Mining Optimization Laboratory

Report Nine –2017/2018

Yashar Pourrahimian
Eugene Ben-Awuah
Hooman Askari-Nasab



School of Mining and Petroleum Engineering
Department of Civil & Environmental Engineering,
University of Alberta, Edmonton, Alberta, CANADA

Pourrahimian, Y., Ben-Awuah E., and Askari-Nasab, H. (2018), Mining Optimization Laboratory (MOL) – Report Nine, © MOL, University of Alberta, Edmonton, Canada, 378 Pages, ISBN: 978-1-55195-414-1.

All rights reserved, all material in this report is, unless otherwise stated, the property of the *Mining Optimization Laboratory (MOL)*.

Reproduction or retransmission of the materials, in whole or in part, in any manner, without the prior written consent of the copyright holder, is a violation of copyright law.

The report may be circulated and disposed at your discretion; however, the following copyright notice must be adhered to.

Copyright © 2018, Mining Optimization Laboratory, University of Alberta

Mining Optimization Laboratory sponsors may utilize and disclose the report material and software within their organization with no prior permission of MOL.

Contact information for requests for permission to reproduce or distribute materials available through this report is:

Hooman Askari-Nasab, PhD, PEng
Professor of Mining Engineering
Department of Civil & Environmental Engineering
University of Alberta, Edmonton, AB, Canada T6G 1H9
Phone: +1 (780) 492-4053
Cell: +1 (780) 893-9365
Email: hooman@ualberta.ca
MOL Webpage: www.ualberta.ca/mol

Executive Summary

This year, we have prepared a report including 20 papers. We continue to update all the research results on the MOL webpage www.ualberta.ca/mol on the members section. Sponsors have access to current and past research results, publications, prototype software, and source code. Let's review the contributions in the MOL Report Nine (2017/2018) by considering some of the main contributors.

In paper 101, ***Mohammad*** presents new variations of his block clustering algorithm which incorporate geological uncertainties into the clustering procedure. The first of the four variations of agglomerative hierarchical clustering algorithm is based on deterministic estimates of properties and the other three are based on possible worlds approach which use Geostatistical realizations to form aggregates with regard to the geological properties and the existing uncertainties. It is shown, through case studies, that uncertainty based algorithms can result in aggregates that are less susceptible to uncertainties, and at the same time, the proposed algorithm can produce aggregates that are within a controlled size and have minable shapes.

Shiv presents a short term production scheduling model which allocates shovels over continuous time frames, thus controlling the model size from growing with the resolution. This approach also models the mine operating environment in a more representative way to create practical and achievable schedules. A verification study of the model is presented, in paper 102, using an iron ore mine case study. A comparison of solutions in the pareto optimal space also justifies the goal optimization as an efficient approach to attain best outcomes from among multiple conflicting objectives of the problem.

Ahlam has developed an uncertainty-based mathematical programming model based on Mixed Integer Linear Goal Programming (MILGP) for oil sands production scheduling and waste management. The production schedule is integrated simultaneously with in-pit and ex-pit dyke construction scheduling. She has integrated a limited duration stockpiling strategy for the ore material that exceeds the processing capacity. To minimize the financial risk from grade uncertainty, kriging estimates with a variance penalty scheme are used for mine planning. The topmost layer of the overburden is stockpiled for land reclamation. The model determines the reclamation strategy for the stockpiled ore and the destination of reclamation and dyke materials to minimize costs and maximize the Net Present Value (NPV) of the project. The results are presented in paper 103. In paper 106, Ahlam also presents a different scenario for the MILGP model that uses two new robust constraints for managing tonnage fluctuations for material mined and processed over the mine life. Organic Rich Solids (ORSs) negatively affect the bitumen recovery and are used as a predictor for ore processability during optimization. The model generates an integrated mine-to-mill production plan with a waste management and stockpiling strategy over the mine life that maximizes the NPV of the operation.

Ali, in paper 104, develops a truck dispatching decision making model based on the transportation problem approach. Although most of the factors affecting the truck dispatching decisions are random in behavior such as truck travel time to the next destination they are rarely treated as stochastic parameters in the literature. The developed model is a stochastic integer programming model that makes decisions on the trucks dispatching problem under condition of trucks' travel time uncertainty. Besides, a simulation model of an open pit mining operation has been developed to evaluate the developed stochastic truck dispatching model. Results of the implementation of the developed model proves impacts of the randomness in the operational parameters on the truck dispatching decisions.

Obinna has been conducting a literature review on geo-metallurgical and geochemical properties of oil sands and its incorporation in oil sands mining and waste management in paper 105. The presence of ultra-fines and organic-rich solids reduce the quality of bitumen during extraction. Current tools and methodology in the oil sands industry have not been able to adequately assess and quantify oil

sands geo-metallurgical and geochemical properties due to uncertainties such as grade and rock type uncertainties which are associated with the quantity and quality of bitumen recovered and tailings produced. Thus, his research focuses on the development of an integrated framework for improving the bitumen recovery as well optimize the mine production schedule for a long term. Mathematical formulations for mine planning will be developed to take into consideration the geo-metallurgical and geochemical properties during oil sands mine planning. Pseudo penalties will be assigned to any material that affects the recovery of bitumen to facilitate ore blending for processing.

In paper 201, ***Ali*** presents a mixed integer linear programming model to solve truck dispatching problem in surface mines. To account for the parameters' inherent uncertainties, he implements a fuzzy approach to solve the developed mathematical model. The developed model is implemented on a surface mining case study and results are compared against a benchmark model implementation on the same case. The results show that the model developed in this paper requires smaller fleet of trucks to meet the production requirement of the mine in comparison to the benchmark model.

Eduardo develops a theoretical framework for equipment planning based on a detailed analysis of mine operations data from an oil sands mine in northern Alberta. The framework is built on static simulation principles and provides better results than existing in-house tools, which rely solely on manufacturer data, and thus aids in efficient equipment planning for life of mine plans. The use of this framework to modify existing productivity curve estimation methods currently in use at the mine site is also proposed. This method replaces "loaded flat haul" with "effective loaded flat haul" in the estimation of productivity. Validation of the model, in paper 202, presents an over estimation of productivity by 4% against an underestimation of over 10% by the existing in-house method.

Bright has been working on open pit to underground mining transition complexes. He has developed mixed integer linear programming (MILP) models to determine the best mining strategy for a deposit amenable to either open pit mining, underground mining or simultaneous open pit and underground mining option with optimal crown pillar location. The models presented in paper 301 have been discussed and implemented on two case studies to investigate the best mining option that maximizes the net present value (NPV) of the mining project. Both underground and simultaneous open pit and underground mining option models further determine the 3D location of the crown pillar required for the safe and profitable operation of the mine. Bright is currently working on incorporating geological uncertainty, and underground development and operational layouts into the models.

Firouz has been carrying out research on block-cave production scheduling using mathematical programming (papers 302 and 303). He looks at block cave production scheduling under material flow uncertainty. One of the main differences between block caving and other mining methods is the influence of the material flow on production, and draw control in general. Achieving an optimum production schedule without consideration of the cave rate and material flow could be unrealistic and impractical as the movements of material between drawpoints will result in unexpected production grades and tonnages. He has developed, a stochastic mixed-integer optimization model to optimize the production schedule during the life of the mine. The uncertainties of production grades and tonnages are captured by defining a number of scenarios that represent the probable movements of fragmented rock between drawpoints in the same neighborhood. The decision variables in the formulation are based on the slice model, means that the mathematical solution determines which slices are extracted from drawpoints in each period of production. The goal is to maximize the net present value of the project during the life of the mine and minimize the deviations of production grades and tonnages from the defined goals in all probable scenarios resulting from the movements of the fragmented rock between drawpoints. Application of the proposed model in caving operations not only can improve the profitability of the project but also increase the confidence of the production schedule. The designed graphical user interface, with the capability of adding different technical and operational constraints, will be a flexible tool for mine planners to control the draw based on the company's goals during the life of the mine.

Roberto has been carrying out research on determination of the optimum undercut level elevation in block caving (305). He has developed a mathematical programming framework to determine the best undercut elevation, by formulating a simplified Linear Integer Programming (IP) model that captures the discounting of the profits from the vertical draw rate extraction and the horizontal advance direction that can be applied at the early stages of a caving project. The IP model comprises a simplified block caving scheduling algorithm that considers "Mining Units" (MU), which are groupings of blocks within each column based on the minimum draw rate condition and the undercut elevation considered. The formulation considers the following constraints: mining capacity, minimum and maximum draw rate, vertical precedence within columns and a horizontal precedence based on a concave mining advancement front. The model is set on an iterative loop over the different possible levels.

Shuwei has developed and implemented an optimization framework for underground cut-and-fill mining production scheduling using mixed integer linear programming (MILP) in paper 304. The objective function of the model is to maximize the net present value (NPV) of the operation. In addition to the general productivity and sequencing constraints, the use of geotechnical constraints and multiple operational constraints make the MILP model more practical, flexible and expandable to suit a variety of applicable scenarios. The MILP model is verified and validated with two case studies from an existing mine and the results compared with the actual mining strategy. The comparison shows the potential benefits of the model in terms of maximizing NPV and generating a practical and operational long-term cut-and-fill mining schedule.

Zeinab has been working towards the development of a methodology to determine the optimal underground stope layout and production scheduling in sublevel stoping (paper 306). The stopes layout and production scheduling optimization are applied based on the total blocks in the block model altogether (LOT and SOT methods) and based on the separated levels of block model (LOL and SOL methods) to see the impact of leveling in the stopes layout and production scheduling optimization which has not been clarified in the previous researches. The proposed methodologies maximize the economic value (EV) to determine the stopes layout. The presented production scheduling algorithm maximizes the NPV over the life of mine by using the Binary Integer Programming (BIP) while honoring the constraints such as only one-time mining, stopes adjacency, the connection between mining the stopes and activation of levels, concurrent active levels, and the delay between activation of the levels.

Determining stope boundaries is one of the critical steps to be taken when an underground mining method is selected; because of their significant impact on the profitability of the mining project, the stope boundaries have to be optimum to achieve maximum profits. ***Vahid*** has been working on the stope boundary optimization. He has developed a new hybrid algorithm that is a combination of dynamic programming and greedy algorithm (paper 307). Although this proposed algorithm may fail to provide a true optimum solution, it generates better solutions than existing algorithms do. The new proposed algorithm and three existing algorithms are used to find the optimal stope boundaries on a real case ore body. The results demonstrate that the proposed algorithm can improve the profit by 117.78%, 16.86% and 0.42% compared to Floating Stope, Maximum Value Neighborhood (MVN), and Greedy algorithm solutions, respectively, on a real case study at a reasonable CPU time.

Emmanuel in paper 308 presents a literature review that focuses on identifying research opportunities in the areas of underground stope layout/limit and production schedule optimization. Current algorithms and models have been developed separately, for either stope layout/limit optimization or for production scheduling optimization. It is evident that underground stope layout/limit and production scheduling optimization remains a vital area in the underground mining industry and opportunity for researching into improving and obtaining global optimality is still open. His paper reveals that a global look at the underground mine optimization problem in the areas of integrated stope layout/limit and production scheduling, and incorporating stochastic variables such

as grade and price into the optimization framework is imperative to achieving global optimality. This inevitably calls for the formulation of robust and tactical mathematical programming models, capable of solving the problem of global optimality in 3D space by maximizing NPV while satisfying all available technical and production constraints.

Magreth has investigated the effects of fragmentation size distribution on truck-shovel productivity (paper 401). Fragmentation size distribution affects shovel productivity through digability and fill factor and truck productivity through payload, loading time and waiting time at the loader. The reviewed models indicated that shovel productivity depends on P80 (fragment size at which 80 percent of material passes) and uniformity index. Suggested models are based on post-blasting assessment and since they were prepared for fragmentation distribution, they do not consider all factors influencing productivity such as material looseness, swell and equipment parameters. Since productivity is affected by several factors other than fragmentation distribution, its prediction (establishment) requires more involving models. A collective approach to increase productivity and lower operations cost should focus on increasing blasting effectiveness. Blasting should be designed to provide a fragmentation size distribution that ensures the efficiency of downstream processes. This can be done by manipulating blasting set up if the target fragmentation is known.

William in paper 402 presents his research on emergent properties of MoFrac generated discrete fracture networks (DFNs). Discrete fracture networks allow for real structural features to be represented when considering a specific volume of rock. Through the construction of valid fracture network models in combination with blast planning, emergent properties can be identified that are useful for blast optimization. MoFrac discrete fracture network modelling software is used to develop realistic models of fracture networks in selected rock masses that are to be blasted. By including statistically derived stochastic fractures into fracture network models, where mapping has not occurred, a complete representation of the structural discontinuities in a specific volume of rock is achieved. Emergent properties of fracture networks that are useful for characterization have been identified. These properties include in situ block size distributions, orientation of structures with respect to planned free faces, and heterogeneity of the rock mass character. A method to determine these properties is presented and the heterogeneity of these characteristics is considered for two DFN models. A fragmentation model is proposed that incorporates the identified emergent properties of a discrete fracture network model that apply to blast fragmentation optimization by means of a rock engineering systems solution.

Shahrokh has reviewed the state-of-the-art in short-term underground mine planning, which is critical for future mining activities (paper 309). According to the literature, the research is conducted in the categories of mathematical programming optimization and discrete event simulation. Mathematical programming aims at minimizing an objective with the functionality of cost, makespan, deviation from a specific target such as tonnage and fleet management and scheduling. Mining simulation is widely applied in case of short-term mine planning, mostly based on modelling the internal mining process and parameters and on implementing a model for different case studies. In paper 403, he also presents an open-source programming framework for automatic database management for discrete event simulation using Python.

Yashar Pourrahimian
Eugene Ben-Awuah
Hooman Askari-Nasab
September 2018

Annual Research Report Nine
Mining Optimization Laboratory
(MOL) 2017-2018

Table of Contents

Paper	Page	Title
100		**Surface Mine Planning Optimization**
101	1	Clustering mining blocks in presence of Geological uncertainty *Mohammad Tabesh and Hooman Askari-Nasab.*
102	17	A multi-objective multi-stage mining fleet management system linking dynamic operation to short-term plan *Shiv Upadhyay, Hooman Askari-Nasab, and John Doucette.*
103	32	Uncertainty-based mine planning framework for oil sands production scheduling and waste management *Ahlam Maremi and Eugene Ben-Awuah.*
104	66	A transportation problem based stochastic integer programming model to dispatch surface mining trucks under uncertainty *Ali Moradi Afrapoli and Hooman Askari-Nasab*
105	75	Incorporation of geo-metallurgical and geochemical properties in oil sands mine planning and waste management *Obinna Mbadozie and Eugene Ben-Awuah.*
106	105	Integration of mine-to-mill production planning strategy for oil sands mining and waste disposal *Ahlam Maremi and Eugene Ben-Awuah*
200		**Simulation & Optimization of Mining Systems**
201	131	A fuzzy logic approach towards truck dispatching problem in surface mines *Ali Moradi Afrapoli, Shiv Upadhyay, and Hooman Askari-Nasab.*
202	142	Improvements to production planning in oil sands mining through analysis and simulation of truck cycle time *Eduardo Cervantes, Shiv Upadhyay, and Hooman Askari-Nasab.*
300		**Underground Mine planning and Optimization**
301	157	Optimizing the extraction strategy for open pit and underground mining transitions using mathematical programming *Bright Oppong Afum and Eugene Ben-Awuah.*

302 179 Application of mathematical modelling for draw control under material flow uncertainty
Firouz khodayari and Yashar Pourrahimian.

303 186 Production scheduling with horizontal mixing simulation in block cave mining
Firouz Khodayari and Yashar Pourrahimian.

304 205 An optimization framework for cut-and-fill mining production scheduling using mixed integer linear programming
Shuwei Huang and Eugene Ben-Awuah

305 224 Determination of optimum undercut level elevation in block caving
Roberto Noriega and Yashar Pourrahimian.

306 235 Application of mathematical programming for stope layout and production schedule optimization in sublevel stoping
Zeinab Basiri and Yashar Pourrahimian.

307 257 Stope boundary optimization using a 3D approximate hybrid algorithm
Vahid Nikbin, Majid Ataee-pour, Kourosh Shahriar, and Yashar Pourrahimian.

308 271 Underground mining stope layout optimization and production scheduling: A review of existing solvers and algorithms
Emmanuel John Andrew Appianing and Eugene Ben-Awuah

309 305 Short-term underground mine planning: A review
Shahrokh Paravarzar, Yashar Pourrahimian, and Hooman Askari-Nasab

400 **Miscellaneous**

401 335 Effects of fragmentation size distribution on truck-shovel productivity
Magreth Dotto and Yashar Pourrahimian.

402 343 Emergent properties of MoFrac generated discrete fracture networks
William Junkin and Eugene Ben-Awuah

403 362 Open source programming (OSP) framework for automatic database management for discrete event simulation
Shahrokh Paravazar, Mohammad Tabesh, Yashar Pourrahimian, and Hooman Askari-Nasab

Mining Optimization Laboratory (MOL) Researchers / Graduate Students

Following are researchers and students affiliated with Mining Optimization Laboratory in September 2018.

1.	**Hooman Askari-Nasab**	Professor - Director of MOL
2.	**Yashar Pourrahimian**	Assistant Professor, University of Alberta
3.	**Eugene Ben-Awuah**	Associate Professor, Laurentian University
4.	**Kwame Awuah-Offei**	Associate Professor, Missouri S&T

5.	**Mohammad Tabesh**	PhD, Research Associate – 2015/09
6.	**Shiv Prakash Upadhyay**	PhD, Research Associate – 2016/09
7.	**Saleh Balideh**	PhD, Research Associate – 2016/09
8.	**Mahdi Badiozaman Tari**	PhD, Research Associate – 2017/12
9.	**Firouz Khodayari**	PhD Student – 2014/01
10.	**Ali Moradi Afrapoli**	PhD Student – 2014/09
11.	**Magreth Dotto**	PhD Student – 2018/01
12.	**Ahlam Maremi**	PhD Student – 2015/09
13.	**Bright Oppong Afum**	PhD Student – 2017/01
14.	**Emmanuel Appianing**	PhD Student – 2018/05
15.	**Nasib Al-Habib**	PhD Student – 2018/09
16.	**Alireza Kamrani**	PhD Student – 2018/09
17.	**Shuwei Huang**	PhD Visiting Student – 2017/10
18.	**Shahrokh Parvarzar**	PhD Visiting Student – 2018/03
19.	**Efrain Ugarte-Zarate**	MSc Student (CCG) – 2015/09
20.	**Zeinab Basiri**	MSc Student – 2016/09
21.	**Roberto Noriega**	MSc Student – 2017/09
22.	**Hongshuo Gong**	MSc Student – 2017/09

23. **Obinna Mbadozie** MSc Student – 2018/01

24. W**illiam Junkin** MSc Student – 2018/01

Clustering Mining Blocks in Presence of Geological Uncertainty

Mohammad Tabesh and Hooman Askari-Nasab
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

A major trend in mine production planning research is incorporating geological uncertainty in the processes of planning. Many mathematical models and heuristic approaches are proposed to deal with the uncertainty. However, incorporating geological uncertainty in planning, usually through Geostatistical realizations, significantly increases the problem sizes and forces researchers to use heuristics or aggregation techniques to be able to obtain solutions in reasonable times. In this paper, we present four variations of agglomerative hierarchical clustering algorithm, one based on deterministic estimates of properties and three based on possible worlds approach which use Geostatistical realizations to form aggregates with regard to the geological properties and the existing uncertainties. We show, through case studies, that uncertainty based algorithms can result in aggregates that are less susceptible to uncertainties, and at the same time, the proposed algorithm can produce aggregates that are within a controlled size and have minable shapes.

1. Introduction

Open pit mine production scheduling problem (OPMPSP) is a complicated procedure which has attracted many researchers in the past decades. Many of the research is centred around formulating the problem using mixed integer linear programming and solving it through exact and heuristics methods. Bienstock and Zuckerberg (2010) proposed a technique (BZ) to solve the LP-relaxation of large instances of the OPMPSP in a few minutes. Similarly, Chicoisne, Espinoza, Goycoolea, Moreno, and Rubio (2012) proposed a topological sort algorithm to solve the LP-relaxation in a reasonable time. This encouraged many other researchers to use the algorithms to obtain the LP-relaxation solution and use heuristics to obtain integer solutions. Samavati, Essam, Nehring, and Sarker (2017), Samavati, Essam, Nehring, and Sarker (2017), Liu and Kozan (2016), Lamghari, Dimitrakopoulos, and Ferland (2015) can be named among those who took advantage of these advancements and developed heuristic solution techniques. However, most of the work in the literature is limited to the production planning problem in the absence of grade blending and stockpiling constraints as they change the structure of the model. Kumral (2011) propose a two stage modelling approach to incorporate geological uncertainty in production. Their approach aims at maximizing the minimum NPV using multiple geological and price uncertainty scenarios. However, the authors implement the approach on a very small block model with only 3,115 blocks in the final pit and do not provide processing times. Solving such model on a real-size block containing hundreds of thousands of blocks will not be possible.

On the other hand, when it comes to incorporating uncertainty in production planning, most of the researchers have to use heuristics to obtain good solutions. (e.g. Lamghari and Dimitrakopoulos (2016), Montiel and Dimitrakopoulos (2015)). Another approach that can help solve the production planning problem with presence of grade blending, stockpiling and uncertainty is to use

aggregation techniques to reduce the size of the problem and increase the practicality of the generated solutions.

Boland, Dumitrescu, Froyland, and Gleixner (2009) propose a solution procedure based on aggregation and disaggregation. In their work, excavation decisions are made based on some given aggregates through an MIP. Afterwards, disaggregation techniques are proposed to enable the model make processing decisions in block level by introducing the concept of bins. Jélvez, Morales, Nancel-Penard, Peypouquet, and Reyes (2016) propose an aggregation-disaggregation technique to solve the OPMPSP. Their technique starts by aggregating (reblocking) the model into larger units. Then, they use a sliding time window heuristic to solve the OPMPSP for the aggregated block model and use the solution to categorize the aggregates into two groups. The first group consists of the blocks which all their direct predecessor and successor blocks are extracted in the same period. The rest are called border blocks. Afterwards, they fix the variables corresponding to non-border blocks, disaggregate the border blocks and solve the problem in disaggregated level. The authors implement their technique on MineLib (Espinoza et al., 2013) datasets and show that their technique can solve all the instances to near-optimality in a reasonable time period.

Goodfellow and Dimitrakopoulos (2016) propose a stochastic formulation for OPMPSP with uncertainty. Their model aims at optimizing the production schedule and downstream processes simultaneously. The authors incorporate uncertainty by minimizing deviations from target while maximizing the NPV. First, they use k-means clustering to group blocks into clusters with similar grades. Next, they formulate a non-linear mathematical formulation to maximize the NPV of the operation with respect to various constraints on capacities, grade control and stockpiling. Since the model is non-linear they use three different meta-heuristic algorithms to solve it. They implement their technique on a gold-copper deposit with one stockpile and six processing streams. They first solve the deterministic case with the three algorithms and then apply the same strategy to the stochastic model. They conclude that incorporating uncertainty in decision making results in higher NPV and less risk.

Although it is possible to cluster blocks using the expected attributes and use the clusters in production planning in presence of uncertainty, as done by Goodfellow and Dimitrakopoulos (2016), it is a major step forward to consider the uncertainty in attributes at the clustering stage. The literature in computing science shows good potential in treating object (block) attributes as uncertain variables and perform clustering with respect to this uncertainty. It has been shown that the outcome of such clustering technique is more reliable compared to the ones based solely on expected values or averages (Aggarwal, 2007). There are various methods for clustering uncertain objects proposed in the computing science literature. However, applying such techniques to blocks in a mining operation requires significant modifications and innovations. A clustering algorithm for mining blocks requires control over size, shape and number of created clusters, unlike general purpose clustering algorithms where the ultimate goal is to find the underlying similarities and dissimilarities between the data points. Moreover, techniques such as information-theoretic approach (Gullo et al., 2008), mixture model clustering (Hamdan and Govaert, 2005), FOPTICS (Kriegel and Pfeifle, 2005), FDBSCAN (Donghua and Lilei, 2011) and UK-means (Aggarwal et al., 2016) can be applied to blocks ignoring the strong dependency between attributes of one block and the adjacent one. However, the geographical dependency of block attribute distribution prohibits us from using these algorithms as they share the independency assumption. Therefore, another approach called possible worlds (Volk et al., 2009) has been chosen in this paper. Combining this approach with the deterministic approach in Tabesh, Mieth, and Askari–Nasab (2014) resulted in a clustering algorithm that, using sequential Gaussian simulation realizations, provides aggregates that incorporate uncertainty of block attributes and can be used in future research on open pit and underground mining simulation and production planning in presence of uncertainty.

2. Theoretical framework

Clustering is the process of grouping similar objects together using measures of similarity and dissimilarity. The similarity of mining blocks is usually defined using the grade and rock type attributes of the blocks. However, these attributes are estimated based on the drillhole data collected and include uncertainty.

The possible worlds approach as proposed by Volk, et al. (2009) starts by using a value generator function (VG-function) to generate the possible worlds. In a mining block model, due to the geographical dependencies between the rock type and grades of the of the blocks, the best generator function to use is the Sequential Gaussian Simulation (SGS) (Rossi and Deutsch, 2014). The next step is to apply any clustering algorithm on every possible world and obtain a clustering model for every world. The clustering processes are independent, and therefore, they can run on parallel processes. Finally, an aggregation technique is required to derive the final clustering model from the generated clustering models for every possible world.

In this paper, we propose four variations of the agglomerative hierarchical clustering algorithm with shape and size control (AHCA) from Tabesh, et al. (2014). The first algorithm uses the expected values of parameters and runs the AHCA once to obtain the clustering model. The second algorithm runs the AHCA on every realization obtained from SGS and aggregates blocks into clusters if they happen to be grouped together in more than 50% of the single realization cluster models. The third algorithm runs the AHCA on every realization and uses the frequency of two blocks being grouped together as the similarity between the blocks and runs AHCA again to form the final clusters. The forth variation uses k-means to cluster every realization and aggregates the results using AHCA.

First three variations run the AHCA using the similarity index calculated based on equation (1). The equation is a simplified version of the original similarity index defined in M. Tabesh and Askari-Nasab (2013) and is divided into two parts for computation economics. The first part is the distance measure which is independent of the realizations and does not vary by sampled values of grade and rock type. The second part, however, is calculated for every realization in the realization based variations of the algorithm. $\tilde{D}_{ij}$ represents the normalized Euclidean distance between blocks i and j (equation (1)) and $\tilde{G}_{ij}^{k}$ represents the normalized difference between the grade values of blocks i and j in realization k. Although, a block model may have multiple elements, the formulation here, without losing generality, is presented for one major element grade only. Moreover, we can assign weights to these parameters and include more parameters in the similarity index definition. The effects of weights and other parameters are studied in Tabesh, et al. (2014) and Tabesh and Askari-Nasab (2013) and they are removed from this paper for brevity. R_{ij}^{k} is calculated based on equation (5), which considers a penalty value of $r \in [0,1]$ if the two blocks have different rock types and 1, if the blocks are of the same rock type in realization k. Finally, we need to determine the adjacency between the block using an adjacency threshold as in equation (6) in order to avoid forming fragmented clusters.

$$\tilde{D}_{ij} = \frac{D_{ij}}{\max(D_{ij})} \tag{1}$$

$$S_{ij} = \frac{1}{\tilde{D}_{ij}} \tag{2}$$

$$S_{ij}^{k} = \frac{R_{ij}^{k}}{\tilde{G}_{ij}^{k}} \tag{3}$$

$$S_{ij}^{k} = \frac{1}{\tilde{D}_{ij}} \times \frac{R_{ij}^{k}}{\tilde{G}_{ij}^{k}} \tag{4}$$

$$R_{ij}^{k} = \begin{cases} 1 \; if \; blocks \, i \, and \; j \, are \; from \, the \, same \, rock \, type \\ r \qquad\qquad\qquad\qquad\qquad otherwise \end{cases} \tag{5}$$

$$A_{ij} = (D_{ij} < AdjacencyTreshold) \tag{6}$$

2.1. Algorithm 1 - Expected value algorithm (EVA)

This algorithm is very similar to the AHCA proposed in Tabesh, et al. (2014). Since the only uncertain parameters are rock type and grade we will determine the expected values of these attributes first and use them in forming the clustering model. The grade attributes are numerical, and therefore, we can use arithmetic average of all the realizations to determine the expected values. However, rock types are categorical attributes and arithmetic averages cannot be applied. Hence, we set the most frequent rock type from all the realizations as the expected rock type of each block. Since the realizations are preprocessed and we have assigned single values to grade and rock type, we only need to calculate the similarity index in equation (1) once and use it to run the AHCA. The resulted clustering model will be the final clustering scheme. Note that this algorithm is not based on possible worlds and is presented as a basis for comparison. The pseudo code of the algorithm is presented in Table 1.

Table 1.Expected Value Clustering Pseudo Code

FOR EACH *i* **in** Blocks
  FOR EACH *j* **in** Blocks
    $S_{ij} = \frac{1}{\tilde{D}_{ij}} \times \frac{R_{ij}}{\tilde{G}_{ij}}$
    $A_{ij} = (D_{ij} < AdjacencyTreshold)$
  NEXT
NEXT
FUNCTION Clusters = *Clustering(S,A)*
  NumClusters = NumBlocks
  WHILE NumClusters > DesiredNumofClusters
    $(i, j) = Max(S)$
    IF *length*(Clusters(*i*)) + *length*(Clusters(*j*)) <= MaxClusterSize **THEN**
      $S_{i:} = Min(S_{i:}, S_{j:})$
      $S_{j:} = 0$
      $A_{i:} = Max(A_{i:}, A_{j:})$
      $A_{j:} = 0$
      Clusters(*i*) = Clusters(*i*) + Clusters(*j*)
      NumClusters = NumClusters – 1
    ELSE
      $A_{ij} = 0$
    ENDIF
  ENDWHILE

Clusters = *ShapeRefinementProcedure(*Clusters*)*
END FUNCTION

2.2. Algorithm 2 - Simple realization aggregation (SRA)

As explained earlier, possible world clustering works by sampling possible worlds from the uncertain data using a generator function, clustering every sampled set and aggregating the generated clustering models. Both following algorithms start by running the AHCA on every realization and obtain one clustering model per realization. The difference between the two algorithms is in the aggregation step. The aggregation step in the SRA algorithm is summarized in equations (7) and (8). First, we calculate the common cluster matrix (C_{ij}^k) for every realization cluster model. Next, the common cluster values are summed up over all the realizations and we calculate the frequency of two blocks being in the same cluster (F_{ij}). Finally, every pair of blocks with $F_{ij} \geq 0.5$ are grouped together in the final clustering model. The pseudo code of the algorithm is presented in Table 2.

$$C_{ij}^k = \begin{cases} 1 & if\ blocks\ i\ and\ j\ are\ from\ the\ same\ cluster \\ 0 & otherwise \end{cases} \tag{7}$$

$$F_{ij} = \frac{\sum_{r=1}^{R} C_{ij}^k}{R} \tag{8}$$

Table 2.Simple Realization Aggregation Pseudo Code

FOR EACH *k* **in** Realizations
  FOR EACH *i* **in** Blocks
    FOR EACH *j* **in** Blocks
      $S_{ij}^k = \frac{1}{\tilde{D}_{ij}} \times \frac{R_{ij}^k}{\tilde{G}_{ij}^k}$
      $A_{ij} = (D_{ij} < AdjacencyTreshold)$
    NEXT
  NEXT
  $Clusters^k = Clustering\,(S^k, A)$
  FOR EACH *i* **in** Blocks
    FOR EACH *j* **in** Blocks
      $C_{ij}^k = \begin{cases} 1 & if\ blocks\ i\ and\ j\ are\ from\ the\ same\ cluster \\ 0 & otherwise \end{cases}$
    NEXT
  NEXT
NEXT
FOR EACH *i* **in** Blocks
  FOR EACH *j* **in** Blocks

$$F_{ij} = \frac{\sum_{r=1}^{R} C_{ij}^{k}}{R}$$

NEXT
NEXT
FOR EACH *i* **in** Blocks

$$Clusters(i) = F_{i:} \geq 0.5$$

NEXT

2.3. Algorithm 3 - Hierarchical realization aggregation (HRA)

In this algorithm, we run the AHCA on every realization and calculate C_{ij}^{k} and F_{ij} as explained earlier in (7) and (8). Afterwards, the F_{ij} values are used as similarity indices in the original AHCA instead of S_{ij} and the algorithm is run using the same settings as the original AHCA used on every realization. The pseudo code of the algorithm is presented in Table 3.

Table 3.Hierachical Realization Aggregation Pseudo Code

FOR EACH *k* **in** Realizations
FOR EACH *i* **in** Blocks
FOR EACH *j* **in** Blocks

$$S_{ij}^{k} = \frac{1}{\tilde{D}_{ij}} \times \frac{R_{ij}^{k}}{\tilde{G}_{ij}^{k}}$$

$$A_{ij} = (D_{ij} < AdjacencyTreshold)$$

NEXT
NEXT

$$Clusters^{k} = Clustering(S^{k}, A)$$

FOR EACH *i* **in** Blocks
FOR EACH *j* **in** Blocks

$$C_{ij}^{k} = \begin{cases} 1 & if\ blocks\ i\ and\ j\ are\ from\ the\ same\ cluster \\ 0 & otherwise \end{cases}$$

NEXT
NEXT
NEXT
FOR EACH *i* **in** Blocks
FOR EACH *j* **in** Blocks

$$F_{ij} = \frac{\sum_{r=1}^{R} C_{ij}^{k}}{R}$$

NEXT
NEXT

$$Clusters = Clustering(F, A)$$

2.4. Algorithm 4 - K-Means with Hierarchical realization aggregation (KHRA)

K-means is a famous clustering algorithm which is known to be faster but less accurate than hierarchical clustering (Feng et al., 2010). Moreover, k-means, in its original form, is not designed to deal with categorical variables such as rock type (Mastrogiannis et al., 2009) and it does not provide control over shape and size of the generated clusters. Since block models have a mixture of numerical and categorical attributes we have modified the k-means algorithm to be able to apply it on block models.

In order to apply k-means clustering we first need to create the feature matrix. We use the same set of attributes, weights and penalties as AHCA to be able to compare the results. The first two columns of the feature matrix are X and Y coordinates of the blocks on each bench. The coordinates are powered to the distance weight and normalized by dividing all the values by the maximum values of each column. Similarly, the major element grade values are powered to the grade weight, normalized and added as the third column. Next, in order to add rock types to the feature matrix, we add as many columns as the number of categories i.e. number of rock types. If a block is of rock type *i*, the i^{th} column will get a value of *1-r* (one minus rock type penalty). After preparing the feature matrix for every realization, k-means clustering is performed and C_{ij}^{k} and F_{ij} are calculated as explained earlier in (7) and (8). Finally, the F_{ij} values are used as similarity indices in AHCA to aggregate the clustering results and form the final clustering scheme. The pseudo code of the algorithm is presented in Table 4.

Table 4. K-Means with Hierachical Realization Aggregation Pseudo Code

FOR EACH *k* **in** Realizations
  S^k = *BuildFeatureMatrix()*
  $Clusters^k = KMeans(S^k)$
  FOR EACH *i* **in** Blocks
    FOR EACH *j* **in** Blocks

$$C_{ij}^{k} = \begin{cases} 1 & \text{if blocks } i \text{ and } j \text{ are from the same cluster} \\ 0 & \text{otherwise} \end{cases}$$

    NEXT
  NEXT
NEXT
FOR EACH *i* **in** Blocks
  FOR EACH *j* **in** Blocks

$$F_{ij} = \frac{\sum_{r=1}^{R} C_{ij}^{k}}{R}$$

  NEXT
NEXT
$Clusters = Clustering(F, A)$

2.5. Result evaluation

Since the goal of block clustering is to create mining polygons which are homogenous in grade, rock type and destination, Tabesh and Askari-Nasab (2013) define three performance measures for a block clustering algorithm: rock unity (RU), destination dilution factor (DDF) and grade coefficient of variation (CV). Moreover, the generated clusters will be used as planning units, and

thus, need to be of controlled size and shape. In addition, since all the created Geostatistical realizations reproduce the spatial continuity of the variables and have equal probability of occurrence (Rossi and Deutsch, 2014), a good clustering scheme is the one that produces better results over more realizations. Therefore, we evaluate every clustering result by looking at the average and standard deviations of the three performance measures, introduced by Tabesh and Askari-Nasab (2013), over all the realizations. In other words, after obtaining a clustering scheme from every algorithm we apply the scheme to every realization and calculate the performance measures. Moreover, we look at the cluster size variations and cluster shapes resulted from every algorithm.

3. Case studies

In order to evaluate the proposed algorithms we implement them on two case studies provided by Malaki, Khodayari, Pourrahimian, and Liu (2017) and Cabral Pinto (2016). The first dataset is a copper deposit with three different rock types. The dataset block dimensions are $10\times10\times10$ meters and there are a total of 189,000 blocks in the model. The dataset contains 20 realizations of rock types and element grade estimates created using SGS (Malaki et al., 2017). The second dataset is a multi element dataset where the element names and coordinates are distorted for confidentiality. The block model dimensions are $300\times300\times20$ feet and there are a total of 570,770 blocks in 176 benches. The element grades and rock types are estimated from the drill hole data and 100 realizations of geological uncertainty are created through SGS. The case studies are tested on machine with Intel® Xeon® CPU with eight cores with 2.8 GHz speed and 24 GB of ram.

3.1. Case study 1

The first case study is a Copper deposit borrowed from Malaki, et al. (2017). Although the dataset is a block-caving block model, the results can be generalized to other open-pit and underground methods that follow the same procedure of creating block model realizations and grouping the blocks prior to planning. A 3D presentation of the ore body is shown in Fig. 1.

Every bench of the block model contains 7,200 blocks which are clustered using the three algorithms. The first algorithm uses the expected values for rock type and grade while the other algorithms use every realization in the block model. Sample plan views of expected rock type (Fig. 2), expected copper grade (Fig. 3), realization 1 rock type (Fig. 4) and realization 1 copper grade (Fig. 5) are presented here to provide a sense of attribute variations within the block model and between the realizations.

In order to have simple and comparable results, the four algorithms are implemented using the same set of weights and control parameters as summarized in Table 5. The first group of runs are set to aggregate 30 blocks into one cluster and the second group are set to aggregate 60 blocks into one cluster with maximums set to 35 and 70 respectively. Minimum cluster size is not applied and the number of shape refinement iterations is set to 3 for all the runs. Afterwards, the three algorithms are run and the resulted clustering scheme is applied to every realization of the block model. Since all the realizations are equiprobable, we look at the average values of grade CV and rock unity among all the realizations. Destination dilution factor is omitted as blocks do not have a predetermined destination assigned to them.

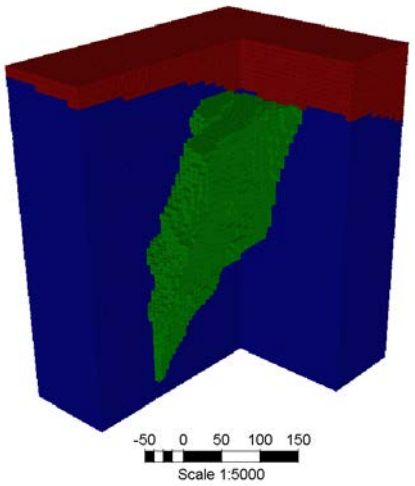


Fig. 1 3D presentation of the ore body

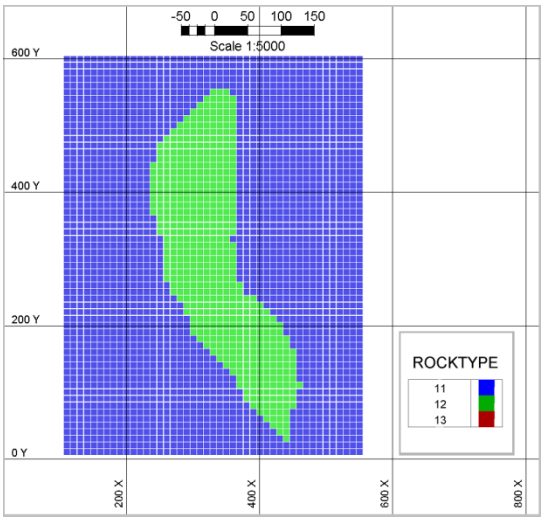


Fig. 2 Expected rock type at level 40

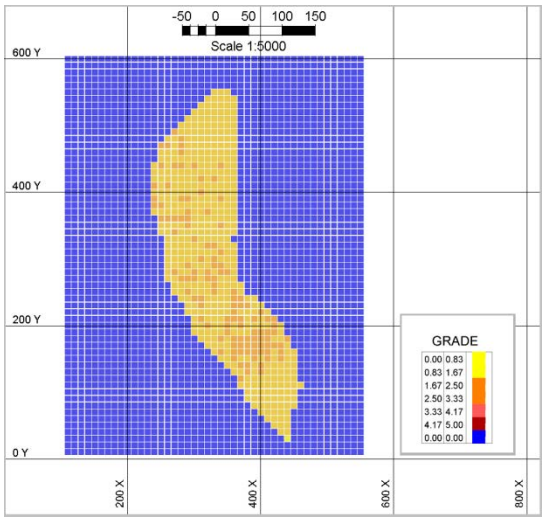


Fig. 3 Expected grade at level 40

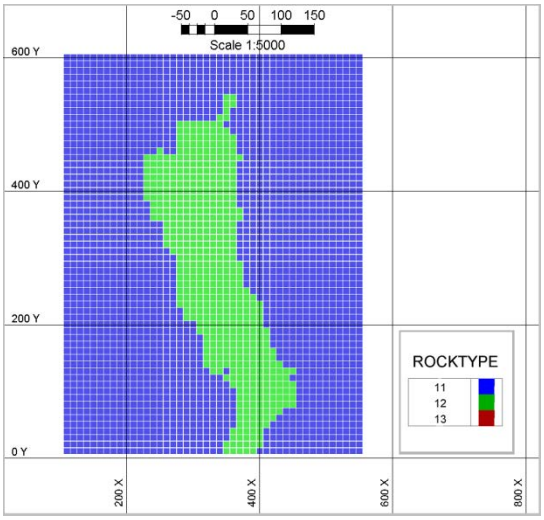


Fig. 4 Realization 1 rock type at level 40

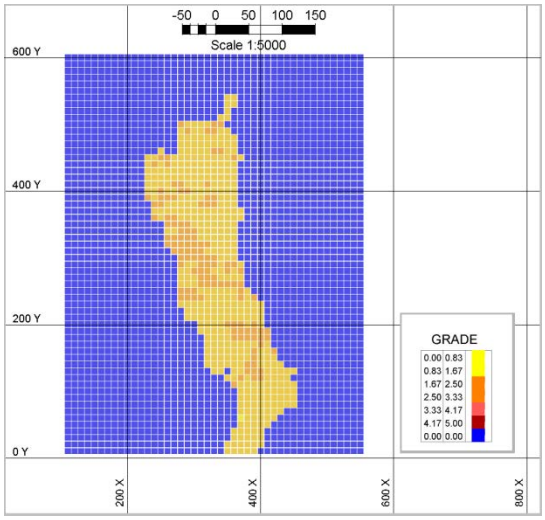


Fig. 5 Realization 1 grade at level 40

Table 5.Case study 1 clustering runs settings

ID	Algorithm	Avg. Block per cluster	Max. Block per cluster	Shape Refin. Iter.
1-EVA-30-35	Expected	30	35	3
1-SRA-30-35	SRA	30	35	3
1-HRA-30-35	HRA	30	35	3
1-KHRA-30-35	KHRA	30	35	3
1-EVA-60-70	Expected	60	70	3
1-SRA-60-70	SRA	60	70	3
1-HRA-60-70	HRA	60	70	3
1-KHRA-60-70	KHRA	60	70	3

The clustering performance measures are presented in Table 6. The first column presents the average copper grade CV over the realizations. A lower grade CV means a better clustering scheme since it illustrates the variation of block grades within a cluster. In contrast, rock unity measures the homogeneity of block rock types within a cluster and a larger value means a better clustering scheme. We are testing the algorithms on two different size settings. The question of the proper size of clusters is thoroughly discussed in Tabesh and Askari-Nasab (2013) and the goal of this case study is to compare the proposed algorithms. According to Table 6, we can conclude that the SRA is performing best while EVA is performing worst. However, a major requirement in clustering blocks in a mining operation is to have clusters of the same size to able to use them as planning units. In this sense, SRA results are significantly poorer than the other algorithms when comparing the size variations. Moreover, comparing the shapes of the generated clusters show that clusters created using SRA are not minable and cannot be used as units of production planning. The sample plan views in Fig. 6 and Fig. 7 show the difference between the resulted clusters from SRA and HRA respectively. EVA and KHRA results have cluster shapes similar to HRA and are presented in Fig. 8 and Fig. 9 respectively. In addition to unacceptable size and shape properties of SRA results, looking at the realization by realization statistics shows that EVA, HRA and KHRA results are more consistent over different realizations while the SRA is varying significantly. Fig. 10 shows the variations in average cluster grade CVs over different realizations and Fig. 11 shows the variations in average cluster rock unities. The two Fig.s show that EVA, HRA and KHRA follow the same patterns with KHRA performing better over all the realizations. In contrast, SRA performance, especially regarding grade variations, is highly sensitive to variations in realizations. In addition to better results, KHRA is significantly faster which can be an important algorithm selection criterion even on a small scale case study as presented here.

Table 6.Case study 1 clustering Results

ID	Average Grade CV	Average Rock Unity	Cluster Size CV	Time (min)
1-EVA-30-35	41.3%	95.7%	20.8%	14
1-SRA-30-35	24.1%	98.6%	357.7%	171
1-HRA-30-35	34.9%	96.4%	32.3%	181
1-KHRA-30-35	28.6%	97.0%	35.6%	13
1-EVA-60-70	75.3%	95.1%	21.4%	13
1-SRA-60-70	50.7%	97.9%	245.4%	167
1-HRA-60-70	63.3%	95.9%	35.3%	176
1-KHRA-60-70	54.2%	96.3%	38.5%	13

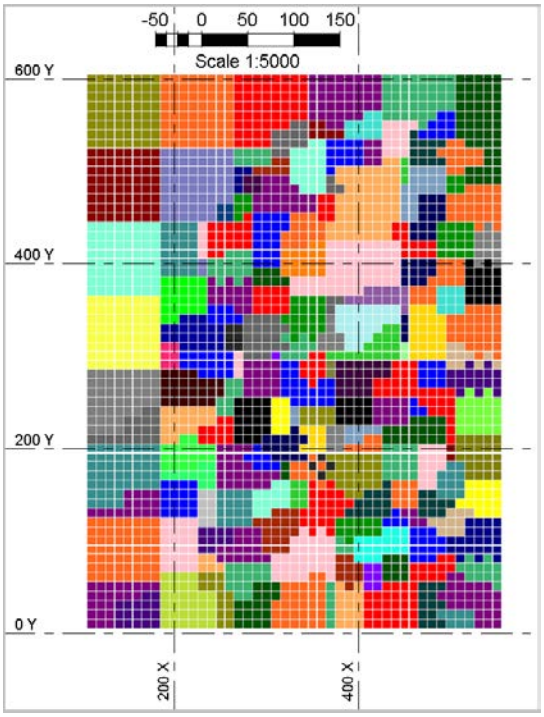


Fig. 6 1-SRA-60-70 results at level 40

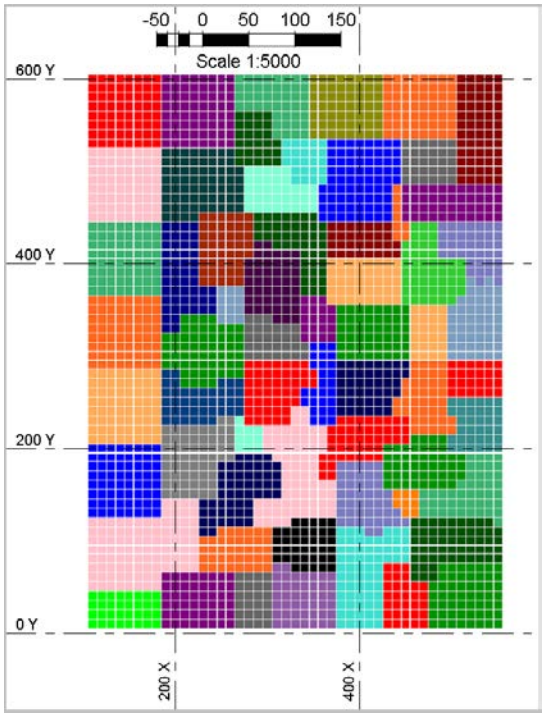


Fig. 7 1-HRA-60-70 results at level 40

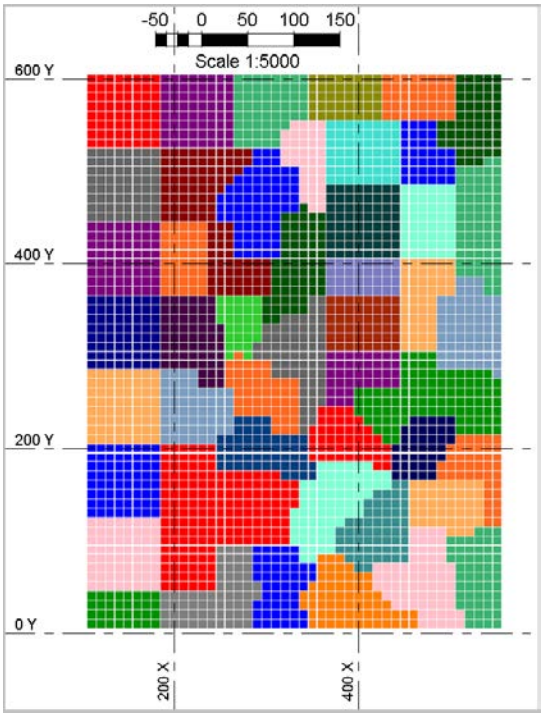


Fig. 8 1-EVA-60-70 results at level 40

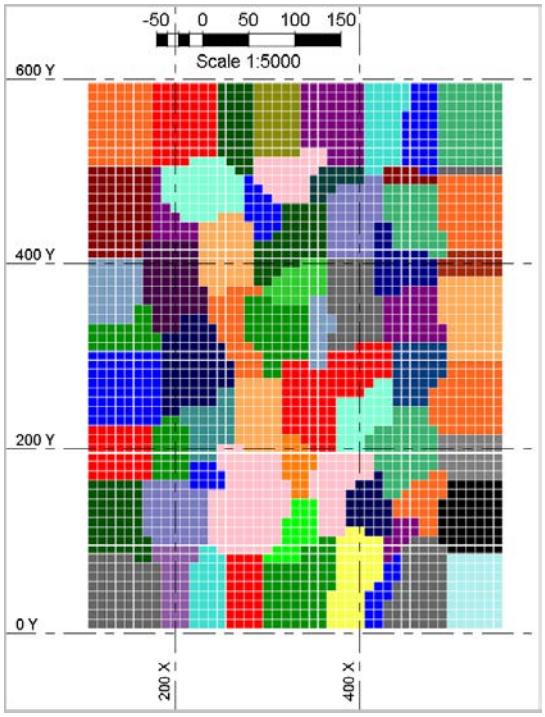


Fig. 9 1-KHRA-60-70 results at level 40

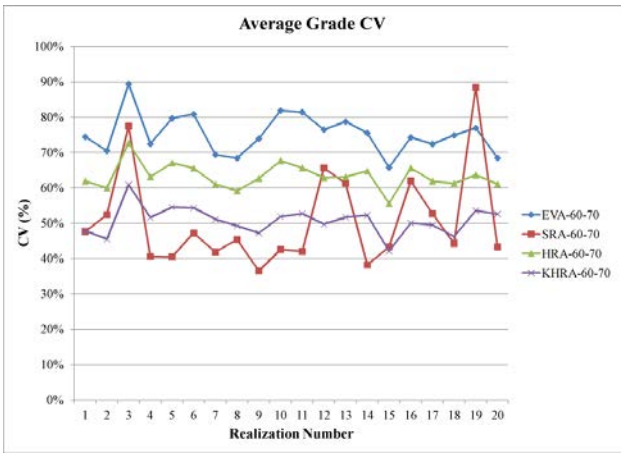


Fig. 10 Average grade CV over realizations

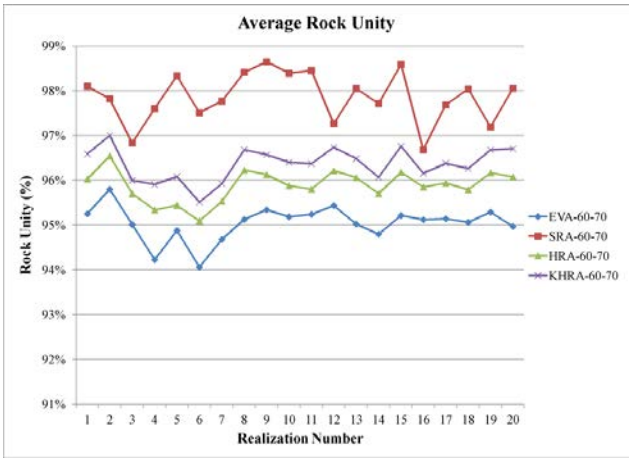


Fig. 11 Average rock unity over realizations

3.2. Case study 2

The second case study is borrowed from Cabral Pinto (2016) with three elements and nine rock types. The element and rock type names are masked for confidentiality as well as the block coordinates and dimensions. However, the large number of blocks and realizations in the block model make it a good measure for evaluating the performance of the proposed algorithms. We applied the same clustering settings as the first case study except we did not perform the SRA settings as it was shown that the results would not be acceptable and they would take a long time to process. The clustering settings for the second case study are summarized in Table 7.

Table 7.Case study 2 clustering runs settings

ID	Algorithm	Avg. Block per cluster	Max. Block per cluster	Shape Refin. Iter.
2-EVA-30-35	Expected	30	35	3
2-HRA-30-35	HRA	30	35	3
2-KHRA-30-35	KHRA	30	35	3
2-EVA-60-70	Expected	60	70	3
2-HRA-60-70	HRA	60	70	3
2-KHRA-60-70	KHRA	60	70	3

As can be seen in Fig. 12 this deposit is more variable in rock types. Fig. 13 and Fig. 14 are two sample plan views of expected grade and rock type distribution in the deposit. The clustering

results of HRA and KHRA for the same plan view are presented in Fig. 15 and Fig. 16 for comparison. The summary of running the clustering algorithms on the second case study is presented in Table 8. Similar to the previous case study, KHRA is performing better regarding grade variation and it performs slightly worse regarding rock unity compared to HRA. However, considering the processing time required for running HRA, KHRA is the more reasonable choice of algorithm. Finally, it can be seen that both uncertainty based algorithms outperform the deterministic algorithm over multiple realizations.

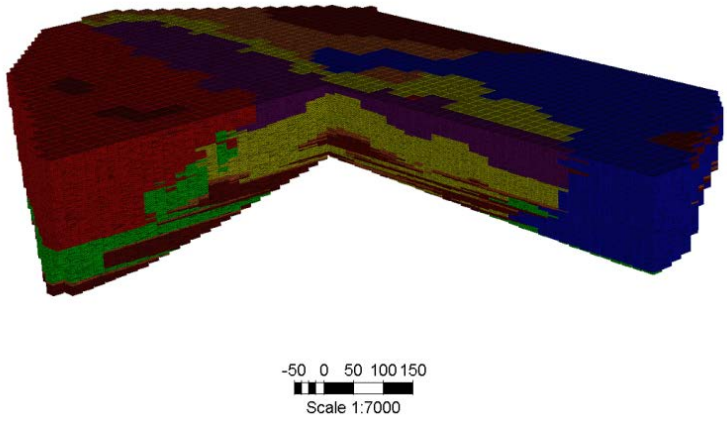


Fig. 12 3D presentation of the ore body

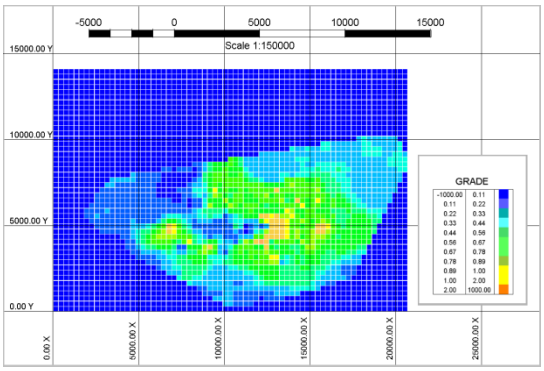


Fig. 13 Expected rock type at level 40

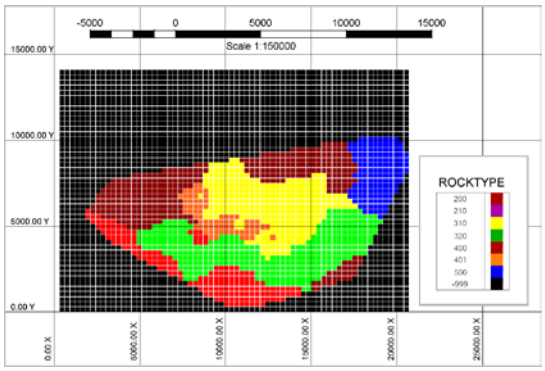


Fig. 14 Expected grade at level 40

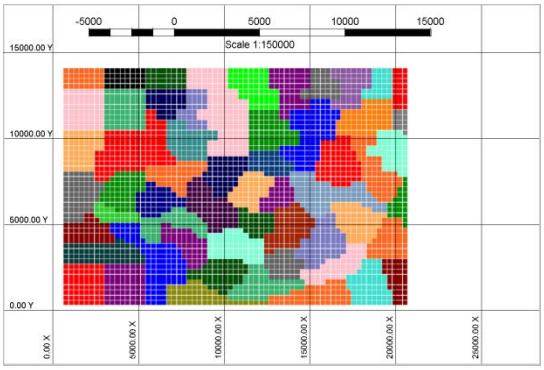


Fig. 15 2-HRA-60-70 results at level 40

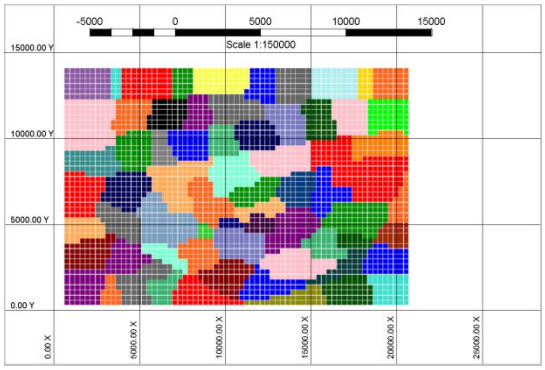


Fig. 16 2-KHRA-60-70 results at level 40

Table 8.Case study 2 clustering Results

ID	Average Grade CV	Average Rock Unity	Cluster Size CV	Time (min)
2-EVA-30-35	34.8%	93.5%	24.0%	54
2-HRA-30-35	35.5%	93.7%	23.6%	3,452
2-KHRA-30-35	27.8%	95.8%	30.1%	70
2-EVA-60-70	38.7%	92.8%	23.8%	53
2-HRA-60-70	39.5%	93.1%	23.7%	3,363
2-KHRA-60-70	29.6%	95.5%	33.4%	69

4. Conclusion

In this paper we presented a hierarchical agglomerative clustering technique based on possible worlds uncertainty approach to aggregate mining blocks into larger units in a mining operation. The clustering technique deals with geological uncertainties in rock type and grade estimates. The first step is to create the possible worlds using SGS realizations as the generator function. Next, we proposed four different algorithms that use these realizations and form one clustering scheme to apply to the block model. Afterwards, we defined quality measures to evaluate the result of each algorithm and choose the one that provides the best results. Finally, we implemented the algorithms on two case studies and showed how considering uncertainties in clustering can improve the results and provide clustering schemes that have higher quality measures over more possible realizations. Moreover, we showed that the hybrid algorithm using k-means and hierarchical aggregation can be implemented on real-size block models in a reasonable processing time to create reliable clustering schemes in presence of geological uncertainty.

5. References

[1] Aggarwal, C. C. (2007). *On Density Based Transforms for Uncertain Data Mining*. in Proceedings of 2007 IEEE 23rd International Conference on Data Engineering, pp. 866-875.

[2] Aggarwal, S., Agarwal, N., and Jain, M. (2016). *Uncertain data mining: A review of optimization methods for UK-means*. in Proceedings of 2016 3rd International Conference on Computing for Sustainable Global Development (INDIACom), pp. 3672-3677.

[3] Bienstock, D. and Zuckerberg, M. (2010). Solving LP Relaxations of Large-Scale Precedence Constrained Problems. in *Integer Programming and Combinatorial Optimization: 14th International Conference, IPCO 2010, Lausanne, Switzerland, June 9-11, 2010. Proceedings*, F. Eisenbrand and F. B. Shepherd, Eds., Berlin, Heidelberg, Springer Berlin Heidelberg, pp. 1-14.

[4] Boland, N., Dumitrescu, I., Froyland, G., and Gleixner, A. M. (2009). LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity. *Computers & Operations Research, 36* (4), 1064-1089.

[5] Cabral Pinto, F. A. (2016). Advances in data spacing and uncertainty.Thesis, University of Alberta, Edmonton, Pages 177.

[6] Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno, E., and Rubio, E. (2012). A New Algorithm for the Open-Pit Mine Production Scheduling Problem. *Operations Research, 60* (3), 517-528.

[7] Donghua, P. and Lilei, Z. (2011). *Uncertain data cluster based on DBSCAN*. in Proceedings of 2011 International Conference on Multimedia Technology, pp. 3781-3784.

[8] Espinoza, D., Goycoolea, M., Moreno, E., and Newman, A. (2013). MineLib: a library of open pit mining problems. *Annals of Operations Research, 206* (1), 93-114.

[9] Feng, L., Qiu, M.-H., Wang, Y.-X., Xiang, Q.-L., Yang, Y.-F., and Liu, K. (2010). A fast divisive clustering algorithm using an improved discrete particle swarm optimizer *Pattern Recogn. Lett. , 31* (11), 1216-1225

[10] Goodfellow, R. C. and Dimitrakopoulos, R. (2016). Global optimization of open pit mining complexes with uncertainty. *Applied Soft Computing, 40* 292-304.

[11] Gullo, F., Ponti, G., Tagarelli, A., and Greco, S. (2008). *A Hierarchical Algorithm for Clustering Uncertain Data via an Information-Theoretic Approach*. in Proceedings of 2008 Eighth IEEE International Conference on Data Mining, pp. 821-826.

[12] Hamdan, H. and Govaert, G. (2005). *Mixture Model Clustering of Uncertain Data*. in Proceedings of The 14th IEEE International Conference on Fuzzy Systems, 2005. FUZZ '05., pp. 879-884.

[13] Jélvez, E., Morales, N., Nancel-Penard, P., Peypouquet, J., and Reyes, P. (2016). Aggregation heuristic for the open-pit block scheduling problem. *European Journal of Operational Research, 249* (3), 1169-1177.

[14] Kriegel, H. P. and Pfeifle, M. (2005). *Hierarchical density-based clustering of uncertain data*. in Proceedings of Fifth IEEE International Conference on Data Mining (ICDM'05), pp. 4 pp.

[15] Kumral, M. (2011). Incorporating geo-metallurgical information into mine production scheduling. *Journal of the Operational Research Society, 62* (1), 60-68.

[16] Lamghari, A. and Dimitrakopoulos, R. (2016). Network-flow based algorithms for scheduling production in multi-processor open-pit mines accounting for metal uncertainty. *European Journal of Operational Research, 250* (1), 273-290.

[17] Lamghari, A., Dimitrakopoulos, R., and Ferland, J. A. (2015). A hybrid method based on linear programming and variable neighborhood descent for scheduling production in open-pit mines. *Journal of Global Optimization, 63* (3), 555-582.

[18] Liu, S. Q. and Kozan, E. (2016). New graph-based algorithms to efficiently solve large scale open pit mining optimisation problems. *Expert Systems with Applications, 43* 59-65.

[19] Malaki, S., Khodayari, F., Pourrahimian, Y., and Liu, W. V. (2017). An application of mathematical programming and sequential Gaussian simulation for block cave production scheduling. in *Proceedings of the First International Conference on Underground Mining Technology*, M. Hudyma and Y. Potvin, Eds. Sudbury: Australian Centre for Geomechanics, pp. 323-337.

[20] Mastrogiannis, N., Giannikos, I., Boutsinas, B., and Antzoulatos, G. (2009). CL.E.KMODES: a modified k-modes clustering algorithm. *Journal of the Operational Research Society, 60* (8), 1085-1095.

[21] Montiel, L. and Dimitrakopoulos, R. (2015). Optimizing mining complexes with multiple processing and transportation alternatives: An uncertainty-based approach. *European Journal of Operational Research, 247* (1), 166-178.

[22] Rossi, M. E. and Deutsch, C. V. (2014). Recoverable Resources: Simulation. in *Mineral Resource Estimation*, Dordrecht, Springer Netherlands, pp. 167-192.

[23] Samavati, M., Essam, D., Nehring, M., and Sarker, R. (2017). A local branching heuristic for the open pit mine production scheduling problem. *European Journal of Operational Research, 257* (1), 261-271.

[24] Samavati, M., Essam, D., Nehring, M., and Sarker, R. (2017). A methodology for the large-scale multi-period precedence-constrained knapsack problem: an application in the mining industry. *International Journal of Production Economics, 193* 12-20.

[25] Tabesh, M. and Askari-Nasab, H. (2013). Automatic Creation of Mining Polygons using Hierarchical Clustering Techniques. *Journal of Mining Science, 49* (3), 426-439.

[26] Tabesh, M., Mieth, C., and Askari-Nasab, H. (2014). A multi–step approach to long–term open–pit production planning. *International Journal of Mining and Mineral Engineering, 5* (4), 273-298.

[27] Volk, P. B., Rosenthal, F., Hahmann, M., Habich, D., and Lehner, W. (2009). *Clustering Uncertain Data with Possible Worlds*. in Proceedings of 2009 IEEE 25th International Conference on Data Engineering, pp. 1625-1632.

Short-term Production Scheduling in Open-pit Mines with Shovel Allocations over Continuous Time Frames

Shiv Prakash Upadhyay, Hooman Askari-Nasab and John Doucette
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

Short term mine production scheduling is essential to attain the desired capacity utilization of equipment, meet blending targets and adhere to the strategic schedules. Although, several optimization models exist in the literature, size and complexity remains a problem which poses a limitation to solve and generate the schedules at smaller resolutions for larger time frames. This paper presents a short term production scheduling model which allocates shovels over continuous time frames, thus controlling the model size from growing with the resolution. This approach also models the mine operating environment in a more representative way to create practical and achievable schedules. A verification study of the model is presented in this paper using an iron ore mine case study. A comparison of solutions in the pareto optimal space also justifies the goal optimization as an efficient approach to attain best outcomes from among multiple conflicting objectives of the problem.

1. Introduction

Mining is the backbone of world economy. The origin of most of the products that we use in our regular life can be traced to mining of its constituent ingredients. The market price and demand makes an operation profitable and hence viable, however, the dynamic price fluctuations in the market may pose a threat to the entire investment. A detailed planning is thus carried out before making the investment to realize a profitable operation. Through a long term production planning, it is desired to maximize the profit in the early years of the production so that breakeven can be achieved early and maximum NPV can be attained. Although, strategic planning process is significant, it is at the mine operation level that the actual profit can be realized. Efficient utilization of equipment and human resources coupled with the realization of long term planned schedules and adjusting to the existing market demand, bears a significant potential to realize or even exceed projected returns. Short term production scheduling and planning should therefore be considered an important step in the mine planning process. Attaining even marginal improvements at the operation level using efficient short term plans can lead to major gains at the strategic level.

Short term mine production scheduling has started to gain attention of the researchers in recent years and very few researchers have addressed the problem so far. Bjørndal et al. (2012) attributes the size and complexity of the problem for the lack of research in this area. These types of problems belong to NP-hard class as proved by Souza et al. (2010). A comprehensive review of the existing models and solution techniques is presented by Blom et al. (2018). The review (Blom*, et al.*, 2018) presents a vast

This paper is submitted as is for publication in the international Journal of Mining and Mineral Engineering

difference in various existing models, in terms of mine operation features/activities considered, varying level of details and the objectives of the problem. Eivazy and Askari-Nasab (2012) propose a MILP model for multi-destination short term production scheduling model to minimize mining, processing, haulage, re-handling and rehabilitation costs, incorporating ramps, horizontal directional mining and buffer and blending stockpiles. Gholamnejad (2008) proposes a binary integer programming model to solve the problem at the block resolution, emphasizing on the accessibility of blocks for mining. L'Heureux et. al. (2013), Liu et. al.(2013), Kozan and Liu (2016; 2014) propose multi-stage MIP models to incorporate drilling, blasting and extraction operations. Topal and Ramazan (2010) provide a MIP model for scheduling a fixed fleet of mining trucks in a mine operation to minimize the maintenance costs. Due to the vastness of open pit mine operations, there exists numerous objectives and elements of the operations that need to be accounted for at the short term planning time resolution. For the short term production scheduling purposes, this paper assumes four major objectives as to maximize the production, meet tonnage and grade requirements of the processing operations and minimize the shovel movements (Upadhyay and Askari-Nasab, 2017, 2018).

Short term mine production scheduling problem requires to determine optimal sequence of faces to be drilled, blasted and mined by the available equipment resources for the strategically scheduled faces in the given time horizon and the amount of material to be transported from the faces to their respective destinations to achieve production and grade blend requirements of the processing plants over daily or weekly time frames. A detailed short term production scheduling model is required to account for equipment resources and corresponding drilling/charging/production and movement capabilities between faces; throughput rates of the processing operations; precedence among faces for drilling, blasting and excavation operation; production between faces and various dump destinations; and discrete haulage capacities based on the available haul truck resources and road network architecture of the mine. L'Heureux et. al. (2013) also incorporates the selection of blocks for each blast in their model. Mousavi et. al.(2016a) incorporates drop cuts in the predetermined periods and maximum vertical distance between two blocks extracted in a period to consider bench access design (haul roads).

Although, a detailed short term planning model is desired, it poses a limitation on solvability. Number of faces considered in the optimization time frame is one major reason for the size and complexity of the problem. The definition of faces is debatable in the existing literature where an excavator resource can be allocated. Block aggregation is a common technique to group similar blocks together and considered as a face. L'Heureux et. al.(2013) defines a face as an aggregate of 1 to 4 adjacent blocks having similar characteristics and considers a single block as a face while solving the model. Kozan and Liu (2016) considers a set of several same-grade block units on the same bench in the same pit as a face. Mousavi et al. (2016a, 2016b) solves the problem at the block resolution.

Uncertainty in mine operations is another problem that leads to deviations between planned and reality at this resolution. Dimitrakopoulos and Jewbali (2013) proposes a joint stochastic optimization approach to link the variability in short term with long term production schedules. Matamoros & Dimitrakopoulos (2016) proposes a stochastic integer programming formulation to simultaneously optimize fleet and production schedule accounting for uncertainty in grade and fleet parameteres and availability. Simulation and optimization approaches have also been used to capture the uncertainties of the mine operations to generate short term production schedules (Fioroni*, et al.*, 2008; Upadhyay and Askari-Nasab, 2018).

One important contribution of this paper is the use of continuous time variables to control shovel allocations and movements while capturing the production and quality of material produced in discrete periods. It serves two purposes. First, as shovel allocation and movement variables are independent of the periods, model size does not increase drastically with increasing resolution. We can solve a problem at larger or smaller resolutions without having big impacts on the size of the model. The second purpose is related to practicality. It is essential at this planning resolution that precedence are accurately modeled and that the production plans are practical. Allocating shovels within discrete periods (L'Heureux*, et al.*, 2013; Mousavi*, et al.*, 2016a, 2016b) does not model the

exact time production finished or started at a face. Thus a shovel, in such models, can be allocated to a face in the beginning of a period even if the precedence requirements are met towards the end of the period as shown in Fig. 1. Such a solution would not be practical and the planned production could not be achieved in reality, especially when the resolution of the model is larger. Kozan and Liu (2016) propose a framework strictly for equipment allocations at the various operational stages using continuous time variables, however, they do not capture production and quality at discrete time resolutions, rendering the model not usable for blending and other objectives which are measured over discrete time frames.

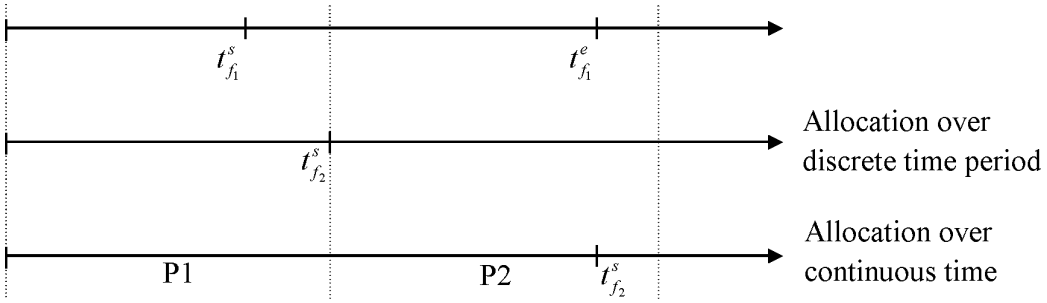


Fig. 1. Comparison between allocations over discrete and continuous time frames for the mining start time at a face f_2 based on the finish time of its precedence face f_1.

Another drawback observed with the models which allocate shovels within discrete time periods with single possible assignment in a period, is an induced, but not desired, characteristic to the model that tries not to assign a shovel to a face based on the remaining tonnage at the face. If the remaining tonnage is small compared to capacity or minimum production constraints at the model resolution, a different assignment will be preferred by the model and the optimality of the solution would be questionable. Shovel movements can be allowed within periods for such models (Upadhyay and Askari-Nasab, 2018), but it becomes hard to capture the movements across periods.

A mixed integer programming (MIP) model is presented in this paper to formulate the short term production scheduling problem by providing shovel allocations over continuous time frames, and discrete production between faces and destinations to achieve production and grade blend objectives. Model verification and a comparison of solution times is presented with an iron ore mine case study. The model is solved following a goal optimization approach due to the conflicting nature of model objectives. A comparison of solution for each objective within the pareto optimal space is also presented to justify the goal programming approach and obtain best solution based on the decision makers (DMs) preference.

2. Model characteristics and assumptions

In this paper a face is an aggregate of multiple blocks together based on their similarity (Tabesh and Askari-Nasab, 2013) to reduce the size of the problem and have practical solutions for an open pit mining framework. The characteristics of a face are the average values of the blocks constituting the face. Shovels are allocated to the faces to mine and send the material to various destinations based on their grade and tonnage requirements and capacity constraints in each period.

The model presented in this paper assumes that a shovel will move to a new face only after the current face is completely mined out. The validity of this assumption is dictated mainly by the size of the faces. Using large faces poses a problem on the validity of using the average characteristics of the constituting blocks and neglecting possible and may be significant variations. A very small face, on the other hand, loses its purpose to reduce the problem size and practically minable dimension requirements. This assumption is different from some existing models where shovel may move from and to a face multiple times and that a face may be mined intermittently, and not continuously, over multiple periods. In the model presented, decreasing the size increases the number of faces and allows

for more possible shovel movements if desired. Thus possible movements can be controlled by the size of the faces in this model.

3. Optimization model

3.1. Goals

Indices, variables and parameters used in the model are described in Table 2, 3 and 4 respectively in the Appendix section. This model considers four main operational objectives as goals of the short term production scheduling model: (1) maximize production by minimizing the negative deviation in production by shovels compared to their capacities, (2) minimize the deviation in ore tonnage received at processing plants compared to their capacities, (3) minimize the deviation in grades delivered to ore destinations compared to desired grades and (4) minimize the movement times of shovels (Upadhyay and Askari-Nasab, 2017).

$$\Psi_1 = \sum_p \sum_s x^-_{s,p} \tag{1}$$

$$\Psi_2 = \sum_{d^c} \sum_p (\delta^-_{d^c,p} + \delta^+_{d^c,p}) \tag{2}$$

$$\Psi_3 = \sum_p \sum_{d^o} \sum_k (g^-_{k,d^o,p} + g^+_{k,d^o,p}) \tag{3}$$

$$\Psi_4 = \sum_s \sum_{f_1} \sum_{f_2} \Gamma_{s,f_1,f_2} \times a_{s,f_1,f_2} \tag{4}$$

3.2. Objective function

The model optimizes these objectives as a goal programming function by carefully choosing priority weights for each objective as per the requirements of the decision maker (Romero, 2004). The objectives are first normalized in the pareto optimal space and then combined using weighted sum approach to form the objective function (Grodzevich and Romanko, 2006; Tamiz*, et al.*, 1998) as given in Eq. (5).

$$\Psi = W_1 \times \overline{\Psi}_1 + W_2 \times \overline{\Psi}_2 + W_3 \times \overline{\Psi}_3 + W_4 \times \overline{\Psi}_4 \tag{5}$$

Where:

$$\overline{\Psi}_i = \left(\Psi_i - Utopia_i\right) / \left(Nadir_i - Utopia_i\right) \tag{6}$$

3.3. Constraints

$$\sum_{f_1} \sum_s a_{s,f_1,f_2} = 1 \qquad \forall f_2 \neq f^0 \tag{7}$$

$$\sum_{f_1} \sum_s a_{s,f_1,f_2} = \overline{S} \qquad f_2 = f^0 \tag{8}$$

$$\sum_{f_1} a_{s,f_1,f} - \sum_{f_2} a_{s,f,f_2} = 0 \qquad \forall s, f \tag{9}$$

$$t^s_{f_1} - t^s_{f_2} + \sum_s \left(\Pi_{s,f_1} + \Gamma_{s,f_1,f_2}\right) \times a_{s,f_1,f_2} + B \times \sum_s a_{s,f_1,f_2} \leq B \qquad \forall f_1 \,\&\, \forall f_2 \neq f^0 \tag{10}$$

$$t^s_{f_1} + \sum_s \left(\Pi_{s,f_1} + \Gamma_{s,f_1,f_2}\right) \times a_{s,f_1,f_2} + B \times \sum_s a_{s,f_1,f_2} \leq 2 \times B \qquad \forall f_1 \,\&\, f_2 = f^0 \tag{11}$$

$$t_f^s + \sum_s \sum_{f_1} \frac{O_f}{X_s} \times a_{s,f,f_1} \le t_f^e \qquad \forall f \qquad (12)$$

$$t_{f_1}^e + \sum_s \Gamma_{s,f_1,f_2} \times a_{s,f_1,f_2} \le t_{f_2}^s + (1 - \sum_s a_{s,f_1,f_2}) \times B \qquad \forall f_1, \forall f_2 \ne f^0 \qquad (13)$$

The first set of constraints model the shovel allocations and corresponding movement between faces. These constraints are formulated similar to the modeling of a multiple travelling salesman problem or vehicle routing problem. Given a set of faces and a starting dummy face where all shovels are initially located, it is required to visit each face exactly ones by the shovels and return to the starting dummy face. The distance between all the faces and the dummy face is zero along with its tonnage. Constraint (7) restricts only one possible movement to a face, i.e. only one shovel can move to a face from only one of the available faces. Constraint (8) models the return of all shovels to the dummy face. Constraint (9) is the flow balancing equation which states that if a shovel has moved to a face, it must move out from that face. The sub-tour elimination constraint is modeled using a continuous time variable in equations (10) and (11), which captures the start time of the allocated shovel at each face. Constraint (10) dictates that start time of the face f_2 must be greater than or equal to the summation of start time and mining time of face f_1, and movement time of the corresponding shovel from face f_1 to f_2. If there is no movement between the faces, the difference between the start times will always be lesser than the maximum time B. When there is a movement to the dummy face, Constraint (11) models the same equation (10) considering the start of the dummy face at the maximum time B, preserving its actual start time variable as zero. Constraint (12) ensures that finish time of a face is always greater than or equal to the start time plus the minimum mining time of the face. Coupled with constraint (12), constraint (13) restricts start time of a face to be greater than the finish time of its predecessor face plus the movement time of the corresponding shovel between them.

$$\left(1 - m_{f,p}^s\right) \times B + m_{f,p}^s \times p \times T \ge t_f^s \qquad \forall f, p \qquad (14)$$

$$\left(1 - m_{f,p}^e\right) \times B + m_{f,p}^e \times p \times T \ge t_f^e \qquad \forall f, p \qquad (15)$$

$$\left(1 - m_{f,p}^s\right) \times p \times T \le t_f^s \qquad \forall f, p \qquad (16)$$

$$\left(1 - m_{f,p}^e\right) \times p \times T \le t_f^e \qquad \forall f, p \qquad (17)$$

$$m_{f,p}^s \ge m_{f,p-1}^s \qquad \forall f \,\&\, \forall p \ne 1 \qquad (18)$$

$$m_{f,p}^e \ge m_{f,p-1}^e \qquad \forall f \,\&\, \forall p \ne 1 \qquad (19)$$

$$\sum_s \sum_d x_{s,f,d,p} \le m_{f,p}^s \qquad \forall f, p \qquad (20)$$

$$m_{f,p}^e \le \sum_s \sum_d x_{s,f,d,p} \qquad \forall f, p \qquad (21)$$

$$m_{f,p}^e \le m_{f,p}^s \qquad \forall f, p \qquad (22)$$

$$m_{f_2,p}^s \le \sum_s \sum_d x_{s,f_1,d,p} + 1 - \sum_s a_{s,f_1,f_2} \qquad \forall f_1, f_2, p \qquad (23)$$

$$\sum_d \left(x_{s,f,d,p} - x_{s,f,d,p-1}\right) \times O_f \le \begin{pmatrix} (m_{f,p}^s - m_{f,p-1}^s) \times p \times T + \\ \left(1 - (m_{f,p}^s - m_{f,p-1}^s)\right) \times B - t_f^s \end{pmatrix} \times X_s \qquad \forall s, f, p \qquad (24)$$

$$\sum_{d}\left(x_{s,f,d,p}-x_{s,f,d,p-1}\right)\times O_f \le \left(t^e_f-(m^e_{f,p}-m^e_{f,p-1})\times(p-1)\times T\right)\times X_s \qquad \forall s,f,p \qquad (25)$$

$$t^e_{f'} \le t^s_f \qquad \forall f \,\&\, f' \in F_f \qquad (26)$$

As shovel movement variable is not indexed over periods, constraints (14) to (25) are used to relate the continuous time variable with discrete periods from which production can start from a face. Constraint (14) and (15) coupled with constraints (16) and (17) restricts the mining start and mining end variables to be true for the periods corresponding to mining start and end times respectively. Constraints (18) and (19) make sure mining start and end variables remains true after they first became true. Constraint (20) states that no production is possible from a face until mining start variable becomes true, i.e. until a shovel has reached to the face. Constraint (21) on the other hand restricts the mining end variable to become true until the face is completely mined. Constraint (22) simply states that mining end variable can be true only if mining start variable is true. Constraint (23) ensures zero production from a face until its predecessor, from where shovel has moved, is completely mined. Constraint (24) determines the maximum production possible from the face since the shovel arrived at the face in that period. It states that if mining start variable became true in a period, i.e. a shovel arrived to the face, the maximum production possible from the face in that period will be limited by the remaining time. Similarly constraint (25)ensures that mining end time of a face is accurately captured based on the amount of material mined in that period. Precedence requirements are modeled using constraint (26) which ensures that a shovel cannot start mining a face before its precedence faces are finished.

$$\sum_{d} x_{s,f_2,d,p}-\sum_{f_1} a_{s,f_1,f_2} \le 0 \qquad \forall s,f_2,p \qquad (27)$$

$$x_{s,f,d,p} \le x_{s,f,d,p+1} \qquad \forall s,f,d,p \qquad (28)$$

$$\sum_{d}\sum_{f}(x_{s,f,d,p}-x_{s,f,d,p-1})\times O_f \le X^+_s \qquad \forall s,p \qquad (29)$$

$$\sum_{d}\sum_{f}(x_{s,f,d,p}-x_{s,f,d,p-1})\times O_f \Big/ X^+_s + x^-_{s,p} = 1 \qquad \forall s,p \qquad (30)$$

$$\sum_{s}\sum_{d^c} x_{s,f,d^c,p} \le Q_f \qquad \forall f,p \qquad (31)$$

$$\sum_{s}\sum_{d^w} x_{s,f,d^w,p} \le (1-Q_f) \qquad \forall f,p \qquad (32)$$

$$\sum_{s}\sum_{f}(x_{s,f,d^c,p}-x_{s,f,d^c,p-1})\times O_f \Big/ \left(Z_{d^c}\times T\right)+\delta^-_{d^c,p}-\delta^+_{d^c,p}=1 \qquad \forall d^c,\forall p \qquad (33)$$

$$\delta^-_{d^c,p} \le \Lambda^-_{d^c} \Big/ Z_{d^c} \qquad \forall d^c,p \qquad (34)$$

$$\delta^+_{d^c,p} \le \Lambda^+_{d^c} \Big/ Z_{d^c} \qquad \forall d^c,p \qquad (35)$$

$$\sum_{s}\sum_{f}(x_{s,f,d^c,p}-x_{s,f,d^c,p-1})\times O_f \times \bar{G}_{f,k}+g^-_{k,d^c,p}-g^+_{k,d^c,p}= \sum_{s}\sum_{f}(x_{s,f,d^c,p}-x_{s,f,d^c,p-1})\times O_f \times G_{k,d^c} \qquad \forall k,d^c,p \qquad (36)$$

Constraint (27) allows no production by a shovel from a face, if there is no movement of the shovel to the face. Constraint (28) insures that production from a face by a period is always greater than previous period. Constraint (29) is the capacity constraint restricting total production in a period to the maximum production capacity of shovels. Negative deviation in production compared to the

maximum shovel capacities are modeled by constraint (30). Constraints (31) and (32) restrict ore production to be sent to crushers and waste production to the waste dumps. Negative or positive deviation in tonnage received at crushers compared to desired rate is modeled using constraint (33). Constraint (34) and (35) provides lower and upper limits on the deviation in tonnage received at crushers. Grade deviations are captured using constraint (36) as the negative or positive deviation in tonnage of received metal content as compared to desired metal content based on the received tonnage of ore at the crushers.

4. Model Implementation

The model is verified by implementing it over an iron ore open pit mine case study with 5 shovels (2 ore and 3 waste mining shovels). There exists two crushers (C1 and C2) and a waste dump. The crushers are required to be fed at-least at half of its desired capacity with a MWT grade of 65% and75%. The model is implemented with IBM ILOG CPLEX solver version12.7.1 through Matlab, on a Windows 7 environment with Intel(R) Core(TM) i7-3770 processor of 3.40 GHz and 16 GB RAM.

Table 1. Comparison of models, sizes, solution time and relative MIP gap

Problem	Faces	Periods	Variables	Binary variables	Constraints	Solution time - Goal (s)	Relative MIP gap (%)
P1	24	4	4963	3325	7628	12	0
P2	24	20	12099	4125	33980	185	0
P3	43	8	15924	10384	33252	36000	0.31%
P4	62	3	23251	20223	27189	-	-

Four set of problems (P1-P4) are formulated with increasing time frame and resolution to study the impact on model size and solution time which is presented in Table 1. It shows that number of binary variables does not grow drastically with increase in the resolution, which is not the case if shovel allocation variables are indexed over periods.

To assist the model runtime, initial faces for shovel assignments are fixed in this implementation. Moreover, based on the prior knowledge that ore shovels should work only on ore faces and waste shovels on the waste faces, and no movement is possible from a face to any of its precedence faces, corresponding variables are fixed prior to solving the model. Also, based strictly on the precedence requirements and shovel capacities, a minimum possible start time of all the faces is determined to fix the mining start and mining end binary variables of the faces for the relevant periods to be zero.

Table 1 shows the run time and MIP gap of the goal function obtained by running the model for each problem with a stopping criteria of 36000 s or 10 h. P1 and P2 could be easily solved to optimality within a reasonable time, however a parito optimal space could not be determined for P4 within the stopping criteria. For problem P3, all individual objectives could be solved to optimality within 45 minutes combined, however goal function could not converge to optimality within the stopping criteria after achieving a 0.36% relative MIP gap within half an hour.

Solutions for problems P1 and P2 are compared in Fig. 2 to 4 for the major objectives considered in the optimization. Fig. 2 and 3 show ore and waste productions sent to two crushers (C1 and C2) and the waste dump over four periods in P1 and 20 periods in P2 respectively. As P2 correspond to the same time frame as P1 with a smaller resolution, one period in P1 is equivalent to 5 periods in P2. The capacity utilization values are presented with respect to desired feed. It can be observed that the two solutions are not the same entirely. The reason for the difference is the grade blend objective function, which evaluates grade as an average over a larger time resolution in P1, but a smaller and practical resolution in P2.

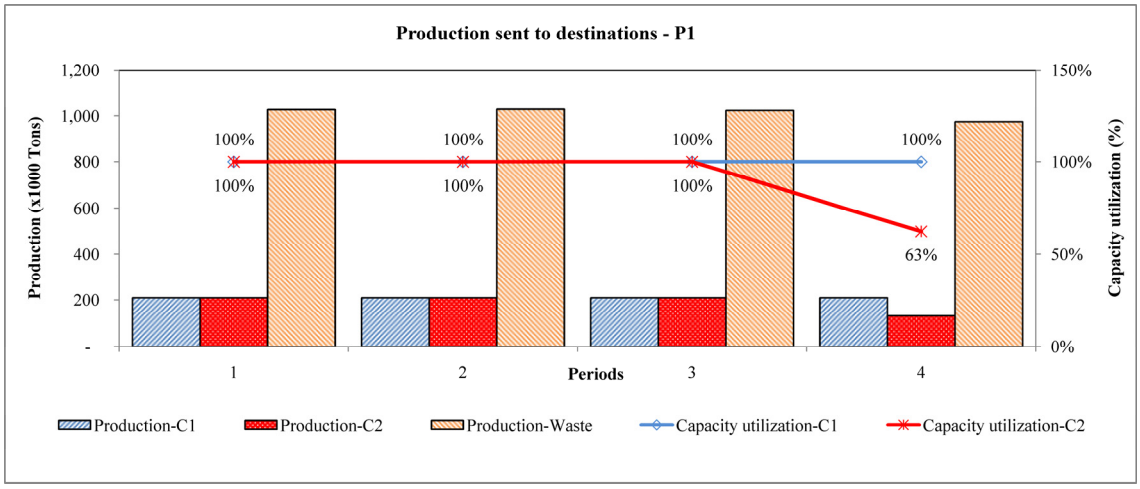


Fig. 2. Production and capacity utilizations for problem P1

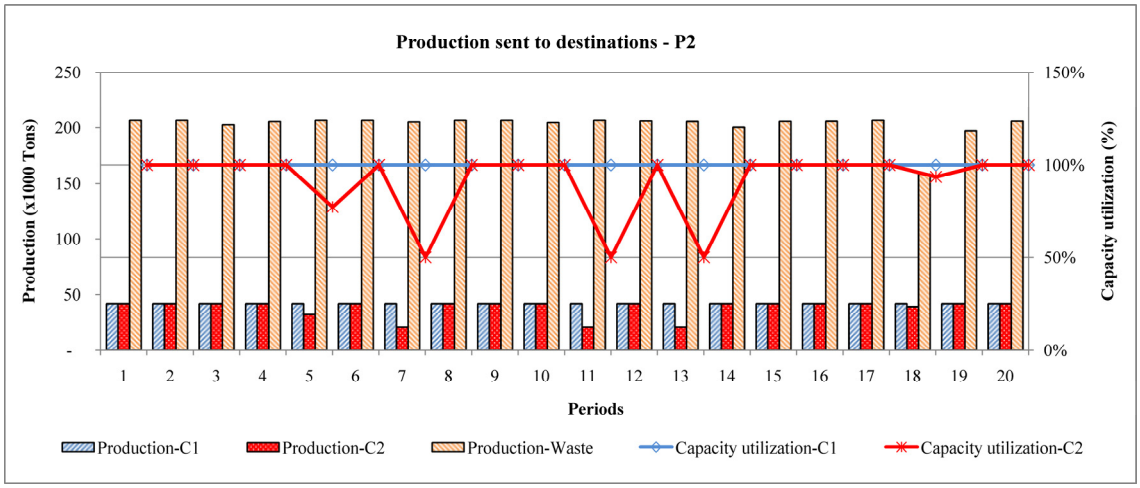


Fig. 3. Production and capacity utilizations for problem P2

Fig. 2 and 3 show that crusher 1 operates at its desired capacity in both the solutions for P1 and P2. However capacity utilization of crusher 2 drops to 63% of desired capacity in the last period for P1. In comparison, crusher utilization for P2 drops to the minimum limit at three occasions. The reason for this behavior can be attributed to the grade blend objective which sends less material to crusher 2 whenever desired grade blend cannot be achieved as observed from Fig. 4. A comparison of grade blend delivered over both resolutions (P1 and P2) is shown in Fig. 4, which categorically suggests the practicality of solution for P2 in comparison to P1, by showing the variations in grade blend deliverable at a higher resolution.

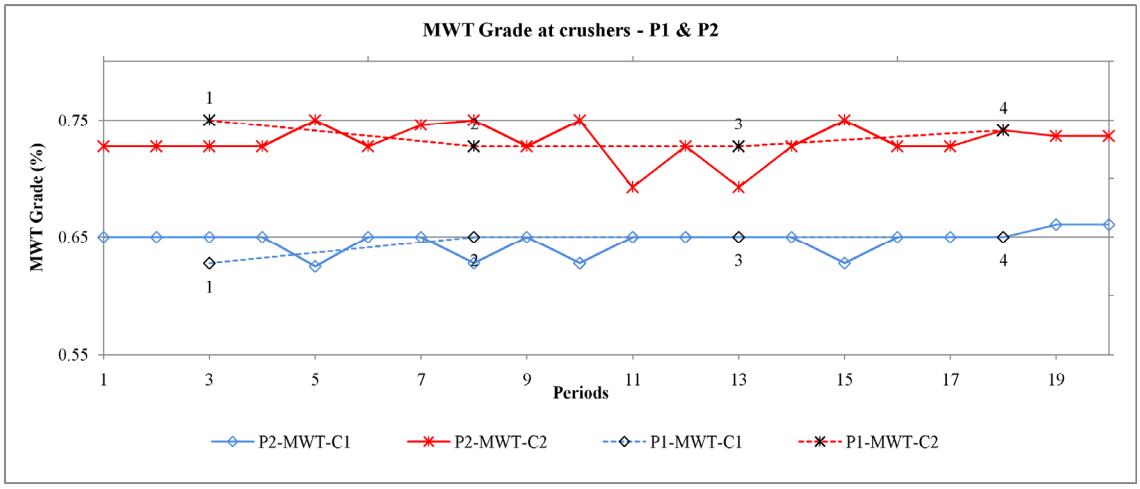


Fig. 4. MWT Grade delivered to crushers for problems P1(4 periods) and P2 (20 periods)

Similarly, solutions for problem P3 are presented in Fig. 5 and 6. The first four periods in this problem correspond to P1, however the results are observed to be different. The obvious reason is the inclusion of more available faces in the problem, which provides more options to achieve the objectives spread over more periods. The production and capacity utilizations of the crushers show promising results, however the grade blend objective for crusher 2 performed poorly.

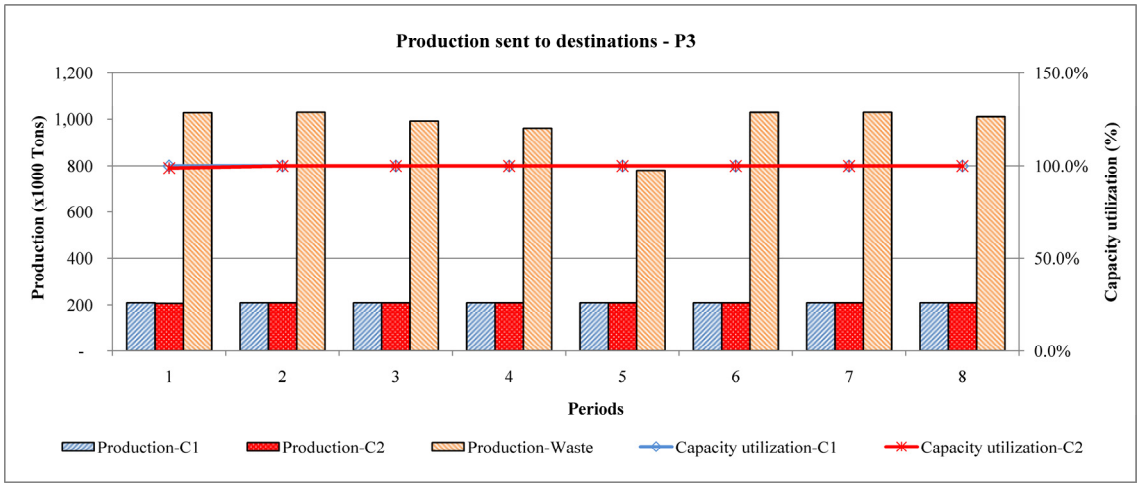


Fig. 5. Production and capacity utilizations for problem P3

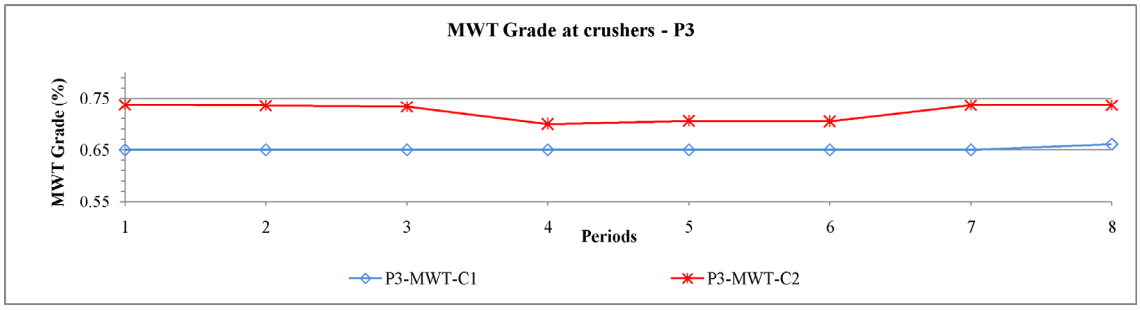


Fig. 6. MWT grade delivered to crushers for problem P3

Although goal function provides the desired solution based on decision makers (DMs) preference to each objective, it is imperative to understand and analyze the best solution for individual objectives as well. Such an analysis supports the DMs judgment of preference weights used in the goal function. A

similar analysis is presented for problem P3 through Fig. 7, 8, 9 and 10. The solutions are recorded during the determination of parito optimal space which involved optimizing each objective separately.

Best possible production scenario is presented in Fig. 7, which falls short by 3.5% of the maximum production capacity of the mine. The lesser material handling capacity of the crushers in comparison to production capacity of the ore shovels could be one major reason for the short fall, coupled with lost production in movement and idling due to precedence requirements. Moreover, goal function schedule's 0.8% less production in comparison to best production case scenario to compensate for other objectives. It can be observed from Fig. 8 that by having a 0.8% production loss, a much better grade blend is achieved by the goal function (Fig. 6). Also, crushers are overfed in best production case scenario and made to operate at their peak capacities, more than desired, in comparison to schedule provided by goal function. Similarly, the best grade blend scenario can provide an improvement in the grade blend delivered to crushers (see Fig. 9), however, it cannot feed the plants at their desired capacity (see Fig. 10).

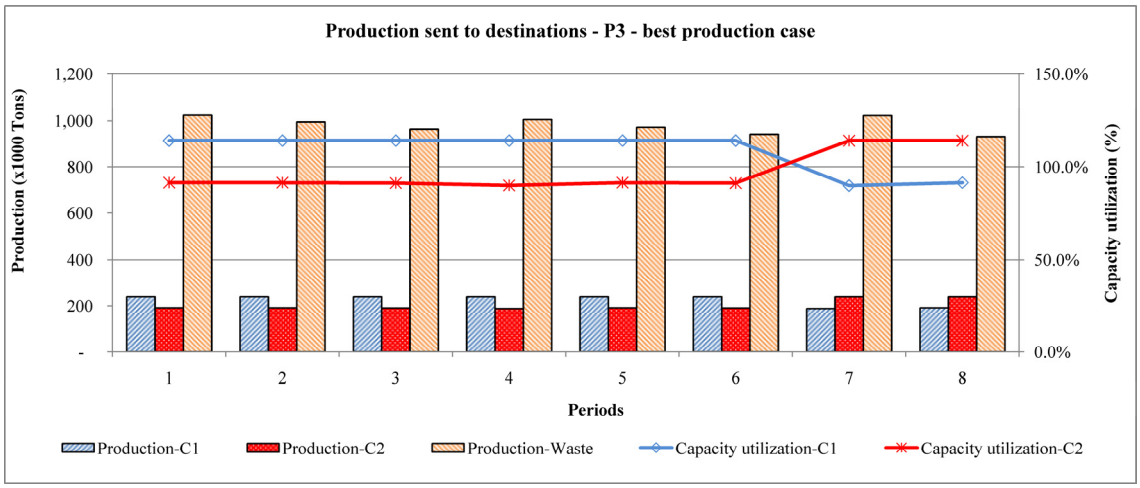


Fig. 7. Production and capacity utilizations for problem P3 - best production case

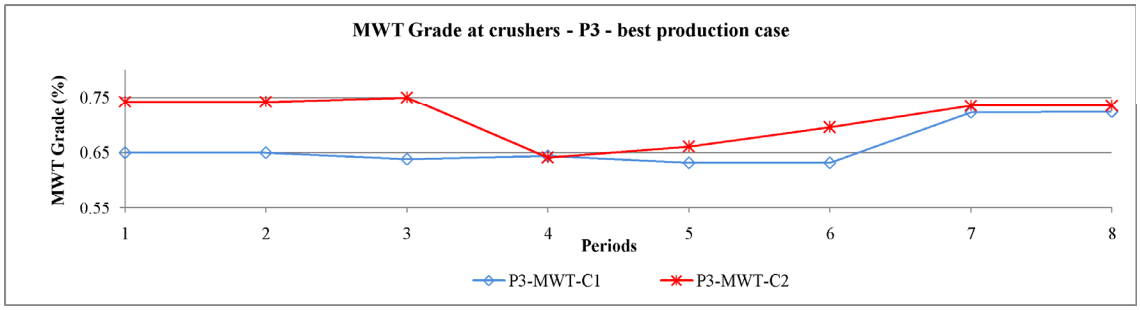


Fig. 8. MWT grade delivered to crushers for problem P3 - best production case

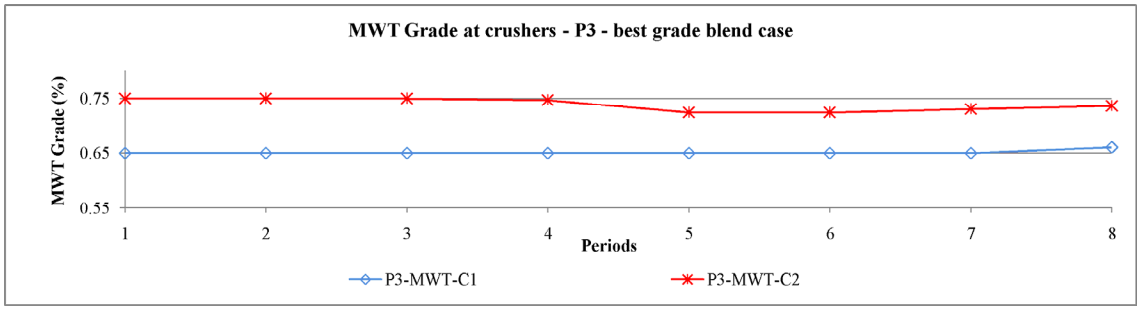


Fig. 9. MWT grade delivered to crushers for problem P3 - best grade blend case

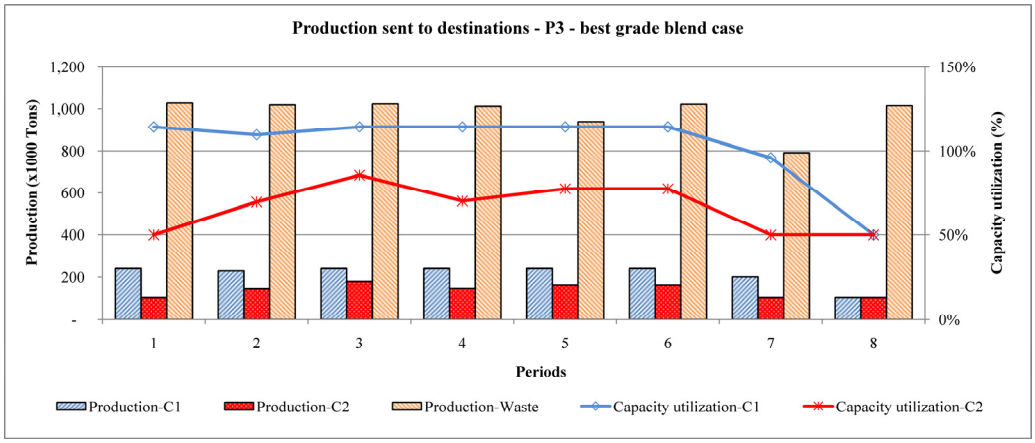


Fig. 10. Production and capacity utilizations for P3 - best grade blend case

To summarize and justify the goal function, production values obtained by the optimization of each objective is compared against the best production scenario in Fig. 11. A similar comparison for crusher feed objective is presented in Fig. 12. Negative values in Fig. 12 correspond to overfeeding the crushers above desired feed. It can be seen that production deviation can be as large as 5.4% when only grade blend is optimized, whereas goal function could reduce it to 0.8% (Fig. 11) while maintaining a satisfactory grade blend as shown in Fig. 6. Crusher feed is also compromised by only 0.7% in the optimal goal solution in comparison to best crusher scenario as shown in Fig. 12.

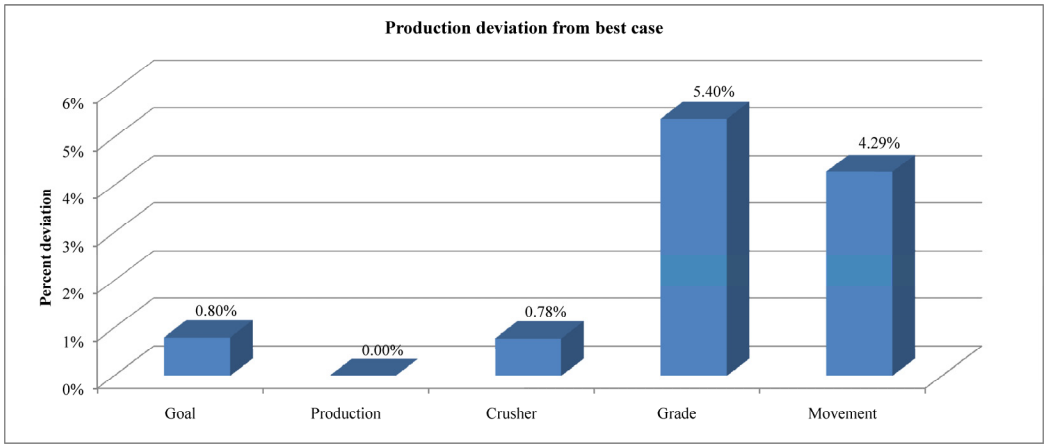


Fig. 11. Production deviation from best production scenario for the optimal solution of each objective function

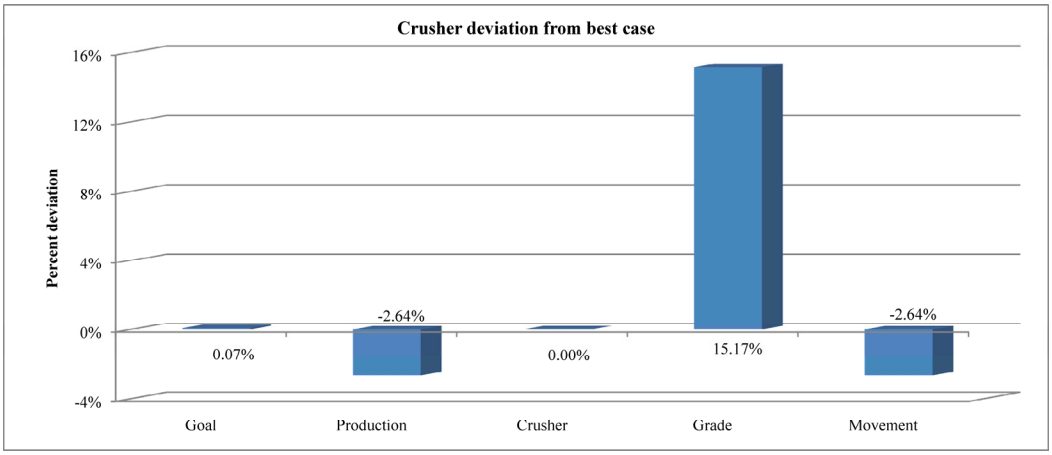


Fig. 12. Crusher feed deviation from best crusher feed scenario for the optimal solution of each objective function

Due to the complexity of the mine operations and multiple objectives to be taken care of at the short term scheduling stage, it is imperative to optimize all the objectives simultaneously to have a reasonable solution. Optimizing the goal function in the pareto optimal space is also essential to obtain non-dominated solution for each objective based on DMs preferential weights as shown in Fig. 11 and 12.

5. Conclusions and recommendations

This paper presented a model for short term production scheduling of open pit mines. Through a comparison in modeling approach and the selection of variables, the presented model can be observed to be superior in the context of modeling the problem correctly. The use of continuous time variables for shovel allocations and capturing productions and grades over discrete periods models the system precisely. The modeling approach also shows its capability to control the size of the problem at increasing resolutions which is necessary for practical solutions especially for grade blend deliverability. An implementation of the model on a verification case study shows its strength in solving the short term production scheduling model at the size of problem P3 in a reasonable time, and capturing the practical deliverance of the objectives of the model.

A comparison of all objective function values in the pareto optimal space is also presented to compare the final solution against the best possible solution for each objective considered. The comparison presented can be used to justify the DMs preference weight of each objective and can be subsequently adjusted as desired. However, in the case presented, the solution obtained fairly justifies the weights used.

Run time for larger problems still remains a major hurdle for this model. Selection of blast areas, drilling and blasting processes will also be included in the model as a future work for more practical production schedules.

6. Reference

[1] Bjørndal, T., Herrero, I., Newman, A., Romero, C., and Weintraub, A. (2012) 'Operations research in the natural resource industry', *International Transactions in Operational Research,* Vol. 19 No. 1-2, pp.39-62.

[2] Blom, Michelle, Pearce, Adrian R., and Stuckey, Peter J. (2018) 'Short-term planning for open pit mines: a review', *International Journal of Mining, Reclamation and Environment*, pp.1-22.

[3] Dimitrakopoulos, R., and Jewbali, A. (2013) 'Joint stochastic optimisation of short and long term mine production planning: method and application in a large operating gold mine', *Mining Technology,* Vol. 122 No. 2, pp.110-123.

[4] Eivazy, H., and Askari-Nasab, H. (2012) 'A mixed integer linear programming model for short-term open pit mine production scheduling', *Mining Technology,* Vol. 121 No. 2, pp.97-108.

[5] Fioroni, M. M., Franzese, L. A. G., Bianchi, T. J., Ezawa, L., Pinto, L. R., and de Miranda, G. (2008), 'Concurrent simulation and optimization models for mining planning' in WSC 2008, pp.759-767.

[6] Gholamnejad, J. (2008) 'A zero-one integer programming model for open pit mining sequences', *Journal of the Southern African Institute of Mining and Metallurgy,* Vol. 108, pp.759-762.

[7] Grodzevich, Oleg, and Romanko, Oleksandr. (2006), 'Normalization and Other Topics in Multi-Objective Optimization' in Fields-MITACS Industrial Problems Workshop, Toronto, pp.89-101.

[8] Kozan, E., and Liu, S. Q. (2016) 'A new open-pit multi-stage mine production timetabling model for drilling, blasting and excavating operations', *Mining Technology,* Vol. 125 No. 1, pp.47-53.

[9] Kozan, Erhan, and Liu, Shi Qiang. (2014), 'An open pit multistage mine production scheduling model for drilling, blasting and excavating operations' in Orebody Modelling and Strategic Mine Planning Symposium, Perth, Western Australia, pp.329-334.

[10] L'Heureux, G., Gamache, M., and Soumis, F. (2013) 'Mixed integer programming model for short term planning in open-pit mines', *Mining Technology,* Vol. 122 No. 2, pp.101-109.

[11] Liu, Shi Qiang, Kozan, Erhan, and Wolff, Rodney. (2013), 'A short-term production scheduling methodology for open-pit mines' in International Symposium on the 36th Applications of Computers and Operations Research in the Mineral Industry (36th APCOM), Brazil, pp.465-469.

[12] Matamoros, Martha E. Villalba, and Dimitrakopoulos, Roussos. (2016) 'Stochastic short-term mine production schedule accounting for fleet allocation, operational considerations and blending restrictions', *European Journal of Operational Research,* Vol. 255 No. 3, pp.911-921.

[13] Mousavi, Amin, Kozan, Erhan, and Liu, Shi Qiang. (2016a) 'Comparative analysis of three metaheuristics for short-term open pit block sequencing', *Journal of Heuristics,* Vol. 22 No. 3, pp.301-329.

[14] Mousavi, Amin, Kozan, Erhan, and Liu, Shi Qiang. (2016b) 'Open-pit block sequencing optimization: A mathematical model and solution technique', *Engineering Optimization,* Vol. 48 No. 11, pp.1932-1950.

[15] Romero, Carlos. (2004) 'A general structure of achievement function for a goal programming model', *European Journal of Operational Research,* Vol. 153 No. 3, pp.675-686.

[16] Souza, M. J. F., Coelho, I. M., Ribas, S., Santos, H. G., and Merschmann, L. H. C. (2010) 'A hybrid heuristic algorithm for the open-pit-mining operational planning problem', *European Journal of Operational Research,* Vol. 207 No. 2, pp.1041-1051.

[17] Tabesh, M., and Askari-Nasab, H. (2013) 'Automatic creation of mining polygons using hierarchical clustering techniques', *Journal of Mining Science,* Vol. 49 No. 3, pp.426-440.

[18] Tamiz, Mehrdad, Jones, Dylan, and Romero, Carlos. (1998) 'Goal programming for decision making: An overview of the current state-of-the-art', *European Journal of Operational Research,* Vol. 111 No. 3, pp.569-581.

[19] Topal, Erkan, and Ramazan, Salih. (2010) 'A new MIP model for mine equipment scheduling by minimizing maintenance cost', *European Journal of Operational Research,* Vol. 207 No. 2, pp.1065-1071.

[20] Upadhyay, Shiv Prakash, and Askari-Nasab, Hooman. (2017) 'Dynamic shovel allocation approach to short-term production planning in open-pit mines', *International Journal of Mining, Reclamation and Environment*, pp.1-20.

[21] Upadhyay, Shiv Prakash, and Askari-Nasab, Hooman. (2018) 'Simulation and optimization approach for uncertainty-based short-term planning in open pit mines', *International Journal of Mining Science and Technology,* Vol. 28 No. 2, pp.153-166.

7. Appendix

Table 2**.** Indices for variables, parameters and sets.

Indices	Description
s	Index for set of shovels (s = 1, … $\overline{S}$)
f	Index for set of faces (f = 1, … $\overline{F}$)

k	Index for set of material types (k = 1, … $\overline{K}$)
d	Index for set of destinations (crushers, waste dumps)
d^c	Index for set of crushers/processing plants (d^c = 1, … $\overline{C}$)
d^w	Index for waste dumps (d^w = 1, … $\overline{W}$)
p	Index for periods (p=1,… $\overline{P}$)

Table 3. Variables considered in the model

Variables	Description
a_{s,f_1,f_2}	Movement variable if shovel s moved from face f_1 to face f_2 (binary)
$m^s_{f,p}$	Mining start variable if any shovel reached to face f by period p (binary)
$m^e_{f,p}$	Mining end variable if any shovel finished a face f by period p (binary)
t^s_f	Continuous time variable at which production start from face f
t^e_f	Continuous time variable at which production end at face f
$x_{s,f,d,p}$	Fraction of tonnage at face f sent by shovel s, to destination d by period p
$x^-_{s,p}$	Fraction of maximum capacity of shovel to model negative deviation in production by shovel s compared to its capacity in period p
$\delta^-_{d^c,p}, \delta^+_{d^c,p}$	Negative and positive deviations in production received at crusher destinations d^c in period p, as a fraction of processing plant capacities
$g^-_{k,d^c,p}, g^+_{k,d^c,p}$	Negative and positive deviations in tonnage equivalent of grade of material type k compared to desired grade at crusher destinations d^c in period p

Table 4. Parameters of systems considered.

Parameters	Description
T	Time horizon of a period (hr)
B	Time horizon in which all faces can be mined out completely (hr)
X_s	Shovel hourly production capacity (tonne/hr)
X^+_s	Maximum possible shovel production in decision time frame 'T' (tonne)
Γ_{s,f_1,f_2}	Movement time of shovel s from face f_1 to face f_2 (hr)
$\Pi_{s,f}$	Mining duration of face f by shovel s at maximum production capacity (hr)
$\overline{\Pi}_f$	Minimum mining duration possible for face f (hr)
Fi_s	Face where shovel is initially located (start of the optimization time frame)
f^0	Dummy face, which has zero tonnage and distance to all the faces
F_f	Set of precedence faces to be mined before starting face f
z_{d^c}	Maximum capacity of the crushers/processing plants (tonne/hr)

$\Lambda^{+}_{d^c}$	Maximum positive deviation in tonnage accepted at crushers/processing plants (tonne/hr)
$\Lambda^{-}_{d^c}$	Maximum negative deviation in tonnage accepted at crushers/processing plants (tonne/hr)
G_{k,d^c}	Desired grade of material types at the crushers
$\overline{G}_{f,k}$	Grade of material type k at face f
O_f	Tonnage available at face f at the beginning of optimization (tonne)
Q_f	1 if material at face is ore, 0 if it is waste (binary parameter)
M_s^{ore}	0 if shovel s is locked to an ore face, 1 for waste face, 2 otherwise.
W_i	Normalized weights of individual goals (i = 1, 2, 3) based on priority

Uncertainty-Based Mine Planning Framework for Oil Sands Production Scheduling and Waste Management

Ahlam Maremi and Eugene Ben-Awuah
Mining Optimization Laboratory (MOL)
Laurentian University, Sudbury, Canada

ABSTRACT

For oil sands mining, the production schedule must be integrated simultaneously with in-pit and ex-pit dyke construction scheduling. The mined ore that exceeds the plant capacity is stockpiled for a limited duration. The topmost layer of the overburden is also stockpiled for land reclamation. Uncertainty is always present in the presence of sparse geological data. Kriged estimates with a variance penalty scheme are used to minimize the financial risk from grade uncertainty associated with the production schedule. An uncertainty-based mathematical programming model is developed based on Mixed Integer Linear Goal Programming (MILGP) for oil sands production scheduling and waste management. The model aims to generate a life of mine plan that maximize the NPV of the project. In addition, the model determines the rehandling strategy for the stockpiled ore and the destination of reclamation and dyke materials to minimize costs. The uncertainty-based model is implemented with two different scenarios. Scenario 1 generates an integrated mine plan with a waste management and stockpiling strategy that maximizes the NPV of the operation and minimizes dyke construction cost. Scenario 2 generates a range of total NPV and estimates the production scheduling risk associated with grade uncertainty.

1. Introduction

In open pit mining, the goal is usually to maximize the Net Present Value (NPV) of the project by providing the plant with ore at full capacity while satisfying physical, operational and economic constraints. The very first and highly important step in the mine planning process is modeling the ore body appropriately. All other activities throughout the mine life starting from evaluating the economic viability of the entire mining operation to undertaking all the processes of mine planning will be based on the ore body model (Hustrulid and Kuchta, 2006). The different phases of the mine planning process include: 1) Block model determination that consists of drilling in different locations and depths of the mine; and obtaining samples of material for grade and density interpolation, dividing the orebody into blocks of equal sizes and assigning estimated tonnage and mineral grades to each block. As a result, the estimated extraction profit or loss for each block in the model needs to be computed. This generates what is referred to as an economic block model; 2) Definition of the Ultimate Pit Limit (UPL) which is the area in which extraction will take place. Before any block can be extracted, all blocks immediately above and at certain angles must be extracted. To determine the UPL, it is necessary to determine the slope angles. This depends on the structural composition of the rocks and the location and depth of each block; 3) Production planning, which involves the decision of which blocks and when and how they should be extracted (Chicoisne, et al., 2012). Extracting mining blocks from an open pit mine in a specific sequence to give the highest NPV is known as

open pit mine planning optimization. This is subject to a variety of production, grade blending and pit slope constraints (Whittle, 1989).

The focus of this paper is Long-Term Production Planning (LTPP) optimization with integrated waste management. Oil sands mining of the McMurray formation was studied and used as a case study. The paper seeks to develop an uncertainty-based theoretical framework that maximizes the NPV of an oil sands mining operation and minimizes waste management cost using Mixed Integer Linear Goal Programming (MILGP) model. The model incorporates multiple material types with multiple elements for multiple destinations in oil sands long-term production planning. Although mathematical programming models have been applied in mine production scheduling, "very little work has been carried out in terms of oil sands mine planning, which has a unique scenario when it comes to waste management" (Ben-Awuah and Askari-Nasab, 2013). The proposed model integrates waste management strategy into the production plan as required by recent regulations from Alberta Energy Regulator (AER) Directive 085 (formerly interim directive ID 2001-7) (Alberta Energy Regulator, 2017). Muskeg, the topmost layer of the overburden, is stockpiled and will be used for land reclamation at the end of the mine life. The model also includes a limited duration stockpiling strategy for the mined ore that exceeds the processing capacity. Financial risk is defined as the deviation from expected historical returns within a specific period (Mangram, 2013). The optimal solution to the LTPP problem is affected by uncertainties related to the input parameters. To minimize the financial risk (risk) associated with grade uncertainty, kriging estimates with a variance penalty scheme are used for the production scheduling optimization.

Subsequently, a case study with two different scenarios is examined using the proposed MILGP model. The first scenario, Scenario 1, uses goal functions with a limited duration stockpiling strategy for ore and stockpiling strategy for reclamation material. The second scenario, Scenario 2, evaluates the risk associated with the production schedule based on grade uncertainty and a variance penalty scheme. MATLAB (Mathworks, 2017) is used for coding the mathematical programming formulation and the resulting optimization problem is solved with a large-scale optimization solver IBM/CPLEX (ILOG, 2012). This solver uses a branch and cut algorithm which is a hybrid of branch-and-bound algorithm and cutting plane methods to solve the optimization problem (Horst and Hoang, 1996; Wolsey, 1998).

The next section of this paper gives details of the problem definition. Section 3 covers a summary of the literature review on LTPP optimization problems based on deterministic and uncertainty approaches, clustering and paneling in mine planning, and stockpiling. The process of oil sands mining and waste management is explained in Section 04. Section 5 highlights the concepts of block modeling, variography and kriged estimates with a variance penalty scheme to manage risk. Section 6 presents the theoretical mathematical programming formulation. The implementation of the MILGP model is in Section 7. A case study is presented in Section 8. Finally, Section 9 documents the research conclusions and recommendations.

2. Problem Definition

Oil sands mining operations result in different types of material: ore, interburden (IB), overburden (OB), reclamation material (RM) and waste. Material with a bitumen grade of 7% or more will be classified as ore Directive 082: (Alberta Energy Regulator, 2016). Processing the ore results in a huge amount of tailings. Based on the fines content, the tailings are divided into Tailings Coarse Sands (TCS) and tailings slurry. TCS is used for dyke construction and tailings slurry is deposited in the disposal areas created with dykes. Any ore material that has a bitumen grade less than 7%, known as interburden, will be reclassified based on the fines content. Material with fines content less than 50% will be used for dyke construction; otherwise, it will be sent to the waste dump. Overburden (OB) comprises of the Pleistocene unit and Clearwater formation. OB will be used either for road construction or for dyke construction if it meets the fines requirement. Muskeg, the topmost layer of

the overburden, will be extracted, stockpiled and used to reclaim the land at the end of the mine life. Any material that does not meet the requirements of ore, dyke materials or reclamation material is classified as waste and will be sent to the waste dump.

A schematic representation of the problem definition is presented in Fig. 1**.** The final pit block model is divided into pushbacks. The material intersecting a pushback and a bench is known as a mining-panel. Each mining-panel contains a set of mining-cuts and is used to control the mine production operation sequencing. Mining-cuts are clusters of blocks within the same mining bench that are similar in terms of location, grade, rock type and the shape of mining-cuts created on the lower bench. The figure depicts the scheduling of an oil sands ultimate pit block model containing K mining-cuts within P mining-panels. Each mining-cut k, could be made up of one or more of the following materials: ore, O_k, interburden and overburden dyke materials, ib_k and ob_k, muskeg reclamation material, mu_k, and waste, w_k. The material in each mining-cut is to be scheduled over T periods based on the goals and constraints associated with the mining operation. The mined ore extracted from mining-cut k within mining-panel p in period t will be sent to the processing destination a. Any material that exceeds the processing capacity will be sent to the stockpile sp in period t and will be reclaimed in period $t + ts$, where ts is the stockpiling duration limit controlled by the planner to minimize oxidation of the stockpiled material. Oxidized ore reduces processing recovery. The ore extracted in the current period t and the ore that has been sent to the stockpile in period $t - ts$ together will be sent to the processing destination to extract the bitumen. The generated TCS material together with the OB and IB dyke materials will be used for constructing $dyke_i$ at site i. Muskeg material will be sent to the muskeg stockpile area for reclamation. This is referred to as operational material scheduling.

These strategic and operational schedules to be developed are subject to a variety of economic, technical and physical constraints. The constraints control the mining extraction sequence and ore and dyke material blending requirements. The constraints also control mining, processing, muskeg and dyke material goals that specify the quantities of allowable material for the mining operation, processing plant, reclamation works and dyke construction.

Considering uncertain input variables will minimize the difference between the theoretical and actual NPV, and will result in a high degree of confidence for the mining project. Based on this, production risk from grade uncertainty will be minimized using the variance and a penalty scheme. The strategic and operational schedules determine the profitability and sustainability of the project. The schedules control the NPV of the operation and enable a robust waste management planning strategy. Improper waste management planning can lead to environmental issues, resulting in immediate mine closure by regulatory agencies. It is assumed that when a mining-panel is scheduled, all the mining-cuts, blocks or parcels within this mining-panel are extracted uniformly. Also, it is assumed that when modeling the relationship between the mining-panels and mining-cuts, the planner has access to all the mining-cuts within each mining-panel. The stockpiling strategy is considered in the optimization problem for extra ore that exceeds the mill capacity and there are stockpile bins available for each period. It is considered that the exact amount of ore sent to the stockpile in period t will be reclaimed after the stockpiling duration ts controlled by the planner.

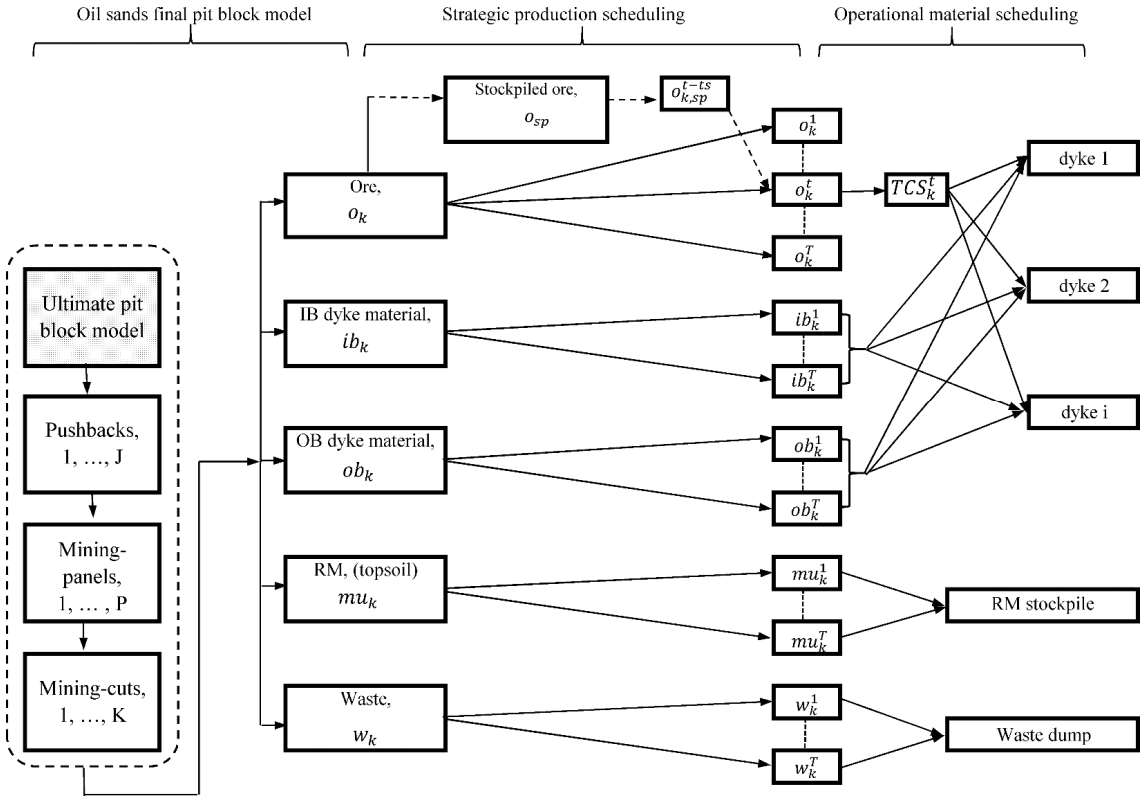


Fig. 1. Schematic representation of the problem definition showing strategic production and operational material scheduling.

3. Summary of Literature Review

Long-term production planning (LTPP) focuses mainly on ore reserves, stripping ratio and major annual investment plans (Newman, et al., 2010). The geologic block model is the backbone of open pit mine design and scheduling processes. Assigning the geological characteristics of each block and their grade can be done using available estimation techniques. Using financial and metallurgical data, the economic value of each block is also calculated (Osanloo, et al., 2008). Since the 1960s, researchers have studied and applied Mathematical Programming Models (MPMs) such as Linear Programming (LP), Integer Programming (IP), and Mixed Integer Linear Programming (MILP). These are commonly used in addition to Dynamic Programming (DP) and Goal Programming (GP) for mine production scheduling (Osanloo, et al., 2008).

Johnson (1969) introduced linear programming as a MPM to the mine planning research area. The author's model was for a long-term multi-destination open pit production planning problem. The results were not optimum and the size of the problem was computationally intractable. Subsequently, the initial LP model was modified by Gershon (1983) and Dagdelen (1985) to a MILP model. The authors considered a set of binary variables to satisfy the precedence of block extraction. The modified models could handle multiple ore processing options and multiple grades. However, their formulations could not ensure feasible solutions for all cases. Also, the number of binary variables makes the model intractable for real-size mine planning projects and difficult to be solved with the current state of hardware and software.

Ramazan and Dimitrakopoulos (2004a) developed MILP formulations to reduce the number of binary variables and solution times. They set certain variables as binary and others as continuous. Their model resulted in partial mining of blocks that have the same ore value affecting the NPV generated. Askari-Nasab, et al. (2011) developed MILP models that use block clustering

techniques. The models use a combination of continuous and binary integer variables and were applied to a large-scale problem. The authors stated that they successfully implemented the models for some basic large-scale production scheduling problems. A Dynamic Programming (DP) model that maximizes the NPV, subject to production and processing constraints was presented by Osanloo, et al. (2008). This model considers both the time value of money and block sequencing to determine the UPL. However, it cannot be applied to large-scale problems and there is no guarantee that mining and processing constraints will be satisfied. In general, applying MPMs to the LTPP result in large scale optimization problems with many integer and continuous variables which are difficult to solve with the available software and hardware and might need lengthy solution time. The efforts that have been made in reducing the solution time were inefficient for large-scale problems or could not generate integrated practical mining strategies.

One of the deterministic approaches used to solve long-term production planning and scheduling problems is Goal Programming (GP). It is a popular deterministic approach for solving multiple objective optimization problems. The main idea of GP is that the optimizer provides results for the objective very close to the required goals, regardless of whether the goals are achievable or not. GP minimizes the deviations between the target values of the objectives and the satisfying solution (Orumie and Ebong, 2014). Ben-Awuah and Askari-Nasab (2013) formulated the oil sands LTPP and waste disposal planning problem using a combination of MILP and GP formulations. The hybrid termed as Mixed Integer Linear Goal Programming (MILGP) has an objective function, goal functions and constraints. These goals are prioritized according to the impact of a deviation from their targets on the entire mining operation. The authors stated that using MILGP is appropriate for their framework because, based on the importance of the goals, the MILGP structure will allow the planner to achieve some goals while others are traded off. According to the authors, solutions with known optimality limits are generated when using exact solution methods for LTPP problems. For the resulting production schedule, a higher NPV is achieved as the solution gets closer to optimality.

In LTPP, the size of the problem grows exponentially as the number of blocks increases resulting in insufficient computer memory during optimization. Researchers have tried to classify the large amount of data into relatively few classes of similar entities (cluster) by maximizing both intra-cluster similarity and inter-cluster dissimilarity. This classification is known as aggregation or clustering. Clustering will minimize the number of integer decision variables as well as maintaining the minimum mining width for large mining equipment (Askari-Nasab and Awuah-Offei, 2009). Clustering algorithms can be categorized into hierarchical, partitional or overlapping clustering (Tabesh and Askari-Nasab, 2011). In mine planning, only hierarchical and partitional clustering can be used because all blocks must belong to a single cluster. Tabesh and Askari-Nasab (2011) developed a new clustering approach more suitable to the mining industry. Paneling is another technique that has been introduced in production scheduling to maintain practical mining widths and reduce the size of the optimization problem. The intersections of pushbacks and mining benches generate mining panels (Ben-Awuah and Askari-Nasab, 2013). Each mining-panel contains a set of mining-cuts and is used to control the mine production operation sequencing. For this paper, hierarchical clustering algorithm developed by Tabesh and Askari-Nasab (2011) was used.

In the implementation of most LP and MILP models, the material flow post-extraction is not considered (Moreno, et al., 2017). “In particular, the use of stockpiling to manage processing plant capacity, and the interplay of material flows from the mine to a stockpile, the mine to a processing plant, and a stockpile to a plant, have not been treated as an integrated part of mine extraction sequence optimization” (Moreno, et al., 2017). Stockpiling can be used in mine operations for many reasons such as the blending of material, storage of overproduced ore or low-grade ore for future processing, and storage of waste material for reclamation purposes. Asad (2005) cautioned that long-term stockpiling could result in problems such as leaching, deterioration of material and oxidation, which might result in poor recovery in the treatment process. For oil sands mining, the stockpiled material must also be processed within a limited duration due to oxidation that affects efficiency of

the processing recovery process. For this paper, the stockpiling duration is limited to a maximum of two years to ensure there is no significant effect on the ore recovery.

The geologic block model is the main input data for optimizing LTPP problems. Some random input variables such as grades, rock types, costs, prices, and recoveries might affect the optimal solution. In deterministic approaches, the best-estimated values of these random variables available at the time are used for the optimization run. A rerun is required when new data become available. Uncertainty can be reduced only by getting more data over time. Most researches have focused on minimizing the negative impacts of grade, geological, and market uncertainties on production schedules. The uncertainties involved in mine planning can be classified as: 1) orebody model and in-situ grade uncertainty and material type distribution; 2) technical mining specification uncertainty, for example, mining and processing capacities and slope consideration; and 3) financial uncertainties including prices and costs (Dimitrakopoulos, 1998). Results have shown that significant differences might exist between actual production and theoretical expectations due to geological and economic uncertainties (the most important sources of risk in mining operations), especially in the first years of production. A loss of $1.4 billion to the Canadian mining industry in early 1991 has been reported due to geological and economic uncertainties (Sabour and Dimitrakopoulos, 2011). A survey of mining operations in the early production years show that 60% of mines had 70% less production than designed capacity (Osanloo, et al., 2008). As hardware, software, and solution techniques evolve, more accurate models are expected (Osanloo, et al., 2008).

Gholamnejad and Osanloo (2007) presented an uncertainty-based model for production scheduling. They considered grade uncertainty in which each block has a probability distribution function obtained using geostatistical simulation. However, they do not give details or examples. Koushavand, et al. (2009) presented two different methodologies based on grade uncertainty for open pit mine production schedules. They evaluated the output parameters such as NPV, ore tonnage, head grade, stripping ratio, amount of final production and annual target production. They stated that there is significant uncertainty in the long-term production schedules. Also, the long-term schedule based on one particular simulated ore body model is not optimal for other simulated geological models. Dimitrakopoulos and Ramazan (2008) presented a stochastic integer programming model to generate the optimal production schedule using multiple realizations as input. They used a penalty scheme for the deviation from the target production. The function is calculated from a Geological Discount Rate (GDR). They use linear programming to maximize NPV minus penalty costs. They stated that the production schedule is the optimum solution. However, they did not show how to define the GDR parameter. Sabour and Dimitrakopoulos (2011) built their model based on metal price and exchange rate uncertainties. They developed a system for mine planning and scheduling that instantly reacts to new information. They claimed that their results showed a significant difference and improved the design-ranking process.

Solving MPMs with deterministic approaches generate solutions within known limits of optimality. Though, they result in large-scale optimization problems that are difficult to solve with the current computing software and hardware and may have lengthy solution times. In order to decrease the size of the optimization problem, clustering is used in mine planning. It generates practical schedules, the formulations are faster to solve and it is easy to implement. Stockpiling is introduced for material blending, storage of overproduced ore or low-grade ore for future processing, and storage of waste material for reclamation purposes. Geological and economic uncertainties are also used in mine planning to minimize the differences between actual production and theoretical expectations that will result in a high degree of confidence in the NPV of the mining project. The aim of this research is to propose a MILGP optimization framework that integrates oil sands production planning and waste management for multiple material types, elements and destinations with limited duration stockpiling. The research also aims to introduce an uncertainty-based MILGP model that evaluates production schedule financial risk associated with grade uncertainty.

4. Oil Sands Mining and Waste Management

Oil sands mining is one of the most rapidly developing industries in North America. This industry started in the 1960s with surface mining operations that used Clark Hot Water Extraction (CHWE) to extract bitumen from the McMurray formation (Masliyah, 2010 ; Sanford, 1983). A truck-shovel system is used to extract the oil sands from the Athabasca Wabiskaw-McMurray deposit, which is located in the northeastern part of the province of Alberta. It is the largest deposit in the world and mostly located near the surface. The Pleistocene unit is the topmost layer of the formation. It contains muskeg, also known as bog peat, which is comprised mainly of organic matter. The Clearwater formation overlying the McMurray formation is comprised of marine clay, fine sand and siltstone. Both the Pleistocene and Clearwater formation are known as Overburden (OB). The McMurray formation contains bitumen, the element of interest. It is informally subdivided into Upper, Middle and Lower formations based on the environment of sediments deposition. Devonian carbonates mark the end of the oil sands deposit (Masliyah, 2010). Fig. 2 shows a sketch of the vertical soil profile for an oil sands formation.

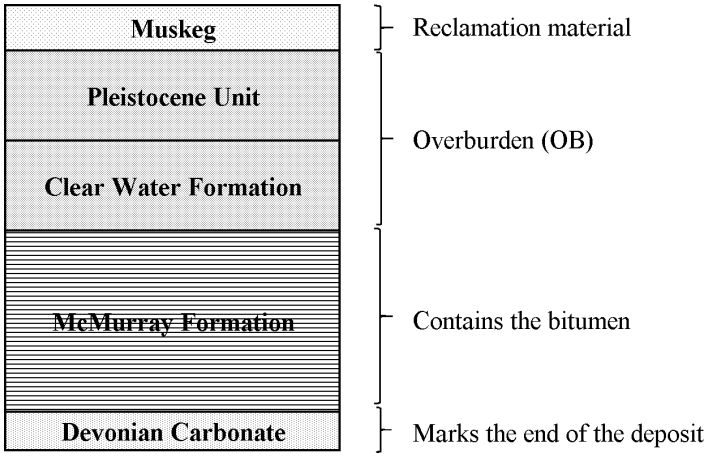


Fig. 2. Schematic view of the soil profile for an oil sands formation.

During oil sands mining, huge amounts of bituminous sands are sent to the processing plant, which results in a huge amount of a mixture of water, fine materials, sands and residual bitumen known as tailings (Masliyah, 2010). There are three significant aspects in dealing with oil sands tailings, the most unwanted by-product of oil sands processing. First, the greenhouse gas emissions resulting from the CHWE process (Devenny, 2009). Second, the environmental challenges due to the toxicity of the tailings resulting in the contamination of the fresh water table by polluted tailings' water leaks. Third, space limitations increases the need for in-pit tailings containment, and storage space since more mining processes lead to additional volume of tailings slurry (Devenny, 2009).

Presently, plans for tailings deposition and mine reclamation are prepared after the optimization of the long-term mine production plans (Ben-Awuah and Askari-Nasab, 2013). Directive 085 (Alberta Energy Regulator, 2017) issued by the Alberta Energy Regulator (AER) requires oil sands operators to periodically publish their waste disposal and tailings plans publically (McFadyen, 2008; Ben-Awuah and Askari-Nasab, 2013).

It should be mentioned that according to Directive 082, any ore with a bitumen grade of 7% or more should not be left behind (Alberta Energy Regulator, 2016). Air flotation technique is used to separate bitumen from the fines results in tailings. Using the hydro-cyclone, tailings is classified into Tailings Coarse Sands (TCS) and Fine Sands (Kalantari, et al., 2013). The ore with a bitumen grade of less than 7%, known as interburden (IB), will be used together with Overburden (OB) and (TCS) for dyke construction if they meet the dyke material requirements. It is important that the sequence of extracting the ore and the supply of material for dyke construction be integrated to guarantee a uniform material supply to the plant and for dyke construction throughout the mine life (Fauquier

and al., 2009). Fig. 3 shows a conceptual mining model, which includes an oil sands deposit area to be mined and simultaneously used as an in-pit tailings storage facility. As mining advances in the specified direction, the in-pit tailings dyke footprints are released for dyke construction.

Waste management is a significant part of oil sands mining operations. It requires special geotechnical considerations and tailings management techniques that may lead to economic liabilities and delayed reclamation if not well managed (Boratynec, 2003; Ben-Awuah and Askari-Nasab, 2013; Azam and Scott, 2005). Additional documentation on oil sands solid waste and tailings management can be found in Ben-Awuah, et al., (2012); Badiozamani and Askari-Nasab, (2014); Badiozamani and Askari-Nasab, (2016).

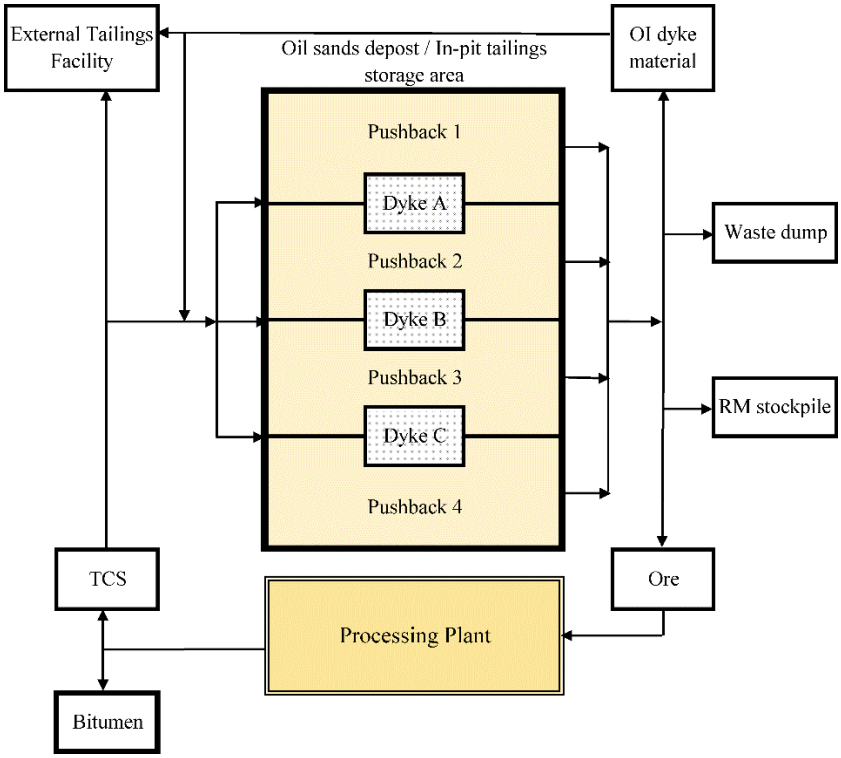


Fig. 3. Conceptual model for mining and waste management strategy modified after Ben-Awuah and Askari-Nasab (2013).

