Mining Optimization Laboratory

Report Six –2014/2015

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Executive Summary

This year, we have prepared a report including 14 papers. We continue to update all the research results on the MOL webpage www.ualberta.ca/mol on the members section. Sponsors have access to current and past research results, publications, prototype software, and source code.

Mohammad Mahdi Badiozamani defended his PhD thesis in June 2014. Eugene Ben-Awuah, who was working with Snowden consulting company (Perth, Australia), was appointed to a tenure-track assistant professor position in the Bharti School of Engineering at the Laurentian University in September 2014. Mohammad Tabesh successfully completed his PhD in September 2015. Let's review the contributions in the MOL Report Six (2014/2015) by considering some of the main contributors.

In paper 101, ***Mohammad*** presents a multi-step approach that starts form a pushback design procedure that is able to create pushbacks with controlled ore and waste tonnage. This is a hybrid algorithm that incorporates mathematical programming with heuristics to tackle large-scale problems. It benefits from the special structures of the mathematical formulation to calculate relaxation bounds and uses heuristics to provide near-optimal solutions. Also, a hierarchical clustering algorithm that creates mining units (polygons) with minable shapes and homogeneity in rock-type and grade within the boundaries of generated pushbacks is presented. Finally, a mathematical model that provides a long-term mining schedule based on the generated mining units as well as the created pushbacks is introduced. The proposed formulation considers various mining and processing constraints and is able to include stockpiling in long-term plans to improve the blending. Mohammad also has developed a proto-type software application which is introduced in paper 402. The application has a graphical user interface for easy use. The application is designed to handle all the steps of importing blocks data from various sources, clustering blocks with two different techniques, setup MILP parameters and interpret and plot results.

Shiv presents a mixed-integer linear goal programming (MILGP) model to optimize the open-pit mine production operation in paper 201. The optimization is based on four desired goals of the company to: a) maximize production, b) minimize deviations in head grade, c) minimize deviations in tonnage feed to the processing plants from the desired target feed, and d) minimize operating cost. The model provides shovel assignments and the target productions; as an input to the dispatching system while meeting the desired goals and constraints of the mining operation. This paper, also presents results of the model implementation with an iron ore mine data. To develop a simulation model, exercising control on the truck movements and capturing interactions, first the haul road network and its characteristics within the simulation model should be modeled and then the trucks should be moved through the road network based on truck characteristics and the haul road gradient, rolling resistance, turning angle and interactions with other trucks. In paper 403, ***Shiv*** describes a method to generate the haul road network within Arena using a MATLAB application with GUI. Then he discusses a MATLAB application to generate the speeds of trucks on haul roads based on rimpull characteristics of the trucks and various haul road gradients.

Firouz has been carrying out research on block-cave production scheduling using mathematical programming (paper 306). He has reviewed the mathematical models and algorithms in block-caving scheduling (paper 302). He also presents a method for determination of development precedence for drawpoints in block-cave in paper 304. ***Firouz***, also models the production scheduling in block-cave mining to maximize the net present value of the project using MILP and also implements MIQP as non-linear tool to minimize the difference between the objective and the target tonnage of the mining project considering the related constraints of the operations (Paper 306). The models have the same constraints with different objective functions. This paper also, presents a model application of a production schedule for 102 drawpoints over 5 periods. Results show that the MILP model tries to produce more in first years with higher grades. This will result in ununiformed extraction profile with high probability of dilution. The MIQP model extracts from the drawpoints smoothly with a very low fluctuation of tonnage and grade during the life of mine.

Saha has been working towards development of a methodology to find the best extraction level under grade uncertainty for block-cave mining (paper 305). The main goal of the study is to develop a framework to find the best level of extraction under grade uncertainty. In this paper, several realizations are modelled by using geostatistical studies to consider the grade uncertainty. After determining the best extraction level, the production schedule is generated for the given advancement direction and in presence of some constraints at the extraction level using a mixed-integer linear model.

Ali has started his research in truck shovel allocation and dispatching. He has done a literature review on open-pit mine production optimization (paper 202). This paper shows that from applicability point of view there are two major groups of algorithms: a) the group are being widely used in the mining projects, and b) the group of algorithms developed in academia. The paper first discusses main industrial algorithms, then after reviews well-known academically developed algorithms. Strengths and weaknesses of the algorithms are discussed and suggestions for the future researches are presented.

Eduardo, ***David***, and ***Miga*** have joined us as MSc students in September 2015 with Mining Engineering background.

Yashar Pourrahimian
Eugene Ben-Awuah
Hooman Askari-Nasab
September, 2015

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Table of Contents

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Mining Optimization Laboratory (MOL) Researchers / Graduate Students

Following are researchers and students affiliated with Mining Optimization Laboratory in September 2015.

1.	**Hooman Askari-Nasab**	Associate Professor - Director of MOL
2.	**Yashar Pourrahimian**	Assistant Professor, University of Alberta
3.	**Eugene Ben-Awuah**	Assistant Professor, Laurentian University
4.	**Kwame Awuah-Offei**	Associate Professor of Mining Engineering

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5.	**Mohammad Tabesh**	PhD – 2009/09
6.	**Shiv Prakash Upadhyay**	PhD Candidate – 2011/09
7.	**Enrique Jelvez Montenegro**	PhD Student – 2012/03
8.	**Cuiwei Feng**	MSc Student – 2013/09
9.	**Firouz Khodayari**	PhD Student – 2014/01
10.	**Ali Moradi Afrapoli**	PhD Student – 2014/09
11.	**Nisarg Tripathi**	MSc Student – 2014/09
12.	**Saha Malaki**	MSc Student – 2014/09
13.	**Myagmarjav Batsukh**	MSc Student – 2015/09
14.	**David Omane**	MSc Student – 2015/09
15.	**Eduardo Cervantes**	MSc Student – 2015/09

Production Scheduling Optimization with Stockpiling

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ABSTRACT

Strategic mine planning is a complicated process that is usually broken down into smaller problems in order to get more practical solutions in shorter time. In this paper, we present a multi-step approach that starts form a pushback design procedure that is able to create pushbacks with controlled ore and waste tonnage. This is a hybrid algorithm that incorporates mathematical programming with heuristics to tackle with large-scale problems. It benefits from the special structures of the mathematical formulation to calculate relaxation bounds and uses heuristics to provide near-optimal solutions. Next, we propose a hierarchical clustering algorithm that creates mining units with minable shapes and homogeneity in rock-type and grade within the boundaries of generated pushbacks. The third step of our solution procedure is to form a mathematical model that provides a long-term mining schedule based on the generated mining units as well as the created pushbacks. Our proposed formulation considers various mining and processing constraints and is able to include stockpiling in long-term plans to improve the blending. Finally, we use the idea of piecewise linearization to modify the model to be able to solve it with mixed integer linear programming solvers. The model is tested on a synthetic dataset to evaluate the performance of the model and show the errors introduced by linearization.

1. Introduction

Open-pit mining is the most common and the oldest method of mining valuable material from the ground. It has attracted many researchers to study various aspects of the operation such as production planning, truck-shovel allocations, risk analysis and grade blending. Various heuristic, meta-heuristic and mathematical programming techniques have been implemented on these areas to improve the operation. Their goals are to maximize profit, minimize costs and to optimize the utilization of the resources or the outcome of the mining operation. The mine planning problem has also been studied in different time frames. Long- to short-term plans are usually determined based on different levels of details. Long-term plans usually deal with larger units of production and decide when to extract material and where to send them. Short-term plans, on the other hand, deal with smaller units and make more detailed decisions on the production levels, blending, truck-shovel allocations etc. In this paper, we present a multi-step hybrid approach to deal with the long-term multi-destination open-pit production planning problem by creating controlled pushbacks and

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Some parts of this paper are direct excerpts from the paper presented in APCOM 2015 (Tabesh M., et al., 2015) aggregated mining units and using mathematical programming to solve the problem. We incorporate the blending constraints and stockpiling in the long-term mine planning decisions to improve the operation and help the mine planners decide if they want to use stockpiles in the operation.

Mathematical programming is not new to mine planning researchers. Johnson (1969) introduced mathematical programming and in particular linear programming to the mine planning research area. He proposed a linear programming model for the long-term multi-destination open-pit production planning problem along with a decomposition approach to solve the problem. However, this initial model was using continuous variables to control precedence constraints which would result in partial extraction of blocks and infeasible solutions (Gershon, 1983). Although, introducing binary variables to control the block extraction precedence can solve this problem, it will create another obstacle on the way: curse of dimensionality. In other words, introducing binary variables will make the problem NP-Hard and impossible to solve for real size block models. Therefore, the focus of mine planners in the past few decades has been on breaking the problem into smaller problems, reducing the size of the problem or finding near-optimal solutions to the problem. Interested readers are referred to Osanloo et. al (2008) and Newman et. al (2010) for a complete review on the applications of operations research and mathematical programming on the mine planning problem. On the other hand, most of the proposed models incorporate mining, processing and precedence constraints and do not include grade blending and stockpiling constraints.

2. Pushback design

Pushback design is an important step in LTOPP in which the phases of production, pushbacks, are determined. The intersection of pushbacks and mining benches are called bench-phases. These are the most common units of long-term planning in open-pit mines. From manual methods such as fixed lead to more advanced heuristics such as Milawa (Geovia, 2012) use the bench-phases to optimize the long-term open-pit production plan. Therefore, how the pushbacks are defined can significantly affect the output. In this paper, we are using a hybrid heuristic-binary programming approach from Mieth (2012) to create the pushbacks. The pushback design procedure is explained in details in Mieth (2012) and Tabesh et al. (2014). The generated bench-phases are then used as units of mining and as boundaries for clustering.

3. Clustering

Clustering is the process of grouping similar objects together in a way that maximizes the similarity between the objects of the same cluster and the dissimilarity between the objects of different clusters. However, the clustering algorithm we used here not only accounts for the similarities but also respects the size and shape constraints. The clustering algorithm mentioned is a variation of hierarchical agglomerative clustering and is thoroughly explained in Tabesh and Askari-Nasab (2011) and Tabesh and Askari-Nasab (2013). We use the clustering algorithm to create processing units within the boundaries of bench-phases. Therefore, the bench-phases are divided into smaller units with similar rock-type and grade which are the basis for making processing and stockpiling decisions.

4. Mathematical Formulations

As mentioned earlier, various LTOPP mathematical models have been proposed in the literature. However, none of them incorporate stockpiling in long-term planning. One major reason is that calculating the reclamation grade of the stockpiles introduces non-linearity into the model. Bley, Boland, Froyland, & Zuckerberg (2012) model the LTOPP with stockpiling by adding the non-linear constraints and proposing a problem-specific solution method. In this paper, we tried to avoid the non-linear constraints by benefiting from the piecewise linearization technique. We introduce multiple stockpiles with different acceptable grades to be able to assign fixed reclamation grades to each stockpile. These input grade ranges, as well as reclamation grades, are determined based on histograms of grades to be representative of data.

4.1. Original Model

We first present the original LTOPP mathematical model without the stockpile. The model is a multi-destination LTOPP which uses two different sets of units for making mining and processing decisions. Two sets of variables are defined for bench-phases: $y_m^t \in [0,1]$ is the portion of bench-phase extracted in each period and $b_m^t \in \{0,1\}$ is the binary variable to control the precedence. Since the number of bench-phases is less than number of blocks and clusters, controlling the precedence with bench-phases results in less binary variables and less resource consumption for solving the model. Moreover, using bench-phases as mining units is the common practice in the mining industry. However, making material destination decisions requires more accurate units with distinction between ore and waste. This is achieved by dividing every bench-phase into smaller units using clustering algorithm.

- ***Sets***

S^m For each bench-phase m, there is a set of bench-phases (S^m) that have to be extracted prior to extracting bench-phase m to respect slope and precedence constraints

U^m Each bench-phase m is divided into a set of clusters. U^m is the set of clusters that are contained in bench-phase m

- ***Indices***

$d \in \{1,...,D\}$ Index for material destinations

$m \in \{1,...,M\}$ Index for bench-phases

$p \in \{1,...,P\}$ Index for clusters

$c \in \{1,...,C\}$ Index for processing plants

$e \in \{1,...,E\}$ Index for elements

$t \in \{1,...,T\}$ Index for scheduling periods

- ***Parameters***

D Number of material destinations (including processing plants and waste dumps)

M Total number of bench-phases

P Total number of clusters

E Number of elements in the block model

T	Number of scheduling periods
$\overline{MC}^t$	Upper bound on the mining capacity in period t
$\underline{MC}^t$	Lower bound on the mining capacity in period t
$\overline{PC}_c^t$	Maximum tonnage allowed to be sent to plant c in period t
$\underline{PC}_c^t$	Minimum tonnage allowed to be sent to plant c in period t
$\overline{G}_c^{t,e}$	Upper limit on the allowable average grade of element e at processing plant c in period t
$\underline{G}_c^{t,e}$	Lower limit on the allowable average grade of element e at processing plant c in period t
s_m	Number of predecessors of bench-phase m (members of S^m)
o_m	Total ore tonnage in bench-phase m
w_m	Total waste tonnage in bench-phase m
o_p	Total waste tonnage in cluster p
w_p	Total waste tonnage in cluster p
c_m^t	Unit discounted cost of mining material from bench-phase m in period t
$r_{p,c}^t$	Unit discounted revenue of sending material from processing unit p to processing destination c in period t minus the processing costs
g_p^e	Average grade of element e in cluster p

- ***Decision Variables***

$y_m^t \in [0,1]$	Continuous decision variable representing the portion of bench-phase m extracted in period t
$x_{p,c}^t \in [0,1]$	Continuous decision variable representing the portion of ore tonnage in cluster p extracted in period t and sent to processing plant c
$b_m^t \in \{0,1\}$	Binary decision variable indicating if all the predecessors of bench-phase m are completely extracted by or in period t

- ***Objective Function***

$$\max \sum_{t=1}^{T} \left(\sum_{p=1}^{P} \sum_{c=1}^{C} (r_{p,c}^t \times o_p \times x_{p,c}^t) - \sum_{m=1}^{M} (c_m^t \times (o_m + w_m) \times y_m^t) \right) \quad (1)$$

- ***Constraints***

$$\underline{MC}^t \le \sum_{m=1}^{M}\left((o_m + w_m)\times y_m^t\right) \le \overline{MC}^t \qquad \forall t\in\{1,...,T\} \qquad (2)$$

$$\underline{PC}_c^t \le \sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right) \le \overline{PC}_c^t \qquad \forall t\in\{1,...,T\}, \forall c\in\{1,...,C\} \qquad (3)$$

$$\sum_{p\in U^m}\sum_{d=1}^{D}\left(o_p \times x_{p,d}^t\right) \le \left(o_m + w_m\right)\times y_m^t \qquad \forall t\in\{1,...,T\}, \forall m\in\{1,...,M\} \qquad (4)$$

$$\underline{G}_c^{t,e} \le \frac{\sum_{p=1}^{P}\left(o_p \times g_p^e \times x_{p,c}^t\right)}{\sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right)} \le \overline{G}_c^{t,e} \qquad \forall t\in\{1,...,T\}, \forall c\in\{1,...,C\}, \forall e\in\{1,...,E\} \qquad (5)$$

$$\sum_{t=1}^{T} y_m^t = 1 \qquad \forall m\in\{1,...,M\} \qquad (6)$$

$$\sum_{i=1}^{t} y_m^i \le b_m^t \qquad \forall m\in\{1,...,M\}, \forall t\in\{1,...,T\} \qquad (7)$$

$$s_m \times b_m^t \le \sum_{i\in S^m}\sum_{j=1}^{t} y_i^j \qquad \forall m\in\{1,...,M\}, \forall t\in\{1,...,T\} \qquad (8)$$

$$b_m^t \le b_m^{t+1} \qquad \forall m\in\{1,...,M\}, \forall t\in\{1,...,T-1\} \qquad (9)$$

The objective function (Eq. (1)) is summation of discounted revenue made from sending material to the processing plants minus the total cost of mining material from the ground. Eqs.(2) and (3) are responsible for controlling the minimum and maximum extraction and processing capacity in each period. Eq. (4) controls the relation between the tonnage mined from each bench-phase and the tonnage processed from the clusters within that bench-phase. Note that the difference between the tonnage extracted and the tonnage processed is the waste extracted and sent to the waste dump. However, if we have a waste dump with an extra haulage cost the dump can be defined as a destination with negative revenue. Eq. (5) controls the average head grade of material sent to processing plants in each period. However, to avoid non-linearity the equations are rearranged before putting into matrix format. Eq. (6) ensures that all the material within the ultimate pit is extracted during mine life. Eqs. (7) to (9) are the precedence control constraints with the binary variables.

4.2. Non-linear Model

We can modify the original LTOPP model to account for stockpiling by adding stockpiles as material destinations and introducing $f_c^t \ge 0$ variables. These variables are the tonnages reclaimed from the stockpile and sent to processing plants in each period. The stockpile is added as a destination with the index of c'. $G^{t,e}$ is the reclamation grade of element e in period t and $r_c^{t,e}$ is the unit discounted revenue of processing one unit of element e from stockpile in processing destination c in period t minus the processing and re-handling costs. Accordingly, we can rewrite the LTOPP model be replacing Eqs.(1), (3) and (5) with Eqs.(10), (11) and (12) respectively. Note that the objective function is not linear anymore. Moreover, we have to add a constraint for calculating $G^{t,e}$ as in Eq. (13) which has a nonlinear term. Eq. (14) ensures that the summation of tonnages reclaimed from stockpile from the first period to the current period does not exceed the summation of tonnages sent to the stockpile by the current period.

$$\max \sum_{t=1}^{T}\left(\sum_{p=1}^{P}\sum_{c=1}^{C}(r_{p,c}^{t}\times o_p \times x_{p,c}^{t}) - \sum_{m=1}^{M}(c_m^t \times (o_m + w_m)\times y_m^t) + \sum_{e=1}^{E}\sum_{c=1}^{C}\left(f_c^t \times G^{t,e}\times r_c^{t,e}\right)\right) \quad (10)$$

$$\underline{PC}_c^t \le \sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right) + f_c^t \le \overline{PC}_c^t \qquad \forall t\in\{1,...,T\}, \forall c\in\{1,...,C\} \quad (11)$$

$$\underline{G}_c^{t,e} \le \frac{\sum_{p=1}^{P}\left(o_p \times g_p^e \times x_{p,c}^t\right) + f_c^t \times G^{t,e}}{\sum_{p=1}^{P}\left(o_p \times x_{p,c}^t\right) + f_c^t} \le \overline{G}_c^{t,e} \qquad \forall t\in\{1,...,T\}, \forall c\in\{1,...,C\}, \forall e\in\{1,...,E\} \quad (12)$$

$$G^{t,e} = \frac{\sum_{p=1}^{P}\left(o_p \times g_p^e \times x_{p,c'}^t\right) - \sum_{t'=1}^{t-1}\sum_{c=1}^{C} f_c^{t'} \times G^{t',e}}{\sum_{p=1}^{P}\left(o_p \times x_{p,c'}^t\right) + \sum_{t'=1}^{t-1}\sum_{c=1}^{C} f_c^{t'}} \qquad \forall t\in\{1,...,T\}, \forall e\in\{1,...,E\} \quad (13)$$

$$\sum_{t'=1}^{t}\sum_{c=1}^{C} f_c^{t'} \le \sum_{t'=1}^{t-1}\sum_{p=1}^{P}\left(o_p \times x_{p,c'}^{t'}\right) \qquad \forall t\in\{2,...,T\} \quad (14)$$

4.3. Linearized Model

In order to have a linear LTOPP model with stockpiling, we assume that there are multiple stockpiles with tight ranges for the acceptable element grades. Therefore, we can assign an average reclamation grade and the corresponding reclamation revenue to each stockpile. The more stockpiles defined the smaller error is introduced into the model. However, more stockpiles sacrifices the complete blending assumption present in most stockpiling scenarios. Therefore, making reasonable assumptions regarding the number of stockpiles to define and the acceptable element grade ranges is crucial to obtaining reasonable results.

In order to create the linear LTOPP model with stockpiling we define S stockpiles. G_s^e is the average reclamation grade of element e from stockpile s and $r_{s,c}^t$ is the unit discounted revenue of reclaiming material from stockpile s with the average grade and processing them in plant c in period t minus the processing and re-handling costs. $\underline{G}_s^e$ and $\overline{G}_s^e$ are the lower and upper bounds on the acceptable element grade e for stockpile s. $f_{s,c}^t \ge 0$ is the set of variables representing the tonnage of material reclaimed from stockpile s in period t and sent to processing destination c. Now we can rewrite the model by replacing the objective function with Eq. (15) and Eqs.(11) to (14) with Eqs.(16) to (19) respectively.

$$\max \sum_{t=1}^{T}\left(\sum_{p=1}^{P}\sum_{c=1}^{C}(r_{p,c}^{t}\times o_{p}\times x_{p,c}^{t})-\sum_{m=1}^{M}(c_{m}^{t}\times\left(o_{m}+w_{m}\right)\times y_{m}^{t})+\sum_{s=1}^{S}\sum_{c=1}^{C}\left(f_{s,c}^{t}\times r_{s,c}^{t}\right)\right) \quad (15)$$

$$\underline{PC}_{c}^{t}\leq\sum_{p=1}^{P}\left(o_{p}\times x_{p,c}^{t}\right)+\sum_{s=1}^{S}f_{s,c}^{t}\leq\overline{PC}_{c}^{t} \qquad \forall t\in\{1,...,T\},\forall c\in\{1,...,C\} \quad (16)$$

$$\underline{G}_{c}^{t,e}\leq\frac{\sum_{p=1}^{P}\left(o_{p}\times g_{p}^{e}\times x_{p,c}^{t}\right)+\sum_{s=1}^{S}f_{s,c}^{t}\times G_{s}^{e}}{\sum_{p=1}^{P}\left(o_{p}\times x_{p,c}^{t}\right)+\sum_{s=1}^{S}f_{s,c}^{t}}\leq\overline{G}_{c}^{t,e} \quad \forall t\in\{1,...,T\},\forall c\in\{1,...,C\},\forall e\in\{1,...,E\} \quad (17)$$

$$\underline{G}_{s}^{e}\leq\frac{\sum_{p=1}^{P}\left(o_{p}\times g_{p}^{e}\times x_{p,s}^{t}\right)}{\sum_{p=1}^{P}\left(o_{p}\times x_{p,s}^{t}\right)}\leq\overline{G}_{s}^{e} \qquad \forall t\in\{1,...,T\},\forall s\in\{1,...,S\},\forall e\in\{1,...,E\} \quad (18)$$

$$\sum_{t'=1}^{t}\sum_{c=1}^{C}f_{s,c}^{t'}\leq\sum_{t'=1}^{t-1}\sum_{p=1}^{P}\left(o_{p}\times x_{p,s}^{t'}\right) \qquad \forall t\in\{2,...,T\},\forall s\in\{1,...,S\} \quad (19)$$

5. Case Study

Marvin is a well-known test dataset used as the demo dataset in Whittle™ mine planning software (Geovia, 2012) and presented as a standard dataset in MineLib (Espinoza et al., 2013). Marvin dataset consists of 53,271 blocks with four different rock-types and two element grades. Each block is 30 meters in all dimensions. The main rock-types are Mixed (MX), Oxide (OX) and Primary (PM) along with the undefined waste denoted here with UND. Marvin mine is a synthetic gold and copper mine with a thin layer of overburden which makes planning easier and provides access to ore in the very first periods of extraction.

As mentioned earlier, Marvin dataset exists in both Whittle™ and MineLib (Espinoza et al., 2013). Despite using the same cost and profit parameters the optimum pit determined by Whittle™ is different from the optimum pit determined in MineLib (Espinoza et al., 2013). The former has 9,381 blocks (575 million tons) in the final pit compared to the latter with 8,516 blocks (527 million tons) in the final pit. We have used the outputs from Whittle™ in our case studies to be able to compare our results to commercial software used by many companies in the mining industry.

In all cases, the block economics are calculated based on a mining cost of $1.5 per ton and a processing cost of $6.25 per ton for all rock-types. The gold and copper recoveries in the processing plant, selling costs and prices are summarized in Table 1. Sample plan views of rock-type and grade distribution within the final pit are presented in Fig 1 to Fig 6. For the purpose of calculating clustering measures, we use the initial destinations determined by Milawa NPV in Whittle™ as the destination of blocks in the clustering step.

First, we compare our MILP schedules based on processing blocks and clusters against Whittle™ Milawa algorithms. Next, we add stockpiles and compare the stockpiling scenarios and present the actual versus estimated cash flows for our model. Moreover, we show how we can decrease the error by restricting stockpiles to specified grade values which is not an uncommon practice in real mining operations. Finally, we use the stockpiles and restrict the head grade of material sent to the processing plant to show the flexibilities that come with using MILP models instead of heuristics for scheduling.

Table 1. Marvin Element Economics

Element	Unit	Recovery	Selling Cost	Price
Au	gram	0.6	4.80	38.6
Cu	%m	0.8	11.03	33.1

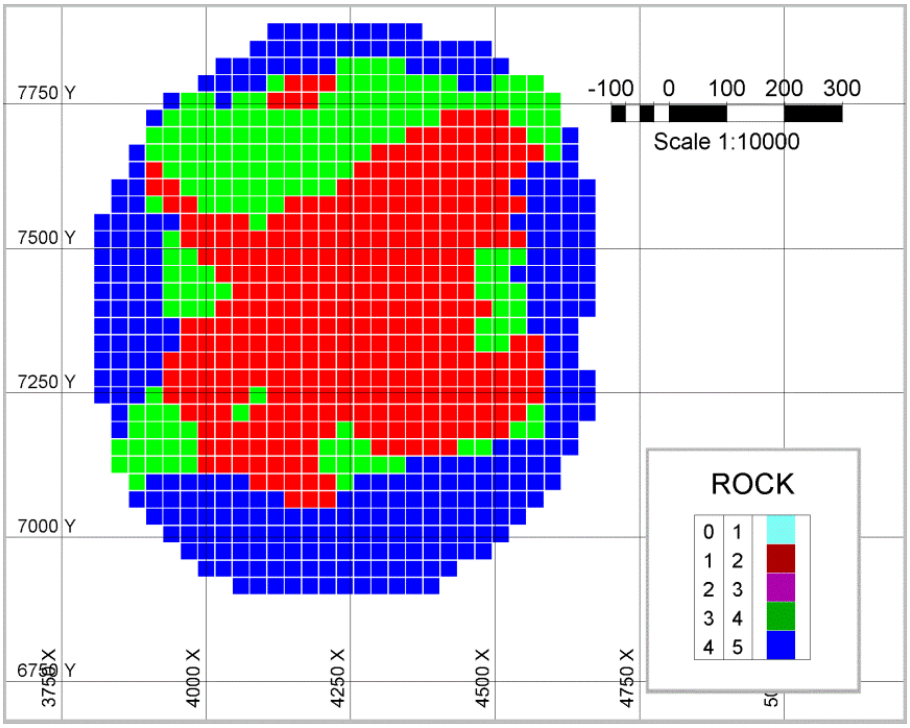

Fig 1. Rock-type Distribution Plan View at 600m Elevation

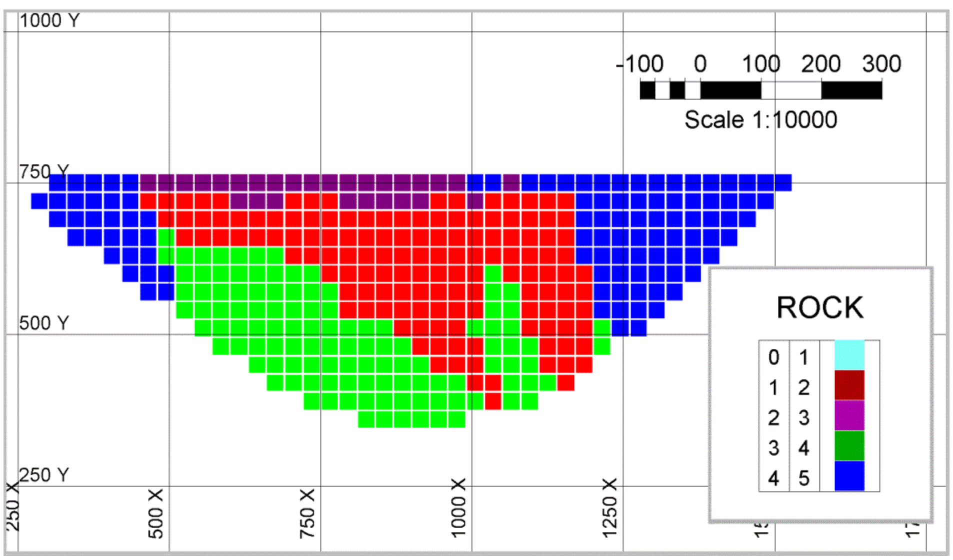

Fig 2. Rock-type Distribution at 4100m Easting

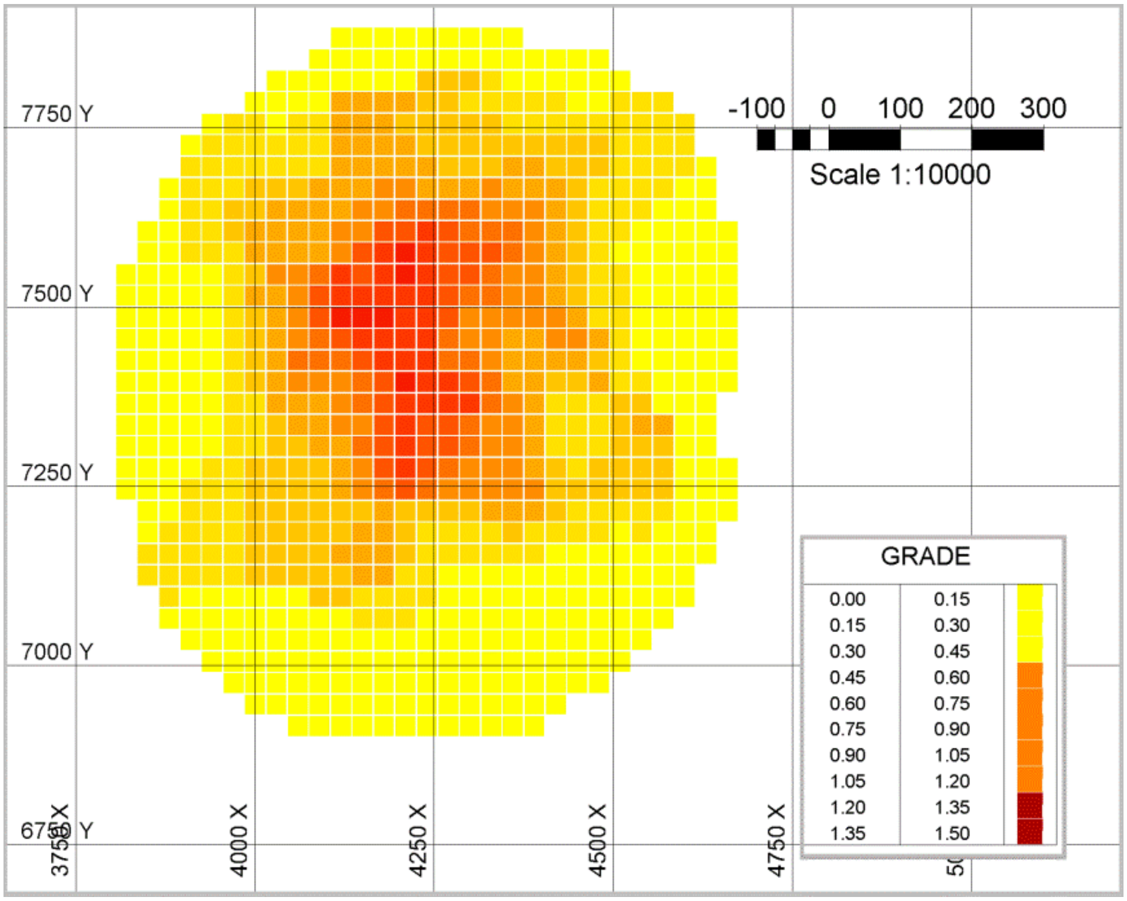

Fig 3. Gold Grade Distribution Plan View at 600m Elevation

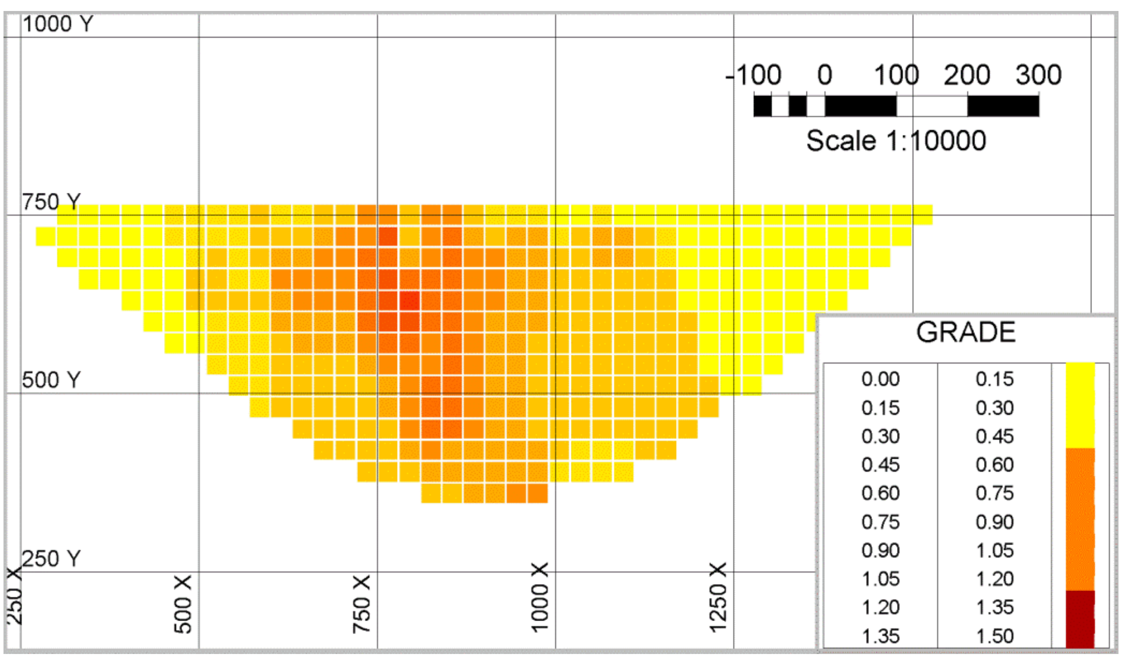

Fig 4. Gold Grade Distribution at 4100m Easting

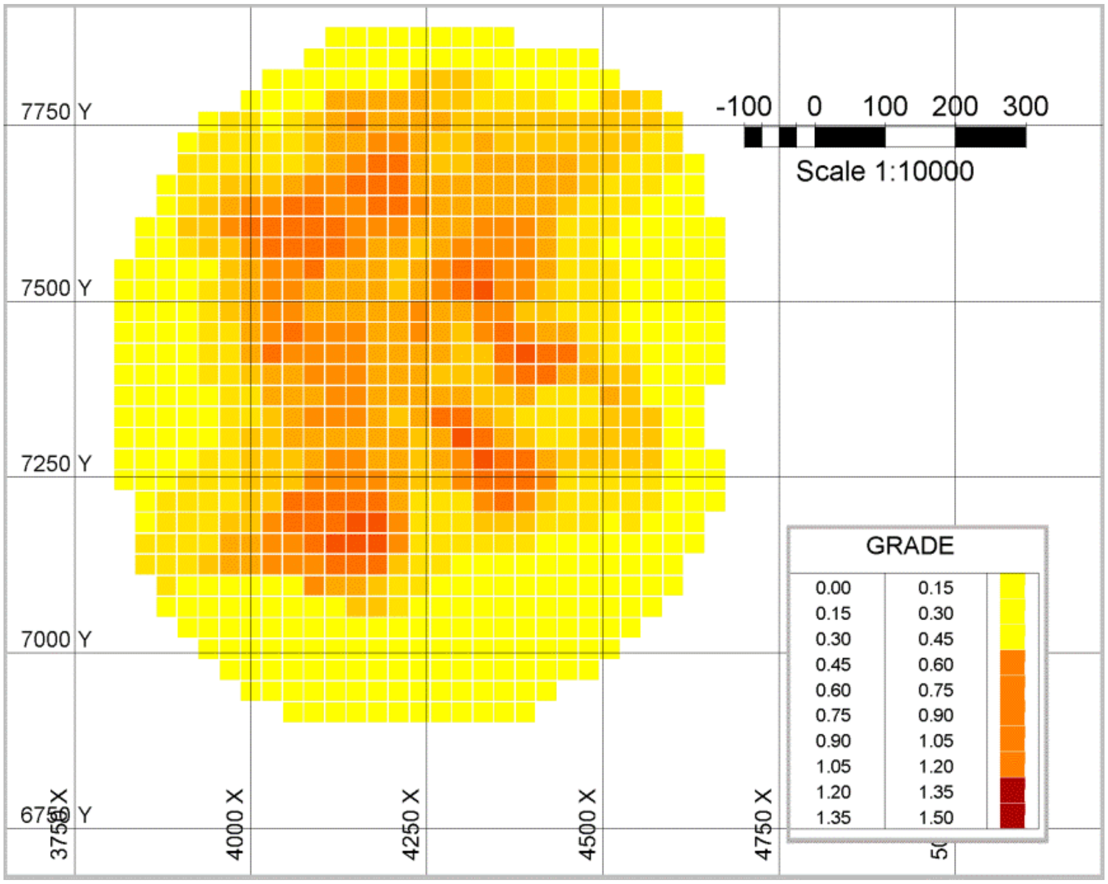

Fig 5. Copper Grade Distribution Plan View at 600m Elevation

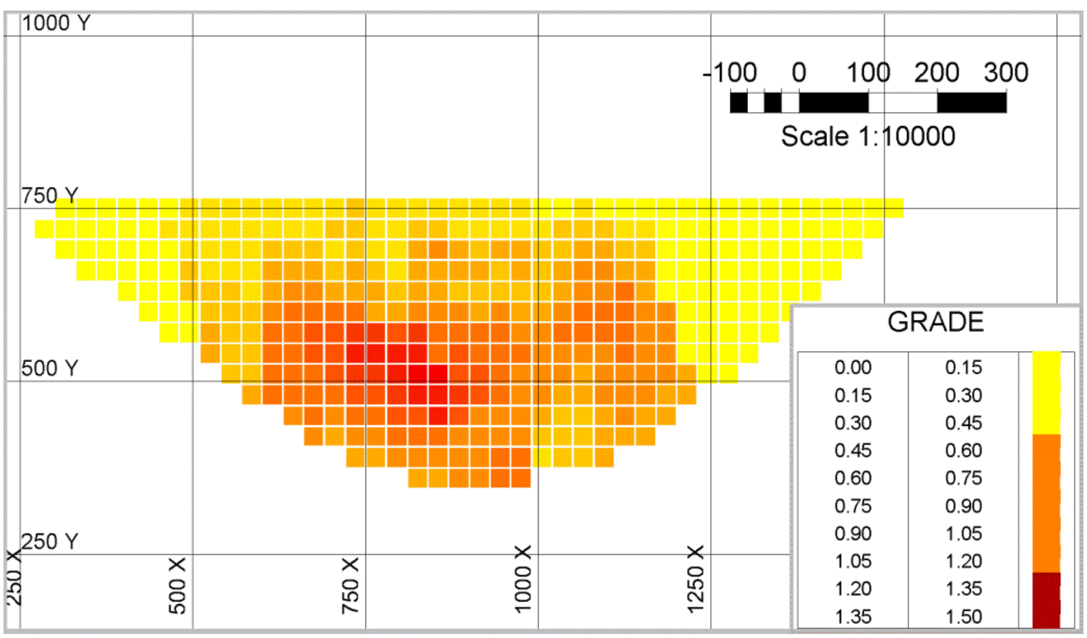

Fig 6. Copper Grade Distribution at 4100m Easting

5.1. GEOVIA Whittle™ Schedule

We obtained two production schedules based on four pushbacks from Whittle™. The first schedule is based on Milawa NPV algorithm and the second one is based on Milawa Balanced. These algorithms are heuristics deigned to use pushbacks as mining units and maximize NPV of the operation. Milawa NPV focuses on maximizing NPV where Milawa Balanced looks into maximizing NPV and having a balanced schedule at the same time. The Milawa balanced algorithm results in $2,166M of NPV and the schedule in Fig 10. The Milawa NPV algorithm results in $2,240M of NPV and the schedule in Fig 7. As expected, the Milawa NPV algorithm resulted in higher NPV by sacrificing the balance in utilizing resources.

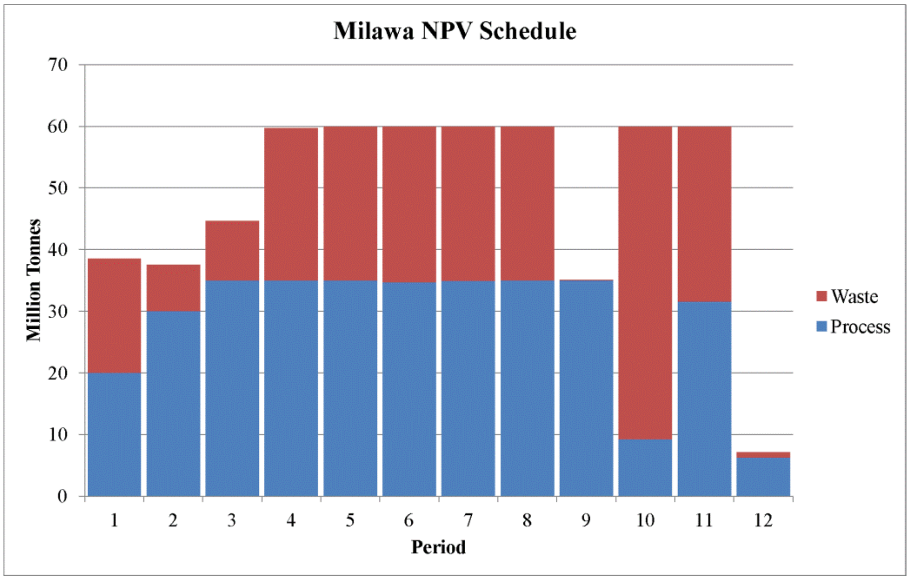

Fig 7. Milawa NPV Schedule

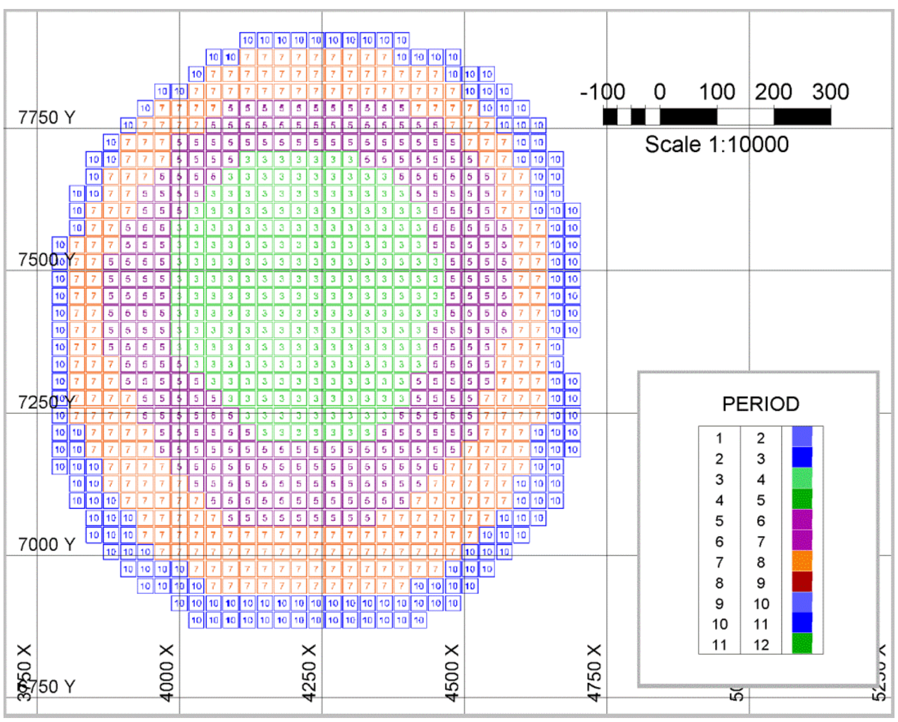

Fig 8. Milawa NPV Schedule Plan View at 600m Elevation

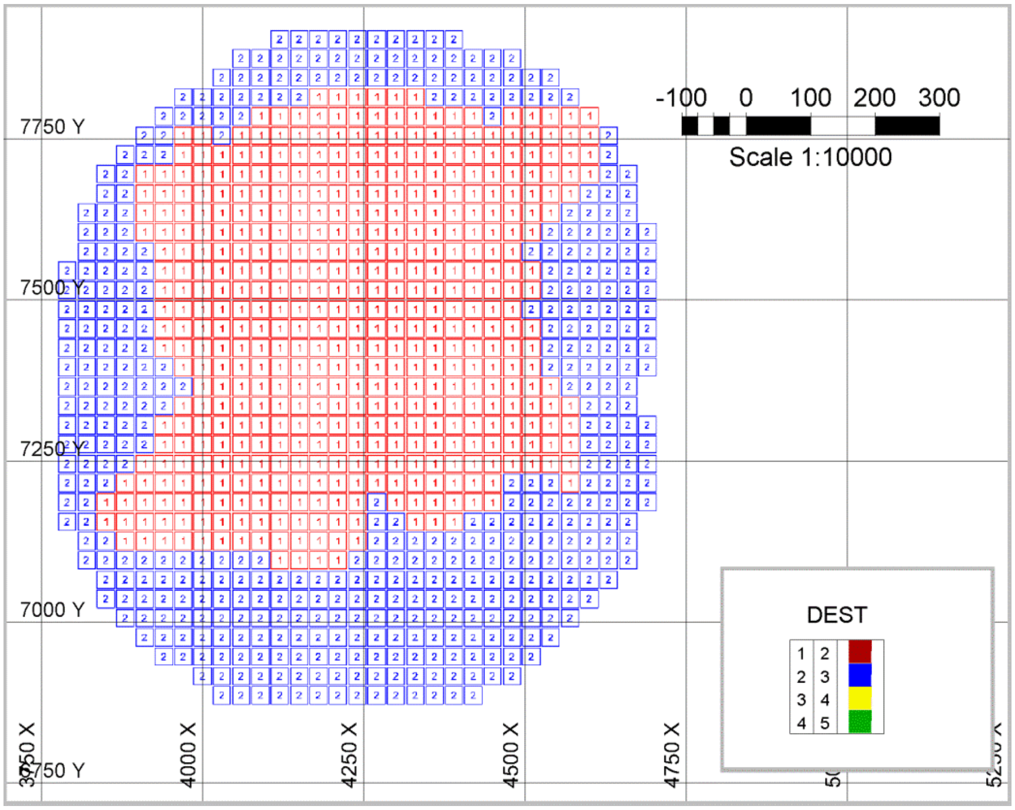

Fig 9. Milawa NPV Destination Plan View at 600m Elevation

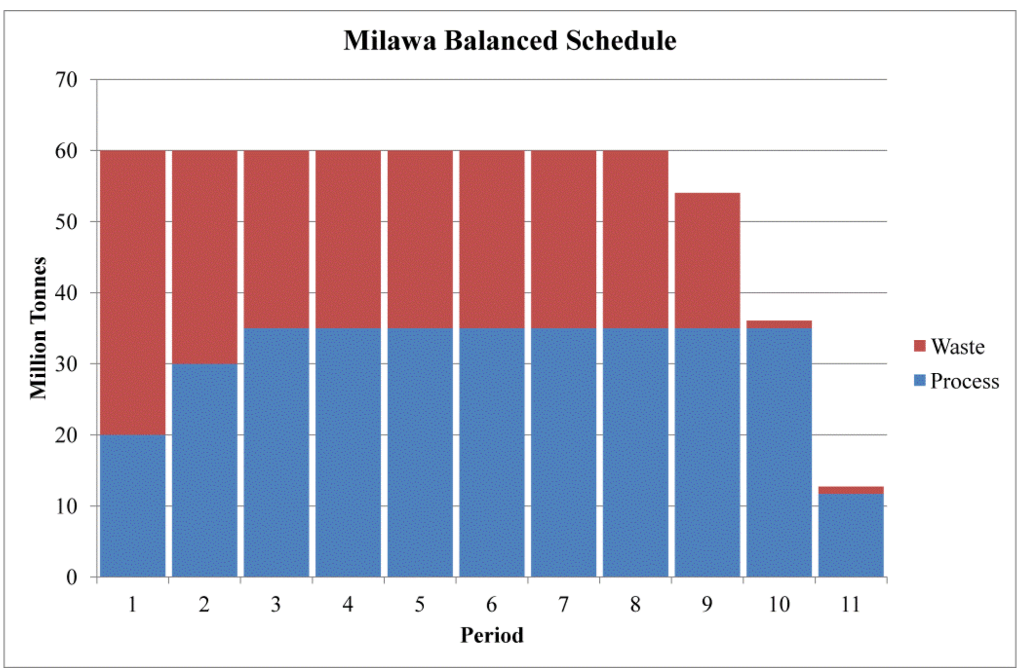

Fig 10. Milawa Balanced Schedule

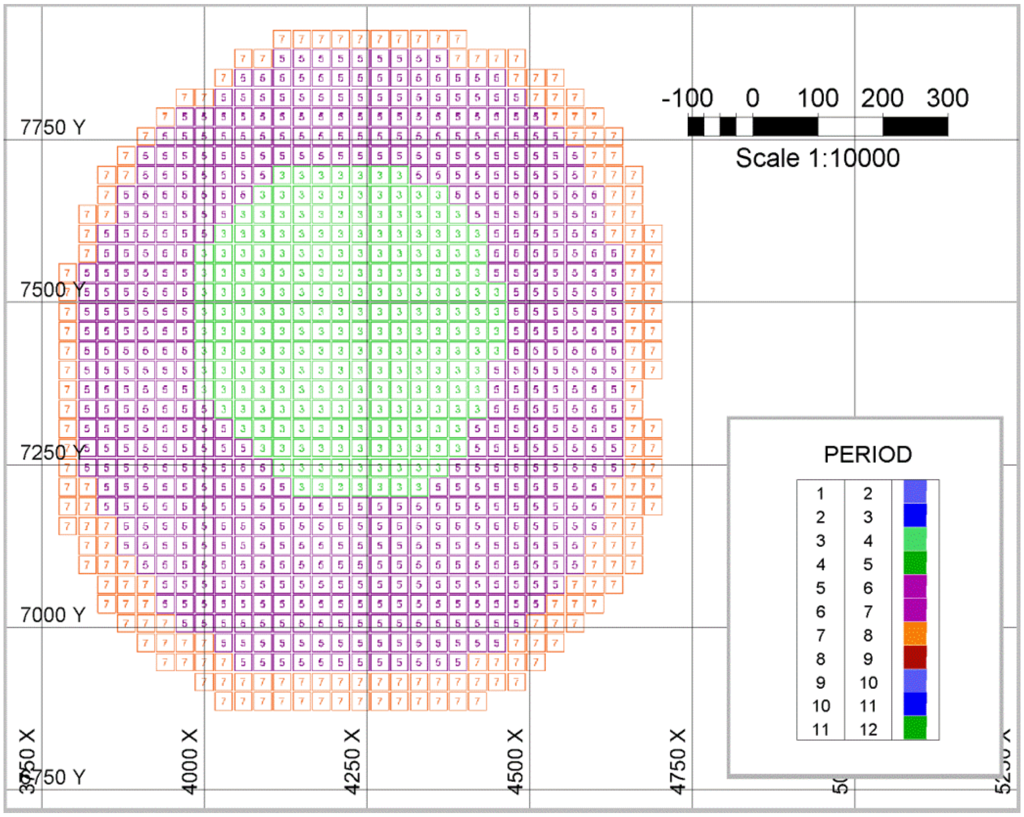

Fig 11. Milawa Balanced Schedule Plan View at 600m Elevation

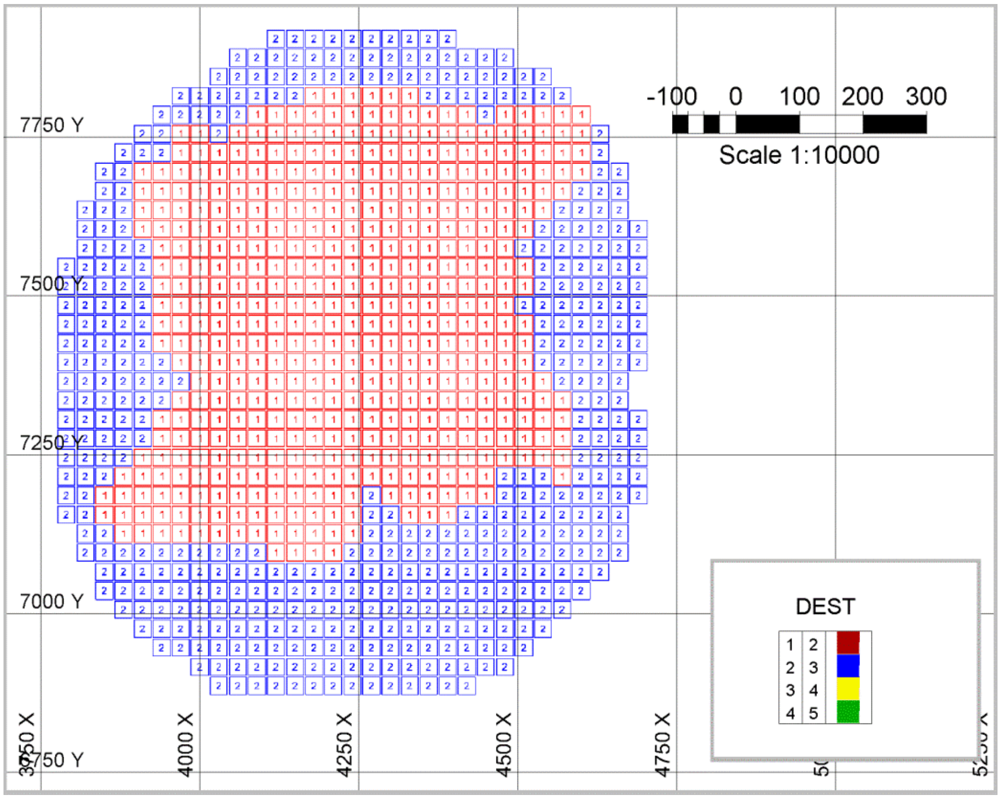

Fig 12. Milawa Balanced Destination Plan View at 600m Elevation

5.2. MILP Schedule, Mining Units: Bench-phases, Processing Units: Blocks

Since the Whittle™ scheduling is performed based on using bench-phases as mining units and blocks as processing units, we used the same resolution for the first case-study. We first ran the MILP model to 5% optimality gap. The results were obtained in 7 seconds and an NPV of $2,653M is reached. Since the runtime was short we ran the model to optimality and obtained the optimal solution in this resolution in 322 seconds. The optimal NPV is $2,660M and the corresponding schedule graph and plan views are presented in Fig 13 to Fig 15. Although the block destinations are very similar to the Whittle™ results, the NPV of the operation shows 18.8% improvement over the highest NPV obtained by implementing Mila NPV scheduling algorithm.

