International, 2008). The input parameters and the mining strategies in Whittle and MILP scheduler were inspected cautiously to make sure an unbiased comparative study was undertaken. The goal was to maximize the NPV at a discount rate of 10%, while assuring a constant uniform feed to the processing plant. We aimed at generating a practical schedule taking into account the minimum operational room required, the number of active benches in each period, the number of benches added to the pit in each period, uniformity of processing plant feed, and variability of the stripping ratio. The pit includes 427.33 Mt of rock where, 116.29 Mt is iron ore with an average magnetic weight recovery grade of 72.9%. The discounted cash flow of the MILP scheduler was \$1929.4 million dollars; whereas the discounted cash flow generated by Whittle Milawa Balanced is \$1879.0 million dollars. The difference between the cumulative discounted cash flow of the MILP scheduler and the Whittle Milawa balanced results is \$50.4 million dollars. This is a substantial amount considering the relatively small size of the open pit.

Production scheduling optimization techniques are still not widely used in the mining industry. There is a need to improve the practicality and performance of the current production scheduling optimization tools used in mining industry. Also, to gain more common recognition in industry, there is a need to highlight the considerable achievable economic gains that are possible through production scheduling optimization. Further focused research is underway to develop and test different clustering techniques that would generate an optimized clustering approach for the mining-cuts. Also the next step is to extend the mixed integer linear programming framework into stochastic mathematical programming domain to address the grade uncertainty issue.

## 6. References

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