5. Block Modeling, Variography and Kriging

For this paper, a block model was created using Ordinary Kriging (OK) as the best linear unbiased estimator (Isaaks and Srivastava, 1989). For oil sands resource modeling, it has been found that OK with a large number of search data results in a low observed mean squared error (Deutsch, et al., 2014). This makes OK a preferred resource estimation technique. With Geovia GEMS (Gemcom Software International, 2015a), the block model was used to define the orebody for reserve estimation and mine planning. The creation of interpolation profiles was required to determine the spatial correlation between the observations. A semi-variogram was used for this purpose. Variogram models are used in kriging estimation procedures, which can be used further to construct search parameters for interpolation techniques. To create a variogram model, an experimental variogram was calculated for the oil sands McMurray Formation being used as case study. Omnidirectional variograms for bitumen grades were prepared to identify the sill while vertical variograms were used to identify the nugget effect. Primary variogram maps were calculated to determine the orientation of the major axis in the presence of anisotropy. The directional horizontal variogram (major axis) was calculated and modeled. The secondary variogram maps (semi-major axis) were calculated and

modeled. The variograms were extracted along the ellipse axis. The major, semi-major and minor axes were selected for final modeling. Search ellipse profiles and semi-variogram profiles were updated from the semi-variogram models and used for interpolation within the ore rock types for bitumen and fines grades.

Ordinary Kriging estimates are calculated for each block in the block model. Using the concept of Block Kriging (Gringarten and V. Deutsch, 2001), these kriging estimates are used together with the sill, the nugget effect and the range to calculate the Ordinary Kriging grade and variance for each mining-cut and mining-panel in the final pit limit. The kriged estimates together with a variance penalty scheme are used for mine planning to estimate the financial risk (risk) associated with grade uncertainty as explained in Section 5.2.

5.1. Kriged estimates and a variance penalty scheme

Ordinary Kriging (OK) estimates the expected value of a spatial variable (the grade in this case) and the surrounding uncertainty at a given location (Isaaks and Srivastava, 1989). It is the best linear unbiased estimator since it tries to have the mean residuals equal to zeros and aims to minimize the variance of the errors. OK uses statistical distance instead of geometric distance, which controls redundancy of estimation samples. As mentioned in Section5, the spatial continuity for bitumen and fines grades was modeled with semi-variograms, so the pattern and anisotropy are incorporated into the estimation using OK. The general equation for anisotropic spherical variogram is given by Eq. (1). The variogram models for bitumen and fines consists of two nested spherical models Table 1.

$$\gamma(h)=\begin{cases} C_0 & if\ \ h=0.0 \\ C_0+C\times\left(1.5\left(\frac{h}{r}\right)-0.5\left(\frac{h}{r}\right)^3\right) & if\ \ 0<h<r \\ C_0+C & if\ \ h\geq r \end{cases} \quad (1)$$

Table 1. Variogram model for bitumen and fines grades.

Element	Nugget Effect, C_0	Spherical 1		Spherical 2	
		Sill, C	Range, r	Sill, C	Range, r
Bitumen	0.026	0.673	46.34	0.292	46.61
Fines	0.000	0.600	29.15	0.400	44.13

Using the semi-variograms, the kriged grade and kriged variance are determined using OK. The kriged variance is robust to most errors likely to be made in the semi-variogram model selection for spherical models. However, the nugget effect should be carefully selected to avoid understating the kriged variance (Brooker, 1986).

The kriged block estimates (bitumen grades), sill, nugget effect and the range are used to determine the mining-cut and mining-panel kriged grades and kriged variances following the concept of Block Kriging. The estimated kriged variances for all mining-panels are used to calculate the frequency and probability of occurrence of the kriged variance. The latter is used to calculate the expected variance of the mining-panels.

5.2. The effect of grade uncertainty

Grade uncertainty affects the metal content in the material sent to the plant which will subsequently affects the NPV of a project. This uncertainty exists because of our ignorance or lack of knowledge. It is not an inherent feature of the deposit (Isaaks and Srivastava, 1989).

For this study, the effect of grade uncertainty on the mine plan is investigated through the grade variance. A more robust method is to minimize the grade-variance effect while the expected value

of NPV is maximized. It is called the mean-variance method. The grade uncertainty can affect the Economic Block Values (EBV) that will affect the production plan due to the variation in ore tonnages sent to the mill. Consequently, this will affect the NPV of the project. This approach works based on the mean-variance method referred to as the Modern Portfolio Theory (MPT) for risk-based portfolio optimization (Markowitz, 1952). MPT requires calculating the expected value and expected variance of a portfolio's return considering a weighted average combination of the assets' return (Fig. 4). MPT is a quadratic optimization problem that maximizes the expected return and minimizes the standard deviation. The decision variables are the weights or the portions of each asset's contribution towards the objective function.

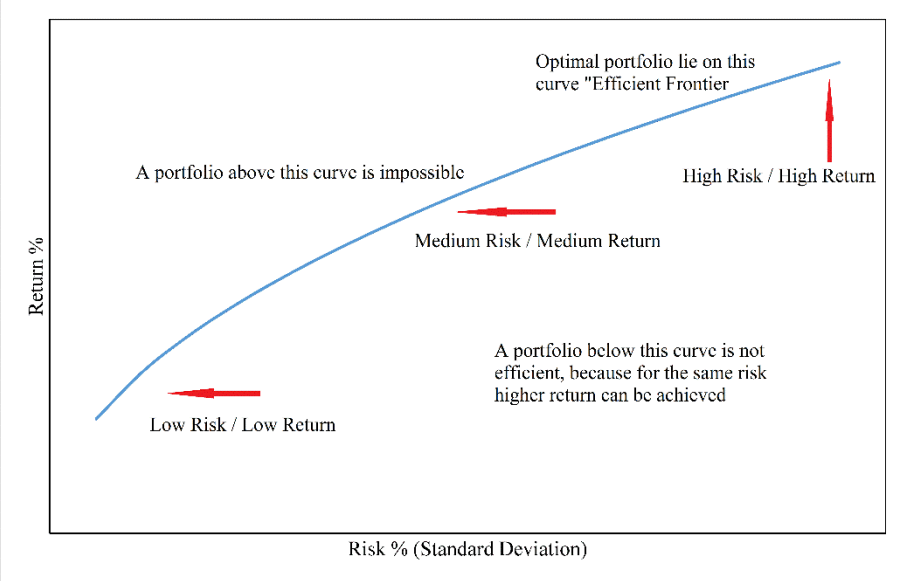


Fig. 4. Return versus standard deviation, modified after dos Santos and Brandi (2017).

In this paper, the mean-variance method is adapted and applied to production planning optimization in the presence of grade uncertainty using an uncertainty-based MILGP model to reduce project risk. The kriged mean and kriged variance for each mining block, mining-cut and mining-panel are calculated as well as the EBVs. The main concept of the mean-variance method deployed for long-term mine planning is to find the low variance mining blocks, mining-cuts or mining-panels to be extracted earlier, such that the average project NPV is maximized and the mining of high-variance blocks, cuts or panels is deferred to later years.

In the uncertainty-based MILGP model, the kriged variance for mining-panels together with a penalty scheme is used to minimize the risk associated with the production schedule. Applying the penalty postpones mining of high variance mining-panels to later years. Fig. 5 shows the kriged variance for mining-panels within the UPL for a case study. This paper investigates two options in applying the variance penalty for mining-panels. The first option is to apply the penalty for high variance mining-panels only. In this case, the expected variance of the deposit is the threshold above which the variance of a mining-panel is classified as high. The expected variance is used to determine whether a mining-panel should be penalized or not. Mining-panels with kriged variance greater than the expected variance will be penalized. The second option is to apply the penalty for all mining-panels. The optimizer will give preference to the mining of low variance mining-panels earlier in the mine life to reduce the risk associated with achieving the production schedule. The term "grade uncertainty cost" is used in this study to represent a pseudo cost for each mining-panel calculated as a product of a penalty value and the mining-panel kriged variance. Subsequently, the grade uncertainty cost is used to enforce the mining of low variance mining-panels early in the mine life to reduce project risk. The "overall uncertainty cost" is therefore a quantitative parameter which estimates the difference in NPV of the production schedule from the MILGP model and uncertainty-

based MILGP model due to grade uncertainty. The grade uncertainty cost is calculated from Eq. (2) .

$$\text{Grade uncertainty cost } \left(PN_u\right) = \text{Penalty cost} \times \text{mining-panel kriged variance} \quad (2)$$

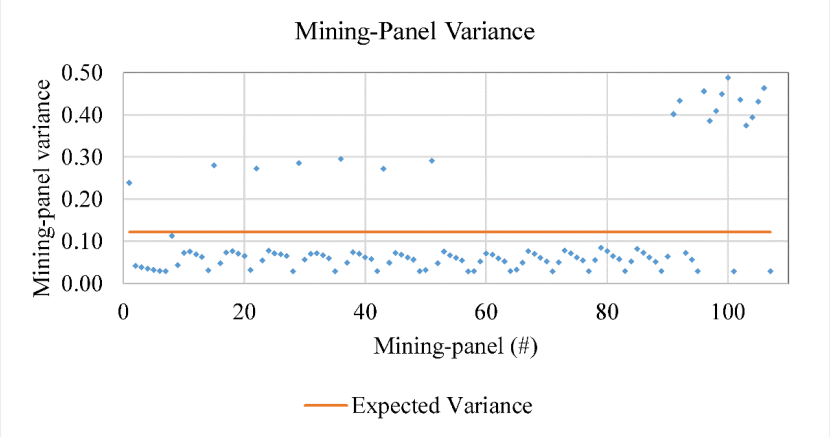


Fig. 5. Kriged variance for the mining-panels within the ultimate pit limit.

5.3. Penalty cost determination

The penalty cost is controlled by the planner. It starts from zero and increases gradually until the production schedule starts to change and consequently the NPV. As the penalty applied increases, the optimizer changes the production schedule, looking for low-variance mining-panels to be mined earlier to minimize the risk associated with the production schedule. At some point, increasing the penalty cost has no effect on the production schedule and subsequently the NPV (Fig. 6). Low penalty means high NPV and high risk, and vice versa. Increasing the penalty will minimize the risk that results in high degree of confidence in the mining project.

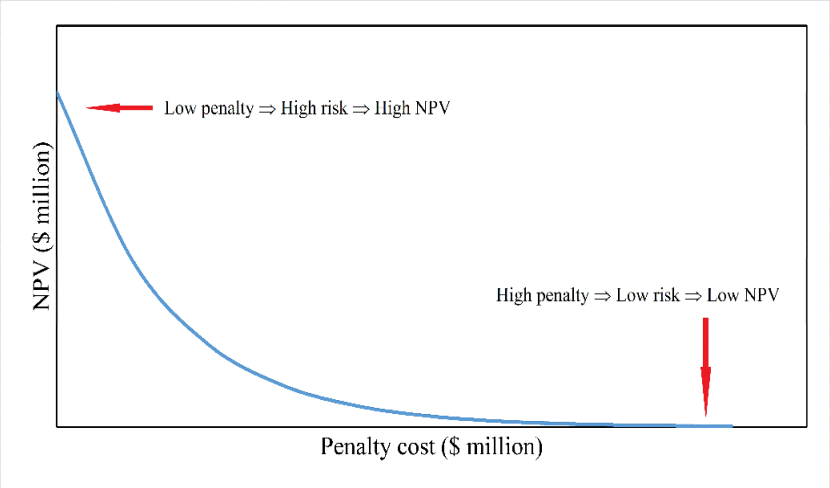


Fig. 6. Net present value versus penalty cost.

6. MILGP Theoretical Model Formulation

The strategic production schedule considers the time and sequence of extracting the ore, muskeg, overburden, interburden and waste blocks, as well as their destinations from a predefined UPL. The proposed MILGP model is capable of considering multiple mining locations, multiple pushbacks and different types of materials. The stockpiled ore can be reclaimed after open pit mining is completed or simultaneously during active open pit mining with a pre-determined reclamation duration.

However, long-term stockpiling could result in problems such as leaching, the deterioration of material and oxidation, which can affect the processing recovery efficiency. For oil sands mining, to avoid the risk of oxidation, the ore will be reclaimed in a pre-determined period controlled by the planner. The proposed oil sands production scheduling model integrates waste management through dyke construction and stockpiling for a limited duration. Stockpiling is for the mined ore that exceeds the plant capacity in any given year. The MILGP model is subject to economic, technical and physical constraints that control the mining operation. The notations used in the formulation of the oil sands long-term production planning and waste management framework have been classified as indices, sets, parameters and decision variables. The details of these notations can be found in the list of nomenclature in Appendix.

6.1. Modeling of economic mining-cut value

Based on the value of the mining-cut and the costs incurred during mining and processing operations, the discounted economic value of each mining-cut can be calculated after applying a discount rate to calculate its present value since the mining-cuts are extracted in different periods. The discounted economic mining-cut value can be calculated using the following formula:

Discounted Economic Mining-Cut Value = Discounted Revenue – Discounted Costs

For a mining-cut, if there are valuable elements, its discounted economic value if it is sent from the mine to the processing plant ($dm_k^{d,t}$) or from the stockpile to the processing plant ($ds_{k,sp}^{d,t}$) is given by Eqs. (3) and (4), respectively.

$$dm_k^{d,t} = rm_k^{a,e,t} - dw_k^{l,t} - dmu_k^{d,t} - dob_k^{d,t} - dib_k^{d,t} - dt_k^{d,t} \tag{3}$$

$$ds_{k,sp}^{d,t} = rs_k^{a,e,t} - dw_k^{l,t-ts} - dmu_k^{d,t-ts} - dob_k^{d,t-ts} - dib_k^{d,t-ts} - dt_k^{d,t-ts} \tag{4}$$

Eqs. (5) to (11) define the parameters used in Eqs. (3) and (4). Eq. (5) defines the discounted revenue generated from selling the final product within each mining-cut k minus the discounted processing cost; for mining-cuts sent directly from the mine to the processing plant. Eq. (6) defines the discounted revenue generated from selling the final product within each mining-cut k minus the discounted processing cost, minus the extra discounted re-handling cost; for mining-cuts sent from the mine through the stockpile to the processing plant. Eq. (7) defines the discounted cost for extracting mining-cut k as waste. Eq. (8) shows the extra discounted cost of re-handling reclamation material (Muskeg unit). Eqs. (9) to (11) show the extra discounted cost of mining the OB, IB and TCS dyke materials from mining-cut k, respectively.

$$rm_k^{a,e,t} = \sum_{e=1}^{E} o_k \times g_k^e \times rp_{avg}^{a,e} \times \left(p^{e,t} - sc^{e,t} \right) - \sum_{e=1}^{E} o_k \times pc^{a,e,t} \tag{5}$$

$$rs_k^{a,e,t} = \sum_{e=1}^{E} o_k \times g_k^e \times rp_{avg,sp}^{a,e} \times \left(p^{e,t} - sc^{e,t} \right) - \sum_{e=1}^{E} o_k \times pc^{a,e,t} - \sum_{e=1}^{E} o_k \times pc_{sp}^{a,e,t} \tag{6}$$

$$dw_k^{l,t} = (o_k + mu_k + ob_k + ib_k + w_k) \times mc^{l,t} \tag{7}$$

$$dmu_k^{d,t} = mu_k \times muc^{d,t} \tag{8}$$

$$dob_k^{d,t} = ob_k \times obc^{d,t} \tag{9}$$

$$dib_k^{d,t} = ib_k \times ibc^{d,t} \tag{10}$$

$$dt_k^{d,t} = t_k \times tc^{d,t} \tag{11}$$

6.2. Uncertainty-based MILGP objective function

The MILGP model objective function for oil sands long-term production planning and waste management is developed based on deterministic and risk-based approaches. The objective functions in general aim to:

- Maximize the NPV of the project;
- Minimize financial risk (risk) associated with the production schedule;
- Minimize reclamation material mining cost;
- Minimize dyke material mining cost;
- Minimize deviations from the production goals.

To develop the models, the concepts presented in Ben-Awuah and Askari-Nasab, (2013) are used as the starting point. The formulation uses continuous decision variables, $y_p^{l,t}$, $x_k^{a,t}$, $c_{k,sp}^{a,t}$, $v_k^{d,t}$, $z_k^{d,t}$, $u_k^{d,t}$, and $q_k^{d,t}$ to model mining, processing from mine, processing from stockpile, RM, and OB, IB and TCS dyke material requirements, respectively; for all mining locations, and processing and dyke construction destinations. Continuous decision variables are used to allow for fractional extraction of mining-panels and mining-cuts in different periods for different locations and destinations. The model features the parameter grade uncertainty cost, PN_u, which is a pseudo cost used to enforce preferential mining of low variance mining-panels. This is used to estimate risk associated with the generated production schedule. Continuous deviational variables, $dv_1^{-,l,t}$, $dv_2^{-,a,t}$, $dv_3^{-,d,t}$, $dv_4^{-,d,t}$, $dv_5^{-,d,t}$, and $dv_6^{-,d,t}$ are defined to support the goal functions that control mining, processing, RM, OB, IB and TCS dyke materials, for all mining locations and processing, reclamation and dyke construction destinations. They provide a continuous range of units (tonnes) that can be determined by the optimizer to satisfy the set goals. In the objective function, these deviational variables are minimized. There are also deviational penalty cost and priority parameters in the objective function used to model the focus of mine management in the presence of multiple conflicting goals. The deviational penalty cost parameters PN_1, PN_2 , PN_3, PN_4, PN_5, and PN_6 penalize the NPV for any deviation from the set goals. The priority parameters, P_1 , P_2, P_3, P_4, P_5, and P_6 are used to place emphasis on the most important goals. In general, the deviational penalty cost and priority parameters are set up to penalize the NPV if the set goals and the most important goals are not met. When setting up these parameters, the planner has to monitor how continuous mining proceed period by period, the uniformity of tonnages mined per period, and the corresponding NPV generated, to keep track of how parameter changes affect these key performance indicators. More weight should be assigned to a goal that has a higher priority for mine management.

The uncertainty-based objective function presented in Eq. (12), aims to maximize the NPV of the project, minimize grade uncertainty cost, minimize reclamation and dyke material mining cost, and minimize deviations from the production goals. The production schedule risk is estimated through the grade uncertainty cost calculated by applying a mining-panel grade-variance penalty scheme to generate varying production schedules with different NPVs.

$$Max \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{d=1}^{D}\sum_{sp=1}^{SP}\sum_{a=1}^{A}\sum_{e=1}^{E}\sum_{t=1}^{T}\left(\sum_{\substack{k\in MK_p \\ p\in Mp_j}}\begin{bmatrix}\left(rm_k^{a,e,t}\times x_k^{a,t}+rs_{k,sp}^{a,e,t}\times c_{k,sp}^{a,t}-(dw_p^{l,t}+PN_u)\times y_p^{l,t}\right)- \\ \left(dmu_k^{d,t}\times v_k^{d,t}\right)-\left(dob_k^{d,t}\times z_k^{d,t}+dib_k^{d,t}\times u_k^{d,t}+dt_k^{d,t}\times q_k^{d,t}\right) \\ -P_1\left(PN_1\times dv_1^{-,l,t}\right)-P_2\left(PN_2\times dv_2^{-,a,t}\right)-P_3\left(PN_3\times dv_3^{-,d,t}\right) \\ -P_4\left(PN_4\times dv_4^{-,d,t}\right)-P_5\left(PN_5\times dv_5^{-,d,t}\right)-P_6\left(PN_6\times dv_6^{-,d,t}\right)\end{bmatrix}\right) \quad (12)$$

6.3. MILGP model goal functions

There are tonnage targets for all mining locations and processing destinations as well as tonnage targets for muskeg as reclamation material and dyke material for dyke construction destinations. Eqs. (13) to (18) represent all required goal functions. Eq. (13) defines the mining goal function that controls the total amount of material mined from mining-panel p within pushback j in each period. The negative allowable deviation from the set mining goal is controlled by the planner using the $dv_1^{-,l,t}$ decision variable. Eq. (14) defines the processing goal function that controls the total amount of ore from mining-cut k within mining-panel p sent from both the mine and stockpile in each period to the processing destination. It should be mentioned that the amount of ore sent to the processing destination from the stockpile in period t and the amount of ore sent to the stockpile from the mine in period $t-ts$ must be equal. The negative allowable deviation from the set processing goal is controlled by the planner using the $dv_2^{-,a,t}$ decision variable. Eq. (15) defines the RM tonnage goals that control the total amount of muskeg material to be mined from mining-cut k within mining-panel p in each period. The negative allowable deviation from the set RM goal is controlled by the planner using the $dv_3^{-,d,t}$ decision variable. OB, IB and TCS dyke material goal functions control the dyke material production targets for different dyke construction destinations. These are defined by Eqs. (16) to (18), respectively. These functions provide a feasible schedule for dyke construction. The negative allowable deviation from the set OB, IB and TCS dyke material goals are controlled by the planner using $dv_4^{-,d,t}$, $dv_5^{-,d,t}$ and $dv_6^{-,d,t}$ decision variables.

$$\sum_{j=1}^{J}\left(\sum_{p\in MP_j}\left(o_p+mu_p+ob_p+ib_p+w_p\right)\times y_p^{l,t}\right)+dv_1^{-,l,t}=Mg^{l,t} \tag{13}$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}}\left(o_k\times x_k^{a,t}\right)+\sum_{sp=1}^{SP}\sum_{\substack{k\in MK_p\\ p\in MP_j}}\left(o_k\times c_{k,sp}^{a,t}\right)\right)+dv_2^{-,a,t}=Pg^{a,t} \tag{14}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(mu_k\times v_k^{d,t}\right)\right)+dv_3^{-,d,t}=MUg^{d,t} \tag{15}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(ob_k\times z_k^{d,t}\right)\right)+dv_4^{-,d,t}=OBg^{d,t} \tag{16}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(ib_k\times u_k^{d,t}\right)\right)+dv_5^{-,d,t}=IBg^{d,t} \tag{17}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(cs_k\times q_k^{d,t}\right)\right)+dv_6^{-,d,t}=CSg^{d,t} \tag{18}$$

6.4. MILGP model constraints

6.4.1. Stockpiling capacity constraints

Constraints that control ore stockpile tonnages are presented in Eqs. (19) and (20). These equations control the amount of ore sent from mining-cut k to stockpile sp in period t. Material sent to the stockpile in period t are reclaimed in period $t+ts$, where ts is the stockpiling duration controlled by the planner. The planner also controls the upper and lower capacity limits for stockpile bins.

$$\sum_{sp=1}^{SP}\sum_{k=1}^{K} o_k \times s_k^{sp,t} \le \overline{os}^{sp,t} \tag{19}$$

$$\sum_{sp=1}^{SP}\sum_{k=1}^{K} o_k \times s_k^{sp,t} \ge \underline{os}^{sp,t} \tag{20}$$

6.4.2. Bitumen and fines grade blending constraints

The MILGP bitumen and fines grade blending constraints ensure that the quality requirements of the processing plant, stockpile and dyke construction destinations are achieved. These constraints are formulated using Eqs. (21) to (30). Ore bitumen grade blending constraints ensure the extracted ore from mining-cut k within mining-panel p sent to either processing destination a or to stockpile sp in period t meets the grade quality requirements. Ore bitumen grade blending constraints are formulated using Eqs. (21) to (24). Eqs. (21) and (22) represent inequality constraints that control the limiting ore bitumen grade sent from the mine and stockpile to the processing plant. Eqs. (23) and (24) represent inequality constraints that control the limiting ore bitumen grade sent from the mine to the stockpile.

$$\sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) - \overline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) \le 0 \tag{21}$$

$$\underline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) - \sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) \le 0 \tag{22}$$

$$\sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times s_k^{a,t} - \overline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times s_k^{a,t} \le 0 \tag{23}$$

$$\underline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times s_k^{a,t} - \sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times s_k^{a,t} \le 0 \tag{24}$$

Ore fines grade blending constraints ensure the extracted ore from mining-cut k within mining-panel p sent to either processing destination a or to stockpile sp in period t meets the fines requirements. Interburden fines grade blending constraints also ensure that the interburden fines for dyke construction are within the upper and lower limits required. Fines grade blending constraints are formulated using Eqs. (25) to (30). Eqs. (25) and (26) represent inequality constraints used to control the limiting grade of ore fines sent from the mine and stockpile to the processing plant. Eqs. (27) and (28) represent inequality constraints used to control the limiting grade of ore fines sent from the mine to the stockpile. Eqs. (29) and (30) represent inequality constraints used to control the limiting grade of interburden dyke material fines sent from the mine to dyke construction destinations.

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p \\ p\in MP_j}} o_k \times fn_k^e \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) - \overline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p \\ p\in MPj}} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) \le 0 \tag{25}$$

$$\underline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p \\ p\in MPj}} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) - \sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p \\ p\in MP_j}} o_k \times fn_k^e \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) \le 0 \tag{26}$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} o_k \times fn_k^e \times s_{k,sp}^{a,t}\right) - \overline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MPj}} o_k \times s_{k,sp}^{a,t}\right) \le 0 \tag{27}$$

$$\underline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MPj}} o_k \times s_{k,sp}^{a,t}\right) - \sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} o_k \times fn_k^e \times s_{k,sp}^{a,t}\right) \le 0 \tag{28}$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} ib_k \times fn_k^{ib} \times u_k^{d,t}\right) - \overline{fn}^{d,t,ib}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MPj}} ib_k \times u_k^{d,t}\right) \le 0 \tag{29}$$

$$\underline{fn}^{d,t,ib}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MPj}} ib_k \times u_k^{d,t}\right) - \sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} ib_k \times fn_k^{ib} \times u_k^{d,t}\right) \le 0 \tag{30}$$

6.4.3. Mining-panels extraction precedence constraints

Five precedence constraints presented in Eqs. (31) to (35) are used to define the precedence extraction sequence for each mining panel p based on its spatial location. These equations use the binary integer decision variable b_p. This variable is equal to one if the extraction of mining-panel p has started by or in period *t*; otherwise, it is zero. Specifically:

- Eq. (31) defines the vertical mining precedence. Prior to the extraction of a specific mining-panel, all the mining-panels above it must be extracted so that the mining-panel is accessible. The set $IP_p(Z'')$ represents the set of immediate mining-panels that are above mining-panel p.
- Eq. (32) defines the horizontal mining precedence. Prior to the extraction of a specific mining-panel, all the mining-panels in a specified horizontal mining direction on a level must be extracted. The set $IH_p(Z')$ represents the set of immediate mining-panels in the specified horizontal mining direction.
- Eq. (33) defines the pushback mining precedence. Eq. (33) checks all the mining-panels within the immediate predecessor pushback that must be extracted prior to the extraction of mining-panels in pushback *j*. The set $MP_j(H'')$ represents the set of mining panels in the predecessor pushback.
- Eq. (34) ensures that mining-panel p can only be extracted if it has not been extracted before.
- Eq. (35) ensures that once the extraction of a mining-panel starts in period t, this mining-panel is available for extraction during the subsequent periods.

$$b_p^t - \sum_{c=1}^{c}\sum_{m=1}^{t} y_{u_1}^{c,m} \le 0, \quad u_1 \in IP_p(Z'') \tag{31}$$

$$b_p^t - \sum_{c=1}^{c}\sum_{m=1}^{t} y_{u_2}^{c,m} \le 0, \quad u_2 \in IH_p(Z') \tag{32}$$

$$b_p^t - \sum_{c=1}^{c}\sum_{m=1}^{t} y_{u_3}^{c,m} \le 0, \quad u_3 \in MP_j(H'') \tag{33}$$

$$\sum_{c=1}^{c}\sum_{m=1}^{t} y_p^{c,m} - b_p^t \le 0 \tag{34}$$

$$b_p^t - b_p^{t+1} \le 0 \tag{35}$$

6.4.4. Decision variables' control constraints

In the MILGP model, all decision variables used to control mining, processing, stockpiling, reclamation material, dyke materials and goal deviations are continuous variables. Inequality Eq. (36) makes sure that all the material mined as ore (sent to either the processing destination a or the stockpile sp), and all reclamation and dyke materials extracted from the mining-cuts belonging to mining-panel p in period t are less than or equal to the total material mined from mining-panel p in period t from any mining location.

Eq. (37) ensures that the total fractions of ore mined from a mining-cut (sent to either the processing destination a or the stockpile sp) is less than or equal to one. Eq. (38) ensures that the fraction of ore extracted from mining-cut k and sent to the stockpile sp in period $t-ts$ must be equal to the fraction of ore reclaimed from the stockpile sp and sent to the processing plant a in period t; where ts is the stockpiling duration.

Eq. (39) ensures that the fractions of TCS dyke material produced from processed ore is less than or equal to the fractions of ore sent from the mine and stockpile to the processing plant in each period. Eq. (40) ensures that the fractions of mining-panel p extracted and sent to different destinations in different periods is less than or equal to one. Eq. (41) ensures that the total fractions of reclamation material extracted from the mine and sent to its destinations in different periods is less than or equal to one. Eqs. (42) to (44) ensure that the total fractions of dyke materials extracted from the mine (OB and IB) or generated from the processing plant (TCS) and sent to all destinations in different periods is less than or equal to one.

$$\left[\begin{array}{l} \sum_{d=1}^{D}\sum_{sp=1}^{SP}\sum_{a=1}^{A}\sum_{\substack{k\in MK_p \\ p\in MP_j}} \left(o_k \times x_k^{a,t} + o_k \times s_{k,sp}^{a,t} + mu_k \times v_k^{d,t} + ob_k \times z_k^{d,t} + ib_k \times u_k^{d,t}\right) \\ \le \sum_{l=1}^{L}\sum_{p\in MP_p}\left[y_p^{l,t}\left(o_p + mu_p + ob_p + ib_p + w_p\right)\right] \end{array}\right] \tag{36}$$

$$\sum_{a=1}^{A}\sum_{sp=1}^{SP}\sum_{t-1}^{T}\left(x_k^{a,t} + s_{k,sp}^{a,t}\right) \le 1 \tag{37}$$

$$\sum_{a=1}^{A}\sum_{sp=1}^{SP}\sum_{t=1}^{T}\left(c_{k,sp}^{a,t} - s_{k,sp}^{a,t-ts}\right) = 0, \quad t - ts \ge 0 \tag{38}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} q_k^{d,t} \le \sum_{a=1}^{A}\sum_{t=1}^{T} x_k^{a,t} + \sum_{sp=1}^{SP}\sum_{t=1}^{T} c_{k,sp}^{a,t} \tag{39}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} y_p^{d,t} \le 1 \tag{40}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} v_p^{d,t} \le 1 \tag{41}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} z_p^{d,t} \le 1 \tag{42}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} u_p^{d,t} \le 1 \tag{43}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} q_p^{d,t} \le 1 \tag{44}$$

6.4.5. Non-negativity constraints

Eq. (45) ensures that the decision variables for mining, processing, stockpiling (ore sent and reclaimed), RM, OB, IB and TCS dyke material are non-negative. Eq. (46) ensures that the deviational decision variables that support the goal functions are non-negative as well.

$$\mathrm{y}_p^{l,t},\ x_k^{a,t},\ \mathrm{s}_{k,sp}^{d,t},\ \mathrm{c}_{k,sp}^{a,t},\ v_k^{d,t},\ z_k^{d,t},\ u_k^{d,t},\ q_k^{d,t} \quad \ge 0 \tag{45}$$

$$dv_1^{-,l,t},\ dv_2^{-,a,t},\ dv_3^{-,d,t},\ dv_4^{-,d,t},\ dv_5^{-,d,t},\ dv_6^{-,a,t} \quad \ge 0 \tag{46}$$

7. Implementation of the MILGP Framework for an Oil Sands Deposit

A kriged block model is developed using Geovia GEMS Software (Gemcom Software International, 2015a). The ultimate pit limit is generated using Geovia Whittle Software (Gemcom Software International, 2015b) which is based on 3D LG algorithm (Lerchs and Grossmann, 1965). The optimized pit shell from Whittle is used to design the final pit in GEMS software (Gemcom Software International, 2015a). The mining blocks within the pit limit are used as input data for the MILGP model. An agglomerative hierarchical clustering algorithm is used in clustering blocks within each intermediate pushback into mining-cuts (Tabesh and Askari-Nasab, 2011). The intersection between benches and intermediate pushbacks are used in creating mining-panels. MATLAB (Mathworks, 2017) application is used as the programming platform to define the MILGP framework and IBM/CPLEX (ILOG, 2012) solver which uses a branch and cut optimization algorithm is employed to solve the MILGP problem. This section documents the application and results from the developed MILGP model for an oil sands dataset.

Two implementation scenarios highlighting different aspects of the developed MILGP model are outlined in Fig. 7. These scenarios are designed to highlight features of the MILGP model including: 1) integrating waste management, reclamation and limited duration stockpiling strategy into oil sands mine planning (Scenario 1); and 2) estimating the effect of grade uncertainty on the generated NPVs of the project (Scenario 2).

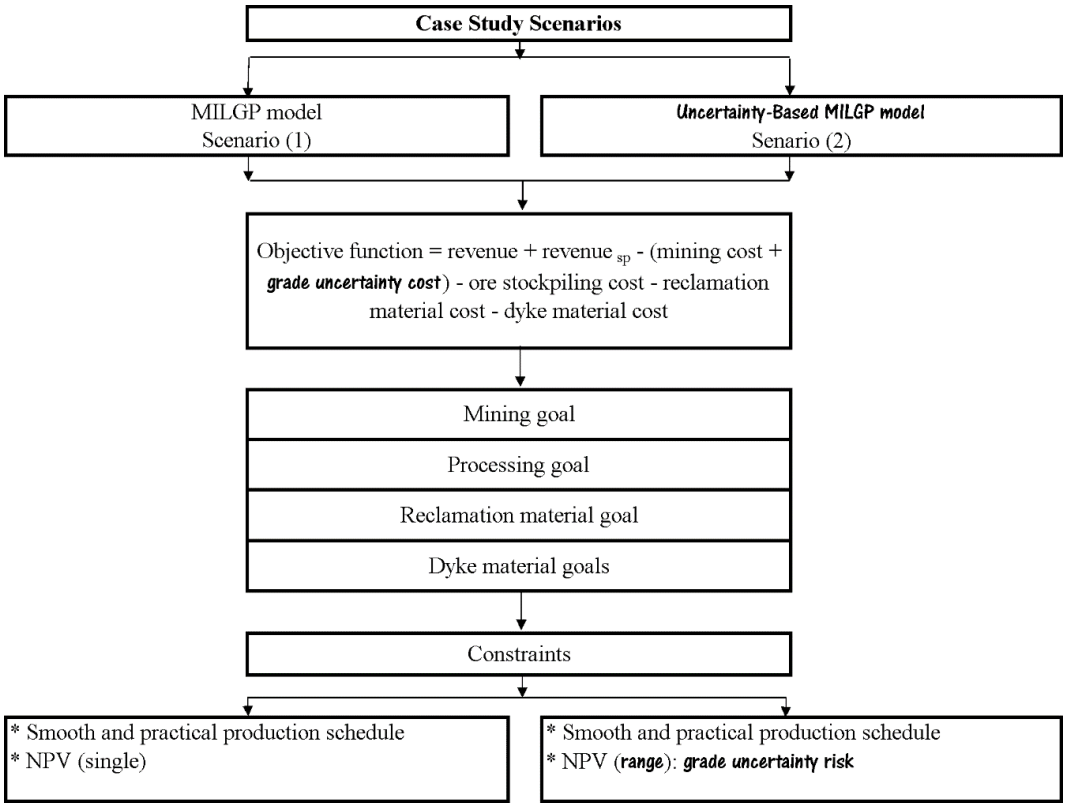


Fig. 7. Case study scenarios.

7.1. MILGP model: Scenario 1

The developed MILGP model for Scenario 1 features goal functions for mining, processing, dyke material, reclamation material and a stockpiling strategy for ore. The ore that exceeds the processing capacity in each period will be stockpiled for a limited stockpiling duration controlled by the mine planner. The revenue generated from processing the stockpiled ore is calculated. The grade uncertainty cost is set to zero. This MILGP model generates a smooth and practical production schedule, and an NPV with known limits of optimality. To model the mining, ore processed from mine, ore processed from stockpile, ore sent to stockpile, RM, OB, IB and TCS dyke material, the following continuous decision variables $y_p^{l,t}$, $x_k^{a,t}$, $c_{k,sp}^{a,t}$, $s_{k,sp}^{d,t}$, $v_k^{d,t}$, $z_k^{d,t}$, $u_k^{d,t}$ and $q_k^{d,t}$ are used respectively in the proposed MILGP model, for all mining locations l, processing destination a and other possible destinations d. To control the precedence of mining-panel extraction, the binary integer decision variable b_k^t is used.

For Scenario 1, continuous deviational variables $dv_1^{-,l,t}$, $dv_2^{-,a,t}$, $dv_3^{-,d,t}$, $dv_4^{-,d,t}$, $dv_5^{-,d,t}$, and $dv_6^{-,d,t}$ are used to support the goal functions that control mining, processing, reclamation material, OB, IB and TCS dyke materials for all locations and destinations. The deviational variables make a continuous range of tonnes available to the optimizer to choose from to satisfy the set goals. These deviational variables are minimized in the objective function. Deviational penalty cost and priority parameters are included in the objective function. Deviational penalty cost parameters $PN_1, PN_2, PN_3, PN_4, PN_5$ and PN_6 are used to penalize the NPV for any deviation from the set goals. The priority parameters P_1, P_2, P_3, P_4, P_5 and P_6 are used to place emphasis on the most important goals. These parameters also penalize the NPV if the most important set goal is not met. While setting up the deviational variables, penalty and priority cost parameters, the smoothness of the mining schedule from one period to the next and the uniformity of tonnages mined per period needs to be monitored together with the corresponding NPV generated. Setting up the priority and

penalty cost depends on whether the planner needs to trade off NPV to meet some set goals consistent with mine management's objective.

7.2. Uncertainty-based MILGP model: Scenario 2

The uncertainty-based MILGP model for Scenario 2 features the developments for the MILGP model in Scenario 1. In addition, the uncertainty-based MILGP model is implemented with a mining-panel grade variance penalty scheme through the grade uncertainty cost to estimate the risk associated with the production schedule. This is implemented using two different risk-based techniques: 1) applying the grade uncertainty cost for high variance mining-panels, and 2) applying the grade uncertainty cost for all mining panels. In applying either technique, different production schedules are generated as the pseudo penalty cost is varied resulting in a range of NPVs. An estimated overall risk value associated with the production schedule can then be calculated. This uncertainty-based MILGP model generates a smooth and practical production schedule, an NPV with known limits of optimality, is easy to setup with more flexibility for the optimizer and provides a quantified risk value associated with the production schedule.

8. Case Study

For this case study, no pushbacks prior to the UPL were considered. However, to create mining-panels, the ultimate pit was divided into four pseudo pushbacks. Blocks in each mining-panel were clustered into mining-cuts using hierarchical clustering algorithm (Tabesh and Askari-Nasab, 2011). Two implementation scenarios as outlined in Section 7 are investigated with the oil sands data. The deposit is to be scheduled for 8 years for the processing plant, reclamation and dyke construction destinations. Summarized information on the oil sands deposit final pit design is presented in Table 2. Table 3 shows the economic parameters and operational capacities for production scheduling. The economic data are extracted and compiled based on (Ben-Awuah and Askari-Nasab, 2013). Table 4 shows the upper and lower bounds of material quality requirements for ore and interburden dyke material. The model is implemented on a Lenovo Think Pad computer with i5 Core at 2.2 GHz, and 8.0 GB of RAM.

8.1. MILGP model: Scenario 1

In Scenario 1 for this case study, the mining operation is limited by mining and processing capacities using the operational capacities in Table 2 and material quality requirements in Table 3. The main target is to achieve a smooth processing rate throughout the mine life and generate a uniform production schedule that generates the highest NPV. The overall NPV generated is $ 18,108.0 M and the results of the production schedule are shown in Table 5 and **Error! Reference source not found.** and **Error! Reference source not found.**.

Table 2. Oil sands deposit final pit design characteristics.

Description	Value
Total tonnage of material (Mt)	5,735.60
Total ore tonnage (Mt)	1,906.90
Total TCS dyke material tonnage (Mt)	1,351.60
Total OB dyke material tonnage (Mt)	1,515.50
Total IB dyke material tonnage (Mt)	1,925.50
Total RM tonnage (Mt)	218.74
Total waste tonnage (Mt)	168.96
Number of blocks	79,095
Number of mining-cuts	4,494
Number of mining-panels	107
Number of benches	8

Table 3. Economic parameters and operational capacities.

Parameter (unit)	Value	Parameter (unit)	Value
Mining cost ($/tonne)	4.60	Processing capacity (Mt/year)	82.59
Processing cost ($/tonne)	5.03	RM capacity (MT/year)	220.00
Ore re-handling cost ($/tonne)	0.50	OB capacity (MT/year)	230.00
Selling price ($/bitumen %mass)	4.50	IB capacity (MT/year)	200.00
TCS dyke material cost ($/tonne)	0.92	TCS capacity (MT/year)	75.00
OB dyke material cost ($/tonne)	1.38	Mining recovery fraction (%)	100.00
IB dyke material cost ($/tonne)	1.38	Processing recovery (%)	90.00
RM extra mining cost ($/tonne)	0.50	Discount rate (%)	10.00
Mining capacity (Mt/year)	239.76		

Table 4. Material quality requirements.

Parameter	Value
Upper bound of ore bitumen grade (wt %)	16.0
Lower bound of ore bitumen grade (wt %)	7.0
Upper bound of ore fines percent (wt %)	30.0
Lower bound of ore fines percent (wt %)	0.0
Upper bound of IB dyke material fines percent (wt %)	50.0
Lower bound of IB dyke material fines percent (wt %)	0.0

Table 5. Production schedule using goal functions with a 2-year ore stockpiling duration from the MILGP model.

Period	Average bitumen grade (wt %)	Material mined (Mt)	Material processed (Mt)
1	8.33	239.76	13.49
2	10.33	239.76	62.43
3	9.96	239.76	67.08
4	10.32	239.76	58.24
5	10.44	239.76	82.59
6	10.51	239.76	82.59
7	10.16	239.76	82.59
8	10.74	239.76	82.59
9	10.80	239.76	82.59
10	11.13	239.76	82.59
11	10.94	239.76	82.59
12	10.44	239.76	82.59
13	9.58	239.76	82.59
14	9.60	239.76	82.59
15	10.64	239.76	82.59
16	11.28	239.76	82.59
17	10.67	221.39	82.59
18	10.58	209.76	82.59
19	10.84	209.76	82.59
20	10.38	209.76	82.59
21	10.65	209.76	82.59
22	9.79	209.76	82.59
23	10.23	209.76	82.59
24	8.89	209.76	82.59
25	9.37	209.76	53.89

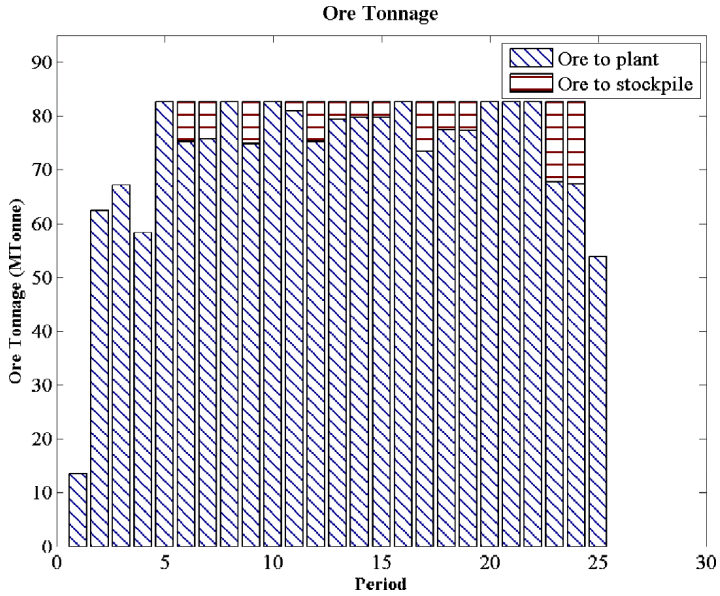


Fig. 8. Processing schedule using goal functions with a 2-year ore stockpiling duration from the MILGP model.

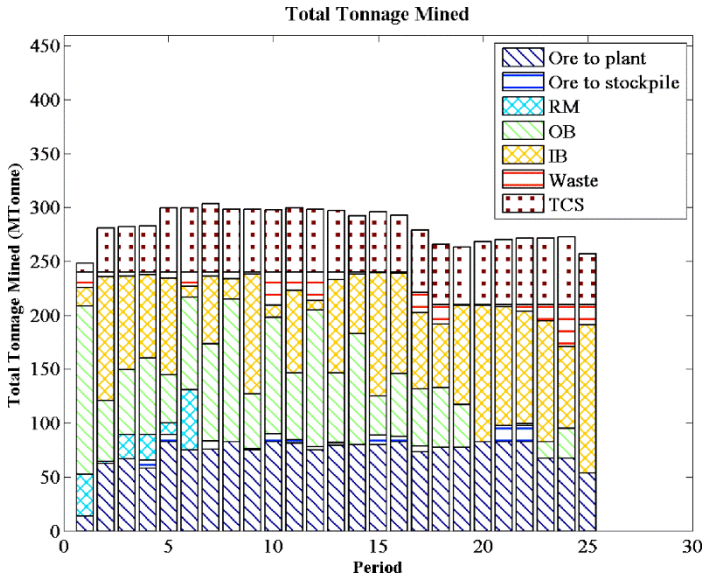


Fig. 9. Production schedule using goal functions with a 2-year ore stockpiling duration from the MILGP model.

8.2. Uncertainty-based MILGP model: Scenario 2

In Scenario 2 for this case study, the mining operation is limited by mining and processing capacities. To investigate in detail the features of the Uncertainty-based MILGP model, the scenario was setup with goal functions and no stockpiling, reclamation material or dyke materials are considered. First, this scenario is run with no penalty applied for mining-panels (Scenario 2A). Two different risk-based techniques are examined: 1) applying penalty cost for only high-variance mining-panels (Scenario 2B) and; 2) applying penalty cost for all mining panels variance (Scenario 2C). The main focus is to achieve a smooth processing rate throughout the mine life and generate a range of NPVs corresponding to the varying penalty cost values. As the penalty cost values increases, the grade uncertainty cost increases, which forces the optimizer to preferentially mine lower variance mining-panels thereby affecting the NPV. The relationship between NPV and penalty cost can be seen in Fig. 10 with selected data points. The selected results of the production schedule using both techniques are shown in Table 6. Fig. 11 shows the production schedule using Scenario 2A which is

similar to 2B and 2C. It can be noticed that the extraction sequence is postponed to later years for some of the high variance mining-panels such as mining-panels number 15, 43, 51, 91, 96, 98, 100 and 103. At the same time, the extraction sequence for some low variance mining-panels are extracted earlier such as mining-panel number 12, 35, 38, 45, 64, 65, 70, 83 and 93. In addition, there is an exceptional effect of penalty cost on the low variance mining-panels such as mining-panels 26, 44, 52, 55 and 87 as the will give access to lower variance mining-panels. It is important to note that, the primary objective function of the uncertainty-based MILGP model is to maximize NPV and hence the optimizer looks for high grade mining-cuts in addition to low mining-panels variance.

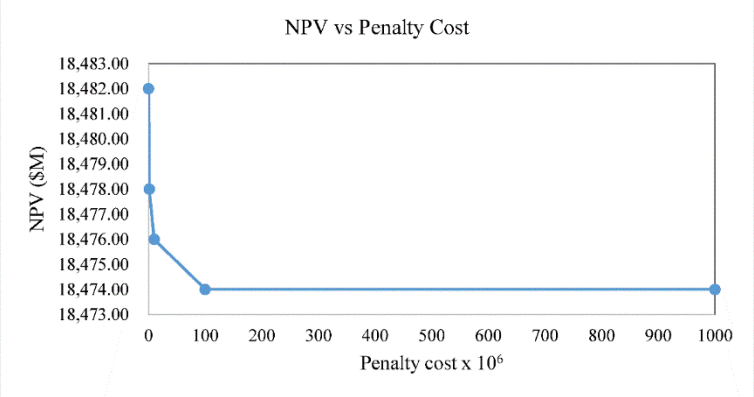


Fig. 10. Relationship between net present value and penalty cost.

Table 6. Production schedule for selected mining-panels using goal functions from the uncertainty-based MILGP model.

Mining-Panel #	Mining-Panel Variance	Extraction Sequence (year)		
		Scenario 2A	Scenario 2B	Scenario 2C
12	0.0691	4	3	3
15	0.2809	3	4	4
26	0.0697	5	6	6
35	0.0291	10	10	8
38	0.0743	7	6	6
43	0.2721	6	7	7
44	0.0487	6	7	7
45	0.0724	8	7	7
51	0.2911	7	8	8
52	0.0485	7	8	8
55	0.0613	10	11	11
64	0.0291	21	17	17
65	0.0328	25	22	22
70	0.0526	18	17	17
83	0.0291	22	21	21
87	0.0617	18	19	19
91	0.4013	17	18	18
93	0.0723	22	19	20
96	0.4564	19	20	20
98	0.4090	20	23	22
100	0.4883	23	24	23
103	0.3747	23	24	24

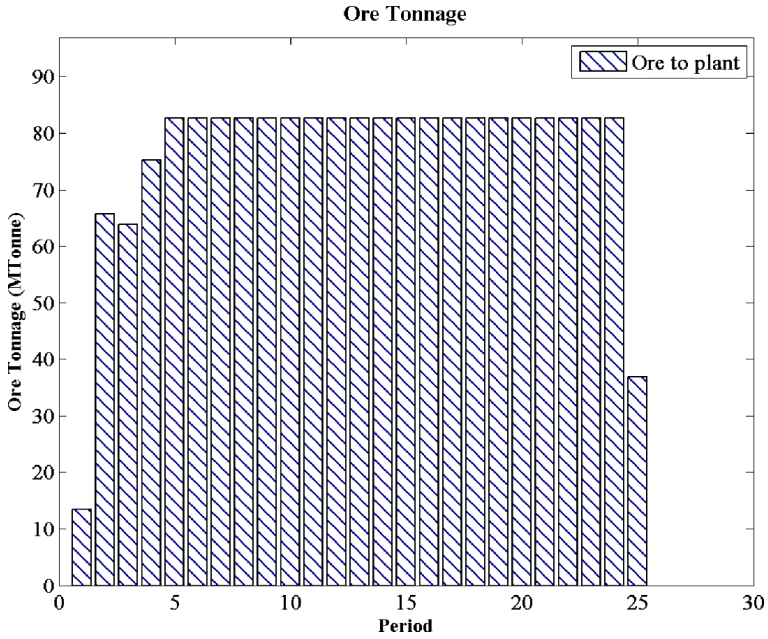


Fig. 11. Processing schedule using goal functions from the uncertainty-based MILGP model (Scenario 2A).

8.3. Discussion of the results

The performance of the MILGP models in Scenarios 1 is analyzed based on: NPV, mining and processing production targets and smoothness and practicality of the generated schedules. Scenario 1 generates smooth schedules for mining, processing, reclamation and dyke materials. The total material mined is 5,735.60 Mt. This is made up of 1,906.9 Mt of ore, 218.74 Mt of RM, 1,515.50 Mt and 1,925.50 Mt of OB and IB dyke materials, whilst 1,351.60 Mt of TCS dyke material is generated. The model generated a uniform production schedule for OB, IB and TCS dyke material over the 25 periods. This ensures the effective utilization of the mining fleet and processing plant throughout the mine life.

The uncertainty-based MILGP model, Scenario 2, is developed based on grade uncertainty using the mining-panels grade variance. The first run (Scenario 2A) no penalty is applied and the NPV obtained is $ 18,482.00 M. For (Scenario 2B), the optimizer penalizes mining-panels with grade variance higher than the expected grade variance, calculated as 0.1219. The penalty cost applied starts from zero (no grade uncertainty cost) and increases gradually until the production schedule starts to change by postponing the mining of high variance mining-panels to later years. As the penalty cost increases, the NPV of the project decreases due to the changes in the production schedule. At some point, increasing the penalty cost has no effect on the production schedule and subsequently the NPV. The NPV obtained ranges from $ 18,482.00 M to $ 18,474.00 M. Thus, the potential risk associated with the production schedule can be estimated as a loss of $ 8.00 M in the most pessimistic case. For (Scenario 2C), the optimizer penalizes all mining-panels to give preference to low variance mining panels to be mined earlier. The NPV obtained ranges from $ 18,482.00 M to $M 18,475.00 M. Thus, the potential risk associated with the production schedule can be estimated as a loss of $ 7.00 M in the most pessimistic case. Among other things, by taking into consideration the NPV range and the production schedule risk, investors can make more pragmatic choices when managing their mining investment risk profile. Table 7 shows the production schedule and grade profile over 8 periods for Scenarios 2A, 2B, and 2C. Fig. 12 shows the grade profile comparisons when no penalty applied and when it is applied for the two risk-based techniques.

Table 7. Production schedule and grade profile for the two risk-based techniques.

Period	Scenario 2A			Scenario 2B			Scenario 2C		
	Mining Schedule (Mt)	Processing Schedule (Mt)	Grade Profile (%)	Mining Schedule (Mt)	Processing Schedule (Mt)	Grade Profile (%)	Mining Schedule (Mt)	Processing Schedule (Mt)	Grade Profile (%)
1	239.76	13.49	8.33	13.49	13.49	8.33	239.76	13.49	8.33
2	239.76	65.71	10.37	65.71	65.71	10.37	239.76	65.71	10.37
3	239.76	63.80	9.89	63.80	63.80	9.89	239.76	63.80	9.89
4	239.76	75.24	10.41	75.24	75.24	10.41	239.76	75.24	10.41
5	239.76	82.59	10.37	82.59	82.59	10.37	239.76	82.59	10.37
6	239.76	82.59	10.34	82.59	82.59	10.34	239.76	82.59	10.34
7	239.76	82.59	10.44	82.59	82.59	10.44	239.76	82.59	10.44
8	239.76	82.59	10.77	82.59	82.59	10.77	239.76	82.59	10.77
9	239.76	82.59	10.74	82.59	82.59	10.74	239.76	82.59	10.74
10	239.76	82.59	10.86	82.59	82.59	10.89	239.76	82.59	10.86
11	239.76	82.59	11.11	82.59	82.59	11.07	239.76	82.59	11.11
12	239.76	82.59	10.43	82.59	82.59	10.43	239.76	82.59	10.43
13	239.76	82.59	9.64	82.59	82.59	9.66	239.76	82.59	9.66
14	239.76	82.59	10.22	82.59	82.59	10.16	239.76	82.59	10.21
15	239.76	82.59	10.86	82.59	82.59	10.23	239.76	82.59	10.85
16	239.76	82.59	10.17	82.59	82.59	10.63	239.76	82.59	10.19
17	209.76	82.59	9.88	82.59	82.59	10.07	209.76	82.59	9.87
18	211.52	82.59	10.70	82.59	82.59	10.79	209.76	82.59	11.07
19	209.76	82.59	11.15	82.59	82.59	11.08	209.76	82.59	10.77
20	209.76	82.59	10.93	82.59	82.59	10.92	209.76	82.59	10.93
21	209.76	82.59	10.67	82.59	82.59	10.64	209.76	82.59	10.66
22	209.76	82.59	10.03	82.59	82.59	10.06	209.76	82.59	10.05
23	209.76	82.59	9.98	82.59	82.59	9.93	210.92	82.59	9.93
24	209.76	82.59	9.25	82.59	82.59	9.29	209.76	82.59	9.29
25	209.76	36.88	8.93	36.88	36.88	8.93	209.76	36.88	8.93

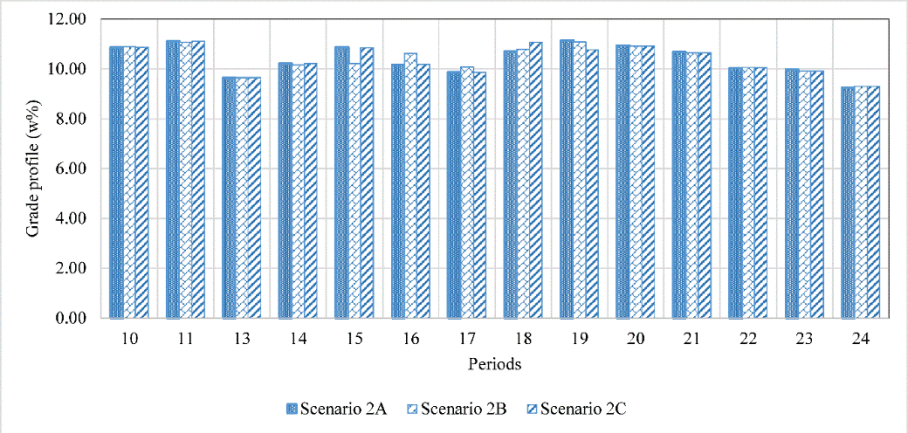


Fig. 12. Grade profile comparison.

9. Conclusions

The proposed uncertainty-based MILGP model for oil sands long-term production planning involves the interactions of their three main subcomponents: the objective function, the goal functions and the constraints in an optimization framework to achieve the research objectives. The model generates a strategic production schedule for the ore, reclamation material and dyke materials using two different scenarios. In Scenario 1, using goal functions for mining and processing with uncertainty penalty cost set to zero, the MILGP model illustrates how production scheduling with limited duration stockpiling strategy for ore can be effectively integrated with waste disposal planning and reclamation material stockpiling in oil sands mining. Based on dyke construction requirements, schedules are generated to provide the required dyke materials to support engineered dyke construction that will help in reducing environmental impacts. This schedule gives the planner good control over dyke materials and provide a solid platform for effective dyke construction and waste management planning.

The effect of grade uncertainty on the NPV of a mining project is investigated using the uncertainty-based MILGP model (Scenarios 2A, 2B, and 2C). The research is based on the concept of mean-variance analysis, which is a process of weighing risk (variance) against expected NPV. The main goal in Scenario 2 is to feed the plant with ore that has less potential grade variation especially in the early years of mine life. By deferring ore with highly uncertain grades to later years, the risk associated with generating the estimated mineral content will be reduced; as well as the potential uncertainty cost associated with the production schedule. The cost of uncertainty is a good indicator of the level of risk associated with generating the expected NPV from the mine plan.

The future work for this research will focus on integrating the optimization of pushback size for waste management in long-term production planning and scheduling.

10. References

[1] Directive 082-Operating Criteria: Resource Recovery Requirements for Oil Sands Mine and Processing Plant Operations, (2016).

[2] Directive 085-Fluid Tailings Management for Oil Sands Mining Projects, (2017).

[3] Asad, M. W. A. (2005). Cutoff grade optimization algorithm with stockpiling option for open pit mining operations of two economic minerals. *International Journal of Surface Mining, Reclamation and Environment, 19*(3), 176-187.

[4] Askari-Nasab, H., and Awuah-Offei, K. (2009). *Mixed integer linear programming formulations for open pit production scheduling*. Edmonton: University of Alberta, Mining Optimization Laboratory Report One.

[5] Askari-Nasab, H., Pourrahimian, Y., Ben-Awuah, E., and Kalantari, S. (2011). Mixed integer linear programming formulations for open pit production scheduling. *Journal of Mining Science, 47*(3), 338-359.

[6] Azam, S., and Scott, J. D. (2005). *Revisiting the ternary diagram for tailings characterization and management*. Edmonton: The Oil Sands Tailings Research Facility.

[7] Badiozamani, M. M., and Askari-Nasab, H. (2014). Integration of reclamation and tailings management in oil sands surface mine planning. *Environmental Modelling & Software, 51*, 45-58.

[8] Badiozamani, M. M., and Askari-Nasab, H. (2016). Integrated mine and tailings planning: a mixed integer linear programming model. *International Journal of Mining, Reclamation and Environment, 30*(4), 319-346.

[9] Ben-Awuah, E., and Askari-Nasab, H. (2013). Incorporating waste management into oil sands long term production planning. *Transactions of the Institution of Mining and Metallurgy, Section A, 122*(1), 33-45.

[10] Ben-Awuah, E., Askari-Nasab, H., and Awuah-Offei, K. (2012). Production scheduling and waste disposal planning for oil sands mining using goal programming. *Journal of Environmental Informatics, 20*(1), 20-33.

[11] Boratynec, D. J. (2003). *Fundamentals of rapid dewatering of composite tailings.* Unpublished MSc, University of Alberta, Edmonton.

[12] Brooker, P. I. (1986). A Parametric Study of Robustness of Kriging Variance as a Function of Range and Relative Nugget Effect for a Spherical Semivariogram. *Mathematical Geology, 18*(5), 12.

[13] Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno, E., and Rubio, E. (2012). A new algorithm for the open-pit mine production scheduling problem. *Operations Research, 60*(3), 517-528.

[14] Dagdelen, K. (1985). *Optimum multi-period open pit mine production scheduling by Lagrangian parameterization.* Unpublished PhD, University of Colorado, Colorado.

[15] Deutsch, J.L., Szymanski, J., and Deutsch, C.V. (2014). Checks and measures of performance for kriging estimates. *Journal of the Southern African Institute of Mining and Metallurgy, 114*, 223-223.

[16] Devenny, D. W. (2009). *Overview of oil sands tailings*. Calgary: Alberta Energy Research Institute.

[17] Dimitrakopoulos, R. (1998). Conditional simulation algorithms for modelling orebody uncertainty in open pit optimisation. *International Journal of Mining, Reclamation and Environment, 12*, 173-179.

[18] Dimitrakopoulos, R., and Ramazan, S. (2008). Stochastic integer programming for optimising long term production schedules of open pit mines: Methods, application and value of stochastic solutions. *Institute of Materials, Minerals and Mining, 117*(4), 155-160.

[19] dos Santos, Silvio Francisco, and Brandi, Humberto Siqueira. (2017). Selecting portfolios for composite indexes: application of Modern Portfolio Theory to competitiveness. *Clean Technologies and Environmental Policy, 19*(10), 2443-2453.

[20] Fauquier, R., and al., et. (2009). *In-pit dyke construction planning*. Ft. McMurray: Shell Upstream Americas.

[21] GemcomSoftwareInternational. (2015a). Gemcom GEMS (Version 6.7.1). Vancouver: Gemcom Software International Inc.

[22] GemcomSoftwareInternational. (2015b). Whittle strategic mine planning software (Version 4.5.5). Vancouver: Gemcom Software International Inc.

[23] Gershon, M. E. (1983). Optimal mine production scheduling: evaluation of large scale mathematical programming approaches. *International Journal of Mining Engineering, 1*(4), 315-329.

[24] Gholamnejad, J., and Osanloo, M. (2007). Using chance constrained binary integer programming in optimising long term production scheduling for open pit mine design. *Institute of Materials, Minerals and Mining, 116*(2), 58-66.

[25] Gringarten, E., and V. Deutsch, C. (2001). *Teacher's Aide Variogram Interpretation and Modeling* (Vol. 33).

[26] Horst, R., and Hoang, T. (1996). *Global optimization: deterministic approaches* (3rd ed.). New York: Springer.

[27] Hustrulid, W. A., and Kuchta, M. (2006). *Open pit mine planning and design* (2nd ed.): Taylor and Francis/Balkema.

[28] ILOG, IBM. (2012). CPLEX reference manual and software (Version 12.5). Pullman, WA, USA: ILOG S.A. and ILOG Inc.

[29] Isaaks, Edward H., and Srivastava, R. Mohan. (1989). *An Introduction to Applied Geostatistics*. New York: Oxford University Press, Inc.,.

[30] Johnson, T. B. (1969). *Optimum open-pit mine production scheduling.* Paper presented at the 8th International Symposium on the Application of Computers and Operations Research in the Mineral Industry, Utah.

[31] Kalantari, Samira, Ben-Awuah, Eugene, and Askari-Nasab, Hooman. (2013). Towards an integrated oil sands mine plan and composite tailings plan. *International Journal of Mining, Reclamation and Environment, 27*(2), 103-126.

[32] Koushavand, B., Askari-Nasab, H., and Deutsch, C.V. (2009). Transfer geological uncertainty through mine planning. *MPES. MPES-International Symposium of Mine Planning/Equipment Selection*.

[33] Mangram, M. (2013). *A Simplified Perspective of the Markowitz Portfolio Theory* (Vol. 7).

[34] Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance, 7*(1), 77-91.

[35] Masliyah, J. (2010). *Fundamentals of oilsands extraction*. Edmonton: University of Alberta.

[36] Mathworks, Inc. (2017). MATLAB Software (Version 9.2 (R2017a)) [Software]: Mathworks Inc.

[37] McFadyen, D. (2008). *Directive 074*. Calgary: Energy Resources Conservation Board.

[38] Moreno, E., Rezakhah, M., Newman, A., and Ferreira, F. (2017). Linear models for stockpiling in open-pit mine production scheduling problems. *European Journal of Operational Research, 260*(1), 212-221.

[39] Newman, Alexandra M., Rubio, Enrique, Caro, Rodrigo, Weintraub, Andres, and Eurek, Kelly. (2010). A review of operations research in mine planning. *Interfaces, 40*(3), 222-245.

[40] Orumie, U. C., and Ebong, D. (2014). A Glorious Literature on Linear Goal Programming Algorithms *American Journal of Operations Research, 4*, 59-71.

[41] Osanloo, M., Rashidinejad, F., and Rezai, B. (2008). Incorporating Environmental Issues into Optimum CutOff Grades Modeling at Porphyry Copper Deposits. *Resources Policy, 33*(4), 222-229.

[42] Ramazan, S., and Dimitrakopoulos, R. (2004a). *Recent applications of operations research and efficient MIP formulations in open pit mining.* Paper presented at the SME Annual Meeting, Cincinnati, Ohio.

[43] Sabour, S. A., and Dimitrakopoulos, R. (2011). Incorporating Geological and Market Uncertainties and Operational Flexibility into Open Pit Mine Design. *Journal of Mining Science, 47*(2).

[44] Sanford, E. C. (1983). Processibility of athabasca oil sand: Interrelationship between oil sand fine solids, process aids, mechanical energy and oil sand age after mining. *The Canadian Journal of Chemical Engineering, 61*(4), 554-567.

[45] Tabesh, M., and Askari-Nasab, H. (2011). Two stage clustering algorithm for block aggregation in open pit mines. *Transactions of the Institution of Mining and Metallurgy, Section A, 120*(3), 158-169.

[46] Whittle, J. (1989). *The facts and fallacies of open pit design*. North Balwyn, Victoria, Australia: Whittle Programming Pty Ltd.

[47] Wolsey, L. A. (1998). *Integer programming*. New York: J. Wiley.

11. Appendix

Indices

$a \in A, A = \{1.....,A\}$	index and set for all possible processing destination in the model.
$d \in D, D = \{1,.....,D\}$	index and set for all possible destinations for materials in the model.
$e \in E, E\{1,....,E\}$	index and set for all elements of interest in each mining-cut.
$j \in J, J = \{1,.....,J\}$	index and set for all pushbacks in the model.
$k \in K, K = \{1,.....,K\}$	index and set for all mining-cuts in the model.
$l \in L, L = \{1,.....,L\}$	index and set for all possible mining locations (pits) in the model.
$p \in P, P = \{1,.....,P\}$	index and set for all mining-panels in the model.
$sp \in SP, SP = \{1,....,SP\}$	index and set for all possible stockpiles in the model.
$t \in T, T = \{1,....,T\}$	index and set for all the scheduling periods, years.
$ts \in TS, TS = \{1,....,TS\}$	index and set for all possible stockpiling durations, years.

Sets

$IH_p(Z')$ For each mining-panel *p*, there is a set $IH_p(Z') \subset P$ defining the immediate predecessor mining-panels in a specified horizontal mining direction that must be extracted prior to extraction of mining-panel *p* at the specified level, where *Z′* is the total number of mining-panels in the set $IH_p(Z')$.

$IP_p(Z'')$ For each mining-panel *p*, there is a set $IP_p(Z") \subset P$ defining the immediate predecessor mining-panels above mining-panel *p* that must be extracted prior to extraction of mining-panel *p*, where *Z″* is the total number of mining-panels in the set $IP_p(Z'')$.

$MK_p(H')$ For each mining-panel *p*, there is a set $MK_p(H') \subset K$ defining the mining-cuts that belongs to the mining-panel *p*, where *H′* is the total number of mining-cuts in the set $MK_p(H')$.

$MP_j(H'')$ For each phase *j*, there is a set $MP_j(H'') \subset P$ defining the mining-panels within the immediate predecessor pit phases (pushbacks) that must be extracted prior to extracting phase *j*, where *H″* is an integer number representing the total number of mining-panels in the set $MP_j(H'')$.

Decision Variables

$b_p^t \in [0,1]$ Binary integer variable controlling the precedence of extraction of mining-panels. b_p^t is equal to one if the extraction of mining-panel *p* has started by or in period *t*, otherwise it is zero.

$c_k^{a,t+ts} \in [0,1]$ Continuous variable representing the ore portion of mining-cut *k* to be reclaimed and processed at destination *a* in period *t+ts*.

$dv_1^{-,l,t}$ Negative deviation from the mining goal (tonnes) in period *t* at location *l*.

$dv_2^{-,a,t}$ Negative deviation from the processing goal (tonnes) in period *t* at processing destination *a* (tonnes).

$dv_3^{-,d,t}$ Negative deviation from the reclamation material goal (tonnes) in period *t* at destination *d* (tonnes).

$dv_4^{-,d,t}$ Negative deviation from the overburden dyke material goal (tonnes) in period *t* at destination *d* (tonnes).

$dv_5^{-,d,t}$ Negative deviation from the interburden dyke material goal (tonnes) in period t at destination d (tonnes).

$dv_6^{-,d,t}$ Negative deviation from the tailings coarse sand dyke material goal in (tonnes) period *t* at destination *d* (tonnes).

$q_k^{d,t} \in [0,1]$ Continuous variable representing the tailings coarse sands dyke material portion of mining-cut *k* to be extracted and used for dyke construction at destination *d* in period *t*.

$s_k^{sp,t} \in [0,1]$ Continuous variable representing the ore portion of mining-cut *k* to be extracted and sent to the stockpile *sp* in period *t*.

$u_k^{d,t} \in [0,1]$ Continuous variable representing the interburden dyke material portion of mining-cut *k* to be extracted and used for dyke construction at destination *d* in period *t*.

$v_k^{d,t} \in [0,1]$ Continuous variable representing the muskeg reclamation material portion of mining-cut *k* to be extracted and stockpiled at destination *d* in period *t*.

$x_k^{a,t} \in [0,1]$ Continuous variable representing the ore portion of mining-cut *k* to be extracted and processed at destination *d* in period *t*.

$y_p^{l,t} \in [0,1]$ Continuous variable representing the portion of mining-panel *p* to be mined in period *t* from location *l*, which includes ore, overburden and interburden dyke material, muskeg reclamation material and waste from the associated mining-cuts.

$z_k^{d,t} \in [0,1]$ Continuous variable representing the overburden dyke material portion of mining-cut *k* to be extracted and used for dyke construction at destination *d* in period *t*.

Parameters

$CSg^{d,t}$ Tailings coarse sand dyke material goal in period *t* at destination *d* (tonnes).

cs_k Tailings coarse sand dyke material tonnage in mining-cut *k*.

$dib_k^{d,t}$ Extra discounted cost of mining all the material in mining-cut *k* in period *t* as interburden dyke material for dyke construction at destination *d*.

$dm_k^{d,t}$ Discounted economic mining-cut value obtained by extracting mining-cut *k* and sending it to destination *d* in period *t*.

$dmu_k^{d,t}$ Extra discounted cost of mining all the material in mining-cut *k* in period *t* as muskeg reclamation material at destination *d*.

$dob_k^{d,t}$ Extra discounted cost of mining all the material in mining-cut *k* in period *t* as overburden dyke material for dyke construction at destination *d*.

$ds_{sp,k}^{d,t}$ Discounted economic mining-cut value obtained by extracting mining-cut *k* and sending it to stockpile *sp* and reclaiming it to destination *d* in period *t*.

$dt_k^{d,t}$ Extra discounted cost of mining all the material in mining-cut *k* in period *t* as tailings coarse sand dyke material for dyke construction at destination *d*.

$dw_k^{l,t}$ Discounted cost of mining all the material in mining-cut *k* in period *t* as waste from location *l*.

fn_k^e The average percent of fines in ore portion of mining-cut *k*.

$\overline{fn}^{a,t,e}$ Upper bound on the required average fines percent of ore in period *t* at processing destination *a*.

$\underline{fn}^{a,t,e}$ Lower bound on the required average fines percent of ore in period *t* at processing destination *a*.

fn_k^{ib} The average percent of fines in interburden dyke material portion of mining-cut *k*.

$\overline{fn}^{d,t,ib}$ Upper bound on the required average fines percent of interburden dyke material in period *t* at dyke construction destination *d*.

$\underline{fn}^{d,t,ib}$ Lower bound on the required average fines percent of interburden dyke material in period *t* at dyke construction destination *d.*

g_k^e The required average head grade of element *e* in ore portion of mining-cut *k.*

$\underline{g}^{a,t,e}$ Lower bound on the required average head grade of element *e* in period *t* at processing destination *a.*

$\overline{g}^{a,t,e}$ Upper bound on the required average head grade of element *e* in period *t* at processing destination *a*.

i	The discount rate.
$IBg^{d,t}$	Interburden dyke material goal in period t at destination d (tonnes).
$ibc^{d,t}$	Cost in present value terms per tonne of interburden dyke material for dyke construction at destination d in period t.
ib_k	Interburden dyke material tonnage in mining-cut k.
ib_p	Interburden dyke material tonnage in mining-panel p.
$Mg^{l,t}$	Mining goal (tonnes) in period t at location l.
$mc^{l,t}$	Cost in present value terms of mining a tonne of waste in period t from location l.
$MUg^{d,t}$	Muskeg, reclamation material goal (tonnes) in period t at destination d.
mu_k	Reclamation material tonnage in mining-cut k.
mu_p	Reclamation material tonnage in mining-panel p.
$muc^{d,t}$	Cost in present value terms per tonne of reclamation material at destination d.
o_k	Ore tonnage in mining-cut k.
o_p	Ore tonnage in mining-panel p.
$OBg^{d,t}$	Overburden dyke material goal in period t at destination d (tonnes).
$obc^{d,t}$	Cost in present value terms per tonne of overburden dyke material for dyke construction at destination d in period t.
ob_k	Overburden dyke material tonnage in mining-cut k.
ob_p	Overburden dyke material tonnage in mining-panel p.
$\overline{os}^{sp,t}$	The upper bound of ore tonnage sent to stockpile sp from mining-cut k in period t that exceeds the processing capacity.
$\underline{os}^{sp,t}$	The lower bound of ore tonnage sent to stockpile sp from mining-cut k in period t that exceeds the processing capacity.
$p^{e,t}$	The selling price of element e in present value terms per unit of product.
P_1	Priority level associated with minimizing the deviations from the mining goal.

P_2 Priority level associated with minimizing the deviations from the processing goal.

P_3 Priority level associated with minimizing the deviations from the reclamation material goal.

P_4 Priority level associated with minimizing the deviations from the overburden dyke material goal.

P_5 Priority level associated with minimizing the deviations from the interburden dyke material goal.

P_6 Priority level associated with minimizing the deviations from the tailings coarse sand dyke material goal.

$pc^{a,e,t}$ Extra cost in present value terms per tonne of ore for mining and processing at processing destination *a* in period *t*.

$pc_{sp}^{a,e,t}$ Extra cost in present value terms per tonne of ore for stockpiling at stockpile *sp* and processing at destination *a* in period *t*.

$Pg^{d,t}$ Processing goal in period *t* at destination *d* (tonnes).

PN_1 Penalty paid per tonne in deviating from the mining goal.

PN_2 Penalty paid per tonne in deviating from the processing goal.

PN_3 Penalty paid per tonne in deviating from the reclamation material goal.

PN_4 Penalty paid per tonne in deviating from the overburden dyke material goal.

PN_5 Penalty paid per tonne in deviating from the interburden dyke material goal.

PN_6 Penalty paid per tonne in deviating from the tailings coarse sand dyke material goal.

PN_u Grade uncertainty cost, a pseudo cost for each mining-panel calculated as a product of a penalty value and the mining-panel kriged variance

$rp_{avg}^{a,e}$ Proportion of element *e* recovered (processing recovery) if it is sent from the mine to processing destination *a*.

$rp_{avg,sp}^{a,e}$ Proportion of element *e* recovered (processing recovery) if it is sent from the stockpile to processing destination *a*.

$rm_k^{a,e,t}$ Discounted revenue obtained by selling the final products within mining-cut *k* in period *t* if it is sent to processing destination *a*, minus the extra discounted cost of mining all the material in mining-cut *k* as ore from location *l* and processing at destination *d*.

$rs_k^{a,e,t}$ Discounted revenue obtained by selling the final products within mining-cut k from stockpile sp in period t if it is sent to destination a in period t, minus the extra discounted cost of processing and re-handling

$sc^{e,t}$ Selling cost of element e in present value terms per unit of product.

$tc^{d,t}$ Cost in present value terms per tonne of tailings coarse sand dyke material for dyke construction at destination d in period t.

w_k Waste tonnage in mining-cut k.

w_p Waste tonnage in mining-panel p.

A Transportation Problem Based Stochastic Integer Programming Model to Dispatch Surface Mining Trucks under Uncertainty

Ali Moradi Afrapoli, Mohammad Tabesh and Hooman Askari-Nasab
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

Over the last 50 years several truck allocation and dispatching decision making models have been developed in the field of surface mining operation optimization. Despite the stochastic approaches towards truck allocation problem, the researchers have not considered effects of random parameters on truck dispatching problem. However, most of the factors affecting the truck dispatching decisions are random in behavior such as truck travel time to the next destination. This paper develops a truck dispatching decision making model based on the transportation problem approach. The developed model is a stochastic integer programming model that makes decisions on the trucks dispatching problem under condition of trucks' travel time uncertainty. Besides, a simulation model of an open pit mining operation has been developed to evaluate the developed stochastic truck dispatching model. Results of the implementation of the developed model proves impacts of the randomness in the operational parameters on the truck dispatching decisions.

1. Introduction

In surface mining operations, truck dispatching is the process of determining the best assignment of trucks to the right destinations. All thus far published research have introduced a two stage decision making model to solve truck dispatching problem with the exception of one proposed by Hauck (1973). The two stages are called upper stage or production optimization stage and lower stage or truck dispatching stage (Alarie & Gamache 2002). The set of decisions made in the lower stage is a set of dynamic operational outlines to meet the upper stage targets. Several decision-making models have been developed to address the two sets of decisions.

Researchers have applied different operational research approaches to solve the upper stage problem. White and Olson (1986) developed a two segment linear programming (LP) model that in its first segment it maximizes shovels' dig rate by minimizing total material handling costs (White & Olson 1986). Then, it determines optimum flow rate for each path in the mine. They developed the most popular currently available truck dispatching decision maker tool in the market (Modular Mining DISPATCH (2017)) based on their two segment LP model.

In 1989 a combination of mixed integer LP and none linear programming (NLP) operational research approach was introduced by Soumis et al. (1987) and (1989). The developed model

This paper, in its entirety, has been submitted to be published in the proceedings of 27th International Symposium on Mine Planning & Equipment Selection (MPES), Santiago, Chile, Nov 19-23, 2018.

schedules trucks' travel between any source and destination in the mine in two steps. One important advantage of the model developed by Soumis et al. (1987) and (1989) is that before implementing an NLP model to allocate trucks to shovels, it first runs an MILP to assign shovels to the right working faces.

Following that, Li (1990) published an LP model that was called transportation model for truck dispatching in mines. The model tries to minimize transportation work required for each path. Implementing the developed LP model, optimum number of trucks to meet the path production requirement is determined.

In late 1990s Temeng et al. (1998) developed a truck allocation model that works based on goal programming approach. With respect to ore grade, shovel dig rate, stripping ratio, and dumping points' capacities, the model maximizes shovel production.

A chance-constrained stochastic approach was applied to the upper stage (truck allocation or production optimization) problem by Ta et al. (2005). The developed model determines optimum number of trucks of a kind to be allocated to a path to meet its production requirement in presence of truck cycle time and its capacity uncertainties.

By adding shovel assignment to the mining faces, Gurgur et al. (2011) developed an LP truck allocation model that is capable of minimizing deviation from the strategic level production requirements. Ta et al. (2013) developed a mixed integer LP model that solves truck allocation problem based on the probability of shovel idle time. The objective of the developed model is to minimize total number of trucks required to meet the production target. In the same year, Mena et al. (2013) introduced an availability based mixed integer LP model that tries to solve upper stage problem using knapsack problem approach. The developed model maximizes cumulative truck fleet production within a fixed time horizon. Chang et al. (2015) developed a mixed integer LP model to solve truck allocation problem. The model maximizes transportation revenue with respect to priority of shovels. A heuristic algorithm was developed to solve the model for each operation shift. Readers are encouraged to read Alarie and Gamache (2002) and Moradi Afrapoli and Askari-Nasab (2017) for detailed information regarding the approaches and solution methodologies.

Despite all the aforementioned efforts in developing mathematical models to solve the upper stage problem, a limited number of models can be found in the literature that deal with the lower stage truck dispatching problem. White and Olson (1986) and Soumis et al. (1989) applied assignment problem approach towards making decisions on trucks next destination. Li (1990) developed a model that minimizes the relative difference between the actual time that next truck will arrive to a destination and the time that next truck must be in that destination based on upper stage results. Lizotte et al. (1989) developed a simulation based semi-automated model that assigns trucks to the destinations using three different heuristic models. Temeng et al. (1997) implemented a transportation-based approach towards dealing with the truck dispatching problem. However, there is a limitation in the above-mentioned models. Although most of the input parameters are uncertain and show stochastic behavior, the developed models use deterministic input parameter values. One of the input parameters that is used in many models is truck cycle time. The truck cycle time in a mining operation is subject to fluctuations from its deterministic value due to different factors including road conditions, traffic conditions, intersection blockages, trucks' bunching effects, etc.

The aim of this paper is to minimize the idle time of shovels and trucks through improved dispatch logic that quantifies the impact of empty haulage travel time uncertainty on the haulage cycle time calculations and truck assignments. To achieve the objective of the paper, we present a stochastic integer programming model that makes truck dispatching decisions while considering the uncertainties in truck travel time. The travel time has been selected due to its higher contribution in the material handling cycle time than any other components of a cycle time such as spot time, load time and dump time. To assess the developed model, we implemented it in an open pit mine operation simulation and results are presented here.

2. Model Formulation

The model development consists of two main steps. At the first step of the model development, a deterministic model was developed. Then, implementing the recourse approach (Birge & Louveaux 2011) we developed a stochastic model based on the model developed in the first step to capture uncertainty in truck travel time.

The model presented in this section is a deterministic model with all its input parameters taking deterministic values. It can also be categorized as a mixed integer linear programming model based on transportation problem. The objective function of the model, presented in Eq. (1), minimizes the cumulative absolute time difference between the times truck t will reach shovel s after dumping its at dump d (tts) and the time shovel s will be available to load the next truck (nas). The second part of the objective function tries to maximize the adjustment factor (AF) encouraging the model to maximize a balanced material delivery to all destinations. AF will be explained later on. Finally, VBN stands for very big number.

$$\min Z = \sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{s=1}^{S} C_{tds}x_{tds} + VBN(mf - AF) \quad \forall t \in \{1,...,T\}, \forall d \in \{1,...,D\}, and \; \forall s \in \{1,...,S\} \tag{1}$$

The objective function coefficient for each of the variables is calculated using Eq.(2):

$$\begin{aligned} C_{tds} &= \left|t_{ts} - na_s\right| \\ &= \left|T(t,3+s) - S(4,s)\right| \\ &= \left| ltt_{td} + qt_{td} + dt_{td} + ett_{tds} - \sum_{t'=1}^{TT}\left(tinq_{t's} + tenr_{t's}\right)\times\left(st_{t's} + lt_{t's}\right)\right| \\ &\forall t \in \{1,...,T\} \;\&\; \forall s \in \{1,...,S\} \;\&\; \forall d \in \{1,...,D\} \end{aligned} \tag{2}$$

Where:

ltt_{td}	loaded travel time from current truck t position to dump d
qt_{td}	time truck t must spend in queue at dump d to dump its material
dt_{td}	time truck t spends at dump d to back up and dump its material
ett_{ts}	time truck t spends to travel empty from the dump location d to shovel s
$tinq_{t's}$	time a truck of type t' that is already in queue must spend in shovel s queue
$tenr_{t's}$	time a truck of type t' must travel from its current position to reach shovel s
$st_{t's}$	spot time for a truck of type t' at shovel s
$lt_{t's}$	loading time for a truck of type t' at shovel s

Moreover, the decisions need to meet operational constraints such as trucks' and shovels' supply (Eq. (3) and Eq.(4)), destination demand constraint (Eq. (5)), balancing truck distribution over the paths (Eq. (6)), and binary constraints (Eq. (7)).

$$\sum_{d=1}^{D}\sum_{s=1}^{S} x_{tds} \le 1 \forall t \in \{1,...,T\} \tag{3}$$

$$\sum_{t=1}^{T}\sum_{d=1}^{D} tc_t x_{tds} \le sc_s \;\forall s \in \{1,...,S\} \tag{4}$$

$$\sum_{t=1}^{T}\sum_{s=1}^{S} tc_t x_{tds} \geq AF \times pc_d \;\forall\, d \in \{1,\ldots,D\} \tag{5}$$

$$0 \leq AF \leq mf \tag{6}$$

$$x_{tds} \in \{0,1\}\;\forall t \in \{1,\ldots,T\},\;\; \forall d \in \{1,\ldots,D\}, and\; \forall s \in \{1,\ldots,S\} \tag{7}$$

Where:

x_{tds}	binary integer variable to assign truck t to the path connecting shovel s to dump d
tc_t	capacity of truck t
sc_s	capacity of shovel s
pc_d	capacity of dump d (ton)
AF	adjustment factor that forces model to evenly distribute extra available trucks among all the possible destinations
mf	proportion of the cumulative available trucks' capacity to the cumulative required path flow rate that can be met using the available trucks
pf_{sd}	required path flow rate for path from shovel s to dump d based on upper stage decisions
$pmsf_{sd}$	met so far path flow rate for path from shovel s to dump d

Constraint (3) makes sure that truck t cannot be assigned to more than one shovel. Constraint (4) ensures that summation of nominal capacity of all the trucks assigned to shovel s does not exceed the shovel's nominal digging rate (capacity). AF in constraint (5) is defined as adjustment factor. The adjustment factor is a variable that is forcing the model to evenly distribute the truck fleet capacity between all the destinations and is constrained by mf as in Eq. (6). mf is a matching factor that is calculated based on cumulative available truck capacity and cumulative path material handling requirement. This factor is equal to 1 when the total truck fleet capacity is less than the required path flow rate and is equal to the proportion of the available truck capacity to the total path requirements when there is extra fleet capacity available. The adjustment factor is constrained by mf in order to uniformly distribute the extra truck fleet capacity among all the needy paths to balance ore and waste production.

The presented model uses expected (deterministic) values for the input parameters. However, most of the parameters affecting the truck dispatching decisions are random variables. In this paper, we formulated our model as a stochastic integer programming model with recourse (Birge & Louveaux 2011) to capture uncertainty of one of the major parameters affecting the operation (trucks' empty travel time). Reason to capture uncertainty in trucks' empty travel time is that more than 90% of trucks' cycle time in each cycle is spent in traveling. From that time, about 50% is spent in travel empty. As most of the time a truck needs to be dispatched has already passed some portion of its loaded travel or even completed its loaded travel, the most important parameter where the uncertainty associated with it needs to be captured is empty travel. Thus, the objective function of the stochastic model that captures empty travel time uncertainty is (Eq. (8)):

$$\begin{aligned} Minimize\, Z = & \overbrace{\sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{s=1}^{S} C_{tds} x_{tds}}^{1st} \\ & + \overbrace{VBN(1-AF)}^{2nd} \\ & + \overbrace{\frac{1}{nR}\sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{s=1}^{S}\sum_{r=1}^{nR}\left| ltt_{td} + qt_{td} + dt_{td} + ett_{tds}^{r} - \sum_{t'=1}^{TT}\left(tinq_{t's} + tenr_{t's}\right)\times\left(st_{t's} + lt_{t's}\right)\right| x_{tds}}^{3rd} \end{aligned} \tag{8}$$

Where:

ett^{r}_{tds} time truck t spends to travel empty from the dump location d to shovel s in rth realization

r is an index referring to a scenario in the stochastic integer model

nR number of realizations implemented to generate random variables for empty travel time from its distribution.

In the developed model, the first two components of the objective function are the same as the deterministic model. The third component is the minimization of the truck or shovel idle time in material handling given the uncertainty in trucks empty travel time. The model is constrained with Eq. (3) to Eq. (7). For each of the realizations r in the stochastic model with nR number of realizations, a random value is being sampled from the fitted distribution of the historical data of the empty truck velocity. The sample is then imported into the model after preprocessing procedure that calculates required travel time and is used during the decision making procedure.

The developed model was implemented in a simulation study of an iron ore mine and the results of Key Performance Indicators (KPI) were compared against model developed by White & Olson (1986) which is used as the backbone of the operation optimization in Modular Mining DISPATCH (2017) fleet management system. There are two small size shovels in the operation serving two active processing plants with an input feed rate requirement of 2,300 ton per hour of the operation. Another small shovel alongside with two large ones are digging waste material to meet the stripping ratio requirement.

3. Results

Some key performance indicators (KPI) were chosen to compare results of implementation of the developed stochastic truck dispatching model against the benchmark truck dispatching model as listed in Table 1.

Results of the simulation of the case study show that the required plant feed rates are met when using the developed model as the dispatching logic while plant feed rates will be short by 5 to 10 percent when using the benchmark dispatching logic (Fig. 1).

Table 1. Key performance indicators to assess the developed stochastic model

No.	KPI	Description
1	Plant Feed Rate (t/hr)	Amount of material delivered to each processing plant in an hour of operation
2	Shovels' Utilization (%)	Percentage of shovels' available time being used in the operation
3	Queue Length	Number of trucks lining up in front of a resource when a truck reaches there

Using benchmark truck dispatching model, none of the plants are fed with their required hourly feed rate. Plant 1 is on average 12% short on its hourly feed target rate , whereas plant 2 is on average of 8% short. However, by replacing the benchmark truck dispatching model with the stochastic truck dispatching model, both plants' target rate is met. Fig. 1 also shows that implementing benchmark truck dispatching, plant 2 is fed 4% more than plant 1. It is due to a critical drawback of the benchmark model. The benchmark model dispatches trucks based on minimum distance between trucks and destinations. However, it does not account for the queue that might happen after a truck reaches to the assigned destination. Thus, if we have multiple destinations similar to what we have in this case study, the benchmark model dispatches trucks to the destinations with the shorter distance. Here in this case study, plant two is located about 400

meters closer to the ore shovels than plant one. As a result, plant one is served more by the benchmark model comparing to plant two.

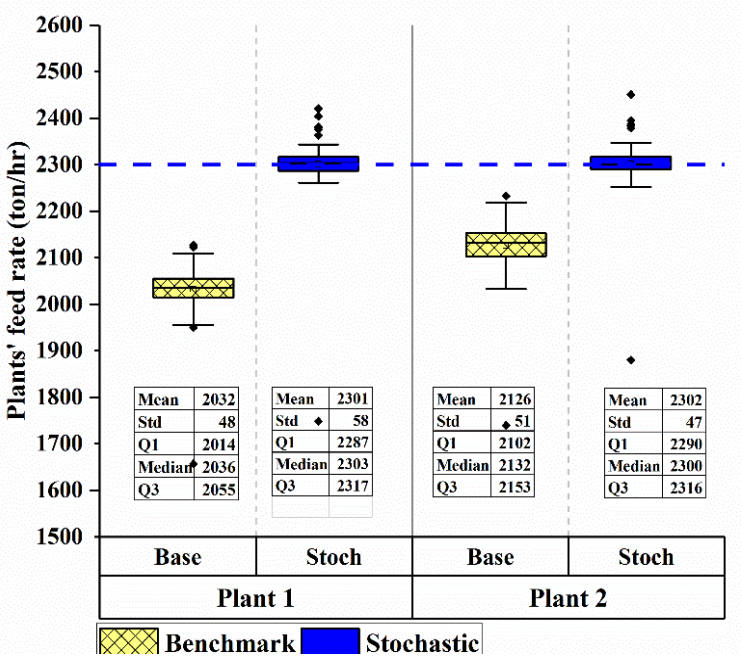


Fig. 1. Ore delivered to processing plants; the horizontal dashed line on 2300 (t/hr) stands for the required plant feed rate for each of the active processing plants in the operation. For each processing plant, the left hand side box plot (hatched yellow) shows plant feed rate using benchmark model and the right hand side box plot (blue) shows the plant feed rate using the proposed model.

This difference in feeding rate is not happening when the benchmark truck dispatching model is replaced by the stochastic truck dispatching model. As one of the advantages of the stochastic model, it does account for all of the possible queueing that will happen when the truck is assigned. Thus, it is capable of feeding the plants equally in a multi plant mining operation.

Another important KPI is the utilization of the shovels as the second most expensive equipment after the processing plant in mining operation. A comparison of shovels' utilization when using the developed and the benchmark dispatching logic is presented in Table 2. Shovel 1 and Shovel 2 feed the processing plants by extracting ore material and Shovel 3, Shovel 4, and Shovel 5 remove waste materials. Shovels working on the waste mining faces are approximately 10% more utilized than the ore shovels when using the benchmark truck dispatching decision making model.

Table 2. Comparison of utilization of active shovels in the operation

	Utilization (%) (benchmark)		**Utilization (%) (stochastic model)**		**Difference (%)**
Shovel	Mean	Standard deviation	Mean	Standard deviation	
1	88.9	1.1	96.4	2.9	8
2	84.7	1.2	95.5	3	11
3	97.5	1.7	75.8	2.8	-22
4	96.4	1.4	76.7	2.9	-20
5	99.9	5.1	90.5	2.6	-9

However, the trend is reversed when using the stochastic model: the ore shovels are utilized more than 90% of their available time whereas the waste shovels are utilized about 80% of their

available time. The difference is mainly due to the fact that the benchmark truck dispatching model makes decisions based on the distance between trucks and the shovels and it does not include the time trucks have to spend in queue at each specific shovel when they reach to that shovel. Though, the stochastic model involves the expected queue time in the decision making procedure.

There are two possibilities when a truck reaches a loader. Either it encounters a shovel standing idle and waiting for the next truck to load or it meets a shovel that is currently loading a truck and has to enter a queue. In either case, the mine is not working efficiently. In the former, shovel is utilized less than its available time and in the latter the truck is losing its available time in queue. Fig. 2 represents a graphical and statistical comparison of the histograms of the number of trucks in queue at shovels when a truck arrives to that shovel between the time the base decision making model makes decisions on the truck dispatching problem and the time the base decision making model is substituted by the stochastic model developed in this study.

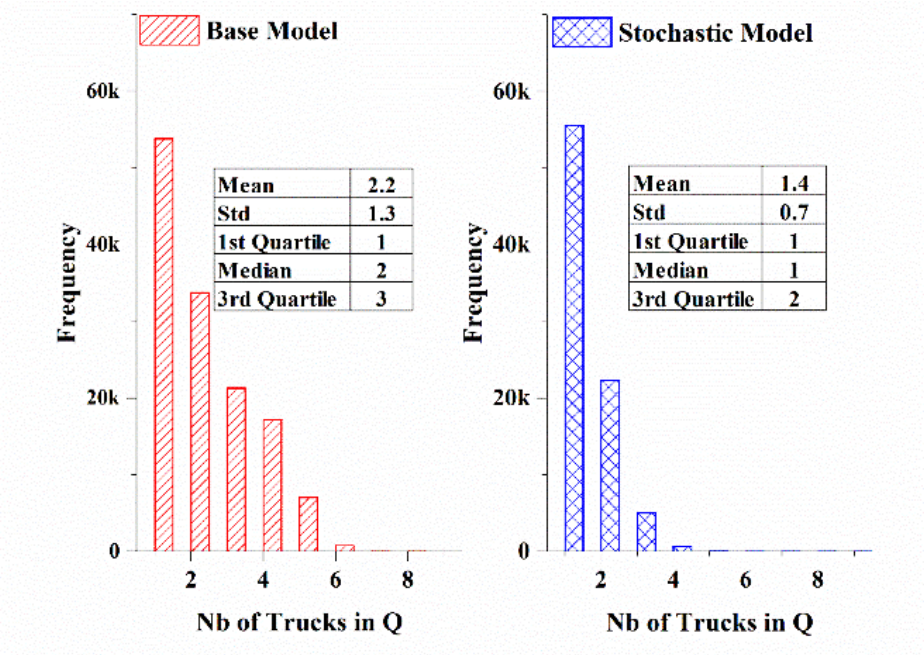


Fig. 2. Histogram of number of trucks waiting in queue when a truck arrives to a loading point when the truck dispatching decisions are made by the base model (red bars) and the time the truck dispatching decisions are made by the stochastic model (blue bars)

Simulation results show that the average number of trucks in queue at shovel when a truck arrives at that shovel decreases by 36% by replacing the benchmark model with the stochastic model. The developed stochastic model also shows a 50% reduction in the median. It means by replacing the base model with the developed model most of the time when a truck reaches to a shovel it encounters with only one truck waiting in queue instead of 2 which happens in the base model. This consequently results in shorter queue time.

4. Conclusion

Truck dispatching problem in the surface mining operations has been addressed in this paper. Despite uncertainties associated with the governing operational parameters, most of the conducted research in the field thus far have ignored random nature of input parameters. However, in this paper, a stochastic integer programming model was introduced that accounts for the uncertainties associated with truck travel times into the truck dispatching problem solving procedure. This helps to make more realistic decisions for trucks optimal destinations. The model was developed based on the transportation problem approach. The model was implemented in a simulated case study and

results of its implementation were compared against results of implementation of the model developed by White & Olson (1986) which is backbone of Modular Mining DISPATCH (2017) as benchmark truck dispatching model from the literature. Results of the comparisons show a promising improvement in all of the measured KPI.

5. References

[1] Alarie, S. & Gamache, M., 2002. Overview of Solution Strategies Used in Truck Dispatching Systems for Open Pit Mines. *International Journal of Mining, Reclamation and Environment*, 16(1), pp.59–76.

[2] Birge, J.R. & Louveaux, F., 2011. Introduction to Stochastic Programming Second., New York, USA.

[3] Chang, Y., Ren, H. & Wang, S., 2015. Modelling and Optimizing an Open-Pit Truck Scheduling Problem. *Discrete Dynamics in Nature and Society*, 2015.

[4] Gurgur, C.Z., 2011. Optimisation of a real-time multi-period truck dispatching system in mining operations Kadri Dagdelen and Songwut Artittong. , 4(1), pp.57–79.

[5] Hauck, R.F., 1973. A Real-Time Dispatching Algorithm for Maximizing Open-Pit Mine Production under Processing and Blending Requirements. *In C. I. of M. and Metallurgy, ed. Seminar on Scheduling in Mining, Smelting and Metallurgy*. Montreal, pp. 1–10.

[6] J. Elbrond & Soumis, F., 1987. Towards integrated production planning and truck dispatching in open pit mines. *International Journal of Mining, Reclamation and Environment*, 1(1), pp.1–6.

[7] Li, Z., 1990. A methodology for the optimum control of shovel and truck operations in open-pit mining. *Mining Science and Technology*, 10(3), pp.337–340.

[8] Lizotte, Y., Bonates, E. & Leclerc, A., 1989. Analysis of truck dispatching with dynamic heuristic procedures. In T. S. Golosinski & V. Srajer, eds. *Proceedings of the International Symposium on Off-Highway Haulage in Surface Mines*. Balkema, Rotterdam, pp. 47–55.

[9] Mena, R. et al., 2013. Availability-based simulation and optimization modeling framework for open-pit mine truck allocation under dynamic constraints. *International Journal of Mining Science and Technology*, 23(1), pp.113–119.

[10] Modular Mining Systems, 2017. DISPATCH Fleet Management. , 2017(December 4th). Available at: http://www.modularmining.com/.

[11] Moradi Afrapoli, A. & Askari-Nasab, H., 2017. Mining fleet management systems: a review of models and algorithms. *International Journal of Mining, Reclamation and Environment.*

[12] Soumis, F., Ethier, J. & Elbrond, J., 1989. Evaluation of the New Truck Dispatching in the Mount Wright Mine. *In 21st APCOM*. pp. 674–682.

[13] Ta, C.H. et al., 2005. A stochastic optimization approach to mine truck allocation. *International Journal of Surface Mining, Reclamation and Environment*, 19(3), pp.162–175.

[14] Ta, C.H., Ingolfsson, A. & Doucette, J., 2013. A linear model for surface mining haul truck allocation incorporating shovel idle probabilities. *European Journal of Operational Research*, 231(3), pp.770–778.

[15] Temeng, V. a., Otuonye, F.O. & Frendewey, J.O., 1998. A non preemptive goal programming approach to truck dispatching in open pit mines. *Mineral Resource Engineering*, 7(2), pp.59–67.

[16] Temeng, V. a., Otuonye, F.O. & Frendewey, J.O., 1997. Real-time truck dispatching using a transportation algorithm. *International Journal of Surface Mining, Reclamation and Environment*, 11(4), pp.203–207.

[17] White, J.W. & Olson, J.P., 1986. Computer-based dispatching in mines with concurrent operating objectives. *Mining Engineering* (Littleton, Colorado), 38(11), pp.1045–1054.

Incorporation of Geo-metallurgical and Geochemical Properties in Oil Sands Mine Planning and Waste Management

Obinna Mbadozie and Eugene Ben-Awuah
Mining Optimization Laboratory (MOL)
Laurentian University, Sudbury, Canada.

ABSTRACT

The primary purpose of oil sands mine planning and waste management is to provide ore from the mine pit to the processing plant and contain the tailings in an efficient manner that will maximize the net present value, and minimize dyke construction cost. However, the presence of mineral solids such as clays, ultra-fines, organic-rich solids and heavy minerals in Alberta's oil sands has been investigated to have an impact on the yield of bitumen recovery and the net present value. These minerals possess certain physical and chemical properties such as particle size, cation exchange, layer charge density, and surface properties, which negatively affect the recovery of the bitumen product during extraction and processing. This paper reports on the ore characterization of oil sands and define the ore types in terms of bulk geochemistry and mineral characteristics that affect the processing recovery. A template model consisting of both the geo-metallurgical and geochemical properties that impact processing recovery or waste management will be developed. Pseudo penalties or rewards will be assigned to any material that affects the recovery of bitumen to facilitate ore blending. The equations that will be formulated in this research will contribute positively to the sustainability of the mine plan through optimization and reclamation strategies.

1. Introduction

One of the biggest challenges for oil sands extraction operations is to minimize the number of undesirable events caused by the variation of ore feed. Information on the properties of the incoming ore feed should be the basis used to establish mitigation and control strategies so that operators can proactively deal with the dynamics of the feed. To predict the bitumen recovery and processability of oil sands, comprehensive characterization of mined oil sands is essential. Common properties such as bitumen concentration indicates that ore processing is governed by the interactions of other factors such as mineralogy, connate water chemistry and, particle size which tend to influence the overall performance of the ore (Osacký et al., 2017).

This paper summarizes a literature review of oil sands, oil sands ore mining and processing operations. The primary aim of this study is to understand the various oil sands ore properties and establish a correlation between these properties with respect to bitumen recovery and ore processability. This review also summarizes how these oil sands properties could be incorporated into long-term oil sands mine planning and production scheduling.

2. Oil Sands

2.1. Brief history of oil sands deposition

Although the origin of oil sands remains disputed, most petroleum geologists believe that this resource was formed in the same way as other fossil fuels (conventional oil, natural gas, and coal). Ancient organic matter died and was covered by layers of sediment that exerted sufficient pressure and temperatures to transform the matter into petroleum. It is speculated that the oil sands that exist today formed as a result of ancient oceans that existed millions of years ago covering the areas where the oil sands exist today. As the microscopic marine life within the oceans died, they decomposed with the help of bacteria removing the oxygen and nitrogen, leaving mainly hydrogen and carbon. Heat and pressure then resulted in the layering of rock, silt, and sand over time and cooked the dead organic material for millions of years at temperatures between 50 and 150°C. This formation of oil is similar to that of conventional oil deposits, however, in the case of oil sands, the oil is absorbed into the existing sand. Although scientists are fairly convinced that this is how the oil sands began, exactly the same way as how conventional oil forms, with the one minor difference that sand is involved. The reason that oil sands contain bitumen and not conventional crude oil is a topic that is more controversial, but there are two main theories. The first of these theories is that the oil sands began as a vast reserve of crude oil, and over long periods of time the lighter crude oil escaped or was destroyed microbiologically, leaving behind bitumen. The second theory is that bitumen was formed immediately in a process similar to the formation of oil shale. In this theory, bitumen was released from shales with a large amount of organic matter (kerogen-rich shales) instead of crude oil being released. In both theoretical bitumen formation methods, the bitumen collects around the particles of sand. Instead of migrating through permeable rock as traditional oil would, this bitumen-soaked sand is rather forced to the surface from the pressure of mountain formation (Hepler et al., 1989).

2.2. Geology of oil sands

Canada's oil sands are concentrated mostly in Alberta, extending slightly into Saskatchewan's border. Oil sands deposits are localized in 3 regions, classified as follow; the Athabasca Basin, the Peace River Basin, and the Cold Lake Basin. The Athabasca basin is by far the largest, spanning an area of about 40,000 km² (Conly et al., 2011). All mineable oil sands are located north of Fort McMurray, Alberta within the Athabasca basin, where the deposit can be found very close to the surface (Mossop, 1980).

The mineral deposit for this study is located in the Lower Cretaceous McMurray formation which consists of a dominant continental sequence of uncemented sands and shales overlying an un-conforming surface of Devonian limestone. The McMurray formation is approximately 38 m thick and it is overlain by about 128 m of strata (Hepler et al., 1989). The Athabasca basin consists of the following stratigraphic formation; Muskeg, Pleistocene Unit, Clearwater Formation, McMurray Formation and Devonian carbonates (Masliyah et al., 2011). The topmost layer of the strata is known as Muskeg and is the wet topmost layer of overburden material that contains the seeds and roots of native plants and is used for the topmost layer of the reclaimed land. Before mining, this layer is removed and stockpiled and later used for reclamation works (Masliyah et al., 2011). The Pleistocene unit overlies the Clearwater formation and both are considered as waste rocks lying above the bitumen-bearing McMurray formation. Overburden materials from these layers are normally used for road and dyke construction in the mine depending on the soil properties and its mineral content. The oil-bearing rock type is the McMurray formation which is further classified into three rock types; the Upper McMurray (UKM), the Middle McMurray (MKM) and the Lower McMurray (LKM) (Masliyah et al., 2011). The McMurray formation is made up of coarse sand, fine sand, water, and bitumen. The main element of interest is bitumen which exists in various grades across the formation. The formation rests with profound unconformity on the Devonian carbonates and is unconformably

overlain by the Clearwater formation. The LKM is made up of gravel, coarse sand, silt and clay with siderite as cement. The UKM and MKM comprise of micaceous, fine-to- medium-grained sand, silt, and clay, with rare siderite as cement and intra-clasts and pyrite nodules up to 10 cm in diameter (Hein and Cotterill, 2006). Lastly, the Devonian Carbonates (DVN) which is a rock layer that sits below the oil sands (McMurray formation) and is made up of numerous limestone outcrops. A sketch of the vertical profile of an oil sands formation is shown in Fig. 1.

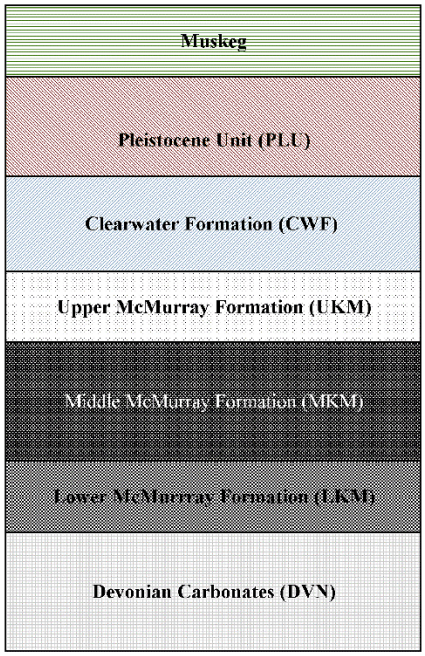


Fig. 1. Sketch of the vertical profile of an oil sands formation.

2.3. Oil sands compositions and properties

Oil sands are a loose sand deposit which contains a very viscous form of petroleum known as bitumen. These unconsolidated sandstone deposits comprise primarily of sand, clay, and water saturated with bitumen. Oil sands are sometimes referred to as tar sands or bituminous sand. Oil sands are composed of 55-80 wt% solid contents (sand, silt, and clays), 0-16 wt% bitumen and 0-7 wt% water. The composition of oil sands is defined based on particle size. For any material passing through a 44 μm (325 US mesh) sieve, it is referred to as fines which include silt, very fine quartz and clay minerals. In addition to this composition, it has been found that heavy minerals are contained in the oil sands as part of the solid contents (Bichard, 1987). These heavy minerals are unique features of the solid contents in oil sands.

The properties of the constituents of oil sands play a vital role in the processability of oil sands ores. Physical and chemical properties such as the nature of the bitumen, the coarse solid size distribution and fine solids and clay content and the salt concentration and type in the formation water all contribute to the efficiency of bitumen floatation and recovery (Masliyah et al., 2011). Furthermore, the surface properties such as surface tension, interfacial tension, and electric potentials of the sand grains and bitumen have an effect on the separation of bitumen from the sand grain with its ultimate floatation. These properties are affected by temperature as well as water chemistry used in the bitumen extraction process (Long et al., 2008). Thus, it is important to understand the compositions and the properties associated with oil sands and bitumen recovery. Each of this composition and properties affecting bitumen recovery will be discussed further in subsequent sections.

2.3.1. Bitumen

Bitumen as an unconventional fossil fuel is an extra heavy oil with high viscosity, density, and heavy metal concentrations, and a low hydrogen to carbon ratio, which makes the oil extraction process more costly, compared to that for conventional oil recovery (Masliyah et al., 2011). Bitumen from oil sands contains high molecular weight and low volatility components, with typical compositions of 83 wt% carbon, 10.6 wt% hydrogen, 4.8 wt% sulphur and, 0.4wt% nitrogen (Basu et al., 1996). The density of bitumen is higher than that of heavy oil and is very close to water. This is an important factor in the bitumen flotation process since this requires a density difference in order for oil to move to the top of the separation vessels. Therefore, air is also required as an additive to effectively reduce the density of the oil and cause the oil droplets to rise during frothing. Bitumen viscosity in-situ varies from several hundred to several tens of thousands Pascal-seconds (Carrigy, 1967). The viscosity of Athabasca bitumen is about 1,000,000 millipascal-seconds (mPa.s) at reservoir temperature, making the bitumen practically immobile; however, this characteristic gives enough material strength to the oil sands to be mineable (Mossop, 1980).

2.3.2. Organic rich solids and ultra-fine clays

Fines are defined as all solid particles smaller than 44 micrometer (μm) in diameter while clays are those fines which are less than 2μm in size, sometimes referred to as ultra-fine. Clays found in oil sands deposits are mostly comprised of kaolinite and illite. Most of the solids in the Athabasca oil sands are composed mainly of quartz sands grains, which are hydrophilic and 99 % water wet. Sand particles with a diameter larger than 44 μm (microns are considered coarse, while those less than 44 μm are considered as fine (Bowman, 1971).

Clays found in oil sands deposits are mostly kaolinite and illite. Also, evidence has shown that a substantial amount of fines are present in the form of thin discontinuous beds (Takamura, 1982). Some other trace minerals, such as mica, rutile, zircon, tourmaline, vanadium, and pyrite, are also found in the sand composition. Fines in the solid component mostly consist of silt and clay. Kaolinite (40-70 wt%) and illite (28-45 wt%) are the dominant clay minerals in Athabasca's oil sands (Chalaturnyk et al., 2002). The other common clay minerals are montmorillonite (1-5 wt%), chlorite (~1 wt%), smectite (~0.3 wt%), and mixed layer clays (~1.7 wt%) such as kaolinite-smectite and illite-smectite. The amount of clays contained in an oil sands deposit is proportional to the number of fines. Therefore, all high fines ores have a high clay content and tend to have a lower bitumen content. These lower grade ores are often described as poor processing since they disrupt the gravity separation process in extraction. Bitumen froth produced from high fines ores tend to be of a lower quality, typically containing less than 50% bitumen and more than 40% water (Masliyah et al., 2011).

Further studies conducted have shown that certain mineral solid fractions are associated with significant amounts Organic-Rich Solids (ORS) (Kotlyar et al., 1988). These organic solids are composed of toluene insoluble organic matter that is physically or chemically adsorbed onto particle surfaces. ORS exist primarily as aggregates of clay, sand, and silt bound together by organic matter that is largely humic in origin. The solids interact strongly with bitumen. During bitumen separation, these solids may report to the bitumen froth, the primary tailings or remain with the middlings, depending on their size and density. As a result of their close association with bitumen, these solids are responsible for significant losses with certain ore types. In the tailings, the solids can contribute to flock structure formation through bonding between particles by free bitumen. In bitumen separation processes, the organic matter associated with various ORS fractions represents an impediment to optimum bitumen separation and upgrading. In this sense, these solids are considered to be active relative to the inactive water wetted quartz particles comprising the bulk of the oil sands ore. Preliminary results indicate that the ORS content of an ore appears to be a better predictor for ore processability than the traditional use of bitumen or fines (~44 μm) contents. Two types of ORS have been recognized. The first type is a coarser fraction, usually less than 44 μm but also occurring as particles greater than 100 μm in diameter. This coarser fraction of ORS typically occurs as

aggregates of smaller particles bound together by humic matter and precipitated minerals. During the bitumen separation process, these heavy aggregates transport any associated bitumen into the aqueous tailings, thus reducing overall bitumen recovery. The second type of ORS comprises very thin, ultra-fine clay particles with a major dimension of <0.3 μm in size. These ultra-fine clays, with a surface coating of organic matter, remain with bitumen during the separation process. In bitumen upgrading, these solids may be entrained with volatile overheads and cause problems in downstream operations (Kotlyar et al., 1988).

2.3.3. Heavy minerals in oil sands

The occurrence of heavy minerals in Alberta's oil sands has long been known and has been documented by many researchers (Kaminsky et al., 2008). The most relevant heavy minerals identified were titanium-bearing, zirconium-bearing, and the rare-earth-bearing minerals in a smaller fraction of mineral solids. These minerals are all important from the point of view of economic potential. In general, the heavy mineral concentration in most of the Athabasca McMurray Formation was found to be consistently low averaging about 0.35% titanium dioxide (TiO_2) and 0.032% zirconium dioxide (ZrO_2) in feed grade oil sands. The TiO2 content varied from 0.08 to 1.6% and the ZrO2 content ranged from 0.0012 to 0.13%. Other valuable elements that were identified in the oil sands included rare earth and trace amounts of palladium, platinum, and gold. However, none of these occurred in high enough grades to warrant economic extraction. The distribution of heavy minerals in different range sizes on the surface mineable area of the Athabasca oil sands was studied. It was observed that titanium and zirconium minerals were concentrated in the fine size fractions, and iron minerals in the coarse size fractions. The silicon minerals seem to distribute evenly throughout the size range tested. In terms of size, heavy minerals are typically fine-grained. The 100 mesh (150 mm) size fraction is probably of most interest to potential users of the heavy minerals. It was reported that the 100 mesh fraction made up approximately 55 to 66% of the solid weight of the oil sands. Although bitumen is the primary target of the commercial extraction plants, there is a selective enrichment of the heavy minerals in the bitumen extraction processes as well. The titanium and zirconium minerals seem to move with the bitumen. As the bitumen froths are treated to remove the mineral solids and water, these heavy minerals are concentrated in the froth treatment tailings. Thus, in order to make use of the heavy minerals contained in the oil sands tailings, the heavy minerals need to be separated and upgraded (Kaminsky et al., 2008).

2.3.4. Connate water in oil sands

The water present in oil sands contains dissolved ions such as sodium, calcium, magnesium, chloride, potassium, sulfate and bicarbonate. Bitumen is not in direct contact with the mineral phase because a thin film of water surrounds individual sands grains. The thickness of the water film is predicted to be about 10 nm (Hall et al., 1982). This water film is assumed to be stabilized by electrostatic forces coming from electrical double layers at the oil/water and water/sand grain interfaces (Hall et al., 1982). A thin layer of water separates bitumen from sand particles, and, as the water content of an ore increases, the bitumen content decreases (Takamura, 1982). For higher concentrations of ions in the water, the bitumen content will be lower; this means that rich oil sands have a low salt content and poor oil sands have a high salt content. The pH of most of the oil sands is between 8 and 9, with some acidic ores at higher depths (Masliyah et al., 2011). Also, the bitumen and water contents vary and depend on the ore variation and the clay mineralogy (Kasperski, 2001). Takamura, (1982) presented a description of the generic microscopic structure of Athabasca's oil sands. A microscopic structure of Alberta Oil Sands could be seen in Fig. 2.

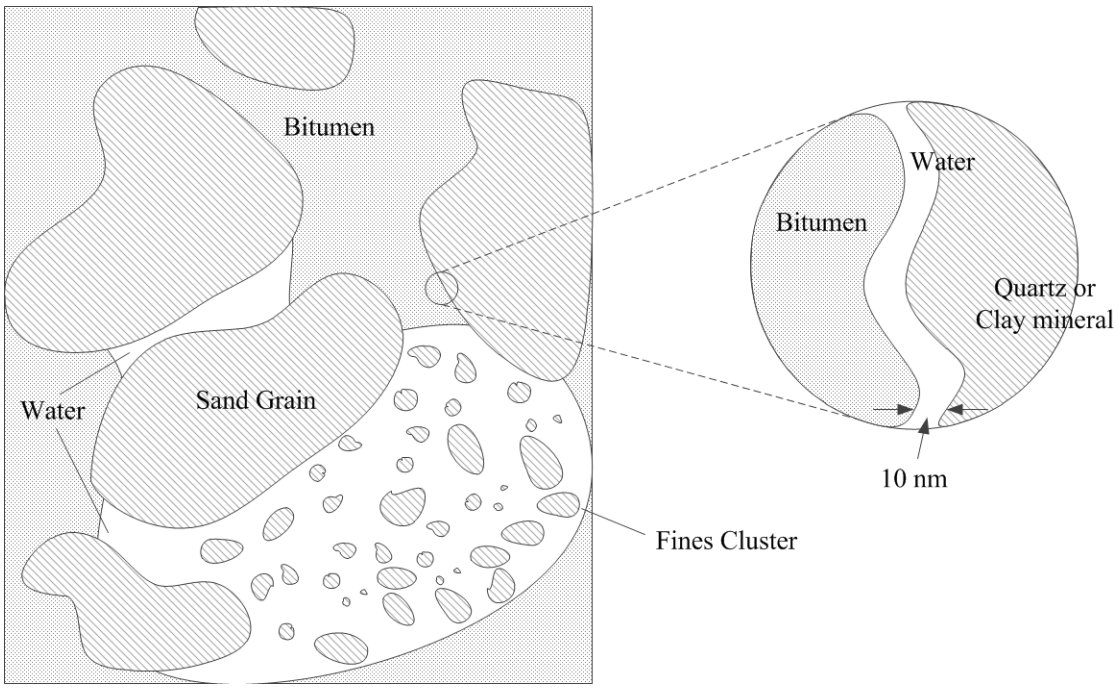


Fig. 2. Microscopic structure of Alberta oil sands modified after Takamura (1982).

3. Oil Sands Classification

There are several ore classification schemes for oil sands based on descriptors such as bitumen content, fines content and ore processability as indicated in Table 1 (Kaminsky, 2008). The most common method of classifying oil sands ore is based on its grade. A high-grade ore is considered to contain more than 10% bitumen, a mid-grade ore contains 8 - 10% bitumen, and a low-grade ore contains less than 8% bitumen. Likewise, a high-fines ore contains more than 18% fines while low-fines ore contains less than 6% fines. Processability indicates the quantity of bitumen that is being recovered in the froth. When more than 80% of bitumen and 90% of total bitumen recovery are obtained from the froth, it is referred to as a good processing ore (Kasperski, 2001). Froth quality and settling behavior are also sometimes considered as factors in processability, but most processability curves report recovery only. A high-quality froth will contain about 66% bitumen, 25% water, and 9% solids or a 7:1 bitumen to solids ratio (Kasongo et al., 2010).

Table 1: Oil sands ore classification.

Methods Classification	Content (wt%)	Ore Classification
Bitumen	>10 Between 8-10 <8	High Medium Low
Fine Content	>18 <6	High Fines Ore Low Fines Ore
Processability	Bitumen >10 Fines < 6 Bitumen < 8 Fines > 18	Good Processing Ore Good Processing Ore Poor Processing Ore Poor Processing Ore

4. The Effect of Oil Sand Properties in Bitumen Recovery

According to the Alberta Energy Regulator (AER) (Alberta Energy Regulator, 2016), there are four criteria used collectively to establish the volume of bitumen that an operator is required to recover each year from its mining and processing operations. The criteria are:

- **Cut off grade**: This is defined as the minimum bitumen content of the oil sands that would be classified as ore. The standard cutoff grade is 7 weight percent bitumen.
- **Minimum mining thickness:** This is the minimum thickness of ore that can be separated from waste or waste that can be separated from ore. It has been set at 3 meters
- **Total Volume to Bitumen in Place ratio (TV: BIP):** This the minimum value for TV/BIP that would be used to determine the pit crest limits. It is set at 12.
- **Processing Plant recovery:** This is a variable factor based on the average bitumen content of the as-mined ore. If the average bitumen content of the as-mined ore is 11 weight percent bitumen or greater, the recovery factor is 90 weight percent. If the average bitumen content of the as-mined ore is less than 11 weight percent bitumen, recovery is determined as shown in Eq. (1), where x is the average weight percent bitumen content of the as-mined ore:

$$Recovery = -202.7 + 54.1(x) - 2.5(x^2) \quad (1)$$

The recovery of bitumen from oil sands ore during extraction depends on certain different process variables. The variables can be classified into three main groups namely; ore properties, water chemistry, and operating conditions. Extensive research works have been conducted in order to recognize and clarify the involved factors that could impact bitumen recovery during bitumen extraction process from oil sands ores (Sanford, 1983; Dai and Chung, 1995; Basu et al., 1996; Masliyah et al., 2004; Long et al., 2008; Wallwork et al., 2008) as shown in Table 2. For this research, the focus is based only on the ore properties since it involves the mine-to- mill concept. Thus, ore properties such as bitumen grade, fines contents, types of fines, and mineralogy of sands and fines will be considered in this review.

Table 2: Factors affecting bitumen recovery and oil sands ore processability.

Ore Properties	Water Chemistry	Operating Conditions
Bitumen grade	pH	Temperature
Fines content	Presence, valence and concentrations of ions	Mechanical mixing and residence time
Types of fines	Presence and concentrations of surfactants	Slurry density
Mineralogy of sands and fines	Presence and concentrations of carbonates	Aeration
Weathering of ores	Presence and concentrations of dispersants and polymers	Bubble size

4.1. Effect of organic-rich solids in bitumen recovery

Bitumen is usually separated from the inorganic matrix by a surface layer of interstitial water which allows bitumen to be liberated from its bulk mineral component. But the humic content associated with organic-rich solids tend to adsorb and bind the bitumen to both internal and external aggregate surfaces thus reducing the recovery of bitumen. The size and density of ORS are vital properties that need to be considered in bitumen recovery. Small particles of ORS report with the froth along with the liberated bitumen. If the particles are not discarded to the tailings during the froth cleaning process, it will remain with the bitumen froth and adversely affect the quality of the bitumen. Coarser and higher density aggregates will not float with the bitumen fraction rather they will remain with the middling or settle with the primary tailings along with the associated bitumen thus reducing the recovery (Kotlyar et al., 1988).

4.2. Effect of ultra-fine clays in bitumen recovery

In the oil sands industry, it has been recognized that the coarse sands do not cause any problem throughout the extraction process. However, the fines (<44 μm) content plays an important role during the oil extraction process. Bitumen recovery decreases with increasing fines content in warm water extraction (Chalaturnyk et al., 2002). It is illustrated that small particle size is a contributing factor affecting bitumen recovery but comparison tests with ultra-fine silica particles (<0.3 μm) showed that mineralogy is a more important parameter (Wik et al., 2008). Nano and micro-sized minerals, mainly clay minerals, are most detrimental (Hooshiar et al., 2012). These minerals may interact with the solvent, bitumen and connate water due to their small particle size, high specific surface area, swelling capacity, cation exchange capacity, layer charge and, specific physicochemical properties. The presence of fine solids in oil sands have been found to adversely interfere with interactions among bitumen droplets. Fine solids also minimize bitumen coalescence thus, producing small-sized bitumen droplets which are more difficult to contact with air bubbles. Coated bitumen droplets form steric barriers between bitumen and air bubbles which makes it difficult for the bitumen to attach and engulf the bubbles. This results to an increase in slurry viscosity, and possibly gelation, which slows the flotation of formed bitumen-bubble aggregates. The bitumen froth quality and its floatation kinetics becomes reduced due to the fine solids appearing in the froth with recovered bitumen and slurry water (Liu et al., 2008).

The mineralogy of clay minerals in the Alberta oil sands has been studied and a summary can be found in Mikula et al. (2008). Based on such studies, the mineralogy of clay minerals in the Alberta oil sands deposit varies across the deposit. Kaolinite, illite, smectite, and chlorite are found to be the major clay minerals in the Alberta oil sands deposits with the first two being the most abundant. In one of the studies conducted by Zhou et al. (1999), to investigate the effects of fine solids with different surface properties on oil extraction, it shows that the addition of fine solids has a negative impact on bitumen extraction. The bitumen recovery decreased with the solids addition, compared with no solids addition. Also, the smaller solids (e.g. < 5 mm silica) reduced bitumen recovery more than the coarser solids (e.g. < 40 mm silica). This could be attributed to the physical barriers built between bitumen and bubbles by suspended fine solids, increasing the difficulty for direct bitumen-bubble contact. Since finer solids are more easily suspended in the slurry, they would be more likely to block direct bitumen-bubble contact than coarser solids, thereby retarding the flotation more severely. In another study conducted by Liu et al. (2008) to investigate the role of fine clays in bitumen extraction from oil sands, it was observed that in the absence of calcium, montmorillonite clays were weakly attached on the bitumen surface. However, with the addition of 1 millimolar (mM) of calcium-montmorillonite clay was attached strongly to the bitumen surface. The reason for the strong attachment was due to a low electrostatic double-layer force and an increased adhesive force between bitumen and montmorillonite clays which acted as a barrier for bitumen - air attachment and bitumen - bitumen coagulation, thus, led to poor bitumen recovery.

Although many efforts have been made to demonstrate the impact of bulk properties of oil sands ore on bitumen recovery, it is equally important to understand the clay characteristics and mineralogy of organic-rich solids and ultrafine clays in the oil sands. Both extraction recovery and product quality are significantly affected by interactions between organic materials and clay minerals. These interactions produce clay-organic matter complexes which make the bitumen extraction process more complicated. Therefore, it is crucial to understand the influence of water, bitumen, and solids, on the extraction process (Hooshiar et al., 2012).

5. Oil Sands Mining and Waste Management

5.1. Oil sands mining and mine planning

The first stage in any typical oil sand mining operation is to remove approximately 30 m of overburden material prior to the oil sand ore can be mined. The overburden is stripped using a truck

and shovel fleet. The overburden stripped material is stored for later reclamation of the mined out area. The ore body thickness generally varies from 20 to 90 meters. The underlying oil sand is mined using large mining shovels, transported in mining trucks, and sized in double roll crushers. The oil sand is then mixed with water in a processing tower to create an oil sand slurry, and this slurry is pipelined to a central extraction plant. After mining and processing the oil sands ore, more than 80% of the tailings are deposited in tailing dams (Masliyah et al., 2011). A significant amount of tailings material has caused several environmental issues. A typical mine plan strategy describes the best order of extraction of mining units. It is assumed that the optimal limits of the mining-pit are already known as inputs to mine planning. The dominant algorithm for open-pit mine design was developed by Lerchs and Grossmann as LG algorithm (Lerchs and Grossmann, 1965).It determines which blocks are within the optimal pit. With the optimal pit limit, the decision to make is the order that the blocks should be extracted so as to generate the maximum NPV.

The implementation of an exact solution to solving mine planning problem is thorough mathematical programming. Mine planning models are divided into short term and long term models. Strategic decisions such as the overall capacity of mining and processing are involved in long-term mine planning framework, while operational considerations such as route selection for trucks within the mining-pit are considered in short-term mine planning (Badiozamani and Askari-Nasab, 2014).

5.2. Waste management

Oil sand tailings are produced as a by-product of the bitumen extraction process and are deposited in large tailings ponds. Tailings are characterized by having poor consolidation and water release properties. The tailings slurry is a mixture of sand particles, dispersed fines, water and residual bitumen. It contains approximately 55 wt% solids, of which 82 wt% is sand, 17 wt% are fines smaller than 44 μm and 1 wt% is residual bitumen. When tailings are pumped into the tailing ponds, the coarse solids settle out very rapidly to form dykes and beaches. Most of the fines and residual bitumen are carried in the run-off slurry that flows over the already formed beach. The run-off slurry stream arrives at the water's edge in the tailings pond with a suspended solids concentration of about 3-8 wt%. These fine tailings solids settle quickly over several days to create a zone with 20 wt% solids content and a "free water zone" that contains little solids. This water is recycled as extraction water. Below the free water zone, solids settling continue within the fine tailings; however, the suspension starts to develop non-Newtonian properties which make further settling an even slower process (Sheeran, 1993). Over 2 to 3 years, fines tailings settle to 30 wt% solids with a stable slurry structure denominated mature fine tailings (MFT). This suspension has very high viscosity and yield stress (MacKinnon, 1989; Kasperski, 1992). Further, de-watering occurs very slowly and it takes several centuries for the MFT to reach the consistency of soft solids. Even after 40 years, the MFT does not have enough strength to support the overburden or soil horizon replacement required for reclamation (Sheeran, 1993; Eckert et al., 1996). A schematic cross-section of an oil sands tailings pond is shown in Fig. 3.

The two main challenges in managing oil sands tailings are environmental challenges and space limitations defined in the lease agreements. The environmental challenges include the toxicity of the tailings pore water and land reclamation. In dealing with this, dewatering techniques have been developed to decrease the volume of water in the tailings and to recycle the water to the processing plant. The regulatory requirement from Energy Resources Conservation Board (ECRB) referred to as Directive 085, requires oil sands companies to reduce the volume of fluid tailings and convert them to trafficable landscapes (Kalantari et al., 2013). Based on these facts, tailings management represents a real environmental impact, primarily related to water usage and recycling. The production of one barrel of bitumen requires approximately 12 barrels of water, where about 70 % of this water is recycled and the remaining 30 % comes from the river, leaving a net requirement of almost 4 barrels of water per barrel of bitumen. This remaining water is tied up in the pore spaces of the sand, silt, and clay in the tailings, and is effectively lost to the process (Mikula et al., 2008).

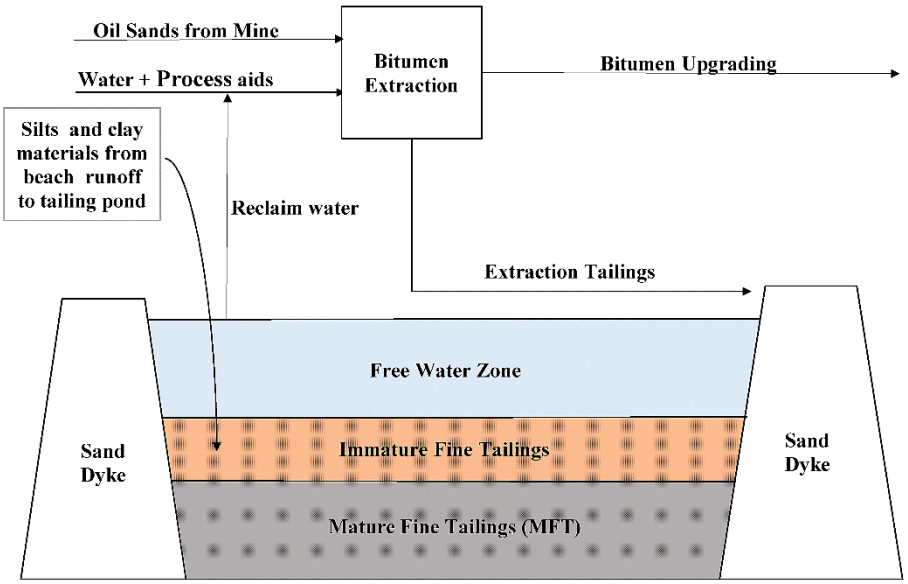


Fig. 3. Cross section of an oil sands tailing basin modified after Beier and Sego (2008).

5.3. Integrating mine planning and waste management in oil sands.

The objective of a typical mine plan is the maximization of Net Present Value (NPV) over the mine-life which is subjected to a number of constraints. Most of the constraints in mine planning models are technical constraints. Examples of constraints are the capacities of mining and processing in each period, the limitations on the grade range as required by processing plants and the precedence constraints to control the vertical order of extraction of the mining-blocks (Badiozamani and Askari-Nasab, 2014). Considering the environmental issues related to the oil sands surface mining, two concepts can be integrated into the long-term mine planning optimization framework, thus tailings management and solid waste management. The solution to such an integrated model provides sustainable production planning in the oil sands mining operation.

The regulatory requirement by the Alberta Energy Regulator (AER) Directive 082 requires oil sands mining companies to integrate their waste management strategy into their long-term production plans. It also requires mining companies not to leave behind any material containing more than 7% bitumen. In order to reduce the environmental footprints, dyke construction should take place simultaneously as the mine advances and each push back become available for dyke construction. The material required for the dyke construction mainly comes from the mining operation. The dyke material includes Overburden (OB), Inter-burden (IB) and Tailings Coarse Sand material (TCS) (Ben-Awuah and Askari-Nasab, 2013). The above mentioned regulatory requirement and environmental issues makes the waste management strategy a necessary part of oil sands long-term production planning. Tailings management in oil sands has been addressed in a number of academic publications (Mikula et al., 1998; Chalaturnyk et al., 2002; Soane et al., 2010). One of the latest works that studied the effect of oil sands mine planning on tailings management, in terms of Composite Tailings (CT) production, is published by Kalantari et al., (2013). The authors investigated the linkage between long-term mine planning and CT production in the presence of uncertainty. A tailings model including the tailings mass-balance relations is developed to map the tonnage of the ore feed to the final CT tonnage. However, the capacity of the tailings facility and the deposition of produced CT are not considered explicitly in the long-term mine planning model. A review of the literature reveals that there are not many research works addressing the linkage between tailings management and mine planning. On the other hand, the ERCB forces Alberta oil sands operators, through Directive 085, to publish their tailings management plans (McFadyen, 2009). For solid waste management, the material required for construction of dykes comes mainly from mining and processing operations, as overburden, inter-burden, and coarse tailings sand (Ben-Awuah, 2013).

With respect to the regulatory requirements from Directive 074, waste disposal planning must be considered in a close relation to the mine planning system (McFadyen, 2009; Ben-Awuah and Askari-Nasab, 2011). However, there are a number of complexities in modeling and solving an integrated optimization framework.

A Mixed Integer Linear Programming (MILP) model for long-term mine planning is known to be an NP-hard problem (Gleixner, 2009). Adding more variables and constraints regarding waste disposal and dyke construction even adds to its complexity. That is one of the reasons why in current practice, the oil sands waste disposal planning is handled as a post-production scheduling optimization (Ben-Awuah and Askari-Nasab, 2011). A team of geologists, geotechnical and mine planning engineers, tailings planners, and operations engineers work on dyke construction plan, yet there is no guarantee that the developed plan meets all the material requirements for dyke construction in all periods and the resulting NPV is maximized. There are three main methods for dyke construction, as upstream construction, downstream construction, and centerline construction. With some modifications, some more dyke construction methods are developed as well. Depending on the dykes' design, different material types are required for construction. Due to the limited space of leas areas, a common approach is preparing the excavated pit for tailings storage, as in-pit tailings facilities. When the mining-pit is cleared to its bottom level, the in-pit tailings containments will be ready by partitioning the empty pit by constructing a number of internal dykes (Ben-Awuah, 2013).

Ben-Awuah and Askari-Nasab, (2013) proposed an integrated optimization framework for long-term open pit mine planning in oil sands surface mining. The authors considered directional mining as a requirement for in-pit tailings containment, and waste disposal planning to ensure that the optimal mine plan provides required material for dyke construction. The author proposed a mixed integer goal programming model and considers two destinations for the extracted material: the overburden and inter-burden will be sent for in-pit and external tailings facilities' construction, while the ore will be sent to the processing plant. The tailings coarse sand resulting from oil sands processing will be used for dyke construction. Fig. 4 illustrates the strategic production planning for an oil sands deposit containing K mining cuts and M push backs. Mining-cuts are made up of blocks within the same level that is grouped together based on their attributes; location, rock type and grade using an agglomerative hierarchical clustering algorithm by Tabesh and Askari-Nasab (2011). The proposed model is a comprehensive mine planning model that covers the solid waste disposal and dyke construction planning.

Reclamation planning is the other aspect in oil sands long-term decisions. According to ERCB, the oil sands operators are required to plan for mine closure and reclamation in advance to make sure that the mine site and the tailings ponds will be restored to their original conditions of pre-mining operations (McFadyen, 2009). A review of reclamation plans published by the oil sands operators proves that the material used for capping the closure phase, such as the overburden and tailings coarse sand, comes from the mining and processing operations. This shows another potential development in typical mine planning models by the inclusion of a material required for reclamation capping in long-term mine planning. Due to the same reasons mentioned for waste disposal and dyke construction, the reclamation planning is done as a post mine planning process in the current practice. Most of the techniques required for reclamation have been developed by the industry itself. However, the industry will benefit from an integrated mine planning framework that includes reclamation material handling as part of the long-term model, because any shipment of material for capping adds to the costs and will cause the NPV to decrease. An integrated model for long-term mine planning, with respect to reclamation material handling and tailings capacity constraints has been proposed and the concept of directional mining is used in modeling to provide capacity for in-pit tailings facility. The model determines the destination for each extracted parcel (dynamic cut-off) in such a way to maximize the NPV over the mine-life. Mining aggregates are used in the model to follow the selective mining units.

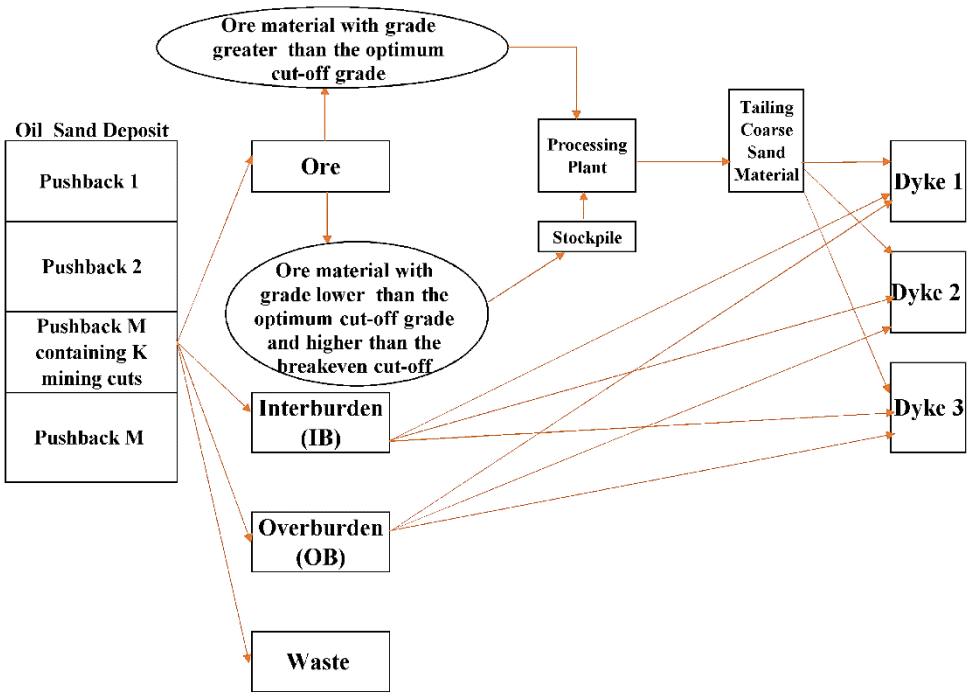


Fig. 4. Oil sands production planning and waste management modified after Ben-Awuah and Askari-Nasab (2013).

The authors reach to integer solutions within 2% optimality gap in less than 10 minutes for the cases with more than 98,000 mining-blocks aggregated to 535 mining-cuts. The resulting schedule generates the maximum NPV, minimizes the material handling cost of reclamation, and the tailings volume produced downstream meets the tailings capacity constraints in each period. The authors took further steps in integrated mine planning by including the tailings management, in terms of composite tailings production and deposition, in the mine planning optimization framework (Badiozamani and Askari-Nasab, 2014).

6. Open Pit Optimization and Production Scheduling

The operation and management of a large open pit mine is an enormous and complex task, particularly for mines having a life of many years. Optimization techniques can be successfully applied to resolve a number of important problems that arise in the planning and management of a mine. These applications include: ore-body modeling and ore reserve estimation; the design of optimum pits; the determination of optimal production schedules; the determination of optimal operating layouts; the determination of optimal blends; the determination of equipment maintenance and replacement policies (Caccetta and Giannini, 1986).

A fundamental problem in mine planning is determining the optimum ultimate pit limit of a mine. The optimum ultimate pit limit of a mine is defined to be that contour which is the result of extracting the volume of material which provides the total maximum profit whilst satisfying the operational requirement of safe wall slopes. The ultimate pit limit gives the shape of the mine at the end of its life. Usually, this contour is smoothed to produce the final pit outline. Optimum pit design plays a major role in all stages of the life of an open pit: at the feasibility study stage when there is a need to produce a whole-of-life pit design; at the operating phase when pits need to be developed to respond to changes in metal prices, costs, ore reserves, and wall slopes; and towards the end of a mine's life where the final pit design may allow the economic termination of a project. At all stages there is a need for constant monitoring of the optimum pit, to facilitate the best long-term, medium-term and short-term mine planning and subsequent exploitation of the reserve. The optimum pit and mine

planning are dynamic concepts requiring constant review. Thus the pit optimization technique should be regarded as a powerful and necessary management tool. Further, the pit optimization method must be highly efficient to allow for an effective sensitivity analysis. In practice, one needs to construct a whole spectrum of pits, each corresponding to a specific set of parameters. The ultimate pit limit problem has been efficiently solved using the Lerchs-Grossmann and graph-theoretic algorithm also known as Picard's network flow method. These methods are based on the block model of an orebody (Lerchs and Grossmann, 1965; Picard, 1976).

Production scheduling in surface mining is defined as the sequence in which blocks of the ore orebody are extracted and moved in order to maximize the net present value subject to mining, economic and processing constraints (Caccetta and Giannini, 1986).Typically, the constraints related to the mining extraction sequence are: mining, milling and refining capacities; grades of mill feed and concentrates; stockpile-related restrictions; a range of logistics issues and various operational requirements such as minimum pit bottom width and maximum vertical depth. The definition of block models gives a starting point for the production scheduling optimization task. Block models divide the orebody into small blocks which are dependent on pit slopes, orebody dip, grade distribution and the type of equipment being used (Caccetta and Giannini, 1986). Each block is assigned certain attributes using either kriging or inverse distance interpolation techniques. These attributes include density, grade, and tonnages. Using the cut-off grade optimization techniques, the blocks are then divided into ore and waste. The size of the blocks created can be in the order of millions thus requiring movement over a long period of time. This movement of blocks demands adequate scheduling considering the physical and operational constraints (Khan and Niemann-Delius, 2014).

The techniques used by Operation Research (OR) in mine production planning are collectively grouped into two main approaches which include (i) deterministic approach and (ii) uncertainty-based /stochastic approach. In the deterministic approach, it assumes all inputs to have fixed known real values from which a sequence of possible solutions are generated. The number of sequence point is defined by the technique that is being used and the technique terminates each sequence of points once a solution has been reached or when the number of iterations has been exhausted. Linear programing, integer programing and mixed integer linear programming use this type of technique. (Hickman, 2014). One of the major drawbacks on the deterministic approach is that the optimal scenario is affected by uncertainties related to input parameters. These uncertainties are categorized as in-situ grade uncertainty, economic uncertainties such as capital and operating costs and technical mining specification uncertainty such as extraction capacities and slope considerations (Dimitrakopoulos and Ramazan, 2008; Hickman, 2014). Meanwhile, in stochastic approach, it uses some form of randomness when searching for feasible solutions (Hickman, 2014). The input to stochastic approach takes into account the variabilities of each input parameter. Their outputs do not give one optimum solution but a range of solutions with some form of distribution to give the user a feel of variability for the solution. The use of stochastic techniques works best when using evolutionary algorithms (Hickman, 2014).