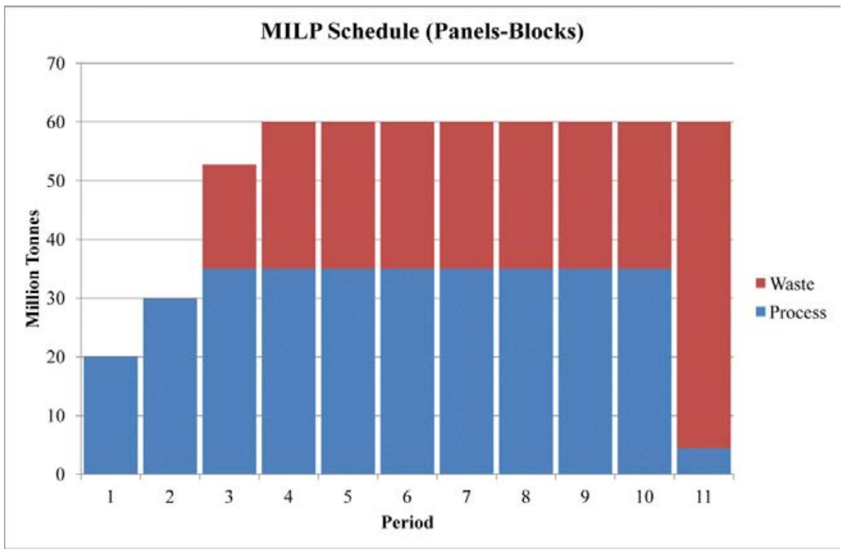

Fig 13. MILP Schedule

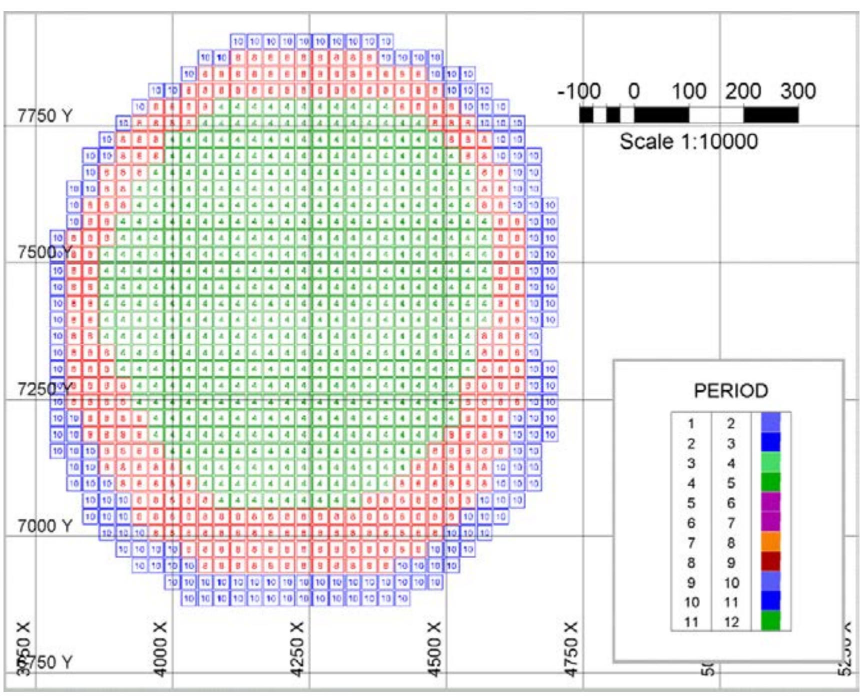

Fig 14. MILP Schedule Plan View at 600m Elevation

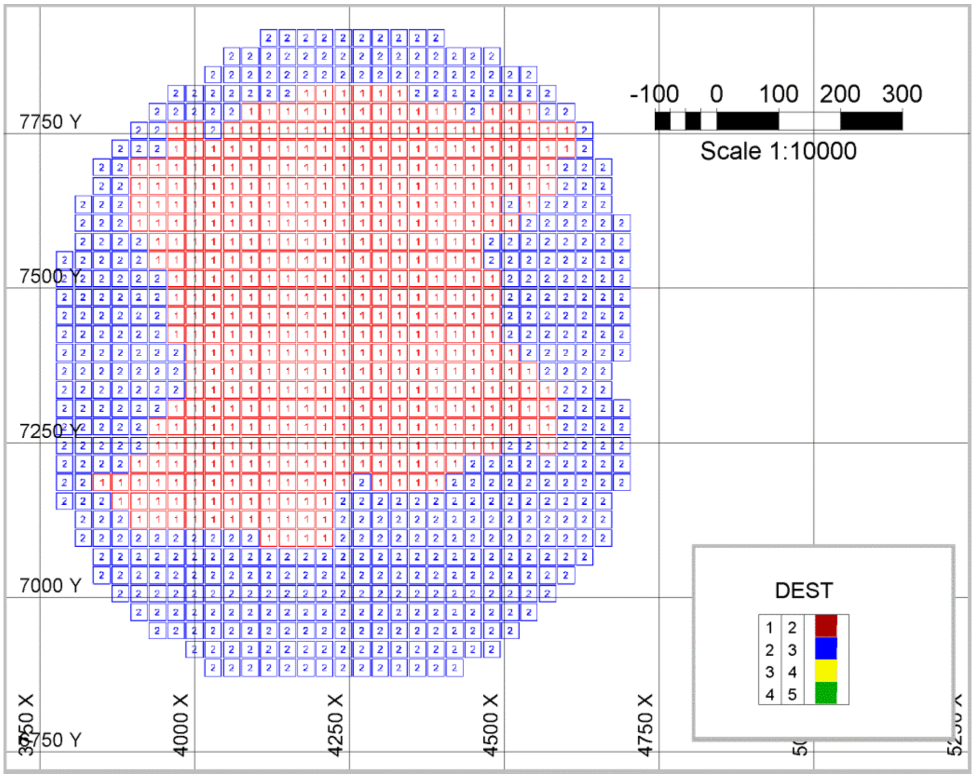

Fig 15. MILP Destination Plan View at 600m Elevation

5.3. MILP Schedule, Mining Units: Bench-phases, Processing Units: Clusters

The focus of this project is on creating clusters of blocks with homogenous grade and rock-type and using the clusters as mining and processing units. Therefore, we use bench-phases as mining units and clusters as processing units in this scenario to evaluate the effects of clustering on the mine planning outcomes. The hierarchical clustering algorithm in this scenario is performed based on the parameters summarized in Table 2. It takes the solver 1.6 seconds to solve the MILP formulation to 10% gap with an NPV of $2,136M. Solving the MILP to optimality takes 30.5 seconds and results in an NPV of $2,185M which is 3.5% less than the Milawa NPV algorithm but with a more balanced schedule. The schedule is presented in Fig 16 and the clusters, extraction periods and destination plan views follow in Fig 17 to Fig 19.

Table 2. Clustering Parameters

Parameter	Value
Distance Weight	0.5
Grade Weight	0
Rock-type Penalty	0.5
Avg. Blocks per Cluster	20
Max. Blocks per cluster	25
Min. Blocks per Cluster	5
Number of Shape Refinement Iterations	3

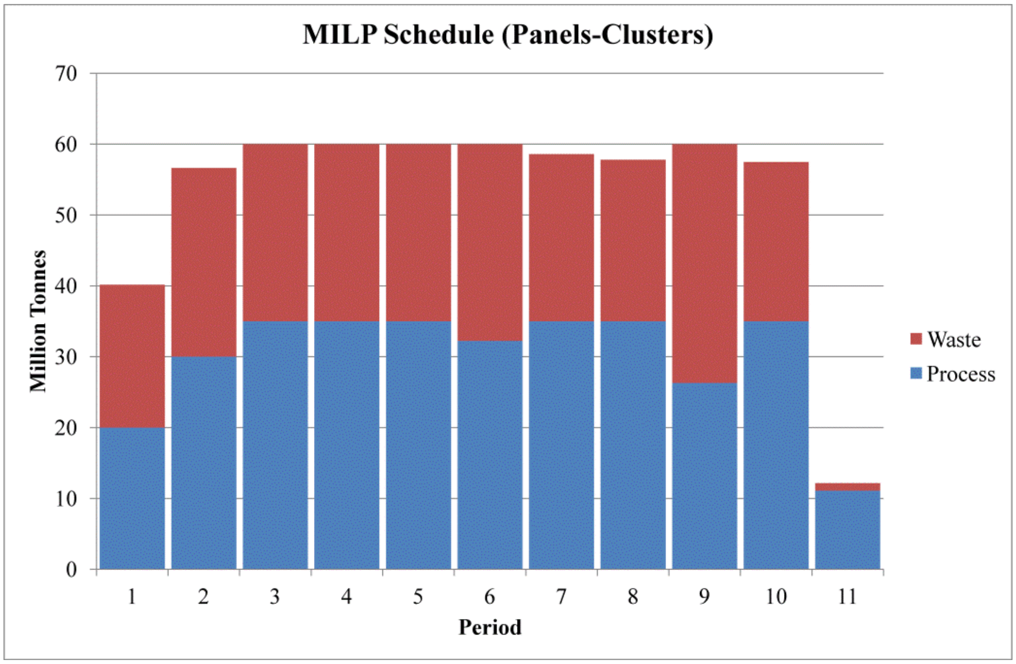

Fig 16. MILP Schedule

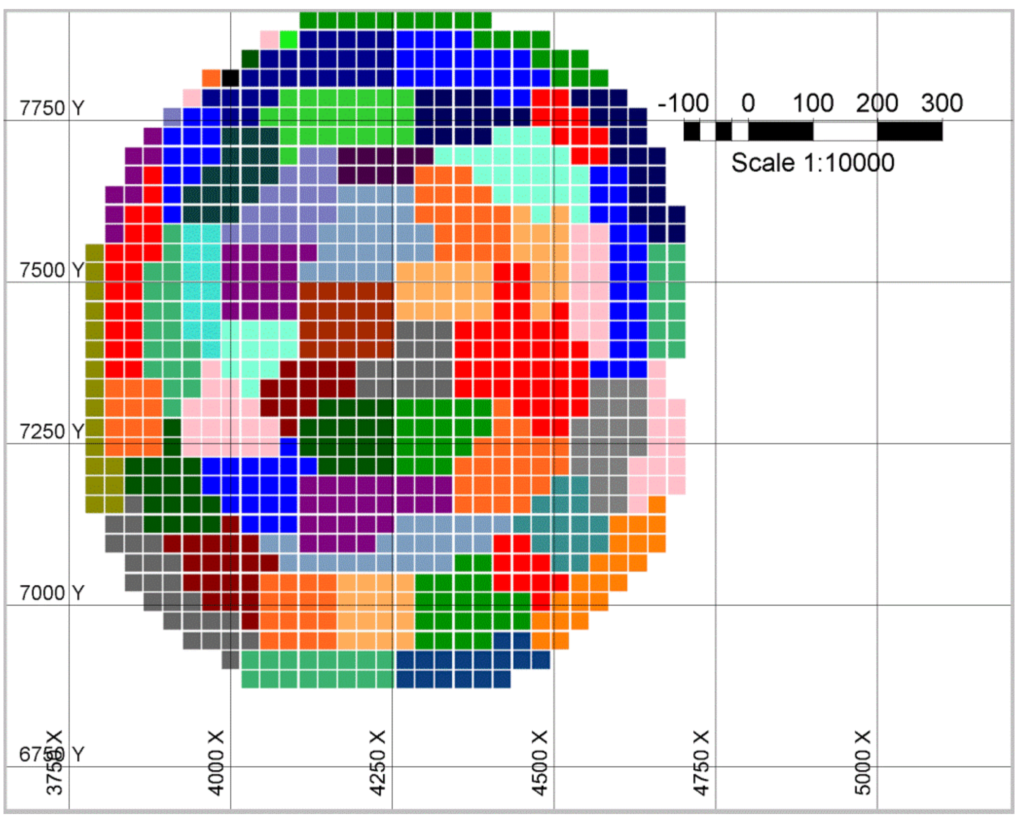

Fig 17. Cluster IDs Plan View at 600m Elevation

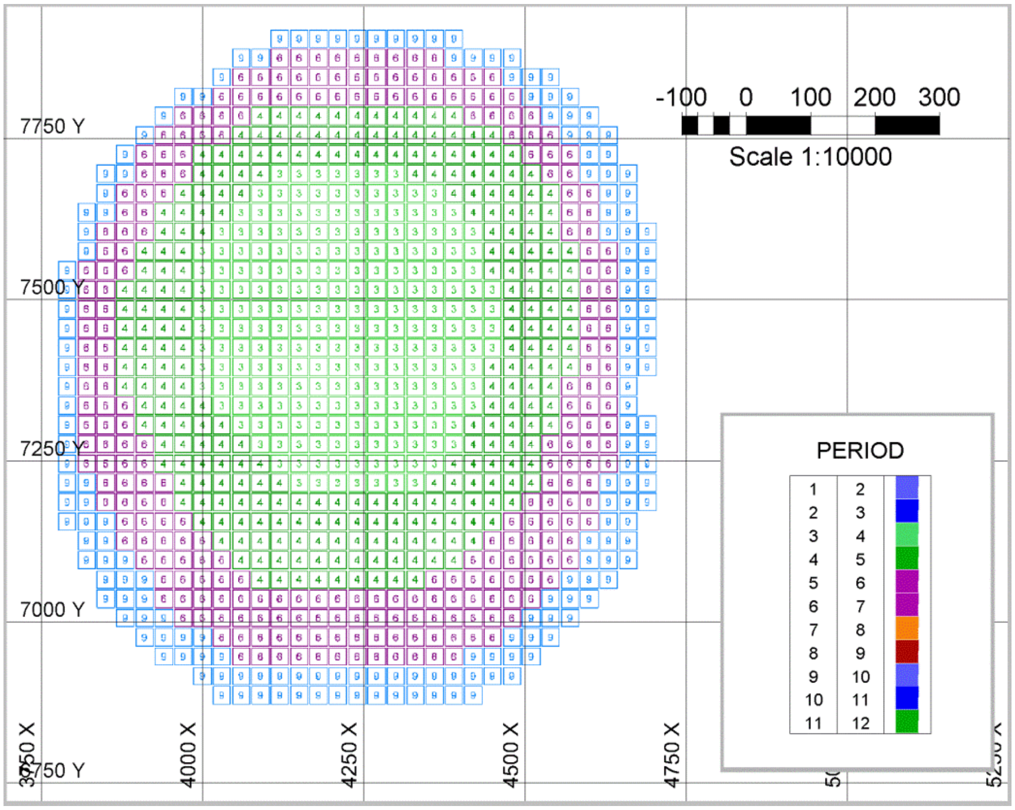

Fig 18. MILP Schedule Plan View at 600m Elevation

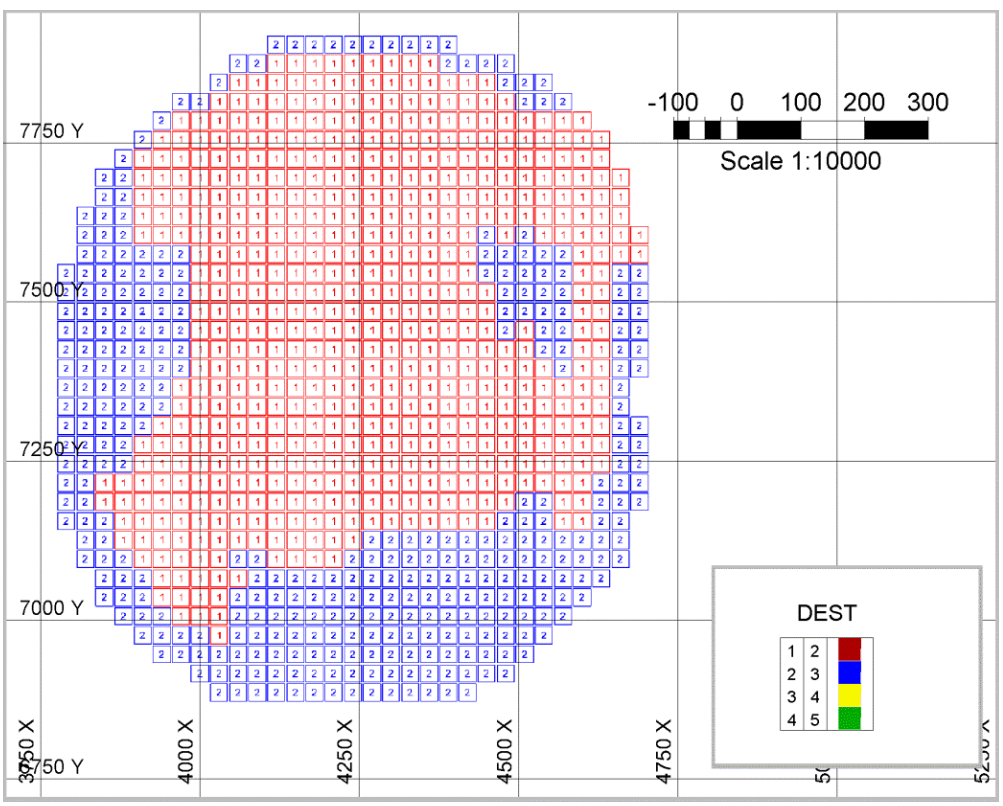

Fig 19. MILP Destination Plan View at 600m Elevation

5.4. Stockpiling

In this section, we study how we can add stockpiles to the case-study. For this purpose, we add three stockpiles with unlimited capacity to Whittle™ scheduler and our own model. Each stockpile is limited to one rock-type. A re-handling cost of 0.4 $/tonne is applied for reclaiming material from stockpiles and sending to the processing plant. Moreover, it is assumed that the recovery of material reclaimed from stockpiles is 2% less than original recoveries. The fleet required to reclaim material from stockpiles and send to the processing plant is considered to be independent of the available mining capacity. Fig 23 shows the schedule generated with Whittle™ based on the listed assumptions. As can be seen in the figure, Whittle™ uses the stockpile to make sure that the plant has enough feed in every period. However, the NPV of the operation based on this schedule is $2,224M which is 0.8% less than the original Milawa NPV algorithm. Fig 24 is the schedule generated from the MILP formulation with the same assumptions. Since the MILP formulation requires a fixed reclamation grade for each stockpile, we averaged the grade values for the clusters with each rock-type and used as the reclamation grade. The reclamation grades presented in Table 3 are used to calculate the revenue generated from reclaiming material from each stockpile and sending to the plant in the MILP formulation.

Table 3. Stockpile Parameters

	Rock-type	Au Grade (gram/tonne)			Cu Grade (%m)		
		Min	Max	Avg	Min	Max	Avg
SP1	PM	0	0.89	0.45	0	0.42	0.20
SP2	MX	0	1.14	0.58	0	1.29	0.50
SP3	OX	0	1.13	0.42	0	1.29	0.58

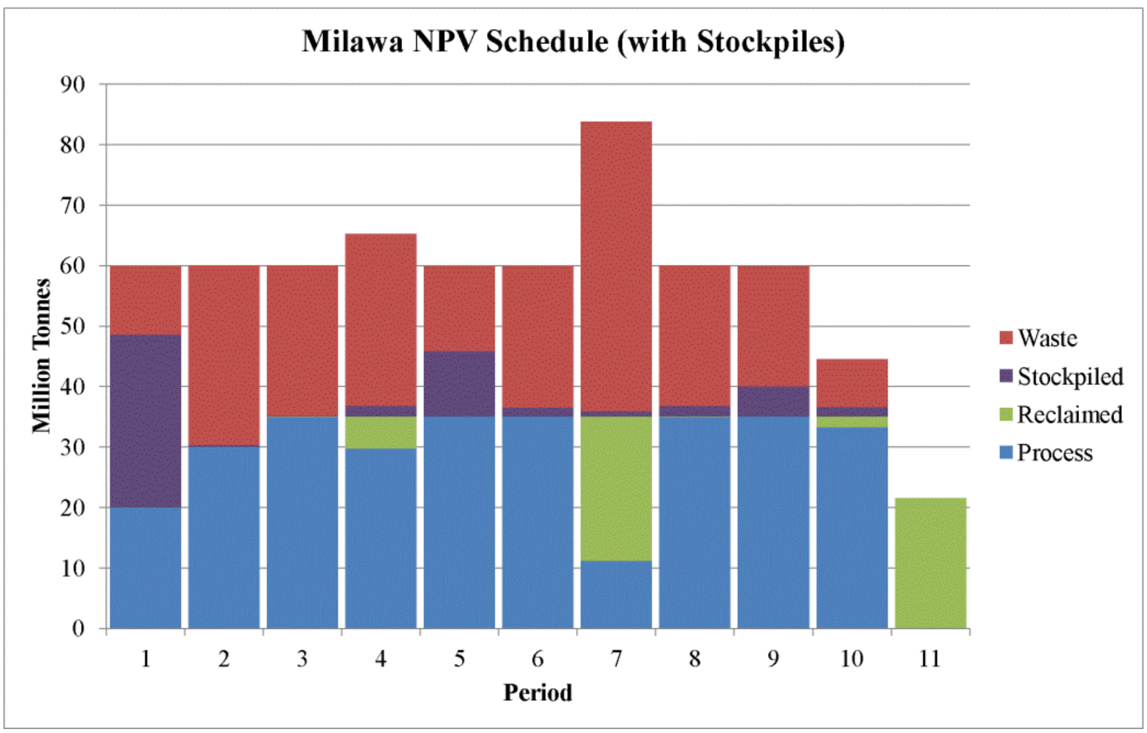

Fig 20. Milawa NPV Schedule (with Unrestricted Stockpiles)

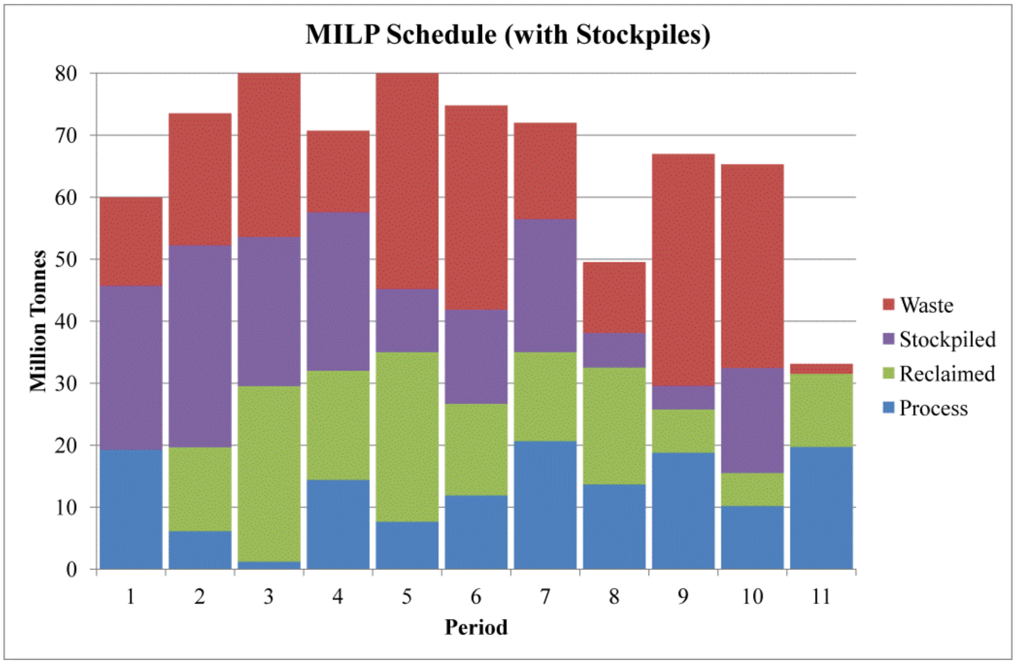

Fig 21. MILP Schedule (with Unrestricted Stockpiles)

The NPV resulted from the MILP schedule is $2,747M which is significantly higher than other scenarios. However, this NPV is resulted from approximating the reclamation grade of the stockpiles with the average grade of the material in the pit. Therefore, we calculated the actual grade of material in the stockpile based on the proposed schedule and the actual revenue generated from reclaiming material from stockpiles and sending to the plant. Fig 22 shows the approximated cash flow based on fixed reclamation grade minus the actual generated revenue for each period. As can be seen in Fig 22, the difference between the approximated revenue and the actual revenue is significant in most of the periods and has resulted in overestimation of the final NPV. In total, the generated NPV is $516M more than the actual NPV that can be generated with this schedule. Therefore, we restricted each stockpile to accept one rock-type with limited grade range to reduce the difference between the assumed average reclamation grade and the actual stockpile grade. The summary of stockpile definitions is provided in Table 4. We calculated the weighted average of grade values in each rock-type within the acceptable ranges and used as the reclamation grades for each stockpile. We used the same ranges for Milawa NPV and compared the outcomes of both schedulers.

As mentioned earlier, Milawa NPV algorithm uses the stockpiles to feed the plant when enough ore cannot be extracted from the mine and extends the mine life to the 11^{th} period in this case. The resulted NPV is $2,155M which is 3.8% less than the original Milawa NPV schedule and 0.6% less than the original Milawa balanced schedule. However, the plant is fully utilized in all periods except than the last period. On the other hand, the MILP model uses stockpiles more frequently and increases the NPV of the operation to $2,432M which is 11.3% more than the original panel-cluster scenario. The plant is also better utilized compared to not using the stockpiles. The generated schedules from Milawa NPV and MILP are presented in Fig 23 and Fig 24 respectively. Similar to the unrestricted stockpile case, we plotted the approximation error in cash flows in Fig 25. The total overestimation in calculating the NPV of the operation is $27M which is a 1.1% error. Moreover, it is possible to decrease the error by using tighter bounds on the stockpile grades or by calibrating the reclamation grades based on the resulted schedule.

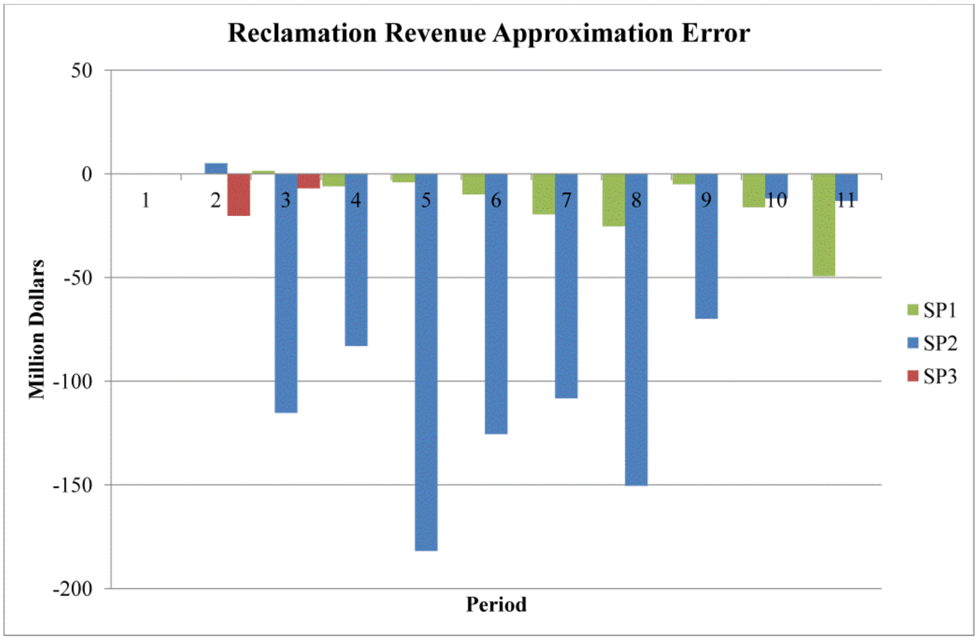

Fig 22. Reclamation Revenue Approximation Error

Table 4. Stockpile Parameters

	Rock-type	Au Grade (gram/tonne)			Cu Grade (%m)		
		Min	Max	Avg	Min	Max	Avg
SP1	PM	0.1	0.3	0.25	0.15	0.45	0.18
SP2	MX	0.2	0.5	0.31	0.1	0.3	0.23
SP3	OX	0.1	0.4	0.15	0.1	0.2	0.17

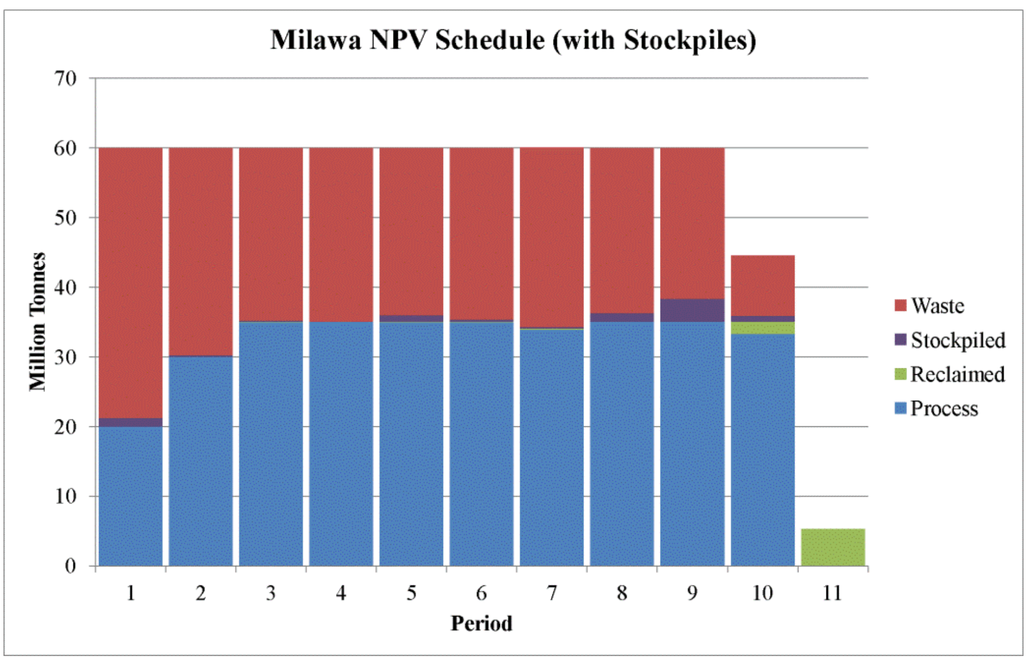

Fig 23. Milawa NPV Schedule (with Stockpiles)

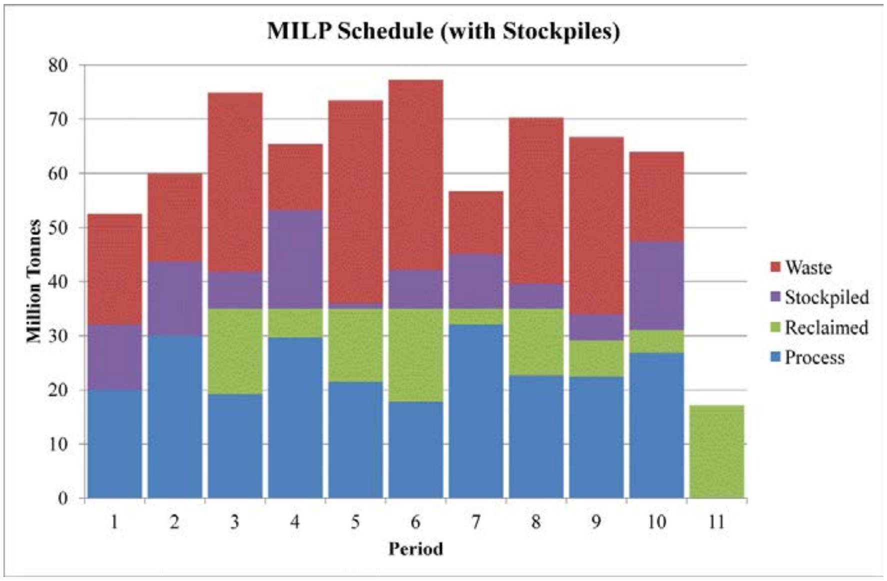

Fig 24. MILP Schedule (with Stockpiles)

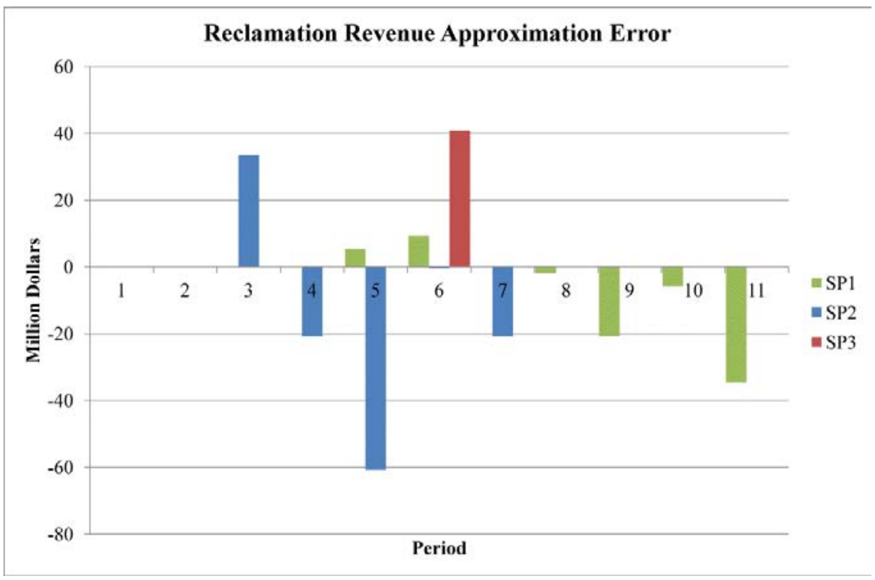

Fig 25. Reclamation Revenue Approximation Error

5.5. Blend Control

Blending is another important aspect of production planning. As mentioned in section 4, our model is capable of controlling the head grade of material sent to processing destinations. This features works for both models with and without stockpiles. In this section, we add lower and upper bounds to the head grade of material sent to the processing plant. The gold and copper grade lower and upper bounds are presented in Table 5. The rest of the parameters are the same as before.

Table 5. Head Grade Control Parameters

Au Grade (gram/tonne)		**Cu Grade (%m)**	
Min	Max	Min	Max
0.3	0.7	0.3	0.7

First, we solve the model by adding grade control constraints and removing stockpiles. The model is solved to optimality in 1,290 seconds and results in an NPV of $2,085M. The production schedule is presented in Fig 26 and the head grade of material sent to the process is presented in Fig 27. Afterwards, we add the same stockpiles as in the previous section to increase the flexibility of the model and test its performance. Although the mathematical formulation uses fixed reclamation grades for stockpiles, we used the actual grade of stockpiles in calculating the head grades. We solved the model by adding the same stockpile settings as the previous section and applying the head grade constraints. Solving the model to optimality takes 613 seconds and results in an NPV of $2,394M which is 1.3% less than not having constraints on the head grades. We expect the NPV to drop more if tighter bounds on the head grade are applied. The production schedule and head grades are plotted in Fig 28 and Fig 29 respectively. As can be seen in Fig 29, the actual head grades violate the upper bounds in only one instance due to approximation of reclamation grade with a fixed number.

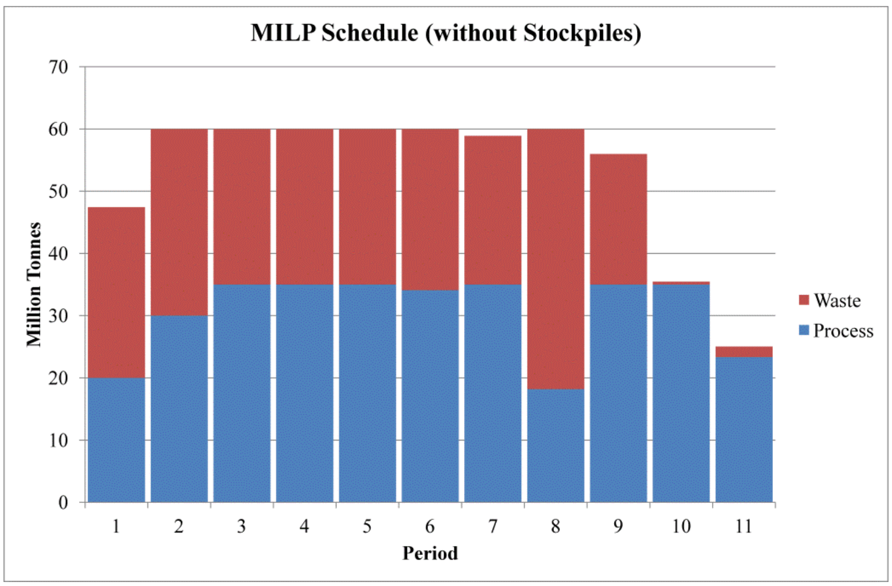

Fig 26. MILP Schedule (without Stockpiles)

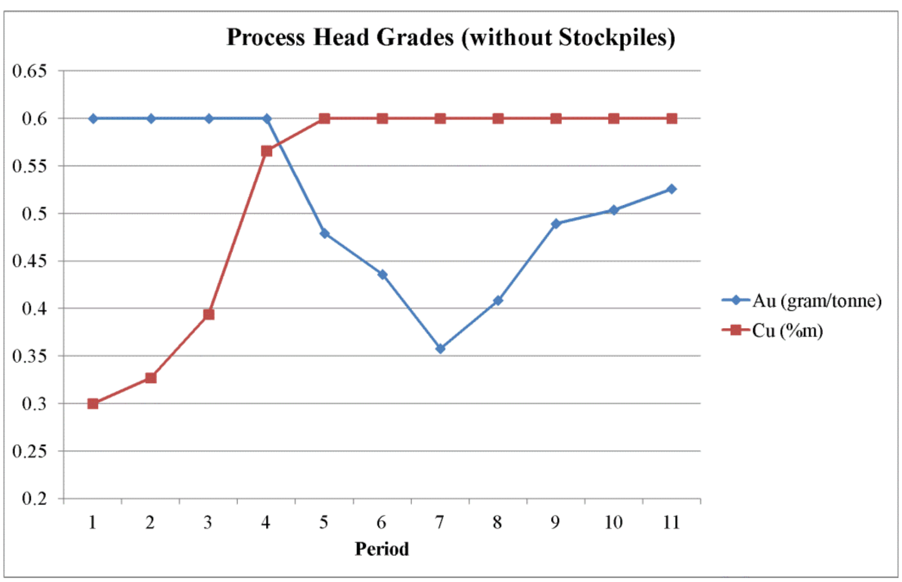

Fig 27. Process Head Grades (without Stockpiles)

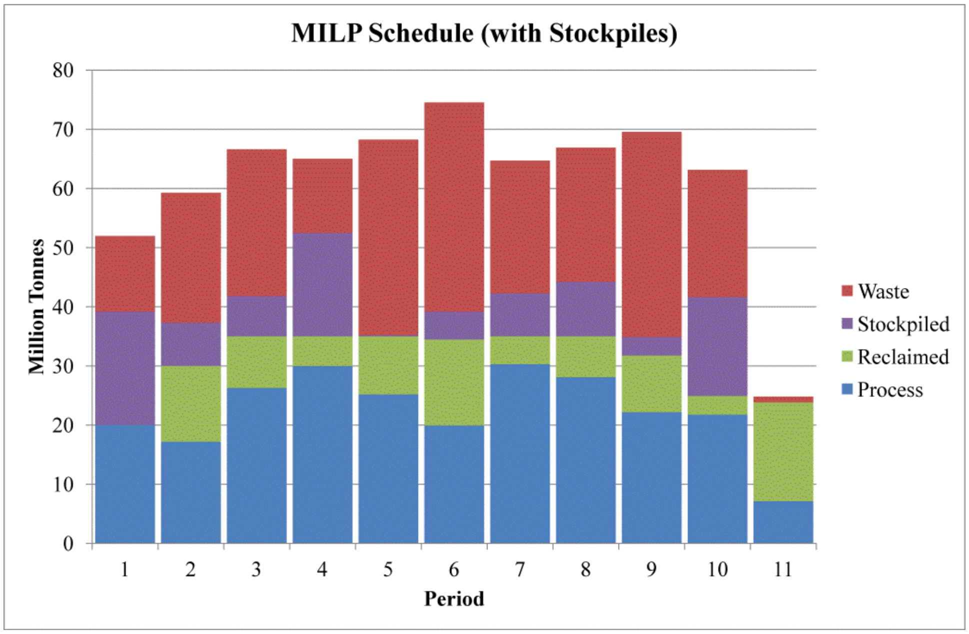

Fig 28. MILP Schedule (with Stockpiles)

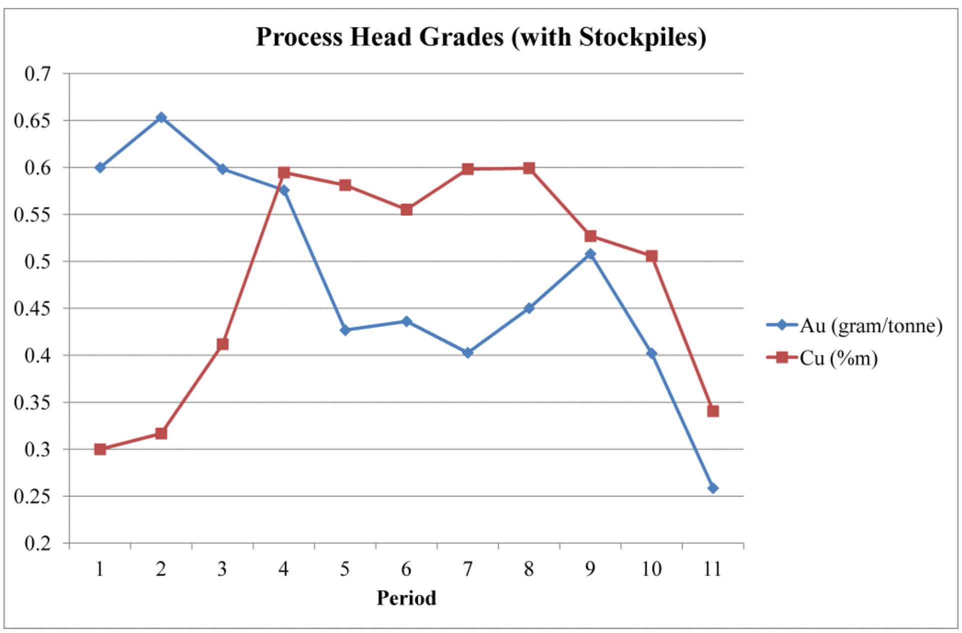

Fig 29. Process Head Grades (with Stockpiles)

6. Conclusion and future work

In this paper, we presented a multi-step approach to long-term open-pit production planning by using different resolutions for making mining and processing decisions. We determine the pushbacks based on a hybrid binary programming-heuristic method and use the intersections of pushbacks and mining benches as mining units. Afterwards, we divide the bench-phases into smaller units with similar rock-type and grade using an agglomerative hierarchical clustering algorithm. These units are then used as processing units. Then, we presented a mathematical model to solve the LTOPP problem with the aggregated units. Finally, we added stockpiling to the model with non-linear and linear objective functions and constraints. In the next step, the linear model was tested on a synthetic small case study to verify the simplification assumptions used for linearization. We concluded that using clusters as processing units results in more practical schedules. Moreover, we showed that we can control the linearization error by restricting stockpiles to predetermined grade values which is aligned with the common practices in mining industry. However, the obtained solution is not optimal for the original non-linear model. Therefore, if we can solve the quadratic model and obtain the optimum solution we can have a better understanding of linearization errors.

7. References

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Oil Sands Concurrent Production Scheduling and Waste Management

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Abstract

Mine planning for oil sands involves the integration of waste management into the long term production planning process. This ensures that while ore is provided for the processing plant, sufficient in-pit tailings containment areas are made available as dedicated disposal areas for backfilling. This enables the creation of a trafficable landscape at the earliest opportunity to facilitate progressive reclamation. Apart from being a regulatory requirement, this integration impacts directly on the profitability and sustainability of oil sands mining operations. This paper introduces a mixed integer linear programming mine planning framework that seeks to simultaneously determine the production schedule, dyke construction schedule and the backfilling schedule. Different waste management strategies were also investigated. The model generated a practical, smooth and uniform schedule for ore, dyke material and backfilling activities. The results show that for the case studies considered, increasing the number of in-pit tailings cells reduces the net present value of the mine as a result of a reduced operational flexibility. However, this strategy makes in-pit tailings storage areas available earlier in the mine life, and ensures an efficient use of in-pit storage areas required for sustainable operations.

1. Introduction

Oil sands mining is usually characterized by large open pits and tailings dams. These operations leave behind large reclamation areas. Over 80% of oil sands ore are ultimately deposited in tailings dams in the form of fine and coarse sand by-products. These sand by-products significantly increase in volume during processing generating environmental and regulatory concerns in terms of their storage. Regulations by Alberta Energy Resources and Conservation Board (Directive 074) (McFadyen, 2008) requires oil sands mining companies to develop integrated mine planning and waste management strategies for their in-pit and external tailings facilities. It is therefore important to develop mine plans that integrate production scheduling with waste management in an optimization framework that generates value and is sustainable. Sustainability for oil sands operations includes ensuring that in-pit storage areas are available on time and making an efficient use of this storage space. This ensures that the operation does not grind to a halt due to unavailable tailings storage areas and reclamation can start early in time. Optimization of this problem is quite

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a challenge in terms of mathematical formulations, computational power and speed. Applying mathematical programming models (MPMs) such as linear programming (LP), mixed integer linear programming (MILP) and goal programming (GP) with exact solution methods have proven to be robust. Solving MPMs with exact solution methods result in solutions within known limits of optimality. As the solution gets closer to optimality, it results in production schedules that generate higher net present value (NPV) than those obtained from heuristic optimization methods.

Though MPMs have been applied in mine production scheduling, little work has been done in terms of oil sands mine planning, which has a challenging scenario when it comes to waste management. It is our objective to develop an MILP model that simultaneously schedules for production material, dyke material and backfilling material in an integrated oil sands mine planning (IOSMP) framework. The MILP formulation maximizes the NPV of the operation, minimizes the dyke construction cost and maximizes the backfilling revenue through the cash flow from mining the production and dyke construction material, and backfilling the in-pit mined areas respectively. The production material cash flow is controlled by the revenue from mining ore and the cost of mining ore and waste. The dyke construction material cash flow is controlled by the extra cost of mining dyke material and sending it to the required destination. The in-pit backfilling cash flow is controlled by a pseudo revenue generated by backfilling the in-pit mined areas. This pseudo revenue is the savings generated from in-pit backfilling as compared to ex-pit waste management. Snowden's Evaluator software (Snowden Mining Industry Consultants, 2013) was chosen as the modeling platform for this research. Evaluator can be used for a wide range of mining scenarios with a user friendly graphical modeling interface that allows for great flexibility. It allows for material flow to be modeled for multiple sources, destinations and materials types while applying the required material stream flow constraints necessary to describe complex problems. Evaluator uses an optimization solver known as Gurobi (Gurobi Optimization, 2013) which is developed based on branch and cut optimization algorithm.

The rest of the paper is organized as follows. Section 2 outlines the general process of oil sands mining and material classification system used. Section 3 defines the IOSMP problem, while section 4 summarizes the literature on the application of mathematical programming models to the long term production planning problem. This is followed by a section on the application of MILP model for IOSMP problem. Section 6 outlines the concepts used in modeling the IOSMP problem and a case study presented in section 7. The paper concludes in section 8.

2. Oil sands mining

The oil sands mining system comprises of the removal of overburden material and the mining of McMurray formation. The overburden material includes muskeg/peat, the Pleistocene unit and the Clearwater formation. The muskeg/peat is barren and very wet in nature and once it is stripped, it is left for about 2 to 3 years to get it dry making it easier to handle. This material is stockpiled for future reclamation works required for all disturbed landscapes. The mining of the Pleistocene and Clearwater formation, which is classified as waste, is to enable the exposure of the ore bearing McMurray formation. Some of this material is used in the construction of dykes and are referred to as overburden dyke material. The dyke construction is for the development of tailings dam facilities constructed in-pit or ex-pit in dedicated disposal areas.

The mining of the oil bearing McMurray formation follows after the removal of the overburden material. By the regulatory and technical requirements, the mineable oil sand should have about 7% bitumen content (Dilay, 2001; Masliyah, 2010). All material satisfying this requirement is classified as ore and otherwise as waste. Some of this class of waste material are used for dyke construction and are referred to as interburden dyke material. The ore is sent directly to the processing plant. After processing the ore to extract bitumen, two main types of tailings are produced; fine and coarse tailings. The coarse tailings which can be used for dyke construction are referred to as tailings coarse sand dyke material. The fine tailings form the slurry which needs to be contained in the tailings facilities.

3. Defining the IOSMP problem

As oil sands mining companies continue to commit themselves to sustainable mining, the urgency of generating and implementing sustainable waste management practices becomes evident. Together with the limitations in lease areas, it has become necessary to look into effective and efficient waste disposal planning system. In oil sands operations, the pit phase mining occurs simultaneously with the construction of in-pit dykes in the mined out areas of the pit and ex-pit dykes in designated areas outside the pit. These dykes are constructed to hold tailings that are produced during processing of the oil sands ore. The materials used in constructing these dykes come from the oil sands mining operation. The dyke materials are made up of overburden (OB), interburden (IB) and tailings coarse sand (TCS). Any material that does not qualify as ore or dyke material is sent to the waste dump.

The integrated oil sands planning problem can be categorized in four main parts:

- Determining the order and time of extraction of ore, dyke material and waste to be removed from the designed pit shell that maximizes the Net Present Value (NPV) of the operation;
- Determining the destination of dyke material that minimizes construction cost based on the construction requirements of the various dykes;
- Determining the number and location of dykes that minimizes waste management cost; and
- Generating a backfilling schedule that maximizes the in-pit tailings disposal strategy.

Prior to IOSMP, it is assumed that the material in the designed pit limit is discretized into a three-dimensional array of rectangular or cubical blocks called a block model. Attributes of the material in the block model such as rock types, densities, grades, or economic data are represented numerically (Askari-Nasab et al., 2011, Ben-Awuah and Askari-Nasab, 2011). Fig. 1 shows the schematic diagram of the scheduling of an oil sands final pit block model containing K mining-cuts. Mining-cuts are clusters of blocks within the same level or mining bench that are grouped based on a similarity index defined using the attributes; location, grade, rock type and the shape of mining-cuts that are created on the lower bench. In this research, an agglomerative hierarchical clustering algorithm which seeks to generate clusters with reduced mining-cut extraction precedences compared with other automated methods is used (Tabesh and Askari-Nasab, 2011). Each mining-cut k, is made up of ore o_k, OB dyke material d_k, IB dyke material n_k, and waste w_k. The material in each mining-cut is to be scheduled over T periods depending on the goals and constraints associated with the mining operation. OB dyke material scheduled d_k^T, IB dyke material scheduled n_k^T, and TCS dyke material from the processed ore scheduled, l_k^T, must further be assigned to the dyke construction sites based on construction requirements. For period t_1, the dyke construction material required by site i is $dyke_i$. In addition, the final pit limit block model is divided into pushbacks. The material intersecting a pushback and a bench is known as a mining-panel. Each mining-panel contains a set of mining-cuts and is used to control the mine production operation sequencing.

The schedules generated for IOSMP drives the profitability and sustainability of an oil sands mining operation. The strategic production schedule controls the NPV of the operation while the dyke material, dyke location and backfilling schedules provide the platform for a robust waste management planning system. Previous attempts in solving the IOSMP problem with mathematical programming did not include a backfilling schedule in the optimization problem (Ben-Awuah, 2013, Ben-Awuah and Askari-Nasab, 2013). This places some limitations on the IOSMP optimization problem which can result in deviations from the optimal mining strategy. In large mining projects, such deviations can lead to major losses in revenue. In this study, we are seeking to optimize the production schedule with material destination being determined based on the mine economics, regulatory and operational requirements. The number and location of dykes will also be investigated as well as an effective backfilling schedule. This way the delicate balance between deciding on tailings dam cell sizes versus maximizing NPV and minimizing waste management cost can be evaluated.