Due to the complexity and size of the problem all these approaches suffer from one or more of the following limitations: (i) they cannot cater for most of the constraints that arise; (ii) they yield only suboptimal solutions and in most cases without a quality measure; (iii) they can only handle small sized problems. The most commonly applicable method is parameterization which was initially introduced by Lerchs and Grossman (1965). In these technique, a set of nested pits is generated, starting with the final pit contour, by varying the economic parameters. For each parameter value, the Lerchs-Grossman algorithm is applied to generate the optimum pit. The most widely used software packages that use this method include: Whittle's Four-D and Four-X and Earthworks NPV Scheduler (Ver. 3.2.5). The latter product has a restricted tree search procedure that is used to re-sequence the pushbacks in an effort to improve the NPV (Meagher et al., 2014).

Stochastic mine planning is a relatively recent development aimed at addressing uncertainty in ore supply from an orebody, commodity prices and metal demand, as well as expanding to other issues of uncertainty in mine planning. The application of stochastic optimization in mine scheduling problems have proven to substantially increase the net present value to about 30% compared to other deterministic approaches used (Ramazan and Dimitrakopoulos, 2018).

6.1. Operation research methods used in mine scheduling problems

Production scheduling requires efficient optimization techniques in order to realize the benefits. Several optimization techniques have been applied. These operation research methods include linear programming (Johnson, 1968), mixed integer programming (Gershon, 1983), branch and cut (Caccetta and Hill, 2003), dynamic programming (Onur and Dowd, 1993), fundamental tree algorithm (Ramazan, 2007), lagrangian parameterization (Dagdelen, 1986), simulated annealing and genetic algorithms (Denby and Schofield, 1995) and neural networks (Denby et al., 1991). The main goal of an open pit mine production scheduling problem in oil sands mining is to specify the sequence in which blocks should be removed from the mine in order to maximize the total discounted profit from the mine subject to a variety of constraints. These constraints include; the mill throughput, the volume of material extracted per period, the blending constraints, and stockpile related constraints, and logistics constraints. In order to achieve this goal, mathematical formulations which satisfy these constraints are developed (Gershon, 1983; Caccetta and Giannini, 1986).

6.2. Linear programming

The Linear Programming (LP) model was proposed by Johnson as an optimization technique in mine scheduling problem (Johnson, 1968). The LP model considered the time value of money, different processing types and also the dynamic cutoff grade strategy. In order to solve the LP model, the large multi-period production planning model is decomposed first into a master problem and a set of sub-problems by using Dantzig - Wolf decomposition principles. Each sub-problem is then solved as a single-period problem that has the same characteristics as the ultimate pit limit problem. This can be done using a maximum network flow algorithm. After solving all sub-problems, solving the master problem is relatively simple. Although Johnson's method generated optimum results for each period individually, however it does not solve totally the mine scheduling problem in a long term., The variables were linearly continuous, which was responsible for fractional block extraction. Also, the LP model provided situations in which some portion of a block was extracted while all the overlying blocks have not been mined. This drawback causes some percentage of overlying blocks to be suspended in air. Another disadvantage of this model was that it had too many constraints (nine slope constraints per block), which itself limits the number of blocks that can be handled by the model (Johnson, 1988).

6.3. Mixed integer programming

The problem of partial mining block associated Johnson's LP model was latter addressed by Gershon (1983) who discussed a Mixed Integer Programming (MIP) model that would allow partial blocks to be mined if all precedent blocks have been completely removed. The key to this formulation was adding additional decision variables to Johnson's LP model. The advantages of the MIP model compared to the linear programming model was that, it provided a more practical extraction sequence in mine scheduling. Partial block mining was allowed on the condition that all blocks preceding the partially mined block have been mined. The net result of this model was that only one constraint per block was required. The main disadvantage of this model was its inability to handle large problems using commercial software because it contains too many binary variables. In addition, because of increasing the size of the model, the dynamic cut-off grade concept could not be considered (Gershon, 1983).

6.4. Integer programming

6.4.1. Lagrangian relaxation approach

This is another research method used in solving mine scheduling problem. A binary linear programming formulation usually involves a large number of zero – one variables, which is beyond the capacity of current commercial packages. Several approaches have been proposed by researchers to solve such models. Dagdelen and Johnson (1986) used a lagrangian relaxation approach to solve the open-pit production planning problem. The objective function is to maximize the NPV, subject to block precedence and production capacity constraints (Dagdelen, 1986). The authors relax the mining capacity constraints and added penalty in the objective function accordingly. The sub-gradient algorithm is used to update the Lagrangian multipliers for small-scale problems. The Lagrangian multipliers are adjusted using the sub-gradient method until the optimum schedule is obtained. At each step, a problem similar to an ultimate pit limit problem should be solved. In cases where there are no multipliers that can result in a feasible solution for the constraints, this method may not converge to an optimum solution. This problem is named the gap problem. Caccetta et al., (1998) tested this method on a real ore body with 20979 blocks and six time periods. The schedule obtained was within 5% of the theoretical optimum. Another drawback of this algorithm is that it does not consider the dynamic cutoff grade concept during scheduling (Caccetta et al., 1998). Akaike and Dagdelen extended the work by Dagdelen and Johnson (1986), using an iterative procedure to update the values of Lagrangian multipliers. Akaike and Dagdelen then transformed the integer programming model by the use of the Lagrangian relaxation method, so that the transformed problem has the same characteristics as the final pit design problem. This problem can then be interactively solved changing the Lagrangian multipliers by using the sub-gradient method to converge it to the optimum solution of the primal problem. The authors also improved the efficiency of the sub-gradient method to reach the optimal solution much faster. The most important advantages of this algorithm are the use of the dynamic cutoff grade concept with the stockpile option with zero – one variables during the scheduling process. This will improve the NPV of a mining project. The disadvantage of this method is the possibility of a gap problem occurring, which means that it may not lead to an optimum solution (Akaike and Dagdelen, 1999).

6.4.2. Clustering Approach

The next approach to solve an Integer Programming (IP) model of production planning in an open pit mine is the clustering approach, which was applied by Ramazan et al. (2005). Clustering means classifying the large amount of data into relatively few classes of similar objects. This is the reason for complexity reduction in the considered application, which allows for improved decisions based on the information gained. Ramazan et al. (2005) combined ore and waste blocks together to decrease the number of binary variables in the IP model. They introduced the fundamental tree as any combination of blocks within the push backs, such that the blocks can be profitability mined and obey the slope constraints so that no sub-set of chosen blocks can be found that meets the above two requirements. This re-blocking (clustering) process is done using an LP mathematical formulation so that the information available for individual blocks is not lost (Ramazan et al., 2005).The authors created three fundamental trees that can be used for the LP model where tree I can be mined first; trees II and III are then feasible to mine in the suggested order. After defining the fundamental trees, their precedence relations should be determined using the cone template. Each fundamental tree is treated as a mining block containing a certain ore tonnage, metal content and quality parameters. Then a binary variable is assigned to each fundamental tree for each production period except the last one. In order for the IP model that uses fundamental trees to be handled by commercial software such as CPLEX*, the material within the final pit limit is divided into smaller volume by determining 3 to 5 push backs. Finally, fundamental trees should be scheduled by an IP formulation that contains all the mining and milling operational constraints and tree sequence requirements. The advantages of using clustering approach is that it reduces the binary variables and eliminates the gap problem. It also generates more NPV than other scheduler software. Its disadvantages are that it needs to generate

pushbacks before scheduling. More than one iteration is needed to generate fundamental trees. Finally, its application is complicated, the optimality of the solution depends on the optimality of the generated pushbacks (Ramazan et al., 2005).

6.5. Mixed integer linear programming

Many combinatorial optimization problems that are formulated as Mixed Integer Linear Programming (MILP) problems can be solved by branch-and-cut methods. These are exact algorithms consisting of a combination of a cutting plane and branch-and-bound algorithms. These methods work by solving a sequence of linear programming relaxations of the IP problem. Cutting plane methods improve the relaxation of the problem to more closely approximate the IP problem. Branch-and-bound algorithms proceed by a sophisticated divide and conquer approach to solve problems. It is usually not possible to solve a general IP problem efficiently using just a cutting plane approach; it is necessary to also use branching, which results in a branch-and-cut approach (Mitchell, 2002).

Caccetta and Hill (2003) outlined their branch-and-cut procedure for solving IP models of Long-Term Production Plan (LTPP) problems. Because of the commercialization of their software, they did not provide full details of their algorithm. Explicit incorporation of all constraints (like maximum vertical depth, minimum pit bottom width and stockpile option) in the optimization procedure is a key advantage of their algorithm. Also, it can produce good solutions for medium mine production planning problems. However, obtaining optimal solutions for large problems is difficult. On a large model containing about 209600 blocks and ten scheduling periods, they could obtain a solution within 2.5% of the optimum within four hours. Another disadvantage of this method is that they did not optimize the cutoff grade during the optimization process. It should be noted that for large and/or hard problems, branch-and-cut methods can be used in conjunction with heuristics or meta-heuristics to obtain a good (possibly optimal) solution, and also to indicate how far from optimal this solution may be obtained. MILP formulations, do not consider the smoothness of the scheduled patterns, which relates to equipment movement in a period and to equipment access. Also, geological uncertainty is ignored in the MILP optimization model. Thus, the schedules that are manually generated, are often outside the specified operating range and certainly far from optimal. The usual approach is to first determine the final pit outline and then through a series of refinements mining schedules are generated. The final pit outline is determined by smoothing the contour produced by solving the ultimate pit limit problem. The ultimate pit limit is the maximum value pit resulting from the mining of ore and waste blocks under the assumption that all mining could be done in one period (Caccetta and Hill, 2003).

6.6. Stochastic integer programming

The complexity of mine scheduling problem is increased by uncertainties due to the sparse variability of geological data. The uncertainty of the ore grade may cause discrepancies between planning expectations and actual production. Various authors presented methodologies to account for grade uncertainty and demonstrate its impact. Godoy and Dimitrakopoulos (2004) presented a new risk-inclusive long-term production plan (LTPP) approach based on simulated annealing. A multistage heuristic framework was presented to generate a schedule that minimizes the risk of deviations from production targets. The authors reported a significant improvement in NPV in the presence of uncertainty; however, heuristic methods do not guarantee the optimality of the results. Also, these techniques can be difficult to implement, and many parameters may need to be chosen in order to get reasonable results. Osanloo et al. (2008) proposed a risk-based algorithm for surface mine planning. A predefined distribution function is used for some variables such as commodity price, mining cost, processing cost, investment required, grade and tonnage. Different schedules are generated for a number of realizations of the grades. The proposed method leads to multiple schedules reflecting the grade uncertainty. Koushavand et al. (2014) used simulated orebodies to show the impact of grade uncertainty on production scheduling. They used simulated orebody models one at a time in

traditional optimization methods; however, this sequential process does not optimize accounting for uncertainty. Ramazan and Dimitrakopoulos (2018) suggested a mixed integer linear programming (MILP) model maximize NPV for each realization. Then, the probability of extraction of a block at each period is calculated. These probabilities are used in the second stage of optimization to arrive at one schedule however the uncertainty is not used directly in the optimization process. Ramazan and Dimitrakopoulos (2018) also presented a Linear Integer Programming (LIP) model to generate optimal production schedules. Multiple realizations of the block model were considered. This model has a penalty function; the cost of deviations from the target production and is calculated based on the geological risk discount rate (GRD), which is the discounted unit cost of deviation from target production. The authors use linear programming to maximize a new function that is NPV fewer penalty costs. It is not clear how to define the GRD parameter (Ramazan and Dimitrakopoulos, 2018).

Stochastic integer programming (SIP) provides a framework for optimizing mine production scheduling considering uncertainty. A specific SIP formulation is briefly shown to generate optimal production schedules, using equally probable simulated orebody models as input, without averaging the related grades (Dimitrakopoulos and Ramazan, 2008). The optimal production schedule is then the schedule that can produce the maximum achievable discounted total value from the project, given the available orebody uncertainty described through a set of stochastically simulated orebody models. The proposed SIP model allows the management of geological risk in terms of not meeting planned targets during actual operation. This is unlike the traditional scheduling methods that use a single orebody model, where risk is randomly distributed between production periods while there is no control over the magnitude of the risks on the schedule. The objective function of the SIP model is constructed as the 'maximization of a profit function'. The profit function is defined as the total expected NPV minus the cost of deviations from planned production targets. The general form of the objective function is expressed in Eq. 2.

$$Max\sum_{t=1}^{P}\left[\sum_{i=1}^{N} v_i^t b_i^t - \sum_{s=1}^{M}\left(c_u^{to} d_{su}^{to} + c_1^{to} d_{sl}^{to} + c_u^{tg} d_{su}^{tg} + c_l^{tg} d_{sl}^{tg} + c_u^{tq} d_{su}^{tq} + c_l^{tq} d_{sl}^{tq}\right)\right] \quad (2)$$

Where P is the total production periods, N is a number of blocks, and b_i^t is the decision variable for when to mine block i (if mined in period t, b_i^t is 1 and otherwise b_i^t is 0). The c variables are the unit costs of deviation (represented by the d variables) from production targets for grades and ore tonnes. The subscripts u and l correspond to the deviations and costs from excess production (upper bound) and shortage in production (lower bound), respectively, while s is the simulated orebody model number, and g and o are grade and ore production targets.

The objective function shown in Eq. 2 is used for maximizing the total discounted economic value and also managing the risk of not meeting production targets using conditionally simulated orebody models. Traditionally, one orebody model, a smooth image of the deposit, is used for maximizing NPV. However, when the expected deviations from the planned amount of ore tonnage having a planned grade and quality in a schedule are high in actual mining operations, the traditional model is unlikely to achieve the resultant NPV of the planned schedule. So, the NPV to be generated from actual mining can be far from optimal even if the schedule is optimized using a traditional true optimizer, MIP model. Therefore, the SIP model is developed to consider the minimum of the deviations together with maximization of NPV to generate achievable NPV (Ramazan and Dimitrakopoulos, 2018).

When constructing the objective function, a constant value is initially assigned for each of the cost parameters representing the cost at time 0 (base cost). Then, the risk discounting parameter (f) is introduced to determine the cost at different time periods by discounting the base cost using f. The risk-discounting concept is then incorporated into the SIP model (Dimitrakopoulos and Ramazan, 2003). If f is set to 0, the deviations in production targets can be expected to result in more or less the same level between different production periods because the cost of a unit deviation will be the

same in all periods. However, the distribution of deviations will also depend on how the variability in grade and ore tonnage is distributed over the deposit and on how the relative magnitude of the costs for the deviations used in the SIP model compare with the economic values of the blocks. The Geological Risk Discount rate (GRD) allows the management of risk to be distributed between periods. If a very high GRD is used, the lowest risk areas in terms of meeting production targets will be mined earlier, and the most risky parts will be left for later periods. If a very small GRD or a GRD of zero is used, the risk will be distributed at a more balanced rate among production periods, depending on the distribution of uncertainty within the mineralized deposit. The *c* variables in the objective function are used to define a risk profile for the production, and the NPV produced is the optimum for the defined risk profile. It is considered that if the expected deviations from the planned amount of ore tonnage having planned grade and quality in a schedule are high in actual mining operations, it is unlikely to achieve the resultant NPV of the planned schedule. Therefore, the stochastic integer programming model contains the minimization of the deviations together with the NPV maximization to generate practical and feasible schedules and achievable cash flows.

In the model constraints, the deviation parameters are calculated within the SIP model by using the related constraints that consider each of the simulated orebody model. Stochastic constraints that use simulated multiple orebody realizations are considered because they are feasible for any value of decision variables. Stochastic constraints related to grade blending are used to satisfy not only the grade requirement at the mill but also the requirements for quality parameters (Dimitrakopoulos and Ramazan, 2003).

7. Geostatistics Approaches in Reserve Estimation

Estimating recoverable reserves is one of the important processes in the mining industry during production for grade control as well as mine planning. The main aim of grade control in mines is to differentiate between material that is above cut-off grade and material that is below cut-off grade through recoverable reserve estimation. Since drilling does not cover the whole area to be mined, recoverable reserve estimation aims at predicting the quality (grade) and quantity (tonnage) over an area where drilling was not done, from a limited number of data. From the estimates, a decision can be made whether to mine the material as ore (above cut-off) or waste (below cut-off). According to Sinclair and Blackwell (2002), the term local/recoverable reserve estimation is not defined rigidly but is used in the general context of point estimation or the estimation of small blocks or units on the scale of a Selective Mining Unit (SMU). The most important goal is to provide estimates that are accurate and reliable. In mining, geostatistical techniques are preferred for recoverable reserve estimation compared to other techniques. Since not all areas where mining is done are drilled and sampled, geostatistical techniques were developed to aid in estimating the grade and tonnage of an area based on nearby sample values. Geostatistics has been described as the application of the theory of regionalized variables to the estimation of mineral deposits (Matheron, 1971). The idea of using geostatistics for local reserve estimation in the mining industry was introduced around 1951 by Krige, (1951).

The use of geostatistics in the mining industry can be dated to as far as the 1970's. Apart from reserve estimation, geostatistical techniques are also used for mine planning as well as testing the importance of using different types of machinery on the output of the mine (Journel and Huijbregts, 1978; Clark, 1979). Currently, the technique is commonly used in other disciplines like hydrogeology, meteorology and contour mapping (Royle et al., 1981). In mining, geostatistics' emphasizes the geological context of the data and the spatial relationship between data. Compared to other methods of resource estimation, geostatistics is a better method for estimating grade and tonnage of the ore reserves. Unlike classic techniques which examine the sample data's statistical distribution, geostatistics incorporates both the statistical distribution of sample data and the spatial correlation between the sample data, hence addressing more earth science problems. Geostatistical techniques use the variogram, which relies on the spatial distribution and internal structure of data and not only

on the actual values. If the variogram is good (robust) it provides estimates that are a good representation of the spatial distribution of the input data (Samal et al., 2008). The technique is based on the theory of regionalized variables, which states that the interpolation from points in space should not be based on a smooth continuous object (Matheron, 1971). Unlike classical statistics that considers grade to be randomly distributed, geostatistical techniques consider the changes of mineralization in relation to the trend direction of the ore body. It also considers the area of influence and continuity or lack of continuity of mineralization within the ore body. Other techniques like inverse distance weighting and polygonal estimation do not provide a measure of the accuracy of estimates, geostatistical techniques provide not only the estimates but also the measure of the accuracy of the estimates. Apart from the estimated mean grade, it also gives the estimated variance or spread of the grade. The estimated variance is used in risk analysis of the reserve estimate, especially in conditional simulation (Samal et al., 2008).

7.1. Conditional simulation

In the context of mining, the simulation means imitation of conditions (Sinclair and Blackwell, 2002). It is conditional if the resulting realizations honor the actual/original data at their locations. The method is mainly used for continuous variables like grade, height and age and its basic principle is to obtain an appropriate simulation of a point and its value must be drawn from its conditional distribution given the values at some nearest points. In the mining industry, it is used in the study of grade continuity, recoverable reserve estimation, optimizing sampling plans for advanced exploration, evaluation of resource estimation and also in mine planning (Sinclair and Blackwell, 2002), mill optimization (Journel and Isaaks, 1984) and financial risk analysis (Ravenscroft, 1992). In mining, it can also be used to assess the variability of the spatial distribution of the mineralization, risk sensitivity analysis in the mine planning process and effect of block size on ore variability. During mine production, conditional simulation can be used for reconciliation by comparing predicted grades in the resource model with the actual grades sampled at the process plant. In the geostatistical simulation, a system of model realizations is generated which present a range of possibilities (Vann et al., 2002). Conditional simulation generates maps of the grades of mineralization at a point honoring the sample grades' histogram, the variogram or spatial continuity of the sample grades as well as the grades at sample locations (Deutsch and Journel, 1992). The calculated histogram and variogram of simulated values and sample grades should be similar. Usually, it is the small-scale ore variation that causes ore misclassification in mining, it is important that these small-scale variations are reproduced like in conditional simulation since they affect the ore-waste selection process. In mineral resource estimation, Conditional Simulation was introduced to correct the smoothing effect that other techniques like Ordinary Kriging (OK) have on estimates (Matheron, 1971). As an estimation technique, it creates a pattern of values with the same statistical and spatial characteristics similar to true grades. In so doing, smoothing of data is minimized. Unlike other estimation techniques, the uncertainty attached to each estimate can be known when conditional simulation has been used. When assessing uncertainty attached to the prediction of a variable, or when realistic scenarios are required for post-processing algorithms, for instance, simulation of production, the spatial model can be used for simulation purpose rather than estimation (Rambert, 2005). Unlike kriging, simulation generates a series of realistic outcomes, having equal probabilities with each outcome honoring the input data, the spatial model and also the distribution model. Simulation has to reproduce statistical and geostatistical characteristics of variability (histogram and variogram) of the data (Vizi, 2008). Consequently, geostatistical simulation generates a set of values forming one of infinite possible 21 realizations which have simulated values having the same model of variogram as the experimental one and simulated values following the same distribution as the experimental ones. It is not uncommon for estimates to have some error or uncertainty since predictions can be inaccurate. Some of the errors can be due to widely spaced data used, geological variability, some approximations made in the estimation process and also the limitations of the models used. Apart from resource estimation, conditional simulation is considered to be the best option in predicting uncertainty for estimates since the uncertainty can be predicted at different scales

by simply averaging up the simulated values (Rossi and Deutsch, 2013). In addition, using a set of simulated realizations obtained by conditional simulation, a model of uncertainty at each location can be provided. The model can be examined by predicting the uncertainty at locations where there is data from drill holes or previous production data. Rossi and Deutsch (2013) further stated that the probability intervals can then be created by counting the number of times that the true values fall within those intervals, thus determining if the predicted percentage is verified. The uncertainty model depends on the Random Function model used and can be used to characterize risk. In grade control risk analysis is used in making economic decisions, through evaluating the consequence of grade uncertainty and the best choice is considered based on the maximum profit or minimum loss choice. Furthermore, the uncertainty model as described by the realizations provides all the information required to optimize decision-making under uncertainty (Rossi and Deutsch, 2013). Techniques like ordinary kriging do not allow the confidence interval to be calculated using its variance. However, simulations can be used to build confidence intervals empirically, since many different simulations of a variable are calculated, there is information at each point, and there is access to a complete empirical distribution; therefore being able to evaluate the probability for a given variable to take a value belonging to a given interval (Vann et al., 2002). The main objective is to reproduce the variance of the input data, both in univariate sense and spatiality through the histogram and the variogram or another covariance model respectively. Consequently, conditional simulation provides an appropriate platform to study any problem relating to variability, in such a way that other techniques like kriging cannot (Vann et al., 2002). Sinclair and Blackwell (2002) summarized the procedure as generating a number of realizations of the same location in space. These individual simulations at each point give a probability distribution for the grade at a specific point. These distributions are then used in many ways in which estimation of the probability that the grade is above a certain cut-off grade at a particular point is one of the uses. One of them is where the distributions in a specified block are combined and averaged to estimate the grade of that particular block or the probability that the block grade is above a certain cut-off grade. Some of the simulation techniques that are used in the mining industry are turning bands, sequential Gaussian and sequential indicator simulation. In this review, sequential Gaussian simulation will be considered.

7.1.1. Sequential Gaussian simulation

There are many simulation algorithms but Sequential Gaussian Simulation (SGS) is one of the simplest and widely used. Kriging gives an estimate of both mean and standard deviation of a variable at a point, hence the variable at each point can be represented as a random variable following a normal distribution. SGS uses a random deviate from the normal distribution selected according to a uniform random number representing the probability level rather than the mean as an estimate (Bohling, 2005).

SGS is an algorithm which simulates nodes after each other sequentially, subsequently using simulated values as a conditioning data. It is necessary to use standard Gaussian values in SGS method, therefore the data are transformed into Gaussian space. SGS creates realizations of normal random variables and performs a Gaussian transformation of the data. A simulated value at a visited point is randomly drawn from the conditional cumulative distribution function, defined by the kriging mean and variance, based on neighborhood values. At a new randomly visited point, the simulated value is conditional to the original data and previously simulated values. Finally, the simulated normal values are transformed back into the simulated values for the original variables. The process is repeated until all points are simulated for each realization throughout the grid (Deutsch and Journel, 1992). An illustrated basic steps in SGS algorithm is shown in Fig. 5 (Deutsch and Journel, 1992):

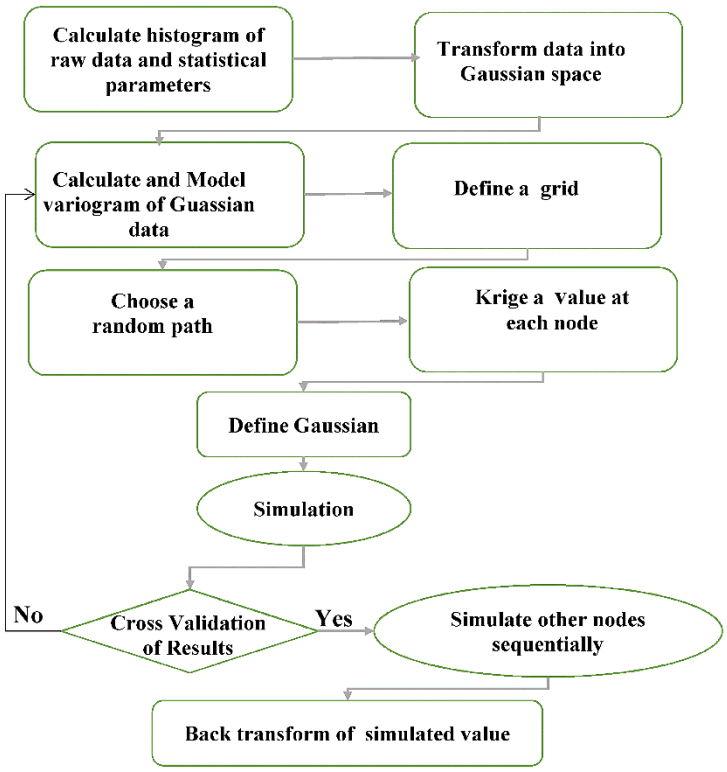


Fig. 5. The basic steps in the SGS algorithm modified after Deutsch and Journel (1992)

7.2. Kriging and ordinary kriging

Kriging is one of the earliest and commonly used techniques in mine operations for grade control purposes especially in estimating grade and tonnage of an area. It has been defined as a generic term that is applied to a range of methods of estimation that depend on minimizing the error of estimation, commonly by a least square procedure (Sinclair and Blackwell, 2002). Just like all geostatistical techniques, the basis of the technique is on the assumption that the variable to be estimated is a regionalized variable i.e. samples are related in space. The technique is applied upon meeting underlying assumptions of second-order stationarity which implies that at a minimum the mean and variance of the sample data remain invariant in space (Journel and Huijbregts, 1978). This means that the mean for the data must be constant at any location. To ensure that the weight functions are based on the variation of the observed variable as a function of its distance, kriging makes use of the variogram (Clark, 1979) . The advantage of using kriging as an estimation technique is that the kriged estimate is accompanied by a corresponding kriging standard deviation. Furthermore, the maps and/or calculations of the standard errors can be produced without actually taking the samples. The technique relies on the spatial relationship of the data. Both kriging and inverse distance weighting rely on distance in assigning weights, however, in kriging, the weights are based on both distances between points and also on the spatial relationship of the points. With kriging estimation, the variogram model, search parameters, and sample number form the basis for assigning kriging weights (Clark, 1979). Kriging uses the variogram, which does not depend on the actual value of data, but rather on its spatial distribution and internal spatial structure. In minimizing the estimation error, it finds a combination that is best for sample weights used in kriging process. The kriging variance obtained from the kriging technique can be used in the classification of resources. Based on how information has been used to estimate each block, the kriging variance can be used to rank the blocks of the resource model. This is done by using relative thresholds since the values do not have any physical or geological meaning. Alternatively, a visual comparison of the kriging variance can be used for defining resource categories (Rossi and Deutsch, 2013). Universal kriging, simple kriging,

indicator kriging, co-kriging and ordinary kriging are some of the types of kriging techniques (Isaaks and Srivastava, 1989). These techniques have been categorized into two, linear kriging of which ordinary kriging is one example and non-linear kriging which includes indicator kriging and disjunctive kriging. However, the choice of technique to use is mainly dependent on the type of the deposit, homogeneity of the data and what one is trying to achieve i.e. is it only the mean of the data or the whole distribution of data values, is it global estimates or local estimate and is it point estimate or block estimate (Isaaks and Srivastava, 1989).

Ordinary kriging is one of the estimation techniques commonly used in mining for grade control for mineral resource estimation. The technique is considered to be linear interpolator since it is based on the linear weighted average and assumes that the mean of the data is constant but also not known (Clark, 1979).

The distance from the estimation point and spatial correlation of the samples greatly influence the sample weight values. In ordinary kriging, the spatial correlation structure of the data influences the weighting of neighboring data points rather than by the power of their inverted distance like the inverse distance weighting method. Upon close evaluation, the kriging approach is similar to the inverse distance weighting method. However, the major difference is that ordinary kriging weights are assigned based on the variogram model thereby taking the spatial relationship into consideration.

Using the variogram in computing weights helps to minimize the expected error in the least square way. It is for this reason that ordinary kriging is sometimes said to produce the best linear unbiased estimate and some authors have associated ordinary kriging with the acronym B.L.U.E.; Best Linear Unbiased Estimator (Isaaks and Srivastava, 1989; Sinclair and Blackwell, 2002). It is considered to be linear since the weighted linear of data being estimated make up the estimate. The unbiased nature of the results is due to the absence of residual and estimation errors. Ordinary kriging is considered to be best since the variance of errors is mostly minimized through a linear combination of surrounding sample that are used to make predictions by assigning weights to the surrounding samples. Some estimation techniques like inverse distance weighting are also considered to be unbiased as well as linear like the ordinary kriging technique, however, ordinary kriging aims at reducing the error variance hence distinguishes it from these techniques. However, kriged estimates tend to be smoother than the input data.

7.3. Comparison between conditional simulation and ordinary kriging

Conditional simulation and ordinary kriging are techniques commonly used in the mining industry for resource estimation. Just like other geostatistical techniques, both techniques use variogram in determining the spatial relationship of data. Estimation errors through the kriging variance and block variance are provided along with the kriged estimates. The kriging variance forms the basis for Conditional Simulation. Although both techniques are used to provide estimates, their main goals are different. Ordinary kriging aims at providing unique estimates which minimize the local estimation variance while Conditional Simulation's purpose is to provide a set of maps of showing the grade, which honors the known reality and is likely to represent unknown reality at any location (Schofield and Rolley, 1997). Its ability to represent the spatial continuity of high grades and local variability of grades makes Conditional Simulation a better technique over ordinary kriging. Whilst the techniques are commonly used for resource estimation, Conditional Simulation also provides a direct way of quantifying the uncertainty in knowing the true grade of mining unit attached to an estimate in the form of a histogram, unlike ordinary kriging. Since conditional simulation generates several simulations of the grade of the deposit, there is direct access to a histogram of the possible grade for any particular point in the deposit. Furthermore, each value generated is independent of other values generated in other simulations and is more likely to be the true grade value at that particular location. This makes conditional simulation a better technique than ordinary kriging in describing the local uncertainty of the grade mineralization through direct access to the grade distribution through averaging of simulated values. Although kriging variance may be used to

provide a description of the uncertainty, it requires an assumption about the shape of the histogram of possible grades which practically is not realistic. Another limitation of kriging variance is that it assumes that the estimation variance is independent of the magnitude of the grade being estimated, contrary to most mineral deposits which have a variability of grades directly related to the magnitude of the grade (Schofield and Rolley, 1997). This becomes a limitation on using kriging variance to describe uncertainty in knowing the true grade of mining unit. It does not allow assessment of risk associated with the resource. Simulations attempt to sample at the unknown location using constraints like statistical moments of data, thereby the requirements of stationarity are stricter than that of kriging. Although ordinary kriging is considered to be the Best Linear Unbiased Estimator, the smoothing effect it has on estimates is one of the technique's main disadvantages. Local variability is not preserved as a result of smoothing and does not reproduce the histograms of the original data well like conditional simulation (Asghari et al., 2009).

On the other hand, conditional Simulation honors the original data and has a less smoothening effect on estimates and preserves local variability. However, with increasing sampling density like that of blast hole data, maps from both ordinary kriging and conditional simulation gradually become similar to the true map of reality because they both honor the sample values at the sample locations (Deutsch and Journel, 1992; Asghari et al., 2009).

8. Summary and Remarks

It is clearly seen from the relevant literature that ore properties such as bitumen grade, amount of fine content, type of fine content, mineralogy of both sand and fines and ore weathering have a significant impact on bitumen recovery and ore processability. If these variables are not controlled, they can decrease the quality of the bitumen and thus reduce the economic value of the mine plan. It could additionally pose an environmental issue in the tailings if this variable is not controlled.

The mine and tailings long-term plans define the complex strategy of the displacement of ore, waste, overburden, and tailings over the mine life. The objective of the long-term mine plans is to minimize the environmental footprint and maximize the cash flow. Limitation of space because of lease conditions, the scale of operations and construction of external and in-pit dyke impoundments add to the complexity of planning in oil sands mining. Oil sands long-term mine plans are driven by the quantity and quality of tailings produced downstream. Production scheduling is an important aspect of mine planning and design. Maximizing the NPV and considering the sequence of material that has to be mined over time, under the defined constraints, can be used to schedule for long-term production. However, oil sands mine planning and waste management are faced with limitations such as risks and uncertainties associated with the tailing process and only relying on deterministic models.

Mine planning and waste management problems in oil sands can be achieved by using a combination of mixed integer, linear and goal programming formulations. The aim of using goal programming is to enable the optimization solution to try to achieve a set of goals where some goals can be balanced off against one another depending on their significance. Hard constraints can also be converted to soft constraints which otherwise could lead to infeasible solutions (Ben-Awuah and Askari-Nasab, 2013). Stochastic integer programming models offer a framework to address uncertainty in key inputs of mine production schedules, in areas such as grade and rock type uncertainties. Mathematical formulations based on stochastic integer programming has shown to generate the optimal production schedule (Dimitrakopoulos and Ramazan, 2008). Hence, its application in the mine plan will allow the management of uncertainties associated with ore grades and waste disposal.

9. Research Methodology

The main goal of this research is to develop, implement, verify and validate an integrated mine planning optimization framework using stochastic integer programming that will address the related domains of bitumen grade uncertainty and oil sands waste management. The application of a stochastic integer programming model in this research will allow the increase of the bitumen recovery which will also maximize the net present value. The implementation of solid waste management into the mine plan will also contribute to an increase in the net present value of the operation over the mine life. Fig. 6 illustrates a summary of the methodology that will be used for this research.

In order to achieve the research goal, the related literature is reviewed, including the applications of the operations research methods in mine planning, oil sands waste management, and factors affecting bitumen recovery. Afterwards, a geological data containing bitumen ore properties such as bitumen, fines, water, organic rich solids and ultrafine clays will be modeled and imported into GEMS software to develop a suitable block model that will be used for this research. The next part of the research will involve finding the final pit limits for a real-case oil sands block model, using the 3-dimensional LG algorithm (Lerchs and Grossmann, 1965) and this will be conducted in Whittle software. A number of scenarios will be investigated in Whittle to generate the final pit shell and pushback design for different mining directions. The block model, the pit limits and the pushbacks corresponding to the directions with higher NPV values will then be used for experimental studies through the proposed stochastic integer programming model.

The required matrices for the deterministic MILP model are prepared in MATLAB, and the TOMLAB/CPLEX has been used to solve the MILP model. CPLEX uses a branch-and-cut algorithm, which is a combination of branch-and-bound and cutting plane algorithms to solve the integer programming. It continues to search for best integer solutions up to the point that finds an integer solution within an acceptable gap to the LP-relaxation optimal solution. Once the deterministic approach is completed, a stochastic approach will be considered where uncertainties such as bitumen grade and rock type will be considered.

Block modelling will be done using conditional simulation in order to meet the objectives of this research. The stochastic approach will be conducted on the developed block model by performing estimation using kriging. Cross validation for the kriging results will be conducted for accuracy of the results. Finally, multiple realizations will be generated using Sequential Gaussian Simulation (SGS). The simulation results obtained will then be used to compare with the kriging results.

A simulation optimization framework/tool would be presented at the end of the research to account for uncertainties in mining operations for robust long term production planning and proactive decision making. This framework/tool will use a discrete event simulation model of mine operations, which interacts with a goal programming based mine operational optimization tool to develop an uncertainty based long term schedule. Using scenario analysis, this framework will allow the planner to make proactive decisions to achieve the mine's operational and long-term objectives.

10. Expected Scientific and Industrial Contribution of the Proposed Research

Oil sands processability and waste management are major environmental concerns in Alberta’s oil sands industry. The presence of clay, fines, ultrafine and organic-rich solids reduces the quality of bitumen during extraction. Also, environmental challenges caused by the oil sands tailings production led to setting regulations for oil sands tailings operations. According to the Directive 074, the oil sands production operators are required to satisfy the conditions and regulations set by this directive (McFadyen, 2009).

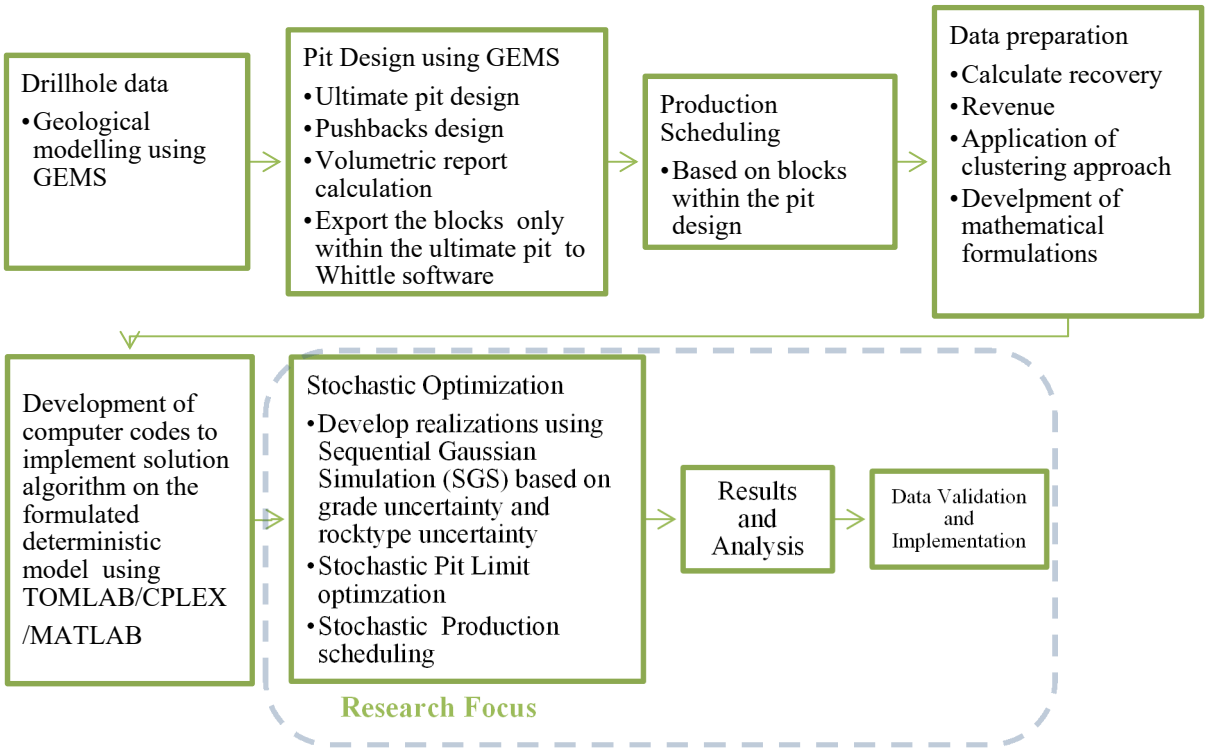


Fig. 6: Summary of research methodology.

The aim of these regulations is to ensure that the fluid tailings produced at the end of the oil sands production process, will become a reclaimable and trafficable landscape in order to overcome the environmental challenges caused by oil sands industry. In addition, the tailings production should meet the limitations on the deposition area specified by the directive (McFadyen, 2009).

Hence, the main contribution of this research is to develop an integrated framework for improving the bitumen recovery and to optimize the mine production schedule with respect to tailings and waste management. Mathematical formulations and simulation models based on uncertainties will be developed to contribute to the oil sands industry in mine planning and waste management by assisting in the improvement of bitumen recovery during extraction as well as the quality of the tailings and waste generated at the end of the oil sands production process. The main limitation of current oil sands mine plans is in over-estimating and under-estimating the quantity of the final tailings produced. Furthermore, the lack of an accurate relation between the oil sands long-term mine plans and the final product can lead to serious problems such as limitations in deposition area. An important industrial contribution of this research is a methodology/tool that will enables mine planners to modify and re-evaluate the oil sands mine plans based on the deposition area constraint, and the amount of tailings which is expected to be produced based on the mine plans. Also, a methodology/tool will be developed based on Sequential Gaussian Simulation (SGS), that will enable mine and tailings planners to assess and quantify uncertainties associated with the quantity and quality of bitumen recovered and tailings produced. The outcome of this research is expected to contribute towards the improvement of bitumen recovery and development of oil sands tailings management strategies, which will result in overcoming some of the environmental challenges associated with the oil sands production process.

11. Conclusion

Conditional simulation such as sequential Gaussian simulation is shown to be an appropriate framework to generate equally probable resource models used for risk analysis in mine planning

(Lantuéjoul, 2010). The combination of multiple simulated models provides the necessary tool to assess uncertainty associated with predictions of the attributes studied. Obtaining the local distribution of values by generating multiple simulations makes it possible to predict the magnitude of the fluctuations of an attribute for a given mining scenario. Through a mine-to-mill framework, fluctuations in geological and engineering parameters and their consequence in mine planning and scheduling can be anticipated thus enabling decisions that help to improve the net present value of the operation over the mine life.

12. References

[1] Akaike, A. and Dagdelen, K. (1999). A strategic production scheduling method for an open pit mine. *proceedings of the 28th Application of Computers and Operation Research in the Mineral Industry*, 729-738.

[2] AlbertaEnergyRegulator (2016). Directive 082. *Operating Criteria: Resource Recovery Requirements for Oil Sands Mine and Processing Plant Operations*,

[3] Asghari, O., Soltni, F., and Bakhshandeh Amnieh, H. (2009). *The Comparison Between Sequential Gaussian Simulation (SGS) of Choghart Ore Deposit and Geostatistical Estimation Through Ordinary Kriging.* Vol. 3,

[4] Badiozamani, M. M. and Askari-Nasab, H. (2014). Integration of reclamation and tailings management in oil sands surface mine planning. *Environmental modelling & software*, *51* 45-58.

[5] Basu, S., Nandakumar, K., and Masliyah, J. H. (1996). A Study of Oil Displacement on Model Surfaces. *Journal of Colloid and Interface Science*, *182* (1), 82-94.

[6] Beier, N. and Sego, D. (2008). The oil sands tailings research facility. *Geotechnical news*, *26* (2), 72-7.

[7] Ben-Awuah, E. (2013). *Oil sands mine planning and waste management using goal programming.* Vol. 74,

[8] Ben-Awuah, E. and Askari-Nasab, H. (2011). Oil sands mine planning and waste management using mixed integer goal programming. *International Journal of Mining, Reclamation and Environment*, *25* (3), 226-247.

[9] Ben-Awuah, E. and Askari-Nasab, H. (2013). Incorporating waste management into oil sands long term production planning. *Mining Technology*, *122* (1), 33-45.

[10] Bichard, J. A. (1987). Oil sands composition and behaviour research. The research papers of John A. Bichard 1957-1965. Alberta Oil Sands Technology and Research Authority, Edmonton, AB, paper Document Number,

[11] Bohling, G. (2005). Introduction to geostatistics and variogram analysis. C and PE 940, 17 October 2005.

[12] Bowman, C. W. (1971). The Athabasca Oil Sands Development—50 Years in Preparation. in *Origin and Refining of Petroleum*, Vol. 103, *Advances in Chemistry*, AMERICAN CHEMICAL SOCIETY, pp. 81-93.

[13] Caccetta, L. and Giannini, L. (1986). *Optimization techniques for the open pit limit problem*. in Proceedings of Proc. Australas. Inst. Min. Metall, pp. 57-63.

[14] Caccetta, L. and Hill, S. P. (2003). An Application of Branch and Cut to Open Pit Mine Scheduling. *Journal of Global Optimization*, *27* (2), 349-365.

[15] Caccetta, L., Kelsey, P., and Giannini, L. (1998). *Open pit mine production scheduling*. in Proceedings of APCOM 98: Computer Applications in the Mineral Industries International Symposium, pp. 65-72.

[16] Carrigy, M. A. (1967). Some sedimentary features of the athabasca oil sands. *Sedimentary Geology*, *1* 327-352.

[17] Chalaturnyk, R. J., Don Scott, J., and Özüm, B. (2002). Management of oil sands tailings. *Petroleum Science and Technology*, *20* (9-10), 1025-1046.

[18] Clark, I. (1979). *Practical geostatistics.* Vol. 3, Applied Science Publishers London,

[19] Conly, F. M., W Crosley, R., and Headley, J. (2011). *Characterizing sediment sources and natural hydrocarbon inputs in the lower Athabasca River, Canada.* Vol. 1, Pages 187-199.

[20] Dagdelen, K. (1986). Optimum open pit mine production scheduling by Lagrangian parameterization. *Proc. of the 19th APCOM*, 127-142.

[21] Dai, Q. and Chung, K. H. (1995). Bitumen—sand interaction in oil sand processing. *Fuel*, *74* (12), 1858-1864.

[22] Denby, B. and Schofield, D. (1995). *The use of genetic algorithms in underground mine scheduling*. in Proceedings of Proceedings of the 25th international symposium application of computers mineral industries, pp. 389-394.

[23] Denby, B., Schofield, D., and Bradford, S. (1991). Neural network applications in mining engineering. *Department of Mineral Resources Engineering Magazine, University of Nottingham*, 13-23.

[24] Deutsch, C. V. and Journel, A. G. (1992). *GSLIB: Geostatistical Software Library and User's Guide. Hauptbd.* Oxford university press,

[25] Dimitrakopoulos, R. and Ramazan, S. (2003). *Managing risk and waste mining in long-term production scheduling of open pit mine*. in Proceedings of 2003 SME Annual Meeting & Exhibit, Feb, pp. 24-26.

[26] Dimitrakopoulos, R. and Ramazan, S. (2008). Stochastic integer programming for optimising long term production schedules of open pit mines: methods, application and value of stochastic solutions. *Mining Technology*, *117* (4), 155-160.

[27] Eckert, W. F., Masliyah, J. H., Gray, M. R., and Fedorak, P. M. (1996). Prediction of sedimentation and consolidation of fine tails. *AIChE Journal*, *42* (4), 960-972.

[28] Gershon, M. E. (1983). Mine scheduling optimization with mixed integer programming. *Min. Eng.(Littleton, Colo.);(United States)*, *35* (4),

[29] Gleixner, A. M. (2009). Solving large-scale open pit mining production scheduling problems by integer programming.

[30] Godoy, M. and Dimitrakopoulos, R. (2004). Managing risk and waste mining in long-term production scheduling of open-pit mines. *SME transactions*, *316* (3),

[31] Hall, A. C., Collins, S. H., and Melrose, J. C. (1982). *Stability of Aqueous Wetting Films in Athabasca Tar Sands.* Vol. 23,

[32] Hein, F. J. and Cotterill, D. K. (2006). The Athabasca Oil Sands — A Regional Geological Perspective, Fort McMurray Area, Alberta, Canada. *Natural Resources Research*, *15* (2), 85-102.

[33] Hepler, L. G., Hsi, C., Alberta Oil Sands, T., and Research, A. (1989). *AOSTRA technical handbook on oil sands, bitumens and heavy oils.* Alberta Oil Sands Technology and Research Authority, [Edmonton, Alta.]

[34] Hickman, D. A. (2014). Particle swarm optimization for energy minimization of molecular systems.Thesis, Colorado School of Mines. Arthur Lakes Library,

[35] Hooshiar, A., Uhlik, P., Ivey, D. G., Liu, Q., and Etsell, T. H. (2012). *Clay minerals in nonaqueous extraction of bitumen from Alberta oil sands: Part 2. Characterization of clay minerals.* Vol. 96, Pages 183-194.

[36] Isaaks, E. H. and Srivastava, R. M. (1989). An introduction to applied geostatistics. Oxford university press, paper Document Number,

[37] Johnson, T. (1988). Application of the Max Flow algorithm to ultimate pit design. *Engineering design: better results through operations research methods*, 518-531.

[38] Johnson, T. B. (1968). Optimum open pit mine production scheduling. CALIFORNIA UNIV BERKELEY OPERATIONS RESEARCH CENTER, paper Document Number,

[39] Journel, A. and Isaaks, E. (1984). Conditional indicator simulation: application to a Saskatchewan uranium deposit. *Journal of the International Association for Mathematical Geology*, *16* (7), 685-718.

[40] Journel, A. G. and Huijbregts, C. J. (1978). *Mining geostatistics.* Academic press,

[41] Kalantari, S., Ben-Awuah, E., and Askari-Nasab, H. (2013). Towards an integrated oil sands mine plan and composite tailings plan. *International Journal of Mining, Reclamation and Environment*, *27* (2), 103-126.

[42] Kaminsky, H. (2008). *Characterization of an Oil Sand Ore and Process Streams.*

[43] Kaminsky, H., H. Etsell, T., G. Ivey, D., and Omotoso, O. (2008). *Characterization of Heavy Minerals in Alberta Oil Sands.* Vol. 21, Pages 264-271.

[44] Kasongo, T., Zhou, Z., Xu, Z., and Masliyah, J. (2010). Effect of clays and calcium ions on bitumen extraction from athabasca oil sands using flotation. *The Canadian Journal of Chemical Engineering*, *78* (4), 674-681.

[45] Kasperski, K. (1992). A review of properties and treatment of oil sands tailings. *AOSTRA Journal of Research*, *8* 11-11.

[46] Kasperski, K. L. (2001). Extraction and water chemistry, Canadian Oil Sands Network for Research and Development (CONRAD) Oil Sands Extraction Fundamentals and Process Water Workshop, 7-8 May 2004. paper Document Number,

[47] Khan, A. and Niemann-Delius, C. (2014). *Research Article Production Scheduling of Open Pit Mines Using Particle Swarm Optimization Algorithm.* Vol. 2014,

[48] Kotlyar, L. S., Ripmeester, J. A., Sparks, B. D., and Montgomery, D. S. (1988). Characterization of organic-rich solids fractions isolated from Athabasca oil sand using a cold water agitation test. *Fuel*, *67* (2), 221-226.

[49] Koushavand, B., Askari-Nasab, H., and Deutsch, C. V. (2014). A linear programming model for long-term mine planning in the presence of grade uncertainty and a stockpile. *International Journal of Mining Science and Technology*, *24* (4), 451-459.

[50] Krige, D. (1951). A Statistical Approach to Some Basic Mine Valuation Problems on the Witwatersrand. *Journal of the Chemical, Metallurgical and Mining Society of South Africa*, *52* (6), 119-139.

[51] Lantuéjoul, C. (2010). *Conditional simulations.*

[52] Lerchs, H. and Grossmann, I. (1965). Optimum Design of Open Pit Mines, Canadian Institute of Mining Bulletin, 58.

[53] Liu, J., Xu, Z., and Masliyah, J. (2008). Interaction between Bitumen and Fines in Oil Sands Extraction System: Implication to Bitumen Recovery. *The Canadian Journal of Chemical Engineering*, *82* (4), 655-666.

[54] Long, J., Drelich, J., Xu, Z., and Masliyah Jacob, H. (2008). Effect of Operating Temperature on Water-Based Oil Sands Processing. *The Canadian Journal of Chemical Engineering*, *85* (5), 726-738.

[55] MacKinnon, M. (1989). Development of the tailings pond at Syncrude's oil sands plant: 1978–1987. *AOSTRA J. Res*, *5* (2), 109-133.

[56] Masliyah, J., Zhou, Z. A., Xu, Z., Czarnecki, J., and Hamza, H. (2004). *Understanding Water-Based Bitumen Extraction From Athabasca Oil Sands.* Vol. 82, Pages 628-654.

[57] Masliyah, J. H., Czarnecki, J., and Xu, Z. (2011). *Handbook on theory and practice of bitumen recovery from Athabasca oil sands - Volume 1: Theoretical basis.* Pages 115-116.

[58] Matheron, G. (1971). The theory of regionalised variables and its applications. *Les Cahiers du Centre de Morphologie Mathématique*, *5* 212.

[59] McFadyen, D. (2009). Directive 074: Tailings Performance Criteria and Requirements for Oil Sands Mining Schemes. *Energy Resources Conservation Board, Calgary, AB, Canada*,

[60] Meagher, C., Dimitrakopoulos, R., and Avis, D. (2014). Optimized open pit mine design, pushbacks and the gap problem—a review. *Journal of Mining Science*, *50* (3), 508-526.

[61] Mikula, R., Munoz, V., Kasperski, K., Omotoso, O., and Sheeran, D. (1998). *Commercial implementation of a dry landscape oil sands tailings reclamation option: consolidated tailings*. in Proceedings of Proceedings 7th UNITAR Conference, pp. 907-921.

[62] Mikula, R. J., Omotoso, O., and Kasperski, K. L. (2008). The chemistry of oil sands treatment: Production to treatment. Edmonton, AB: University of Alberta Geotechnical Center and Oil Sands Tailing Research Facility, pp. 10 pages.

[63] Mitchell, J. E. (2002). Branch-and-cut algorithms for combinatorial optimization problems. *Handbook of applied optimization*, 65-77.

[64] Mossop, G. D. (1980). Geology of the Athabasca Oil Sands. *Science*, *207* (4427), 145.

[65] Onur, A. and Dowd, P. (1993). *Open-pit optimization - Part 2: Production scheduling and inclusion of roadways.* Vol. 102, Pages 105-113.

[66] Osacký, M., Geramian, M., Uhlík, P., Čaplovičová, M., Danková, Z., Pálková, H., Vítková, M., Kováčová, M., Ivey, D. G., Liu, Q., and Etsell, T. H. (2017). Mineralogy and Surface Chemistry of Alberta Oil Sands: Relevance to Nonaqueous Solvent Bitumen Extraction. *Energy & Fuels*, *31* (9), 8910-8924.

[67] Osanloo, M., Gholamnejad, J., and Karimi, B. (2008). *Long-term open pit mine production planning: A review of models and algorithms.* Vol. Reclamation and Environment, Pages 3-35.

[68] Picard, J.-C. (1976). Maximal closure of a graph and applications to combinatorial problems. *Management science*, *22* (11), 1268-1272.

[69] Ramazan, S. (2007). The new fundamental tree algorithm for production scheduling of open pit mines. *European Journal of Operational Research*, *177* (2), 1153-1166.

[70] Ramazan, S., Dagdelen, K., and Johnson, T. (2005). Fundamental tree algorithm in optimising production scheduling for open pit mine design. *Mining Technology*, *114* (1), 45-54.

[71] Ramazan, S. and Dimitrakopoulos, R. (2018). Stochastic optimisation of long-term production scheduling for open pit mines with a new integer programming formulation. in *Advances in Applied Strategic Mine Planning*, Springer, pp. 139-153.

[72] Rambert, F. (2005). *Introduction to Mining Geostatistics*. in Proceedings of 67th EAGE Conference and Exhibition-Workshops,

[73] Ravenscroft, P. J. (1992). Recoverable reserve estimation by conditional simulation. *Geological Society, London, Special Publications*, *63* (1), 289-298.

[74] Rossi, M. E. and Deutsch, C. V. (2013). *Mineral resource estimation.* Springer Science & Business Media,

[75] Royle, A., Clausen, F., and Frederiksen, P. (1981). Practical universal kriging and automatic contouring. *Geoprocessing*, *1* 377-394.

[76] Samal, A. R., Mohanty, M. K., and Fifarek, R. H. (2008). Backward elimination procedure for a predictive model of gold concentration. *Journal of Geochemical Exploration*, *97* (2-3), 69-82.

[77] Sanford, E. C. (1983). Processibility of athabasca oil sand: Interrelationship between oil sand fine solids, process aids, mechanical energy and oil sand age after mining. *The Canadian Journal of Chemical Engineering*, *61* (4), 554-567.

[78] Schofield, N. and Rolley, P. (1997). *Optimisation of ore selection in mining: Method and Case Studies*. in Proceedings of Proceedings Third International Mining Geology Conference,

[79] Sheeran, D. (1993). *An improved understanding of fine tailings structure and behaviour*. in Proceedings of Proceedings fine tails symposium, oil sands–our petroleum future conference. Edmonton: AOSTRA Publications, pp. 19.

[80] Sinclair, A. J. and Blackwell, G. H. (2002). *Applied Mineral Inventory Estimation.* Cambridge University Press, Cambridge,

[81] Soane, D., Ware, W., Mahoney, R., and Kincaid, K. (2010). *Oil sands tailings treatment via surface modifications of solids with polymers*. in Proceedings of Proceedings of the 2nd International Oil Sands Tailings Conference, pp. 5-8.

[82] Tabesh, M. and Askari-Nasab, H. (2011). Two-stage clustering algorithm for block aggregation in open pit mines. *Mining Technology*, *120* (3), 158-169.

[83] Takamura, K. (1982). Microscopic structure of athabasca oil sand. *The Canadian Journal of Chemical Engineering*, *60* (4), 538-545.

[84] Vann, J., Bertoli, O., and Jackson, S. (2002). *An overview of geostatistical simulation for quantifying risk*. in Proceedings of Proceedings of Geostatistical Association of Australasia Symposium" Quantifying Risk and Error, pp. 1.

[85] Vizi, L. (2008). A case study in uniform conditioning of local recoverable reserves estimation for Jelšava magnesite deposit-level 220.

[86] Wallwork, V., Xu, Z., and Masliyah, J. (2008). Bitumen Recovery with Oily Air Bubbles. *The Canadian Journal of Chemical Engineering*, *81* (5), 993-997.

[87] Wik, S., Sparks, B. D., Ng, S., Tu, Y., Li, Z., Chung, K. H., and Kotlyar, L. S. (2008). Effect of process water chemistry and particulate mineralogy on model oilsands separation using a warm slurry extraction process simulation. *Fuel*, *87* (7), 1394-1412.

[88] Zhou, Z. A., Xu, Z., Masliyah, J. H., and Czarnecki, J. (1999). Coagulation of bitumen with fine silica in model systems. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, *148* (3), 199-211.

Integration of Mine-to-Mill Production Planning Strategy for Oil Sands Mining and Waste Disposal

Ahlam Maremi and Eugene Ben-Awuah
Mining Optimization Laboratory (MOL)
Laurentian University, Sudbury, Canada

ABSTRACT

For oil sands mining, the production schedule must be integrated simultaneously with in-pit and ex-pit dyke construction scheduling. In-pit dykes are constructed in the mined-out areas concurrently with the advancement of the pit phase mining. The mined ore that exceeds the plant capacity will be stockpiled for a limited duration. The topmost layer of the overburden will be used for land reclamation. Organic Rich Solids (ORSs) which represents about 5 wt% of the total ore and negatively affect the bitumen recovery will be used as a predictor for ore processability. In this research, a theoretical and conceptual mine planning framework based on Mixed Integer Linear Goal Programming (MILGP) for oil sands production scheduling and waste management is presented. New robust constraints that control the annual tonnage fluctuation for material mined and processed over the mine life are introduced in the model. The model generates an integrated mine plan with a waste management and stockpiling strategy over the mine life that maximizes the Net Present Value (NPV) of the operation. The model also integrates a mine-to-mill production planning strategy that uses ORSs content during optimization.

1. Introduction

Extracting mining blocks from an open pit mine in a specific sequence that maximizes the NPV is known as open pit mine planning optimization. Open pit mine planning optimization aims to provide the plant with ore at full capacity which is subject to a variety of production, grade blending and pit slope constraints (Whittle, 1989). A major aspect of mine planning is optimizing the long term production planning. The very first and highly important step in the mine planning process is modeling the ore body appropriately. All other activities throughout the mine life starting from evaluating the economic viability of the entire mining operation to undertaking all the processes of mine planning will be based on the ore body model (Hustrulid and Kuchta, 2006). The main phases of the mine planning process are: 1) block model determination; 2) ultimate pit limit (UPL) definition; and 3) production planning (Chicoisne, et al., 2012).

The focus of this research is the integration of waste management into Long-Term Production Planning (LTPP) optimization as required by recent regulations from Alberta Energy Regulator (AER) Directive 085 (Alberta Energy Regulator, 2017). Oil sands mining of the McMurray formation is studied and used as a case study. The McMurray formation is the largest deposit in the world and mostly located near the surface. The Pleistocene unit is the topmost layer of the deposit. It contains muskeg (reclamation material), which is comprised mainly of organic matter. The Clearwater formation overlying the McMurray formation is comprised of marine clay, fine sand and siltstone. Both the Pleistocene and Clearwater formation known as overburden. The McMurray formation contains bitumen, the element of interest. It is informally subdivided into Upper, Middle and Lower formations based on the environment of sediments deposition. Devonian carbonates mark

the end of the oil sands deposit (Masliyah, 2010). Fig. 1 shows a sketch of the vertical soil profile for a typical oil sands formation.

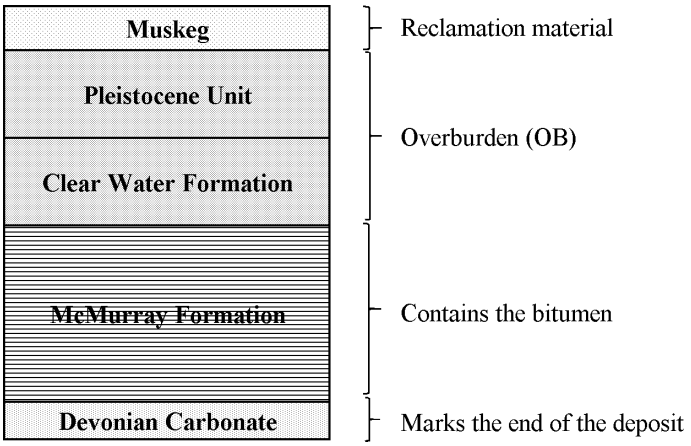


Fig. 1. Schematic view of the vertical soil profile for a typical oil sands formation.

Oil sands mining operations result in different types of material: ore, interburden (IB), overburden (OB), reclamation material (RM) and waste. Material with a bitumen grade of 7% or more will be classified as ore according to Directive 082 (Alberta Energy Regulator, 2016). Air flotation technique is used to separate bitumen from the fines. Processing the ore results in a huge amount of a mixture of water, fine materials, sands and residual bitumen known as tailings, the most unwanted by-product of oil sands processing (Masliyah, 2010). Using the hydro-cyclone, tailings is classified into Tailings Coarse Sands (TCS) and Fine Sands or slurry (Kalantari, et al., 2013). TCS is used for dyke construction while tailings slurry is deposited in the disposal areas created with dykes.

Any ore material that has a bitumen grade less than 7%, known as interburden, will be reclassified based on the fines content. Material with fines content less than 50% will be used for dyke construction; otherwise, it will be sent to the waste dump. Overburden (OB) will be used either for roads or dyke construction if it meets the fines requirement. Muskeg will be stockpiled and used to reclaim the land at the end of mine life. Any material that does not meet the requirements of ore, dyke materials or reclamation material is classified as waste and will be sent to the waste dump.

Waste management requires special geotechnical considerations and tailings management techniques that may lead to economic liabilities and delayed reclamation if not well managed (Boratynec, 2003; Ben-Awuah and Askari-Nasab, 2013; Azam and Scott, 2005). There are three significant aspects in dealing with oil sands tailings. Firstly, the greenhouse gas emissions resulting from the Clark Hot Water Extraction (CHWE) process (Devenny, 2009). Secondly, the environmental challenges due to the toxicity of the tailings resulting in the contamination of the fresh water table by polluted tailings' water leaks. Thirdly, space limitations increase the need for in-pit tailings containment, and storage space since more mining processes lead to additional volume of tailings slurry (Devenny, 2009).

Presently, plans for tailings deposition and mine reclamation are prepared after the optimization of the long-term mine production plans (Ben-Awuah and Askari-Nasab, 2013). Directive 085 (Alberta Energy Regulator, 2017) issued by the Alberta Energy Regulator (AER) requires oil sands operators to periodically publish their waste disposal and tailings plans publicly (McFadyen, 2008; Ben-Awuah and Askari-Nasab, 2013). Additional documentation on oil sands solid waste and tailings management can be found in Ben-Awuah, et al., (2012); Badiozamani and Askari-Nasab, (2014); Badiozamani and Askari-Nasab, (2016). Fig. 2 shows a conceptual mining model, which includes an oil sands deposit area to be mined and simultaneously used as an in-pit tailings storage facility. As mining advances in the specified direction, the in-pit tailings dyke footprints are released for dyke construction Fig. 2.

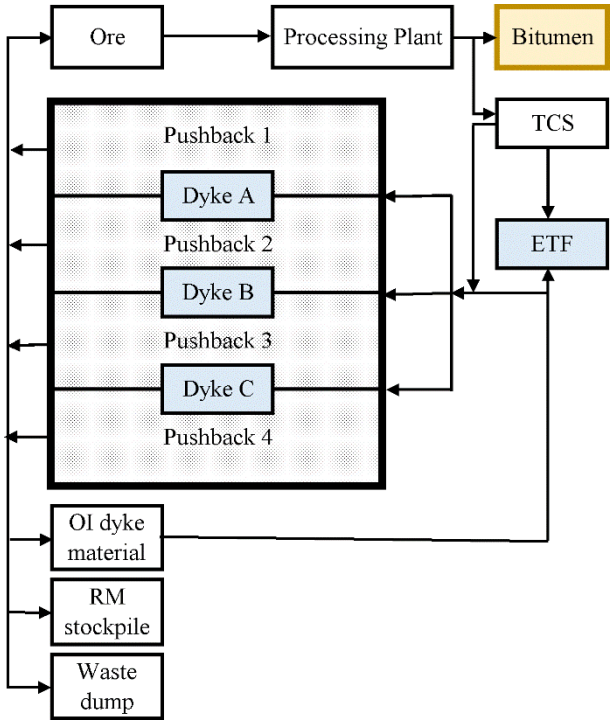


Fig. 2. Conceptual model for mining and waste management strategy modified after Ben-Awuah and Askari-Nasab (2013).

A schematic representation of the problem definition is presented in Fig. 3**.** The final pit block model is divided into pushbacks. The material intersecting a pushback and a bench is known as a mining-panel. Each mining-panel contains a set of mining-cuts and is used to control the mine production operation sequencing. Mining-cuts are clusters of blocks within the same mining bench that are similar in terms of location, grade, rock type and the shape of mining-cuts created on the lower bench. The figure depicts the scheduling of an oil sands ultimate pit block model containing K mining-cuts within P mining-panels. Each mining-cut k, could be made up of one or more of the following materials: ore, O_k, interburden and overburden dyke materials, ib_k and ob_k, muskeg reclamation material, mu_k, and waste, w_k.

The material in each mining-cut is to be scheduled over T periods based on the goals and constraints associated with the mining operation. The mined ore extracted from mining-cut k within mining-panel p in period t will be sent to the processing destination a. Any material that exceeds the processing capacity will be sent to the stockpile sp in period t and will be reclaimed in period $t+ts$, where ts is the stockpiling duration limit controlled by the planner to minimize oxidation of the stockpiled material that reduces processing recovery. The ore extracted in the current period t and the ore that has been sent to the stockpile in period $t-ts$ together will be sent to the processing destination to extract the bitumen. The generated TCS material together with the OB and IB dyke materials will be used for constructing $dyke_i$ at site i. Reclamation material will be sent to the reclamation material stockpile area. This is referred to as operational material scheduling.

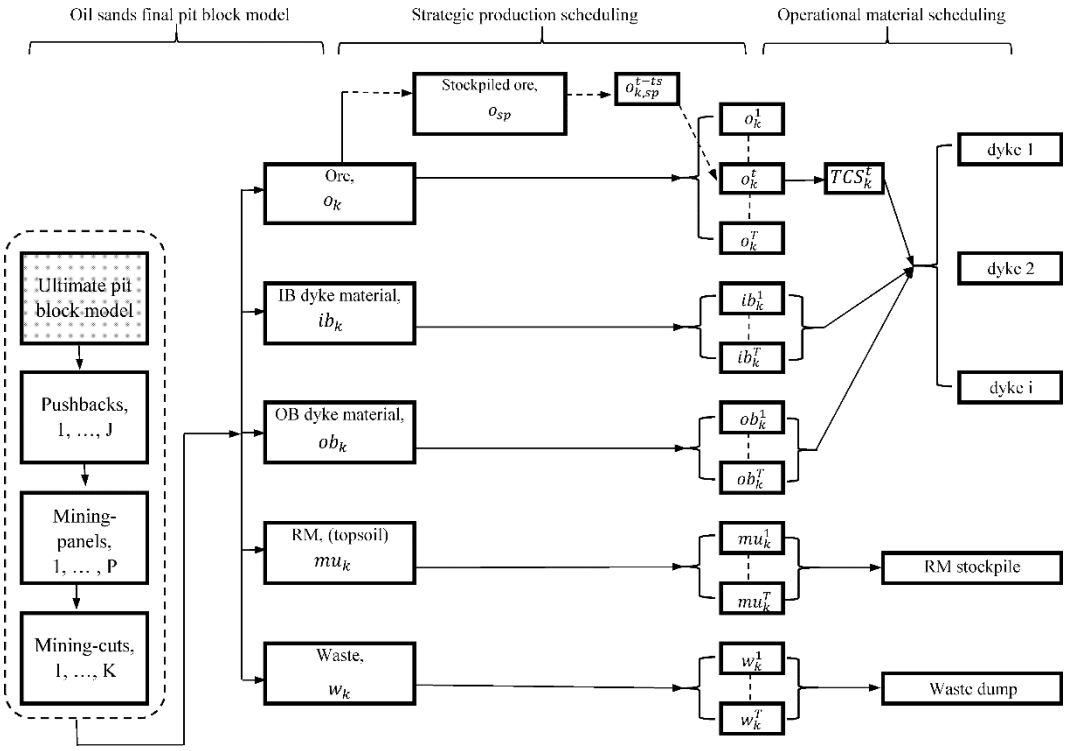


Fig. 3. Schematic representation of the problem definition showing strategic production and operational material scheduling.

These strategic and operational schedules to be developed are subject to a variety of economic, technical and physical constraints. The constraints control the mining extraction sequence, annual fluctuation of the tonnage mined and ore processed, and ore and dyke material blending requirements. The constraints also control reclamation and dyke material goals that specify the quantities of allowable material for reclamation works and dyke construction. The strategic and operational schedules determine the profitability and sustainability of the project. The schedules control the NPV of the operation and enable a robust waste management planning strategy. Improper waste management planning can lead to environmental issues, resulting in immediate mine closure by regulatory agencies.

The research seeks to develop a mine planning theoretical framework that maximizes the NPV of an oil sands mining operation and minimizes waste management cost using Mixed Integer Linear Goal Programming (MILGP) model. The model incorporates multiple material types with multiple elements for multiple destinations in oil sands long-term production planning. The proposed MILGP model aims to generate:

- A strategic schedule that determines the sequence of extracting ore, reclamation material, overburden and interburden from a predefined ultimate pit limit over the life of mine to maximize the NPV of the project;
- An operational schedule that determines the destination of reclamation material to minimize the extra mining cost and the destination of dyke materials to minimize dyke construction costs;
- A reclamation strategy for the ore that is stockpiled for a limited duration to reduce oxidation;
- A mine-to-mill production planning strategy that uses Organic Rich Solids content (ORSs) during optimization;

The next section of this research covers a summary of the literature review on LTPP optimization problems based on deterministic approaches, clustering and paneling in mine planning, and

stockpiling. Section 3 gives details of ORS definition and calculations and Section 4 highlights the theoretical mathematical programming formulation. Section 5 explains the implementation of the MILGP model research methodology. A case study is presented in Section 6. Section 7 documents the research conclusions.

2. Summary of Literature Review

Long-term production planning (LTPP) focuses mainly on ore reserves, stripping ratio and major annual investment plans (Newman, et al., 2010). The geologic block model is the backbone of open pit mine design and scheduling processes. Assigning the geological characteristics of each block and their grade can be done using available estimation techniques. Using financial and metallurgical data, the economic value of each block is also calculated (Osanloo, et al., 2008). In the literature, the LTPP and scheduling methods are divided into deterministic algorithms that consider the input values and parameters as known and fixed, and uncertainty-based algorithms that consider some input parameters as uncertain. Since the 1960s, researchers have studied and applied Mathematical Programming Models (MPMs) for mine production scheduling. A variety of exact optimization methods including Linear Programming (LP), Integer Programming (IP), and Mixed Integer Linear Programming (MILP) are commonly used in addition to Dynamic Programming (DP) and Goal Programming (GP) (Osanloo, et al., 2008).

Johnson (1969) introduced linear programming as a MPM to the mine planning research area. The author's model was for a long-term multi-destination open pit production planning problem. The results were not optimum and the size of the problem was computationally intractable. Subsequently, the initial LP model was modified by Gershon (1983) and Dagdelen (1985) to a MILP model. The authors considered a set of binary variables to satisfy the precedence of block extraction. The modified models could handle multiple ore processing options and multiple grades. However, their formulations could not ensure feasible solutions for all cases. Also, the number of binary variables makes the model intractable for real-size mine planning projects and difficult to be solved with the current state of hardware and software. Other researchers proposed some methods to solve IP models. Dagdelen and Johnson (1986) used the Lagrangian relaxation and sub-gradient optimization algorithm approach. Akaike and Dagdelen (1999) used 4D-network relaxation and sub-gradient optimization algorithm approach. Caccetta and Hill (2003) used branch-and-cut approach which is a combination of cutting plane and branch-and-bound algorithms. They solved sequences of linear programming relaxations of the IP problem by reducing the size of the problem prior to optimization. None of the abovementioned methods to solve IP models could be used for large-scale problems with dynamic cut-off grades.

Ramazan and Dimitrakopoulos (2004a) developed MILP formulations to reduce the number of binary variables and solution times. They set certain variables as binary and others as continuous. Their model resulted in partial mining of blocks that have the same ore value affecting the NPV generated. An MILP model was also developed by Ramazan and Johnson (2005) and Ramazan (2007) based on an aggregation method to reduce the number of integer variables in scheduling. The model was solved based on fundamental tree algorithm. However, it did not guarantee a global optimum solution of the problem. Caccetta and Hill (2003) developed an MILP model and Boland, et al. (2009) developed an LP model to generate mine production schedules with block processing selectivity. They did not provide enough information on the generated schedules to evaluate the practicality of the solutions. Askari-Nasab, et al. (2011) developed MILP models that use block clustering techniques. The models use a combination of continuous and binary integer variables and were applied to a large-scale problem. The portion of a block to be mined is controlled by continuous variables while binary integer variables control the extraction sequence of the blocks. The authors stated that they successfully implemented the models for some basic large-scale production scheduling problems. A Dynamic Programming (DP) model that maximizes the NPV, subject to production and processing constraints was presented by Osanloo, et al. (2008). This model

considers both the time value of money and block sequencing to determine the UPL. However, it cannot be applied to large-scale problems and there is no guarantee that mining and processing constraints will be satisfied. Based on a combination of heuristics and DP, Newman, et al. (2010) proposed their methodology. They state that the ultimate pit limits, the cut-off grade, the mining sequence and production scheduling are related to each other and without the knowledge of one variable, the next variable could not be determined. Their method provides the required simultaneous solution for the variables of the problem. In general, applying MPMs to the LTPP result in large scale optimization problems with many integer and continuous variables which are difficult to solve with the available software and hardware and might need lengthy solution time. The efforts that have been made in reducing the solution time were inefficient for large-scale problems or could not generate integrated practical mining strategies.

One of the deterministic approaches used to solve long-term production planning and scheduling problems is Goal Programming (GP). It is a popular deterministic approach for solving multiple objective optimization problems. The main idea of GP is that the optimizer provides results for the objective very close to the required goals, regardless of whether the goals are achievable or not. GP minimizes the deviations between the target values of the objectives and the satisfying solution (Orumie and Ebong, 2014). GP was used by Chanda and Dagdelen (1995) for the mine planning of a coal deposit. Due to goal functions interactions involved in solving the problem, the optimal solution could not always be achieved. Ben-Awuah and Askari-Nasab (2013) formulated the oil sands' long-term production scheduling and waste disposal planning problem using a combination of MILP and GP formulations. The hybrid termed as Mixed Integer Linear Goal Programming (MILGP) has an objective function, goal functions for mining, processing and dyke construction and constraints. These goals are prioritized according to the impact of a deviation from their targets on the entire mining operation. The authors stated that using MILGP is appropriate for their framework because, based on the importance of the goals, the MILGP structure will allow the planner to achieve some goals while others are traded off. In other words, the model allows for flexible formulation and the specification of priorities among goals. According to the authors, solutions with known optimality limits are generated when using exact solution methods for LTPP problems. For the resulting production schedule, a higher NPV is achieved as the solution gets closer to optimality.

In LTPP, the size of the problem grows exponentially as the number of blocks increases resulting in insufficient computer memory during optimization. More blocks mean more decision variables are used to control the block extraction precedence. Researchers have tried to classify the large amount of data into relatively few classes of similar entities (cluster) by maximizing both intra-cluster similarity and inter-cluster dissimilarity. This classification is known as aggregation or clustering. Applying clustering is an efficient way of dealing with this problem. Clustering will minimize the number of integer decision variables as well as maintaining the minimum mining width for large mining equipment (Askari-Nasab and Awuah-Offei, 2009). Boland, et al. (2009) proposed a solution procedure based on using aggregated blocks for the order of extraction decisions while individual blocks are used for processing decisions. This reduces the number of integer variables in the optimization problem that leads to reduced solution times.

Clustering algorithms can be categorized into hierarchical clustering, partitional clustering or overlapping clustering (Tabesh and Askari-Nasab, 2011). In mine planning, only hierarchical and partitional clustering can be used because all blocks must belong to a single cluster. For this reason, Tabesh and Askari-Nasab (2011) reviewed different clustering algorithms and developed a new clustering approach more suitable to the mining industry. Paneling is another technique that has been introduced in production scheduling to maintain practical mining widths and reduce the size of the optimization problem. The intersections of pushbacks and mining benches generate mining panels (Ben-Awuah and Askari-Nasab, 2013). Each mining-panel contains a set of mining-cuts and is used to control the mine production operation sequencing. For this research, hierarchical clustering algorithm developed by Tabesh and Askari-Nasab (2011) was used.

In the implementation of most LP and MILP models, the material flow post-extraction is not considered (Moreno, et al., 2017). “In particular, the use of stockpiling to manage processing plant capacity, and the interplay of material flows from the mine to a stockpile, the mine to a processing plant, and a stockpile to a plant, have not been treated as an integrated part of mine extraction sequence optimization” (Moreno, et al., 2017). Stockpiling can be used in mine operations for many reasons such as the blending of material, storage of overproduced ore or low-grade ore for future processing, and storage of waste material for reclamation purposes. Gemcom Software International (2015b) has a stockpiling module that considers mixing material with different grades in the stockpile. However, it does not use optimization techniques to model the stockpile, so there is no guarantee of obtaining an optimal solution with respect to the number of stockpiles and/or the grade contained in each stockpile (Moreno, et al., 2017).

Smith and Dimitrakopoulos (1999) used mixed integer programming to solve a short-term production scheduling problem with blending, considering stockpiles both at the mine and mill. They noted that it requires nonlinear constructs to correctly capture the contents of the stockpile. Ramazan and Dimitrakopoulos (2013) used a stochastic framework to incorporate stockpiling. In their model, the authors ignored the mixing of material in the stockpile. Asad (2005) cautioned that long-term stockpiling could result in problems such as leaching, deterioration of material and oxidation, which might result in poor recovery in the treatment process. For oil sands mining, the stockpiled material must also be processed within a limited duration to avoid oxidation that affects the efficiency of the processing recovery process. For this research, the stockpiling duration is limited to a maximum of two years to ensure there is no significant effect on the ore recovery.

3. Oil Sands Properties and Organic Rich Solids Definition

In oil sands mining, Clark Hot Water Extraction process (CHWE) is used to recover bitumen from the ore deposit. HWEP depends on the surface characterization of solid particle in the ore matrix. Measurements of fines (< 45μm) used to predict the processability of the ore, however is not always effective. It has been found that certain solid fractions known as Organic Rich Solids (ORSs) still exist even after the treatment of oil sands by multiple extraction toluene. These ORSs comprise about 5% of the total ore, and potentially affect the processability of oil sands (O'Carroll, 2002; Sparks, et al., 2003). During the bitumen separation process, the ORS carry any associated bitumen into the aqueous tailings, thus reducing overall bitumen recovery. In this sense, these solids are considered to be active and their associated quantity per ore can be estimated, thus, a better predictor for ore processability than the traditional use of bitumen ore fines contents (O'Carroll, 2002 ; Sparks, et al., 2003).

Oil sands analysis shown that the main components are bitumen, water and solids (Masliyah, 2010; O'Carroll, 2002). Solids are further classified into fines, ultra-fines, organic rich solids, clays and mineralogical composition. O'Carroll (2002) noted that loss in bitumen recovery is associated with higher ORS content in the ore. The author calculated bitumen to ORS ratio for each sample and reported that the ratio increases with higher bitumen content and further plotted the primary bitumen recovery against the BIT:ORS ratio. After a raising trend, the recovery levels off at 90% for BIT:ORS ratio values of 20 and more. Based on this result, the author concluded that the ratio has potential for use as an index in the characterization of oil sands ores.

4. Recovery Calculations

4.1. Recovery calculations based on Directive 082: Alberta Energy Regulator

For oil sands mining, the recovered volume of bitumen from the mining and processing operations, is specified by Directive 082 from (Alberta Energy Regulator, 2016). One of the four required

operating criteria for oil sands mining based on Directive 082 is the recovery of the processing plant which is uncertain and changes based on the average bitumen grade as defined by AER.

The recovery equals to 90% if the average bitumen content of the as-mined ore is 11% bitumen or greater. If the average bitumen content of the as-mined ore is less than 11 weight per cent bitumen, recovery is determined by Eq. (1), (Alberta Energy Regulator, 2016), where BIT is the average weight per cent bitumen content of the as-mined ore:

$$RECOV_{AER} = -2.5*(BIT)^2 + 54.1*(BIT) - 202.7 \quad (1)$$

4.2. Recovery calculations based on Organic Rich Solids

To calculate the ORS and BIT:ORS ratio for our dataset the following steps are done:

- BIT against ORS is plotted For O'Carroll's samples and an exponential correlation exists as can be seen in Eq. (2) with the coefficient of determination (R-squared) of 62.6%. (R-squared is a statistical measure of how close the data are to the fitted regression line. It is also known as the coefficient of determination).
- For this small dataset, bitumen, water and fines were given. The rest considered as solids (Solids = 100% - (bitumen% + water% + Fines%)).
- Solids for our dataset contain organic rich solids, ultra-fines and other solids. For each block in the ultimate pit block model ORS is calculated using Eq. (2). ORS ranging from 0.00 to 2.60% with the average of 0.88%.
- For O'Carroll's samples primary bitumen recovery is plotted against BIT:ORS ratio, a raising trend is noted. Recovery levels off at 90% for BIT:ORS ratio values of 20 and more. The relationship is modeled as seen in Eq. (3).
- BIT to ORS ratio is calculated for each block in our dataset. It covers the range from 0.00 to 17.69 with the average of 4.1.
- Eq. (3) is used to calculate the recovery for each block based on ORS content.

$$ORS = 3.8145 \times e^{-0.094 \times Bit} \quad (2)$$

$$RECOV_{ORS} = -0.0219 \times \left(\frac{BIT}{ORS}\right)^2 + 0.3335 \times \left(\frac{BIT}{ORS}\right) - 0.3789 \quad (3)$$

- The recovery is used to calculate the revenue.
- Bitumen recovery is plotted against BIT:ORS ratio. Recovery levels off at 90% for BIT:ORS ratio values of 8 and more (Fig. 4).

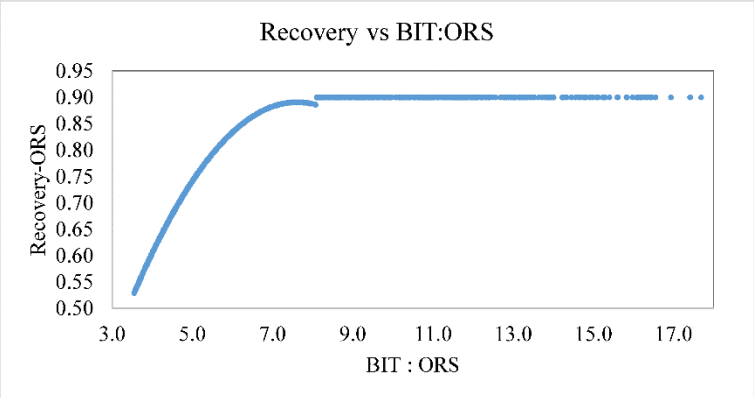


Fig. 4. Recovery vs. BIT to ORS ratio.

Subsequently, a case study with two different scenarios is examined using the proposed MILGP model. Both scenarios use tonnage fluctuation constraints to achieve the mining and processing

targets as part of the production scheduling optimization process with a limited duration stockpiling strategy for ore. However, the first scenario uses recovery calculated from AER to determine the revenue while the second uses recovery calculated from ORS. MATLAB (Mathworks, 2017) is used for coding the mathematical programming formulation and the resulting optimization problem is solved with a large-scale optimization solver IBM/CPLEX (ILOG, 2012). This solver uses a branch and cut algorithm which is a hybrid of branch-and-bound algorithm and cutting plane methods to solve the optimization problem (Horst and Hoang, 1996; Wolsey, 1998). The role of ORS on the ore processability comparing to the recovery calculations recommended by AER is investigated.

5. MILGP Theoretical Model Formulation

The strategic production schedule considers the time and sequence of extracting the ore, muskeg, overburden, interburden and waste blocks, as well as their destinations from a predefined UPL. The proposed MILGP model is capable of considering multiple mining locations, multiple pushbacks and different types of materials. The stockpiled ore can be reclaimed after pit mining is completed or simultaneously during active pit mining with a pre-determined reclamation duration. However, long-term stockpiling could result in problems such as leaching, the deterioration of material and oxidation, which can affect the efficiency of the processing recovery.

For oil sands mining, to avoid the risk of oxidation, the ore will be reclaimed in a pre-determined period controlled by the planner. The proposed oil sands production scheduling model integrates waste management through dyke construction and stockpiling for a limited duration. Stockpiling is for the mined ore that exceeds the plant capacity in any given year. The MILGP model is subject to economic, technical and physical constraints that control the mining operation. It is assumed that: 1) when a mining-panel is scheduled, all the mining-cuts, blocks or parcels within this mining-panel are extracted uniformly; 2) when modeling the relationship between the mining-panels and mining-cuts, the planner has access to all the mining-cuts within each mining-panel; 3) the stockpiling strategy is considered in the optimization problem for extra ore that exceeds the mill capacity and there are stockpile bins available for each period; 4) the exact amount of ore sent to the stockpile in period will be reclaimed after the stockpiling duration controlled by the planner. The notations used in the formulation of the oil sands long-term production planning and waste management framework have been classified as indices, sets, parameters and decision variables. The details of these notations can be found in the list of nomenclature.

5.1. Modeling of economic mining-cut value

The notion of economic block value is based on ore parcels which could be mined selectively. The profit from mining a block is a function of the value of the block and the cost incurred in mining, processing and dyke construction. Based on the value of the mining-cut and the costs incurred during mining and processing operations, the discounted profit of each mining-cut equals to the discounted revenue obtained by selling the final product contained in mining-cut k minus the discounted costs. For a mining-cut, if there are valuable elements, its discounted economic value if it is sent from the mine to the processing plant ($dm_k^{d,t}$) or from the stockpile to the processing plant ($ds_{k,sp}^{d,t}$) is given by Eqs. (4) and (5), respectively.

$$dm_k^{d,t} = rm_k^{a,e,t} - dw_k^{l,t} - dmu_k^{d,t} - dob_k^{d,t} - dib_k^{d,t} - dt_k^{d,t} \tag{4}$$

$$ds_{k,sp}^{d,t} = rs_k^{a,e,t} - dw_k^{l,t-ts} - dmu_k^{d,t-ts} - dob_k^{d,t-ts} - dib_k^{d,t-ts} - dt_k^{d,t-ts} \tag{5}$$

Eqs. (6) to (12) define the parameters used in Eqs. (4) and (5). It should be mentioned that the recovery is calculated using two different approaches: i) based on AER; ii) based on ORS.

$$rm_k^{a,e,t} = \sum_{e=1}^{E} o_k \times g_k^e \times rp_{avg}^{a,e} \times \left(p^{e,t} - sc^{e,t} \right) - \sum_{e=1}^{E} o_k \times pc^{a,e,t} \tag{6}$$

$$rs_k^{a,e,t} = \sum_{e=1}^{E} o_k \times g_k^e \times rp_{avg,sp}^{a,e} \times \left(p^{e,t} - sc^{e,t} \right) - \sum_{e=1}^{E} o_k \times pc^{a,e,t} - \sum_{e=1}^{E} o_k \times pc_{sp}^{a,e,t} \quad (7)$$

$$dw_k^{l,t} = (o_k + mu_k + ob_k + ib_k + w_k) \times mc^{l,t} \quad (8)$$

$$dmu_k^{d,t} = mu_k \times muc^{d,t} \quad (9)$$

$$dob_k^{d,t} = ob_k \times obc^{d,t} \quad (10)$$

$$dib_k^{d,t} = ib_k \times ibc^{d,t} \quad (11)$$

$$dt_k^{d,t} = t_k \times tc^{d,t} \quad (12)$$

Where:

$a \in A, A = \{1....., A\}$	Index for possible processing destination.
$d \in D, D = \{1,....., D\}$	Index for possible destinations for materials.
$e \in E, E\{1,...., E\}$	Index for elements of interest in each mining-cut.
$j \in J, J = \{1,....., J\}$	Index for pushbacks.
$k \in K, K = \{1,....., K\}$	Index for mining-cuts.
$l \in L, L = \{1,....., L\}$	Index for possible mining locations (pits).
$p \in P, P = \{1,....., P\}$	Index for mining-panels.
$sp \in SP, SP = \{1,...., SP\}$	Index for possible stockpiles in the model.
$t \in T, T = \{1,...., T\}$	Index for the scheduling periods, years.
$ts \in TS, TS = \{1,...., TS\}$	Index for possible stockpiling durations, years.
$dm_k^{d,t}$	Discounted economic mining-cut value obtained by extracting mining-cut k and sending it to destination d in period t.
$rm_k^{a,e,t}$	Discounted revenue obtained by selling the final products within mining-cut k in period t if it is sent to processing destination a, minus the extra discounted cost of mining all the material in mining-cut k as ore from location l and processing at destination d.
$dw_k^{l,t}$	Discounted cost of mining all the material in mining-cut k in period t as waste from location l.
$dmu_k^{d,t}$	Extra discounted cost of mining all the material in mining-cut k in period t as muskeg reclamation material at destination d.
$dob_k^{d,t}$	Extra discounted cost of mining all the material in mining-cut k in period t as overburden dyke material for dyke construction at destination d.
$dib_k^{d,t}$	Extra discounted cost of mining all the material in mining-cut k in period t as interburden dyke material for dyke construction at destination d.
$dt_k^{d,t}$	Extra discounted cost of mining all the material in mining-cut k in period t as tailings coarse sand dyke material for dyke construction at destination d.

$ds_{k,sp}^{d,t}$	Discounted economic mining-cut value obtained by extracting mining-cut k and sending it to stockpile sp and reclaiming it to destination d in period t.
$rs_{k}^{a,e,t}$	Discounted revenue obtained by selling the final products within mining-cut k from stockpile sp in period t if it is sent to destination a in period t, minus the extra discounted cost of processing and re-handling.
$dw_{k}^{l,t-ts}$	Discounted cost of mining all the material in mining-cut k in period $t-ts$ as waste from location l.
$dmu_{k}^{d,t-ts}$	Extra discounted cost of mining all the material in mining-cut k in period $t-ts$ as muskeg reclamation material at destination d.
$dob_{k}^{d,t-ts}$	Extra discounted cost of mining all the material in mining-cut k in period $t-ts$ as overburden dyke material for dyke construction at destination d.
$dib_{k}^{d,t-ts}$	Extra discounted cost of mining all the material in mining-cut k in period $t-ts$ as interburden dyke material for dyke construction at destination d.
$dt_{k}^{d,t-ts}$	Extra discounted cost of mining all the material in mining-cut k in period $t-ts$ as tailings coarse sand dyke material for dyke construction at destination d.
o_k , o_p	Ore tonnage in mining-cut k and mining-panel p
g_{k}^{e}	The required average head grade of element e in ore portion of mining-cut k.
$rp_{avg}^{a,e}$	Proportion of element e recovered (processing recovery) if it is sent from the mine to processing destination a.
$rp_{avg,sp}^{a,e}$	Proportion of element e recovered (processing recovery) if it is sent from the stockpile to processing destination a.
$p^{e,t}$	The selling price of element e in present value terms per unit of product.
$sc^{e,t}$	Selling cost of element e in present value terms per unit of product.
$pc^{a,e,t}$	Extra cost in present value terms per tonne of ore for mining and processing at processing destination a in period t.
$pc_{sp}^{a,e,t}$	Extra cost in present value terms per tonne of ore for stockpiling at stockpile sp and processing at destination a in period t.
mu_k , mu_p	Reclamation material tonnage in mining-cut k and mining-panel p.
ob_k , ob_p	OB dyke material tonnage in mining-cut k and mining-panel p.
ib_k , ib_p	IB dyke material tonnage in mining-cut k and mining-panel p.
w_k , w_p	Waste tonnage in mining-cut k and mining-panel p.
$mc^{l,t}$	Cost in present value terms of mining a tonne of waste in period t from location l.
$muc^{d,t}$	Cost in present value terms per tonne of RM at destination d.
$obc^{d,t}$	Cost in present value terms per tonne of OB dyke material for dyke construction at destination d in period t.
$ibc^{d,t}$	Cost in present value terms per tonne of IB dyke material for dyke construction at destination d in period t.
t_k , t_p	TCS dyke material tonnage in mining-cut k and mining-panel p.

$tc^{d,t}$ Cost in present value terms per tonne of tailings coarse sand dyke material for dyke construction at destination d in period t.

5.2. MILGP objective function for optimizing production schedule

To develop the models, the concepts presented in Ben-Awuah and Askari-Nasab, (2013) are used as the starting point. The objective function of the MILGP model for oil sands long-term production planning and waste management that maximizes the NPV of the mining operation is developed using the continuous decision variables $y_p^{l,t}$, $x_k^{a,t}$ and $c_{k,sp}^{a,t}$ to model mining, processing from mine and processing from stockpile respectively; for all mining locations and processing destinations. Continuous decision variables are used to allow for fractional extraction of mining-panels and mining-cuts in different periods for different locations and destinations. The objective function of the MILGP model for maximizing the NPV of the mining operation is stated in Eq. (13):

$$Max \sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{d=1}^{D}\sum_{sp=1}^{SP}\sum_{a=1}^{A}\sum_{e=1}^{E}\sum_{t=1}^{T}\left(\sum_{\substack{k\in MK_p \\ p\in Mp_j}}\left(rm_k^{a,e,t}\times x_k^{a,t} + rs_{k,sp}^{a,e,t}\times c_{k,sp}^{a,t} - dw_p^{l,t}\times y_p^{l,t}\right)\right) \quad (13)$$

Tonnage fluctuation constraints defined by Eqs. (14) and (15) are introduced in the proposed MILGP model to control the mining and processing targets. These constraints control the consecutive periodic fluctuation of the tonnage mined and tonnage processed. For material mined, the sum of deviations between two consecutive years should be less than or equal to a set deviation tonnage, D_m, allowed for mining. For material processed, the sum of deviations between two consecutive years should be less than or equal to a set deviation tonnage, D_p, allowed for processing. The mine planner controls the parameters D_m and D_p. The planner also controls when to start and finish applying these constraints. For instance, the planner might want to allow for a positive deviation in a couple of the first years and negative deviation in the last years. In other words, Eqs. (14) and (15) can be used for a controlled ramping up in the first years and ramping down in the last year. Eq. (15) is valid only when ore reclamation starts. Otherwise, set the ore reclamation variable in Eq. (15) to zero.

The importance of these special constraints is that, the planner does not need to set mining and processing targets. The optimizer uses the set periodic tonnage fluctuation parameters to determine the mining and processing targets. These constraints provide varying practical production schedule options for mine planners.

$$\sum_{t=1}^{T}\left(\sum_{p\in MP_j}\left(o_p + mu_p + ob_p + ib_p + w_p\right)\times\left(y_p^{l,t+1} - y_p^{l,t}\right)\right) \le D_m \quad (14)$$

$$\sum_{t=1}^{T}\left(\left(\sum_{\substack{k\in MK_p \\ p\in Mp_j}}\left(x_k^{a,t+1} + \sum_{sp=1}^{SP} c_{k,sp}^{a,(t-ts)+1}\right) - \left(x_k^{a,t} + \sum_{sp=1}^{SP} c_{k,sp}^{a,t-ts}\right)\right)\times o_k\right) \le D_p \quad (15)$$

Constraints that control ore stockpile tonnages are presented in Eqs. (16) and (17). These equations control the amount of ore sent from mining-cut k to stockpile sp in period t. The continuous decision variable $s_k^{sp,t}$ is used to model the ore sent to the stockpile. Material sent to the stockpile in period t are reclaimed in period $t+ts$, where ts is the stockpiling duration controlled by the planner. The planner also controls the upper and lower capacity limits for stockpile bins.

$$\sum_{sp=1}^{SP}\sum_{k=1}^{K} o_k \times s_k^{sp,t} \le \overline{os}^{sp,t} \tag{16}$$

$$\sum_{sp=1}^{SP}\sum_{k=1}^{K} o_k \times s_k^{sp,t} \ge \underline{os}^{sp,t} \tag{17}$$

The MILGP bitumen and fines grade blending constraints ensure that the quality requirements of the processing plant, stockpile and dyke construction destinations are achieved. These constraints are formulated using Eqs. (18) to (25). Ore bitumen grade blending constraints ensure the extracted ore from mining-cut k within mining-panel p sent to either processing destination a or to stockpile sp in period t meets the grade quality requirements. Ore bitumen grade blending constraints are formulated using Eqs. (18) to (21). Eqs. (18) and (19) represent inequality constraints that control the limiting ore bitumen grade sent from the mine and stockpile to the processing plant. Eqs. (20) and (21) represent inequality constraints that control the limiting ore bitumen grade sent from the mine to the stockpile.

$$\sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) - \overline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) \le 0 \tag{18}$$

$$\underline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) - \sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t-ts}\right) \le 0 \tag{19}$$

$$\sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times s_k^{a,t} - \overline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times s_k^{a,t} \le 0 \tag{20}$$

$$\underline{g}^{a,e,t}\sum_{p=1}^{P}\sum_{k\in MP_p} o_k \times s_k^{a,t} - \sum_{p=1}^{P}\sum_{k\in MP_p} g_k^e \times o_k \times s_k^{a,t} \le 0 \tag{21}$$

Ore fines grade blending constraints ensure the extracted ore from mining-cut k within mining-panel p sent to either processing destination a or to stockpile sp in period t meets the fines requirements. Interburden fines grade blending constraints also ensure that the interburden fines for dyke construction are within the upper and lower limits required. Fines grade blending constraints are formulated using Eqs. (22) to (25). Eqs. (22) and (23) represent inequality constraints used to control the limiting grade of ore fines sent from the mine and stockpile to the processing plant. Eqs. (24) and (25) represent inequality constraints used to control the limiting grade of ore fines sent from the mine to the stockpile.

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p \\ p\in MP_j}} o_k \times fn_k^e \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) - \overline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p \\ p\in MPj}} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) \le 0 \tag{22}$$

$$\underline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p \\ p\in MPj}} o_k \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) - \sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p \\ p\in MP_j}} o_k \times fn_k^e \times \left(x_k^{a,t} + c_{k,sp}^{a,t}\right)\right) \le 0 \tag{23}$$

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p \\ p\in MP_j}} o_k \times fn_k^e \times s_{k,sp}^{a,t}\right) - \overline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p \\ p\in MPj}} o_k \times s_{k,sp}^{a,t}\right) \le 0 \tag{24}$$

$$\underline{fn}^{a,t,e}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MPj}} o_k \times s_{k,sp}^{a,t}\right)-\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} o_k \times fn_k^{e} \times s_{k,sp}^{a,t}\right)\le 0 \tag{25}$$

5.3. The MILGP model for optimizing reclamation and dyke material schedule

The objective function of the MILGP model that minimizes reclamation material cost and the dyke construction cost as part of the waste management operation can be formulated using the continuous decision variables $v_k^{d,t}$, $z_k^{d,t}$, $u_k^{d,t}$, and $q_k^{d,t}$. These variables used to model RM , OB , IB and TCS dyke material requirements, respectively, for all dyke construction destinations. Other continuous deviational variables, $dv_1^{-,l,t}$, $dv_2^{-,a,t}$, $dv_3^{-,d,t}$, and $dv_4^{-,d,t}$ are defined to support the goal functions that control RM, OB, IB and TCS dyke materials, for all reclamation and dyke construction destinations. They provide a continuous range of units (tonnes) that can be determined by the optimizer to satisfy the set goals. In the objective function, these deviational variables are minimized. There are also deviational penalty cost and priority parameters in the objective function used to model the focus of mine management in the presence of multiple conflicting goals.

The deviational penalty cost parameters PN_1 , PN_2 , PN_3 , and PN_4 penalize the NPV for any deviation from the set goals. The priority parameters, P_1 , P_2 , P_3 , and P_4 are used to place emphasis on the most important goals. In general, the deviational penalty cost and priority parameters are set up to penalize the NPV if the set goals and the most important goals are not met. When setting up these parameters, the planner has to monitor how continuous mining proceed period by period, the uniformity of tonnages mined per period, and the corresponding NPV generated, to keep track of how parameter changes affect these key performance indicators. More weight should be assigned to a goal that has a higher priority for mine management. The objective function for minimizing the reclamation material and the dyke construction cost is represented by Eq. (26):

$$Min\sum_{l=1}^{L}\sum_{j=1}^{J}\sum_{d=1}^{D}\sum_{sp=1}^{SP}\sum_{a=1}^{A}\sum_{e=1}^{E}\sum_{t=1}^{T}\left(\sum_{\substack{k\in MK_p\\ p\in Mp_j}}\begin{bmatrix}\left(dmu_k^{d,t}\times v_k^{d,t}\right)-\left(dob_k^{d,t}\times z_k^{d,t}+dib_k^{d,t}\times u_k^{d,t}+dt_k^{d,t}\times q_k^{d,t}\right)\\ -P_1\left(PN_1\times dv_1^{-,l,t}\right)-P_2\left(PN_2\times dv_2^{-,a,t}\right)\\ -P_3\left(PN_3\times dv_3^{-,d,t}\right)-P_4\left(PN_4\times dv_4^{-,d,t}\right)\end{bmatrix}\right) \tag{26}$$

There are tonnage targets for reclamation material and dyke material for dyke construction destinations. Eqs. (27) to (30) represent all required goal functions. Eq. (27) defines the RM tonnage goal $MUg^{d,t}$ that control the total amount of RM to be mined from mining-cut k within mining-panel p in each period. The negative allowable deviation from the set RM goal is controlled by the planner using the $dv_1^{-,d,t}$ decision variable. OB , IB and TCS dyke material goal functions control the dyke material production targets, $OBg^{d,t}$, $IBg^{d,t}$ and $CSg^{d,t}$ for different dyke construction destinations. These are defined by Eqs. (28) to (30), respectively. These functions provide a feasible schedule for dyke construction. The negative allowable deviation from the set OB , IB and TCS dyke material goals are controlled by the planner using $dv_2^{-,d,t}$, $dv_3^{-,d,t}$ and $dv_4^{-,d,t}$ decision variables.

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(mu_p\times v_p^{d,t}\right)\right)+dv_3^{-,d,t}=MUg^{d,t} \tag{27}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(ob_k\times z_k^{d,t}\right)\right)+dv_4^{-,d,t}=OBg^{d,t} \tag{28}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(ib_k\times u_k^{d,t}\right)\right)+dv_5^{-,d,t}=IBg^{d,t} \tag{29}$$

$$\sum_{p=1}^{P}\left(\sum_{k\in MP_p}\left(cs_k\times q_k^{d,t}\right)\right)+dv_6^{-,d,t}=CSg^{d,t} \tag{30}$$

Eqs. (31) and (32) represent inequality constraints used to control the limiting grade of interburden dyke material fines sent from the mine to dyke construction destinations.

$$\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} ib_k\times fn_k^{ib}\times u_k^{d,t}\right)-\overline{fn}^{d,t,ib}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MPj}} ib_k\times u_k^{d,t}\right)\le 0 \tag{31}$$

$$\underline{fn}^{d,t,ib}\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MPj}} ib_k\times u_k^{d,t}\right)-\sum_{p=1}^{P}\left(\sum_{\substack{k\in MK_p\\ p\in MP_j}} ib_k\times fn_k^{ib}\times u_k^{d,t}\right)\le 0 \tag{32}$$

5.4. The MILGP model general constraints

5.4.1. Mining-panels extraction precedence constraints

Five precedence constraints presented in Eqs. (33) to (37) are used to define the precedence extraction sequence for each mining panel p based on its spatial location. These equations use the binary integer decision variable b_p. This variable is equal to one if the extraction of mining-panel p has started by or in period *t*; otherwise, it is zero. Specifically:

- Eq. (33) defines the vertical mining precedence. Prior to the extraction of a specific mining-panel, all the mining-panels above it must be extracted so that the mining-panel is accessible. The set $IP_p\left(Z''\right)$ represents the set of immediate mining-panels that are above mining-panel p.
- Eq. (34) defines the horizontal mining precedence. Prior to the extraction of a specific mining-panel, all the mining-panels in a specified horizontal mining direction on a level must be extracted. The set $IH_p\left(Z'\right)$ represents the set of immediate mining-panels in the specified horizontal mining direction.
- Eq. (35) defines the pushback mining precedence. Eq. (35) checks all the mining-panels within the immediate predecessor pushback that must be extracted prior to the extraction of mining-panels in pushback *j*. The set $MP_j\left(H''\right)$ represents the set of mining panels in the predecessor pushback.
- Eq. (36) ensures that mining-panel p can only be extracted if it has not been extracted before.
- Eq. (37) ensures that once the extraction of a mining-panel starts in period t, this mining-panel is available for extraction during the subsequent periods.

$$b_p^t-\sum_{c=1}^{c}\sum_{m=1}^{t} y_{u_1}^{c,m}\le 0,\ \ u_1\in IP_p\left(Z''\right) \tag{33}$$

$$b_p^t - \sum_{c=1}^{c}\sum_{m=1}^{t} y_{u_2}^{c,m} \le 0, \quad u_2 \in IH_p\left(Z'\right) \tag{34}$$

$$b_p^t - \sum_{c=1}^{c}\sum_{m=1}^{t} y_{u_3}^{c,m} \le 0, \quad u_3 \in MP_j\left(H''\right) \tag{35}$$

$$\sum_{c=1}^{c}\sum_{m=1}^{t} y_p^{c,m} - b_p^t \le 0 \tag{36}$$

$$b_p^t - b_p^{t+1} \le 0 \tag{37}$$

5.4.2. Decision variables' control constraints

In the MILGP model, all decision variables used to control mining, processing, stockpiling, reclamation material, dyke materials and goal deviations are continuous variables. Inequality Eq. (38) makes sure that all the material mined as ore (sent to either the processing destination a or the stockpile sp), and all reclamation and dyke materials extracted from the mining-cuts belonging to mining-panel p in period t are less than or equal to the total material mined from mining-panel p in period t from any mining location.

Eq. (39) ensures that the total fractions of ore mined from a mining-cut (sent to either the processing destination a or the stockpile sp) is less than or equal to one. Eq. (40) ensures that the fraction of ore extracted from mining-cut k and sent to the stockpile sp in period $t - ts$ must be equal to the fraction of ore reclaimed from the stockpile sp and sent to the processing plant a in period t; where ts is the stockpiling duration.

Eq. (41) ensures that the fractions of TCS dyke material produced from processed ore is less than or equal to the fractions of ore sent from the mine and stockpile to the processing plant in each period. Eq. (42) ensures that the fractions of mining-panel p extracted and sent to different destinations in different periods is less than or equal to one. Eq. (43) ensures that the total fractions of reclamation material extracted from the mine and sent to its destinations in different periods is less than or equal to one. Eqs. (44) to (46) ensure that the total fractions of dyke materials extracted from the mine (OB and IB) or generated from the processing plant (TCS) and sent to all destinations in different periods is less than or equal to one.

$$\left[\begin{array}{l} \sum_{d=1}^{D}\sum_{sp=1}^{SP}\sum_{a=1}^{A}\sum_{\substack{k\in MK_p \\ p\in MP_j}}\left(o_k \times x_k^{a,t} + o_k \times s_{k,sp}^{a,t} + mu_k \times v_k^{d,t} + ob_k \times z_k^{d,t} + ib_k \times u_k^{d,t}\right) \\ \le \sum_{l=1}^{L}\sum_{p\in MP_p}\left[y_p^{l,t}\left(o_p + mu_p + ob_p + ib_p + w_p\right)\right] \end{array}\right] \tag{38}$$

$$\sum_{a=1}^{A}\sum_{sp=1}^{SP}\sum_{t-1}^{T}\left(x_k^{a,t} + s_{k,sp}^{a,t}\right) \le 1 \tag{39}$$

$$\sum_{a=1}^{A}\sum_{sp=1}^{SP}\sum_{t=1}^{T}\left(c_{k,sp}^{a,t} - s_{k,sp}^{a,t-ts}\right) = 0, \quad t - ts \ge 0 \tag{40}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} q_k^{d,t} \le \sum_{a=1}^{A}\sum_{t=1}^{T} x_k^{a,t} + \sum_{sp=1}^{SP}\sum_{t=1}^{T} c_{k,sp}^{a,t} \tag{41}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} y_p^{d,t} \le 1 \tag{42}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} v_p^{d,t} \le 1 \tag{43}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} z_p^{d,t} \le 1 \tag{44}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} u_p^{d,t} \le 1 \tag{45}$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} q_p^{d,t} \le 1 \tag{46}$$

5.4.3. Non-negativity constraints

Eq. (47) ensures that the decision variables for mining, processing, stockpiling (ore sent and reclaimed), RM, OB, IB and TCS dyke material are non-negative. Eq. (48) ensures that the deviational decision variables that support the goal functions are non-negative as well.

$$\mathrm{y}_p^{l,t},\ x_k^{a,t},\ \mathrm{s}_{k,sp}^{d,t},\ \mathrm{c}_{k,sp}^{a,t},\ v_k^{d,t},\ z_k^{d,t},\ u_k^{d,t},\ q_k^{d,t} \ge 0 \tag{47}$$

$$dv_1^{-,l,t},\ dv_2^{-,a,t},\ dv_3^{-,d,t},\ dv_4^{-,d,t},\ dv_5^{-,d,t},\ dv_6^{-,a,t} \ge 0 \tag{48}$$

6. Implementation of the MILGP framework

In general, this section documents the application and results from the developed MILGP model for an oil sands dataset. Whittle software (Gemcom Software International, 2015b), which is based on 3D LG algorithm (Lerchs and Grossmann, 1965) is used to generate the optimized pit limit for the oil sands mine. The optimized pit shell from Whittle is used to design the final pit in GEMS software (Gemcom Software International, 2015a). The blocks within the final pit design are used as input data for the MILGP model for subsequent integrated long-term production scheduling and waste disposal planning. An agglomerative hierarchical clustering algorithm is used in clustering blocks within each intermediate pushback into mining-cuts (Tabesh and Askari-Nasab, 2011). The intersection between benches and intermediate pushbacks are used in creating mining-panels.

As mentioned in section **Error! Reference source not found.**, organic rich solids ORS comprising about 5% of the total ore and it might reduce overall bitumen recovery by carrying any associated bitumen into the aqueous tailings (O'Carroll, 2002; Sparks, et al., 2003). In this sense, ORSs are considered to be active and might be a better predictor for ore processability than the traditional use of bitumen ore fines contents. The recovery is calculated based on the BIT to ORS ratio. Directive 082 identifies the recovery as one of the operating criteria used by the AER. It is required that the bitumen recovery must be calculated based on the bitumen content. The recovery equals to 90% if the bitumen content greater than or equal to 11%, otherwise, Eq. **Error! Reference source not found.** will be used. It is noted that the recovery calculated based on AER requirements is always greater than or equal to the recovery calculated based on BIT to ORS ratio. The recoveries are percentage difference between zero and 4% (Fig. 5)

The MILGP model has two new robust constraints, which control the periodic tonnage fluctuation for mining and processing material. The tonnage fluctuation constraints eliminate the need to set a mining and processing target. It requires the mine planner to set an acceptable cumulative periodic tonnage fluctuation throughout the mine life, together with any production ramp up or ramp down requirements. The optimizer then determines the appropriate mining and processing targets that meets the tonnage fluctuation requirements. The cumulative periodic tonnage fluctuation value also controls the maximum possible mining and processing capacities indirectly. The tonnage fluctuation constraints provide varying practical production schedule options for mine planners. This MILGP

model generates a smooth and practical production schedule, a NPV with known limits of optimality and is easy to setup with more flexibility for the optimizer.

Two implementation scenarios highlighting different aspects of the developed MILGP model are outlined in Fig. 6. These scenarios are designed to highlight features of the MILGP model including:

1. Determining the mining and processing annual targets as part of the production scheduling optimization process and not as an input;
2. Determining the NPV based on the revenue generated from AER recovery (Scenario 1);
3. Determining the NPV based on the revenue generated from BIT:ORS recovery (Scenario 2).

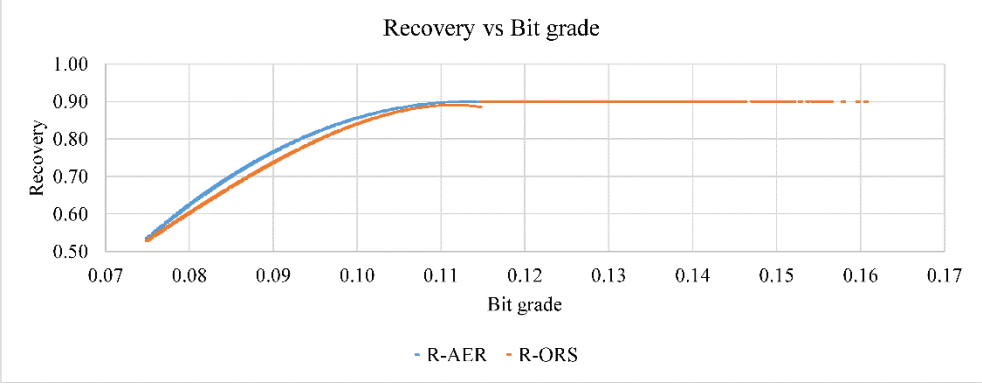


Fig. 5. Recovery vs bitumen grade.

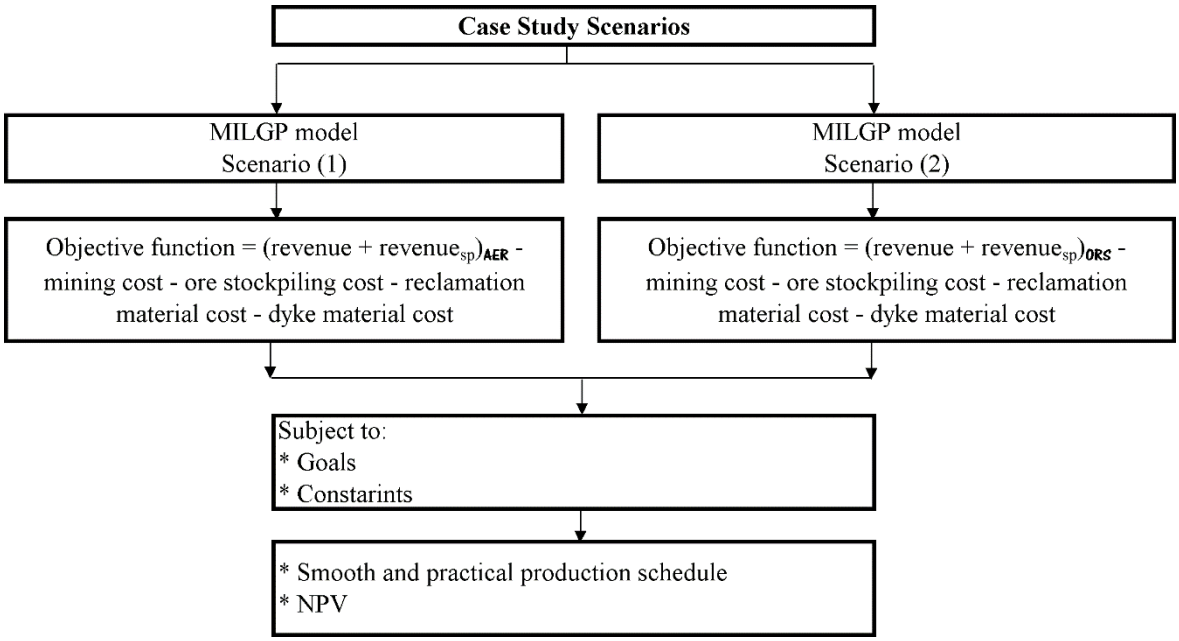


Fig. 6. Case study scenarios.

7. Case Study

For this case study, no pushbacks prior to the UPL were considered. However, to create mining-panels, the ultimate pit was divided into four pseudo pushbacks. Blocks in each mining-panel were clustered into mining-cuts using hierarchical clustering algorithm (Tabesh and Askari-Nasab, 2011). Two implementation scenarios as outlined in Section **Error! Reference source not found.** are investigated with the oil sands data. The deposit is to be scheduled for 8 years for the processing

plant, reclamation and dyke construction destinations. Summarized information on the oil sands deposit final pit design is presented in Table 1.

Table 2 shows the economic parameters and operational capacities for production scheduling. The economic data are extracted and compiled based on (Ben-Awuah and Askari-Nasab, 2013). Table 4 shows the upper and lower bounds of material quality requirements for ore and interburden dyke material. The model is implemented on a Lenovo Think Pad computer with i5 Core at 2.2 GHz, and 8.0 GB of RAM.

Table 1. Oil sands deposit final pit design characteristics.

Description	Value
Total tonnage of material (Mt)	182.23
Total ore tonnage (Mt)	88.44
Total TCS dyke material tonnage (Mt)	66.33
Total OB dyke material tonnage (Mt)	18.02
Total IB dyke material tonnage (Mt)	20.05
Total RM tonnage (Mt)	7.05
Number of blocks	2,523
Number of mining-cuts	155
Number of mining-panels	22
Number of benches	6

Table 2. Economic parameters and operational capacities.

Parameter (unit)	Value	Parameter (unit)	Value
Mining cost ($/tonne)	4.60	Cumulative periodic processing tonnage fluctuation (Mt)	30.00
Processing cost ($/tonne)	5.03	Mining recovery fraction (%)	100.00
Ore re-handling cost ($/tonne)	0.50	Processing recovery (%)	90.00
Selling price ($/bitumen %mass)	4.50	Discount rate (%)	10.00
TCS dyke material cost ($/tonne)	0.92	RM capacity (MT/year)	1.42
OB dyke material cost ($/tonne)	1.38	OB capacity (MT/year)	2.50
IB dyke material cost ($/tonne)	1.38	IB capacity (MT/year)	2.40
RM extra mining cost ($/tonne)	0.50	TCS capacity (MT/year)	9.00
Cumulative periodic mining tonnage fluctuation (Mt)	50.00		

Table 3. Material quality requirements.

Parameter	Value
Upper bound of ore bitumen grade (wt %)	16.0
Lower bound of ore bitumen grade (wt %)	7.0
Upper bound of ore fines percent (wt %)	30.0
Lower bound of ore fines percent (wt %)	0.0
Upper bound of IB dyke material fines percent (wt %)	50.0
Lower bound of IB dyke material fines percent (wt %)	0.0

In Scenario 1 and 2, the mining and processing capacities are not required to be set. The mine planner decides on an acceptable cumulative periodic tonnage fluctuation throughout the mine life for the mining and processing operations and allows the optimizer to determine the mining and processing limits that meets the cumulative periodic tonnage fluctuation value. The planner also controls how

many ramping up years is allowed at the beginning of the operation and how many ramping down years is allowed at the end. In this case study, 1-year ramping up is allowed at the beginning and 1-year ramping down is allowed at the end. The cumulative periodic tonnage fluctuation value used is 50.0 Mt for mining and 30.0 Mt for processing. This means the maximum possible mining and processing capacities allowed is 25 Mt and 15 Mt, respectively. In other words, not more than 50% of the cumulative periodic tonnage fluctuation value. The focus is to achieve a smooth processing rate throughout the mine life and generate a uniform production schedule that generates the highest NPV.

7.1. MILGP model: Scenario 1

In Scenario 1 for this case study, the overall NPV generated, including the reclamation and dyke material costs, is $ 1,499.3 M. The results of the production schedule are presented in Table 4 and Fig. 7 to Fig. 9.

Table 4. Production schedule using cumulative periodic tonnage fluctuation of 50.0 Mt for mining and 30.0 Mt for processing for a 2-year ore stockpiling duration and based on AER recovery (Scenario 1).

Period	Average bitumen grade (wt %)	Material mined (Mt)	Material processed (Mt)
1	8.32	25.00	0.89
2	10.55	25.00	13.85
3	11.53	25.00	13.85
4	10.97	25.00	13.85
5	10.86	25.00	13.85
6	11.10	25.00	13.85
7	11.21	25.00	13.85
8	10.90	6.90	4.43

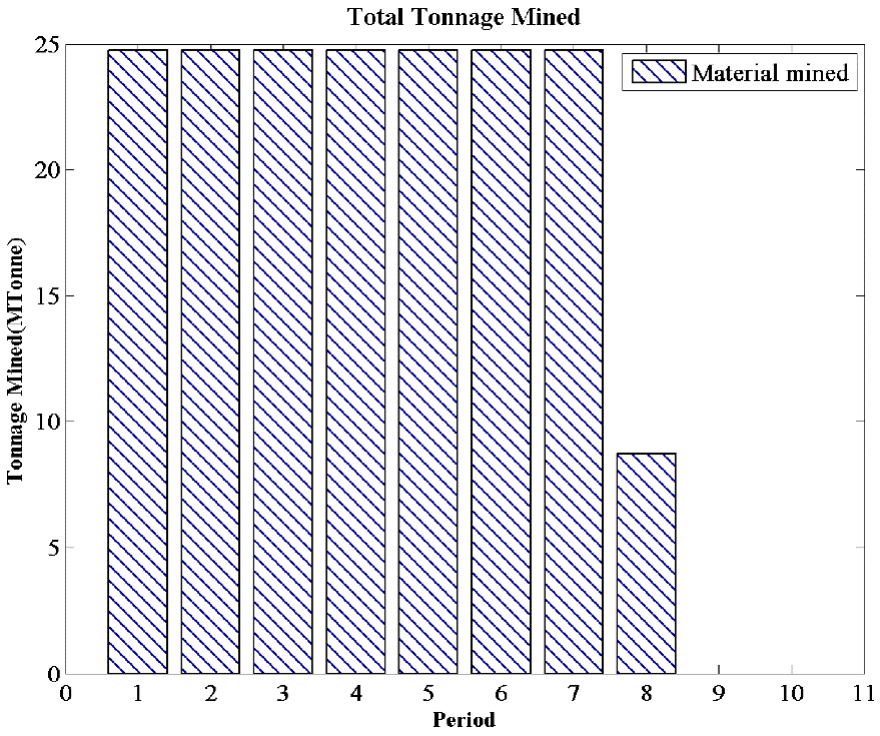


Fig. 7. Mining schedule using tonnage fluctuation constraints based on AER recovery.

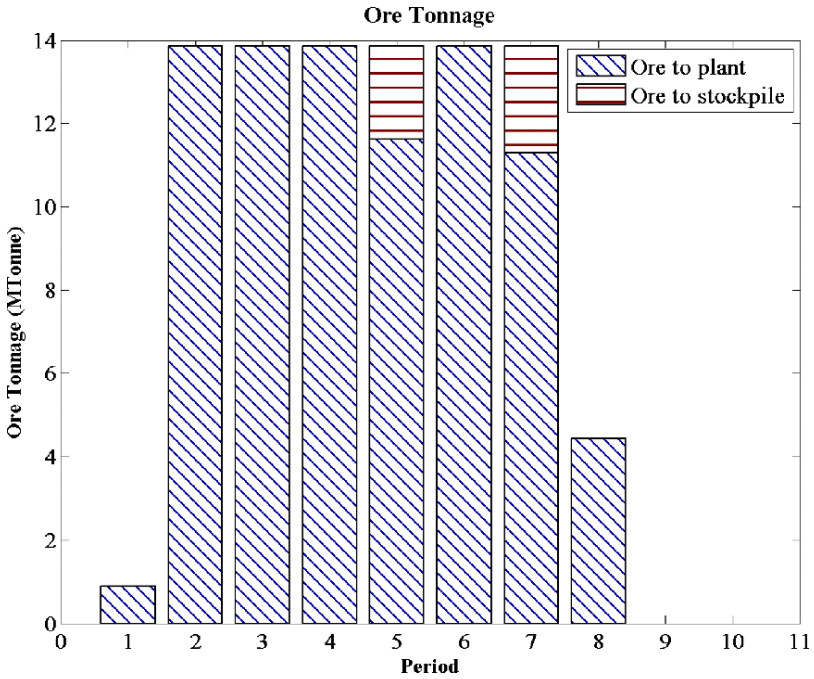


Fig. 8. Processing schedule using tonnage fluctuation constraints with a 2-year ore stockpiling duration based on AER recovery.

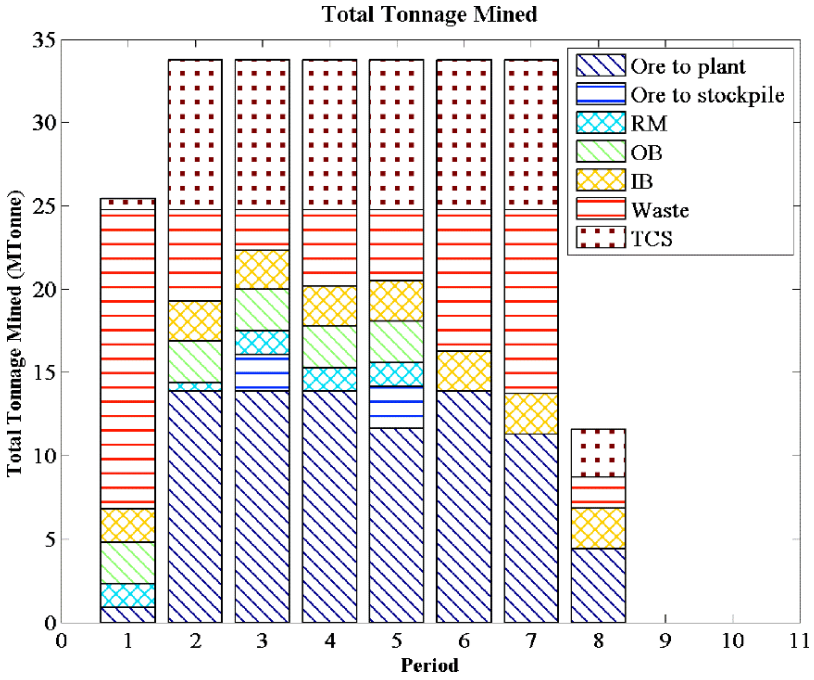


Fig. 9. Ore, RM, OB, IB and TCS dyke materials, and waste schedule using tonnage fluctuation constraints with a 2-year ore stockpiling duration based on AER recovery.

7.2. MILGP model: Scenario 2

In Scenario 2 for this case study, the overall NPV generated, including the reclamation and dyke material cost, is $ 1,468.2 M. The results of the production schedule are presented in Table 4 and Fig. 7 to Fig. 9.

Table 5. Production schedule using cumulative periodic tonnage fluctuation of 50.0 Mt for mining and 30.0 Mt for processing for a 2-year ore stockpiling duration and based on ORS recovery (Scenario 2).

Period	Average bitumen grade (wt %)	Material mined (Mt)	Material processed (Mt)
1	8.32	24.78	0.89
2	10.57	24.78	13.80
3	11.53	24.78	13.80
4	10.98	24.78	13.80
5	10.85	24.78	13.80
6	11.09	24.78	13.80
7	11.25	24.78	13.80
8	10.90	8.43	4.75

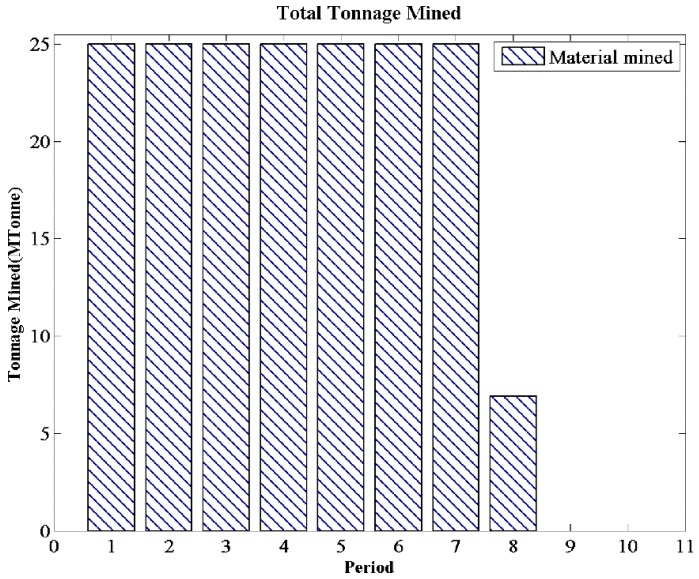


Fig. 10. Mining schedule using tonnage fluctuation constraints based on ORS recovery.

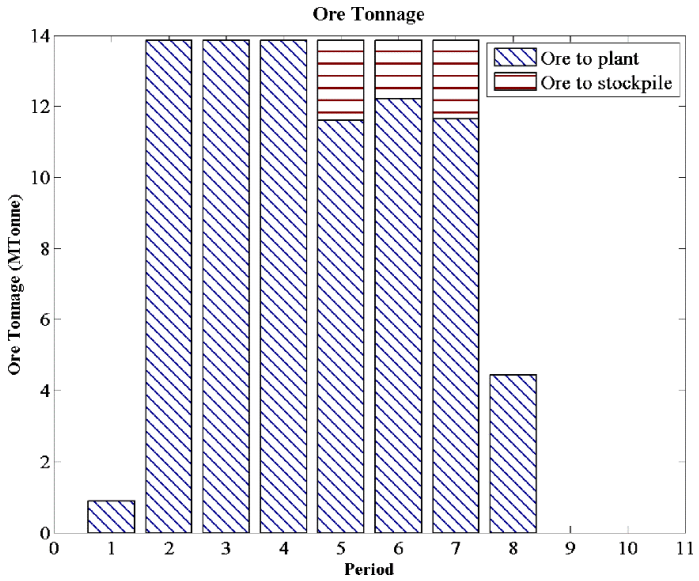


Fig. 11. Processing schedule using tonnage fluctuation constraints with a 2-year ore stockpiling duration based on ORS recovery.

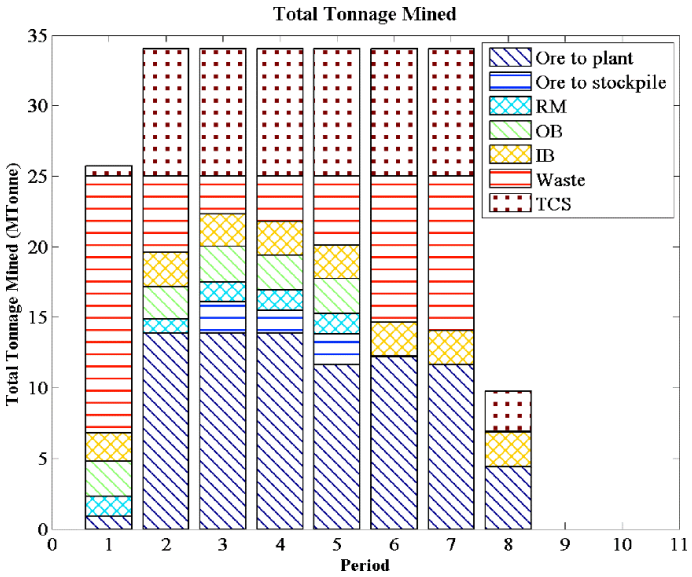


Fig. 12. Ore, RM, OB, IB and TCS dyke materials, and waste schedule using tonnage fluctuation constraints with a 2-year ore stockpiling duration based on ORS recovery.

7.3. Discussion of the results for Scenarios 1 and 2

The performance of the MILGP model in Scenarios 1 and 2 is analyzed based on: NPV, mining and processing production targets, bitumen grade profile and smoothness and practicality of the generated schedules. Both scenarios generate smooth schedules for mining, processing, reclamation and dyke materials However, in Scenario 1, more material is mined and processed all through the years compared to material mined and processed in the scenario 2. The average bitumen head grade is slightly different due to the amount of material processed and the use of the stockpile. The overall NPV generated from scenario 1, including the reclamation and dyke material cost, is 2.05% higher ($ 31.0 M) than the overall NPV generated from scenario 2.

There are two reasons explain the differences in the generated NPV. Firstly, and the primary reason is that the calculated recovery based on AER is always higher or equal to the recovery generated based on BIT:ORS ratio. Secondly, more ore is processed all through the years that means higher NPV. The average bitumen head grade is slightly different due to the difference in mining and processing schedules, and stockpile reclamation in different periods. The total material mined and processed, using Scenario 1 and 2, are the same as 181.9 Mt and 88.44 Mt, respectively. The model generated a uniform production schedule for OB, IB and TCS dyke material over the 8 periods for both scenarios. This ensures the effective utilization of the mining fleet and processing plant throughout the mine life.

The main advantage of using the new robust tonnage fluctuation constraints is that they are easy to set up and there is no need to decide on the periodic mining and processing targets. The only inputs required is how much total deviation is allowed throughout the mine life and if there is any production ramp up or ramp down requirements. These constraints provide varying practical production schedule options for mine planners.

8. Conclusions

The MILGP model for oil sands long-term production planning involves the interactions of their three main subcomponents: the objective function, the goal functions and the constraints in an optimization framework to achieve the research objectives. The MILGP model uses tonnage fluctuation constraints for mining and processing. Tonnage fluctuation constraints are easy to set up

and do not require the periodic mining and processing targets. This provides robust and practical production schedule options for mine planners. The model generates a strategic production schedule for the ore, reclamation and dyke materials using two different scenarios. For both scenarios, the MILGP model illustrates how production scheduling with limited duration stockpiling strategy for ore can be effectively integrated with waste disposal planning and reclamation material stockpiling in oil sands mining. Based on dyke construction requirements, schedules are generated to provide the required dyke materials to support engineered dyke construction that will help in reducing environmental impacts. This schedule gives the planner a satisfactory control over dyke materials and provide a solid platform for effective dyke construction and waste management planning. The MILGP model is extended to integrate a mine-to-mill production planning strategy that uses organic rich solids (ORSs) content during optimization.

9. References

[1] Akaike, A., and Dagdelen, K. (1999). *A strategic production scheduling method for an open pit mine.* Paper presented at the 28th International Symposium on the Application of Computers and Operations Research in the Mineral Industry, Littleton.

[2] Directive 082-Operating Criteria: Resource Recovery Requirements for Oil Sands Mine and Processing Plant Operations. (2016).

[3] Directive 085-Fluid Tailings Management for Oil Sands Mining Projects. (2017).

[4] Asad, M. W. A. (2005). Cutoff grade optimization algorithm with stockpiling option for open pit mining operations of two economic minerals. *International Journal of Surface Mining, Reclamation and Environment, 19*(3), 176-187.

[5] Askari-Nasab, H., and Awuah-Offei, K. (2009). *Mixed integer linear programming formulations for open pit production scheduling*. Edmonton: University of Alberta, Mining Optimization Laboratory Report One.

[6] Askari-Nasab, H., Pourrahimian, Y., Ben-Awuah, E., and Kalantari, S. (2011). Mixed integer linear programming formulations for open pit production scheduling. *Journal of Mining Science, 47*(3), 338-359.

[7] Azam, S., and Scott, J. D. (2005). *Revisiting the ternary diagram for tailings characterization and management*. Edmonton: The Oil Sands Tailings Research Facility.

[8] Badiozamani, M. M., and Askari-Nasab, H. (2014). Integration of reclamation and tailings management in oil sands surface mine planning. *Environmental Modelling & Software, 51*, 45-58.

[9] Badiozamani, M. M., and Askari-Nasab, H. (2016). Integrated mine and tailings planning: a mixed integer linear programming model. *International Journal of Mining, Reclamation and Environment, 30*(4), 319-346.

[10] Ben-Awuah, E., and Askari-Nasab, H. (2013). Incorporating waste management into oil sands long term production planning. *Transactions of the Institution of Mining and Metallurgy, Section A, 122*(1), 33-45.

[11] Ben-Awuah, E., Askari-Nasab, H., and Awuah-Offei, K. (2012). Production scheduling and waste disposal planning for oil sands mining using goal programming. *Journal of Environmental Informatics, 20*(1), 20-33.

[12] Boland, N., Dumitrescu, I., Froyland, G., and Gleixner, A. M. (2009). LP-based disaggregation approaches to solving the open pit mining production scheduling problem with block processing selectivity. *Computers and Operations Research, 36*(4), 1064-1089.

[13] Boratynec, D. J. (2003). *Fundamentals of rapid dewatering of composite tailings.* Unpublished MSc, University of Alberta, Edmonton.

[14] Caccetta, L., and Hill, S. P. (2003). An application of branch and cut to open pit mine scheduling. *Journal of Global Optimization, 27*(2-3), 349-365.

[15] Chanda, E. K. C., and Dagdelen, Kadri. (1995). Optimal blending of mine production using goal programming and interactive graphics systems. *International Journal of Mining, Reclamation and Environment, 9*(4), 203-208.

[16] Chicoisne, R., Espinoza, D., Goycoolea, M., Moreno, E., and Rubio, E. (2012). A new algorithm for the open-pit mine production scheduling problem. *Operations Research, 60*(3), 517-528.

[17] Dagdelen, K. (1985). *Optimum multi-period open pit mine production scheduling by Lagrangian parameterization.* Unpublished PhD, University of Colorado, Colorado.

[18] Dagdelen, K., and Johnson, T. B. (1986). *Optimum open pit mine production scheduling by Lagrangian parameterization.* Paper presented at the 19th International Symposium on the Application of Computers and Operations Research in the Mineral Industry.

[19] Devenny, D. W. (2009). *Overview of oil sands tailings*. Calgary: Alberta Energy Research Institute.

[20] GemcomSoftwareInternational. (2015a). Gemcom GEMS (Version 6.7.1). Vancouver: Gemcom Software International Inc.

[21] GemcomSoftwareInternational. (2015b). Whittle strategic mine planning software (Version 4.5.5). Vancouver: Gemcom Software International Inc.

[22] Gershon, M. E. (1983). Optimal mine production scheduling: evaluation of large scale mathematical programming approaches. *International Journal of Mining Engineering, 1*(4), 315-329.

[23] Horst, Reiner, and Tuy, Hoang. (2018). *Global optimization : deterministic approaches / Reiner Horst, Hoang Tuy*.

[24] Hustrulid, W. A., and Kuchta, M. (2006). *Open pit mine planning and design* (2nd ed.): Taylor and Francis/Balkema.

[25] ILOG, IBM. (2012). CPLEX reference manual and software (Version 12.5). Pullman, WA, USA: ILOG S.A. and ILOG Inc.

[26] Johnson, T. B. (1969). *Optimum open-pit mine production scheduling.* Paper presented at the 8th International Symposium on the Application of Computers and Operations Research in the Mineral Industry, Utah.

[27] Kalantari, Samira, Ben-Awuah, Eugene, and Askari-Nasab, Hooman. (2013). Towards an integrated oil sands mine plan and composite tailings plan. *International Journal of Mining, Reclamation and Environment, 27*(2), 103-126.

[28] Lerchs, H., and Grossmann, I. F. (1965). Optimum design of open-pit mines. *Transactions of the Canadian Mining and Metallurgical Bulletin, 68*, 17-24.

[29] Masliyah, J. (2010). *Fundamentals of oilsands extraction*. Edmonton: University of Alberta.

[30] Mathworks, Inc. (2017). MATLAB Software (Version 9.2 (R2017a)) [Software]: Mathworks Inc.

[31] McFadyen, D. (2008). *Directive 074*. Calgary: Energy Resources Conservation Board.

[32] Moreno, E., Rezakhah, M., Newman, A., and Ferreira, F. (2017). Linear models for stockpiling in open-pit mine production scheduling problems. *European Journal of Operational Research, 260*(1), 212-221.

[33] Newman, Alexandra M., Rubio, Enrique, Caro, Rodrigo, Weintraub, Andres, and Eurek, Kelly. (2010). A review of operations research in mine planning. *Interfaces, 40*(3), 222-245.

[34] O'Carroll, Josephine B. (2002). *Factors affecting bitumen recovery from oil sands.* National Library of Canada = Bibliothèque nationale du Canada, Ottawa.

[35] Orumie, U. C., and Ebong, D. (2014). A Glorious Literature on Linear Goal Programming Algorithms *American Journal of Operations Research, 4*, 59-71.

[36] Osanloo, M., Rashidinejad, F., and Rezai, B. (2008). Incorporating Environmental Issues into Optimum CutOff Grades Modeling at Porphyry Copper Deposits. *Resources Policy, 33*(4), 222-229.

[37] Ramazan, S. (2007). *Large-scale production scheduling with the fundamental tree algorithm: Model, case study and comparisons.* Paper presented at the Orebody Modelling and Strategic Mine Planning, Perth.

[38] Ramazan, S., and Dimitrakopoulos, R. (2004a). *Recent applications of operations research and efficient MIP formulations in open pit mining.* Paper presented at the SME Annual Meeting, Cincinnati, Ohio.

[39] Ramazan, S., and Dimitrakopoulos, Roussos. (2013). Production scheduling with uncertain supply: a new solution to the open pit mining problem. [journal article]. *Optimization and Engineering, 14*(2), 361-380.

[40] Ramazan, S., and Johnson, T. B. (2005). Fundamental tree algorithm in optimising production scheduling for open pit mine design. *Transactions of the Institutions of Mining and Metallurgy: Section A, 114*(1), 45-54.

[41] Smith, Martin, and Dimitrakopoulos, Roussos. (1999). The influence of deposit uncertainty on mine production scheduling. *International Journal of Surface Mining, Reclamation and Environment, 13*(4), 173-178.

[42] Sparks, B. D., Kotlyar, L. S., O'Carroll, J. B., and Chung, K. H. (2003). Athabasca oil sands: effect of organic coated solids on bitumen recovery and quality. *Journal of Petroleum Science and Engineering, 39*(3), 417-430.

[43] Tabesh, M., and Askari-Nasab, H. (2011). Two stage clustering algorithm for block aggregation in open pit mines. *Transactions of the Institution of Mining and Metallurgy, Section A, 120*(3), 158-169.

[44] Whittle, J. (1989). *The facts and fallacies of open pit design*. North Balwyn, Victoria, Australia: Whittle Programming Pty Ltd.

A Fuzzy Logic Approach towards Truck Dispatching Problem in Surface Mines

Ali Moradi Afrapoli, Shiv Upadhyay and Hooman Askari-Nasab
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

This paper presents a mixed integer linear programming model to solve truck dispatching problem in surface mines. Although, all thus far developed models in the literature approach to the truck dispatching as a deterministic problem, we believe that most of the input parameters for truck dispatching are uncertain. To account for the parameters' inherent uncertainties, we implemented a fuzzy approach to solve the developed mathematical model. The developed model is implemented on a surface mining case study and results are compared against a benchmark model implementation on the same case. The results show that the model developed in this paper requires smaller fleet of trucks to meet the production requirement of the mine in comparison to the benchmark model.

1. INTRODUCTION

Decision making tools play important roles in surface mines. These tools are utilized in a varied range of planning horizons. The major mine plans are Long-term plans with a range of 5 to 20 years of mine life, medium-term plans with a range of 1 to 5 years, short-term plans with a range of 1 week to 1 year (Osanloo et al. 2008), and daily or shift by shift operational plans.

In the context of surface mining operational plan, truck dispatching problem was referred as a single-stage or a multi-stage decision making problem (Moradi Afrapoli & Askari-Nasab 2017; Alarie & Gamache 2002). In the single-stage approach, after finding the shortest paths between sources and destinations, optimization of each path flow rate and decision on active trucks dispatching are made by solving a single optimization model. While in the multi-stage approach, after finding the shortest paths in the road network, a decision making model provides the optimum flow rate for each specific path in the upper stage. Then, at the lower stage, another optimization model provides dynamic decisions on where to dispatch the truck in need of immediate assignment including other trucks that would seek assignment subsequently. Aside from the single-stage model developed by Hauck (1973), the rest of the published models are based on multi-stage approach. Major research in this area, as explained by Alarie and Gamache (2002) and Moradi Afrapoli and Askari-Nasab (2017), have been focused on the upper stage. Kappas and Yegulalp (1991), Xi and

This paper, in its entirety, has been submitted to be published in the proceedings of 368th International Conference on Mining, Material, and Metallurgical Engineering (ICMMME), Tallinn, Estonia, Aug 14-15, 2018.

Yegulalp (1993), and Ercelebi and Bascetin (2009) implemented queuing theory approach towards solving the upper stage sub problem. Soumis et. al. (1989) developing a non-linear model. The literature survey shows that other researchers have implemented different versions of linear programming (LP) to solve the upper stage. White and Olson (1986) and Li (1990) implemented general LP to deal with the upper stage problem. The solution of their models provide tonnage of material required to be transported from each loader to a specific destination. Temeng, Otuonye and Frendewey (1998), Gurgur, Dagdelen and Artittong (2011), and Upadhyay and Askari-Nasab (2017) developed upper stage models based on goal programming approach. Ta et al. (2005), Mena et al. (2013), Chang, Ren and Wang (2015), and Matamoros and Dimitrakopoulos (2016) have published different models to solve the upper stage problem. Readers are encouraged to read Moradi Afrapoli and Askari-Nasab (2017) and (Alarie and Gamache (2002) for a detailed overview of the researches conducted in the field.

Despite the fact that the problem of truck dispatching in surface mines contains an important lower stage, there are only a few published optimization models to make the lower stage decisions. White and Olson (1986), Li (1990), Olson, Vohnout and White (1993), Soumis, Ethier and Elbrond (1989), and Temeng, Otuonye and Frendewey (1997) are the only published decision making models to solve the lower stage of the truck dispatching problem. After the work by Temeng, Otuonye and Frendewey (1997) the problem have been abandoned by the researchers of the field for more than two decades. From amongst the published models, none account for the fuzzy behavior of the required input parameters. Most of the input parameters of the lower stage problem are imprecise or uncertain at the time of solving the decision making model. Although most of the uncertainties in these input parameters can be handled by implementing the probability theory, not all of these parameters are random phenomena and thus successful application of the probability theory is not possible (Brito et al. 2009). These types of problem can be solved using fuzzy linear programing (FLP) approach based on the researchers' view point of the definition of the input parameters. Importance of FLP implementation in the mining industry rises when in the decision making procedures it is possible to accept more than one solution (Pendharkar 1997). Giving an example from truck capacity, although the nominal capacity of a Cat 797B truck is 380 short ton, however, loading 390 short ton payload is acceptable as well. Thus, a typical LP constraint limiting the Cat 797B capacity to 380 short ton will ignore this possibility. This means that although there is a capacity limit for a truck payload, it can be violated to some extent.

With the entrance of fuzzy set applications in the field of mining engineering in late 1980s (Nguyen 1985), (Bandopadhyay & Chattopadhyay 1986), (Bandopadhyay 1987), it has been extensively used in the decision making models for mine planning (Rahmanpour & Osanloo 2017) equipment selection and sizing (Bascetin et al. 2007; Aghajani Bazzazi et al. 2008) plant location selection (Yavuz 2008) as well as post-mining land use and reclamation (Bangian et al. 2012).

Lack of models that capture uncertainty and imprecision of the input parameters encouraged us to develop a fuzzy linear programming model to solve the truck dispatching problem. The aim of this paper is to introduce a new fuzzy linear programming (FLP) model that makes decision in the lower stage of truck dispatching problem in surface mines that incorporate imprecision of the input parameters in the solution procedure. The organization of the paper follows: model formulation, defuzzification, implementation, results, and conclusion.

2. MODEL FORMULATION

2.1. Simulation Model

The simulation model consists of eight different sub models. Table 1 lists the sub models developed to perform the simulation of the mining complex operation.

Each of the simulation sub models listed in Table 1 plays a crucial role in the simulation of the mining complex.

Table 1: List of sub models used to develop simulation model

No.	sub model	Task's description
1	Operation starting	Recalls available trucks from the bay and assigns them to the available shovels
2	Loading	Imitates the operation from the time a truck reaches to a shovel up until that truck leaves the shovel
3	Dumping	Imitates the process of dumping truck payloads into the dumping areas
4	Hopper and conveyor	Simulates the stockpile, hopper, and the conveyor that feed processing plants
5	Season change	This sub model simulates change of the season and thus corresponding changes in the parameter distributions affected by it.
6	Shift change	Simulates change of the operation shifts as well as days
7	Path flow rate	Prepares required input parameters to run the decision making model making decisions on the optimum path flow rate
8	Truck dispatching	Prepares required input data to run the truck dispatching decision making model

The procedure that developed simulation model to mimic the operation are listed below:

Step one: at the start of the simulation sub model 1 that handles the start of the operation process is responsible for the process of trucks travel from the bay to the available shovels.

Step two: at each shovel station, trucks are loaded by a shovel already assigned to the polygon in that specific position, all the required information are transferred from the polygon to the material loaded onto the truck, truck leave the shovel and polygon (coming from the short-term schedule) remains there until it is fully depleted.

Step three: the loaded truck travel on the road network taking the shortest path from the shovel to the dump destination.

Step four: if the destination is a waste dump, the truck backs up and dumps its material in the designated area, otherwise, a decision is made based on current line up in front of each hopper, then the truck is assigned to the hopper with the least number of trucks in its queue. Then, as soon as all the trucks in the line are done with the dumping process, the hopper capacity is tested, if it has enough room for the truck, truck can dump its material. Otherwise, the truck needs to wait for hopper to open enough room for its material.

Step five: if the truck is already assigned to a shovel, it leaves the dumping area to start travel to the designated shovel. Otherwise, the simulation model prepares required input data for the fuzzy truck dispatching decision making model. Then, implementing a VBA block, the required data are written into a file that can be read in CPLEX (IBM 2016). Once the decision is made by solving the fuzzy model, the truck is assigned to a shovel.

Step six: the truck travels to the shovel it is assigned.

Step seven: go to step two.

The procedure explained above consists of sub models 1, 2, 3, and 8. Sub model 4 controls stockpiles, hoppers, and conveyors. It accepts discrete truck loads in hopper and by simulating the conveyor that connects the hopper to the downstream processing operation, it continually feeds the plant based on the required hourly feed rate. Sub model 5 runs using a logical entity to change all the required input parameters when the simulation runs over two different seasons. In sub model 6, a logical entity works toward changing shifts of the operation when the simulation time reaches to the end of each shift. The process of production optimization decisions that must be made within a time interval are handled using sub model 7. This sub model collects required data from the status of the mining operation and sends them to an external decision maker tool to make required decisions regarding optimum path flow rate or allocation of trucks.

2.2. Optimization Model

Under the multi-stage truck dispatching approach (Moradi Afrapoli & Askari-Nasab 2017; Alarie & Gamache 2002), we developed a deterministic ILP mathematical model to make decisions on the trucks' next destination. After that, we identified fuzzy parameters and based on those fuzzy parameters we improved the crisp model to a fuzzy model. The sets, parameters, and variables used in the model development are as follows:

Parameters:

ltt_{td}	loaded travel time from current truck t position to dump d
qt_{td}	time truck t must spend in queue at dump d to get the permission to dump its material
dt_{td}	time truck t spends at dump d to back up and dump its load into the dumping area
ett_{ts}	time it takes truck t to travel empty from where its empty travel starts to shovel s
$tinq_{tts}$	time a truck of type tt that is already in queue must spend in shovel s queue
$tenr_{tts}$	time a truck of type tt must travel from its current position to reach shovel s before truck t
st_{tts}	spot time for a truck of type tt at shovel s
lt_{tts}	loading time for a truck of type tt at shovel s
tc_t	capacity of truck t (ton)
TC_t	nominal truck capacity for truck t (ton)
sc_s	capacity of shovel s
pc_d	capacity of dump d (ton per hour)
mf	factor that shows with the available trucks what portion of the current demands can be met by the fleet
pf_{sd}	optimal path flow rate for path from shovel s to dump d based on upper stage decisions
$pmsf_{sd}$	portion of the required path flow rate for current time span that have been met so far

Variables:

x_{tds}	binary integer variable to assign truck t to the path connecting shovel s to dump d
AF	variable factor that adjusts trucks available to be assigned with the demands of dumping locations

In this study we first developed a new deterministic model that makes truck dispatching decisions in a way that minimizes cumulative lost time for the entire active material handling fleet including both the loader fleet and the transporter fleet, accounting for the operational limitations such as truck capacity, shovel digging rate, and processing plants feed rate requirement. The objective function coefficient for each of the variables is calculated using Eq. (1):

$$C_{tds} = \left| \begin{array}{l} ltt_{td} + qt_{td} + dt_{td} + ett_{ts} \\ -\sum_{tt=1}^{TT} \left(tinq_{tts} + tenr_{tts} \right) \times \left(st_{tts} + lt_{tts} \right) \end{array} \right| \quad (1)$$

The developed model is a deterministic model with all its input parameters taking deterministic values. It also can be categorized as mixed integer transportation problem based decision making model. The objective function of the model is presented in Eq. (2). The objective function of the model tries to minimize the cumulative time difference between the times truck *t* will reach to

shovel *s* after dumping its load at dump *d* and the time shovel *s* will be available next time with its first part of objective. The second part of objective function tries to maximize the adjustment factor (*AF*), where *VBN* stands for very big number. This will help to attain higher precision in dispatching decisions.

$$\min Z = \sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{s=1}^{S} C_{tds}x_{tds} + VBN(1-AF) \tag{2}$$

Subject to:

$$\sum_{d=1}^{D}\sum_{s=1}^{S} tc_t x_{tds} \le TC_t \;\forall\, t \in \{1,\ldots,T\} \tag{3}$$

$$\sum_{t=1}^{T}\sum_{d=1}^{D} tc_t x_{tds} \le sc_s \forall\, s \in \{1,\ldots,S\} \tag{4}$$

$$\sum_{t=1}^{T}\sum_{s=1}^{S} tc_t x_{tds} \ge AF \times pc_d \forall\, d \in \{1,\ldots,D\} \tag{5}$$

$$AF \begin{cases} \le mf & \text{if } mf > 1 \\ \le 1 & \text{otherwise} \end{cases} \tag{6}$$

$$x_{tds} \in \{0,1\} \tag{7}$$

$$mf = \frac{\sum_{t=1}^{T} tc_t x_{tds}}{\sum_{s=1}^{S}\sum_{d=1}^{D}(pf_{sd} - pmsf_{sd})} \tag{8}$$

The decisions are made by optimizing the objective function under the operational constraints given by Eq. (3) to Eq. (7). Constraint (3) makes sure that the truck *t* cannot accept loads more than its nominal capacity. Constraint (4) ensures that summation of nominal capacity of all the trucks assigned to shovel*s* does not exceed the shovel's nominal digging rate (capacity). *AF* in constraint (5)is defined as adjustment factor. The adjustment factor is a variable that is constrained by *mf* (Eq. (6)) which is calculated by the formula presented in Eq. (8). The model is also constrained to meet as much targeted plants' capacity as possible by Eq. (5). Eq. (7) guarantees that both sets of the decision variables are binary integer.

Zimmermann (1976) and (1978) for the first time implemented fuzzy set theory in conventional LP models (Madadi & Wong 2014). Then after, several FLP models have been developed to deal with different real-world problems and more specifically in mining industries. The most recent FLP model developed in a mining operation context can be credited to (Rahmanpour & Osanloo 2017) where the authors developed a FLP model to solve surface mines short term planning problem. Even though all the input parameters to solve the optimization models in the truck dispatching problem might behave in a fuzzy behavior, none of the developed models have considered that fuzzy behavior so far. Thus, in this paper, we developed the fuzzy version of our deterministic model as follows:

$$\min \widetilde{Z} = \sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{s=1}^{S} \widetilde{C}_{tds}x_{tds} + VBN(1-AF) \tag{9}$$

Subject to:

$$\sum_{d=1}^{D}\sum_{s=1}^{S} \widetilde{tc}_t x_{tds} \le \widetilde{TC}_t \tag{10}$$

$$\sum_{t=1}^{T}\sum_{d=1}^{D}\widetilde{tc}_t x_{tds} \le \widetilde{sc}_s \tag{11}$$

$$\sum_{t=1}^{T}\sum_{s=1}^{S}\widetilde{tc}_t x_{tds} \ge AF \times \widetilde{pc}_d \tag{12}$$

And Eq. (6) to Eq. (8) where:

$$\widetilde{C}_{tds} = \left| \begin{array}{l} \widetilde{ltt}_{td} + \widetilde{qt}_{td} + \widetilde{dt}_{td} + \widetilde{ett}_{ts} \\ -\sum_{tt=1}^{TT}\left(tinq_{tts} + tenr_{tts}\right) \times \left(\widetilde{st}_{tts} + \tilde{lt}_{tts}\right) \end{array} \right| \tag{13}$$

It is worth noting that $\tilde{x}$ represents fuzzy parameter for x in the deterministic ILP model. In this research we implement defuzzification method developed by Jiménez *et al.*(2007) to rank fuzzy constraints and objectives and solved the problem based on their developed approach.

3. RESULTS AND DISCUSSION

3.1. Case Study

The case study is derived from an iron ore surface mine located in central Iran. The mine has two processing plants that must be fed by a truck and shovel based material handling system. Based on the short-term production schedule of the mine, the fleet must be capable of meeting an hourly feed rate requirement of 2300 tons for each of the active processing plants and meets a stripping ratio of 1.3. The material handling fleet to be implemented in the study consists of two types of shovels (Hitachi Hit 2500 and Hit 5500), and one type of truck (Cat 785C). The study consists of five active faces where two small shovels work on the ore mining faces and serve the processing plants, and two large shovels and one small shovel remove the waste material. To determine minimum truck fleet size to handle the material, we implemented match factor (MF) approach (Burt & Caccetta 2007; Chaowasakoo et al. 2017). The match factor calculation showed that the operation needs a fleet of 37 trucks to have a balanced operation. However, the approach does not account for the effects of the truck dispatching tools on the optimum number of trucks to meet the production requirement. Thus, we developed different operational scenarios by varying the number of trucks in the simulation to determine optimum fleet size which can meet the production requirement. The simulation study of different operational scenarios showed that the production requirement will be met using a fleet of 28 Cat 785C trucks when the trucks are dispatched using the FLP model. For the optimum scenario, the truck dispatching model was replaced with the model developed by (White & Olson 1986; Olson et al. 1993) as a benchmark model. Then, the results of implementing FLP model and the benchmark model were compared against each other.

3.2. Implementation on Case Study

As mentioned in the previous section, result of the deterministic calculation shows that the production requirement will be met by using a fleet of 37 trucks. Adding a fleet management system results in efficient truck allocations and consequently less number of required trucks. Thus, we designed experiments with less number of trucks to perform material handling operation in the case study. The designed experiments are listed in Table 2.

Table 2: Operation Scenarios of the case study ran for evaluation of the developed truck dispatching model

Scenario	S01	S02	S03	S04	S05	S06	S07
Fleet Size	25	26	27	28	29	30	31
Scenario	S08	S09	S10	S11	S12	S13	
Fleet Size	32	33	34	35	36	37	

For the designed experiments, we ran the simulation and optimization model of the case study for a designated operation time of 10 consecutive 12-hour shifts for two times. We named the first time as Case A and the second time as Case B. In Case A, the simulation and optimization model uses the benchmark fleet management system to make required upper and lower stage truck dispatching decisions. However, in Case B, the lower stage decision making model of the benchmark fleet management system is replaced by the fuzzy based decision making model. After running the developed simulation and optimization models for the case study, we plotted the production for different scenarios in Fig.1 to compare results.

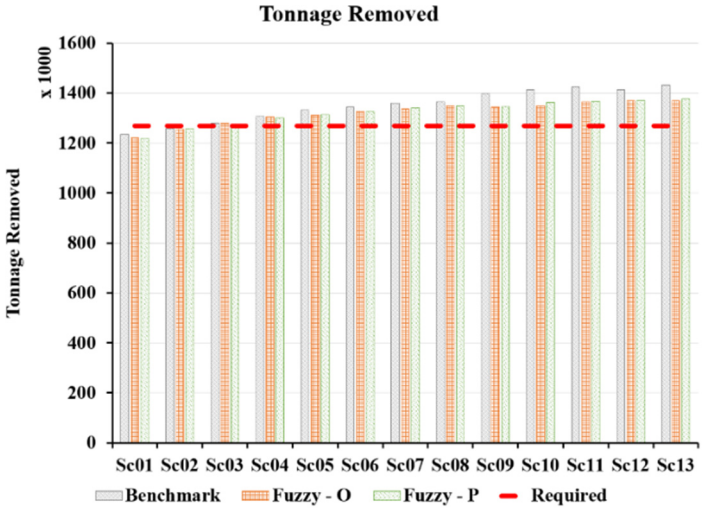


Fig.1. Total material removed from the pit over the simulation run time using different fleet where Fuzzy – O stands for optimistic fuzzy decisions (with degree of optimism equal to 75%) and Fuzzy – P stands for pessimistic fuzzy decisions (with degree of optimism equal to 25%)

Fig.1 shows that from scenario Sc03 with 27 trucks on, both approaches to solve the truck dispatching problem result in meeting the total production (ore + waste) requirement. However, producing as much material as possible is not the only goal of a mining operation. Meeting plants' requirements (tonnage of ore sent to the plants) is another critical objective of the mining operation. Thus, the best fleet is a fleet with the minimum number of truck in it that can meet both total production requirement and ore sent to plants' requirement. Fig.2 represents how various scenarios with the two truck dispatching models can meet the required ore delivery.

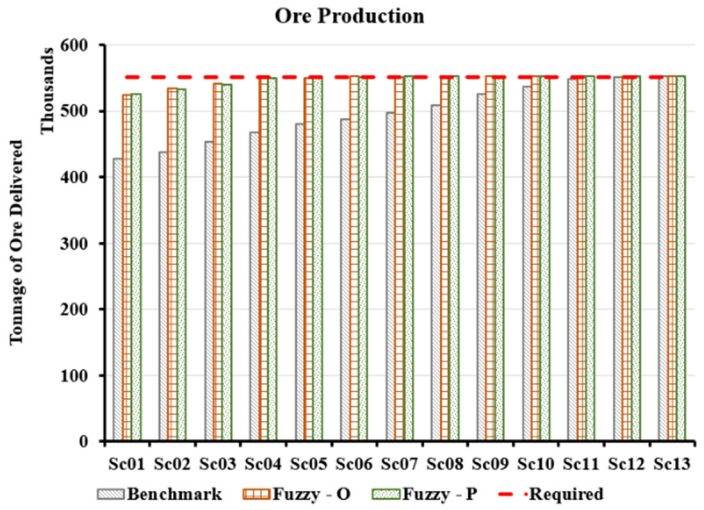


Fig.2. Cumulative tonnage of ore delivered to the processing plants during the simulation run time using each scenario

Although using both benchmark model and the FLP model the production requirement is met in scenario Sc03 with 28 trucks, the plants are fed in their full capacity in scenario Sc04 with 29 trucks when implementing FLP model and in scenario Sc12 with 36 trucks when using benchmark truck dispatching model. Thus, the optimum fleet required to meet the schedule is a fleet of 36 trucks when using benchmark truck dispatching model and a fleet of 29 trucks when using FLP truck dispatching model.

4. CONCLUSIONS

Several mathematical models have been developed to solve the truck dispatching problem in surface mines. The existing and publicly available models are all deterministic mathematical models. However, most of the required input parameters to solve such a problem are stochastic in nature. This paper introduces a new mathematical model to solve the truck dispatching problem in surface mines considering stochastic behavior of the required input parameters. The paper takes a FLP approach to deal with the developed model. To evaluate the performance of the developed model, the paper uses backbone of a currently in the market truck dispatching model as the benchmark. Results of implementation of benchmark truck dispatching model and the developed FLP model in a surface mining case study show that the developed model requires a smaller fleet of trucks to meet the production requirement given by the short-term mine schedule. This consequently results in lower costs and higher profits. The results also show that using the developed model, trucks spend less time in the queue at different destination.

5. REFERENCES

[1] Aghajani Bazzazi, A., Osanloo, M. & Soltanmohammadi, H., 2008. Loading-haulage equipment selection in open pit mines based on fuzzy-TOPSIS method. *Gospodarka Surowcami Mineralnymi Mineral Resources Management; The Journal of Polish Academy of Sciences*, 24(4, Part 3), pp.87–102.

[2] Alarie, S. & Gamache, M., 2002. Overview of Solution Strategies Used in Truck Dispatching Systems for Open Pit Mines. *International Journal of Mining, Reclamation and Environment*, 16(1), pp.59–76. Available at: http://www.tandfonline.com/doi/abs/10.1076/ijsm.16.1.59.3408.

[3] Bandopadhyay, S., 1987. Partial ranking of primary stripping equipment in surface mine planning. *International Journal of Surface Mining, Reclamation and Environment*, 1(1), pp.55–59. Available at: https://doi.org/10.1080/09208118708944102.

[4] Bandopadhyay, S. & Chattopadhyay, A., 1986. Selection of Post-Mining Uses of Land Via Fuzzy Algorithm. In R. V Ramani, ed. *19th International Symposium on the Application of Computers in Mine Planning*. SME of AIME, pp. 321–332.

[5] Bangian, A.H. et al., 2012. Optimizing post-mining land use for pit area in open-pit mining using fuzzy decision making method. *International Journal of Environmental Science and Technology*, 9(4), pp.613–628. Available at: http://link.springer.com/10.1007/s13762-012-0047-5 [Accessed March 1, 2018].

[6] Bascetin, A., Oztas, O. & Kanli, A., 2007. A computer software using fuzzy logic for equipment selection in mining engineering. *Journal of the Southern African Institute of Mining and Metallurgy*, 106(1), pp.63–70. Available at: https://journals.co.za/content/saimm/107/2/AJA0038223X_3299 [Accessed March 1, 2018].

[7] Brito, J., Moreno, J.A. & Verdegay, J.L., 2009. Fuzzy Optimization in Vehicle Routing Problems. In J. P. Carvalho et al., eds. *Joint 2009 International Fuzzy Systems Association*

World Congress and 2009 European Society of Fuzzy Logic and Technology Conference. Lisbon, Portugal: European society for Fuzzy logic and Technology, pp. 1547–1552. Available at: http://www.eusflat.org/proceedings/IFSA-EUSFLAT_2009/.

[8] Burt, C.N. & Caccetta, L., 2007. Match factor for heterogeneous truck and loader fleets. *International Journal of Mining, Reclamation and Environment*, 21(4), pp.262–270.

[9] Chang, Y., Ren, H. & Wang, S., 2015. Modelling and Optimizing an Open-Pit Truck Scheduling Problem. *Discrete Dynamics in Nature and Society*, 2015.

[10] Chaowasakoo, P. et al., 2017. Improving fleet management in mines: The benefit of heterogeneous match factor. *European Journal of Operational Research*, 261(3), pp.1052–1065. Available at: http://www.sciencedirect.com/science/article/pii/S0377221717301789.

[11] Ercelebi, S.G. & Bascetin, a, 2009. Optimization of shovel-truck system for surface mining. *Journal of the Southern African Institute of Mining and Metallurgy*, 109(7), pp.433–439.

[12] Gurgur, C.Z., 2011. Optimisation of a real-time multi-period truck dispatching system in mining operations Kadri Dagdelen and Songwut Artittong. , 4(1), pp.57–79.

[13] Hauck, R.F., 1973. A Real-Time Dispatching Algorithm for Maximizing Open-Pit Mine Production under Processing and Blending Requirements. In C. I. of M. and Metallurgy, ed. *Seminar on Scheduling in Mining, Smelting and Metallurgy*. Montreal, pp. 1–10.

[14] IBM, 2016. IBM ILOG CPLEX Optimization Studio.

[15] Jiménez, M. et al., 2007. Linear programming with fuzzy parameters: An interactive method resolution. *European Journal of Operational Research*, 177(3), pp.1599–1609. Available at: https://www.sciencedirect.com/science/article/pii/S0377221705006454 [Accessed February 5, 2018].

[16] Kappas, G. & Yegulalp, T.M., 1991. An application of closed queueing networks theory in truck-shovel systems. *International Journal of Surface Mining, Reclamation and Environment*, 5(1), pp.45–51.

[17] Li, Z., 1990. A methodology for the optimum control of shovel and truck operations in open-pit mining. *Int J Min Sci Technol*, 10(3), pp.337–340.

[18] Madadi, N. & Wong, K.Y., 2014. A Multiobjective Fuzzy Aggregate Production Planning Model Considering Real Capacity and Quality of Products. *Mathematical Problems in Engineering*, 2014.

[19] Matamoros, M.E.V. & Dimitrakopoulos, R., 2016. Stochastic short-term mine production schedule accounting for fleet allocation, operational considerations and blending restrictions. *European Journal of Operational Research*, 255(3), pp.911–921. Available at: http://www.sciencedirect.com/science/article/pii/S0377221716303873.

[20] Mena, R. et al., 2013. Availability-based simulation and optimization modeling framework for open-pit mine truck allocation under dynamic constraints. *International Journal of Mining Science and Technology*, 23(1), pp.113–119. Available at: http://dx.doi.org/10.1016/j.ijmst.2013.01.017.

[21] Moradi Afrapoli, A. & Askari-Nasab, H., 2017. Mining fleet management systems: a review of models and algorithms. *International Journal of Mining, Reclamation and Environment*, pp.1–19. Available at: https://www.tandfonline.com/doi/full/10.1080/17480930.2017.1336607 [Accessed July 28, 2017].

[22] Nguyen, V.U., 1985. Some fuzzy set applications in mining geomechanics. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 22(6), pp.369–379. Available at: https://www.sciencedirect.com/science/article/pii/0148906285900026 [Accessed March 1, 2018].

[23] Olson, J.P., Vohnout, S.I. & White, J., 1993. On improving truck/shovel productivity in open pit mines. *CIM Bulletin*, 86(973), pp.43–49.

[24] Osanloo, M., Gholamnejad, J. & Karimi, B., 2008. Long-term open pit mine production planning: a review of models and algorithms. *INT J MIN RECLAM ENV*, 22(1), pp.3–35. Available at: http://dx.doi.org/10.1080/17480930601118947.

[25] Pendharkar, P.C., 1997. A fuzzy linear programming model for production planning in coal mines. *Computers & Operations Research*, 24(12), pp.1141–1149. Available at: https://www.sciencedirect.com/science/article/pii/S0305054897000245 [Accessed March 1, 2018].

[26] Rahmanpour, M. & Osanloo, M., 2017. Application of fuzzy linear programming for short-term planning and quality control in mine complexes. , 117, pp.685–694.

[27] Soumis, F., Ethier, J. & Elbrond, J., 1989. Truck dispatching in an open pit mine. *International Journal of Surface Mining, Reclamation and Environment*, 3(2), pp.115–119. Available at: http://dx.doi.org/10.1080/09208118908944263.

[28] Ta, C.H. et al., 2005. A stochastic optimization approach to mine truck allocation. *International Journal of Surface Mining, Reclamation and Environment*, 19(3), pp.162–175.

[29] Temeng, V.A., Otuonye, F.O. & Frendewey, J.O., 1998. A Nonpreemptive Goal Programming Approach to Truck Dispatching in Open Pit Mines. *Mineral Resources Engineering*, 7(2), pp.59–67. Available at: http://www.worldscientific.com/doi/abs/10.1142/S0950609898000092.

[30] Temeng, V. a, Otuonye, F.O. & Frendewey, J.O., 1997. Real-time truck dispatching using a transportation algorithm. *International Journal of Mining, Reclamation and Environment*, 11(4), pp.203–207.

[31] Upadhyay, S.P. & Askari-Nasab, H., 2017. Dynamic shovel allocation approach to short-term production planning in open-pit mines. *International Journal of Mining, Reclamation and Environment*, pp.1–20. Available at: http://dx.doi.org/10.1080/17480930.2017.1315524.

[32] White, J.W. & Olson, J.P., 1986. Computer-based dispatching in mines with concurrent operating objectives. *Mining Engineering*, 38(11), pp.1045–1054.

[33] Xi, Y. & Yegulalp, T.M., 1993. Optimum dispatching algorithm for Anshan open-pit mine. In *24th APCOM*. Montreal, Quebec, Canada: Canadian Institute of Mining, Metallurgy and Petroleum, pp. 426–433.

[34] Yavuz, M., 2008. Selection of plant location in the natural stone industry using the fuzzy multiple attribute decision making method. *Journal of the Southern African Institute of Mining and Metallurgy*, 108(10), pp.641–649. Available at: http://www.scielo.org.za/scielo.php?script=sci_arttext&pid=S2225-62532008000900008 [Accessed March 1, 2018].

[35] Zimmermann, H.-J., 1978. Fuzzy programming and linear programming with several objective functions. *Fuzzy Sets and Systems*, 1(1), pp.45–55. Available at:

https://www.sciencedirect.com/science/article/pii/0165011478900313 [Accessed March 13, 2018].

[36] ZIMMERMANN, H.-J., 1976. Description and Optimization of Fuzzy Systems. *International Journal of General Systems*, 2(4), pp.209–215. Available at: http://dx.doi.org/10.1080/03081077608547470.

Improvements to Production Planning in Oil Sands Mining Through Analysis and Simulation of Truck Cycle Times

Eduardo Goris Cervantes, Shiv Upadhyay and Hooman Askari-Nasab
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

A theoretical framework based on a detailed analysis of mine operations data from an oil sands mine in northern Alberta and the static simulation of truck cycle times is developed, verified and validated in this paper. Implementation of this framework provides better results than existing in-house tools, which rely solely on manufacturer data, and thus aids in efficient equipment planning for life of mine plans. The use of this framework to modify existing productivity curve estimation methods currently in use at the mine site is also proposed. This method replaces "loaded flat haul" with "effective loaded flat haul" in the estimation of productivity. Validation of the model presents an over estimation of productivity by 4% against an underestimation of over 10% by the existing in-house method.

1. Introduction

One area that differentiates oil sands mines in northern Alberta fromconventional hard-rock mining is the environment in which they operate. Due to characteristics of the ground, producers in the region experience very high rolling resistance (RR) that, in turn, negatively affect haul truck performance and greatly increase fuel consumption and emissions. Oil sands mining companies employ some of the largest haul trucks in the world. In addition, typically oil sands mines are much more extensive in area than hard rock pits, resulting in longer haul distances on roads of comparatively more viscous material.

The research presented in this paper has direct application in the areas of predicting truck requirements for budgeting and life of mine (LOM) planning. There have been insufficient advances in the research of shovel-truck simulation or estimation methods that produce reliable results for LOM planning. Whereas there are various software packages dedicated to this purpose, they are limited in their level of detail, as they solely rely on equipment manufacturers' performance data that are usually not representative of the complex nature of large-scale, real-world mining operations. These limitations hinder reliable predictions as these software packages fail to capture varying site specific characteristics. Talpac (RungePincockMinarco, 2015), which is one of the most widely used fleet performance software packages, possess a comprehensive library of equipment and their characteristics as provided by the manufacturers. It captures the variability in truck speeds based on gradients, rolling resistance and adjustment factors based on the haulage path between a source and a destination. However, the amount of site specific details that is

This paper, in its entirety, has been accepted to be published in CIM Journal.

incorporated is limited. Chanda and Gardiner ((2010)) claim that Talpac (RungePincockMinarco, 2015) underestimates and overestimates short and long haul times, respectively. Manyele (2017) claims that loaded haul times are usually longer than empty haul times because of the difference in the weights resulting in slower speeds of loaded trucks, and that empty haul times are usually more variable due to the dispatching logic.

Production rates in an open pit mine operation are affected by the project complexity, traffic flow, accessibility, road condition, gradient, rolling resistance, size of equipment, match factor, operator competency, weather and disruptions, as noted by Kuo (2004). Burt and Caccetta (2014) consider appropriate representation of the associated variability in truck cycle times extremely important as it affects the feasibility of the fleet and the match factor in addition to predicting productivity. Moreover, as noted by Burt (2008), the number of trucks can also affect the truck cycle times due to bunching and excessive queuing. A detailed simulation modeling of the production operations (S. P. Upadhyay*, et al.*, 2013; Shiv Prakash Upadhyay & Askari-Nasab, 2018) bears the capability to account for all the factors and provide more precise results. However, building and running detailed simulation models for site specific operations are time consuming and undesired unless a detailed scenario analysis of the production operations is required.

Bozorgebrahimi, Hall, and Blackwell (2003) emphasize the importance of the haul road network and its characteristics, which have a direct bearing on the truck performance. Rolling resistance of haul roads is an important characteristic affecting cycle times and haulage costs, as observed by Dotto (2014), Thompson and Visser (1997; 2003) and Joseph and Szymanski (2013).

Effective flat haul (EFH) is another metric often used to measure haulage distances and predict haul times (Curi*, et al.*, 2014; Sheremeta, 2015). Curi et al. (2014) define EFH as a "calculated parameter that accounts for both the distance from the source to the destination, and the elevation change from the source to the destination". Campbell and Hagan (2012) used ranges of EFH values for different gradient ranges as modifying factors to develop an equipment selection model. Similarly, Hargroves, Gockowiak, McKeague, and Desha (2014) used EFH to normalize elevation changes. Newmont mining has implemented EFH as one of their key performance indicators (KPIs) in order to better categorize and account for production and equipment usage figures (Newmont, 2014).

Although it has been established that truck cycle time is one of the most important parameters in measuring LOM productivity, efficient prediction tools for estimating this parameter do not exist. A review of existing research shows the need to solve the problem of inaccurate cycle time estimations in open pit mining systems. Poor cycle time estimations may lead to poor LOM planning and, in-turn, deviations from planned production targets. Overestimation of truck requirements may lead to increased costs. On the other hand, underestimation may lead to a shortfall in production. It is thus considered important in this research to develop a tool that can provide cycle time estimations and truck requirements as accurately as possible.

Cycle time is the single most important parameter of a mining operation, as it is a key parameter in estimating maximum achievable production rate between a source and destination. In addition to being dependent on the type and capability of equipment used, cycle time is also impacted by other controllable and external factors. Relevant controllable factors include the mining sequence (schedule), road design, road construction, safety guidelines, maintenance of roads and equipment, as well as operator proficiency and behavior. External factors include unexpected equipment downtime and weather events that affect the characteristics of the road and performance of the equipment. Weather events such as large amounts of rainfall or thawing snow make the material on oil sands haul roads even softer, and can create ruts on the surface that increase rolling resistance and therefore cycle times.

Many companies have ceased using standard software programs, and, in place, have developed site-specific in-house methods for predicting productivity through cycle times, often with

unreliable results due to the low-detail and fast-paced nature of their development. The in-house method evaluated in this paper involves relating the historical loaded haul distances to a productivity performance indicator: tonnes per gross operating hour (TPGOH) through a line of best fit. This line of best fit is then used to predict LOM productivity based on distance between scheduled mining locations and respective dump destinations. TPGOH for each truck cycle is calculated as the ratio of truck payload and the cycle time plus any related delays. The activities that make up the cycle time are: idling at dump, dumping, loading, time in queue, spotting, waiting to spot, loaded hauling and empty hauling. A plot of normalized TPGOH versus loaded haul distance, as shown in Fig. 1 for the mine site evaluated, reveals how inadequate the line of best fit method can be due to high variability in TPGOH. A detailed study of the data revealed that one of the reasons for this variability is the assumption that same distance over different haulage paths shall yield the same haul time, which is not the case. Two paths with same distance but different haul road profiles and surface characteristics cannot be assumed equivalent. Fig. 2 shows how much variability there is in terms of haul time for specific loaded haul distances.

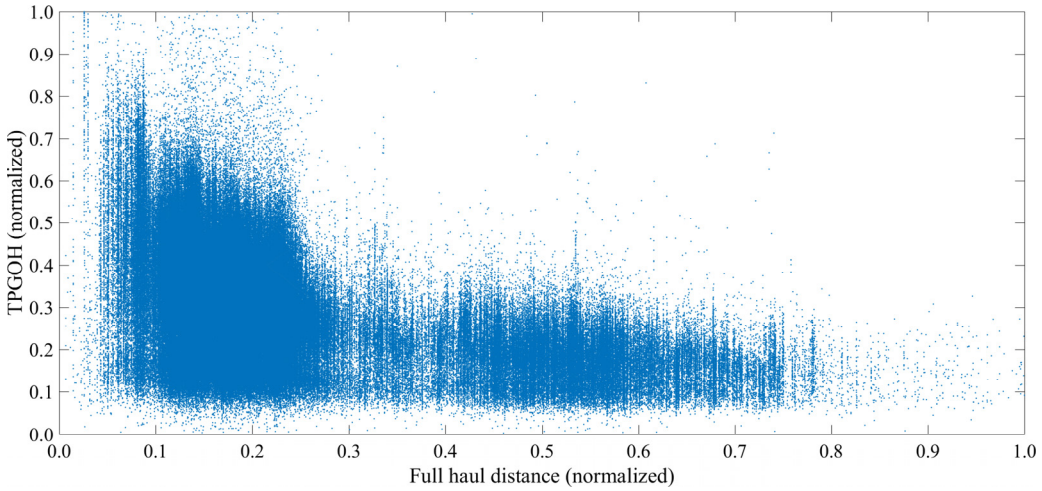


Fig. 1. Tonnes per gross operating hour (TPGOH) vs full haul distance variability (normalized) - company database

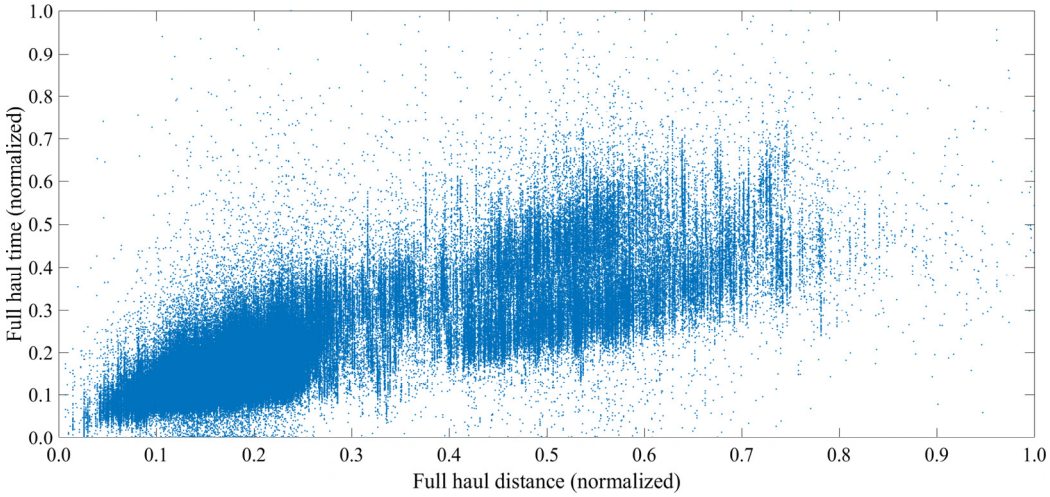


Fig. 2. Full haul time vs. full haul distance variability (normalized) - company database

This paper focuses on generating accurate and reliable estimates of loaded truck travel times through simulation so that productivity figures, such as TPGOH, can be accurately estimated. A comparison of predictions over short, medium and long hauls is presented to show the strength of proposed method (Method 2 in this paper) in comparison to the rimpull method (Method 1 in this paper) which strictly follows the manufacturers' truck performance data. This paper also aims to

improve the company's in-house method of deriving productivity curves by replacing the relation of TPGOH to loaded haul with a relationship of TPGOH to loaded EFH (estimated by proposed Method 2 in this paper) in order to reduce the variability in TPGOH, as shown in Fig. 1. To validate this proposition, a comparison of the output of the current in-house method with that of the proposed framework is also presented.

2. Theoretical framework

The framework/tool proposed in this paper assumes the travel component of trucks as the only component contributing to changes in truck cycle times and productivity over the LOM. Other components of cycle time, such as loading time, dumping time, queuing time and spotting time remain independent of the schedule and should follow a fixed distribution over the LOM. For this study, historical data recorded by the company dispatch system are used to fit distributions on each component of cycle time except loaded haul time. It is important here that the distributions are representative of the system for a true representation of the environment. This framework features full integration with the company's SQL database to access historical dispatch records. Other potentially useful information recorded by the company includes operator ID, equipment ID (both for excavators and trucks) and a timestamp (or shift number), which could be used for further classification at higher resolutions.

A digital model of the mine's haul road network is also required. The accuracy of the outputs produced by this program is, to a large extent, dependent on the accuracy and precision of the road network model. A proper characterization of distances within segments is important, but properly capturing changes in elevation is essential, since these directly affect the speed of the truck. The framework/tool reads in the drawing exchange format (dxf) input of the haul road network from any modeling software to characterize the paths and gradient of segments.

Historical truck speed records on haul roads are also required to model flat haul speed distributions. Usually such data are recorded periodically as velocity maps from the global positioning systems within the trucks. For this study, this information was provided, which included coordinates of trucks with time stamp, velocity, and corresponding shovel and dump names. However, these records were not explicitly segregated, so further manipulation using timestamps and shift identifiers was performed to characterize loaded and empty speed records. A long flat haul section of the road network was chosen to filter out loaded and empty flat haul speeds. A distribution was then derived for flat haul speeds while empty and loaded. This ensures that the variability and characteristics of the truck/network interactions are captured. Moreover, while calculating these data points, an effort was made to exclude areas where the trucks may be accelerating or decelerating. This was accomplished by selecting coordinates of flat haul roads well within the flat section of the road, choosing limits that are not close to where the flat road starts or ends.

To mimic the changes in truck speed with varying resistance values (gradient plus rolling resistance), the rimpull curves for each specific truck type is used in this framework. Rimpull curves are not used to assign speeds as in Talpac (RungePincockMinarco, 2015), but rather to examine the relative changes in velocity based on varying payload and total resistance of segments of haulage paths. It is also essential to properly characterize the rolling resistance of the haul roads, the data for which usually remain unavailable. Joseph, Curley and Anand (2017) provide excellent guidelines and insights into rolling resistance measurements and typical values within oil sands mining operations, stressing that seasonal weather changes affect RR significantly.

The framework/tool presented in this paper is written in MATLAB. This framework presents two distinct methods of calculating haul times. Method 1,based entirely on performance data provided by the truck manufacturer (similar to Talpac and CAT FPC), serves as a benchmark for comparison with proposed Method 2. Method 2 is a data-driven simulation of haul times that incorporates data from the mine site and dispatch, yielding much more accurate and realistic outputs. These two

methods are described in more detail below. The remaining parameters of the complete cycle time are calculated in the same manner, regardless of the method. In addition, there is another important distinction in the algorithm. This code was developed with high integration capability to a company database through direct querying, operating under the assumption that the intended application of this framework is for existing operations with a significant amount of dispatch data recorded and available within the database. Fig. 3 provides an overview of the framework, showing the various steps required to complete the analysis. Step 1provides the assumed rolling resistance of the haul network, truck types and corresponding rimpull curve characteristics and the number of replications required for the desired confidence interval. In Step 2, the framework reads in the road network of the mine and checks for errors. This stage also refines the road network from native resolution to a desired resolution, i.e. it manipulates the segments to have desired lengths to minimize computational burden. Moreover, it eliminates high fluctuations in gradients that may be observed due to the close proximity of two points showing aberrations on haul road characteristics.

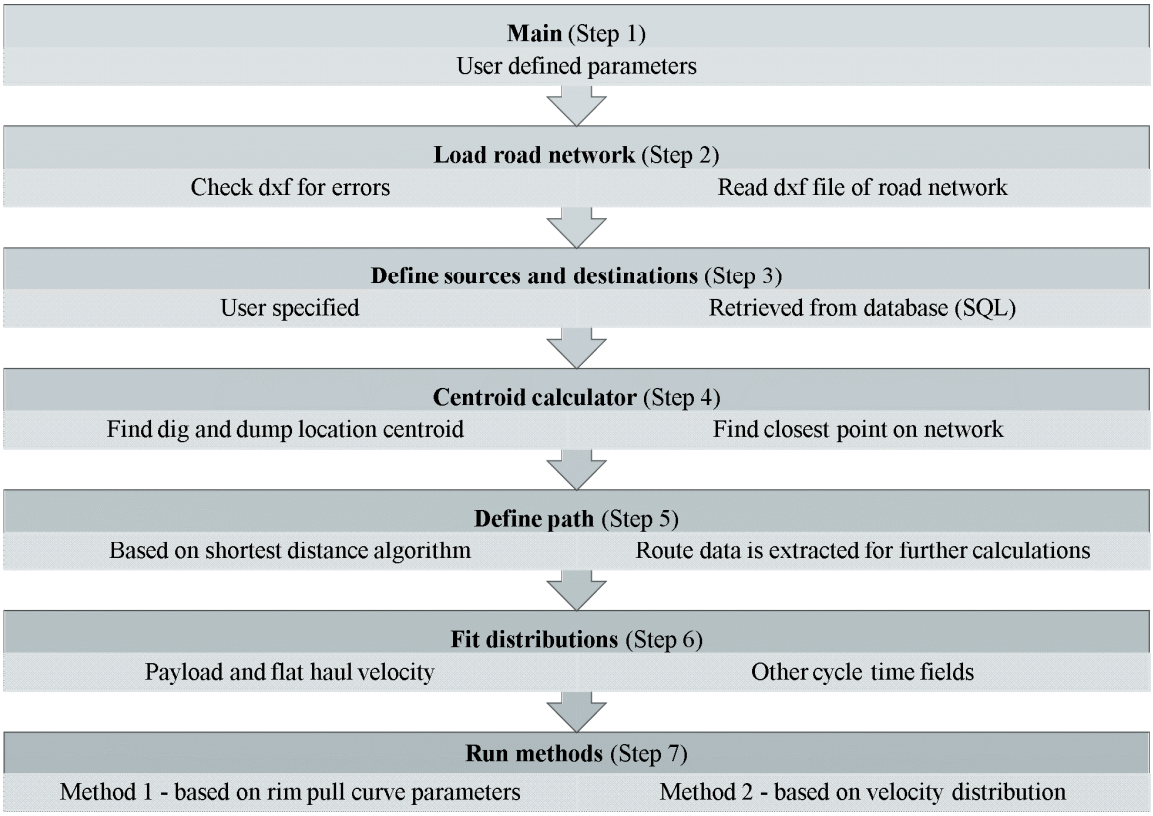


Fig. 3. High level flow chart of the framework

Step 3 provides the dig location and corresponding dump location names. If the tool is used for prediction, this input corresponds to the schedule; otherwise, a SQL query is used to retrieve historical data from a company database for validation and calibration purposes. As the dig and dump locations do not correspond to a single point but an area, Step 4 estimates the centroids for each dig location and dump location and determines the closest point on the haul road network from these centroids. Based on the shortest path algorithm, a path following the haul road network is established in Step 5, constituting segment lengths, gradients and rolling resistance in the direction from dig location to dump location for the loaded haul. Step 6 is a parallel step which provides flat haul velocity distributions for each truck type as well as payload distributions and other cycle time components. Finally the simulation replications are executed in Step 7, using the paths and distributions generated by two methods (Method 1 - benchmark and Method 2 - simulation). The simulation output includes cycle times, payload, EFH and TPGOH. The derived EFH values are proposed to replace corresponding full haul values in company's in-house method

to derive productivity curves. The two methods used in this framework to estimate loaded haul times comprise the following.

2.1. Method 1 –benchmark

This method evaluates the total resistance of each segment, by adding the previously defined rolling resistance to the gradient of the segment. Then, a random sample is generated from the payload tonnage distribution and the rimpull force is calculated, based on the weight of the empty truck plus the payload tonnage. Having calculated a rimpull value for each segment, the algorithm then performs an interpolation of the manufacturer's rimpull characteristic data to assign a speed to each segment which is the maximum possible speed at the given conditions. The segment length is then divided by its corresponding speed to determine the time the truck takes to travel through said segment. The sum of all segment travel times results in the loaded haul time between source and destination. The method then calculates the time it takes for the loaded truck to travel the path assuming it is flat. The EFH factor is found by dividing the rimpull based estimate by the flat haul estimate.

2.2. Method 2 –simulation

This method is similar to Method 1 with the exception that rimpull characteristics are used to estimate only the relative changes in velocity instead of velocity itself. Instead of directly finding the speed corresponding to a rimpull value, for each road segment, this method generates a factor which equals the ratio of the rimpull velocity estimate for a given segment to the rimpull velocity estimate for an equivalent flat haul segment. This factor works as an adjustment factor to the flat haul velocity of the truck. Velocity is then sampled from the flat haul velocity distribution for the truck type and multiplied by the rimpull factor to estimate the velocity corresponding to each segment. The remaining estimation of loaded haul time and EFH remains same as in Method 1.

3. Implementation and results

The framework is implemented using data from a large scale oil sands open pit mine in Northern Alberta. At these operations an in-house method is used to derive a productivity curve, i.e. the relationship between TPGOH and loaded haul distance. This, in-turn, is used to predict truck requirements over the LOM for budgeting purposes. The operation employs Cat 797F trucks to haul material from the mine with large haul distances. Although the rolling resistance varies in different areas of the mine and varies based on changing conditions, seasons etc., it is hard to find. For simplicity and in accordance with the literature, a representative equivalent rolling resistance value of 5.5% is used in this implementation for the entire mine haulage road network. This value is determined during calibration of the model, which provided reasonable results. The segment lengths in the road network are defined as 35 m. The road network consists of 47 separate roads between intersections and the end nodes, representing more than 35 km in total length. Due to confidentiality requirements, most of the data presented in this paper are normalized.

A literature review identified that one of the shortcomings of available commercial programs is that cycle time estimates are not accurate for either short hauls or long hauls. Thus, the verification and validation of the model was performed by examining haul time estimates produced by this framework over several haul routes of varying distances. The outputs from both Methods 1 and 2 are compared against records in the company database for four cases consisting of a short haul of 1.3 km (Fig. 4 and 5), a short/medium haul of 2.6 km (Fig. 6 and 7), a medium haul of 4 km (Fig. 8 and 9) and a long haul of 8 km (Fig. 10 and 11). Corresponding statistics and percentage difference from historical data is presented in Table 1, 2, 3 and 4, respectively.

It is important to note that the minimum haul time value on the histogram generated by Method 2 is equal to the unique estimated haul time generated by Method 1, which is the best-case scenario using the maximum velocity possible at the given total resistance. It should also be noted that,

although the histograms generated from the company database records show values smaller than this theoretical value, this can be attributed to recording errors within the dispatch system. A total of 500 replications were performed with less than 20 seconds of run time for each case.

Table 1. Short haul output summary

Short haul (1.3 km average)	Database	Method 2	Difference (%)	Method 1	Difference (%)
Mean	5.58	5.52	–1	4.19	–24.9
95% confidence interval	0.11	0.12			
Upper bound	5.69	5.64			
Lower bound	5.48	5.40			
Median	5.27	5.16	–2.1		
Standard deviation	1.67	1.35	–19.2		
Variance	2.80	1.82	–35.0		

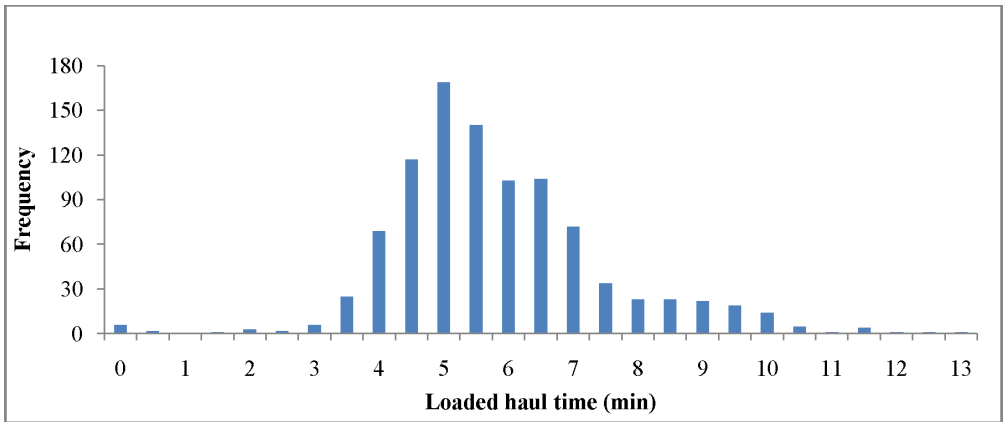


Fig. 4. Short haul histogram, company database

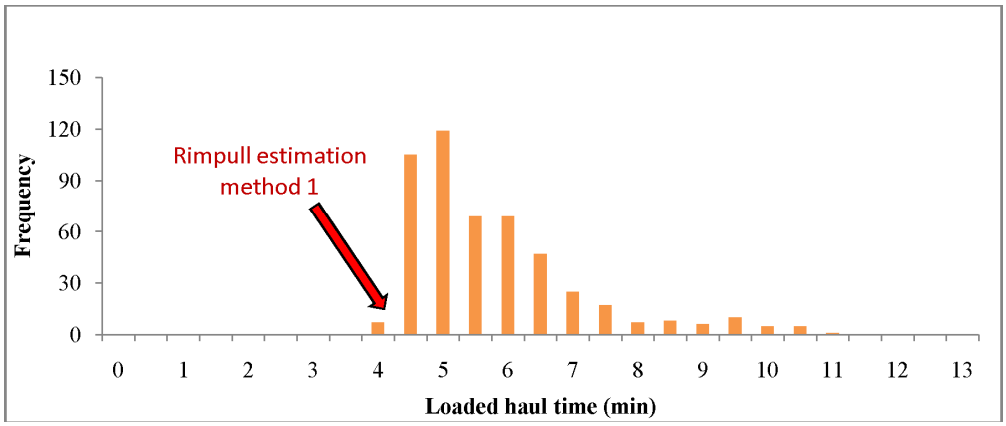


Fig. 5. Short haul simulation output histogram, Methods 1 and 2

Table 2. Short/medium haul output summary

Short/medium haul (2.6 km average)	Database	Method 2	Difference (%)	Method 1	Difference (%)
Mean	8.96	9.08	1.3	6.86	–23.4
95% confidence interval	0.14	0.21			
Upper bound	8.99	9.29			
Lower bound	8.92	8.87			
Median	8.83	8.50	–3.7		
Standard deviation	2.10	2.40	14.3		
Variance	4.42	5.61	26.9		

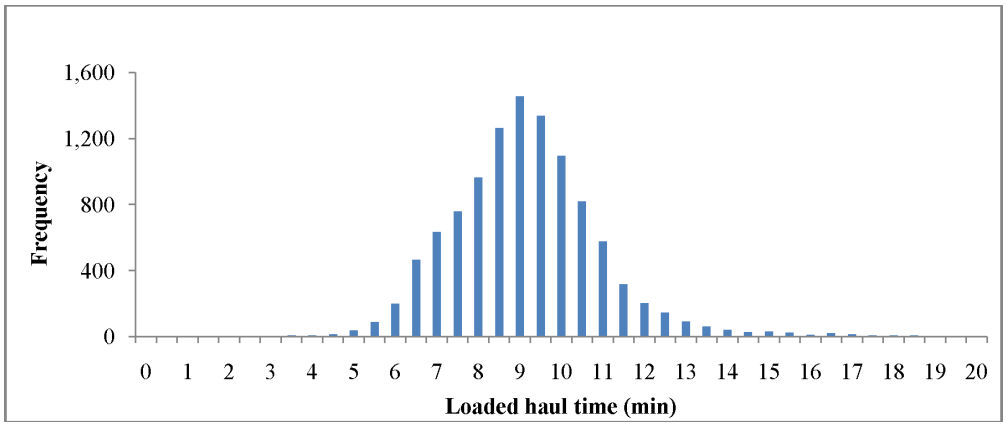


Fig. 6. Short/medium histogram, company database

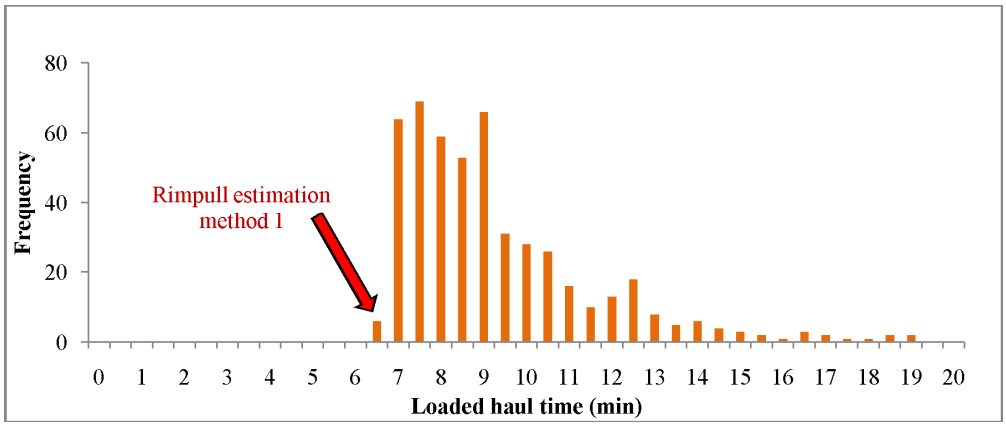


Fig. 7. Short/medium simulation output histogram, Methods 1 and 2

Table 3. Medium haul output summary

Medium haul (4 km average)	Database	Method 2	Difference (%)	Method 1	Difference (%)
Mean	15.62	15.60	–0.12	11.59	–25.8
95% confidence interval	0.09	0.37			
Upper bound	15.71	15.97			
Lower bound	15.52	15.23			
Median	15.58	14.38	–7.7		
Standard deviation	3.24	4.20	29.6		
Variance	10.21	18.01	71.0		

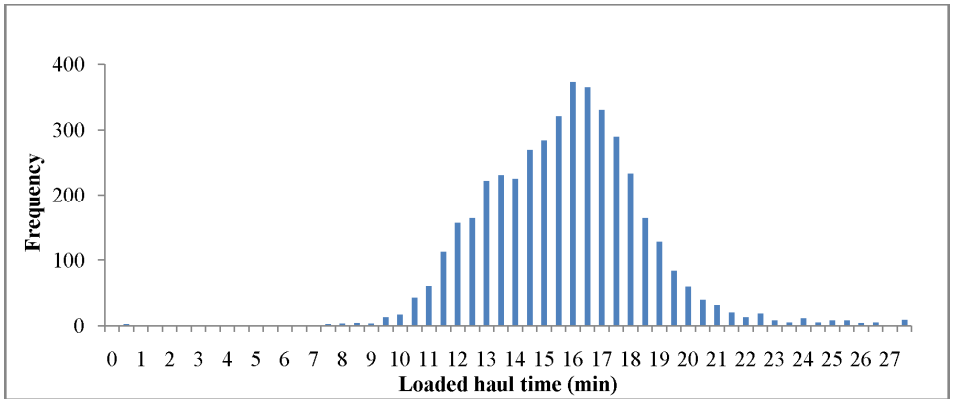


Fig. 8. Medium Haul histogram, company database

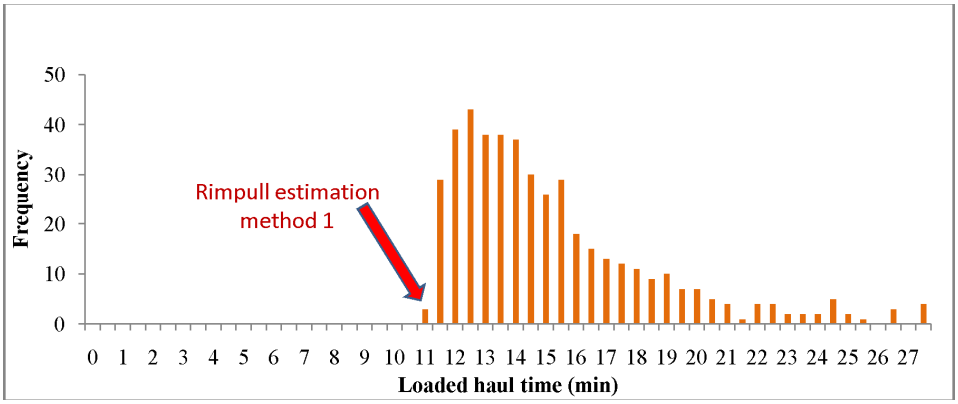


Fig. 9. Medium haul simulation output histogram, Methods 1 and 2

Table 4. Long haul output summary

Long haul (8 km average)	Database	Method 2	Difference (%)	Method 1	Difference (%)
Mean	23.20	23.09	–0.5	17.2	–25.9
95% confidence interval	0.22	0.37			
Upper bound	23.42	23.46			
Lower bound	22.98	22.72			
Median	22.78	21.41	–6.0		
Standard deviation	4.5	6.0	33.3		
Variance	20.0	36.5	82.5		

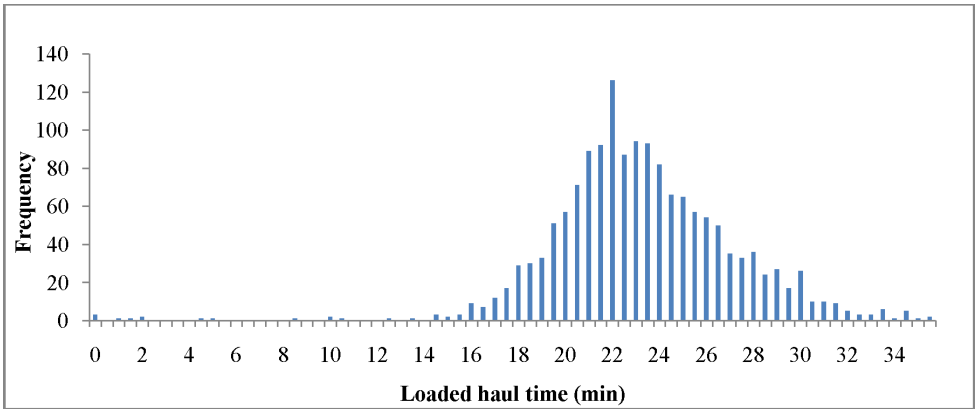


Fig. 10. Long haul histogram, company database

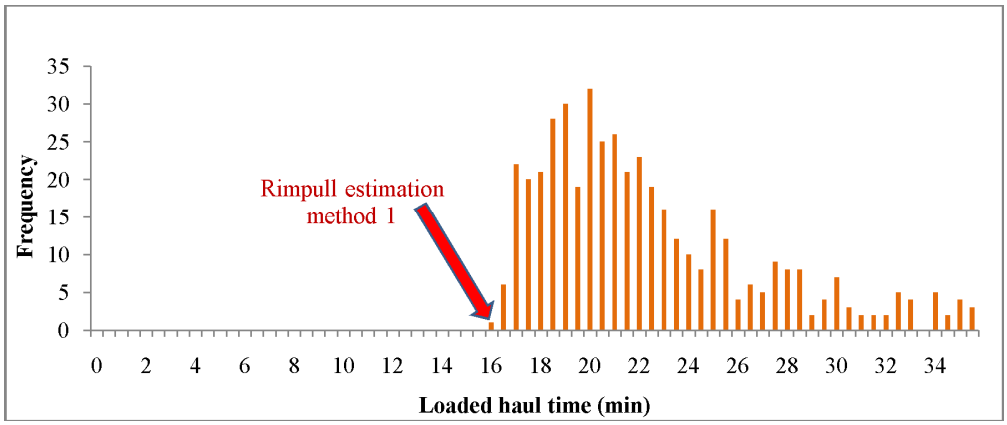


Fig. 11. Long haul simulation output histogram, Methods 1 and 2

It can be concluded from this study that Method 2 is much more accurate in its predictions over all distance ranges in comparison to Method 1. For the four cases, the maximum percentage difference of the mean values of loaded haul times between Method 2 and recorded company data is 1.3% for short/medium haul and the minimum difference is 0.12% for medium haul. For all the cases, Method 2 provided haul times within 1.3% of accuracy compared to the database. As expected, Method 1 overestimates the performance of the trucks by consistently underestimating haul times by about 25%. Comparisons of the histograms generated by the model to those generated from company data for the four cases suggest that the model is simulating the operation correctly.

After the model is verified, it is extended to estimate TPGOH and the results compared against historical data. For this, a quarterly (3-month) simulation of TPGOH was performed. TPGOH is calculated by dividing the payload tonnage by the cycle time. Within the cycle time, the loaded haul time is simulated with Method 1 and 2, and the rest of the parameters are simulated directly from their probability distributions. Since the empty return trip time remains unknown due to the possibility of a truck returning from a combination of various dump locations, two scenarios have been analyzed. The first scenario does not incorporate return trip time to estimate TPGOH (scenario 1) whereas the second scenario does (scenario 2). For the second scenario, return trip time is assumed to follow a probability distribution similar to other cycle time components. This is considered a valid assumption at this stage of the study, as data are taken directly from the company database for the given time period being simulated and thus represent reality. Further study is proposed to eliminate this assumption and develop an accurate prediction model for both loaded and empty haul times.

Eleven distinct dig locations with corresponding scheduled production tonnage over the 3 month quarter from 2016 production year are selected. These combinations of dig locations and corresponding dump locations are fed into the model, the corresponding centroids estimated and connected to the network (Fig. 3) and multiple replications run. The resulted TPGOH values for each source/destination combination are then multiplied by corresponding weighting factors. A weighting factor here is the fraction of tonnage scheduled for the dig location/dump location combination as compared to the total scheduled tonnage in the quarter. This process yielded a single TPGOH value estimate to be compared against the database.

3.1. Scenario 1 (return trip not included)

Fig. 12, 13 and 14 show the histograms of TPGOH values for scenario 1 derived from the historical recorded data and the predictions from Method 1 and Method 2, respectively. A statistical comparison of results is presented in Table 5. It can be observed that Method 2 overestimates the mean value of the TPGOH by only 0.58% as compared to 15% for Method 1. Moreover, standard deviation is observed to be fairly small in the prediction methods in comparison to reality. The reason for this variance may be attributed to the variability in truck speeds due to interactions with

other trucks and systems on the haulage paths. During the development of these results, it was also noted that there is a 2% difference in productivity during day and night shifts, with the night shift being more productive in reality, which is not accounted for in the current scenario comparison.

Table 5. Quarterly TPGOH simulation output summary (normalized) – scenario 1

	Database	Method 2	Difference (%)	Method 1	Difference (%)
Mean	0.5288	0.5319	0.58	0.6081	15.1
95% confidence interval	0.0058	0.0024	–58	0.0027	–52.8
Median	0.5365	0.5399	0.63	0.6068	13.1
Standard deviation	0.1744	0.0857	–50.9	0.0981	–43.6

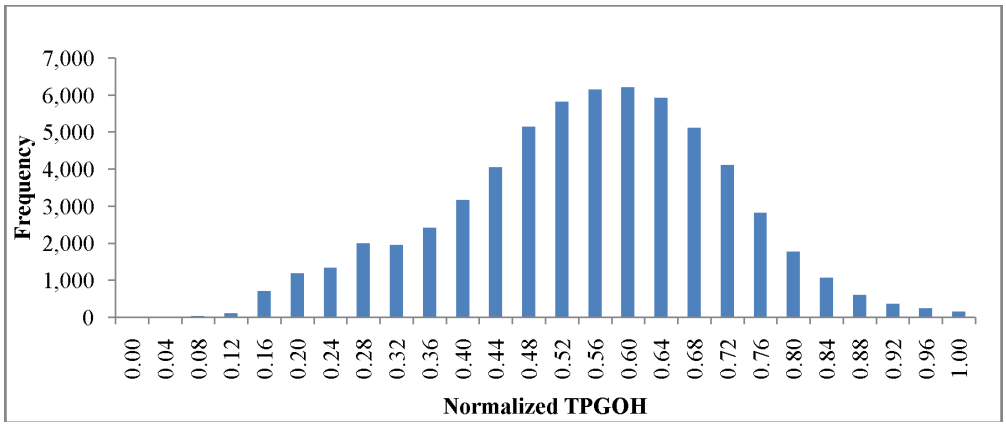


Fig. 12. Quarterly tonnes per gross operating hour (TPGOH)company database histogram – scenario 1

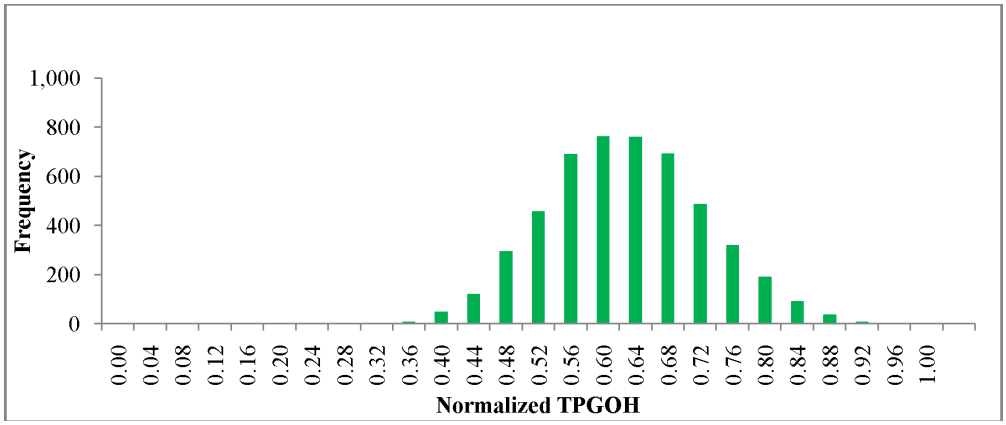


Fig. 13. Quarterly tonnes per gross operating hour (TPGOH) Method 1 output – scenario 1

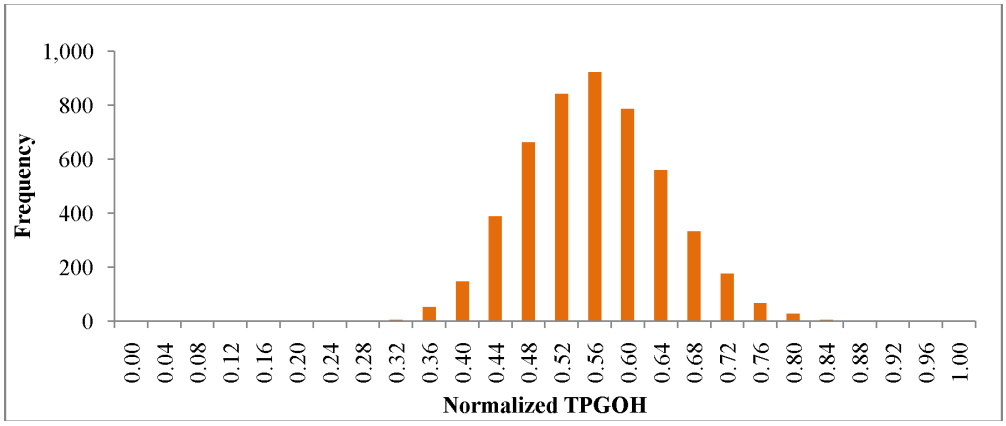


Fig. 14. Quarterly tonnes per gross operating hour (TPGOH) Method 2 output – scenario 1

3.2. Scenario 2 (return trip included)

Fig. 15, 16 and 17 present the histograms of TPGOH values for scenario 2 derived from historical recorded data and the predictions from Method 1 and Method 2, respectively. A statistical comparison of the results is presented in Table 6.This scenario shows more precise estimations of TPGOH compared to scenario 1. Again, Method 2 outperforms method 1 in predicting the mean value of TPGOH with a difference of only 0.09% from recorded data in comparison to 8% for Method 1. Similar to scenario 1, standard deviation of the estimates remain very small in comparison to recorded data for the same reasons outlined above.

Table 6. Quarterly tonnes per gross operating hour simulation output summary (normalized) – scenario 2

	Database	Method 2	Difference (%)	Method 1	Difference (%)
Mean	0.3654	0.3651	–0.09	0.3951	8.1
95% confidence interval	0.0009	0.0014	58.6	0.0015	76.3
Median	0.3739	0.3629	–2.9	0.3929	5.1
Standard deviation	0.1139	0.0491	–56.8	0.0547	–52

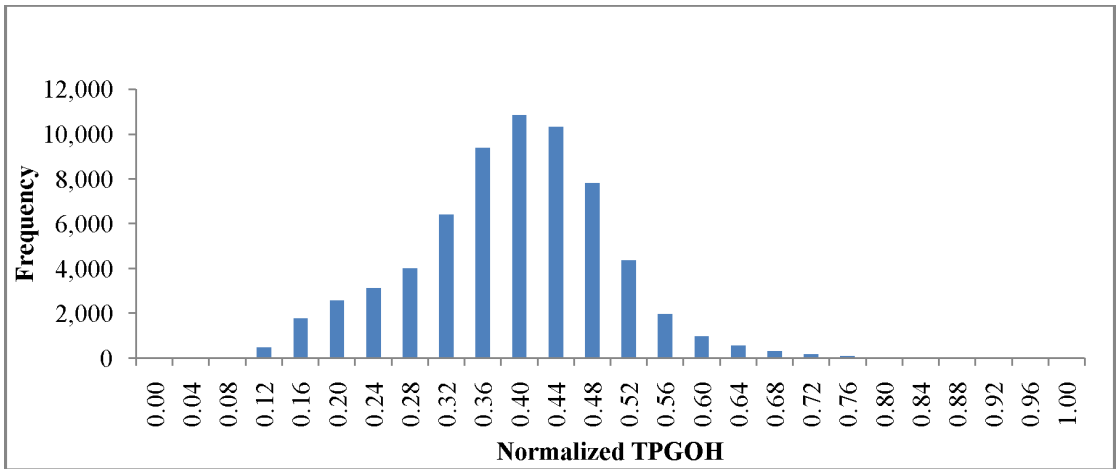


Fig. 15. Quarterly tonnes per gross operating hour (TPGOH) database histogram – scenario 2

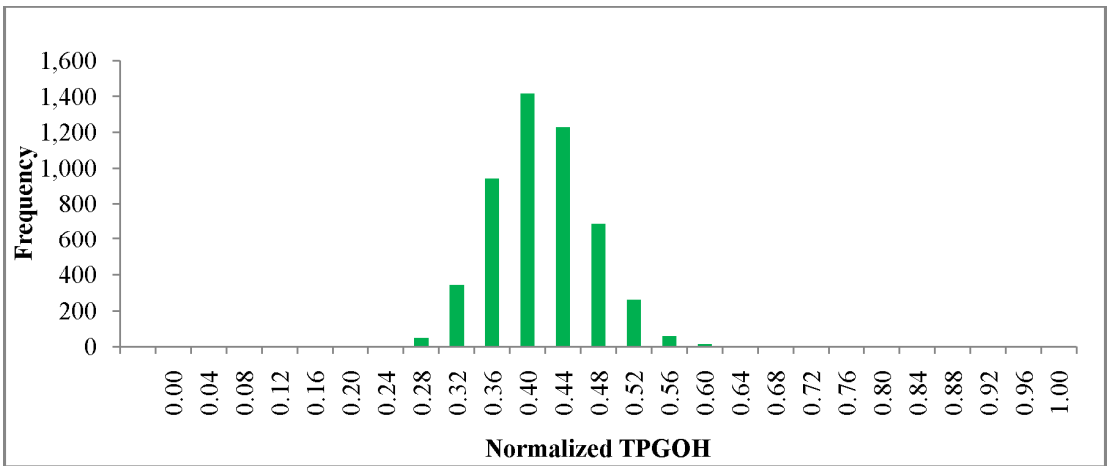


Fig. 16. Quarterly tonnes per gross operating hour (TPGOH) Method 1 averaged output – scenario 2

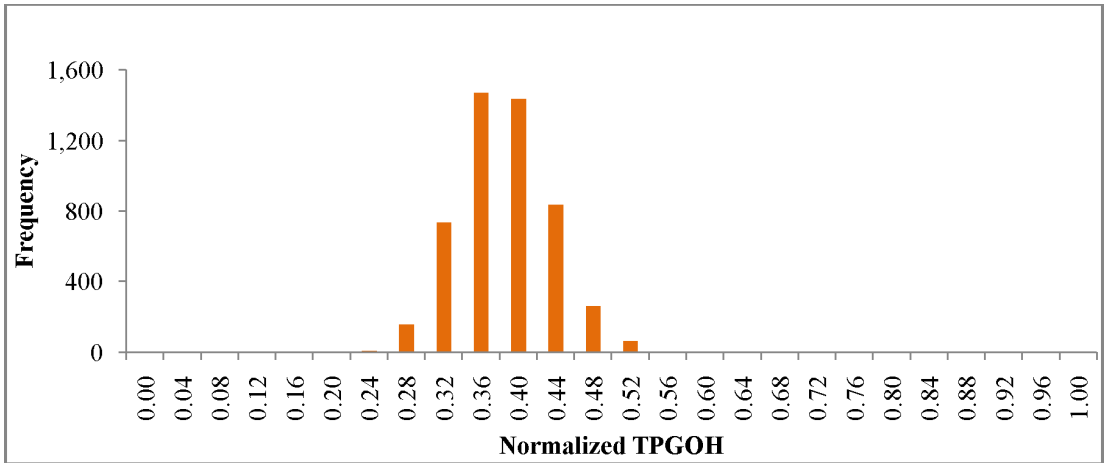


Fig. 17. Quarterly tonnes per gross operating hour (TPGOH) Method 2 averaged output – scenario 2

As expected, the outputs from Method 1 overestimate productivity by underestimating truck cycle times. However, Method 2 provides very close predictions of cycle times and productivity. This analysis proves the validity of the model and Method 2 in estimating the mean value of productivity/TPGOH and that the model is well calibrated and correct.

To improve the current company 'in-house' method, it was compared to Method 2, as proposed in this paper. Using the 'in-house' method, three productivity curves are generated using two years of data, one year of data, and a dataset from the quarter to predict TPGOH for the quarter studied above. A comparison of predictions from the three curves against recorded TPGOH at the mine showed an under-prediction of TPGOH by 12.3,10.3, and 18.2%, respectively. In comparison, Method 2 was applied to replace the full haul distance with EFH while fitting the productivity curves. The resultant curve overestimated the quarterly TPGOH by 4% as compared to recorded data. Although, it is not an exact prediction, this represents a significant improvement in the 'in-house' method. Moreover, an analysis of data used for fitting the productivity curves in both categorizations, i.e. EFH and full haul, showed a reduction in standard deviation by 25% for EFH, which can be regarded as a significant improvement in data variability for fitting amore representative productivity curve.

4. Conclusion

This paper presented a framework for improving the travel time predictions of loaded haul trucks in open pit mines with a focus on large scale oil sands operations, and an improvement to the industrial in-house method of productivity curve estimation for efficient prediction of truck requirements and budgeting. The framework presented here is the result of a thorough investigation of the production data for the mine site used in the case study. The proposed framework was developed to be flexible and easy to implement. During the development of this framework, the main data sources were identified and limited to those that are typically available from mine dispatch systems. Very tight half widths and high confidence intervals were achieved in short computation times for the validation/verification of the case study, adding to the positive features of this framework. The method also shows definite improvement over existing methods which rely solely on rimpull and manufacturer provided data. This improvement is rooted in the fact that the proposed framework is driven by historical data and thus incorporates the characteristics of the specific mine site.

The proposition to replace loaded flat haul with EFH to derive productivity curves also showed reasonable improvements in its prediction. The proposed framework overestimates TPGOH by 4% in comparison to an underestimation of over 10% by the current in-house method used at the case-study mine. However, further improvements to the framework are proposed to improve prediction

methods and to model truck travel times more accurately by capturing the actual spread in the travel times. In addition, as future work, the incorporation of methods to estimate empty truck travel times is also proposed. This would incorporate a realistic empty truck travel times to dig locations from various dump locations, similar to that provided by a dynamic dispatch system, in the proposed static simulation framework. Also, efforts will be made to more accurately characterize dig locations in order to model the variation in travel distances of trucks over the duration of mining at each location.

5. References

[1] Bozorgebrahimi, E., Hall, R. A., & Blackwell, G. H. (2003). Sizing equipment for open pit mining – a review of critical parameters. [doi: 10.1179/037178403225003591]. *Mining Technology, 112*(3), 171-179.

[2] Burt, Christina N., & Caccetta, Lou. (2014). Equipment Selection for Surface Mining: A Review. *Interfaces, 44*(2), 143-162.

[3] Burt, Christina Naomi. (2008). *An Optimisation Approach to Materials Handling in Surface Mines.* Curtin University of Technology.

[4] Campbell, S, & Hagan, P. (2012). *Selection of an optimum truck and shovel fleet based on effective flat haul and capacity constraint modelling*. Paper presented at the Eighth Open Pit Operators' Conference.

[5] Chanda, Emmanuel K., & Gardiner, Steven. (2010). A comparative study of truck cycle time prediction methods in open pit mining. *Engineering, Construction and Architectural Management, 17*(5), 446-460.

[6] Curi, Adilson, Felsch, Walter Schmidt, Rodovalho, E. Cunha, & Meireles, B. Prado. (2014). *Evaluation of Haul Trucks Performance in a CSN Mine*. Paper presented at the MPES Conference.

[7] Dotto, Magreth S. (2014). *Impact of hauler scale in mine planning.* Unpublished Thesis, University of Alberta, Edmonton.

[8] Hargroves, K., Gockowiak, K., McKeague, F., & Desha, C. (2014). *An Overview of Energy Efficiency Opportunities in Mining and Metallurgy Engineering*: The University of Adelaide and Queensland University of Technology (The Natural Edge Project), commissioned by the Australian Government Department of Industry, Canberra.

[9] Joseph, T, & Szymanski, J. (2013). Roads Make Loads – Rolling Resistance Saves Emissions. *Journal of Civil & Environmental Engineering, 3*(1).

[10] Joseph, T. G., Curley, M., & Anand, A. (2017). Operational Methodologies for Rolling Resistance Evaluation. [journal article]. *Geotechnical and Geological Engineering, 35*(6), 2935-2946.

[11] Kuo, Yao-chen. (2004). *Highway earthwork and pavement production rates for construction time estimation.* The University of Texas at Austin, Austin, Texas.

[12] Manyele, Samwel Victor. (2017). Analysis of Waste-Rock Transportation Process Performance in an Open-Pit Mine Based on Statistical Analysis of Cycle Times Data. *Engineering, Vol.09No.07*, 649-679.

[13] Newmont. (2014). Full Potential Newmont Newsroom: http://www.newmont.com/newsroom/newsroom-details/2014/Full-Potential/default.aspx.

[14] RungePincockMinarco. (2015). Talpac (Version 10.1.4). Brisbane, Queensland, Australia.

[15] Sheremeta, Robin. (2015). Truck Shovel Productivity: The Historical and Future Strategic Advantage for Teck Coal (pp. 41): Simon Fraser University.

[16] Thompson, R. J., & Visser, A. T. (1997). An introduction to the integrated design of surface mine haul roads. [doi: 10.1080/09208119708944074]. *International Journal of Surface Mining, Reclamation and Environment, 11*(3), 115-120.

[17] Thompson, R.J.*, Visser, A.T. (2003). Mine haul road maintenance management systems. *Journal of the Southern African Institute of Mining and Metallurgy, 103*(5), 303-312.

[18] Upadhyay, S. P., Suglo, R. S., Lin, Z., Sundararajan, S., & Szymanski, J. (2013). Simulation analysis model and equipment selection in continuous surface mining systems. *CIM Journal, 4*(2), 87-94.

[19] Upadhyay, Shiv Prakash, & Askari-Nasab, Hooman. (2018). Simulation and optimization approach for uncertainty-based short-term planning in open pit mines. *International Journal of Mining Science and Technology, 28*(2), 153-166.

Optimizing the Extraction Strategy for Open Pit and Underground Mining Transitions using Mathematical Programming

Bright Oppong Afum and Eugene Ben-Awuah
Mining Optimization Laboratory (MOL)
Laurentian University, Sudbury, Canada

ABSTRACT

When an orebody extends from the surface to great depths, the decision to exploit such deposit is an interesting challenge in the mining industry. Simplified decisions are usually made to exploit the deposit by open pit mining followed by underground mining which leads to missed financial and sustainable mining opportunities. Optimized mining strategies that maximize the project value and increase resource extraction ratio include several mining variations. This paper investigates the mining strategy for open pit and/or underground mining options using a Mixed Integer Linear Programming (MILP) optimization framework. The model maximizes the Net Present Value (NPV) of the reserve when exploited using open pit mining, underground mining, or simultaneous open pit and underground mining. The model further features the definition of the 3D location and size of the required crown pillar, and the extraction sequence of the open pit cuts and the underground stopes. The models are implemented on two case studies to investigate the best mining option, and the optimal material extraction strategy that maximizes NPV and increase resource extraction ratio for the mining projects.

1. Introduction

When a mineral deposit extends from the surface to great depth, the portion of the orebody close to the surface is usually preferred to be exploited with open pit (OP) mining options to generate early revenue while the deeper portions are exploited with underground (UG) mining options. In open pit mining, the incremental stripping ratio and overall mining cost with depth makes underground mining profitable beyond a certain depth. This depth has been referred by several authors as transition depth or transition point (Bakhtavar et al., 2009; Opoku and Musingwini, 2013; Roberts et al., 2013; Dagdelen and Traore, 2014; Ordin and Vasil'ev, 2014; De Carli and de Lemos, 2015; King et al., 2016; MacNeil and Dimitrakopoulos, 2017).

During the feasibility study, knowledge of the required mining option(s) in developing the mineral resource at the onset of the mine is a major information that influences several mining and investment decisions. Proper decisions made at the onset of the mine gives much confidence to both mine planners and investors. Knowing the extent or limit of the depth of a typical open pit mining operation is an added advantage to the planning of the mine from geotechnical, resource and financial points of views.

A strategic mathematical framework for determining the mining strategy for a deposit that has the potential to be exploited with several variations of the mining methods has been developed. The mining variations include the exploitation of the deposit completely with (1) open pit mining method, (2) underground mining method, or (3) both open pit and underground mining methods. In

the third mining variation, the deposit could be sequentially exploited with an open pit mine before transition into an underground mine or sequentially exploited with an underground mine followed by an open pit mine, or simultaneously exploited with both open pit and underground mining options. In the case where there is the need to exploit the deposit by both open pit and underground mining methods, the location of a required crown pillar is essential for the mining project. The decision to adopt any mining option strategy primarily depends on the economics of the project and the geology of the mining area.

Optimizing the location of a crown pillar is a key factor in the metalliferous mining industry. Finding the most suitable location of the crown pillar in a combined mining method of open pit and underground operations, especially in block caving method, is one of the most interesting problems for mining engineers today (Bakhtavar et al., 2012). The transition from open pit to underground mining involves a complicated geomechanical process.

The paper presents Mixed Integer Linear Programming (MILP) model frameworks for solving the open pit and underground mining transition problems. The model determines an optimized mining strategy for a deposit that is potentially amenable by both open pit and underground mining options. A synthetic copper deposit and a small gold deposit is used as a case study to implement the models for evaluation.

The next section of this research paper covers a summarized literature review on open pit to underground mining transition problem with highlights on research gaps. Section 3 discusses the assumptions and notations used in the proposed MILP models. Section 4 introduces and explains the proposed integrated MILP models for the open pit to underground mining transition complex. Section 5 explains the implementation of the MILP models to the evaluation of two case studied deposits while Section 6 documents the research conclusions and future works.

2. Summary of Literature Review

Strategic open pit and underground mining interface optimization models have been developed based on determining the transition depth between open pit mining and underground mining. These models mainly focus on investigating how an underground mining operation can be exploited after an open pit mine or combined with an existing open pit operation (Ben-Awuah et al., 2016). Acknowledging notable challenges and shortfalls, several researchers have employed techniques, algorithms and/or models to determine the transition depth (Bakhtavar et al., 2009; Opoku and Musingwini, 2013; Roberts et al., 2013; Dagdelen and Traore, 2014; Ordin and Vasil'ev, 2014; De Carli and de Lemos, 2015; King et al., 2016; MacNeil and Dimitrakopoulos, 2017) and the ore block extraction strategy (De Carli and de Lemos, 2015; Ben-Awuah et al., 2016; King et al., 2016; MacNeil and Dimitrakopoulos, 2017).

Kurppa and Erkkilä (1967) assessed the simultaneous mining between open pit and underground (OP-UG) mining during the operations of the Pyhasalmi mine. They indicated that, simultaneous mining was possible due to the geometry of the orebody being worked. Luxford (1997) argued that, cost usually drives the decision to make the transition because as the open pit waste stripping cost keeps increasing with depth, there comes a time when the underground mining cost will be less than the open pit mining cost.

Ben-Awuah et al. (2016) investigated the strategy of mining options for an orebody by using a mathematical programming model – MILP. The research evaluated the financial impacts of applying different mining options separately or concurrently to extract a given orebody. The MILP formulation maximizes the NPV of the reserve when extracted with: (1) open pit mining, (2) underground mining, and (3) concurrent open pit and underground mining. The location of a required crown pillar was not considered in this model.

King et al. (2016) incorporated crown and sill pillar placement into OP-UG transition studies to separate the open pit from the underground mine. In their model, the location of the crown pillar was simulated, and the number of crown pillar locations was selected. Producing several crown pillar locations for a deposit will lead to the difficulty in selecting the optimal crown pillar location that maximizes the value of the mining project.

MacNeil and Dimitrakopoulos (2017) investigated the transition decision at an operating open pit mine that exists within the context of a mining complex comprising five producing pits, four stockpiles and one processing plant. The proposed method improves upon previous developments related to the OP-UG transition problem by incorporating geological uncertainty into the decision-making process while providing a transition depth described in three-dimensions (3D). In their research, MacNeil and Dimitrakopoulos (2017) priori identified the crown pillar envelope for a gold deposit and evaluated four crown pillar locations within this envelope leading to four distinct candidate transition depths.

A mathematical model that solves the transition problem was been developed by Whittle et al. (2018) after modifying the normal pit optimization model based on the maximum graph closure problem (Lerchs and Grossman, 1965). The modifications allow the algorithm to account for the underground mining value of a block, and the requirement for a specified crown pillar thickness and shape above the underground mine. The algorithm of Whittle et al. (2018) is based on what they called the "opportunity cost approach", thus, if a given block is mined by open pit method, its open pit value is gained while its value that would have been obtained by mining blocks using underground mining methods is lost. By applying the opportunity cost approach, the underground value is subtracted from the open pit value for each block before doing pit optimization. This approach compromises the optimal value of the mining project since the optimization process does not fully control the mining sequence with time.

Current formulations for optimizing the open pit to underground transition complexes do not integrate the location and size of a 3D crown pillar into the optimization process and further produce optimal solutions within a known level of confidence. In this research study, we have developed, implemented, and tested a MILP optimization framework capable of evaluating a deposit amenable by either open pit, underground or both open pit and underground mining options. The optimization process of the MILP further decides the best location of the required 3D crown pillar that maximizes the profit of the mining project without human interference on the location of the crown pillar.

3. Assumptions and Notations

The size of the Selected Mining Units (SMUs) of the open pit mining is equal to the stope sizes of the underground mining. In this model, the SMUs are represented by mining block in general or mining-cut for the open pit operation or mining-stope for the underground operation. The location of each mining block or mining-cut or mining-stope is represented by the coordinate of the centroid of the block. It is assumed that a crown pillar is required for the exploitation of the orebody by underground mining. The size of the crown pillar is assumed to be one vertical length of the stope or bench; thus, one bench or level of the block model represents the crown pillar. For the underground mining, ore extraction is achieved by a retreating method of ore extraction. The notation of decision variables, parameters, sets, and constraints are as follows:

3.1. Sets

$K = \{1,\ldots,K\}$ set of all the blocks in the model.

$J = \{1,\ldots,J\}$ set of all the open pit mining-cuts in the model.

$J_s = \{1,\ldots,J_s\}$ set of all the open pit mining-cuts on a level in the model.

$P = \{1,...,P\}$ set of all the underground mining-stopes in the model.

$J_p = \{1,...,J_p\}$ set of all the underground mining-stopes on a particular level in the model.

$C = \{1,...,C\}$ set of all the levels acting as crown pillars (c) in the model.

$O_j(S)$ for each open pit mining-cut, j, there is a set $O_j(S) \subset J$, defining the immediate predecessor mining-cut that must be extracted prior to extracting cut j; where S is the total number of mining-cuts in set $O_j(S)$.

$U_p(S)$ for each underground mining-stope, p, there is a set $U_p(S) \subset P$, defining the immediate predecessor mining-stope that must be extracted prior to extracting stope p; where S is the total number of mining-stopes in set $U_p(S)$.

$C_j(S)$ for each level, there is a set $C_j(S) \subset J$, defining the number of mining-cuts, j, on that level that must be extracted as open pit cut j, or left as unmined level or crown pillar (c); where S is the total number of mining-cuts on a level in set $C_j(S)$.

$C_p(S)$ for each level, there is a set $C_p(S) \subset P$, defining the number of mining-stopes, p, on that level that must be extracted as underground stope p, or left as unmined level or crown pillar (c); where S is the total number of mining-stopes on a level in set $C_p(S)$.

3.2. Indices

A general parameter f can take four indices in the format of $f_{k,l}^{a,t}$. Where:

$t \in \{1,...,T\}$ index for scheduling periods.

$k \in \{1,...,K\}$ index for mining-blocks in the model.

$j \in \{1,...,J\}$ index for open pit mining-cuts in the model.

$p \in \{1,...,P\}$ index for underground mining-stopes in the model.

$c \in \{1,...,C\}$ index for crown pillars in the model.

3.3. Parameter

$v_j^{op,t}$ the open pit (op) discounted revenue generated by selling the final product within mining-cut j in period t minus the discounted extra cost of mining all the mining-cut j as ore and processing it.

$v_p^{ug,t}$ the underground (ug) discounted revenue generated by selling the final product within mining-stope p in period t minus the discounted extra cost of mining all the mining-stope p as ore and processing it.

$q_j^{op,t}$ the open pit (op) discounted cost of mining all the material in mining-cut j in period t as waste.

$q_p^{ug,t}$ the underground (ug) discounted cost of mining all the material in mining-stope p in period t as waste.

g_j average grade of element in ore portion of mining-cut j.

g_p average grade of element in ore portion of mining-stope p.

o_k ore tonnage in mining-block k.

o_j	ore tonnage in mining-cut *j*.
o_p	ore tonnage in mining-stope *p*.
w_k	waste tonnage in mining-block *k*.
w_j	waste tonnage in mining-cut *j*.
w_p	waste tonnage in mining-stope *p*.
$g_{lb}^{op,t}$	lower bound on acceptable average head grade of element for the open pit mining (*op*) in period *t*.
$g_{lb}^{ug,t}$	lower bound on acceptable average head grade of element for the underground mining (*ug*) in period *t*.
$g_{ub}^{op,t}$	upper bound on acceptable average head grade of element for the open pit mining (*op*) in period *t*.
$g_{ub}^{ug,t}$	upper bound on acceptable average head grade of element for the underground mining (*ug*) in period *t*.
$T_{pr,lb}^{op,t}$	lower bound on ore processing capacity from open pit and underground mining in period *t.*
$T_{pr,lb}^{ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period *t*.
$T_{pr,ub}^{op,t}$	lower bound on ore processing capacity from open pit and underground mining in period *t.*
$T_{pr,ub}^{ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period *t*.
$T_{m,lb}^{op,t}$	lower bound on available open pit mining capacity in period *t.*
$T_{m,lb}^{ug,t}$	lower bound on available underground mining capacity in period *t.*
$T_{m,ub}^{op,t}$	upper bound on available open pit mining capacity in period *t*.
$T_{m,ub}^{ug,t}$	upper bound on available underground mining capacity in period *t.*
$T_{pr,lb}^{op,ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period *t.*
$T_{pr,lb}^{op,ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period *t*.
$T_{pr,ub}^{op,ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period *t.*
$T_{pr,ub}^{op,ug,t}$	lower bound on ore processing capacity from open pit and underground mining in period *t*.
r	processing recovery; the proportion of mineral commodity recovered.
sp^t	selling price of mineral commodity in present value terms.

sc^t selling cost of mineral commodity in present value terms.

pc^t extra cost in present value terms per tonne of ore for mining and processing in period *t*.

cm_l^t cost per bench or level *l* in present value terms of mining a tonne of waste by open pit mining in period *t.*

cm^t cost in present value terms of mining a tonne of waste by underground mining in period *t.*

3.4. Decision Variables

$x_j^{op,t} \in \{0,1\}$ continuous variable, representing the portion of mining-cut *j* to be extracted as ore and processed in period *t* for open pit.

$x_p^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of mining-stope *p* to be extracted as ore and processed in period *t* for underground mining.

$y_j^{op,t} \in \{0,1\}$ continuous variable, representing the portion of mining-cut *j* to be mined in period *t* for open pit, fraction of *y* characterizes both ore and waste included in the mining-cut.

$y_p^{ug,t} \in \{0,1\}$ continuous variable, representing the portion of mining-stope *p* to be mined in period *t* for underground, fraction of *y* characterizes both ore and waste included in the mining-stope.

$y_c^t \in \{0,1\}$ binary integer variable; equal to one if a level is left as crown pillar (*c*) in period *t,* otherwise zero.

$y_j^t \in \{0,1\}$ binary integer variable; equal to one if a mining-cut *j* or all mining-cuts *Js* on a level is extracted as open pit in period *t,* otherwise zero.

$y_p^t \in \{0,1\}$ binary integer variable; equal to one if a mining-stope *p* or all mining-stopes *Ps* on a level is extracted as underground in period *t,* otherwise zero.

$b_j^t \in \{0,1\}$ binary integer variable controlling the precedence of extraction of mining-cut for the open pit. b_j^t is equal to one if extraction of cut *j* has started by or in period t, otherwise it is zero.

$b_p^t \in \{0,1\}$ binary integer variable controlling the precedence of extraction of mining-stope for the underground. b_p^t is equal to one if extraction of stope *p* has started by or in period t, otherwise it is zero.

4. Integrated MILP Models

Three separate MILP models are formulated to determine the time and sequence of extraction of ore and waste blocks over the life of the mining project. The proposed MILP models interrogate the orebody and determine the best mining option that produces an optimal extraction sequence to maximize the Net Present Value (NPV) of the mining project. The mining options could either be open pit mining or underground mining or simultaneous open pit and underground mining. The NPV of the production schedule is maximized in the presence of technical, geotechnical, geological, and economic constraints to enforce the mining extraction, blending requirements, and mining and processing capacities.

The objective function of the MILP models maximizes the operation of the mining project. The quantity of ore processed is controlled by the continuous decision variables $x_j^{op,t}$ and $x_p^{ug,t}$ for open pit and underground mining respectively. Similarly, the quantity of rock material extracted by open pit and underground mining are respectively controlled by the continuous decision variables $y_j^{op,t}$ and $y_p^{ug,t}$. The continuous variables ensure fractional extraction of the mining-cut or mining-stope in different periods.

The constraints of the proposed MILP models control the mining, processing, plant head grade, geotechnical slope, crown pillar, and the surface and underground mining precedence. Acceptable upper and lower targets have been defined for the mining, processing, and plant head grade inequality constraints.

The mining capacity is a function of the ore reserve, targeted mine-life, designed processing capacity and available capital for mining fleet acquisition for the mining operation. The processing capacity constraints define the ore production schedule in each period and further ensures the run-of-mine (ROM) material satisfies the quantity specification of the processing plant. Based on the cut-off grades or head grade of the plant, limiting grade requirements for appropriate ore blending from each mining option are defined within a lower and upper grade targets for the mining operation.

4.1. Modeling the economic block value

The economic block values are defined based on the SMU, thus, mining-cut and mining-stope for the open pit and underground mining respectively. The value of the block is a function of the recovered quantity of mineral present in the block, the discounted revenue of selling the commodity, and the discounted costs of mining, processing and selling costs.

The discounted revenues generated by selling the final product within block k being extracted in period t by open pit mining $v_j^{op,t}$ and underground mining $v_p^{ug,t}$ are respectively shown in Eqs. (1) and (2). Similarly, the discounted costs of mining all the material within block k being extracted in period t by open pit mining $q_j^{op,t}$ and underground mining $q_p^{ug,t}$ are respectively shown in Eqs. (3) and (4).

$$v_j^{op,t} = \sum_{j=1}^{J} o_j \times g_j \times r \times (sp - sc) - \left(\sum_{j=1}^{J} o_j \times pc \right) \tag{1}$$

$$v_p^{ug,t} = \sum_{p=1}^{P} o_p \times g_p \times r \times (sp - sc) - \left(\sum_{p=1}^{P} o_p \times pc \right) \tag{2}$$

$$q_j^{op,t} = \sum_{j=1}^{J} (o_j + w_j) \times \sum_{l=1}^{L} cm_l^t \tag{3}$$

$$q_p^{ug,t} = \sum_{p=1}^{P} (o_p + w_p) \times cm^t \tag{4}$$

4.2. Model 01 – open pit mining option

Objective function:

$$Max \left[\sum_{t=1}^{T} \sum_{j=1}^{J} \left(v_j^{op,t} \times x_j^{op,t} - q_j^{op,t} \times y_j^{op,t} \right) \right] \tag{5}$$

Subject to:

$$T_{m,lb}^{op,t} \le \sum_{j=1}^{J}\left[\left(o_j + w_j\right)\times y_j^{op,t}\right] \le T_{m,ub}^{op,t} \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{6}$$

$$T_{pr,lb}^{op,t} \le \sum_{j=1}^{J}\left(o_j \times x_j^{op,t}\right) \le T_{pr,ub}^{op,t} \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}, p \in \{1,\ldots,P\}; \tag{7}$$

$$g_{lb}^{op,t} \le \left[\sum_{j=1}^{J}\left(\frac{g_j \times o_j}{o_j}\right)\right]\times x_j^{op,t} \le g_{ub}^{op,t} \qquad \forall t \in \{t,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{8}$$

$$x_j^{op,t} - y_j^{op,t} \le 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{9}$$

$$b_j^t - \sum_{t=1}^{T} y_s^{op,t} \le 0 \qquad \forall t \in \{1,\ldots,T\}, s \in O_j(S), j \in \{1,\ldots,J\}; \tag{10}$$

$$\sum_{t=1}^{T} y_j^{op,t} - b_j^t \le 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{11}$$

$$b_j^t - b_j^{t+1} \le 0 \qquad \forall t \in \{1,\ldots,T\text{-}1\}, j \in \{1,\ldots,J\}; \tag{12}$$

$$\left(\sum_{j=1}^{Js} b_{j,s}^t \times \frac{1}{Js}\right) - y_j^t \le 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,Js\}, s \in C_j(S), j \in \{1,\ldots,J\}; \tag{13}$$

$$\sum_{t=1}^{T} y_j^{op,t} \le 1 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{14}$$

The formulation of Model 01 follows after Askari-Nasab et al. (2011). The objective function of Model 01 shown in Eq. (5) maximizes the operation of the open pit mining option. The quantity of ore processed, and rock material mined are controlled by the continuous decision variables $x_j^{op,t}$ and $y_j^{op,t}$ respectively.

Eq. (6) defines the mining capacity constraints for the open pit mining. This inequality ensures that the total tonnage of rock material mined in each period is within the acceptable lower and upper limits of the total available equipment capacity for the open pit mining operation.

Eq. (7) is the processing capacity constraint that controls the quantity of mill feed for the open pit mining option. The processing constraints are controlled by the continuous decision variables $x_j^{op,t}$. This inequality ensures that uniform ore is fed to the processing plant throughout the mine life of the open pit mining operation within acceptable lower and upper targets of ore for the processing plant in each period.

Eq. (8) represents the grade blending constraints for the open pit mining option. This inequality is controlled by the continuous variable $x_j^{op,t}$ and further ensures quality ore is delivered to the processing plant in each period. Eq. (9) defines the relationship between the ore and mining-cut tonnages controlling the mining and processing decisions. Thus, the continuous variable $x_j^{op,t}$ is always smaller than or equal to the continuous variable $y_j^{op,t}$.

Eqs. (10), (11) and (12) control the downward precedence relationship of mining-cut extraction and the appropriate geotechnical mining slope for the open pit mining option. For the open pit mining, nine overlying mining-cuts are mined before the underlying mining-cut is mined.

Eq. (13) ensures that a level y_j^t is categorized for open pit mining when a block on that level is extracted. Eq. (14) defines the reserve constraints and ensures that each block is extracted once in the life of the open pit mine.

4.3. Model 02 – underground mining option

Objective function:

$$Max\left[\sum_{t=1}^{T}\sum_{p=1}^{P}\left(v_p^{ug,t}\times x_p^{ug,t}-q_p^{ug,t}\times y_p^{ug,t}\right)\right] \tag{15}$$

Subject to:

$$T_{m,lb}^{ug,t}\le\sum_{p=1}^{P}\left[\left(o_p+w_p\right)\times y_p^{ug,t}\right]\le T_{m,ub}^{ug,t}\quad \forall t\in\{1,...,T\},\ p\in\{1,...,P\}; \tag{16}$$

$$T_{pr,lb}^{ug,t}\le\sum_{p=1}^{P}\left(o_p\times x_p^{ug,t}\right)\le T_{pr,ub}^{ug,t}\quad \forall t\in\{1,...,T\},\ p\in\{1,...,P\}; \tag{17}$$

$$g_{lb}^{ug,t}\le\left[\sum_{p=1}^{P}\left(\frac{g_p\times o_p}{o_p}\right)\right]\times x_p^{ug,t}\le g_{ub}^{ug,t}\quad \forall t\in\{1,...,T\}; \tag{18}$$

$$x_p^{ug,t}-y_p^{ug,t}\le 0\quad \forall t\in\{1,...,T\},\ k\in B_p,\ p\in\{1,...,P\}; \tag{19}$$

$$b_p^t-\sum_{t=1}^{T}y_s^{ug,t}\le 0\quad \forall t\in\{1,...,T\},\ s\in U_p(S_{1,2,3}),\ p\in\{1,...,P\}; \tag{20}$$

$$\sum_{t=1}^{T}y_p^{ug,t}-b_p^t\le 0\quad \forall t\in\{1,...,T\},\ p\in\{1,...,P\}; \tag{21}$$

$$b_p^t-b_p^{t+1}\le 0\quad \forall t\in\{1,...,T\text{-}1\},\ p\in\{1,...,P\}; \tag{22}$$

$$\sum_{t=1}^{T}x_p^{ug,t}\le 1\quad \forall t\in\{1,...,T\},\ p\in\{1,...,P\}; \tag{23}$$

$$y_c^t+y_p^t\le 1\quad \forall t\in\{1,...,T\},\ p\in\{1,...,P\},\ c\in\{1,...,C\}; \tag{24}$$

$$\left(\sum_{p=1}^{Ps}b_{p,s}^t\times\frac{1}{Ps}\right)-y_p^t\le 0\quad \forall t\in\{1,...,T\},\ p\in\{1,...,Ps\},\ s\in C_p(S),\ p\in\{1,...,P\}; \tag{25}$$

$$y_c^t-y_c^{t+1}\le 0\quad \forall t\in\{1,...,T\},\ c\in\{1,...,C\}; \tag{26}$$

$$\sum_{c=1}^{C}y_c^t=1\quad t\in\{1,...,T\},\ c\in\{1,...,C\}; \tag{27}$$

The objective function of Model 02 shown in Eq. (15) maximizes the operation of the underground mining option. The quantity of ore processed, and rock material mined are controlled by the continuous decision variables $x_p^{ug,t}$ and $y_p^{ug,t}$ respectively.

Eq. (16) defines the mining capacity constraints for the underground mining. This inequality ensures that the total tonnage of rock material mined in each period is within the acceptable lower and upper limits of the total available equipment capacity for the underground mining operation.

Eq. (17) is the processing capacity constraint that controls the quantity of mill feed for the underground mining option. The processing constraints are controlled by the continuous decision variables $x_p^{ug,t}$. This inequality ensures that uniform ore is fed to the processing plant throughout the mine life of the underground mining operation within acceptable lower and upper targets of ore for the processing plant in each period.

Eq. (18) represents the grade blending constraints for the underground mining option. This inequality is controlled by the continuous variables $x_p^{ug,t}$ and further ensures quality ore is delivered to the processing plant in each period. Eq. (19) defines the relationship between the ore and mining-stope tonnages controlling the mining and processing decisions. Thus, the continuous variable $x_p^{ug,t}$ is always smaller than or equal to the continuous variable $y_p^{ug,t}$.

Eqs. (20), (21) and (22) control the lateral precedence relationship of mining-stope extraction on each level for the underground mining option. For the underground mining, ore extraction sequence is implemented in a retreating manner towards the main entrance of the mine for each underground level.

Eq. (23) defines the reserve constraints and the ore selective nature of underground mining. This inequality ensures that each stope is extracted once in the life of the underground mine.

Eqs. (24), (25), (26) and (27) define the location and size of the required crown pillar for the underground mining option. Eq. (24) ensures that, a level is either considered for underground mining or left as crown pillar or unmined level. Eq. (25) ensures that a level belongs to an underground mining option when a block or more on that level is extracted by underground mining. Eq. (26) ensures that a level acting as the crown pillar stays the same throughout the life of the mining operation while Eq. (27) ensures that one level always acts as the crown pillar throughout the life of the underground mine.

4.4. Model 03 – simultaneous open pit and underground mining option

Objective function:

$$Max\left[\sum_{t=1}^{T}\left(\sum_{j=1}^{J}\left(v_j^{op,t}\times x_j^{op,t}-q_j^{op,t}\times y_j^{op,t}\right)+\sum_{p=1}^{P}\left(v_p^{ug,t}\times x_p^{ug,t}-q_p^{ug,t}\times y_p^{ug,t}\right)\right)\right] \tag{28}$$

Subject to:

$$T_{m,lb}^{op,t}\le\sum_{j=1}^{J}\left[\left(o_j+w_j\right)\times y_j^{op,t}\right]\le T_{m,ub}^{op,t}\quad \forall t\in\{1,\ldots,T\}, j\in\{1,\ldots,J\}; \tag{29}$$

$$T_{m,lb}^{ug,t}\le\sum_{p=1}^{P}\left[\left(o_p+w_p\right)\times y_p^{ug,t}\right]\le T_{m,ub}^{ug,t}\quad \forall t\in\{1,\ldots,T\},\ p\in\{1,\ldots,P\}; \tag{30}$$

$$\left[o_j\times b_j^t\right]+\left[o_p\times b_p^t\right]\le o_k\quad \forall t\in\{1,\ldots,T\}, j\in\{1,\ldots,J\}, p\in\{1,\ldots,P\}, k\in\{1,\ldots,K\}; \tag{31}$$

$$T_{pr,lb}^{op,ug,t}\le\sum_{j=1}^{J}\left(o_j\times x_j^{op,t}\right)+\sum_{p=1}^{P}\left(o_p\times x_p^{ug,t}\right)\le T_{pr,ub}^{op,ug,t}\quad \forall t\in\{1,\ldots,T\}, j\in\{1,\ldots,J\}, p\in\{1,\ldots,P\}; \tag{32}$$

$$T_{pr,lb}^{ug,t}\le\sum_{p=1}^{P}\left(o_p\times x_p^{ug,t}\right)\le T_{pr,ub}^{ug,t}\quad \forall t\in\{1,\ldots,T\}, p\in\{1,\ldots,P\}; \tag{33}$$

$$g_{lb}^{op,t}\le\left[\sum_{j=1}^{J}\left(\frac{g_j\times o_j}{o_j}\right)\right]\times x_j^{op,t}\le g_{ub}^{op,t}\quad \forall t\in\{t,\ldots,T\}, j\in\{1,\ldots,J\}; \tag{34}$$

$$g_{lb}^{ug,t} \le \left[\sum_{p=1}^{P}\left(\frac{g_p \times o_p}{o_p}\right)\right] \times x_p^{ug,t} \le g_{ub}^{ug,t} \qquad \forall t \in \{1,\ldots,T\}; \tag{35}$$

$$x_j^{op,t} - y_j^{op,t} \le 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{36}$$

$$b_j^t - \sum_{t=1}^{T} y_s^{op,t} \le 0 \qquad \forall t \in \{1,\ldots,T\}, s \in O_j(S), j \in \{1,\ldots,J\}; \tag{37}$$

$$\sum_{t=1}^{T} y_j^{op,t} - b_j^t \le 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{38}$$

$$b_j^t - b_j^{t+1} \le 0 \qquad \forall t \in \{1,\ldots,T\text{-}1\}, j \in \{1,\ldots,J\}; \tag{39}$$

$$\sum_{t=1}^{T} y_j^{op,t} \le 1 \qquad \forall t \in \{1,\ldots,T\},\ j \in \{1,\ldots,J\}; \tag{40}$$

$$x_p^{ug,t} - y_p^{ug,t} \le 0 \qquad \forall t \in \{1,\ldots,T\}, k \in B_p, p \in \{1,\ldots,P\}; \tag{41}$$

$$b_p^t - \sum_{t=1}^{T} y_s^{ug,t} \le 0 \qquad \forall t \in \{1,\ldots,T\}, s \in U_p(S_{1,2,3}), p \in \{1,\ldots,P\}; \tag{42}$$

$$\sum_{t=1}^{T} y_p^{ug,t} - b_p^t \le 0 \qquad \forall t \in \{1,\ldots,T\}, p \in \{1,\ldots,P\}; \tag{43}$$

$$b_p^t - b_p^{t+1} \le 0 \qquad \forall t \in \{1,\ldots,T\text{-}1\},\ p \in \{1,\ldots,P\}; \tag{44}$$

$$\sum_{t=1}^{T} x_p^{ug,t} \le 1 \qquad \forall t \in \{1,\ldots,T\}, p \in \{1,\ldots,P\}; \tag{45}$$

$$y_j^t + y_c^t + y_p^t \le 1 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}, p \in \{1,\ldots,P\}, c \in \{1,\ldots,C\}; \tag{46}$$

$$\left(\sum_{j=1}^{Js} b_{j,s}^t \times \frac{1}{Js}\right) - y_j^t \le 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,Js\}, s \in C_j(S), j \in \{1,\ldots,J\}; \tag{47}$$

$$\left(\sum_{p=1}^{Ps} b_{p,s}^t \times \frac{1}{Ps}\right) - y_p^t \le 0 \qquad \forall t \in \{1,\ldots,T\}, p \in \{1,\ldots,Ps\}, s \in C_p(S), p \in \{1,\ldots,P\}; \tag{48}$$

$$y_c^t - y_{j-1}^t \le 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}, c \in \{1,\ldots,C\}; \tag{49}$$

$$y_j^t - y_{j-1}^t \le 0 \qquad \forall t \in \{1,\ldots,T\}, j \in \{1,\ldots,J\}; \tag{50}$$

$$y_{c-1}^t - y_p^t \le 0 \qquad \forall t \in \{1,\ldots,T\}, p \in \{1,\ldots,P\}, c \in \{1,\ldots,C\}; \tag{51}$$

$$y_c^t - y_c^{t+1} \le 0 \qquad \forall t \in \{1,\ldots,T\}, c \in \{1,\ldots,C\}; \tag{52}$$

$$\sum_{c=1}^{C} y_c^t = 1 \qquad t \in \{1,\ldots,T\}, c \in \{1,\ldots,C\}; \tag{53}$$

The objective function of Model 3 shown in Eq. (28) maximizes the simultaneous open pit and underground mining operation. The quantity of ore processed is controlled by the continuous decision variables $x_j^{op,t}$ and $x_p^{ug,t}$ for open pit and underground mining respectively. Similarly, the

quantity of rock material extracted by both open pit and underground mining options are respectively controlled by the continuous decision variables $y_j^{op,t}$ and $y_p^{ug,t}$. The continuous variables ensure fractional extraction of the mining-cut or mining-stope in different periods.

Eqs. (29) and (30) are the mining capacity constraints respectively for the open pit and underground mining operations. The open pit and underground mining are respectively controlled by the continuous decision variables $y_j^{op,t}$ and $y_p^{ug,t}$. These inequalities ensure the total tonnage of rock material mined in each period is within the acceptable lower and upper limits of the total available equipment capacity for the mining operations.

Eq. (31) represents the interaction of the open pit and underground mining options. The inequality ensures that the mining block is extracted by only one mining option or left us unmined level or bench.

Eqs. (32) and (33) are the processing capacity constraints that control the quantity of the mill feed for each mining option. Eq. (32) represents the contribution of ore production from both open pit and underground mining option to the processing plant. The constraints are controlled by the continuous decision variables $x_j^{op,t}$ and $x_p^{ug,t}$ for open pit and underground mining respectively. These inequalities ensure that uniform ore is fed to the processing plant throughout the mine life of the mining operation within acceptable lower and upper targets of ore for the processing plant in each period.

Eqs. (34) and (35) represent the grade blending constraints for both open pit and underground mining respectively. These inequalities are also respectively controlled by the continuous variables $x_j^{op,t}$ and $x_p^{ug,t}$ for open pit and underground mining and ensure quality ore is delivered to the processing plant in each period. The ore production schedule in each period ensures the run-of-mine (ROM) material satisfies the ore quality specification of the processing plant. Based on the cut-off grades or head grade of the plant, limiting grade requirements for appropriate ore blending from each mining option are defined within a lower and upper grade targets for the mining operation.

Eq. (36) defines the relationship between the ore and mining-cut tonnages controlling the mining and processing decisions. Thus, the continuous variable $x_j^{op,t}$ is always smaller than or equal to the continuous variable $y_j^{op,t}$.

Eqs. (37), (38) and (39) control the downward precedence relationship of mining-cut extraction and the appropriate geotechnical mining slope for the open pit mining option. For the open pit mining, nine overlying mining-cuts are mined before the underlying mining-cut is mined.

Eq. (40) defines the reserve constraints and ensures that a block is extracted once in the life of the open pit mine. Eq. (41) defines the relationship between the ore and mining-cut tonnages controlling the underground mining and processing decisions. Thus, the continuous variable $x_p^{ug,t}$ is always smaller than or equal to the continuous variable $y_p^{ug,t}$.

Eqs. (42), (43) and (44) control the lateral precedence relationship of mining-stope extraction on each level for the underground mining option. For the underground mining, ore extraction sequence is implemented in a retreating manner towards the main entrance of the mine for each underground level.

Eq. (45) defines the reserve constraints and the ore selective nature of underground mining. This inequality ensures that each stope $x_p^{ug,t}$ on a level y_p^t is extracted once in the life of the underground mine.

Eqs. (46), (47), (48), (49), (50), (51), (52) and (53) define the location and size of the required crown pillar for the commencement of the underground mining option. Eq. (46) ensures that, each level or bench is either considered for open pit mining or underground mining or left as crown pillar or unmined level within the block model. Eq. (47) ensures that, a level belongs to an open pit mining option when one block or more on that level is extracted by open pit mining while Eq. (48) ensures that a level belongs to an underground mining option when one block or more on that level is extracted by underground mining. Eq. (49) ensures that a level can be the crown pillar when the levels above are being considered for open pit mining option. Thus, crown pillar immediately follows an open pit mining option.

Equation (50) ensures that when a level is considered for open pit mining, the immediate level below could be considered for open pit mining option. Equation (51) ensures that the crown pillar is always above the level being considered for underground mining. Equation (52) ensures that a level acting as the crown pillar stays the same throughout the life of the mining operation while Equation (53) ensures that one level always acts as the crown pillar.

4.5. Non-negativity constraints

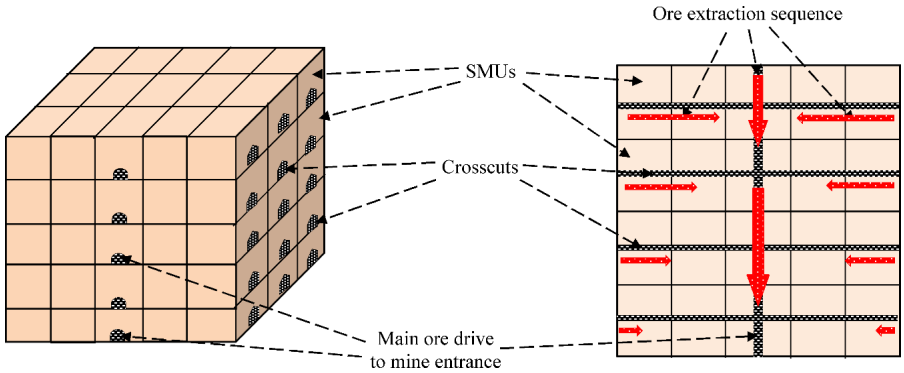
Eq. (54) ensures that the decision variables for open pit and underground mining and processing, crown pillar, open pit mining benches, and underground mining levels are non-negative. The inequality further defines the binary variables controlling the geotechnical and extraction sequence in the open pit and underground mining are integers.

$$x_j^{op,t},\ y_j^{op,t},\ x_p^{ug,t},\ y_p^{ug,t} \geq 0 \text{ and } b_j^t,\ b_p^t,\ y_j^t,\ y_p^t,\ y_c^t \text{ are integers} \tag{54}$$

5. Computational Implementation of the MILP Models

The formulated models were implemented on two experimental case studies. IBM ILOG CPLEX Optimization Studio V12.6.3 (ILOG, 2015) was integrated within MATLAB 2018a V9.4.0.813654 (Mathworks, 2018) environment to define the modelled framework and solve the optimization problem at a MILP gap tolerance of 5%. The CPLEX solver stops at an integer feasible solution when the gap tolerance of optimality is 5%. The model was tested on an Intel(R) Core™ i7-7700HQ CPU Dell computer @ 2.80GHz, with 32 GB RAM.

The block model of the deposit is prepared by dividing it into two halves with an operating ore drive on each level. Crosscuts development extend from this ore drive to the ends of the minefields through each SMU, acting as stope drives. Fig. 1 is a representation of the block model showing the main underground operating developments and the retreating ore extraction sequence on a typical level.



a) Block model showing developments layouts

b) Development layout and ore extraction sequence of a typical underground level

Fig. 1. Isometric view of the block model and development layout of an underground level.

In the open pit mining, the ore is exploited from the top to the bottom with a 45^{o} slope to ensure a geotechnical control of the mine. However, in underground mining, the ore is extracted from the ends of the minefield in a retreating manner towards the main entrance of the mine through the crosscuts and the ore drives.

5.1. Case study 1 – synthetic dataset for copper deposit

The strategic Mixed Integer Linear Programming (MILP) models were implemented and tested on a synthetic copper deposit. The copper deposit consists of 500 mining blocks representing the Selective Mining Units (SMUs). The deposit is represented in a geological block model, in a 3D array of cubical blocks containing 500 SMUs of sizes 250 x 200 x 15. The orebody in the block model is irregularly shaped with a total mineral resource of 69.32 Mt of ore with an average Cu grade of 0.7664 g/t. Table 1 is a description of the copper deposit used to implement the models.

Table 1. Statistical description of the gold deposit.

No.	Description	Value
1	Total material (Mt)	675.00
2	Total mineralized material (Mt)	69.32
3	Minimum value of Cu (g/t)	0.7190
4	Maximum value of Cu (g/t)	0.8160
5	Average value of Cu (g/t)	0.7664
6	Variance	0.0006
7	Standard deviation	0.0251

5.1.1. Economic and mining data for case study 1

Taylor's mine life rule was used to estimate the mine life between 15.0 to 22.5 years. A mine life of 20 years was used to assess three mining options. Based on the quantity of mineralized material present in the deposit, the yearly mining and processing capacities were determined. An incremental bench cost of $ 2.0 per 7.5 m was used as the open pit mining variable cost as the pit transcends downwards. The economic, mining and processing data used for evaluating the gold deposit is shown in Table 2.

5.1.2. Results and discussions for case study 1

The Net Present Value (NPV) of the three mining options were evaluated and the best mining option selected for the deposit. The results of the integrated MILP models for the open pit mining option, simultaneous open pit and underground mining option with a crown pillar and underground mining option with a crown pillar on case study 2 are shown in

Table 3. The copper deposit is best exploited with simultaneous open pit and underground mining option for an operational mine life of 20 years since the generated NPV of $ 102.02 billion is greater than that determined for the other mining options.

From the resource depletion point of view, the investor, mine operator and government regulations have broader understanding to the resource exploitation from the implementation of each mining option. Although simultaneous open pit and underground mining option (OP-UG) yields the maximum discounted revenue, 86.7% of the resource will be depleted as compared to the other mining options. The remaining 13.3% of the available mineral resource per the OP-UG mining option remains in the crown pillar as unmined.

The yearly ore production schedule for the open pit mining (OP), underground mining, and simultaneous open pit and underground mining with a crown pillar option for case study 1 are respectively shown in Fig. 2, Fig. 3 and Fig. 4. The ore production schedules show that the ore

target is attained in the initial years of the mine life before dropping to a stable schedule for the OP mining option (Fig. 2) but fluctuates in the case of the UG (Fig. 3) and OP-UG (Fig. 4) throughout the mine life. The ore fluctuating scenarios are usually impractical and must be controlled to ensure a steady cashflow of the mining project. A practical ore production schedule demonstrates a gradual rise in production tonnages at the beginning of the mine life and a gradual fall in the later years of the life of the mine. The practicality of the ore production is an integral part of the improvement phase of these models.

Table 2. Economic, mining and processing data for evaluating the copper deposit.

No.	Description	Value
1	Open pit mining cost ($/t)	8.0
2	Underground mining cost ($/t)	300.0
3	Processing cost ($/t)	15.0
4	Selling cost ($/lb)	1.5
5	Selling price of copper ($/lb)	3.5
6	Discount rate (%)	10.0
7	Mining recovery (%)	100.0
8	Mining dilution	0.0
9	Max open pit (OP) processing capacity (Mt/year)	4.6
10	Min open pit (OP) processing capacity (Mt/year)	3.1
11	Max open pit (OP) mining capacity (Mt/year)	45.0
12	Min open pit (OP) mining capacity (Mt/year)	30.0
13	Max underground (UG) processing capacity (Mt/year)	3.5
14	Min underground (UG) processing capacity (Mt/year)	1.2
15	Max underground (UG) mining capacity (Mt/year)	45.0
16	Min underground (UG) mining capacity (Mt/year)	0.0
17	Max open pit & underground (OP-UG) processing capacity (Mt/year)	6.0
18	Min open pit & underground (OP-UG) processing capacity (Mt/year)	1.0
19	Max open pit & underground (OP-UG) mining capacity (Mt/year)	45.0
20	Min open pit & underground (OP-UG) mining capacity (Mt/year)	0.0
21	Incremental bench cost ($/7.5 m)	2.0
22	Average mine life (years)	20.0

Table 3. Computed results of the integrated MILP model for case study 1.

No.	Description	Discounted Revenue ($)	Ore Tonnage (Mt)	Rock Tonnage (Mt)	Resource Depletion (%)
1	Open pit mining ($ bn)	99.59	69.32	600.00	100.0
2	Underground mining ($ bn)	43.99	29.09	37.80	42.0
3	Open pit and underground mining (Simultaneous) ($ bn)	102.02	60.10	78.30	86.7

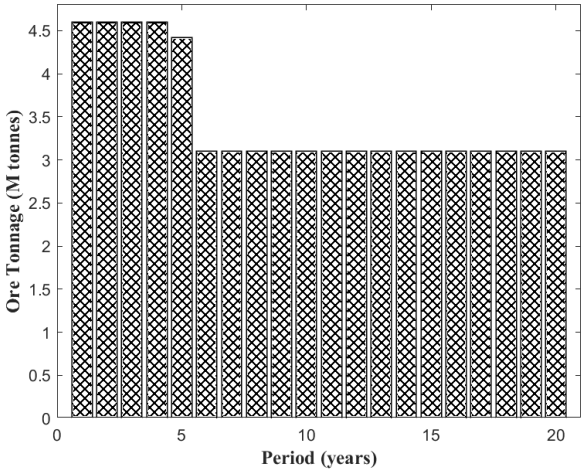


Fig. 2. Yearly ore production schedule for the open pit mining option in case study 1.

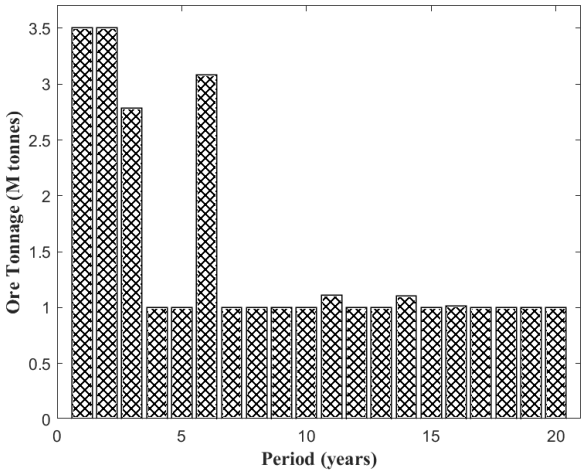


Fig. 3. Yearly ore production schedule for the underground mining option in case study 1.

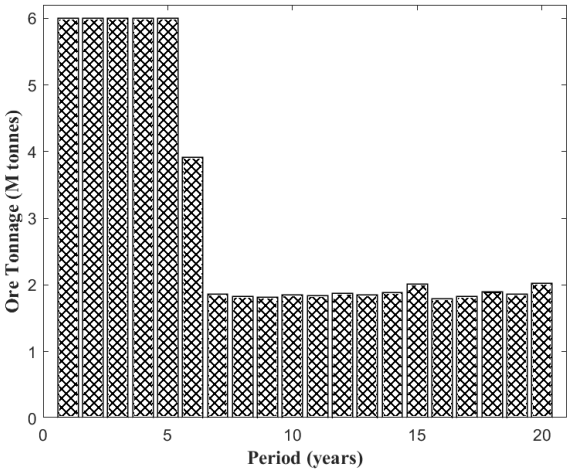


Fig. 4. Yearly ore production schedule for the simultaneous open pit and underground mining option in case study 1.

In the case of the simultaneous open pit and underground mining option (OP-UG) with a required crown pillar, Model 3; when the open pit mining cost is $ 8.0 per tonne and the underground mining cost is $ 300 per tonne, level 2 acts as the crown pillar and the NPV of the mining project is $ 102.02 billion. A representation of the location and size of the crown pillar for exploiting the copper deposit is shown in Fig. 5.

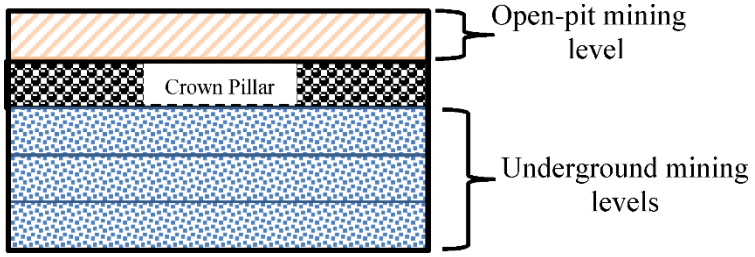


Fig. 5. Representation of the location of the crown pillar for the gold deposit in case study 1.

5.2. Case study 2 – dataset on gold deposit

The strategic Mixed Integer Linear Programming (MILP) models were implemented and tested on the gold deposit. The gold deposit consists of 420 mining blocks representing the Selective Mining Units (SMUs), referred to as mining-cuts in open pit mining and stopes in underground mining. The gold deposit is represented in a geological block model, which is a 3D array of cubical blocks containing 6,720-unit blocks of sizes 15 x 15 x 5. These unit blocks were re-blocked into the 420 SMUs of sizes 30 x 30 x 20 using GEOVIA GEMS 6.8 (Dassault, 2018). The orebody in the block model is irregularly shaped with a total mineral resource of 25.7 Mt of ore with an average Au grade of 0.7689 g/t. Table 4 is a description of the gold deposit used to implement the models.

Table 4. Statistical description of the gold deposit.

No.	Description	Value
1	Total material (Mt)	860.24
2	Total mineralized material (Mt)	25.70
3	Minimum value of Au (g/t)	0.7187
4	Maximum value of Au (g/t)	0.8765
5	Average value of Au (g/t)	0.7659
6	Variance	0.0006
7	Standard deviation	0.0252

5.2.1. Economic and mining data for case study 2

Taylor's mine life rule was used to estimate the mine life between 11.7 to 17.6 years. A mine life of 16 years was used to assess three mining options. Based on the quantity of mineralized material present in the deposit, the yearly mining and processing capacities were determined. With knowledge of a similar gold mining company in Canada (Centerra Gold Inc. and Premier Gold Mines Limited, 2016), an incremental bench cost of $ 2.0 per 10 m was used as the open pit mining variable cost as the pit transcends downwards. The economic, mining and processing data used for evaluating the gold deposit is shown in Table 5.

Table 5. Economic, mining and processing data for evaluating the gold deposit.

No.	Parameter	Values
1	Open pit mining cost ($/t)	5.51
2	Underground mining cost ($/t)	275.5
3	Processing cost ($/t)	8.93
4	Selling cost ($/oz)	48.0
5	Selling price of gold ($/oz)	1500
6	Discount rate (%)	10.0
7	Mining recovery (%)	95.0
8	Mining dilution	0.0
9	Max open pit (OP) processing capacity (Mt/year)	2.0
10	Min open pit (OP) processing capacity (Mt/year)	1.5
11	Max open pit (OP) mining capacity (Mt/year)	55.0
12	Min open pit (OP) mining capacity (Mt/year)	10.0
13	Max underground (UG) processing capacity (Mt/year)	1.40
14	Min underground (UG) processing capacity (Mt/year)	0.80
15	Max open pit & underground (OP-UG) processing capacity (Mt/year)	2.20
16	Min open pit & underground (OP-UG) processing capacity (Mt/year)	0.80
17	Max underground (UG) mining capacity (Mt/year)	50
18	Min underground (UG) mining capacity (Mt/year)	0.0
19	Incremental bench cost ($/10 m)	2.0
20	Average mine life (years)	16

5.2.2. Results and discussions for case study 2

The Net Present Value (NPV) of the three mining options were evaluated and the best mining option selected for the gold deposit. The results of the integrated MILP models for the open pit mining option, simultaneous open pit and underground mining option with a crown pillar and underground mining option with a crown pillar on case study 2 are shown in Table 6. The gold deposit is best exploited with open pit mining option for an operational mine life of 16 years since the generated NPV of $ 460.46 billion is greater than that determined for the other mining options.

Table 6. Computed results of the integrated MILP model for case study 2.

No.	Description	Discounted Revenue ($)	Ore Tonnage (Mt)	Rock Tonnage (Mt)	Resource Depletion (%)
1	Open pit mining ($ bn)	460.46	25.70	740.68	100.0
2	Underground mining ($ bn)	347.62	22.23	392.05	86.5
3	Open pit and underground mining (Simultaneous) ($ bn)	453.67	24.53	481.82	95.4

From the resource depletion point of view, the investor, mine operator and government regulations have broader understanding to the resource exploitation from the implementation of each mining option. The open pit mining option with the highest NPV depletes all the available mineral resource while the OP-UG mining option depletes 95.4% of the available mineral resource. The

remaining 4.6% of the available mineral resource per the OP-UG mining option remains in the crown pillar as unmined.

The yearly ore production schedule for the open pit mining (OP), underground mining (UG), and simultaneous open pit and underground mining (OP-UG) with a crown pillar option are respectively shown in Fig. 6, Fig. 7 and Fig. 8. The ore production schedule for the OP and OP-UG (Fig. 6 and Fig. 8) show a quick rise in the ore production at the initial years of the mine life but falls through the middle sections of the mine life towards the end. This indicates the readily available ore characterized by open pit mining option as opposed to underground mining option shown in Fig. 7. The cashflow is however steady for the UG mining option (Fig. 7) compared to the OP (Fig. 6) and OP-UG (Fig. 7) mining options. The huge drop in the ore production at the initial years compared to the later years of the mine life needs to be controlled to reflect practical mining operations.

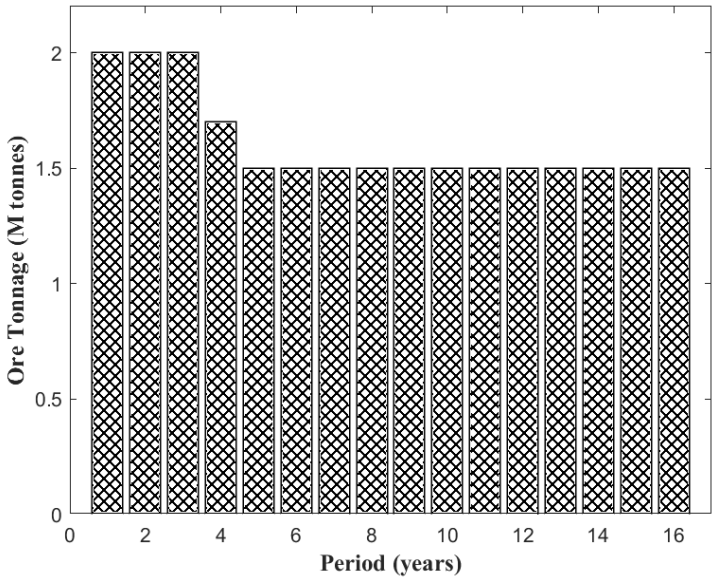


Fig. 6. Yearly ore production schedule for the open pit mining option in case study 2.

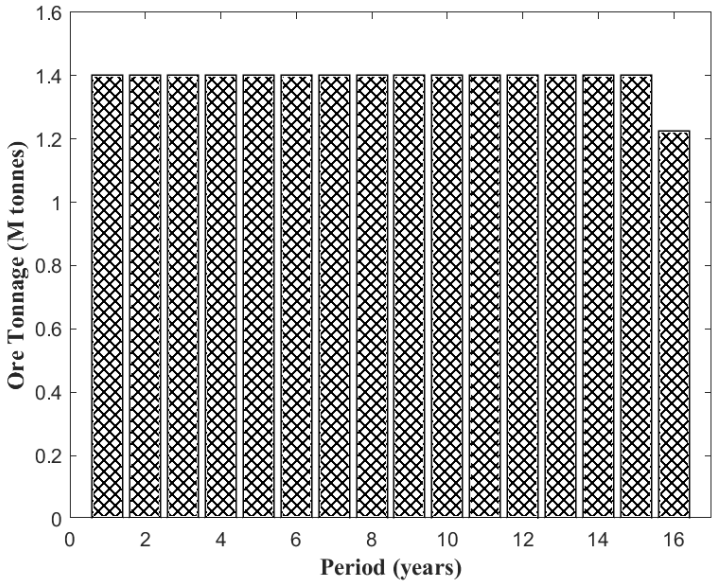


Fig. 7. Yearly ore production schedule for the underground mining option in case study 2.

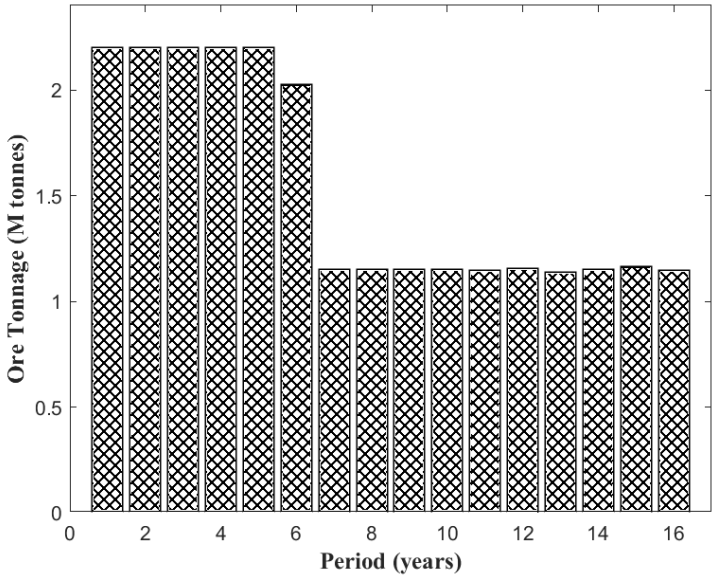


Fig. 8. Yearly ore production schedule for the simultaneous open pit and underground mining option in case study 2.

For the simultaneous open pit and underground mining option (OP-UG) with a required crown pillar, Model 3; when the open pit mining cost is $ 5.51 per tonne and the underground mining cost is $ 275.5 per tonne, level 3 acts as the crown pillar and the NPV of the mining project is $ 453.67 billion. A representation of the location and size of the required crown pillar for the exploitation of the deposit is shown in Fig. 9.

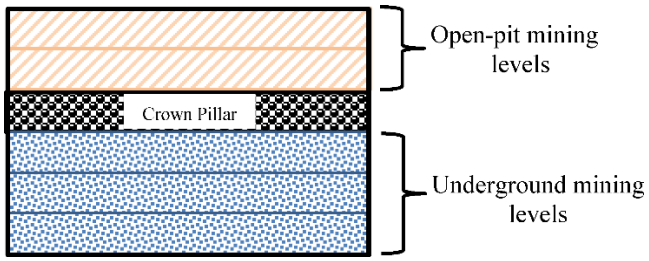


Fig. 9. Representation of the location of the crown pillar for the gold deposit in case study 2.

6. Conclusions and Future Works

In this paper, three Mixed Integer Linear Programming (MILP) models have been developed, implemented, and tested on two datasets. The MILP models produce a deterministic open pit production scheduling (Model 1), underground production scheduling with a required crown pillar (Model 2) and simultaneous open pit and underground mining production scheduling with a required crown pillar (Model 3).

For Models 2 and 3, the block model is organized by determining the location of the underground operating developments and the extraction sequence. The models are applicable to any preferred direction of mineral extraction sequence. All the models require that the cost of open pit is defined per depth (m) of ore extraction, thus, the cost of open pit mining increases with depth. This concept allows the optimization to decide when to stop the open pit mining, introduce a crown pillar and initiates underground mining.

Two novel models have been developed to interrogate a deposit and decide whether the deposit should be extracted by open pit mining or underground mining with a crown pillar or simultaneous open pit and underground mining with a crown pillar. For the underground mining and simultaneous open pit and underground mining options, the location of the 3D crown pillar is decided by the optimization process and not by human interferences.

Further research work is focused on controlling the respective gradual rise and fall in achieving ore production targets at the initial and final years of the mine life to ensure a steady cashflow of the mining project. Similarly, it is necessary to integrate prescheduled underground developments to Models 2 and 3 as required for exploiting the ore to ensure the discounted costs and revenues depicts the practicalities of the mining industry. The models will further be extended from its current deterministic approach to a stochastic mathematical programming domain to address the geological uncertainty of the deposit and the risk associated with the mining project.

7. References

[1] Askari-Nasab, H., Pourrahimian, Y., Ben-Awuah, E., and Kalantari, S. (2011). Mixed Integer Linear Programming Formulations for Open Pit Production Scheduling. *Journal of Mining Science*, *47* (3), 338-359.

[2] Bakhtavar, E., Shahriar, K., and Mirhassani, A. (2012). Optimization of the transition from open-pit to underground operation in combined mining using (0–1) integer programming. *J South Afr Inst Min Metall 112* (12), 1059-64.

[3] Bakhtavar, E., Shahriar, K., and Oraee, K. (2009). Transition from open-pit to underground as a new optimization challenge in mining engineering. *Journal of Mining Science*, *45* (5), 485-494.

[4] Ben-Awuah, E., Otto, R., Tarrant, E., and Pourrahimian, Y. (2016). Strategic mining options optimization: Open pit mining underground mining or both. *International Journal of Mining Science and Technology*, *26* 1065-1071.

[5] Centerra Gold Inc. and Premier Gold Mines Limited. (2016). Centerra Gold and Premier Gold Announce Feasibility Study Results on the Hardrock Project. J. W. Pearson and M. Gollat, Eds.: CenterraGold and Premier Gold Mines Limited, pp. 14.

[6] Dagdelen, K. and Traore, I. (2014). *Open pit transition depth determination through global analysis of open pit and underground mine scheduling*. in Proceedings of Proceedings of Orebody Modelling and Strategic Mine Planning, Australasian Institute of Mining and Metallurgy, Perth, Australia, pp. 195-200.

[7] De Carli, C. and de Lemos, P. R. (2015). Project optimization. *REM: R. Esc. Minas*, *68* (1), 97-102.

[8] ILOG, I. (2015). CPLEX reference manual and software. Ver. 12.6.3.0, Pullman, WA, USA.

[9] King, B., Goycoolea, M., and Newman, A. (2016). Optimizing the open pit-to-underground mining transition. *European Journal of Operational Research*, *257* 297-309.

[10] Kurppa, R. and Erkkilä, E. (1967). *Changing from open pit to underground mining at Pyhasalmi*. in Proceedings of Rock Mechanics Symposium, Helsinki, pp. 239-247.

[11] Lerchs, H. and Grossman, I. F. (1965). Optimum design of open-pit mines. *The Canadian Mining and Metallurgical Bulletin*, *58* 47-54.