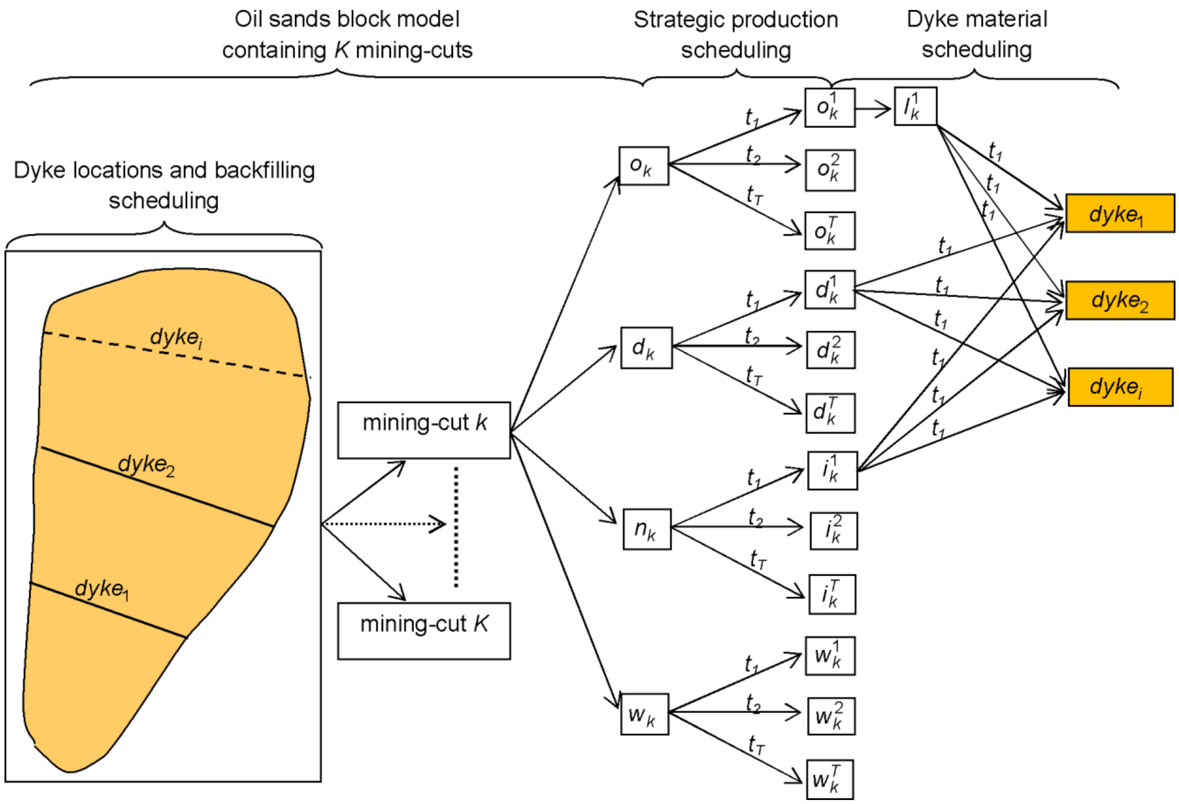

Fig. 1. Schematic diagram of the problem definition showing strategic production, dyke material, dyke location and backfilling scheduling modified after Ben-Awuah and Askari-Nasab (2011)

4. Summary of literature review

The application of mathematical programming models (MPMs) to mining decision making problems has been a major research area since the 1960s. The challenge at the time included the availability of powerful personal computers and robust optimization solvers that could deal with the large problem sizes resulting from these applications. This led to extensive research on the application of MPMs like LP and MILP to the long term production planning problem. The inherent difficulty in implementing these models is that, they result in large scale optimization problems containing many binary and continuous variables. These are difficult to solve and may have lengthy solution times.

Previous researchers have made significant efforts in reducing the solution time associated with solving MPMs. Their models however, were not capable of dealing with large block model sizes or could not generate feasible practical mining strategies (Akaike and Dagdelen, 1999, Caccetta and Hill, 2003, Dagdelen, 1985, Gershon, 1983, Johnson, 1969, Ramazan, 2001, Ramazan and Dimitrakopoulos, 2004a). These publications note that the size of the resulting LP and MILP models is a major problem because it contains too many binary and continuous variables. GP has also been explored in dealing with the long term production planning problem. It permits flexible formulation, specification of priorities among goals, and some level of interactions between the decision maker and the optimization process (Hannan, 1985, Zeleny, 1980). This led to its application to the long term production planning problem by Zhang et al. (1993), Chanda and Dagdelen (1995) and Esfandiri et al. (2004). They were however unable to practically implement their models due to the numerous mining production constraints and size of the optimization problem.

Recent implementation of MILP models with block clustering techniques were successfully undertaken for an iron ore deposit (Askari-Nasab et al., 2010, Askari-Nasab et al., 2011). It however lacks the framework for the implementation of an integrated mine planning and waste management system as is the case required for sustainable oil sands mining. Due to the strategy required for sustainable oil sands mining and the regulatory requirements from Directive 074, waste management is directly linked to the mine planning system (Askari-Nasab and Ben-Awuah, 2011, Ben-Awuah, 2013, Ben-Awuah and Askari-Nasab, 2011, McFadyen, 2008). Currently, oil sands waste disposal planning is managed as a post-production scheduling optimization activity. Consequently, the lack of an integrated sustainable oil sands mine production scheduling and waste disposal planning system in an optimization framework is a challenge. Modeling such an integrated mine planning system even adds more complexity to the long term production planning problem. Ben-Awuah et al. (2012) implemented a MILGP model for an integrated oil sands production scheduling and waste disposal planning system. The model takes into account multiple material types, elements and destinations, directional mining, waste management and sustainable practical mining strategies. The implementation of the MILGP model did not include assessment of backfilling strategies which forms an integral part of the IOSMP problem.

This paper presents scheduling models and tests on how to implement an MILP framework for an IOSMP problem with varying waste management strategies. The tests show that, varying waste management strategies have different impacts on NPV and waste management cost. Depending on the mining operation’s environmental and reclamation policy as compared to its investment strategy, the appropriate IOSMP option may be suitable. An oil sands data set is used for the case study.

5. MILP model for IOSMP

The IOSMP problem can be summarized as finding the time and sequence of extraction of ore, dyke material and waste mining-cuts to be removed from an open pit outline and sent to their respective destinations over the mine life, so that the NPV of the operation is maximized and waste management cost is minimized. The waste management includes dyke construction and backfilling activities. This requires an MILP formulation involving multiple mines, material types and destinations as well as pushbacks which ties into the waste management strategy for the oil sands operations. The production schedule is subject to a variety of technical, physical and economic constraints which enforce mining extraction sequence, mining and dyke construction capacities, blending requirements and backfilling strategy. The notations used in the formulation of the IOSMP problem have been classified as sets, indices, subscripts, superscripts, parameters and decision variables. An exhaustive list of these notations can be found in this section and in the Appendix.

The summary of economic data for each mining-cut known as economic mining-cut value is based on ore parcels within mining-cuts which could be mined selectively. The economic mining-cut value is a function of the value of the mining-cut based on the processing destination and the costs incurred in mining from a designated location and processing, and dyke construction at a specified destination. The cost of dyke construction is also a function of the location of the tailings facility being constructed and the type and quantity of dyke material used. The discounted economic mining-cut value for mining-cut k is equal to the discounted revenue obtained by selling the final product contained in mining-cut k minus the discounted cost involved in mining mining-cut k as waste minus the extra discounted cost of mining OB and IB dyke material, and generating TCS dyke material from mining-cut k for a designated dyke construction destination. This can be summarized by Eqs. (1) to (6). The concepts presented in Ben-Awuah and Askari-Nasab (2013) were used as the starting point of the development.

Discounted economic mining-cut value = discounted revenue - discounted costs

$$d_k^{u,t} = v_k^{u,t} - q_k^{a,t} - p_k^{u,t} - m_k^{u,t} - h_k^{u,t} \tag{1}$$

The variables in Eq. (1) can be defined by Eqs. (2) to (6).

$$v_k^{u,t} = \sum_{e=1}^{E} o_k \times g_k^e \times r^{u,e} \times \left(p^{e,t} - cs^{e,t} \right) - \sum_{e=1}^{E} o_k \times cp^{u,e,t} \tag{2}$$

$$q_k^{a,t} = \left(o_k + d_k + n_k + w_k \right) \times cm^{a,t} \tag{3}$$

$$p_k^{u,t} = d_k \times ck^{u,t} \tag{4}$$

$$m_k^{u,t} = n_k \times cb^{u,t} \tag{5}$$

$$h_k^{u,t} = l_k \times ct^{u,t} \tag{6}$$

Where:

$t \in \{1,......,T\}$ index for scheduling periods.

$k \in \{1,.....,K\}$ index for mining-cuts.

$p \in \{1,.....,P\}$ index for mining-panels.

$e \in \{1,.....,E\}$ index for element of interest in each mining-cut.

$j \in \{1,.....,J\}$ index for phases (pushback).

$u \in \{1,.....,U\}$ index for possible destinations for materials.

$a \in \{1,.....,A\}$ index for possible mining locations (pits).

$d_k^{u,t}$ the discounted economic mining-cut value obtained by extracting mining-cut *k* and sending it to destination *u* in period *t*.

$v_k^{u,t}$ the discounted revenue obtained by selling the final products within mining-cut *k* in period *t* if it is sent to destination *u*, minus the extra discounted cost of mining all the material in mining-cut *k* as ore from location *a* and processing at destination *u*.

$q_k^{a,t}$ the discounted cost of mining all the material in mining-cut *k* in period *t* as waste from location *a*.

$b_p^{a,t}$ the discounted cost of mining all the material in mining-panel *p* in period *t* as waste from location *a*. Each mining-panel *p* contains its corresponding set of mining-cuts.

$p_k^{u,t}$ the extra discounted cost of mining all the material in mining-cut k in period t as overburden dyke material for construction at destination u.

$m_k^{u,t}$ the extra discounted cost of mining all the material in mining-cut k in period t as interburden dyke material for construction at destination u.

$h_k^{u,t}$ the extra discounted cost of mining all the material in mining-cut k in period t as tailings coarse sand dyke material for construction at destination u.

o_k the ore tonnage in mining-cut k.

d_k the overburden dyke material tonnage in mining-cut k.

n_k the interburden dyke material tonnage in mining-cut k.

w_k the waste tonnage in mining-cut k.

l_k the tailings coarse sand dyke material tonnage in mining-cut k.

g_k^e the average grade of element e in ore portion of mining-cut k.

$r^{u,e}$ the proportion of element e recovered (processing recovery) if it is processed at destination u.

$p^{e,t}$ the price of element e in present value terms per unit of product.

$cs^{e,t}$ the selling cost of element e in present value terms per unit of product.

$cp^{u,e,t}$ the extra cost in present value terms per tonne of ore for mining and processing at destination u.

$cm^{a,t}$ the cost in present value terms of mining a tonne of waste in period t from location a.

$ck^{u,t}$ the cost in present value terms per tonne of overburden dyke material for dyke construction at destination u.

$cb^{u,t}$ the cost in present value terms per tonne of interburden dyke material for dyke construction at destination u.

$ct^{u,t}$ the cost in present value terms per tonne of tailings coarse sand dyke material for dyke construction at destination u.

5.1. The MILP model for optimizing production schedule

The objective function of the MILP model that maximizes the NPV of the mining operation can be formulated using the continuous decision variables, $y_p^{a,t}$, and $x_k^{u,t}$ to model mining and processing requirements for all mining locations and processing destinations respectively. Using continuous decision variables allows for fractional extraction of mining-panels and mining-cuts in different periods for different locations and destinations. The objective function of the MILP model for maximizing the NPV of the mining operation is represented by Eq. (7).

$$Max \sum_{a=1}^{A}\sum_{j=1}^{J}\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{\substack{k\in B_p \\ p\in B_j}}\left(v_k^{u,t}\times x_k^{u,t}-b_p^{a,t}\times y_p^{a,t}\right)\right) \tag{7}$$

5.1.1. Related constraints

These constraints are used in controlling the mining and processing targets. They are defined in the form of an upper and lower bound and are controlled by the decision variables, $y_p^{a,t}$ and $x_k^{u,t}$. Eq. (8) defines the mining capacity requirements while Eq. (9) defines the processing capacity requirements. Since ore processing drives the optimization problem, the lower bound for the processing target is usually not defined. The production grade blending constraints control the grade of ore bitumen and ore fines in the mined material for all processing destinations. These constraints are formulated in Eqs. (10) to (13).

$$T_{m,lb}^{a,t} \le \sum_{j=1}^{J}\left(\sum_{p\in B_j}\left(o_p + d_p + n_p + w_p\right)\times y_p^{a,t}\right) \le T_{m,ub}^{a,t} \tag{8}$$

$$T_{pr,lb}^{u,t} \le \sum_{p=1}^{P}\left(\sum_{k\in B_p}\left(o_k \times x_k^{u,t}\right)\right) \le T_{pr,ub}^{u,t} \tag{9}$$

$$\sum_{p=1}^{P}\sum_{k\in B_p} g_k^e \times o_k \times x_k^{u,t} - \overline{g}^{u,t,e}\sum_{p=1}^{P}\sum_{k\in B_p} o_k \times x_k^{u,t} \le 0 \tag{10}$$

$$\sum_{p=1}^{P}\sum_{k\in B_p} g_k^e \times o_k \times x_k^{u,t} - \underline{g}^{u,t,e}\sum_{p=1}^{P}\sum_{k\in B_p} o_k \times x_k^{u,t} \ge 0 \tag{11}$$

$$\sum_{p=1}^{P}\sum_{k\in B_p} f_k^e \times o_k \times x_k^{u,t} - \overline{f}^{u,t,e}\sum_{p=1}^{P}\sum_{k\in B_p} o_k \times x_k^{u,t} \le 0 \tag{12}$$

$$\sum_{p=1}^{P}\sum_{k\in B_p} f_k^e \times o_k \times x_k^{u,t} - \underline{f}^{u,t,e}\sum_{p=1}^{P}\sum_{k\in B_p} o_k \times x_k^{u,t} \ge 0 \tag{13}$$

5.2. The MILP model for optimizing dyke material schedule

The objective function of the MILP model that minimizes the dyke construction cost as part of the waste management operation can be formulated using the continuous decision variables $z_k^{u,t}$, $c_k^{u,t}$, and $s_k^{u,t}$ to model OB, IB and TCS dyke material requirements respectively for all dyke construction destinations. The objective function for minimizing the dyke construction cost is represented by Eq. (14).

$$Min \sum_{a=1}^{A}\sum_{j=1}^{J}\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{\substack{k\in B_p\\ p\in B_j}}\left(p_k^{u,t}\times z_k^{u,t} + m_k^{u,t}\times c_k^{u,t} + h_k^{u,t}\times s_k^{u,t}\right)\right) \tag{14}$$

1.1.1 Related constraints

The constraints used in controlling the OB, IB and TCS dyke material requirements are modeled with Eqs. (15) to (17) respectively. These define the upper and lower bounds and are controlled by the variables $z_k^{u,t}$, $c_k^{u,t}$, and $s_k^{u,t}$. Eq. (18) and Eq. (19) are grade blending constraints which control the grade of interburden fines in the mined material for dyke construction destinations. These constraints ensure that the movement of dyke material and dyke construction scheduling can be well integrated with the mining fleet management plan.

$$T_{d,lb}^{u,t} \le \sum_{p=1}^{P}\left(\sum_{k\in B_p}\left(d_k \times z_k^{u,t}\right)\right) \le T_{d,ub}^{u,t} \tag{15}$$

$$T_{n,lb}^{u,t} \le \sum_{p=1}^{P}\left(\sum_{k\in B_p}\left(n_k \times c_k^{u,t}\right)\right) \le T_{n,ub}^{u,t} \tag{16}$$

$$T_{l,lb}^{u,t} \le \sum_{p=1}^{P}\left(\sum_{k\in B_p}\left(l_k \times s_k^{u,t}\right)\right) \le T_{l,ub}^{u,t} \tag{17}$$

$$\sum_{p=1}^{P}\sum_{k\in B_p} f_k^{d} \times n_k \times c_k^{u,t} - \overline{f}^{u,t,d}\sum_{p=1}^{P}\sum_{k\in B_p} n_k \times c_k^{u,t} \le 0 \tag{18}$$

$$\sum_{p=1}^{P}\sum_{k\in B_p} f_k^{d} \times n_k \times c_k^{u,t} - \underline{f}^{u,t,d}\sum_{p=1}^{P}\sum_{k\in B_p} n_k \times c_k^{u,t} \ge 0 \tag{19}$$

5.3. The MILP model for optimizing in-pit tailings backfilling schedule

The objective function of the MILP model that maximizes the in-pit volume for tailings backfilling as part of the waste management strategy can be formulated using the continuous decision variable, $d_j^{a,t}$, to model the volume of mining phase backfilled in each period. A pseudo mining revenue per meter cube, ps^{rev}, is defined to drive the backfilling operation. The continuous decision variable allows for fractional backfilling of a mining phase. This objective function can be represented by Eq. (20).

$$Max\sum_{a=1}^{A}\sum_{j=1}^{J}\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{\substack{k\in B_p\\ p\in B_j}}\left(ps_j^{rev} \times d_j^{a,t}\right)\right) \tag{20}$$

5.3.1. Related constraint

The constraint used in controlling the in-pit and ex-pit volume filled in each period is modeled with Eq. (21). This defines the available in-pit volume in each mining phase to be backfilled, vp_j, and is controlled by the variable $d_j^{a,t}$; the ex-pit volume, ep^u, to be filled and is controlled by the variable, $i^{a,u,t}$. This constraint cumulatively reconciles the in-pit and ex-pit volume available with the volume of tailings produced, ts_p, waste material mined, wv_p overburden dyke material mined, dv_p, and interburden dyke material mined, nv_p, throughout the mine life.

$$\sum_{j=1}^{J}\left(\sum_{p\in B_j}\left(vp_j \times d_j^{a,t}\right)\right) + \sum_{u=1}^{U}\left(ep^u \times i^{a,u,t}\right) - \sum_{j=1}^{J}\left(\sum_{p\in B_j}\left(ts_p + wv_p + dv_p + nv_p\right)\right) = 0 \tag{21}$$

5.4. The MILP model general constraints

The general constraints that apply to all the MILP models discussed relate to the mining precedence and the logics of the variables during optimization. These have been documented in Ben-Awuah et al. (2012) and Ben-Awuah and Askari-Nasab (2013). These constraints include:

a) Vertical mining precedence: all the immediate predecessor mining-panels above the current mining-panel should be extracted prior to extracting the current mining-panel;
b) Horizontal mining precedence: all the immediate predecessor mining-panels preceding the current mining-panel in the horizontal mining direction are extracted before or together with the current mining-panel. These are referred to as absolute and concurrent precedences respectively;
c) Tailings cells precedence: all the mining phases within the immediate predecessor tailings cell that precedes the current tailings cell are extracted before extraction of the mining phases in the current tailings cell;
d) Variables logic control: the logic of the mining, processing, dyke material and backfilling variables with regards to their limits and definitions are within acceptable ranges.

6. Modeling the IOSMP problem

The IOSMP problem is modeled in Evaluator as a multi-mine, multi-destination and multi-material type optimization problem. A schematic diagram of the scheduling project network can be seen in Fig. 2. The conceptual mining and waste management model applied here is similar to that presented in Ben-Awuah et al. (2012). This includes completely extracting all material in the current tailings cell prior to mining the next tailings cell in the direction of mining. This makes the current tailings cell available for in-pit tailings deposition. The IOSMP problem was modeled with four mines namely; Pit, DykeMat, BackFill and ExWaste (Fig. 2). The Pit node contains all the data relating to the mining-cuts and mining panels to be extracted. The DykeMat node contains the quantity of OB, IB and TCS dyke material required to construct the designed dykes. The dyke locations are fixed prior to each optimization. The BackFill node contains the volume of the mine phases that becomes available as mining proceeds in the defined direction for subsequent backfilling. The ExWaste node contains the available volume at the external waste facility.

Material from the pit can be sent to the processing plant, dyke construction destinations or waste dump based on the material type and mine economics. Material sent to the processing plant results in a product that generates revenue for the mining project. Material sent for dyke construction can be sent to either of the dyke destinations depending on which dyke is immediately needed and has the minimum cost. Material that does not qualify for processing or dyke construction is sent to the waste dump. Material that qualifies for building dykes but is not needed for construction at any point in time will be sent to the waste dump as well. The constraints that are set up to control the pit mining are mainly the mining capacity, processing limits and the ore quality requirements throughout the mine life. The vertical and horizontal mining sequences for the mining-panels which include both absolute and concurrent precedences are defined as well. Complete extraction of the in-pit ore is enforced as required by oil sands mining regulations.

The DykeMat node which contains the designed dyke construction requirements is modeled to send a request for dyke material anytime a dyke needs to be built at a specified location. This request specifies the dyke material type and quantity required. This is done through a constraint which ensures that the dyke material request emanating from DykeMat is equal to the dyke material flowing from the Pit through the DykeM node to the appropriate dyke construction destination. The corresponding destination specific dyke construction cost is then applied.

The request to construct a dyke is issued by the BackFill node. The backfilling activity has been modeled to generate a pseudo revenue for every meter cube backfilled. As mining proceeds in the defined mining direction, once the mining phases making up the first tailings cell are completely extracted, a request for dyke construction material is placed and then subsequently backfilling starts. The model features a constraint which cumulatively reconciles the in-pit and ex-pit volume available with the volume of tailings produced, dyke material placed and waste material mined throughout the mine life. Dyke construction proceeds simultaneously with backfilling until the dyke is fully built and the corresponding tailings cell completely filled. Continuous backfilling is enforced such that once in-pit backfilling starts, this activity must continue until the end of the mine life. This ensures that the installed backfilling pumping or trucking capacity is fully utilized. Any excess waste material is sent to the external waste facility.

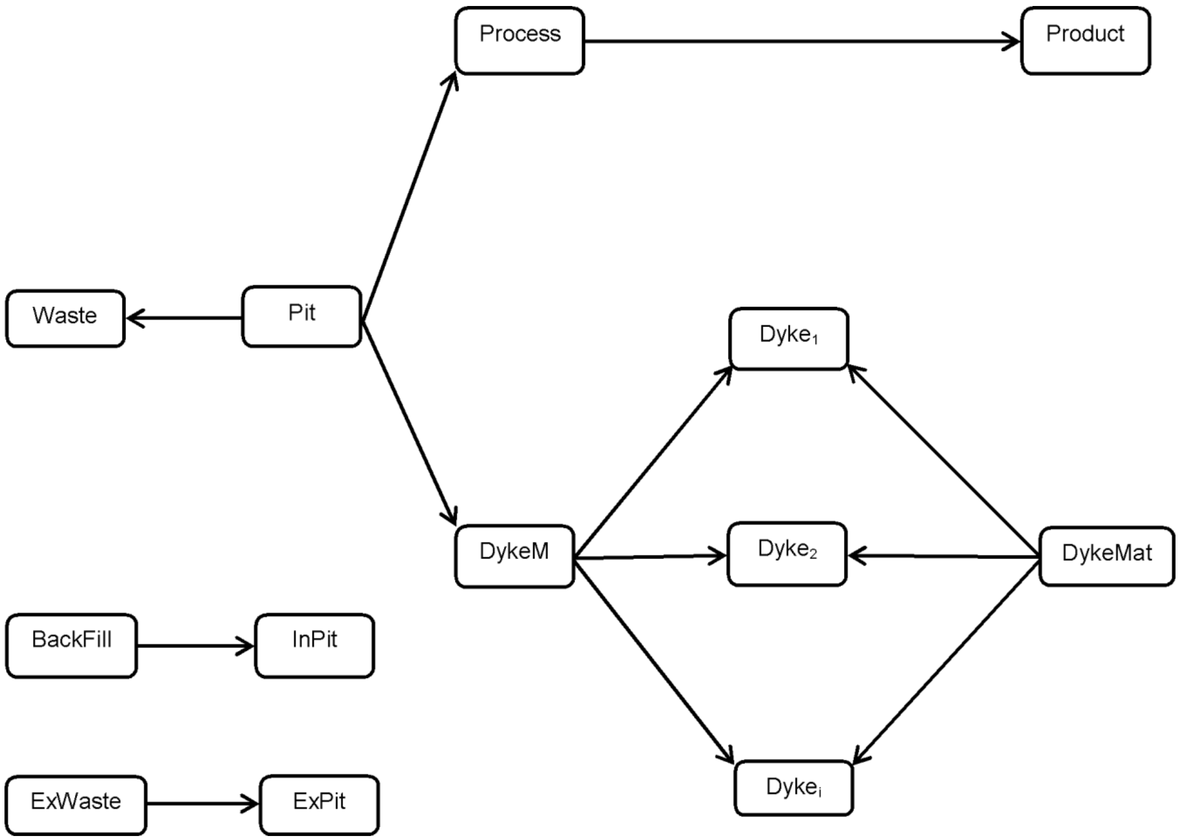

Fig. 2. Schematic diagram of the project scheduling network

7. Case study: results and discussions

The MILP model for the IOSMP problem was implemented on an oil sands deposit with a final pit covering an area of about 3000 ha. The mineralized zone of this deposit occurs in the McMurray formations. The deposit is to be scheduled for 20 periods for the processing plant with an integrated waste management strategy that includes dyke construction and an in-pit tailings disposal scheme. The performance of the proposed MILP model was analyzed based on NPV, mining production targets, smoothness and practicality of the generated schedules and the availability of tailings containment areas. Table 1 provides information about the orebody model within the ultimate pit limit used in the case study. The area to be mined is divided into 15 pushbacks with each holding approximately equal tonnes of material. These pushbacks enable the creation of practical mining-panels to be used in controlling the mining operation. In consultation with tailings dam engineers based on required tailings cell capacities, three scenarios of tailings disposal strategies will be investigated. This relates to the number and location of dykes to be constructed and their impact on the mining operation. The waste management scenarios to be investigated include tailings disposal strategies with four, three and two tailings cells.

A hierarchical clustering algorithm is used in clustering blocks within each pushback into mining-cuts (Tabesh and Askari-Nasab, 2011). Clustering blocks into mining-cuts ensures the MILP scheduler generates a mining strategy at a selective mining unit that is practical from mining operation perspective. In solving the MILP model with Gurobi, the absolute tolerance on the gap between the best integer objective and the objective of the best node remaining in the branch and cut algorithm, referred to as MIPGap, was set at 2% for the optimization of the mining project. The controls for the mining capacity, processing plant feed, dyke construction requirements, bitumen grade and fines percent have been summarized in Table 2. Mining will proceed from pushback 1 to 15 with complete extraction of each tailings cell prior to the next. In addition to the processing plant, tailings backfilling activities and dyke construction requirements will be scheduled. Backfilling of the last tailings cell prior to the end of the mine life is not started since ore processing is assumed to have been completed. Details of the waste management strategy implemented here has been documented by Ben-Awuah et al. (2012).

Table 1. Oil Sands Deposit Characteristics

Characteristic	**Value**
Tonnage of rock (Mt)	6263
Ore tonnage (Mt)	1923
OB dyke material tonnage (Mt)	1866
IB dyke material tonnage (Mt)	1873
TCS dyke material tonnage (Mt)	1350
Waste tonnage (Mt)	601
Average ore bitumen grade (wt%)	13.3
Average ore fines (wt%)	18.1
Number of blocks	81,760
Number of mining-cuts	1773
Number of mining-panels	123
Block dimensions (m^3)	50 x 50 x 15
Number of benches	9

Table 2. Mining and Processing Targets, OB, IB and TCS Dyke Construction Requirements and Ore Grade Limits for the MILP Model

Production scheduling parameter	**Value**
Mining target (Mt) $\left[T_{m,ub}^{a,t} / T_{m,lb}^{a,t}\right]$	350/0
Processing target (Mt) $\left[T_{pr,ub}^{u,t} / T_{pr,lb}^{u,t}\right]$	120/0
Average ore bitumen grade (wt%) $\left[\overline{g}^{u,t,e} / \underline{g}^{u,t,e}\right]$	16/7
Average ore fines (wt%) $\left[\overline{f}^{u,t,e} / \underline{f}^{u,t,e}\right]$	30/0
OB dyke material tonnage required per dyke (Mt)	4.5
IB dyke material tonnage required per dyke (Mt)	4.5
TCS dyke material tonnage required per dyke (Mt)	220

7.1. Analysis

The experiment was carried out on three scenarios of the IOSMP problem with varying waste management strategies at a 10% discount rate. Scenario 1 (Table 3) was chosen for discussions due to its relatively efficient waste management strategy which uses about 80% of the in-pit volume before the end of mine life. After optimization, the NPV generated is $25,211 M at a 1.5%

MIPGap. This excludes the dyke construction cost for all tailings cells and the pseudo revenue from backfilling. The total dyke construction cost is $232 M and the total pseudo revenue from backfilling is $1,405 M. The scenario implemented here focuses on a practically integrated oil sands production planning and waste management strategy that generates value and is sustainable. This includes mining in a specified direction and making completely extracted tailings cells available for in-pit dyke construction and subsequently tailings deposition. This reduces the environmental footprints of the external tailings facility by commissioning in-pit tailings facilities on time. The mining direction was decided on during an initial production schedule run in Whittle (GEOVIA-Dassault, 2015). The mining direction with the best NPV was selected for the MILP model. The mining sequence at level 305 m for all pushbacks with a west-east mining direction and tailings cells dyke locations in Scenario 1 can be seen in Fig. 3. Fig. 3 also shows the complete extraction of each tailings cell prior to mining the next, to support tailings management. The mining sequence shows a progressive continuous mining in the specified direction to ensure least mobility and increased utilization of loading equipment. This is very important in the case of oil sands mining where large cable shovels are used. The size of the mining-cuts and mining-panels also enables good equipment maneuverability and supports multiple material loading operations. It enables mining to proceed with a reduced number of required drop-cuts.

Table 3. Summary of Results for the IOSMP Problem with Different Waste Disposal Strategies

Scenario #	Tonnage mined (Mt)	Ore Tonnage (Mt)	Dyke material tonnage (Mt)	NPV (M$)	Dyke construction cost (M$)	Pseudo backfilled revenue ($M)	No. of tailings cells	In-pit volume backfilled (%)	MIPGap (%)
1	6217	1923	687	25,211	232	1,405	4	81	1.5
2	6211	1923	458	25,363	154	1,175	3	67	1.8
3	6201	1923	229	27,262	68	820	2	54	2.0

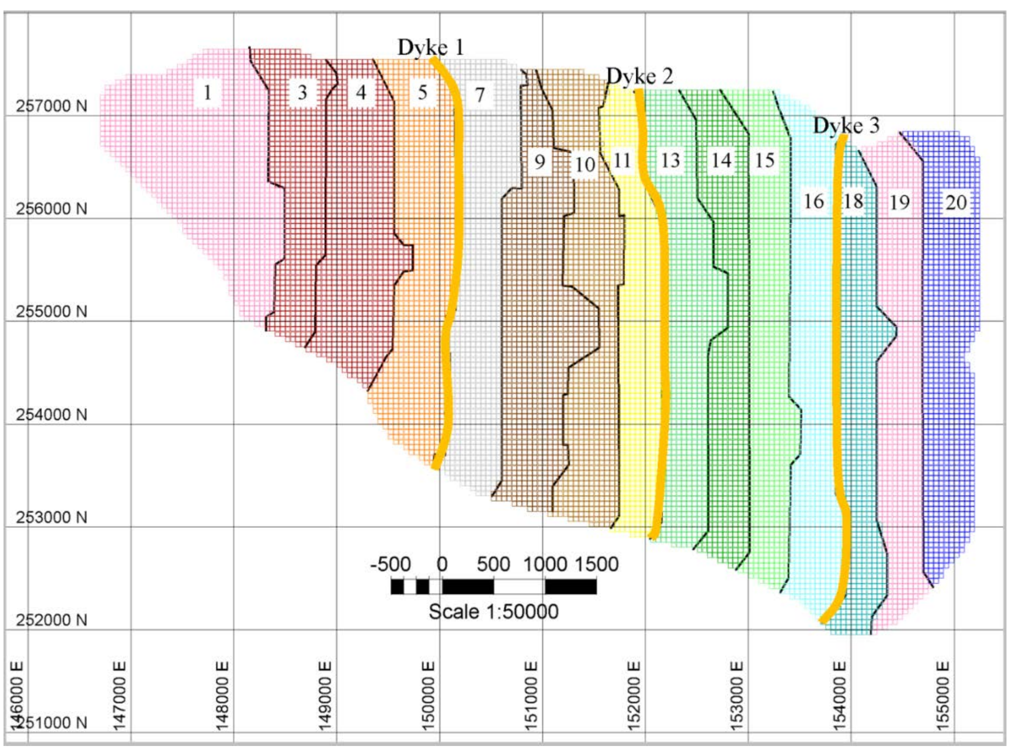

Fig. 3. Scenario 1 mining sequence at level 305m with a west-east mining direction and dyke locations for tailings cells

Fig. 4 illustrates how mining and processing progress uniformly throughout the mine life. This ensures efficient utilization of the mining fleet and processing plant capacity. Pre-stripping of pushbacks 1 and 2 start in the first and second years, resulting in less ore being mined. Subsequently, uniform ore feed is provided at the required processing plant capacity throughout the mine life with a capacity step-down in year 17. The type and quantity of dyke material needed to build the in-pit tailings cells dykes in a timely manner and at a minimum cost can be seen in Fig. 5. The request for dyke material is made anytime all the pushbacks in a tailings cell are completely mined and backfilling operations are ready to take off. At that time, the dyke material mined is sent to the scheduled dyke construction destination. By design the OB and IB dyke material are initially required to construct the dyke foundation and then subsequently TCS dyke material is needed for the main dyke. The tailings backfilling schedule is shown in Fig. 6. This shows that at a continuous backfilling rate of about 140 Mm^3 per year, tailings cell 1 is filled from periods 6 to 11. Tailings cell 2 is filled from periods 12 to 16 at a rate of about 160 Mm^3 per year while tailings cell 3 is filled from period 17 to 20 at a rate of about 190 Mm^3. These backfilling variations are as a result of enforcing continuous backfilling and the volume of tailings, dyke material and waste available for backfilling. After the dyke foundation construction with overburden and interburden (OI) dyke material, the backfilling operation occurs simultaneously as the main tailings cell dyke is being constructed with TCS dyke material. This operation is usually undertaken with a hydro-cyclone that places the TCS dyke material on the dyke and the tailings slurry inside the cell. Table 3 shows the total material mined, ore, OB and IB dyke material tonnage mined and TCS dyke material tonnage placed from the processing plant. The schedules give the planner good control over production forecasting and provides a robust platform for effective dyke construction planning and tailings storage management.

The ore and dyke material quality is obtained by blending the run-of-mine material. The targeted processing plant head grade was successfully achieved in all periods. It was targeted to reduce the periodic grade variability by setting tighter lower and upper grade bounds. The periodic grades in each pushback can be varied depending on the processing plant or dyke construction requirements while ensuring a feasible solution is obtained. Figs. 7 and 8 show the average ore bitumen grades and ore fines percent over the mine life.

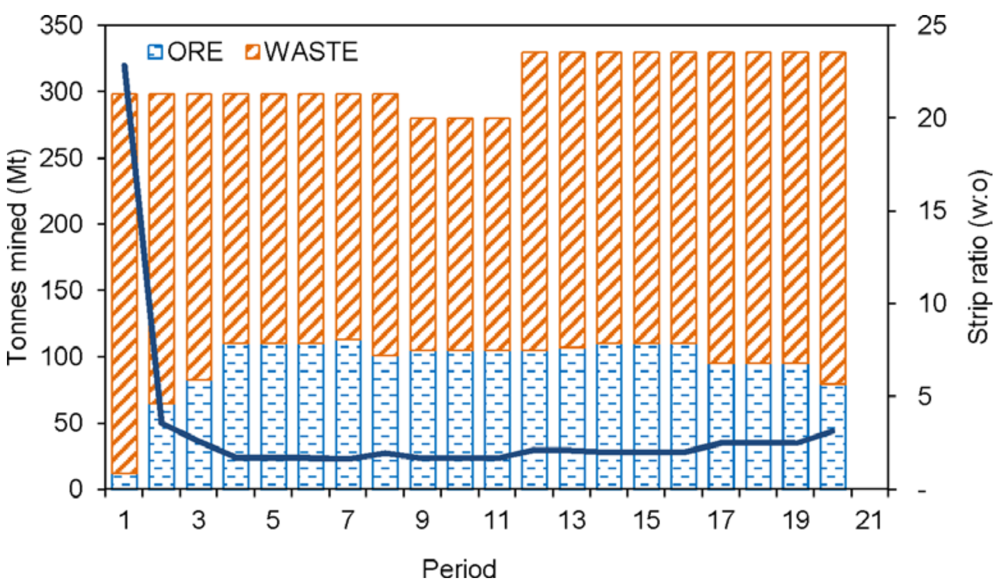

Fig. 4. Production schedule for ore and waste, and stripping ratio

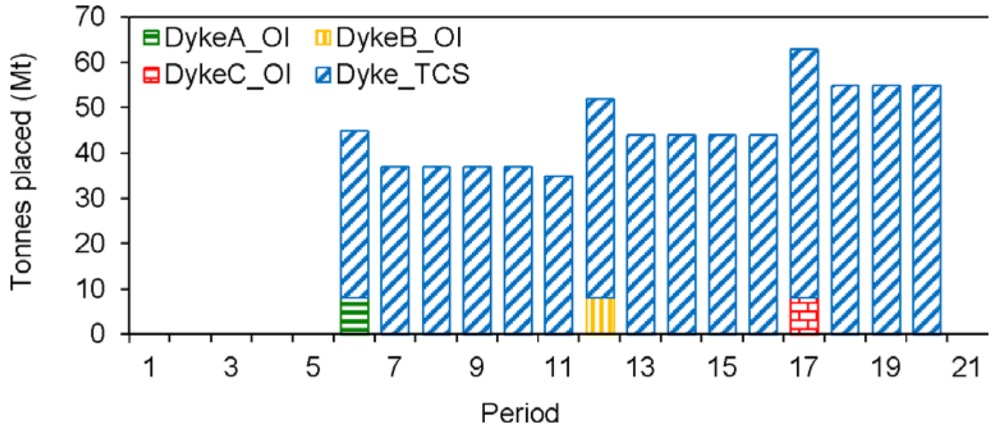

Fig. 5. Dyke material schedule including OB, IB and TCS for Dykes 1, 2 and 3

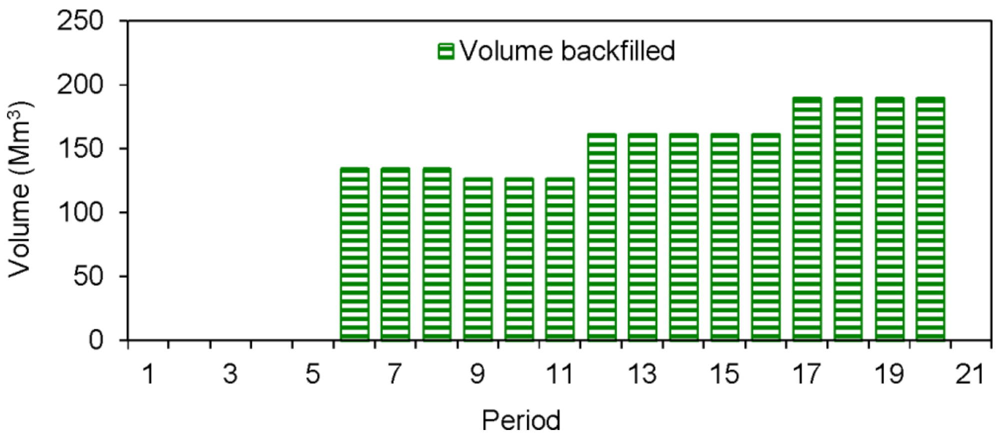

Fig. 6. Backfilling schedule for in-pit tailings cells

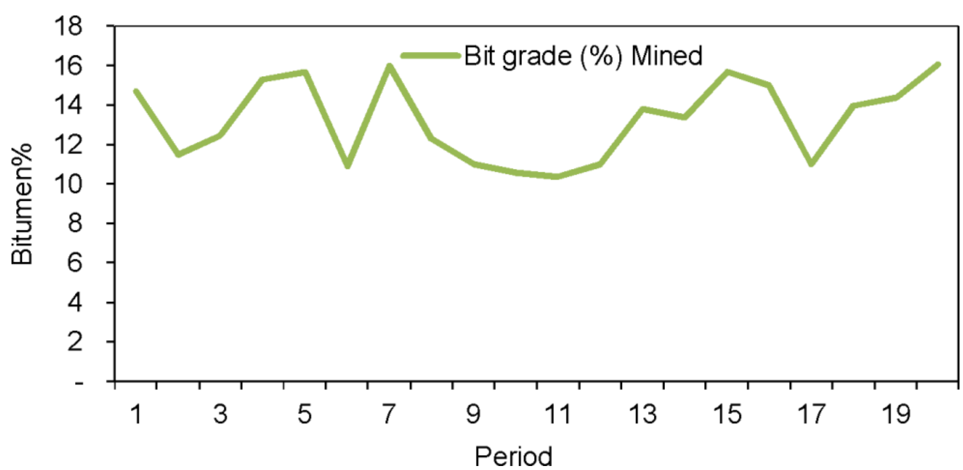

Fig. 7. Average ore bitumen grades

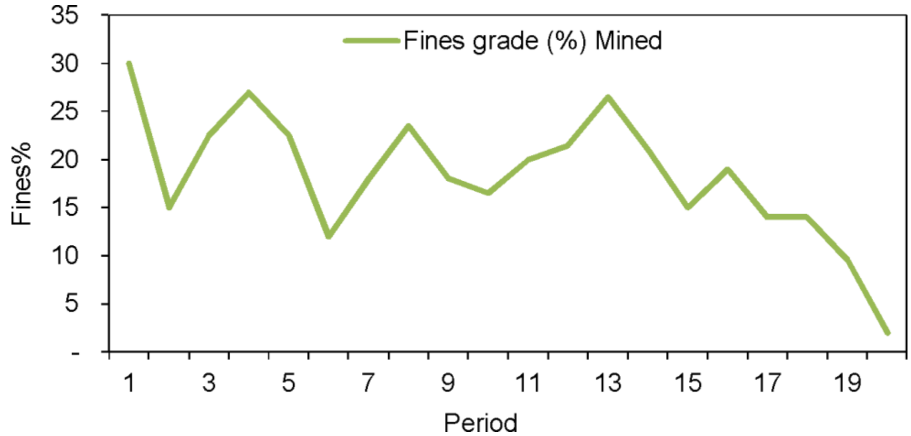

Fig. 8. Average ore fines percent

7.2. Comparison

In implementing the MILP model for the IOSMP problem, three optimization scenarios were executed to assess the effect of different waste disposal strategies on the mining operation in terms of NPV and waste management cost. Table 3 shows a summary of the results of the scenarios with different number of dykes and tailings cells. The results show that Scenario 1 has the lowest NPV, highest dyke construction cost and highest pseudo backfilling revenue. This is due to the fact that with more tailings cells, the production operation is more restricted as each tailings cell must be completely exhausted before mining in the next tailings cell commences. The reduced operational flexibility also comes from the decrease in the size of the tailings cells thereby restricting the optimizer when generating the mining schedules. More tailings cells also mean more dykes being constructed to hold the tailings thereby increasing dyke construction cost. However, this strategy also leads to the availability of in-pit tailings disposal areas quite early in the mine life. This results in an increase in the pseudo revenue from the backfilling operation which in real terms is savings in not sending the tailings to an external tailings facility at a higher cost. The scenario with the least number of tailings cells (Scenario 3) generates the highest NPV due to production scheduling flexibility and a corresponding reduced dyke construction cost. This strategy on the other hand results in delayed in-pit tailings deposition leading to reduced pseudo backfilling revenue. It is also noted that the unfilled tailings cell size at the end of the mine life for Scenario 1 is 571 Mm^3 (19%) compared to 968 Mm^3 (33%) for Scenario 2 and 1336 Mm^3 (46%) for Scenario 3.

These three different waste management strategies have their own inherent advantages and disadvantages depending on conditions at the mine and priorities of the operation. If in-pit tailings deposition must happen soon as part of an environmental policy, reclamation plan or limited immediate availability of an external tailings facility capacity, then Scenario 1 may be preferred. If on the other hand, there is the need to increase NPV and delay in-pit deposition due to availability of capacity at an external tailings facility, then Scenario 3 may be preferred. Another strategy between these two relatively extreme scenarios (Scenario 2) can be considered as well as a hybridized approach which may include lateral splitting of the in-pit area to reduce the unfilled tailings cell size remaining at the end of the operation.

8. Conclusions

The integrated oil sands mine planning problem involves the incorporation of waste management into the production planning process in an optimization framework that maximizes value and is sustainable. This research developed, implemented and verified a MILP formulation which takes into account practical shovel movements by selecting mining-panels and mining-cuts that are comparable to the selective mining units of oil sands mining operations. Different waste management techniques ranging from having two in-pit tailings cells to four in-pit tailings cells have been presented for the MILP model. The model generated a practical, smooth and uniform schedule for ore and in-pit tailings disposal. The schedule gives the planner good control over dyke material and provides a robust platform for effective dyke construction and waste disposal planning.

The results show that increasing the number of in-pit tailings cells reduces the NPV of the operation as a result of a reduced operational flexibility. The reduced operational flexibility comes from the decrease in the size of the tailings cells thereby restricting the optimizer when generating the mining schedules. However, this strategy apart from making in-pit tailings storage areas available early in the mine life, also makes an efficient use of in-pit storage areas which are required for sustainable operations and timely reclamation. This framework for the IOSMP problem results in solutions within known limits of optimization. In general, this integrated mine planning framework can be implemented for various directions of mining, different shapes and sizes of tailings cells, multiple mine pits and phases configurations and final landscape designs.

The total NPV generated for Scenario 1 excluding dyke construction and pseudo backfilling revenue for all tailings cells is $25,211 M. The total dyke construction cost is $232 M and the total pseudo revenue from backfilling is $1405 M. The average bitumen grade and fines percent for the scheduled ore was 13.3% and 18.1% respectively. The total material mined was 6217 Mt, which includes: 1923 Mt of ore, 27 Mt of OB and IB dyke material, while 660 Mt of TCS dyke material was placed.

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10. Appendix

10.1. Notations

10.1.1. Sets

$\boldsymbol{K} = \{1,.....,K\}$ set of all the mining-cuts in the model.

$P = \{1,.....,P\}$ set of all the mining-panels in the model.

$\boldsymbol{J}=\{1,......,J\}$	set of all the phases (push-backs) in the model.
$\boldsymbol{U}=\{1,.....,U\}$	set of all the possible destinations for materials in the model.
$A=\{1,.....,A\}$	set of all the possible mining locations (pits) in the model.
$B_p(V)$	for each mining-panel *p*, there is a set $B_p(V)\subset K$ defining the mining-cuts that belongs to the mining-panel *p*, where *V* is the total number of mining-cuts in the set $B_p(V)$.
$B_j(H)$	for each phase *j*, there is a set $B_j(H)\subset P$ defining the mining-panels that belongs to the pit phase *j*, where *H* is the total number of mining-panels in the set $B_j(H)$.

10.1.2. Parameters

$\underline{g}^{u,t,e}$	the lower bound on the required average head grade of element *e* in period *t* at processing destination *u.*
$\overline{g}^{u,t,e}$	the upper bound on the required average head grade of element *e* in period *t* at processing destination *u*.
f_k^e	the average percent of fines in ore portion of mining-cut *k*.
$\underline{f}^{u,t,e}$	the lower bound on the required average fines percent of ore in period *t* at processing destination *u.*
$\overline{f}^{u,t,e}$	the upper bound on the required average fines percent of ore in period *t* at processing destination *u*.
f_k^d	the average percent of fines in interburden dyke material portion of mining-cut *k*.
$\underline{f}^{u,t,d}$	the lower bound on the required average fines percent of interburden dyke material in period *t* at dyke construction destination *u.*
$\overline{f}^{u,t,d}$	the upper bound on the required average fines percent of interburden dyke material in period *t* at dyke construction destination *u*.
o_p	the ore tonnage in mining-panel *p*.
d_p	the overburden dyke material tonnage in mining-panel *p*.
n_p	the interburden dyke material tonnage in mining-panel *p*.
w_p	the waste tonnage in mining-panel *p*.
$T_{m,ub}^{a,t}$	the upper bound on the mining capacity (tonnes) in period *t* at location *a*.
$T_{m,lb}^{a,t}$	the lower bound on the mining capacity (tonnes) in period *t* at location *a*.
$T_{pr,ub}^{u,t}$	the upper bound on the processing capacity in period *t* at destination *u* (tonnes).

$T_{pr,lb}^{u,t}$ the lower bound on the processing capacity in period t at destination u (tonnes).

$T_{d,ub}^{u,t}$ the upper bound on the overburden dyke material requirement in period t at destination u (tonnes).

$T_{d,lb}^{u,t}$ the lower bound on the overburden dyke material requirement in period t at destination u (tonnes).

$T_{n,ub}^{u,t}$ the upper bound on the interburden dyke material requirement in period t at destination u (tonnes).

$T_{n,lb}^{u,t}$ the lower bound on the interburden dyke material requirement in period t at destination u (tonnes).

$T_{l,ub}^{u,t}$ the upper bound on the tailings coarse sand dyke material requirement in period t at destination u (tonnes).

$T_{l,lb}^{u,t}$ the lower bound on the tailings coarse sand dyke material requirement in period t at destination u (tonnes).

ps^{rev} a pseudo mining revenue per metre cube backfilled.

10.1.3. Decision variables

$x_k^{u,t} \in [0,1]$ a continuous variable representing the ore portion of mining-cut k to be extracted and processed at destination u in period t.

$z_k^{u,t} \in [0,1]$ a continuous variable representing the overburden dyke material portion of mining-cut k to be extracted and used for dyke construction at destination u in period t.

$c_k^{u,t} \in [0,1]$ a continuous variable representing the interburden dyke material portion of mining-cut k to be extracted and used for dyke construction at destination u in period t.

$s_k^{u,t} \in [0,1]$ a continuous variable representing the tailings coarse sand dyke material portion of mining-cut k to be extracted and used for dyke construction at destination u in period t.

$y_p^{a,t} \in [0,1]$ a continuous variable representing the portion of mining-panel p to be mined in period t from location a, which includes both ore, overburden and interburden dyke material and waste.

$d_j^{a,t} \in [0,1]$ a continuous variable representing the portion of mining phase j to be backfilled in period t from location a.

$i^{a,u,t} \in [0,1]$ a continuous variable representing the portion of ex-pit volume at destination u to be filled in period t from location a.

Oil Sands Integrated Mine Planning and Tailings Management

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Abstract

A strategic mine planning model determines the best order of extraction and destination of material over the mine-life, to maximize the net present value (NPV) of the produced minerals. In oil sands open-pit mining, further processing of the extracted oil sands generates massive volumes of tailings. To save space, the tailings are deposited in in-pit tailings containments constructed by internal dykes using mine waste material. In this paper, an integrated mine planning framework is proposed and implemented using mixed-integer linear programming to optimize the production schedule with respect to dyke construction and in-pit tailings deposition. A case study is carried out to verify the performance of the proposed optimization model. The results approves that the produced tailings is being deposited in the excavated mining-pit as the mining operations proceed and the in-pit dykes are constructed using mine waste material.

1. Introduction

An oil sands deposit is a mixture of bitumen and water in sands and clay. It is a thick, sticky, heavy and viscous material and needs rigorous extraction treatment to refine its bitumen. The oil sands is one of the fastest growing industries in North America. Though in recent times oil prices are relatively low, there have been considerable investments in the past that can keep this industry vibrant for some decades. It is also more relevant now that further research aimed at improving the profitability of these operations in the long-term is pursued.

A mine production plan determines the best schedule for extraction and the destination of the extracted material, in a way that maximizes the net present value of the production. In oil sands operations, the material mined is sent to the processing plant for extraction of bitumen through hot water extraction process, which produces tailings. About 80% of the material sent to the processing plant ends up in the tailings dam.

Solid waste management is a related concept to long-term mine planning. Mining operations generate considerable volumes of solid waste mostly as overburden and interburden (OI) to access the mineralized zone. The current practice is to dump the waste material for later use mostly in dyke construction and reclamation. The dykes may be constructed either in-pit or ex-pit depending on the waste management strategy in place at the time. The main source of the required material for dyke construction is OI material coming from mining operations, and the tailings coarse sand (TCS)

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coming from processing plant (Ben-Awuah, 2013, Fauquier et al., 2009). Ben-Awuah et al. (2012) provide a detailed description of an integrated oil sands mining operation including material flows, solid waste and tailings management. Hence, waste disposal, reclamation planning and dyke construction planning can be integrated with the mine planning framework. In the literature, few works have addressed such integration, but none of them has covered the mentioned domains completely (Badiozamani and Askari-Nasab, 2013, Badiozamani and Askari-Nasab, 2014a, Badiozamani and Askari-Nasab, 2014b, Ben-Awuah, 2013, Ben-Awuah and Askari-Nasab, 2011, Ben-Awuah et al., 2012).

Since 1960s, operations research techniques such as linear programming, integer programming, mixed-integer linear programming (Johnson, 1969) and dynamic programming (Tan and Ramani, 1992) have been used to find the optimized pattern of extraction and determine a destination for the extracted material in open-pit mining and block caving (Newman et al., 2010). The most common way to control the precedence order of extraction for mining blocks is to define integer variables, which makes the mine planning problem a non-deterministic polynomial-time hard for large-scale problems (Gleixner, 2008). Due to the large number of integer variables corresponding to mining blocks over large number of periods, it takes considerably a long time for the current solvers to solve the problem.

The mixed-integer linear programming (MILP) is a powerful tool extensively used in the literature for mine planning optimization. Typical mine planning models maximize the NPV over the mine-life, with respect to the mining and processing capacities, ore blending constraints, and spatial precedence among mining blocks (Askari-Nasab and Awuah-Offei, 2009, Askari-Nasab et al., 2011b, Johnson, 1969). Further than the pure long-term mine planning models, few works are published addressing the linkage between mine planning and tailings production (Kalantari et al., 2013). The solid waste disposal management and dyke construction planning in oil sands are also integrated into the long-term mine planning framework (Ben-Awuah, 2013, Ben-Awuah and Askari-Nasab, 2011, Ben-Awuah et al., 2012).