[12] Luxford, J. (1997). Surface to Underground-making the transition, in Proceedings of the International Conference on Mine Project Development. in *The Australasian Institute of Mining and Metallurgy*, E. Barnes, Ed. Sydney: AusIMM, pp. 79-87.

[13] MacNeil, J. A. L. and Dimitrakopoulos, R. (2017). A stochastic optimization formulation for the transition from open pit to underground mining. *Springer*, 21.

[14] Mathworks, I. (2018). MATLAB Software Ver. 9.4.0.813654 (R2018a).

[15] Opoku, S. and Musingwini, C. (2013). Stochastic modelling of the open pit to underground transition interface for gold mines. *International Journal of Mining, Reclamation and Environment*, *27* (6), 407-424.

[16] Ordin, A. A. and Vasil'ev, I. V. (2014). Optimized depth of transition from open pit to underground coal mining. *J Min Sci*, *50* (4), 696-706.

[17] Roberts, B., Elkington, T., van Olden, K., and Maulen, M. (2013). Optimising combined open pit and underground strategic plan. in *Mining Technology*, vol. 122: Maney on behalf of the Institute and The AusIMM, pp. 94-100.

[18] Whittle, D., Brazil, M., Grossman, P. A., Rubinstein, J. H., and Thomas, D. A. (2018). Combined optimisation of an open-pit mine outline and the transition depth to underground mining. *European Journal of Operational Research*, *000* 1-11.

Application of Mathematical Modelling for Draw Control under Material Flow Uncertainty

Firouz Khodayari and Yashar Pourrahimian
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

Production scheduling is one of the key steps in the decision-making process of any mining operations; in block caving, it is the choice in the amount of caved rock to extract from drawpoints in different periods. One of the main differences between block caving and other mining methods is the influence of the material flow on production, and draw control in general. Achieving an optimum production schedule without consideration of the cave rate and material flow could be unrealistic and impractical as the movements of material between drawpoints will result in unexpected production grades and tonnages. In this paper, a stochastic mixed-integer optimization model is proposed to optimize the production schedule during the life of the mine. The uncertainties of production grades and tonnages are captured by defining a number of scenarios that represent the probable movements of fragmented rock between drawpoints in the same neighborhood. The decision variables in the formulation are based on the slice model, means that the mathematical solution determines which slices are extracted from drawpoints in each period of production. The goal is to maximize the net present value of the project during the life of the mine and minimize the deviations of production grades and tonnages from the defined goals in all probable scenarios resulting from the movements of the fragmented rock between drawpoints. Application of the proposed model in caving operations not only can improve the profitability of the project but also increase the confidence of the production schedule. MATLAB was used for programming and CPLEX for solving the model. The designed graphical user interface, with the capability of adding different technical and operational constraints, will be a flexible tool for mine planners to control the draw based on the company's goals during the life of the mine.

1. Introduction

As the number of block cave mines in the world is increasing, the challenges of this mining method are the subject of increased investigation and investment. Draw control plays a critical role both for operational and financial purposes; good draw control can improve safety outcomes, result in improved fragmentation, which results in fewer hang-ups, all of which result in improved efficiencies at the processing plant. These will result in improved financial outcomes for the owners. Mathematical modelling can be used to optimize the production from drawpoints while considering operational constraints. As a result, many researchers have used mathematical models to optimize the production of block cave mining. Rubio (2002) maximized the net present value (NPV) of the project and the life of the mine; a similar goal was achieved by Diering (2004) using a linear programming model, he later applied quadratic programming to control the shape of the cave. Rahal et al. (2008) minimized the deviations from production targets using a mixed integer linear programming model. In other research, the production schedule of Northparkes E48 mine was optimized by Rahal et al. (2008).

This paper, in its entirety, has been accepted to be published in the proceedings of 4th International Symposium on Block and Sublevel Caving (Caving 2018), Vancouver, Canada, Oct 15-17, 2018.

Optimization of production schedule at three different levels of resolution (cluster level, drawpoint level, and drawpoint-and-slice level) was first introduced by Pourrahimian et al. (2013). This model made it possible to spend a reasonable solution time for investigating the financial viability of block cave operations based on the level of required precision. Khodayari and Pourrahimian (2014) included the calculation of best height of draw (BHOD) in the production scheduling optimization model to be an output of the mine plan and not an input. A more detailed review of the mathematical models and their applications in block cave mining production scheduling was presented by Khodayari and Pourrahimian (2015b).

Almost none of the mentioned mathematical models have considered material flow uncertainties as part of the optimization models. A production scheduling model was proposed by Khodayari and Pourrahimian (2017) to maximize the NPV of the project and minimize the production-grade deviations. They considered only horizontal movements of the fragmented rock during the caving operations. It was shown that stochastic optimization can be a strong tool for the purpose of material movement modelling. The uncertainties of material flow can change the outputs of the production in a block cave mining operation; unlike open pit mining, the production grades and tonnages can vary from the expected values of the mine plan. In such a situation, any strategic decision should be made with the consideration of movements of the fragmented rock within the cave. Since the processing plant relies on the mine production, the production tonnages and grades should be realistically considered for the draw control purposes during the life of the mine. Stochastic optimization can play a critical role to model the material movements and its uncertainties during the production. In this paper, a draw control strategy for block cave mining is proposed in which the material flow and its uncertainties are modelled within the mine plan. In the following section, the concept of the proposed methodology is presented. Because of the importance of the concept, this paper focuses on explaining the underlying logic and theory behind the model rather than the details of the explicit mathematical formulation.

2. Methodology

In order to capture the uncertainties of material flow during the production, it is important to consider the possible movements of the fragmented rock as it moves down into the drawpoints, this could be done by defining a number of scenarios and considering them in the production scheduling optimization model. Based on the full-scale marker experiments that Alvial (1992, as cited in Laubscher (2000)) carried out at El Teniente block caving mine, the horizontal displacement of the markers can be up to 42 meters within a slip angle of 60 degrees. In this paper, these parameters are used to define the Cone of Movement (CoM): the cone in which the fragmented rock can possibly move within its neighborhood (Fig 1).

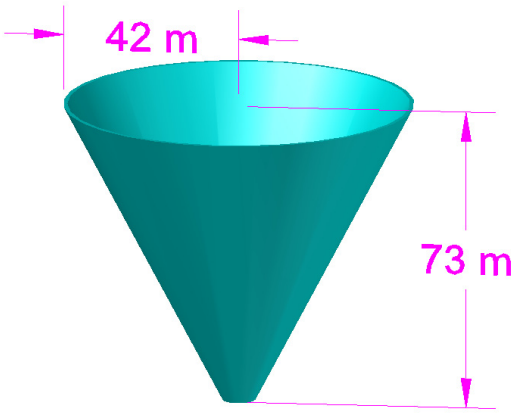


Fig 1. The CoM dimensions based on the experimental results

For each drawpoint in each slice horizon (elevation of the bottom of the slice), a CoM is defined to be used for the model. While extracting the material, the fragmented rock can move from the draw column above or the other ones in the neighborhood into the drawpoint (Fig 2).

The slices that fall inside the CoM are considered to define the scenarios for that specific drawpoint in that horizon.

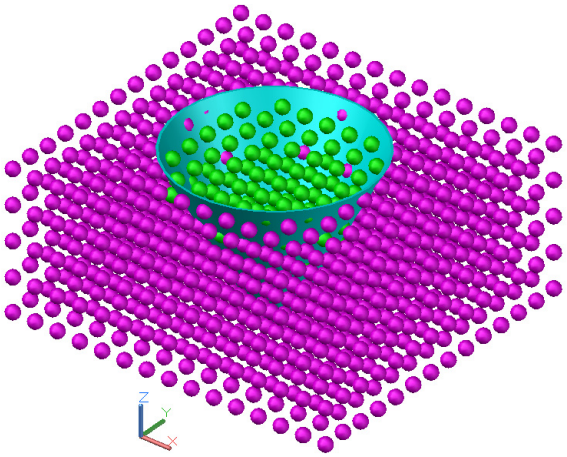


Fig 2. Defining the boundary in which the slices in the neighborhood can fall into a specific drawpoint at the time of extraction, each ball represents one slice

Fig 3 presents the cones in the same neighborhood and how they can overlap based on the distance of the drawpoints. The shape of the CoM and number of scenarios depend on the nature of the orebody, geometry of the drawpoints and how they are located in the layout compared to each other, and the rock properties. Each scenario represents one possible set of movements that can occur during the extraction; using this concept, both vertical and horizontal movements are considered in the optimization model. The goal is to maximize the NPV of the project with consideration of material movements (horizontal and vertical) and to achieve the production targets with respect to the defined mining and geotechnical constraints while meeting the processing plant's metallurgical requirements. A simplified description of the model is as follows:

$$\text{for all defined scenarios} \rightarrow \left\{ \begin{array}{c} \textit{Maximise}\ \ \text{NPV} \\ \& \\ \text{Minimise Ore and Grade Deviations} \end{array} \right\}$$

$$\rightarrow \text{while considering material movements, production targets are achieved}$$

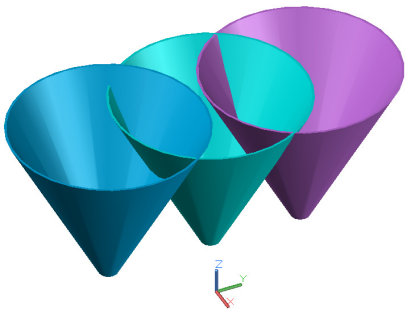


Fig 3. Different CoMs in the same neighborhood and their overlap

3. Application and results

The optimization model with the proposed mixing methodology was tested for a copper block cave mining operation that isbased on 410 drawpoints (Fig 4). The mathematical formulation, including the objective function and constraints, is a modified version of the model that was presented by Khodayari and Pourrahimian (2017). The updated version considers both horizontal and vertical mixing rather than only horizontal mixing that the previous model was

based on. Additionally, it minimizes the deviations of production tonnage and grade while only grade deviations were minimized in the last version. The number of slices in the drawpoints ranges from 16 to 60 with the height of 15 meters, Fig 5 and Fig 6 show the grade and tonnage distributions among the slices.

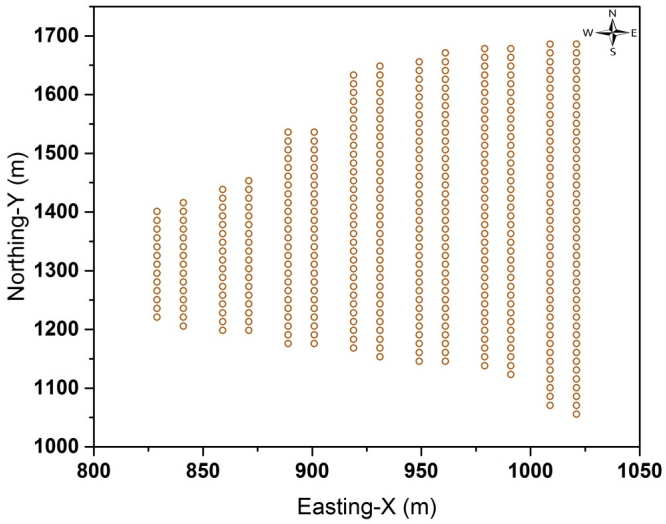


Fig 4. Layout of drawpoints

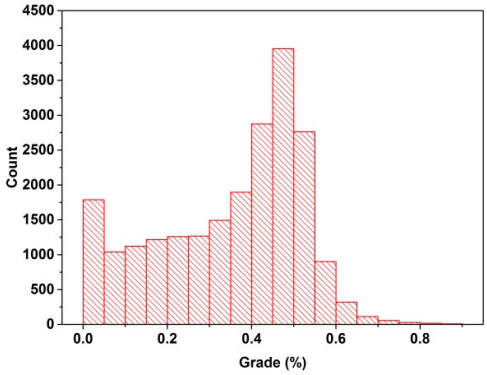


Fig 5. Grade distribution among the slices

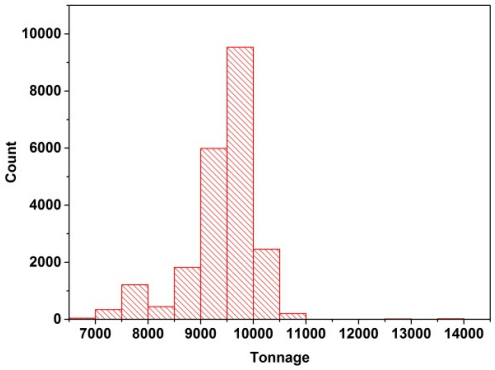


Fig 6. Tonnage distribution among the slices

The best mining direction in the block cave layout is defined based on the methodology that was proposed by Khodayari and Pourrahimian (2015a). Fig 7 shows the optimum mining direction for achieving maximum profit during the life of the mine. This direction is used as a guideline to develop the sequence of extraction among the drawpoints within the layout.

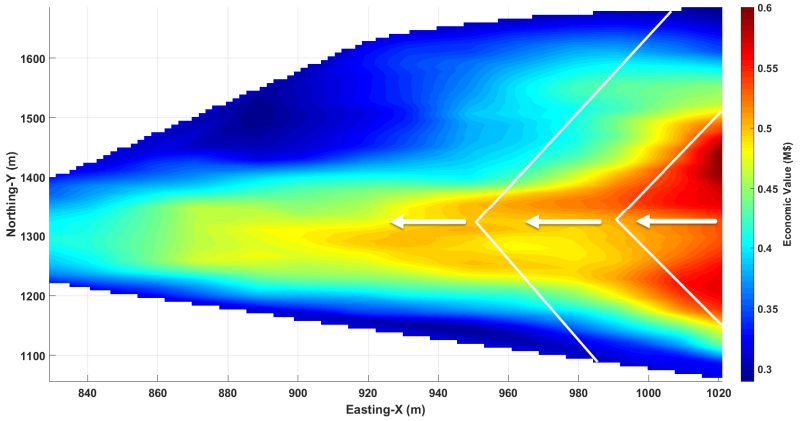


Fig 7. The best potential mining direction in the block cave layout

The goal is to achieve both production tonnages and grades while considering material flow uncertainties. Production tonnages, production grades, the number of active drawpoints each period, and their sequence of extraction are the main constraints to be considered for this example. Running the model for this case study indicated that 127 million tonnes of ore is produced with an NPV of 279.8 million dollars over 20 years of operations; in addition, the production tonnages and grades are to the target values. The production starts with 3 million tonnes in the first year and increases to 7 million tonnes within a 3-year ramp-up period (Fig 8).

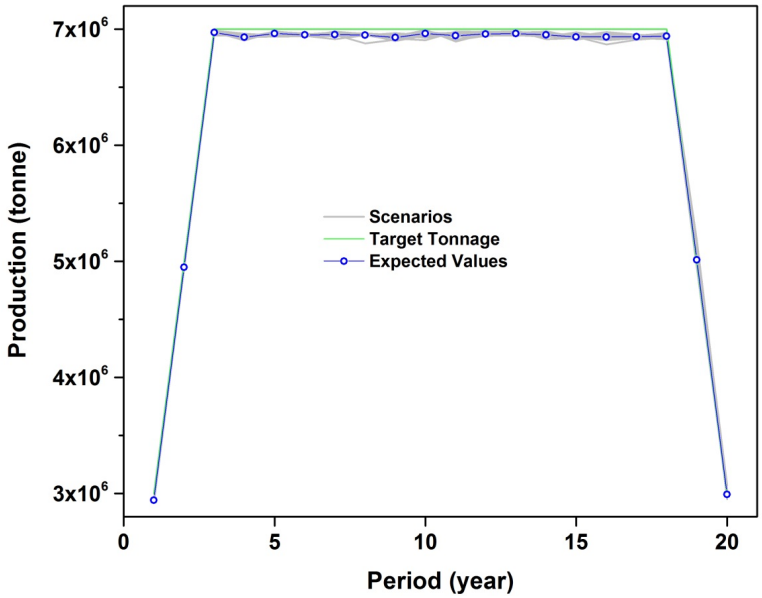


Fig 8. Ore production during the life of the mine

Fig 9 shows the production-grade profile which deviates from the target grade of 0.45% of Cu. In addition, the expected sequence of extraction based on the V-shape mining direction is achieved (Fig 10). The sequence of extraction and NPV maximization explain the higher and lower production grades at the first and last few years of the mine life.

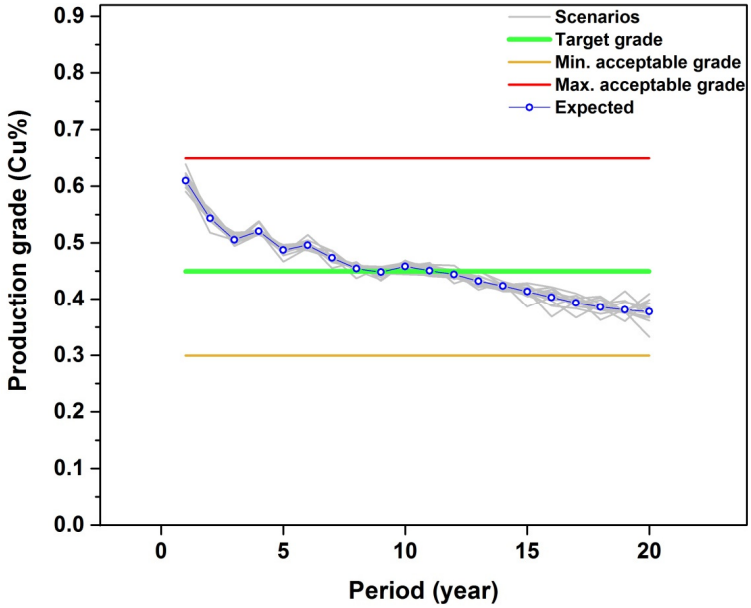


Fig 9. Production grade during the life of the mine

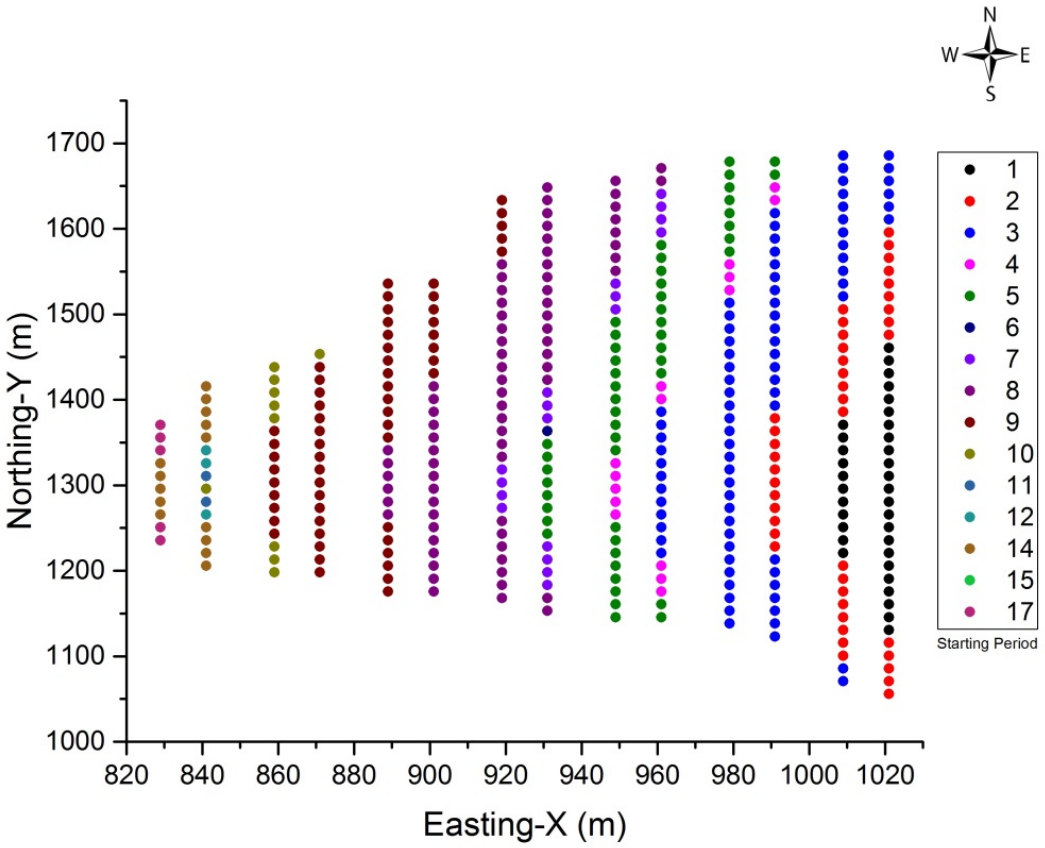


Fig 10. Sequence of extraction for drawpoints

4. Conclusion

In this paper, a methodology was proposed to optimize the production schedule in block-cave mining operations in presence of material flow uncertainties. Defining the CoM helps to understand the material movements within the cave area during the production, as a modelling tool, it facilitates the process of generating different possible mixing scenarios. Using this concept in the optimization model can lead us to improved production schedules and as a result higher efficiencies. Testing the model for a block cave mining project showed that the NPV of the project during the life of mine is maximized and the production and grade targets are achieved with respect to the defined constraints. Each scenario represented a series of movements and the model proposed a schedule in which the deviations from the target production tonnages and grades are minimized for all scenarios. Production targets are closely related to the mineral resource, operational limitations, and economic considerations. Based on the preferences of the management team, the model can be adapted for achieving satisfactory results.

5. References

[1] Diering, T. (2004). Computational considerations for production scheduling of block cave mines. *Paper presented at the MassMin 2004*, Santiago, Chile, Aug 22-25.

[2] Khodayari, F., & Pourrahimian, Y. (2014). Determination of the best height of draw in block cave sequence optimization. *Paper presented at the 3rd International Symposium on Block and Sublevel caving (CAVING 2014)*, Santiago, Chile, Jun 5-6.

[3] Khodayari, F., & Pourrahimian, Y. (2015a). Determination of development precedence for drawpoints in block-cave mining. *Paper presented at the 5th International Symposium Mineral Resources and Mine Development (AIMS 2015)*, Aachen, Germany, May 27-28.

[4] Khodayari, F., & Pourrahimian, Y. (2015b). Mathematical programming applications in block-caving scheduling: a review of models and algorithms. *International Journal of Mining and Mineral Engineering (IJMME)*, 6(3), 234-257.

[5] Khodayari, F., & Pourrahimian, Y. (2017). Production scheduling in block caving with consideration of material flow. *Aspects in Mining and Mineral Science (AMMS)*, 1(1).

[6] Laubscher, D.A. (2000). A practical manual on block caving. Prepared for International Caving Study (1997-2000), 525 pages.

[7] Pourrahimian, Y., Askari-Nasab, H., & Tannant, D. (2013). A multi-step approach for block-cave production scheduling optimization. *International Journal of Mining Science and Technology*, 23(5), 739-750.

[8] Rahal, D., Dudley, J., & Hout, G.v. (2008). Developing an optimized production forecast at Northparkes E48 mine using MILP. *Paper presented at the 5th International Conference and Exhibition on Mass Mining*, Luleå, Sweden, Jun 9-11.

[9] Rubio, E. (2002). Long term planning of block caving operations using mathematical programming tools. M.Sc. Thesis, The University of British Columbia.

Production Scheduling with Horizontal Mixing Simulation in Block-Cave Mining

Firouz Khodayari and Yashar Pourrahimian
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

High production rates and low operating costs highlight block caving as one of the favorable underground mining methods. However, the uncertainties involved in the material flow make it complicated to optimize the production schedule for such operations. In this paper, a stochastic mixed integer linear optimization model is proposed in order to capture horizontal mixing that occurs among the draw columns within the production scheduling optimization. The goal is to not only consider the material above each drawpoint for extraction from the same drawpoint, as traditional production scheduling does, but also to capture the horizontal movements among the adjacent draw columns. In this approach, different scenarios are generated to simulate the horizontal mixing among adjacent slices within a neighborhood radius. The best height of draw for draw columns is also calculated as part of the optimization. The model is tested for a block-cave mine with 640 drawpoints to feed a processing plant for 15 years. The resulting NPV is 473M$ while the deviations from the targets in all scenarios during the life of the mine are minimized. Using the proposed model will result in more reliable mine plans as it takes the horizontal mixing into account in addition to achieving the production goals. Using different penalties for grade deviations shows that the model is a flexible tool in which the mine planners can achieve their goals based on their priorities.

1. Introduction

Gravity is the main driver in block-cave mining: an undercut is developed beneath the orebody, the rock fractures because of the created empty space, the caved material is extracted using the designed drawpoints, and finally, the ore is transported to the surface for processing (Fig 1). For scheduling purposes, the resulting block model from the resource estimation is used to create a slice model in which each slice is an aggregation of several blocks (Fig 2). Likewise the blocks, each slice is represented by its grade, density, volume, and a decision variable (for the mathematical modeling). Production scheduling for this type of operation determines the mining direction in the layout as well as the amount of material to extract from each of the drawpoints in each period.

Similar to open pit mining, production scheduling has significant impacts on the feasibility of the project as it directly controls the cash flow. However, in block caving, because of the uncertainties involved in the flow of the caved material, the material movement influences the production (Fig 3). As a result, the production scheduling optimization is more complicated, and an optimum mine plan without consideration of the material flow can be impractical.

In this paper, a stochastic model is presented that aims to capture some of the uncertainties involved in material flow to achieve production schedules that can be applied in real operations. To do this, the Height of Interaction Zone (HIZ) is used to consider the material flow within the optimization model (Fig 3). The slices that fall into the HIZ are directly extracted from their associated drawpoint which means no horizontal mixing occurs. Different scenarios are generated to simulate the potential horizontal movements of the material within an adjacency radius from each slice. The aim is to achieve production targets and maximize the net present value (NPV) of the project as well as consider the horizontal mixing between draw columns as the caved rock is extracted from the

This paper, in its entirety, has been submitted for publication in the international Journal of Mining Science

drawpoints. Integer and continuous decision variables in the stochastic mixed-integer optimization model (SMIOM) represent slices, drawpoints, and deviations. In addition, the best height of draw (BHOD) is calculated as part of the optimization, which means that this part of optimization is not separated from the production scheduling.

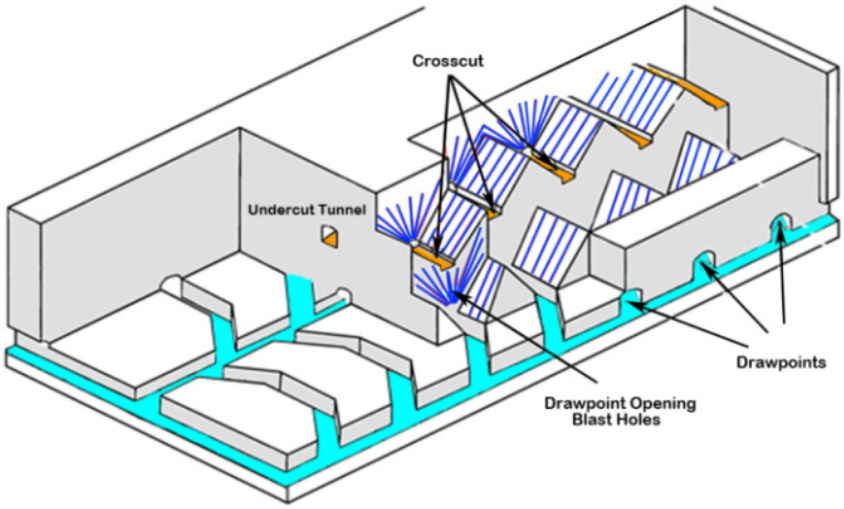


Fig 1. Block-cave mining

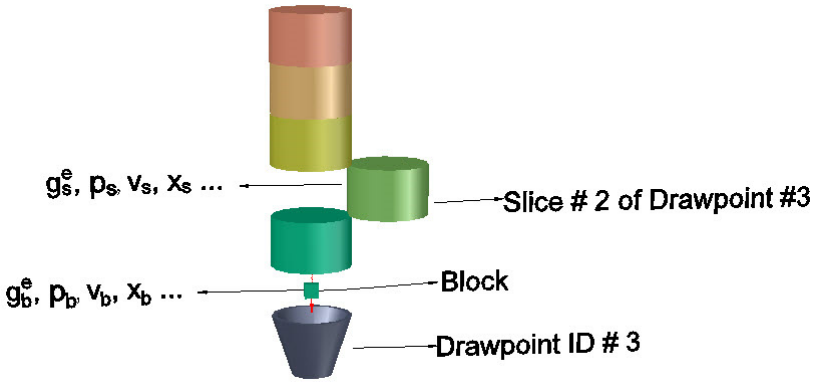


Fig 2. Slice model

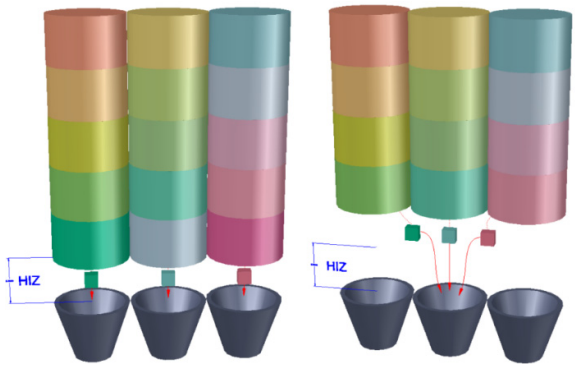


Fig 3. Horizontal mixing and its impact on production: below HIZ (left) and above HIZ (right)

2. Literature review

Since the proposed model includes horizontal mixing, literature on both mine planning optimization and material flow is presented in the following sections.

2.1. Production scheduling optimization in block caving

Mine planning optimization for both open pit and underground mining has been a focus of many studies. For block-cave mining, researchers have mostly used mixed-integer linear programming (MILP); also, there are some models with application of Linear programming (LP) and quadratic programming (QP) for optimizing the production scheduling problem (Khodayari & Pourrahimian, 2015b). A summarized review of the existing research in this area is discussed in the following.

Song (1989) was one of the first researchers who used mathematical programming for block-cave operations. He used MILP for sequence optimization at the Tong Kuang Yu mine in China. Guest et al. (2000) formulated a long-term production scheduling model for a diamond mine in South Africa using LP and MILP to maximize the NPV of the project. Rubio (2002) had two strategic goals in his model: to maximize the NPV and to achieve an optimum mine life for the project. Rahal et al. (2003) developed an MILP goal program to minimize deviation from ideal draw coupled with accomplishing a production target. In their model, material-mixing impact on the long-term plan was assumed insignificant. Rahal (2008) used MILP to control monthly production and cave shape, he tried to integrate geotechnical constraints in the production schedule by draw rate constraint; it was shown that strict relative draw rates (RDR) result in more uniform cave shape but less production rates. Using an MILP model by Rahal et al. (2008) to optimize the production schedule of Northparkes E48 mine showed that mathematical programming models could generate optimum results while being practical. A mixed-integer programming model (MIP) was developed by Weintraub et al. (2008) to determine the times in which each block is extracted within the draw columns. The production schedule was based on a five-year plan. In their model, they used aggregation methods to reduce the size of the model and as a result the solution time. Smoljanovic et al. (2011) adapted a model from open pit mining to find the optimum sequence of drawpoints in block-cave layout. Testing the model for a case study with 322 drawpoints, they proved that an optimum sequence could improve the objective function up to 50%.

Three integer programming models were developed by Parkinson (2012) to optimize the sequence of extraction. Using the models for two datasets, she concluded that the proposed models could help to find the opening sequence but it was not easy to find the best one among different models. Pourrahimian et al. (2012) developed MILP models to maximize the NPV of the block caving project during the life of the mine. They also proposed three formulations to optimize the production schedule in three levels of resolution: cluster level, drawpoint level and drawpoint-and-slice level (Pourrahimian et al., 2013). The goal was to provide the mine planner with different alternatives based on their expected solution time and level of resolution. Alonso-Ayuso et al. (2014) tried to capture the uncertainty of copper price to maximize the NPV of a block-cave mining project in Chile using an MIP model. Their results showed the advantage of stochastic models in mining projects. Khodayari and Pourrahimian (2014) proposed a model in which the best height of draw (BHOD) is determined as part of the sequence optimization in block caving. In other words, the mining reserve was an output of the optimization, not an input.

Nezhadshahmohammad et al. (2017) proposed an MILP model to optimize the mine plan with consideration of draw rate curve as a geotechnical constraint. Nezhadshahmohammad et al. (2017) used a clustering model to optimize a long-term production schedule for block caving operations. They also used a draw control system within the neighborhood while maximizing the NPV of the project (Nezhadshahmohammad et al., 2017). Malaki et al. (2017) applied sequential Gaussian simulation and MILP to find the best level of extraction in block caving while considering the grade estimation uncertainties. An LP model was developed by Diering (2004) to determine the tonnages to extract from each drawpoint; his goal was to maximize the NPV. He also proposed a QP model to control the shape of the cave by minimizing the deviations between the production and target tonnages (Diering, 2012). The interaction between neighboring drawpoints was not considered in his

model. Khodayari and Pourrahimian (2016) proposed a quadratic programming model for production scheduling, the goal was to achieve a uniform extraction profile in order to reduce the potential dilutions in the production.

Among the above-mentioned models, the following were the most common constraints: allowable output per shift, number of working drawpoints per shift, grade limitations, tonnage limitations, draw rate, draw ratio, draw life, and mining precedence. In addition to what presented here, the authors of this paper have published a detailed literature review on application of mathematical programming in block-caving in (Khodayari & Pourrahimian, 2015b).

2.2. Material flow

The literature on material flow can be divided into three categories: numerical models, pilot tests, and full-scale experiments. Castro et al. (2009), as part of the development of FlowSim software, used numerical methods to simulate the gravity flow of caved rock. They calibrated the proposed model with real data from two mining operations in Chile. Another numerical model was developed by Pierce (2010), he used distinct element method to identify different zones of movement (or isolations) in the caved material. A stochastic approach was proposed by Gibson (2014) to study the material flow in block-cave operations. He used Pascal cone theory to assess the probabilities of material that move within their neighborhood when empty space is created because of extraction from drawpoints. He found it useful to use stochastic models instead of Finite Element methods although it needs a certain amount of information about the rock mass. The cell size was one of the main criteria in the Gibosn's model, and any changes can remarkably affect the results.

Castro et al. (2014) designed a test setup to study the flow mechanisms of cohesionless material if extracting from a single drawpoint. It was shown that the fragmentation size, the diameter of the opening, and the vertical load (the amount of material above) are the factors that affect the material flow. Jin et al. (2017) studied the shape of the ellipsoid draw by some pilot tests; they proposed a new methodology to predict the extraction and movement zone shapes and their change trends. Brunton et al. (2016) performed a full-scale experiment in Ridgeway Deeps and Cadia East block caving mines in Australia. They used markers to quantify and assess the development and shape of the extraction zone and find the mechanisms that control the flow behavior. It was shown that marker experiments can provide a good understanding about the development of the extraction zone both during undercutting and production stages. Garcés et al. (2016) applied smart marker technology in block 2 of Esmeralda mine, El Teniente to study the gravity flow. The main objectives were to check the interaction between extraction zones, estimate the mining recovery, evaluate the flow characteristics of caved rock, and develop a model for flow. Results showed that the extraction zone mainly depends on the fragmentation condition and the mine plan. It was also observed that the horizontal mixing is higher in the case of a non-concurrent draw.

In summary, a significant number of research exist in production scheduling optimization and material flow; however, both concepts have been investigated separately, and there is a gap of optimizing the mine plan with the material flow consideration. An optimized mine plan will provide a guide towards a successful operation, and a material flow simulation model can give us a valuable understanding of the caving process; however, these two will not be helpful enough if they are not studied simultaneously. The goal of this research is to consider the material flow uncertainties as part of the production scheduling optimization; in other words, to achieve an optimum production schedule in which not only the technical constraints are satisfied but also the horizontal mixing is modeled within the optimization.

3. Problem statement and formulation

In this section, the formulations of the proposed model are presented. The model includes technical constraints that are common among block caving operations; other project-specific constraints can be added to the model as it is needed.

3.1. Notation

- ***Indices***

$t \in \{1,...,T\}$	Index for scheduling periods
$s \in \{1,...,S\}$	Index for individual slices
$d \in \{1,...,D\}$	Index for individual drawpoints
$n \in \{1,...,N\}$	Index for individual scenarios
$a \in \{1,...,A\}$	Index for the adjacent drawpoints

- ***Variables***

$x_{s,t} \in \{0,1\}$	Binary decision variable that determines if slice *s* is extracted in period *t* ($x_{s,t} = 1$) or not ($x_{s,t} = 0$)
$y_{d,t} \in \{0,1\}$	Binary decision variable which determines whether drawpoint *d* in period *t* is active ($y_{d,t} = 1$) or not ($y_{d,t} = 0$)
$z_{d,t} \in \{0,1\}$	Binary decision variable which determines whether drawpoint *d* at period *t* has started its extraction ($z_{d,t} = 1$) or not ($z_{d,t} = 0$)
$dev^{uo}_{n,t} \in [0,\infty)$	Excessive amount from the target production ore in scenario n at period *t*
$dev^{lo}_{n,t} \in [0,\infty)$	Deficient amount from the target production ore in scenario n at period *t*
$dev^{ug}_{n,t} \in [0,\infty)$	Excessive amount from the target production grade (the metal content) in scenario *n* at period *t*
$dev^{lg}_{n,t} \in [0,\infty)$	Deficient amount from the target production grade (the metal content) in scenario *n* at period *t*

- ***Model Parameters***

g^e_s	Grade of element *e for* slice *s*
Eg^e_s	Expected grade of element *e* for slice *s* based on all scenarios
ρ_s	Density of slice *s* (t/m^3)
$E\rho_s$	Expected density of slice *s* (t/m^3) based on all scenarios
V_s	Volume of slice *s* (m^3)
t'	Current period
N_{Sd}	Number of slices associated with drawpoint *d*
d_p	Drawpoint that its extraction should be started before drawpoint d based on the defined mining direction of extraction
s_p	Slice which is located below the slice *s* and must be extracted before *s*

- ***Input Parameters***

$\hat{g}^e_t$	The target grade of production for element *e* in period *t,* which is defined based on the production goals and processing requirements
$o_{s,t}$	The ore production resulting from extraction slice *s* in period *t*
$\hat{o}_t$	The target ore production in period *t* which is defined based on the production goals and processing requirements
$\min Act_t$	Minimum number of active drawpoints in period *t*
$\max Act_t$	Maximum number of active drawpoints in period *t*

M	A big number, this is chosen based on the maximum number of slices in draw columns
DL_{min}	Minimum drawpoint life
DL_{max}	Maximum drawpoint life
DR_{min}	Minimum draw rate
DR_{max}	Maximum draw rate
i	Discount rate
$Ramp-up$	Ramp-up time
$ScenNum$	Number of scenarios
$DP_{d,t'}$	Drawpoint depletion percentage: the portion of draw column associated with drawpoint d that has been extracted from $t=1$ to t'
p^e	Price of element e ($/tonne)
c	Operating costs ($/tonne)
c^{lo}	Cost (penalty) for deficient amount of ore production from the target ($)
c^{uo}	Cost (penalty) for excessive ore production from the target ($)
c^{lg}	Cost (penalty) for deficient production grade from the target ($)
c^{ug}	Cost (penalty) for excessive production grade from the target ($)
r^e	Recovery of the processing plant for element e (%)
R	Adjacency radius

3.2. Preliminaries

The scenarios are generated based on the neighborhood concept within the caved material. For each slice, a horizontal neighborhood is defined in a radius (adjacency radius) from the center of the slice; all of the slices that fall into that neighborhood (the distance from their center to the center of slice s is less than or equal to R) are called adjacent slices for that slice (Fig 4). Therefore, a population (P_s) is created for slice s that includes slice s and all of its adjacent slices. In the next step, random samples are generated from the population to simulate different scenarios for each slice in terms of horizontal movement while extracting from each drawpoint during the operations.

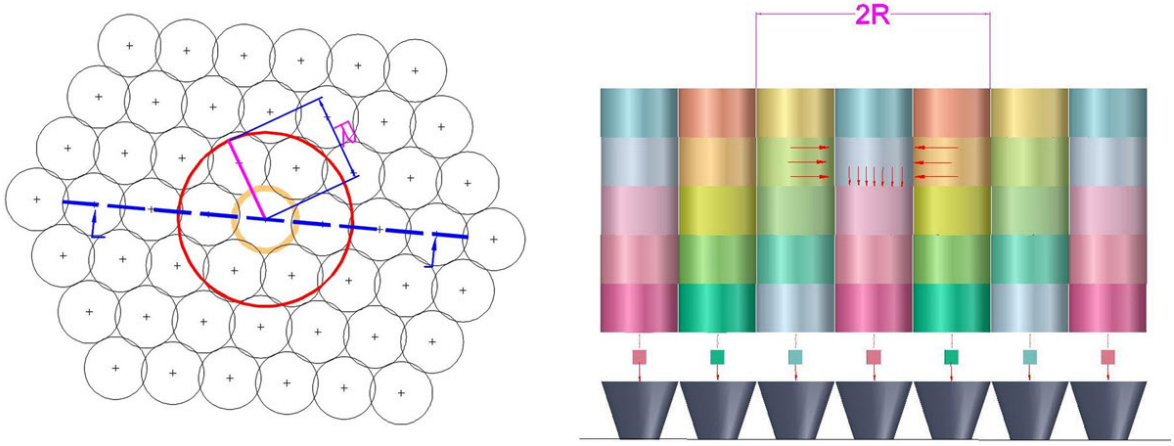


Fig 4. Adjacency concept: plot view of adjacent drawpoints (left Fig) and adjacent slices (right Fig); in the plan view, the black circles represent the drawpoints with the cross sign as their center point, the red circle is the adjacent neighborhood for the orange-colored drawpoint

In this research, only the horizontal movements are considered as the neighborhood is defined in two dimensions (2-D). The ore tonnage ($P_{s,o}$) and grade ($P_{s,g}$) populations for each slice are defined based on the adjacent slices, and then the scenarios are generated by sampling from those populations. The following equations present the process:

$$P_{s,o}=\{o_1,...,o_A\} \qquad s\in\{1,...,S\} \tag{1}$$

$$o_{s,n}=\text{sample}(P_{s,o}) \qquad \forall n\in\{1,...,N\},\ s\in\{1,...,S\} \tag{2}$$

$$P_{s,g}=\{g_1,...,g_A\} \qquad s\in\{1,...,S\} \tag{3}$$

$$g_{s,n}=\text{sample}(P_{s,g}) \qquad \forall n\in\{1,...,N\},\ s\in\{1,...,S\} \tag{4}$$

As an example, for slice *s* with an original grade of 0.51% and considering the closest 19 slices that are located in its adjacency (based on the adjacency radius), the population is defined as a set of the original grades of all adjacent slices and slice s itself (Table 1). In the next step, the scenarios, in this case 15 scenarios, are generated (Table 1).

As shown in Fig 3, this model assumes that in each draw column, the material that is located in the interaction zone is extracted from the same drawpoint (the drawpoint associated with that draw column); in other words, horizontal mixing will not occur for the slices that are located within the HIZ. HIZ can be calculated based on the curve that was presented by Laubscher (1994).

Table 1. An example of creating a population within a slice's neighborhood and Generating 15 scenarios

Population ID	Grade (%)	Scenarios	Grade (%)
1	0.51	1	0.45
2	0.71	2	0.51
3	0.61	3	0.64
4	0.71	4	0.51
5	0.6	5	0.71
6	0.64	6	0.44
7	0.56	7	0.47
8	0.41	8	0.56
9	0.49	9	0.43
10	0.41	10	0.51
11	0.46	11	0.49
12	0.44	12	0.41
13	0.45	13	0.49
14	0.47	14	0.49
15	0.45	15	0.71
16	0.49		
17	0.44		
18	0.49		
19	0.44		
20	0.43		

3.3. Objective function

The objective function is defined as follows:

$$Maximize \ \sum_{t=1}^{T}\sum_{s=1}^{S} E\{(NPV_{s,t})\}\,x_{s,t} - \sum_{t=1}^{T}\sum_{n=1}^{N}\{Ore\ deviations\}_{n,t} - \sum_{t=1}^{T}\sum_{n=1}^{N}\{Grade\ deviations\}_{n,t}$$

$$= \sum_{t=1}^{T}\sum_{s=1}^{S}\left(\frac{\left(p^{e}\times r^{e}\times Eg_{s}^{e}-c\right)\times E\rho_{s}\times V_{s}}{(1+i)^{t}}\right)x_{s,t}$$
$$-\sum_{t=1}^{T}\sum_{n=1}^{N}\frac{1}{N}\left(\frac{c^{lo}\times dev_{\mathrm{n},t}^{lo}+c^{uo}\times dev_{n,t}^{uo}}{(1+i)^{t}}\right)-\sum_{t=1}^{T}\sum_{n=1}^{N}\frac{1}{N}\left(\frac{c^{\mathrm{lg}}\times dev_{\mathrm{n},t}^{\mathrm{lg}}+c^{ug}\times dev_{n,t}^{u\mathrm{g}}}{(1+i)^{t}}\right) \quad (5)$$

The first part of the equation (5) maximizes the NPV of the project; this is calculated based on the smallest units of production, which are the slices. Decision variable $x_{s,t}$ is a member of the first set of decision variables and represents slice s in period t; it is 1 if slice s is extracted in period t or zero if it is not. The second and third parts minimize the deviations of the production from target tonnages and grades, respectively (during the life of the mine). The expected values of tonnage and grade are considered for calculation of NPV and both the revenue and deviations are discounted by i. Four penalties control the deviations in the objective function: two for excessive and deficient amounts of tonnage (c^{lo}, c^{uo}) and two for excessive and deficient amounts of grade (c^{lg}, c^{ug}) from the targets.

3.4. Constraints

Two types of constraints, logical and technical constraints, are defined and discussed in this section.

- *Logical constraints*

Logical constraints connect the decision variables. The first set of decision variables, $x_{s,t}$, is associated with the slices and the second set, $y_{d,t}$, is associated with the drawpoints. The following equations show the logic:

$$y_{d,t}-M\times\sum_{s=1}^{N_{Sd}}x_{s,t}\le 0 \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \quad (6)$$

$$\sum_{s=1}^{N_{Sd}}x_{s,t}-M\times y_{d,t}\le 0 \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \quad (7)$$

The third set of decision variables is used to define the continuous mining and precedence of extraction constraints. This set is defined using the draw percentage concept. $z_{d,t}$ represents drawpoint d in period t, which is 0 if draw percentage of drawpoint d at period t is zero and 1 if not; equations (8) to (10) connect the second and third set of decision variables.

$$DP_{d,t'}=\sum_{t=1}^{t'}y_{d,t} \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \quad (8)$$

$$DP_{d,t'}\le M\times z_{d,t} \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \quad (9)$$

$$z_{d,t}\le M\times DP_{d,t'} \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \quad (10)$$

where t' is the current period and the constraints have to be satisfied for all periods ($t\in\{1,...,T\}$).

Technical constraints model the practical restrictions of the operations and are as follows:

- *Production targets*

Production tonnage is limited by the mining equipment, requirements of the processing plant, the market demand, and the goals of the management team. This constraint ensures that the extractions in different scenarios are as close as possible to the production targets during the life of the mine ($\hat{o}_t$).

$$\left\{\sum_{s=1}^{S}\rho_{s,n}\times V_{s}\times x_{s,t}\right\}-\hat{o}_{t}+dev_{n}^{lo}-dev_{n}^{uo}=0 \qquad \forall n\in\{1,...,N\},\ t\in\{1,...,T\} \quad (11)$$

- *Grade targets*

Production grade is limited by the requirements of the processing plant and it is difficult to predict in block-cave mining because of the material flow. This constraint ensures that the production grade of element *e* is as close as possible to the target grade in different scenarios during the life of the mine ($\hat{g}_t^e$).

$$\sum_{s=1}^{S}(g_{s,n}-\hat{g}_t^e)\times\rho_{s,n}\times V_s\times x_{s,t}+dev_n^{lg}-dev_n^{ug}=0 \qquad \forall n\in\{1,...,N\},\ t\in\{1,...,T\} \tag{12}$$

- *Reserve*

This is a control constraint that ensures the model does not extract more than the existing mining resource. It can also force the model to extract the whole resource (in case of equality) if maximum ore extraction is the goal.

$$\sum_{t=1}^{T}x_{s,t}\le 1 \qquad \forall s\in\{1,...,S\} \tag{13}$$

- *Active drawpoints*

Because of operational considerations, only a certain number of drawpoints can be extracted (active) at the same time.

$$\min Act_t\le\sum_{d=1}^{D}y_{d,t}\le\max Act_t \qquad t\in\{1,...,T\} \tag{14}$$

- *Mining direction*

The decision of the starting point and the mining direction within the designed layout of block-cave drives the development and operational priorities during the life of the mine. The mining direction, which is the horizontal sequence of extraction among drawpoints, can be chosen based on economic or geotechnical criteria. If there are no geotechnical limitations that dictate the direction, the economic value of the draw columns is the main driver of the starting point and the direction. Khodayari and Pourrahimian (2015a) proposed a methodology to find the optimum mining direction based on the economic value of the draw columns. This methodology is used in this paper to find the best starting point and the optimum mining direction, and then the sequence of extractions between the drawpoints in the given layout is defined. Equation (15) ensures that the sequence of extraction is achieved in the production schedule.

$$z_{d,t}\le y_{d_p,t} \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \tag{15}$$

- *Slice sequence (vertical)*

This constraint controls the sequence of extraction between the slices in each of the draw columns during the life of mine.

$$x_{s,t}\le\sum_{t=1}^{t}x_{s_p,t} \qquad \forall d\in\{1,...,D\},t\in\{1,...,T\} \tag{16}$$

This equation ensures that in each period of *t*, slice *s* (in the draw column associated with drawpoint *d*) is extracted only if slice s_p, which is located beneath it, is already extracted in the periods before or at the same period *t*.

- *Continuous mining*

In block-cave mining, when a drawpoint is opened, its extraction has to be continued until the end of its life. Any discontinuation can cause compaction resulting in the secondary blasting or the loss of

remaining ore in the draw column. In other words, if a drawpoint is opened, it is active in consecutive years with at least a minimum draw rate of DR_{min} utill it is closed.

$$y_{d,t} \le y_{d,t-1} + (1 - z_{d,t}) \qquad \forall d \in \{1,...,D\}, t \in \{1,...,T\} \tag{17}$$

- *Draw rate*

The draw rate, which is the total extraction from each drawpoint in each period of *t*, is limited to a minimum and maximum amount of material. The geomechanical parameters, the geometry of drawpoints and the ore passes, the mining equipment, and the production targets will dictate this constraint. This constraint keeps the draw rate of each drawpoint within a defined range (DR_{min} to $DR_{\max}$); however, it is possible to consider a production rate curve.

$$DR_{min} \times y_{d,t} \le \sum_{s=1}^{s_d} \rho_s \times V_s \times x_{s,t} \le DR_{max} \qquad \forall d \in \{1,...,D\}, t \in \{1,...,T\} \tag{18}$$

- *Draw life*

Drawpoints can be in production during a certain time which is called draw life. Draw life is dictated by geomechanical, operational, and economic parameters. This constraint ensures that each drawpoint is active during certain periods.

$$DL_{\min} \le \sum_{t=1}^{T} y_{d,t} \le DL_{\max} \qquad \forall d \in \{1,...,D\} \tag{19}$$

3.5. Solving the optimization model

MATLAB R2017a (2017) was used to build the model based on the mentioned objective function and the constraints. Then CPLEX IBM12.7.1 is used to solve the model. CPLEX uses branch-and-cut algorithm to solve the MILP problem. In this algorithm, a search tree consisting of nodes is created; the nodes represent LP subproblems to be solved and analyzed further. Nodes are processed until either no more active nodes are available or some limit has been reached. Creation of two nodes from a parent is called a branch. A cut is a constraint added to the model in order to reduce the size of the solution domain (IBM, 2017).

4. Numerical results

The proposed model was applied on a block-cave mining project with a production layout of 640 drawpoints (Fig 5). According to the designed layout, each draw column above a drawpoint is divided into a number of slices, and the total number of slices is 5,260. For the first set of decision variables, one decision variable has to be assigned to each slice in each period. Fig 6 and Fig 7 show tonnage and grade of the slice model, respectively. The mine life is 15 years with a starting production of 3 million tonnes and a ramp-up period of 3 years to reach the full production of 7 million tonnes per year as the target. The “datasample” function in MATLAB is used to generate 15 mixing scenarios. The sequence of extraction between the drawpoints was defined using the distribution of the economic value of the ore in the layout. In this case study, the materials with higher economic value are located on the northwest of the layout (the red area in Fig 8). The model was set in a way to start the extraction from the area with higher economic value, and then propagation of the cave occurs as the caved block is expanded through the rest of the mining layout (lateral extension). Also, a V-shape mining advancement was considered to have a concave face for the undercut level which provides better control of major structures and more secure undercut. Therefore, the extraction starts from location A (x=1962, y=1434) and moves towards southeast and northwest by two V-shapes at the same time (Fig 8). The penalties for deviations are set based on the target grades and tonnages. For the current caving operations, based on the targets, two different sets of penalties (case A and B) were tested to study the impact of the penalties on the results (Table 2). More details of the input parameters are presented in Table 3.

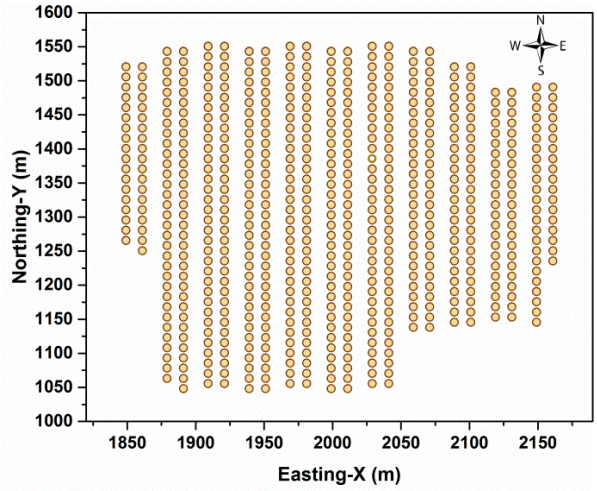


Fig 5. Layout of the drawpoints

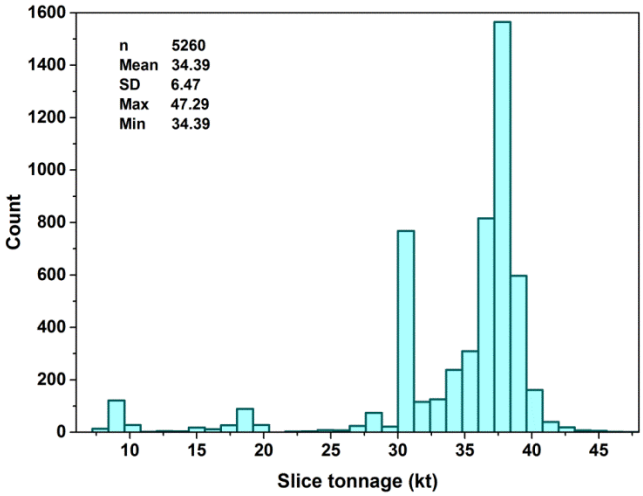


Fig 6. Histogram of the tonnage for the slice model

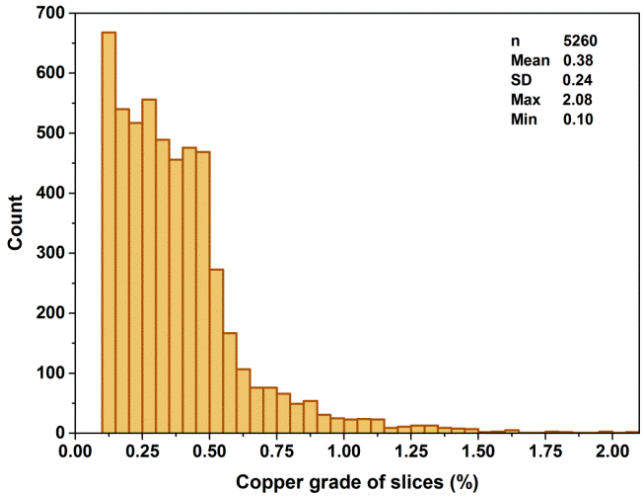


Fig 7. Histogram of the copper grade for the slice model

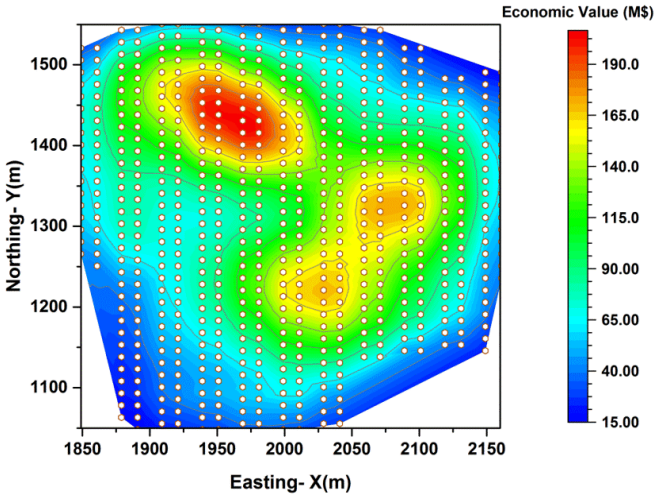


Fig 8. Distribution of economic value of ore in the mining layout

Table 2. Applying two sets of penalties for the case study

Case/Penalties	Deviations from grade ($)	Deviations from tonnage ($)
A	10	10
B	50	10

Table 3. Scheduling parameters for the case study

Parameter	Value	Unit	Description
T	15	Year	Production schedule timeline (the life of the mine)
$\hat{g}_t^{cu}$	0.52	%	Target production grade for copper (Cu) $\forall t \in \{1,...,T\}$
$\hat{o}_1$	3	Mt	The ore target at the first year of production
$\hat{o}_2$	5	Mt	The ore target at the second year of production
$\hat{o}_t$	7	Mt	The ore target $\forall t \in \{3,...,T\}$
Ramp-up time	3	Year	The time in which the production is increased from starting amount to the full capacity
minAct	70	-	Minimum number of active drawpoints per period
maxAct	200	-	Maximum number of active drawpoints per period
MIPgap	5	%	Relative tolerance on the gap between the best integer objective and the objective of the best node remaining
DL_{min}	0	Year	Minimum life of drawpoints
DL_{max}	6	Year	Maximum life of drawpoints
DR_{min}	30,000	Tonne/year	Minimum draw rate
DR_{max}	50,000	Tonne/year	Maximum draw rate
p^{cu}	5,000	$/tonne	Copper price
c	15	$/tonne	Operating costs per tonne of ore (Mining+Processing)
i	10	%	Discount rate
r^{cu}	85	%	Recovery of the processing plant for copper (cu)
N	15	-	Number of scenarios
R	50	meter	Adjacency radius
HIZ	76	meter	Height of Interaction Zone

The case study was solved with CPLEX 12.7.1 (Academic license) in a MATLAB environment on a computer with two Intel Xeon CPU E5-2630 version 0 @ 2.3 GHz processors and 64 GB RAM. The results show that both production tonnages and grades are as close as possible to the defined targets with the NPV of 473 M$ for the project (considering the first part of the objective function). The productions for the 15 generated scenarios follow the target production line with minor deviations in some periods (Fig 9).

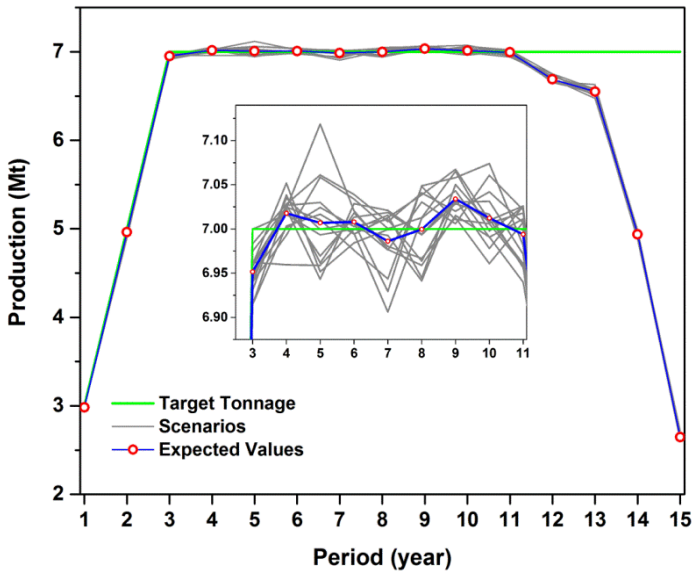


Fig 9. Production during the life of mine (case A)

However, compared to tonnage deviations, the resulting production grade shows visible deviations in some periods (Fig 10); this is because of the grade distribution in the orebody and the defined mining direction. In other words, the extraction starts from high-grade parts of the orebody, which has high economic values, and then moves to the low-grade area. Therefore, the production grade is higher than the target in the first few years and lower in the last few years of production. The model is defined in a way that the penalties control the deviations; a reduction in the deviations is expected by increasing the penalties although the NPV might decrease.

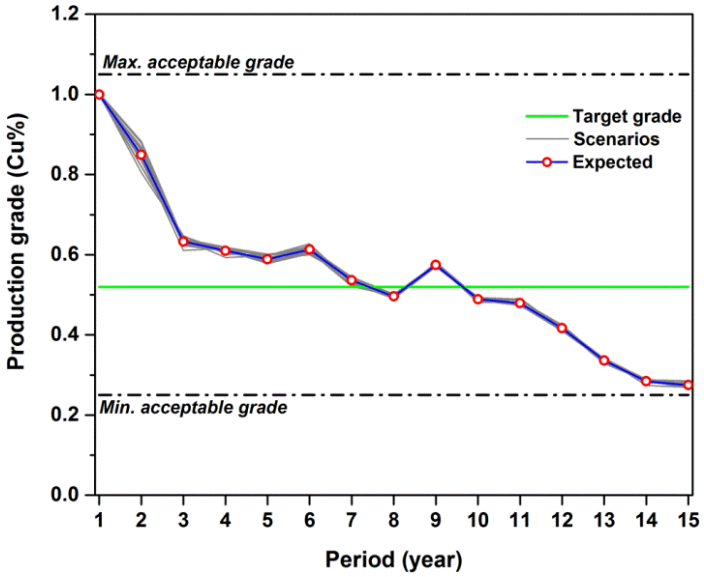


Fig 10. Production grade compared to the target grade (case A)

The model was run for the same case study and input parameters but with higher penalties for grade deviations. The results show a decrease in deviations of production grade compared to the target grade among all scenarios during the life of mine (Fig 11). On the other hand, the deviations of production tonnages from the targets have increased because, in this case, the weight of grade in the objective function is higher than tonnage (Fig 12). In addition, as expected, the NPV of the project decreased to 450 M$. As a result, the model is flexible and the mining engineer, or management team, can make the decision based on their priorities: less deviation from the target grades with less NPV or accepting some deviations for achieving a higher NPV.

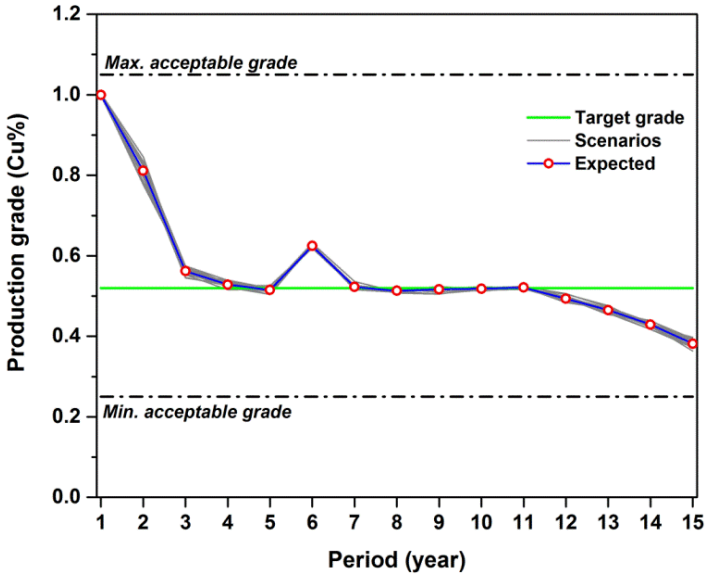


Fig 11. Production grade compared to the target grade (case B)

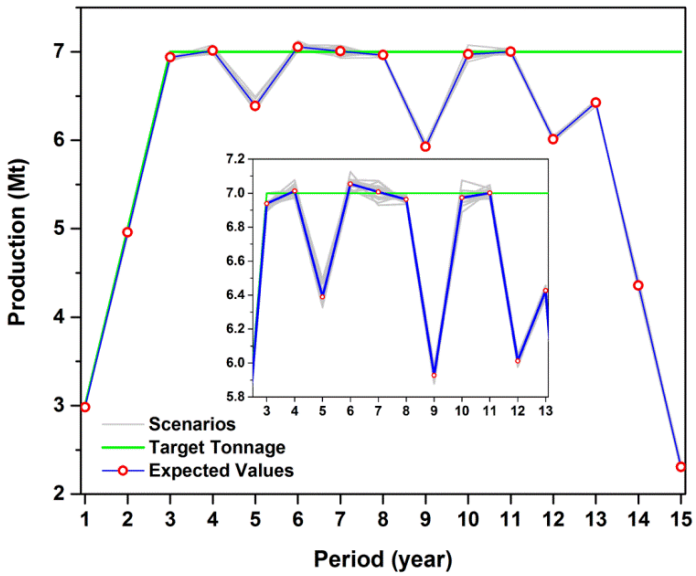


Fig 12. Production during the life of mine (case B)

The period in which extraction from each drawpoint is started (starting period) shows the resulting sequence of extraction in the mining layout (Fig 13). Comparing the defined mining direction in Fig 8 and the resulting sequence of extraction shows that the production schedule follows the desired mining direction in addition to satisfying the number of active drawpoints constraint (Fig 14). The

BHOD and as a result the mining reserve is calculated as one of the outputs of the production schedule. Fig 15 shows the BHOD among draw columns compared to their initial heights. Additionally, the mine reserve can be determined according to the BHODs as an output of optimization.

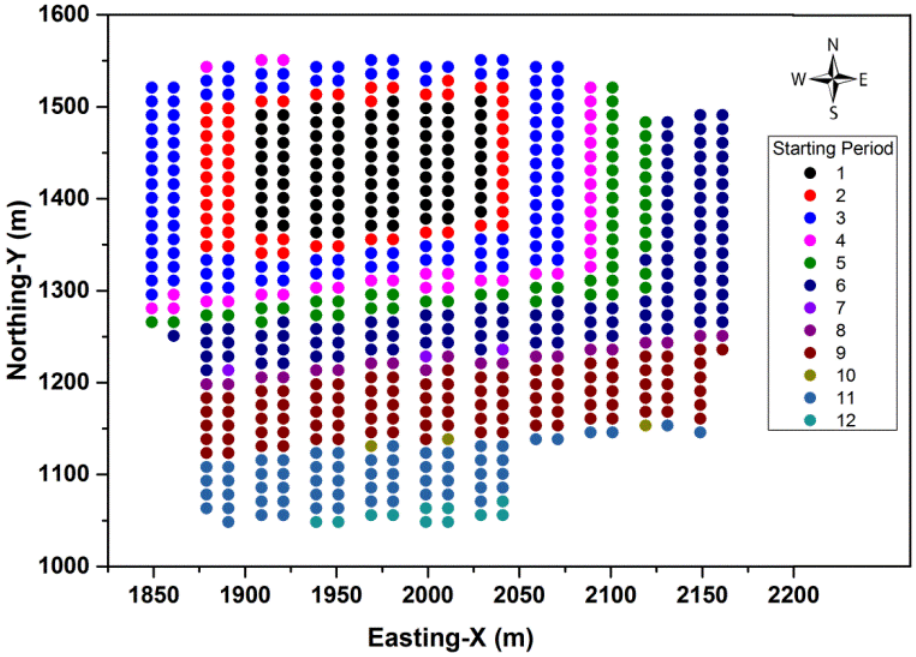


Fig 13. Sequence of extraction for drawpoints based on the defined mining direction

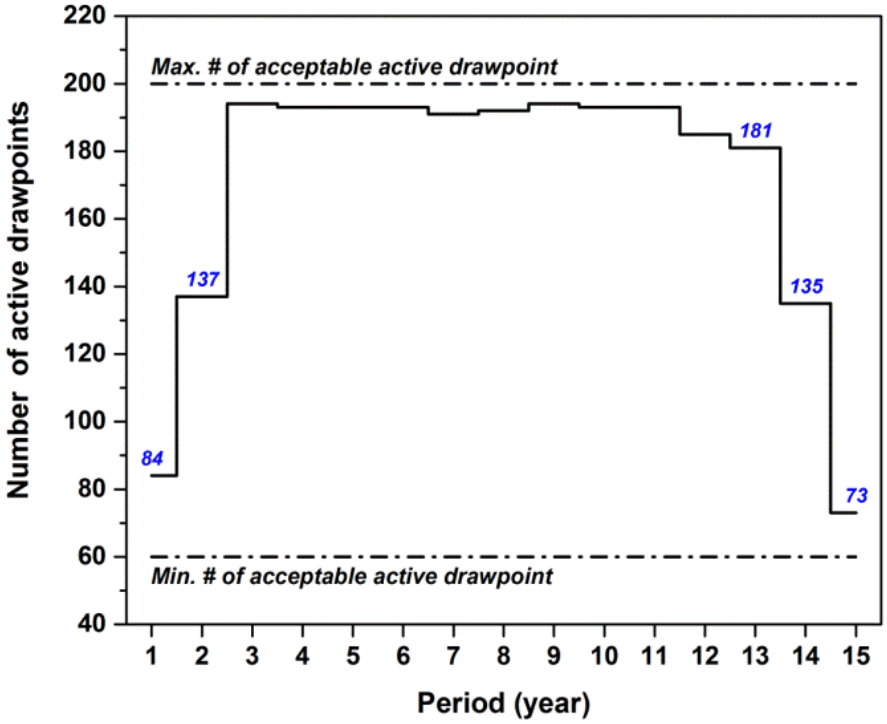


Fig 14. Active drawpoints during the life of the mine

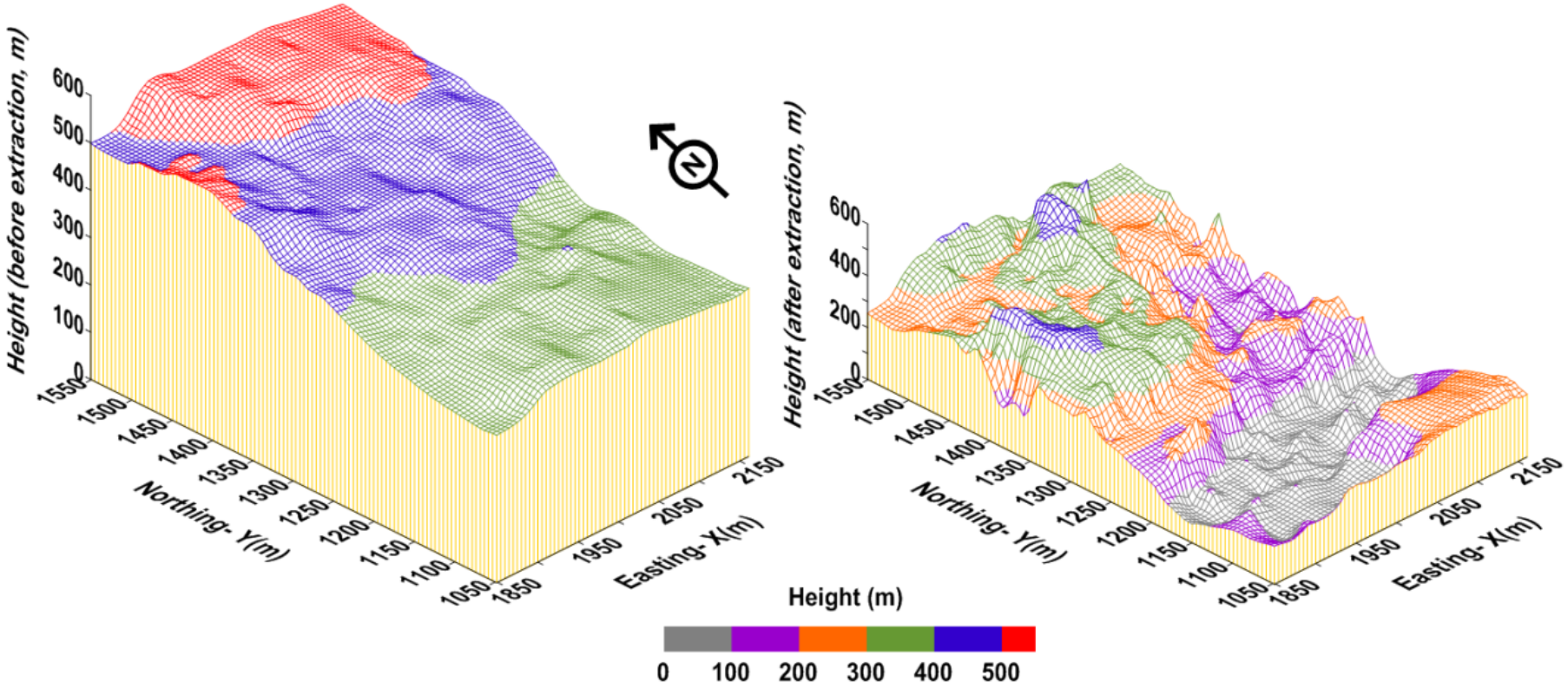


Fig 15. Height of the material after extraction compare to initial height of the draw columns

5. Discussion and future research direction

Despite its significant impact on production scheduling, material flow and its uncertainties have not been part of production scheduling models. The existing uncertainties can be one of the main reasons for gaps between the production goals and the actual operations resulting in an inefficient mine to mill system. In this paper, using stochastic optimization, an MILP model was proposed to capture the horizontal mixing that occurs in material flow when extracting from drawpoints in block-cave mining. Adjacent slices located within the same neighborhood were used to generate scenarios in order to simulate the horizontal movements of the material. To address the processing plant's objectives, in addition to technical and operational constraints, the model was built based on two production targets: tonnage and grade. For the presented case study, the resulting tonnages show only minor deviations from the targets in all scenarios. There are visible deviations between the target grade and the resulting production grade which is mainly because of the defined mining direction (extracting high grade material in the first years of the mine life). The BHOD calculation was also brought into the optimization, which means that the mine reserve is the output of the production scheduling and not an input; this takes us one step closer towards overall optimization. The model was run for two different sets of penalties for grade deviations and proved that increasing the penalties will provide us with closer results to the targets but lower NPVs. As a result, mine planners can use the proposed stochastic model as a flexible tool; they can define the penalties for deviations from their specified targets based on their priorities: higher penalties will result in less deviation from the targets but lower NPVs, and vice versa. Our future work will be developing models that can capture vertical mixing in addition to horizontal mixing.

6. References

[1] Alonso-Ayuso, A., Carvallo, F., Escudero, L.F., Guignard, M., Pi, J., Puranmalka, R., & Weintraub, A. (2014). Medium range optimization of copper extraction planning under uncertainty in future copper prices. *European Journal of Operational Research, 233*(3), 711-726.

[2] Brunton, I., Lett, J., Sharrock, G., Thornhill, T., & Mobilio, B. (2016). *Full Scale Flow Marker Experiments at the Ridgeway Deeps and Cadia East Block Cave Operations.* Paper presented at the Massmin 2016, Sydney, Australia, May 9-11.

[3] Castro, R., Gonzalez, F., & Arancibia, E. (2009). Development of a gravity flow numerical model for the evaluation of drawpoint spacing for block/panel caving. *Journal of the Southern African Institute of Mining and Metallurgy, 109*(7), 393-400.

[4] Castro, R.L., Fuenzalida, M.A., & Lund, F. (2014). Experimental study of gravity flow under confined conditions. *International Journal of Rock Mechanics and Mining Sciences, 67*, 164-169.

[5] Diering, T. (2004). *Computational considerations for production scheduling of block cave mines.* Paper presented at the MassMin 2004, Santiago, Chile, Aug 22-25.

[6] Diering, T. (2012). *Quadratic Programming applications to block cave scheduling and cave management.* Paper presented at the Massmin 2012, Sudbury, Ontario, Canada, June 10-14, 2012.

[7] Garcés, D., Viera, E., Castro, R., & Meléndez, M. (2016). *Gravity Flow Full□scale Tests at Esmeralda Mine's Block□2, El Teniente.* Paper presented at the Massmin 2016, Sydney, Australia, May 9-11.

[8] Gibson, W. (2014). *Stochastic Models For Gravity Flow: Numerical Considerations.* Paper presented at the Caving 2014, Santiago, Chile, Jun 5-6.

[9] Guest, A.R., Van Hout, G.J., & Von Johannides, A. (2000). *An Application of Linear Programming for Block Cave Draw Control.* Paper presented at the MassMin 2000, Brisbane, Australia, Oct 29- Nov 2.

[10] IBM. (2017). IBM ILOG CPLEX Optimization Studio V12.7.1. New York: IBM Corporation.

[11] Jin, A., Sun, H., Wu, S., & Gao, Y. (2017). Confirmation of the upside-down drop shape theory in gravity flow and development of a new empirical equation to calculate the shape. *International Journal of Rock Mechanics and Mining Sciences, 92*, 91-98.

[12] Khodayari, F., & Pourrahimian, Y. (2014). *Determination of the best height of draw in block cave sequence optimization.* Paper presented at the 3rd International Symposium on Block and Sublevel caving (CAVING 2014), Santiago, Chile, Jun 5-6.

[13] Khodayari, F., & Pourrahimian, Y. (2015a). *Determination of development precedence for drawpoints in block-cave mining.* Paper presented at the 5th International Symposium Mineral Resources and Mine Development (AIMS 2015), Aachen, Germany, May 27-28.

[14] Khodayari, F., & Pourrahimian, Y. (2015b). Mathematical programming applications in block-caving scheduling: a review of models and algorithms. *International Journal of Mining and Mineral Engineering (IJMME), 6*(3), 234-257.

[15] Khodayari, F., & Pourrahimian, Y. (2016). *Quadratic Programming Application in Block-cave Mining.* Paper presented at the 1st International Conference of Underground Mining (U-Mining 2016), Santiago, Chile, Oct 19-21.

[16] Laubscher, D.A. (1994). Cave mining-the state of the art. *The Journal of The South African Institute of Mining and Metallurgy, 94*(10), 279-293.

[17] Malaki, S., Khodayari, F., Pourrahimian, Y., & Liu, W.V. (2017). *An application of mathematical programming and sequencial Gaussian simulation for block-cave production scheduling.* Paper presented at the 1st International Conference on Underground Mining Technology (UMT 2017), Sudbury, Ontario, Canada, October 11-13.

[18] Nezhadshahmohammad, Aghababaei, H., & Pourrahimian, Y. (2017). Conditional draw control system in block-cave production scheduling using mathematical programming. *International Journal of Mining, Reclamation and Environment*(1748-0930), 1-24.

[19] Nezhadshahmohammad, F., Khodayari, F., & Pourrahimian, Y. (2017). *Draw rate optimization in block cave production scheduling using mathematical proframming.* Paper presented at the 1st International Conference on Underground Mining Technology (UMT 2017), Sudbury, Ontario, Canada, October 11-13.

[20] Nezhadshahmohammad, F., Pourrahimian, Y., & Aghababaei, H. (2017). Presentation and application of a multi-index clustering technique for the mathematical programming of block-cave production scheduling. *International Journal of Mining Science and Technology, https://doi.org/10.1016/j.ijmst.2017.11.005*.

[21] Parkinson, A. (2012). *Essays on Sequence Optimization in Block Cave Mining and Inventory Policies with Two Delivery Sizes.* Ph.D. Thesis, The University Of British Columbia.

[22] Pierce, M.E. (2010). *A Model for Gravity Flow of Fragmented Rock in Block Caving Mines.* Ph.D. Thesis, The University of Queensland.

[23] Pourrahimian, Y., Askari-Nasab, H., & Tannant, D. (2012). Mixed-Integer Linear Programming formulation for block-cave sequence optimisation. *Int. J. Mining and Mineral Engineering, 4, No. 1*, 26-49.

[24] Pourrahimian, Y., Askari-Nasab, H., & Tannant, D. (2013). A multi-step approach for block-cave production scheduling optimization. *International Journal of Mining Science and Technology, 23*(5), 739-750.

[25] Rahal, D. (2008). *Draw Control in Block Caving Using Mixed Integer Linear Programming.* Ph.D. Thesis, The University of Queensland.

[26] Rahal, D., Dudley, J., & Hout, G.v. (2008). *Developing an optimised production forecast at Northparkes E48 mine using MILP.* Paper presented at the 5th International Conference and Exhibition on Mass Mining, Luleå, Sweden, Jun 9-11.

[27] Rahal, D., Smith, M., Van Hout, G., & Von Johannides, A. (2003). *The use of mixed integer linear programming for long-term scheduling in block caving mines.* Paper presented at the 31st International Symposium on Application of Computers and Operations Research in the Minerals Industries, Cape Town, South Africa, 14-16 May.

[28] Rubio, E. (2002). *Long term planning of block caving operations using mathematical programming tools.* M.Sc. Thesis, The University of British Columbia.

[29] Smoljanovic, M., Rubio, E., & Morales, N. (2011). *Panel Caving Scheduling Under Precedence Constraints Considering Mining System.* Paper presented at the 35th APCOM Symposium, Wollongong, NSW, Australia, Sept 24-30.

[30] Song, X. (1989). *Caving process simulation and optimal mining sequence at Tong Kuang Yu mine, China.* Paper presented at the 21st International Symposium on Application of Computers and Operations Research in the Mineral Industry, Las Vegas, USA, 27 February-2 March.

[31] TheMathWorksInc. (2017). MATLAB. Massachusetts, United States.

[32] Weintraub, A., Pereira, M., & Schultz, X. (2008). A Priori and A Posteriori Aggregation Procedures to Reduce Model Size in MIP Mine Planning Models. *Electronic Notes in Discrete Mathematics 30*, 297–302.

An Optimization Framework for Cut-and-Fill Mining Production Scheduling using Mixed Integer Linear Programming

Shuwei Huang and Eugene Ben-Awuah
Mining Optimization Laboratory (MOL)
Laurentian University, Sudbury, Canada

ABSTRACT

Mixed integer linear programming models are recognized to have significant potential for optimizing production scheduling problems. A review of general optimization studies that have been proposed for underground mining shows that previous works lack flexibility, operability and practicality in relation to cut-and-fill mining production scheduling. This paper presents a mixed integer linear programming (MILP) formulation for cut-and-fill mining. The objective function of the model is to maximize the net present value (NPV) of the operation while meeting all mining and processing operational and technical constraints. In addition, stope development and extraction sequencing constraints, and the use of mining and processing tonnage fluctuation, extraction duration and active levels control constraints make the MILP model more practical and expandable. The MILP model is verified and validated with two case studies from an existing mine and the results are compared with the actual mining strategy. The comparison shows a 5% to 7% improved NPV in both case studies resulting from mining higher grades and processing less tonnes thereby generating a better cash flow.

1. Introduction

Mine production scheduling is the determination of the best mining strategy that defines extraction sequence at a given time, and the movement of mineralized material to maximize the overall net present value (NPV) while optimizing the available resource utilization based on identified economics, technical, environmental and geological constraints (Ben-Awuah et al., 2016; Ebbels, 2016). Mine production scheduling is grouped into long-term (usually three to five years or more) and short-term (daily, weekly or monthly plans) production scheduling mainly determined based on various degrees of information provided and the planned working duration. In underground mining, long-term production scheduling specifies the stope extraction sequence and the connected mine developments required to achieve the production targets and maximize the NPV usually from three to five years or more. Short-term production scheduling defines a highly detailed basic operation or guidance for operating over a time frame of days, weeks and months (Alford et al., 2007) to minimize the deviations from the pre-defined production capacities that can maintain a constant plant feed rate (Nehring et al., 2010).

The application of operation research techniques in mining started in the early 1960s (Musingwini, 2016). During the past decades, many researchers have made efforts to use operation research techniques in the mining industry (Osanloo et al., 2008). These works are mainly focused on open pit mining. Although operations research techniques have been extensively used in open pit mining, its use is limited in underground mine planning and scheduling. This is because of the complexities

associated with the optimization of development layouts, which are directly related to the underground mining methods used.

This research seeks to develop, implement and verify an optimization framework for an underground cut-and-fill mining production scheduling using mixed integer linear programming (MILP). The objective of the framework is to maximize the net present value (NPV) of an underground mining operation and generate a practical, operational, and long-term cut-and-fill mining schedule.

2. Literature Review

Optimization methods in underground mining were introduced in the 1980s (Lizotte and Elbrond, 1985; Chatterjee and Sridhar, 1986), and a common optimization method applied to underground problems is Linear programming (LP). Jawed and Sinaha (1985), Pendharkar (1997), and Li (2009) used linear programming for optimizing production planning and scheduling of a group of underground mines. However, all these initial models lack generality to some extent and are specific to localized problems, for instance, optimal blending or fleet management problem. Meanwhile, these types of mathematical models do not allow the utilization of integer or binary decision variables that are required to meet precedence needs (Brickey, 2015).

Linear programming is completely dependent on the linear equation in theoretical logic and program implementation. The decision variables are allowed to be fractional, but that is a certain limitation for the practical stopes sequencing problem of the production scheduling of underground mines, which are discrete in nature. Usually, the majority of these activities encountered in mine production management are discrete. Therefore, linear programming sometimes gives results that are not practical.

Mixed integer linear programming (MILP) approaches is a well-suited modeling method developed in the 1990s. Chanda (1990) presented a computerized integer programming model for short-term production scheduling in a copper mine based in Zambia. Trout (1995) developed a general mixed integer programming (MIP) model to solve a 55-stope underground mine scheduling problem with sequencing, production and backfilling constraints. Topal (2003) developed a MIP model and implemented on a small block database for underground sublevel caving operation. These MIP models are unable to handle realistic industrial mining scenarios and solution time may exponentially increase when more integer variables are added. That will make the model very difficult or even impossible to be solved in an actual problem (Ben-Awuah and Askari-Nasab, 2011). More recently, Nehring and Topal (2007) presented a MIP model for a small conceptual sublevel stoping operation. Binary and integer variables were defined to represent four separate mining phases, including preparation, extraction, void and backfilling. To improve on the computational time, a new MIP model was developed by assigning a single binary decision variable for the entire production to reduce the number of variables (Nehring et al., 2010). But, it was necessary to assign earliest and latest possible start times for each production block in advance. Pourrahimian et al. (2012), Pourrahimian (2013), and Pourrahimian and Askari-Nasab (2014) developed, implemented, and verified a theoretical optimization framework based on a MILP model for a long-term production scheduling in underground block-caving with the maximum economic return measured by NPV, and within acceptable technical and operational constraints. However, the introduced optimization framework is suitable only for block-cave mining.

Heuristic approaches have depicted to be effective for large-scale Non-deterministic Polynomial (NP) hard problems, especially in the wider field of mine planning and scheduling. O'Sullivan and Newman (2014) proposed an integer-programming model for underground mine scheduling which aims to maximize metal production subject to resource constraints while considering sequencing of extraction and backfilling. When the determined optimization methods mentioned above are not sufficiently effective, authors have used a heuristic technique to obtain a suboptimal solution within an acceptable time range. Heuristic algorithms generate candidate solutions for the model constraints

randomly. Nevertheless, they have their own selecting ways and searching directions to produce better children candidate solutions for the optimization of a model objective function. Although the number of iterations may increase for a larger-scale problem, the consumption increases in computational costs are usually substantially lower than it would be for traditional methods. A terminal problem for these heuristic methods is that there are no mathematical proving methods for the provided solutions to compare to the theoretical optimum. Jie et al. (2016) constructed a MIP model considering underground mining condition of equilibrium and continuity and calculated by an algorithm based on the artificial bee colony model. Similar research has been conducted by Denby and Schofield (1995), Kumral (2004), Yao et al. (2011), and Hu et al. (2013).

Mixed integer programming models are recognized as having significant potential to suit the discrete problem in certain conditions (Ben-Awuah and Askari-Nasab, 2013). The integer variables can represent entities that cannot be divided. However, the downside is that MILP problems are non-convex, they must be solved by some kinds of systematic and possibly exhaustive search. Therefore, the literatures so far report few MILP formulation applications to large-scale underground mine production problems. In addition to the optimization methods and techniques, the current formulations focusing on the implementing optimization procedures in underground mining mostly lack flexibility, operability and practicality. The lack of operational constraints will possibly result in a widely spread mining pattern for various periods. It is sometimes infeasible to incorporate equipment mobility and access requirement into long-term underground production scheduling and ore extraction.

Underground mine production scheduling has developed over the last two decades but proven to be an intractable and a challenging task. There is currently no suitable underground mine production scheduling optimization model available to the application of cut-and-fill mining. Hence, based on the critical review of the optimization of mine production planning and scheduling studies that have been proposed for underground hard rock mining, we present a MILP model for cut-and-fill mining that seeks to maximize the project NPV and provide a feasible, practical and flexible ore extraction and development scheduling.

3. Assumptions, Definitions and Notations

Cut-and-fill mining is applied to steeply dipping orebodies, good to moderate geological strength and stability, and a comparatively high-grade mineralization. It provides better selectivity and is preferred to orebodies with irregular shape, discontinuities and scattered mineralization (Hamrin et al., 2001).

Cut-and-fill mining method extracts the ore in horizontal slices, starting from a bottom undercut, advancing upward. The ore is drilled, blasted, loaded and removed from a stope, which is then backfilled with tailings from the dressing plant combined with cement, or waste rock obtained from any part of the underground working. The backfilling body serves both to support stope walls, and as a working platform for the equipment when the next slice is mined.

There are horizontal development levels to work mines from the main shaft at regular intervals along the orebody. In a plan view, the orebody is broken-up into a series of stopes which are generally square or rectangular in shape. A crosscut is a horizontal opening driven from the shaft and near right angles to the orebody, which is connecting a level with the main shaft. A horizontal drift is a necessary access for mine equipment transporting broken materials from the stopes. Fig. 1 shows a schematic diagram of stopes layout.

Generally, some mining exploration developments, such as the main shaft, auxiliary shaft, main return airways and dewatering systems are selected and designed considering future development. These are often completed at one time as a permanent facility for subsequent development and operation. However, these infrastructure projects are not within the scope of this research.

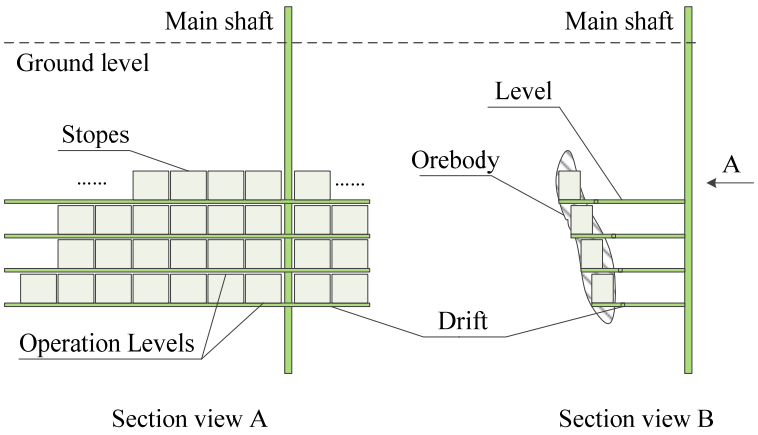


Fig. 1. A schematic diagram of stopes layout.

This study is focused on the optimization of mining production phases for cut-and-fill underground mining. Fig. 2 shows the stope production phases modeled in three separate phases including advancement, extraction, and backfilling. In cut-and-fill mining, all work is done inside the stope, therefore, advancement, including necessary crosscuts, transport drifts, and proper ventilation rises, must be provided to the worker. After these activities, stope extraction activities such as drilling, blasting and mucking, can begin. Finally, every mined-out stope is backfilled for ground control purpose and to provide a surface to work from for future stopes. The advancement, extraction and backfilling are repeated until mining has progressed along the entire strike of the stope.

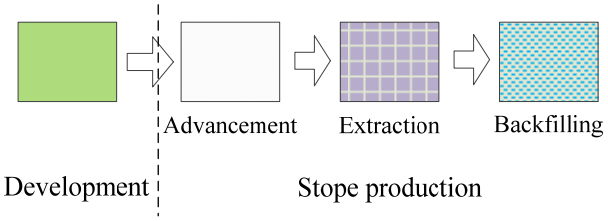


Fig. 2. Stope production phases.

3.1. Notations

The following notations, sets, parameters, and definitions are used to construct the objective function and constraints for formulation of the mathematical mixed integer linear programming model.

t schedule time periods: $t = 1, 2, \ldots, T$, where T is the schedule duration.

n stope identification: $n = 1, 2, \ldots, N$, where N is the total number of stopes.

l level identification: $l = 1, 2, \ldots, L$, where L is the total number of levels.

3.2. Decision variables

$a_n^t \in \{0,1\}$ binary integer variable; equal to one if the advancement for the stope n is to be scheduled in time period t, otherwise zero.

$e_n^t \in \{0,1\}$ binary integer variable; equal to one if the extraction for the stope n is to be scheduled in time period t, otherwise zero.

$b_n^t \in \{0,1\}$ binary integer variable; equal to one if the backfilling for the stope n is to be scheduled in time period t, otherwise zero.

$x_n^t \in [0,1]$ continuous variable, representing the portion of the stope *n* is to be extracted in time period *t*. This variable is linked to the stope extraction variable *e* and only provides a non-zero value when *e* is non-zero.

3.3. Sets

$A(J)$ for each stope *n*, there is a set $A(J)$ defining the adjacent stopes that cannot be mined simultaneously with the extraction or backfilling of the stope *n*; where *J* is the total number of stopes in set $A(J)$.

$D(I)$ for each stope *n*, there is a set $D(I)$ defining the development that must be completed prior to the advancement and extraction of the current stope *n*. where *I* is the total number of advancement activities in set $D(I)$.

$L(M)$ For each level *l*, there is a set $L(M)$ defining all the stopes in this level, where *M* is the total number of stopes in set $L(M)$.

3.4. Parameters

R_n^t discounted revenue, generated by selling the final product.

i discounted rate.

o_n ore tonnage in stope *n*.

g_n average grade of mineral in ore portion of stope *n*.

r^t processing recovery, the proportion of mineral recovered in period *t*.

p^t selling price in present value terms obtainable per unit of product.

CA_n^t cost in present value terms of advancement for stope *n* in period *t*.

ca_n^t variable cost in present value terms of advancement in period *t*.

ad_n^t advancement tonnage of stope *n* in period *t*.

CE_n^t cost in present value terms of extraction in period *t*.

ce_n^t variable cost in present value terms of extraction in period *t*.

w_n waste tonnage in stope *n*.

CF_n^t cost in present value terms of backfilling in period *t*.

cf_n^t variable cost in present value terms of backfilling in period *t*.

f_n^t backfilling amount of stope *n* in period *t*.

CP_n^t cost in present value terms of processing in period *t*.

cp_n^t variable cost in present value terms of processing in period *t*.

al^t lower bound on advancement capacity in period *t*.

au^t upper bound on advancement capacity in period *t*.

h hoist rate, representing the portion of material to be hoisted in period *t*.

hl^t lower bound on hoisting capacity in period *t* (tonnes).

hu^t upper bound on hoisting capacity in period *t* (tonnes).

pl^t lower bound on processing capacity of ore in period *t* (tonnes).

pu^t upper bound on processing capacity of ore in period *t* (tonnes).

gl^t lower bound on acceptable average grade of mineral in period *t*.

gu^t upper bound on acceptable average grade of mineral in period *t*.

D_m deviation for mining tonnage fluctuation constraints.

D_P deviation for processing tonnage fluctuation constraints.

l_l^t binary integer variable; when it is equal to one, it means there are activities in progress in this level in period *t*.

N_{ed} the maximum length of the extraction duration of a stope.

N_{al} the maximum number of the active levels in period *t*.

4. Mathematical Formulation

4.1. Objective function

The objective function of the MILP formulation seeks to maximize the NPV of the mining operation. The profit from mining a stope depends on the value of that stope and the costs incurred in advancement, extraction, backfilling and processing. Hence, the discounted profit from mining a stope can be represented by the discounted revenue minus the discounted costs. The objective function is expressed by Eq. (1), composed of the revenue, discounted rate, costs of advancement, extraction, backfilling and processing, two binary decision variables indicating the time of advancement and backfilling to be scheduled, and a continues decision variable indicating the portion of a stope which is extracted in each period. The component equations of the discounted revenue and costs are shown in Eqs. (2) to (6).

$$Max \sum_{t=1}^{T}\sum_{n=1}^{N}\left[\frac{\left(R_n^t - CE_n^t - CP_n^t\right)\times x_n^t - CA_n^t \times a_n^t - CF_n^t \times b_n^t}{\left(1+i\right)^t}\right] \tag{1}$$

$$R_n^t = o_n \times g_n \times r^t \times p^t \tag{2}$$

$$CA_n^t = ca_n^t \times ad_n^t \tag{3}$$

$$CE_n^t = ce_n^t \times \left(o_n + w_n\right) \tag{4}$$

$$CF_n^t = cf_n^t \times f_n^t \tag{5}$$

$$CP_n^t = cp_n^t \times o_n \tag{6}$$

4.2. Productivity constraints

The constraints can generally be classified into three categories, namely productivity constraints, sequencing and geotechnical constraints and operational constraints. Advancement capacity, grade blending, and hoisting capacity are considered as productivity constraints and a vital part in the development of an optimal production schedule. Sequencing and geotechnical constraints are required to satisfy geotechnical and technical conditions. Operational constraints include the mining and processing tonnage fluctuation constraints, extraction duration constraints and active levels control constraints. They are expressed in mathematical form as follows.

4.2.1. Advancement capacity constraints

Eq. (7) ensures that the total amount of advancement in each period cannot exceed the predefined and acceptable range of advancement capacity of the equipment in that period. al^t and au^t are the lower and upper bounds for the advancement capacity respectively.

$$al^t \le \sum_{n=1}^{N} (ad_n^t \times a_n^t) \le au^t \qquad \forall t \in \{1,...,T\} \tag{7}$$

4.2.2. Hoisting capacity constraints

Eq. (8) ensures that the total tonnage of material hoisted in each period cannot exceed the predefined and acceptable range of hoist capacity in that period. The inequality accounts for the total rock material from the stopes and advancement that must be hoisted from the mine. hl^t and hu^t are the lower and upper bounds for the hoisting capacity respectively.

$$hl^t \le h \times \sum_{n=1}^{N} \left[(o_n + w_n) \times x_n^t + ad_n^t \times a_n^t \right] \le hu^t \quad \forall t \in \{1,...,T\} \tag{8}$$

4.2.3. Grade blending constraints

Eq. (9) ensures that the average grade of the ore or production is within the desired range in each period. It enforces the production scheduling to achieve the desired quality of material for the processing plant. gl^t and gu^t are the lower and upper bounds for the desired grade range.

$$gl^t \le \frac{\sum_{n=1}^{N} g_n \times o_n \times x_n^t}{\sum_{n=1}^{N} o_n \times x_n^t} \le gu^t \qquad \forall t \in \{1,...,T\} \tag{9}$$

4.3. Sequencing and geotechnical constraints

4.3.1. Advancement sequencing constraints

If a stope is scheduled to be mined in a period, advancement must be ready in or before that period. Thus, all the advancements prior to the current advancement necessary to exploit a stope should be completed first. To meet these requirements, we use a set *D(I)* representing the development that must be completed prior to the advancement of the current stope *n*. The same with a_n^t, a_i^t.is a binary integer variable representing the development in *D(I)* to be scheduled or not in time period *t*. Eq. (10) guarantees the reasonable advancement sequence.

$$a_n^t \le \sum_{k=1}^{t} a_i^k \qquad i \in D(I), \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{10}$$

4.3.2. Stope status and sequencing constraints

Sequencing constraints were required to satisfy geotechnical and technical conditions. These constraints define where to start the advancement, extraction and backfilling for a particular level and how to progress specifically. Eq. (11) ensures that an individual stope could only be in one extraction or backfilling phase during any specific period. These two activities cannot be in progress at the same time for a particular stope. Eqs. (12) to (13) ensure that production mining phases progress sequentially through all mining phases including advancement, extraction and backfilling. Eq. (14) ensures that each stope is in either the extraction or backfilling phase immediately after a non-zero stope extraction period. The stope status constraints and sequencing constraints together function to ensure discrete mining activities progress sequentially, through all phases for all the periods. On the other hand, these constraints limit the exposure time of the void area, that potentially leads to excessive geotechnical stresses, increasing the stability to a certain extent.

$$e_n^t + b_n^t \le 1 \qquad \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{11}$$

$$e_n^t - \sum_{k=1}^{t} a_n^k \le 0 \qquad \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{12}$$

$$b_n^t - \sum_{k=1}^{t} x_n^k \le 0 \qquad \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{13}$$

$$e_n^t \le e_n^{t+1} + b_n^{t+1} \qquad \forall n \in \{1,...,N\}, \forall t \in \{1,...,T-1\} \tag{14}$$

4.3.3. Non-adjacent stope production constraints

Non-adjacent stop production constraint is a kind of geotechnical constraints, which focuses on limiting the size of unsupported void areas. Fig. 3 is a vertical view of nine adjacent stopes showing two kinds of spatial constraints required to ensure geotechnical stability of the mine. When a stope is being extracted, the adjacent stopes cannot be mined. For instance, as shown in Fig. 3 (1), if the central stope *S5* is in the extraction state, all kinds of activities are forbidden in the near adjacent stopes (*S2, S4, S6, S8*). Similarly, when a stope is being backfilled, the adjacent stopes cannot be exploited. Eqs. (15) and (16) ensure that all forms of simultaneous adjacent stope production must be avoided to prevent excessively large unsupported voids.

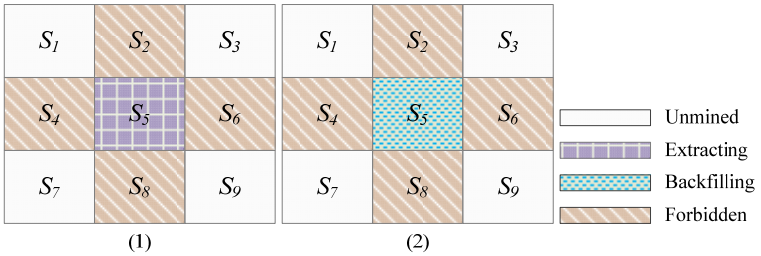


Fig. 3. Schematic representation of activities around the stopes.

$$e_n^t + e_j^t \le 1 \qquad j \in A(J), \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{15}$$

$$b_n^t + e_j^t \le 1 \qquad j \in A(J), \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{16}$$

4.4. Operational constraints

4.4.1. Mining and processing tonnage fluctuation constraints

In previous works, the upper and lower bounds (mining and processing target/capacity constraints) are usually used to ensure the total tonnage of material or ore mined in each period is within the

acceptable range of production targets or mining equipment capacity. Unfortunately, this kind of constraints cannot control the production fluctuations with time and have no motivation to make full use of the mining capacity to enforce higher ore production in the early periods of the mining operation. Tonnage fluctuation constraints on mining and processing shown in Eqs. (17) and (18) respectively are used instead of production target/capacity constraints to ensure that the total rock tonnage and ore tonnage mined increase steadily and stable during the production periods for an improved cash flow of the mining operation. D_m and D_p are function of the maximum mining and processing capacity respectively.

$$\sum_{t=1}^{T}\left(\sum_{n=1}^{N}\left[\left(o_n + w_n\right)\times(x_n^{t+1}\text{-}x_n^t)\right]\right) \le D_m \tag{17}$$

$$\sum_{t=1}^{T}\left(\sum_{n=1}^{N}\left[o_n \times(x_n^{t+1}\text{-}x_n^t)\right]\right) \le D_P \tag{18}$$

4.4.2. Extraction duration constraints

Eq. (19) ensures that the extraction duration of a stope cannot exceed the predefined periods. The inequality helps the model to provide an implementable schedule that better reflects the actual mining environment to control the maximum length of the extraction duration. According to the variety of applicable scenarios, the length of the extraction duration can customize depending on inter alia, stope size, production rate and mining strategies. Therefore, it provides added flexibility and functionality to the formulation.

$$\sum_{t=1}^{T} e_n^t \le N_{ed} \qquad \forall n \in \{1,...,N\} \tag{19}$$

4.4.3. Active levels control constraints

Eqs. (20) to (22) control the maximum number of simultaneous active levels at each period in the schedule. For the inequalities (20) and (21), when an advancement activity is active or a portion of a stope is being extracted in period t in this level, the relevant binary variable l_l must be equal to one. N_{al} is given as an input to the algorithm. In each period, the number of simultaneous active levels must not exceed the allowable number N_{al}, shown in Eq. (22). A large number of active levels might lead to serious operational problems. Therefore, a customized maximum number of simultaneous active levels allow mine planners to adjust constraints that adapt to different application scenarios depending on available infrastructure, production rates, equipment capacities, and the stability of the surrounding rocks. Accordingly, the active levels control can also be considered as a kind of geotechnical constraints. It takes into account the equipment mobility and access required for the underground mine production scheduling optimization framework. Overall, the active levels control constraints help the optimization model to generate a more operational mining pattern, indirectly leading to a practical maximum NPV.

$$\sum_{k=1}^{t}\left(\frac{1}{M}\sum_{m=1}^{M} a_m^k - l_l^k\right) \le 0 \qquad m \in L(M), \forall l \in \{1,...,L\}, \forall t \in \{1,...,T\} \tag{20}$$

$$\sum_{k=1}^{t}\left(\frac{1}{M}\sum_{m=1}^{M} x_m^k - l_l^k\right) \le 0 \qquad m \in L(M), \forall l \in \{1,...,L\}, \forall t \in \{1,...,T\} \tag{21}$$

$$\sum_{l=1}^{L} l_l^t \le N_{al} \qquad \forall t \in \{1,...,T\} \tag{22}$$

4.4.4. Variables constraints

In the proposed model, the variables constraints control the logics and interrelationships of the binary and continuous variables that define advancement, extraction and backfilling to ensure all of them are within desired ranges. Eq. (23) links the continuous extraction variable *x* to the binary extraction variable *e*, and ensures to only provide a non-zero value *x* when *e* is non-zero. Eqs. (24) and (25) ensure that all stopes are going to be advanced and backfilled once in the mine life. Eq. (26) ensures that the total fractions of stopes that are extracted over the scheduling periods are going to be sum up to one, which means all the underground stopes are going to be scheduled. Some other variables defining the deviation for mining and processing tonnages are not written here, they should be non-negative.

$$x_n^t - e_n^t \le 0 \qquad \forall n \in \{1,...,N\}, \forall t \in \{1,...,T\} \tag{23}$$

$$\sum_{t=1}^{T} a_n^t = 1 \qquad \forall n \in \{1,...,N\} \tag{24}$$

$$\sum_{t=1}^{T} b_n^t = 1 \qquad \forall n \in \{1,...,N\} \tag{25}$$

$$\sum_{t=1}^{T} x_n^t = 1 \qquad \forall n \in \{1,...,N\} \tag{26}$$

5. Case Study

5.1. Case study 1 - Forty stopes

For the purpose of evaluating and demonstrating the potential benefits of optimized production scheduling, the proposed model is implemented on a small underground cut-and-fill gold mining operation containing forty stopes. Fig. 4 shows the layout of stopes in the mine. The size of the stope is 50 m in length, 40 m in height, and the width is the same as orebody's width ranging from 10 to 30 m. The total mineral resource in forty stopes is 4.3 Mt with an average Au grade of 2.73 g/t. The goal is to obtain an ore production scheduling over eight periods. A discount factor of 8 percent per annum is applied.

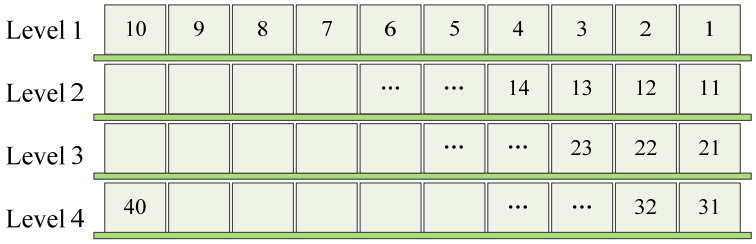


Fig. 4. Forty stopes layout.

Each stope is available for scheduling from the first year period onwards. Advancement and backfilling for each stope is expected to take one time period and extraction of each stope is expected to take no more than a predefined time period length (e.g., three time periods). Therefore, a total of three to five time periods are required for full stope production from the commencement of advancement to the completion of backfilling. The non-adjacent extracting and backfilling production and sequencing constraints are applied to ensure that mining activities progress sequentially through all stope production phases and maintain stability. The technical and economic parameters used for testing the models are shown in Table 1.

Table 1. The technical and economic parameters for the case study 1.

No.	Description	Value
1	Mining cost ($/t)	33.97
2	Processing cost ($/t)	9.82
3	Backfilling cost ($/t)	6.59
4	Advancement cost ($/t)	32.12
5	Price of gold ($/g)	45.19
6	Discount rate (%)	8.00
7	Mining recovery (%)	92.47
8	Mining dilution (%)	4.50
9	Mining loss (%)	8.00
10	Maximum grade (g/t)	2.90
11	Minimum grade (g/t)	2.60
12	Maximum mining capacity (Mt/Period)	0.55
13	Maximum processing capacity (Mt/Period)	0.55
14	Maximum advancement capacity (Mt/Period)	0.07
15	Maximum hoisting capacity (Mt/Period)	0.62
16	Maximum extraction duration (Periods)	2
17	Maximum active levels	3

Note: Period = two months.

The models were formulated in Matlab (2017a) (MATLAB and Release, 2016) and CPLEX environment (IBM., 2015). All stopes are extracted while meeting all mining operational constraints. Solution times for the model during all the testes are less than 10 seconds.

Results are shown in Fig. 5 to Fig. 8. It can be seen that the ore production increases to the maximum capacity in the second period and kept stable during all the production periods before the end of schedule periods. The mining and processing fluctuation constraints help to avoid peaks and dips often found in other conventional target/capacity constraints production schedules. The average ore grade can reasonably feed the processing. All the stope production phases including advancement, extraction and backfilling execute sequentially. Advancement is indicated by the color blue. It is followed by several periods of extraction (green) then backfilling (yellow). The stopes extraction duration and the number of simultaneous active levels during all the schedule periods fall within the boundary of the related constraints. In comparing the actual mining strategy of the underground gold mining operation in 2016 to the MILP production schedule, it is apparent that the MILP model optimized schedule improved NPV by $ 9.61 M (approximately 7.19% in this case).

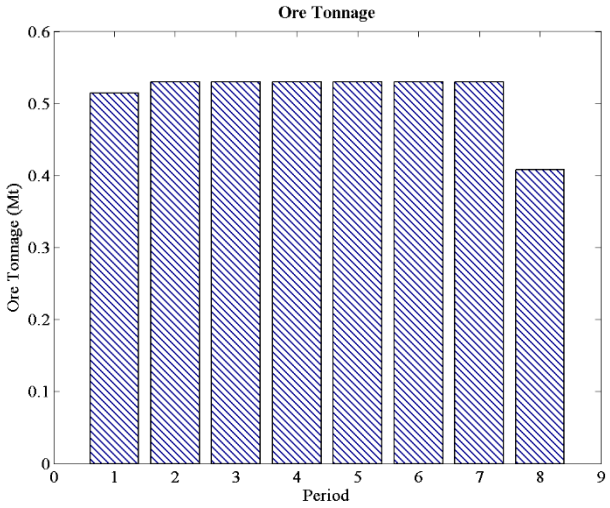


Fig. 5. Ore tonnage over 8 periods.

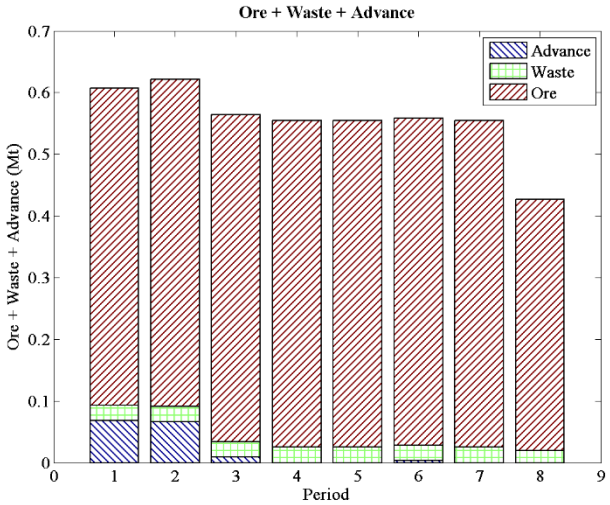


Fig. 6. Total tonnage over 8 periods.

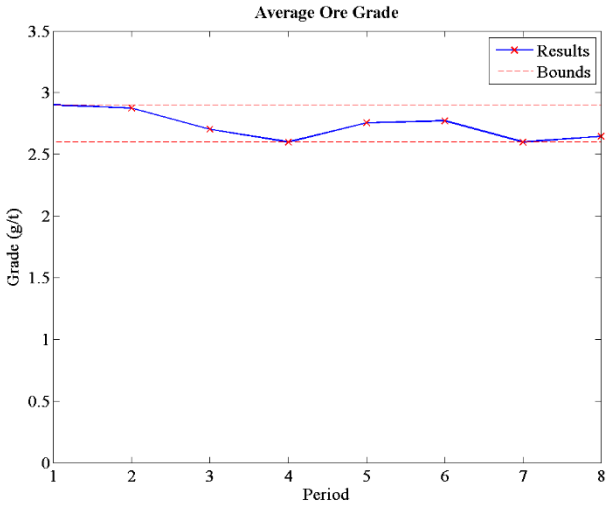


Fig. 7. Average ore grade over 8 periods.

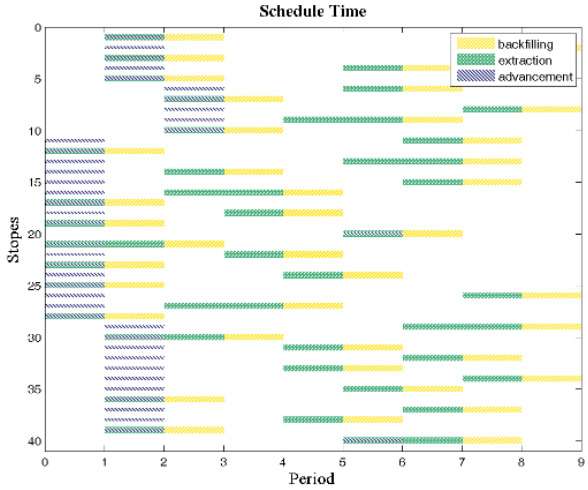


Fig. 8. Stopes mining schedule.

5.2. Case study 2

The case study 2 is a large-scale underground gold mining operation using cut-and-fill mining method. Annual production is approximate 3.6 million tons of ore, and 8000 kg of gold. The case study 2 is carried out using the actual stopes data as well as production parameters in order to facilitate the comparison research with actual mining strategy. The case study area contained 104 stopes, distributed in 8 levels. The size of the stope is 50 m in length, 40 m in height, and the width varies according to the width of the orebody ranging from 10 m to 30 m. the total tonnage of material in this area is almost 11 Mt with an average density of 2.78 t/m^3. Almost all the rock mass is ore with an average grade of 2.35 g/t. The technical and economic parameters used for evaluating the models are shown in Table 2.

The objective function of the model is to maximize the NPV at a discount rate of 0.08, while all the constraints are satisfied during a three years mining operation. The MILP model was solved by using the Matlab (2017a) (MATLAB and Release, 2016) programming and CPLEX solver (IBM., 2015) on a machine with Intel(R) Xeon(R) CPU E5-1650 v4 processor and 64 Gigabytes of RAM. The performance of the proposed MILP model is analyzed on the basis of the NPV, stability as well as practicality of the generated schedules. Comparisons between the actual mining strategy and the optimized schedule are presented herein.

The results of the optimized MILP model are shown in Fig. 9 to Fig. 12. Production schedule is produced per six monthlies instead of yearly in this case study. The tonnage and average grade of ore within each period are shown in Fig. 9, Fig. 10 and Fig. 11. It is obvious that the model can keep the mining capacity at the upper bound; moreover, it avoids production fluctuations benefiting from the mining and processing fluctuation constraints so as to provide a more stable ore and metal supply for the plant.

Fig. 12 illustrates the stope production phases is sequential in specific order of advancement, extraction and backfilling, in the meantime, extraction phrases are all within two time periods. The maximum number of the simultaneous active levels is limited to four. When considering control of active levels, the optimized scheduling production is more concentrated and requires significantly less equipment investment and movement.

The graphs shown in Fig. 13, Fig. 14 and Fig. 15 show the cumulative metal, rock production, and cash flow comparison between optimized results and actual mining strategy over six time periods. The MILP optimized schedule improved NPV by $ 17.15M, about 5.29 percent on the actual mining strategy. Apparently, the results show a significant improvement in metal production and NPV while

less total tonnages mined. This reflects that, the MILP model has the motivation to extract high-grade stopes earlier to generate better cash flow.

Table 2. The technical and economic parameters for case study 2.

No.	Description	Value
1	Mining cost ($/t)	33.97
2	Processing cost ($/t)	9.82
3	Backfilling cost ($/t)	6.59
4	Advancement cost ($/t)	32.12
5	Price of gold ($/g)	45.19
6	Discount rate (%)	8.00
7	Mining recovery (%)	92.47
8	Mining dilution (%)	4.50
9	Mining loss (%)	8.00
10	Maximum grade (g/t)	2.50
11	Minimum grade (g/t)	2.10
12	Maximum mining capacity (Mt/Period)	1.80
13	Maximum processing capacity (Mt/Period)	1.80
14	Maximum advancement capacity (Mt/Period)	0.15
15	Maximum hoisting capacity (Mt/Period)	1.95
16	Maximum extraction duration (Periods)	2
17	Maximum active levels	4

Note: Period = six months.

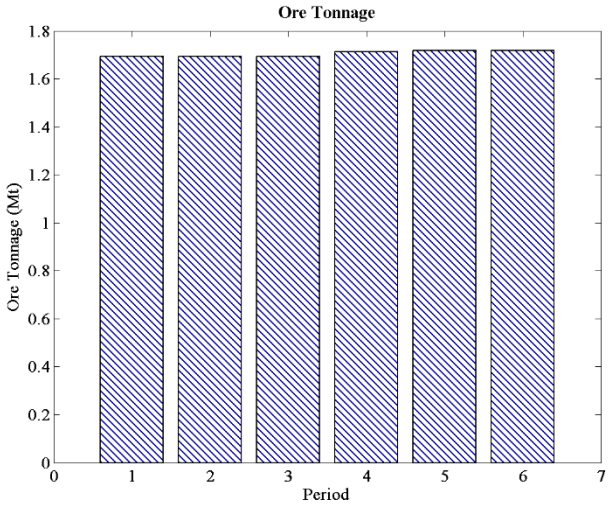


Fig. 9. Ore tonnage over 6 periods.

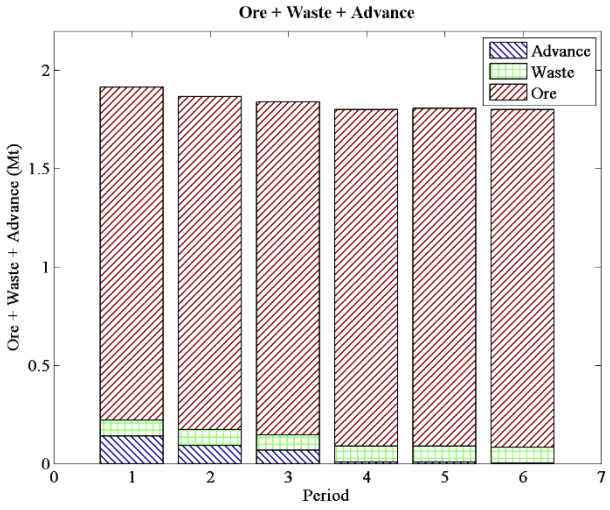


Fig. 10. Total tonnage over 6 periods.

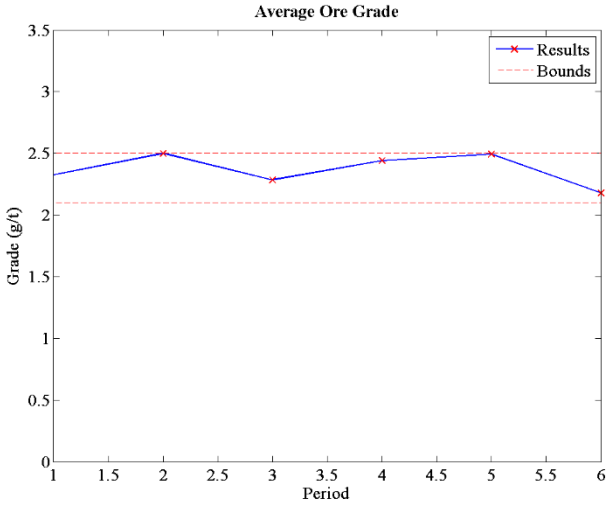


Fig. 11. Average ore grade over 6 periods.

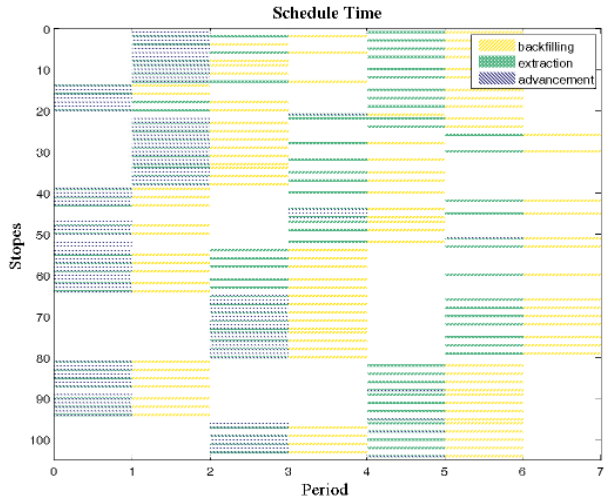


Fig. 12. Stopes mining schedule.

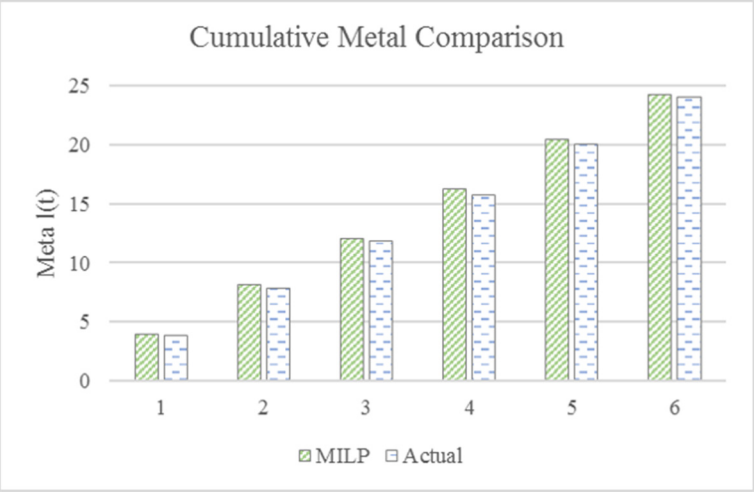


Fig. 13. Cumulative metal comparison.

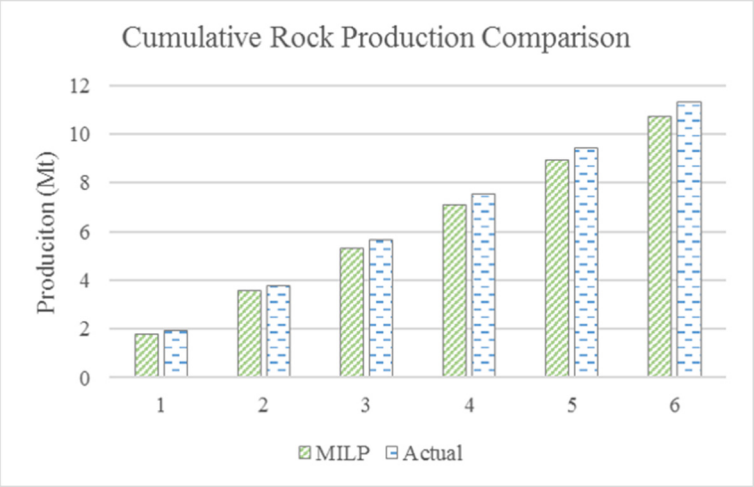


Fig. 14. Cumulative rock production comparison.

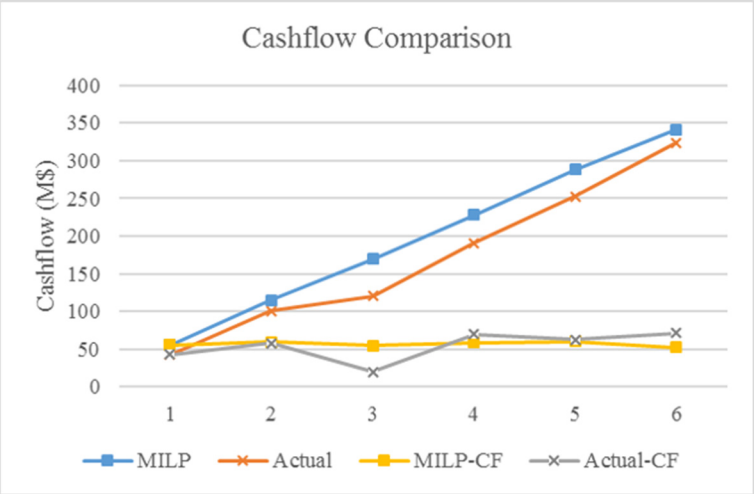


Fig. 15. Cumulative cash flow comparison.

6. Conclusions

This research presented an optimization framework for cut-and-fill mine production scheduling using mixed integer linear programming. The objective function of the model is to maximize the NPV of the operation while meeting all mining resource and operational constraints.

In addition to the mining and processing productivity constraints, and stope development and extraction sequencing constraints, the use of mining and processing tonnage fluctuation constraints ensure full utilization of mining equipment and further avoid excessive fluctuations during the production periods. Since the extraction duration and simultaneous active levels in each time period are controlled in the optimization, the production schedule obtained from the MILP model is more concentrated and requires significantly less equipment movement. All these operational constraints make the new MILP model more flexible and adjustable to different solutions and processes in relation to cut-and-fill mining production scheduling, as well as other underground mining variations.

The optimization formulation was implemented in a Matlab/CPLEX environment. Apart from the first and the last periods in two case studies, the main periods all reached the processing capacity at the upper bound (0.55 Mt/Period in case study 1 and 1.8 Mt/Period in case study 2). The average grade is reasonably within the desired range (2.6g/t to 2.9 g/t in case study 1 and 2.1g/t to 2.5 g/t in case study 2) in each period. The results compared with the actual mining strategy demonstrate the potential benefits of production schedule optimization for cut-and-fill mining using MILP. The comparison shows a 5% to 7% improved NPV resulting from mining higher grades and processing less tonnes thereby generating a better cash flow.

7. References

[1] Alford, C., Brazil, M., and Lee, D. (2007). Optimisation in underground mining. *Handbook of operations research in natural resources*, 561-577.

[2] Ben-Awuah, E. and Askari-Nasab, H. (2011). Oil sands mine planning and waste management using mixed integer goal programming. *International Journal of Mining, Reclamation and Environment*, *25* (3), 226-247.

[3] Ben-Awuah, E. and Askari-Nasab, H. (2013). Incorporating waste management into oil sands long term production planning. *Mining Technology*, *122* (1), 33-45.

[4] Ben-Awuah, E., Richter, O., Elkington, T., and Pourrahimian, Y. (2016). Strategic mining options optimization: Open pit mining, underground mining or both. *International Journal of Mining Science and Technology*, *26* (6), 1065-1071.

[5] Brickey, A. J. (2015). *Underground production scheduling optimization with ventilation constraints.* Colorado School of Mines.

[6] Chanda, E. (1990). An application of integer programming and simulation to production planning for a stratiform ore body. *Mining Science and Technology*, *11* (2), 165-172.

[7] Chatterjee, P. and Sridhar, C. (1986). Computer-aided mine design and planning for underground mines. *CIM BULLETIN*, *79* (893), 55-60.

[8] Denby, B. and Schofield, D. (1995). *The use of genetic algorithms in underground mine scheduling*. in Proceedings of Proceedings of the 25th international symposium application of computers mineral industries, pp. 389-394.

[9] Ebbels, A. M. (2016). Optimising underground mining strategies. *Australasian Institute of Mining and Metallurgy Publication Series*, p 60-65.

[10] Hamrin, H., Hustrulid, W., and Bullock, R. (2001). Underground mining methods and applications. *Underground mining methods: Engineering fundamentals and international case studies*, 3-14.

[11] Hu, N. L., Li, Y., Li, G. Q., and Yao, X. L. (2013). Optimization of open-pit-mining operational planning by using a particle swarm algorithm. *Journal of University of Science & Technology Beijing*, *35* (4), 537-543.

[12] IBM. (2015). IBM ILOG CPLEX optimization studio CPLEX user's manual, Version 12 Release 6. IBM Corp Armonk, NY.

[13] Jawed, M. and Sinha, D. (1985). System approach to production planning and scheduling with special reference to coal mines-A case study. *Journal of Mines, Metals and Fuels, February*, 37-41.

[14] Jie, H., Nai-Lian, H. U., Guo-Qing, L. I., Zhao-Yang, M. A., Di, L. I., and Jian, G. (2016). Dynamic optimization of production plans for multi-metal underground mines. *Chinese Journal of Engineering*.

[15] Kumral, M. (2004). Genetic algorithms for optimization of a mine system under uncertainty. *Production Planning & Control*, *15* (1), 34-41.

[16] Li, S. (2009). Application of Linear Programming Approach in Mining Schedule Planning. *Metal Mine*.

[17] Lizotte, Y. and Elbrond, J. (1985). Optimal layout of underground mining levels. *CIM BULLETIN*, *78* (873), 41-48.

[18] MATLAB and Release, S. T. (2016). The MathWorks, Inc. United States Natick, Massachusetts.

[19] Musingwini, C. (2016). Optimization in underground mine planning-developments and opportunities. *Journal of the Southern African Institute of Mining and Metallurgy*, *116* (9), 809-820.

[20] Nehring, M. and Topal, E. (2007). Production schedule optimisation in underground hard rock mining using mixed integer programming. *Proceedings-Project Evaluation. The Australasian Institute of Mining and Metallurgy, Melbourne*, 169-175.

[21] Nehring, M., Topal, E., and Knights, P. (2010). Dynamic short term production scheduling and machine allocation in underground mining using mathematical programming. *Mining Technology*, *119* (4), 212-220.

[22] Nehring, M., Topal, E., and Little, J. (2010). A new mathematical programming model for production schedule optimization in underground mining operations. *Journal of the Southern African Institute of Mining and Metallurgy*, *110* (8), 437-446.

[23] O'Sullivan, D. and Newman, A. (2014). Extraction and backfill scheduling in a complex underground mine. *Interfaces*, *44* (2), 204-221.

[24] Osanloo, M., Gholamnejad, J., and Karimi, B. (2008). Long-term open pit mine production planning: a review of models and algorithms. *International Journal of Mining, Reclamation and Environment*, *22* (1), 3-35.

[25] Pendharkar, P. C. (1997). Fuzzy linear programming model for production planning in coal mines. *Computers and Operations Research*, *24* (12), 1141-1149.

[26] Pourrahimian, Y. (2013). *Mathematical programming for sequence optimization in block cave mining.* University of Alberta (Canada).

[27] Pourrahimian, Y. and Askari-Nasab, H. (2014). An application of mathematical programming to determine the best height of draw in block-cave sequence optimisation. *Mining Technology*, *123* (3), 162-172.

[28] Pourrahimian, Y., Askari–Nasab, H., and Tannant, D. (2012). Mixed–Integer Linear Programming formulation for block–cave sequence optimisation. *International Journal of Mining and Mineral Engineering*, *4* (1), 26-49.

[29] Topal, E. (2003). Advanced underground mine scheduling using mixed integer programming.

[30] Trout, L. (1995). *Underground mine production scheduling using mixed integer programming*. in Proceedings of 25th International APCOM Symposium, Melbourne, Australia, pp. 395-400.

[31] Yao, X. L., Hu, N. L., Zhou, L. H., and Li, Y. (2011). Ore blending of underground mines based on an immune clone selection optimization algorithm. *Journal of University of Science & Technology Beijing*, *33* (5), 526-531.

Determination of the Optimum Undercut Level Elevation in Block Caving

Roberto Noriega and Yashar Pourrahimian
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

The selection of an elevation for the placement of the undercut level is a decisive initial step in the planning and design of block caving mines. It is key for the success of a caving project to define the undercut elevation that will most likely yield the highest possible Net Present Value (NPV) of the operation, considering the discounting periods from both the vertical draw rate extraction of the ore and the horizontal mining direction advancement.

This paper outlines a mathematical programming framework to determine the best undercut elevation, by formulating a simplified Linear Integer Programming (IP) model that captures the discounting of the profits from the vertical draw rate extraction and the horizontal advance direction that can be applied at the early stages of a caving project. The IP model comprises a simplified block caving scheduling algorithm, that considers "Mining Units" (MU), which are groupings of blocks within each column based on the minimum draw rate condition and the undercut elevation considered. The formulation considers the following constraints: mining capacity, minimum and maximum draw rate, vertical precedence within columns and a horizontal precedence based on a concave mining advancement front. The model is set on an iterative loop over the different possible levels.

The model is tested on a case study, where it provides a tool to evaluate the potential mineable reserves, define the optimal undercut elevation, and a starting schedule for future detailed engineering design.

1. Introduction

One of the initial steps in the long-term planning of Block Caving mines is the selection of an undercut elevation and the definition of a mining footprint, after which a draw point layout is built, and the production schedule is generated (Diering, 2000; Rubio, 2002). The selection of a specific undercut elevation constraints the downstream planning process having a direct influence in the potential NPV of the project and can be thought as an optimization problem on its own.

The current industry standard for the selection of an undercut elevation is the Footprint Finder module in GEMS PCBC software. As explained by (Diering et al., 2008), Footprint Finder evaluates the different levels considering each block within a specific level as a draw point from which the discounted economic value of the column above is calculated, with the capability of applying Laubscher's vertical mixing model (Laubscher, 1994) as well. The level economic value is then

This paper, in its entirety, has been accepted to be published in the proceedings of 4th International Symposium on Block and Sublevel Caving (Caving 2018), Vancouver, Canada, Oct 15-17, 2018.

obtained as the sum of each column within. A method described by (Elkington, 2012) finds the optimal cave outline for a given undercut elevation that maximizes the metal content, by following an approach similar to that of pit outline optimization, using an Integer Programming (IP) method considering operational constraints. This method works at a block level scale and provides an approximation to the potential final cave shape for a given elevation, however it does not involve economic parameters. Another proposed method by (Vargas et al., 2014) considers the caving outline definition as an inversed ultimate pit problem, and for each level calculates the vertically discounted economic value for the caving envelope considering precedence constraints on the inverted block model.

The economic value of the extracted ore is not only subject to vertical discounting on each individual column, as each column is opened at a certain period based on the mining direction and the starting point in the footprint. The starting point and direction of mining are also usually a geotechnical decision, as it is desirable to start extraction in weaker rocks to achieve the hydraulic radius required and steady production rates early in the mine life (Bartlett, 1992; Tukker et al., 2016). Therefore, the horizontal discounting due to the undercut layout sequence has to be considered in order to evaluate and select the best possible undercut elevation. The sequencing and scheduling problem in Block Caving is a complex optimization problem that involves millions of possible combinations for a given set of draw points on a specific undercut level (Khodayari and Pourrahimian, 2015). While the use of mathematical programming techniques can guarantee an optimal solution for Block Caving sequencing and scheduling (Chanda, 1990; Rahal et al., 2008; Pourrahimian et al., 2012; Pourrahimian et al., 2013; Pourrahimian and Askari-Nasab, 2014), it is often at the expense of large and prohibiting processing times.

The following method provides a simplified scheduling Integer Programming (IP) formulation applicable at the block model scale that can be repeated over multiple levels at reasonable processing times, to find the optimal undercut elevation for Block Caving mines considering time discounting over the vertical direction within each column, and the horizontal advancement direction.

Two assumptions are made to simplify the schedule optimization problem that are in line with block caving operational practices. The first assumption is that instead of considering each individual block as a single draw point, a grouping of individual adjacent blocks is performed to represent a draw column or draw zone. Within each draw zone, the individual blocks are then vertically aggregated into representative mining units based on the minimum draw rate required. These mining units are then scheduled based on a concave horizontal mining advancement front, that is desirable to provide a better control of structures and a more secure undercutting (Brown, 2002), and mining capacities constraints to evaluate the economic potential of each undercut level. These assumptions greatly reduce the number of variables and possible combinations while still being representative of the extraction practice in Block Caving operations and provide some useful insight for the planning process at early stages of the project.

2. Methodology

The first is to generate the projected draw column areas on the undercut footprint by grouping the individual block model columns, to generate a more representative scenario of the ore extraction in Block Caving mines where the extraction is carried from draw columns over draw points at the undercut level. The representative columns are selected in order to account for the potential draw point layout geometry configurations, such as the different spacing measures between the draw points and the minimum required hydraulic radius to sustain mining (Malaki et al., 2017), that are selected on a case-by-case basis depending on the properties of the rock mass. Figure 1 show a schematic representation of the aggregation of multiple individual block–model columns into a single representative draw column. This zone is parametrized by two dimensions, a size in X and a size in Y to define a rectangular area of influence to use to the block model information for calculation of

tonnages and metal content within it. The dimensions of this representative column can be adjusted to match the required spacing between draw points, A and B in Figure 1, but is constrained by the dimensions of the underlying block model.

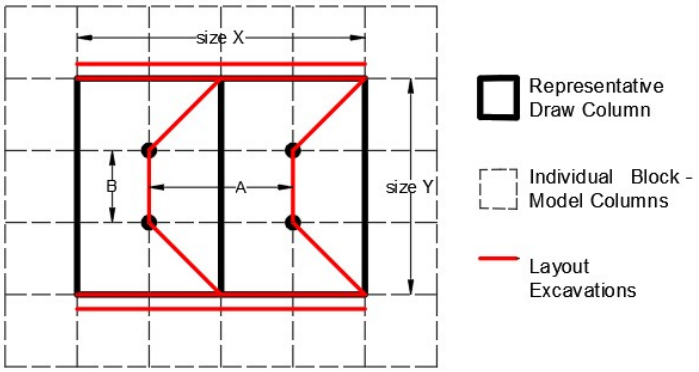


Fig. 1. Schematic of the draw column aggregation from the individual block-model columns.

The grouping of the columns into the representative units for a given undercut is then an optimization problem on its own, and its solution can provide useful insight on the draw point layout design, and a systematic way to evaluate different levels automatically. To optimize the layout, the key parameter considered is to maximize the total metal content of the units. The metal content was selected to not include the economic parameters yet, but still guarantee a priority on the selection of high grade areas. The metal content of each unit is calculated as the summation of the metal contents of each individual block model columns that are part of it. Each column is considered as continuous and the metal summation is carried until a first block of waste is found or a specific maximum column height is reached, to avoid waste material within the cave profile and to consider the operational maximum draw height.

Based on the representative unit dimensions in X and Y, all possible units are generated on a given undercut elevation plan view and the associated total metal content for each representative column is calculated as described above.

The optimization problem is then to find the combination of column units that yield the highest metal content from the deposit. The optimal column units must be adjacent and not include units with individual columns that do not the meet a minimum height operational constraint. To achieve this, all column units that do not meet the operational criteria are assigned a zero-metal content value, and the optimization model includes a constraint to avoid overlapping. This initial IP optimization model is described below.

Indices

$p \in \{1,...,P\}$ Index for all possible column units

$b \in \{1,...,\mathrm{B}\}$ Index for all possible ore blocks in a given undercut level

Sets

O^b Set containing all column units that overlap with block b, with number of elements $N\left(O^b\right)$

Decision Variables

$x_p \in \{0,1\}$ Binary variable controlling the decision to include unit p in the layout

$y_b \in \{0,1\}$ Binary variable used to ensure that all ore blocks in the undercut are covered with no gaps in the layout

Parameters

M_p Metal content of column unit p

Objective Function

$$Max \sum_{1}^{P} M_p \times x_p \quad (1)$$

Constraints

$$\sum_{1}^{N(O^b)} x_p \leq y_b \qquad \forall b \in \{1,...,B\} \quad (2)$$

$$y_b = 1 \qquad \forall b \in \{1,...,B\} \quad (3)$$

Equation (1) represents the objective function to maximize the metal content of the selected units, with the consideration that units with any individual column that does not meet the minimum column height (continuous ore from the undercut) are assigned a 0 value. Equation (2) and Equation (3) represent the overlapping constraint to assure that only one column unit from all those that share at least a single block is selected, while also guaranteeing that all ore blocks in the undercut level are covered to avoid gaps in the layout.

After this initial step a layout is generated for any given undercut, that simplifies the number of variables involved for the next scheduling optimization step while still being representative of the operating conditions in block caving operations.

The second assumption mentioned above is the grouping of block slices along the vertical section of each of the column units based on the minimum draw rate. The decision to group the blocks on these mining units is based on the condition that on each period at least the minimum tons, or inches, must be drawn continuously from each unit. The tonnage of each mining unit is the summation of the tonnage of the individual blocks within it, and the grade of the mining unit is the tonnage-weighted average grade of the individual blocks. As observed in Figure 1 this minimum draw rate would apply to each pair of draw points on the same excavation and extracting ore from the same draw zone. While resolution is lost for the scheduling along the vertical section of each column unit due to the grouping, it further reduces the number of variables and possible combinations to achieve an optimal solution for the scheduling problem under reasonable processing times.

The simplified IP model then is formulated to maximize the discounted profit, or NPV, of the extraction of each mining units over several periods. The model is constrained by the total mining capacity, the maximum draw rate from each column unit, the vertical precedence within each column and the horizontal precedence across the layout based on a concave advancement front.

Indices

$u \in \{1,...,U\}$ Index for all mining units

$t \in \{1,...,T\}$ Index for all periods

$c \in \{1,...,C\}$ Index for all column units

Sets

C^c Set containing the mining units that are within column unit c. Each set has a total number of elements C^c.

$V^{u,c}$ Single element set containing the mining unit directly below unit u of column c.

$H^{u,c}$ Set containing the column unit that must be opened previously to the extraction of unit u of column c based on a horizontal concave mining direction. Each set has a total number of elements H_u.

Decision Variables

$z^t_{u,c} \in \{0,1\}$ Binary variable controlling the decision to extract mining unit u of column c.

Parameters

$P^t_{u,c}$ Discounted profit for the extraction of unit u of column c.

$\overline{M}$ Maximum mining capacity (tons/period)

$\underline{M}$ Minimum mining capacity (tons/period)

T_u Tonnage of mining unit u.

$\overline{DR}$ Maximum draw rate from each column unit (tons/period)

$\underline{DR}$ Minimum draw rate from each column unit (tons/period)

Objective Function

$$Max \sum_{1}^{T}\sum_{1}^{P} P^t_{u,c} \times z^t_{u,c} \tag{4}$$

Constraints

$$\underline{M} \le \sum_{1}^{P} T_u \times z^t_{u,c} \le \overline{M} \qquad \forall c \in \{1,...,C\} \quad \forall t \in \{1,...,T\} \tag{5}$$

$$\sum_{1}^{T} z^t_{u,c} \le 1 \qquad \forall u \in \{1,...,U\} \quad \forall \mathrm{c} \in \{1,...,C\} \tag{6}$$

$$z^t_{u,c} \le z^t_{V^{u,c}} + z^{t-1}_{V^{u,c}} \qquad \forall u \in \{2...,U\} \quad \forall c \in \{1,...,C\} \quad \forall t \in \{2,...,T\} \tag{7}$$

$$\sum_{1}^{C_c} T_{u,c} \times z^t_{u,c} \le \overline{DR} \qquad \forall u \in \{1,...,U\} \quad \forall \mathrm{c} \in \{1,...,C\} \quad \forall t \in \{1,...,\mathrm{T}\} \tag{8}$$

$$H_u \times z^t_{1,c} \le \sum_{1}^{T}\sum_{1}^{H_u} z^t_{1,c} \qquad \forall u \in \{1,...,U\} \quad \forall \mathrm{c} \in \{1,...,C\} \quad \forall t \in \{1,...,\mathrm{T}\} \tag{9}$$

Equation (4) shows the objective function of the optimization model, that maximizes the total NPV of the operation by considering the discounted profit from the extraction of each block. The profit is calculated as the difference between the revenues and the costs associated to mining each block and is then discounted based on the period of extraction. Equation (5) corresponds to the mining capacity constraint, and forces that the total ore tonnage extracted from all columns over each period is within the minimum and maximum established ranges. Equation (6) guarantees that no mining unit is extracted more than once. Equation (7) establishes the vertical precedence within mining units of the same column, and guarantees that to extract a unit, it's directly below unit must be extracted on the

same or previous period to assure a continuous draw of the column. Equation (8) forces that the total ore tonnage extracted from each column unit on each period is less or equal than the maximum draw rate. Since the mining units are built based on the minimum draw rate volume there is no need to include a minimum draw rate constraint. Finally, Equation (9) forces the horizontal precedence constraint. The horizontal precedence constraint is based on a concave mining advancement front and defined by three angles, the azimuth of the main advancement direction a, and the two diagonal angles $a1$ and $a2$, as shown in Figure 2. The constraints guarantee that a column unit can initiate extraction only after all the directly preceding columns, based on the mentioned direction, have started extraction.

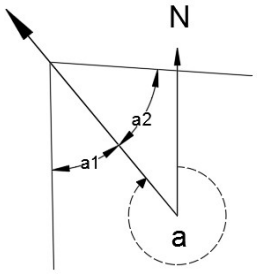


Fig. 2. Concave advancement front for the establishment of the horizontal precedence constraint.

The optimization framework then corresponds that for a given undercut elevation, the column unit layout that maximizes the metal content is found first, then the mining units are built based on the optimized layout and the minimum draw rate to be scheduled considering the vertical and horizontal profit discounting, and operating constraints as well. The process is then carried over the different possible levels of the deposit to select the level that yields the highest NPV. The model then can be used to individually evaluate multiple scenarios, such as different mining directions or operational parameters, on any specific undercut level of interest.

3. Case Study

The optimization model was tested on a copper deposit case study. A block model of the deposit is built with blocks of $10\ m \times 10\ m \times 10\ m$, as shown in Figure 3. The deposit has a total of 58.5 Mtons of ore with a weighted average grade of 1.46% copper.

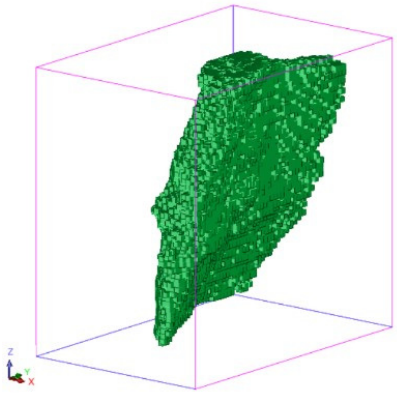


Fig. 3. Isometric view of the block model for the case study.

The IP optimization framework is solved using a MATLAB environment (MathWorksInc, 2018) and the IBM ILOG CPLEX engine (IBM, 2015) with a 5% gap (the feasible integer solution found is proven to be within the five percent of the optimal), on an Intel Core i7 machine with 3.40 GHz and 16 GB of RAM.

The dimensions considered for the column unit layout optimization of each undercut sections are 20m in the X direction and 30m in the Y directions, to mimic some common spacing measures of 10m between draw points on the same drawbell and 20m between different draw point excavations.

The operational and economic parameters considered are shown in Table 1.

Table 1. Operational and economical parameters for the application of the optimization framework.

Parameter	Value	Unit	Description
T	15	Year	Number of periods
$\overline{M}$	1200	kton/period	Maximum mining capacity per period
$\underline{M}$	500	kton/period	Minimum mining capacity per period
$\overline{DR}$	150	kton/column/period	Maximum draw rate per column unit per period
$\underline{DR}$	60	kton/column/period	Minimum draw rate per column unit per period
$\overline{H}$	300	meters	Maximum column height
$\underline{H}$	100	meters	Minimum column height
Recovery	85	%	Metal process recovery
Price	6000	$/ton	Selling price of the metal
Cost	26.1	$/ton	Extraction cost (Mining + Processing)
Discount Rate	10	%	Discount rate
a	290	degree	Main horizontal mining direction
a1	45	degree	Concave face angle
a2	45	degree	Concave face angle

The minimum draw rate from which the mining units are built on this test model was 60 kton/column/period, which considering the dimensions of the column units used is roughly equivalent to drawing about 5 inches per day from each column unit. This value is in line with some reported operational parameters. The optimization model is run for an interval of levels between 30 (elevation 705 meters) and 45 (elevation 555 meters) from the block model z index. Figure 4 shows the NPV and metal content obtained by the optimization model from each level.

The optimization model yielded level 38, at an elevation of 625 m, as the optimal undercut elevation for the case study, with a NPV of 587.85 MUSD and a total of 318 kton of Cu. This level is selected by analyzing not only the total blocks above each undercut but considering operating constraints and a mining advancement direction, with the mining reserves and NPV obtained subjected to operational constraints. The undercut layout consists of 64 column units, which are representative of 128 draw points. Overall, the computing time for each level was about 30 – 60 minutes depending on the number of ore blocks in each undercut plan section. The undercut ore areas varied between 38,000 m^2 to 57,000 m^2. On larger undercuts from massive deposits, the processing times can be expected to be longer, however an optimal solution can still be reached in a reasonable time.

Figure 5 shows the optimal layout for level 38. This layout maximizes the metal content of the deposit based on the described aggregation of individual columns into the column units to represent the

extraction practice in block caving, considering operational constraints. This layout also gives some insight on the potential drawpoint layout for detailed engineering studies at a later stage of the project.

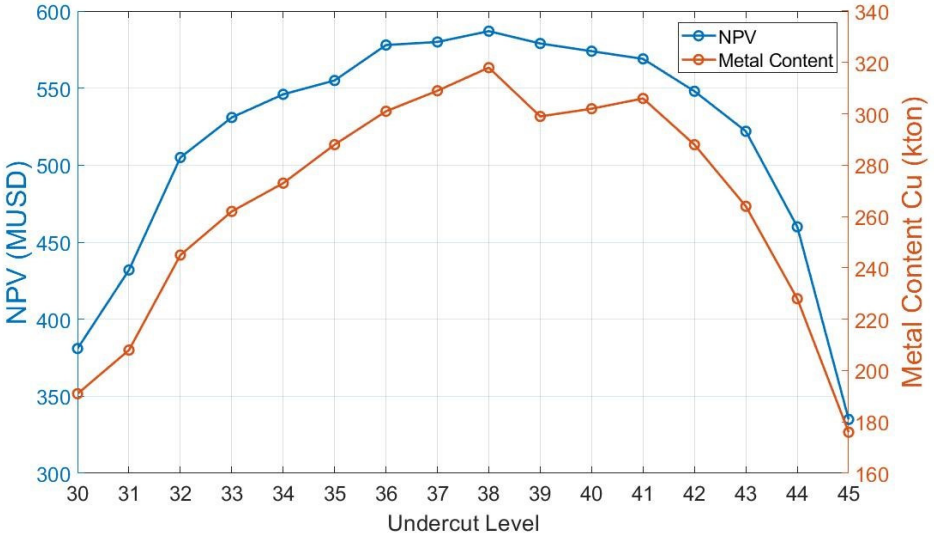


Fig. 4. NPV and metal content obtained from the optimization model for each level. Level 38 appears as the optimal undercut level.

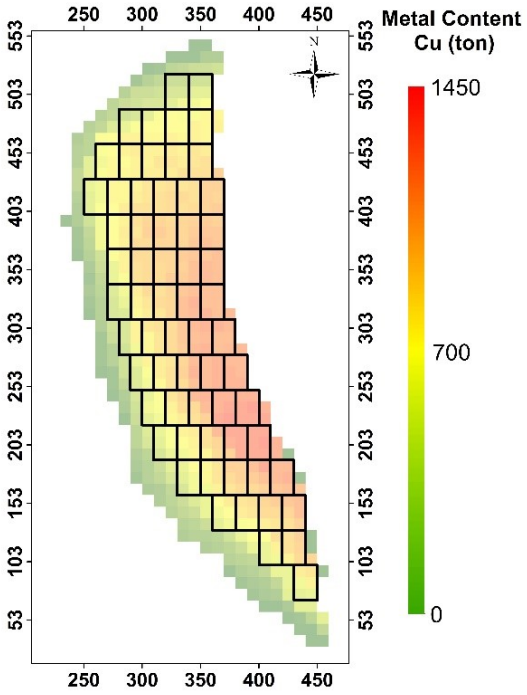


Fig. 5. Draw layout for the optimal undercut level 38, showing the metal content.

Figure 6 shows the extraction opening sequence of the column units in the optimal undercut level. This sequence achieves the maximum NPV considering the horizontal concave mining direction. At the moment of the development of this model, the starting point is selected to be the farthest column unit based on the direction advancement.

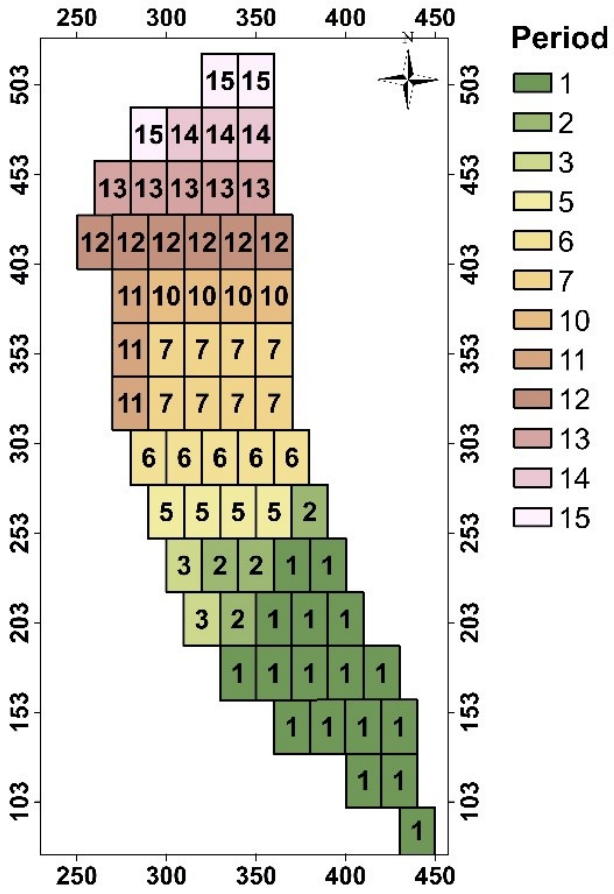


Fig. 6. Sequence of column opening to apply the horizontal discounting on the NPV at undercut level 38.

One of the main concerns in mine planning is the risk of building an initial design based on a set of economic assumptions that are uncertain in the future. To deal with this uncertainty, a sensitivity analysis is carried to understand the behavior of a decision or design over multiple scenarios. The model proposed can be easily set up to evaluate multiple scenarios for any required economical or technical parameter.

As an example, a sensitivity analysis on the price of the metal is presented. The price of the metal is a decisive factor on the profitability over the life of a mine, but it is also one of the most uncertain parameters as it is dictated by global market conditions.

For the model presented, the response of the NPV and, most importantly, the most profitable level is evaluated over different price scenarios. The selection of an undercut elevation is not a flexible decision, once development has started it not possible or very costly to start over. Therefore, it is important to evaluate multiple economic conditions to make a decision. There is also geotechnical uncertainty, however the model takes as input parameters a starting point and mining direction as well as draw rates, all of which are assumed to be defined considering the proper geotechnical environment. Table 2 shows the results for the sensitivity analysis on different prices. Two lower cases and two upper cases were considered at 10% intervals each. The model was set up to run on the same technical parameters as specified previously.

The optimum level remains at level 38, at an elevation of 625 m, for both the upper cases and up to a -10% variation in the price. That means that starting the operation with the undercut placed at that level will maximize the profitability of the mine even over optimistic scenarios, and up to 10% metal price value reductions. The optimum level does change when a -20% reduction in the copper price is considered, and moves up to level 39, at an elevation of 635 m.

Table 2. Sensitivity analysis for different price scenarios.

	Lower Cases		Base Case	Upper Cases	
Variation	-20%	-10%	-	+10%	+20%
Price ($/ton)	4800	5400	6000	6600	7200
Optimum Level	39	38	38	38	38
NPV(M$)	327.34	430.95	587.85	712.32	824.61

On a pessimistic scenario, with the price set at 4800 $/ton, a better NPV while scheduling based on the mining direction is found at a different level. However, setting the undercut at level 38 seems as a robust decision, as it maximizes the NPV over multiple price scenarios and would likely be the most profitable decision for this case study.

4. Conclusions

The optimization framework achieves an optimal solution in under 20 minutes for each level and gives insight on the possible alternatives for footprint layout design and development sequence. The fast processing times allow for an iterative process to find the most profitable undercut elevation considering operational constraints and guaranteeing an optimal solution via an IP model. While the assumptions used to simplify the problem cause a loss of resolution for the long-term scheduling of the caving mine, the final goal of this model is not to find and optimal schedule but to include the mining direction discounting on the NPV evaluation for the selection of an undercut elevation.

Although it would be expected for the computing time to increase for massive deposits with a larger footprint area, the model could still perform on reasonable times as on this particular case at a footprint of 57,000 m^2 the optimal solution was reached after 20 minutes of processing, and the iterating process over multiple levels is a computational parallelisable process.

The model could be still improved by considering dilution mixing models such as Laubscher's (non-linear mixing models would require another type of mathematical formulation), considering grade uncertainty via multiple geostatistical simulations of the deposit, and considering specific starting points for the extraction sequence. Multiple metals can also be considered when building the objective function, that is when calculating the economic block values. Considering multiple economic metals, common in many porphyry deposits suitable to block caving, would not add to the computing times.

The mining direction and starting point in the sequence still play a major role in the NPV of a caving project. The model takes as an input a mining direction, usually defined considering the geotechnical conditions of the deposit and evaluates the profitability of setting the undercut level at different elevations considering the economic discounting from this direction. Once an elevation or a small number of undercut levels have been found to yield the highest NPV for the deposit, the model could be applied to evaluate different mining directions and scenarios.

The initial layout optimization step is also of interest as it could be improved to account for the different geometrical constraints of the common draw point layout schemes, and possible underground infrastructure placement, in order to develop a tool to optimize the draw point locations in the caving footprint.

5. References

[1] Bartlett, P. (1992). The Design and Operation of a Mechanized Cave at Premier Diamond Mine. in *Proceedings of MassMin 1992*, H. W. Glen, Ed. Johannesburg: SAIMM

[2] Brown, E. T. (2002). *Block Caving Geomechanics*. Julius Kruttschnitt Mineral Research Centre, Queensland, Australia,

[3] Chanda, E. (1990). An application of integer programming and simulation to production planning for a stratiform ore body. *Mining Science and Technology*, *11* (2), 165-172.

[4] Diering, T. (2000). PC-BC: A Block Cave Design and Draw Control System. in *Proceedings of MassMin 2000*. Melbourne, Australia: The Australasian Institute of Mining and Metallurgy

[5] Diering, T., Richter, O., and Villa, D. (2008). *Block Cave Production Scheduling Using PCBC*. in Proceedings of Proceedings of the 5th International Conference and Exhibition on Mass Mining, Lulea, Sweden,

[6] Elkington, T. B., Lee; Richter, Otto. (2012). Block Caving Outline Optimisation. in *MassMin 2012*. Sudbury, Ontario, Canada

[7] IBM (2015). *IBM ILOG CPLEX Optimization Studio CPLEX User's Manual Ver. 12 Rel. 6.* IBM Corp.,

[8] Khodayari, F. and Pourrahimian, Y. (2015). Mathematical Programming Applications in Block-Caving Scheduling: A Review of Models and Algorithms. *International Journal of Mining and Mineral Engineering*, *6* (3), 234 - 257.

[9] Laubscher, D. (1994). Cave Mining: State of the Art. *Journal of the SAIMM*, 279-293.

[10] Malaki, S., Khodayari, F., Pourrahimian, Y., and Liu, W. (2017). *An Application of Mathematical Programming and Sequential Guassian Simulation for Block Cave Production Shceduling*. in Proceedings of Proceedings of the First International Conference on Underground Mining Technology, Australian Centre for Geomechanics ACG, Sudbury, Canada,

[11] MathWorksInc (2018). *MATLAB Primer R2018a.* MathWorks, Inc.,

[12] Pourrahimian, Y. and Askari-Nasab, H. (2014). An Application of Mathematical programming to Determine the Best Height of Draw in Block-Cave Sequence Optimisation. *Mining Technology*, *123* (3), 162-172.

[13] Pourrahimian, Y., Askari-Nasab, H., and Tannant, D. (2012). *Block Caving Production Scheduling Optimization Using Mathematical Programming*. in Proceedings of Proceedings of the Sixth International Conference and Exhibition on Mass Mining, Sudbury, Canada,

[14] Pourrahimian, Y., Askari-Nasab, H., and Tannant, D. (2013). A multi-step approach for block-cave production scheduling optimisation. *International Journal of Mining Science and Technology*, *23* (5), 739-750.

[15] Rahal, D., Dudley, J., and Hout, G. (2008). Developing and optimised production forecast at Northparkes E48 mine using MILP. in *Proceedings of the Fifth International Conference and Exhibition on Mass Mining* H. Schunesson and E. Nordlund, Eds. Lulea, Sweden: Lulea University of Technology

[16] Rubio, E. (2002). Long Term Planning of Block Caving Operations Using Mathematical Programming Tools. MSc. Thesis Thesis, University of British Columbia,

[17] Tukker, H., Holder, A., Swarts, B., van Strijp, T., and Grobler, E. (2016). The CCUT block cave design for Cullinan Diamond Mine. *Journal of the SAIMM*, *116*

[18] Vargas, E., Morales, N., and X, E. (2014). *Footprint and economic envelope calculation for Block/Panel caving mines under geological uncertainty*. in Proceedings of Proceedings of the Third International on Block and Sublevel Caving, Santiago, Chile, pp. 449 - 456.

Application of Mathematical Programming for Stope Layout and Production Schedule Optimization in Sublevel Stoping

Zeinab Basiri and Yashar Pourrahimian
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

The economics of today's mining industry is such that the major mining companies are increasing the use of massive underground mining methods. Follow to this attraction, mine planning in underground mining and its optimization have been considered more seriously in recent years. Two aspects of mine planning optimization in underground mining are stope layout and production scheduling optimization. Few algorithms have been presented to optimize the stope layout and stope production scheduling for underground mining; however, those are not able to provide an optimum solution. Most of the presented algorithms are heuristic so they cannot guarantee to achieve the optimum solution. The objective of this study is to present the mathematical formulations to find the optimum stopes layout and production scheduling in sublevel stoping method. The stopes layout and production scheduling optimization are applied based on the total blocks in the block model altogether (LOT and SOT methods) and based on the separated levels of block model (LOL and SOL methods) to see the impact of leveling in the stopes layout and production scheduling optimization which has not been clarified in the previous researches. The proposed methodologies maximize the economic value (EV) to determine the stopes layout. The presented production scheduling algorithm maximizes the NPV over the life of mine by using the Binary Integer Programming (BIP) while honoring the constraints such as only one-time mining, stopes adjacency, the connection between mining the stopes and activation of levels, concurrent active levels, and the delay between activation of the levels. The methods have been applied to a block model to check the application of the model in real size problems. Achieved EV by LOT method is 4% higher than LOL method; however, the running time of LOT method is 418 times more than LOL method. Also, from the accessibility of production levels to all the stopes during mining point of view, LOL presented the practical stops layout. The NPV of SOT method is 22% higher than the NPV of SOL method; however, the running time of SOT method is 3.4 times more than SOL method.

1 Introduction

The economics of today's mining industry is such that the major mining companies are increasing the use of massive underground mining methods. They expect that approximately fifty percent of the world's copper production will come from underground mines by 2020. It is a step change for the industry, move from the traditional open-pit mining to the underground mining. Because of attraction to the underground mining, the mine planning in underground mining and its optimization have been considered more seriously in recent years. Compare to the open-pit mine, limited techniques and algorithms are available for underground mining because of the complexity and less flexibility due to geotechnical and operational constraints in underground mining. In fact, to model the problems in underground mining, more constraints and variables are required. The number of variables at mix integer programming (MIP) may exceed hundred thousand in the planning of underground mining (Little et al., 2011). Even, this complexity can be higher in

sublevel stoping mining method because of some condition such as the alignment of the extraction level, and non-concurrent production (Copland et al., 2016).

Based on Topal (2008) the optimization of three aspects of the underground mine planning is the center of the attention. These aspects are stope layout, production scheduling and infrastructure. The stope layout optimization determines the dimension and the locations of the stopes. Production scheduling is defined as the sequential order of the preparation, extraction and backfilling of the stopes. The goal of stope layout and production scheduling optimization is profit maximization. In fact, stope layout and production scheduling optimization are tools to maximize the profitability of mining over mine life while the operational and geotechnical constraints are met.

The existing algorithms for underground stope optimization are divided into two sets level-based and field-based. Level-based algorithms of stope optimization implement the optimization on the different levels or panels of the block model; however, field-based stope optimization algorithms are applied on the block model before dividing into levels or panels (Sotoudeh et al., 2017).

Since different mining methods obtain the different geotechnical constraints, it is not reasonable to define a general purpose optimization algorithm suited for all underground mining methods (Bai et al., 2012). As a result, in this study in contrast with the majority of previous researches, the optimization of stopes layout and production scheduling specifically in sublevel stoping method, which is suitable for the wide vein-type steeply dipping deposits, is investigated.

The focus of this study is on two aspects of three aspects of optimization in underground mine planning presented by Topal (2008). Create the mathematical optimization models to find the optimum stopes layout and optimum production scheduling in sublevel stoping are the purposes of this study. Besides, the impact of applying the level-based and field-based algorithms is evaluated. In this research, MATLAB (Math Work Inc., 2017) is employed as a numerical modeling platform and IBM ILOG CPLEX Optimization Studio (12.7.1) as a solver to optimize stopes layout.

2 Literature Review

Sublevel stoping which also is referred to as long-hole stoping or blast-hole stoping is a vertical large-scale underground method. This method is a proper underground mining technique for wide vein-type deposits with stable host rock and competent steeply dipping ore-bodies (Haycocks et al., 1992; Lawrence, 1998). After finishing the development of declines, shafts, raises, orepasses and production levels, a raise slot or a winze is operated into one corner of the stope from one sublevel to the next sublevel, and drawpoints and funnels are provided. After completing the infrastructures and constructing the slot, the drilling, blasting and extraction in the stope can be started. Extracted ore in drawpoints is transported to the crusher or the surface. After extracting the ore within the stope, stope is backfilled by a mixture of mill tailings and cement (Hartman, 1992; Haycocks et al., 1992; Nehring, 2011).

The goal of designing stopes in sublevel stoping is achieving the highest profit by defining the best location, size and number of stopes within an ore-body, while the geotechnical stability concerns are met. The stope size depends on the size and shape of the ore-body (Nehring, 2011). The main consideration with planning and scheduling of sublevel stoping method is geotechnical nature of the ore-body such as faults and principal stress directions. Beside the geotechnical nature of the ore-body, other parameters related to the ore-body including shape, continuity and grade distribution of the ore-body are important parameters in designing and scheduling of sublevel stoping method (Mann, 1998).

Optimization techniques for defining the stope layout back to the more than forty years ago. Riddle (1977) presented the first algorithm, called “Dynamic Programming Algorithm” to find optimum stope layout in block-caving mining method. This method solves the 3D problems by using 2D north-south sections and east-west sections. Although the presented algorithm can optimize the

sections, it fails to find the optimum stope in 3D because it does not consider all necessary constraints simultaneously.

Deraisme et al. (1984) used the Downstream Geostatistical Approach to determine optimal stope. This model is a 2D sectional numerical model. Generally, the Downstream Geostatistical Approach is based on a combination of conditional simulation with underground mining simulation to compare selectivity, productivity, and profitability in cut-and-fill and block caving methods.

Cheimanoff et al. (1989) described a heuristic approach with binary-tree division technique, called "Octree Division Approach", to move from geological resources to mineable reserves based on the mining constraints and provides a 3D solution to find optimum stope. In fact, this model is based on the removing the non-desired mining blocks to define minimum stope size. Since this algorithm does not control the amount of the waste in the final mine layout, it cannot guarantee to reach the optimum stopes layout.

Ovanic et al. (1995) developed one-dimensional "Branch and Bound Technique" to optimize outline of the stope based on the optimizing of starting and ending points of mining locations within each row of the blocks. In contrast with previous algorithms, having regular or uniform the shapes blocks is not required in their algorithm.

Alford (1996) described a heuristic model called "Floating Stope Algorithm", which is similar to the "Moving Cone" method in open-pit optimization, to set up the optimal stope boundary. This algorithm applies to use in all underground mining method. The main constraint in this algorithm is the geometry of the stope. Inner and outer envelopes are the model's outputs, and the solution is located between these two envelopes which are defined by users. It means finding the solution relays on the users' experience that can be led to the error. This algorithm is the based algorithm on the Datamine software.

Ataee-pour (2000) and Ataee-pour (2005) presented a heuristic algorithm and called it "Maximum Value Neighborhood" (MVN). He defined the neighborhood concept based on the number of mining blocks equivalent to minimum stope size. The MVN algorithm is applied to all underground mining method. MVN algorithm failed to determine the optimal stope layout. However, it guarantees the optimum value neighborhood for each block.

Topal et al. (2010) proposed a heuristic algorithm to find optimum stopes layout in the case of single as well as variable stope sizes in 3D. Their proposed methodology can be used in all underground mining method. Their algorithm works based on two assumptions. Firstly, all stopes have a fixed start-up time, and the production and backfilling time have a linear relation with the stope volume. Secondly, the calculation of NPV is based on the mining of single stope at a given time.

Bai et al. (2012) suggested a new 3D method using "Network Flow Algorithm" to design stopes layout. This model is based on a cylindrical coordinate. They introduced an optimization algorithm that was suitable for sublevel stoping method. Since their algorithm is based on the cylindrical coordinate system with vertical raise, this algorithm is not acceptable in the case of sub-vertical or sub-horizontal deposits that need inclined raise. Furthermore, this approach is based on the small ore-body with single raise parameters, and it is not useful for larger ore-bodies that need many contiguous stopes.

Sandanayake (2014) and Sandanayake et al. (2015a) offered the algorithm to maximizes the economic value regarding the physical and geotechnical constraints. They claim that the algorithm is flexible enough for varying underground mining situations. Sandanayake et al. (2015b) continued their work. The new algorithm considers the fixed and variable stope sizes. They defined an upper bound to limit the number of possible solutions to decrees the solution time.

Nikbin et al. (2018) introduced a Hybrid Algorithm that is a one-dimensional polynomial-time algorithm. This algorithm is a combination of 'Dynamic Programming Algorithm and Greedy

algorithm. The algorithm includes three main steps. This algorithm guarantees the optimal solution for a selected row or column of a block model although may fail to reach the optimum solution.

The principal of the work to production scheduling presented by Manchuk (2007) is creating the sequence decisions based on information about the stopes and timing of operations and calculating the probabilities, which dictate the sequence based on this information. He presented the simulated Annealing and a Logic driven algorithm as the optimization techniques. Based on his research, the simulated Annealing technique achieves a better solution in a shorter time.

Nehring (2011) executed the research based on the short and medium term production and activity scheduling for sublevel stoping mining method. His research presented three mathematical optimization models using mixed integer programming to evaluate the relationship between medium and short term scheduling.

Little (2012) presented two approaches to define the stopes layout and production scheduling in sublevel stoping method called integrated approach and isolated approach. In an integrated approach, the optimization of stope layouts and production schedules are done simultaneously. In an isolated approach, the optimum stope layout is defined as the first step, and then the production scheduling is applied.

Copland et al. (2016) proposed a model to maximize the profit from mining the stopes minus the cost of the level development. The model is employed for sublevel stoping underground method. Their primary focus is on using the binary integers programming to help the model in order to decrease the solution time compare to the previously presented production scheduling models.

Generally, the significant limitations of the current stope layout and production scheduling optimization in sublevel stoping are as followed:

- Not any algorithms consider the level-based and field-based optimization with the same data set to be able to compare the results.
- Most of the algorithms are applicable for all mining methods although different mining methods obtain the different geotechnical constraints.
- Only a few numbers of researches are presented the mathematical algorithm with the optimum solution.
- Not all the defined constraints are practical for the case of sublevel stoping method. Also, none of the studies covers all the required constraints for the real situation, for instance, concurrent active levels, and the delay between activation of the levels constraints are ignored in those algorithms.
- Not all the algorithms are able to solve the large-scale problems.

3 Proposed Algorithm Methodology

This study present two methods of stope layout optimization. The first method called LOT method considers total blocks of the block model altogether. The second method called LOL, the block model is divided into the levels, and possible stopes are created for each level independently.

3.1 Layout Optimization Based on Total Blocks (LOT)

The overall process of the proposed algorithm is generated from five main steps (Fig 1). At the first step, based on the blocks' information, economic parameters, and cut-off grade the economic block model is prepared. Block information includes block ID, coordinates or indexes, grade, rock type, the tonnage of each block and economic parameters contains the metal price, cost of selling, mining cost, processing costs, and recovery.

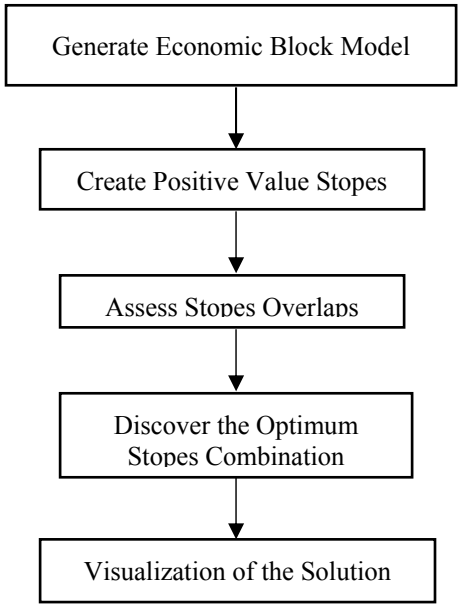


Fig 1. The overall process of the algorithm of layout optimization in LOT method

Next step is creating the positive value which starts by defining the dimension of the stopes. The dimension of the stopes is based on the number of the blocks in three directions X, Y and Z. Then, the stope with this dimension is floated along axes to find all stope possibilities. Then, the economic value of each stope should be calculated, which is the summation of all blocks value in each stope, and stopes with the positive value are separated. Discovering the overlaps between positive possible stopes, which is positive possible stopes with at least one common block is the next step. In reality, the positive stopes combination is not acceptable when there is the overlap between its stopes. After running the optimization model, the best stopes combination with no overlap and the highest economic value is determined. This combination of stopes generates the optimum stopes layout. Finally, the optimum stope layout as the output of the presented algorithm is displayed.

A BIP formulation presents for finding the optimum stopes layout for LOT method. The purpose is to maximize the economic value, while selected stopes in the layout do not have any overlaps. The following assumptions are used in the BIP formulations for optimum stopes layout in LOT method.

- The numerical data such as tonnage, grade, coordinates, and economic value are applied to identify the ore-body attributes in each block.
- In order to create stope, no partial block is considered. In other words, a stope consists of the number of complete blocks.
- The present EV of stopes is considered to find the optimum solution. In fact, the factor of time is not considered.

This problem is formulated as a knapsack problem with conflict graph (see Pferschy et al. (2009)). The required indices, set, decision variable, and parameters to formulate the problem are as follows:

Indices

$s \in \{1,...,S\}$ Index for stopes

Set

Os Set of all stopes overlaps contains all overlaps for each stope

Decision variable

$x_s \in \{0,1\}$	Binary variable controlling the selection of stope s. It is equal to 1, if the stope s is selected in the stopes combination; otherwise is 0. (Decision variable indicating whether item s is picked in knapsack)

Parameters

EV_s	Economic value of stope s (Utility of items)
Ns	Total number of positive stopes (Capacity of knapsack)
S	Maximum number of positive stopes (Number of items)

Objective function

$$Max \sum_{s=1}^{S} EV_s \times x_s \tag{1}$$

Constraints

$$x_s + x_{s'} \leq 1, \qquad \forall (s, s') \in Os \tag{2}$$

$$\sum_{s=1}^{S} x_s \leq Ns, \qquad \forall s \in \{1,...,S\} \tag{3}$$

The objective function, equation (1) consists of the stopes EV and a binary decision variable that indicates the selection or not the selection of each stope in the combination. The stope combination with highest EV is the output of this objective function. The stopes overlap constraint, equation ensures that not two stopes, s and s' with overlap can be in the same stope combination. The number of selected stopes constraint, equation (3), presents the maximum allowable number of stopes in the combination that is equal to the total number of positive stopes.

3.2 Layout Optimization Based on Levels (LOL)

Fig 2 shows the seven main steps of the process. First step, preparing the block model is same as LOT method. Next Step is defining the possible levels which are any elevation in Z direction that can be the elevation as the base for creating the stopes. The process of creating positive value stopes and assessing stopes overlap in LOL method is same as creating possible positive stopes in LOT method. The only difference is in the LOL method the process should be repeated for every single possible level. As a result, stopes with positive economic value and their overlaps in all possible levels are generated. Next step is determining the best stopes combination with the highest economic value and without stopes overlap in each possible level. The algorithm needs to be run for all possible levels separately. At the next step, the presented algorithm compares the economic value of possible levels, which is the value of best stopes combination and finds the best set of levels with the highest value among all possible sets. It is obvious that the difference between elevations of levels in each set should be more than the dimension of stope in the Z direction. Based on the optimum levels set, the optimum stope layout generates. In fact, optimum stope layout is the combination of selected stopes in selected levels. The last step is displaying the solution.

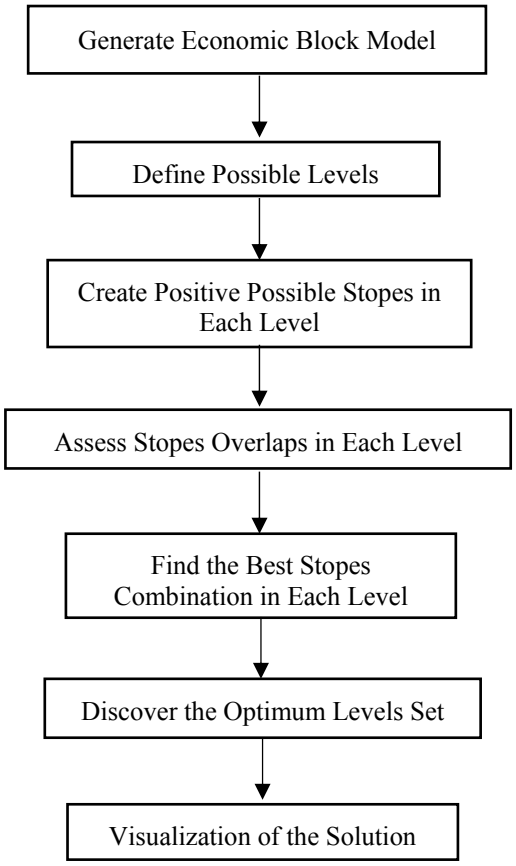


Fig 2. The overall process of the algorithm of layout optimization in LOL method

The formulation of finding the optimum stopes in each level in LOL method is same as LOT method. Nevertheless, the problem is not solved only once. The problem should be formulated for every single level, and the optimum stopes should be found at all levels. To find the optimum levels in LOL method, the knapsack problem with conflict graph is applied as well. Beside the mentioned assumption for LOT method, the following assumptions are applied in the BIP formulations for finding the optimum stope layout in LOL method.

- In order to create a level, no partial stope is considered. In fact, levels must create only at top or bottom of the stopes, not middle.
- A level covers of all possible stopes with the same base elevation, not some of those stopes.

In the LOL method, the objective function of the BIP formulation is to maximize the EV of stopes in each level, and the EV of levels compose the objective function. The required indices, set, decision variable, and parameters to formulate the problem are as followed.

Indices

$l \in \{1,...,L\}$ Index for levels

Set

O_l Set of all levels overlaps which contain all overlaps for each levels

Decision variable

$x_l \in \{0,1\}$	Binary variable controlling the selection of level l. It is equal to 1, if the levels l are selected in the levels set; otherwise is 0. (Decision variable indicating whether item l is picked in knapsack)

Parameters

EV_l	Economic value of level l (Utility of items)
Nl	Total number of levels (Capacity of knapsack)
L	Maximum number of levels (Number of items)

Objective function

$$Max \sum_{l=1}^{L} EV_l \times x_l \tag{4}$$

Constraints

$$x_l + x_{l'} \leq 1, \qquad \forall l \,\&\, l' \in Ol \tag{5}$$

$$\sum_{l=1}^{L} x_l \leq Nl, \qquad \forall l \in \{1,...,L\} \tag{6}$$

The objective function, equation (4) consists of the levels EV of levels and a binary decision that indicates the selection or not the selection of each level in the optimum level set. The optimum level set with the highest EV is the output of this objective function. The levels overlap constraint, equation (5), ensures that two levels l and l' with overlap cannot be in the same level set. The number of selected levels, equation (6), presents the maximum allowable number of levels in a set that is equal to the total number of levels.

After defining the optimum stopes layout by LOL method and discovering the best stopes to mine, finding the optimum mining sequence of those stopes during the mine life is a next goal, which is called production scheduling optimization. In this research, two methods are applied to produce the production scheduling, SOT and SOL. SOT considers total selected stopes as a one set and employs the optimization process on those stopes and SOL method applies the optimization process for each selected level separately.

3.3 Production Scheduling Optimization Based on Total Stopes (SOT)

The overall process of the proposed algorithm to implement the production scheduling includes seven steps (

Fig 3). At the first step based on the tonnage of stopes in optimum stopes layout, the life of mine is defined. By using Long's rule, the production rate can be calculated and then based on the production rate and tonnage of material, the life of mine is determined. Next step is determining the discounted economic value of selected stops to generate the objective function.

In order to decrease the chance of geotechnical failures, adjacent stopes are not allowed to be mined concurrently during any periods. In SOT method, adjacent stopes are defined as the stopes with the shared boundaries in any coordinate planes. Also, in order to determine practical production scheduling, mining the stopes from different levels should follow the reasonable sequence.

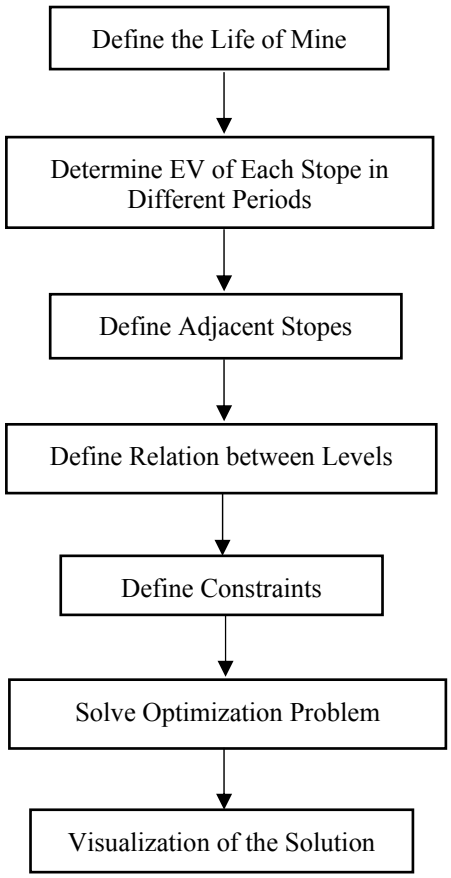


Fig 3. The overall process of the algorithm of production scheduling optimization in SOT method

In addition, the reasonable delay between activation of subsequent levels needs to be defined. Also, the maximum number of active levels at the same time should be defined. Defining other constraints such as mining capacity, grade blending and desirable direction of mining between levels are necessary. After creating the objective function and all the constraints, the optimum stope production schedule with the highest NPV is generated. Finally, the last step is displaying the solution.

The BIP formulation to optimize the production scheduling based on SOT method is presented. The purpose is to maximize NPV while considering mining capacity, grade blending, only one-time mining, stope adjacency, the connection between mining the stopes and activation of levels, concurrent active levels and delay between activation of the levels conditions. The following assumptions are used in the BIP formulations for production scheduling optimization in SOT method.

- No partial stope is considered in production scheduling. In other words, a stope must thoroughly be mined in a period or not be mined at all.
- When the model selects a stope in a period, it includes all the required preparation, extraction and backfilling of that stope during that period. In this research, the combination of these three functions called mining.
- The market fluctuations during mine life are not considered.
- There is no material mixing between stopes and within a stope during mine operation.

The required indices, sets, decision variables, and parameters to formulate the problem are as followed. Since the stopes and levels in this section are the output of stope layout optimization by LOL method and are shown by * (s^* and l^*).

Indices

$s^* \in \{1,...,S^*\}$	Index for selected stopes as the output of LOL and input of SOT
$l^* \in \{1,...,L^*\}$	Index for selected levels as the output of LOL and input of SOT
$t \in \{1,...,T\}$	Index for scheduling period in SOT method

Sets

A_{s^*}	Set of all stopes adjacency which contains all adjacency for every stopes
$B_{l^*}^{s^*}$	Set of the stopes s^* in level l^*

Decision variables

$x_{s^*}^t \in \{0,1\}$	Binary variable controlling mining of stope s^* in period t. It is equal to 1, if the stope s* is mined in period t; otherwise is 0.
$y_{l^*}^t \in \{0,1\}$	Binary variable controlling the activation of level l^* in period t. It is equal to 1, if the level l^* is active in period t; otherwise is 0.

Parameters

EV_{s^*}	Economic value of stope s^* in SOT method
S^*	Maximum number of stopes
T	Maximum number of scheduling periods (Mine life)
L^*	Maximum number of levels l^*
Ton_{s^*}	Tonnage of stope s^*
Cu_t	Upper bound of mining capacity in period t
Cl_t	Lower bound of mining capacity in period t
G_{s^*}	Average grade of stope s^*
Gu_t	Upper bound of the acceptable average grade in period t
Gl_t	Lower bound of the acceptable average grade in period t
$N_{l^*}^{s^*}$	Number of selected stopes s^* in level l^*
Mcl	Maximum number of concurrent active levels
D	The required delay between activation of levels
R_t	Metal processing recovery in period t

Objective function

$$Max\sum_{t=1}^{T}\sum_{s^*=1}^{S^*}\frac{EV_{s^*}}{(1+\mathrm{i})^{\mathrm{t}}}\times x_{s^*}^{t} \tag{7}$$

Constraints

$$Cl_t \le \sum_{s^*=1}^{S^*} Ton_{s^*}\times x_{s^*}^{t} \le Cu_t, \qquad \forall t\in\{1,...,T\} \tag{8}$$

$$Gl_t \le \sum_{s^*=1}^{S^*} Ton_{s^*}\times x_{s^*}^{t}\times G_{s^*}\times \mathrm{R_t} \le Gu_t, \qquad \forall t\in\{1,...,T\} \tag{9}$$

$$\sum_{t=1}^{T} x_{s^*}^{t}=1, \qquad \forall s^*\in\{1,...,S^*\} \tag{10}$$

$$x_{s^*}^{t}+x_{s^{*'}}^{t}\le 1, \qquad \forall s^* \,\&\, s^{*'}\in A s^* \tag{11}$$

$$\sum_{s^*\in B_{l^*}^{s^*}} x_{s^*}^{t} \le N_l^{s^*}\times y_{l^*}^{t}, \qquad \forall t\in\{1,...,T\}, l^*\in\{1,...,L^*\} \tag{12}$$

$$\sum_{l^*}^{L^*} y_{l^*}^{t}\le Mcl, \qquad \forall t\in\{1,...,T\} \tag{13}$$

$$\mathrm{y}_{l^*}^{t}\le \sum_{t'=1}^{t-D} y_{l^*+1}^{t'}, \qquad \forall t\in\{1,...,T\}, l^*\in\{1,...,L^*\} \tag{14}$$

The objective function, equation (7) consists of the EV of selected stope, discount rate, and a binary decision that indicates mining or not mining of each stope in each period. The stope with the highest EV is chosen to be part of the production in order to maximize the NPV. The required constraints are presented by equations (8) to (14). Equation (8) represents the mining capacity. The binary variable $x_{s^*}^{t}$ controls this constraint. One constraint should be defined for each period. Equation (9) ensures that the production's average grade is in the acceptable range. The binary variable $x_{s^*}^{t}$ controls this constraint. One constraint per period is required. Equation (10) shows every single stope s* must be mined only once. The binary variable $x_{s^*}^{t}$ controls this constraint. One constraint should be defined for each stope. Equation (11) is related to the adjacent stopes. This constraint is controlled by the binary variable $x_{s^*}^{t}$. This constraint indicates stopes s^* and $s^{*'}$ must not be mined at the same period, if s^* and $s^{*'}$ belong to A_{s^*}. Equation (12) presents the connection between mining the stopes and activation of levels. This constraint is controlled by binary variables $x_{s^*}^{t}$ and $y_{l^*}^{t}$. $x_{s^*}^{t}$ controls stopes that belong to level l^* and $y_{l^*}^{t}$ controls the levels. One constraint should be defined for each level in each period. Equation (13) is defined for concurrent active levels. This constraint is controlled by the binary variable $y_{l^*}^{t}$. One constraint should be defined for each period. Equation (14) keeps the desired delay between the activation of levels. The binary variable $y_{l^*}^{t}$ controls this constraint. One constraint should be defined for each level in each period.

3.4 Production Scheduling Optimization Based on Levels (SOL)

As Fig 4 indicates, the production scheduling optimization in SOL method includes seven steps.

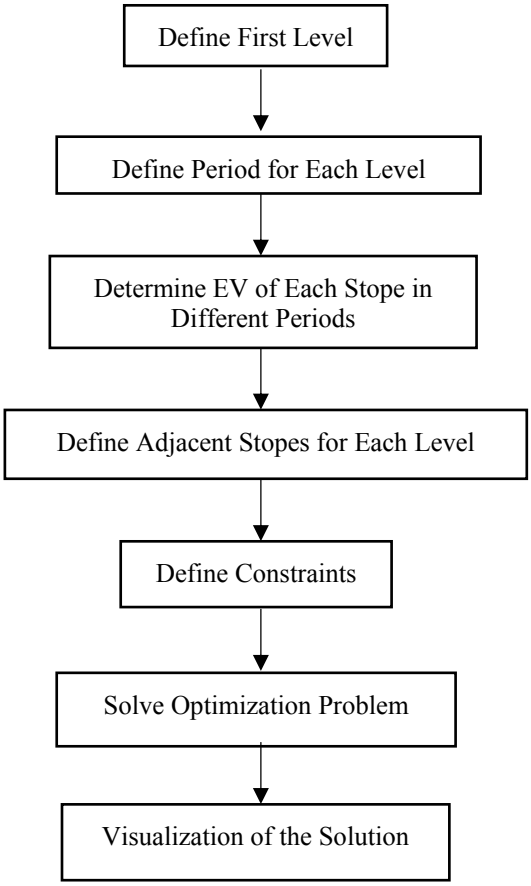


Fig 4. The overall process of the algorithm of production scheduling in SOL method

Since starting production scheduling optimization in a level occurs after finishing schedule optimization at the previous level, selecting the first level is critical. In order to choose the first level, a desired direction of mining must be known. Based on the information of the selected levels and selected stopes by LOL method, the required mining period is defined for each level. In SOL method since all stopes are in the same level with the same Z coordinate, adjacent stopes are defined as the stopes with the shared boundaries in X-Y coordinate planes. Mining capacity, grade blending and precedence between levels also are considered as the constraints. After considering all conditions, the optimum stope production scheduling with the highest NPV is created for each level separately. The last step is displaying the result of optimum stope production scheduling in SOL method. Comparison of the results of SOT and SOL methods is the last step in production scheduling optimization.

The BIP formulation for optimizing the production scheduling for SOL method is presented. The purpose is to maximize NPV of mining stopes in each level separately while considering mining capacity, grade blending, only one-time mining, stope adjacency. In addition to assumptions in the SOT method, the following assumptions are used in the BIP formulations for production scheduling optimization in SOL method.

- Only based on the proportion of material tonnage in each level out of total material tonnage, the proportion of mine life is assigned to each level and other factors are not considered.

- Production scheduling is done at each level separately, and the connection between levels is not considered.
- Mining capacity and required average grade in each period in SOL method are assumed same as SOT method other factors are not considered.

The required indices, sets, decision variables, and parameters to formulate the problem are as follows.

Indices

$s^*_{l*} \in \{1,...,S^*_{l*}\}$	Index for selected stopes $s*$ in selected level $l*$ as the output of LOL method
$t_{l*} \in \{1,...,T_{l*}\}$	Index for scheduling period in level $l*$ in SOL method

Set

A^{s*}_{l*}	Set of all stopes adjacency which contains all adjacency for every stopes

Decision variable

$x^{t_{l*}}_{s*,l*} \in \{0,1\}$	Binary variable controlling mining of stope $s*$ of level $l*$ in period t_{l*} in SOL method. It is equal to 1, if the stope s*is mined in period t; otherwise is 0.

Parameters

EV^{s*}_{l*}	Economic value of stope s* of level l* in SOL method
S^*_{l*}	Maximum number of stopes in level $l*$
T_{l*}	Maximum number of scheduling periods for level $l*$
Ton^{s*}_{l*}	Tonnage of stope $s*$ in level $l*$
$Cu^{t_{l*}}_{l*}$	Upper bound of mining capacity in level $l*$ in period t_{l*}
$Cl^{t_{l*}}_{l*}$	Lower bound of mining capacity in level $l*$ in period t_{l*}
G^{s*}_{l*}	Average grade of stope $s*$ in level $l*$
$Gu^{t_{l*}}_{l*}$	Upper bound of the acceptable average grade in level $l*$ in period t_{l*}
$Gl^{t_{l*}}_{l*}$	Lower bound of the acceptable average grade in level $l*$ in period t_{l*}
R^t_{l*}	Metal processing recovery in period t_{l*}

Objective function

$$Max \sum_{t_{l*}=1}^{T_{l*}} \sum_{s^*_{l*}=1}^{S^*_{l*}} \frac{EV^{s*}_{l*}}{(1+i)^{t_{l*}}} \times x^{t_{l*}}_{s*,l*} \tag{15}$$

Constraints

$$Cl_{l^*}^{t_{l^*}} \le \sum_{s_{l^*}^*=1}^{S_{l^*}^*} Ton_{l^*}^{s^*} \times x_{s^*,l^*}^{t_{l^*}} \le Cu_{l^*}^{t_{l^*}}, \qquad \forall t_{l^*} \in \{1,...,T_{l^*}\} \tag{16}$$

$$Gl_{l^*}^{t_{l^*}} \le \sum_{s_{l^*}^*=1}^{S_{l^*}^*} Ton_{l^*}^{s^*} \times x_{s^*,1^*}^{t_{l^*}} \times G_{l^*}^{s^*} \times R_{l^*}^{t_{l^*}} \le Gu_{l^*}^{t_{l^*}}, \qquad \forall t_{l^*} \in \{1,...,T_{l^*}\} \tag{17}$$

$$\sum_{t_{l^*}=1}^{T_{l^*}} x_{s^*,1^*}^{t_{l^*}} = 1, \qquad \forall s_{l^*}^* \in \{1,...,S_{l^*}^*\} \tag{18}$$

$$x_{s^*,l^*}^{t_{l^*}} + x_{s'^*,l^*}^{t_{l^*}} \le 1, \qquad \forall s_{l^*}^* \,\&\, s_{l^*}^{*'} \in A_{l^*}^{s^*} \tag{19}$$

The objective function, equation (15) consists of the economic value of selected stopes in selected levels, discount rate, and a binary decision that indicates mining or not mining of each stope of each level in each period. The constraints are presented by equations (16) to (19). Equation (16) represents the mining capacity. This constraint is controlled by the binary variable $x_{s^*,l^*}^{t_{l^*}}$. One constraint should be defined for each period in a level. Equation (17) ensures that the production's average grade in each level is in the acceptable range. This constraint is controlled by the binary variable $x_{s^*,l^*}^{t_{l^*}}$. One constraint per period in a level is required. Equation (18) indicates every single stope $s_{l^*}^*$ in each level must be mined only once. The binary variable $x_{s^*,l^*}^{t_{l^*}}$ controls this constraint. One constraint should be defined for each stope in a level. Equation (19) is related to the adjacent stopes. This constraint is controlled by the binary variable $x_{s^*,l^*}^{t_{l^*}}$. This constraint indicates stopes $s_{l^*}^*$ and $s_{l^*}^{'*}$ must not be mined at the same period, if $s_{l^*}^*$ and $s_{l^*}^{'*}$ belong to $A_{l^*}^{s^*}$.

In this research, MATLAB (Math Work Inc., 2017) is employed as a numerical modeling platform and IBM ILOG CPLEX Optimization Studio (12.7.1) as a solver to optimize stopes layout. MATLAB is a high-level language to analyze data, develop algorithms and models. CPLEX solver is a tool to solve a large-scale mixed-integer linear and quadratic programming (Pourrahimian, 2013).

4 Implementation of the Algorithm

In this section, the mathematical formulations will be implemented on a drillhole data set in order to demonstrate how the presented methodologies work. The block model belongs to a silver (AG) deposit. Table 1 indicates the block model information. The total tonnage of ore-body in the block model is 37.9 (Mt). Fig 5 shows different range of grade of silver. Also, Fig 6 indicates the grade distribution of ore-body.

Table 1. Block model information

Number of blocks	48,362
Blocks size (m3)	10×10×10
Blocks tonnage (tonne)	2700
Blocks grade (g/tonne)	0 - 1580
X Coordinate (X index)	1-60
Y Coordinate (Y index)	1-13
Z Coordinate (Z index)	1-62

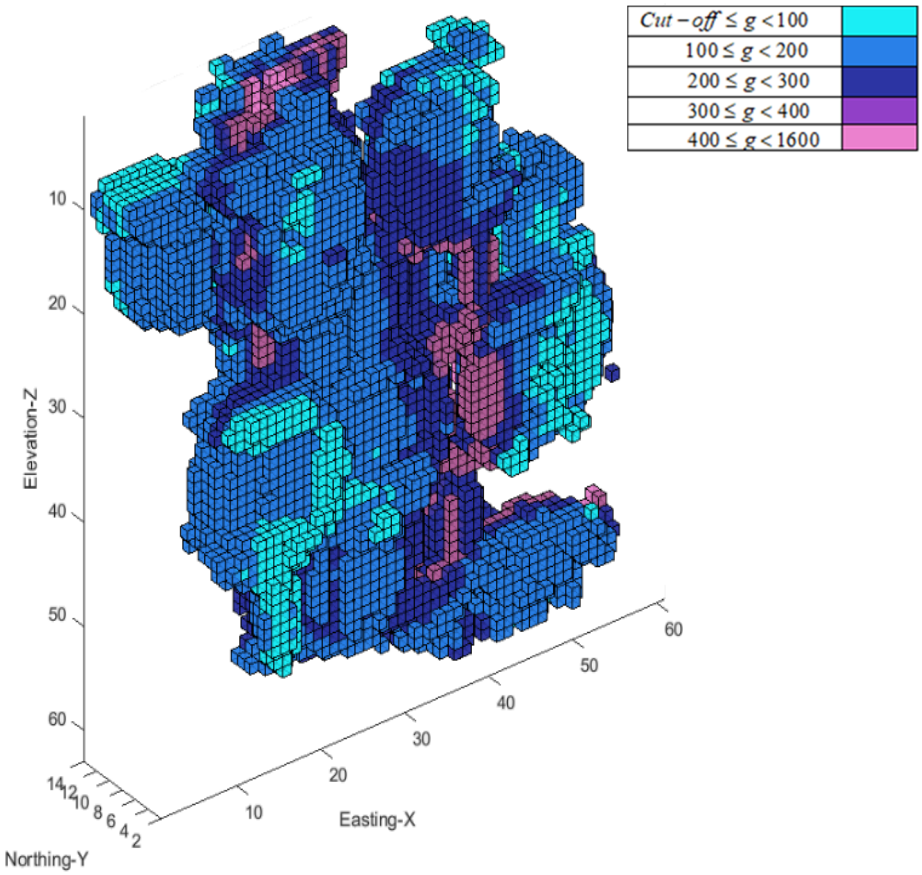


Fig 5. Grade of Ag (g/ton) in the ore blocks

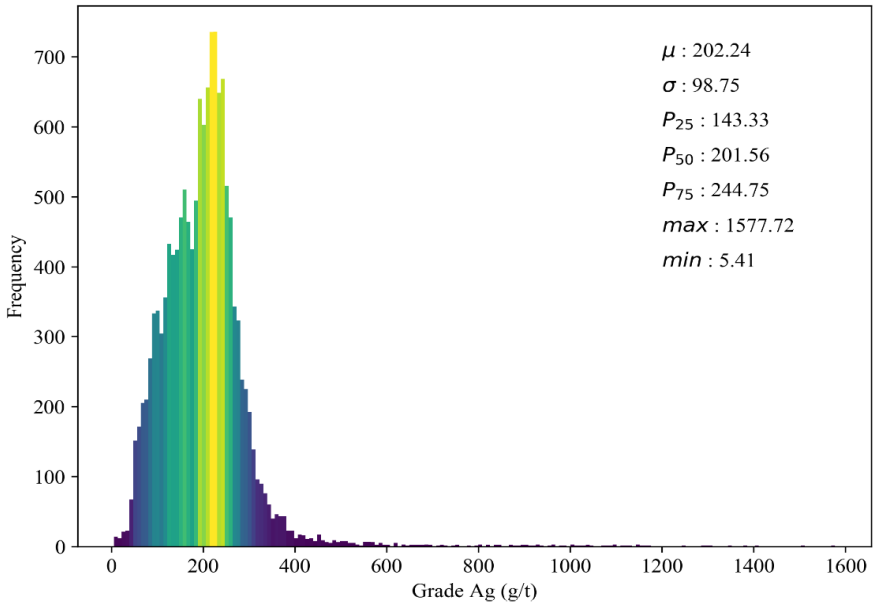


Fig 6. Grade distribution of the ore-body

4.1 Layout Optimization Based on Total Blocks (LOT)

The economic value (EV) of the blocks was calculated based on the parameters listed in Table 2. After calculating the cut-off grade, waste or ore blocks were determined and the BEV was calculated. The calculated cut-off grade for this block model was 66.7 (g/t), and the block values were between $ -0.64M and $ +2,20M.

Table 2. Economic parameters

Metal price ($/g)	0.6
Cost of mining ($/t)	24
Cost of processing ($/t)	12
Recovery	90%

According to (Hartman, 1992; Haycocks et al., 1992), stope width at least must be 6 meters, and the range for length and height of the stopes is between 45 and 120 meters. In order to have a practical condition, the stope dimension 40×40×120 cubic meters (4×4×12 blocks) was chosen in this study. However, there is the possibility of changing this dimension and scanning the impacts of changing. Creating the stopes and separating the positive ones were the following steps in this section. In LOT method, 29,070 initial stopes were generated which 13,403 of those stopes had positive economic value. By running the LOT algorithm and removing all overlaps, the combination of 85 stopes was discovered. In fact, this combination of stopes was the optimum stope layout. The EV of stopes in this layout was $ 2,341.1M. Also, the total solution time was 62(hr):02(min):07(sec). Fig 7 illustrates the stopes in the result of the LOT method. The selected stopes in the optimum stopes layout are demonstrated with the different colors.

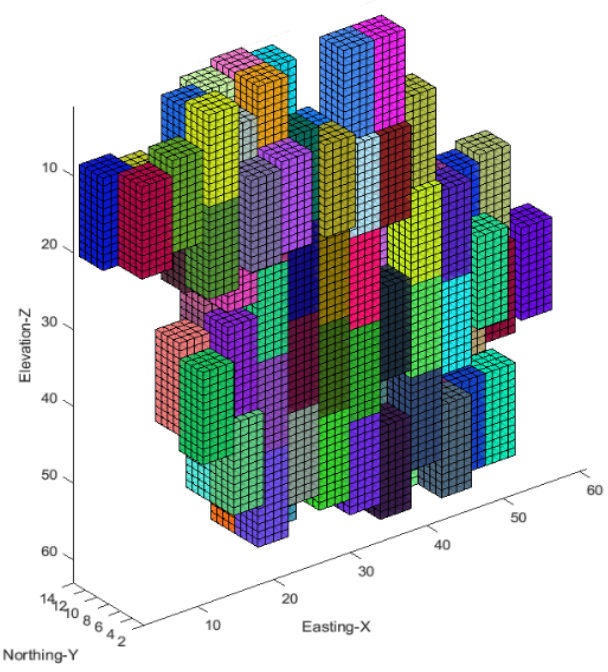


Fig 7. Optimum stopes layout based on the LOT method

4.2 Layout Optimization Based on Levels (LOL)

In LOL method, the primary block model was divided into a number of possible levels that was the base elevation for creating the stope. In the case study, the number of blocks in direction Z is 62 and dimension of stope in the Z direction is 12, so number of possible levels is equal to 51. For all 51 possible levels, the process of creating stopes was applied, and initial and positive stopes were

defined in each level. For instance, in the first level, 597 initial stopes were determined which 165 of those have positive EV. In total, 30, 447 initial stopes and 13,403 positive stopes were generated in LOL method. A set of possible levels 2 (Z=13 of block model), 14 (Z=25 of block model), 26 (Z=37 of block model), 38 (Z=49 of block model) and 50 (Z=61 of block model) was determined as the optimum set with the highest EV. Table 3 demonstrates a number of initial stopes, positive stopes and the selected stopes in each selected level. In addition, the EV of the selected levels are indicated in this table. Generally, 5 levels including 93 stopes and total value of $ 2,252.2M were the result of LOL method. The LOL method reached the answer in 00:08(min):54(sec). Fig 8 indicates the stopes layout based the result of the LOL method.

Table 3. Information of selected levels

Level ID	Possible level ID	Initial stopes (#)	Positive stopes (#)	Selected stopes (#)	Value (M$)
Level 1	2	597	202	12	263.6
Level 2	14	597	283	20	434.8
Level 3	26	597	265	21	599.6
Level 4	38	597	294	22	630.1
Level 5	50	597	232	18	324.1

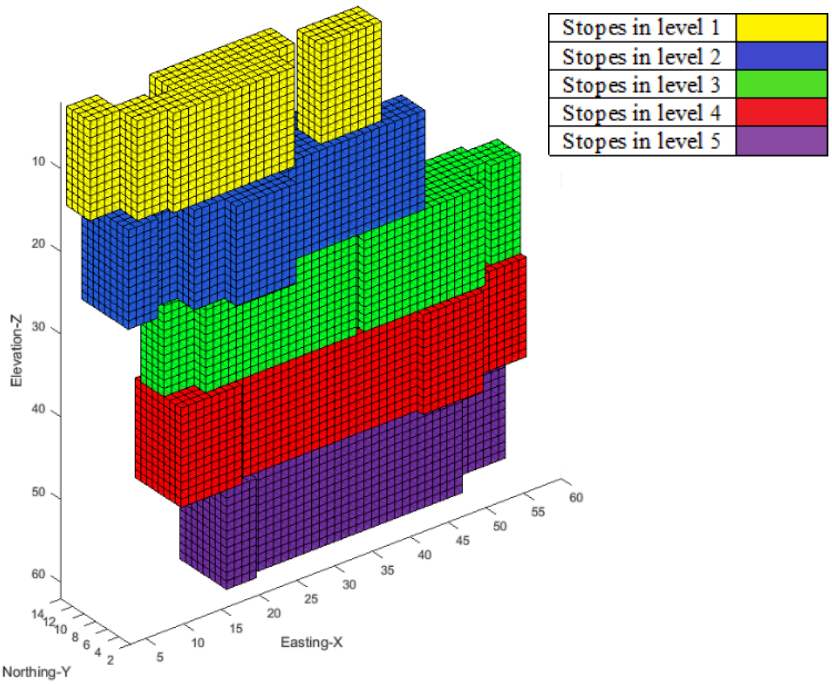


Fig 8. Optimum stopes layout based on the LOL method

Number of selected stopes in the optimum layout defined by LOL method, is eight stopes more than LOT method and total mined tonnage in LOL method is 8.5% higher than LOT method. The achieved value by LOT method is 4% more than that value for LOL method. However, there is a world of difference between the running times the methods. As in LOT method all the stopes are evaluated at one set, some steps of the algorithm including assessing stopes overlaps, creating the overlap constraint and running optimization model are time-consuming processes. Nevertheless, in LOL method the stopes divided to the smaller sets, levels, so the processing time drops significantly. As indicated in Fig 8, in LOL method, selected stopes in the optimum stopes layout were in the same levels. As a result, this method achieved to the practical solution. However, in

LOT method Fig 7, selected stopes in the optimum stopes layout were in different elevations so it is not possible to design production levels to have access to all the stopes. Therefore, the achieved solution by LOT method was not practical.

4.3 Production Scheduling Optimization Based on Total Stopes (SOT)

The total tonnage of 93 stopes was 48.21Mton. The life of mine was 22 years. A discount rate of 10% was used in this case study for production scheduling. Table 4 provides other required parameters for production scheduling based on the SOT method.

The achieved NPV by running SOT method was $ 1,288.1M. Running time for SOT method was 00:46(min):14(sec) and the optimality gap is zero percent. Fig 9 illustrates the sequence of mining the stopes based on SOT method. Different colors show the different periods.

Table 4. Production scheduling parameters in SOT method

Life of mine (year)	22
Discount rate (%)	10
Minimum mining capacity (Mt)	1.55
Maximum mining capacity (Mt)	3.1
Minimum average grade (g/t)	55
Maximum average grade (g/t)	220
Delay between activation (period)	2
Maximum number of active levels at a period	3
Direction of mining the levels	Upward

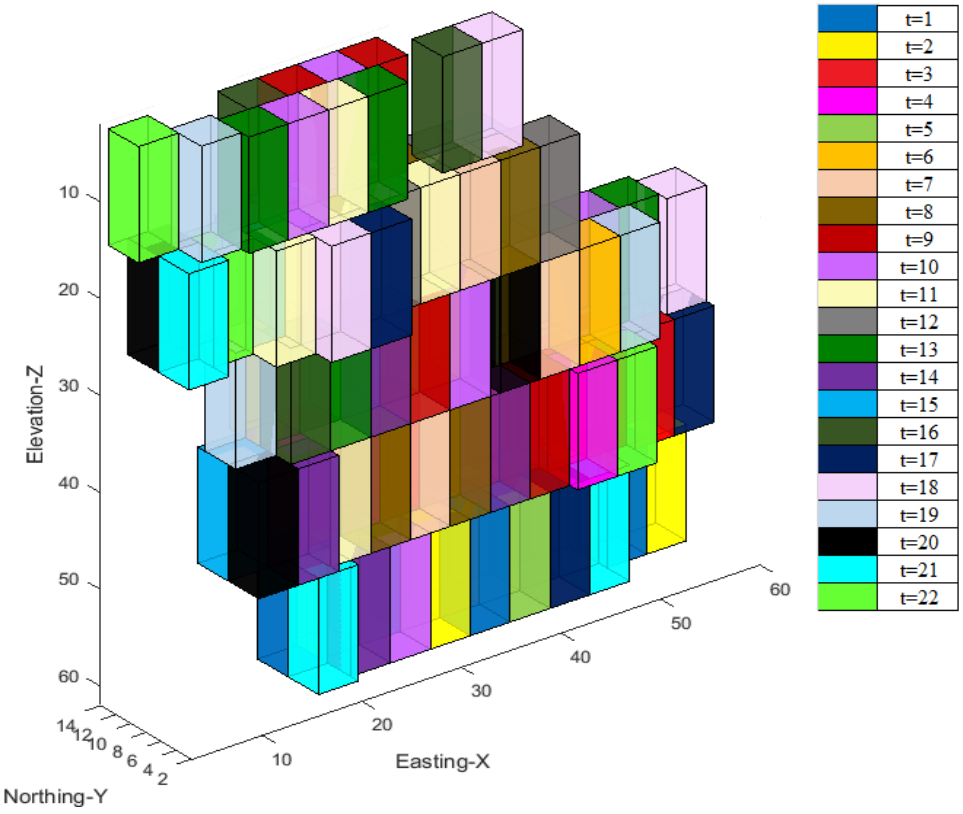


Fig 9. Optimum production scheduling based on the SOT method

4.4 Production Scheduling Optimization Based on Levels (SOL)

22 years was considered as life of the mine. By distributing the mine life to levels based on the total tonnage of stopes in each level, mining period for each level was determined. Table 5 shows the number of stopes in each level, total tonnage of each level and the assigned period for each level.

Table 6 provides the required parameters for production scheduling based on the SOL method. The total obtained NPV and the running time for the SOL method with the optimality gap of zero percent were $ 1,052.5 M and 00:13(min):34(second), respectively. Table 7 indicates the NPV achieved by applying the formulation at each level. According to this table, Level 4 has the highest NPN, and Level 1 has the lowest NPV. Since Level 5 contains less number of stopes than Level 4, Level 4 reaches to the highest NPV. Fig 10 displays the solution in levels 1, 2, 3, 4 and 5 respectively. Different colors show the different mining periods.

Table 5. The assigned period for each level

Level ID	Number of Stope	Average grade of the level (g/t)	Tonnage (Mt)	Period
Level 1	12	138.3	6.2	3
Level 2	20	137.8	10.4	5
Level 3	21	163	10.9	5
Level 4	22	162.9	11.4	5
Level 5	18	120.7	9.3	4
Total	93	146.2	48.2	22

Table 6. Production scheduling parameters in SOL method

Life of mine (year)	22
Discount rate (%)	10
Minimum mining capacity (Mt)	1.55
Maximum mining capacity (Mt)	3.1
Minimum average grade (g/t)	55
Maximum average grade (g/t)	220
Direction of mining the levels	Upward

Table 7. Maximum achieved value of each level in SOL method

Level ID	NPV (M$)
Level 1	39.7
Level 2	102.2
Level 3	230.9
Level 4	388.7
Level 5	291.0
NPV	1,052.5

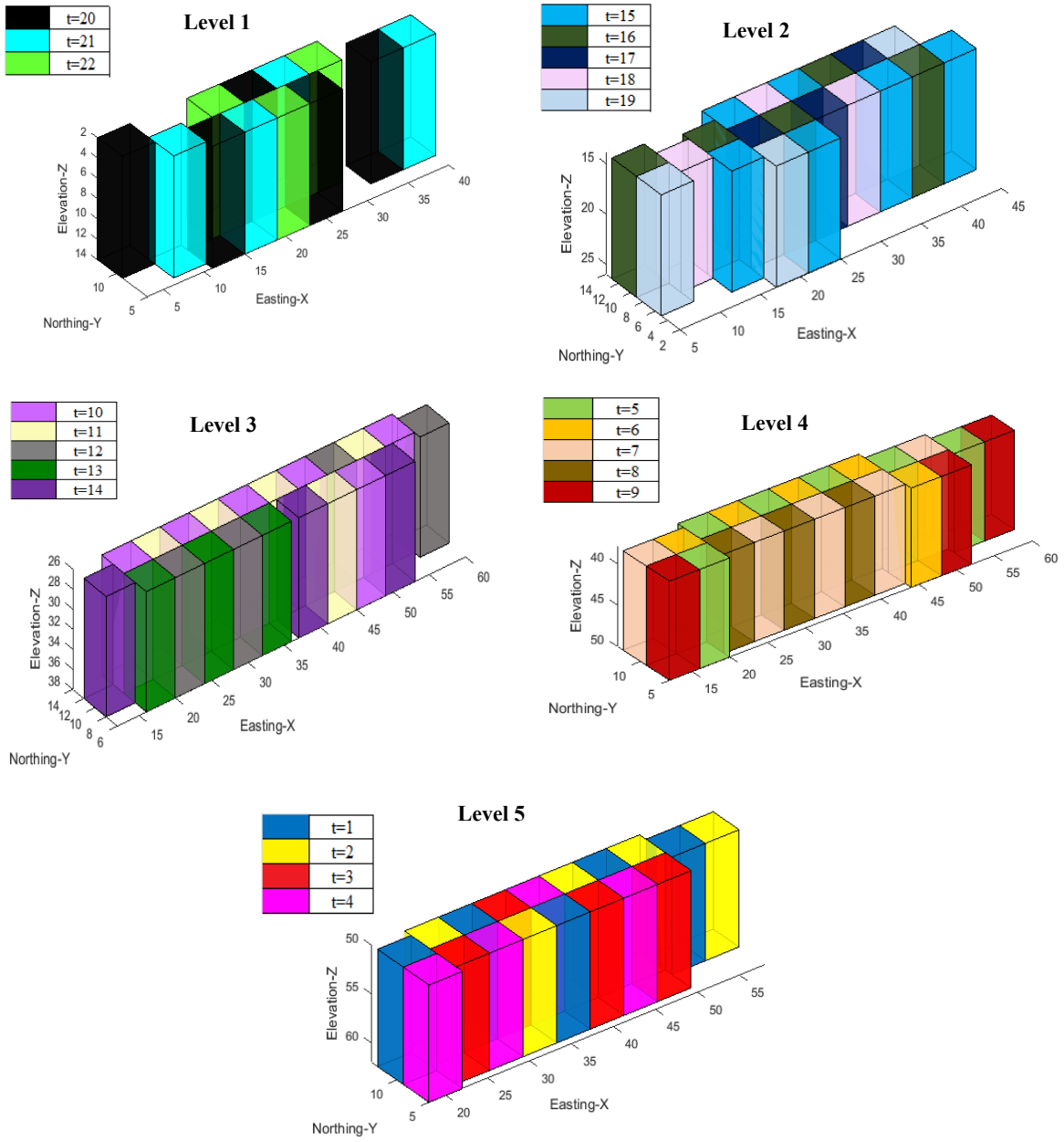


Fig 10. Optimum production scheduling in all levels based on the SOL method

The achieved NPV by SOT method is 22% more than what is got by SOL method. However, the running time of SOT method is 241% more than SOL method. Since in SOL method, stopes are divided to the few sets, levels, and then the optimization algorithm is applied for each set separately, the running time of SOT method is higher than the running time of SOL method.

5 Summary and Conclusion

The presented methods are able to find the optimum stopes layout in sublevel stoping based on the total blocks and based on the levels while considering the constraints. Also, the proposed methodologies are verified regarding both feasibility and optimality on a case study and the optimality gap for all four methods are zero percent. According to the results, for stope layout optimization, it is reasonable to use the leveling to save the time although the achieved economic value is a little bit lower. Also, from a practicality point of view, LOL reaches the better solution.

However, for production scheduling, the model does not show a specific advantage of using leveling method. Fig 11 indicates the comparisons between achieved NPVs and running times. One of the assumptions in this study was to create stope, no partial block and to create a level, no partial stope were considered and considering those for future studies is recommended. Also, this study presented the deterministic model, so it was not able to capture the uncertainty. It is good to consider this matter in future.

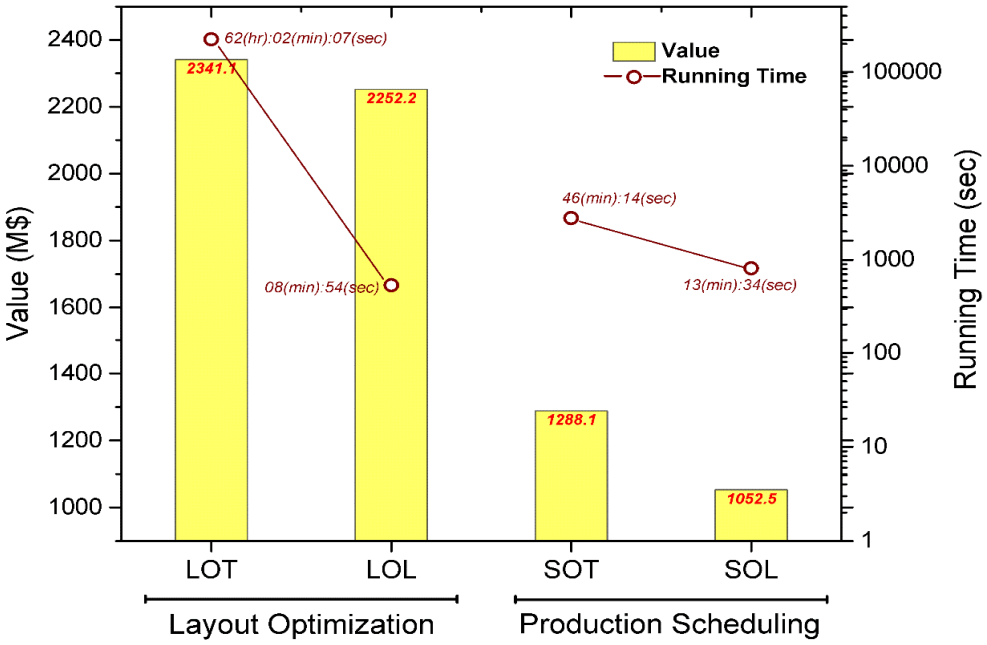


Fig 11. Summary of results of all presented method

6 References

[1] Alford, C. (1996). Optimization in Underground Mine Design. *International Journal of Rock Mechanics and Mining Sciences and Geomechanics 220A* 561.

[2] Ataee-Pour, M. (2000). A Heuristic Algorithm to Optimise Stope Boundaries.Thesis, University of Wollongong,

[3] Ataee-Pour, M. (2005). A Critical Survey of the Existing Stope Layout Optimization Techniques *mining science*, *41, No.5* (UDC 519.256:622.2),

[4] Bai, X., Marcotte, D. and Simon, D. (2012). Underground Slope Optimization with Network Flow Method. *Computers and Geosciences*, 361-371.

[5] Cheimanoff, N. M., Deliac, E. P. and Mallet, J. L. (1989). GEOCAD: An Alternative CAD and Artificial Intelligence Tool That Helps Moving from Geological Resources to Mineable Reserves. *21st Application of Computers and Operations Research in the Mineral Industry:21st International Symposium*, Page 471.

[6] Copland, T. and Nehring, M. (2016). Integrated optimization of stope boundary selection and scheduling for sublevel stoping operations. *The Southern African Institute of Mining and Metallurgy*, 1135-1142.

[7] Deraisme, J., De Fouquet, C. and Fraisse, H. (1984). Geostatistical Orebody Model for Computer Optimization of Profits from Different Underground Mining Methods. in *Proceedings of the 18th APCOM Symposium*. London, England, pp. 583–590.

[8] Hartman, H. L. (1992). *Sublevel stoping in SME Mining Engineering Handbook.* Society for Mining, Metallurgy and Exploration Inc.: Littleton,

[9] Haycocks, C. and Aelick, R. C. (1992). *Sublevel Stoping.* Littleton,

[10] Lawrence, B. W. (1998). Considerations for sublevel stoping techniques in underground mining. *Society for Mining, Metallurgy and Exploration Inc.: Littleton*,

[11] Little, J. (2012). Simultaneous optimisation of stope layouts and production schedules for long-term underground mine planning.Thesis, University of Queensland,

[12] Manchuk, J. (2007). Stope Design and Sequencing.Thesis, University of Alberta,

[13] Mann, C. (1998). Sublevel Stoping. *RE Gertsch & RL Bullock (ed.), Techniques in underground mining: selections from underground mining methods handbook society for Mining, Metallurgy and Exploration Inc*, 223-224.

[14] Nehring, M. (2011). Integrated Production Schedule Optimisation for Sublevel Stoping Mines.Thesis, The University of Queensland,

[15] Nikbin, V., Ataee-Pour, M., Shahriar, K. and Pourrahimian, Y. (2018). A 3D approximate hybrid algorithm for stope boundary optimization. *Computers and Operations Research*, *https://doi.org/10.1016/j.cor.2018.05.012* 1–9.

[16] Ovanic, J. and Young, D. S. (1995). Economic Optimisation of Stope Geometry Using Separable Programming with Special Branch and Bound Techniques in *Third Canadian Conference on Computer Applications in the Mineral Industry, Rotterdam, Balkema*, pp. 129–135.

[17] Pferschy, U. and Schauer, J. (2009). The Knapsack Problem with Conflict Graph. *Journal of Graph Algorithms and Applications*, *vol. 13, no. 2* 233–249.

[18] Pourrahimian, Y. (2013). Mathematical programming for sequence optimization in block cave mining.Thesis, University of Alberta,

[19] Riddle, J. M. (1977). A Dynamic Programming Solution of A Block-Caving Mine Layout. *Application of Computer Methods in the Mineral Industry: Proceedings of the Fourteenth Symposium*, 767–780.

[20] Sandanayake, D. S. S. (2014). Stope Boundary Optimisation in Underground Mining Based on A Heuristic Approach.Thesis, Curtin University

[21] Sandanayake, D. S. S., Topal, E. and Asad, M. W. A. (2015a). A Heuristic Approach to Optimal Design of An Underground Mine Stope Layout. *Applied Soft Computing*, *1568-4946*

[22] Sandanayake, D. S. S., Topal, E. and Asad, M. W. A. (2015b). Designing An Optimal Stope Layout for Underground Mining Based on A Heuristic Algorithm. *International Journal of Mining Science and Technology*, *25* (5), 767-772.

[23] Topal, E. and Sens, J. (2010). A New Algorithm for Stope Boundary Optimization. *Journal of Coal Science and Engineering (China)*, *16* (2), 113-119.

Stope Boundary Optimization Using A 3D Approximate Hybrid Algorithm

Vahid Nikbin, Majid Ataee-pour, Kourosh Shahriar and Yashar Pourrahimian
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada
Amirkabir University of Technology, Tehran, Iran

ABSTRACT

Determining stope boundaries is one of the critical steps to be taken when an underground mining method is selected; because of their significant impact on the profitability of the mining project, the stope boundaries have to be optimum to achieve maximum profits. This paper introduces a new hybrid algorithm that is a combination of dynamic programming and greedy algorithm. Although this proposed algorithm may fail to provide a true optimum solution, it generates better solutions than existing algorithms do. The new proposed algorithm and three existing algorithms are used to find the optimal stope boundaries on a real case ore body. The results demonstrate that the proposed algorithm can improve the profit by 117.78%, 16.86% and 0.42% compared to Floating Stope, Maximum Value Neighborhood (MVN), and Greedy algorithm solutions, respectively, on a real case study at a reasonable CPU time.

1. Introduction

Due to the increase in stripping ratios and haulage distances in open pit mines, mining costs increase. As a result, switching from open pit mining to underground mining may be more beneficial. Also, underground mining can be the only alternative for deposits with large overburdens.

When the underground mining method is selected, it is necessary to determine the optimal workable layout as stope boundaries. The main purpose of optimizing stope boundaries is to determine an appropriate plan to select the best combination of blocks in the block model. This plan should maximize the overall profit subject to geometrical, geotechnical, and safety constraints. Obviously, this plan should be optimal because even a minor deviation from the exact plan's value may result in wasting millions of dollars of mining capitals.

The existing algorithms are either heuristic and therefore do not guarantee the optimal solution or else they suffer from over simplifications that restrict their applications. Despite its importance, because of the inherent complexities of the problem, a comprehensive algorithm has not been reported. Most of the presented algorithms are heuristic, and true optimality is not guaranteed. Also, other algorithms with solutions that are assumed to be optimal, either fail to run on 3D problems or their applications are limited to a specific method. In sum, no efficient algorithm has yet been reported.

Most of the mining problems are large scale while exact optimization methods are usually not applicable to solve these problems. Inevitably, they should be solved by low computational time (meta) heuristic algorithms. There exist a large number of strong heuristic and metaheuristic algorithms in the literature to solve a variety of mining problems such as prediction of hangingwall stability, slope stability, waste tailing recycling potential and so on (Qi et al.,

[1]This paper, in its entirety, is available online from 14 May 2018 in Computer and Operations Research. https://doi.org/10.1016/j.cor.2018.05.012

2017, Qi et al., 2018a, Qi et al., 2018b, Qi and Tang, 2018). Before describing next sections, stope boundary optimization problem is defined as follows:

Let P an economic block model and $p_{i,j,k}$ be the obtained value from extraction of block $B_{i,j,k}$. Each stope is a subset of P that has a cube shape and its sizes along the main coordination axis are determined based on the minimum space required for labors and operating machines in each production cycle. Each combination of these stopes creates a feasible boundary which is denoted by q. Boundary value is equal to summation value of those blocks which are inside of the set q i.e., $value(q) = \sum_{(i,j,k);B_{i,j,k} \in q \cap p} p_{i,j,k}$. Our goal is maximizing boundary value among the all possible boundaries i.e., $\max\{\text{value(q)} : \text{q} \in \text{Q}\}$. Where, Q is a set which consists of all feasible boundaries.

The next section of this paper reviews relevant literature related to stope boundary optimization. Section 3 highlights the development of the algorithm for stope boundary optimization. Section 4 focuses on the application of the presented algorithm, and compares the results with other existing methods. The paper concludes in Section 5.

2. Methodology

Even four decades after the presentation of the first algorithm for optimizing stope boundaries, the growth rate of these algorithms has been slow. The reported works on the stope boundaries optimization problem are either sub-optimal or over simplified. For instance, although the Riddle's (1977) algorithm was designed based on the powerful dynamic programming approach, it does not consider the constraints simultaneously; this weakness may lead to a sub-optimal solution. Also, it is a two-dimensional algorithm that can only be used for the block caving method. The mixed-integer programming model that was presented by Ovanic and Young, 1995, Ovanic and Young, 1999) guarantees the optimal starting and ending points for stopes in block model rows. It is a one-dimension model and it is not clear how the model can be used to find the optimal stope boundaries in 2D or 3D ore bodies.

Floating Stope is a famous stope boundary algorithm that was proposed by Alford (1995) and implemented on the Datamine software package. In this algorithm, two envelopes (the inner and outer envelopes) are introduced as the output, and the final solution is located between the two. The final solution is determined by users, which means it is greatly dependent on their experience. Although, this algorithm is simple to understand and implement, its solution remains sub-optimal due to its heuristic logic.

The MVN algorithm, another heuristic algorithm, was proposed by Ataee-pour, 2000, Ataee-pour, 2005). It investigates all stopes that cover a block with a positive value and selects the stope with the highest value. It then updates the solution by adding this stope to the stope boundaries; this is continued until all positive blocks are checked. MVN is a fast algorithm and easy to understand; however, it is also a heuristic algorithm (similar to the Floating Stope), and the optimality of the solution is not guaranteed. The Optimum Limit Integrated Probable Stope (OLIPS), another dynamic programming algorithm, was designed by Jalali et al. (2004). Despite its strong mathematical background, the method uses complex logic and its application has been limited to inclined vein deposits. Also, no analysis has been reported for its solution time, but because it is complex it is expected to have a significant solution time. Therefore, applying this algorithm to large-sized problems may lead to restrictions due to hardware limitations.

Topal and Sens (2010) presented a heuristic algorithm that can use different strategies to optimize stope boundaries. At first, location, size and profit of all possible stopes are listed in a table if they have a positive value. In the next step, the maximum value stope is selected from this table and it is flagged as a minable stope. Then found stope and its neighboring overlapped stopes which have common blocks with this selected stope are eliminated from the aforementioned table. This procedure continues while all stopes are removed from the table. Low computation time and application on 3D problems are the main advantages of this algorithm. However, by removing overlapping stopes without performing any more accurate analysis on them some feasible combinations of stopes are rejected from the problem which may have good

potentials to yield a higher stope layout (Sandanayake, 2014, Nhleko et al., 2017). Therefore, due to simplification of the problem by eliminating the overlapping stopes, its optimality is not guaranteed.

Bai et al., 2014, Bai et al., 2013 have used an intelligent method to find the optimal boundaries around the vertical raises based on the network flow algorithm. By converting the conventional orthogonal block model to a cylindrical block model with this method, the complicated underground mining problem is converted to an analogous, simpler, and well known open pit problem. However, the raise locations are determined by heuristic methods and the application of this technique is restricted to the sub-level stoping method.

Sandanayake et al., 2015a, Sandanayake et al., 2015b) developed two analogous heuristic algorithms which obtained a better solution than MVN. Both these reported works track the following five main steps sequentially in order to find the highest value stope layout. These steps are: (1) standardize the irregular block model (2) create all possible stopes (3) assign a value to each stope (4) define non-overlapping stopes set (5) find the maximum value combination among the mentioned set. These methods are 3D and their validation was successful compared to other alternatives (Erdogan et al., 2017). However, similar to Topal and Sens approach, these algorithms do not consider the overlapping assumptions and because of this simplification, they fail to guarantee the optimum solution.

Nikbin et al. (2017) introduced a greedy algorithm to find the highest value stope boundary subject the minimum stope sizes. At first, the algorithm finds the most valuable probable stope between the non-investigated stopes set. Then all of the blocks located in this stope are added to the optimal stope boundaries set if the stope value is positive. The found stope is removed from the non-investigated set. This procedure will iterate untill the stope value is positive. The validation was successful compared to MVN on a 2D hypothetical economic block model. However, this algorithm is not only heuristic but also it has not been employed on a real large-size ore body.

Advantages, application domains and some other related features of these explained algorithms have been summerized in Table 1.

Although the heuristic algorithms for the stope boundary problem mainly generate sub-optimal solutions, they usually have low processing time and include fewer simplifications. Consequently, they are more applicable than rigorous alternatives for solving the real world problems. The heuristic algorithms' solutions differ from each other. Any modification to these algorithms to decrease the optimality gap is very valuable. The contribution of this paper is to propose a new hybrid algorithm for stope boundaries optimization to achieve better results than what is possible with current algorithms. A new polynomial-time dynamic programming algorithm is proposed in this paper that guarantees the optimal solution for a given row or column. It is a 1D algorithm and in order to make it capable of solving 3D problems, it is combined by a greedy algorithm as a hybrid approach. Since dynamic programming optimality has a direct effect on the hybrid algorithm, highest value results was recorded by this proposed approach and its validation was completely successful on both hypothetical and real cases.

3. Methodology

In this paper, a new hybrid algorithm is proposed to optimize stope boundaries. This combines greedy and dynamic programming algorithms. The main problem is solved using the proposed greedy algorithm. In the greedy algorithm, by converting specific two-dimensional subsets of the original economic block model to single blocks, the initial complicated 3D problem is converted to a simple secondary one-dimensional sub-problem. Then, a new dynamic programming algorithm is applied to find the optimal boundaries on the aforementioned secondary sub-problem. Of the solutions to these simple sub-problems, the best is chosen based on greedy approach criterion. Then corresponding blocks to this solution are added to the initial problem solution.

Table 1. Comparison of common existing algorithms to solve stope boundary optimization problem

Algorithm	Model type	Mining method	Dimension	Mathematical formulation	Partial blocks	True optimality
Dynamic Programming	Fixed blocks	Block caving	2D	Yes	No	No
MIP model	Irregular blocks	All	1D	Yes	Yes	Yes
Floating stope	Fixed blocks	All	3D	No	Yes	No
MVN	Fixed blocks	All	3D	No	No	No
OLIPS	Fixed blocks	All	2D	Yes	No	Yes
Topal and Sens	Irregular blocks	All	3D	No	No	No
Network Flow	Cylindrical	Sublevel stoping	3D	No	No	No
Sandanayake et al.	Irregular blocks	All	3D	No	No	No
Greedy	Fixed blocks	All	3D	No	No	No

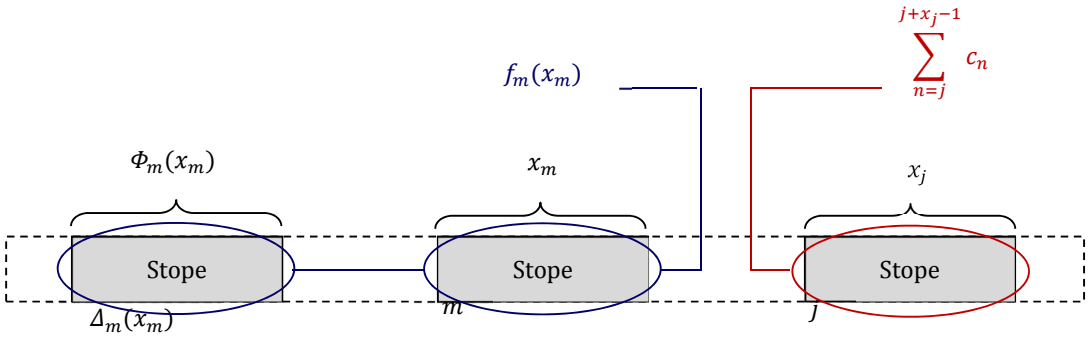


Fig 1. Relative position of algorithm's variables to each other.

3.1. One-dimensional exact dynamic programming algorithm (ODEDPA)

Dynamic programming is a well-known technique that can be used to solve a large number of optimization problems. Using this technique, a complicated problem is broken into smaller sub-problems and instead of solving the complex original problem, the sub-problems are solved and their solutions stored. After solving all of the sub-problems, the algorithm investigates the sub-problems solutions and combines the solutions to solve the original problem.

The new proposed algorithm is designed based on the dynamic programming procedure, and is suitable for finding the optimal stope boundaries in a given row or column of a block model. It is a one-dimensional polynomial-time algorithm and guarantees the optimal solution.

All dynamic programming algorithms almost use the same approach. They calculate the cumulative value for each state at each stage; find the highest cumulative value and track it to find other states located in the optimal solution. The algorithms' steps are:

1. Calculate the cumulative value for all states and store their best previous states.
2. Find the highest cumulative value at the final stage; its value presents the optimal stope boundaries value.
3. Track this highest value state at the final stage; dimensions, positions, and the number of the stopes that make the optimal boundaries.

The algorithm's variables and parameters are as follows:

d_j	Minimum stope size along j axis in terms of blocks
C	A 1D economic block model consists of J blocks along the j axis; input of dynamic programming algorithm
c_n	Value obtained by extracting n^{th} block of C
j	Current stage number
m	Previous stage number
x_j	Discrete state variable at the j^{th} stage; length of a stope that its left corner is located at j
x_m	Discrete state variable at the m^{th} stage; length of a stope that its left corner is located at m
$f_j(x_j)$	Cumulative value of x_j
$f_m(x_m)$	Cumulative value of x_m
$F_j(x_j)$	Cumulative blocks set of x_j; a subset of C that makes $f_j(x_j)$
Γ	State set; set of all feasible stope sizes along the j axis
$H_{j,m}(x_j, x_m)$	Overlap function; a function to recognize overlapping of two neighborhood stopes
$\Delta_j(x_j)$	The best previous stage ($\bar{m}$) that maximizes the cumulative value; previous stope position
$\Phi_j(x_j)$	The best previous state ($\bar{x}_m$) that maximizes the cumulative value; length of previous stope
$\pi_j(x_j)$	Optimal path of x_j; the union of all states that are created $F_j(x_j)$
$\lvert\pi_j(x_j)\rvert$	Optimal path's size, the number of states that are created $F_j(x_j)$
z^{dp}	Dynamic programming value
J^{dp}	Left corner position of stopes that are located at the dynamic programming solution

X^{dp}	Length of stopes that are located at the dynamic programming solution
$g(a,b)$	A function for comparing two numbers, 1 if a is greater than b, otherwise 0
$ge(a,b)$	A function for comparing two numbers, 1 if a is greater than or equal to b, otherwise 0
M^-	A large negative number
C^*	A 1D binary matrix consists of J blocks along j axis; output of dynamic programming algorithm
c_j^*	Value of j^{th} block of C^*, 1 if the block is selected, otherwise 0

Input and output matrices are denoted by capital letters and their elements are defined by lowercase letters.

The stage and state are the position (left corner) and dimension (length along j axis) of stopes respectively (Fig. 1). Since, each stope should has a minimum size, the state set (Γ) consists of all sizes equal to zero or greater than d_j (Eq. (1)).

$$\Gamma = \{0\} \cup \{d_j, \dots, 2d_j - 1\} \tag{1}$$

If x_j is zero the state set is empty and no block is located in the stope. Therefore, the cumulative value is equal to the highest cumulative value among the previous states. This is expressed by Eq. (2).

$$\begin{matrix} f_j(x_j) = \max\{f_m(x_m) : m < j\} \\ x_j = 0 \end{matrix} \tag{2}$$

In other situations (i.e., x_j is not zero), the right corner of stope is located at $j + x_j - 1$. Similar to the left corner, the right corner should not be located out of the block model boundaries. The right corner is forced to be located in C (i.e., it should be less than or equal to J). Therefore, the cumulative value is expressed by the following two-conditional equation (Eq. (3)):

$$\begin{matrix} f_j(x_j) = \\ x_j \neq 0 \end{matrix} \begin{cases} -\infty & j + x_j - 1 > J \\ \max\{f_m(x_m) + M^-.H_{j,m}(x_j, x_m) : m < j\} + \sum_{n=j}^{j+x_j-1} c_n & j + x_j - 1 \leq J \end{cases} \tag{3}$$

The current cumulative value ($f_j(x_j)$) is maximized by a specific combination of m and x_m. This combination is denoted by $\bar{m}$ and $\bar{x}_m$ symbols and they are stored in two independent functions (EQs. (4, 5)).

$$\Delta_j(x_j) = \bar{m} \tag{4}$$

$$\Phi_j(x_j) = \bar{x}_m \tag{5}$$

Also, the cumulative block set is defined by recursive Eq. (6). It is the union of the cumulative block set of $\bar{x}_m$ and the current state set.

$$F_j(x_j) = \begin{cases} F_{\bar{m}}(\bar{x}_m) \cup \emptyset & x_j = 0 \\ F_{\bar{m}}(\bar{x}_m) \cup \{B_n \in C : n \in \{j, \dots, j + x_j - 1\}\} & x_j \neq 0 \end{cases} \tag{6}$$

The union of all the state values from 1^{th} stage to j^{th} stage that create the $F_j(x_j)$, define the optimal path of x_j which is denoted by $\pi_j(x_j)$. The mathematical representation of $\pi_j(x_j)$ is

illustrated in Eq. (7). This path consists of all C blocks that create $F_j(x_j)$. In order to track these paths, it is necessary to determine the path size. Therefore, after solving each sub-problem and finding the best value of previous stage and state, the current path size is updated by Eq. (8). The only difference between $\pi_j(x_j)$ and $\pi_{\bar{m}}(\bar{x}_m)$ is the combination of (j, x_j). Therefore, the relationship between the sizes of these two path sets is represented by Eq. (8).

$$\pi_j(x_j) = \left\{ \begin{pmatrix} \Delta_{\Delta_{\bar{m}}(\bar{x}_m)(\Phi_{\bar{m}}(\bar{x}_m))}\left(\ldots \Phi_{\Delta_{\bar{m}}(\bar{x}_m)}(\Phi_{\bar{m}}(\bar{x}_m))\right) \\ \Phi_{\Delta_{\bar{m}}(\bar{x}_m)(\Phi_{\bar{m}}(\bar{x}_m))}\left(\ldots \Phi_{\Delta_{\bar{m}}(\bar{x}_m)}(\Phi_{\bar{m}}(\bar{x}_m))\right) \end{pmatrix}, \ldots, \begin{pmatrix} \Delta_{\Delta_{\bar{m}}(\bar{x}_m)}(\Phi_{\bar{m}}(\bar{x}_m)) \\ \Phi_{\Delta_{\bar{m}}(\bar{x}_m)}(\Phi_{\bar{m}}(\bar{x}_m)) \end{pmatrix}, \begin{pmatrix} \Delta_{\bar{m}}(\bar{x}_m) \\ \Phi_{\bar{m}}(\bar{x}_m) \end{pmatrix}, \begin{pmatrix} \bar{m} \\ \bar{x}_m \end{pmatrix}, \begin{pmatrix} j \\ x_j \end{pmatrix} \right\} \tag{7}$$

$$\left|\pi_j(x_j)\right| = 1 + |\pi_{\bar{m}}(\bar{x}_m)| \tag{8}$$

This algorithm does not allow the two adjacent states to overlap, without any negative effects on the global optimality.

After identifying the overlap between the current state value and previous state path $\pi_m(x_m)$), selection of x_m as the best previous state of x_j is prevented by penalizing the recursive cumulative value equation, $((\bar{m}, \bar{x}_m) \neq (m, x_m))$. Overlapping is recognized by using a two-conditional equation (Eq. (9)) which is denoted by $H_{j,m}(x_j, x_m)$ and its output is binary; if two states overlap then the output is one; otherwise it is zero. If its output is one, the previous state has overlapped with the current state. By adding a large negative number to the cumulative value equation, the previous state has no chance of being selected as the best previous state of the current state. When x_j is zero, the state set is empty and obviously the overlap set is also empty. In the other situation, when x_j is not zero, overlapping is determined by comparing the right corner of previous stopes on the $\pi_m(x_m)$ to the left corner of current stope (j), the overlapping is determined.

$$H_{j,m}(x_j, x_m) = \begin{cases} 0 & x_j = 0 \\ \sum_{n=1}^{|\pi_m(x_m)|} g(\lambda_n + \varphi_n - 1, j) & x_j \neq 0 \end{cases} \tag{9}$$

Where λ_n and φ_n are respectively the left corner and the length of stopes that are located on $\pi_m(x_m)$. By knowing the size and the final element of this path, all other elements are determined by backward tracking Eqs. (10) and (11).

$$\lambda_n = \begin{cases} m & n = |\pi_m(x_m)| \\ \Delta_{\lambda_{n+1}}(\varphi_{n+1}) & n < |\pi_m(x_m)| \end{cases} \tag{10}$$

$$\varphi_n = \begin{cases} x_m & n = |\pi_m(x_m)| \\ \Phi_{\lambda_{n+1}}(\varphi_{n+1}) & n < |\pi_m(x_m)| \end{cases} \tag{11}$$

After calculating all the cumulative values, it is necessary to determine the optimal path ($\pi_J(\tilde{x}_J)$) and its value ($f_J(\tilde{x}_J)$). This optimal path is represented the dynamic programming solution. Therefore, the value of the optimal path is the same as the dynamic programming value (z^{dp}). This value is calculated by Eq. (12).

$$z^{dp} = \max\{f_J(x_J): x_J \in \Gamma\} = f_J(\tilde{x}_J) \tag{12}$$

Also, the positions and dimensions of all the stopes that are located on the $\pi_J(\tilde{x}_J)$, are determined by backward tracking Eqs. (13) and (14).

$$J_n^* = \begin{cases} J & n = \left|\pi_J(\tilde{x}_J)\right| \\ \Delta_{J_{n+1}^*}(X_{n+1}^*) & n < \left|\pi_J(\tilde{x}_J)\right| \end{cases} \quad (13)$$

$$X_n^* = \begin{cases} \tilde{x}_J & n = \left|\pi_J(\tilde{x}_J)\right| \\ \Phi_{J_{n+1}^*}(X_{n+1}^*) & n < \left|\pi_J(\tilde{x}_J)\right| \end{cases} \quad (14)$$

To determine C blocks that are located on the $\pi_J(\tilde{x}_J)$, each block index is compared to the right and left corners of stopes on the $\pi_J(\tilde{x}_J)$. If this index becomes greater than or equal to the left stope's corner and also less than or equal to the right stop's corner simultaneously, Eq. (15) returns one; otherwise c_j^* is zero.

$$c_j^* = \sum_{\substack{n=1 \\ X_n^* \neq 0}}^{|\pi_J(\tilde{x}_J)|} ge(j, J_n^*) ge(J_n^* + X_n^* - 1, j) \quad (15)$$

Algorithm 1 Pseudo-code of the One-Dimensional Exact Dynamic Programming Algorithm

ODEDPA(C, d_j)

$\Gamma = \{0\} \cup \{d_j, \ldots, 2d_j - 1\}$
for ($j = 1$ to J){
  for ($x_j \in \Gamma$){
    if ($x_j = 0$) {
      $f_j(x_j) = \max\{f_m(x_m) : m < j\}$
    }
    else{
      if ($j + x_{j-1} > J$){
        $f_j(x_j) = -\infty$
      }
      else ($j + x_{j-1} \leq J$){
        $f_j(x_j) = \max\{f_m(x_m) + M^-.H_{j,m}(x_j, x_m) : m < j\} + \sum_{n=j}^{j+x_j-1} c_n$
      }
    }
    $H_{j,m}(x_j, x_m)$ in the above equations is calculated using Eq. (9)
    Store the best previous stage and state: $\Delta_j(x_j) = \bar{m}$ and $\Phi_j(x_j) = \bar{x}_m$
    Update the path size: $\left|\pi_j(x_j)\right| = 1 + |\pi_{\bar{m}}(\bar{x}_m)|$
  }
}
Find the highest cumulative value (z^{dp}) and its corresponding state ($\tilde{x}_J$):
$z^{dp} = \max\{f_J(x_J)\} = f_J(\tilde{x}_J)$
Track the highest valued path (J^{dp}, X^{dp}) using Eqs. (13, 14)
Calculate values of C^*elements using Eq. (15)
Return $C^*$and z^{dp}

3.2. Three-dimensional approximate hybrid algorithm

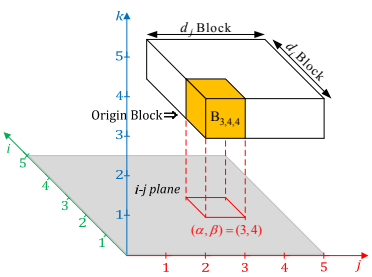
The proposed hybrid algorithm uses a combination of greedy and dynamic programming approaches to optimize stope boundaries. The dynamic programming is optimal but the greedy approach does not consider its previous and future decisions in order to obtain the current decision: therefore its optimality is not guaranteed. Consequently, a combination of these two approaches as a hybrid algorithm remains sub-optimal. The symbols that are used in this hybrid algorithm are as follows:

d_i Minimum stope size along i axis; in terms of blocks

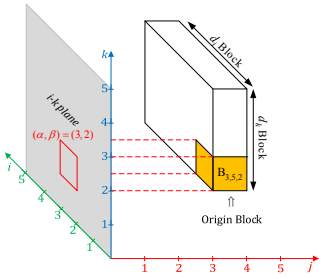
d_j Minimum stope size along j axis; in terms of blocks

d_k	Minimum stope size along k axis; in terms of blocks
P	A 3D economic block model; Input of Hybrid algorithm
I	The number of P blocks along i axis
J	The number of P blocks along j axis
K	The number of P blocks along k axis
$\mathrm{B}_{m,n,o}$	A P block; m, n, o present its position along i, j, k axes, respectively
$p_{i,j,k}$	Net value obtained by extracting $\mathrm{B}_{i,j,k}$
z^h	Value of hybrid algorithm
P^*	Solution of hybrid algorithm; a subset of P that presents the optimal or near-optimal stope boundaries as the output of Hybrid algorithm
s	Scenario number
$\bar{I}$	The maximum allowable index for origin blocks along i axis
$\bar{J}$	The maximum allowable index for origin blocks along j axis
$\bar{K}$	The maximum allowable index for origin blocks along k axis
Ω	Non-investigated set
C^*	solution of dynamic programming

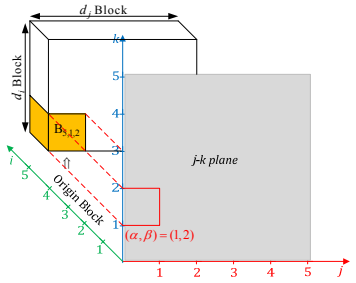
In this algorithm, by converting a subset of P blocks to a single block, a 1D matrix is built and is denoted by C. These subsets are 2D slices from P that own specific attributes which are defined as follows: the slices are fixed-size rectangles that are located in planes parallel to one of the $i-j$, $i-k$, or $j-k$ planes. They are identified by their origin blocks. Among the blocks of each slice, a unique block exists that has a minimum index on the both two slice directions which is named as origin block. Origin blocks have two indices on their parallel planes. The first index is α and the second one is denoted by β. Parallel slices with the same α and β are set behind each other in a specific direction. Three possible scenarios are defined to identify these directions and parallel planes. In the scenario 1, slices are parallel to the $i-j$ plane and their direction is along k axis (Fig 2.a). In the two other scenarios, No. 2 and No 3, slices are located on the planes parallel to $i-k$ and $j-k$ planes (Fig 2.a and Fig 2.b).



(a) Scenario 1(s=1)



(b) Scenario 2 (s=2)



(C) Scenario 3 (s=3)

Fig 2. Three possible scenarios for slice positions

In these scenarios, slices are arranged in the j and i directions, respectively. Dimensions of these slices depend on the minimum stope size at each plane. For instance, in scenario 1, the length and width of the slice is equal to d_i and d_j. The total value of $P \backslash P^*$ blocks which are located inside γ^{th} slice is denoted by c_γ. For each (α, β, s) combination located inside Ω, C is built and then Algorithm 1 is called. Algorithm 1 returns a binary 1-D matrix named as C^*. Based on the dynamic programming solution for each combination, the best combination is selected and denoted as $(\bar{\alpha}, \bar{\beta}, \bar{s})$. Our criteria for selecting the best combination is the absolute ratio of positive to negative blocks in the dynamic programming solution, i.e. $\max\{\sum_{\gamma=1}^{\Gamma} c_\gamma^* c_\gamma^+ / |\sum_{\gamma=1}^{\Gamma} c_\gamma^* c_\gamma^-|\}$. The objective function value at each iteration is called the best ratio. The dynamic programming solution that is obtained from the $(\bar{\alpha}, \bar{\beta}, \bar{s})$ combination is denoted as $\bar{C}^*$. In the next step, the $(\bar{\alpha}, \bar{\beta}, \bar{s})$ combination is removed from Ω. Then all of the blocks located in the γ^{th} slice are added to P^* if $\bar{c}_\gamma{}^*$ is equal to one. This procedure is iterated until the dynamic programming solution becomes positive. The pseudo-code of this algorithm has been illustrated as follows:

Algorithm 2 Pseudo-code of the Three-Dimensional Approximate Hybrid Algorithm

TDAHA (P, d_i, d_j, d_k)

$z^h = 0$
$P^* = \emptyset$
$(\bar{I}, \bar{J}, \bar{K}) = (I - d_i + 1, J - d_j + 1, K - d_k + 1)$
$\Omega = \{(i,j,1) \cup (i,k,2) \cup (j,k,3) : i \in \{1, \dots, \bar{I}\}, j \in \{1, \dots, \bar{J}\}, k \in \{1, \dots, \bar{K}\}\}$
$Best\ Ratio = 0$
$(\bar{\alpha}, \bar{\beta}, \bar{s}, \bar{C}_{1\times\Gamma}) = Null$
for (s = 1 to 3){
$$(A, B, \Gamma, l) = \begin{cases} (\bar{I}, \bar{J}, K, d_k) & s = 1 \\ (\bar{I}, \bar{K}, J, d_j) & s = 2 \\ (\bar{J}, \bar{K}, I, d_i) & s = 3 \end{cases}$$
for $(\alpha = 1$ to $A)$**{**
for $(\beta = 1$ to $B)$**{**
if $((\alpha, \beta, s) \in \Omega)$**{**
$C_{1\times\Gamma} = Null$
for $(\gamma = 1$ to $\Gamma)$**{**
$$c_\gamma = \begin{cases} \sum_{m=\alpha}^{\alpha+d_i-1} \sum_{n=\beta}^{\beta+d_j-1} p_{m,n,\gamma} : B_{m,n,\gamma} \notin P^* & s = 1 \\ \sum_{m=\alpha}^{\alpha+d_i-1} \sum_{n=\beta}^{\beta+d_k-1} p_{m,\gamma,n} : B_{m,\gamma,n} \notin P^* & s = 2 \\ \sum_{m=\alpha}^{\alpha+d_j-1} \sum_{n=\beta}^{\beta+d_k-1} p_{\gamma,m,n} : B_{\gamma,m,n} \notin P^* & s = 3 \end{cases}$$
}
Call Algorithm 1: ODEDPA$(C_{1\times\Gamma}, l)$
if $(\frac{\sum_{\gamma=1}^{\Gamma} c_\gamma^* c_\gamma^+}{|\sum_{\gamma=1}^{\Gamma} c_\gamma^* c_\gamma^-|} > Best\ Ratio)$**{**
$Best\ Ratio = \sum_{\gamma=1}^{\Gamma} c_\gamma^* c_\gamma^+ / |\sum_{\gamma=1}^{\Gamma} c_\gamma^* c_\gamma^-|$
$(\bar{\alpha}, \bar{\beta}, \bar{s}, \bar{\Gamma}, \bar{C}_{1\times\Gamma}, \bar{C}^*_{1\times\Gamma}) = (\alpha, \beta, s, \Gamma, C_{1\times\Gamma}, C^*_{1\times\Gamma})$
}
}
}
}
}
Remove $(\bar{\alpha}, \bar{\beta}, \bar{s})$ from non-investigated set: $\Omega = \Omega \backslash (\bar{\alpha}, \bar{\beta}, \bar{s})$
while $(\sum_{\gamma=1}^{\bar{\Gamma}} \bar{c}_\gamma^* \bar{c}_\gamma > 0)$**{**
Update TDAHA value: $z^h = z^h + \sum_{\gamma=1}^{\bar{\Gamma}} \bar{c}_\gamma^* \bar{c}_\gamma$
Update TDAHA solution:

29 $P^* =$

$$P^* \cup \begin{cases} \{B_{m,n,\gamma} : m \in \{\{\bar{\alpha}, \dots, \bar{\alpha} + d_i - 1\}\}, n \in \left\{\{\bar{\beta}, \dots, \bar{\beta} + d_j - 1\}\right\}, \gamma \in \{1, \dots, \bar{\Gamma}\}\} & \bar{s} = 1, \bar{c}_\gamma^* \\ \{B_{m,\gamma,n} : m \in \{\{\bar{\alpha}, \dots, \bar{\alpha} + d_i - 1\}\}, n \in \left\{\{\bar{\beta}, \dots, \bar{\beta} + d_k - 1\}\right\}, \gamma \in \{1, \dots, \bar{\Gamma}\}\} & \bar{s} = 2, \bar{c}_\gamma^* \\ \{B_{\gamma,m,n} : m \in \left\{\{\bar{\alpha}, \dots, \bar{\alpha} + d_j - 1\}\right\}, n \in \left\{\{\bar{\beta}, \dots, \bar{\beta} + d_k - 1\}\right\}, \gamma \in \{1, \dots, \bar{\Gamma}\}\} & \bar{s} = 3, \bar{c}_\gamma^* \end{cases}$$

30

31 Go to Line 5

32 }

33 **Output** z^h and P^*

A graphical visualization for converting 2D slices to 1D matrix (C) is illustrated in Fig 3. The green blocks are located in the P^* set and they are not considered in the calculation of C elements (Algorithm 2).

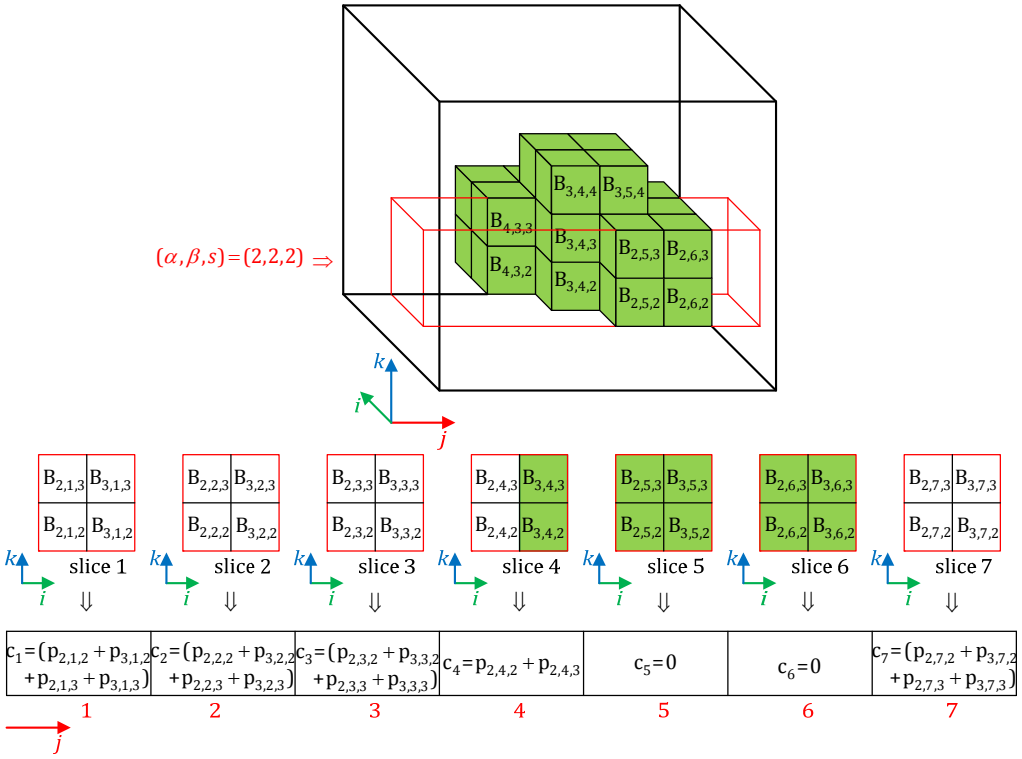
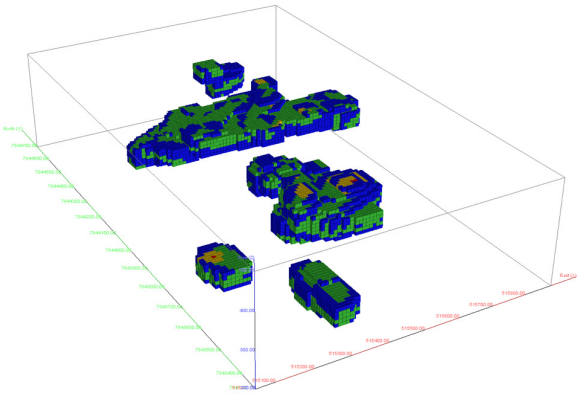
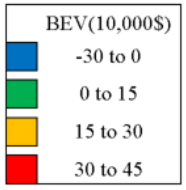


Fig 3. Converting 2D slices of P blocks to a single block in C

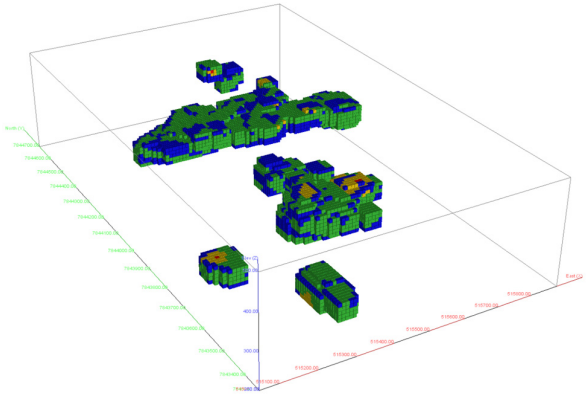
4. Application of algorithm in a case study

A poly-metal silver-zinc-lead deposit was modeled using the proposed algorithm. This deposit consists of 1,036,800 blocks. Block dimensions along the all main coordinate axes are 10 m and the average grade of silver, zinc and lead is 139.21 gr/ton, %3.21 and %0.26, respectively. Due to the high volume of the deposit's overburden, underground mining methods generate more profit than open pit method. Also, feasibility studies show that the sublevel stoping method is more compatible than other underground mining methods for extracting this deposit. The minimum length, width and height of the stopes are assumed to be 50 m. Fig. 4 shows the stope layouts generated by the Floating Stope, MVN, Greedy and proposed hybrid algorithms. The codes are written in C# programming language and the algorithms are run on a personal computer with an Intel(R) Core(TM) i5-2430 M CPU @ 2.40 GHz and 4.00GB of RAM. Table 2 shows the net profits and solution times of these algorithms.

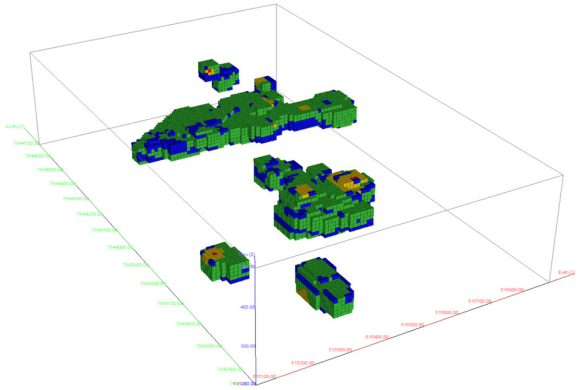




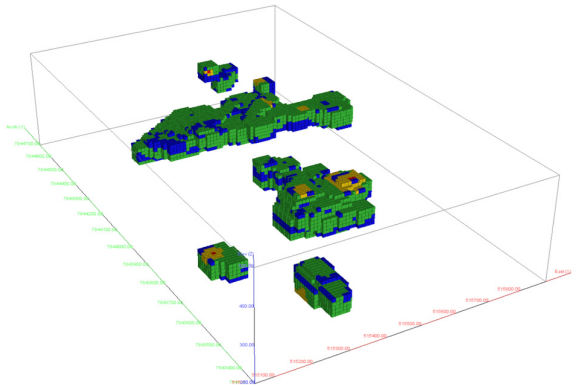
(a) A 3D view of stopes obtained by Floating Stope algorithm (Inner Envelope)



(b) A 3D view of stopes obtained by MVN algorithm



(c) A 3D view of stopes obtained by greedy algorithm



(d) A 3D view of stopes obtained by new proposed hybrid algorithm

Fig 4. Visualization of stope layouts from (a) Floating Stope, (b) MVN, (c) greedy, (d) proposed hybrid algorithm

Table 2. Results from the Floating Stope, MVN, greedy and new hybrid algorithms

Algorithm	Net profit ($)	Solution times	Number of minable blocks
Floating Stope	268,525,412	00:00:03	16,925
MVN	500,423,614	00:00:01	12,898
Greedy	582,352,644	00:02:08	11,678
Hybrid Algorithm	584,809,384	00:22:54	11,541

The new hybrid algorithm improves the profit by 117.87%, 16.86% and 0.42% compared to the Floating Stope, MVN and Greedy algorithms, respectively. Although the new algorithm requires more run time compared to its alternatives, its solution time still falls within a reasonable range given the size of the problem.

5. Conclusion

Undoubtedly, stope boundary optimization is one of the main mining problems which plays an important role in the economy of the mining industry. However, four decades after the first algorithm was presented for this problem, there is yet to be a comprehensive algorithm. Most of the algorithms are heuristic and their optimality is not guaranteed. Also, due to many simplifications in designing rigorous algorithms, they are not appropriate to solve real case problems. In this research, a new hybrid algorithm was introduced for stope boundary optimization. Although it may provide a locally optimum solution, it can find a better solution compared to its alternative algorithms at a reasonable CPU time. This hybrid algorithm and three existing algorithms were employed to find optimal stope boundary in a silver-zinc-lead deposit. The minimum and maximum improvements by this algorithm were 0.42% and 117.87%, respectively. The obtained results confirm the introduced algorithm's ability to find a better solution to stope boundary optimization problems. Better solution and better boundaries mean less waste of mining project capital, while help mining companies to develop sustainable projects that boost their profit. At the end, it should be noted that substitution of the proposed greedy algorithm by one of its alternatives such as genetic algorithm (GA), particle swarm optimization (PSO), simulated annealing (SA) and so on may yield to better solutions for the stope boundary optimization problem. For further investigation, it is suggested to check these meta-heuristics instead of the new proposed greedy algorithm.

6. References

[1] Alford, C., 1995. Optimisation in underground mine design. *In: 25th Interna-tional Symposium on the Application of Computers and Operations Research in the Mineral Industry*. The Australasian Institute of Mining and Metallurgy, pp. 213–218.

[2] Ataee-pour, M., 2000. A Heuristic Algorithm to Optimize Stope Boundaries Ph.D Thesis. University of Wollongong, New South Wales.

[3] Ataee-pour, M., 2005. A critical survey of the existing stope layout optimization techniques. *J. Min. Sci.* 41 (5), 447–466.

[4] Bai, X., Marcotte, D., Simon, R., 2013. Underground stope optimization with network flow method. Comput. *Geosci.* 52, 361–371.

[5] Bai, X., Marcotte, D., Simon, R., 2014. A heuristic sublevel stope optimizer with multiple raises. *South. Afr. Inst. Mining Metall*. 114, 427–434.

[6] Erdogan, G., Cigla, M., Topal, E., Yavuz, M., 2017. Implementation and comparison of four stope boundary optimization algorithms in an existing underground mine. *Int. J. Min. Reclam. Environ.* 31 (6), 389–403.

[7] Jalali, S.E., Ataee-pour, M., Shahriar, K., 2004. A 2D dynamic programming algo-rithm to optimize stope boundary. *In: Proceedings of the Thirteenth Interna-tional Symposium on Mine Planning and Equipment Selection*, Wroclaw, Poland, pp. 45–52.

[8] Ovanic, J., Young, D., 1995. Economic optimisation of stope geometry using separable programming with special branch and bound techniques. *In: Third Canadian Conference on Computer Applications in the Mineral Industry*. Quebec, pp. 129–135.

[9] Ovanic, J., Young, D., 1999. Economic optimisation of open stope geometry. *In: 28th International APCOM Symposium. Colorado school of Mines*, Colorado, pp. 855–862.

[10] Riddle, J., 1977. A dynamic programming solution of a block-caving mine layout. *In: 14th International Symposium on the Application of Computers and Opera-tions Research in the Mineral Industry*. Society for Mining, Metallurgy and Exploration, Colorado, pp. 767–780.

[11] Sandanayake, D.S.S., 2014. Stope Boundary Optimization in Underground Mining Based on a Heuristic Approach Ph.D Thesis. Curtin University, Western Australian School of Mines.

[12] Sandanayake, D.S.S., Topal, E., Asad, M.W.A., 2015a. A heuristic approach to optimal design of an underground mine stope layout. *Appl. Soft Comput*. 30, 595–603.

[13] Sandanayake, D.S.S., Topal, E., Asad, M.W.A., 2015b. Designing an optimal stope layout for underground mining based on a heuristic algorithm. *Int. J. Min. Sci. Technol*. 25, 767–772.

[14] Topal, E., Sens, J., 2010. A new algorithm for stope boundary optimization. *Coal Sci. Eng*. 16 (2), 113–119.

[15] Nhleko, AS., Tholana, T., Neingo, PN., 2017. A review of underground stope boundary algorithms. *Resour. Policy* doi: 10.1016/j.resourpol.2017.12.004.

[16] Nikbin, V., Ataee-pour, M., Shahriar, K., Pourrahimian, Y., 2017. A Greedy Algorithm for Stope Boundaries Optimization. *8th Annual Report Mining Optimization Laboratory (MOL)*. University of Alberta, pp. 246–252. Report Eight.

[17] Qi, C., Fourie, A., Ma, G., Tang, X., Du, X., 2017. Comparative study of hybrid artificial intelligence approaches for predicting hangingwall stability. *J. Comput. Civil Eng*. 32 (2), 04017086.

[18] Qi, C., Fourie, A., Chen, Q., Zhang, Q., 2018a. A strength prediction model using artificial intelligence for recycling waste tailings as cemented paste backfill. *J. Cleaner Prod*. 183, 566–578.

[19] Qi, C., Fourie, A., Chen, Q., 2018b. Neural network and particle swarm optimization for predicting the unconfined compressive strength of cemented paste backfill. *Constr. Build. Mater*. 159, 473–478.

[20] Qi, C., Tang, X., 2018. Slope stability prediction using integrated metaheuristic and machine learning approaches: a comparative study. *Comput. Ind. Eng*. 118, 112–122.

Underground Mining Stope Layout Optimization and Production Scheduling: A Review of Existing Solvers and Algorithms

Emmanuel John Andrew Appianing and Eugene Ben-Awuah
Mining Optimization Laboratory (MOL)
Laurentian University, Sudbury, Canada

ABSTRACT

Planning of an underground mine poses considerable difficulties in the areas of safety, environment, ground control and production scheduling. To maximize the profitability of these operations, an integrated optimized stope layout and production schedule is required to coordinate underground mining activities such as development, extraction, haulage, dumping, stockpiling and processing. An optimal mine life is detailed in proper scheduling of all available resources and more importantly the mining sequence. The process of preparing such strategic mine plan is an optimization decision-making process which entails a mining sequence that takes into consideration the physical limitations, resource and production constraints, financial performance and uncertainties of the mine, and meets the required quantities of ore type at each period throughout the life of the mine. The current underground stope layout optimization and production scheduling algorithms in literature lack the ability to deal with real-world large-scale complex underground mine planning problems. Most of the algorithms and solvers available do not integrate production scheduling with stope layout optimization. A few that integrate this two important mine planning processes do not have the ability to address: i) the broad range of mining methods (exhaustiveness), ii) practical problem sizes, and iii) stochastic parameters. This paper reviews relevant literature on existing solvers and algorithms for underground stope layout and production scheduling optimization, and outlines their limitations and potential research opportunities.

1. Introduction

Underground mining has aided the extraction of deep occurring hard rock minerals such as gold, copper, silver, and iron for centuries. The extraction of these minerals generally require extensive development work that is time-consuming. Fig. 1 is a section through an underground mine showing some developments. Underground mining has a low level of flexibility in its operations. To maximize the profitability of these operations, an integrated optimized mine plan and schedule are required to coordinate underground mining operations including development, stoping, haulage, dumping, stockpiling and processing. To determine the optimal mine life, proper scheduling of all available resources and a strategic mine plan is required. Preparing such strategic mine plan is an optimization decision-making process that involves a mining sequence that takes into consideration the physical limitations, available resources and production constraints, financial performance, and uncertainties in the mine to meet the required quantities and quality of ore type throughout the mine life. Planning, optimization and production scheduling of underground operations are relevant to ensuring the efficient utilization of resources which in effect reduces production cost.

Planning an underground mine is a complex procedure consisting of five stages (Kuchta et al., 2004): i) determining the geometry and grade (or quality), ii) deciding how to mine the ore by underground mining, iii) designing the mine infrastructure, or layout of the mine to efficiently exploit the ore, iv) planning how to mine and process the ore, and v) decommissioning the mine and restoring the site to an environmentally acceptable state.

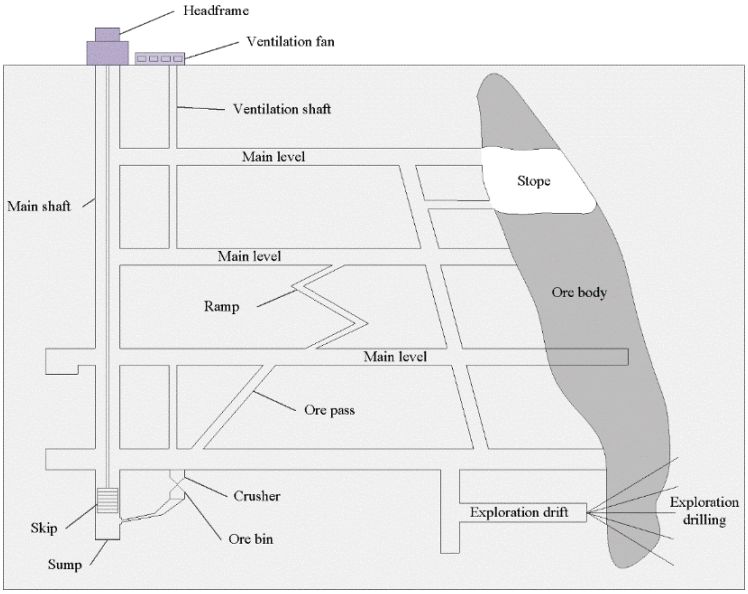


Fig. 1. A cross-section through a typical underground mine modified after Erdogan and Yavuz (2017).

The mining and processing (or production) phase require detailed scheduling. Specifically, a production schedule that must provide a mining sequence that considers the physical limitations of the mine and, meets the demanded quantities of each ore type at each period throughout the life of the mine. Mines, therefore, employ schedules as long-term strategic planning tools to determine when to start mining a production area and as short-term operational guides.

According to Hamrin et al. (2001) and Topal and Sens (2010), a stope is an underground opening from which ore has been excavated. O'Sullivan and Newman (2015) describe stope layout as analogous to the ultimate pit limit in open pit mining, and that it defines the design in underground mining. The optimal stope layout is defined by the size, location and number of stopes within an orebody model. This model is described as consisting of a series of layers for which each is composed of a number of rows referred to as panels, where the panels are made up of a series of rings. Selecting the best combination of available stope layout directly affect the profitability of an underground mining operation (Dimitrakopoulos and Grieco, 2009; Topal and Sens, 2010).

Production scheduling can be defined as the allocation of resources and mined out ore reserves over a period of time with particular sets of constraints. Mine production scheduling is an optimization process which assigns the extraction sequence of mining blocks based on constraints which incorporate, among other things, a method of mining, slope size in other to maximize the Net Present Value (NPV) of a mine (Martinez and Newman, 2011; Gangawat, 2014). Pourrahimian (2013) explains that production scheduling defines the tonnages and grades to be mined throughout the mine-life. Scheduling in any mining system has an enormous effect on the operation's economics. It is becoming essential to generate production schedules that will provide optimal operating strategies while meeting technical and environmental constraints. Improvements in computing power and scheduling algorithms over the past years have allowed planning engineers to develop models to schedule more complex mining systems. Production scheduling problems are usually complex due to the nature and variety mining constraints.

Production scheduling is known to significantly influence the viability of mining operations and scheduling techniques with the capacity to provide optimal results within a reasonable time frame. It presents mining operations with the ability to examine a variety of situations and quickly respond to revised and updated data. This helps to reduce uncertainty and assists in the decision-making process, for future strategies which may result in less volatile operating performance and often higher profitability (Nehring et al., 2010).

Nehring et al. (2012) classified mine production scheduling into two main categories: long-term production scheduling and short-term production scheduling. Long-term production scheduling is done over the life of the mine. In short-term production scheduling, the result generated from the long-term production scheduling is broken down according to task or time period that need to be studied, and different sets of constraints active in short-term are applied to minimize the deviations from the pre-defined capacities.

Long-term underground mine production scheduling problems have been studied by researchers (Jawed, 1993; Nehring et al., 2010; Pourrahimian et al., 2012). These studies have been based on the development of algorithms using operation research methods such as Linear Programming (LP), Mixed Integer Programming (MIP) and Mixed Integer Linear Programming (MILP). The current optimization algorithms in the literature lack the ability to deal with real world large scale and complex, underground mine planning problems. They do not have the ability to address: i) the broad range of mining methods (exhaustiveness) ii) practical problem sizes and iii) stochastic parameters.

This paper reviews relevant literature on existing models and algorithms for underground stope layout and production scheduling optimization for further integration and improvements. The next section of this paper covers summary of the literature review on underground optimization. Section 3 gives details of underground stope layout/limit optimization models and algorithms. Section 4 covers limitations of underground stope layout/limits optimization models and algorithms. Section 5 highlights the underground production scheduling models and algorithms. Section 6 presents the integrated stope layout/limit and production scheduling optimization mathematical programming models and algorithms. In Section 7, stochastic mine planning approaches in underground mining are presented. Research opportunity is presented in section 8. Section 9 documents the rationale behind the proposed PhD research approach and finally, Section 10 presents the summary and conclusions.

2. Summary of Literature Review

Generally, optimization is best described as a process that finds the best or optimal solution for a given problem (Chakraborty, 2010). Mine optimization is an inherent iterative process since all facets of mine design and scheduling are interrelated. It is not possible to change e.g. the level spacing of a mine, without affecting the scheduling, or the stoping efficiencies and number of crews without considering the development requirement (Smit and Lane, 2010). For this reason, changes to design and schedule are usually tested using graphic designs and scheduling applications such as Datamine, Surpac, MineShed, MineSight, NPV Scheduler, Deswik, Maptek Vulcan, Gems and Whittle (Musingwini, 2016; Erdogan et al., 2017).

Optimization in terms of mine planning has also been defined as a descriptive set of techniques that introduce analytic mathematical methods to arrive at an option out of multiple choices; thus bringing it into planning activities. The technique embodies three stages (Shahriar et al., 2007):

- Creation of a mathematical model of the activity;
- Adoption of a criterion; and
- Development of an algorithm.

Underground optimization is considered as a constrained optimization problem, which is aimed at finding the highest profitable stope layout subject to geometric constraints (Bai, 2013). Chakraborty (2010) and Ataee-Pour (2000) further explain that optimization problems are centered around three main factors which are formal procedures of Operations Research (OR):

- An objective function which is either to be maximized or minimized;
- A set of unknowns or variables that affect the objective function; and
- A set of constraints that allow the unknowns to take on certain values but exclude others.

The idea of an optimization problem is therefore to find values of the variables that maximize or minimize the stated or defined objective function while satisfying the stated constraints. The optimal mine layout is greatly influenced by the chosen optimization criteria. The following criteria have been used in mine geometry optimization (Ataee-Pour, 2000):

- Maximization of the total mine economic value;
- Maximization of value per tonne of the saleable product;
- Maximization of the life of the mine provided that the value per tonne does not fall below a certain figure; and
- Maximization of the metal content within the mine.

It is worthy of note that in stope geometry optimization, maximization of the total mine economic value or NPV is frequently used.

Ataee-Pour (2000) asserts that mine geometry or layout is one of the most important stages of mine planning for both surface and underground operations. Outlining the mineable ore assists in determining the amount of reserve, as well as mine life and production scheduling. There is a relationship between mine layout and other mine planning stages such as equipment selection and haulage routes. In effect, mine design and optimization is an interactive process with inconsistency between mine layout definition and production scheduling. However, the optimal layout cannot be determined until the values of the blocks are known or defined. The definition of the block values also depends on factors such as commodity prices and mining costs that in turn depend on when the blocks are to be exploited. Conceptualizing a stope optimizer as a solid box, the input data to the box is the ore block model quantifying the mineral content or economic profit of block volumes on regular grids and the output is the geometry of stope indicating the blocks to be mined (Bai, 2013).

It is a common practice for underground mine plans to be created sequentially, where results from one planning process form the input data for another. The disadvantage to this method is that optimizing an individual mine process, such as stope layouts introduces a likelihood of increasing costs or decreasing revenues associated with other areas such as production scheduling, as harmful decisions must be balanced. This is practical for manual methods; computerized optimization techniques should consider an integrated approach to creating the global mine plan. Considering the interaction and influence that individual underground mine planning processes have on each other during optimization will provide more profitable results than if ignored (Little et al., 2013).

Due to the significant nature of optimization in the designing and planning of mines, researchers have carried out several works. However, little has been done in the formulation, optimization of design and the long-term production scheduling problems for underground mines. In addition, limited algorithms are available for the optimization and scheduling of underground mine production. It is worth noting that until the last few years, comprehensive computer-based designing, planning, and scheduling tools for underground mining did not exist hence process was manual, tedious and inefficient. Mining software developers were able to tackle underground mine design, planning and production problems after their success in applying computer techniques that have been embedded in commercial software packages to open pit mining. This can be attributed to the large

number of underground mining methods, lack of versatility and complexity of underground mining economic block modeling (Ataee-Pour, 2000; Newman et al., 2010).

Little et al., (2013) reported that software tools for strategic underground mine planning and scheduling are lacking and that the demand for tools will increase as deeper deposits amenable to underground mining continue to grow in value. A number of optimization techniques have been developed for stope layout design and progress continues to be made in the area of production scheduling, both of these mine planning areas are amenable to improve in terms of guaranteeing optimal results.

3. Underground Stope Layout/Limit Optimization Models and Algorithms

A stope is an underground opening from which ore has been or is to be extracted. Selection of the best combination of available stope boundary will directly affect the profitability of the operation (Topal and Sens, 2010).

From 1965-2000, out of 62 publications that contributed to mine geometry optimization, only 10 papers focused on underground mining related. Generally, algorithms for open pit limit optimization are numerous, amounting to approximately two and a half times more than that for underground methods. True optimum solution for the open pit limit optimization is often guaranteed and, additionally, several computer packages are available to the industry. However, only few algorithms have been developed for optimization of ultimate stope layout or boundaries in underground mining (Ataee-Pour, 2005). Nikbin and Ataee-Pour (2016) reported that the first algorithms were presented about five decades ago for optimization of mining limits. Most of the algorithms presented are applied to optimization of open pit limits. Few algorithms have been presented for underground cases, most of which have been tailored for a specific mining method or based on heuristic techniques. These heuristic algorithms do not guarantee the true optimum. There are, however; some rigorous algorithms, which provide a true optimum, but fail to provide a 3D solution. Each of the algorithms developed for ultimate underground layout has only considered few constraints. Consequently, the Floating Stope of Datamine, Maximum Value Neighborhood (MVN), can be run on 3D block models, but they are heuristic and cannot find the true optimum layout. Although Branch and Bound technique and Dynamic programming also called Optimum Limit Integrated Probable Stope (OLIPS) are rigorous, they have been presented for 1 and 2-dimension problems only. Underground mine optimization has attracted more attention in the last 15 years. Less effort has been committed to the field of underground mining and only a few algorithms are available for economic optimization of underground stope boundaries (Sotoudeh et al., 2017).

The available algorithms for underground stope boundaries or layout limit optimization are illustrated in Fig. 2 (Ataee-Pour, 2000; Ataee-Pour, 2005; Grieco and Dimitrakopoulos, 2007; Shahriar et al., 2007; Bai, 2013; Little et al., 2013; Sandanayake, 2015; Erdogan et al., 2017; Sotoudeh et al., 2017; Nhleko et al., 2018): These optimization techniques are discussed in the subsequent sessions.

3.1. Stope boundary/layout limit algorithms based on heuristic logic

Porumbel (2012) explains that a practical approach to solving many intractable problems of large size is by using heuristic search algorithms or simply, heuristics. They are generally used to explore only a small part of a very vast search space. Heuristic algorithms use reasonable resources and are able to produce acceptable solutions, but without any theoretical guarantee. They have the ability to produce competitive results not only on well-known Nondeterministic Polynomial (NP) complete problems, but on any computational problem for which exact or rigorous algorithms require prohibitive time.

Cheimanoff et al. (1989) developed the Octree Division Approach. The technique evolved from the prototype production scheduling tool BONANZA which forms part of GEOCAD package used for

the development of geological resources to mining reserves. The features of CAD and artificial intelligence modules are used to develop a rule-based system to generate the shape of the mineable ore while imposing the underground mining constraints. The program uses the octree space division algorithm to remove undesired mining blocks based on the minimum size.

BONANZA is designed in three steps, the first step is gathering data, including borehole data or geological underground sampling, results from geostatistical analysis or interpreted contours supplied by the geologist and intuition from geoscientist of the mining engineer such as the shapes of geological objects. These data are used to build a geometrical model. The second step is transforming the modelled geological resources into mineable reserves. In this step different production factors are simulated to build possible workable volumes, using the available geological resources. In the final step the geological resources are economically evaluated to determine workable reserves and mining sequence. The mine layout is determined by using a rule-based programming method. The two main constraints are geometric and economic ones (Topal and Sens, 2010). According to Sandanayake (2015) the algorithm consequently divides the reserves into sub-volumes for further economic evaluations. These sub-volumes are either stored or removed from the model, if their mineral content or dimensions violates the constraints set by the algorithm.

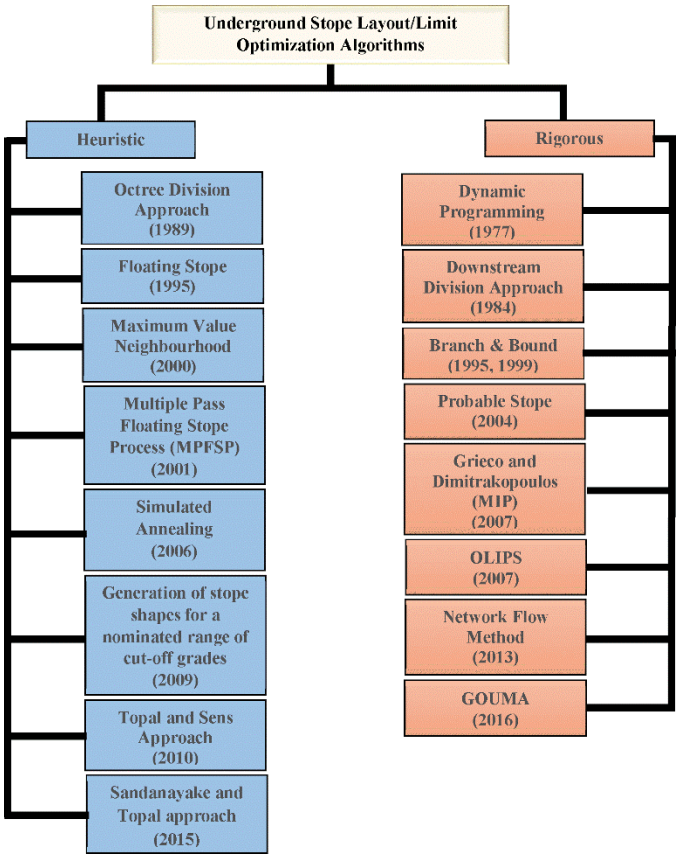


Fig. 2. Taxonomy of underground stope layout/limit optimization algorithms and models.

Alford (1995) explains that the mineable volumes, well-adapted to these constraints are determined through two main modules: Objective Manipulator and Shape Generator. The first phase "Objective Manipulator" gathers mineralized veins into convex blocks distinguishing "large" veins that justify a mineable block by themselves, merges those "close enough" into a single block, and separates those "too far from one another" into two separate blocks. A second phase, "Shape Generator", progressively subdivides a boundary volume in an octree until the smallest subdivision matches the smallest unit. The Octree division algorithm generates a 3D feasible solution for the stope geometry, but it leads to more waste being added to the final mine layout because the algorithm does not analyze

the sub-volumes jointly. It includes individual sub-volumes that consider the minimum allowable dimensions in the final layout but not the amount of waste in these sub-volumes. As such, the algorithm does not guarantee the optimality of stope layouts hence, heuristic.