Badiozamani and Askari-Nasab (2014a) proposed an integrated model for long-term mine planning, with respect to reclamation material handling and tailings capacity constraints. The concept of directional mining is used in modeling to provide capacity for in-pit tailings facility. The model determines the destination for each extracted parcel (dynamic cut-off grade) in such a way to maximize the NPV over the mine-life. Mining aggregates are used in the model to follow the selective mining units. The authors reach to integer solutions within 2% optimality gap in less than 10 minutes for the cases with more than 98,000 mining-blocks aggregated to 535 mining-cuts. The optimality gap refers to the absolute tolerance on the gap between the best integer objective and the objective of the best node remaining in the branch and cut algorithm. The resulting schedule generates the maximum NPV, minimizes the material handling cost of reclamation, and the tailings volume produced downstream meets the tailings capacity constraints in each period. The authors take a further step in integrated mine planning, by including the tailings management in terms of composite tailings (CT) production and deposition, in the mine planning optimization framework (Badiozamani and Askari-Nasab, 2014b).

The gap in current literature is the integration of all these areas: maximization of profit in pure mine planning, minimization of dyke construction costs, and minimization of tailings disposal costs. The proposed MILP model in this research maximizes the net present value (NPV) and at the same time minimizes the costs of dyke construction and CT deposition. The optimization is subject to a number of constraints, including the mining, processing, and tailings storage capacities and extraction precedence constraints. This integrated model will reduce the re-handling cost of dyke construction and reclamation material. The model schedules these material types when they are needed both in quality and quantity directly to the appropriate destination.

2. The mathematical model

The proposed mathematical model includes both tailings management, in terms of CT deposition, and waste management in terms of dyke construction planning. The objective function includes three parts as: (1) maximization of NPV, (2) minimization of Dyke construction costs, and (3) minimization of CT deposition costs. Mining-panels (intersections of bench faces and pushbacks) are used as the units for mining operations, while mining-cuts (aggregated blocks) are used for processing. The detailed structure of the MILP model is as follows:

2.1. Objective function

The objective function maximizes the net present value of the profit gained from processing of each mining-panel. The revenue from each mining-panel consists of two terms: the revenue from selling each tonne of bitumen, and a summation of operational costs. The operational costs include the material extraction costs, the extra costs for mining ore material, and the cost of selling the ore. The other two operational costs are the extra costs of mining and preparing material for dyke construction, and the cost of CT deposition in the CT cells.

The economic value of mining panels is calculated through Eqs. (1) to (5):

$$d_p^{a,u,t} = \sum_{k \in p_a} \left(r_k^{u,t} - n_k^{u,t} - m_k^{u,t} \right) - q_p^{a,t} \qquad \forall t \in \mathbf{T},\ u \in \mathbf{U},\ p \in \mathbf{P},\ a \in \mathbf{A} \qquad (1)$$

Where:

$$r_k^{u,t} = \sum_{e=1}^{E} o_k \times g_k^e \times r^{u,e} \times \left(p^{e,t} - cs^{e,t} \right) - \sum_{e=1}^{E} o_k \times cp^{u,e,t} \qquad \forall t \in \mathbf{T}, u \in \mathbf{U}, k \in \mathbf{K} \qquad (2)$$

$$q_p^{a,t} = \sum_{k \in p} \left(o_k + d_k + w_k \right) \times cm^{a,t} \qquad \forall t \in \mathbf{T}, p \in \mathbf{P}, a \in \mathbf{A} \qquad (3)$$

$$n_k^{u,t} = d_k \times cl^{u,t} \qquad \forall t \in \mathbf{T}, u \in \mathbf{U}, k \in \mathbf{K} \qquad (4)$$

$$m_k^{u,t} = l_k \times cu^{u,t} \qquad \forall t \in \mathbf{T}, u \in \mathbf{U}, k \in \mathbf{K} \qquad (5)$$

And the cost of CT deposition is calculated as in Eq. (6):

$$i_c^t = h_c \times ct^{c,t} \qquad \forall t \in \mathbf{T}, c \in \mathbf{C} \qquad (6)$$

The objective function is defined as Equation (7):

$$Max \sum_{t=1}^{T} \left(\sum_{u=1}^{U} \sum_{a=1}^{A} \sum_{j=1}^{J} \sum_{\substack{p \in B_j \\ k \in B_p}} \left[r_k^{u,t} \times x_k^{u,t} - q_p^t \times y_p^{a,t} - \left(n_k^{u,t} \times w_k^{u,t} + m_k^{u,t} \times v_k^{u,t} \right) \right] - \sum_{c=1}^{C} i_c^t \times z_c^t \right) \qquad (7)$$

2.2. Constraints

The optimization is subject to the constraints stated by Eqs. (8) to (45). Eqs. (8) and (9) present the mining and processing capacity constraints. Eqs. (10) and (11) ensure that the material sent for dyke construction are within the range of minimum and maximum requirements. Eqs. (12), (13) and (14) control the balance of material tonnages extracted and used for different purposes. The blending constraints for ore and OI material are presented in Eqs. (15), (16), and (17). Eqs. (18), (19), (20) and (21) add up the total tonnage of different components of tailings and ensure that the tonnages are not exceeding the corresponding capacity ranges. Eqs. (22), (23) and (24) ensure that the total CT produced and deposited in CT cells does not exceed the capacity of CT containments in each period. Mining precedence constraints are presented in Eqs. (25) to (31). The precedence order of CT cells construction and CT deposition is controlled through Eqs. (32) to (39). Finally, Eqs. (40) to (45) ensure that the summation of decision variables adds up to one.

$$T_{Ml}^{a,t} \le \sum_{j=1}^{J}\left(\sum_{p\in B_j}\sum_{k\in B_p}\left(o_k + w_k + d_k\right)\times y_p^{a,t}\right) \le T_{Mu}^{a,t} \qquad \forall t\in\mathbf{T}, \forall a\in\mathbf{A} \tag{8}$$

$$T_{Pl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(o_k \times x_k^{u,t}\right)\right) \le T_{Pu}^{u,t} \qquad \forall t\in\mathbf{T},\ u\in\mathbf{U} \tag{9}$$

$$T_{Cl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(d_k \times w_k^{u,t}\right)\right) \le T_{Cu}^{u,t} \qquad \forall t\in\mathbf{T},\ u\in\mathbf{U} \tag{10}$$

$$T_{Nl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(l_k \times v_k^{u,t}\right)\right) \le T_{Nu}^{u,t} \qquad \forall t\in\mathbf{T},\ u\in\mathbf{U} \tag{11}$$

$$\sum_{u=1}^{U}\sum_{k\in B_p}\left(o_k \times x_k^{u,t} + d_k \times w_k^{u,t}\right) \le \sum_{a=1}^{A}\sum_{k\in B_p}\left(o_k + d_k\right)\times y_p^{a,t} \qquad \forall t\in\mathbf{T},\ p\in\mathbf{P} \tag{12}$$

$$\sum_{u=1}^{U}\left(l_k \times v_k^{u,t}\right) \le \sum_{u=1}^{U}\left(o_k \times x_k^{u,t}\right) \qquad \forall t\in\mathbf{T},\ k\in\mathbf{K} \tag{13}$$

$$\sum_{d=1}^{D}\left(k_d \times u_d^{t}\right) \le \sum_{u=1}^{U}\sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(d_k \times w_k^{u,t} + l_k \times v_k^{u,t}\right)\right) \qquad \forall t\in\mathbf{T},\ u\in\mathbf{U} \tag{14}$$

$$\underline{g}^{u,t,e} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j} g_k^{e}\times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \le \overline{g}^{u,t,e} \qquad \forall t\in\mathbf{T},\ u\in\mathbf{U}, e\in\mathbf{E} \tag{15}$$

$$\underline{f}^{u,t,o} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^{o}\times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \le \overline{f}^{u,t,o} \qquad \forall t\in\mathbf{T},\ u\in\mathbf{U} \tag{16}$$

$$\underline{f}^{u,t,c} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^{c}\times d_k \times w_k^{u,t} \Big/ \sum_{k\in B_j} d_k \times w_k^{u,t}\right) \le \overline{f}^{u,t,c} \qquad \forall t\in\mathbf{T},\ u\in\mathbf{U} \tag{17}$$

$$T_{Tl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(t_k \times x_k^{u,t}\right)\right) \le T_{Tu}^{u,t} \qquad \forall t \in \mathbf{T},\ u \in \mathbf{U} \tag{18}$$

$$T_{Fl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(f_k \times x_k^{u,t}\right)\right) \le T_{Fu}^{u,t} \qquad \forall t \in \mathbf{T},\ u \in \mathbf{U} \tag{19}$$

$$T_{Sl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(s_k \times x_k^{u,t}\right)\right) \le T_{Su}^{u,t} \qquad \forall t \in \mathbf{T},\ u \in \mathbf{U} \tag{20}$$

$$T_{Wl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(r_k \times x_k^{u,t}\right)\right) \le T_{Wu}^{u,t} \qquad \forall t \in \mathbf{T},\ u \in \mathbf{U} \tag{21}$$

$$T_{Xl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(h_k \times x_k^{u,t}\right)\right) \le T_{Xu}^{u,t} \qquad \forall t \in \mathbf{T},\ u \in \mathbf{U} \tag{22}$$

$$T_{Yl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(p_k \times x_k^{u,t}\right)\right) \le T_{Yu}^{u,t} \qquad \forall t \in \mathbf{T},\ u \in \mathbf{U} \tag{23}$$

$$\sum_{c=1}^{C}\left(h_c \times z_c^t\right) \le \sum_{j=1}^{J}\sum_{k\in B_j}\sum_{u=1}^{U}\left(p_k \times x_k^{u,t}\right) \qquad \forall t \in \mathbf{T} \tag{24}$$

$$b_p^t - \sum_{a=1}^{A}\sum_{i=1}^{t} y_s^{a,i} \le 0 \qquad \forall t \in \mathbf{T},\ p \in \mathbf{P},\ s \in N_p(L) \tag{25}$$

$$b_p^t - \sum_{a=1}^{A}\sum_{i=1}^{t} y_r^{a,i} \le 0 \qquad \forall t \in \mathbf{T},\ p \in \mathbf{P},\ r \in O_p(L) \tag{26}$$

$$\sum_{a=1}^{A}\sum_{i=1}^{t} y_p^{a,i} - b_p^t \le 0 \qquad \forall t \in \mathbf{T},\ p \in \mathbf{P} \tag{27}$$

$$b_p^t - b_p^{t+1} \le 0 \qquad \forall t \in \{1,\ldots,T-1\},\quad p \in \mathbf{P} \tag{28}$$

$$H \times c_j^t - \sum_{a=1}^{A}\sum_{i=1}^{t} y_h^{a,i} \le 0 \qquad \forall t \in \mathbf{T},\ j \in \mathbf{J},\ h \in B_j(H) \tag{29}$$

$$\sum_{a=1}^{A}\sum_{i=1}^{t} y_h^{a,i} - H \times c_j^t \le 0 \qquad \forall t \in \mathbf{T},\ j \in \mathbf{J},\ h \in B_{j+1}(H) \tag{30}$$

$$c_j^t - c_j^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\},\; j \in \mathbf{J} \tag{31}$$

$$a_c^t - \sum_{i=1}^{t} z_r^i \le 0 \qquad \forall t \in \mathbf{T},\, c \in \mathbf{C},\, r \in Q_c(R) \tag{32}$$

$$\sum_{i=1}^{t} z_c^i - a_c^t \le 0 \qquad \forall t \in \mathbf{T},\, c \in \mathbf{C} \tag{33}$$

$$a_c^t - a_c^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\},\; c \in \mathbf{C} \tag{34}$$

$$q_d^t - \sum_{i=1}^{t} u_m^i \le 0 \qquad \forall t \in \mathbf{T},\, d \in \mathbf{D},\, m \in S_d(G) \tag{35}$$

$$\sum_{i=1}^{t} u_d^i - q_d^t \le 0 \qquad \forall t \in \mathbf{T},\, d \in \mathbf{D} \tag{36}$$

$$q_d^t - q_d^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\},\; d \in \mathbf{D} \tag{37}$$

$$q_d^t - \sum_{a=1}^{A} \sum_{i=1}^{t} y_f^{a,i} \le 0 \qquad \forall t \in \mathbf{T},\, d \in \mathbf{D},\; f \in X_d(P) \tag{38}$$

$$a_c^t - \sum_{i=1}^{t} u_n^i \le 0 \qquad \forall t \in \mathbf{T},\, c \in \mathbf{C},\, n \in T_c(D) \tag{39}$$

$$\sum_{u=1}^{U} \sum_{t=1}^{T} x_k^{u,t} \le 1 \qquad \forall k \in \mathbf{K} \tag{40}$$

$$\sum_{u=1}^{U} \sum_{t=1}^{T} w_k^{u,t} \le 1 \qquad \forall k \in \mathbf{K} \tag{41}$$

$$\sum_{u=1}^{U} \sum_{t=1}^{T} v_k^{u,t} \le 1 \qquad \forall k \in \mathbf{K} \tag{42}$$

$$\sum_{t=1}^{T} z_c^t \le 1 \qquad \forall c \in \mathbf{C} \tag{43}$$

$$\sum_{t=1}^{T} u_d^t \le 1 \qquad \forall d \in \mathbf{D} \tag{44}$$

$$\sum_{t=1}^{T} y_p^{a,t} \leq 1 \qquad \forall p \in \mathbf{P}, a \in \mathbf{A} \tag{45}$$

3. Case study: mine planning with consideration of dyke construction and composite tailings (CT) deposition

This case study is designed to show how tailings management, in terms of CT production, and the solid waste management, in terms of dyke construction can be integrated in the production schedule and how the overall NPV is sensitive to such an integration. The specification of the material contained in the block model, which is the input to Whittle for pit-limit optimization is presented in Table 1. The parameters used for optimization in the case study are presented in Table 2. The mineralized material is defined by a regulatory cut-off grade of 7% bitumen content; and the cut-off size between fines and coarse sand is 44 µm (Masliyah, 2010). The overall stripping ratio for this deposit is 2.3:1.

Table 1. Specifications of the block model used in Whittle for pit-limit optimization

Block model data	Value
Devonian rock type	4,526 Mt
McMurray Formation (MMF)	642 Mt
Overburden	446 Mt
Bitumen Content in MMF	54 Mt
Average Bitumen Grade in MMF	8%
Fines Content in MMF	86 Mt
Average Fines Grade in MMF	13%
Water Content in MMF (Mt)	26 Mt

Table 2. MILP Input parameters used in case study

Input parameters	Value	Input parameters	Value
Recovered barrel of bitumen per tonne of Bit.	0.65	Extra OI dyke mining cost ($/t)	0.92
Ore Price ($/t of Bitumen)	450	Extra TCS dyke mining cost ($/t)	1.38
Mining Cost ($/t)	4.60	CT deposition cost ($/m3)	0.50
Processing Cost ($/t)	5.03	Ore cut-off grade	7%
Total material (Mt)	1,237	Upper bound on fines grade in ore	18%
Mineralized material (Mt)	374	Upper bound on fines grade in OI	30%
OI material (Mt)	597	Interest rate	10%
TCS material (Mt)	278	Recovery	90%
Mining direction	W-E	Number of mining-panels	70
Number of periods (years)	10	Number of mining-cuts	972

In the proposed model, it is assumed that the produced CT will be deposited mainly in a number of in-pit CT cells shaped by internal dykes and pit walls. The external tailings facility (ETF) also acts as a buffer to accommodate the excess of the mature fine tailings (MFT) when the CT production has not yet started or when the internal dykes are not available. The internal dykes ensure that both mining and tailings deposition can occur simultaneously in the pit during the mine life.

Before raising the internal dyke walls, the first step is to choose the dykes' footprints. To guarantee a feasible schedule, dyke footprints are selected from among pushback footprints. This selection is made based on the volume of material in pushbacks and the potential volume of CT to be produced from processing the extracted material. Since the pushbacks are extracted following a precedence order, no material will be left behind before constructing a dyke, and the dyke footprint has been cleared already. Fig. 1 illustrates a plan view of the dyke footprints and the schematic ETF used in the case study. The in-pit colors represent the mining panels used to control material extraction on this level.

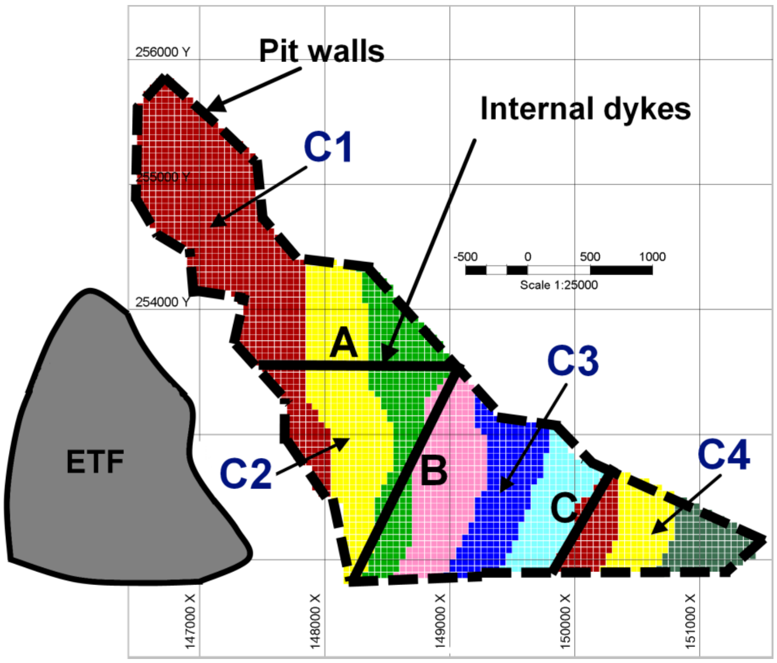

Fig. 1. Dykes' footprints (A & B), CT cells (C1 to C4), and the ETF for the case study

4. Discussion of results

In this case study, the ETF works only as a buffer, and since it has a limited capacity, the in-pit CT cells must be prepared for CT storage. In order to meet such a requirement, the OI and TCS material must be produced and used for the construction of in-pit dykes. Solving the MILP generates an NPV of $3,959M over 10 years. It has resulted in the extraction of 1,237 Mt of material, including 314 Mt of mineralized material, 264 Mt of OI, and 659 Mt of waste (Table 3).

Processing the mineralized material generates 37 Mt of bitumen and 92 Mt of TCS. A total of 227 Mm^3 of CT is produced, from which 148 Mm^3 (65%) is deposited in the ETF and the rest (79 Mt) is deposited in the in-pit CT cell C1. The total material usable for dyke construction is 356 Mt (OI and TCS), from which 159 Mt is used to construct Dyke A and the rest are sent to the waste dump. The resulting production schedule is presented in

Fig. 2. The production schedule generated ensures a uniform mill feed and OI material for dyke construction. There is however some fluctuations in the waste material mined which may require contract mining or equipment lease options during certain periods to ensure efficient utilization of the owner mining fleet.

Table 3. Numerical results of the case study

Total material extracted	Mineralized material Extracted	Processed ore	Recovered bit.	Extracted OI
1,237 Mt	374 Mt	313.7 Mt	37.03 Mt	264 Mt
Produced TCS	Optimality Gap	Run time	NPV	
92 Mt	0.0%	63 s	3,959 M$	

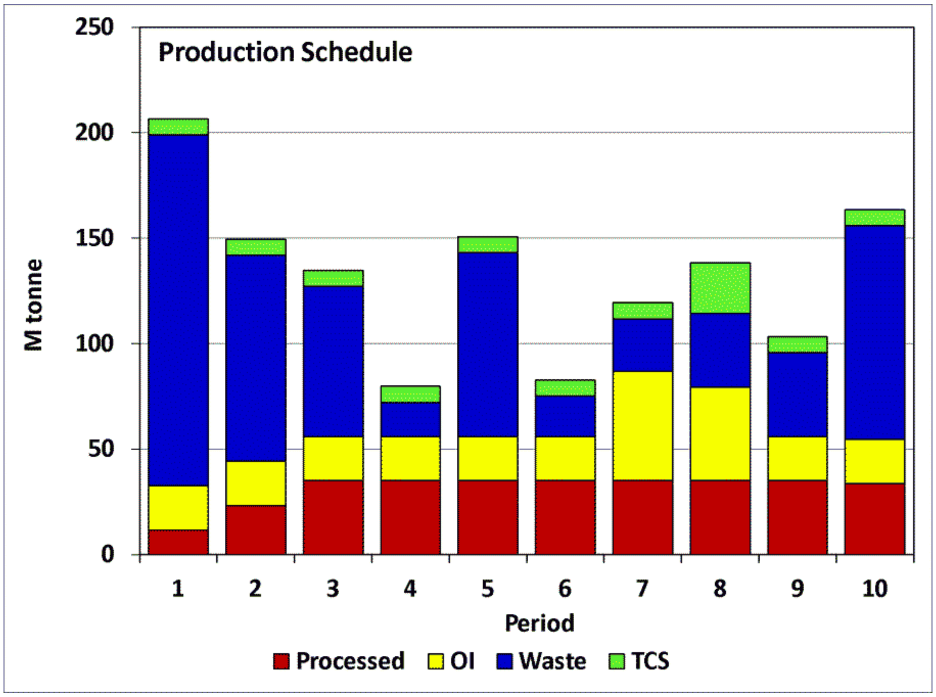

Fig. 2. Production schedule

Fig. 3 illustrates how the nine pushbacks are extracted. Pushback mining follows the West-to-East direction, and the generated schedule ensures that before mining starts in one pushback, the previous pushback has already been extracted. In this way, the footprint of Dyke A as the first in-pit dyke is cleared after pushback three has been completely extracted (in the sixth period).

The only in-pit dyke being constructed is Dyke A and its construction begins after pushback three has been completely extracted in period six (Fig. 4). During periods one to seven when the in-pit cell is not yet ready for tailings deposition, the produced CT is sent to the ETF, as illustrated in Fig. 5. After CT cell 1 is completed in period 8, the CT is deposited in this CT cell over periods 8 to 10.

Fig. 6 illustrates the periods in which the eight lifts of Dyke A are constructed, as well as the start and end periods of CT deposition in the ETF and in CT cell 1. Implementation of the MILP model generates an NPV of $3,959M.

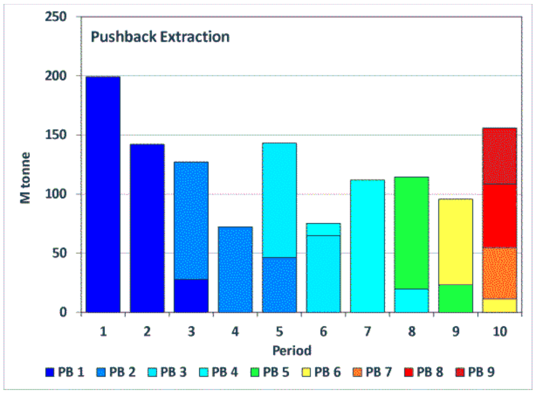

Fig. 3. Pushback extraction schedule

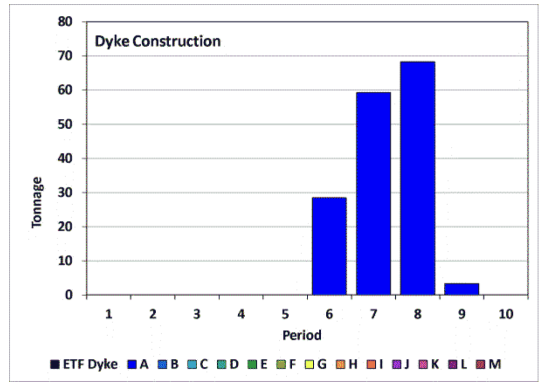

Fig. 4. Dyke construction schedule

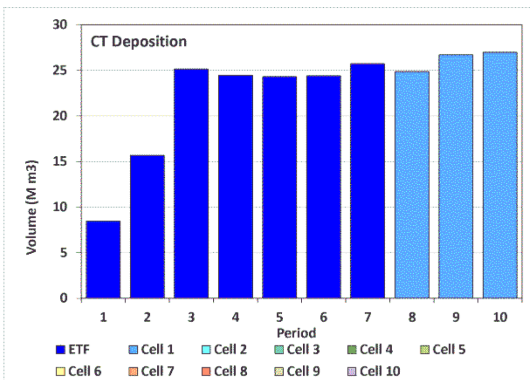

Fig. 5. CT deposition schedule

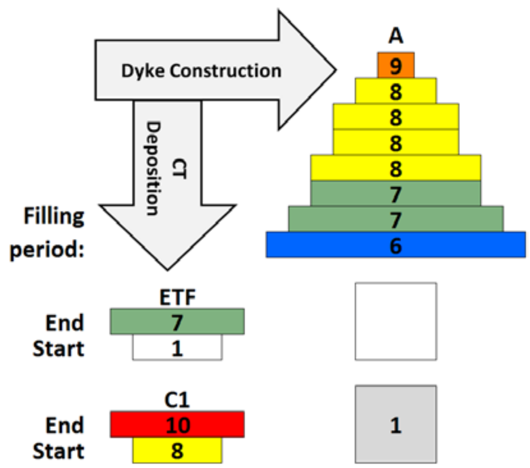

Fig. 6. Construction of Dyke A and CT deposition in the ETF and CT cell

5. Conclusions

The literature related to mine planning and waste management is reviewed. The current literature lacks integration between mine planning and waste management in terms of in-pit deposition of solid waste material and tailings. The implemented framework is a novel topic that fills the current literature gap in strategic open-pit mine planning. An integrated long-term mine production plan has been developed to solve the optimal mine production schedule, with respect to dyke construction and tailings deposition. The model is verified through a case study on a real oil sands data set. The generated schedule is practically mineable, follows the chosen direction, provides a smooth feed for the oil sands processing plant, provides the material required to construct in-pit dykes, and accommodates the produced CT in the ETF and in-pit CT cells. The value of this model to the mining industry can be quantified directly from the savings made by avoiding re-handling of the dyke construction material and indirectly from the reduced mining footprint.

It is recommended to consider efficient methods to reduce the problem size for large-scale problems, through preprocessing and period aggregation technique. The other area for development of the research is to consider other means of tailings dewatering, such as atmospheric fine drying (AFD) used in ETFs, or non-segregated tailings technology (NST) for in-pit impoundment of tailings products.

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7. Appendix

7.1. Decision variables

$x_k^{u,t} \in [0,1]$ A continuous variable representing the portion of ore from mining-cut k to be extracted and processed at destination u in period t.

$w_k^{u,t} \in [0,1]$ A continuous variable representing the portion of OI material from mining-cut k to be extracted and used for dyke construction at destination u in period t.

$v_k^{u,t} \in [0,1]$ A continuous variable representing the portion of TCS from mining-cut k to be extracted and used for dyke construction at destination u in period t.

$y_p^{a,t} \in [0,1]$ A continuous variable representing the portion of mining-panel p to be mined in period t from location a, which includes ore, OI material, tailings sand and waste.

$z_c^t \in [0,1]$ A continuous variable representing the portion of CT cell c to be filled with CT in period t.

$u_d^t \in [0,1]$ A continuous variable representing the portion of dyke unit d to be constructed in period t.

$b_p^t \in \{0,1\}$ A binary integer variable controlling the precedence of extraction of mining-panels. b_p^t is equal to one if the extraction of mining-panel p has started by or in period t, otherwise it is zero.

$c_j^t \in \{0,1\}$ A binary integer variable controlling the precedence of mining phases. c_j^t is equal to one if the extraction of phase j has started by or in period t, otherwise it is zero.

$a_c^t \in \{0,1\}$ A binary integer variable controlling the precedence of filling of CT cells. a_c^t is equal to one if the filling of CT cell c has started by or in period t, otherwise it is zero.

$q_d^t \in \{0,1\}$ A binary integer variable controlling the precedence of Constructing dyke units. q_d^t is equal to one if the construction of dyke unit d has started by or in period t, otherwise it is zero.

7.2. MILP sets and indices

$a \in \mathbf{A}, \mathbf{A} = \{1,.....,A\}$ Index and set of all the possible mining locations (pits) in the model.

$c \in \mathbf{C}, \mathbf{C} = \{1,......,C\}$ Index and set of all CT cells in the model.

$d \in \mathbf{D}, \mathbf{D} = \{1,......,D\}$ Index and set of all dyke units in the model.

$e \in \mathbf{E}, \mathbf{E} = \{1,.....,E\}$ Index and set of all the elements of interest in the model.

$j \in \mathbf{J}, \mathbf{J} = \{1,......,J\}$ Index and set of all the phases (push-backs) in the model.

$k \in \mathbf{K}, \mathbf{K} = \{1,.....,K\}$ Index and set of all the mining-cuts in the model.

$p \in \mathbf{P}, \mathbf{P} = \{1,.....,P\}$ Index and set of all the mining panels in the model.

$t \in \mathbf{T}, \mathbf{T} = \{1,.....,T\}$ Index and set of all the scheduling periods in the model.

$u \in \mathbf{U}, \mathbf{U} = \{1,.....,U\}$ Index and set of all the possible destinations for materials in the model.

$B_j(H)$ For each phase j, there is a set $B_j(H) \subset \mathbf{P}$ defining the mining panels within the immediate predecessor pit phases (push-backs) that must be extracted prior to extracting phase j, where H is an integer number representing the total number of mining panels in the set $B_j(H)$.

$B_p(V)$ For each mining panel p, there is a set $B_p(V) \subset \mathbf{K}$ defining the mining-cuts that belongs to the mining panel p, where V is the total number of mining-cuts in the set $B_p(V)$.

$N_p(L)$ For each mining panel p, there is a set $N_p(L) \subset \mathbf{P}$ defining the immediate predecessor mining panels above mining panel p that must be extracted prior to extraction of mining panel p, where L is the total number of mining panels in the set $N_p(L)$.

$O_p(L)$ For each mining panel p, there is a set $O_p(L) \subset \mathbf{P}$ defining the immediate predecessor mining panels in a specified horizontal mining direction that must be extracted prior to extraction of mining panel p at the specified level, where P is the total number of mining panels in the set $O_p(L)$.

$Q_c(R)$ For each CT cell *c*, there is a set $Q_c(R) \subset \mathbf{C}$ defining the immediate predecessor CT cells below the CT cell *c* that must be filled in prior to filling of CT cell *c*, where *R* is the total number of CT cells in the set $Q_c(R)$.

$S_d(G)$ For each dyke unit *d*, there is a set $S_d(G) \subset \mathbf{D}$ defining the immediate predecessor dyke units that must be constructed in prior to constructing of dyke cell *d*, where *G* is the total number of dyke units in the set $S_d(G)$.

$T_c(D)$ For each CT cell *c*, there is a set $T_c(D) \subset \mathbf{D}$ defining the immediate predecessor dyke units that must be constructed in prior to filling of CT cell *c*, where *D* is the total number of dyke units in the set $T_c(D)$.

$X_d(P)$ For each dyke unit *d*, there is a set $X_d(P) \subset \mathbf{P}$ defining the immediate predecessor mining panels that must be extracted in prior to construction of dyke unit *d* to guarantee that the dykes foot print is cleared, where *P* is the total number of panels in the set $X_d(P)$.

7.3. MILP parameters

$cl^{u,t}$ Extra cost in present value terms for mining, shipping, and using a tonne of OI material for dyke construction at destination u.

$cm^{a,t}$ Cost in present value terms of mining a tonne of waste in period t from mine a.

$cp^{u,e,t}$ Discounted extra cost for mining and processing one tonne of ore at destination u.

$cs^{e,t}$ Selling cost of element e in present value terms per unit of product.

$ct^{c,t}$ Cost in present value terms of sending a volume unite of CT in period t to cell c.

$cu^{u,t}$ Extra cost in present value terms for mining, shipping, and using a tonne of tailings sand for dyke construction at destination u.

$d_p^{a,u,t}$ Discounted profit obtained by extracting mining panel p from location a and sending it to destination u in period t.

d_k OI dyke material tonnage in mining-cut k.

f_k Fines tonnage produced from extracting all of the ore from mining-cut k.

f_k^c Average percentage of fines in the OI dyke material portion of mining-cut k.

$\underline{f}^{u,t,c}$ Lower bound on the required average fines percentage of OI dyke material in period t at destination u.

$\overline{f}^{u,t,c}$ Upper bound on the required average fines percentage of OI dyke material in period t at destination u.

f_k^o Average percentage of fines in the ore portion of mining-cut k.

$\underline{f}^{u,t,o}$ Lower bound on the required average fines percentage of ore in period t at processing destination u.

$\overline{f}^{u,t,o}$ Upper bound on the required average fines percentage of ore in period t at processing destination u.

g_k^e Average grade of element e in the ore portion of mining-cut k.

$\underline{g}^{u,t,e}$ Lower bound on the required average head grade of element e in period t at processing destination u.

$\overline{g}^{u,t,e}$ Upper bound on the required average head grade of element e in period t at processing destination u.

h_c Total volume of CT cell c.

h_k MFT volume produced from extracting all of the ore from mining-cut k.

k_d Total volume of dyke unit d.

l_k Tailings coarse sand tonnage in mining-cut k.

$m_k^{u,t}$ Extra discounted cost of producing tailings sand from mining-cut k in period t and sending it for dyke construction in destination u.

$n_k^{u,t}$ Extra discounted cost of mining the OI material of the mining-cut k in period t and sending it for dyke construction in destination u.

o_k Ore tonnage in mining-cut k.

$p^{e,t}$ Price of element e in present value terms per unit of product.

p_k CT volume produced from extracting all of the ore from mining-cut k.

p_p Mining panel p.

$q_p^{a,t}$ Discounted cost of mining all the material in mining panel p in period t as waste from location a.

r_k Water tonnage produced from extracting all of the ore from mining-cut k.

$r^{u,e}$ Proportion of element e recovered if it is processed at destination u.

$r_k^{u,t}$ Discounted revenue obtained by selling the final products within mining-cut k in period t if it is sent to destination u, minus the extra discounted cost of mining all the material in mining-cut k as ore from location a and processing at destination u.

s_k Sand tonnage produced from extracting all of the ore from mining-cut k.

t_k Tailings tonnage produced from extracting all of the ore in mining-cut k.

$T_{Mu}^{a,t}$ Upper bound on mining capacity (tonnes) in period t at location a.

$T_{Ml}^{a,t}$ Lower bound on mining capacity (tonnes) in period t at location a.

$T_{Pu}^{u,t}$ Upper bound on processing capacity (tonnes) in period t at destination u.

$T_{Pl}^{u,t}$ Lower bound on processing capacity (tonnes) in period t at destination u.

$T_{Cu}^{u,t}$ Upper bound on OI material required for dyke construction (tonnes) in period t at destination u.

$T_{Cl}^{u,t}$ Lower bound on OI material required for dyke construction (tonnes) in period t at destination u.

$T_{Nu}^{u,t}$ Upper bound on tailings sand required for dyke construction (tonnes) in period t at destination u.

$T_{Nl}^{u,t}$ Lower bound on TCS required for dyke construction (tonnes) in period t at destination u.

$T_{Tu}^{u,t}$ Upper bound on capacity of tailings facility (tonnes) in period t at destination u.

$T_{Tl}^{u,t}$ Lower bound on capacity of tailings facility (tonnes) in period t at destination u.

$T_{Fu}^{u,t}$ Upper bound on capacity of fine material (tonnes) in period t at destination u.

$T_{Fl}^{u,t}$ Lower bound on capacity of fine material (tonnes) in period t at destination u.

$T_{Su}^{u,t}$ Upper bound on capacity of tailings sand (tonnes) in period t at destination u.

$T_{Sl}^{u,t}$ Lower bound on capacity of tailings sand (tonnes) in period t at destination u.

$T_{Wu}^{u,t}$ Upper bound on capacity of tailings water (tonnes) in period t at destination u.

$T_{Wl}^{u,t}$ Lower bound on capacity of tailings water (tonnes) in period t at destination u.

$T_{Xu}^{u,t}$ Upper bound on capacity of MFT (tonnes) in period t at destination u.

$T_{Xl}^{u,t}$ Lower bound on capacity of MFT (tonnes) in period t at destination u.

$T_{Yu}^{u,t}$ Upper bound on capacity of CT (tonnes) in period t at destination u.

$T_{Yl}^{u,t}$ Lower bound on capacity of CT (tonnes) in period t at destination u.

w_k Waste tonnage in mining-cut k.

Open-Pit Mine Production Operation Optimization

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Abstract

Decision making in mining is a challenging task. Optimal decisions regarding shovel and truck allocations, in consideration to the short-term production schedule, are very important to keep the operations in line with the planned objectives of the company in long-term. This paper presents a mixed integer linear goal programming (MILGP) model to optimize the operations based on four desired goals of the company: a) maximize production, b) minimize deviations in head grade, c) minimize deviations in tonnage feed to the processing plants from the desired feed, and d) minimize operating cost. The model provides shovel assignments and the target productions; as an input to the dispatching system while meeting the desired goals and constraints of the mining operation. The model implementation with an iron ore mine case study provided average plant utilization above 99%, average truck utilization above 92% and average shovel utilization above 95%.

1. Introduction

Mining is a highly capital intensive operation and the major objective of any mining company remains to maximize the profit by extracting the material at lowest possible cost over the mine-life (Askari-Nasab et al., 2007). Since truck and shovel operations account for approximately 60% of total operating costs in open-pit mines, optimal use of these equipment is essential for the profitability of the mine. It is also important that operations achieve the production targets set by the long-term mine plans. As mining operations are highly stochastic, it is practically impossible to accurately predict the production figures and deliver on them. Amongst many, the main reason of such variability is due to the uncertain uptime of truck-and-shovel fleet in surface mines. The variability of truck-and-shovel availability and utilization may become a cause of deviation from the short-term and in-turn long-term production plans. The operational production plans, therefore, must incorporate two objectives, optimize the usage of mobile assets and meet the strategic production schedule.

Fig. 1 presents various mine production planning stages. Tactical plans are linked with strategic plans through short-term production schedules. The literature reviewed showed that though sufficient attention has been given to optimization of the operations, very few try to link the production operations with the mine strategic plans by providing optimal shovel allocations, which often lead to deviations from the short-term and in-turn long-term production targets. The problem

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of shovel assignments to mining faces has not received sufficient attention in the literature. Optimal shovel assignment to available mining-faces over the shift-by-shift operations can act as a link between the production operations and the mine strategic plans.

Hence, two major problems have been identified for this study: 1) the production optimization problem and, 2) the link between operations and the strategic production schedule. Production operations can have a number of problems, out of which four major problems have been identified in this paper:

- Underutilization of shovels due to in-efficient operational plans,
- Deviation of quantity of processing plant feed with respect to desired feed,
- Deviation of quality of material feed to the processing plants and stock-piles compared to the desired quality, and
- Operational cost escalation due to improper resource allocation.

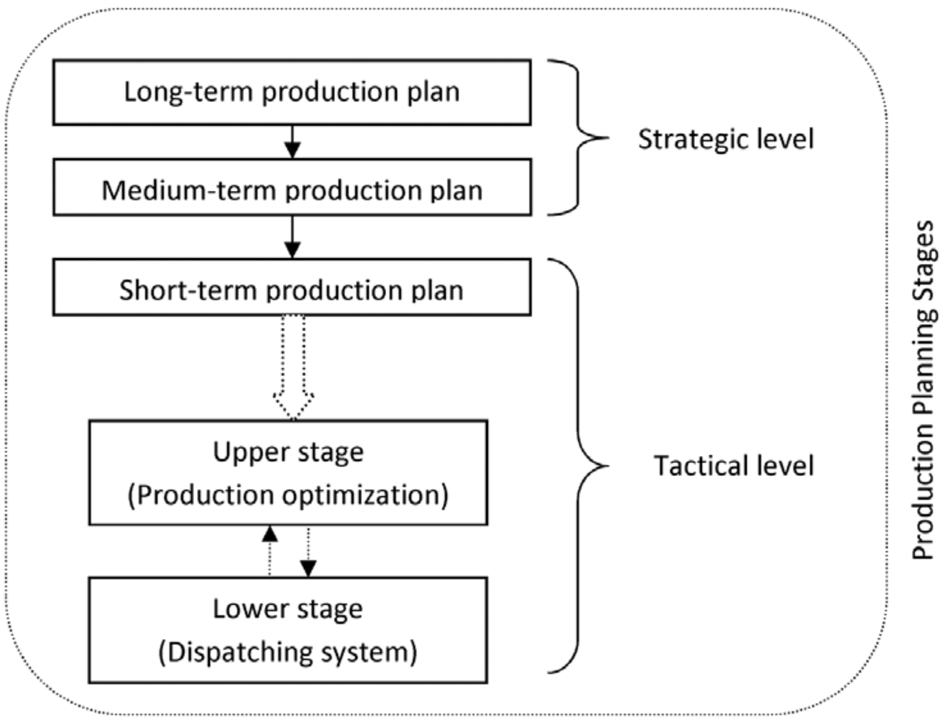

Fig. 1. Mine production planning stages

These four problems can be optimized by combining them into a single objective function and formulating a mixed integer linear goal programming (MILGP) model. To link the production operations with the strategic schedule, shovel assignments can be incorporated into the MILGP model.

The objective of this study is to formulate, implement and verify a mixed integer linear goal programming (MILGP) model for optimal production and, truck-shovel allocation at the operational level.

We present a MILGP model, as an upper stage in a two stage dispatching system, to overcome the limitations of the models described in the literature review section. Taking into account the short-term production plan of the mine, this model assigns the shovels to the available faces and determines their production by maximizing shovel utilization, minimizing the deviation of quality and quantity of the processing plants' feed from the set targets, and minimizing the operational cost.

The model presented in this paper can help improve the automation in operations by removing the need for manual assignment of shovels, to meet the long-term production schedule. The MILGP model is proposed to act as the upper stage in a two stage dispatching system, where upper stage provides the shovel assignments and target productions over a fixed time horizon, and lower stage (dispatching algorithm) achieves those targets in real time.

This paper is structured as follows; a review of research on production scheduling with emphasis on operations is presented in literature review section. Model development section describes the parameters and mathematical formulations that have been used to develop the MILGP model. Subsequently, a case study is presented and the results of application of the model is presented and discussed. Finally the conclusion and future scope of research are presented.

2. Literature Review

Over the years operations research techniques have evolved and found applicability for decision making purposes in mining. Topuz & Duan (1989) mention some of the potential areas in mining such as equipment selection, production planning, maintenance, mineral processing and ventilation, where operations research techniques can act as a helping tool for decision making purposes. Newman *et al.* (2010) provides a comprehensive review of the application of operations research in mining.

Production scheduling in mining has seen a good development over the years. Most of the research in mine production scheduling has remained confined to long-term; and short-term production scheduling has seen very little development in this area (Eivazy and Askari-Nasab, 2012). Eivazy and Askari-Nasab (2012) provides a mixed integer linear programming model to generate short-term open-pit mine production schedule over monthly resolution.

Long-term strategic plans can only be realized with efficient operational production planning. Literature provides broadly two approaches for the optimization of shovel – truck systems at the operational level. Early researches were mostly using queuing theory for studying and optimizing the shovel – truck systems. Koenigsberg (1958) can be considered as the first person who applied queuing theory in mining. With the evolution in computing capability and optimization techniques, mathematical optimization models started to gain more attention. Discrete simulation is another technique which has evolved over time and is now frequently being used for understanding the behavior of the systems and for decision making purposes.

Truck and shovel operations, now days, are primarily optimized by employing truck dispatching algorithms. Munirathinam and Yingling (1994) provide a review of truck dispatching in mining. Elbrond and Soumis (1987) emphasize on a two-step optimization proposed by White and Olson (1986); where the first stage chooses the shovels, the sites and the production rates. The second stage also determines the rates of the shovels but this time it considers the operation in more detail. Soumis *et al.* (1989) proposed a three stage dispatching procedure, namely equipment plan, operational plan and dispatching plan. Based on the overall approach, similar procedures have evolved as multi-stage dispatching systems. Bonates and Lizotte (1988) emphasizes on the accuracy of the model in the upper stage in terms of the true representation of the mining system, so that realistic targets could be fed to the dispatching model in the lower stage.

White and Olson (1986) describe the need of a model which could concurrently maximize the production, minimize the re-handle, meet blending limits and feed the plant. The major limitation of their model is the weak link between the two LP segments proposed. As transportation, in truck-shovel based mining system, occurs as discrete function of number of truck trips and their capacities, modeling it as continuous flow rate is inappropriate, which is another major limitation of their model. Not accounting for mixed fleet poses another limitation on its applicability in mixed fleet mining systems. The MILGP model proposed in this paper is similar in its applicability to the

LP segments proposed by White and Olson (1986) which is solved every time the system state changes.

Soumis *et al.* (1989) proposed a three stage model, which also included shovel assignments. The mixed integer programming model in the first stage, through man-machine interaction, provides 10 best alternatives for the shovel assignments to choose from in the reasonable time. The increased human intervention at this stage poses a limitation on the optimality of the decisions regarding shovel assignments. The second stage determines the production rates of the shovels and truck assignments using non-linear programming with three objectives: maximize shovel productions, minimize the squared difference between computed and available truck hours and minimize the grade deviations (blending). One unique characteristic of the proposition is the use of queuing theory to calculate the truck waiting time so as to compute the truck hours.

Li (1990) proposed a three stage methodology for automated truck dispatching system, by determining the target tonnage to be produced along a path in the network using linear programming as haulage planning stage, truck dispatching based on maximum inter-truck-time deviation, and equipment matching using a least square criterion. Temeng *et al.* (1998) developed a goal programming formulation as an upper stage of a two-stage dispatching system and implemented it with a dispatching system developed by Temeng, Otuonye, & Frendewey (1997). Their paper describes goal programming to be better compared to linear programming using the results obtained. The major limitation of the models in both papers is that they do not take into account the short-term production schedule and do not provide any information regarding shovel assignments. Shovel assignment is an important decision making problem which has a direct impact on achieving the production targets and thus need to be accounted by the upper stage of the dispatching system. Although the model developed by Temeng *et al.* (1998) account for mixed fleet, it does so by taking the average payload of trucks, which would not be a realistic way of modeling this system. A better approach would be to optimize the operation by considering the actual capacities of every truck in the system and their respective payload.

Gurgur *et al.* (2011) proposes an LP model for the shovel and truck allocations with an objective to minimize the deviation of the mine progress from the target provided by the MIP model. Although the model provides shovel assignments, it does so solely on strategic considerations (MIP model). The economic feasibility related to shovel movement cost and production lost during movement is not included in the model, which makes the shovel assignments not optimal. The continuous variables also pose a limitation on modeling the discrete nature of the production. Another model proposed by Subtil *et al.* (2011) does not consider objectives such as grade blending, constant desired feed to plants etc. and do not provide shovel assignments as well.

With the exception of Gurgur *et al.* (2011), to the best of the author's knowledge, no literature in the multi-stage dispatching discussed try to link the operational plans with the strategic plans of the mine. All those models try to improve the efficiency of the mining operations but miss to incorporate an important objective of production operations i.e. to meet the long-term strategic schedule by optimal shovel assignments. None discusses in detail the shovel assignments to faces which still remain a manual task of a planner. Most of the published work focuses on developing mathematical models for maximizing production or minimizing the grade deviation or both. But there can be a number of conflicting objectives of any mining operation, such as steady and desired feed of ore to the processing plants, minimizing the operating costs etc.

The review of literature in the area of multi-stage dispatching at the operational level revealed that:

1. The shovel allocation problem did not receive sufficient attention,
2. Existing models are not equipped sufficiently to handle mixed fleet systems,
3. Optimization models do not incorporate all the major objectives of a production operation,

4. Models do not bridge the existing gap between the production operations and the strategic schedules.
5. Modeling of truck-shovel production operation, in terms of flow rate, seams inappropriate.

The proposed MILGP model provides improvement over the existing mathematical optimization models for production operations by incorporating the abovementioned major limitations identified.

3. Problem Statement

Fig. 2 shows a schematic view of an open-pit mining system, modeled in this paper, consisting of $\hat{F}$ number of available faces to be mined within a predefined time period and $\hat{S}$ number of shovels to be assigned to the available faces. The excavated material is transported from the face to its respective destination, through the pit exit, using $\hat{T}$ haul trucks. A typical open-pit mine can have $\hat{K}$ different elements, consisting of one major element and by-products. The destinations consist of $\hat{O}$ *ore* destinations and $\hat{W}$ waste destinations. Ore destinations consist of $\hat{P}$ processing plants and rest as stockpiles *sp*.

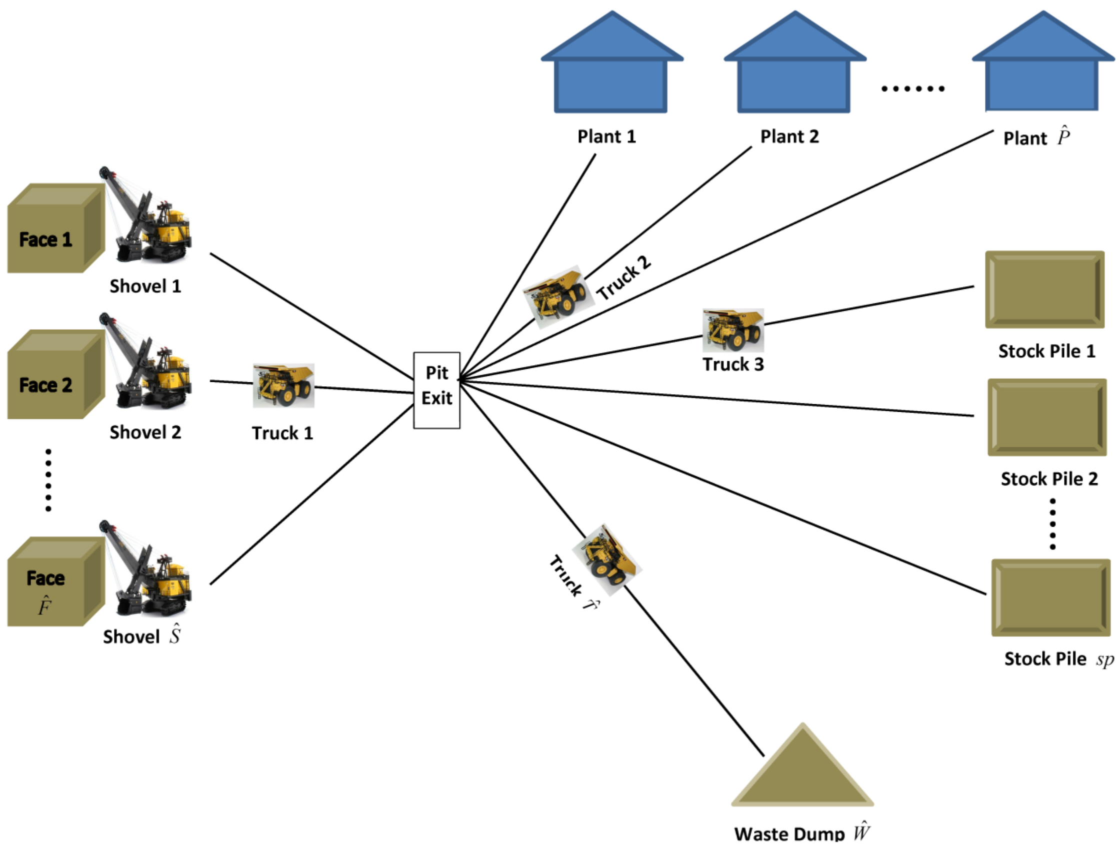

Fig. 2. A typical layout of an open-pit mining system

There is cost associated with truck and shovel operation: shovel movement cost as $ per meter of shovel movement, including additional man hours, when reassigned to a different face; and truck operating costs as $/Km while empty and loaded.

The assumptions and characteristics of the developed MILGP model for shovel allocation and optimal production are:

- Each ore destination can receive material with a specific grade range. The desired grade can be achieved by blending the ore coming from different ore faces.
- Grade range requirements could be applied to multiple elements present in the ore.
- Processing plants are desired to have supply of material at a steady feed but cannot receive material at a rate above the specified limits.

This MILGP model optimizes the multi-destination open-pit mine production and shovel allocation problem subject to available shift time, truck and shovel availability, processing capacity and stripping ratio constraints. The four goals, considered, are to:

1. Maximize the shovel utilization (maximize production),
2. Minimize the grade deviations at ore destinations compared to desired grade,
3. Minimize the deviation in tonnage supplied to the processing plants compared to desired tonnage feed, and
4. Minimize the operating cost of the mine (truck and shovel movement cost)

First goal is to maximize the shovel utilization, which is achieved by minimizing the negative deviation in the production of each shovel compared to its maximum production capacity in a shift. The second goal is to minimize the deviation in grade of each material type compared to the desired grades at the ore destinations. These two goals are similar to those presented by Temeng *et al.* (1998). The third goal optimizes the utilization of processing plants by minimizing the positive and negative deviation in total tonnage supplied, compared to desired, to the processing plants. The fourth goal minimizes the truck and shovel movement cost. It should be noted that, including operating cost as a goal in this model becomes necessary to keep a check on abnormal shovel movement to very far off faces and to achieve the production targets with minimum truck and shovel movement. A flow chart representing the applicability of the proposed MILGP model in dynamic decision making for optimal production operations is given in Fig. 3.

4. Model development

The following section elaborates the preliminary Eqs. and the MILGP model formulation along with required inputs for the model. The parameters and variables considered in the model are described in the Appendix.

4.1. Parameter calculations

Some of the parameters, described as calculated parameters in Appendix, are determined using Eq. (1) to Eq.(6), which are not provided directly as an input to the model. Eq. (1) calculates the distance between available faces, which is primarily used for predicting the shovel movement time and cost in the model. The distance between faces is calculated as straight line distance using the coordinates of the faces.

Eq. (2) calculates the total haul distance between a face and the destinations by summing up the distance to the pit exit from the face and distance of the destination from the pit exit. Eq. (3) determines the shovel movement time based on the distance between faces and the average travel speed of the shovel. Eq. (4) and (5) determines the maximum and minimum production limits for the shovels based on the maximum and minimum desired utilizations, shovel capacities, availabilities and the shift time.