The Floating Stope algorithm was developed by Alford (1995) as an optimization tool used by Datamine to define optimal limit for mineable ore or stope envelope which can be economically extracted by underground stoping methods and was implemented on a fixed economic block model of an orebody (Shahriar et al., 2007). This tool is analogous to the Moving Cone method of open pit limit optimization. The main constraint in this algorithm is the geometry of the stope, which is translated into a minimum stope dimension in three orthogonal directions. The term floating stope is derived from the technique of floating a stope shape of the minimum stope dimensions around any block to locate the stope position of the highest stope grade. Different optimization objectives offered by the floating stope algorithm include the maximization of ore tons, the grade, and accumulated (dollar) value and minimization of the cutoff wastes. The problem is then to determine if any block above the specified cutoff grade can be included into a stope. The Floating Stope technique and its open pit analogue have different constraints. The Moving Cone technique examines cones independently. In fact, the mutual support between blocks is ignored in this method. As a result, two different cones may individually be uneconomical but when they are jointly considered the cones may be profitable. The Floating Stope approach has the opposite limitation. Two overlapping best stopes considered individually may be economical due to a number of high grade blocks. However, when they are considered together, the combination of the stopes may be uneconomical. Sandanayaka (2015) illustrates the issue using a hypothetical 2D scenario. The two stopes measure 5×5 blocks, i.e., five blocks along the x axis and five blocks along the y axis, overlap and share six mining blocks. The two overlapping stopes are designed as stope 1 and stope 2 and the economic values of their shared mining blocks are indicated.

The total economic value of the combination of stopes differs from the actual economic value. Therefore, the algorithm does not guarantee an optimal solution to the problem. The most significant advantage of this algorithm is its simplicity. A new version of the algorithm, the Multiple Pass Floating Stope Process (MPFSP) employs multiple-optimization process. It provides a 3D evaluation over the optimization and sensitivity analysis for mineable reserves and stope geometry while considering partial blocks in the final layout. Moreover, the algorithm benefits from generality. It is not specialized for a certain mining method. Finally, a commercial software package has been developed for the Floating Stope algorithm and it is has been reported by Shahrair et al. (2007) to give true optimality among the list of stope limit optimizers. However, it is a heuristic approach and lacks the rigorous mathematical support (Ataee-Pour, 2000).

The Maximum Value Neighbourhood (MVN) algorithm is another heuristic approach that was developed and implemented by Ataee-Pour (2000). He developed a Stope Limit Optimizer (SLO) on a Fortran 90 based program with Winteracter software developer to implement MVN algorithm. He proposed this algorithm in 1997 to optimize stope boundaries using a 3D dimensional fixed economic block model to locate the best neighborhood of a block which guarantees the mine geometry constraints (Shahriar et al., 2007). According to Ataee-Pour (2005), the neighborhoods are restricted by the mine geometry constraints. The neighborhood concept is based on the number of mining blocks equivalent to the minimum stope size (stope/block ratio). Since several neighborhoods are available for each block, the one that provides the maximum net value is located for inclusion in the final stope. Sandanayake (2015) explains with an example using a hypothetical row of six mining blocks. The six mining blocks are labelled a-f in alphabetical order for easy reference. This algorithm can also be applied to any underground mining method although it does not guarantee the true optimum stope layout (Shahriar et al., 2007).

The Multiple Pass Floating Stope Process (MPFSP) was developed by Cawrse (2001) to extend and improve the functionality of Datamine software. The process of the algorithm relies on the same principals as the floating stope algorithm. The MPFSP process is divided into three steps: input

parameter definition, file generation and file management. Input parameters (multiple), such as the head grade, cutoff grade and maximum waste, are defined by the user. During the file generation step, economic stope envelopes are created for each set of parameters. Finally, the data files or the statistical files generated in the process are converted into a Microsoft Excel compatible (CSV) format. The generated envelopes based on input parameters can provide extra information during the mine design process helping the designers to improve the efficiency of their mine design. Although the method can assist in stope boundary selection and design, it does not generate optimum stope layouts (Topal and Sens, 2010; Sandanayake, 2015).

The Simulated Annealing (SA) approach is an algorithm that was developed by Manchuk (2006) and Manchuk and Deutsch (2008) for stope geometry and sequencing optimization. In this method, a stope is parameterized as a geometric object consisting of a set of vertices and edges that formed a triangulated mesh. This notably facilitated the manipulation of stope geometric constraints in optimization. The rationale behind the optimization is to randomly adjust the shape of stopes, respecting the geometric constraints, in order to find the shell enclosing maximum profits. The algorithm offers a general 3D solution to engineers, integrating full geometry constraints regardless of the mining methods selected. One of the limitations of the simulated annealing process is that it can be long. This is seen especially when the number of vertices is large to construct a complex geometry. Another is that the time of convergence to optimality can be unrealistic. Therefore, in practice, SA is regarded as a heuristic for which the quality of the approximation to the real optimal solution is difficult to assess (Bai, 2013).

Alford and Hall (2009) developed a method for automated stope design. In this method, the stope optimization is run at a sequence of cut-off grades to generate a series of 'nested stopes' which is a similar concept in the pit optimization where 'nested pits' are generated. The method is able to define the best set of extraction levels and stope heights. However, it does not take into account mining cost as a function of the size and the shape of the stop. It specifies a fixed cutoff grade in the design process, which does not allow an optimum solution (Erdogan et al., 2017).

Topal and Sens (2010) heuristic approach is a methodology that consists of three basic elements: the block converter, stope optimizer and visualizer (Topal and Sens, 2010). During initialization, the given mining block model is converted to a regularized block model, i.e., a block model that constitutes mining blocks with consistent sizes. Given their dimensions (height, length and width) stopes are generated from the regularized block model. Finally, a heuristic stope optimization algorithm is implemented in Matlab software based on the economic value of stopes and the results are visualized with respect to different user-defined parameters. The advantage is that it locates stope boundaries using different stope sizes and selection strategies in 3D. To its disadvantage, the algorithm selects the stopes in descending order of the stope economic values while removing the overlapping stopes. To clarify, this process discards the possibility of multiple stope combinations that can be derived from a given stope set. Among these multiple combinations, there may be combinations with higher total economic values (Topal and Sens, 2010; Sandanayake, 2015).

Sandanayake (2015) also proposed and developed a new 3D heuristic algorithm for the stope layout optimization. It finds a unique solution that maximizes the economic value of the stope layout under physical and geotechnical constraints. The suggested algorithm recommends a unique solution for stope layout and generates non-overlapping stopes. Moreover, it includes variable stope sizes with or without pillars and satisfies the mining and geotechnical constraints. However, in order to find the optimal solution, the algorithm needs to evaluate all the possible unique combinations that require significant computational power in large-scale applications. This limits the algorithm to find the optimal solutions in large-scale data-sets (Erdogan et al., 2017).

3.2. Stope boundary/layout limit algorithms based on rigorous logic

Rigorous algorithms can be described as the algorithms based on a mathematical model and hence they guarantee an optimum solution. Such algorithms are robust, possess objectivity, they are

tractable, provide model solutions and facilitate sensitivity. Furthermore, optimal solutions provided by refined rigorous algorithms can often be reached more rapidly with basic meta-heuristics (Martinich, 2008; Porumbel, 2012; Nhleko et al., 2018).

Dynamic Programming (DP) algorithm was proposed by Riddle (1977) to undertake optimization of stope layout of block caving mining method. It was developed as an extension of 3D dynamic programming method for ultimate open pit optimization that had been developed by Johnson and Sharp in 1971. The algorithm was developed in Fortran and implemented on hypothetical economic block models. The algorithm is a multi-section 2D solution for 3D problems. This simply implies that the approach is provided as an optimum stope in 2D section but fails to determine the actual optimal stope in 3D (Ataee-Pour, 2005; Shahriar et al., 2007; Sotoudeh et al., 2017). In running the algorithm, it assumes that there is no footwall in the optimization process and that the maximum profit is achieved irrespective of the footwall design. It then defines a footwall in operational regions and examines the profitability of all feasible solutions for mining and non-mining regions. In the subsequent steps, the process divides the defined footwall region into two sub-regions if the profit is greater than or equal to an assumed profit. However, the process terminates when there are no more profitable footwalls to examine, or no further feasible footwalls can be introduced into the operational region (Sandanayake, 2015). The algorithm is a rigorous mathematical solution to the problem. This method is known to be limited to block caving mines and for that matter unable to function in optimizing the layout of other underground stopping methods (Ataee-Pour, 2005; Shahriar et al., 2007).

The downstream geostatistical approach also known as the mathematical morphology approach was proposed by Deraisme et al. (1984). The introduction of this approach was to determine the outline of the mineable orebody shape in underground mines. This approach was constructed for cut-and-fill and sublevel stoping methods and was based on a 2D sectional block models of the deposit. Mathematical morphology was used to transform the image of ore blocks above cut-off grade to another image satisfying the stope geometry constrains (Erdogan et al., 2017). The main disadvantage of the algorithm is its complexity. The traditional algorithm has been developed for specific mine (project) and tailored for two specific mining methods (cut and fill and sublevel stoping methods). The algorithm benefits from a mathematical support and is expected to provide the real optimum solution at least in 2D (Erdogan and Yavuz, 2017). However, reported by the developers that the economically optimized images do not necessarily respect the geometrical constraints. Further modifications of the images are required to make them meet the constraints and this makes the results non-optimum (Ataee-Pour, 2005). The approach takes into account grade uncertainty in the optimization process. It controls only the stope geometry so it does not consider the economic profit related. This limits the optimality of results in 3D (Erdogan et al., 2017).

Branch and Bound Technique is a mathematical programming technique for economic optimization of stope boundary. It was developed by Ovanic and Young (1995). An optimal economic stope boundary was developed by optimizing the starting and ending locations for mining within each row of blocks. To determine these locations, two piecewise linear, cumulative functions known as Type-Two Special Ordered Sets (SOS2) were used for each row (Ataee-Pour, 2005; Shahriar et al., 2007). SOS2 technique, referred to a separate programming, is used to optimize piecewise linear functions. It is defined as an ordered set of special variables for which the solution allows at most two variables to be non-zero, they must be adjacent. The first function sums block values along the row for inclusion within the stope boundary, while the second function sums block values for exclusion. It is an apparent advantage that the branch and bound algorithm removes some restrictions from mine layout design and, in particular, stope geometry optimization. There are no restrictions for blocks to be treated as a whole, but rather partial blocks can be included in the optimal stope. Moreover, blocks are not limited to be regular or uniform in shape. Whatever their shape or size it does not influence the optimization since the block cumulative function is used in modeling. In other words, branch and bound method allows regular or orthogonal block geometry so blocks can be formed following the

geologic variation and discontinuities. However, it optimizes the stope boundary in one dimension and neglects the wall slope constrains hence lacks a 3D implementation (Shahriar et al., 2007; Sandanayake, 2015; Erdogan et al., 2017; Sotoudeh et al., 2017).

Ovanic and Young (1999) introduced a Mixed Integer Programming (MIP) model to optimize stope boundaries. The model works by locating the optimal starting and ending points for mining within a row (mining panel) in a block model and in effect, it establishes the optimum stope boundaries. In order to determine the optimum starting and ending location of each panel, two piece-wise linear cumulative functions are calculated. The major advantage of this technique compared to the others is that the block geometry is not required to be regular or orthogonal. A disadvantage, however, is that the algorithm optimizes the stope boundary along the row of blocks in only 1D. Therefore, examples and results for using the algorithm in 3D do not exist. Another disadvantage is that the algorithm also only allows the user to optimize the design of extraction rows only, as the locations of these rows are determined earlier in the design process the algorithm only partially optimizes the mine design (Topal and Sens, 2010).

The Probable Stope (PS) algorithm was established by Jalali and Ataee-Pour (2004) based on Riddle's Dynamic Programming (DP) algorithm to optimize the stope limits of the mining methods which are feasible for vein deposits. The main difference between PS technique and the others is that, the algorithm is implemented on a particular economic block model including the constraints of stope dimensions. In effect, the most significant constraints within the objective function are eliminated. It has resulted in the algorithm being simple in concept, easy to program and reaches a solution quickly (Shahriar et al., 2007).

A probabilistic algorithm based on MIP was developed by Grieco and Dimitrakopoulos in 2007 and the application of the method at Kidd Creek Mine, Canada has been reported (Grieco and Dimitrakopoulos, 2007). In this approach, the ore body is initially divided into layers. Each layer is then subdivided into a number of panels and each panel is further subdivided into a series of rings. Each ring is assigned to a binary variable of the mixed integer model. The objective function of the algorithm maximizes the metal content at a given time. The minimum and maximum mining rings, and the sizes of the pillars that are to be left unmined between two primary stopes, are limited by the constraints of the model. Although the proposed method is the first stope design methodology that has acknowledged uncertainty and greatly assists stope design by considering geological uncertainty, it has potential drawbacks. Because the methodology is based on rings that have been predefined in terms of location and size to determine the most profitable stopes, it finds optimum stopes layout based on those rings. This approach does not allow accurate examination of the orebody over smaller areas and examination of stopes in different locations. In addition, as each ring is represented as a binary variable within the MIP model, it encourages a very long solution time as the number of rings increases within the model (Grieco and Dimitrakopoulos, 2007; Dimitrakopoulos and Grieco, 2009; Topal and Sens, 2010; Sandanayake, 2015).

Optimum Limit Integrated Probable Stope (OLIPS) is an algorithm developed in 2007 based on the Dynamic Programming method by Jalali et al. (2007). The algorithm complies with all technical and geometric constraints, provides mathematical proof and optimality. OLIPS algorithm has two major steps. In the first step, a conventional economic model of mining panel is constructed and in the second step, the probable stope economic model and integrated probable stope economic model are derived from a conventional model. Based on OLIPS algorithm, a computer program named Stope Boundary Optimizer (SBO) was developed and validated by 2D hypothetical models (Sotoudeh et al., 2017).

Network Flow method is an optimization algorithm that was introduced by Bai (2013) to optimize stope design based on graph theory and specifically applicable for the sublevel stoping method. The algorithm is based on a cylindrical coordinate system, which is defined around a specified vertical raise (initial) (Bai, 2013; Sandanayake, 2015). In the process, the graph is constructed using vertical

arcs for the footwall and hanging wall slope constraints, with horizontal arcs for the stope width constraint. Selecting a particular mining block for a stope should aim to maximize the stope value subject to two additional constraints: the maximum distance of a block from the raise and the horizontal width required to bring the most distant mining block to the raise. The stope profit is optimized as a function of location and height of the raise and consequently, the best location and height for raises are identified. Finally, the performance of the algorithm is compared with the floating stope algorithm and obtaining better solution values. However, the algorithm is limited to relatively small sub-vertical deposits mined by the sublevel stoping method (Sandanayake, 2015; Erdogan et al., 2017).

Global Optimization for Underground Mining Area (GOUMA) algorithm was presented as a new comprehensive algorithm that proves optimality in 2016 (Sotoudeh et al., 2017). The algorithm has two important characteristics, which makes it very useful. Firstly, it is executed on the Variable Value Economic model (VVEM). Secondly, constraints and technical limitations of all conventional underground mining methods used for tabular deposits are applied in this algorithm. Therefore, the main difference between this computer program and other programs and software tools is the type of algorithm used. For easy use of this algorithm on large□scale problems, a computer program called GOUMA□CP was written in C++ programming language (Jalali et al., 2016).

3.3. Comparative implementation and performance of existing models and algorithms

Intrinsically, underground mine planning is more complex because there are numerous permutations of the direction of mining, such as advance or retreat mining, depending on the mining method chosen. These contribute to the reasons why there is a lack of extensive optimization algorithms and commercial software packages for underground mining, and why most of the work on optimization in underground mine planning is largely academic (Musingwini, 2016).

One of the most effective ways to identifying the strengths and weaknesses of existing models and algorithms for further research is by assessing their comparative implementation and performance. A number of software packages and optimization tools that have the algorithms embedded in them have been developed to enable the implementation of the algorithms. As a result, most of the existing stope layout/limit models and algorithms have been implemented and their results assessed (Bai et al., 2013; Musingwini, 2016; Erdogan and Yavuz, 2017).

The Floating stope algorithm (heuristic) and its extension, the Multiple pass floating stope have been implemented in some commercial software packages, such as Datamine, Vulcan stope optimizer by Maptek Snowden's Stopesizer mining software package, used internally by Snowden Consultants to produce a single mining outline for a selected cut-off grade and the Anglo Platinum Optimization Tool (APMOT) (Smit and Lane, 2010; Bai, 2013; Musingwini, 2016). In Bai (2013), a new algorithm, the Network flow algorithm to optimize stope design for sublevel mining method was proposed and implemented by a comparative work with the Floating stope technique. In the work, the floating stope technique and the network flow technique were applied on three synthetic cases and one real case study. In the three synthetic cases, the profits obtained with respect the Network flow were larger than with the inner and outer envelopes obtained with the Floating stope algorithm. On the real case study based on a metal deposit located in Canada (name and location withheld), the Network flow approach provided an equivalent profit to the inner envelope and more profit than the outer envelope (+53%). It is worth noting also that neither of the floating stope envelopes met the slope angle constraints, as these constraints are simply not considered in the Floating stope algorithm.

Nikbin and Ataee-pour (2016) described a new Integer Programming (IP) model for optimization of underground stope layout. Their model is run on a specific block model, which is called secondary block model. The objective function of this model has been presented based on maximizing the profit. Almost all of the important and critical constraints in this scope have been considered and even some of them such as the minimum width of rib pillars are taken into account for the first time. The value of stopes found by the new model is the same as that obtained from OLIPS algorithm by Jalali et al.,

(2007). Results show that the new model provided more stope value compared to using Dynamic Programming of Riddle (1977). Also, the wide range of application and mathematical formulation of this model are its advantage. However, it is a 2D model and it is recommended that a 3D model is presented in the future researches.

Nelis et al. (2016) proposed a new algorithm for the optimal stope design problem. The algorithm is based on the methodology developed by Bai et al. (2013) where a cylindrical coordinate system is used to define geomechanical restrictions and to find the optimal stope around an initial raise. The proposed algorithm extends this work based on an Integer Programming (IP) formulation incorporating a new set of constraint, directed to solve geomechanical issues present on the original methodology. The results of a test of the new formulation on two synthetic and one real deposits were reported. An economic, geomechanical and feasibility analysis was performed and results compared with Bai's approach. Results indicated that the new approach allows the determination of optimal stope designs, fulfilling geomechanical requirements. It also generates feasible stopes in disseminated orebodies, avoiding unstable and irregular geometries. The generated designs comply with technical requirements such as the stope drilling pattern. The model can be solved in reasonable times for real life cases, and it can be extended to allow further regularization of stope design or faster resolution times with heuristics.

In the work by Erdogan et al. (2017), the capabilities and limitations of four heuristic algorithms are evaluated and compared to each other and then, compared with the stope optimization results from an existing underground mine. The algorithms and the corresponding commercial software packages selected were Floating Stope (Datamine), Maximum Value Neighbourhood (MVN) (Minesight), and two special applications that were developed by Sens and Topal and Sandanayake and Topal. Mineable Shape Optimizer (MSO), an optimization tool introduced by DATAMINE software it was also included in the study to compare with other algorithms. For the success of the comparison, a section of an economic block model that represents a gold deposit located in South Australia and a relatively deep, extending from 150 m below the surface to 600 m was used. The research aimed to find optimal solutions and maximize the value of the operation based on the defined constraints and rules. Results reflected that in terms of net profit values, MSO tool produced the highest followed by Sandanayake and Topal approach, MVN approach, the real mine case, Sens and Topal approach and the Floating Stope approach respectively. However, none of them can give a true optimum solution; they however provide only approximate solutions in 3D.

Sotoudeh et al. (2017) reviewed some existing underground stope layout algorithms for optimization and developed a computer program called Stope Layout Optimizer 3D (SLO3D) with C sharp user interface to implement a heuristic algorithm for optimization of underground stope boundaries on data from an actual copper mine located in southeast Iran. The developed optimizer provides an interactive environment to define and edit important parameters related to the stope layout optimization, including block model parameters, stope geometry, cutoff grade and economic factors. Finally, an example was presented to demonstrate the implementation of the algorithm with different stope limits and selection type strategies. At the end of the implementation, the SLO3D optimizer generated an optimized non-overlapping layout with 29 stopes

Nikbin et al. (2018) introduced a new hybrid algorithm that is a combination of dynamic programming and greedy algorithm. Although this proposed algorithm fails to provide a true optimum solution, it generates better solutions than some existing algorithms. The new proposed algorithm and three existing algorithms were used to find the optimal stope boundaries on a real case ore body. The results demonstrate that the proposed algorithm improved the profit by 117.78%, 16.86% and 0.42% compared to Floating Stope, Maximum Value Neighborhood (MVN), and Greedy algorithm solutions, respectively, on a real case study at a reasonable CPU time.

4. Limitations of Existing Underground Stope Layout/Limit Models and Algorithms

A variety of optimization techniques have been developed to generate optimal stope layouts. These techniques use the term optimization to describe their approach, but all fail to truly generate optimal results in 3D (Little et al., 2013). The complexity of underground stope optimization problem generally necessitates solution by heuristic approaches, i.e. search algorithms. However, the few algorithms that have been developed for optimizing underground mining stope envelopes fail to guarantee an optimum solution in 3D space. For real 3D stope definition, earlier approaches employed include: mathematical morphology tools, floating stope technique, maximum value neighborhood method, octree division, and network flow approach. These approaches share two major drawbacks: they are heuristic approaches which do not guarantee true optimality in stope layout and unable to directly incorporate specific geotechnical constraints, such as load carrying capacity of rock mass, wall stability of stopes etc., which influence not only the stope boundary but also production scheduling. Some of these methods may incorporate an additional step to test the stability of the current solution in order to generate stable designs, but they cannot address the geotechnical restrictions directly. Although the Simulated Annealing-based algorithm, had some mining constraints incorporated, it is very slow and the convergence to a global optimum is impractical. In addition, the restriction of disturbance to movements respecting all constraints on slopes in 3D setting can seriously halter the capacity of the algorithm to finding a good solution. To this effect, the mining engineer has to adjust the stope solution in other to obtain a feasible stope (Manchuk, 2005; Bai, 2013; Bai et al., 2014; Musingwini, 2016; Erdogan et al., 2017; Nhleko et al., 2018).

In the case of rigorous algorithm, works on stope optimization relied mostly on strong simplifications of the initial problem. For example, the 3D problem was simplified by considering optimization along only 1D or 2D. The dynamic programming method, and branch and bound techniques which find optimality were developed in this manner. Although the simplifications decrease the complexity of the optimization, it precludes incorporating realistic geotechnical constraints into the optimization (Ataee-Pour, 2000; Ramazan et al., 2005; Bai et al., 2013; Musingwini, 2016; Nelis et al., 2016).

Another limitation identified in almost all the optimization algorithms was grade uncertainty and the contributions of individual ore types to stope value. Consideration of grade uncertainty should be evaluated in the stope optimization procedure. For example, in Bai et al., (2014) the true grade values of the deposit used were assumed known everywhere or obtained using a conditionally unbiased estimator for simplicity sake and so the effect of the uncertainty on grades with regard to the stope design is not considered in this study. With the exception of downstream geostatistical approach or mathematical morphology and the probabilistic optimization algorithm based on MIP developed by Grieco and Dimitrakopoulos that considers grade uncertainty and geological uncertainty respectively in the optimization process, the others do not incorporate uncertainty (Grieco and Dimitrakopoulos, 2007; Dimitrakopoulos and Grieco, 2009; Topal and Sens, 2010; Sandanayake, 2015; Erdogan et al., 2017; Erdogan and Yavuz, 2017).

Again, a limitation identified in the algorithms is the problem of mining methods to which they can be applied. The existing algorithms cannot be applied to all the mining methods. Algorithms like Network flow can only be applied to Sublevel stoping method. The existing algorithms therefore require some improvements in terms of design criteria in order to take into account different mining or stoping methods, which could affect the economics of the mining operation (Erdogan et al., 2017).

Lastly, Musingwini (2016), Erdogan et al. (2017) and Erdogan and Yavuz (2017) assert that, the underground optimization techniques should consider an integrated approach for stope boundary optimization, development and production schedule optimization. This approach will provide more realistic and profitable results than individual evaluations of the stope boundary optimization methods.

5. Underground Production Schedule Optimization Models and Algorithms

Production scheduling is said to specify the mining sequence for economic stopes and the associated mine development required to bring the specified stopes into production. The decision to optimize centres on variables that represent the time at which to mine each stope in order to maximize NPV subject to operational constraints such as mining infrastructure, production capacity constraints, milling capacity constraints, grade and geometallurgical constraints, and rules on precedence relationships between specified stopes (Musingwini, 2016). The complexity of underground mining has resulted in the delay of software for underground operations and hence a lot of the scheduling concepts and algorithms produced for surface mining operations have eventually found their way into underground mining. The stoping methods employed in underground mining are branded by complex decision combinations, conflicting goals and interaction between production constraints (Kuchta et al., 2004; Pourrahimian, 2013).

As chronicled by Askari-Nasab et al. (2010); Askari-Nasab et al. (2011); Pourrahimian et al. (2012); and Pourrahimian (2013), production scheduling optimization models and algorithms available in literature are not just limited to, but can be divided into two main areas: heuristic and exact algorithms. However, the main disadvantage of heuristic methods is that there is no quality measure to solutions provided compared to the optimum. In addition, most of the results are not reproducible. Generally, heuristic methods are employed when there is no available approach to establishing an optimal solution under given constraints though they are widely known to generate good solutions within reasonable times. Currently, underground mine scheduling practice has inclined toward the use of simulation and heuristic software to determine feasibility rather than optimal draw schedules. Notwithstanding the flaws associated with heuristics such as frequently required intervention and lack of ways to improve optimality, simulations and heuristics are capable of handling non-linear relationships as part of the scheduling procedure. It was concluded by Gershon (1987) that schedules generated with heuristic techniques should only be considered as useful guide. Other methods such as queuing theory, network analysis, and dynamic programming have also been used to schedule production and/or material transport (Pourrahimian, 2013).

Within the past few decades many researchers have employed the use of Operations Research (OR) techniques in the mining industry. The application of OR techniques to optimization in the mining industry started to emerge in the early 1960s. Since then, optimization techniques have been applied to solve widely different mine planning problems. One of such OR techniques is mathematical programming. Mathematical programming techniques have been employed for many open pit optimization and scheduling and has therefore been investigated for similar applications in the underground mine environment. Mathematical models are capable of providing mathematically provable optimum schedules. To this effect, mathematical programming models that have been used for underground mine planning production scheduling, (short and long term) problems include: Linear Programming (LP), Integer Programming (IP), Mixed Integer Programming (MIP), Mixed Integer Linear Programming (MILP), Goal Programming (GP), and Quadratic Programming (QP) approaches (Alford et al., 2006; Nehring et al., 2010; Newman et al., 2010; Pourrahimian, 2013; Musingwini, 2016).

The application of LP in underground mine planning was first reported by Williams et al. (1973). LP was used to produce a schedule for the Nchangua Consolidated Copper Mines, limited which used to undertake production scheduling manually based on the experience of the mine planner for three underground ore sources. The mine planning system had already employed a heuristic solution but had no favorable results. The results from the LP schedule for one out of the three mines showed reduction in deviations from set targets subject to hoisting capacity, tramming capacity, profile constraints and manpower requirements. The application of LP on the other two shafts was less effective due to their much smaller size and the complicated mining sequencing resulting from the folded nature of the orebody.

Another application of LP was reported by Jawed (1985). In a case study on a group of coal mines, LP was employed to optimize production planning and scheduling. The application of LP resulted in the minimization of deviation from prescribed targets subject to operational constraints, manpower requirements, extraction capacity, plant capacity and lower bounds on the quality of extraction.

In 1997, Pendharkar (1997) developed and used a fuzzy LP model which incorporated fuzzy measures of quality to evaluate different production alternatives in the context of the coal industry. The authors applied the developed model on a hypothetical problem. The goals of the formulated model were to determine production outputs by coal mines, the satisfactory quality level of the coal to be delivered to the market and profitability. Results from the research indicated that the model had potential for solving decision making problems (production schedule problems) in the coal mining industry and could solve both simple to very complex problems that were dynamic in nature. They further proposed that the model could be extended to consider Non-Linear objective function since Non-Linear Programming has an advantage over LP because it considers certain factors like economies of scale and economies of scope which are ignored by LP.

An application of mathematical programming (Integer Programming/Mixed Integer Programming) in short-term and long-term production scheduling was reported by Guest et al. (2000) in block caving. the objective function in the work was defined as, to maximize draw-control behavior. In his conclusion, he iterated the benefits and advantages of using LP/MIP options and added that in comparison with spreadsheet manual systems, LP and MIP results had improved by 20%. However, Pourrahimian (2013) attests that the objective was to optimize Net Present Value (NPV). He further identifies two major problems associated with the approach: maximization of tonnage or minimization of reserves will not necessarily lead to maximization of NPV and draw control is a planning constraint rather than an objective function. He concluded that the objective function will therefore be to maximize tonnage, minimize dilution, or maximize mine life.

The application of integer programming to optimize underground mine production scheduling was first attempted by Trout (1995). The author uses MIP to develop an optimal production schedule with maximization of Net Present Value (NPV) as an objection function. The work was carried out on a sublevel stoping underground copper ore operation situated at Mt Isa, Australia. The objective function was subject to stope sequencing, stope extraction and backfilling quantities, equipment capacities and production grade as constraints. Due to an interruption from lack of memory, the proof of optimality could not be realized. Solution after 1.6 hours indicated a 25% improvement over the NPV generated by current operational policies. The model was not implemented at the mine due to additional improvements. Nevertheless, the study indicated the relative merit of employing MIP techniques over manual techniques for scheduling stopes (Little et al., 2008; Pourrahimian, 2013).

Carlyle and Evans (2001) developed a large MIP model which factors several constraints such as planned mine layout, projected ore quality, and projected costs for basic mining activities. The model presented maximized discounted ore revenue when applied to only a part of the sublevel stoping platinum and palladium mine in Stillwater, Montana. The system obtained a near optimal schedule of various planning scenarios (Pourrahimian, 2013).

An application of mathematical programming in block caving was also presented by Rubio (2002). The author presented a methodology and subsequently, an application that would enable mine planners to compute production schedules in block caving. The research demonstrated an integration and formulation of two main planning concepts as potential goals to optimize the long-term planning process, thereby maximizing NPV and mine life.

Smith et al. (2003) constructed a large-scale, time-dynamic, life-of-complex mixed integer program production schedule to optimize copper and zinc cash flow based on a detailed life of project production-scheduling at a copper and zinc underground mine at Mount Isa, Australia. The objective was to maximize NPV subject to operational constraints such as ore availability, mill capacity, mine infrastructure production capacity, grade limits, continuous production rules, and precedence

relationship between production blocks. However, the authors were unable to solve all instances of their problem in a reasonable amount of time (Newman et al., 2010).

A mathematical programming model to determine the levels of extracted ore from several different underground copper mines to maximize profit was presented by Epstein et al. (2003) at El Teniente, the largest underground copper mine in the world. In the formulated MIP model, the geological area of interest is strategically outlined by profitable extraction points, resulting from vertical aggregation of blocks. Mined material is sent through a network of alternative technologies and infrastructure investments. This capacitated network contains hundreds of thousands of variables and constraints. A rounding heuristic, developed and implemented by the authors gave a resulting solution indicating an improvement of more than 5% on the current operations of the mine (Newman et al., 2010).

Rahal et al. (2003) also described a Mixed Integer Goal Program (MIGP) model with dual objectives of minimizing the deviation from the ideal draw while achieving a production target in block caving. The developed algorithm assumes that the optimal draw strategy is known. In addition, a life-of-mine draw profiles for notional scenarios were developed and showed that by using the results from their integer program, they greatly reduced deviation from ideal draw point depletion rates while adhering to a production target.

Rubio and Diering (2004) in their work expand the analysis to include cost considerations based on previous work by Rubio (2002) by using his operational constraints. They described the application of mathematical programming to formulate optimization problems in block-cave production planning by formulating two main planning strategies: maximization of NPV and maximization of mine life (Newman et al., 2010; Pourrahimian, 2013).

Sarin and West-Hansen (2005) developed a MIP model for generating mine production schedules with constraint set consisting primarily of enforcing precedence, smoothing quality, and production levels and limiting the quantity of sections simultaneously mined. The authors in their presentation maximize NPV for an underground coal mine that consists of sections mined with longwall, room-and pillar, and retreat mining. The model is based on the definition of the mine layout as a precedence network, with the nodes representing mining sections. Binary variables track whether a section is scheduled to start being mined by a given set of equipment at a given time, whereas continuous variables track the quality and production volume of the material extracted. They tailored and developed a general solution methodology based on the Benders' decomposition model to successfully solve the original problem in mining (Newman et al., 2010).

McIssa (2005) also employed MIP to generate a long-term mine plan for a polymetallic narrow vein mine. The objective function was to maximize the cash flow of the operation. His model determined the sequence of development and stoping activities of eleven separate sectors based on three month intervals for a four-year period set at a production rate of 500 tonnes per day. Production schedules were generated for each zone rather than individual stopes. Failure on the part of the formulated model to schedule individual stopes but rather just specify zones, however, provides the mine with only a very general indication of where production should take place. Nehring and Topal (2007) reported that if the schedule model was applied to weekly or monthly periods, this general specification may not actually assist in generating optimal results. Though the model was solved in 30 minutes with 1,200 variables, it is not stated how many of these variables are integer variables and there is no mention of a previously generated manual production schedule with which to compare results.

Nehring (2006) further advances on the work of Trout (1995) by formulating a new constraint to limit multiple fill mass exposures without breaking other operational constraints. The model contained a total of 315 integer variables and 774 binary variables. His research also demonstrated the benefits of using MIP for generating production schedules over a common manual method of selecting production from the next highest available cash flow stope. It is reported by Nehring and

Topal (2007) that based on a conceptual nine stope operation, results showed an improvement in NPV.

Newman and Kuchta (2007) formulated a multi-period mixed integer program model for both short and long-term production scheduling of iron ore production at Kiruna, Sweden. They describe how they modify the model to consider several different levels of time resolution in the short versus long-term, and provided guidance for increasing model tractability. They then demonstrate numerically the increase in schedule quality and model tractability as a result of these modifications aggregating the periods and then solved the original model using information gained from the aggregated model (Pourrahimian, 2013).

Similarly, Nehring and Topal (2007) presented a new constraint formulation for limiting multiple exposures of fill masses for a small conceptual sublevel stoping operation. Comparative results of a schedule generated by the MIP production scheduling model and a manually generated model on a nine stope example was presented. Results from the work indicated that the potential benefits of the MIP production scheduling model for the purpose of maximizing NPV are significant. Finally, the new constraint was implemented in conjunction with other constraints without violating or limiting a mixed integer programming production schedule's capacity to optimize production from a series of stopes.

Little et al. (2008) and Nehring et al. (2010) in their research presented a classical MIP model that generates optimal production schedules for sublevel stoping operations. They further proposed a new formulation that significantly reduces the number of integer variables, specifically binary variables, and solution times without altering results while maintaining all constraints. The proposed model proved to have a better ability to provide all sublevel stope mining operations with the capacity to quickly produce new schedules in response to updated data and examined a variety of situations. Their work was based on two theories relating to natural sequence and natural commencement. However, they did not recognize stope grades over smaller volumes within each stope hence assumed an average grade for the entire stope. Therefore, in situations where a particular metal quantity must be satisfied, this requirement could violate the constraint where stope extraction occurs over more than one-time period. Also, timing constraints were not formulated to limit stope production to the extent of development. Generally, not all stopes are available over all time periods for production. This is particularly relevant if a great deal of development is required to access a stope because development costs could outweigh the benefits of mining that specified stope in that particular time period. Other researchers like Bley and Terblanche (2012) and King (2016) also worked to reduce the solution times while maximizing the mine's discounted value.

Pourrahimian et al. (2012) presented two Mixed Integer Linear Programming (MILP) formulations for long-term production scheduling of block caving mining operations. They first solved the problem at the draw point level and then aggregated the draw points into larger units referred to as clusters. Their formulations were developed, implemented and verified in the TOMLAB/CPLEX environment. The objective of the production scheduler is to maximize the Net Present Value (NPV) of the mining operation. The models also provided the mine planner the flexibility to have control over the development rate, vertical mining rate, lateral mining rate, mining capacity, maximum number of active draw point and advancement direction.

As an improvement of the previous work by Pourrahimian et al. (2012), Pourrahimian (2013) in his study developed, implemented, and verified a theoretical optimization framework based on a mixed integer linear programming (MILP) model for block cave long-term production scheduling to generate near-optimal life-of-mine production schedules for block cave mining operations. He introduced three MILP formulations for three levels of problem resolution; cluster level, draw point level, and draw point-and-slice level. These formulations could be applied in two ways; i) as a single-step method in which each of the formulations is used independently; ii) as a multi-step method in which the solution of each step is used to reduce the number of variables in the next level and

consequently to generate a practical block cave schedule in a reasonable amount of CPU runtime for large-scale problems. His model however was based on deterministic data and hence did consider attribute uncertainties, stochastic variables, ore dilution and other mine planning parameters.

O'sullivan and Newman (2014) employed mathematical optimization technique to determine a production schedule of extraction and subsequent filling of resultant voids for a complex underground mine in Ireland. The goal of the mining operation was to maximize metal production over the life of the mine, subject to constraints on maximum monthly extraction and backfilling quantities, maximum and minimum monthly metal production, and sequencing between extraction and backfilling operations. The authors solved the integer programming model with a heuristic to produce a schedule that adds value to the mining operation by: shifting metal production forward, reducing waste mining and backfilling delays, avoiding expensive mill-halting drops in ore production, and enabling smoother workforce management. The formulated module produced near-optimal production schedules for the mine that is more consistent with managerial goals than the mine's previous manual scheduling method was. Although both the manual and the IP schedules added value to the mining operation. It is reported by Brickey (2015) that the authors utilized aggregation techniques to reduce variables and developed methods for assigning precedence to the various activities; however, the disadvantage of aggregation is that it is limited as a result of the complicated precedence structures.

Brickey (2015) presented a generalized, mathematical formulation that results in a large-scale integer optimization model. His model maximizes NPV and determines the optimal or near-optimal sequence of activities related to the development, extraction and backfilling of an underground mine. Defined constraints in his work include physical precedence and resource capacities. He used data from an existing underground mine. Interestingly, the formulated model had the ability to serve other underground mines with similarly structured data and provides the ability to customize constraints. He included a constraint that treats available mine ventilation as a consumable resource.

Terblanche and Bley (2015) proposed a MIP model which is in relation to similar works by Little et al. (2008); Nehring et al. (2010) and Newman and Kuchta (2007). The rationale behind the approach is that the number of variables in the formulation was reduced by introducing a lower time period resolution while maintaining enough information to enable the optimization model to boost profits through selective mining. In addition, a generic formulation of the mine scheduling optimization problem cast within a resource production framework is introduced, with the purpose of simplifying notation used in formulating underground mine scheduling optimization problems. The presented MIP model is applicable to underground selective mining but not limited to a specific mineral where the ore grade is highly variable and by selecting out only high-grade areas to mine a reduction in cost can be achieved, thus improving profitability.

Ben-Awuah et al. (2016) developed, implemented and verified a MILP formulation and methodology with the objective to maximize the Net Present Value (NPV) of an orebody using different mining options with complex production requirements. The mining options considered were: i) open pit mining; ii) underground mining; and iii) concurrent open pit and underground mining. The MILP optimization framework proved to be robust in providing a global optimization solution when assessing the different mining options. In addition, the framework could also be extended to determine the change-over point between an open pit mining operation and an underground mining operation. The different mining options were evaluated based on the assumption of a high preproduction capital investment with low operating cost. The authors concluded that implementing open pit mining generated a higher NPV than underground mining but considering the investment required for these mining options, underground mining generated a better return on investment than open pit mining. For concurrent open pit and underground mining scenario, the optimizer preferred extracting blocks using open pit mining. Although the underground mine could access ore sooner, the mining cost differential for open pit mining was more than compensated for by the discounting benefits associated with earlier underground mining. They further recommended that an additional

study could be done to investigate the mining options including their pre-production capital expenditure requirements.

6. Mathematical Programming Models and Algorithms for Integrated Underground Stope Layout/Limit and Production Schedule Optimization

Available literature as discussed under Sections 3 and 4 of this paper indicates that there has been several optimization techniques that have been developed for stope layouts and progress is being made in the area of production scheduling. However, both areas are still open to improvement.

It is reported by Little et al. (2011) that stope layout methods struggle to produce true optimal results whereas scheduling techniques are hindered by excessive solution times. The available techniques aim to provide local optimal solutions that do not consider simultaneously how other designs or planning areas will affect the optimality of results. Clearly, to date, no method has been proposed, or given consideration, to expanding the scope of underground mine optimization to integrating key areas of long term underground mine design and planning such as to merge stope layouts and production scheduling into one technique. Chadwick (2009) states that, optimization has the ability to add significant value to a mining operation but only if the whole process is considered. However, if concentration is based solely on one part of the process there is a possible risk of transferring cost or reducing revenue elsewhere. Simultaneous optimization would allow for the interaction and influence of one area on another to be assessed and mine plans appropriately improved, in effect, providing more profitable mine plans than currently possible (Little et al., 2011).

Poniewierski et al. (2003) in his investigation into the relationship between stope layouts, production schedule and cut-off grade optimization acknowledges and establishes the complex interaction that exist between the three aforementioned areas. His work describes aspects of the detailed investigation into estimating a practical static cutoff grade for the two main orebodies in Mount Isa Mine's Enterprise Mine, a deep underground copper mine. The approach described by Poniewierski et al. (2003) shows the first step towards achieving a practical application of Lane's theory on optimum cutoff grade in selective underground mining. He iterates in his work that, the key to determining an optimum cutoff grade is the ability to rapidly perform complex mine layout designs combined with rapid output of multiple potential schedules. For the over 200 schedules that were generated for 13 stope layouts, he examined each case using mining, concentrating and smelting net cash flows and the NPV. For each orebody, a cutoff grade was selected based on the cutoff grade which achieved the maximum NPV while allowing for an appropriate production rate. However, due to the limitations of the commercially available software package, the process did not consider all possible designs and schedules together. The process in effect illustrated the connection and complex interaction between stope design and production scheduling and established the need to take into consideration stope design and production scheduling simultaneously if globally optimal results are to be achieved in underground mine planning. However, work completed by Smith and O'Rourke (2005) concludes that using cutoff grades to drive mine design and planning is unnecessary as the optimization process will mine the portion of the deposit which is most economical within the limits of the operation.

As an attempt to address the issue of obtaining globally optimal results by simultaneous optimization of stope layout and production scheduling, Little et al. (2011) and Little (2012) proposed a model that simultaneously optimizes stope layouts and production schedules using integer programming technique mixed integer programming for sublevel stoping operations. They emphasized that MIP techniques are used because it has been shown that they can successfully handle multi-constrained problems while providing a multi-period production schedule that satisfies a required objective. The authors successfully applied the MIP model to 64 443 blocks over a 23-year mine life, which demonstrated the model's ability to produce optimal long term mine plans for a sublevel stoping operation with a $22.7m maximized NPV. The formulated model works by selecting the best stope

sizes, identifying the best stope locations and generating a corresponding optimal long term production schedule simultaneously for a given block model. In solving the optimization problem, the model did not necessarily select the most profitable stope to be produced, it rather resulted to producing different stopes in different configurations, while identifying the best timing for preparation, extraction and backfilling for each stope, which eventually maximizes the mine's NPV. As part of the future works and recommendations, the authors reiterate the need to revise the model formulation to reduce the solution time and memory requirements due to increases in problem sizes. This will make the model more applicable because the reduction in the solution time will make the model more acceptable for practicable mining purposes. They emphasized the need to employing stochastic techniques to geological and economic uncertainties, such as grade and metal prices as these will increase the reliability and success of the mine plan than using deterministic assumptions only. They conclude that poor consideration of these uncertainties recurrently becomes the main reason why many mine sites consistently do not achieve their forecasted performance.

To compare the strength of an integrated stope layout and production schedule optimization with the isolated optimization, Little et al. (2013) developed an Integer Programming (IP) model that allows for either integrated or isolated optimization. He applied both approaches separately to a block model of a gold deposit. Results demonstrate the formulated model's ability to produce optimal long-term sublevel stoping mine plans and the benefits of using an integrated approach. Though the optimization of a part of a mine plan generally tend to yield better results than that of the manual planning methods, the integrated optimization approach provides better operational and financial advantages owing to its ability to assess the interaction and influence between planning areas. It is realized that the isolated approach has a narrow, outlook because its initial focus is solely on the selection of stope layouts without consideration for the impacts on production scheduling. Invariably, this limits the scope in which production scheduling optimization can work due to the various constraints specified, and this ultimately harms the NPV because costs are either transferred or revenues reduced elsewhere in the mine plan. However, the integrated approach, rather takes a holistic look at the operation. This approach predicts and manages issues between the mine planning areas and can accommodate all constraints. As an advantage over the isolated approach, it ensures generating a more considerable and comprehensive mine plan, hence achieving a better NPV. The integrated approach further allows for risks and opportunities to be identified more readily, hence risks can be reduced or opportunities capitalized on more easily in the planning process. Musingwini (2016) highlights the fact that this approach of integrated optimization means a more realistic and sound comparative evaluation of projects within reasonable time frames, facilitating better informed operational and financial decisions is to be expected. The model however did not incorporate development optimization. It only has one development consideration; the location of extraction levels this does not assess if other development requirements, such as shaft or decline construction time or their placement underground, can satisfy the proposed production schedule, or if the proposed stope layouts and schedule works well with development requirements both technically and financially. It is worthy to note that optimizing development along with stope layouts and the production schedule will provide a globally optimal strategic mine plan.

Gangawat (2014) developed an integer programming formulation and solved using CPLEX solver for a single stope. His work is divided into two parts; optimal designing of stopes for open stoping method based on constraints of stope extraction angle and stope height and production scheduling of the generated stopes employing heuristic approaches. The algorithm was solved using part of the zinc mine of India with data containing 4,992 blocks. After successfully solving and eliminating the first stope and respective crown pillar data from the data set, the algorithm was solved again for the second and subsequently a third stope from the remaining data set. In all, a total of 3 stopes were designed. After stope design, production scheduling of the stope blocks was carried out heuristically satisfying extracting angle, mining and processing constraints. However, heuristic approaches are known to give solutions which are just close to the optimal but not an optimal solution though they have the ability to significantly reduce computational time. A flaw in his work, however, was that

consideration was not given to the cost of development in the objective function. Considering it could bring the generated NPV close to the real time solution.

7. Stochastic Mine Planning Approaches in Underground Mining

There are two approaches of conducting mine planning, namely; deterministic and stochastic approaches. Deterministic mine planning uses historical data collected from previous mine production activities and assumes that this trend will continue in future processes. Conversely, stochastic mine planning can be described as a premeditated course of action based on data collected from historical mine production figures and also incorporating uncertainty into the modelling processes. Deterministic approaches are known to ignore variations; hence stochastic mine planning is promoted because it incorporates variations considering the fact that mine production activities are erratic in nature and not static. Stochastic mine planning is a complex scheduling process due to its ability to incorporate uncertainties such as geological, technical and economic, inherent in mining operations (Magagula, 2016).

The application of stochastic mine planning in production scheduling has been reported by various researchers in open pit mining taking into consideration grade uncertainty and associated risk (Dimitrakopoulos et al., 2002; Dimitrakopoulos and Ramazan, 2004; Leite and Dimitrakopoulos, 2007), orebody uncertainty, in situ grade and geological uncertainty (Ramazan and Dimitrakopoulos, 2004; Dimitrakopoulos et al., 2007; Dimitrakopoulos and Ramazan, 2008; Godoy and Dimitrakopoulos, 2011; Silva et al., 2015), geological and market uncertainty (Sabour and Dimitrakopoulos, 2011) and NPV (Ramazan and Dimitrakopoulos, 2012) to mention a few.

Conceptual developments in open pit mining have gradually led to the application of stochastic mine planning in underground mine planning notwithstanding the intricate nature of underground mine planning. The uncertainty related to orebody is a critical aspect affecting the forecasted performance of designs and is linked to the failing of meeting production targets and project financial expectations in mine planning (Grieco and Dimitrakopoulos, 2007).

Grieco and Dimatrakopoulos (2007) developed and explored a new probabilistic mixed integer programming model to optimise stope designs, including size, location and number of stopes under consideration of grade uncertainty and predefined levels of acceptable risk. The model was applied on data from Kidd Creek Mine, Ontario, Canada to demonstrate its practicality. The application exhibited aspects including risk quantification for contained ore tonnes, grade and economic potential. The authors reported that unlike any conventional stope optimization approach, the stope designs generated based on the concept of acceptable risk gives the mine planner control over the final stope layout and its potential future performance while considering grade uncertainty. The application of the proposed approach is based on the ability to stochastically simulate equally probable representations of the deposit.

Martinez (2009) gives a classic presentation on why accounting for uncertainty and risk leads to an improvement in decision making so far as mine evaluation is concerned. He introduced a new mine evaluation framework; the Integrated Valuation/Optimization Framework (IVOF). He presented this as an alternative tool for mine project evaluation where uncertainty and risk are incorporated in the evaluation process. His paper highlights two main objectives; i) it shows what problems can arise when single estimated values are substituted for a distribution of values when evaluating a mine project in the face of uncertainty and ii) it shows how the ability to deal with uncertainty and risk in mine project evaluation can have a significant impact on the owners' and stakeholders' investment decision-making. The author asserts that the complexity of mine projects makes it a business that requires constant assessment of risk due the fact that the value of a mine project is typically influenced by many underlying economic and physical uncertainties, such as metal prices, metal grades, costs, schedules, quantities and environmental issues, among others, which are not known with absolute certainty. He identifies the main sources of uncertainty that arise at the commencement

of a mine project evaluation to be uncertainty in orebody modelling, uncertainty in metal prices and costs, and uncertainty and risk in mine planning and design.

Dimitrakopoulos and Grieco (2009) adopted risk-based concepts developed in open pit mining to the underground stoping environment and shows examples employing data from Kidd Creek Mine. The example illustrates how conventional technologies cannot quantify risk since they are unable to foresee a significant upside potential and/or downside risk for the conventionally produced designs. The work quantified risk is in terms of the uncertainty a conventional stope design has in expected: contained ore tones, grade and economic potential. They further outlined a new probabilistic mathematical formulation that optimizes the size, location and number of stopes in the presence of grade uncertainty with an additional constraint introduced as the minimum acceptable risk allowed in a design. The model was applied to demonstrate the advantages of a user-defined level of acceptable risk. It is worth mentioning, that the authors recommended that further developments of the work could be to include: i) the formulation of a stope optimization formulation that replaces the probability of grades above cutoff with the direct use of all available simulated orebodies, integrating more geological information; ii) consider sequencing and thus accommodate risk management and/or geological risk discounting as part of the stope design process; and iii) extend to integrate geotechnical uncertainties starting from over-breaking and under-breaking.

Dimitrakopoulos (2011) proposed a risk-based Stochastic Integer Programming (SIP) optimization model which incorporates uncertainties from both the geological and economic factors while minimizing cost. The model integrates two elements: stochastic simulation and stochastic optimization. These elements provide an extended mathematical framework that allows modelling and direct integration of orebody uncertainty to mine design, production planning, and valuation of mining projects and operations. This stochastic framework increases the value of production schedules by 25%. Case studies also show that stochastic optimal designs i) can be about 15% larger in terms of total tonnage when compared to the conventional design, while ii) adding about 10% of NPV comparing to the traditional scheduling using a determined averaging orebody model designs. Results suggest a potential new contribution to the sustainable utilization of natural resources. However, Magagula (2016) reports that the author indicated the difficulties surrounding creating such planning process.

To integrate ore/metal uncertainty into the optimization of mine production scheduling, a Stochastic Integer Programming (SIP) formulation is proposed and tested at a copper deposit by Leite and Dimitrakopoulos (2014). The stochastic solution maximizes the economic value of a project and minimizes deviations from production targets in the presence of ore/metal uncertainty. Unlike the conventional approach, the SIP model accounts and manages risk in ore supply, leading to a mine production schedule with a 29% higher NPV than the schedule obtained from the conventional, industry-standard optimization approach, thus contributing to improving the management and sustainable utilization of mineral resources.

Macneil (2015) uses stochastic mine planning methods to identify the optimal open pit to underground mining transition depth by identifying a series of candidate scenarios where it is feasible to make an OP-UG transition. The author evaluated the economic viability of each member of the set of candidate transition depths by producing uncertainty-based life-of-mine production plans that were used to outline expected yearly cash flows. The benefits of using stochastic mine planning to provide well-informed long-term strategic decision-making criteria are observed. Results of the application of the stochastic approach produced operational schedules with an increase NPV compared to the corresponding deterministic framework. An application of the proposed method at Geita gold mine, a large gold mine in Eastern Africa indicate that future ore production is forecasted to fall well below the mill's capacity, and to supplement this deficiency a transition from open pit to underground mining is being considered. Interestingly, results of the analysis from the proposed stochastic framework reflects that the most profitable decision favored continuing production

through solely open pit mining for the foreseeable future. Valuable insights towards the risk associated with the proposed mine design are gained through stochastic risk analysis.

As the need to incorporate multiple components of the mining value chain increased a number of methods were developed over the past decades. Efforts have been made by these new methods to incorporate more decisions and flexibility to the mining optimization of a mining complex, however, they either ignore uncertainties associated with the mining project or consider decisions taken before optimization (Montiel et al., 2016). A method that optimizes mining complexes comprised of multiple open-pits, underground operations and processing destinations was presented by Montiel et al. (2016). The proposed method simultaneously optimizes mining, blending, processing and transportation decision variables while accounting for geological uncertainty. The method employs a simulated annealing algorithm at different decision levels in order to generate a stochastic-based extraction sequence and processing policies. An application based on a case study shows the methods ability to generate a higher NPV while facing a reduced amount of risk when compared to traditional optimization methods.

Malaki (2016) employs the application of grade uncertainty in block cave mining. The author presents a methodology to find the best extraction level and the optimum sequence of extraction for that level under grade uncertainty. The work uses stochastic sequential simulation to address this problem by modelling a set of simulated realizations of the average mineral grade. MILP model was formulated to obtain the maximum NPV given some constraints such as mining capacity, production grade, extraction rate and precedence. Finally, risks associated with grade uncertainty are investigated and analyzed, considerably helping the decision makers in better understanding of various cases and conditions. The author concludes among other things that more uncertain attributes other than grade should be added to the optimization problem. Hence, the MILP model should be extended to take stochastic variables into account during optimization.

A new SIP model incorporating geological uncertainty to optimize long-term scheduling of an underground project extension was introduced by Carpentier et al. (2016). In order to represent a deposit integrating the uncertainty, they stochastically generated a set of simulations and put them into the optimization model. The results show that the schedule generated has a higher expected value when considering and managing grade risk. They also demonstrate the benefits of risk control, allowed by the approach.

MacNeil and Dimitrakopoulos (2017) provide an approach to determine an optimal depth at which a mine should transition from open pit to underground mining, based on managing technical risk. The proposed approach is tested on a gold deposit. This work aims to approve on previous attempts to solve this problem by jointly considering geological uncertainty and describing the optimal transition depth effectively in 3D. Results show the benefits of managing geological uncertainty in long-term strategic decision-making frameworks. The stochastic result produces a 9% increase in NPV over a similar deterministic formulation. The risk-managing stochastic framework also produces operational schedules that reduce a mining project ‘s susceptibility to geological risk. The authors direct future studies to aim at improving the method by considering more aspects of financial uncertainty such as inflation and mining costs. Table 1 presents the summary of current stope layout/limit optimization algorithms and their limitations.

Table 1. Limitations of stope layout/limit optimization models and algorithms.

Classification	Algorithm/Author (s)	Mining method	Dimensional shape	Optimality	Stochastic Consideration	Production Scheduling
Heuristic	Octree Division (1989)	All	3D	No	No	No
	Floating Stope (1995)	All	3D	No	No	No
	Maximum Value Neighbourhood (MVN) (2000)	All	3D	No	No	No
	Multiple Pass Floating Stope Process (2001)	All	3D	No	No	No
	Simulated Annealing (2006)	All	3D	No	No	No
	Generation of Stope shapes for nominated range of Cut-off grades (2009)	Not indicated	Not indicated	No	No	No
	Topal and Sens (2010)	Not indicated	3D	No	No	No
	Sandanayake and Topal (2015)	Sublevel stoping	3D	No	No	No
Rigorous	Dynamic Programming (1977)	Block caving	2D	No	No	No
	Downstream Division (1984)	Cut and fill; Sublevel stoping	2D	No	Yes (Grade uncertainty)	No
	Branch and Bound (MIP) (1995, 1999)	All	1D	Yes	No	No
	Probable Stope (2004)	All	2D	No	No	No
	Grieco and Dimitrakopoulos (MIP) (2007)	Blasthole/Longhole stoping with cemented backfill	Not indicated	No	Yes (Geological uncertainty)	No
	Network Flow (2013)	Sublevel stoping	3D	No	No	No
	OLIPS (2007)	All	2D	Yes	No	No
	GOUMA (2015)	All	2D	Yes	No	No

8. Research Opportunities

It is evident from literature that underground stope layout/limit and production scheduling optimization remains a vital area in the underground mining industry and opportunity for research into improving and/or obtaining global optimality is still open. Most of the research carried out in the area has been towards developing algorithms and models separately for stope layout/limit optimization and for production scheduling optimization. According to Erdogan et al. (2017), Sotoudeh et al. (2017), and Nhleko et al. (2018), various algorithms and models have been developed to solve the problem of underground stope layout/limit optimization. The authors further explain that majority (70%) of these algorithms have gradually managed to define stope boundary in 3D, this used to be the problem with earlier developed algorithms and models due to the fact that mining is a 3D problem. Almost, about 70% of these algorithms are heuristic based and hence true optimality is not guaranteed with these algorithms even in 3D. A majority of the algorithms and models are found to be applicable to all mining methods except the Dynamic Programming by Riddle (1977) and Downstream Geostatistical Algorithm by Deraisme et al. (1984). However, Nhleko et al. (2018) concludes that all these algorithms are based on deterministic orebody models and, therefore, fail to consider the existence of uncertainty in ore deposits. Consequently, there is a need for further research in the field of stope boundary optimization.

Erdogan and Yavuz (2017) mentioned that there is a clear need for improved algorithms and software that guarantee optimal solutions and that the underground optimization techniques should be considered as a whole and techniques should be developed that covers all three areas of optimization: stope boundary optimization, development and production schedule optimization in the future. Currently, the only exception can be mentioned in the case of Little (2012) and Gangawat (2014) who considered a holistic look at simultaneously optimizing stope layout and production scheduling.

In the report by Musingwini (2016), it is mentioned that the idea behind Little's algorithm was to integrate the two and optimize them simultaneously. He highlighted on the fact that the algorithm was developed using the Integer Programming (IP) technique and that previously, most underground optimization techniques focused on optimizing stope layouts and production schedules separately.

The flaw in the attempt by Little (2012) was the inability of the IP model formulation to reduce the solution time and memory requirements due to increase in problem sizes. This would have made the model more applicable because the reduction in the solution time will make the model more acceptable for practicable mining purposes. The model did not also employ stochastic techniques to geological and economic uncertainties, such as grade and metal prices that would help increase the reliability and success of mine plans. The consideration of these uncertainties will help mine sites to achieve their forecasted performance. Finally, the model was developed and tested for sublevel stoping mining method. No mention was made of its applicability on other underground mining methods.

Similarly, the failure on the part of Gangawat (2014) after using an IP formulation to undertake stope design was the use of a heuristic approach to undertake production scheduling of the stope blocks satisfying extracting angle, mining and processing constraints. The flaw in his work was that heuristics are known to give solutions that are just close to the optimal but not an optimal solution though they have the ability to significantly reduce computational time. In addition, he gave no consideration to the cost of development in the objective function which could have brought the generated NPV close to the real time solution. Again, the author did not make mention of the applicability of the model to other underground mining methods in addition to open stoping.

Recently, numerous researchers have explored and advanced industrial knowledge on the use of Mixed Integer Programming (MIP)/Mixed Integer Linear Programming (MILP) for the purpose of generating optimal mine production schedules from defined stope boundaries (Trout, 1995; Kuchta et al., 2004; McIsaac, 2005; Nehring and Topal, 2007; Pourrahimian, 2013). The account by these

authors clearly demonstrate the strength of mathematical programming techniques when applied in the area of production schedule and stope sequence optimization compared to manual production scheduling. Scheduling underground mining operations is primarily characterized by discrete decisions to mine blocks of ore, along with complex sequencing relationships between blocks. Since LP models cannot capture the discrete decisions required for scheduling, MIPs are generally the appropriate mathematical programming approach to scheduling. The advantage of using these methods for production scheduling is that they can provide a mathematically provable optimum schedule. These methods are able to approximate some non-linear systems, though not as flexible as simulation.

In the MILP method, any feasible schedule produced has an associated gap that provides a measure of how far the feasible schedule is from its linear relaxation. This gap is based on the difference between the best node and best integer feasible value found within the branch-and-bound search tree. In a typical mine production scheduling problem, the number of integer and continuous decision variables and the number of constraints determine the complexity and solution time. The model decision variables need to assume integer variables to suit the discrete problems in certain conditions. When this restriction as well as the binary restriction is added to the problem, i.e., IP and MIP, the mine production scheduling problems are nicely addressed. Mixed integer programming models are recognized as having significant potential to optimize production scheduling for underground mines. The integer variables can represent entities that cannot be divided. And the binary variables can be used to represent Yes/No decisions such as to extract or not to extract the ore in a given time period (Topal, 2008; Pourrahimian et al., 2012; Little et al., 2013; Ben-Awuah et al., 2016)

It can be deduced from Musingwini (2016) that in order to meaningfully interpret and communicate results for decision-making it is always important to understand the optimization processes in mine planning. This implies: i) developing more robust mine planning through stochastic optimization by considering probability underground excavation design of stopes and development in order to improve confidence in the placement and sizing of excavations; ii) integrating stochastic optimization within the four broad areas in underground mine planning i.e. development layout, sizing stope envelopes, production scheduling, and equipment selection and utilization so that these can be executed simultaneously and enable planning of just-in-time development. Finally, the integrated optimization should guarantee true optimality in 3D space and incorporate uncertainty, thus making a case for integrated 3D stochastic optimization.

9. Rationale behind PhD and Proposed Research Approach

Mine planning should be geared towards producing an optimized plan. An optimized mine plan is expected to be sufficiently robust to ensure that actual outcomes are close or equal to planned targets, provided that variances arising from poor performance are minimal. However, due to the geological, technical, and economic uncertainties inherent in mining operations, this does not always happen in practice; hence the gulf that mine plans tend to be based on deterministic frameworks, while actual mining operations are stochastic in nature. This observation explains the emerging paradigm shift towards stochastic mine planning. The failure to have actual outcomes close to or the same as planned targets is widely acknowledged in the mining industry due to the way the industry models its systems (Magagula, 2016; Musingwini, 2016).

Literature review has shown that a holistic look at underground mine optimization problem in the areas of stope layout/limit and production scheduling and integrating stochastic considerations is imperative. Few attempts have proved unsuccessful. This therefore calls for the need to formulate robust and tactical models capable of solving the problem of global optimality by maximizing NPV while satisfying all available constraints and not affecting the function of other constraints. To solve such a task will require employing Operations Research (OR) techniques capable of solving such complex real-world modern optimization problems. One of such techniques to employ would be an

optimization model; a mathematical formulation that is solved using an exact algorithm to produce a single optimal solution through a process of either maximization or minimization. To this effect, Mixed Integer Linear Programming (MILP) with its advantages in the formulation and solving of such complex real-world optimization problems should be employed.

Long-term underground mine production scheduling problems have been studied by researchers including Jawed (1993), Nehring et. al. (2010) and Pourrahimian et al. (2012). These studies have been based on the development of algorithms using operation research methods such as Linear Programming (LP), Mixed integer Programming (MIP) and Mixed Integer Linear Programming (MILP). The current optimization algorithms in the literature lack the ability to deal with real□world large□scale complex underground mine planning problems. They do not have the ability to address i) the broad range of mining methods (exhaustiveness) ii) practical problem sizes, and iii) stochastic parameters.

The research objective will be to investigate the application of algorithms or mathematical programming optimization for strategic underground mine planning in the case of both selective and bulk mining methods. Subsequently, the research will develop efficient algorithms to reduce the size and complexity of the real□world large□scale problems. Finally, the research will investigate techniques to incorporate stochastic variables such as grade and price into the optimization framework. The research will result in optimization models and methodology that can be used to generate a comprehensive stope designs and life of mine plan for a given orebody. The optimization framework will consider the economical mineable sections of the deposit and optimal sequence of extraction to maximize the Net Present Value (NPV). To validate the optimization framework, the research project will be applied to case studies of different orebodies that will be provided during the project. The research will therefore focus on the following tasks:

- Investigate the application of algorithms or mathematical programming optimization for strategic underground mine planning;
- Develop efficient algorithms to reduce the size and complexity of the real□world large□scale problems;
- Investigate techniques to incorporate stochastic variables including grade and price into the optimization framework; and
- Formulate optimization model and methodology that can be used to generate stope designs and a comprehensive life of mine plan for a given orebody.

Appropriately scheduling the long-term production of a typical underground mine is very critical to the overall success of the mine. However, the major problem in long-term production scheduling for underground ore bodies generally relate to the substantial number of variables which makes it too complex to solve (Nehring and Topal, 2007). The numerous constraints and uncertainties especially with the geomechanics of an underground mine makes long-term underground mine production scheduling very challenging. A long-term plan must solve three main problems: i) Investment: determine the selection and timing of investments; ii) Extraction: determine the production in the mine; and iii) Processing: determine the operation of the plants. Solving each one of these problems, even separately, is a complex task. However, the need for a strategic underground production schedule approach cannot be dismissed. For instance, the real value of an investment is appreciated only once its coherence is verified with respect to the extraction and processing decisions (Epstein et al., 2012).

The shortfalls of the available models in literature with underground mine production scheduling include: 1) considered constraints for the models are not exhaustive; 2) models are mostly applicable to only one underground mining method; 3) they cannot deal with practical and complex problem sizes; 4) they cannot handle stochastic variables such as grade and price, and 5) limited total schedule quality in terms of tractability (ease to apply), realism (applying to deployment settings) and generality (applying to many protocols). These main factors would often make real-world

underground production scheduling problems very difficult to solve by models in existing literature. In most cases, either the existing models cannot deliver optimal solutions in a reasonable time frame or they do not have the framework to overcome these limitations. Experimental based solutions have therefore been extensively relied upon in managing these challenges in the real-world underground mine settings. These have often affected the overall NPV, equipment and labor utilization, increased ore losses and dilutions, and increased mining costs for the operation. Therefore, finding an optimization framework that is devoid of the above-mentioned shortfalls would be of great benefit in the mining environment. Fig. 3 is a schematic of the proposed research protocol.

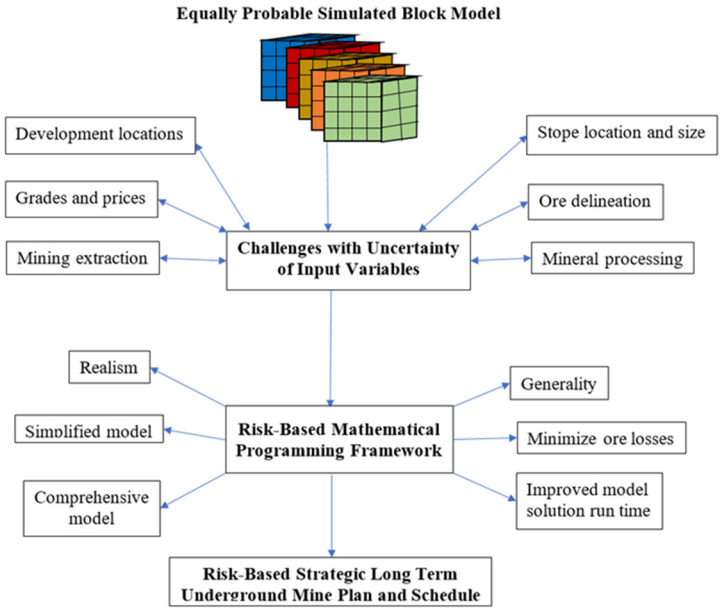


Fig. 3. Schematic diagram of the proposed research protocol.

10. Summary and Conclusions

Discussions so far indicate that underground mine planning optimization work completed to date has focused on solving an individual mine planning area with little consideration for its flow-on effects to other planning areas. It is seen and generally accepted that the optimized solution for the first problem (stope layout/limit) forms the input for the following problem, and so on until a mine plan has been completed. Consequently, the acknowledgement of the importance of an integrated approach towards optimization is growing. Recent studies have shown that decisions made in individual planning areas naturally influence the overall outcome, and focusing on local sub-objectives can be counterproductive to the overall objective (Little et al., 2013).

Current literature shows that optimization in underground mine planning still remains largely fertile for new developments because the direction of mining in underground mines has numerous permutations, depending on the mining method used, making the underground mining optimization problem intrinsically more complex to solve. This complexity explains the isolated piecemeal developments for solving parts of the overall optimization problem in underground mine planning in four key interdependent areas, namely: development layout, sizing stope envelopes, production scheduling, and equipment selection and deployment. Many opportunities therefore, exist for the development of integrated 3D stochastic optimization models for underground mine planning. The failure to obtain actual outcomes close to our targets is widely acknowledged in the mining industry. This is due to the way the industry models its systems (Musingwini, 2016; Nhleko et al., 2018). Sabour and Poulin (2010) and Musingwini (2016) proposed that mining companies ought to develop adaptable mine plans to manage the impact of uncertainties such as commodity price volatility, thus

implicitly advocating stochastic mine planning. In effect, mine plans can incorporate uncertainty such as price volatility if a shift is made away from deterministic to stochastic optimization.

11. References

[1] Abdel Sabour, S. and Poulin, R. (2010). Mine expansion decisions under uncertainty. *International Journal of Mining, Reclamation and Environment*, *24* (4), 340-349.

[2] Alford, C. (1995). Optimisation in underground mine design. *in Proceedings APCOM XXV Symposium*, (The Australasian Institute of Mining and Metallurgy: Melbourne), 213-218.

[3] Alford, C., Brazil, M., and Lee, D. H. (2006). Optimisation in underground mining. in *Handbook of operations research in natural resources*, Springer, pp. 561-577.

[4] Alford, C. and Hall, B. (2009). *Stope optimisation tools for selection of optimum cut-off grade in underground mine design*. in Proceedings of Proceedings of project evaluation conference, Melbourne, pp. 137-144.

[5] Ataee-Pour, M. (2000). A heuristic algorithm to optimise stope boundaries. *Unpublishe PhD Thesis Report, University of Wollongong, Australia*, 319 pp.

[6] Ataee-Pour, M. (2005). A critical survey of the existing stope layout optimization techniques. *Journal of Mining Science*, *41* (5), 447-466.

[7] Bai, X. (2013). Optimization of underground stope with network flow method.Thesis, École Polytechnique de Montréal.

[8] Bai, X., Marcotte, D., and Simon, R. (2013). Underground stope optimization with network flow method. *Computers & geosciences*, *52* 361-371.

[9] Bai, X., Marcotte, D., and Simon, R. (2014). A heuristic sublevel stope optimizer with multiple raises. *Journal of the Southern African Institute of Mining and Metallurgy*, *114* (6), 427-434.

[10] Ben-Awuah, E., Richter, O., Elkington, T., and Pourrahimian, Y. (2016). Strategic mining options optimization: Open pit mining, underground mining or both. *International Journal of Mining Science and Technology*, *26* (6), 1065-1071.

[11] Bley, A. and Terblanche, S. (2012). An improved formulation of the underground mine scheduling optimization problem when considering selective mining. *Optimization Online*,

[12] Brickey, A. J. (2015). *Underground production scheduling optimization with ventilation constraints.* Colorado School of Mines.

[13] Carlyle, W. M. and Eaves, B. C. (2001). Underground planning at stillwater mining company. *Interfaces*, *31* (4), 50-60.

[14] Carpentier, S., Gamache, M., and Dimitrakopoulos, R. (2016). Underground long-term mine production scheduling with integrated geological risk management. *Mining Technology*, *125* (2), 93-102.

[15] Cawrse, I. (2001). *Multiple pass floating stope process*. in Proceedings of Strategic Mine Planning Conference, pp. 87-94.

[16] Chadwick, J. (2009). Mine Opimisation. *International Mining Magazine*, 18-28.

[17] Chakraborty, R. (2010). Fundamentals of genetic algorithms. *Reproduction*, *(22)* 35.

[18] Cheimanoff, N., Deliac, E., and Mallet, J. (1989). *GEOCAD: an alternative CAD and artificial intelligence tool that helps moving from geological resources to mineable reserves*. in Proceedings of 21st International Symposium on the Application of Computers and Operations Research in the Mineral Industry, pp. 471-478.

[19] Deraisme, J., De Fouquet, C., and Fraisse, H. (1984). Geostatistical orebody model for computer optimization of profits from different underground mining methods. *18th APCOM IMM*.

[20] Dimitrakopoulos, R. (2011). Stochastic optimization for strategic mine planning: a decade of developments. *Journal of Mining Science*, *47* (2), 138-150.

[21] Dimitrakopoulos, R., Farrelly, C., and Godoy, M. (2002). Moving forward from traditional optimization: grade uncertainty and risk effects in open-pit design. *Mining Technology*, *111* (1), 82-88.

[22] Dimitrakopoulos, R. and Grieco, N. (2009). Stope design and geological uncertainty: quantification of risk in conventional designs and a probabilistic alternative. *Journal of Mining Science*, *45* (2), 152-163.

[23] Dimitrakopoulos, R., Martinez, L., and Ramazan, S. (2007). A maximum upside/minimum downside approach to the traditional optimization of open pit mine design. *Journal of Mining Science*, *43* (1), 73-82.

[24] Dimitrakopoulos, R. and Ramazan, S. (2004). Uncertainty based production scheduling in open pit mining. *SME transactions*, *316.*

[25] Dimitrakopoulos, R. and Ramazan, S. (2008). Stochastic integer programming for optimising long term production schedules of open pit mines: methods, application and value of stochastic solutions. *Mining Technology*, *117* (4), 155-160.

[26] Epstein, R., Gaete, S., Caro, F., Weintraub, A., Santibanez, P., and Catalan, J. (2003). *Optimizing long term planning for underground copper mines*. in Proceedings of Proc. Copper, pp. 265-279.

[27] Epstein, R., Goic, M., Weintraub, A., Catalán, J., Santibáñez, P., Urrutia, R., Cancino, R., Gaete, S., Aguayo, A., and Caro, F. (2012). Optimizing Long-Term Production Plans in Underground and Open-Pit Copper Mines. *Operations Research*, *60* (1), 4-17.

[28] Erdogan, G., Cigla, M., Topal, E., and Yavuz, M. (2017). Implementation and comparison of four stope boundary optimization algorithms in an existing underground mine. *International Journal of Mining, Reclamation and Environment*, *31* (6), 389-403.

[29] Erdogan, G. and Yavuz, M. (2017). Application of Three Existing Stope Boundary Optimisation Methods in an Operating Underground Mine. *IOP Conference Series: Earth and Environmental Science*, *95.*

[30] Gangawat, J. (2014). Optimization of production planning in underground mining.Thesis,

[31] Gershon, M. (1987). Heuristic approaches for mine planning and production scheduling. *International Journal of Mining and Geological Engineering*, *5* (1), 1-13.

[32] Godoy, M. and Dimitrakopoulos, R. (2011). A risk quantification framework for strategic mine planning: Method and application. *Journal of Mining Science*, *47* (2), 235-246.

[33] Grieco, N. and Dimitrakopoulos, R. (2007). Managing grade risk in stope design optimisation: probabilistic mathematical programming model and application in sublevel stoping. *Mining technology*, *116* (2), 49-57.

[34] Guest, A., Van Hout, G., Von Johannides, A., and Scheepers, L. (2000). An application of linear programming for block cave draw control. *Proceedings of massmin*, 461-468.

[35] H. Askari-Nasab, K. Awuah-Offei, and Eivazy, H. (2010). Large Scale Open Pit Production Scheduling Using Mixed Integer Linear Programming. *International Journal of Mining and Mineral Engineering 2*(3).

[36] H. Askari-Nasab, Y. Pourrahimian, E. Ben-Awuah, and Kalantari, S. (2011). Mixed integer Linear Programming Formulations for Open Pit Production Scheduling. *Journal of Mining Science*, *47* (3).

[37] Hamrin, H., Hustrulid, W., and Bullock, R. (2001). Underground mining methods and applications. *Underground mining methods: Engineering fundamentals and international case studies*, 3-14.

[38] Jalali, S. and Ataee-pour, M. (2004). *A 2D dynamic programming algorithm to optimize stope boundaries*. in Proceedings of 2004 Proceedings of the 13th symposium on Mine Planning and Equipment Selection, pp. 45-52.

[39] Jalali, S., Ataee-pour, M., Shahriar, K., Elahi-Zeyni, E., and Nikbin, V. (2016). Computer based optimisation of underground mining area. *Journal of Civil & Environmental Engineering and Science Technology*, *48* (4), 475-489.

[40] Jalali, S. M. E., Ataeipour, M., Shahriar, K., and Elahi, Z. (2007). A computer program to optimize stope boundaries using probable stope algorithm.

[41] Jawed, M. (1985). Systems Approach to Production Planning and Scheduling with Special Reference to Coal Mining - A Case Study *Journal of Mines, Metals and Fuels.*, *33* (2), 37-40.

[42] Jawed, M. (1993). Optimal production planning in underground coal mines through goal programming: A case study from an Indian mine. *24th Internat. Appl. Comput. Oper. Res. in Mineral Indust.(APCOM) Sympos. Proc*, 43-50.

[43] King, B. W. (2016). *Using integer programming for strategic underground and open pit-to-underground scheduling.* Colorado School of Mines.

[44] Kuchta, M., Newman, A., and Topal, E. (2004). Implementing a production schedule at LKAB's Kiruna Mine. *Interfaces*, *34* (2), 124-134.

[45] Leite, A. and Dimitrakopoulos, R. (2007). Stochastic optimisation model for open pit mine planning: application and risk analysis at copper deposit. *Mining Technology*, *116* (3), 109-118.

[46] Leite, A. and Dimitrakopoulos, R. (2014). Stochastic optimization of mine production scheduling with uncertain ore/metal/waste supply. *International Journal of Mining Science and Technology*, *24* (6), 755-762.

[47] Little, J. (2012). Simultaneous optimisation of stope layouts and production schedules for long-term underground mine planning.

[48] Little, J., Knights, P., and Topal, E. (2013). Integrated optimization of underground mine design and scheduling. *Journal of the Southern African Institute of Mining and Metallurgy*, *113* (10), 775-785.

[49] Little, J., Nehring, M., and Topal, E. (2008). *A new mixed-integer programming model for mine production scheduling optimisation in sublevel stope mining*. in Proceedings of Proceedings-Australian Mining Technology Conference, Twin Waters. The Australasian Institute of Mining and Metallurgy.

[50] Little, J., Topal, E., and Knights, P. (2011). Simultaneous optimisation of stope layouts and long term production schedules. *Mining Technology*, *120* (3), 129-136.

[51] MacNeil, J. A. and Dimitrakopoulos, R. G. (2017). A stochastic optimization formulation for the transition from open pit to underground mining. *Optimization and Engineering*, *18* (3), 793-813.

[52] MacNeil, J. A. L. (2015). Determining optimal open pit to underground mine transition depth using stochastic mine planning techniques.Thesis, McGill University.

[53] Magagula, N. S. (2016). Stochastic characterisation of a mining production system.Thesis,

[54] Malaki, S. (2016). Block-Cave Extraction Level and Production Scheduling Optimization under Grade Uncertainty.Thesis, University of Alberta.

[55] Manchuk, J. (2005). "A Progress Report on Stope Optimization"

[56] Manchuk, J. (2006). A Framework for Stope Sequence Optimization.

[57] Manchuk, J. and Deutsch, C. (2008). Optimizing stope designs and sequences in underground mines. *SME Transactions*, *324* 67-75.

[58] Martinez, L. (2009). *Why accounting for uncertainty and risk can improve final decision making in strategic open pit mine evaluation*. in Proceedings of Proceedings of the 2009 Project Evaluation Conference, The Australian Institute of Mining and Metallurgy.

[59] Martinez, M. A. and Newman, A. M. (2011). A solution approach for optimizing long-and short-term production scheduling at LKAB's Kiruna mine. *European journal of operational research*, *211* (1), 184-197.

[60] Martinich, J. S. (2008). *Production and operations management: An applied modern approach.* John Wiley & Sons.

[61] McIsaac, G. (2005). Long-term planning of an underground mine using mixed-integer linear programming. *CIM Bulletin*, *98* (1089), 1-6.

[62] Montiel, L., Dimitrakopoulos, R., and Kawahata, K. (2016). Globally optimising open-pit and underground mining operations under geological uncertainty. *Mining Technology*, *125* (1), 2-14.

[63] Musingwini, C. (2016). Optimization in underground mine planning-developments and opportunities. *Journal of the Southern African Institute of Mining and Metallurgy*, *116* (9), 809-820.

[64] Nehring, M. (2006). Stope sequencing and optimisation in underground hardrock mining.

[65] Nehring, M. and Topal, E. (2007). Production schedule optimisation in underground hard rock mining using mixed integer programming. *Proceedings-Project Evaluation. The Australasian Institute of Mining and Metallurgy, Melbourne*, 169-175.

[66] Nehring, M., Topal, E., Kizil, M., and Knights, P. (2012). Integrated short-and medium-term underground mine production scheduling. *Journal of the Southern African Institute of Mining and Metallurgy*, *112* (5), 365-378.

[67] Nehring, M., Topal, E., and Little, J. (2010). A new mathematical programming model for production schedule optimization in underground mining operations. *Journal of the Southern African Institute of Mining and Metallurgy*, *110* (8), 437-446.

[68] Nelis, G., Gamache, M., Marcotte, D., and Bai, X. (2016). Stope optimization with vertical convexity constraints. *Optimization and Engineering*, *17* (4), 813-832.

[69] Newman, A. M. and Kuchta, M. (2007). Using aggregation to optimize long-term production planning at an underground mine. *European Journal of Operational Research*, *176* (2), 1205-1218.

[70] Newman, A. M., Rubio, E., Caro, R., Weintraub, A., and Eurek, K. (2010). A Review of Operations Research in Mine Planning. *Interfaces*, *40* (3), 222-245.

[71] Nhleko, A., Tholana, T., and Neingo, P. (2018). A review of underground stope boundary optimization algorithms. *Resources Policy*, *56* 59-69.

[72] Nikbin, V. and Ataee-pour, M. (2016). An Integer Programming Algorithm for Stope Layout Optimization.

[73] Nikbin, V., Ataee-pour, M., Shahriar, K., and Pourrahimian, Y. (2018). A 3D approximate hybrid algorithm for stope boundary optimization. *Computers & Operations Research.*

[74] O'Sullivan, D. and Newman, A. (2014). Extraction and backfill scheduling in a complex underground mine. *Interfaces*, *44* (2), 204-221.

[75] O'Sullivan, D. and Newman, A. (2015). Optimization-based heuristics for underground mine scheduling. *European Journal of Operational Research*, *241* (1), 248-259.

[76] Ovanic, J. and Young, D. (1995). *Economic optimisation of stope geometry using separable programming with special branch and bound techniques*. in Proceedings of Third Canadian conference on computer applications in the mineral industry, pp. 129-135.

[77] Ovanic, J. and Young, D. (1999). *Economic optimization of open stope geometry*. in Proceedings of 28th international APCOM symposium. Colorado school of Mines, Golden, Colorado. USA, pp. 855-862.

[78] Pendharkar, P. C. (1997). A fuzzy linear programming model for production planning in coal mines. *Computers & operations research*, *24* (12), 1141-1149.

[79] Poniewierski, J., MacSporran, G., and Sheppard, I. (2003). Optimisation of cut-off grade at Mount Isa Mines Limited's enterprise mine. *Proc. 12th Internat. Sympos. Mine Planning and Equipment Selection, Australasian Institute of Mining and Metallurgy, Carlton South, Australia*, 531-538.

[80] Porumbel, D. C. (2012). Heuristic algorithms and learning techniques: applications to the graph coloring problem. *4OR*, *10* (4), 393-394.

[81] Pourrahimian, Y. (2013). *Mathematical programming for sequence optimization in block cave mining*. University of Alberta (Canada).

[82] Pourrahimian, Y., Askari–Nasab, H., and Tannant, D. (2012). Mixed–Integer Linear Programming formulation for block–cave sequence optimisation. *International Journal of Mining and Mineral Engineering*, *4* (1), 26-49.

[83] Rahal, D., Smith, M., Van Hout, G., and Von Johannides, A. (2003). *The use of mixed integer linear programming for long-term scheduling in block caving mines*. in Proceedings of Proceedings of the 31st International APCOM Symposium, Cape Town, South Africa, pp. 123-131.

[84] Ramazan, S., Dagdelen, K., and Johnson, T. (2005). Fundamental tree algorithm in optimising production scheduling for open pit mine design. *Mining Technology*, *114* (1), 45-54.

[85] Ramazan, S. and Dimitrakopoulos, R. (2004). Traditional and new MIP models for production scheduling with in-situ grade variability. *International Journal of Surface Mining*, *18* (2), 85-98.

[86] Ramazan, S. and Dimitrakopoulos, R. (2012). Production scheduling with uncertain supply: a new solution to the open pit mining problem. *Optimization and Engineering*, *14* (2), 361-380.

[87] Riddle, J. M. (1977). *A dynamic programming solution of a block-caving mine layout*. in Proceedings of Proceeding of the 14th International Symposium on Application of Computers and Operations Research in the Minerals Industries. Society for Mining, Metallurgy and Exploration, pp. 767-780.

[88] Rubio, E. (2002). Long term planning of block caving operations using mathematical programming tools.Thesis, University of British Columbia.

[89] Rubio, E. and Diering, T. (2004). Block cave production planning using operation research tools. *Proceedings of MassMin*, 141-149.

[90] Sabour, S. A. and Dimitrakopoulos, R. (2011). Incorporating geological and market uncertainties and operational flexibility into open pit mine design. *Journal of Mining Science*, *47* (2), 191-201.

[91] Sandanayake, D. (2015). Stope Boundary Optimization in Underground Mining Based on a Heuristic Approach. *Doctor of Philosophy, University of Curtin*.

[92] Sarin, S. C. and West-Hansen, J. (2005). The long-term mine production scheduling problem. *IIE Transactions*, *37* (2), 109-121.

[93] Shahriar, A. P. K., Oraee, K., and Bakhtavar, P. S. E. (2007). *A Study on the Optimization Algorithms for Determining Open-Pit and Underground Mining Limits*. in Proceedings of 7th International Scientific Conference, SGEM 2007, International Multidisciplinary Scientific GeoConferences (SGEM).

[94] Silva, M. d. F., Dimitrakopoulos, R., and Lamghari, A. (2015). Solving a large SIP model for production scheduling at a gold mine with multiple processing streams and uncertain geology. *Mining Technology*, *124* (1), 24-33.

[95] Smit, A. and Lane, G. (2010). *Mine optimization and its application using the Anglo Platinum Mine Optimisation Tool (APMOT)*. in Proceedings of Platinum in Transition "Boom or Bust." Presented at the 4th International Platinum Conference, The Southern African Institute of Mining and Metallurgy, pp. 317-328.

[96] Smith, M. and O'Rourke, A. (2005). The connection between production schedule and cut-off optimization in underground mines.

[97] Smith, M., Sheppard, I., and Karunatillake, G. (2003). *Using MIP for strategic life-of-mine planning of the lead/zinc stream at Mount Isa Mines*. in Proceedings of Proceedings of the 31st International APCOM Symposium, Cape Town, South Africa, pp. 465-474.

[98] Sotoudeh, F., Kakaie, R., and Ataei, M. (2017). *Development of a computer program for underground mine stope optimisation using a heuristic algorithm*. in Proceedings of Proceedings of the First International Conference on Underground Mining Technology, Australian Centre for Geomechanics, pp. 689-700.

[99] Terblanche, S. and Bley, A. (2015). An improved formulation of the underground mine scheduling optimisation problem when considering selective mining. *ORiON*, *31* (1), 1-16.

[100] Topal, E. (2008). Early start and late start algorithms to improve the solution time for long-term underground mine production scheduling. *Journal of the Southern African Institute of Mining and Metallurgy*, *108* (2), 99-107.

[101] Topal, E. and Sens, J. (2010). A new algorithm for stope boundary optimization. *Journal of Coal Science and Engineering (China)*, *16* (2), 113-119.

[102] Trout, L. P. (1995). Underground mine production scheduling using mixed integer programming. *Proceedings of 25th international symposium, Application of computers in the mineral industry (APCOM), Brisbane, Australia,*, *25* 395-400.

[103] Williams, J., Smith, L., and Wells, P. (1973). Planning of underground copper mining. *10th Internat. Appl. Comput. Oper. Res. in Mineral Indust.(APCOM) Sympos. Proc*, 251-254.

Short-Term Underground Mine Planning: A Review

Shahrokh Paravarzar, Yashar Pourrahimian, and Hooman Askari-Nasab
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

This paper reviews the state-of-the-art in short-term underground mine planning, which is critical for future mining activities. According to the literature, the research is conducted in the categories of mathematical programming optimization and discrete event simulation. Mathematical programming aims at minimizing an objective with the functionality of cost, makespan, deviation from a specific target such as tonnage and fleet management and scheduling. Mining simulation is widely applied in case of short-term mine planning, mostly based on modelling the internal mining process and parameters and on implementing a model for different case studies. A comprehensive review is provided for the aforementioned issues, commercial software packages and programming languages. Finally, research limitations and future area investigation are discussed. The studies reveal a deficiency of research based on the type of mining methods and incorporation of uncertainties.

1. Introduction and scope of the work

Mining is related to a series of operations that result in the extraction of materials from an orebody that is situated in a shallow deep up to a deep level from the surface. Although the conventional open-pit mining method is still used nowadays, in the near future the demand for mineral resources will increase and, consequently, underground mining will be the future challenge for mine industry as surperficial mines will be depleted (Camus, 1992; Nilsson, 1992; Fuentes, 2004; Fuentes and Caceres, 2004; Bakhtavar et al., 2009; Brown, 2012). As a result, more exploration will develop for providing access to deeper mineral resources. Deeper mining means more challenge in production rate, heat, more ventilation demand and higher production costs (Kaiser and Kim, 2009; Brown, 2012; Thrybom et al., 2015; Morrison, 2017).

Research has been conducted and many methods expended for open-pit mine planning problem in different levels of details during the past decades (Bozorgebrahimi et al., 2003; Osanloo et al., 2008; Newman et al., 2010; Moradi Afrapoli and Askari-Nasab, 2017; Blom et al., 2018). This is due to ease of access to these resources with a lower cost of extraction for the industry. Underground mining will be the next target of governments and the mineral industry in the next years, which are not accessible in a way that open-pit mining is. Accessing to more mining resource means deeper level for underground mining. Deeper underground mining operation due to its complexity and variety in extraction methods will be more complicated for production scheduling and extraction process (Hamrin et al., 2001; Hustrulid et al., 2001), so the mining companies will focus on decreasing the operating cost more than in the past.

There are several aspects of underground mining for which researchers provide a literature review. Vagenas et al. (1997) studied the mobile machine automation for underground mining. Logie and Matheson (1982) provided a review in the context of state of the art for pillar design. Peng and Tang (1984) research focused on underground support issues, providing a review of roof bolting support. Diamond and Schatzel (1998) examined the coalbed gas content determination methods for underground coal mining. Schnakenberg and Bugarski (2002) reviewed the new technologies for controlling the diesel emission for an underground

mine. Webber et al. (2003) assessed the heat stress problem using local and international indices. Alford et al. (2007) provided a study of the underground mine design problem considering infill drilling optimization, determining the cut-off grade, stop optimization, mine development network design, decline design and stop scheduling. Altun et al. (2010) provided a short analysis of the environmental impact of underground mining. Song et al. (2013) reviewed the real-time optimization of hard rock underground mining method using the tracking and communication tools for mine production and scheduling. Ralston et al. (2014) focused on mining automation area and investigated the role of automation technologies on safety, environmental sustainability, and production of underground coal mining process. Kang (2014) reviewed the recent development in the area of coal mining transportation and support. Four types of support were studied in their research including a high stressed roadway 1,000 m below the surface, a roadway surrounded by severely weak and broken rocks, a chamber surrounded by weak and broken rocks, and a roadway with very soft and swelling rocks. Khodayari and Pourrahimian (2015) studied mathematical programming optimization methods for block caving model with the purpose of production scheduling. Musingwini (2016) reviewed applied optimization techniques for underground mining method in four area of layout development, stop envelop size, production scheduling, and equipment selection and provided some suggestions for the development of 3D stochastic optimization models.

Surface mining is mostly restricted to open-pit while there are several underground mining methods with their specifications, which means a higher potential for conducting the research in this area for both long and short-term mine planning and the related challenges. In this paper, the aim is to review underground short-term mine planning.

The mining process will continue by the selection of an extraction method after data collection from an orebody. The next stage after geological, mineralogical and structural studies and combining them for resource/reserve estimation leads to the preliminary selection of potential mining methods and the scale of mining operations. Mine planning steps are completely affected by those stages as each planning has its considerations (Darling, 2011). The mine plan is highly dependent on the nature of the orebody; the uniquely most important parameter for mine selection method will always be the style and geological characteristics of the mineral deposit such as thin, steeply dipping tabular vein gold, or large, disseminated porphyry copper deposits. Other parameters are as follows:

- Engineering properties of the mineral deposit and host rock mass
- The required rate of production from mine (underground and open-pit)
- Forecasts of mineral product value
- Comparative study of mine operational cost and capital investment and mineral
- Processing activities necessary to implement mine method
- Availability, cost and skill level required for mining
- Governmental and environmental and mine closure costs.

Two main parameters that classify underground mining methods are:

- Basic deposit geometry
- Support requirement necessary to mine stable stop or produce caving and ground problem.

Other factors such as environment, health and safety, cost, and output rate could be considered. Underground mining is divided into three broad classes: caving, stoping and other methods. Caving is referred to the controlled fragmentation of rocks due to the flow gravity, whereas the term stoping is the excavation of a stable opening of small or large dimensions. Underground mining methods are categorized by the following methods: room and pillar, stop-and-pillar, sublevel stope, shrinkage, cut-and-fill, timbered square set, longwall, sublevel caving (top slicing), block caving and panel caving (Hamrin, 1980; Brown, 2002; Brady and Brown, 2013).

In the next sections, a comprehensive review based on the mentioned methods is presented. In general, there has been a milestone in late 1990's in this research area due to the increase of the computational capacity of the computers, so we divided the methods for the literature to before and after that date. In the first part, the focus is on the mathematical programming models and in the second part simulation approach is reviewed. In the third part, the simulation methods implemented for mine planning is reviewed.

2. Mathematical programming models

In mathematics, computer science and operations research, mathematical optimization refers to finding the best answer with mathematical modelling and programming with a purpose of providing a robust guideline for decision making using current information. In general, mathematical models consist of an objective function which must be optimized (in a sense that the function will be maximum or minimum) (Hillier, 2014). Furthermore, the method for solving a particular class of problem or model is known as an algorithm. In each model also some constraints refer to the availability of sources or natural constraints of the underlying problems (Williams, 2013).

The linear problem (LP), also called linear optimization, is a type of problem where an objective function is minimized or maximized by a linear function of decision variables (Dantzig and Thapa, 2006). Integer linear programming (ILP) or Integer programming is a mathematical programming model where all the variables are restricted to be integers (Karlof, 2005). Quadratic programming allows the objective function to have a quadratic form with linear constraints, that is a type of convex programming (Hildreth, 1957). Non-linear programming is a type of optimization problem whose objective function and/or constraints and inequalities or equalities are non-linear. Mixed-integer nonlinear programming (MINLP) refers to optimization problems with continuous and discrete variables and nonlinear functions in the objective function and/or the constraints (Bazaraa et al., 2013). Stochastic programs are mathematical programs where some of the data incorporated into the objective or constraints are uncertain. Uncertainty is usually characterized by a probability distribution of the parameters (Birge and Louveaux, 2011). Dynamic programming is referring to simplifying a complicated problem by breaking it down into simpler sub-problems in a recursive manner (Bellman, 2013). Multi-objective optimization (or a multi-criterion optimization problem) is a type of problem where the optimization depends on several objective functions (Deb, 2014). There are several other heuristic and metaheuristic methods, the definition of which is out of the scope of this paper. We refer the readers to Gendreau and Potvin (2010), Yang (2010) and Glover and Kochenberger (2006).

2.1. Mathematical programming (MP) for underground short-term mine planning

Mathematical programming is one of the most common approaches for solving mining problems by the minimization or maximization of an objective function. In this section, the mathematical optimization models that have purposed in literature are reviewed.

Muge and Pereira (1979) applied dynamic programming for sub-level stoping. The objective of the model was to minimize the deviation of the element (here working points) for optimal mining sequence. Geometric constraints within a level (the mutual relationship between the blocks) and mutual relationships between the levels are considered in the model.

Yun and Yegulalp (1983) simulated the transportation system using a network flows model for sublevel caving mining system to obtain the optimum system transportation cost (minimum-cost flow). The problem consists of two parameters that are costs and capacity. The cost included the cost of establishing the arc (opening and set-up cost) and the cost of the ore movement.

Chanda (1990) combined mixed integer programming and simulation for minimization of fluctuation between shifts in the extracted average grade. The method applied to continuous block cave mining.

Maximum production from raises in each shift, number of raises, profile constraint, slasher capacity and its availability is considered as a constraint for the model, but economic parameters are not considered in the model. The model output was the number and location of drawpoints for a shift, the tonnage that should be extracted, geology and depletion of the reserve.

LHD fleet management addresses three decision problems: dispatching, routing, and scheduling. Dispatching aims to choose, for one or many vehicles, a new destination (loading or dumping point). Routing consists in selecting the best route (road segments) from the origin to the destination. Finally, scheduling consists of deciding speeds and the waiting times of vehicles on road segments of a route to avoid conflicts between vehicles.

Vagenas (1991) solved the problem of dispatching and traffic control for remote-controlled (automatic) LHD using a graph theory for underground mining. The problem solved by considering the (two-way and one-way) unidirectional lane-segment considering the uncertainty of human operation. Data-driven search strategy applied to a solution. The graph theory problem was solved using Dijkstra's shortest path for origin and destination considering the solution for collision and crossing for traffic zone. For dispatching process, the discrete simulation was implemented to model the operational fleet.

Sepulveda et al. (2005) modeled the LHD allocation with a constraint programming to provide support for task sequencing and vehicle scheduling problem in tunnel network in copper underground mine. Loading activates precede dumping activities (precedence constraint), and working side using that LHD should complete the tasks by one side and then start the task from other side are the model constraints.

Gamache et al. (2005) solved the fleet management problem in an underground mine with the shortest-path algorithm. The problem was solved for single-lane with bi-directional road and operational constraint for LHD statue for their buckets. The model restricted to deterministic parameters for service and travel time and just for one vehicle at a time, but integrated the three significant aspects of dispatching, routing and scheduling at the same time.

Beaulieu and Gamache (2006) applied an enumeration algorithm based on dynamic programming for solving fleet management. Enumeration algorithm presented based on dynamic programming for optimally solving the fleet management problem in underground mines. The objective was to find the best route and schedule for each vehicle in order to reach their destination in the shortest time while also avoiding conflicts on the haulage network. The focus of the study was on routing and scheduling bidirectional vehicles on a haulage network composed of one-lane road segments. The authors' approach is capable of managing a fleet of four vehicles in less than a minute. Significant differences that an underground fleet management system must have in comparison to open-pit operations are also discussed. Vasquez et al. (2011) implemented an integer programming for LHD allocation to minimize the makespan of drift workload subject to operational constraints for underground copper mine (El-Teniente). The objective of the model resulted in a decision-making process for LHD operator. Constraints could be classified into two groups. The first category of constraints which forced the model to obey the logical path and other category satisfy the flow balancing such as drift entrance, side of drift, dumping site and the turning point of drift.

Irem (2011) developed a mathematical program using mixed integer programming to solve schedule problem of LHD vehicle for underground block caving mine. The objective of the model was minimizing the makespan. Problem modelled as a parallel machine scheduling problem that LHD considered as a parallel machine and LHD cycle to each drawpoint is a job schedule. Vargas Pincheira (2011) implemented a mixed integer programming (MIP) mathematical programming for medium to the short-term planning of room and pillar extraction to optimize the extraction sequence. As road-header used for extraction in mine, then the manoeuvre displacement for this kind of machine is difficult in production tunnel then consequently travel time for moving to another front for continue extraction is high. The following objectives were satisfied due to this optimization: (i) increase tons per day of material extracted from the deposit, (ii) decrease transfers for continuous excavation equipment, and (iii) decrease the extraction times of each panel.

Nehring et al. (2010) presented a dynamic mathematical model using mixed integer programming (MIP) for short-term production scheduling with the objective of minimizing the deviation from production target for each period on scheduling horizon for sublevel stoping operation. The model met the process plant feed for stable production quality as long as machine allocation constraints were satisfied for the sublevel stoping mine system. The objective of the mathematical model was minimizing deviation from target metal production. The model satisfied the shift base schedule and also machine allocation and reassignment while operational condition change. Model constraint consisted of target production rate balanced with the tonnage of metal produced across all machines from the primary blast, ore movement, blast block capacity, haulage shaft capacity, machine assignment limitation, machine capacity, ore movement throughput and orepass limit.

Leite-Corthésy (2016) presented a mixed integer linear program model for optimization of the production and development of a sub-level stoping underground mine on the basis of shifts of one week. The model minimized the deviation between the weekly mine plan all quarters of that same week. Besides this model in order to consider the uncertainty and probability simulation model run to assess these constraints. In the same way that short-term planning (week) aims to achieve the objectives of the medium-term planning (3 months), the very short-term model (shift, 10h) aims to achieve the goals of the weekly model. Thus, each quarter is planned to achieve the objectives of the week that depend on the medium-term objectives. They also provided another model with the same constraints but a different objective function (Leite-Corthésy et al., 2016). The objective function of the model defined as the minimization of operation hours in each site subject to each crew cannot provide more than 10 hours of operation, operation and non-operation time should not exceed more than weekly plan. Restrictions are considered between operation hours and weekly plan hours. Operation cannot be continued during the blasting hours. The model forced to finish the operation at each site completely in order to start the other. Table 1 summarizes the mathematical programming models that have been used for underground short-term mine planning. Table 2 shows the considered parameters for each model.

3. Underground mining simulation

Another approach for evaluating the short-term planning in an underground mine is a simulation of the mining process. Simulation is the imitation of the operation of a real-world process or system. It is a general term, and more specifically it could be directed to (Rossetti, 2015):

- physical simulation for the physical objects,
- continuous simulation which time evolves continuously,
- discrete event simulation where specific events occur in a critical moment of time in a system,
- stochastic simulation based on variables and process function based on Monte Carlo estimation using pseudo-random numbers.

Mining systems and activities are more fit to discrete events. For example, the LHD movement in underground mining process for haul and dump and loading a material could be simulated as a discrete event that occurs in a specific moment of time which may cause a change in a system.

The simulation method is used in the mining area for three general purposes:

1- Assessing the operational functionality of the system and system enhancement
2- Evaluate the development rate
3- Feasibility study and equipment selection.

As underground mining is a complicated process, discrete event simulation can be implemented in several mining processes with different goals. The following section summarizes the application of simulation in the mining operations.

Table 1. Summary of applied MP models in underground short-term mine planning

Author	Model	Model Objective(s)	Mining Method	Constraint
Muge and Pereira (1979)	Dynamic programming	Minimizing the deviation sequence	Sub-level stoping	• Block precedence in extraction sequence • Precedence between levels
Yun and Yegulalp (1983)	Graph Theory	Minimizing transportation cost	Sub-level caving	• Opening and set-up cost • Capacity
Chanda (1990)	Simulation and MIP	Minimizing the deviation in the average production grade between operation shift	Block Caving	• Maximum allowable output per shift • Maximum allowable number of working drawpoints per shift • Declaration of exhaustion for exhausted drawpoints • Required grade for each shift (equality) • Tonnage blended ore in each shift
Sepulveda et al. (2005)	Constraint Programming (CP)	Minimizing makespan in working shift	Block Caving	• Loading and dumping activities • Working side
Gamache et al. (2005)	Shortest Path and Dijkstra's algorithm	Fleet management	Conceptual model	• Route traffic • Dispatching • Vehicle orientation and deadlock • Production target
Beaulieu and Gamache (2006)	Dynamic Programming (Enumerate tree)	Fleet management	Conceptual model	• Layout network • Homogeneous vehicles • Route and traffic condition and Collision • Vehicle orientation and deadlock
Vasquez et al. (2011)	Integer Programming (IP)	Minimizing the makespan of drift workload	Block Caving	• Layout network • Extraction side • Flow balance • Vehicle orientation • Dispatching

Table 2. Continued

Irem (2011)	Mixed integer Programming (MIP)	Minimizing the makespan	Block Caving	• Job schedule • Number of trucks • Number of LHD • Number of positions (Drawpoints)
Vargas Pincheira (2011)	Mixed integer Programming (MIP)	Minimizing the roadheader replacement	Room and Pillar	• Layout (Pillar's position) • The direction of access to the blocks • Precedence of blocks • Translation cost of materials
Nehring et al. (2010)	Mixed integer Programming (MIP)	Minimizing the deviation from production target	Sub-level caving	• Machine capacity • Shift availability • Production capacity • Reserve availability • Ore grade
Leite-Corthésy (2016)	Mixed integer Programming (MIP)	Maximizing mined tonnage on a given time horizon	Backfilled long-hole and cut-and-fill.	• Work hours available for a period of time • Available crew • Number of hours for backfill site • Available rock tonnage • Maximum possible tonnage extraction from mine, level, vein, site
Leite-Corthésy et al. (2016)	Mixed integer Programming (MIP)	Minimizing the deviation of short-term (shift) plan from the weekly plan	Backfilled long-hole and cut-and-fill.	• Number of crew • Crew working hours • Weekly operation hours • Shift operational hours • Exceeding and restriction of working operation hours

Table 3. Comparison of features present in short-term underground mathematical programming models

Authors	Status of drawpoints or face of extraction and their availability	Mine network layout	Geometrical limitations	Block extraction precedence	Ore grade, target, and blending	Material flow balance	Working and extraction side	Shift availability and schedule	Dispatching	Machine and transportation capacity (type, number, capacity)	Route traffic	Vehicle orientation	Production target and capacity	Material re-handling cost
Muge and Pereira (1979)		✓		✓										
Yun and Yegulalp (1983)		✓							✓	✓				
Chanda (1990)	✓				✓									
Sepulveda et al. (2005)							✓		✓					
Gamache et al. (2005)									✓		✓	✓	✓	
Beaulieu and Gamache (2006)		✓								✓	✓	✓		
Vasquez et al. (2011)		✓				✓	✓		✓			✓		
Irem (2011)	✓							✓		✓				
Vargas Pincheira (2011)		✓		✓			✓							✓
Nehring et al. (2010)	✓				✓			✓		✓			✓	
Leite-Corthésy (2016)	✓	✓			✓			✓					✓	
Leite-Corthésy et al. (2016)	✓	✓			✓			✓						

3.1. Elderly computer-based simulation of the mining process

Touwen and Joughin (1972) presented a paper with the aim of simulating underground stoping and transport operation for a gold mine. Mining activities were simulated for large and small operation units. Stopes were simulated as a drilling and blasting process and transportation was simulated as a rail material movement process using scrapers. Simulation run for months in a unit of minutes.

Beckett et al. (1979) designed an LHD simulator called LHDSIM based on event-scheduling approach on Fortran computer language program. In this approach, a simulator searched for collision point specifically on intersection nodes on room and pillar mine layout. If the program determined the conflict, then LHD should wait on queue otherwise the time-lapse will be calculated for the machine. The time-lapse for LHD and non-LHD machine were different based on their intuitive movements. The LHSSIM program run for both deterministic and stochastic options. A physical layout such as cut depth width and distance between headings, cut height, density of material, tramming breakthrough, LHD operational parameters (bucket capacity, load, and discharge rate), continuous miner rate was considered in the program.

Steiker (1982) simulated the underground mining process using the system capacity (tons per day), track layout design, train fleet configuration and dispatching procedure using GPSS simulation computer language. Mine layout consisted of two independent production levels with rubber tired load-haul-dump and one haulage level operating using a train. Crusher and orepass locations were fixed in the system. Delays consisted of delay caused by queueing the trucks to the same train, truck repair or accessing the route by crew due to the right-of-way. Simulation run for one week, and the output consisted of tonnage production for each day, a number of trains dump, an average of queuing time per train, average round trip time per train, crusher utilization, and average queueing at the crusher.

Oberholzer and Hardman (1987) did a review for computer simulation to evaluate the production and conveyor belt transportation process for coal mining system. Programs were categorized by three generations of programs as historical databank data, simulation programs for planning, design or control purpose and tools for planning or management control. Based on databank historical data machine capacity and availability were responsible for production losses. It was also concluded that the mining operation method was a more valuable key factor rather than machine breakdown time in a system. FACESIM and COMSIM in 1984 applied for simulating the continuous room and pillar mining method. BELTSIM program was written for belt simulation and optimization. CUTSIM was another simulation part for cutting drum of the continuous miner, longwall shearer or another similar machine. The program predicted the pattern that particular cutting design cut the coal face by showing the average force level and the cyclic force level.

Litke et al. (1993) implemented SLAMII (a Simulation Language for Alternative Modeling) to determine the influence of mining factors and their interaction in a mining system on a scale of daily production. System infrastructure consisted of in-the-hole (ITH) drills, load-haul-dump (LHD), one train with different capacity (4 cars with 20% of availability, six cars with 80% of availability and 10 for tramming development) and one truck for backfilling. In a system drilling and blasting cycle occurred in a production stage then ore mucking and hauling carried on using a ramp system and orepass/ chute system and the material tramming using a train in a system. The boundary of the system was from extraction face up to crusher while crusher and hoisting system were not modelled. They assessed the bottleneck of the system, mucking and hauling and miscellaneous scenarios. The results showed that LHD return hauling system, number of drills and machine availability of the drill influenced the system.

Hunt (1994) modelled the ore transportation process using a computer simulation called SLAMSYSTEM using the SLAMII Fortran based language for Amax's Henderson molybdenum mine in Empire, Colorado. Ore movement and handling occurred by LHD and train system from drawpoints to deliver to the mill. The capacity of the LHD bucket, trains capacity and cycle time for

LHD and train, and crusher considered in the model. The model simulated for 24 hours (1440 min). Based on their process the simulation results suggested decreasing the number of LHD and increasing the number of trains.

Karami and Szymanski (1997) developed a simulation model called SLAM II and implemented it to evaluate the performance of an underground discontinuous hard rock mining transportation system. The objective of the simulation was determining the key components of the system that is controlled by the system output and performance. The SLAM II obtains a solution and considering the various scenarios also for those. Three stops fed one orepass in the model. Mine layout included only two ore bins and work cycle times of ore haulage activities were considered in the model. An optimum number of mine cars in order to reach a production target was obtained through the model.

Runciman et al. (1997) compared two systems for loading the haulage truck: a chute system and Load-Haul-Dump (LHD) loading system to load a haulage truck assessed in order to understand if the chute system could exceed the throughput of the haulage system or not. The results showed that the chute system could pass the current throughput when a system works with one or two trucks. Haulage distance bay pass area and location for the repaired area was considered in the model. The model was included in three categories of the LHD data, haulage track data and system data. For LHD, parameters were type of LHD, loading time, dumping time, load size per bucket and availability; for haulage truck type of haulage truck, chute loading time, dumping time, payload capacity per trip and availability and, for system data, chute capacity, operation schedule, number of track layout roadway grades on the ramps and tramming speed in each segment were the parameters considered in the model.

Runciman (1999) implemented the discrete simulation model for evaluating the developed automated mine equipment system. The purposed model (automate mine equipment) compared with a current model for 30 days based on possible shift for several objectives (short and long-term). Short-term objectives consisted of the limitation of future automated mining equipment, their throughput and analysis the effectiveness and long-term objectives included the future research for development and system analysis simulation of a subsystem such as navigation and material supply and usage of simulation of mine engineering issue. Eight different model compared with a benchmark mining equipment system consisting of cycle mining equipment system and semi-continuous mining equipment concept. The results illustrated that the throughput of the mining system increases up to 45% using an automated system.

3.2. Simulation for short-term underground mine planning

Bayer and Nienhaus (2000) designed a theoretical computer model written in C++ to simulate the capacity of a room and pillar operation. Geological and layout parameters were considered as an input for the model in addition to equipment specific data. The program calculated the best fitting design and mine sequence. Sensibility analysis performed for productivity versus seam height, length of cut, number of entries, specific cutting energy. The model reflected the real mining situation fairly.

Mcnearny and Nie (2000) modelled the conveyer belt haulage system for longwall underground coal mine. The model included the gathering belts, transfer belts, spillage, belt breakdown, time between belt breakdowns, and belt utilization, The model input included the length of each belt, the width of each belt, the cross-section of each belt, the speed of each belt, the density of coal, parameter for distribution of coal flow rate at each section, distribution of shuttle car arrival time, crusher discharge time, belt working time, belt down times, actual work time of each shift and number of replicate simulation. Using the simulation, a quantitative prediction for belt size and surge bin capacity were determined.

Roberts (2002) applied a computer simulation model for sub-level stoping mine in order to predict the operational performance. Considering the truck type, a number of truckloads and decline layouts, size of pass bay, queuing pattern, production capacity, simulation performed for preferred and non-preferred pass bay logic. Operational incidence such as unscheduled stoppages, vehicles breakdowns

or drawpoint blockages and interference to the passage of trucks in decline by another mobile plant such as graders, jumbos, and utility vehicles were not considered in the model constraints. Haulage shaft capacity, reserve capacity, machine trimming, and a number of allocated LHD are other constraints taken into account in the model. The results of the simulation showed that the pass bay size and spacing and high-payload were crucial parameters for the haulage system performance.

Saayman (2006) simulated a dispatching system with Matlab programming for an underground mine. The boundary of system defined from drawpoint which material handles by LHD until dumped into the crusher. Simulation run by the objective of analyzing the different scenarios for dispatching problem. Service time also considered for truck and LHD on workshop area. Collision rules and traffic condition considered in a system as just one LHD was allowed to travel at the same time in a tunnel. Their research followed several objectives. The first objective was maximizing the total tons produced and at the same time keeping the level of each drawing point extraction column equal. Another objective was to decrease the crusher shutdowns as few as possible. For this approach, it would be necessary that the trucks arrival time be equal in event space. They considered the hang-up frequency in the model according to historical data, and maintenance plan was not considered in simulation because the simulation just run for one week, but fuel consumption and refilling were considered in the model. Five strategies were considered for modeling the dispatching system. The base case was the common rule for visiting the drawpoints sequentially if the drawpoint was active as same as normal mining situation. In the second strategy if the LHD in one tunnel started from the first drawpoint, at the adjacent tunnel LHD would start from the furthest drawpoint. The third strategy was regarding the cost function and attempted to minimize the difference between the level of each drawpoint with the adjacent ones and keep the difference minimum. In the fourth strategy, the movement of LHD was similar to the base case, but just one truck moved around. This resulted in less shut down in the crusher and minimized the truck waiting time. In the last strategy, the LHD was not assigned just to the specific tunnel. The decision to assign LHD to each drawpoints was taken by minimization of the cost function. The available tunnel was the one such that there were not any LHD on that tunnel.

Rubio and Troncoso (2008) utilized the discrete event simulation to show how the operational interruption on drawpoints (secondary breakage and hang up frequency), production cross cut's closure strategy and a number of secondary breakage crews could influence the productivity. The simulation model consisted of drawpoints, production cross cuts, orepass, chutes and haulage crosscuts and the operational entities were minerals, LHD, dump trucks, secondary crews and chutes reparation crews. Failure in each drawpoints and ore pass and chutes between the tonnage event distribution were independent of others. Mechanical failure was not considered in the model and just one LHD was allowed for work on each crosscut. The results showed that a higher number of secondary reduction crew would cause higher productivity and haulage crosscut productivity had a reverse relation with drawpoint failures.

Maldonado Meza (2009) modelled the integrated short-term production considering the mine, transport and plant production for calculating the capacity production on Division El Teniente of Codelco Chile. The model designed for a probabilistic production for isolated mine network flow. To this end, the model would be able to illustrate the effect of the maintenance program and a nominal capacity of mine equipment on integrated mining system. Operational failure of mine, transport, and mill were not considered in the model.

Pereira et al. (2010) developed a simulation model for room and pillar mining system with the objective of maximizing production from all coal faces from a given plan. The model validated for forecasting the daily ROM production. Complete production cycle system considered for modeling including providing support for the ceiling, cutting the coal layer, drilling the face, charging and exploitation, ventilation and cleaning the face by ore removal.

Fjellström (2011) studied the transportation cost of ore and waste material to the crusher and backfilling rooms in the underground Renström mine. Mine truck and highway trucks were two alternatives that they considered for the simulation model as highway trucks had less truck cycle but more failure time and mine trucks had a higher cycle time and less failure time. Simulation results demonstrated that using a mine truck would decrease the cost of production by 10-20%. The input of model data consisted of detailed schedule hours, machine working hours consisted of drilling, charger, loader, highway and mine truck, scaler, shotcrete, resin and cement bolt ring cement mixer and water truck. Delay and idle time were also considered for machines. Unscheduled maintenance times (for failures) and scheduled maintenance time (for service time) were considered in the model. A number of vehicles and capacities such as driving speed for loaded and unloaded in three positions of the flat surface, ramp up and down were set for the model.

Pop-Andonov et al. (2012) applied Arena simulation software to analyze the transportation cost for a hypothetical underground mine. Rail and truck transportation system were considered as a haulage system for their goal of taking into account the queuing rule for trucks.

Haviland and Marshall (2015) designed a fundamental model of mine ramp productivity for a haulage system. Under the fundamental assumption that there was not any randomness, the system converged to periodic behavior. The model also showed how with a less haulage capacity the system could have achieved the same productivity. The model did not consider the light vehicle traffic and identical vehicle assumed in the model. Vehicle speed, loading, and dumping were assumed fixed so declaration and acceleration were not considered. Passing bay is allowed and the delay associated with pulling in and out was not simulated. Discrete event simulation was used for this purpose.

Usmani et al. (2014) performed the discrete event simulation for LHD optimization in extraction level for block caving model. Extraction drifts, drawpoints, production rate, daily draw rate, LHD operation and orepass were considered as the simulation elements. Base case scenario along with four other scenarios modeled in their case in order to obtain the optimum number of LHD to meet the production target. Hang-ups were considered fix, and traffic network and road conditions were not considered in their model.

Sjödin (2015) implemented the SimMine as a simulation software to simulate and evaluate the development process and machinery in mine. Average face utilization determined by the number of available faces to be sure that enough face is available if mechanical failure or face block occur. Hours per block indicated the development rate and also showed the machine effective hours. A number of blasts were related to measuring production in mine. The total worked hours divided by the total shift hours showed the utilization of planned time. The total worked hours divided by total hours per year defined the utilization of total mine. The old dataset was used in order to be able to compare the results with the old data. Failure for the automate machine imported to the model as a probability distribution of occurrence for the model. Simulation runs for a Drill rig, Simba, bolters, and LHD's. Average face utilization, an hour per blast, number of blasts, utilization of planned time and utilization of total time were the indicators of simulation and measured parameter.

Skawina et al. (2015) estimated the rate of ore handling from drawpoint to ore pass using the DSE on sublevel mining process. Two types of LHD (Diesel and electric Load-Haul-Dump (LHD) machine) with different capacities were considered as a loader in the model. Rock breaker was modelled as a waiting time in the dumping operation. In a case that LHD did not find a task then it would wait in a parking space for waiting for the next assignment. Lunches, times when the machine is functional but not working and movement of the machine to parking when going off for the break, and extension of drift after each blasting were included. Restriction for working in the production area was considered one or two for each. The simulation was performed by considering the machine size, tramming speeds, acceleration, deceleration, delay and shift break. The aim of simulation was to identify the fleet configuration with the closest production rate.

Greberg et al. (2016) considered a problem of ore transportation via orepass in comparison with the truck for deep mining system on sub-level cave mining system as production disturbances using orepass result into high cost and time for ore pass rehabilitation. Two simulation models were run. One for two sub-levels with two main production areas in each level which were located furthest away from the crusher, which required the largest number of trucks and longer cycle time; and the second one for analyzing the further increasing distance to the existing crusher at the main level. The model was run for seven different scenarios. Electric and diesel trucks were used for the model. LHD, truck and area availability and traffic disturbance were considered for the model as an input. A truck parameter consisted of truck types, capacity, loading time, dumping time, speed when empty and average speed when loaded. The results showed that haul truck could be a replacement for orepass when mining went deeper.

Fukui et al. (2016) modelled a distributed system by implementation of simulation with the aim of comparing the distributed system and LHD system to evaluate the productivity of the system. As Fukui et al. (2016) mentioned LHD was a de-facto standard machine as it was an excavator and transporter at the same time, for this reason, the load and unload heavy weights caused a lot of fuel consumption. Also increasing the capacity for and at the same time smoother operation required wider drifts. Installing the separated loader and transportation system could have a large impact on production. Productivity, cost, the rate of operation and maintainability were examined for the model. Multiple agents simulated transports and excavators motion and internal states on Matlab (MathWorks Inc., 2015).

Park et al. (2016) implemented a simulation model on GPSS/H simulation language to optimize a number of trucks dispatch with the objective of maximizing the production of an underground limestone mine. Parameters considered in the simulation were truck cycle time, number of trucks, crusher availability, working hours, dispatching time, the capacity of train and loader and waiting for discharge yard. According to the model, the optimized number of trucks was obtained.

Anani et al. (2017) implemented a simulation method to evaluate an optimal panel width selection that minimized unit mining costs and maximized the productivity considering the number of haulage units assigned to each continuous miners. One-way traffic network with stations that were representative of each face-cutting and feeder-breaker was considered in the model as a stationary source. In addition to that payload, the empty speed of LHD loading time, dumping time, battery change, loaded speed, the time between cuts and spotting time were considered in the model. The model run with two shifts, one for making cuts and repair the conveyer belt and continuous miner repair and the second one just for production. The simulation result could be summarizing in this way that for a specific operation there was an optimal panel width that maximized productivity. Also, an optimal panel width existed with a minimum cost.

Shelswell and Labrecque (2017) implemented the discrete event simulation for assessing the impact of a number of operators on the performance of truck haulage fleet. The boundary of simulation area consisted of ore from stopes and the generation of lateral waste from up-stream and dumping of material at the run-of-mine (ROM) pad and stockpile site on the surface. Sublevels were not considered in the simulation. Non-production days, maintenance events and random shutdowns were the parameters that suspended the haulage and underground activity. Daily base schedule considered for ore production and a schedule based shifts defined for trucks, underground operation, and surface activities. For haulage trucks traffic, queuing, load delay and shop bays and breakdowns were considered. Truck dispatching, loading, dumping and availability of trucks were also included in the model. Haulage productivity was simulated with and without a number of operators and truck availability.

Table 3 summarizes the simulation models that have been used for underground short-term mine planning. Table 4 shows the considered parameters for each simulation model.

Table 4. Summary of simulation models applied to underground short-term mine planning

Author	Mining method	Model Objective (s)	Resolution and details	Model parameter(s)
Touwen and Joughin (1972)	Sub-level stoping	Transport system evaluation	Short-term (monthly)	• Scraping operation • Drilling and blasting • Support installation
Beckett et al. (1979)	Room and pillar	Productivity on mine haulage system	Short-term (shift)	• Extraction face physical parameter • Material density • LHD parameter, capacity, and cycle-time • Non-LHD parameter capacity and cycle-time • Dispatching and traffic rules
Steiker (1982)	-----	Productivity and performance	Short-term (weekly)	• System capacity (tons per day) • Track layout design, • Train fleet configuration • Dispatching procedure • Mine layout • Delay
Litke et al. (1993)	-----	Assessing the influence of system drivers	Short-term (daily)	• In-the-hole (ITH) drills, • Load-haul-dump (LHD), • Train

				• Haulage system capacity • Machines availability
Hunt (1994)	Block Caving	Ore transportation process	Short-term (daily)	• LHD transportation and capacity • Orepass • Train • Crusher
Karami and Szymanski (1997)	Sub-level stoping	Measuring output performance for transportation	Short-term (daily)	• Mine layout (stops, orepass, etc.) • Transportation and haulage configuration
Runciman et al. (1997)	Conceptual model	System performance evaluation	Short-term (weekly)	• Type of LHD • Cycle time • Load size per bucket • Haulage availability • Operation schedule • Road condition
Runciman (1999)	Crown pillar	System performance od chute and orepass system	Long and Short-term (monthly)	• Orepass configuration (Distance, location, and capacity) • Transportation capacity
Bayer and Nienhaus (2000)	Room and pillar	Production capacity	Short-term (no-data)	• Geological parameters • Layout parameters (seam height, length of cut, number of entries, specific cutting energy)
Mcnearny and Nie (2000)	Underground Coal	Performance of conveyer belt as haulage system	Short-term (14 days)	• Belt capacity and parameters • Cross section of each belt

				• Belt speed • Coal density • Distribution of coal flow rate at each section • Distribution of shuttle car arrival time • Crusher discharge time, • Belt working time • Belt down times, actual work time of each shift
Roberts (2002)	Sub-level stoping	Operational performance	Short-term (5 Shifts)	• Truck type • Number of truck loads • Decline layouts • Size of pass bay • Queuing pattern • Production capacity
Saayman (2006)	Block Caving	Maximization of total production tonnage Minimization of crusher shutdowns	Short-term (weekly)	• LHD and truck numbers • Drawpoint availability • Mine layout • Dispatching • Traffic rules • Crusher availability • LHD and trucks availability • Cave profile shape
Rubio and Troncoso (2008)	Block Caving	Operational interruption	Short-term (yearly)	• Drawpoints and hang-ups • Production cross cuts,

				• Orepass • Chutes and haulage • Haulage system (LHD and Dump trucks)
Maldonado Meza (2009)	Block Caving	Integrated production (Mine to Mill)	Short-term (yearly)	• Mine network flow • Nominal capacity of mine equipment
Pereira et al. (2010)	Room and Pillar	Maximization of a production from all coal faces	Short-term (shift)	• Support process cycle time • Cutting the coal layer • Drilling the face • Charging • Exploitation, • Ventilation • Cleaning the face by ore removal
Fjellström (2011)	Room and Pillar	Transportation cost	Short-term (two months)	• Detailed schedule hours and availability • Machine working hours • Cycle time for blasting and production • Production capacity • Activity times (Lunch break, preparation, run, etc.) • Backfilling • Ore and waste transport
Pop-Andonov et al. (2012)	Conceptual model	Transportation cost	Short-term (no data)	• Truck dispatching system

				• Train for transport and flow of material movement • Ore bine and chute system
Haviland and Marshall (2015)	Sub-level caving	Sensibility analysis for traffic pattern	Short-term (Limited periods hundreds of second)	• Haulage ramp parameters • Traffic flow policy • Delays, queuing, piggybacking • Priority entrance
Usmani et al. (2014)	Block Caving	Optimization of LHD proficiency	Short-term (yearly)	• Extraction drifts • Drawpoints • Production rate • Daily draw rate • LHD operation • Orepass
Sjödin (2015)	Cut and Fill	Evaluation of development process	Short-term (shifts)	• Average face utilization • hours per blast • Number of blasts • Utilization of planned time • Utilization of total time
Skawina et al. (2015)	Sublevel caving	Estimation of the rate of ore handling	Short-term (shift)	• Haulage and loading equipment capacity • Ore pass capacity • Rock breaker • Mine layout
Greberg et al. (2016)	Sublevel caving	Sensibility analysis for ore pass and truck ore transport system	Short-term (daily)	• Truck type • Truck parameter • Cycle time

				• Traffic flow policy
Fukui et al. (2016)	Conceptual model	Evaluation of the productivity, rate, and cost of a system based on a distributed system	Distributed system	• Static driving performance of excavators • Concise drift path • Quasi-static driving performance of transporters • Static excavating and loading work • Queuing at drawpoints and ore passes • Dispatch of transporters to multiple drawpoints • Production
Park et al. (2016)	Room and Pillar	Maximization of the production	Short-term (1000 days)	• Truck dispatch interval • Working hours • The capacity of each truck • Capacity of loader • Cycle time
Anani et al. (2017)	Room and Pillar	Minimization of unit mining costs and maximization of the productivity	Short-term (shift)	• Dispatching • Number of haulages • Maintenance time
Shelswell and Labrecque (2017)	Sublevel caving	Measuring the performance of truck haulage fleet	Daily-shifts	• Non-production days • maintenance events and random shutdowns • Shift schedule • Mine layout • Material handling • Dispatching

Table 5. Comparison of features present in simulation models in underground short-term mine planning

Authors	Mine layout (Distance, the position of extraction points, etc.)	Hang-ups and extraction point availability (failures)	Geological parameter (grade, density)	Mine flow network and Material flow and extraction rate	Transportation and haulage configuration	Transportation capacity (type, Number, Load size, nominal capacity for LHD and trucks)	Dispatching and production	Traffic rules (Queuing pattern, pass bay, road and ramps condition)	Availability of haulage and transport system (working hours, availability and performance)	Operational schedule, rate, and capacity	Belt and conveyor availability and failures	Belt and conveyor capacity and speed	Ore pass and chute Crusher discharge time and capacity	Support and backfilling	Face blasting or secondary fragmentation crew performance	Ore and waste material re-handling and stockpiling	Ventilation	Extra activity times (lunch time, of days, crew delay, fuel refilling etc.,)	Operator experience	Maintenance time	Obj. Maximizing production	Obj. Minimizing cost	Measuring operational and system performance	Measuring material handling and transport performance	Sensibility analysis	Development process
Touwen and Joughin (1972)															✓											
Beckett et al. (1979)	✓		✓		✓			✓													✓					
Steiker (1982)	✓				✓		✓		✓	✓														✓		
Litke et al. (1993)	✓					✓	✓		✓	✓													✓			
Hunt (1994)						✓							✓											✓		
Karami and Szymanski (1997)	✓				✓																			✓		
Runciman et al. (1997)						✓	✓	✓	✓	✓													✓			
Runciman (1999)	✓					✓																		✓		
Bayer and Nienhaus (2000)	✓		✓																		✓					

4. Other related research

Topal and Ramazan (2010) implemented a mixed integer programming (MIP) for minimizing the maintenance cost for the available truck fleet for an underground gold mining operation in Western Australia. Truck age (or age bins), period and critical bin hours were considered in the model. Total operation hours, total available hours, the age of truck at the end of each period and age bins category were considered as a constraint for the model.

Lashkari (2014) measured the performance of shuttle cars on two room and pillar and longwall mining system by measuring the duty cycle and driving cycle time for shuttle car. The goal of the research was to measure power and energy consumption of the various shuttle cares to work in different haulage operations. Parameters such as rolling resistance, grade resistance, utilization considered to determine haulage operation condition. The required power density for each duty cycle operation, and peak power density and peak power and the parameters of the haulage system for each part of the cycle were obtained by simulation.

Shelswell and Labrecque (2014) compared series and parallel conveyor-based material handling system using discrete simulation and static spreadsheet models. Average daily ore flow rates were calculated for both cases. The results showed that the random failures decreased the effectiveness of parallel conveyor. The static separated sheet generated a lower average daily ore flow rate for the series case and higher results for the parallel case in comparison with the discrete simulation.

Peng and Vayenas (2014) simulated the LHD failure using a genetic algorithm during the period of interest for underground mining. Simulations were generated for three and six months interval using the dataset collected from an underground mine in Sudbury, Ontario, Canada. The simulated data were validated by real-life data to evaluate the compatibility of algorithm dealing with the real-life data set.

Xiao-ping et al. (2015) simulated a surge bin capacity and its belt as a real-time continuous dynamic model. There were several failures for surge bin and, among them, only mechanical and electrical failure and empty bin were considered. A probability density function was obtained by the samples as a random variable and the maximum likelihood estimation method was used for making parameter estimation. According to the parameters, random variables were generated and the simulation run based on the generated random variables.

5. Material flow simulation

For those underground mining methods that the extraction occurs due to the draw and caving of materials such as block caving and sublevel caving the draw mechanism and the way that material moves inside the cave area could be critical. Movement of material and draw process play a major role on grade control and dilution entry, which could be forecast by different approaches.

6. Commercial tools and programming languages

6.1. Commercial tools

Underground Coal TALPAC simulates the process of longwall mining. It is programmed by RPM Global Mining Group. The software generates the shearer productivity, Armoured face conveyor (AFC) performance benchmarks and productivity performance (TALPAC, 2017).

MineSched™ aims to maximize productivity and profits for underground and surface mining for both long-term and short-term planning. MineSched generates a schedule for improving the site operational productivity and probability. In the case of underground mining MineSched planning the stop production considering the following constraints (MineSched, 2018):

- Model material flow through ore passes, and material transport levels.
- Drilling and blasting process and backfilling activities
- Considering the scheduling production and development simultaneously

PC-BC (Personal Computer Block Caving) is a mine planning third-party software that generates a long to short-term mine plan schedule for block caving mining method. Cave Management System (CMS) developed for the daily production cave management system. CMS has an ability to monitor the system and LHD dispatch system by storing data in SQL database and providing an Excel interface. Beside the CMS, LSQ tools are designed for data reconciliation between the operational mine sample analysis and the slice files forecast (PCBC, 2018).

Pitram is fleet management and real-time mine control tools for underground and open-pit mining designed by MICROMINE Corporate. Material management tools use the real data such as mine design, survey, and production data in order to report the stockpile balance and metal stock throughput. Online and real-time data for fleet gathered through the network data. The objective of Pitram is to maximize the productivity (Micromine, 2018).

SimMine is a simulation tool for underground mine simulation, scheduling, and planning. Via importing the layout and mine design, time limits, predecessor, and priority the following parameters will obtain from simulation (SimMine, 2018):

- Project scheduling, daily, weekly, monthly and yearly
- Production parameters such as development rate and advance, for ore and waste
- Resource management such as working time and utilization, breakdowns, etc.
- Material handling issues such as material types and destinations
- Cost per advance and developments, per mine area, per shift, per ton waste, etc.

Carlson underground mine software is suitable for room and pillar and longwall mining system. The software tries to reduce the idle time for the haulage system in order to maximize the productivity (Carlson, 2018).

Promine is a mining and geology software based on AutoCAD interface. It supports several mining methods such as Open stope, Room and pillar, Cut and fill, Block caving, Long hole, Shrinkage stoping, and Alimak Mining. In case of short-term planning, Promine generates a forecast for production using iGantt schedules (Promine, 2016).

Deswik is a software developer mining company which is provided in several extensions for long to short-term planning. In the case of short-term planning Deswik.Blend extension has an ability to manage and optimizes the martial handling and blending problem using CPLEX optimizer. Deswik.OPS provides a short-term schedule. The process can be controlled and captured using real-time data receiving from the operations (Deswike, 2018).

6.2. Simulation software and programming languages

General Purpose Simulation System (GPSS) developed by Geoffrey Gordon in IBM at the 1960s. The world is a model using the entities called Transactions moving Block in which each block is a line of code theta affects the transactions (Schriber, 1974). Later on, GPSS/H produced by Wolverine software cooperation. Use external routines in FORTRAN made execution of GPSS faster (Crain, 1997).

SLAM II is an Interactive Execution Environment (IEE) released in 1981 as a first simulation language to formulate a stem description implementing the event; the processor continues worldview (Reilly, 1991).

SIMAN is a general purpose simulation analysis program for modelling combined discrete-continuous systems introduced in 1982. Continuous model and change in a system model with algebraic or differential equations (Pegden, 1984).

OPTIMINE ^{TM} is a simulation software for underground mining. The software can model the mining process by considering the limitations and critical points in the layout. It uses the traffic-controlling tool TRAM^{TM} a program developed by Cybercube. The software works based on AutoCAD design and also can be created by importing the mine design software such as Surpac. The software considers the operational interruptions such as hang-ups on forecasting the system (Puhakka, 2000).

AutoMod as one of the powerful general-purpose software on at its time introduced by AutoSimulations Inc. located in Bountiful, Utah, USA. This software can generate a three-dimensional simulation environment using animation. It is one of the software that is changing the hardcoding of the model to the CAD environment. This software specially generated with the aim of material-handling as long as it can implement for robotics, mechanical tool transfer system and special machinery (AutoMod, 2012).

WITNESS is a visual interactive simulation software the Lanner Group's simulation software package for discrete and continuous simulation modelling. WITNESS provides access to Oracle, SQL Server, Access, etc., direct spreadsheet link in/out, XML. In addition to that this Windows-based provides a link to Microsoft VISIO and graphical CAD linkage application with a 3D visualization (Al-Aomar et al., 2015).

ProModel was introduced by ProModel Cooperation, with a capacity of adding the C or Pascal subroutine. ProModel simulation uses the point-and-click approach in a Windows environment by offering a virtually unlimited size of the model (Harrell and Price, 2003).

Arena produced by System Modeling Cooperation that is user-friendly simulation environment combining the modelling power and flexibility of SIMAN simulation language. The models create by drag and drop action using the module objects, which contains all the logic and process actions at the same time. The modules are accessible through the templates which are a group of the modules use for a specific approach. Arena contains a several third-party software such as input, output and process analyzer that provides more ability for data processing and generating several replications in some specific ways (Rossetti, 2015).

SimEvents is an event-driven discrete simulation extension package produced by MathWorks company as a part of Matlab software. It can be used for task timing, resource usage on the performance of systems for planning capacity, forecasting, and supply-chain management (MathWorks Inc., 2015).

Here we just reviewed those language programming or software which are used more frequently in the literature. For more information, we refer readers to Banks (1998) and Robinson (2004).

7. Shortcomings of the current methods and suggestion for future research

Based on the literature, short-term underground mine planning could be divided into two general areas: (i) mathematical programming optimization and (ii) simulation.

In the case of mathematical programming, most of the models are restricted to exact algorithms and more specifically mixed integer programming, graphs theory, dynamic programming and constraint programming. Models mostly aim to solve the fleet management problem in the context of mathematical programming. The model constraints are mostly restricted to items such as the location of face extraction, the capacity of production or dispatching layout and traffic rules, which are assumed static with a fixed value.

For the second group of research, discrete event simulation (DES) models are restricted to some simplified models to some extent. These models often consider the process from the station(s)

(extraction or drawpoints, bin or chutes) to the destination that is conveyor entrance. The objective is mostly an evaluation of the system and measuring the performance rather than optimization of an objective function. The main advantages of DES are that it can generate a model with a different level of details and complexity considering the stochasticity for a dynamic system (Sharma, 2015). Solving and optimization of such models (systems) where the status of the system changes over time is a difficult task by using mathematical programming.

Underground mining consists of a wide range of extraction methods, each with their intrinsic complexity in their system due to the method of extraction. In some cases, such as the block caving method, the extraction point remains fixed until the end of the operation, but in some others such a cut-and-fill method the layout will not remain fixed and, consequently, the distance between the stations will change. This dynamical inheritance of mine operation could have a direct effect on the effectiveness of mining operation and haulage system performance.

Geological or geometallurgical uncertainty plays a significant role while the quantity and quality of materials are important. The existence of deleterious materials affects the production parameters. As a result, geological uncertainty should be incorporated into the models, considering that ore blending and ore inventory will take more significant place on the models to meet the specific characteristic of the materials that are required as a feed for mineral processing.

During the operation, there are several plans and operational actions that are commenced or run in parallel to the short-term plan which may link to short-term plan or affect the implementation of the plan, which results on increasing the Project OPEX (operating expenditure). The maintenance plan is an essential plan that is connected directly to operational activities. Part of the maintenance plan is a fix time inspection of the haulage, transportation or material handling. There are random failures in different parts of the system, which directly require attention to the maintenance section and their crews. The failure of the system in different parts could be evaluated with different parameters finding initial application in manufacturing and industrial engineering. The tools include process capability and performance indices, six-sigma, Taguchi's Quality Loss Function (QLF) and more (Taguchi et al., 2005; Kenzap and Kazakidis, 2013). Different kinds of risks could be quantified based on methods such as Internal Failure Cost (IFC) and External Failure Cost (EFC) processes (Kenzap and Kazakidis, 2013).

In addition to geological, layout change and haulage system uncertainties, the effect of external uncertainties on downstream could aggregate to the mine plan. Several failures in mill processing or simulation of material processing could incorporate with the mine output results. Supply and demand in mining chain is another parameter in downstream which may affect the production plan from small-scale range up to long range scale.

8. Conclusions

Short-term mine planning was reviewed for underground mining. The existing literature in the area of mine simulation illustrated that the underground mining area is an environment with several dynamic parameters. These dynamic parameters eventuate from a verity of sources into the model. These sources are random such as LHD failures, or scheduled such as the periodic inspection of the conveyor belt. The linkage between a low-resolution plan (long-term scheduling) to a very high-resolution plan (daily schedules) is an essential area of research that is mostly neglected in the literature. It can be seen that mathematical programming is not dynamic enough to deal with a dynamic system. Therefore, one solution could be the implementation of mathematical modelling in combination with simulation models to improve the ability of models for using them in the simulation model. Each model focuses on simulating a specific boundary of the system (mostly inside the mine). The more extended boundary for the mining system could be more comprehensive and include more aspects of the real operations chain. Mining operation is not only about discrete events, which means that a combination of discrete and continuous simulation can result in more realistic results.

9. Acknowledgements

The authors acknowledge the funding by the Chilean Commission for Scientific and Technological Research, through Project CONICYT PIA Anillo ACT1407, and also extend their thanks to the Mining Optimization Laboratory (MOL) and Mining and Rock Science Development and Innovation Lab (MRDIL), University of Alberta teams and researchers for providing the resources and guidelines to developing this research.

10. References

[1] Al-Aomar, R., Williams, E. J., and Ulgen, O. M. (2015). Process simulation using witness. John Wiley & Sons,

[2] Alford, C., Brazil, M., and Lee, D. H. (2007). Optimisation in underground mining. *in Handbook of operations research in natural resources*, Springer, pp. 561-577.

[3] Altun, A. O., Yilmaz, I., and Yildirim, M. (2010). A short review on the surficial impacts of underground mining. *Scientific research and essays*, 5 (21), 3206-3212.

[4] Anani, A., Awuah-Offei, K., and Hirschi, J. (2017). Application of discrete event simulation in optimising coal mine room-and-pillar panel width: a case study. *Mining Technology*, 126 (1), 1-9.

[5] AutoMod (2012). AutoModDatasheet.

[6] Bakhtavar, E., Shahriar, K., and Oraee, K. (2009). Transition from open-pit to underground as a new optimization challenge in mining engineering. *Journal of Mining Science*, 45 (5), 485.

[7] Banks, J. (1998). Handbook of simulation: principles, methodology, advances, applications, and practice. John Wiley & Sons.

[8] Bayer, A. and Nienhaus, K. (2000). Capacity model for room and pillar operations with continuous miners. University of Technology, Aachen.

[9] Bazaraa, M. S., Sherali, H. D., and Shetty, C. M. (2013). Nonlinear programming: theory and algorithms. John Wiley & Sons.

[10] Beaulieu, M. and Gamache, M. (2006). An enumeration algorithm for solving the fleet management problem in underground mines. *Computers & Operations Research*, 33 (6), 1606-1624.

[11] Beckett, L., Haycocks, C., and Lucas, J. (1979). LHDSIM—A Load-Haul-Dump Simulator for Room-and-Pillar Mining Operations. *in Proceedings of Proceedings 16th APCOM Symposium*, pp. 408-413.

[12] Bellman, R. (2013). Dynamic programming. Courier Corporation.

[13] Birge, J. R. and Louveaux, F. (2011). Introduction to stochastic programming. Springer Science & Business Media.

[14] Blom, M., Pearce, A. R., and Stuckey, P. J. (2018). Short-term planning for open-pit mines: a review. *International Journal of Mining, Reclamation and Environment*, 1-22.

[15] Bozorgebrahimi, E., Hall, R., and Blackwell, G. (2003). Sizing equipment for open-pit mining–a review of critical parameters. *Mining Technology*, 112 (3), 171-179.

[16] Brady, B. H. and Brown, E. T. (2013). Rock mechanics: for underground mining. Springer science & business media.

[17] Brown, E. T. (2002). Block caving geomechanics.

[18] Brown, E. T. (2012). Progress and challenges in some areas of deep mining. *Mining Technology*, 121 (4), 177-191.