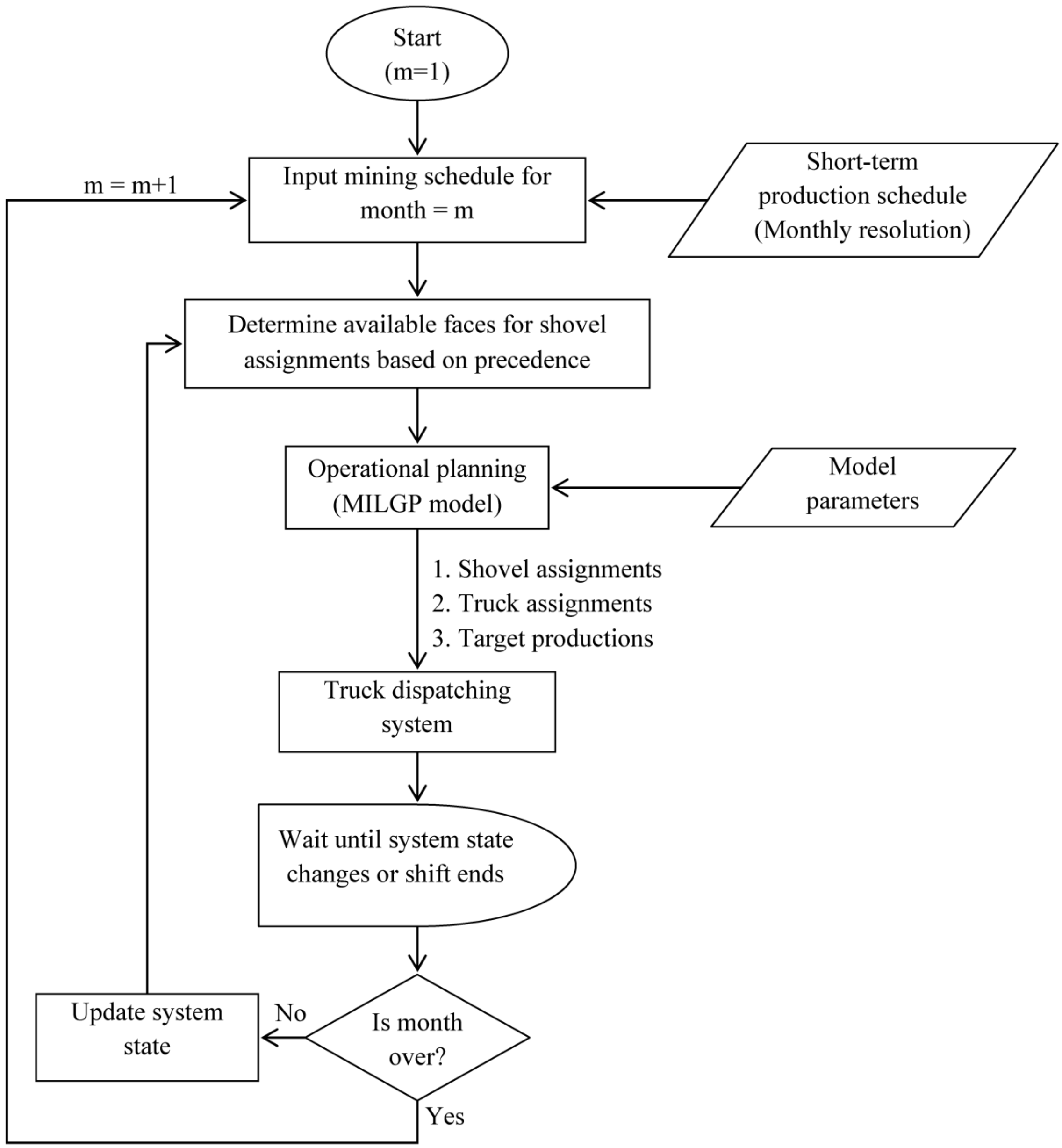

Fig. 3. Flow chart representing the dynamic decision making process for optimal production operations

Eq. (6) calculates the cycle time of trucks of each type between the faces and the destinations by adding the travel time, dumping time, spotting time and the loading time. To keep the model linear and to limit the number of integer variables ($n_{t,f,d}$), cycle time calculation only involves the truck type, the face and the destination. It does not include the shovel working on the face, posing a limitation onto the calculation of truck loading time based on shovel characteristics. Thus an average loading time of all the shovels is used for calculating the total cycle time of trucks.

$$\Gamma^F_{f^1,f^2} = \sqrt{\left(F^x_{f^2} - F^x_{f^1}\right)^2 + \left(F^y_{f^2} - F^y_{f^1}\right)^2 + \left(F^z_{f^2} - F^z_{f^1}\right)^2} \quad (1)$$

$$\Gamma^D_{f,d} = D^{FE}_f + D^{ED}_d \quad (2)$$

$$\tau_{s,f} = \Gamma^F_{F_s,f} \big/ S_s \quad (3)$$

$$X_s^+ = U_s^+ \times \alpha_s^S \times X_s \times T \times 36 / L_s \tag{4}$$

$$X_s^- = U_s^- \times \alpha_s^S \times X_s \times T \times 36 / L_s \tag{5}$$

$$\overline{T}_{t,f,d} = 0.06 \times \Gamma_{f,d}^{d} \times \left(\frac{1}{V_t} + \frac{1}{\overline{\overline{V}}_t} \right) + \left(\frac{D_t + E_t}{60} \right) + \frac{H_t}{\hat{S} \times 60} \times \sum_s \left(\frac{L_s}{X_s} \right) \tag{6}$$

4.2. MILGP formulation

The mixed integer linear goal programming model has been formulated to optimize the goals represented by Eqs. (7), (8), (9) and (10).

4.2.1. Goals

$$\Psi_1 = \sum_s x_s^- \tag{7}$$

$$\Psi_2 = \sum_{d^o} \sum_k (g_{k,d^o}^- + g_{k,d^o}^+) \tag{8}$$

$$\Psi_3 = \sum_{d^p} (\delta_{d^p}^- + \delta_{d^p}^+) \tag{9}$$

$$\Psi_4 = \sum_s \sum_f \Gamma_{F_s,f}^F \times A_s \times a_{s,f} + \sum_t \sum_f \sum_d n_{t,f,d} \times \Gamma_{f,d}^D \times (C_t + \overline{C}_t) \tag{10}$$

Eq. (7) represents the difference between the maximum target production and production achieved by the shovels over a shift. Eq. (8) represents the difference between the material content received at the ore destinations and the material content based on desired grade. Eq. (9) represents the difference between the quantities of ore supplied to the processing plants compared to the target quantities desired over the optimization period. Eq. (10) represents the total cost of shovel movement (if any shovel is reassigned to a new face) and truck operating cost.

4.2.2. Objective

The objective of the model is formulated by combining all the goals and applying a non-preemptive goal programming approach. It should be noted here that, as the goals have different dimensions, it is necessary to normalize them into dimensionless objectives before combining them together. Normalization is carried out by determining the Utopia and Nadir values for individual goals (2006). Normalized goals are then multiplied with weights to achieve the desired priority. The final objective function, thus obtained, is given by Eq. (11).

$$\Psi = W_1 \times \overline{\Psi}_1 + W_2 \times \overline{\Psi}_2 + W_3 \times \overline{\Psi}_3 + W_4 \times \overline{\Psi}_4 \tag{11}$$

Where

$$\overline{\Psi}_i = (\Psi_i - Utopia_i) / (Nadir_i - Utopia_i) \qquad i \in 1,2,3 \,\&\, 4 \tag{12}$$

4.2.3. Constraints

$$\sum_s a_{s,f} \le 1 \qquad \forall f \tag{13}$$

$$\sum_f a_{s,f} \le 1 \qquad \forall s \tag{14}$$

$$\sum_d \sum_f x_{s,f,d} + x_s^- = X_s^+ \qquad \forall s \tag{15}$$

$$\sum_{d}\sum_{f} x_{s,f,d} \geq X_s^- \qquad \forall s \qquad (16)$$

$$\sum_{s} x_{s,f,d} \leq \sum_{t} n_{t,f,d} \times H_t \qquad \forall d \,\&\, \forall f \qquad (17)$$

$$\sum_{s} x_{s,f,d} + J \geq \sum_{t} n_{t,f,d} \times H_t \qquad \forall d \,\&\, \forall f \qquad (18)$$

$$\sum_{d^o} x_{s,f,d^o} \leq a_{s,f} \times O_f \times Q_f \qquad \forall s \,\&\, \forall f \qquad (19)$$

$$\sum_{d^w} x_{s,f,d^w} \leq a_{s,f} \times O_f \times (1-Q_f) \qquad \forall s \,\&\, \forall f \qquad (20)$$

$$\sum_{d} n_{t,f,d} \times H_t \leq \sum_{s}\left(\sum_{d} x_{s,f,d} + a_{s,f} \times J\right) \times \overline{A}_{t,s} \qquad \forall t \,\&\, \forall f \qquad (21)$$

$$\sum_{f}\sum_{d} n_{t,f,d} \times \overline{T}_{t,f,d} \leq T \times 60 \times N_t \times \alpha_t^T \qquad \forall t \qquad (22)$$

$$\sum_{d} x_{s,f,d} \leq \left(T \times 60 - \tau_{s,f}\right) \times 60 \times X_s \times \alpha_s^S \times a_{s,f} \big/ L_s \qquad \forall s \,\&\, \forall f \qquad (23)$$

$$\sum_{s}\sum_{f} x_{s,f,d^p} + \delta_{d^p}^- - \delta_{d^p}^+ = Z_{d^p} \times T \qquad \forall d^p \qquad (24)$$

$$\delta_{d^p}^- \leq \Lambda_{d^p} \times T \qquad \forall d^p \qquad (25)$$

$$\delta_{d^p}^+ \leq \Lambda_{d^p} \times T \qquad \forall d^p \qquad (26)$$

$$\sum_{f}\sum_{s} x_{s,f,d^o} \times \overline{G}_{f,k} + g_{k,d^o}^- - g_{k,d^o}^+ = \sum_{s}\sum_{f} x_{s,f,d^o} \times G_{k,d^o} \qquad \forall k \,\&\, \forall d^o \qquad (27)$$

Constraints (13) and (14) assure that only one shovel is assigned to any face and also that any shovel is assigned to only one face. Constraint (15) is a soft constraint on the production by any shovel with a deviational variable that is minimized in the objective function. Constraint (16) is a hard constraint that puts a lower limit on the production by any shovel. Constraint (17) assures that total production by any shovel from its face to a destination is less than or equal to the total material hauled by trucks between the face and the destination, which in turn is equal to the product of number of trips between the face and destination, and the truck capacity. The inequality constraint makes sure that total material hauled may not be an integer multiple of truck capacity and so some trips may have slightly less load hauled. This constraint enables the model to excavate the faces completely and reduces infeasibility of the model to a great extent due to the tight equality constraint. To counter the effect caused by the inequality, constraint (18) has been included which puts a lower limit on production deviation as equal to a predefined value *J*. To optimize the objective function, *J* is considered as the minimum of the truck capacities in the truck fleet. It means, at the end of the shift, the maximum allowed difference between the shovel production from a face to a destination and the material hauled based on number of truck trips is *J*. In other words, constraints (17) and (18) allow the shovels to load the trucks slightly less than the capacity of the trucks if required. Constraints (19) and (20) make sure that total ore or waste production by any shovel from its assigned face cannot exceed the total available ore or waste material at that face. This constraint also makes sure that no production is possible by the shovel from the face it is not assigned to. Constraint (21) assures that a particular truck type will have zero trips from any non-matching shovel. Part of the right hand side of the inequality is included to incorporate what is modeled in constraint (18). Constraint (22) limits the maximum possible trips by any truck type considering the truck availability and optimization time. Constraint (23) limits the total production possible by a shovel taking into account its availability and the movement time to the face (if assigned to a different face from where it initially was). Constraints (24), (25) and (26) are the processing constraints on the desired tonnage feed to the processing plants and maximum allowable deviation in tonnage accepted at the plants. Constraint (27) tries that the average grade sent to the processing plants is of the desired grade and deviation is within the upper and lower acceptable limits.

4.3. Normalization of goals

The goals considered in this model are conflicting and incomparable in dimensions. Also a non-preemptive approach is adopted for the optimization. Such type of goal programming models need normalization of the goals before the optimization process. Grodzevich and Romanko (2006) provides different normalization strategies that can be adopted for optimization of similar models. Normalization has been carried out by determining the Nadir and Utopia points for individual goals. The goals are then normalized by the differences of optimal function values in the Nadir and Utopia points. This difference is the length of the interval where the optimal objective function vary within the pareto-optimal set (Grodzevich and Romanko, 2006).

Utopia point (z^U) for individual goals is obtained by considering only one goal in the objective and optimizing the system (minimization). This provides the lower bound on the values of individual goals in the Pareto optimal space.

Nadir point sets an upper bound on individual goals. This is the maximum possible value of any goal in the objective space. So, if $z_i^U = f_i(x^{[i]})$ represents Utopia point for goal *i* with solution vector $x^{[i]}$, Nadir point can be obtained for *K* number of goals using Eq. (28) (Grodzevich and Romanko, 2006).

$$z_i^N = \max_{1 \le j \le K}(f_i(x^{[j]})) \qquad \forall i \in goals \qquad (28)$$

Once the Nadir and Utopia points have been determined, goals can be normalized using Eq. (29) (Grodzevich and Romanko, 2006) to range between 0 and 1, and multiplied with respective weights to give priority to desired goals over others.

$$\overline{f_i}(x) = \frac{f_i(x) - z_i^U}{z_i^N - z_i^U} \qquad \forall i \in goals \qquad (29)$$

Weighted sum method, given by Eq. (30), has been used to assign the priority weights to be multiplied to goals.

$$\sum_i w_i = 1 \qquad i \in goals \qquad (30)$$

4.4. Model inputs

The MILGP model presented is proposed to work with a short-term production schedule at the block aggregate level, where mining cuts (faces) are provided to be excavated within a given period of one month. The short-term production schedule is generated using the clustering and scheduling algorithm proposed by Tabesh et al.(Tabesh et al., 2014). Incorporating mining cuts directly from the short-term schedule, with precedence mining cuts, help to remove the block precedence constraints from the current optimization problem. The most significant contribution that the short-term schedule provides is a link between the tactical and the strategic plan, by providing the available faces for shovel assignment in the given period of a month.

Model takes two types of input. All the face characteristics are obtained using the short-term mine production schedule. Information received includes mining cuts (face) IDs, coordinates of faces (for approximating the shovel movement distances from face to face), tonnage of material, fraction to be mined in the given period, minimum haul road distance from the face to the mine exit, precedence cut's IDs and average grades of different material.

Other inputs include:

1. Shovel: shovel ID's, bucket capacities, loading cycle time, availability, cost of shovel movement as per meter moved, movement velocity of shovel and the face where the shovel is initially located.
2. Trucks: truck types ID's, number of trucks of each type, capacities, dump time, spot time, availability, average speed of trucks when empty and when loaded, cost of truck operation per meter moved when empty and when loaded.
3. Destinations: maximum rate of processing at processing plants (tonne/hr), maximum allowed deviation in tonnage supplied to the processing plants per hour, desired grade of each material type at processing plants
4. Optimization duration (hours), 0 or 1 parameter inputs to match trucks with shovels and weights for different goals in the objective function.

5. Case Study

The case study of Gol-E-Gohar iron ore complex, located in south of Iran, has been considered to verify the model presented in this paper. Iron is the main element of interest in the deposit. As the mine employs magnetic separators for recovering the iron, magnetic weight recovery (percent MWT) is the main criterion for selecting the ore to be sent to the processing plants. The ore contains phosphor and sulphur as contaminants or secondary elements.

A life of mine extraction schedule, presented in Fig. 4, is obtained using Whittle software. Year six was selected to run a clustering algorithm and generate a short term production schedule over monthly resolution using the model of Tabesh et al. (Tabesh et al., 2014).

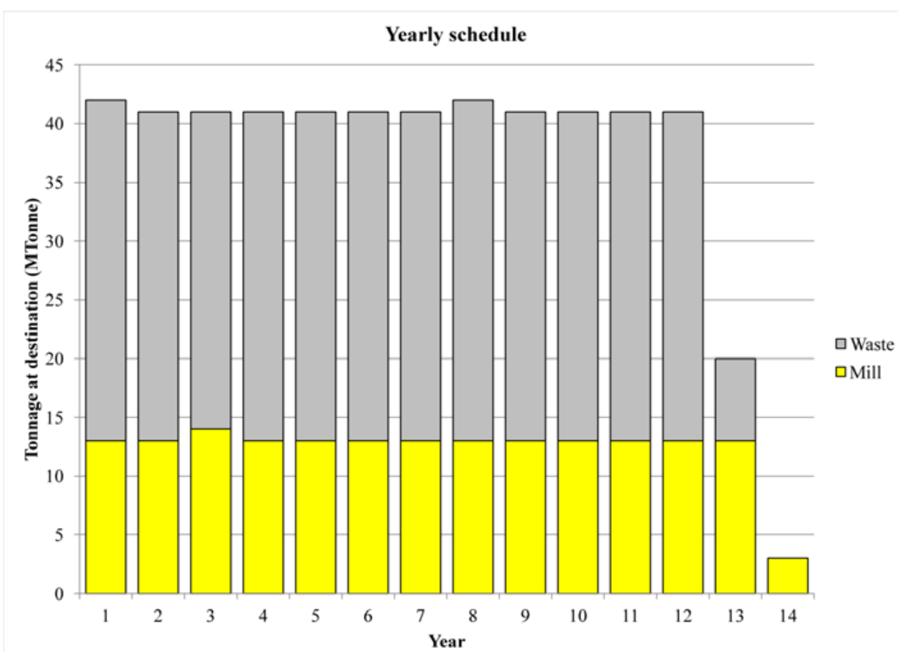

Fig. 4. Life of mine schedule for Gol-E-Gohar mine obtained through Whittle

We have used scheduled fractions of 2904 blocks, located over 3 benches, to be mined in the fourth month to run the optimization model. The schedule requires 1,023 ktonnes of Ore and 2,373 ktonnes of waste to be mined, working a 12 hour shift daily over 30 days. The grade distribution, over several mining cuts scheduled in the fourth month, is presented in Fig. 5.

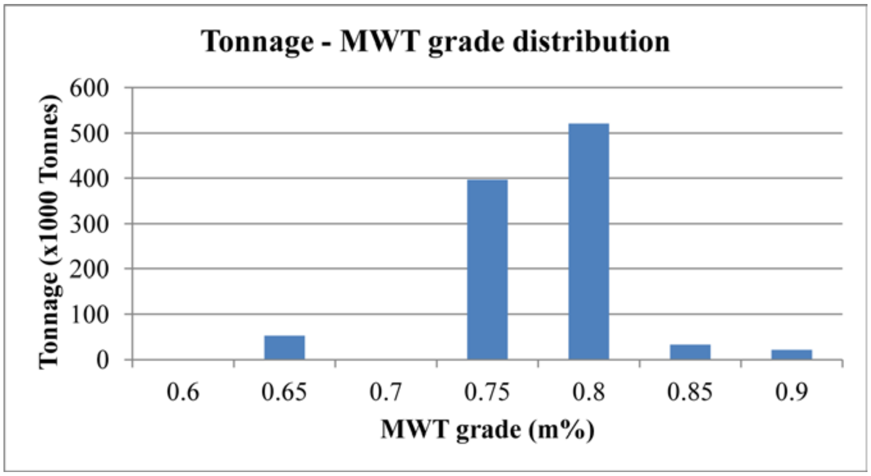

Fig. 5. Graph of grade distribution of MWT (magnetic weight recovery) in the scheduled cuts for month four.

The open-pit is designed to have only one exit. The distance from pit exit to dump destinations are given in Table 1. The distance from pit exit to mining faces (mining cuts) are calculated as distance from pit exit to bench following the gradient of the ramps, and a straight line distance from the ramp access point to mining face.

Table 1. Desired grades, targets, limits and distances to pit exit for the dump destinations

Destinations	Desired MWT grades (m%)	Processing target (t/h)	Processing limit (t/h)	Distance to pit exit (m)
Plant 1	72	1500	1700	1000
Plant 2	78	1500	1700	750
Waste Dump	-	-	No limit	1000

This scenario considers two processing plants and a waste dump as dump destinations in the mine. Table 1 provides the desired grades of MWT, target and maximum processing limits for processing plants and corresponding distances from the pit exit.

The mine employs two Hitachi ZAXIS-650H and two Hitachi Ex 1900-6 hydraulic excavators. ZAXIS-650H excavators work mostly with ore with an average shovel availability of 0.68, and they load on an average 15 tonne per bucket with 24 second bucket cycle time. The two Hitachi Ex 1900-6 excavators load on an average 30 tonne per bucket with 25 second bucket cycle time and an average shovel availability of 0.78. Cost of shovel movements from one face to other is considered to be $1 per meter with an average speed of 50m/min considering all required manpower and moving related equipment. Mine employs 15 Hitachi EH1100-5 haul trucks, with nominal capacity of 90 tonne, and 19 Hitachi EH1700-3 trucks with nominal capacity of 120 tonnes. The 90 tonne nominal capacity trucks are compatible to work only with 15 tonne bucket capacity shovels and move at an average speed of 36 Km/h when empty and 18 km/h when loaded. 120 tonne nominal capacity trucks are compatible to work only with 30 tonne bucket capacity shovels and move at an average speed of 34 Km/h when empty and 17 km/h when loaded. The empty and loaded movement cost for EH1700-3 trucks is considered to be $ 0.22 and $ 0.32 per Km and for EH1100-5 trucks is $ 0.2 and $ 0.3 per Km respectively.

6. Model implementation and results

The MILGP model is used to carry out the case study, optimizing the system over half hour durations up to 12 hours daily, and recording the daily production indices for one month. At the beginning of the month ore shovels 1 and 2 are scheduled to be working on bench 1, shovel 3 on bench 2, and shovel 4 on bench 3. Bench 1 contains mostly ore whereas bench 2 and 3 contains waste scheduled for the month. Fig. 6 represents part of the second bench scheduled and worked upon during the month. Fig. 6 presents the start of working day for every face in numerals, working shovels in color and mining cuts (faces) by solid boundaries.

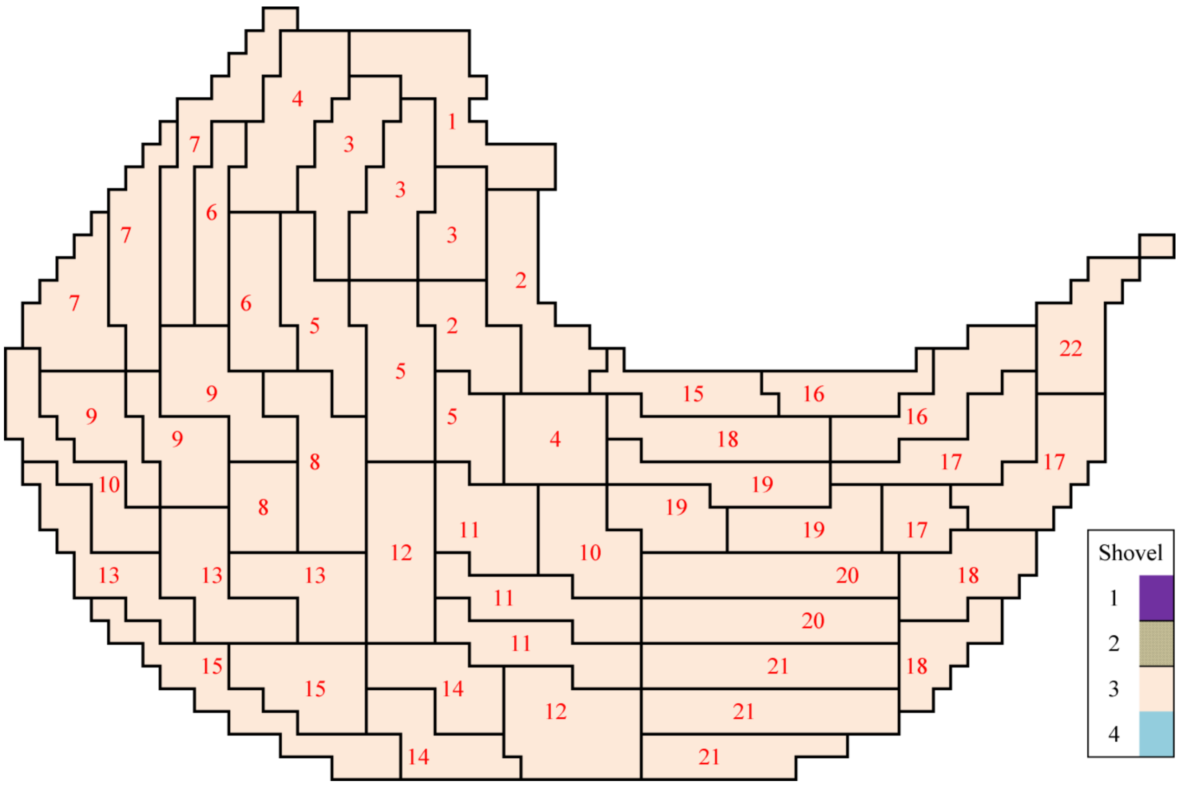

Fig. 6. Start day of working (represented by numerals) of mining blocks forming mining cuts (represented by solid boundaries) by shovels (represented by color) for part of bench 2 at elevation 1655

Shovel 1 and 2 were observed to be mining ore on bench 1 throughout the month. Shovel 3 mines out the scheduled faces on bench 2 and move to bench 3 at the end of the month, whereas shovel 4 remains on bench 3 throughout the month.

Fig. 7 presents the daily production received at processing plants and the waste dump for 20 days. Very small variations observed are attributed to the loss in production due to shovel movements from face to face, or from bench to bench. Capacity utilization curve presents the variation in percentage of production capacity utilization of the mine, including all four shovels.

Fig. 8 and Fig. 9 show the performance of the operational objectives included in the MILGP model for minimizing the deviations in feed and grade to processing plants compared to desired feed and grade. Referring to Table 1, daily desired feed to processing plants is 18,000 tonne and desired grade is 72 and 78 m% at plant 1 and 2 respectively. Fig. 8 shows that daily tonnage fed to processing plants is almost constant at 18,000 tonnes as desired. The small variation in tonnage fed to plants is attributed to the movement time of shovels between faces causing loss in ore production coupled with other optimization objectives of desired feed and grade at the plants. Comparing to available grades distribution in mining faces scheduled (Fig. 5), the obtained results of average grade fed to processing plants shown in Fig. 9 is satisfactory. It should be noted that exact grade requirements are met at the plants from day 16 to 20, where shovel 1 and 2 mine out faces with grades 70 and 80 m% respectively.

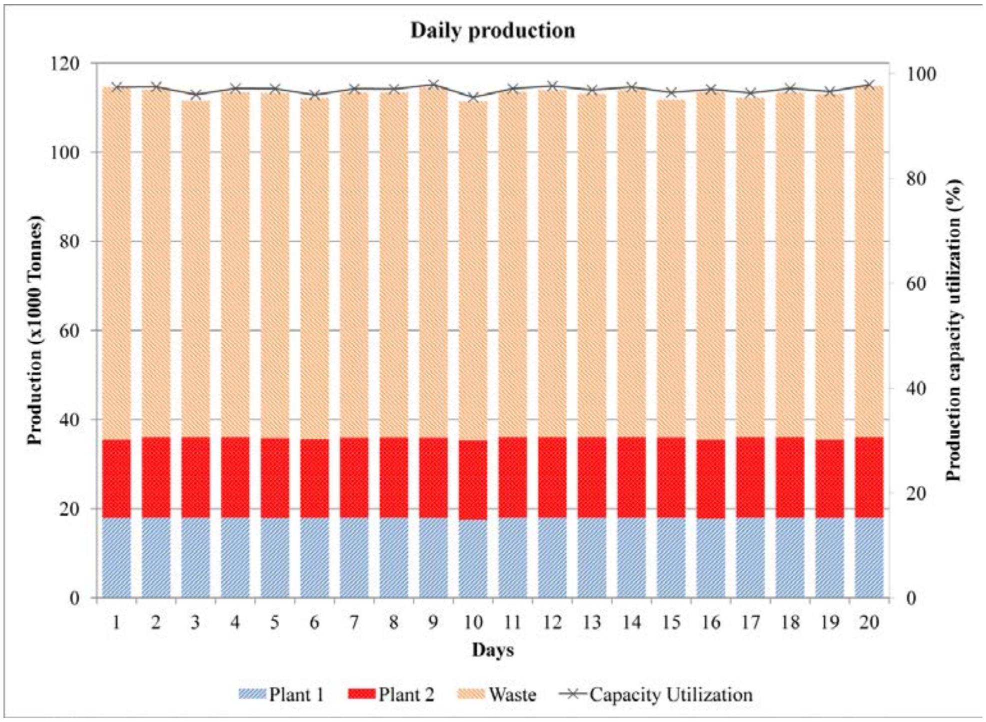

Fig. 7. Daily production sent to processing plants and waste dump

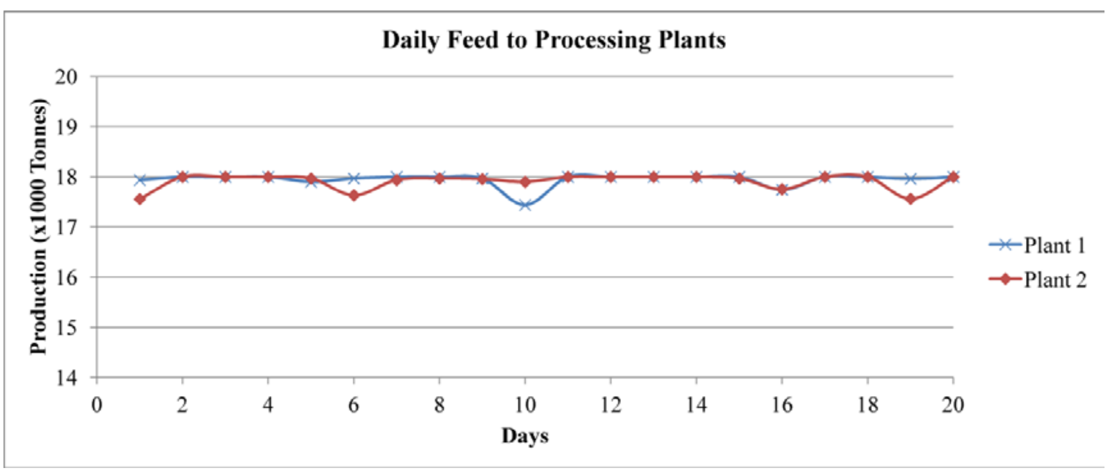

Fig. 8. Daily feed to processing plants

Truck fleet utilizations are presented in Fig. 10, where truck type 2 (EH1100-5) works only with ore shovels 1 and 2, and truck type 1 (EH1700-3) works only with shovels 3 and 4. Truck utilizations are obtained to be above 90% with the variations accounted towards the varying distances of shovels from destinations.

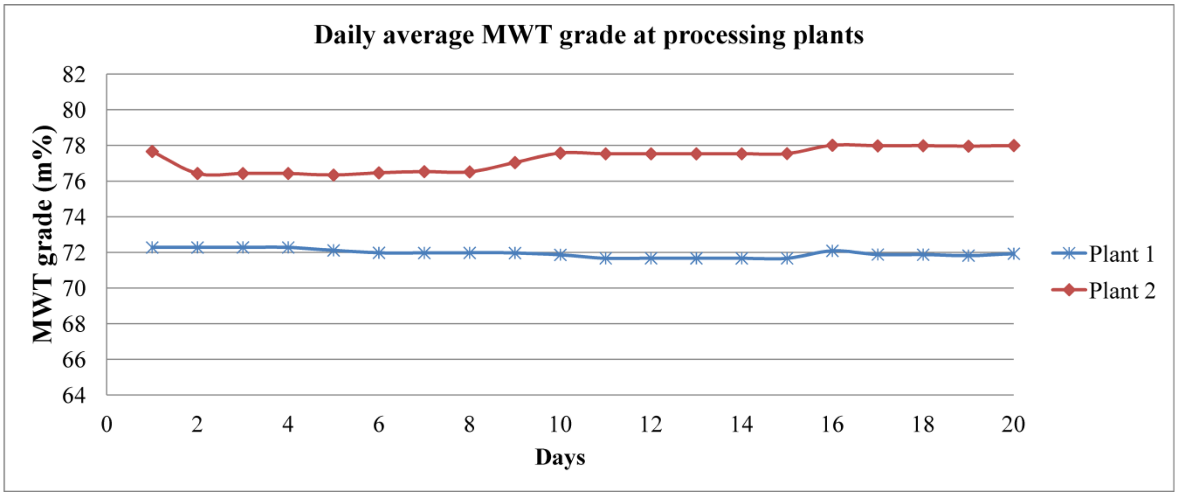

Fig. 9. Average MWT grade received at processing plants

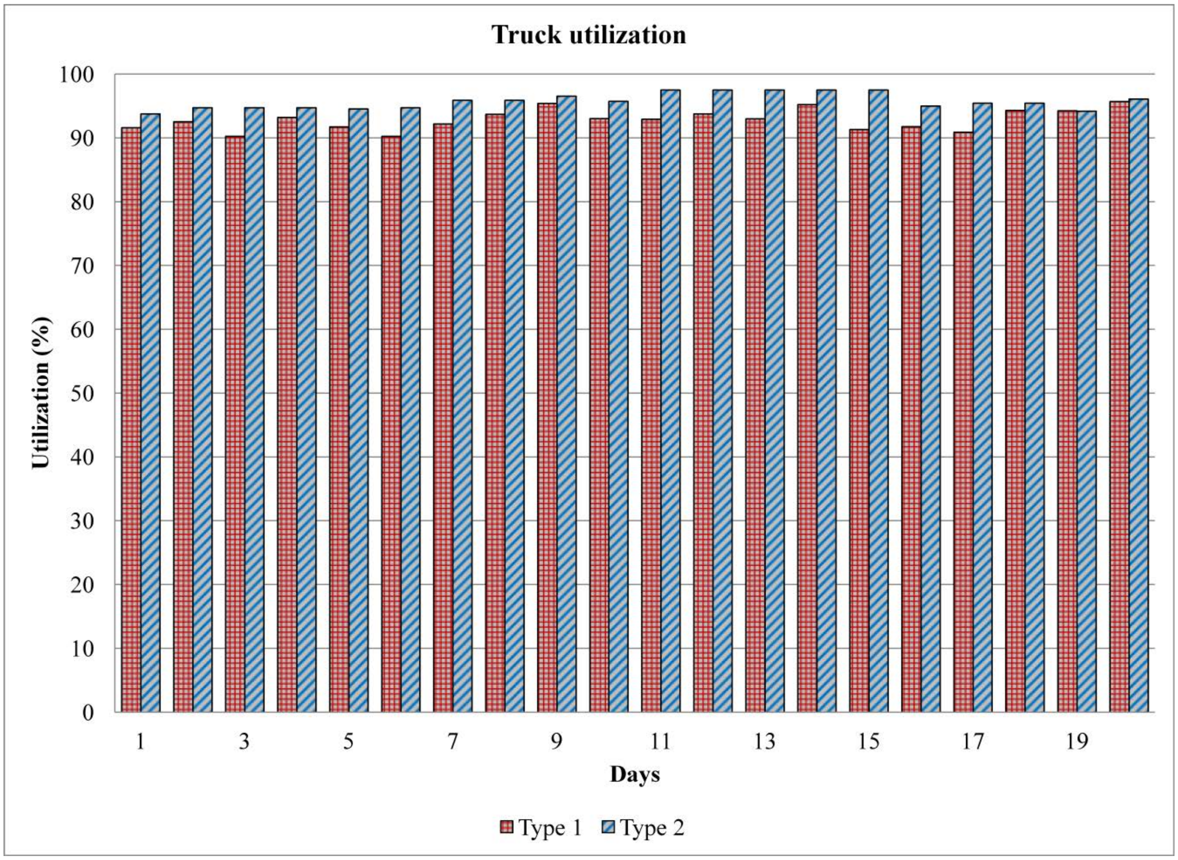

Fig. 10. Daily average truck utilizations for truck type 1 (EH1700-3) and truck type 2 (EH1100-5)

Table 2 presents the key performance indices (KPIs) recorded over 20 days to demonstrate the applicability of the MILGP model while meeting the desired production goals included.

Table 2. Observed production KPIs over 20 days

		Min	Max	Average	Summation
Plant Utilization (%)	Plant 1	96.9	100.0	99.7	-
	Plant 2	97.6	100.0	99.5	-
Truck Utilization (%)	Type 1	90.2	95.7	92.9	-
	Type 2	93.8	97.5	95.8	-
Shovel Utilization (%)	Shovel 1	93.4	98.2	96.7	-
	Shovel 2	95.6	100.0	98.7	-
	Shovel 3	90.7	98.8	95.4	-
	Shovel 4	93.0	99.0	97.0	-
Shovel Movement time (min)	Shovel 1	0.0	6.6	0.0	10.8
	Shovel 2	0.0	4.0	0.0	12.3
	Shovel 3	0.0	7.5	0.2	89.6
	Shovel 4	0.0	6.1	0.1	46.9

7. Conclusion

The solution of the MILGP model for the iron ore mine case study provided average plant utilizations above 99%, average truck utilizations above 92% and average shovel utilizations above 95%. The operational objectives of minimizing the deviations in feed and grade to processing plants compared to desired feed and grade are also met satisfactorily. Also because model provides shovel assignments based on short-term production schedule, it helps realize the strategic production schedule. The results obtained prove the applicability of the model for providing shovel and truck allocations in open-pit mine operations and work as upper stage in a two stage dispatching system.

The MILGP model also provides a scope to work with simulation models for analyzing the operations over longer time horizons. An integrated system with simulation will provide better opportunities for efficient equipment planning and strategic decision making towards achieving long term mining objectives. The future research in this area includes developing an integrated simulation optimization model for understanding the production operations and thus aligning the system towards achieving better compliance with strategic plans.

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9. Appendix

9.1. Notations

Index for variables, parameters and sets

s index for set of *shovels*($s = 1, \ldots \hat{S}$)

f index for set of *faces*($f = 1, \ldots \hat{F}$)

t index for set of truck types *trucks* ($t = 1, \ldots \hat{T}$)

k index for set of material types *MatType* ($k = 1, \ldots \hat{K}$)

d index for set of *destinations* (processing plants, stockpiles, waste dumps)

d^p index for set of *processing plants* ($d^p = 1, \ldots \hat{P}$)

d^o index for ore destinations (processing plants and stockpiles)

d^w index for waste dumps ($d^w = 1, \ldots \hat{W}$)

9.2. Decision variables

To formulate all the system constraints and to represent the system as precisely as possible, while keeping the model linear, following decision variables have been considered.

$a_{s,f}$ Assignment of shovel *s* to face *f* (binary)

$n_{t,f,d}$ Number of trips made by truck type *t*, from face *f*, to destination *d* (integer)

$x_{s,f,d}$ Tonnage production sent by shovel *s*, from face *f*, to destination *d*

x_s^- Negative deviation of shovel production from the maximum capacity in a shift

$\delta_{d^p}^-, \delta_{d^p}^+$ Negative and positive deviation of production received at the processing plants d^p

g_{k,d^o}^-, g_{k,d^o}^+ Negative and positive deviation of tonnage content of material type *k* compared to tonnage content desired, based on desired grade at the ore destinations d^o

9.3. Parameters

D_t Dumping time of truck type t (minutes)

E_t Spotting time of truck type t (minutes)

N_t Number of trucks of type t

H_t Tonnage capacity of truck type t

J Flexibility in tonnage produced, to allow it not to be an integral multiple of truck capacity (tonne)

V_t Average speed of truck type t when empty (Km/hr)

$\bar{V}_t$ Average speed of truck type t when loaded (Km/hr)

C_t Cost of empty truck movement ($/Km)

$\bar{C}_t$ Cost of loaded truck movement ($/Km)

$\bar{A}_{t,s}$ Binary parameter, if truck type t can be assigned to shovel s

X_s Shovel bucket capacity (tonne)

L_s Shovel loading cycle time (seconds)

U_s^+ Maximum desired shovel utilization (%)

U_s^- Minimum desired shovel utilization (%)

A_s Cost of shovel movement ($/meter)

S_s Movement speed of shovel (meter/minute)

α_t^T Truck availability (fraction)

α_s^S Shovel availability (fraction)

F_s Face where shovel is initially located (start of the shift)

D_f^{FE} Distance to exit from face f

D_d^{ED} Distance to destination d from the pit exit

Z_{d^p} Maximum capacity of the processing plants (tonne/hr)

Λ_{d^p} Maximum acceptable deviation in tonnage received at processing plants (tonne/hr)

G_{k,d^o} Desired grade of material types at the ore destinations

G_{k,d^o}^- Lower limit on grade of material type k at ore destinations

G_{k,d^o}^+ Upper limit on grade of material type k at ore destinations

F_f^x, F_f^y, F_f^z x, y, z coordinates of the faces available for shovel assignment (meters)

$\bar{G}_{f,k}$ Grade of material type k at face f

O_f Tonnage available at face f (tonne)

Q_f 1 if material at face is ore, 0 if it is waste (binary parameter)

T Shift duration (hr)

Π^- Lower limit on desired stripping ratio

Π^+ Upper limit on desired stripping ratio

W_i Normalized weights of individual goals (i = 1, 2, 3, 4) based on priority

9.4. Calculated parameters

Γ_{f^1,f^2}^F Distance between available faces (meters)

$\Gamma_{f,d}^D$ Distance of destinations from faces, based on the haulage profile in short-term schedule (meters)

$\tau_{s,f}$ Movement time of shovel s from initial face to face f (minutes)

X_s^+ Maximum shovel production calculated using availability and maximum desired utilization (tonne)

X_s^- Minimum shovel production calculated using availability and minimum desired utilization (tonne).

$\bar{T}_{t,f,d}$ Cycle time of truck type t from face f to destination d (minutes)

Open-Pit Mine Production Optimization: A Review of Models and Algorithms

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Abstract:

Plan and schedule based on which material of an open pit mine are being handled includes two main steps: static scheduling and real-time dispatching. Earlier is done statically at the beginning of the shift and by the time status of mine changes significantly. Along with, the later one is a dynamic decision making procedure running all over the shift. Open pit mines operation costs highly depends on material handling. 50 to 60 percent of operating costs in an open pit mine is spent on digging and transporting the material. So, reducing a small portion of it will save billions of dollars in a large open pit mine. Generally there are two ways to reduce this cost: transferring higher amount of material in each payload and optimizing both static and dynamic operation schedules. To achieve the later goal different algorithms have been developed since late 1960s. This paper shows that from applicability point of view there are two major groups of algorithms: a) the group are being widely used in the mining projects, and b) the group of algorithms developed in academy. The paper first discusses main industrial algorithm, then after reviews well-known academically developed algorithms. Strengths and weaknesses of the algorithms are discussed and suggestions for the future researches are presented.

1. Introduction

Mining projects and more especially surface mines are known as high cost expenditures that need millions of dollars or in the large ones billions of dollars to be expend on them in both capital and operating parts. Material handling procedure as the main consumer of the operating cost plays a critical role in the mining projects decision making procedure. A large portion of total mining costs in an open pit mine must be allocated to excavating and transporting the excavated materials from the mining faces to different destinations out of the pit rim. As it is believed by many researchers, 50% of operating costs in open pit mines (Alarie and Gamache, 2002) and even in some cases especially in large open pit mines up to 60% of the operation costs is to be spent on material handling (Alarie and Gamache, 2002; Oraee K. and Goodarzi A., 2007; Akbari et al., 2009; Ahangaran et al., 2012; Upadhyay and Askari-Nasab, 2015). So, improving the transportation operation and subsequently decreasing expenses of this part of the operation even by 2 or 3 percent will save stockholders a huge amount of money. There are two principle way to improve material transportation efficiency in open pit mines. The first way is to implement large size trucks in the truck fleet with the capacity of transporting more material in each payload, the point current truck

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manufacturers have been reaching to the maturity. The second principle way to improve the transportation operation reduce cost per ton of material transported is to implement operations research techniques to enhance productivity of the operation. Although as Alarie and Gamache (2002) considers there is a single stage approach like the one was presented by (Hauck, 1973) which implement a continuous algorithm to maximize productivity of the operation and send trucks to the destination in a way that minimize deviation from the production target simultaneously, there is a multistage approach of the open-pit operation optimization that is of the most interest under which the problem is divided into two sub problems. In the first sub problem a static scheduling algorithm is implemented to determine the optimal loaders configuration over the mining faces as well optimum production rate for each route connecting loading points to discharge points and also allocation of truck resources to meet production target. This stage called upper stage runs at the beginning of the shift and when the mine status changes. As the lower stage a dynamic algorithm mostly based on assignment problem or rarely based on transportation problem assigns the trucks to a proper destination by the time they asks for a destination in the way that minimize deviation from the production target.

There are two basic categories for open pit mines' operation optimization. Industrial groups who present the software packages to the mining projects without disclosing algorithms behind the software; the academic groups who, although, disclose all logics behind the algorithms never implement the methods world widely.

2. Definition of the Operational Planning Problem in Surface Mines

A mine's production schedule include three time range plans: 1 – Long-term (20 – 30 years length, describe feasibility of the mining adventure and cash flow distribution, and is the major input of the medium- and short-term plan); 2 – Medium-term (1 – 5 years length, provides more detail information for extraction of mining areas specifically, presents more information about fleet expansion or equipment replacement); 3 – Short-term (1 – 12 months, detailed information about faces to be extracted and feeds to be sent to the plant) (Osanloo et al., 2008). Short-term schedule itself is broken down to operational plans. Operational plan is the shift base stage of open pit mine production scheduling which covers dynamic real-time decision making procedure in surface mine operation that includes: finding the shortest paths between loading and discharge points, operation optimization that means finding optimum productivity rate of each route and allocate truck payload to each route in a way that cover the production target (upper stage), and dynamic truck assignment (lower stage) which is illustrated in Fig 1. Indeed, the operational planning tasks challenge with the equipment allocation problem tries to make decision on allocation and dispatching of the trucks based on the routes capacity and equipment capabilities (Newman et al., 2010).

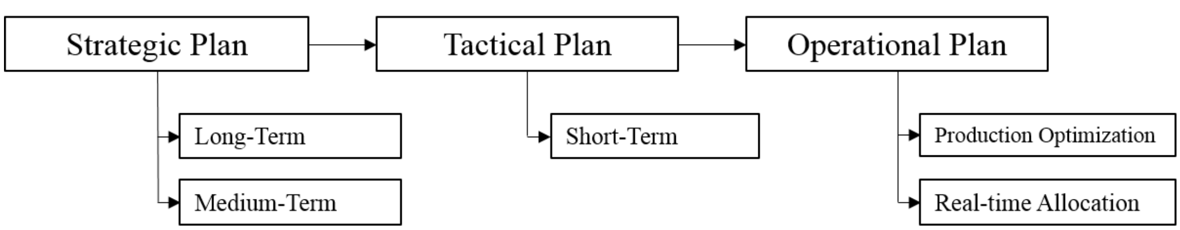

Fig 1. Stages of Making Decisions in Mine Production Scheduling (Upadhyay and Askari-Nasab, 2015).

Operation plan as it is shown in Fig. 2 runs all over the mine life. As it is illustrated it accepts schedule of each period from the short-term schedule and available mining faces. Then it runs until the end of the shift or the time mine status changes.

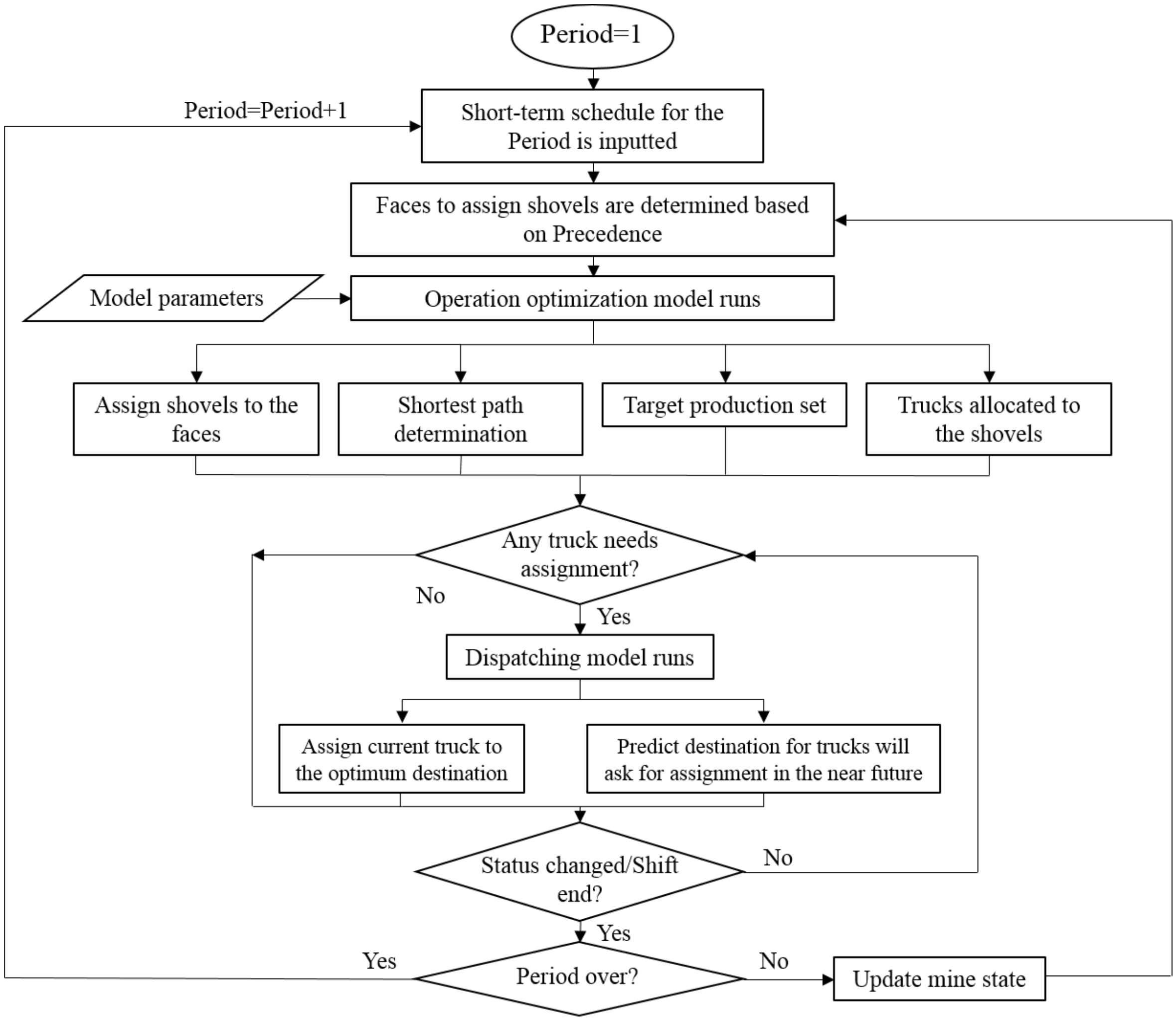

Fig 2. schematic illustration of Open Pit mine operational plan (Upadhyay and Askari-Nasab, 2015).

Researchers in both academic institutes and industrial sectors have been trying to improve the algorithms of open pit mine operational plans to reduce the cost of the project or increase the production level. Herein, we reviewed both sides of the development including industrial algorithms and academically improved ones, though for the former one there is not sufficient revealed algorithms.

3. Fixed Truck Allocation

In this type of mine operation at the beginning of each shift a group of trucks are locked to each transportation route. The trucks allocated to the paths are to work on the same path over the shift period based on some criteria such as production requirement, availability of the trucks in the fleet, and so on (Lizotte and Bonates, 1987; Lizotte et al., 1987). The paths to which trucks have been allocated to will not change until a shovel breaks down or a critical event happens. Some efforts to modify this method has been seen in the literature. Firstly, Bogert (1964) suggested using of radio communication between equipment operators and mine control center. Late 1970s Mueller (1977) introduced implementation of the dispatching boards installed in the control center. This method of operation scheduling is the least productive method and From Kolonja and Mutmansky (1993) to Hashemi and Sattarvand (2015) it has been always being used as the base method to study other algorithms and approaches.

4. Flexible Truck Allocation

In this type of mine operation scheduling a portion of available trucks of the fleet are assigned to a specific working shovel at the beginning of the shift. But these trucks instead of being in the service of only a single shovel or a single route whole the shift, they ask for a new assignment each and every time they loaded the material at the loader or discharge it in a dump area. This method of taking equipment into the work, researcher claim that improves productivity of the operation with a high percentage. Olson et al. (1993) enclosed a 13% increase in the production Bougainville Copper Mine, 10 to 15 % improvement in the productivity of the Barrick Goldstrike Gold mine, 10 % of growth in Iron ore production of LTV steel mining, and 10% increase in the production of the Quintette Coal mine. Furthermore, Hashemi and Sattarvand (2015) in a simulation study of the Sungun Copper Mine's operation showed that by implementing a flexible allocation the productivity of the mine increased by 8% in comparison with the fixed allocation. Also, Kolonja and Mutmansky (1993) study of different types of open pit mine production management heuristics including minimize shovel waiting time (MSWT), minimize shovel saturation (MSS), minimize shovel production requirement (MSPR), minimize truck waiting time (MTWT), minimize truck cycle time (MTCT), and logic behind DISPATCH shows that no matter what type of the flexible truck allocation algorithm be used it always improves productivity in comparison with the fixed allocation (FA) (Table 1).

Table 1: Effect of flexible allocation on open pit mine production (Kolonja and Mutmansky, 1993)

	Percentage difference in production					
System Comparison	13 trucks	Significant difference	16 trucks	Significant difference	18 trucks	Significant difference
FA vs. MSWT	2.97	no	5.67	yes	3.35	yes
FA vs. MSS	3.84	no	5.62	yes	2.89	yes
FA vs. MSPR	2.94	no	4.52	yes	1.33	no
FA vs. MTWT	4.82	yes	6.88	yes	1.47	no
FA vs. MTCT	1.96	no	4.30	yes	1.63	no
FA vs. DISPCH	4.46	yes	7.15	yes	3.15	yes

Since late 1960s number of research have been done in both industries and academies to enhance productivity and reduce cost of mining operation by developing flexible allocation models and algorithms based on different strategies.