[19] Camus, J. (1992). Open-pit optimization considering an underground alternative. *in Proceedings of Proceedings of 23th International APCOM Symposium*, pp. 435-441.

[20] Carlson (2018). Carlson Underground Mining — For Precise Design, Scheduling, and Mapping.

[21] Chanda, E. C. K. (1990). An application of integer programming and simulation to production planning for a stratiform ore body. *Mining Science and Technology*, 11 (2), 165-172.

[22] Crain, R. C. (1997). Simulation using GPSS/H. *in Proceedings of Proceedings of the 29th conference on Winter simulation*, IEEE Computer Society, pp. 567-573.

[23] Dantzig, G. B. and Thapa, M. N. (2006). Linear programming 1: introduction. Springer Science & Business Media.

[24] Darling, P. (2011). SME mining engineering handbook. Vol. 1, SME.

[25] Deb, K. (2014). Multi-objective optimization. *in Search methodologies*, Springer, pp. 403-449.

[26] Deswike (2018). Deswike.

[27] Diamond, W. P. and Schatzel, S. J. (1998). Measuring the gas content of coal: a review. *International Journal of Coal Geology*, 35 (1-4), 311-331.

[28] Fjellström, N. (2011). Simulation of an Underground Haulage System, Renström Mine, Boliden Mineral. Student thesis Thesis, Pages 75.

[29] Fuentes, S. (2004). Going to an underground (UG) mining method. *in Proceedings of Proceedings of MassMin Conference*, pp. 633-636.

[30] Fuentes, S. and Caceres, S. (2004). Block/panel caving pressing final open-pit limit. *CIM Bulletin*, 97 (1082), 1-3.

[31] Fukui, R., Kusaka, K., Nakao, M., Kodama, Y., Uetake, M., and Kawai, K. (2016). Production analysis of functionally distributed machines for underground mining. *International Journal of Mining Science and Technology*, 26 (3), 477-485.

[32] Gamache, M., Grimard, R., and Cohen, P. (2005). A shortest-path algorithm for solving the fleet management problem in underground mines. *European journal of operational research*, 166 (2), 497-506.

[33] Gendreau, M. and Potvin, J.-Y. (2010). Handbook of metaheuristics. Vol. 2, Springer.

[34] Glover, F. W. and Kochenberger, G. A. (2006). Handbook of metaheuristics. Vol. 57, Springer Science & Business Media.

[35] Greberg, J., Salama, A., Gustafson, A., and Skawina, B. (2016). Alternative Process Flow for Underground Mining Operations: Analysis of Conceptual Transport Methods Using Discrete Event Simulation. *Minerals*, 6 (3), 65.

[36] Hamrin, H. (1980). Guide to Underground Mining Methods and Applications. Atlas Copco.

[37] Hamrin, H., Hustrulid, W., and Bullock, R. (2001). Underground mining methods and applications. Underground mining methods: *Engineering fundamentals and international case studies*, 3-14.

[38] Harrell, C. R. and Price, R. N. (2003). Simulation modelling using promodel technology. *in Proceedings of Proceedings of the 35th conference on Winter simulation: driving innovation, Winter Simulation Conference*, pp. 175-181.

[39] Haviland, D. and Marshall, J. (2015). Fundamental behaviours of production traffic in underground mine haulage ramps. *International Journal of Mining Science and Technology*, 25 (1), 7-14.

[40] Hildreth, C. (1957). A quadratic programming procedure. *Naval Research Logistics* (NRL), 4 (1), 79-85.

[41] Hillier, F. (2014). Introduction to Operations Research.

[42] Hunt, C. (1994). Simulation model of ore transport at the Henderson mine. *Computers & Geosciences*, 20 (1), 75-84.

[43] Hustrulid, W. A., Hustrulid, W. A., and Bullock, R. C. (2001). Underground mining methods: Engineering fundamentals and international case studies. SME.

[44] Irem , O. P., Mizrak Ozfirat;M. Kemal, Ozfirat;. (2011). A Mathematical Modeling Approach for Scheduling Problem of Load-Haul-Dump Vehicles in Underground Mining. *in Proceedings of Northeast Decision Sciences Institute Meeting*, pp. 1136-1143.

[45] Kaiser, P. and Kim, B. (2009). Rock mechanics challenges in underground construction and mining.

[46] Kang, H. (2014). Support technologies for deep and complex roadways in underground coal mines: a review. *International Journal of Coal Science & Technology*, 1 (3), 261-277.

[47] Karami, A. and Szymanski, J. (1997). A simulation model of an underground mine haulage system. *Mineral Resources Engineering*, 6 (04), 229-238.

[48] Karlof, J. K. (2005). Integer programming: theory and practice. CRC Press.

[49] Kenzap, S. A. and Kazakidis, V. N. (2013). Operating risk assessment for underground metal mining systems: overview and discussion. *International Journal of Mining and Mineral Engineering*, 4 (3), 175-200.

[50] Khodayari, F. and Pourrahimian, Y. (2015). Mathematical programming applications in block-caving scheduling: a review of models and algorithms. *International Journal of Mining and Mineral Engineering*, 6 (3), 234-257.

[51] Lashkari, H. (2014). Duty Cycle Study of Coal Mine Shuttle Cars.

[52] Leite-Corthésy, R. (2016). Vérification d'un plan de production minier à court terme par simulation.Thesis, Ecole Polytechnique, Montreal (Canada).

[53] Leite-Corthésy, R., Gamache, M., and Baptiste, P. (2016). Validation D'un Plan De Prodution Minier À Cour Terme Par Simulation. *in 11th International Conference on Modeling, Optimization and Simulation - MOSIM'16*. Montréal, Québec, Canada.

[54] Litke, L., Laroche, D., Strong, D., and Szymanski, J. (1993). A computer simulation model of underground production mining: A tool for benchmarking. *Proc 24th APCOM*.

[55] Logie, C. V. and Matheson, G. M. (1982). A critical review of the current state-of-the-art design of mine pillars. *in Proceedings of Proceedings of the 1st International Conference on Stability in Underground Mining*, Vancouver, BC, Canada, pp. 16-18.

[56] Maldonado Meza, J. L. (2009). Modelo de Planificación Integrado Mina-Transporte-Planta División El Teniente Codelco Chile.

[57] MathWorks Inc. (2015). SimEvents Toolbox: User's Guide (R2015a).

[58] Mcnearny, R. L. and Nie, Z. (2000). Simulation of a conveyor belt network at an underground coal mine. *Mineral Resources Engineering*, 9 (03), 343-355.

[59] Micromine (2018). Pitram Fleet Management and Mine Control Solution.

[60] MineSched (2018). GEOVIA MineSched™ production scheduler - great results with proven technology.

[61] Moradi Afrapoli, A. and Askari-Nasab, H. (2017). Mining fleet management systems: a review of models and algorithms. *International Journal of Mining, Reclamation and Environment*, 1-19.

[62] Morrison, D. (2017). Meeting the productivity challenge below 2.5 kilometres. *in Proceedings of Proceedings of the First International Conference on Underground Mining Technology*, Australian Centre for Geomechanics, pp. 3-8.

[63] Muge, F. and Pereira, H. G. (1979). Short-term planning in sublevel stoping methods. *in Proceedings of 16 th International Symposium on the Application of Computers and OR in the Mineral Industry*. Ed TJ O'Neil, pp. 323.

[64] Musingwini, C. (2016). Optimization in underground mine planning-developments and opportunities. *Journal of the Southern African Institute of Mining and Metallurgy*, 116 (9), 809-820.

[65] Nehring, M., Topal, E., and Knights, P. (2010). Dynamic short-term production scheduling and machine allocation in underground mining using mathematical programming. *Mining Technology*, 119 (4), 212-220.

[66] Newman, A. M., Rubio, E., Caro, R., Weintraub, A., and Eurek, K. (2010). A review of operations research in mine planning. *Interfaces*, 40 (3), 222-245.

[67] Nilsson, D. (1992). Surface vs. underground methods. SME Mining Engineering Handbook.

[68] Oberholzer, J. and Hardman, D. (1987). A Review of Computer Simulation and Other Programs Developed and Used by the Coal Mining Laboratory, Chamber of Mines Research Organization. *in Proceedings of APCOM*, pp. 358-366.

[69] Osanloo, M., Gholamnejad, J., and Karimi, B. (2008). Long-term open-pit mine production planning: a review of models and algorithms. *International Journal of Mining, Reclamation and Environmen*t, 22 (1), 3-35.

[70] Park, S., Choi, Y., and Park, H.-s. (2016). Optimization of truck-loader haulage systems in an underground mine using simulation methods. *Geosystem Engineering*, 19 (5), 222-231.

[71] PCBC, (2018). GEOVIA PCBC Leading block cave planning and scheduling application.

[72] Pegden, C. D. (1984). Introduction to SIMAN. *in Proceedings of the 16th conference on Winter simulation*. Dallas, TX: IEEE Press, pp. 34-41.

[73] Peng, S. and Vayenas, N. (2014). Maintainability analysis of underground mining equipment using genetic algorithms: Case studies with an LHD vehicle. *Journal of Mining*.

[74] Peng, S. S. and Tang, D. H. Y. (1984). Roof bolting in underground mining: a state-of-the-art review. *International Journal of Mining Engineering*, 2 (1), 1-42.

[75] Pereira, S. P., Costa, J. F. C. L., Salvadoretti, P., and Koppe, J. C. (2010). Simulação de produção em mina subterrânea de carvão com uso de conjuntos mecanizados. Rem: Revista Escola de Minas, 63 581-589.

[76] Pop-Andonov, G., Mirakovski, D., and Despodov, Z. (2012). Simulation Modeling and Analyzing in Underground Haulage Sistems with Arena Simulation Software. *International Journal for Science, Technics and Innovations for the Industry MTM (Machines, Technologies, Materials)*, 48-50.

[77] Promine (2016). Promine mining and geology software.

[78] Puhakka, T. k., V (2000). Use of OPTIMINETM Simulation Tool for Mobile Fleet Selection. *in MassMin 2000*. brisbane,Qld, pp. 145-148.

[79] Ralston, J., Reid, D., Hargrave, C., and Hainsworth, D. (2014). Sensing for advancing mining automation capability: A review of underground automation technology development. *International Journal of Mining Science and Technology*, 24 (3), 305-310.

[80] Reilly, J. J. O. (1991). SLAM II tutorial (simulation language). *in Proceedings of 1991 Winter Simulation Conference*., pp. 112-117.

[81] Roberts, B. (2002). Computer simulation of underground truck haulage operations. *Mining Technolog*y, 111 (2), 123-128.

[82] Robinson, S. (2004). Simulation: the practice of model development and use. Wiley Chichester.

[83] Rossetti, M. D. (2015). Simulation modelling and Arena. John Wiley & Sons.

[84] Rubio, E. and Troncoso, S. (2008). Discrete events simulation to integrate operational interruption events in block cave production scheduling. *in Proceedings of Proceedings of the 3rd International Conference on Mining Innovation.*

[85] Runciman, N., Vagenas, N., and Corkal, T. (1997). Simulation of Haulage Truck Loading Techniques in an Underground Mine Using WITNESS. *SIMULATION*, 68 (5), 291-299.

[86] Runciman, N. A. (1999). Evaluation of underground mining equipment systems using discrete-event simulation with animation. Laurentian University, Sudbury, Canada.

[87] Saayman, F. (2006). Optimization of an autonomous vehicle dispatch system in an underground mine. *Journal of the Southern African Institute of Mining and Metallurgy*, 106 (2), 77-86.

[88] Schnakenberg, G. H. and Bugarski, A. D. (2002). Review of technology available to the underground mining industry for control of diesel emissions.

[89] Schriber, T. J. (1974). Simulation using GPSS. MICHIGAN UNIV ANN ARBOR.

[90] Sepulveda, J. M., Córdova, F., Quezada, L., Olivares, V., Hernandez, L., and Atero, L. (2005). A constraint-programming model for scheduling vehicles in underground mining operations. *in Proceedings of 18th International Conference on Production Research*.

[91] Sharma, P. (2015). Discrete-event simulation. *International journal of scientific & technology research*, 4 (4), 136-140.

[92] Shelswell, K. and Labrecque, P. (2014). Discrete Simulations Quantifying the Effects of Material Handling Conveyors in Series or Parallel Ore flow Streams. *in Proceedings of Society for Mining, Metallurgy and Exploration Annual Meeting*, Salt Lake City, Utah, USA, pp. 7.

[93] Shelswell, K. J. and Labrecque, P. O. (2017). Discrete simulations modelling the impact of operator numbers and truck availability on haulage fleet productivity. *in Proceedings of the First International Conference on Underground Mining Technology*, M. Hudyma and Y. Potvin, Eds. Sudbury: Australian Centre for Geomechanics, pp. 677-688.

[94] SimMine (2018). SimMine® Development Package.

[95] Sjödin, E. (2015). Evaluation of Mining Automation with Production Simulation.

[96] Skawina, B., Salama, A., Greberg, J., and Schunnesson, H. (2015). Production rate comparison using different Load-Haul-Dump fleet configurations: Case study from Kiirunavaara Mine. *in Proceedings of Mine Planning and Equipment Selection*.

[97] Song, Z., Rinne, M., and Van Wageningen, A. (2013). A review of real-time optimization in underground mining production. *Journal of the Southern African Institute of Mining and Metallurgy*, 113 (12), 889-897.

[98] Steiker, A. (1982). Simulation of an underground haulage system. *Application of Computers and Operations Research in the Mineral Industry (APCOM)*, Denver.

[99] Taguchi, G., Chowdhury, S., and Wu, Y. (2005). Taguchi's quality engineering handbook. Vol. 1736, Wiley Online Library.

[100] TALPAC (2017). Underground Coal TALPAC v3.0 c.

[101] Thrybom, L., Neander, J., Hansen, E., and Landernas, K. (2015). Future Challenges of Positioning in Underground Mines. IFAC-PapersOnLine, 48 (10), 222-226.

[102] Topal, E. and Ramazan, S. (2010). A new MIP model for mine equipment scheduling by minimizing maintenance cost. *European Journal of Operational Research*, 207 (2), 1065-1071.

[103] Touwen, F. and Joughin, N. (1972). The simulation of underground stoping and transport operations in gold mining. Application of computer methods in the mineral industry: *proceedings of the tenth international symposium*. Johannesburg–South Africa, 10-14.

[104] Usmani, T., Szymanski, J., and Apel, D. (2014). Optimisation of LHDs operation at the extraction level in a block cave project. *International Journal of Mining and Mineral Engineering*, 5 (2), 138-151.

[105] Vagenas, N. (1991). Dispatch control of a fleet of remote-controlled/automatic load-haul-dump vehicles in underground mines. *International Journal of Production Research*, 29 (11), 2347-2363.

[106] Vagenas, N., Scoble, M., and Baiden, G. (1997). A review of the first 25 years of mobile machine automation in underground hard rock mines. *CIM Bulletin*, 90 (1006), 57-62.

[107] Vargas Pincheira, F. J. (2011). Desarrollo de un Modelo de Programación Matemática para la Planificación de la Extracción Minera de Corto Plazo en una Empresa de la Mediana Minería.

[108] Vasquez, O. C., Sepúlveda, J. M., and Córdova, F. (2011). Modeling and optimization of vehicle operations in underground copper mining. Rem: Revista Escola de Minas, 64 365-371.

[109] Webber, R., Franz, R., Marx, W., and Schutte, P. (2003). A review of local and international heat stress indices, standards and limits with reference to ultra-deep mining. *Journal-South African Institute of Mining and Metallurg*y, 103 (5), 313-324.

[110] Williams, H. P. (2013). Model Building in Mathematical Programming. Wiley.

[111] Xiao-ping, B., Yu-hong, Z., and Ya-nan, L. (2015). A novel approach to study real-time dynamic optimization analysis and simulation of complex mine logistics transportation hybrid system with belt and surge links. Discrete Dynamics in Nature and Society.

[112] Yang, X. S. (2010). Engineering optimization. Wiley Online Library.

[113] Yun, Q. and Yegulalp, T. (1983). Optimization of underground transportation systems using a network flows model. *International Journal of Mining Engineering*, 1 (3), 267-275.

Effects of Fragmentation Size Distribution on Truck-Shovel Productivity

Magreth S. Dotto and Yashar Pourrahimian
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

Fragmentation size distribution affects shovel productivity through digability and fill factor and truck productivity through payload, loading time and waiting time at the loader. The reviewed models indicated that shovel productivity depends on P_{80} (fragment size at which 80 percent of material passes) and uniformity index. Suggested models are based on post-blasting assessment and since they were prepared for fragmentation distribution, they do not consider all factors influencing productivity such as material looseness, swell and equipment parameters. Since productivity is affected by several factors other than fragmentation distribution, its prediction (establishment) requires more involving models.

Truck-shovel productivity just like other downstream operations (crushing and grinding) are affected by fragmentation distribution which is a product of blasting. A collective approach to increase productivity and lower operations cost should focus on increasing blasting effectiveness. Blasting should be designed to provide a fragmentation size distribution that ensures the efficiency of downstream processes. This can be done by manipulating blasting set up if the target fragmentation is known.

1. Introduction

Fragmentation distribution is an important parameter in productivity of the mine. Blasting is the first stage of fragmentation; its efficiency affects the downstream activities such as loading, crushing, milling and refining. Coarser material lead to higher energy consumption, increased wear rates as well as lowering load and haul productivity and crushing and milling throughputs. Different researches have been conducted on the effects of fragment sizes on downstream operations; this paper is going to review approaches and models established from the studies of fragmentation size distribution effects on truck-shovel productivity and identify research gap. The review aims at improving prediction models to improve the productivity of the mines and minimize operations costs.

The main objective of drilling and blasting in mining is to generate a muck pile with a suitable size distribution of rock fragments that can be handled efficiently on loading, transporting, crushing and milling. Initially, optimum drill and blast designs considered getting the rock to a manageable size for trucks and shovels and minimize both drilling and explosives costs (Singh, R., 2006). With 'Mine to Mill' concept, the focus has shifted whereby blasting practices are modified to achieve energy optimization in all downstream activities. Blasting is now designed to increase load and haul productivity and ensure mill feed size which saves up on energy and increase throughput (McKee, 2013).

Jethro (2016) determined the effects of fragmentation on loading using the Kuz-Ram Model and Split Desktop Software; he observed that with every increase of 10 cm in fragmentation size, loading time

increases by 2 seconds. Singh (2006) conducted a study to determine factors affecting the productivity of loading equipment in mines; the conclusion was, bucket fill factor and rate of production decrease with increasing mean particle size and index of uniformity. Tosun, Konak, Karakus, Onur, and Ongen (2012) observed that the increase in pile density (good fragmentation) increases loading efficiency. Doktan (2001) observed increased productivity of 22% for truck shovel system with better fragmentation.

This paper is organized in two sections which are literature review and discussion. Literature review is divided into three parts. The first part defines fragmentation size distribution and different ways used to assess and quantify fragmentation distribution. Effects of fragmentation size on truck-shovel productivity are briefly described. The second part describes characteristics of rock fragments at different size distributions, and through these characteristics, the effects of size distribution on productivity is explained. The third part reviews models describing effects of fragmentation size distribution to truck-shovel productivity. Discussion section, analyses findings from the review and propose future areas of research on fragmentation size aiming at maximizing productivity and minimize overall mine operations costs.

2. Literature review

Truck shovel productivity can be affected by several factors including machine design which define how efficiently the energy is used and mine plan and design relating to dig methods. Others include shovel-truck matching, operator's skills and practices and operating conditions; rock fragmentation and material digability (Awuah-Offei, 2017). On the other hand, truck productivity depends on the capacity of shovel, operator's skills, material characteristics (swell factor, density, fragmentation), payload and fill factor, truck size, engine power, drive system, fleet management efficiency and haul roads conditions. The capacity of the truck should not exceed 6 to 8 times shovel bucket capacity and shovel should have appropriate height and reach.

Apart from the load and haul productivity, fragmentation size distribution affects energy consumption and throughput in crushing and milling. According to Napier-Munn (2015), most of the energy consumer in the base and precious metal mines is attributed to size reduction, and it is used inefficiently. Blasting is cheaper and more energy efficient as compared to milling. Efficient blasting can save up on energy and increase plant throughput.

2.1. Review on fragment size distribution

Fragmentation size distribution can be computed in two ways; using Rosin Rammler equation (Vesilind, 1980) and cumulative size distribution curve which is plotted with size (X) on x-axis and percentage (P or R_x) in y-axis. The distribution R_x represents the weight of material that is smaller than the given particular size. R_x varies between 0 to 100% or 0 to 1 as shown in **Error! Reference source not found.**. Two important parameters that define the characteristics of the distribution are mean particle size, X_{50} and uniformity index, n. From Rosin Rammler equation they are represented as follows:

$$R_x = 1 - \exp\left[-0.693\left(\frac{X}{X_{50}}\right)^n\right] \quad (1)$$

Where R_x is mass fraction passing on the screen of size X, X_{50} is mean particle size, and n is uniformity index usually between 0.5 and 2.

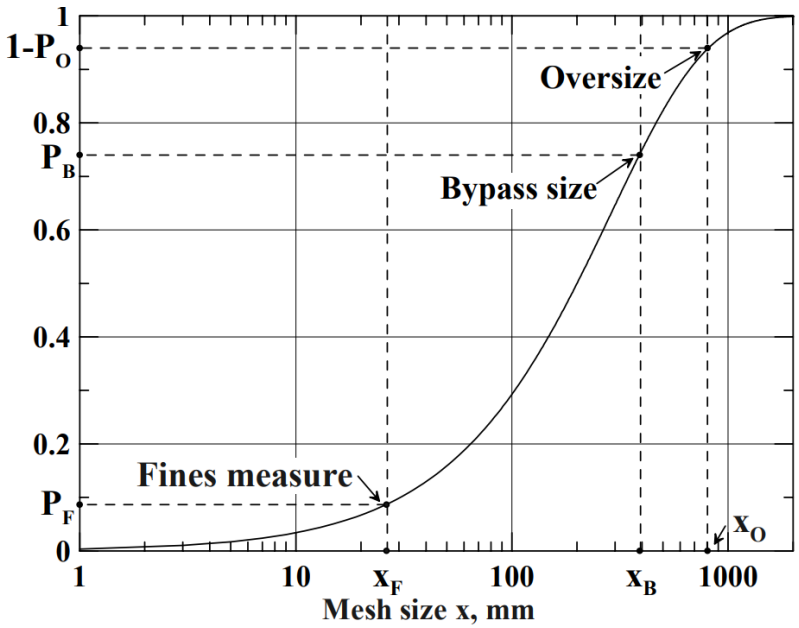


Fig. 1: Fragment size distribution (Ouchterlony, 2003)

Description of quantities on distribution curves can be made referring measure of average fragmentation X_{50}, percentage of fragments larger than a certain size P_o, percentage of fragments smaller than a certain size P_F or any other percentage, P_y.

Shovel productivity is influenced by uniformity index and fragmentation size (Brunton, Thornton, Hodson, & Sprott, 2003; Cottee, 2001). Uniformity index has a great influence in fill factor; a decrease from 1.5 to 0.5 uniformity index, increases fill factor by 15% (Cottee, 2001). The slope of cumulative distribution relates to uniformity index; as the curve gets steeper, uniformity index increases and shovel productivity decreases due to low fill factor as described in **Error! Reference source not found.**. The bottom row in **Error! Reference source not found.** (left) has low uniformity index, where, due to the presence of different sizes, the particles fit together better and improves bucket fill factor.

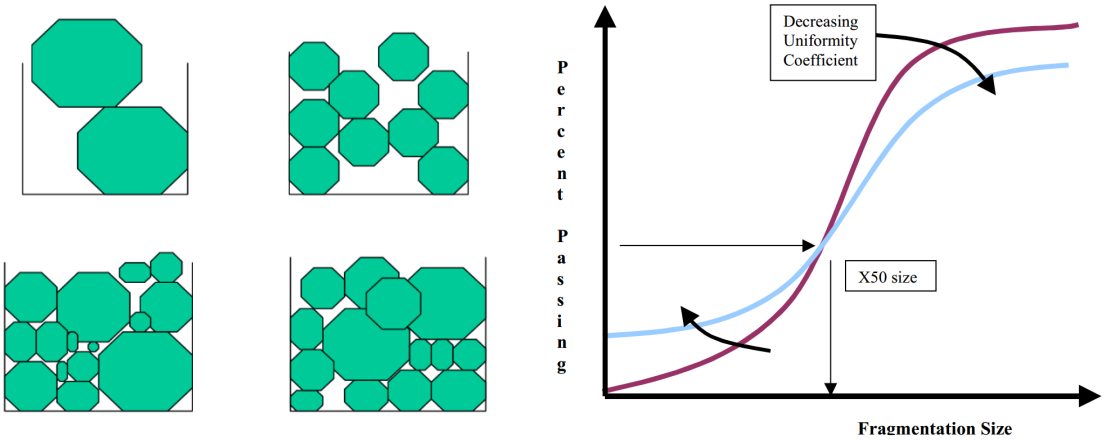


Fig. 2: Diagram and graph showing effect of n in fill factor (Cottee, 2001).

Productivity of shovel truck system is efficient on size range1/6 to 1/8 of shovel bucket capacity (Jimeno, Jimeno, & Carcedo, 1995). In mining, oversize materials are termed depending on loading equipment or crusher capacity. Jimeno, et al. (1995) determined that oversize material are the ones larger than 0.7 times

the smallest dimension of loading bucket. According to Kanchibotla, Valery, and Morrell (1999) loading and hauling operations are affected by fragmentation bigger than 250mm.

Factors affecting fragmentation distribution and properties of blasted muck pile are categorized in two groups; the first group is controllable parameters; such as blasting design parameters and explosive related parameters and second group are uncontrollable parameters, containing physical and geomechanical properties of intact rock and rock mass (Kulatilake, Qiong, Hudaverdi, & Kuzu, 2010). There are several models which have been established to predict blasting fragment size distribution. The most popular model is Kuz-Ram developed by C. V. B. Cunningham (1987).

$$X_n = AK^{-0.8}Q^{\frac{1}{6}} \times \left(\frac{115}{RWS}\right)^{\frac{19}{20}} \quad (2)$$

Where X_n is the mean particle size (cm), A is rock factor (varying from 0.8 to 22 depending on hardness and structure), K is powder factor (kg/m^3), Q is mass of explosive per hole (kg) and RWS is the weight strength relative to ANFO; 115 is RWS of TNT.

Two common methods are used to assess fragmentation size distribution in mining; sieving and photographic techniques. The challenge has been obtaining a representative sample for the whole blasted muck pile. Sieving is more accurate but it is costly and impractical for routine implementation in surface mines due to high volumes involved (Beyglou, 2016). Split desktop image processing is one commonly used programs in computing fragmentation distribution. The downside of image analysis systems is, they cannot resolve particles smaller than the size determined by pixel resolution and obtaining a representative sample. (Brunton, et al., 2003)

2.2. Characteristics of rock fragments at different size distributions

Shovel digability depends on fragmentation size and muck pile swell and looseness. Presence of boulders in muck pile necessitates more time to negotiate the boulders and hence increase loading time. Fill factor and payloads are affected by fragmentation size, distribution of size ranges and material angle of repose.

Void ratio is defined as the ratio of the void volume to the total volume of bulk material including the voids. Void ratio is directly related to the fragmentation level, and it is an indication of how the available room in a bucket have been used (Doktan, 2001). Void ratio determines bulk density; increase in fines lowers void ratio, increases bulk density and therefore improve bucket load and payload.

The angle of repose is the angle at which blasted rock fragments can remain stable. The angle of repose increases with the increase in mean particle size of a muck pile. Looseness and angle of repose affects the bucket fill factor and digging cycle time of the loader and therefore the productivity (Singh, 2006).

Fines result from over crushing of the rock during blasting. Fine materials act as a lubricant between coarser materials in a muck pile, they improve bucket penetration and reduce loading time. From this discussion, a reasonable percentage of fines reduces void ratio, increase fill factor and eventually payload. On the other hand, a higher percentage of fines lowers the angle of repose and cause the material to flow out of the bucket without heaping. Poor fragmentation results into production of boulders that can be too large to be handled by truck and shovel or affects loading operations by reducing productivity and increase the overall operations costs.

2.3. Impacts of fragmentation size distribution on productivity

The most common relationship used to calculate shovel productivity relates production rate to bucket capacity, material characteristics and operations efficiency.

$$P_R = \frac{B_c \times O_e \times B_f \times S_f}{C_t} \tag{3}$$

Where P_R is shovel productivity, B_c is bucket capacity, B_f is bucket fill factor, S_f is material swell factor O_e is the percentage of time that a machine actually operates and C_t is the cycle time. Fragmentation size affects the fill factor and cycle time, but it is not the only factor. From this expression, the effect of fragmentation size distribution on shovel productivity is not studied.

Beyglou et al. (2017) conducted a study to determine fragmentation for efficient loading and crushing. They came out with three observations; first, ore type does not have an impact on loading productivity. Second, fill factor is affected by the percentage of fines in the distribution; very fine fragmentation has lower fill factor compared to medium fine fragmentation and third, fill factor decreases with the increase in fragmentation size. These relationships were presented, but they did not establish any expression which relates them to shovel productivity.

Cottee (2001) conducted a study to determine the impact of fragmentation on truck and loader productivity. He determined that blast design like holes spacing and burden and explosive properties such as explosive strength and detonator timing affect fragmentation and muck pile shape. Fragmentation size affects loader digability and fill factor. Trucks productivity is affected by fragmentation through fill factor, payload and cycle time resulting from loading time and waiting time at the loader. He did not establish expression for truck-shovel productivity in his work.

Sanchidrián and Ouchterlony (2017) established a formula to predict productivity of excavators from rock mined by drilling and blasting in iron and copper open pit mines. Data from 20 blasts with rock ranging from medium to very high strength were used with three blasting agents; ANFO, water-gel and emulsion blends. Powder factor used ranged from 0.88 to 1.45kg/m^3. The formula developed was;

$$Q = Q_0 e^{-k \times \frac{f_s}{B_p}} \left[\frac{\sigma^2}{\left(E - E^0\right)^2 + \sigma^2} \right] \tag{4}$$

Where; Q_0 is the maximum excavator productivity, $e^{-k \times \frac{f_s}{B_p}}$ accounts for the effect of rock characteristics and dipper capacity where, k is a coefficient, B_p is bucket nominal payload, and f_s is rock strength factor. $\left[\sigma^2 / \left(\left(E - E^0\right)^2 + \sigma^2 \right) \right]$ is blast factor including the influence of blast design such as rock properties, fragmentation and resultant heave. E_0 is energy powder factor at which excavator efficiency is maximum (E_0 depends on rock properties and loader type), E is the energy powder factor used, and σ is a scale factor.

They concluded that the productivity of shovel is an indicator of blast performance. As energy powder factor increases also productivity increases until it reaches critical factor E_0 where increase provides no further improvements. The shortcoming in this model is that the detonation delay between rows which influences muck pile property was not considered in the model.

Osanloo and Hekmat (2005) conducted a study at Gol-e-Gohar mine to predict shovel productivity. Fragmentation in Gol-e-Gohar mine is mainly affected by water table, many discontinuities and improper use of explosives. They established that the main factor affecting shovel productivity is bucket fill factor which is influenced operator skills and fragmentation size; other factors are swing period and angle, cycle time and job efficiency. Fragmentation composition has an effect on bucket fill factor, swell factor, job efficiency and rock density. They developed a model for shovel productivity (SP) as:

$$SP = 1769 - 9.63d_{50} + 444.45n - 3.37nd_{80} \quad (5)$$

Where, d_{50} is the average particle size, n is the uniformity coefficient, and d_{80} represent 80% weight of material less than a certain size. They concluded that the larger the particles, the lower is shovel productivity. From this relationship, they established d_{80} that ensure higher shovel productivity is between 20 and 40 cm.

Although several factors are indicated to affect shovel productivity such as operator's efficiency, blasting effectiveness, swing angles and others, they are not included in the model. The model assesses the effects of fragmentation after blasting; it does not consider the effects of blast design and explosive properties on resulting fragmentation.

Brunton et al. (2003) conducted a study to determine the effect of fragmentation on hydraulic excavator dig time. They studied the effect of several fragmentation parameters on dig time such as P_{20}, P_{50}, and P_{80}, uniformity index, top size and percentage of material passing 250 mm. They determined that dig time responds more to P_{80}; as fragmentation becomes coarser, dig time increases. Fragmentation size and number of passes affect loading time. Lowering P_{80} from 600mm to 200mm resulted into 26% improvement on dig time and therefore increased productivity.

Limitations of this study is, it did not include the effects of other muck pile parameters such as muck pile looseness and material ability to flow, fill factor and operators digging strategy. The study is a post-blasting assessment.

Doktan (2001) studied the impact of fragmentation size on truck shovel fleet performance. He determined that, fragmentation impacts load and haul on digability (dig time) and bucket payload (void ratio and fill factor). He established the relationship of dig times with Rosin-Rammler parameters X_{50} and uniformity index as;

$$Dig\ time = a - b \times X_{50} \times n \quad (6)$$

Where, a and b are constant equal to 8.9942 and $-6.8706e^{-2}$ respectively, X_{50} is mean fragmentation and n is uniformity index. Better fragmentation resulted into reduced dig times, improved shovel productivity and increase truck payload.

Similar to previous models, Doktan did not include the effects of other muck pile properties and operators digging strategies in estimating digging time. The model deals with post-blast assessment; it does not consider the effects of blasting to resulting fragmentation.

3. Discussion

Shovel productivity is influenced by number of bucket loads, fill factor and cycle time. From reviews, it is clear that these are affected by fragmentation distribution. Productivity declines with an increase in particle

size at which average dig times increase and fill factor decreases. Similarly, dig times and fill factor are affected by material looseness, operator's skills, loading strategies as well as bucket capacity although in the discussed models they were not included. Since shovel productivity is affected by many other factors than fragmentation distribution, it requires more involving models to establish.

Although it is clear that fragmentation affects productivity, it is not clear how and to what extent and therefore difficult to determine the target for drill and blast operations (Beyglou, 2016). So far trial and error and empirical experimentation are used to describe the influence of fragmentation in shovel productivity similar to the ones discussed. Further studies on the effects of fragmentation on productivity is required to develop a predicting model that can be modified depending on site specifics.

In estimating payload, the interaction between different fragmentation sizes and its effect on void ratio are not discussed. There are models developed to predict void ratio and payloads from mixed fragmentation sizes as the one discussed by (Doktan, 2001) considering relative quantities of fragmentation size, effective size and specific packing density of each fragment. If they are applied, they will offer a better prediction of payloads.

The developed models are focused on post-blasting assessment, (Brunton, et al., 2003; Osanloo & Hekmat, 2005). The collective effort towards improving shovel productivity and other downstream operations should focus on improving blasting effectiveness. Studies conducted by (Cunningham, 2005; Singh, et al., 2016; Singh and Narendrula, 2010) which describe relationships of blasting parameters with uniformity index and fragmentation size distribution and can be used to design blast for targeted results.

4. Conclusion

Fragmentation size distribution affects shovel production on digability and fill factor. Digability impacts loading time and energy used in loading. Truck productivity is also affected on payload, loading time and waiting time at the loader.

There are several models which have been developed to define the relationship between fragmentation sizes and shovel productivity; most of them are developed from post blasting assessment and exclude significant factors affecting productivity. Establishment of more inclusive models requires a further study into the factors affecting productivity and how they influence each other.

A more collective study should include blast effectiveness because fragmentation distribution is the product of blasting. Blasting should be able to provide fragmentation size distribution which ensures efficiency of downstream processes. This can be done by manipulating blasting set up if the target fragmentation distribution is known.

5. References

[1] Awuah-Offei, Kwame. (2017). *Energy Efficiency in the Minerals Industry Best Practices and Research Directions* (Vol. 1): Springer International Publishing.

[2] Beyglou, A., Johansson, D., & Schunnesson, H. (2017). Target fragmentation for efficient loading and crushing - the Aitik case. *Journal of the Southern African Institute of Mining and Metallurgy, 117*, 1053-1062.

[3] Beyglou, Ali. (2016). *On the Operational Efficiency in Open Pit Mines.* Luleå University of Technology Luleå, Sweden.

[4] Brunton, I., Thornton, D., Hodson, R., & Sprott, D. (2003). *Impact of blast fragmentation on hydraulic excavator dig time*. Paper presented at the 5th Large Open Pit Conference.

[5] Cottee, Stuart. (2001). *Impact of fragmentation on truck and loader productivity.* Unpublished B.Sc Thesis, The University of Queensland.

[6] Cunningham, C. (2005). *The kuz-ram fragmentation model - 20 years on.* Paper presented at the Brighton Conference Proceedings.

[7] Cunningham, C.V.B (1987). *Fragmentation estimations and the Kuz-Ram model - four years on.* Paper presented at the Proceedings of Second International Symposium on Rock Fragmentation by Blasting, Keystone, Colorado.

[8] Doktan, M. (2001). *Impact of Blast Fragmentation on Truck Shovel Fleet Performance*. Paper presented at the 17th International Mining Congress and Exhibition of Turkey - IMCET2001.

[9] Jethro, M.A, Shehu, S.A and Kayode, T.S. (2016). Effect of Fragmentation on Loading at Obajana Cement Company Plc, Nigeria. *International Journal of Scientific & Engineering Research, 7*(4), 608-620.

[10] Jimeno, C., Jimeno, E., & Carcedo, F. (1995). Drilling and Blasting of Rocks.

[11] Kanchibotla, SS, Valery, W, & Morrell, S (1999). *Modelling fines in blast fragmentation and its impact on crushing and grinding.* Paper presented at the Proceedings Explo-99 Conference, Kalgoorlie.

[12] Kulatilake, P. H. S. W., Qiong, Wu, Hudaverdi, T., & Kuzu, C. (2010). Mean particle size prediction in rock blast fragmentation using neural networks. *Engineering Geology, 114*(3), 298-311.

[13] McKee, D. (2013). *Understanding mine to mill* Brisbane: St Lucia, Qld. The Cooperative Research Centre for Optimising Resource Extraction.

[14] Napier-Munn, Tim. (2015). Is progress in energy-efficient comminution doomed? *Minerals Engineering, 73*, 1-6.

[15] Osanloo, M., & Hekmat, A. (2005). Prediction of Shovel Productivity in the Gol-e-Gohar Iron Mine. *Journal of Mining Science, 41*(2), 177-184.

[16] Sanchidrián, José A., & Ouchterlony, Finn. (2017). A Distribution-Free Description of Fragmentation by Blasting Based on Dimensional Analysis. *Rock Mechanics and Rock Engineering, 50*(4), 781-806.

[17] Singh, P. K., Roy, M. P., Paswan, R. K., Sarim, Md, Kumar, Suraj, & Ranjan Jha, Rakesh. (2016). Rock fragmentation control in opencast blasting. *Journal of Rock Mechanics and Geotechnical Engineering, 8*(2), 225-237.

[18] Singh, S. P, & Narendrula, R. (2010). Causes implication and control of oversize during blasting. *Taylor and Francis Group, London*.

[19] Singh, S.P., Narendrula, R., (2006). Factors Affecting the Productivity of Loaders in Surface Mines. *International Journal of Mining, Reclamation and Environment, 20*(1), 20-32.

[20] Tosun, Abdurrahman, Konak, G., Karakus, Dogan, Onur, Ahmet, & Ongen, Tugce. (2012). *Investigation of the relationship between blasting pile density and loader productivity*.

[21] Vesilind, P. Aarne. (1980). The Rosin-Rammler particle size distribution. *Resource Recovery and Conservation, 5*(3), 275-277.

Emergent Properties of MoFrac-Generated Discrete Fracture Networks

William Junkin and Eugene Ben-Awuah
Mining Optimization Laboratory (MOL)
Laurentian University, Sudbury, Canada

ABSTRACT

Discrete fracture networks allow for real structural features to be represented when considering a specific volume of rock. Through the construction of valid fracture network models, emergent properties can be identified that can be useful for engineering applications. MoFrac discrete fracture network modelling software can be used to develop realistic models of fracture networks in selected rock masses that are to be blasted. By including statistically derived stochastic fractures into fracture network models, where mapping has not occurred, a complete representation of the structural discontinuities in a specific volume of rock can be achieved. Emergent properties of fracture networks that are useful for characterization have been identified. These properties include in situ block size distributions and heterogeneity of the rock mass character. A method to determine these properties is presented and these characteristics are considered for two DFN models. These models are generated and analyzed using the MoFrac code. For future research, a fragmentation model is proposed that incorporates the identified emergent properties of a discrete fracture network model for blast optimization by means of a rock engineering systems solution.

1. Introduction

A rockmass consists of the rock matrix and the network of discontinuities within it. Rockmass characterization is dependent on both of these qualities. Discrete fracture network (DFN) modelling allows for the discontinuities in a rockmass to be mapped and analyzed. The modelling of discontinuities is independent of the rock matrix characteristics and can be considered separately.

DFN models are generated using software that is designed to create statistically viable fracture networks based on fracture size and orientation distributions. A *deterministic* DFN model is based on mapped data, fractures are generated in the model that honour mapped traces and facets of known fractures. A *stochastic* DFN model is based on probability distributions with fractures seeded randomly within a volume. A hybrid DFN model incorporates both deterministic and stochastic fractures (Lei *et al.*, 2017). A deterministic DFN model is useful when considering individual discontinuities and rockmass failure at known locations whereas a stochastic DFN model is more useful when considering the model as a whole as any particular fracture is likely not located where a discontinuity actually exists. A hybrid DFN model allows for known data to be included in a more complete representation of a rockmass that includes fractures seeded in areas where mapping could not take place. By having stochastic fractures defined using the size and orientation distributions expected to be encountered allows for constraints to be imposed that agree with known data.

A realistic representation of a fracture network is useful for several engineering applications. Hydrogeological modelling, rockmass failure studies, ore reserve analysis and blast optimization are areas of study that are enhanced with knowledge of existing fracture networks.

2. DFN Modelling

The representation of fracture networks in three dimensions is of great interest to engineers but difficult to achieve due to the limitations of mapping. True three dimensional mapping is a destructive process and thus not applicable to most projects. Hybrid DFN models containing both fractures seeded from deterministic traces as well as stochastically derived fractures are useful when considering the DFN model as a whole (Dershowitz and Einstein, 1988). By including deterministically seeded fractures the disaggregate approach of analysis becomes more applicable and thus heterogeneity in a DFN can have meaning other than as derived from the statistical rules used to generate the DFN model.

All DFN models for this study were generated using the Beta v. 0.8 build of MoFrac (MIRARCO, 2018). MoFrac is a DFN modelling tool that generates fractures deterministically from mapped data or stochastically as sampled from size and orientation distributions. Fractures are optionally generated with undulation through tessellation of the fracture mesh to yield a realistic looking DFN model. Priority is given to deterministic fractures as they come first in the modelling process. The software has been verified through validation studies modelling the fracture networks for a tunnel at the SKB's Äspö Hard Rock Laboratory and for the Canadian Shield Dataset further described in Section 2.1 (Junkin, *et al.*, 2017; 2018). These studies validated MoFrac-generated DFN models by considering orientation, intensity and trace maps as Miyoshi *et al.* (2018) describes as necessary for validation.

The goal of this study is to consider a method of measuring the heterogeneity of DFN attributes such as fracture intensity. A voxelization of DFN models allows for a spatial analysis at multiple scales. The subdivision of a DFN into component sections allows for the variability in an attribute such as fracture intensity to be measured. The P_{32} intensity of DFN models will be considered on a deterministic and a stochastic DFN model with subdivisions of 1 (1^3), 8 (2^3), 125 (5^3), 1 000 (10^3), 3375 (15^3) and 15625 (25^3) voxels. Fig. 1 shows an example of voxelization of a simple cube into 1000 component voxels (10^3). P_{32} intensity is a measure of the sum of the surface areas of fractures within a given volume divided by that volume, calculated as shown in Eq. (1).

$$P_{32} = \frac{\sum_{i}^{n} A}{V} \tag{1}$$

The benefit of using P_{32} intensities to characterize a DFN is that this measure is scale independent and there is no bias related to the direction of measurement as occurs with P_{21} values (Dershowitz and Herda, 1992; Alghalandis and Elmo, 2018). By considering fracture intensity by layer and subsections of a DFN model a new method of visualizing DFN models is proposed that honours the location of discrete elements but does not require the elements to be explicitly contained within the model.

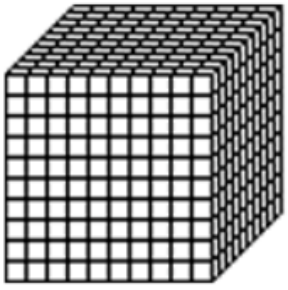


Fig. 1. Example of 10 x 10 x 10 voxelization of a simple cube.

2.1. Canadian Shield dataset

The Canadian Shield dataset developed by R. Mohan Srivastava was used as the input for a deterministic model to be generated. The Canadian Shield dataset was developed in 2002 for the Third Case Study of the Deep Geological Repository technical program of Ontario Power Generation (Gierszewski *et al.*, 2004). The dataset incorporates discontinuities mapped from an aerial photograph of a typical Canadian Shield setting with simulated fracture traces added to have the fracture intensity match expected values for the Canadian Shield (Srivastava, 2002). The dataset covers 200 km^2 of surface area as shown in Fig. 2 and for the purpose of this study a DFN model with a depth of 200 m was used. The total volume of the DFN model being roughly 40 km^3.

This dataset has been used for validation studies on the MoFrac code and the orientations for fracture sets from these studies have been used again here (Junkin *et al.*, 2018). Two major fracture sets are identified with strike angles of 298° and 240° and a third random group identified with strike angle of 8°. These orientations are used with a standard deviation of 10° for the sampling of strike values for the fractures modelled. Dip angle is controlled through a dip undulation scheme with fractures set to dip vertically +/- 10°.



Fig. 2. Integrated Canadian Shield dataset shown with surficial water features as seen in original aerial photograph (Srivastava, 2002).

2.2. Stochastic DFN models

A stochastic DFN model was also designed in order to consider the proposed characterization technique. This model was designed with the eventual goal of blast optimization. The model was set up with 25 rectangular regions in a 5 x 5 pattern as viewed from surface. These rectangular columns are 10 x 10 x 50 units, in combination they form a 50 x 50 x 50 cube. Fig. 3 shows the defined regions in the stochastic DFN model.

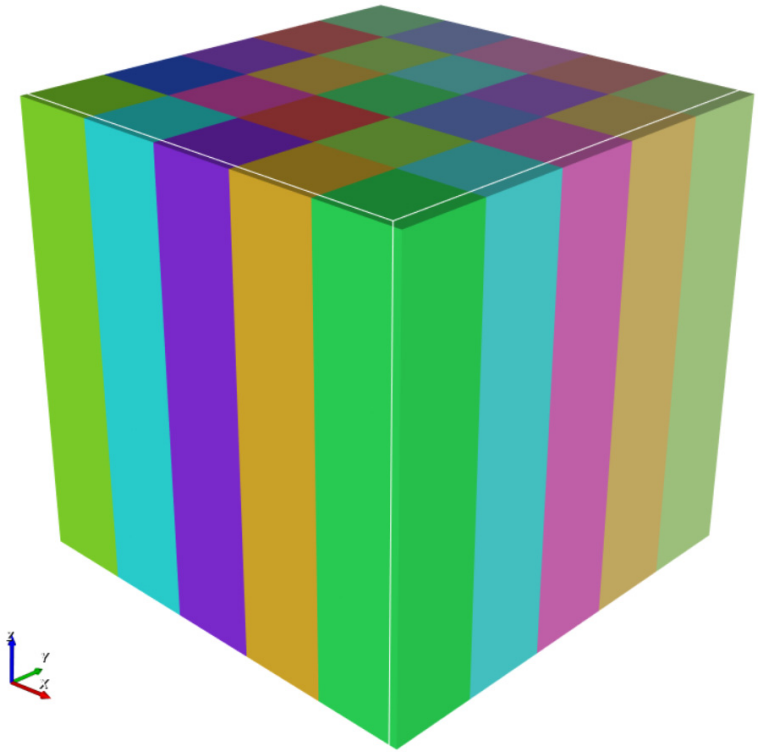


Fig. 3 Location of defined regions in stochastic DFN models. Unique colours represent individual regions and simulated deterministic traces shown in black.

Three fracture groups are defined for the model with orientation distributions as given in Table 1. Stochastic fractures also have size defined through a Cumulative Area Distribution (CAD). The input CAD values define a straight line on a log-log plot of fracture size and the number of fractures greater than the threshold divided by the model volume. These values are shown in Fig. 4 along with the distribution of actual fracture surface areas as measured from a resulting DFN model. The resultant orientation distributions for each group are shown in Wulff stereonets in Fig. 5 with values calculated using Dips v. 6.017 (Rocscience, 2016) included in Table 1 for comparison with input values.

Stochastic fractures are seeded in regions individually allowing for variability in orientation and intensity between regions if desired. Size distributions define that stochastic fractures will have surface areas less than 100 m^2. The size of each region is 10 x 10 x 50 m^3 thus all stochastic fractures will be non-persistent through the model and their region of origin. Fractures are defined not to terminate against other fractures nor to terminate at a regional boundary. There is an expected boundary effect in terms of intensity as regions in the center of the DFN model. This is because regions will have fractures propagate into them from neighbouring regions and thus final fracture intensity will be related to the number of neighbouring regions.

Ten realizations of the stochastic DFN model were generated for analysis. The model was also designed with an external region 1 unit thick surrounding the 25 stochastic regions. This external region allows for the inclusion of deterministic traces. By including deterministic traces, studies to determine the minimum size of fracture that needs to be included in mapping can be determined. The size should reflect the minimum sized feature that is required to have a representative distribution of fracture intensity and blocks. A second set of smaller, non-persistent fractures can be included into the model in order to achieve the predicted overall intensity.

Table 1. Comparison of input orientation distributions with realized orientations for a single stochastic DFN model.

Fracture group	Input dip (°)	Standard deviation of dip (°)	Input dip direction (°)	Standard deviation of dip direction (°)	Resultant dip (°)	Resultant dip direction (°)	Fisher K statistic
1	90	15	90	15	88.54	89.6	15.8
2	90	15	180	15	89.98	179.14	16.59
3	0	5	125	15	0.44	141.74	158.973

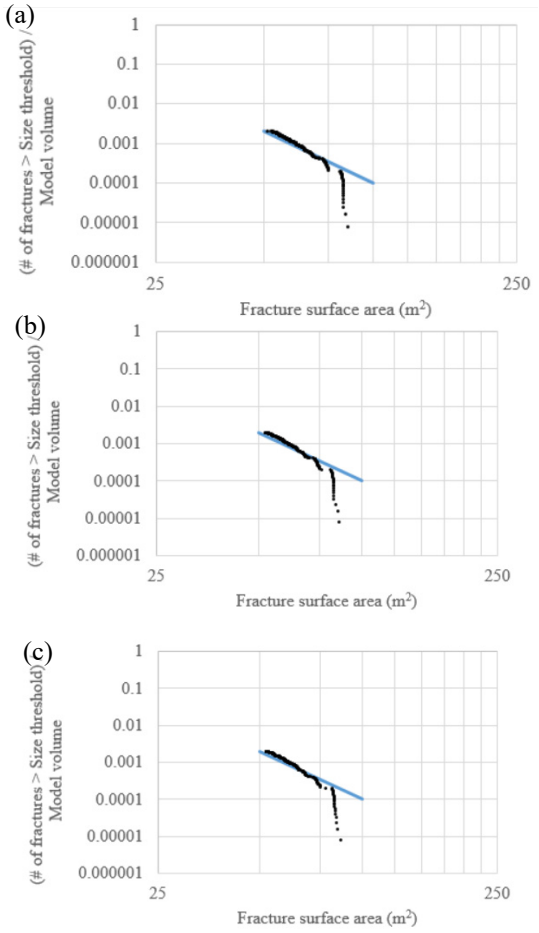


Fig. 4. Input CAD parameters shown as straight line and data points for realized fractures in (a) group 1, (b) group 2, and (c) group 3.

3. Emergent Properties of DFN Models

An emergent property of a system can be defined as a property resulting from the collective effects of the interaction of the component parts of a system. For DFN modelling, emergent properties are only observable when considering the DFN as a whole and cannot be determined by simply examining the input values. For the purpose of DFN modelling with MoFrac several input variables are required. Fracture orientation distributions are required for each defined fracture group, size distributions are also required which when considered with respect to the model size can define fracture persistence. Fracture intensity is also required as an input, MoFrac accepts fracture

intensities and size distributions as two points and a slope on a Cumulative Length Distribution (CLD) or CAD curve.

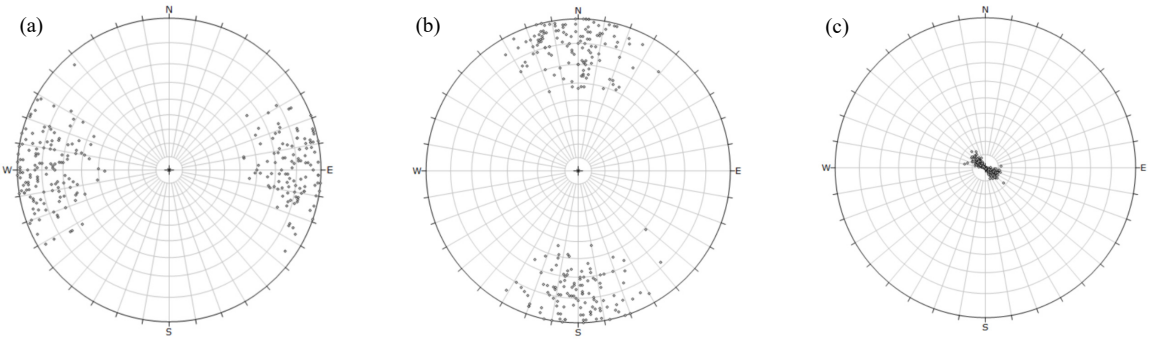


Fig. 5. Stereonets showing fracture orientation distributions in a single DFN realization for fractures in (a) group 1, (b) group 2, and (c) group 3.

Following the generation of a DFN model based on the required inputs several measures of emergent properties can be considered. The *in situ* Block size Distribution (ISBD) of a rock mass has been identified as vital for optimization of processes such as rock blasting and rock slope angle determination (Mavrouli, *et al.*, 2015). Fracture intensity is also important to these two areas of optimization and also is significant for hydraulic modelling of a rockmass (Lei, *et al.*, 2017; Appleyard, *et al.*, 2018) and ore reserve estimation (Mooney and Boisvert, 2016).

This study identifies a simple method of querying blocks within a DFN model for fracture intensity (P_{32}) at different scales. This method allows for a characterization of the variability of intensity within a DFN model as well as for the measurement of the changes in the coefficient of variation (CV %) when considering different sample block sizes. The data collected also allows for an estimation of ISBD within the DFN model.

Where DFN models consisting of non-persistent fractures have defined boundaries where fractures will not be seeded a boundary effect is to be expected. By considering the fracture intensity in the layer of exterior blocks compared to the interior blocks for different block sizes an estimation of the boundary effect based on fracture size and orientation is possible, again using the data collected to characterize fracture intensity.

In summary four emergent properties will be explored using the method of querying sample blocks of a DFN model for fracture intensity. The four emergent properties under study are:

- Measuring heterogeneity of fracture intensity;
- Measuring the coefficient of variation for different inspection block sizes;
- Estimation of *in situ* block size distribution;
- Estimation of the area of influence of any boundary effect.

These properties of a DFN model are considered as well as regular validation of DFN models. For stochastic fractures, these include honouring fracture orientation and size distributions with respect to inputs; for deterministic fractures, validation is based on location and orientation errors with respect to input data of mapped fractures.

There is ongoing work considering the mapping of rock bridges that occur within a rock mass. A rock bridge can be defined as the competent sections of rock between discontinuities. Alghalandis and Elmo (2018) demonstrate a means of quantifying the degree of rock bridging in a rock mass in an analogous way to determining fracture intensity as presented by Dershowitz and Herda (1992). For example a P_{32} intensity quantifies the sum surface area of fractures within a defined volume and a RB_{32} value considers the surface area of rock bridges within a volume of rock. The method outlined

in this study is based on calculations of P_{32} intensity in sample blocks, it should be noted that a new set of emergent properties could evolve from similar analysis of RB_{32} values in the same sample blocks. A relation between fracture intensity and rock bridge formation could also arise through this type of analysis.

3.1. Fracture intensity

Fracture intensity in a DFN model is often described as a P_{10} (number of fractures per linear unit), P_{21} (sum length of fractures per aerial unit), or P_{32} (sum area of fractures per volumetric unit) value (Dershowitz and Herda, 1992). Fracture intensity can also be described using CLD and CAD curves that separate size and intensity as separate factors. Fractures can be seeded in DFN models according to either type of input intensity (Ryan, *et al.*, 2000). Input intensities can be easily verified against resulting DFN models, as in Fig. 4, or by calculating the appropriate intensity for comparison to input values.

The use of inspection blocks to analyze the DFN infers that using the P_{32} intensity measure represents information representative of the entire cube. Having a single value allows for a consideration of the change in intensity between sampled blocks in the DFN model. The degree of spatial variability of fracture intensities is a characteristic that is useful for optimization of blast design. By having fracture intensity data for all component cubes of the DFN it is possible to view the DFN in new ways. The area of influence for a blast hole can be considered independently, layers of a DFN can be assessed for connectivity of high intensity blocks, and boundary effects of the DFN can be quantified.

The mean intensity is a useful measure to characterize a sampled volume as a whole and the standard deviation gives a measure of variance. The coefficient of variation (CV%) is defined as the ratio of the standard deviation to the mean and is useful in normalizing data for comparison, for example, DFN models of different scales. By sampling a DFN several times with inspection cubes of different volumes the change in the CV% is considered as a characteristic and emergent property of the DFN models. This characteristic parameter shows potential to be useful in optimizing the size of blast holes during blast design, namely by limiting the area of influence to areas of low variance (CV%<1).

3.2. In situ block size distribution (ISBD)

The advancements in the development of methods to determine *in situ* block size distributions in undisturbed rock has been outlined by Elmouttie and Poropat (2012). Methods have become more sophisticated over time but four limitations were identified in their review:

- The ability to account for non-persistent fractures;
- The ability to account for more than three fracture groups;
- The use of fully detailed DFN models in Monte Carlo simulations;
- Simplifications and approximations required (ie. convex blocks).

Generally these limitations were found to result in over estimation of fragmentation, thus an over estimation of fracture intensity. Rogers *et al.* (2007) documents the first attempts to derive an ISBD from a fully defined DFN model. It was noted that using this approach blocks can be formed from the interaction of an unlimited number of fractures. The difficulties in determining block size distributions when fractures are smaller relative to intensity (less persistent) was also highlighted in the work, showing that smaller fractures can often intersect a block but do not cross it. This leads to concave blocks to be modelled. The significance between the occurrence of convex and concave blocks would affect different optimization problems in different ways.

The method of estimating *in situ* block size distribution suggested by Elmouttie and Porpat (2012) address the limitations as listed above. A DFN model that consists of polygonal approximations is used to calculate the ISBD thus allowing for non-persistent joints and blocks formed from multiple joints to exist within a DFN model that is analyzed. The fractures included in the models of Elmouttie and Poropat are planar, there is no undulation of fractures as there is with MoFrac-generated

fractures. The inclusion of undulating fractures in a MoFrac DFN further complicates the potential geometry of block shapes that can be encountered. It is accepted that a three dimensional DFN model represents the possibilities of block formation in a rock mass to a greater degree of accuracy than is possible from analysis of two dimensional slices taken from a DFN and analyzed for closed polygons (Miyoshi, *et al.*, 2018). A new method of using data obtained when considering fracture intensity is proposed that directly queries the DFN model for a variety of block sizes for the presence of any fractures. This allows for block sizes to be determined from any number of fracture groups with any degree of persistence as they are measured directly from the DFN model. From this data an estimate of ISBD can be obtained as demonstrated in Section 6.4.

3.3. Rock bridges

Rock bridges are defined as the parts of intact rock surrounded by fractures (Alghalandis and Elmo, 2018). A definition that is very similar to what one would consider an *in situ* block. Whereas block size distributions are significant to fragmentation and the magnitude of potential slope failure, due to the quantity of rock that may fail, rock bridges are important in that they can define a route that existing fractures will coalesce during rock failure. Both of these factors are integral to a subject such as blast design as the block size will determine the amount of explosives to be used and the knowledge of rock bridges could be used to determine blasthole location that will optimize the use of the blast energy in the area that is to be fragmented. Aghlandis and Elmo propose the use of a measurement analogous to fracture intensity measures. In two dimensions an RB_{21} measurement gives the minimum linear length of intact rock between fracture traces needed to contact two boundaries of a DFN model.

The RB_{21} value could be useful as an additional emergent property of DFN models useful for design optimization. An RB_{32} measurement can also be made in three dimensions that could be calculated using the same approach of voxelation as proposed in this paper. The degree to which rock bridge intensities change with sample size would be a useful parameter to determine how energy will flow through a rock mass during failure or blasting.

3.4. Orientation and fracture size distributions

Fracture group orientation and size distributions are required to model fractures with MoFrac. Being the main inputs to the DFN recipe it is important to verify that the inputs assigned have in fact been realized. Figs. 4 and 5 visually show the process of verifying orientation and size distributions. This is the main method used to verify a stochastic DFN model compared to measuring the degree to which traces and assigned orientations are honoured in a deterministic DFN. There are additional tests and comparisons that can be accomplished once a DFN has been generated that would qualify as an emergent property. For example in Table 1 the input orientations for the stochastic model were given as a mean and standard deviation for strike and dip separately. An alternative method is to give the strike and dip along with the Fisher k statistic which is a measurement of clustering when visualized on a stereonet. A larger Fisher k value is associated with a tighter cluster of orientations. The stereonets show that fracture group three is much more clustered than groups one and two. This is due to the lower standard deviation assigned to the dip value for the third group. This clustering is verified by calculating the Fisher k value as shown in Table 1. In a similar way the fracture intensities for the stochastic models are defined using a CAD curve, once a DFN model has been generated it is then possible to consider P_{32} values as a verification and comparison. There is a need for future work to consider the relation between both types of intensity and orientation measurements specifically related to MoFrac-generated DFN models.

4. DFN Characterization Technique

DFN models have shown to demonstrate opportunities in modelling heterogeneity in a rock mass when compared to equivalent medium techniques. DFN models that represent actual rock masses are generally considered to be a combination of fractures seeded from mapped fractures with stochastic

fractures derived from statistics associated with what has been mapped (Elmouttie and Poropat, 2012). Stochastic fracture size distributions are generally shown to be power law when the effect of sampling bias is minimized and thus power law distributions are used for determining fracture size through use of a Cumulative Area Distribution (CAD) (Tonon and Chen, 2007).

A DFN characterization is proposed that allows for a DFN model to be analyzed after discretization through voxelization as shown in Fig. 2. The P_{32} values of component voxels is determined and mapped by layer using a heat map designed using Excel Visual Basic for Applications (VBA). The goal of this method is to create a heat map that characterizes the DFN model implicitly where fracture location can be identified by increases in fracture intensity, the second goal is to identify the distribution of intensities throughout the DFN establishing a measure of heterogeneity of intensity within the DFN models generated. The characterization of these regions will be verified by using simulated boreholes in the DFN models used to measure P_{10} values that will show the same trends as predicted using the analysis of individual voxels.

5. Analysis of Canadian Shield DFN Model

Ten realizations of the Canadian Shield deterministic DFN were generated to be analyzed at different scales of voxelization. All five models were verified using a metric for the location error of the fracture traces. Results were similar to those found in previous validation studies using the dataset (Junkin *et al.*, 2018). Longitudinal Root Square Mean Error (LRSME) values comparing the input traces to traces from an inspection slice on the surface averaged 4.09 m with a standard deviation of 11.91 m. The method followed that of previous validation studies (Junkin *et al.*, 2017; 2018) Variability in the strike to dip ratio and Trace Area Constraint (TAC) factor resulted in randomness to the geometry of fractures thus yielding a variety of different realizations for analysis. The location error remained constant meaning that fracture traces were honoured to the same degree despite the geometry of individual fractures changing. One realization of the Canadian Shield DFN is shown in Fig. 6, the model extends to 200 m depth with a surface area of 200 km^2. The trace map and an inspection plan on surface from a single realization are shown superimposed on each other in Fig. 7 to visualize the degree of fit between fractures and input traces.

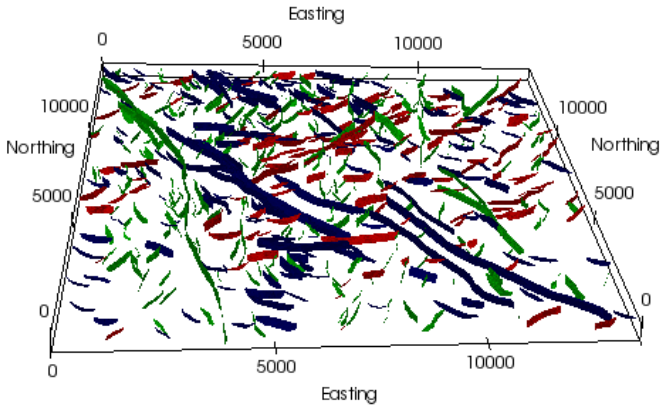


Fig. 6. Single DFN realization of the Canadian Shield dataset, coloured by fracture group.



Fig. 7. Comparison of input traces to an inspection plane of a single realization of the Canadian Shield DFN model cut on the same plane as input traces.

5.1. Coefficient of variation (CV %) plot

In order to characterize heterogeneity of fracture intensity at multiple scales the coefficient of variation (CV %) is considered for each size of inspection cube. This is a useful measure of fracture intensity variation in a DFN as multiple scales are considered and the trends in changes in CV% dependent on sample size can be easily visualized. It is proposed that the curve resulting from plotting CV% against block size is characteristic to a representative DFN model. This method can be used to verify that multiple realizations of the same DFN are demonstrating similar characteristics over the scales used for sampling. A CV% value is considered low variance when below 1, meaning the standard deviation is less than the mean. A high variance occurs when CV% is greater than 1. Fig. 8 shows the plot of the CV% vs block size for the Canadian Shield model as well as the stochastic model which will be discussed in Section 6. Based on the Canadian Shield curve it can be seen that when considering the whole model high variance occurs when a block size of approximately 200 000 m^2 is used. These curves will be dependent both on the DFN and the geometry of the sample blocks.

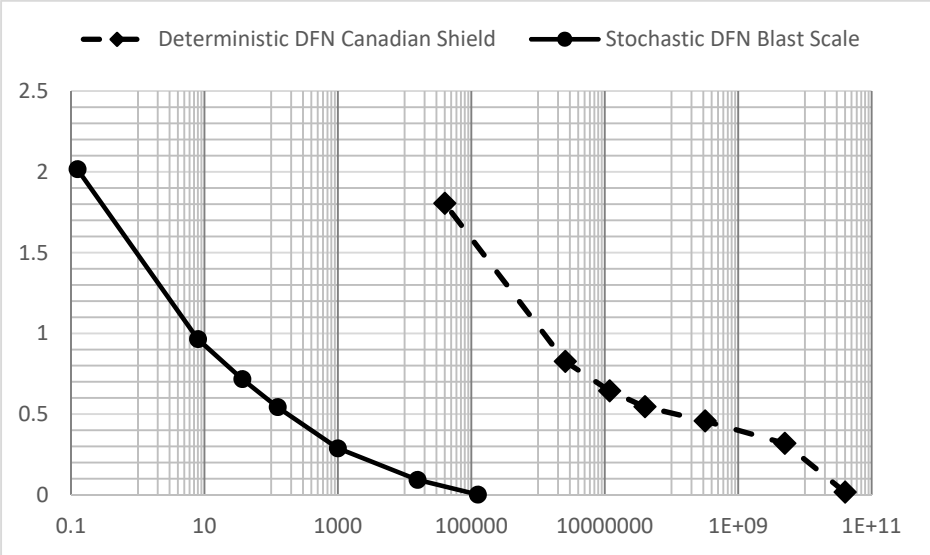


Fig. 8. Coefficient of variation plotted against inspection block size for both the Canadian Shield and stochastic DFN models.

5.2. *In situ* block size estimation

A benefit of calculating the P_{32} fracture intensity for a variety of sizes of blocks within a DFN and to calculating the intensity for all blocks is that an estimate of the ISBD can be achieved by counting null blocks, blocks that have a fracture intensity of 0. For this case ISBD curves were achieved by considering the number of null blocks. The largest block encompasses the entire DFN and thus a fracture intensity of 0 will not occur if any fractures are present. The largest null cubes are encountered when using a voxelization of 10 x 10 x 10 for analysis. This equates to a block size of 200 000 m^3. As progressively more null blocks are found and higher resolutions the volume of blocks accounted for by larger blocks are subtracted from the new sum volume of null blocks. This ensures that null blocks are not counted twice when creating this plot. It can be seen that the resulting distribution follows the power law as is expected with a block size distribution. As there is no association of null blocks with neighbouring null blocks it is expected that this estimate will overestimate small blocks and underestimate large blocks. This will result in an overestimation of fracturing in the model but still provides an indication as to the actual ISBD for the DFN and could be incorporated into optimization models. The ISBD estimate associated with a single DFN realization of the Canadian Shield DFN models is presented in Fig. 9.

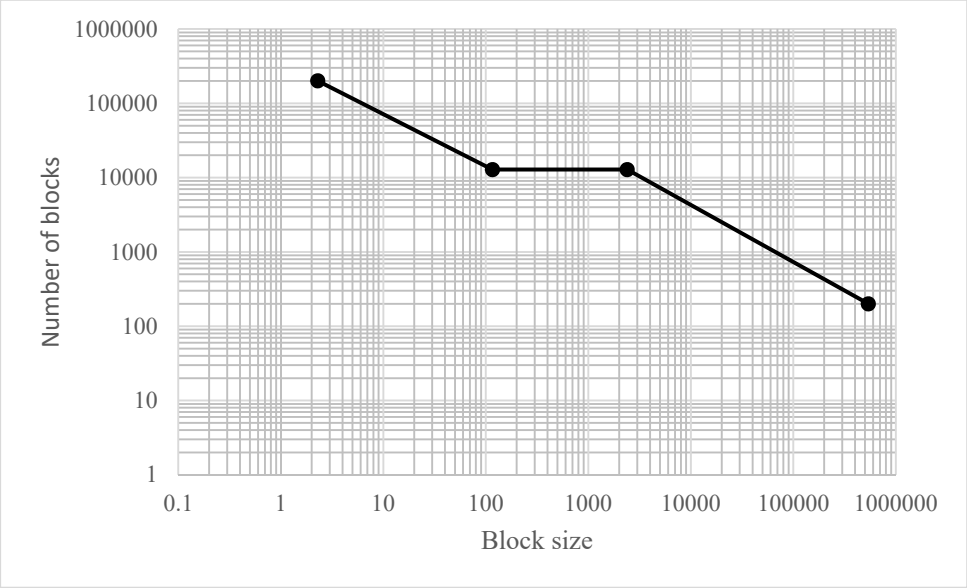


Fig. 9. Estimated *In situ* block size distribution from counting null cubes for a single realization of the Canadian Shield DFN model as measured over ten DFN realizations.

6. Analysis of Stochastic DFN Models

Ten realizations of the stochastic DFN model were generated for analysis. By including both a deterministic and stochastic DFN in this study the methods of analysis presented are shown to be highly versatile, it is also inferred that these methods will be applicable to analysis of hybrid models and will be integral for determining the scale of geological mapping that should be included in DFN models used for engineering design. Fig. 10 shows two views of a single realization of the stochastic model, one view showing the fractures coloured by group and another showing the fractures coloured by region. Although all regions are defined as having the same intensity and the result is that each region is composed of 30 fractures it can be seen that they differ greatly when sampled individually. A boundary effect is expected on exterior blocks that will result in a slightly lower intensity as no fractures are able to leak into these regions from the exterior of the model which would not be representative of the natural case. The orientations and intensities of these models were validated as shown in Fig. 4 and Fig. 5.

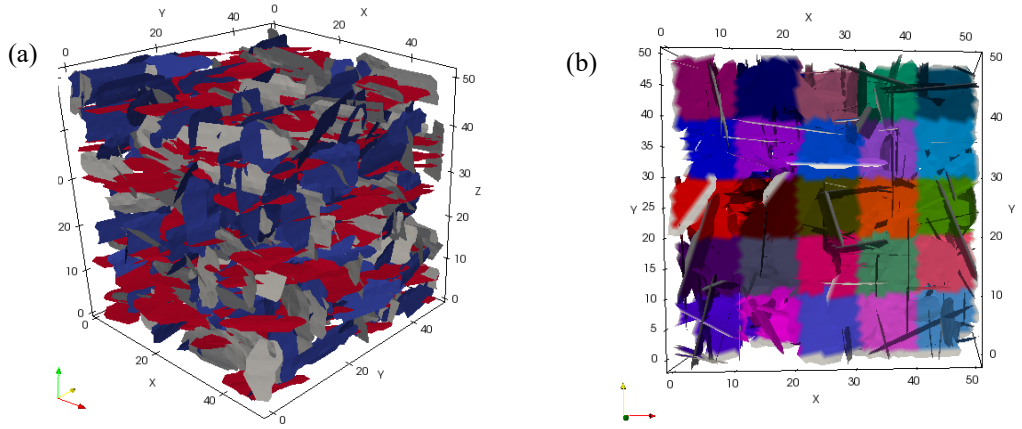


Fig. 10. A single realization of the stochastic model including deterministic fractures (a) coloured by fracture group and (b) coloured by region.

6.1. Fracture intensity mapping

By considering the intensities of all cubes in a DFN model it is possible to generate maps showing changes in intensity throughout a DFN. As an example Fig. 11 shows data from ten layers of a single realization of a stochastic model. From top down, left to right, layers 1 to 10 are shown as a trace map, a fracture intensity heat map and a histogram showing the distribution of intensities specific to the layer. The boundary effects can be seen by comparing the slices from the top and bottom of the model to the interior slices. It can be seen visually that there is a larger number of low intensity blue squares for these layers. This method of visualization can be applied both to layers in a DFN or defined volumes. This allows for the area of influence of a blast hole to be isolated and considered alone. By changing the amount of explosive used the area of influence is changed and thus can be optimized to incorporate only the desired range of fracture intensity in the area of influence for a single blast hole.

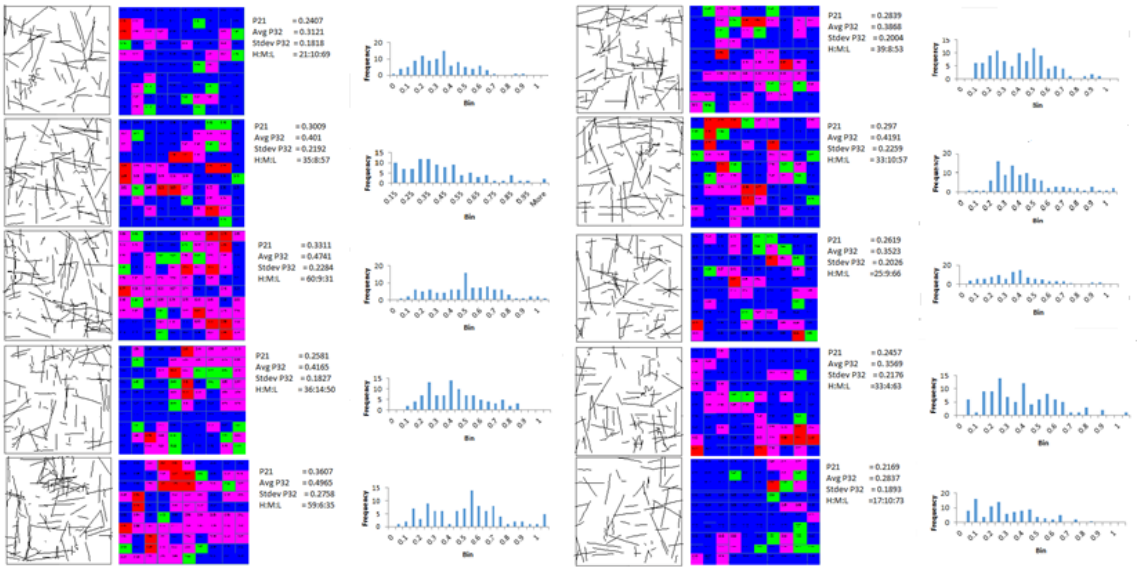


Fig. 11. Comparison of fracture intensities graphed on a heat map with histogram showing intensity distribution by layer of a purely stochastic DFN model (green-low intensity, green-medium intensity, pink-high intensity, red-very high intensity).

The mapping of fracture intensity also allows for a visual validation of the method of visualization. By superimposing fracture traces on the heat map generated a shown in Fig. 12 the location of high fracture intensities can be seen to match the high intensity squares (pink and red). This shows that if

the location of specific fractures is not required but it is still desired to consider a disaggregate representation of a rock mass that a three dimensional heat map can be used. This would require significantly less memory for visualization and for incorporating DFN data into other engineering computer models.

6.2. Boundary effects

As fractures can propagate from one region to another depending on their seed point within a region a boundary effect is expected that will result in lower intensities on exterior blocks. This is due to the fact that no fractures are seeded out of bounds that could leak into a region that is within the DFN. For example a block in the center of a DFN would have 6 neighbouring blocks but a block on the corner of a DFN would only have 3 neighbouring blocks. It would be expected that the corner blocks would have half as many fractures propagate into them as would blocks in the center of the DFN.

The P_{32} fracture intensity is plotted by layer in Fig. 13 with the standard deviation of intensity for each layer. This graph shows that there is an observable decrease in intensity for the top and bottom layers. The standard deviation for these layers does fall into the range of intensities of the interior layers. This demonstrates that although the overall intensity is lower that there will still be some areas of high intensity on the exterior of the DFN models as can be seen by comparing layers in Fig. 11.

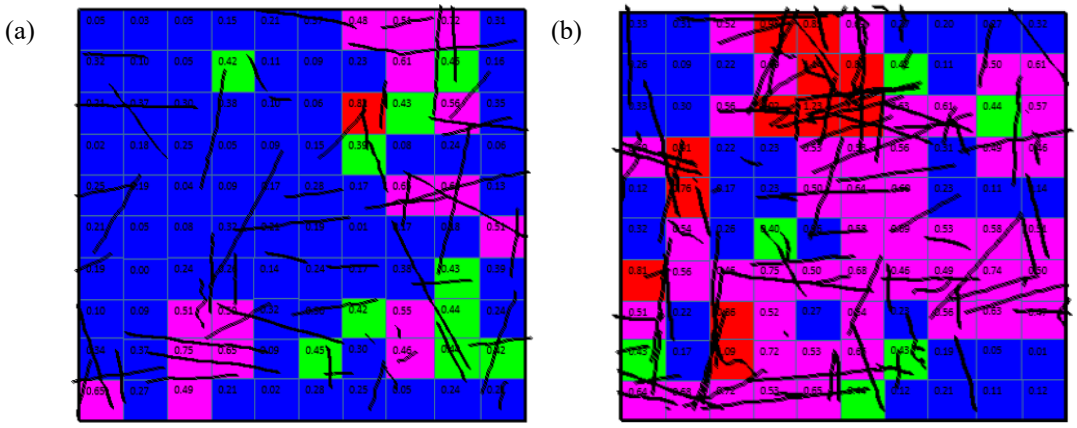


Fig. 12. Close up of heat map for layer 5 and layer 10 of a purely stochastic DFN model with superimposed fracture traces from the midpoint of the layer (green-low intensity, green-medium intensity, pink-high intensity, red-very high intensity).

Based on the fact that a boundary effect is expected and the fact that it is demonstrated through visual inspection and quantitatively when comparing fracture intensities, it was deemed necessary to further investigate this phenomenon. Fig. 14 shows a curve that considers the difference between the mean fracture intensity for exterior and interior blocks in a DFN model based on block size. It is proposed that the extent of the boundary effect can be determined through this analysis. A peak is noted in the difference in intensities for a block size of 20 m^3. This equates to a boundary effect of approximately 2.75 m, or one quarter of a 10 x 10 m^2 region with fractures from 50 to 100 m^2 being seeded.

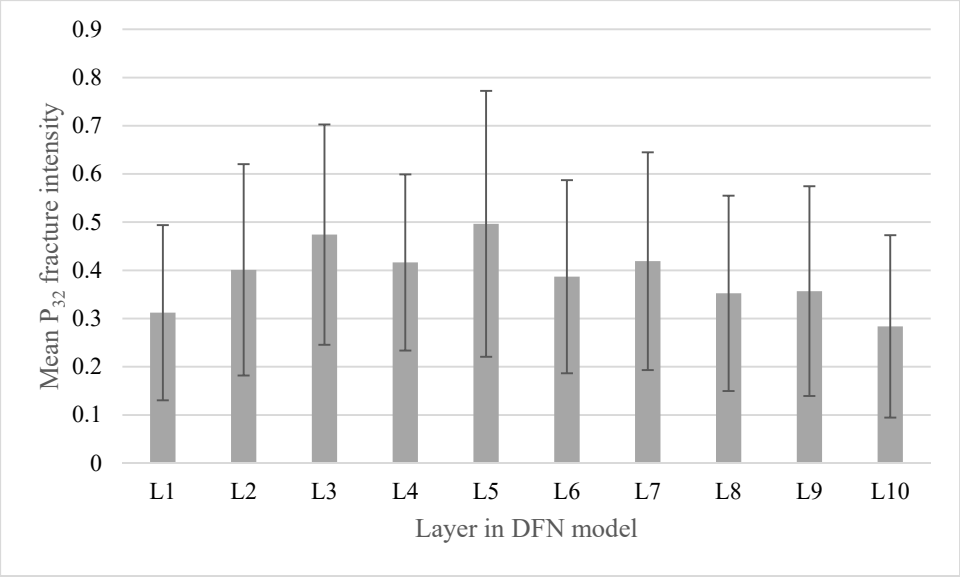


Fig. 13. Variability in P_{32} in a single realization of a stochastic DFN model by layer (5 m thick, numbered from bottom to top), error bars show standard deviation.

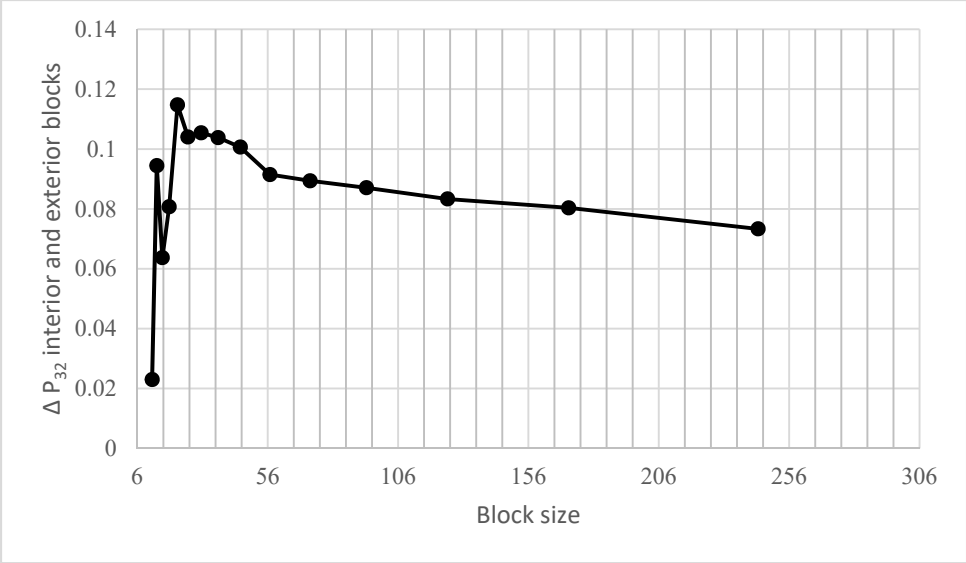


Fig. 14. Difference between interior and exterior P_{32} intensities vs. sample block size

6.3. Fracture intensity distributions and CV% plot

In order to view the distributions of fracture intensities that result from stochastic seeding Fig. 15 was created. By considering how the distribution of fracture intensities changes with sample size and estimation can be made as to what size of block should be used for sampling and what distribution would be expected. This allows for a prediction of fracture intensity if there is a limited number of sampled cubes. For the case of Fig. 20 only 10 000 blocks were sampled for the 100 x 100 x 100 voxelation. The distribution has appeared to break down at this point with peaks at P_{32} of 2, 4, and potentially 6. These peaks need to be further investigated to determine if they are representative of the entire DFN and if so to determine the cause.

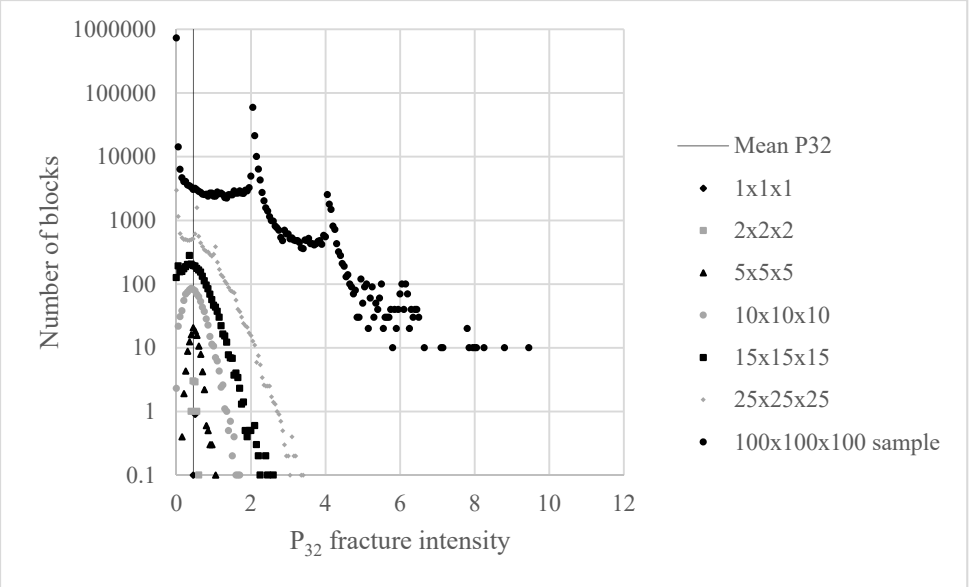


Fig. 15. Fracture intensity distributions with respect to voxelation.

Through the generation of this figure it is possible to view the fracture intensity distribution, estimation of ISBD (y axis values) and the CV% by considering the deviation of the distribution from normal. A CV% curve was plotted for all ten realizations of the stochastic DFN model and is included in Fig. 8. When compared directly to the Canadian Shield curve it can be seen that both the location and shape of the curve has changed. It is proposed that this shape is characteristic of a DFN. The shift in the curve to the left means that there is a lower degree of variance at smaller scales than the Canadian Shield which is likely a function of fracture size and orientation with respect to the experimental volume.

6.4. *In situ* block size estimation

Using the method of ISBD estimation presented in Section 5.2 a block size distribution for a realization of the stochastic was created and is presented in Fig. 21. This curves again is indicative of a power law relation as is expected. The largest blocks are predicted to be about 125 m^3 or 5 x 5 x 5 m^3. By using progressively smaller sample blocks it is possible to define blocks using this method to a very small size given the computing power. The smallest block size shown in Fig. 21 is a result of a 22 x 22 x 22 m^3 inspection of the DFN.

7. Discussion

A new method of analysis is presented in this study that applies directly to MoFrac-generated DFN models. MoFrac DFN models contain non-persistent, undulating fractures and thus estimates of fracture intensity and block size distribution must be derived from generated models. DFN models are both generated and analyzed using the MoFrac code.

By dividing the DFN into component blocks at different scales and measuring fracture intensity it is possible to derive several tools for analysis. Three of these tools are:

- The CV% plot
- Intensity distribution analysis
- ISBD estimation

The values used to determine values that enable these three tools are all visible in Fig. 16. Further to these three tools a new method of visualizing MoFrac fractures is presented that replaces actual fractures with inspection cubes that shows fracture intensity in a sense analogous to a heat map. This

visualization allows for understanding of any boundary effect with MoFrac that was characterized in further detail in this study. It would be necessary to calculate the boundary effect for any new DFN until relation between fracture intensity and orientation in a MoFrac DFN can be used to generate a function that predicts block size. The use of CV% is a measure of fracture intensity variation over different scales and is useful to determine if fractures of the correct size are included in DFN models for engineering design and also of use to optimize such designs. AN estimation of ISBD has been presented that utilizes the data collected to obtain the CV% plot. Although this method serves as an estimate only there is indication that with refinement the method can be used to measure actual block sizes within a DFN by querying null blocks for neighbouring null blocks and summing the volume of connected blocks. This would mean that only one analysis would be required with a block size equivalent to the smallest block volume desired for consideration.

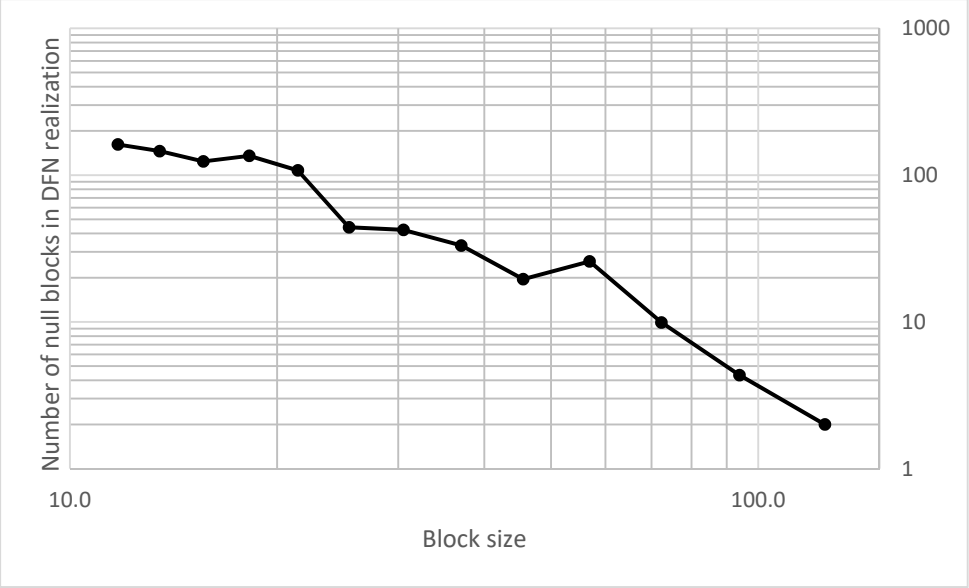


Fig. 16. Estimated *In situ* block size distribution from counting null cubes for a single realization of a stochastic DFN model.

7.1. Applications to Rock Engineering Systems solutions

Rock Engineering Systems (RES) solutions to civil engineering problems were developed by Hudson (1992) as part of research to develop a new methodology for civil engineering design that considers the interaction of a variety of parameters that are identified to be pertinent to a specific project (Hudson , 1994). This approach to developing designs for civil projects has been applied to blasting by a variety of researchers. This application was initiated by Lu in 1997 which was followed up with papers to characterize the blastability of a rock mass using a RES approach (Lu and Latham, 1999).

A rock engineering systems approach involves identifying the parameters of interest to a specific problem and formulating an interaction matrix that describes the relationship between each parameter. This approach is a total systems approach that develops an analytic model defining the environment for a particular system that is to be analyzed (Hudson, 1992). Interaction matrices developed for determining blastability have been previously developed (Lu, 1997; Lu and Latham, 1999). These interaction matrices involve the parameters listed below:

- Strength
- Resistance to fracturing
- Sturdiness of rock
- Elasticity of rock
- Resistance to dynamic loading

- Hardness of rock
- Deformability
- Resistance to breaking
- *In-situ* block sizes
- Fragility of the rock mass
- Integrity of the rock mass
- Discontinuity plane's strength

It is suggested that this model can be used with modifications to incorporate emergent variables derived from MoFrac-generated models. Parameters that could be included in a modified interaction matrix would include the coefficient of variability for a DFN, mean fracture intensity for the area of influence of a blast hole, and *in situ* block size distributions as calculated by MoFrac. Together with the ability to visualize rock surfaces and determine exact burdens and fracture locations and orientations through laser scanning, a new blast optimization technique using the emergent properties in calculating optimized hole locations and powder factors can be developed. Improvements will also be available for the visualization of the blast and for exact calculations of rock volumes (Jian, *et al*., 2018). It is also suggested to develop an interaction matrix that is friendly to the skill set of the blaster and not the rock engineer. Often it is the blaster that makes decisions about blast design based on previous blasting results at the same site. To have a method to verify changes in a blast plan or to develop entirely new blast plan strategies, such as irregular patterns that utilize information available to the blaster would prove to be very useful under a variety of conditions where blast results are unpredictable.

8. Acknowledgements

Appreciation is given to MIRARCO, who develop and provide access to MoFrac. In particular, Scott McGarvey, the lead developer for MoFrac at MIRARCO and Dr. Lorrie Fava who oversees the MoFrac project.

9. References

[1] Alghalandis, Y and D Elmo (1988) Application of graph theory for robust and efficient rock analysis. *Proc. The 2nd International Discrete Fracture Network Engineering Conference (DFNE), American Rock Mechanics Association.* Seattle, Washington. USA. 18-733.

[2] Appleyard, P, P Jackson, S Joyce and L Hartley. (2018) Conditioning discrete fracture network models on intersection, connectivity and flow data. *Swedish Nuclear Fuel and Waste Management Co (SKB)*. Report R-17-11.

[3] Dershowitz, WS and HH Einstein. (1988) Characterizing rock joint geometry with joint system models. *Rock Mechanics and Rock Engineering.* 21(1988) pp. 21-51.

[4] Dershowitz, WS and HH Herda. (1992) Interpretation of fracture spacing and intensity. *Proc. The 33rd U.S. Symposium on Rock Mechanics (USRMS), American Rock Mechanics Association.* Santa Fe, New Mexico. USA. pp. 757-766.

[5] Elmouttie, MK and GV Poropat. (2012) A method to Estimate In Situ Block Size Distribution. *Rock Mechanics and Rock Engineering*. 45(3) pp. 401-407.

[6] Gierszewski, P, J Avis, N Calder, A D'Andrea, F Garisto, C Kitson, T Melnyk, and L Wojciechowski, 2004. Third Case Study - Postclosure Safety Assessment Deep Geological *Repository Technology Program Report No: 06819-REP-01200-10109-R00*, Ontario Power Generation, Toronto, Ontario.

[7] Hudson JA. (1992) *Rock Engineering Systems: theory and practice.* Ellis Horwood Series in Civil Engineering, Chichester, England.

[8] Hudson JA. (1994) Development of a new methodology for rock engineering design. *Final technical report AD-A285 996*. European Research Office of the U.S. Army, London, England.

[9] Jian, X, M Elmouttie, B Zhou, and P Dean. (2018) A novel 3D visualization method for stochastic structural data with geophysical data for slope stability analysis and blast design. Proc. The 2nd International Discrete Fracture Network *Engineering Conference (DFNE), American Rock Mechanics Association.* Seattle, Washington. USA. 18-887.

[10] Junkin WR, L Fava, E Ben-Awuah, and RM Srivastava. (2018) Analysis of MoFrac Generated Deterministic and Stochastic Discrete Fracture Network Models. *Proc. The 2nd International Discrete Fracture Network Engineering Conference (DFNE), American Rock Mechanics Association.* Seattle, Washington. USA. 18-114.

[11] Junkin WR, D Janeczek, S Bastola, X Wang, M Cai, L Fava, E Sykes, R Munier, and RM Srivastava. (2017) Discrete Fracture Network Generation for the Aspo TAS08 Tunnel using MoFrac *Proc. The 51st U.S. Rock Mechanics / Geomechanics Symposium, American Rock Mechanics Association.* San Francisco, California. USA. 17-339.

[12] Latham J-P and P Lu. (1999) Development of an assessment system for the blastability of rock masses. *International Journal of Rock Mechanics and Mining Sciences 36 pp. 41-55.*

[13] Lu P. (1997) The characterization and analysis of in-situ and blasted block size distributions and the blastability of rock masses. *Phd thesis.* Univeristy of London (Queen Mary and Westfield College).

[14] Lei Q, J-P Latham, C-F Tsang. (2017) The use of discrete fracture networks for modelling coupled geomechanical and hydrological behaviour of fractured rocks. *Computers and Geotechnics* 85 pp. 151-176.

[15] Mavrouli OC, J Corominas, and M Jaboyedoff. (2015) Size Distribution for Potentially Unstable Rock Masses and In Situ Rock Blocks using LIDAR-Generated Digital Elevation Models. *Rock Mechanics and Rock Engineering*. 48(4) pp. 1589-1604.

[16] MIRARCO. (2018) MoFrac DFN Modelling Software, Beta v. 0.8. *MIRARCO, Mining Innovation, Rehabilitation and Applied Research Corporation¸* Sudbury, Ontario, Canada. Available at http://www.MoFrac.com.

[17] Miyoshi T, D Elmo, and S Rogers. (2018) A Discrete Fracture Network approach to study the variability of the Geological Strength Index. *Proc. The 2nd International Discrete Fracture Network Engineering Conference (DFNE), American Rock Mechanics Association.* Seattle, Washington. USA. 18-626.

[18] Mooney CR and JB Boisvert, (2016) Using a Discrete Fracture Network and Spatial Point Processes to Populate Veins and Model Grade in a Coarse Gold Deposit. *Natural Resources Research* 25(3) pp. 255-268.

[19] RocScience Inc. (2016) Dips v. 6.017. *Rocscience Inc.¸* Toronto, Ontario, Canada. Available at http://www.rocscience.com.

[20] Rogers SF, DK Kennard, WS Dershowitz and A van As. (2007) Characterizing the In Situ Fragmentation of a Fractured Rock Mass Using a Discrete Fracture Network Approach. In *Rock Mechanics: Meeting Society's Challenges and Demands* (eds. Eberhardt, Stead, and Morrison). pp. 137-143.

[21] Ryan J-L, L Lonergan and RJH Jolly. (2000) Fracture spacing and orientation distributions for two-dimensional data sets. *Journal of Geophysical Research.* 105(B8) pp. 19 305-19 320.

[22] Srivastava, R.M. 2002. The discrete fracture network model in the local-scale flow system for the third case study. Deep Geological Repository Technology Program Report 06819-01300-10061, Ontario Power Generation, Toronto, Ontario.

[23] Tonon F and S Chen. (2007) Closed-form and numerical solutions for the probability distribution function of fracture diameters. *International Journal of Rock Mechanics and Mining Sciences* 44 pp. 332-350.

Open-source Programming (OSP) Framework for Automatic Database Management for Discrete Event Simulation

Shahrokh Paravarzar, Mohammad Tabesh, Yashar Pourrahimian, Hooman Askari-Nasab
Mining Optimization Laboratory (MOL)
University of Alberta, Edmonton, Canada

ABSTRACT

Discrete event simulation is a simulation method has been used in forecasting and analysis the mining process and scheduling during last years. Discrete event simulation requires the pre- and post-processing of data and the results. In this paper, our focus is on pre-processing of data for simulation goal. Capturing the system behaviour and more precisely the event occurrence based on random variables and distributions is the goal of the pre-processing and data analysis in Arena simulation. The simulation process requires pre-processing steps that are time-consuming due to the huge amount of dispatching data that is providing by dispatching system during the mining process and retrieving from databases. Implementation of those stages is (semi)manual in available software. For this reason, Python open-source programming language implemented to provide an automatic procedure from database management and to retrieve data and input data preparation with the aim of discrete event simulation.

1. Introduction

System control and management are intensively software dependent process. These processes are handled mostly by commercial software. Commercial software refers to that kind of software which is produced for selling to the customers for the commercial purposes. Providing commercial software and programming is time, and labour-intensive process and control over this type of software can be achieved by copyright, contract law and software patent (Liberman, 1995). Technically control on commercial software based on the specific approach of end-users and development on the software structure is not possible most of the times; achieving to this objective, usually, clients have to spend more additional cost in addition to the original software price to having a suitable framework based on their system requirements.

Open-source Software (OSS) is the type of computer software that is released under a license that the users have a right of study, change, and distribute the software to several users and for any purposes (Laurent, 2004). Open-source software has properties of free redistribution, by means of having no restrictions for selling or giving away any component of that software or program code. The source code is available for this software and can be used for making any changes and derived works and it can be modified by all the users (Feller and Fitzgerald, 2002). The first open-source program was written by Ada Lovelace for calculating Bernoulli's Number using the Analytical Engine in earliest of the 18^{th} century. In the 1950s the first high-level programming language designed and developed by Germans to communicate instructions to the computer (Ceruzzi, 2003).

There are several programming languages for open-source programming such as Java, PHP, Python, Perl. Each language has its benefits and disadvantages from implementation and programming point

of view (RAO, 2014). Source code for these types of programs can be customized efficiently based on user needs and requirements and the bugs presented in the program can be found easily and fixed, and users from different countries and backgrounds can participate and collaborated. The codes from other languages can be translated whereas in case of licensed programming language it is not allowed (St. Amant, 2007). The source code utilization is unlimited for open-source code while for commercial software trial version is just available for few days only for it can be provided with a restriction on usage. The main drawback for open-source programming language is the version of the code which older versions may not be compatible with a new version of libraries and the original code should be updated after a while and messy lines of codes can be extremely confusing. Another weak point is that the solution for specific problems is highly related to the support provided by the user community. In the area of process and systems simulation there is a verity of simulation software such as AnyLogic, Arena, AutoMod, GPSS, Flexsim, ProModel, Simcad Pro, VisualSim that are commercial software and SimPy, Simula, Salabim, Simmer and etc. which are open-source software (Dagkakis and Heavey, 2016).

The Arena simulation commercial software package is the software provided by Rockwell software (2018) with the aim of simulation of processes and system performance analysis. This commercial software is widely used for in mine engineering with a purpose of mining process simulation. Each simulation process requires the pre- and post-processing stages. For achieving to this goal, Rockwell Company provided the third party packages as an add-on to Arena simulation software. Process analyzer is a tool for scenario analysis by execution of different simulation models and scenarios as useful tools for decision making. Output analyzer is a user interface that provides an easy and quick way for reviewing the simulation outputs. The main stage of every simulation process is providing the data from the database and the probability of their occurrence during the time that will be used in modelling the process. In discrete event simulation occurrence of the events are dependent on the probability of the occurrence of that specific event which is introduced to the simulation process by series of probability distribution function. There exist more than eighty different probability distribution functions that are shared in some properties with each other. Obtaining those functions require the process of retrieving data from the system of database and distribution fitting for each set of data or a process in a system. The input analyzer is a versatile tool that could provide the facility for doing this process. Arena takes the input file and tries to find the best fit for the data twelve different distribution model provided on input analyzer package and provide the best fit according to the least square error, chi-square test and Kolmogorov Smirnov test if they are available for the underlying dataset.

Real simulation process with high precision and detailed modelling requires a detailed analysis of the data that should be prepared from available databases or sampling on the real process. After data preparation, the files in *.csv or *.txt format take as an input for the analysis. In case of having several files, all these processes should be done manually, and all the expressions must be saved in an excel archive files and call by using VBA programming or manually to their place in the software which in case of having hundreds of data sets will be a time-consuming process.

In mining process simulation data for discrete simulation process are usually obtain from dispatching database. These databases consist of thousandths of records for several tables containing multiple columns. Processing these data and preparation is a complex task for modellers (Fig 1).

The most proper way of retrieving these data is the implementation of a database management system and link them to the input analyzer in order to have a rapid data derivation from the database management system. A relational database management system (RDBMS) usually use for this purpose. Retrieving data from RDBMS requires using the database abstraction layer (DBAL) and writing the quarries directly on the command line or indirectly on database management software (Coronel and Morris, 2016). According to DB-Engines, in June 2018, the most widely used systems were Oracle, MySQL (Free software), Microsoft SQL Server, PostgreSQL(Free software), IBM DB2, Microsoft Access, and SQLite (Free software).

There is not any coherent and direct link between database management software and input analyzer in Arena simulation software. Making this link requires the establishing the multiple links between database and Arena simulation. Stages consist of connecting to the database, running the queries, retrieving the data, data manipulation and saving them in a required format (asci, *.txt, *.csv) as an input for input analyzer and obtaining the probability distribution function the underlying variables or datasets.

The common process consists of generating the database from dispatching data, linking the database to Matlab database analyzer and writing the queries using a Matlab mathematical programming language and saving the files and manually import them to input analyzer. After fitting the distribution to each data set by retrieving from queries the user should save the expressions in a file for using in the simulation process. The process consists of several manual procedures in the whole of the process.

In this paper, the open-source Python programming language is used with the aim of generating a comprehensive and coherent process as a replacement for the input analyzer having the capability of processing the several datasets automatically. Our objective is to retrieve the required data from the database and directly obtain the distribution function expression file that is required for the simulation model. In this process, all the intermediate stages combine or omitted. The process is faster and the outputs can be used directly for simulation modelling process in Arena software. The comparison between the Matlab and Python process illustrated in Fig 1.

2. Methodology

Python as an open-source programming language provided different libraries for different purposes. As our goal is to design a process for retrieving the data and then obtain the most fitted distribution function to the data, it is necessary to use the following packages.

NumPy is the fundamental package for scientific computing in Python providing the arrays and matrices along with a large high-level mathematical function to operate on those arrays. This package can be replaced with the computational abilities of Matlab as their functions are C optimized. *Pandas* is a package for open-source data analysis and manipulation tools for tabular data. Dataframe and time series object is the main data structure on panda libraries. *Matplotlib* is the plotting library for the Python programming language that is synchronized with NumPy package. *Pyodbc* is an open-source Python module that provides access to the ODBC databases. These packages would be a replacement for Matlab Database explorer package. *SciPy* it is part of NumPy array object that is designed for scientific computing and technical computing. Some of the key algorithms of SciPy are hierarchical clustering, vector quantization, K-means, Discrete Fourier Transform algorithms, interpolation tools, optimization algorithms including linear programming, etc.

In our research, the SciPy statistic module is the main class that is used for our distribution fitting goal. More than eighty continuous random variable (RVs) and ten discrete random variables have been implemented in SciPy package. The process of retrieving data and distribution fitting is based on pseudo code illustrated in Table 1.

One of the major goals is to test how the obtained data from Python can be feed to Arena simulation as a distribution function as their parameters and the way to obtain them is a black box in Arena simulation as commercial software. For this reason, using the similar synthetic dataset, the fitted distribution in each case should compare with results in both Python and Arena software outputs. For this aim random variable for each distribution function generated with parameters mostly similar to the typical and standard shape of each distribution. For instance, in case of normal distribution synthetic normal data generated using Arena and then based on normal synthetic dataset normal distribution fitted on data using Python and Arena input analyzer package assuming that our generated random variable statistically accepted and match with a theoretical distribution. Parameters

for each model registered and compared with each other in order to match the parameters. Summary of the parameters and test results are shown in Table 2.

Table 1 Pseudo code for retrieving data and Python approach

Pseudo code in Python

```
Import Libraries
For i in a number of queries:
    Connect to database or server using Pyodbc
    Input SQL queries
    Execute SQL queries
    Save data in a temporary variable
    Close the Connection
    If data continuous:
      If data sufficient for draw distribution:
      Call best Fit Function:
            Generate a histogram of data obtain classes and their bin values
             Set best fit as normal and error infinity
            For j in the distribution model
                Try:
                Obtain fit parameters corresponding to the model
                Obtain theoretical PDF of the model
                 Compare Experimental Pa DF with a theoretical model
                Calculate squared error
                IF Error is less than the last one
                    Set the best distribution as the current
                    Set the parameter as the current parameter
                    Set error as current error
                Except:
      Calculate the Chi-square and Kolmogorov-Smirnov test
      Return best distribution, Errors, Test results
    elif:
       Consider Mean of data as a random exponential occurrence EXPO ()
    else:
      Calculate Discrete Empirical Function
    Save temporary model
    Plot and save the Figs
Write-Output Files
```

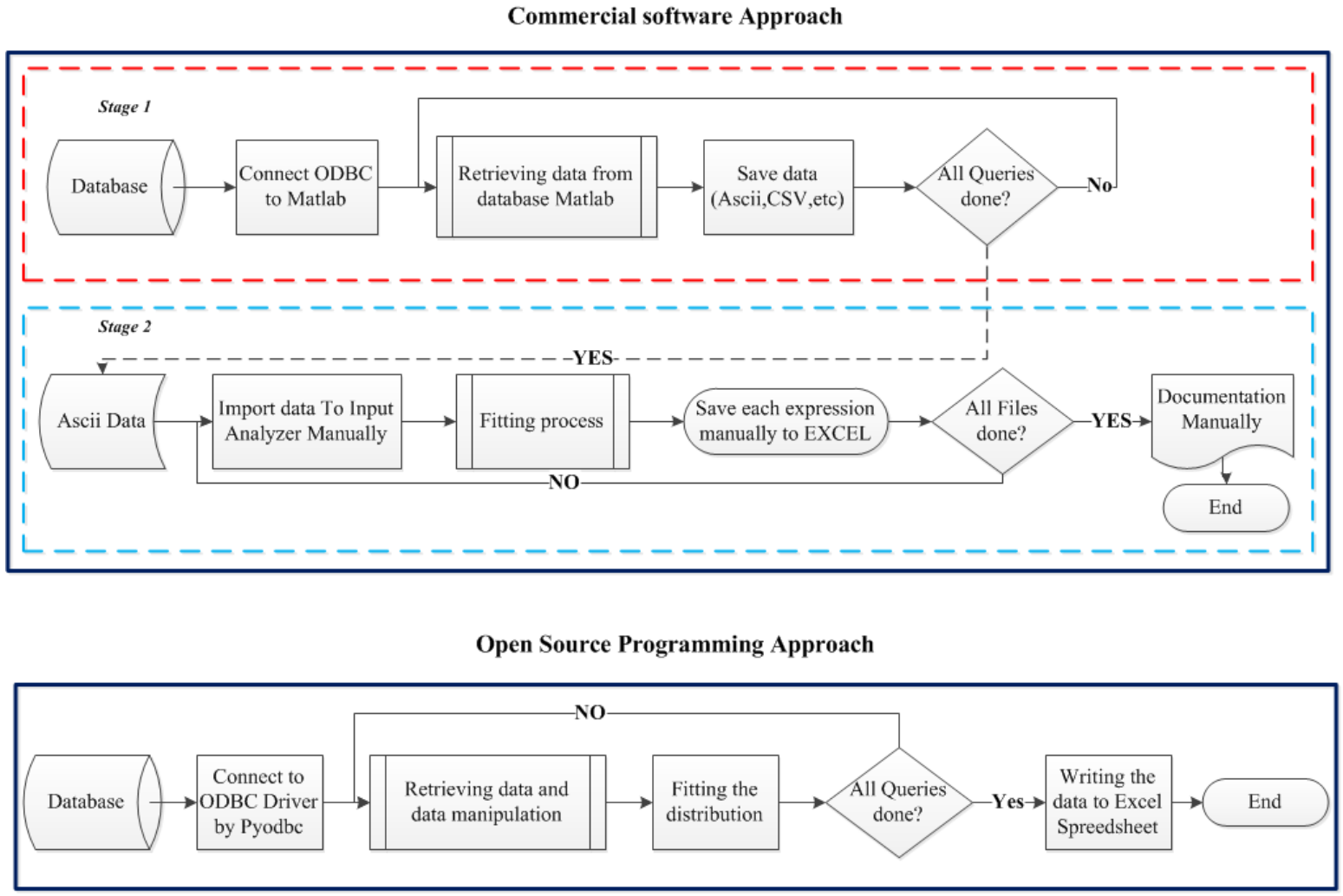


Fig 1 Comparison of data extraction and distribution model fitting for Matlab and Python

Table 2 Comparison of the distribution model output parameters for Arena input analyzer and Python

Distribution	Arena model	Python model	Link Python output to Arena
BETA	Min+Max*BETA(Alpha1, Alpha2)	beta(a, b, loc, scale)	Loc+Scale*BETA(a,b)
Erlang	Offset+Erlang(Exponential Mean, Erlang parameter k)	erlang(a, loc, scale)	Loc + erlang(scale,a)
Exponential	Offset + EXPO(Mean)	expon(loc, scale)	Loc + EXPO(scale)
Gamma	Offset + GAMM(Scale Parameter Beta, Shape Parameter Alpha)	gamma(a, loc, scale)	Loc + gamma(scale,a)
Normal	NORM(Mean, Standard Deviation)	norm(loc, scale)	NORM(loc,scale)
Triangular	TRIA(Minimum Value, Most Likely Value, Maximum Value)	triang(c, loc, scale)	TRIA(loc, loc+(c*scale), scale)
Uniform	UNIF(Minimum Value , Maximum Value)	uniform(loc, scale)	UNIF(loc,scale)
Weibull	WEIB(Scale Parameter Beta, Shape Parameter Alpha)	weibull_min(c, loc, scale)	loc + WEIB(scale,c)
Johnson	JOHN(Shape parameter 1(Gamma), Shape parameter 2(Delta),Scale Parameter Lamda, Location Parameter Xi)	johnsonsb(a, b, loc, scale)	JOHN(-2.19, 1.35, 2.60, 0.21)
Lognormal	Offset +LOGN(LogMean ,LogSTD)	Lognormal(s, scale, location)	No direct conversion

Table 3 Fitted model for normal distribution using synthetic data

Distribution	Parameter	Arena model	Arena Square Error	Python model	Python Square Error	Link Python output to Arena
Normal	Mean = 2.0, Std Dev = 10	NORM(1.71,9.98)	0.000077	NORM(1.746,9.712)	0.00006	NORM(1.746,9.712)

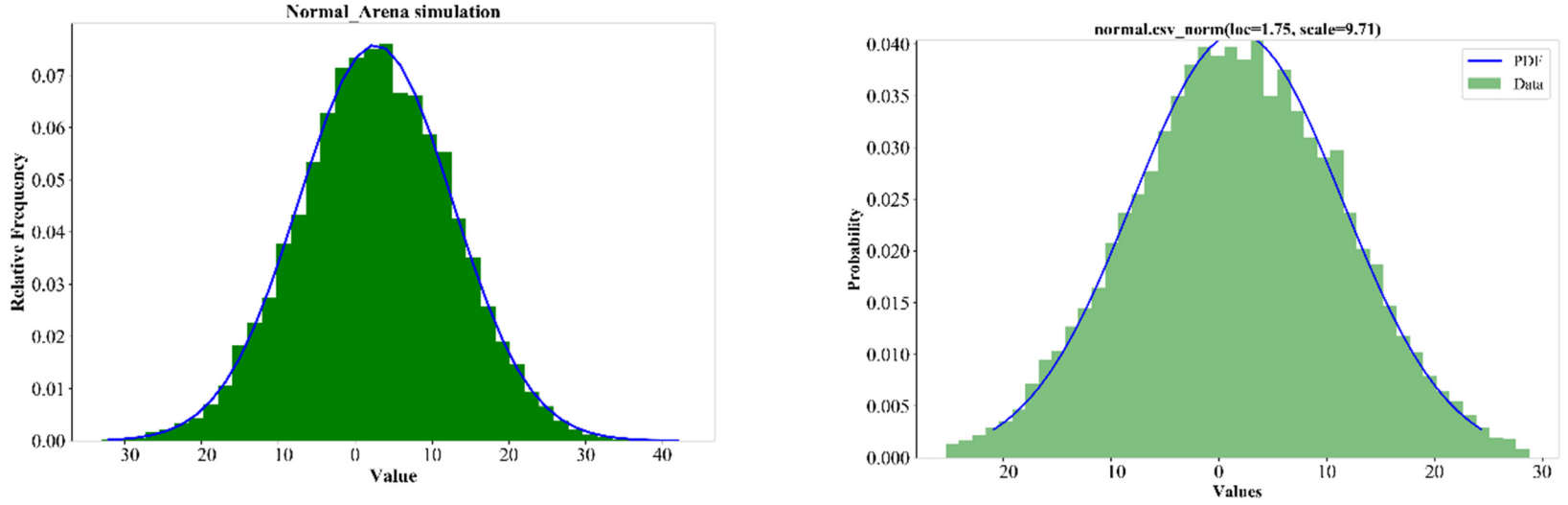


Fig 2 Comparison between input analyzer output and Python for normal distribution using synthetic data

Table 4 Fitted model for lognormal distribution using synthetic data

Distribution	Parameter	Arena model	Arena Square Error	Python model	Python Square Error	Link Python output to Arena
Lognormal	LogMean = 2, logSTD =0.3 , Offset = 5	6+LOGN(1,0.328)	0.000770	Lognormal(s=0.16,loc=5.07,scale=1.90)	0.12866	4.258 + LOGN(2.73,0.285)

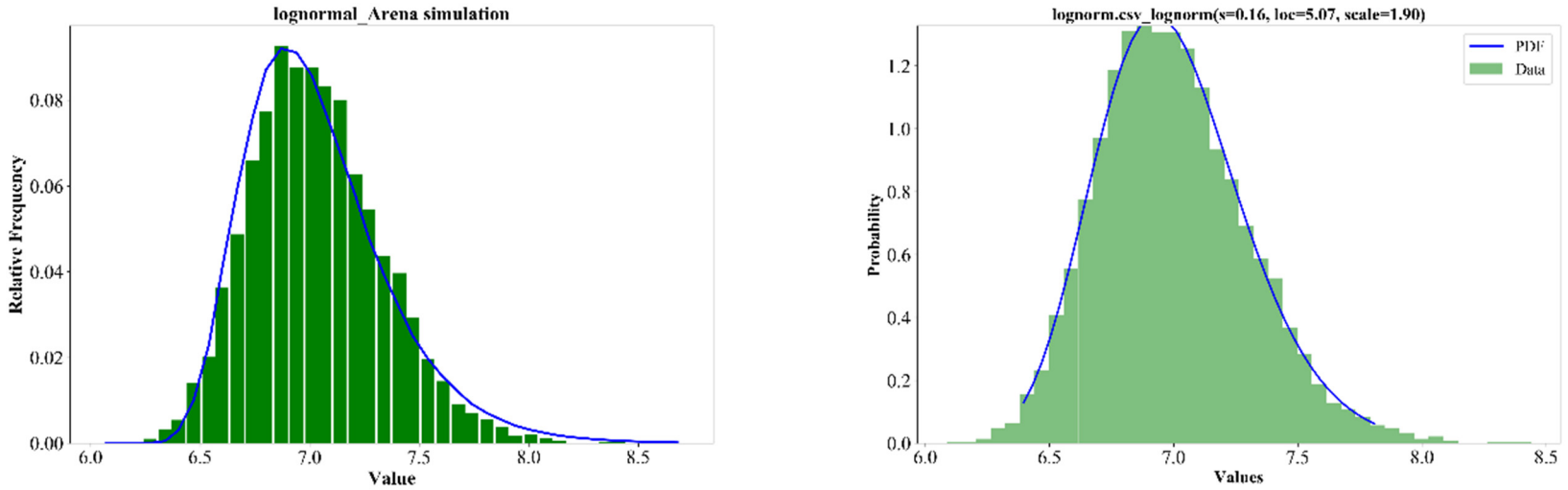


Fig 3 Comparison between input analyzer output and Python for lognormal distribution using synthetic data

Table 5 Fitted model for beta distribution using synthetic data

Distribution	Parameter	Arena model	Arena Square Error	Python model	Python Square Error	Link Python output to Arena
Beta	Alpha1= 0.5, Alpha2=0.5, Min=0, Max=2	2 * BETA(0.547,0.59)	0.001042	Beta(a=0.54,b=0.54,loc=0,scale=2.0)	1.00973	-0.002+ 2.002 * BETA(0.539,0.54)

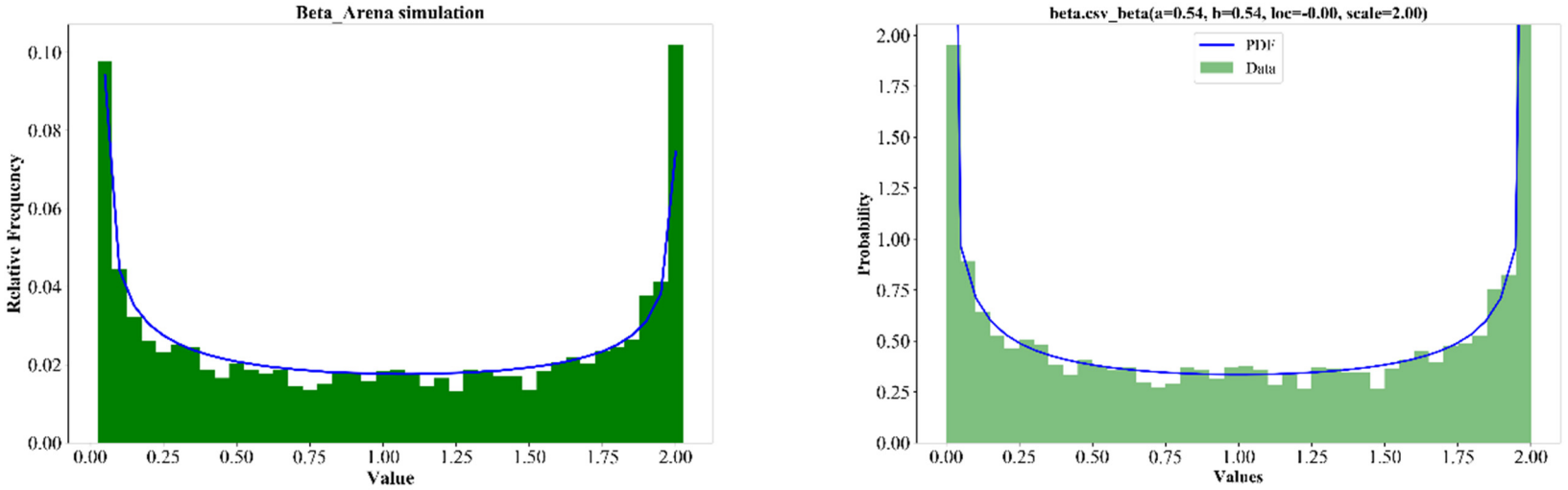


Fig 4 Comparison between input analyzer output and Python for beta distribution using synthetic data

Table 6 Fitted model for Gamma distribution using synthetic data

Distribution	Parameter	Arena model	Arena Square Error	Python model	Python Square Error	Link Python output to Arena
Gamma	Alpha = 2 beta=4 offset=5	5 + GAMM(4.04, 1.98)	0.000121	Gamma(a=1.97, loc=5.01,scale=4.05)	0.000725	5.011 + GAMM(4.055,1.971)

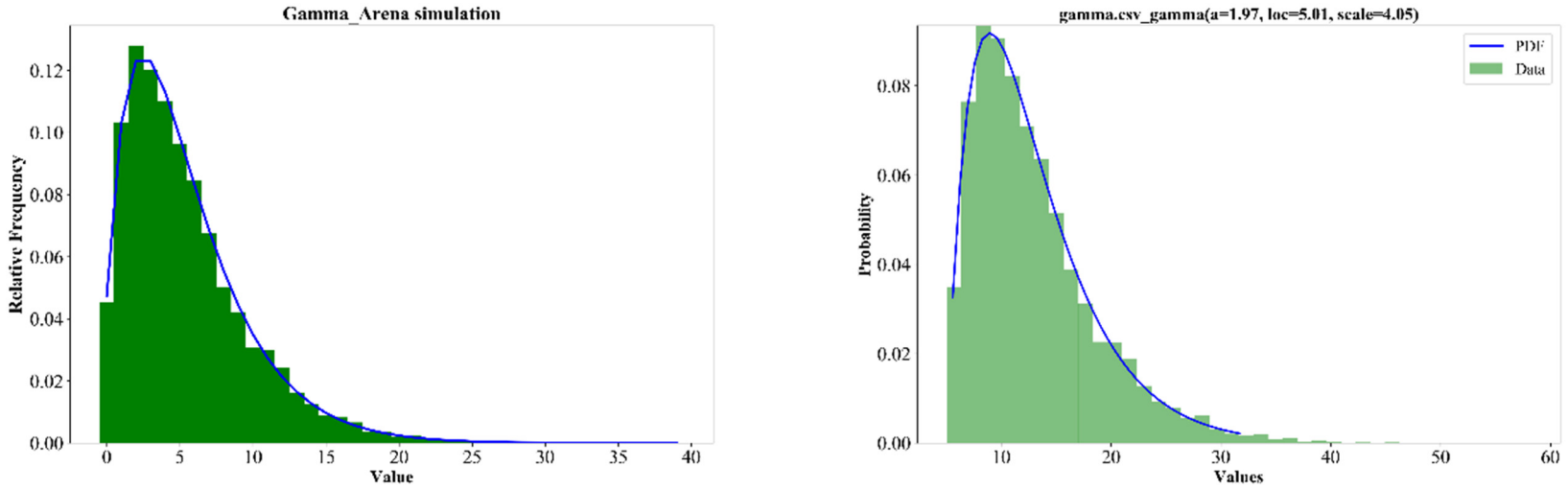


Fig 5 Comparison between input analyzer output and Python for gamma distribution using synthetic data

2.1. Model accuracy and goodness of fit

The goodness of fit is the statistical model provide the numbers for expert's judgment to assess how good our distribution fitted model is matched with the theoretical model. These values show the discrepancy between the observed values and the values obtained from the theoretical model. There are several ways of performing these tests. In Arena simulation, the tests provided by software are a square error, Chi-square test, Kolmogorov-Smirnov method. In the next section, a brief description of these tests is provided. According to the Arena's manual, the distribution fitting (curve fitting) is based on maximum likelihood estimators, and the beta, triangular and uniform distributions are exceptions. The beta distribution is fitted first with maximum likelihood estimators and then moments, and then the best result among them will accept as the main result. Triangular and uniform distributions use empirical rules to fit distributions on data. According to SciPy reference fitting distributions is based on maximizing a log-likelihood function and the outlier's values receive the penalty. The answer is not globally optimal, and it is only locally optimal or optimization may fail altogether.

2.1.1. Square error measurement

The quality of the distribution fitting primarily is based on the square error criterion. This condition can be defined as a sum of the square difference between the relative frequency of the data for ith interval and relative frequency of the fitted probability distribution function. This relationship can be defined as follows:

$$LSQ = \left[f_i - f(x_i)\right]^2 \tag{1}$$

Where f_i is relative frequency of data on interval ith and $f(x_i)$ is the relative frequency of the fitted probability distribution. $f(x_i)$ is obtained from cumulative distribution function by the difference of the two intervals $F(x_i) - F(x_{i-1})$ in a case that the cumulative distribution function is not available the $f(x_i)$ will be determine by numerical integration (Corder and Foreman, 2014; Bethea, 2018).

2.1.2. Kolmogorov-Smirnov Goodness-of-fit test

The Kolmogorov-Smirnov test is used to decide that if the sample under study comes from the same population for a specific distribution or not. The test is based on the empirical distribution function. The Kolmogorov-Smirnov test can be defined according to the following formulation

H_0: the data follow a specified distribution

H_1: the data do not follow the specific distribution

The Kolmogorov-Smirnov test statistic is defined as:

$$D = \max_{1 \le i \le N} \left(F(Y_i) - \frac{i-1}{N}, \frac{i}{N} - F(Y_i)\right) \tag{2}$$

Where F is the theoretical cumulative distribution function of the underlying data which should be in the form of the continues data and N is the number of samples. The test is a nonparametric test which Gaussian distribution assumption is not required (Corder and Foreman, 2014; Bethea, 2018).

2.1.3. Chi-square test

The goal of the test is same as the Kolmogorov-Smirnov test; this test can be applied to any univariate distribution if the cumulative distribution function existed. In our case, the test applied to the binned data which is calculated from the histogram of the experimental data. So the value that is obtained for the test depends on the number of binned data, and for this reason, sufficient data should be available in order to the chi-square test be approximately valid. This test can be applied to discrete distribution.

H_0: the data follow a specified distribution

H_a: the data do not follow the specific distribution

The test is performed on the data that are divided into k bins a, d the test is defined as **Error! Reference source not found.**):

$$\chi^2 = \sum_{i=1}^{k} (o_i - \mathrm{E}_i)^2 / \mathrm{E}_i \tag{3}$$

Where o_i is the observed frequency for bin i_{th} and $\mathrm{E}_i$is the expected frequency for bin i^{th} and the expected frequency is calculated from the interval between the lower and upper bound of i^{th} bin and n is number of data. As explained before the test is sensitive to number of bins and there is not any optimal value for the test output (Bethea, 2018).

3. Case studies

3.1. Synthetic data

In order to test the compatibility of using SciPy as a replacement of input analyzer synthetic data generated to performing the tests and comparing the results. Both input analyzer and SciPy have an ability to generating the random distribution data based on specific distribution. For our goal, the synthetic data generated using the input analyzer and for each specific distribution typical parameters assigned in order to generate the typical distribution shapes for each specific distribution types. After generating the data fitting and corresponding shape parameters obtained and the fitting tools forced to fit the corresponding model that is directly related to the data. The same process performed by SciPy package. On the next step, the shape parameters compared with those which obtained from an input analyzer. The results show the difference for lognormal distribution as the Arena software's conversion for mean and standard deviation is different from SciPy libraries. According to SciPy documentation, the parametrization of random variable X with mean μ and standard deviation of σ is expressed as a $expo(Y) = X$ then the S as a shape parameter is equal to σ and the scale is $scale =$ $\exp(\mu)$. On the other hand, in Arena input analyzer conversions are as **Error! Reference source not found.**) to **Error! Reference source not found.**):

$$LogMean = \mu \, , LogStd = \sigma \tag{4}$$

$$\mu = \ln\left(\frac{\mu^2}{\sqrt{(\sigma^2 + \mu^2}}\right) \tag{5}$$

and

$$\sigma = \sqrt{\frac{\ln(\sigma^2 + \mu^2)}{\mu^2}} \tag{6}$$

Solving the system of equations, the values obtained by SciPy can be converted to Arena formulation as **Error! Reference source not found.**):

$$\sigma = \sqrt{e^{2S} e^{2scale} (e^{s2} - 1)} \tag{7}$$

$$\mu = \sqrt{\mathrm{e}^{\mathrm{s}2} \, \mathrm{e}^{2\mathrm{scale}}} \tag{8}$$

3.2. Mining real dataset

Real dispatching data from an open pit mining system used as a database. Dispatching database contains a table for production, Activity and Shovel_In_Opereation and Trucks_In_Operation retrieved from the Production table which contains 1,468,959 number of rows and retrieving final necessary data from those tables. The logic for the database is illustrated in Fig 6.

Data and queries write for two different seasons (summer and winter) based on shovels and trucks group consist of six shovel types and six truck types. Number of passes for each truck and shovel, average bucket tonnage for trucks, average loading cycle for shovels, the total cycle time for trucks are the parameters which are retrieving from the database.

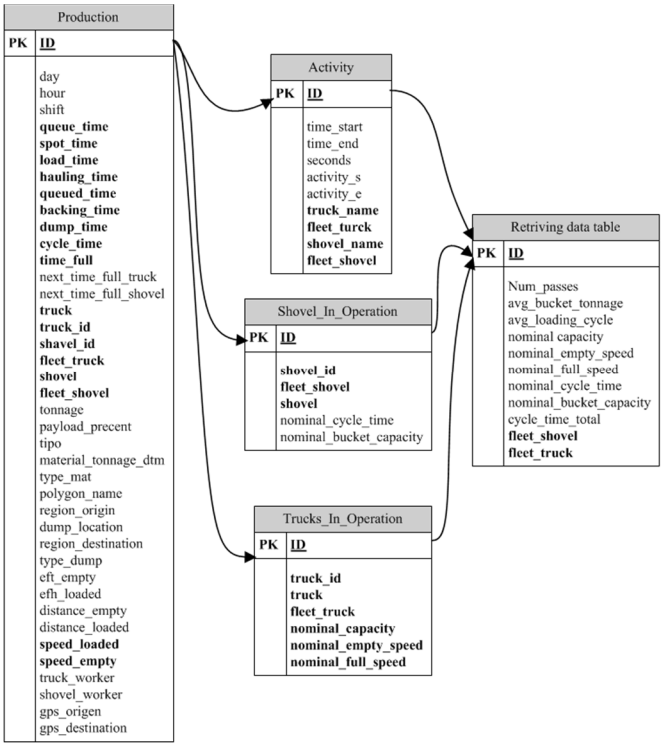


Fig 6 database diagram and relationships between each table for mining database

From the database and running related queries more than 270 series of data obtained that are showing the behaviour of shovel and trucks and their related parameters in two different seasons. The simulation model will recognize these behaviours based on a random distribution function. So it would be necessary to find the best-fitted distribution function model for the data. The process using Arena input analyzer is not time efficient, so data retrieving from database analyzed with the code that is provided by Python library, and final results write into CSV file. The fitted parameters acceptable as an input for Arena software are shown in Table 7 for discrete and continues variables with a related fitted distribution on Figs 7 and 8.

Table 7 Retrieved data and their fitted distribution with its parameters

Retrieved data	Best distribution	Expression	Least Square Error
Average_ Load_CycleTime _summerData_992GHL_CAT77D	gamma	-21.103 + GAMM(4.976,11.239)	0.000517865
Average_Load_CycleTime_winterData_Hit 5500EX_CAT 793D	norm	NORM(53.792,13.388)	0.000445379
backing_time_summerData_CAT 793D	lognorm	-11.024 + LOGN(24.36,6.468)	0.017501618
backing_time_winterData_CAT 793DT	lognorm	-9.064 + LOGN(24.701,7.607)	0.006585836
speed_Empty_winterData_CAT 793DT	norm	NORM(36.952,8.073)	0.000889691
speed_Empty_winterData_Cat 793B	norm	NORM(32.726,9.802)	0.000368787
SpotTime_summerData_CAT 785C_Hit 5500EX	lognorm	0.025 + LOGN(47.374,54.748)	0.000209112
SpotTime_summerData_CAT 793B_Hit 5500EX	lognorm	-0.269 + LOGN(64.171,77.815)	7.91092E-05
Avg_Bucket_Ton_summerData_Hit 5500EX_CAT 785C	lognorm	45.228 + LOGN(26.351,5.923)	0.00641712
Avg_Bucket_Ton_winterData_Hit 5500EX_CAT 793DT	norm	NORM(80.001,3.894)	0.085005238
NumPass_winterData_CAT 994F_CAT 793B	discrete cumulative	DISC(0.0,0,0.007,1,0.035,2,1.0,3)	
NumPass_summerData_Hit 2500_CAT 793D	discrete cumulative	DISC(0.005,0,0.075,1,1.0,2)	

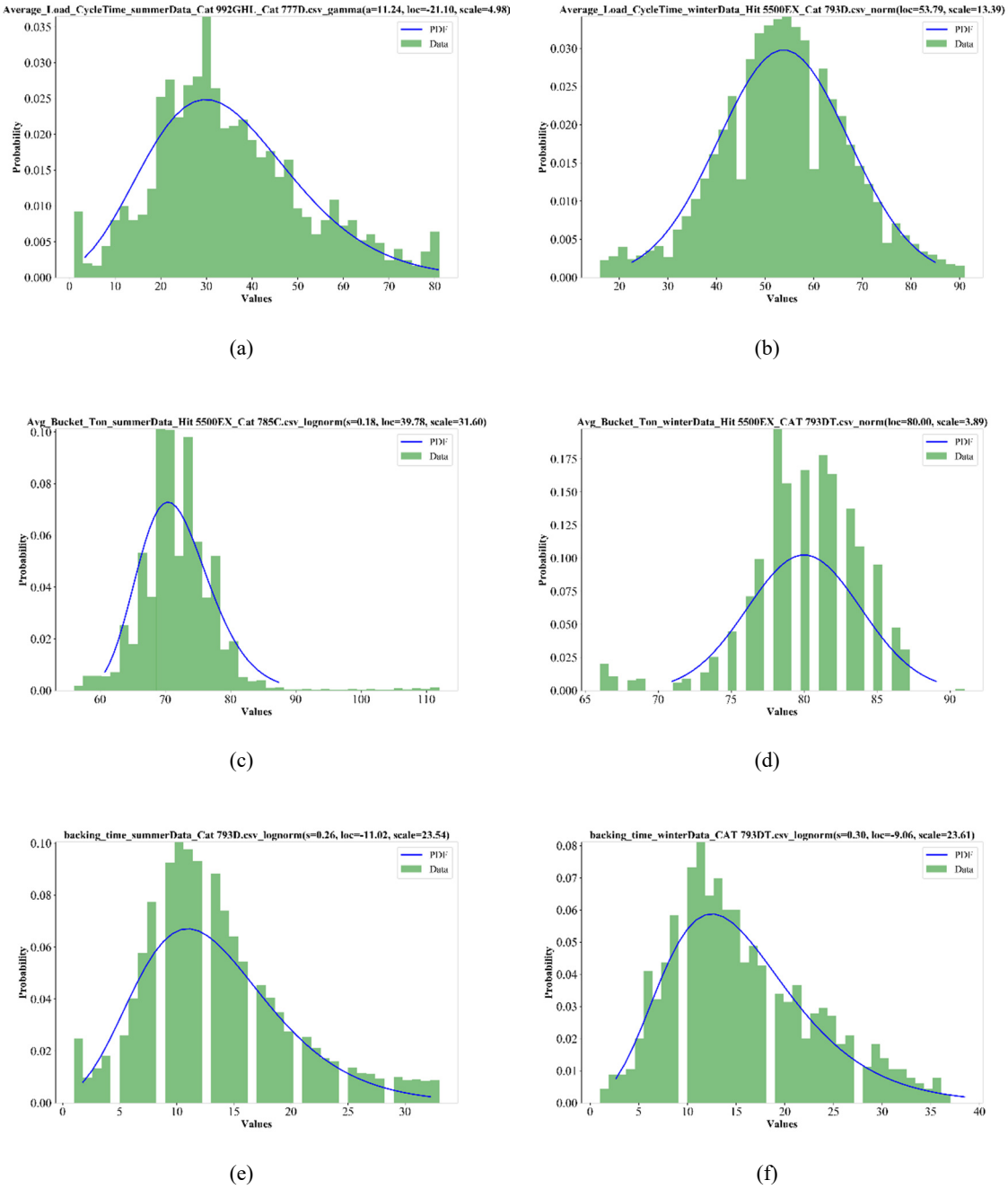


Fig 7 fitted distribution for average load cycle time for shovel CAT 992GHL and truck CAT 777D summer (a) and average load cycle time for winter data for shovel Hit 5500EX truck CAT 793D (b), average bucket tonnage for summer data shovel Hit 5500EX truck CAT 785C (c) and average bucket tonnage for winter data Hit 5500EX CAT 793DT (d), backing time for summer data truck CAT 793D (e) and backing time for winter data CAT 793DT (f).

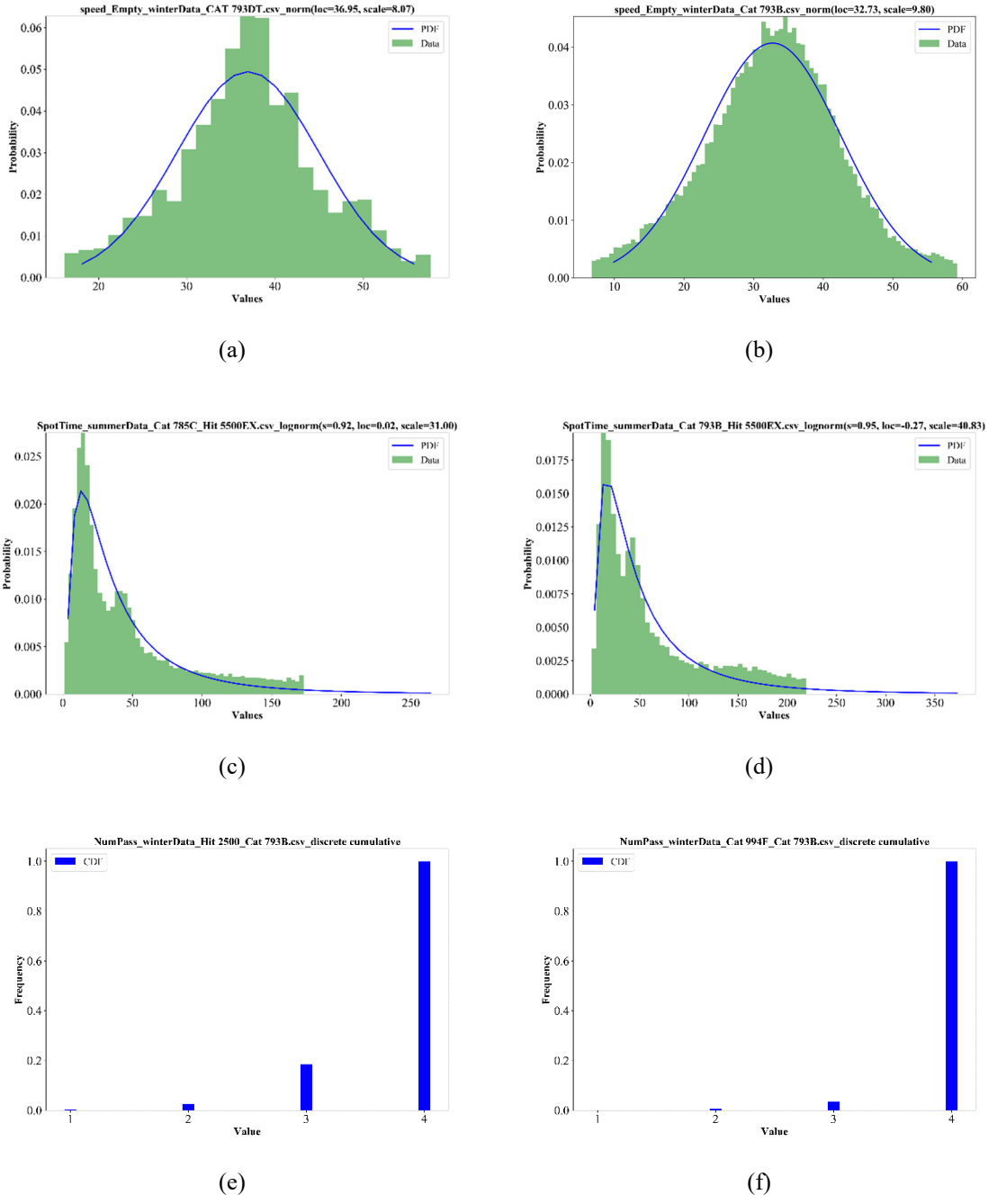


Fig 8 Fitted distribution for speed empty travel on winter for CAT 793DT (a) and for CAT 793B (b), spot time for summer data on shovel Hit 5500EX for truck CAT 785C (c) and truck CAT 793B (d), number of passes for winter data on shovel Hit 2500 truck CAT 793B (e) and shovel CAT 994F truck CAT 793B (f)

4. Conclusion

Open-source programming using Python programming language implemented for pre-processing analysis of data retrieving from dispatching database. The code provided has an ability to retrieving the data from the database and fit the distributions automatically to the data and generate the file consist of distribution expressions which are required for performing the simulation. The results show that the data manipulation such as removing outliers, duplicate values, bimodality, etc. can have a great effect on output results. Arena simulation software supports a restricted range of distributions and is not able to fit and use bimodal distributions model for the simulation process.

For this reason, after retrieving data from the database, the data should belong to the same population and check for the corresponding hypothesis. In case of not having a sufficient number of data the number of bins can change to the critical parameters as the current methods for automatically choosing the number of bins will be failed to obtain the optimum number of the bins and expert's ideas will be required and the process will be performed by trial and error for choosing the suitable number of bins. The advantage of the method, using Python versus Matlab programming language, is providing more coherent and faster process for pre-processing of the data with the lowest cost for time and money.

5. References

[1] Bethea, R. M. (2018). *Statistical methods for engineers and scientists.* Routledge,

[2] Ceruzzi, P. E. (2003). *A history of modern computing.* MIT press.

[3] Corder, G. W. and Foreman, D. I. (2014). *Nonparametric Statistics: A Step-by-Step Approach.* Wiley.

[4] Coronel, C. and Morris, S. (2016). *Database systems: design, implementation, & management.* Cengage Learning.

[5] Dagkakis, G. and Heavey, C. (2016). A review of open source discrete event simulation software for operations research. *Journal of Simulation*, *10* (3), 193-206.

[6] Feller, J. and Fitzgerald, B. (2002). *Understanding Open Source Software Development.* Addison-Wesley.

[7] Laurent, A. M. S. (2004). *Understanding Open Source and Free Software Licensing: Guide to Navigating Licensing Issues in Existing & New Software.* O'Reilly Media.

[8] Liberman, M. (1995). Overreaching Provisions in Software License Agreements. *Richmond Journal of Law & Technology*, *1* (1), 4.

[9] RAO, M. N. (2014). *FUNDAMENTALS OF OPEN SOURCE SOFTWARE.* PHI Learning,

[10] Rockwell software (2018). Arena simulation. Rockwell software, Retrieved 2018 from: https://www.arenasimulation.com/

[11] St. Amant, K. (2007). *Handbook of Research on Open Source Software: Technological, Economic, and Social Perspectives: Technological, Economic, and Social Perspectives.* Information Science Reference.