4.1. Algorithms Implemented in Industrial Packages

There are many companies across the world providing mine operation management system. From them Mudular mining system with 12% improvement in productivity and accompanying in 200 mines around the world is the leader. Jigsaw with 130 mine is in the second place. However, Wenco by presenting FleetControl claims of 11% improvement in system productivity. They currently have 65 mine sites using their system. CMC introduces Dynamine with a range of productivity improvement of 10% to 15%, Micromine with Pitram system and Caterpillar are the next leader of mine operation management system. Commercial companies who supports mine fleet management do not have willingness of disclosing the logics behind their fleet manager software, though. However, in 1980s and early 1990s Mudular Mining System revealed the models and algorithms based on which DISPATCH mine fleet management system works. Thus, in this section we try to review the algorithms behind DISPATCH from finding the shortest paths to real-time dispatching. Fig 3 and Fig 4 illustrate the procedure DISPATCH goes through to find the solution and the algorithms implements to complete the tasks, respectively.

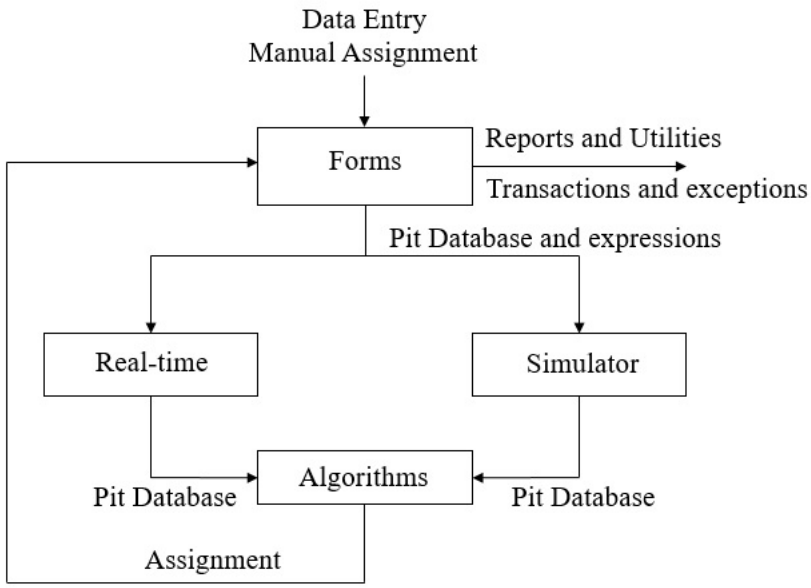

Fig 3. Schematic representation of DISPATCH block diagram (Olson et al., 1993)

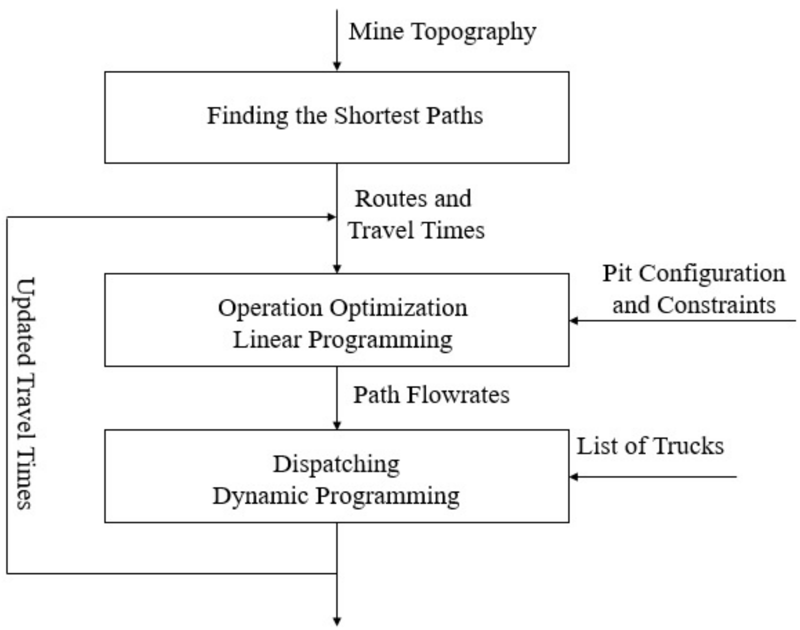

Fig 4. Procedure through with DISPATCH assigns trucks (Olson et al., 1993)

4.1.1. Finding the Shortest Path

In graph theory, the shortest path problem is the problem of finding a path between two vertices (or nodes) in a graph such that the sum of the weights of its constituent edges is minimized. Among different algorithms to find the shortest path in the literature of operations research such as Dijkstra, Bellman – Ford, A* search, Floyd – Warshall, Johnson, Viterbi, and so on, DISPATCH uses Dijkstra's algorithm with the objective of minimizing travel time between each pair of starting and ending points. After solving the shortest path problem in DISPATCH following information is presented to the operation optimization model: 1- total minimum distance and travel time for each specific transport. 2- The nodes truck must passes from to reach destination.

4.1.2. Production Optimization and Truck Allocation

DISPATCH uses linear programming (LP) approach to optimize production target of the time horizon by dividing it into two separated but weakly coupled models from which the first one (Eq. (1)) optimizes total operation including mine sector, plant sector and stockpile, and the second part (Eq. (5)) maximizes the fleet production by minimizing total required volume to be handled. White and Olson (1986) and Olson et al. (1993) describe the model as follow:

$$\min C = \sum_{i=1}^{N_m}(C_m \times Q_i) + C_p \times (P_t - \sum_{i=1}^{N_m+N_s} Q_i) + \sum_{i=1}^{N_s}(C_s \times Q_i) + \sum_{i=1}^{N_m+N_s}\sum_{j=1}^{N_q}(L_j \times C_q \times X_{ij} \times Q_i) \quad (1)$$

Subject to:

$$0 \le Q_i \le R_i \quad (2)$$

$$P_t \ge \sum_{i=1}^{N_m+N_s} Q_i \quad (3)$$

$$X_jL \le X_jA + \sum_{i=1}^{N_m+N_s}\left(X_{ij} - X_jA\right) \times Q_i \times T_c / \left(M_c / SG\right) \le X_jU \quad (4)$$

Where:

C	is functional dimensionless pseudo cost
N_m	is number of shovels at mining faces
C_m	is material transportation pseudo cost (hr/m^3)
Q_i	is material being transported per hour (m^3/hr) that should be determined in the procedure
N_s	is number of shovels working at stockpile
C_s	is stockpile material handling pseudo cost (hr/m^3)
N_q	is number of quality constraints
L_j	is quality director: 1 for low crit and -1 for high crit
C_q	is quality pseudo cost (hr/m^3)
X_{ij}	is jth quality factor at ith shovel
C_p	is pseudo cost of low feed to plant (hr/m^3)
P_t	is target rate of plant feed
R_i	is digging rate at ith shovel
M_c	is 1^{st} in/1^{st} out average control mass, kg
SG	is specific gravity
T_c	is base control interval, hr
X_jL	is lower limit for quality factor j
X_jA	is running average value of quality factor j
X_jU	is upper limit for quality factor j

All pseudo costs are being chosen arbitrarily with respect to ($C_m < C_q < C_s < C_p$).

As the second segment of the LP model, DISPATCH tries to minimize total haulage capacity needed to meet shovel production coverage:

$$\min \mathrm{V} = \sum_{i=1}^{N_p}\left(P_i \times T_i\right) + \sum_{j=1}^{N_d}\left(P_j \times D_j\right) + N_e \times T_s \tag{5}$$

Subject to:

$$\sum_{k=1}^{N_{pi}} P_k = \sum_{k=1}^{N_{po}} P_{k'} \tag{6}$$

$$R_j = \sum_{k=1}^{N_{po}} P_{k'} \quad \text{in the mine} \tag{7}$$

$$R_j \le \sum_{k=1}^{N_{po}} P_{k'} \quad \text{at the stockpile} \tag{8}$$

$$P_j = Q_i \tag{9}$$

$$0 \le P_i \tag{10}$$

Where:

V	is total mine haulage (m^3)
N_p	is number of feasible haul routes
P_i	is haulage on path i which should be determined (m^3/hr)
T_i	is path i travel time (hr)
N_d	is number of dumps for mine haulage
P_j	is net haulage input to dump j (m^3/hr)
D_j	is the average dump time at dump j (hr)
N_e	is number of operating shovels
T_s	is fleet average truck size (m^3)
N_{pi}	is number of feasible input paths at node j
N_{po}	is number of feasible output paths at node j
P_k	is input path haulage (m^3/hr)
$P_{k'}$	is output path haulage (m^3/hr)
R_j	is limiting rate at node j (m^3/hr)

The model introduce first segment of the operation optimization as a pseudo cost based LP which is established on summation of costs in all four operational sector of the mine. The solution of the first segment present shovels production rates with respect to maximum digging rate of shovel (Eq. (2)), maximum capacity of the plant (Eq. (3)), and lower and upper bounds of the blending grade (Eq. (4)). The second segment LP maximizes production of the operation by allocating minimum number of trucks to each active route (Eq. (5)) to meet the routes productivity rate. Eq. (6) makes sure that input and output flow at each shovel and each dumping point are equal. Eqs.(7) and (8) guarantee material handling to meet grade requirement at plant cannot exceed shovels' digging rate working at stockpile. Coupling between two segments of the operation plan is attained by constraining total productivity of all routes servicing a shovel to be greater or equal to the shovel productivity (Eq.(9)). It should be mentioned here that both P and Q in Eq. (9) are vector. Finally, Eq. (10) ensures that all haul rate in the mine are nonnegative. The model follows current status of the mine. An advantage of the model is that the optimum production rate of each route is based on the volume of material not on number of trucks. That helps the dispatching step to send proper truck to cover the shortage. Major drawback of the model is that it does not consider stripping ratio (SR) limitation in the operation. However, most of the drawbacks of DISPATCH will arise in the real-time dispatching model which will be explained in more detail later on.

4.1.3. Real-time Dispatching

After solving upper stage (operation optimization) LP problem by implementing simplex method resulting optimum material flow rate on routes, White and Olson (1986) employs dynamic programming (DP) approach to send trucks to the proper destination. To do so, two list and three parameters are defined. List of needy shovels or LP-selected paths and list of trucks dumping material at discharge points or en route from a loading point to a destination are provided. Also, need-time (Eq.(11)) which is defined as the expected time for each path's next truck requirement formulated as follow:

$$need-time_i = L_j + F_{ij} \times \left(A_j - R_j\right) / P_i \qquad (11)$$

Where:

- L_j is time last truck was allocated to the shovel j
- F_{ij} is flow rate of path i over the total flow rate into shovel j
- A_j is total haulage allocated by time L_j to shovel j
- R_j is haulage requirement of shovel j
- P_i is path flow rate (ton/hr or m^3/hr)

So, the neediest path which is on the top of the neediest shovels list will be the one with the shortest need-time. Then lost-ton is defined and formulated as a criterion to find the best truck for the neediest path from the truck list with Eq. (12):

$$lost-ton = \frac{truck\,size \times total\,rate}{required\,trucks} \times \left(truck\,idle + excess\,travel\right) + shovel\,rate \times shovel\,idle \qquad (12)$$

Where:

Truck size is size of truck being assigned; Total rate is total digging rate of all shovels in mine; Required trucks is total required trucks in LP solution; Truck idle is expected truck idle time for this assignment; Excess travel is extra empty travel time to neediest shovel; Shovel rate is sum of all path rates into neediest shovel; Shovel idle is expected shovel idle time for this assignment.

Considering the lost-ton definition, best truck is the truck covering lost-ton of neediest shovel the most. After finding the best truck and assigning it to the neediest shovel, it is moved to the last position on the needy paths' list and the procedure is repeated for the second neediest which is now the neediest until all trucks on the list are assigned.

Defining a rolling time horizon when a sequence of assignment is needed is a benefit of the model. Because the information of the mine status which is being used in the model always is up to the minute. However, it does not consider effect of current truck assignment on the forthcoming truck matching, though all trucks previously sent to the shovels are considered. Another drawback of the model is that despite the authors claim, the solution method is not a DP. It is a heuristic rule solving each sub problem based on the best solution of previous sub problems and based on Alarie and

Gamache (2002), maybe it is because of the authors misunderstanding of Bellman's principal of optimality.

4.2. Academic Algorithms

Algorithms and models have been presented to solve open pit mine real-time operational plan in academic institutes are enclosed to the public. Herein, we reviewed the models those effects on the real-time open pit mines' operation optimization is undeniable.

4.2.1. Finding the Shortest Path

One of the first appearance of the operational problem in the literature of open pit mining is (Hauck, 1973), in which the shortest path was defined as the closet route from loading to discharge point time-wisely and based on the previous experience. In their non-linear model of solving upper stage problem as a network problem, Elbrond and Soumis (1987) and Soumis et al. (1989) solved a non-linear programming (NLP) network problem to find shortest path between all loading and discharge points. Among all, the most famous and interesting dispatching algorithm up until now which had been presented by Temeng et al. (1997) and Temeng et al. (1998) uses Dijkstra's algorithm of finding shortest path between source and sink to select the best route of connecting shovels to their destination.

4.2.2. Production Optimization and Truck Allocation

Most of the models presented in operational planning of open pit mines are focusing on upper stage or shovel and truck allocation part. The model developed by Soumis et al. (1989) performs the upper stage in two steps. As the first step, it fixes shovels' location by implementing combinatory mixed integer linear programming (MILP) model with respect to available trucks and the objective of maximizing the production and subject to quality constraints. By solving the MILP model it suggests some location for shovels to be seated on the computer screen, and it needs a human to make a decision on the shovels siting locations. Then after, as the second step of the algorithm Soumis et al. (1989) represents truck travel plan between shovels and dumping points by solving a NLP. The model's objective function consists of three major factors: 1- shovel production objective (computed shovel production); 2- available truck hours (computed truck hours) which includes truck waiting time as well; and 3- penalty for the deviation of the produced ore material from the blending objectives. Munirathinam and Yingling (1994) claim that there is an advantage of using NLP versus LP. The point is paths will not be on extreme. Because solution methods for solving LP models always look for the optimum solution on the corner of the feasible regions whereas NLP solution methods search for the optimum solution over the entire feasible region. As a result of implementing NLP model the flow rate will be split over paths which helps to achieve blending goals easier. Beside the advantage of the model, it is assumed that all trucks in the fleet are from the same capacity called homogenous truck fleet. However, generally truck fleet in mine is heterogeneous including different types and capacity of trucks. Second drawback of the model is assumption of fixed grade material in each mining faces. Whereas, stochastic nature of the ore material quality even in a single block is not ignorable (Osanloo et al., 2008).

However, the model was not presented clearly in the paper. Herein, other approaches of optimizing mine truck allocation for each of them there is at least one disclosed mathematical model are presented in following subsections: first queuing theory implementation is studied. As the second subsection algorithm based on transportation work is introduced. Then, models based on linear programming are overviewed before studying goal programming approach as a separated subsection. Finally, stochastic nature of the operation problem in open pit mine is considered in the last subsection.

4.2.2.1. Queuing Theory Approach

The first use of queuing theory in mining context is referred to (Koenigsberg, 1958) in which a room and pillar underground mine and a surface mine haulage system were modeled by using of queuing theory. The model presented by aforementioned author has a computational difficulties by the time fleet size increases. Afterwards, Barnes et al. (1978), Dallaire et al. (1978) and Carmichael (1987) applied queuing theory to solve truck – shovel problem in surface mines which has been followed by (Kappas and Yegulalp, 1991). Dallaire et al. (1978) defined mining operation as a system of several networks. After that, capacity of the transportation system and cycle time of each transportation unit (truck) is calculated by implementing mean value analysis method and based on recursive relations between waiting times. Major drawback of this model is production rate underestimation due to the approach under which it does not consider traveling time as infinite server queuing system. Second drawback of the model presented by Elbrond is that to implement the model in the operation a significant engineering judgment is needed. The model developed by Barnes et al. (1978) has the same drawback of Dallaire et al. (1978) model which does not consider traveling time as infinite server system, as well as disadvantage of using Erlang queuing model. Because Erlang distribution can approximate actual distribution with the coefficient variation of interval times less than one that can easily be violated in a real mining operation.

Kappas and Yegulalp (1991) offered a queuing theory model by considering truck – shovel system as a production network with regard to trucks as customer and shovels, crushers, waste dump, roads and maintenance service areas as servers. In their model, it is assumed that a mining system is a stochastic system with Markovian nature. Although it is stochastic, because of some parameters like service time distribution in different service areas, it is not Markovian (Newman et al., 2010).

Najor and Hagan (2006) applied queuing theory to analyze equipment (trucks and shovels) utilizations in the stochastic environment. Application of the model in an Australian case study shows that ignoring queue of trucks at hoppers (or plant capacity) causes overestimation of the production.

Later, Ercelebi and Bacetin (2009) represent a queuing theory model to allocate trucks in an open pit mine which can estimate some of mining systems performance parameters including number of trucks, throughput of the processing plant and waiting time. The model is presented below:

$$\left(\frac{N+M-1}{N}\right)=\frac{(N+M-1)!}{(M-1)N!} \tag{13}$$

$$P(n_1,n_2,\mathrm{k},n_M)=\frac{\mu_1^{N-n_1}}{\mu_2^{n_2}\mu_3^{n_3}\Lambda\mu_M^{n_M}}P(N,\mathrm{O},K,\mathrm{O})=\left(\frac{\mu_1}{\mu_1}\right)^{n_1}\left(\frac{\mu_1}{\mu_2}\right)^{n_2}\Lambda\left(\frac{\mu_1}{M_1}\right)^{n_M}P(N,\mathrm{O},K,\mathrm{O}) \tag{14}$$

$$\sum P(n_1,n_2,K,n_M)=1 \tag{15}$$

$$P(N,O,...,O)=\left[\sum\left(\frac{\mu_1}{\mu_1}\right)^{n_1}\left(\frac{\mu_1}{\mu_2}\right)^{n_2}\Lambda\left(\frac{\mu_1}{M_1}\right)^{n_M}\right]^{-1} \tag{16}$$

$$\sum_{i=1}^{M}n_i=N \tag{17}$$

$$\Pr[\textit{phase i is working}]=\eta_i=1-\sum P(n_1,n_2,K,n_{i-1},\mathrm{O},\mathrm{n}_{i+1},K,n_M) \tag{18}$$

$$L_{qi}=\sum n_iP(n_1,n_2,K,n_M)-\sum P(n_1,n_2,K,n_M) \tag{19}$$

$$W_i=W_{qi}+\frac{1}{\mu_i} \tag{20}$$

$$LCT = \sum_{i=1}^{M} (W_{qi} + \frac{1}{\mu_i}) \tag{21}$$

$$\text{Production} = \frac{\textit{time period of } \text{interest}}{\textit{average cycle time}} \times N \times \textit{truck capacity} \tag{22}$$

$$\text{Production} = \text{time period of interest} \times \eta_{shovel} \times \mu_{shovel} \times \textit{truck capacity} \tag{23}$$

$$C = \frac{C_1 + C_2 N}{\textit{unit production} \times \textit{truck capacity}} \tag{24}$$

Where:

N	is total number of trucks
M	is total number of service centers (herein: loaders, loaded haul roads, empty haul roads, dump sites)
n_i	is the number of trucks in ith service center
P	is the steady state probability (Eq.(16))
μ_i	is service rate at ith service center
η_i	computes the probability that service center ith is working (utilization) (Eq.(18))
L_{qi}	calculates the expected number of trucks in the queue at the ith service center (Eq.(19))
W_{qi}	is the expected time a truck spends at service center (= $L_{qi}/ \eta_i \mu_i$)
W_i	estimates the expected time that a truck spends in the ith service center Eq.(20)
LCT	is the average total cycle time for a truck to complete M service centers (Eq.(21))
C_1	is the cost per unit of shovel (including capital and operating costs)
C_2	is the cost per unit time of truck (including capital and operating costs)
C	is total cost for unit production

Average cycle time is sum of load time, dump time, queuing time at the shovel, queuing time at the dump, loaded haul time, and empty haul time. Eqs.(13), (14), (15), (16), and (17) show the procedure from which probability of each phase utilization is being account. Eq.(22) or (23) are implemented to find production per unit of time and Eq.(24) computes total cost per tonne of material extracted.

Their model has some disadvantages such as: they assumed all stochastic procedures in the operation are Markovian which is not true, the fleet is consisting of the same size (homogenous fleet), and truck cycle time is calculated based on locked-in allocation which does not care about the time a truck needs to reach the route.

To sum up, although queuing theory is a powerful approach, but by advances in the simulation, researchers prefer to use simulation as a tool to cover stochastic nature of the problems instead of queuing theory in the field of mine operation optimization.

4.2.2.2. Transportation Approach

Li (1990) says that an optimum material flow on a path should minimize total transportation work (Eq.(25)) with respect to Eqs.(26) and (27) those ensure the model will meet stripping ratio, Eq.(28) to meet grade requirement and Eq.(29) to ensure that number of trucks input in a loading or discharge point is equal to number of trucks come out of that point. Transportation work is defined as the distance material is transported multiply by the amount of the material. The transportation model was presented by Li (1990) for five shovels is as follow:

$$\min W = \sum_{i \in S_1} \sum_{j \in S_2 \cup S_3} X_{ij} (Z_1 + Z_2) \sum_{k=1}^{K_{ij}} f_{ij}^{(k)} D_{ij}^{(k)} + \sum_{i \in S_4} \sum_{j \in S_5} X_{ij} (Z_1 + Z_3) \sum_{k=1}^{K_{ij}} f_{ij}^{(k)} D_{ij}^{(k)} + \sum_{i \in S_2 \cup S_3 \cup S_5} \sum_{j \in S_1 \cup S_4} X_{ij} Z_1 \sum_{k=1}^{K_{ij}} f_{ij}^{(k)} D_{ij}^{(k)} \quad (25)$$

Subject to:

$$P_i / T \le \sum_{j \in S_2 \cup S_3} X_{ij} Z_2 \quad for\, i \in S_1 \quad (26)$$

$$P_i / T \le \sum_{j \in S_5} X_{ij} Z_3 \quad for\ i \in S_4 \quad (27)$$

$$\sum_{i \in S_1} \alpha_i^{(q)} \sum_{j \in S_2} X_{ij} = \alpha^{(q)} \sum_{i \in S_1} \sum_{j \in S_2} X_{ij} \quad for\ q = 1, 2, ..., Q \quad (28)$$

$$\sum_{i \in S_{.j}} X_{ij} = \sum_{k \in S_{j.}} X_{jk} \quad for\ j \in \bigcup_{i=1}^{5} S_i \quad (29)$$

Where:

S_1	is set of ore shovels
S_2	is set of ore discharge points
S_3	is set of stockpile points
S_4	is set of waste shovels
S_5	is set of waste disposing points
X_{ij}	is the truck flow over path from ith loading point to jth discharge point
K_{ij}	is total number of segments on path ij
$D_{ij}^{(k)}$	is the length of kth segment on ijth route
$f_{ij}^{(k)}$	is the road resistance factor of kth segment of ijth path
Z_1	is net truck weight
Z_2	is ore payload
Z_3	is waste payload
T	is planning period over which number of loading and dumping points do not change
P_i	is amount of material to be transported from ith loading point in T time
Q	is total number of ore quality indicator
$\alpha_i^{(q)}$	is ore quality of indicator q at ith loading point
$\alpha^{(q)}$	is required ore quality of indicator q at processing plant
$S_{.j}$	is set of all loading and discharging points which have path to jth discharge point

$S_{j.}$ is set of all loading and discharge points constitute feasible paths from j

The method implements aforementioned LP model to allocate optimal number of trucks to a route meeting its productivity rate. The model presented is based on five shovel fleet but author claims that the model can be implemented in a mine with higher number of loading point as well. It consider productivity of each shovel and also blending requirement. One major drawback of the model is that total model of operational plan including upper and lower stages are based on homogenous fleet. However, it will not guarantee optimality in real projects where the fleet is heterogeneous because it allocate trucks to each shovel based on assumption of the same capacity whereas they are not. Another major drawback is that the model does not consider truck breakdown as a major event that changes mine status. However, truck breakdown will not allow the operation to achieve the maximum production planned.

4.2.2.3. Linear Programming Approach

LP and specially MILP has been implemented in the open pit mine operation optimization more than any other approaches. The general LP model implemented in mine operation optimization was developed by Bonates (1992) who introduced an LP model to maximize shovel productivity (Eq.(30)) as follow:

$$\max Z = \sum_{i=1}^{n} P_i X_i + \sum_{j=1}^{m} Q_j X_j \tag{30}$$

Subject to:

$$\sum_{i=1}^{n} X_i \le CC \tag{31}$$

$$\sum_{i=1}^{n} \left[G_u - G_i \right] X_i \ge 0 \tag{32}$$

$$\sum_{i=1}^{n} \left[G_i - G_l \right] X_i \ge 0 \tag{33}$$

$$X_k < MAXP_k \quad for \;\; k = 1, 2, ..., n+m \tag{34}$$

$$X_k > MINP_k \quad for \; k = 1, 2, ..., n+m \tag{35}$$

$$\sum_{k=1}^{n+m} \left[X_k / B_k \right] \le TT \tag{36}$$

$$R_u \sum_{i=1}^{n} X_i - \sum_{j=1}^{m} X_j \ge 0 \tag{37}$$

$$R_l \sum_{i=1}^{n} X_i - \sum_{j=1}^{m} X_j \le 0 \tag{38}$$

Where:

i is index of shovels in ore

j is index of shovels in waste

n is total number of shovels in ore

m is total number of shovels in waste

k	is general shovel index
CC	is crusher capacity
X_i	is ore production per period of ith shovel
X_j	is waste production per period of jth shovel
P_i	is priority of ith shovel for production
Q_j	is priority of jth shovel for production
G_u	is material quality upper limit
G_l	is material quality lower limit
G_i	is material grade at ith shovel
$MAXP_k$	is maximum digging rate at kth shovel
$MINP_k$	is minimum production rate at kth shovel
B_k	is linear approximation for trucks working with kth shovel between $MINP_k$ and $MAXP_k$
TT	is total number of available trucks over the time horizon
R_l	is lower limit of SR
R_u	is upper limit of SR

Constraint (31) makes sure that total production of shovels working in ore area do not exceed maximum capacity of crusher. Eq.(32) and (33) guarantee that ore quality is within the prescribed limits. Constraints (34) and (35) ensure total production of each shovel over the time period will not deviate from minimum and maximum digging rate of the shovel. Eq.(36) ensures total number of truck is being used over the time horizon do not proceed total number of available trucks. Constraints (37) and (38) ensure stripping ratio requirement will be met.

The LP model was presented to be employed in small to medium size mines. The objective is to maximize the production of all shovels. The model consider required grade interval for feeding the plant. It also account for stripping ratio and relative priority of shovels specially ones working on ore faces. Nevertheless, it was assumed that shovels production will increase linearly by increasing the number of trucks. However, in heterogeneous fleet by adding trucks with different sizes to the available fleet production rate will increase nonlinearly up to its maximum production rate. Another major drawback of the model is that it is necessary to add stockpiling (re-handling) to the objective as well.

Gurgur et al (2011) proposed an LP model of operation optimization that helps to minimize deviation of the operation from the strategically set targets in short- and long-term schedules. To link operation plan to the strategic ones the model provides shovel assignment. Advantage of model is that it account for available trucks of fleet in each time period. Second advantage of the model is that it is a lifelong model which considers the mine as a multi period task. As a result, effects of current operations on the next ones are taken into account. There is a major disadvantage of the model presented by (Gurgur et al., 2011) that pushs it away from optimality. Costs and lost tons associated with the shovel movement during the operation is not considered. Another drawback of the model is using continuous variables in the discrete production operation, which provides the rates of material transported using various trucks. The only constraint relating the flow rate with the capacities of the trucks is the available fleet constraint, which though limits the total production transported by trucks with the maximum transportation possible, cannot provide exact measure of the number of truck trips required.

Ta et al. (2013) developed a mixed integer linear programming (MILP) model to allocate trucks of a fleet to different shovels based on probability of shovels' idle time. The probability of the ide time approximated by defining shovel as a server of the mine as a G/G/1 /y finite-source system. The objective of the model is to minimize total number of trucks. The model was implemented in a simulation mode of an oil sand mine. Results of the simulations show that in some cases idle probability of some shovels goes up to 40% to 60% illustrating that the model does not provide a reliable open-pit mine equipment allocation. Second drawback of the model is: Although it is claimed by authors that the model has the ability of being used in heterogeneous fleet, regarding to the simulation results, it does not offer a realistic combination of the trucks with different sizes available at the fleet.

Mena et al. (2013) defined a knapsack problem which tries to maximize cumulative truck fleet production by a fixed time horizon. Their main aim was to allocate available trucks to the route requesting for a truck. To do so, they used equipment availability function as a part of objective function coefficient. They multiplied productivity of truck on each route by the availability of the truck and tried to maximize the problem in this way. Then by implementing simulation procedure they solve the model for a period horizon of one week. Then compared results of their model with the result of general model without considering availability. The comparison showed decrease in the productivity of the fleet because the enhanced model was more accurate. Advantage of the model is that it uses each truck with its own availability and in this model there is no equipment with 100% availability. Major drawback of the model was that at the time a certain number of trucks fail or go out of performance for maintenance repair, then the system becomes infeasible and the optimizer is not able to find an optimal solution problem. Another disadvantage of the model is that only availability of the trucks is inputted in the optimization problem. However, priority in mining system is with bigger equipment and it is needed to add availability of all equipment who plays a role in the production procedure. Along with above cons, the blending requirement of the plant feed is not considered in the model as well.

The most resent model based on the LP has been presented by Chang et al. (2015). The model schedules trucks over a shift by implementing MILP with the objective of maximizing transportation revenue. Then a heuristic rule is implemented to solve the model. They also take into account transport priority. The model is based on homogenous truck fleet which is far from reality and cause non-optimality of the model results on a real system. The model does not consider stripping ratio requirement as well as ignoring stochastic nature of grade distribution. Plant capacity and feed head grade are ignored as well.

One of the major drawbacks of all models based on linear programming is that: to consider limitations of operation such as stripping ratio and required feed grade the models have to define an acceptable range. However, it pushes the operation far behind optimality especially if plant feed grade requirement changes.

4.2.2.4. Goal Programming Approach

The Goal Programming (GP) first introduced by Charnes and Cooper (1955) and (1961). In the simplest version of GP, the designer prepares some goals he or she wishes to achieve for each objective function. Then, the optimum solution is the set which minimize deviations from the goals has been set which means that it does not maximize or minimize an specific objective, it tries to find an specific goal value of those objectives, though (Caramia and Dell'Olmo, 2008). The general GP model for multi-objective problems (Eq.(39)) is as follow (Rao, 2009):

$$\min\left[\sum_{j=1}^{k}(d_j^{+}+d_j^{-})^{p}\right]^{1/p} \quad , \; p\geq 1 \tag{39}$$

Subject to:

$$g_j(X) \leq 0 \qquad for\ j = 1, 2, ..., m \tag{40}$$

$$f_j(X) + d_j^+ - d_j^- = b_j \quad for\ \ j = 1, 2, ..., k \tag{41}$$

$$d_j^+ \geq 0 \qquad for\ \ j = 1, 2, ..., k \tag{42}$$

$$d_j^- \geq 0 \qquad for\ \ j = 1, 2, ..., k \tag{43}$$

$$d_j^+ d_j^- = 0 \qquad for\ \ j = 1, 2, ..., \mathrm{k} \tag{44}$$

Where:

b_j	is the set of goals
$d_j^+\ and\ d_j^-$	are the underachievement and over achievement of the jth goal
p	is the value chosen by designer based on utility function

Eq. (40) shows general format of constraints. By algebraic summation of optimum results and deviations the goals will be achieved (Eq.(41)). Eq.(42), (43) and (44) are defining negative and positive deviation from each goal.

In the mining operation optimization there exist variety of goals to be achieved such as production maximization and maintenance of ore quality between the desired limits (Temeng et al., 1998), optimization of the processing plant utilization and minimization of trucks and shovels movement costs (Upadhyay and Askari-Nasab, 2015).

Temeng et al. (1998) formulated a model of open pit mine operation optimization based on GP which is presented below:

$$\min P_1 \sum_{i=1}^{n_s} d_i^- + P_2 \sum_{k=1}^{n_q} \sum_{j=1}^{n_c} (c_{ij}^+ + c_{kj}^-) \tag{45}$$

Subject to:

$$\sum_{j=1}^{n_d} x_{ij} + d_i^- = M_i \qquad for \quad i = 1, ..., n_s \tag{46}$$

$$\sum_{j=1}^{n_d} x_{ij} \geq B_i \qquad for \quad i = 1, ..., n_s \tag{47}$$

$$\sum_{i=1}^{n_s} x_{ij} \leq C_j \qquad for \quad i = 1, ..., n_d \tag{48}$$

$$\sum_{j=1}^{n_d} y_{ji} = \sum_{j=1}^{n_d} x_{ij} \qquad for \quad i = 1, ..., n_s \tag{49}$$

$$\sum_{i=1}^{n_s} x_{ij} = \sum_{i=1}^{n_s} y_{ji} \qquad for \quad i = 1, ..., n_d \tag{50}$$

$$\sum_{i=1}^{n_{os}} G_{ik} x_{ij} + c_{kj}^- - c_{kj}^+ = Q_{kj} \sum_{i=1}^{n_{os}} x_{ij} \qquad for \quad k = 1, ..., n_q \quad j = 1, ..., n_c \tag{51}$$

$$c_{kj}^{-} \le (Q_{kj} - L_{kj})\sum_{i=1}^{n_{os}} x_{ij} \qquad for \quad k = 1,\ldots,n_q \quad j = 1,\ldots,n_c \tag{52}$$

$$c_{kj}^{+} \le (Q_{kj} - U_{kj})\sum_{i=1}^{n_{os}} x_{ij} \qquad for \quad k = 1,\ldots,n_q \quad j = 1,\ldots,n_c \tag{53}$$

$$R_L \le \frac{\sum_{i=n_{os}+1}^{n_s} \sum_{j=n_c+1}^{n_d} x_{ij}}{\sum_{i=1}^{n_{os}} \sum_{j=1}^{n_c} x_{ij}} \le R_U \tag{54}$$

$$\sum_{i=1}^{n_s}\sum_{j=1}^{n_d} H_{ij} x_{ij} + \sum_{i=1}^{n_s}\sum_{j=1}^{n_d} D_j x_{ij} + \sum_{j=1}^{n_d}\sum_{i=1}^{n_s} R_{ji} y_{ji} + \sum_{j=1}^{n_d}\sum_{i=1}^{n_s} S_i y_{ji} \le N.T \tag{55}$$

$$d_i^{-}, x_{ij}, y_{ij}, c_{kj}^{+}, c_{kj}^{-} \ge 0 \tag{56}$$

Where:

P_1	is priority factor for production
P_2	is priority factor for grade control
d_i^{-}	is ith shovel production negative deviation variable
c_{kj}^{+} *and* c_{kj}^{-}	are positive and negative deviation from ore grade indicator k at jth crusher
n_s	is number of shovels
n_q	is number of quality identifiers
n_c	is number of the crushers
n_d	is total number of destinations
n_{os}	is number of shovels working at ore faces
x_{ij}	is the production to be assigned to the ijth path connecting ith shovel to jth discharge point in each shift
y_{ij}	is capacity of truck which is to be assigned from jth dumping point to ith shovel per shift
M_i	is the maximum production of ith shovel per shift
B_i	is the minimum production of ith shovel per shift
C_j	is the maximum available capacity of jth discharge point per shift
G_{ik}	is the average ore quality indicator k at ith shovel
Q_{kj}	is the target ore quality indicator k at jth crusher
L_{kj}	is the prescribed lower limit of ore quality indicator k at jth crusher
U_{kj}	is the prescribed upper limit of ore quality indicator k at jth crusher
R_L and R_U	are prescribed lower and upper bounds of required stripping ratio

H_{ij}	is the average travel time from ith shovel to jth discharge point
D_j	is the average dumping time at jth destination including spot time
R_{ji}	is the average travel time from jth discharge point to ith shovel
S_i	is the average loading time at ith shovel including spot time
N	is number of trucks
T	is weighted average truck payload

The model maximize shovel production and ensure ore grade requirement achieved as much as possible (Eq.(45)). Eqs.(46) and (47) ensures that total material transported from ith shovel cannot exceed shovel's digging rate and will not be less than its minimum digging rate. Eq.(48) makes sure that total material dumped in each discharge point cannot surpass its maximum capacity. Eqs.(49) and (50) ensure that number of trucks travels into a point is equal to number of trucks come out of the point. Eqs. (51), (52) and (53) guarantee ore quality requirement at plant. Eq.(54) conserves the production between required stripping ratio. Eq.(55) ensures that total production cannot exceed total truck capacity available. The main advantage of GP model developed by (Temeng et al., 1998) is that it optimizes two major goals of the open pit operation simultaneously without neglecting any of them. Besides covering the objective function drawbacks of previous models it covers another disadvantage of LP models which is defining upper and lower limits for the target grade of material are being sent to the plant. As it was introduced before, in LP models it is usual to control the grade by imposing it between upper and lower limit. Let us assume that objective is to maximize the production. Then truck assignment to the shovel closer to the crusher which results shorter truck cycle time will be higher. If the average grade at these closer faces are pretty close to one of the allowed grade boundaries, then whatever the dispatching algorithm is controlling the feed grade within the interval is difficult. As a result, existing of stockpile and subsequently re-handling cost associated with it is undeniable. However, the model has some disadvantages. It does not consider all the goals are supposed to be met in an open pit mine operation such as equipment movement costs, of which some of them are covered by Upadhyay and Askari-Nasab (2015). The model the mining operation as a multi-period operation which needs to meet strategic goals of the project. It does not consider stochastic nature of the grade of material are feeding to the plant as well. The most resent open pit operation optimization model based on GP can be found in (Upadhyay and Askari-Nasab, 2015) where the authors enhanced aforementioned model's objective with adding two new goals. The newly added goals are minimizing the deviation of calculated plant feed to desire feed and minimizing cost of both trucks and shovels operation, respectively.

4.2.2.5. Stochastic Approach

Ta et al. (2005) implemented a chance-constrained stochastic optimization to allocate trucks in an open pit mines as a part of upper stage in mine operational plan. They also used an updater to renew the model and parameters by the time shift or status of the mine changes. The presented model considers truck load and its cycle time as stochastic parameters. The decision variables in the model are number and types of trucks allocated to the shovels. Authors claim that stochastic model they presented can be solved by converting it to a quadratic deterministic model and implementation of mixed integer nonlinear programming techniques and solvers but it is time consuming. So the initial model was divided into two sub model. The sub models were solved to allocate discrete number of trucks to each loader. The main model is as follow:

$$\text{Minimize Truck Resource} = \sum_{s}\sum_{d}\sum_{g} K(g)X(s,d,g) \qquad (truck\ units) \tag{57}$$

Subject to:

$$\text{Prob}\left\{V_o + H\left[V_{Truck} - V_{Extraction}\right] \geq V_{Min}\right\} \geq \alpha \quad (58)$$

$$V_{Truck} = \sum_s \sum_d \sum_g \frac{60}{\tau_o(s,d,g)} L_o(\text{s,d,g})\, \text{X(s,d,g)} \qquad \text{(tonnes/ hr)} \quad (59)$$

$$\sum_d \sum_g \frac{60}{\overline{\tau}_o(s,d,g)} \overline{L}_o(\text{s,d,g})\, \text{X(s,d,g)} \leq C_{Shovel}(s) \qquad \text{(tonnes/ hr)} \quad (60)$$

$$\sum_s \sum_d \text{X(s,d,g)} \leq R(g) \quad (61)$$

$$X(s,d,g) \geq 0 \quad (62)$$

Eq.(58) ensures confidence level of the model is more than or equal to predefined level (α). Eq.(59) calculates total volume a truck can transport in a unit of time (hr). Eq.(60) aims to limit trucks at shovel based on the shovel capacity. Eqs.(61) and (62) limit number of trucks in use to the available trucks in the fleet. The first sub model which is a probabilistic chance-constrained model is as follow:

$$\text{Minimize Truck Resource}(1) = \sum_s \sum_d \sum_g K(g) X(s,d,g) \quad (63)$$

Subject to:

$$V_{Truck} = \sum_d \sum_g \frac{60}{\overline{\tau}_o(s,d,g)} \overline{L}_o(\text{s,d,g})\, \text{X(s,d,g)} \quad (64)$$

$$\text{Prob}\left\{V_o + H\left[V_{Truck} - V_{Extraction}\right] \geq V_{Min}\right\} \geq \alpha \quad (65)$$

$$V_{Truck} \leq C_{Shovel}(s) \quad (66)$$

$$V_{Truck} \geq mC_{Shovel}(s) \quad (67)$$

$$\sum_s \sum_d \text{X(s,d,g)} \leq R(g) \quad (68)$$

$$X(s,d,g) \geq 0 \quad (69)$$

First sub problem is almost the same as the general problem. Except for constraint (67) which maintains the solution from assignment of zero truck to shovels is the only difference. Also minimum ore throughput from the shovels is maintained. The model must be simplified as a nonlinear deterministic model and be solved by using of nonlinear techniques. The model provides a continuous amount for number of trucks which must be a discrete number. To do so, second sub problem as follow was presented:

$$\text{Minimize Truck Resource}(2) = \sum_s \sum_d \sum_g K(g) Y(s,d,g) \quad (70)$$

Subject to:

$$\sum_s \sum_d \sum_g K(g) Y(s,d,g) \geq \text{Truck Resource}(1) \quad (71)$$

$$\sum_d \sum_g \frac{60}{\overline{\tau}_o(s,d,g)} \overline{L}_o(\text{s,d,g})\, \text{Y(s,d,g)} \leq C_{Shovel}(s) \quad (72)$$

$$\sum_{d}\sum_{g}\frac{60}{\overline{\tau}_o(s,d,g)}\overline{L}_o(\mathrm{s,d,g})\,\mathrm{Y}(\mathrm{s,d,g}) \geq mC_{Shovel}(s) \tag{73}$$

$$\sum_{s}\sum_{d}\mathrm{Y}(\mathrm{s,d,g}) \leq R(g) \tag{74}$$

$$\begin{aligned} &i=1: Y^{(i)}(x,d,g) \geq 0 \\ &i=2,3,...: Y^{(i-1)}(x,d,g)-1 \leq Y^{(i)}(x,d,g) \leq Y^{(i-1)}(x,d,g)+1 \end{aligned} \tag{75}$$

$$Y(s,d,g) \geq 0 \tag{76}$$

Where:

s	is shovel type; d is type of discharge point
g	is truck type
K(g)	is cost coefficient of truck type g (for the truck type g with the smallest capacity K(g)=1 and for the rest it is calculated based on the smallest truck capacity. For example, in a fleet consisting of 240 ton and 320 ton capacity trucks K(240)=1 and K(320)=1.33)
X(s,d,g)	is number of truck type g assigned to shovel s and dump d (fractional or theoretical)
Y(s,d,g)	is number of truck type g assigned to shovel s and dump d (discrete)
L_o(s,d,g)	is the truck type g capacity working on route connecting shovel s to dump d
$\tau_o(s,d,g)$	is ore truck cycle time (minute)
V_o	is initial surge volume
V_{Truck} & $V_{Extraction}$	are ore production rate that goes in and out of surge per hour
C_{Shovel}(s)	is capacity of shovel s (tonnes/hr)
D_w	is amount of waste needs to be handled per hour
R(g)	is tha available number of type g truck
H	is number of hours in each period of concern
m	is used to specify the minimum amount of ore to be mined by the working shovels $(0 \leq m \leq 1 \quad ton/hr)$

Constraint (71) defines the lower bound of the objective function. Eqs.(72), (73) and (74) are the same as (66), (67) and (68) in the first sub model with the exception of number of trucks being discrete. Eq.(75) helps to move to the next time period realistically.

The objective function value of the first sub problem helps to define a lower limit for the objective function value of the second sub problem. To move to the next time horizon, constraint (75) is defined to ensure gradual transition of allocation from the current period of time. Although the model provides a good conceptual background for stochastic optimization approach to solve the multi-stage optimization problem, it takes into account the probabilistic nature of truck travel times only. Also the model formulation is very much specific to a mining case and cannot be generalized to other mining systems.

4.2.3. Real-time Dispatching

Real-time decision making on the destination of trucks in a mining operation was first used in early 1960s with implementation of radio communication tools to link between dispatcher and trucks operators in a fixed truck allocation mine. However, based on utilization of computer real-time fleet management in mining operation systems are divided into three major categories: locked-in or fixed allocation, semi-automated and fully automated systems. In the locked-in method there is no effort for dispatching the transportation units. Semi-automated dispatching which has been developing by increasing the computer usage in mining sector is divided into two different classes: passive and active. In the earlier class computer just displays current mine operation information and does not have any role on decision making procedure. However, in the later class computers use current mine status information as inputs and process them based on predefined models and suggest list of assignments to the dispatcher and leave the decision to be make for humans. In the automated dispatching data of the current mine status and condition and position of the equipment within the operation are collected into a main computer server and it sends the assignment to trucks after solving some heuristics or mathematical programs. What we review here is the last class where computers receive data, process them and assign the trucks to the next destinations.

There are two major approaches governing dispatching procedure: Assignment problem approach and transportation problem approach from those the first one is a subcategory of the transportation problem in the operations research context.

4.2.3.1. Algorithms based on Assignment Problem

A general assignment problem is a balanced transportation problem in which all demands and sources have capacity of one unit. In each assignment problem there is a cost matrix that consists of the costs associated with assigning each supply to each demand. The objective of each assignment model is to minimize cost of allocating supplies to demands. In mining context assignment problem has been used mostly to dispatch trucks as supply to shovels or dumping points as demand. The objective in mining truck dispatching based on assignment model is to minimize shovel idle time, truck waiting time, inter-truck time, and so on. In comparison with the other approach, almost all real-time truck dispatching models in both industrial and academic research area are based on assignment problem.

Hauck (1973) implemented a sequence of assignment problem to dispatch the trucks need destination. The objective function of his model is to minimize total idle time of shovels which minimize lost ton of the operation subsequently. The sub problem that is solved in each assignment request is as follow:

$$\min \sum_i \sum_j W_{ij}(t_k) X_{ij}(t_k) \tag{77}$$

Subject to:

$$\sum_j X_{ij}(t_k) \le 1 \qquad for \quad i = 1,...,m \tag{78}$$

$$\sum_i X_{ij}(t_k) = 1 \qquad for \quad j = 1,...,n \tag{79}$$

$$X_{ij}(t_k) \in D_k \tag{80}$$

Where:

$$X_{ij}(t_k) = \begin{cases} 1 \ \ if\ truck\ i\ is\ loaded\ by\ shovel\ \ j\ and\ departed\ at\ time\ t_k \\ 0 \ \ otherwise \end{cases}$$

$W_{ij}(t_k)$ is lost ton due to idle time caused by assigning ith truck to jth shovel at time t_k; D_k is representative of a situation will be explained later on.

The model tries to minimize lost ton due to idle periods. Constraint (78) guarantees that each truck is assigned to at most one shovel and constraint (79) ensures that each shovel is assigned exactly one truck. Eq.(80) ensures that a truck to be assigned meets all requirements

Two main disadvantages of the dispatching part of Hauck's model are: firstly, assignment is not as accurate as possible because the decisions are made now will not be recomputed unless number of available trucks change. As a result, the assignment decision is not up to the minute. Secondly, the model is a sub model of a large model which uses result of last stage of the total model above dispatching. The last stage above dispatching decision making model itself is an optimum result of its previous sub model. So, the dispatching model is not able to use DP to solve assignment problem because it does not have possibility of using all possible solutions of previous stages and only uses the optimum solution of those stages.

Soumis et al. (1989) developed an assignment model that consider 10-15 forthcoming trucks and their effects on current assignment. The objective of the model is to minimize sum of squared deviation of estimated waiting time of trucks from the planned waiting time. The model finds 10-15 next trucks based on average travel time, discharge time, and loading time and shovel inter-truck waiting time. After assignment of current truck, all 10-15 trucks which had been used for the assignment are erased. The procedure will repeat when next assignment is requested. The main advantage of the method is that it considers effects of forthcoming trucks on the current assignment. However, assumption of homogeneous fleet is a drawback of the model. Assuming homogenous fleet of trucks in a multistage model of truck dispatching cause a considerable deviation from the reality. The reason behind such a deviation is to use homogenous fleet in the lower stage (real-time dispatching level) it is necessary to model upper stage (operation optimization level) considering homogenous truck fleet as well. Consequently, optimized production rate resulted from upper stage is far from the one in reality because in reality trucks in the fleet are from different size in most of the fleets (Alarie and Gamache, 2002). But, based on Lizotte et al. (1987) to implement a multistage dispatching algorithm for an open pit mine operation the production plan should represents the mine as close to reality as possible to have an optimal plan. The second major drawback of the model which happens in almost all of the dispatching models based on assignment problem is that, although they account for upcoming trucks for current assignment request, effects of current assignment on forthcoming trucks are not accounted for.

Ercelebi and Bascetin (2009) after providing optimum truck allocation by using of queuing theory implemented assignment problem approach based on the model was presented by White and Olson (1986) to dispatch trucks requesting a new destination. Lizotte et al. (1987) in their semi-automated model first provided a simulation model of the case study where by the time a truck needs assignment, three dispatching heuristic based on assignment problem are solved and the results of the simulation are presented on the board in a table beside the result of fixed allocation method and leave the decision for the dispatcher.

All dispatching heuristic rules in the literature that are grounded on maximize truck utilization in which a truck is sent to the shovel where it is supposed to be loaded first follow assignment problem. Although such an objective improves production in comparison with locked-in non-dispatching operation, they have some drawback including: ore quality and stripping ratio are not taken into account. Another major drawback of these types of algorithms is that it tries to send trucks to the shorter routes and as a result shovels sitting on further mining faces will idle more (Tu and Hucka, 1985; Lizotte et al., 1987). All dispatching rules in the literature based on maximum utilization of the shovels in which truck is sent to the shovel that is supposed to idle longer by the time truck reaches the face are following assignment problem as well.

To sum up, although implementing assignment problem provides fast solution to for real-time truck dispatching in mining operations, it has two major drawbacks arising from the nature of the assignment problem: The main drawbacks of algorithms based on assignment problem is that at each time just one truck is assigned to each shovel even if a shovel is far behind its production target and needs more than one truck. As the second drawback it can be say that despite claims of some authors, it is not able to consider effects of forthcoming trucks.

4.2.3.2. Algorithms based on Transportation Problem

A transportation problem in the optimization context is described as follow (Winston, 2003):

1- A set of supply points (m);

2- A set of demand points (n);

3- Cost associated with transporting material from the supply point i to the demand point j.

Let x_{ij} is number of units shipped from the supply point i to the demand point j, then general formulation of the transportation problem is:

$$\max \text{or} \min \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij} \tag{81}$$

Subject to:

$$\sum_{j=1}^{m} x_{ij} \le s_i \qquad for \quad i=1,...,n \qquad \text{Supply constraints} \tag{82}$$

$$\sum_{i=1}^{n} x_{ij} \ge d_j \qquad for \quad j=1,...,m \qquad \text{Demand constraints} \tag{83}$$

$$x_{ij} \ge 0 \qquad for \quad i=1,...,n \quad j=1,...,m \tag{84}$$

To have a feasible solution, each transportation model must be constrained as:

$$\sum_{i=1}^{m} s_i \ge \sum_{j=1}^{n} d_j \tag{85}$$

The model tries to minimize total costs of the decision to be made (Eq.(81)). Constraint (82) makes sure that total material sent to different sink points cannot exceed ith source capacity. Constraint (83) ensure that jth sink will meet its demand. Constraint (84) limits the material to be handled to non-negativity. The most reliable algorithm of the real-time truck dispatching in open pit mine is the model was developed based on transportation problem by Temeng et al. (1997). The procedure of truck dispatching by using of Temeng et al. (1997) transportation algorithm is as follow:

Firstly a needy shovel is defined as a shovel uses a route that up until now has a cumulative production behind its production target. Or in the other word, a non-needy shovel is a shovel that cumulative production of all routes ending to it are above or equal to the target.

To find the needy shovels we first calculate current mean of tonnage ratio by using of Eq.(86):

$$R = \frac{1}{nm}\sum_{i=1}^{n}\sum_{j=1}^{m} R_{ij} \tag{86}$$

Where:

$R_{ij} = x_{ij} / T_{ij}$; x_{ij} is current cumulative tonnage on path ij; T_{ij} tonnage be assigned to the path ij which links ith shovel to jth dump.

Set current mean as the target ratio of each route. Then for each route define d_{ij} (Eq.(87)) as deviation of the route ij from the target production:

$$d_{ij} = R_{ij} - R \tag{87}$$

Now, a needy shovel is a shovel with $d_{ij} < 0$ (negative deviation).

Secondly, number of trucks each needy shovel requires is determined. To do so, at first Eq.(88) or (89) are being implemented to calculate y_{ij} as the tonnage behind the target of the route ij:

$$\frac{x_{ij} + y_{ij}}{T_{ij}} = R \tag{88}$$

$$y_{ij} = RT_{ij} - x_{ij} \tag{89}$$

Then, a basic truck capacity (small, large, or an average of them) is chosen based on some statistical analysis. Before, the demand of each route is found by using of Eq.(90):

$$M_{ij} = \left\lceil \frac{y_{ij}}{C_1} \right\rceil \tag{90}$$

Where:

M_{ij} is the demand of each route ij; C_1 is the larger truck capacity in the fleet consisting of two different truck sizes; $\lceil x \rceil$ is the smallest integer ≥ x.

Finally, the demand for each shovel will be Eq.(91):

$$D_i = \sum_{j=1}^{m} M_{ij} \qquad i = 1,...,n \tag{91}$$

And if the demand of ith shovel is D_i, then Eq. (92) is being used to calculate total demand of the operation at current status:

$$D = \sum_{i=1}^{n} D_i \tag{92}$$

In which D must be less than or equal to number of trucks available for the assignment and if it is not, a cut-off value for required tonnage should be used that selects those shovels as needy ones with relatively higher negative tonnage.

Finally Eq.(93) presents the model to assign trucks that tries to minimize total cumulative waiting time associated with the assignment:

$$\min \sum_{k=1}^{l} \sum_{i=1}^{n} W_{ik} X_{ik} \tag{93}$$

Subject to:

$$\sum_{i=1}^{n} X_{ik} \le S_k \qquad for \quad k = 1,...,l \tag{94}$$

$$\sum_{k=1}^{l} X_{ik} \geq D_i \qquad for \quad i=1,...,n \tag{95}$$

$$X_{ik} \geq 0 \tag{96}$$

Where:

W_{ik} $= L_i(N_i + E_i) - (t_k + d_j + e_j + r_{ij})$ is waiting time associated with assigning truck k to shovel i

X_{ik} is the decision on assigning truck k to ith shovel

S_k is supply of truck k

D_i is the demand of ith shovel

L_i is the mean loading time of ith shovel

N_i is the number of trucks at ith shovel

E_i is the number of trucks en route to ith shovel

t_k is the expected travel time of truck k to reach discharge point

d_j is the expected waiting time of truck at discharge point j

c_j is the average dumping time of truck at discharge point j

r_{ij} is average empty travel time from discharge point j to ith loader

Eq.(94) ensures that total number of trucks assigned cannot exceed number of available trucks. Eq.(95) makes sure that trucks are sent to the ith shovel will cover its lost ton as much as possible. And, Eq.(96) ensures that number of type k trucks assigned to ith shovel is non-negative. The model assumes heterogeneous truck fleet, as a result it will be as close to reality as the upper stage model is. It also considers the situation a shovel is far behind its target production and needs to be assigned more than one truck. In such a situation the model easily assign more than a single truck to those needy shovel further behind the schedule without any limitation occurs by implementing assignment model. However, there are two major drawbacks with the model. The first major drawback is that mean of production rate for all routes is the basis for calculating the deviation of routes. But based on upper stage plan, sometimes it is required to extract much more of some specific materials that makes production rate of the routes of transporting those material be maximized and for some other be as less as possible. Then during the assignment it will send more trucks to those with higher negative deviation. The second major drawback is in transportation problem cost of transporting unit of material is constant and independent of supplier centers. Whereas, each truck waiting time at shovel or crusher is depending on the trucks previously assigned especially in over-truck systems. Also the waiting time accounting for in transportation method is based on trucks currently at destination or en route to the destination and there is no way to account for the waiting time will be caused by trucks will be assigned in the future but will reaches the destination earlier (Alarie and Gamache, 2002).

4.2.4. Single Stage Approach

In academic manners, one of the first algorithms introduced to solve truck allocation and dispatching problem in open pit mines is a single stage algorithm presented by Hauck (1973). The main feature of presented algorithm is combination of operation plan and real-time scheduling in a single model. The model is based on solving a sequence of assignment problem by using of DP. The model considers stripping ratio, blending objectives, capacity of the plant and stockpile. Objective of the model is to maximize the production by minimizing the lost ton caused by shovels idle time:

$$\min \sum_{j}\sum_{i}\sum_{q(j)=1}^{Q(j)} W_{ij}(t_{ij}(p(i),q(j)))X_{ij}(t_{ij}(p(i),q(j))) \tag{97}$$

Subject to:

$$r_L \le \frac{\sum_{l=1}^{k}\sum_{i}\sum_{j\in J_1} C_i X_{ij}(t_l)+a}{\sum_{l=1}^{k}\sum_{i}\sum_{j\in J_2} C_i X_{ij}(t_l)+b} \le r_U \qquad \text{for each } k \tag{98}$$

$$\sum_{l=1}^{k}\sum_{i}\sum_{j\in J_2} C_i X_{ij}(t_l)+(V(t_0)-V(t_k)) \le R_U t_k \qquad \text{for each } k \tag{99}$$

$$\sum_{l=1}^{k}\sum_{i}\sum_{j\in J_3} C_i X_{ij}(t_l)+(\sum_{l=1}^{k}\sum_{i}\sum_{j\in J_2} C_i X_{ij}(t_l)+(V(t_0)-V(t_k))) \ge R_L t_k \qquad \text{for each } k \tag{100}$$

$$\sum_{i}\sum_{j\in J_3} C_i X_{ij}(t_k) \le V(t_k) \qquad \text{for each } k \tag{101}$$

Where:

m	is number of available trucks
n	is number of shovels
C_i	is average haulage capacity of truck i
J_1	is the set of shovels j working at waste
J_2	is the set of shovels j working at ore mining faces
J_3	is the set of shovels j working at stockpile
t_k	is the time a shovel has just loaded a truck (assuming discrete points in time to keep track of the process)
Q(j)	is total number of loads completed by jth shovel in T working cycle
p(i)	is pth load of truck i
q(j)	is qth load of shovel j
$t_{ij}(p(i),q(j))$	is the earliest time pth load of truck i which is qth load of shovel j is loaded by shovel j on truck i

$$W_{ij}(t_{ij}(p(i),q(j))) = \begin{cases} E_j \Gamma_{ij}(t_{ij}(p(i),q(j))) & j \in J_{12} \\ 0 & j \in J_1 \end{cases}$$

E_j	is the loading rate of jth shovel (ton/time)
$\Gamma_{ij}(t_{ij}(p(i),q(j)))$	is the idle time incurred by jth shovel when it loads its q(j) load as truck's p(i) load into the truck
r_L and r_U	are the lower and upper limits of SR
b	is a suitable quantity of ore

$a = b(r_L + r_U)/2$

R_L *and* R_U	are minimum and maximum processing plant rate
$V(t_o)$ and $V(t_k)$	are stockpile inventory at the beginning of the cycle and at time t_k

For each k^{th} decision an assignment problem is solved as a sub problem by implementing DP which has been presented in Eq.(77) to Eq.(80).

Eq.(98) ensures meeting SR requirement; Eqs.(99) and (100) guarantee that processing plant is always being fed; Eq.(101) ensures that total material handling at stockpile cannot exceed the amount of current stockpile inventory. D_k is assignment domain satisfying Constraints (99) to (101) also are the criteria defined for assignment in D_k domain. The algorithm presents optimal combinatorial intractable assignment procedure. Although it is a complex algorithm containing all limitation satisfaction criteria, it runs fast. However, assuming the problem as a completely deterministic procedure shows that stochastic properties of truck waiting time is ignored. Meeting all the production requirements is not the goal of the operation for each assignment and if they can be satisfied in a longer period of the time, their short term violation is acceptable. As previously be mentioned DP tries to find optimal solution from all of the feasible solutions of previous sub problems not from the best solution of them.

4.2.5. Some Other Efforts

Krause and Musingwini (2007) used machine repair analogy to analyze and choose truck fleet size for an open pit mine. They chose Arena for the simulation part "because it can be programmed with any number of probability distribution fitted to an unlimited number of cycle variables and is therefore a very flexible model for use in analyzing several variables in shovel-truck analysis".
The analogy is as follow: "The Machine Repair Model equivalents are shown in parenthesis. A truck is sent for loading (repair) every cycle with the number of shovels or shovel loading sides or number of tipping bins (repair bays) being equal to R and the inter-arrival and service times both assumed to have an exponential distribution. Therefore, a shovel-truck system can be described as M/M/R/GD/K/K, where the first M is truck arrival rate, the second M is loader service rate, R is the number of shovels or shovel loading sides that are loading K trucks drawn from a population of size K, whereby the loading follows some general queue discipline, GD".

He et al. (2010) implement GA to optimize truck dispatching problem in open pit mines. They tried to find a route and assign upcoming truck to it based on minimized transportation and maintenance costs. In that model it has been assumed that velocity of trucks in both loaded and empty condition are the same that is a drawback for their model. Although, their major focus was on minimizing the costs, by assuming same velocity for both loaded and unloaded trucks they underestimated costs. Another major drawback which is similar to almost all other models is assignment of trucks to routes not to shovel-destination. They claimed that truck maintenance cost get higher with the age of the truck by a constant coefficient, whereas Topal and Ramazan (2012) revealed that maintenance cost behaves in a fluctuated manner during its life and by each main repair the equipment's maintenance cost will decrease dramatically.

5. Limitations of Current Algorithms and Future Research Directions

5.1. Linking between strategic level and operational level plans

Many researchers and companies have been work on open pit mine operational planning, there are still many restrictions in the algorithms and models, though. The main objective in mining projects is to maximize the net present value (NPV). To achieve the main goal of the expenditure the operational plan has to be tied to both short-term and long-term plans. But with the current models of operation optimization there is no guarantee of meeting the main goal. Short-term production schedule which is the closest part of strategic planning to the operation planning provides destination of material from mining cuts, but in reality it will not be followed up to the minute.

5.2. Accounting for Uncertainty

Most of the models for operation optimization are deterministic and also current simulation models do not cover all the mine life. However, the nature of mining operation is stochastic and it is a multi-period task in which each period effects on later ones up until end of the mine life.

Beside the stochastic operation, material quality in each mining face is stochastic as well. But most of the models assume constant average grade for each mining face which causes lack of optimality.

5.3. Modeling Close to Reality

Although most mines are using heterogeneous trucks, mixed fleet is ignored in most of the dispatching models.

All models developed for mine operational plan optimization have been validated by using of a simulation model of an actual mine. In almost all the simulation models presented for validation of the models modeling the processing plant and hoppers have been ignored. To evaluate the models it is suggested that a simulation model of a complete open pit mine operation be used to be as close as possible to the reality.

5.4. Dynamic Best Path Determination

In small mines with a limited number of route segments and small fleet a fixed shortest path between all pair of loaders and destination is sufficient. However, for very large open pit mines there exist a vast network of haul roads and a large fleet of which trucks travel in the operation area. A large fleet of trucks usually consists of variety of truck types with different speed limits and averages which causes traffic mass on some route segments. Consequently, it will cause lost production. To fix this problem it is shortest paths can be determined dynamically. In the other word, by keeping track of trucks working in the system determine the shortest path between the current location of the truck and its next destination based on the time it will take to reach the destination regarding current traffic jam on approaching route segments.

5.5. Real-time Dispatching based on Transshipment Problem

In dispatching procedure it is recommended to implement transshipment problem instead of transportation or assignment approaches. In a transshipment problem in addition to supplier and demand points there exist transshipment points through which material can be transported from suppliers to demand points. In mining system stockpiles can be assumed as transshipment points.

6. Conclusion

Open pit mine operational plan algorithms and models first have been divided into two major classes of industrial and academic algorithms and then have been reviewed in this paper. The planning problem has been broken down to three major sub problems (1- finding the shortest paths, 2- operation optimization, and 3- real-time truck dispatching). Then for each sub problem existing algorithms have been reviewed. Limitation of current models introduced and suggestions for the future works presented.

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An Application of Mathematical Programming to Determine the Best Height of Draw in Block-Cave Sequence Optimization

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Abstract

Planning block-caving operations poses complexities in different areas such as safety, environment, ground control, and production scheduling. Production schedules that provide optimal operating strategies while meeting technical constraints are an inseparable part of mining operations. Applications of mathematical programming in mine planning have proven very effective in supporting decisions on sequencing the extraction of materials in mines. The objective of this paper is to develop a practical optimization framework for production scheduling of block-caving operations. A mixed-integer linear programming (MILP) formulation is developed, implemented and verified in the TOMLAB/CPLEX environment. In this formulation, the slices within each draw column are aggregated into selective units using a hierarchical clustering algorithm and the mining reserve is computed as a result of the optimal production schedule for each advancement direction. This paper presents a model application of a production schedule for 102 drawpoints with 3,457 slices over 14 periods. The results show in order to obtain the maximum net present value, only 88% of the reserve is extracted. Also, the solving time for the presented method is 78 times faster than method without slice aggregation.

1. Introduction

Production scheduling of any mining system has an enormous effect on the operation's economics. A production schedule must provide a mining sequence that takes into account the physical and technical constraints and, to the extent possible, meets the demanded quantities of each raw ore type at each time period throughout the mine life. As the mining industry is faced with more marginal resources, it is becoming essential to generate production schedules which will provide optimal operating strategies while meeting technical and environmental constraints.

Most of the common production scheduling methods in the industry rely only on manual planning methods or computer software based on heuristic algorithms. These methods cannot guarantee the optimal solution. They lead to mine schedules that are not the optimal global solution (Pourrahimian et al., 2012a). On the other hand, the height of draw (HOD) is determined before

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production scheduling without considering the advancement direction. Improvements in computing power and scheduling algorithms over the past years have allowed planning engineers to develop models to schedule more complex mining systems (Alford et al., 2007; Caccetta, 2007). Consequently, it is now possible to formulate a mixed-integer linear programming (MILP) scheduling model that captures the essential components of a caving mine to generate a robust, practical, near-optimal schedule. The caving industry is now moving towards the next generation of caving geometries and scenarios: super caves (Chitombo, 2010). This requires a new approach to looking at scheduling block-cave operations.

The objective of this study is to develop, implement, and verify a theoretical optimization framework based on a MILP model for block-cave long-term production scheduling. The objective of the theoretical framework is to maximize the net present value (NPV) of the mining operation and determine the best height of draw (BHOD), while the mine planner has control over the planning parameters. The planning parameters considered in this study are: (i) mining capacity, (ii) draw rate, (iii) mining precedence, (iv) maximum number of active drawpoints, (v) number of new drawpoints to be opened in each period, (vi) continuous mining, and (vii) reserve. The production scheduler defines the opening and closing time of each drawpoint, the draw rate from each drawpoint, the number of new drawpoints that need to be constructed, the sequence of extraction from the drawpoints, and the BHOD for each draw column.

The resulting formulation and methodology generate a practical, long-term block-cave schedule in a reasonable CPU time and compute the mining reserve based on the cave advancement direction as a result of the optimal production schedule.

The following general workflow for a block-cave operation is proposed in this research:

1. The slices within each draw column are aggregated into selective units using a modified hierarchical clustering algorithm developed based on an algorithm presented by Tabesh and Askari-Nasab (2011). Aggregation is necessary to reduce the number of variables, especially binary variables in the MILP formulation, to make it tractable and generate near-optimal realistic schedules in a reasonable CPU time.
2. The optimal life-of-mine multi-period schedule is generated for the clustered slices.

The optimization formulation is implemented in the TOMLAB/CPLEX (Holmstrom, 2011) environment. A scheduling case study with real mine data is carried out over 14 periods to verify the MILP model.

Pourrahimian (2013) used a multi-step method to overcome the size problem of the mathematical programming model. In his method, the problem is first solved at the cluster level. At the cluster level, the draw columns are aggregated into practical scheduling units using a hierarchical clustering algorithm. Then, the result of the cluster-level formulation is used to reduce the number of variables in the drawpoint-level formulation. Finally, using the result of the drawpoint-level formulation, the problem is solved at the drawpoint-and-slice level. The combination of the method presented here and the multi-step approch (Pourrahimian, 2013) can solve large-scale problems in reasonable CPU time. The main contributions of the paper are (i) proposition of using a hierarchcal clustering algorithm to aggregate slices within each draw column into selective units, (ii) introduction of the concept of multi-directional clustering in which the result of the drawpoints aggregation (horizontal clustering) and the slices aggegation (vertical clustering) are used to production scheduling of block-caving operations, and (iii) computation of the mining reserve as a result of the optimal production schedule for each advancement direction.

The rest of the paper is organised as follows: the section on summary of literature review summarizes the literature on the block-cave production scheduling problem. Then the problem definition, methodology and assumptions are discussed. Afterwards, the problem's MILP formulation is explained and a case study about implementing the MILP model is presented. Finally, the last section presents the conclusions followed by a reference list.

2. Summary of literature review

In spite of the difficulties associated with applying mathematical programming to production scheduling in underground mines, the authors have attempted to develop methodologies to optimize production schedules. These difficulties could be due to the complicated nature of underground mining (Kuchta et al., 2004; Topal, 2008). On the other hand, there is a wide range of underground mining strategies that makes it difficult to develop a general framework for optimizing production scheduling in underground mines (Alford et al., 2007). Newman et al. (2010) presented a comprehensive review of operations research in mine planning. They summarized authors' attempts to use different methods to develop methodologies for optimizing production scheduling in underground and surface mines using different methods.

The manual draw charts were used to avoid early dilution entry at the beginning of block-caving (Rubio, 2006). Over time, different methods and objective functions have been used to present a good production schedule and optimized outline for block caving. Chanda (1990) implemented an algorithm to write daily orders and developed the interface between mathematical programming and simulation by integrating the two into a short-term planning system for a continuous block-cave. The objective function was defined to minimize the fluctuation in the average grade drawn between shifts. The production schedule given by the integer program was used as input to a simulation model that considered constraints such as production capacity. Winkler and Griffin (1998) described a production-scheduling model to determine the amount of ore to mine in each period from each production block. They used linear programming to solve a corresponding single-period model, and simulation to fix the current period's decisions and optimize over the successive period. Song (1989) also attempted to account for material movement within the panel by using simulation with mathematical programming. He used simulation to determine the effect of undercut parameters, drawpoint spacing, caving probability, and drift stability on production. A MILP formulation was then developed using regression equations for the restrictions revealed within the simulation study. Guest et al. (2000) applied mathematical programming to long-term scheduling in block-caving. In this case, the objective function was explicitly defined to maximize draw-control behavior. Rubio (2002) developed a methodology that would enable mine planners to compute production schedules in block-cave mining. He proposed new production process integration and formulated two main planning concepts as potential goals to optimize the long-term planning process, thereby maximizing the NPV and mine life. Rahal et al. (2003) described a mixed-integer goal program. The model had the objective of minimizing the deviation from the ideal draw. This algorithm assumes that the optimal draw strategy is known. The authors developed life-of-mine draw profiles for notional scenarios and showed that by using the results from their integer program, they greatly reduced deviation from ideal drawpoint depletion rates while adhering to a production target. Diering (2004) presented a non-linear optimization method to minimize the deviation between a current draw profile and the target defined by the mine planner. He emphasized that this algorithm could also be used to link the short-term plan with the long-term plan. The long-term plan is represented by a set of surfaces that are used as a target to be achieved based on the current extraction profile when running the short-term plans. Rubio and Diering (2004) described the application of mathematical programming to formulate optimization problems in block-cave production planning. They formulated two main planning strategies: maximization of NPV and maximization of mine life. They used the operational constraints presented by Rubio (2002). Weintraub et al. (2008) developed and successfully used MIP models for El Teniente, a large Chilean block-caving mine. They used *a priori* and *a posteriori* aggregation procedures to reduce the model size in their model. Parkinson (2012) developed three integer programming models: Basic, Malkin, and 2Cone. All of the models share three basic constraints.

The start-once constraint ensures that each drawpoint is opened once and only once. The global-capacity constraint ensures that the number of active drawpoints does not exceedthe downstream-processing capacity. The last constraint, that the opened drawpoints must form a single, contiguous group, or cave, is the source of the model variations. Pourrahimian (2013) presented a theoretical optimization framework based on a MILP model for block-cave long-term production scheduling. He introduced three MILP formulations for three levels of problem resolution: (i) cluster level, (ii) drawpoint level, and (iii) drawpoint-and-slice level. These formulations can be used in two ways: (i) as a single-step method in which each of the formulations is used independently; (ii) as a multi-step method in which the solution of each step is used to reduce the number of variables in the next level and consequently to generate a practical block-cave schedule in a reasonable amount of CPU runtime for large-scale problems.

Although simulation and heuristics are able to handle non-linear relationships and effects as a part of the scheduling procedure, they cannot guarantee the optimal solution. Applying mathematical programming models such as linear programming (LP) and MILP with exact solution methods for optimization has proved to be robust. Solving these models with exact solution methods, results in solutions within known limits of optimality. As the solution gets closer to optimality, production schedules generate higher NPV than those obtained from heuristic optimization methods. The literature has shown that both surface and underground mining systems can adapt to formulations as a set of linear constraints. This has resulted in extensive research on the application of mathematical programming models to the long-term production planning problem.

The inherent difficulty in applying these models to the long-term production planning problem is that they result in large-scale optimization problems containing many binary and continuous variables. These are difficult to solve with the current available computing software and hardware, and may require lengthy solution times. On the other hand, defining the draw height of each drawpoint before optimization, and using this height for optimization without considering the advancement direction, lead to mine schedules that are not the optimal global solution. These limitations can affect the viability as well as other aspects of mining projects, emphasizing the need for optimization tools that take into consideration these deficiencies.

This paper will introduce a MILP mine-scheduling framework for block-caving in which solving a large-scale problem in a reasonable CPU time and optimal mining reserve based on advancement direction will be addressed to generate a near-optimal production schedule with higher NPV.

3. Problem definition, methodology, and assumption

The production schedule of a block-cave mine is subject to a variety of physical and economic constraints. The production schedule defines the amount of the material to be mined from the drawpoints in every period of production, the opening and closing time of each drawpoint, the draw rate from each drawpoint, the number of new drawpoints that need to be constructed, the sequence of extraction from the drawpoints to support a given production target, and the best height of draw to achieve a given planning objective.

Several assumptions are used in the proposed MILP formulation. The ore-body is represented by a geological block model. The column of rock above each drawpoint, which is referred as a draw column, is vertical. Each draw column is divided into slices that match the vertical spacing of the geological block model. Numerical data are used to represent each slice's ore-body attributes, such as tonnage, density, grade of elements, elevation, percentage of dilution, and economic data. These attributes vary by slice throughout the height of the draw column (Fig 1). It is assumed that the physical layout of the production level is offset herringbone (Brown, 2003). There is selective mining, meaning that in order to maximize the NPV, all the material in the draw column or some part of that can be extracted. In other words, the mining reserve will be computed as a result of the optimal production schedule. Extraction precedence for drawpoints and clusters is used to control the horizontal and vertical mining advancement direction.

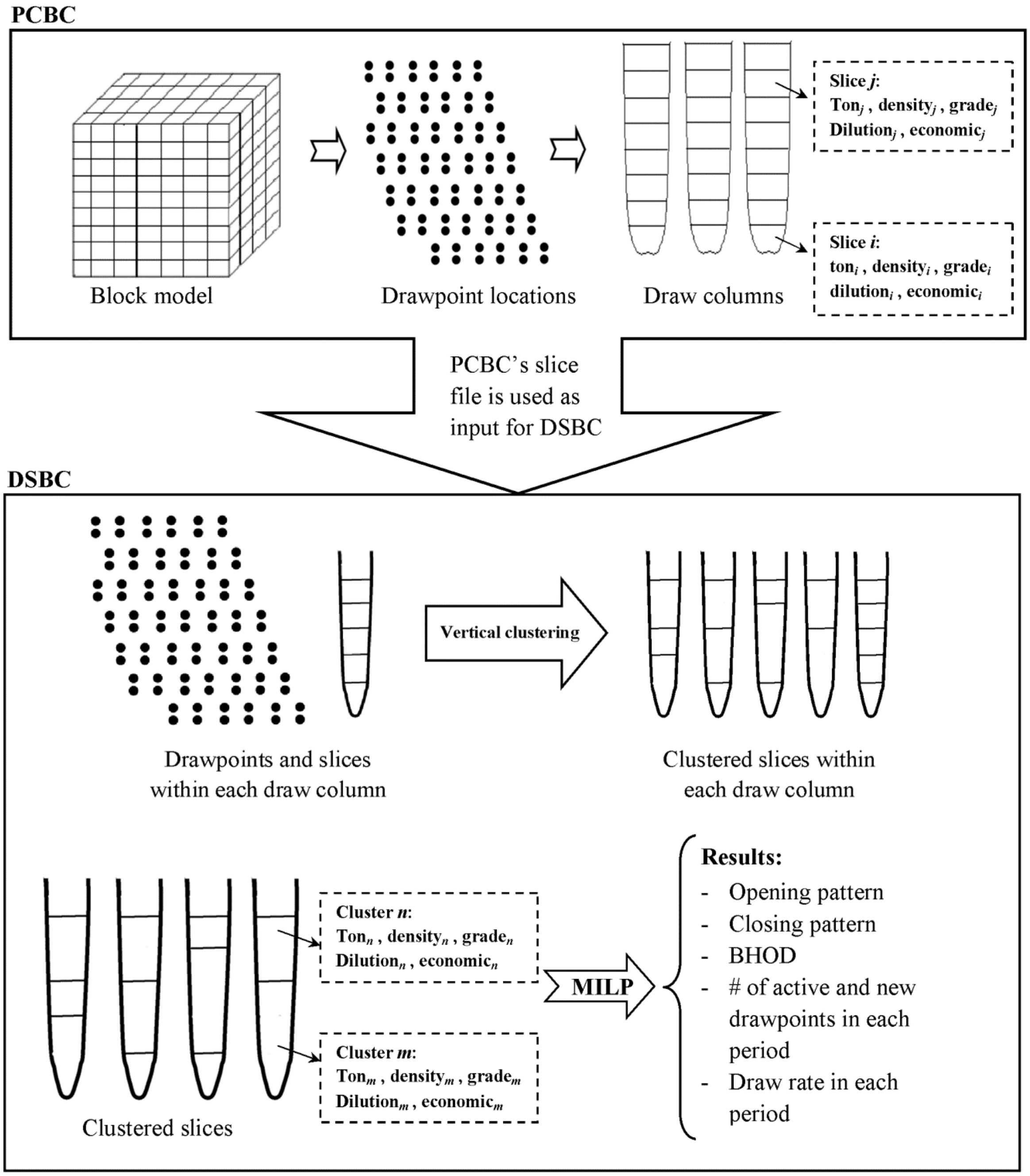

Fig 1. Required steps for block-cave production scheduling using the developed MILP model

Fig 1 shows the workflow that has to be followed to schedule a block-cave mine using the developed MILP model. The developed MILP model uses PCBC's (GEOVIA-Dassault, 2012) slice file as input. The first step is to create a block model in which each block represents an attribute of the geological deposit. The second step is to create a slice file. Afterwards, slices within each draw column are aggregated based on the similarity of the slices. The similarity index is defined based on economic value, dilution percentage, and physical location. All the clustering and optimization steps are carried out by a prototype software developed in-house for drawpoint scheduling in block-caving (DSBC) (Pourrahimian, 2013).

In practice, formulating a real-size mine production planning problem by including all the slices as integer variables will exceed the capacity of the current commercial mathematical optimization solvers. An efficient way of overcoming the large number of decision variables and constraints is to apply a clustering technique. Clustering can be referred to as the task of grouping similar entities together so that maximum intra-cluster similarity and inter-cluster dissimilarity are achieved.

Various methods of aggregation have been used to reduce the number of integer variables that are required to formulate the mine-planning problem with mathematical programming (Epstein et al., 2003; Newman and Kuchta, 2007; Weintraub et al., 2008; Askari-Nasab et al., 2011; Tabesh and Askari-Nasab, 2011; Pourrahimian et al., 2012a; Pourrahimian, 2013).

In order to reduce the number of binary variables in the formulation presented here, the algorithm presented by Tabesh and Askari-Nasab (2011) was modified to aggregate slices within each draw column. The general procedure of the algorithm is as follows:

1. Define the maximum number of required clusters and the maximum number of allowed slices within each cluster.
2. Each slice is considered as a cluster. The similarities between clusters are the same as the similarities between the objects they contain.
3. Similarity values are calculated.
4. The most similar pair of clusters is merged into a single cluster.
5. The similarity between the new cluster and the rest of the clusters is calculated.
6. Steps (2) and (3) are repeated until the maximum number of clusters is reached or there is no pair of clusters to merge because of the maximum number of allowed slices within each cluster.

Similarity value between slices i and j, S_{ij}, is calculated by

$$S_{ij} = \frac{1}{(Dis_{ij})^{W_{Dis}} \times (EV_{ij})^{W_{Ev}} \times (Dil_{ij})^{W_{Dil}}} \tag{1}$$

Where Dis_{ij} represents the normalized distance value between slices i and j, EV_{ij} represents the normalized economic value difference between slices i and j, and Dil_{ij} represents the normalized dilution difference between slices i and j. W_{Dis}, W_{Ev}, and W_{Dil} are weighting factors for distance, economic value, and dilution, respectively. The weights are defined by the mine planner.

The economic value of each cluster (CLSEV) is equal to the summation of the economic value of the slices within the cluster and the costs incurred in mining. The CLSEV is a constant value for each cluster.

According to the advancement direction, the precedence between drawpoints is defined. For each drawpoint d there is a set S^d which defines the predecessor drawpoints among the adjacent drawpoints that must be started before drawpoint d is extracted. The set S^d is created in each advancement direction based on the presented method by Pourrahimian et al. (2012a; 2012b)

4. MILP model for block-cave production scheduling

The MILP model for block-cave production scheduling optimization is explained in this section. The notation used to formulate the problem is classified as indices, parameters, sets, and decision variables. To solve the problem using the developed MILP model, one continuous decision variable and one binary variable for clusters and two binary variables for drawpoints are employed. The continuous decision variable indicates the portion of extraction from each cluster in each period. The binary variables control the number of active drawpoints, precedence of extraction between drawpoints, the opening and closing time of each drawpoint, the extraction rate from each drawpoint, the number of new drawpoints that need to be constructed in each period, and precedence between clusters.

4.1. Notation

4.1.1. Indices

$cl \in \{1,...,CL\}$	Index for clusters.
$e \in \{1,...,E\}$	Index for elements of interest in each cluster.
l	Index for a drawpoint belonging to set S^d .
n	Index for a cluster belonging to set S^{dcl} .
p	Index for a cluster belonging to set S^{dlcl} .
q	Index for a cluster belonging to set S^{cl} .
$t \in \{1,....,T\}$	Index for scheduling periods.

4.1.2. Parameters

CL	Maximum number of clusters in the model.
$CLSEV_{cl}$	Economic value of cluster cl .
D	Maximum number of drawpoints in the model.
$\underline{DR}_{d,t}$	Minimum possible draw rate of drawpoint d in period t .
$\overline{DR}_{d,t}$	Maximum possible draw rate of drawpoint d in period t .
i	Discount rate.
G_{ecl}	Average grade of element e in the ore portion of cluster cl .
$\overline{G}_{e,t}$	Upper limit of the acceptable average head grade of element e in period t .
$\underline{G}_{e,t}$	Lower limit of the acceptable average head grade of element e in period t .

$\underline{M}_t$	Lower limit of mining capacity in period t.
$\overline{M}_t$	Upper limit of mining capacity in period t.
$N_{Ad,t}$	Maximum allowable number of active drawpoints in period t.
Ncl_d	Number of clusters within the draw column associated with drawpoint d.
$\underline{N}_{Nd,t}$	Lower limit for the number of new drawpoints, the extraction from which can start in period t.
$\overline{N}_{Nd,t}$	Upper limit for the number of new drawpoints, the extraction from which can start in period t.
T	Maximum number of scheduling periods.
Ton_{cl}	Total tonnage of material within cluster cl.
Ton_d	Total tonnage of material within the draw column associated with drawpoint d.
Ton_{hd}	Tonnage of material related to the minimum height of draw h within the draw column associated with drawpoint d.

4.1.3. Sets

S^d	For each drawpoint d, there is a set S^d defining the predecessor drawpoints that must be started prior to extracting drawpoint d.
S^{dcl}	For each drawpoint d, there is a set S^{dcl} defining the clusters in the draw column associated with drawpoint d.
S^{dlcl}	For each drawpoint d, there is a set S^{dlcl} defining the lowest cluster within the draw column associated with drawpoint d.
S^{cl}	For each cluster cl, there is a set S^{cl} defining the predecessor clusters that must be extracted prior to extracting cluster cl.

4.1.4. Decision variables

$B_{cl,t} \in \{0,1\}$	Binary variable controlling the precedence of the extraction of clusters. It is equal to 1 if the extraction of cluster cl has started by or in period t; otherwise it is 0.
$C_{d,t} \in \{0,1\}$	Binary variable controlling the closing period of drawpoints. It is equal to 1 if the extraction of drawpoint d has finished by or in period t; otherwise it is 0.
$E_{d,t} \in \{0,1\}$	Binary variable controlling the starting period of drawpoints and precedence

of extraction of drawpoints. It is equal to 1 if the extraction of drawpoint d has started by or in period t; otherwise it is 0.

$X_{cl,t} \in [0,1]$ Continuous decision variable representing the portion of cluster cl to be extracted in period t.

Objective function

$$\text{Maximize} \quad \sum_{t=1}^{T}\sum_{cl=1}^{CL}\left(\frac{CLSEV_{cl}}{(1+i)^{t}}\right)\times X_{cl,t} \tag{2}$$

Constraints

$$\underline{M}_t \le \sum_{cl=1}^{CL}(Ton_{cl})\times X_{cl,t} \le \overline{M}_t \qquad \forall t\in\{1,...,T\} \tag{3}$$

$$\sum_{cl=1}^{CL}\left(Ton_{cl}\times\left(\underline{G}_{e,t}-G_{ecl}\right)\right)\times X_{cl,t}\le 0 \qquad \forall t\in\{1,...,T\},\quad e\in\{1,...,E\} \tag{4}$$

$$\sum_{cl=1}^{CL}\left(Ton_{cl}\times\left(G_{ecl}-\overline{G}_{e,t}\right)\right)\times X_{cl,t}\le 0 \qquad \forall t\in\{1,...,T\},\quad e\in\{1,...,E\} \tag{5}$$

$$X_{p,t}-E_{d,t}\le 0 \qquad \forall t\in\{1,...,T\},\quad d\in\{1,...,D\},\quad p\in S^{dlcl} \tag{6}$$

$$E_{d,t}-E_{d,(t+1)}\le 0 \qquad \forall t\in\{1,...,T\},\quad d\in\{1,...,D\} \tag{7}$$

$$\begin{aligned}&E_{d,t}-C_{d,t}\le L\times\sum X_{n,t} \qquad \forall t\in\{1,...,T\},\quad d\in\{1,...,D\},\quad n\in S^{d\,cl}\\ &L\ge\left(\frac{\max\{Ton_d\}}{\text{minimum draw rate}}\right)\end{aligned} \tag{8}$$

$$C_{d,t}-C_{d,(t+1)}\le 0 \qquad \forall t\in\{1,...,T\},\quad d\in\{1,...,D\} \tag{9}$$

$$\sum_{d=1}^{D}(E_{d,t}-C_{d,t})\le N_{Ad,t} \qquad \forall t\in\{1,...,T\} \tag{10}$$

$$E_{d,t}-E_{l,t}\le 0 \qquad \forall d\in\{1,...,D\},\quad t\in\{1,...,T\},\quad l\in S^{d} \tag{11}$$

$$\sum_{j=1}^{t}X_{cl,j}-B_{cl,t}\le 0 \qquad \forall cl\in\{1,...,CL\},\quad t\in\{1,...,T\} \tag{12}$$

$$B_{cl,t} - \sum_{j=1}^{t} X_{q,j} \le 0 \qquad \forall cl \in \{1,...,CL\},\ \ t \in \{1,...,T\},\ \ q \in S^{cl} \tag{13}$$

$$B_{cl,t} - B_{cl,(t+1)} \le 0 \qquad \forall cl \in \{1,...,CL\},\ \ t \in \{1,...,T\} \tag{14}$$

$$\frac{\sum X_{n,t}}{Ncl_d} \le E_{d,t} - C_{d,t} \qquad \forall d \in \{1,...,D\},\ \ t \in \{1,...,T\},\ \ n \in S^{dcl} \tag{15}$$

$$\underline{N}_{Nd,t} \le \sum_{d=1}^{D} E_{d,t} - \sum_{d=1}^{D} E_{d,(t-1)} \le \overline{N}_{Nd,t} \qquad \forall t \in \{2,...,T\} \tag{16}$$

$$\sum_{d=1}^{D} E_{d,1} \le N_{Ad,1} \tag{17}$$

$$\left(E_{d,t} - C_{d,t}\right).\underline{DR}_{d,t} \le \sum \left(Ton_n\right).X_{n,t} \le \overline{DR}_{d,t} \quad \forall d \in \{1,...,D\},\ t \in \{1,...,T\},\ n \in S^{dcl} \tag{18}$$

$$Ton_{hd} \le \sum_{t=1}^{T} \sum \left(Ton_n\right).X_{n,t} \le Ton_d \qquad \forall d \in \{1,...,D\},\ \ t \in \{1,...,T\},\ \ n \in S^{dcl} \tag{19}$$

Profit from mining a drawpoint depends on the value of the clusters and the costs incurred in mining. The objective function, equation (2), is composed of the CLSEV, discount rate, and a continuous decision variable that indicates the portion of the cluster extracted in each period. The objective function seeks to mine clusters with higher economic value earlier than other clusters.

The constraints are presented by equations (3) to (19). Equation (3) represents the mining capacity which ensures that the total tonnage of material extracted from clusters in each period is within the acceptable range that allows flexibility for potential operational variations. The constraints are controlled by the continuous variable $X_{cl,t}$. There is one constraint per period.

Equations (4) and (5) control the production's average grade. They force the mining system to achieve the desired grade. The average grade of the element of interest has to be within the acceptable range and between the certain values.

Each draw column is divided into slices. Then, slices are aggregated based on the presented clustering method. The lowest cluster in each draw column controls the starting period of extraction from the associated drawpoint. This means that the extraction from the draw column associated with drawpoint d is started by the extraction from the relevant lowest cluster. Equation (6) controls this concept and forces variable $E_{d,t}$ to change to 1 when a portion of the lowest cluster of the draw column is extracted in period t. Equation (7) ensures that when variable $E_{d,t}$ changes to 1, it remains 1 until the end of the mine life.

When the extraction of the last portion of a cluster is finished in period t, extraction of the cluster above can start in period t or $t+1$. In other words, the extraction of a cluster can start if the cluster below has been totally extracted. If the extraction of a cluster is not started after finishing the extraction of the cluster below in period t or $t+1$, the relevant drawpoint must be closed. The concept is applied using equation (8). This ensures that when drawpoint d is open, at least a portion of one of the clusters within the draw column associated with drawpoint d is extracted. This means extraction must be continuous; otherwise, the drawpoint will be closed. Equation (9) ensures that when variable $C_{d,t}$ changes to 1, it remains 1 until the end of the mine life.

As mentioned, when variables $E_{d,t}$ and $C_{d,t}$ change to 1, they remain 1 until the end of the mine life. This helps us to control the maximum number of active drawpoints in each period using equation (10).

The mining precedence is controlled in vertical and horizontal directions. The precedence between drawpoints is controlled in a horizontal direction while the precedence between clusters is controlled in a vertical direction. Equation (11) ensures that all drawpoints belonging to the relevant set, S^d, are started before drawpoint d is extracted. This set is defined based on the selected mining advancement direction. The set can be empty, which means the considered drawpoint can be extracted in any time period in the schedule. Equation (11) also ensures that only the set of the immediate predecessor drawpoints needs to start prior to starting the drawpoint under consideration.

Extraction of cluster cl can be started if the cluster below it has been totally extracted. For each cluster within the draw column except the lowest, there is a set S^{cl} defining the predecessor cluster that must be extracted prior to the extraction of cluster cl. The extraction precedence of the clusters within each draw column is controlled by equations (12), (13), and (14). Equation (12) forces variable $B_{cl,t}$ to change to 1 if extraction from cluster cl is started in period t. Equation (13) ensures that variable $B_{cl,t}$ can change to 1 only if the cluster below it has been extracted totally. In other words, this ensures that the extraction of the slice belonging to the relevant set, S^{cl}, has been finished prior to the extraction of cluster cl. Equation (14) ensures that when variable $B_{cl,t}$ changes to 1, it remains 1 until the end of the mine life. Equation (15) guarantees that cluster cl is extracted when the relevant drawpoint is active.

The drawpoint opening is controlled by the variable, $E_{d,t}$, which takes a value of 1 from the opening period to the end of the mine life. From period two to the end of the mine life, the difference between the summation of opened drawpoints until and including period t, and the summation of opened drawpoints until and including previous period $t-1$, indicates the number of new drawpoints that need to be opened in each period. Equation (16) ensures that the number of new drawpoints opened in each period except period one is within the acceptable range. At the beginning and in period one, the number of new drawpoints is equal to the maximum number of active drawpoints, equation (17).

Equation (18) ensures that the draw rate from each drawpoint is within the desired range in each period. Equation (18) imposes upper and lower bounds for the draw rate. When drawpoint d is not active, $\left(E_{d,t} - C_{d,t}\right)$ is equal to zero and this relaxes the lower bound of the equation.

In this formulation the mining reserve is computed as a result of the optimal production schedule. Equation (19) ensures that the amount of the extracted material from drawpoint d is equal to or less than the total tonnage of the material within the draw column associated with drawpoint d. The lower bound of equation (19) is the tonnage related to the minimum height of the draw in each draw column associated with drawpoint d. The minimum height of the draw is defined by the mine planner.

5. Solving the optimization problem

The proposed MILP model has been developed, implemented, and tested in the TOMLAB/CPLEX environment (Holmstrom, 2011). A prototype software with a graphical user interface has been developed in-house (DSBC) in the MATLAB environment. DSBC integrates all the steps of the optimization including setting up the input parameters, clustering, creating the objective function and constraints, and calling the CPLEX optimization engine in one environment.

Using a branch-and-bound algorithm to solve MILP problem formulations guarantees an optimal solution if the algorithm is run to completion. We have used the gap tolerance (EPGAP) as an optimization termination criterion. The gap tolerance sets an absolute tolerance on the gap between the best integer objective and the objective of the best node remaining.

The application of the model was implemented on a Dell Precision T7500 computer at 2.7 GHz, with 24GB of RAM. The goal was to maximize the NPV at a discount rate of 12% and determine the mining reserve as a result of the optimal production schedule, while assuring that all constraints were satisfied during the mine-life.

6. Case study: implementation of MILP model

The performance of the proposed model is analyzed based on NPV, mining production, and practicality of the generated schedules. A real data set containing 102 drawpoints and 3,470 slices with the slice height of 10 meters is considered. The minimum and maximum numbers of slices within draw columns are 33 and 36, respectively. Fig 2 illustrates a plan view of the drawpoint configuration based on the relevant coordinates and distance between the centre-lines of draw columns. Fig 3 illustrates a 3D view of the draw columns. The total tonnage of available material is 22.5 Mt. The tonnage of draw columns varies from 203.5 kt to 355.5 kt. The deposit is scheduled over 14 periods.

To aggregate the slices within each draw column, the modified clustering method was applied. The weight factors of the distance, economic value, and dilution were set to 5, 3, and 3, respectively. The maximum number of slices in each cluster could not be more than five. One-thousand clusters were created based on the presented algorithm. Fig 4 illustrates examples of grade distribution within two different draw columns.

A capacity of 900 kt/yr is considered as the upper bound on the mining capacity. The maximum number of active drawpoints in each period was set to 40. The maximum number of new drawpoints which could be opened in each period was set to 15. The lower and upper bounds of the draw rate for drawpoints were set to 10 kt/yr/per drawpoint and 40 kt/yr/per drawpoint. The lower and upper bounds of the average grade of Copper were set to 0.8% and 1.7%.The height of draw is limited to not less than 50 m. This means at least 50 m of the drawpoints must be extracted. An EPGAP of 5% was set for the optimization run. The problem was solved for two directions, west to east (WE) and south to north (SN). Table 1 and Table 2 show the number of decision variables, the number of constraints, and numerical results for both the WE and SN directions. The resulting NPVs are $133.73 M and $132.0 M in the WE and SN directions, respectively.

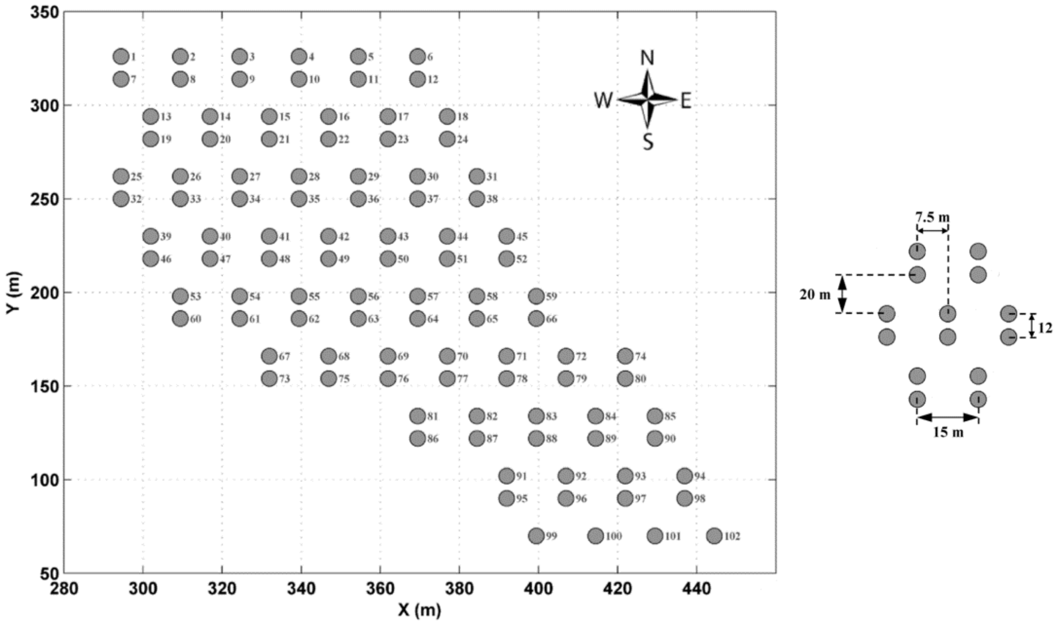

Fig 2. Plan view of 102 drawpoints

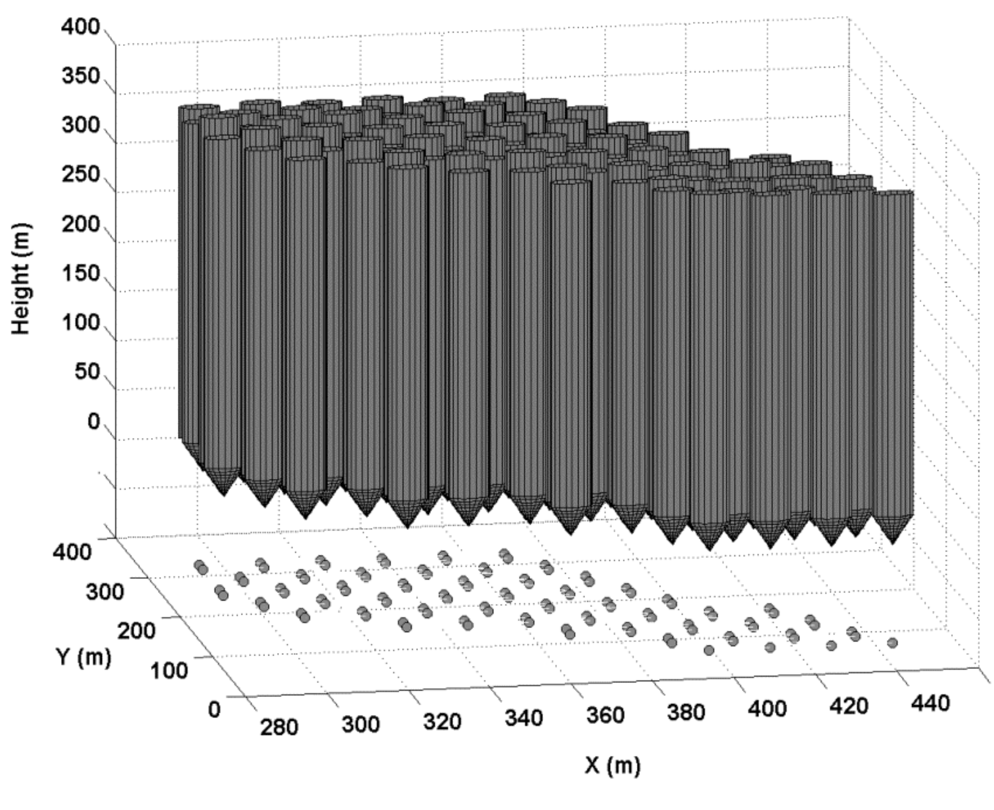

Fig 3. 3D view of draw columns (102 draw columns)

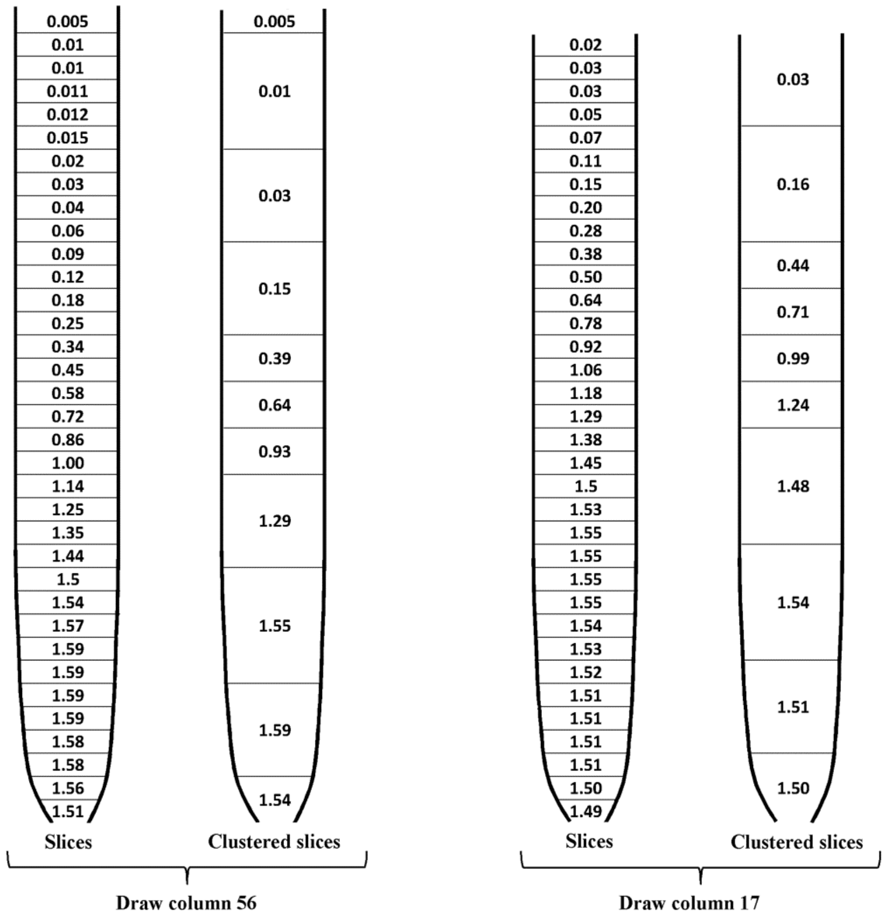

Fig 4. Grade distribution (Cu%) within draw columns 56 and 17, before and after clustering

Table 1. Number of variables and constraints for the proposed formulation with 102 drawpoints and 1,000 clusters

Direction	Number of DPs/CLs	Number of constraints	Decision Variables		
			Total	Continuous	Binary
WE	102 / 1,000	59,546	30,856	14,000	16,856
SN	102 / 1,000	61,086	30,856	14,000	16,856

Table 2. Numerical results for the proposed formulation with 102 drawpoints and 1,000 clusters

Direction	CPU time 8 CPUs @ 2.7 GHz	EPGAP (%)	Optimality GAP (%)	NPV ($M)	Reserve (Mt)
WE	01:21:19	5	4.99	133.73	11.93
SN	02:09:05	5	4.43	132.0	11.94

Figures 5 to 7 show that all assumed constraints were satisfied in the considered directions. Fig 5 illustrates the production tonnage in each period. If mining reserve was calculated based on the BHOD (Diering, 2000) for each draw column, the total tonnage of material that could be extracted was almost 13.5 Mt, which was independent of direction. In other words, in each considered direction all the 13.5 Mt must be extracted. But in the proposed formulation, the mining reserve is computed as a result of the optimal production schedule for each advancement direction. The total tonnage of material that must be extracted in the WE and SN directions is 11.9 Mt.

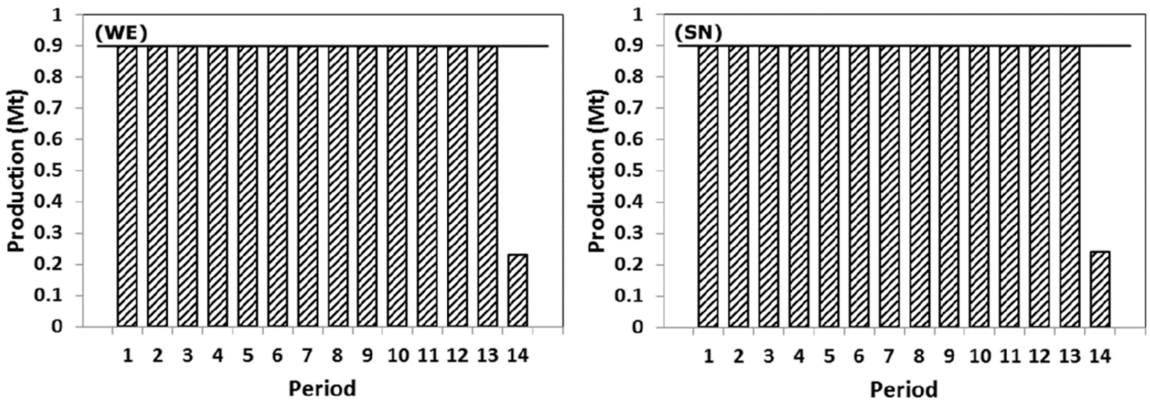

Fig 5. Production tonnage in the WE and SN directions

Fig 6 illustrates the number of active drawpoints and the number of drawpoints that must be opened in each period. In the WE direction, the mine works with the maximum number of active drawpoints from periods two to ten. Then, the number of active drawpoints reduces. In the SN direction, the mine works with the maximum number of active drawpoints from periods two to 13 except period nine. In both directions, the number of new drawpoints from periods two to 15 is less than 15 except in period six of the WE direction, in which 15 new drawpoints must be opened. In the WE direction, the last drawpoints are opened in period 11 while a number of new drawpoints are opened in period 12 in the SN direction.

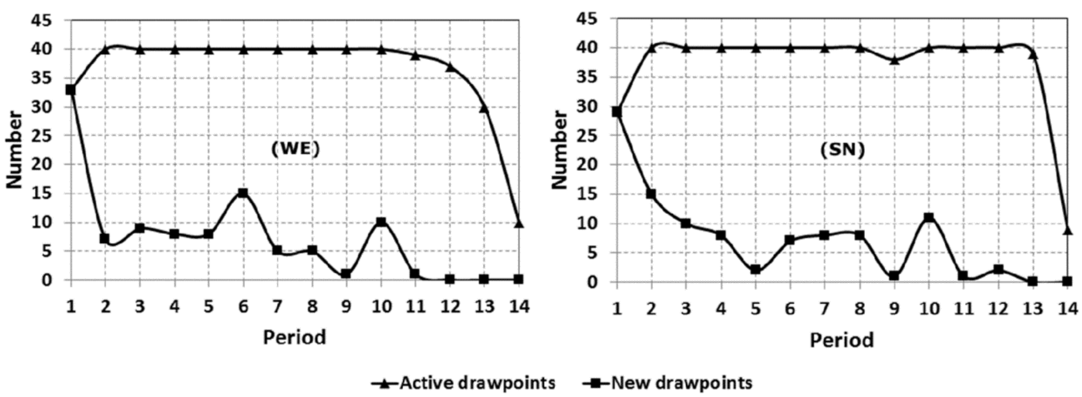

Fig 6. Number of active drawpoints and number of new drawpoints that must be opened in the WE and SN

Fig 7 illustrates the average grade of production. In the WE direction, during the first two periods the average grade of the production is higher than the SN direction. In both directions, during the mine life the average grade of the production was higher than 0.9 %. In the SN direction, the average grade of production between periods 11 and 14 is higher than the WE direction.

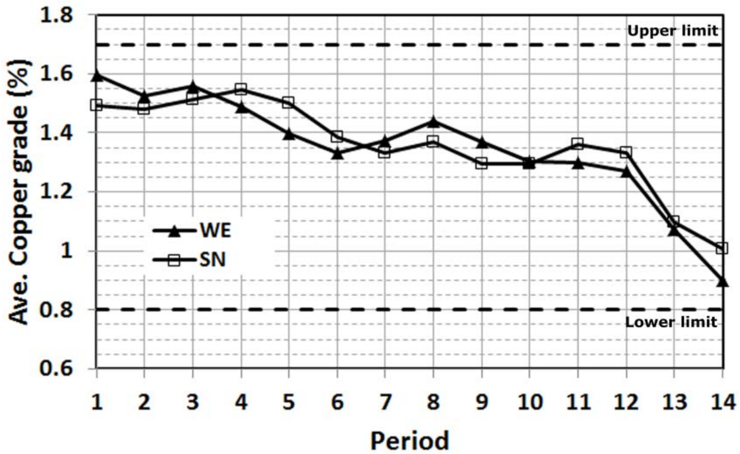

Fig 7. Average grade of production in the WE and SN directions

Fig. 8 shows the opening pattern of the drawpoints in the WE and SN directions. In the WE direction, 83% of drawpoints will be opened in the first seven years and the rest, most of which are located at the southwest area of the mine, will be opened after period seven. In the SN direction, if the mine is divided into two sections, north and south, most of the drawpoints in the south section are opened during the first four periods.

The possible advancement directions are defined based on geotechnical conditions. Then, using the proposed formulation, the best advancement direction and related mining reserve are computed. In the presented case study, the results are based on an optimization termination criterion (EPGAP) of 5%. In the presented formulation, the model uses drawpoints-and-slices (Pourrahimian, 2013) in which the slices are aggregated vertically in each draw column. The considered case study was also solved without vertical clustering. The solving time for the clustered slices was 78 times faster than for the other method.

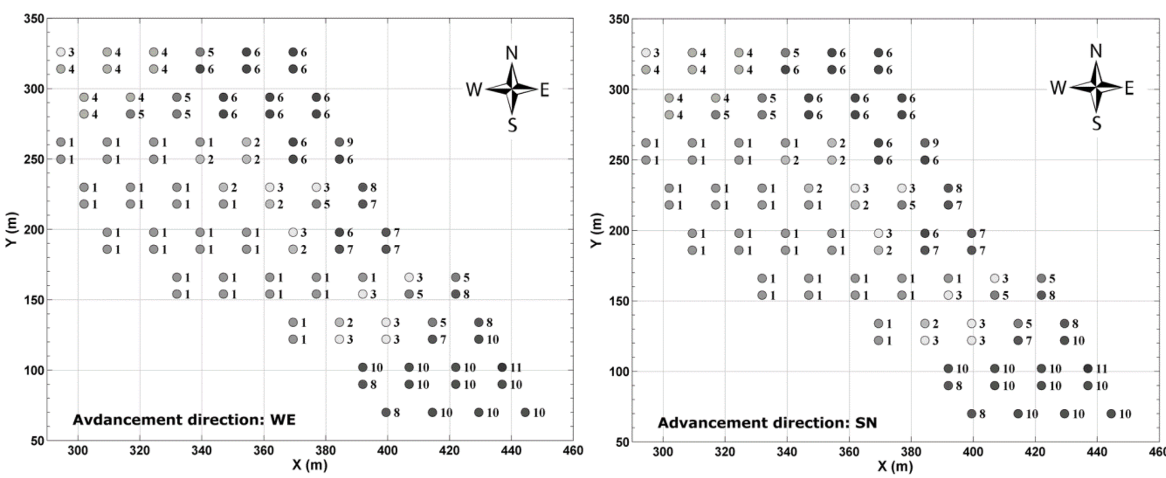

Fig 8. Opening pattern in the WE and SN directions

Table 3 shows the obtained NPVs and CPU times for different EPGAPs. It is obvious that when the EPGAP decreases the CPU time dramatically increases.

Table 3. Effect of the EPGAP on NPV and CPU time

	WE direction			SN Direction		
EPGAP (%)	NPV ($M)	CPU time (hr:min:sec)	NPV Diff. From the best (%)	NPV ($M)	CPU time (hr:min:sec)	NPV Diff. From the best (%)
3	135.13	04:48:50	0	132.91	20:31:04	0
4	134.67	02:49:43	- 0.34	132.36	02:27:48	- 0.41
5	133.73	01:21:19	- 1.04	132.0	02:09:05	- 0.68

7. Conclusion

This paper investigated the development of a mixed-integer linear programming (MILP) formulation for block-cave production scheduling optimization. The presented MILP formulation developed, implemented, and tested for block-cave production scheduling in the TOMLAB/CPLEX environment. The formulation maximizes the NPV subject to several constraints and the mining reserve is computed as a result of the optimal production schedule. To reduce the number of binary variables and to solve the problem within a reasonable CPU time, slices within each draw column were aggregated based on the similarity index that was defined based on the slices' distance, economic value, and dilution.

The proposed formulation can be used in different advancement directions which are selected based on geotechnical considerations. Consequently, the mining reserve, which is a result of optimization, also varies from one direction to another. The large-scale problems can be solved in a reasonable CPU time by applying the presented method here on the drawpoint-and-slice level of the multi-step method presented by Pourrahimian (2013). The concept of different cave advancement directions presented here helps planners to find the best single operation direction or combination thereof, and the best starting location to reach the maximum NPV.

Production scheduling optimization techniques are still not widely used in the mining industry. There is a need to improve the practicality and performance of the current production scheduling optimization tools used by the mining industry. Future research will focus on modifying the approach for handling multiple-lift and multiple-mine scenarios. In addition, other efficient mathematical formulation techniques will be explored in an attempt to reduce the execution time for large-scale block-cave production scheduling.

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A Review of Mathematical Models and Algorithms in Block-Caving Scheduling

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Abstract

Production scheduling is one of the most important problems in a mining operation, as it has a significant impact on a project's profitability. As open-pit mines go deeper, because of the massive amount of waste removal which is required to extract the ore as well as high operational costs per tonne, underground mining has become more attractive. Among underground mining methods, block caving could be a good alternative because its high rate of production is similar to that of open-pit mining, and it has the added advantage of low operational costs. Relying only on manual planning methods or computer software based on heuristic algorithms will lead to mine schedules that are not the optimal global solution. Block-caving scheduling has been the subject of a lot of research. Most studies have applied mathematical programming, simulation, and stochastic approaches. This paper reviews mathematical programming applications in block-caving scheduling, highlights, and suggestions for future works.

1. Introduction

These days, most surface mines work in a higher stripping ratio than in the past. In the following conditions, a surface mine can be less attractive to operate and underground mining is used instead. These conditions are (i) too much waste has to be removed in order to access the ore (high stripping ratio), (ii) waste storage space is limited, (iii) pit walls fail, or (iv) environmental considerations could be more important than exploitation profits (Newman et al., 2010).

Among underground methods, block-cave mining, because of its high production rate and low operation cost, could be considered an appropriate alternative. Projections show that 25 percent of global copper production will come from underground mines by 2020. Mining companies are looking for an underground method with a high rate of production, similar to that of open-pit mining. Therefore, there is an increased interest in using block-cave mining to access deep and low-grade ore bodies.

Production scheduling is one of the most important steps in the block-caving design process. Optimum production scheduling could add significant value to a mining project. The goal of long-term mine production scheduling is to determine the mining sequence, which optimizes the company's strategic objectives while honoring the operational limitations over the mine life. The

Pourrahimian, Y., Ben-Awuah, E., and Askari-Nasab, H. (2015), Mining Optimization Laboratory (MOL), Report Six, © MOL, University of Alberta, Edmonton, Canada, Pages 250 , ISBN: 978-1-55195-356-4, pp. 130-148

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production schedule defines the management investment strategy. An optimal plan in mining projects will reduce costs; increase equipment use; and lead to optimum recovery of marginal ores, steady production rates, and consistent product quality (Chanda, 1990; Chanda and Dagdelen, 1995; Dagdelen and Johnson, 1986; Winkler, 1996; Wooller, 1992). Although manual planning methods or computer software based on heuristic algorithms are generally used to generate a good solution in a reasonable time, they cannot guarantee mine schedules that are the optimal global solution.

Mathematical programming with exact solution methods is considered a practical tool to model block-caving production scheduling problems; this tool makes it possible to search for the optimum values while considering all of the constraints involved in the operation. Solving these models with exact solution methodologies results in solutions within known limits of optimality.

In this paper, block caving and production scheduling in cave mining have been introduced, and mathematical programming methods which can be used as a tool for production scheduling have been discussed. Finally, the related research in this area is presented, as are some conclusions and new ideas for future studies.

2. Block caving

Generally speaking, underground mining methods can be classified in three categories: (i) caving methods such as block caving, sublevel caving, and longwall mining; (ii) stoping methods such as room-and-pillar, sublevel stoping, and shrinkage; and (iii) other methods such as postpillar cut-and-fill, and Avoca (Carter, 2011).

Block caving is usually appropriate for low grade and massive ore bodies in which natural caving could occur after an undercut layer is made under the ore-body. Laubscher (1994) refers block caving "to all mining operations in which the ore-body caves naturally after under cutting its base. The caved material is recovered using drawpoints."

Depending on the ore-body dimensions, inclination, and rock characteristics, block caving could be implemented as block caving, panel caving, inclined drawpoint caving, and front caving. The low-cost operation could be understood from the natural caving. In other words, during the extraction period, there is no cost for caving unless some small blasting is needed to deal with hang-ups. In block caving (Fig 1), the pre-development period can last for more than five years. This is a significant period of time with no cash back. But when the production starts, the extraction network can be used for the life of the drawpoint, so the operating cost is low and production rate can be remarkable. To sum up, block caving has the lowest operating cost of all underground mining methods. In some cases its cost is comparable to that of open-pit mining.

There are three methods of block caving. In the grizzly or gravity system, the ore from the drawpoints flows directly to the transfer raises after sizing at the grizzly, and then is gravity-loaded into ore cars. In the slusher system, slusher scrapers are used for the main production unit. In the load-haul-dump (LHD) system, rubber tired equipment are used for ore handling in production level (Hustrulid, 2001). Table 1 shows some examples for each method. Caterpillar jointly with the Chilean mining company Codelco has developed a continuous haulage technology for block caving operation. In this method, the LHD at the drawpoint is replaced by a rock feeder. This device pushes the rock into the haulage access, where it drops onto a hard rock production conveyor.

The size of the caved material, the mine site location, availability of labor, and economics are some aspects which determine the block-caving system (Julin, 1992). Factors that have to be considered in block caving include caveability, fragmentation, draw patterns for different types of ore, drawpoint or drawzone spacing, layout design, undercutting sequence, and support design (Laubscher, 2011). Some large-scale open-pit mines will be transferred to underground mining as they go deeper; they need to produce in a similar rate to open-pit mines to provide their processing plants with feed, so block caving with a high production rate could be an attractive alternative. Around the world, more than 60 mines have been closed, are operating or are planned to be mined by block caving (Woo et al., 2009).

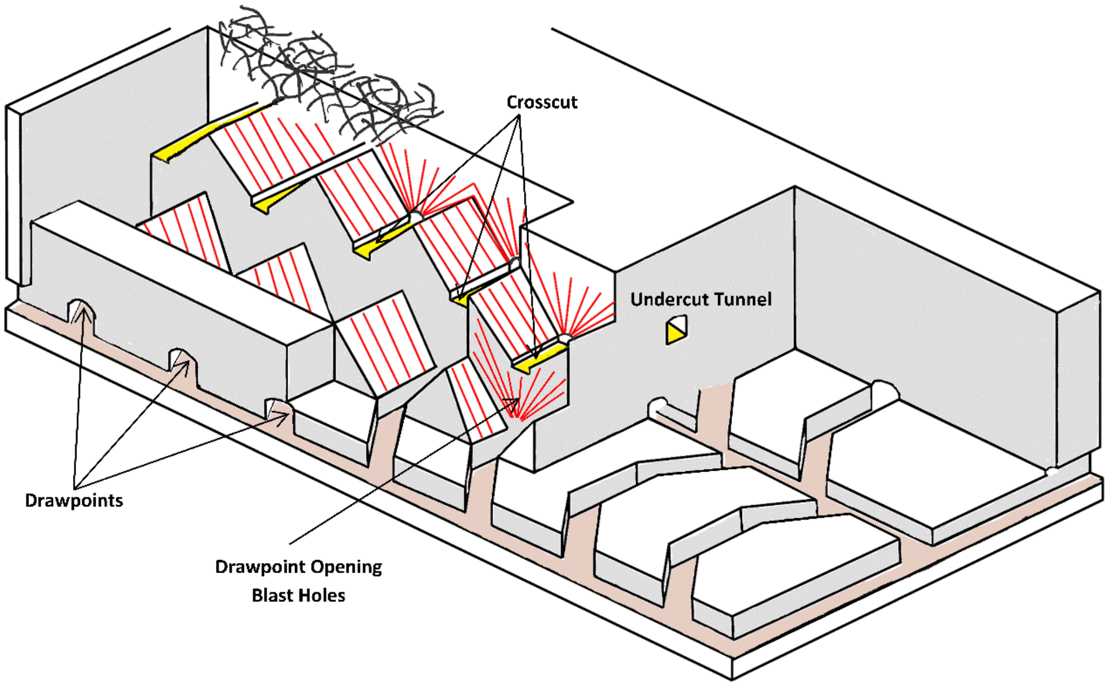

Fig 1. Block-caving mining

Table 1. Some real cases for different block-caving methods (Bergen et al., 2009; Inc., 2012; Julin, 1992; Song, 1989)

Method	Mine	Ore Type	Location
Gravity (Grizzly)	San Manuel	Copper	Arizona
	Andina	Copper	Chile
Slusher	Climax	Molybdenum	Colorado
	Tong Kuang Yu	Copper	China
LHD	Henderson	Molybdenum	Colorado
	Ertsberg	Copper	Indonesia
	El Teniente	Copper	Chile
	New Afton	Copper-Gold	Canada

Laubscher (2000) identified 10 different horizontal LHD layouts as having been use in block caving mines. Fig 2 and Fig 3 illustrate two of them. Fig 2 shows offset Herringbone. In this layout, the drawpoints on opposite sides of a production drift are offset. This helps improve both the stability conditions and the operational efficiency. This layout was used initially at the Henderson Mine, USA, and Bell Mine, Canada. Fig 3 shows the layout developed at the El Teniente Mine, Chile. In this layout, the drawpoint drifts are developed in straight lines oriented at 60o to the production drift (Brown, 2003).

One of the most critical processes in block-cave mining is undercutting. The undercutting strategy can have a significant influence on cave propagation and on the stresses induced in, and the performance of the extraction level installations (Brown, 2003; 2007). The three mostly used undercutting strategies are post, pre, and advanced undercutting. In the post-undercutting strategy, undercut drilling and blasting takes place after the production level has been developed. In the pre-undercutting method, no development or construction takes place on the production level before the undercut has been blasted. In the advanced-undercutting strategy, the production level is developed in advance of the blasting of the undercut. This method was introduced to reduce the drawpoints' exposure to the abutment stress zones, which were induced as a result of the undercutting process. The next section presents production scheduling in mining operations, and particularly in block-cave mining.

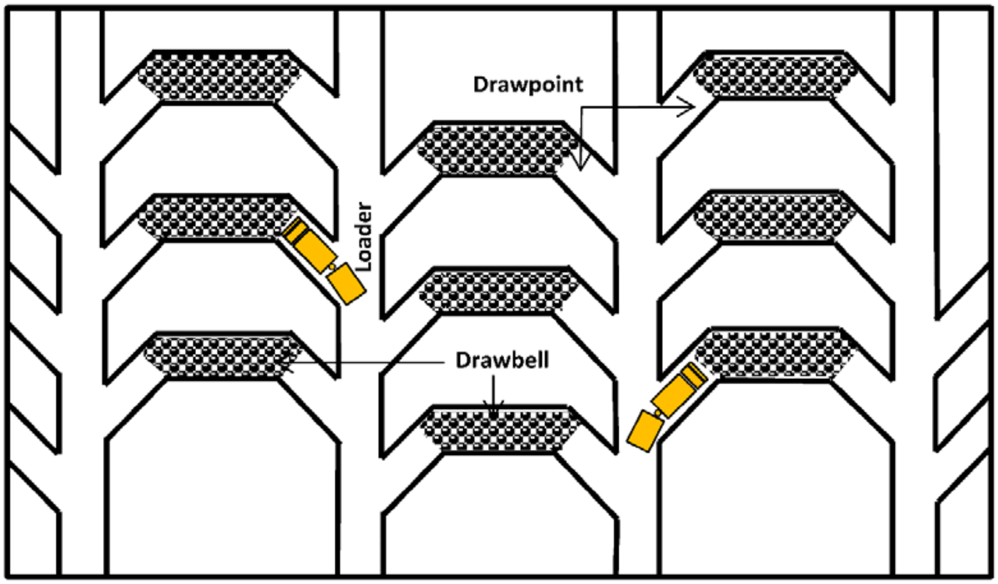

Fig 2. Typical offset Herringbone layout (after Brown, 2007)

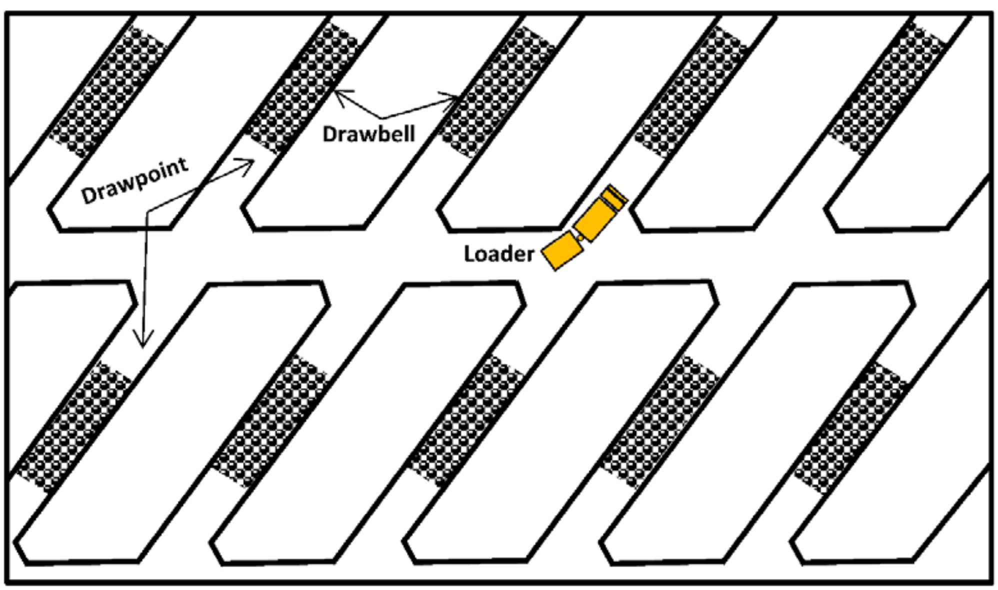

Fig 3. Typical El Teniente layout (after Brown, 2007)

3. Underground mines production scheduling

The production scheduling in mining operations requires determining which blocks should be extracted and the time of their extraction during the life of mine, while considering geomechanical, operational, economic, and environmental constraints. Production scheduling for any mining system has an enormous effect on the operation's economics. Some of the benefits expected from better production schedules include increased equipment use, optimum recovery of marginal ores, reduced costs, steady production rates, and consistent product quality (Chanda, 1990; Chanda and Dagdelen, 1995; Dagdelen and Johnson, 1986; Winkler, 1996; Wooller, 1992).

There are three time horizons for production scheduling: long-, medium-, and short-term. Long-term mine-production scheduling provides a strategic plan for mining operations, whereas medium-term scheduling provides a monthly operational scheme for mining while tracking the strategic plan. Medium-term schedules include more detailed information that allows for a more accurate design of ore extraction from a special area of the mine, or information that allows for necessary equipment substitution or the purchase of necessary equipment and machinery. The medium-term schedule is also divided into short-term periods (Osanloo et al., 2008).

The majority of scheduling publications to date have been concerned with open-pit mining applications. As a result, the software development for underground operations has been delayed and many of the scheduling concepts and algorithms developed for surface mining have found their way into underground mining. Underground mining methods are characterized by complex decision combinations, conflicting goals and interaction between production constraints.

Current practice in underground-mine scheduling has tended toward using simulation and heuristic software to determine feasible, rather than optimal, schedules. A compromise between schedule quality and problem size has forced the use of mine design and planning models, which incorporate the essential characteristics of the mining system while remaining mathematically tractable. Different types of methods have been applied to underground mine scheduling. Similar to open-pit mines, production scheduling algorithms and formulations in literature can be divided into two main research areas: (i) heuristic methods and (ii) exact solution methods for optimization. Heuristic methods are generally used to generate a good solution in a reasonable amount of time. These methods are used when there is no known method to find an optimal solution under the given constraints. Despite shortcomings such as frequently required intervention, and the lack of a way to prove optimality, simulation and heuristics are able to handle non-linear relationships as part of the scheduling procedure.

In addition to these categories, other methods such as queuing theory, network analysis, and dynamic programming have been used to schedule production and/or material transport. This paper reviews mathematical programming applications in block-caving production scheduling. In block-cave mining, production scheduling determines the amount of material which should be mined from each drawpoint in each period of production, the number of new drawpoints that need to be constructed, and their sequence during the life of mine (Pourrahimian, 2013). The same concerns in deep open-pit mining can be applied to block-cave mining; possibility of value changes of the project through scheduling is remarkable.

4. Mathematical programming methods

Mathematical programming (MP) is the use of mathematical models, particularly optimizing models, to assist in making decisions. The MP model comprises an objective function that should be maximized or minimized while meeting some constraints which determine the solution space and a set of decision variables whose values are to be determined. Objectives and constraints are functions of the variables and problem data. Mathematically, a MP problem can be stated as,

$$\begin{aligned} &\text{Maximize} && f(X) \\ &\text{Subject to} && g_i(X) \le 0 \quad i = 1, 2, ..., m. \\ & && X \ge 0 \end{aligned}$$

Where $X = (x_1, x_2 ... x_n)^T \in R^n$, $f(X), g_i(X), i = 1, 2, ... m$ are real valued functions of X. If the functions $f(X)$ and $g_i(X)$ are all linear, the problem is known as a linear programming problem (LP). Otherwise it is said to be a non-linear programming (NLP) problem (Sinha, 2006).

The modeling process in mathematical programming has eight steps (Eiselt and Sandblom, 2010): problem recognition, authorization to model, model building and data collection, model solution, model validation, model presentation, implementation, and monitoring and control. The mathematical programming models which are considered for production scheduling are linear programming (LP), mixed-integer linear programming (MILP), non-linear programming (NLP), dynamic programming (DP), multi-criteria optimization, network optimization, and stochastic programming (Shapiro, 1993). In a LP problem, when some or all of the variables must be integers, the problem is called pure integer (IP) and mixed-integer programming (MIP, MILP) respectively. A linearly constrained optimization problem with a quadratic objective function is called a quadratic program (QP).

The tractability of the mathematical models depends on the size of the problem, in terms of the number of variables and constraints, and the structure of the constraint sets. In the integer programming, as the size of an integer program grows, the time required for solving the problem increases exponentially. The most common problem in the MILP formulation is size of the branch and cut tree. The tree becomes so large that insufficient memory remains to solve the LP sub-problems. The size of the branch and cut tree can actually be affected by the specific approach one takes in performing the branching and by the structure of each problem. So, there is no way to determine the size of the tree before solving the problem (Pourrahimian et al., 2012). For instance, Pourrahimian et al (2013) presented three MILP formulations at three different levels of resolution: (i) aggregated drawpoints level; (ii) drawpoint level, and (iii) drawpoint-and-slice level. Table 2 summarize the number of decision variables and constraints if the proposed formulations are applied on a mine with 3000 drawpoints over 25 years.

Table 2. Number of decision variables and constraints for the models presented by Pourrahimian et al (Pourrahimian et al., 2013)

Level of resolution	Description	Number of continuous variables	Number of binary variables	Number of constraints
Aggregated DPs	3000 DPs aggregated into 200 clusters	5000	10000	20,675
DPs	3000 DPs	75000	150000	309,075
DP-and-slice	3000 DPs and 60530 slices	1,513,250	1,663,250	640,185

5. Production scheduling optimization in block-cave mining

Using mathematical programming optimization with exact solution methods to solve the long-term production planning problem has proved to be robust, and results in answers within known limits of optimality (Pourrahimian, 2013). Lerchs and Grossmann (1965) applied mathematical programming in mine planning (open-pit mining) for the first time. Since the 1960's, considerable research has been done in mine planning using mathematical programming, both in open-pit and underground mining. Newman et al. (2010) and Osanloo et al. (2008) have mentioned many of the studies related to open-pit mining. Alford (1995) listed problems which have the potential of being considered optimization problems in underground mining. These problems are: (i) primary development (shaft and decline location); (ii) selection from alternative mining methods; (iii) mine layout (i.e., sublevel location and spacing, stope envelope); (iv) production sequencing; (v) product quality control (material blending); (vi) mine ventilation; and (vii) production scheduling (ore transportation and activity scheduling).

Among these problems, product quality control and production scheduling have received the greatest consideration for optimization (Rahal, 2008). Production scheduling optimization is so important because its impact on a project's net present value (NPV) is critical. Therefore, it should be updated periodically. Scheduling underground mining operations is primarily characterized by discrete decisions regarding mine blocks of ore, along with complex sequencing relationships between blocks. To optimize block-caving scheduling, most researchers have used mathematical programming, LP (Guest et al., 2000; Hannweg and Van Hout, 2001; Winkler, 1996), MILP (Alonso-Ayuso et al., 2014; Chanda, 1990; Epstein et al., 2012; Guest et al., 2000; Parkinson, 2012; Pourrahimian, 2013; Rahal, 2008; Rahal et al., 2008; Rahal et al., 2003; Rubio, 2002; Rubio and Diering, 2004; Smoljanovic et al., 2011; Song, 1989; Weintraub et al., 2008; Winkler, 1996), and QP (Diering, 2012; Rubio and Diering, 2004). LP is the simplest method for modeling and solving. Since LP models cannot capture the discrete decisions required for scheduling, MIP is generally the appropriate MP approach to scheduling (Pourrahimian, 2013). Solving a MILP problem can be difficult when production system is large, but MILP is a useful methodology for underground scheduling (Rahal, 2008). This section includes reviews of MP applications in block-caving scheduling and some features for each methodology.

Song (1989) used simulation and a MILP model to find the optimal mining sequence in the block-cave operations at the Tong Kuang Yu mine in China. To obtain an optimal mining sequence, Song first simulated the caving process dependent on undercut parameters. Then, he determined ore-draw spacing and pressure distribution during ore-draw. Finally, he used caving simulation and analysis results to obtain the optimal mining sequence. He optimized the production schedule using total mining cost minimization while considering the geometrical and operational limitations which guarantee caveability and stability demands. Defining linear functions was an advantage of his methodology. The disadvantage, especially in long-term scheduling, was the solution time.

Chanda (1990) combined a simulation with MIP to model the problem of scheduling drawpoints for production at the Chingola Mine, in Zambia. He computerized a model for short-term production scheduling in a block-caving mine. The model used MIP to determine the production rate in finger raises in each production drift considering some quality and quantity constraints. The objective was to minimize the deviation in the average production grade between operating shifts.

Guest et al. (2000) developed LP and MILP models to maximize the NPV of block-caving scheduling (long-term scheduling) over the mine life of a diamond mine in South Africa. This model tried to consider, as constraints, related aspects of mining: mining capacity, metallurgical issues, economic parameters, grades and geotechnical limitations. Applying this wide range of constraints is a remarkable advantage of this model. However, there were two problems with this approach; maximizing tonnage or mining reserves will not necessarily lead to maximum NPV; and draw control is a planning constraint and not an objective function. The objective function in this case would be to maximize tonnage, minimize dilution or maximize mine life (Rubio, 2002).

Rubio (2002) formulated two strategic goals; maximization of NPV and optimization of the mine life in block caving. As constraints, he considered geomechanical aspects, resource management, the mining system and metallurgical parameters involved in the mining operation. One of the main advantages of his model was that it integrated estimates of mine reserves and the development rate that resulted from the production scheduling. Traditionally these parameters were computed independent of production scheduling. Rubio also formulated a relationship between the draw control factor and the angle of draw. This relationship was built into the actual draw function to compute schedules with high performance in draw control. Opportunity cost in block caving was defined as the financial cost of delaying production from newer drawpoints; a drawpoint will stay active at any given period of the schedule, if it has enough remaining value to pay the financial cost of delaying production from newer drawpoints that may have a higher remaining value.

Rahal et al. (2003) described a MILP goal program with dual objectives of minimizing deviation from the ideal draw profile while achieving a production target. They performed a schedule optimization using a-life-of-mine approach in which all production periods were optimized simultaneously. They assumed that material mixing in the short-term has a minimal effect on the panel's long-term state. The model's constraints were deviation from ideal practice, panel state, material flow conservation, production quality, material flow capacity, and production control. They applied the model to De Beers kimberlite mine. The results showed how different production control constraints regulate production from individual drawpoints, as well as recovery of the ideal panel profile by implementing an optimized draw schedule.

Diering (2004) described the basic problem in block-caving scheduling as trying to determine the best tonnages to extract from a number of drawpoints for various periods of time. Those periods could range a day to the life of the mine. Diering singled out NPV as the overall objective to maximize, subject to some constraints: minimum tonnage per period, maximum tonnage per period, maximum total tonnage per drawpoint, maximum total tonnage per period, ratio of tonnage from current drawpoint compared with neighbors, height of draw of current drawpoint with respect to neighbors, percentage drawn for current drawpoint with respect to neighbors, and maximum tonnage from selected groups of drawpoints in a period (usually the groups of drawpoints are referred to as production blocks or panels). Diering emphasized that it would be better to formulate the problem as a LP instead of a NLP because of solution time and the size of problems. He applied a multi-step non-linear optimization model to minimize the deviation between a current draw profile and a defined target. It was shown that this algorithm could also be used to link the short-term plan with the long-term plan.

Rubio and Diering (2004) applied MP to maximize the NPV, optimize the draw profile and minimize the gap between long- and short-term planning. They integrated the opportunity cost into PC-BC (GEOVIA) for computing the best height of draw in a dynamic manner. To solve their problem, they used different mathematical techniques such as direct iterative methods, LP, a golden section search technique, and integer programming. In their formulation, mining reserves were not part of the set of constraints; the mining reserves were computed as a result of the optimal production schedule. They also used QP to minimize the differences between actual heights of draw versus a desired target.

Rahal (2008) presented a draw control model that indirectly increases resource value by controlling production based on geotechnical constraints. He used MILP to formulate a goal programming model with two strategic targets: total monthly production tonnage and cave shape. This approach increased value by ensuring that reserves are not lost due to poor draw practice. The model's advantage was that it allows any number of processing plants to feed from multiple sources (caves, stockpiles, and dumps). There were three main production control constraints in the MILP: the draw maturity rules, minimum draw rate, and relative draw rate (RDR). Rahal used MILP to quantify production changes caused by varying geotechnical constraints, limiting haulage capacity, and reversing mining direction. He showed that tightening the RDR constraint decreases total cave production. He applied his model for three case studies and illustrated how the MILP can be used by a draw control engineer to analyze production data and develop long-term production targets both before and after a cave is brought into full production. Rahal et al. (2008) used MILP to develop an optimized production schedule for Northparkes E48 mine. They described the system constraints as minimum and maximum draw tonnage, the permissible relative draw rate difference between adjacent drawpoints, drawpoint availability, and the capacity of the materials handling system. The impact of different production constraints on total cave capacity was examined. It was shown that the strength of using

MILP lies in its ability to generate realistic production schedules that require little manual manipulation.

Weintraub et al. (2008) developed an approach to aggregate the reduced models (which have been derived from a global model) using the original data for a MIP mine planning model in a large block-caving mine. The aggregation was based on clustering analysis. The MIP model was developed to support decisions for planning extraction of blocks and the decisions of exact timing for each block in the extraction columns. The final model was developed to integrate all mines for corporate decisions, to determine extraction from each sector, in each mine, for each period (for a five-year horizon). Weintraub et al. used two types of aggregation: *Priori* aggregation and *Posteriori* approach. Comparing the original model with the disaggregation, the first approach reduced execution time by 74% and the model dimension by 90%. The second approach reduced solve time by 88% and the model dimension by 15%.

Smoljanovic et al. (2011) presented a model to optimize the sequence of the drawpoint opening over a given time horizon. They incorporated sequencing and capacity constraints. Their model was based on an open-pit model (BOS2) adapted to underground mining. Binary variables were used to indicate whether or not a specific drawpoint had been opened. The real numbers represent the percentage of the column that was extracted. The model was applied in a panel caving mine in which the studied layout included 332 drawpoints. It was shown that the sequencing can change the value of objective function by as much as 50%. Smoljanovic (2012) applied MILP to optimize NPV and the mining material handling system in a panel caving mine. The model output selected the best sequence after considering different mining systems. Results showed that the out-coming NPV of the objective function for different systems could vary by up to 18%. The importance of the mining system and capacity constraints in the sequencing was shown in comparison to different scenarios.

Parkinson (2012) developed three integer programming models for sequence optimization in block-cave mining: Basic, Malkin, and 2Cone. The research was carried out to help provide a required input to a PC-BC program to find an optimized sequence in which the drawpoints are opened in an automated manner. The models were applied on the two data sets. A simple answer was not found. A combination of the presented models was proposed to help the planner to optimize the sequence. Parkinson demonstrated that integer programming models can generate opening sequences but that the process can be complicated.

Epstein et al. (2012) presented a methodology for long-term mine planning based on a general capacitated multi-commodity network flow formulation. They considered underground and open-pit ore deposits sharing multiple downstream processing plants over a long horizon. The model's target was the optimization of several mines as an integrated problem. LP and IP with a customized procedure were applied to solve the combined model. For the production phase in underground mine, which it was block caving, constraints were production per sector, product and period, production cost, extraction times for each block (at most once), block and period priority, minimum blocks for each column, order of drawpoints, maximum duration of a drawpoint, extraction rate of each column, the column in each period, similarity of heights in neighboring columns, bounds on the area, extracted rock per period, and each sector extraction within its time window. The model developed by Epstein et al. has been implemented at Codelco. Production plans for a single mine and integrating multiple mines increased the NPV.

Diering (2012) used QP techniques for block-caving production scheduling. He focused on single-period formulations. He explained that the block caving process is non-linear (the tons which you mine in later periods will depend on the tons mined in earlier periods), so it would not be appropriate to use LP for production scheduling in block caving. The objective function was the shape of the cave. Three sets of constraints were applied in the model: mandatory, modifying, and grade-related. This formulation omitted the sequence of drawpoint development (interaction between neighboring drawpoints) as a constraint.

Pourrahimian et al. (2012) presented two MILP formulations at two different levels of resolution: (i) drawpoint level, and (ii) aggregated drawpoints (cluster level). The objective function was to maximize the NPV. Pourrahimian et al. used PCBC's slice file as an input into their model, but their models treat the problem in the drawpoint or cluster level as a strategic long-term plan, and the slices are not used in the presented formulations. To reduce the number of binary integer variables, Pourrahimian et al. used Fuzzy c-means clustering to aggregate the drawpoints into clusters based on similarities between draw columns and the physical location of the drawpoint and its tonnage. They used same data for both models and solved the problem for four different advancement directions. The execution time for aggregated drawpoints was reduced by more than 99%.

Pourrahimian et al. (2013) developed a theoretical optimization framework based on a MILP model for block-cave long-term production scheduling. The objective function was to maximize the NPV. Pourrahimian et al. formulated three MILP models for three levels of problem resolution: cluster level, drawpoint level, and drawpoint-and-slice level. They showed that the formulations can be used in both the single-step method, in which each of the formulations is used independently; and as a multi-step method, in which the solution of each step is used to reduce the number of variables in the next level and consequently to generate a practical block-cave schedule in a reasonable amount of CPU runtime for large-scale problems. They considered mining capacity, grade blending, the maximum number of active clusters or drawpoints, the number of new clusters or drawpoints, continuous mining, mining precedence, reserves, and the draw rate as constraints which were involved in the all three levels of resolutions. Using such a flexible formulation is very helpful because depending on the level of studies — prefeasibility studies (PFS), feasibility studies (FS) or detailed feasibility studies (DFS) — a mine planner can use the appropriate level of solution and the related runtime. Pourrahimian et al. developed and tested their methodology in a prototype open-source software application with the graphical user interface DSBC (Drawpoint Scheduling in Block-Caving).

Alonso-Ayuso et al. (2014) considered uncertainty in copper prices along a given time horizon (five years) using a multistage scenario tree to maximize the NPV of a block-cave mine in Chile. The stochastic model then was converted into a MIP model. Alonso-Ayuso et al. applied the stochastic model in both risk-neutral and risk-averse environments. Results showed the advantage of using the risk-neutral strategy over the traditional deterministic approach, as well as the advantage of using any risk-averse strategy over the risk-neutral one.

Rubio (2014) introduced the concept of portfolio optimization for block caving. In this method, every decision related to mine design and mine planning could be a component of a set that defines a feasible portfolio. This set is optimized for different production targets to maximize return subject to a given level of reliability. Using this approach a frontier efficient is proposed as a boundary to display different strategic designs and planning options for the set of variables under study. By this method, the decision makers can define a point along the frontier efficient where they want to place a given project.

Table 3 shows the summary of the aforementioned MP applications in block-caving scheduling.

The available tools for block-cave production scheduling can be divided into two categories: (i) commercial, and (ii) in house tools. One of the commercial software is Geovia PCBC. The program is integrated into a general purpose geological modeling and mine planning system so that it can be used for studies ranging from pre-feasibility to daily draw control. The simulation of mixing is an important part of the program. PCBC simulates the extraction from each active drawpoint period-by-period subject to a range of constraints and inputs (Diering, 2000).

Table 3. Summary of applied MP models in block-caving production scheduling

Author	Model	Model objective(s)	Constraint
Song (1989)	Simulation and MILP	Minimization of total mining cost	• Geometrical limitations • Operational limitations
Chanda (1990)	Simulation and MIP	Minimization of the deviation in the average production grade between operating shifts	• Maximum allowable output per shift • Maximum allowable number of working drawpoints per shift • Declaration of exhaustion for exhausted drawpoints • Required grade for each shift (equality) • Tonnage of blended ore in each shift
Guest et al. (2000)	LP and MILP	Maximization of NPV	• Geotechnical constraints → Column draw rates → Precedence of accumulated tons drawn → Limits in differences of accumulated tons drawn between columns within time horizons → Limits in ratios of tons drawn between columns (neighbors) within time horizons • Mining constraints → Ore flow constraints (tunnels, ore passes, haulage, underground accumulation areas, shaft systems) • Metallurgical constraints → Treatment plant (capacities per period) • Economic constraints (revenue, costs) • Geological constraints (grade, size)
Rubio (2002)	MILP and NLP	Two models (a) maximization of NPV and (b) optimization of the mine life	• Development rate • Undercut sequence • Drawpoint status • Maximum opened production area • Draw rate • Period constraints • Mining reserves
Rubio and Diering (2004)	MILP and QP	Maximization of NPV, optimization of draw profile, and minimization of the gap between long- and short-term planning	• Development rate • Undercut sequence • Maximum opened production area • Draw rate • Draw ratio • Period constraints
Diering (2004)	NLP	Maximizing NPV for M periods and minimization of the deviation between a current draw profile and a defined target	• Minimum and maximum tonnage per period • Maximum total tonnage per drawpoint and per period • Ratio of tonnage from current drawpoint compared with neighbors. • Height of draw of current drawpoint with respect to neighbors • Percentage drawn for current drawpoint with respect to neighbors • Maximum tonnage from selected groups of drawpoints in a period

Table 3. Summary of applied MP models in block-caving production scheduling (continued)

Author	Model	Model objective(s)	Constraint
Rahal (2008)	MILGP	Minimizing deviation from the ideal draw profile while achieving a production target	• Deviation From Ideal Plan → Ideal depletion → Panel production rate → Production from external sources • Contents of material sources • Material flow conservation (blocks, tunnels, ore pass, haulage, accumulation, shaft, plant) • Material flow and capacity (source flow, block, externals, ore pass, haulage, accumulation, shaft, plant) • Production Control → Block available for draw → Relative draw rate → Block flow bounds → Draw maturity rules (Lower Depletion Bound/Upper Production Bound) • Product quality and quantity • Economics
Weintraub et al. (2008)	MIP	Maximization of profit	• Each cluster can be extracted only once • Sequence of extractions • The allowable speed • Capacity of extraction • Conservation of flows and logical relationships between variables
Smoljanovic et al. (2011)	MILP	Optimization of NPV and mining material handling system	• Production Constraints → Max and min amount of tonnage to be extracted per time period → The overall mine capacity → Total number of drawpoints to be opened at each time period → Capacity per drawpoint → Min percent of extraction for each drawpoint → Lifetime of a drawpoint → Capacity of haulage system • Geometric constraints → Connectivity and shape constraints
Epstein et al. (2012)	MIP	Maximization of NPV	• Production per sector (product and period) • Production cost • Extraction times for each block • Block and period priority • Minimum blocks for each column • Order of drawpoints • Maximum duration of a drawpoint • Extraction rate of each column • The column in each period • Neighboring columns heights similarity • Bounds on the area • Extracted rock per period • Each sector extraction within its time window

Table 3. Summary of applied MP models in block-caving production scheduling (continued)

Author	Model	Model objective(s)	Constraint
Diering (2012)	QP	Objective tonnage (to optimize the shape of the cave)	• Mandatory constraints → Production capacity → A maximum tonnage for each drawpoint based on the drawpoint maturity curve → A minimum tonnage for each drawpoint. • Modifying constraints → Maximum tonnage from production tunnels. → Maximum tonnage from an orepass or crusher. → Maximum tonnage from an entire sector • Grade-related constraints
Parkinson (2012)	IP	Finding an optimal opening sequence in an automated manner	• Each drawpoint starts once • Global capacity (processing plant capacity) • Tunnel development • Additional constraints: → Within-tunnel contiguity → Across-tunnel contiguity
Pourrahimian et al. (2013)	MILP	Maximization of NPV	• Mining capacity • Grade blending • Maximum number of active clusters or drawpoints (according to the model resolution) • Number of new clusters or drawpoints (according to the model resolution) • Continuous mining • Mining precedence → Slice → Drawpoint → Cluster • Reserves • Draw rate → Draw column → Cluster
Alonso-Ayuso et al. (2014)	MILP	Maximization of NPV while considering uncertainty in copper price	• Each cluster is processed at most once • If a cluster is processed at a given period then all predecessor clusters are also processed by that period • The clusters in each set would be extracted simultaneously in each sector • Number of tons processed in each sector at each period • Flow conservation constraints for the processing stream • Number of tons processed in each period • Upper and lower bounds for the total area processed in each sector • Number of tons processed in each period • Upper bound due to the capacity of processing stream • The maximum increase and decrease of tons in each sector in each period

In mathematical programming, we look for values of variables which are allowed and which do not violate the constraints. This defines what is called a solution space, in which the edges of this space are the constraints. In case of an LP formulation, the solution must be on a boundary of this space. In the case of block-cave scheduling, an LP formulation will always seek to take the maximum tons from the highest value drawpoints and the least tons from the lower valued drawpoints (Diering, 2012). As a result, this kind of scheduling may result in high levels of horizontal mixing between drawpoints because the draw columns have different heights. This is a potential disadvantage of LP application in block-caving scheduling. Table 4 summarizes the advantages and disadvantages of methodologies examined in previous studies.

Table 4. Advantages and disadvantages of applied mathematical methodologies in block-caving production scheduling

Methodology		Features
LP	Advantage	• LP method has been used most extensively (Rahal, 2008) • It can provide a mathematically provable optimum schedule (Rahal, 2008)
	Disadvantage	• Straight LP lacks the flexibility to directly model complex underground operations which require integer decision variables (Winkler, 1996) • Mine scheduling is too complex to model using LP and the only possible approach is to use some combination of theoretical and heuristic methods to ensure a good, if not optimal schedule (Scheck et al., 1988)
MILP	Advantage	• Computational ease in solving a MIP problem (and MILP) is dependent upon the formulation structure (Williams, 1974) • MILP could be used to provide a series of schedules which are marginally inferior to a provable optimum (Hajdasinski, 2001) • MILP is superior to simulation when used to generate sub-optimal schedules, because the gap between the MILP feasible solution and the relaxed LP solution provides a measure of solution quality (Rahal, 2008) • MILP can provide a mathematically provable optimum schedule (Rahal, 2008)
	Disadvantage	• It is often difficult to optimize large production systems using the branch-and-bound search method (Rahal, 2008) • The block-caving process is non-linear (the tons which you mine in later periods will depend on the tons mined in earlier periods), so it would not be appropriate to use LP for production scheduling in block caving (Diering, 2012)
QP	Advantage	• Since the block-caving process is non-linear, QP could be an appropriate option to model it • It can find solutions in the interior of the solution space, which results in an even height of drawpoints as well as lower horizontal mixing between drawpoints (Diering, 2012)
	Disadvantage	• Solving this kind of problem could be a challenge. It must be changed to LP and then be solved, to ensure conversion errors

6. Conclusion

Increasing the use of block-cave mining in new-world mining environments has led many researchers to focus on this area to make mining operations as optimal as possible. Production scheduling in block caving, because of its significant impact on the project's value, has been considered a key issue to be improved. The problem is complex, unique for each case, large-scale, and non-linear. Researchers have applied different methods to model production scheduling in block caving, for short-term and long-term periods of mining, some for real case studies as industry projects and others as academic research projects.

Generally speaking, confronting future challenges in block-cave mining can be divided into two categories: (i) operational and (ii) economical. Block caving is known as a low-cost mining method which makes it possible to mine the low grade ore-bodies, therefore, optimal production schedule with lower cost is required. Block-cave mining is one of the best solutions for continuing the operation after shutting down the mine in deep open-pit mines. The new operation (block caving) has to feed the processing plant which used to be fed by the open-pit mine. Therefore, the production rate in the block-cave operation has to be as high as the open-pit mining. Although some semi auto mining equipment has been introduced for block caving, but it is just the starting point to reach the full automated operations. Also, making decisions about the geometry of drawpoints, best height of draw, undercut level, and the production level are critical and challenging. Block-cave mining usually requires much more development compare to other mining methods which needs a long period of time before starting the production, so the high capital cost is needed to run the project. High capital cost increases the risk of project. The operational costs of block cave mining is low but if the rock mass caveability is not achieved as it expected, the costs for additional drilling and blasting can be definitely challenging.

Most of researchers have applied MILP to model production scheduling; it can be useful because both the integer variables (whether a block, slice, or draw column should be extracted) and continues variables (the constraints and mining operation details) can be modeled so that the optimal values can be achieved while considering the system's constraints. Basically, the block-caving operation is non-linear. Therefore, linear programming could be an inappropriate method to model production scheduling. Quadratic programming as a non-linear methodology has been applied by a few researchers for block-cave mining scheduling.

As computer base algorithms are improved, we expect to see the development of more detailed models with more complexity, models that try to be more practical and include all aspects of mining systems, with new algorithms for faster solutions. Using non-linear methodologies for multi-period scheduling with a reasonable solution time would improve block-caving scheduling. In block-caving operations, decisions about current actions are often based on how those actions affect future actions. For this reason, a real options technique can be properly used in production scheduling optimization.

There are some uncertainties in block-cave mining that should be involved in production scheduling. Grade uncertainty is one of the most common, because of the nature of ore-body, but in block-caving operations grade uncertainty is more critical and complicated, due to the vertical and horizontal mixing which occurs during the caving and production processes. Once the rock is fragmented the particles of the rock flow towards the production level in different ways depending on the fragmentation profile and distribution. Price uncertainty is another variable which should be considered when attempting to achieve realistic optimal production scheduling.

Blasting operations in block caving has a critical impact on fragmentation and, as a result, on material flow. The gravity flow of fragmented rock plays an important role in the production rate and grade in block-caving operations. When considering the material flow in the presence of blasting parameters, production scheduling could result in more realistic plans. Since rock mass is broken by caving the actual fragmentation expected at the drawpoints is uncertain. Therefore, drawpoint productivity is uncertain and the amount of area that needs to be developed and undercutted also becomes uncertain.

Geotechnical aspects of ore-body and its surrounding rocks determine caveability and the efficiency of block-caving operations. Using more geotechnical constraints in production scheduling modeling helps the mine planners to have more confidence about scheduling results. If the rock mass is high stress and competent the cave propagation could be uncertain and triggered erratic dilution and non-uniform grade extraction.

Developing accurate clustering methods, with more flexible levels of problem resolution, could lead to better options for mine planners during the different stages of planning.

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Mining Options Optimization: Open Pit to Underground

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Abstract

Near-surface deposits that extend to considerable depths are often amenable to both open pit mining and/or underground mining. This paper investigates the strategy of mining options for an orebody using a Mixed Integer Linear Programming (MILP) optimization framework. The MILP formulation maximizes the net present value (NPV) of the reserve when extracted with i) open pit mining, ii) underground mining, and iii) concurrent open pit and underground mining. The NPV generated at a 5% discount rate when the orebody is extracted with i) open pit mining is $2103 M, ii) underground mining is $822 M and iii) concurrent open pit and underground mining is $2154 M. Comparatively, implementing open pit mining generates a higher NPV than underground mining. However considering the investment required for these mining options, underground mining generates a better return on investment than open pit mining. The results also show that in the concurrent open pit and underground mining scenario, the optimizer prefers extracting blocks using open pit mining. This is due to the fact that although the underground mine could access ore sooner, the mining cost differential for open pit mining is more than compensated for by the discounting benefits associated with earlier underground mining.

1. Introduction

Mining is the process of extracting a beneficial naturally occurring resource from the earth (Newman et al., 2010) and historical assessment of mineral resource evaluations has demonstrated the sensitivity of project profitability to decisions based on mine planning. A major aspect of mine planning is the optimization of long-term production scheduling. The aim of long-term production scheduling is to determine the time and sequence of extraction and displacement of ore and waste in order to maximize the overall discounted net revenue from a mine within the existing economic, technical and environmental constraints. Long-term production schedules defines the mining and processing plant capacity, and expansion potential as well as management investment strategy. In mining projects, deviations from optimal mine plans may result in significant financial losses, future financial liabilities, delayed reclamation and resource sterilization.

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The problem of optimizing reserve exploitation depends largely on the mining option used in the extraction. Some mineral deposits have orebodies that extend from near the surface to several meters in depth. Such deposits can be amenable to both open pit mining and/or underground mining. Significant value can be generated by rigorously investigating these mining options using optimization tools to arrive at the appropriate strategic plan that maximizes the overall net present value of the deposit (Roberts et al., 2009). Open pit mining usually features a relatively lower mining cost, higher stripping ratio and longer time to access ore. Underground mining on the other hand features a higher mining cost, higher grade and earlier access to ore (Anthony, 2012). There are currently limited tools or methods to directly optimize this interface. It is our objective to develop a Mixed Integer Linear Programming (MILP) framework and methodology to evaluate the financial impact of applying different mining options separately or concurrently to extract a given orebody.

The next section of this paper presents the mining options problem definition and section 3 outlines a MILP model framework for strategic mining options optimization. Section 4 covers the modeling and material flow network of the mining options problem. Details of a numerical experiment to be conducted are outlined in section 5 and the application of the MILP formulation to a case study is discussed in section 6. The paper concludes in section 7.

2. The mining options problem

A strategic plan is to be developed for a moderate dipping gold-silver-copper orebody that is amenable to both open pit and underground mining. It is required that in addition to these mining options, a combined case of concurrent open pit and underground mining is investigated as well. From preliminary underground mining studies, a selective underground mining method known as long hole open stoping was identified as a potentially viable underground mining method. The production schedule for such a combined case requires that both mining options compete for the same reserve during optimization. The problem presented here involves scheduling of N different ore and waste blocks: i) within the final pit limit over T different periods of extraction – OP mining, ii) within the economic stope outlines over T different periods of extraction – OS mining, and iii) within the combined final pit limit and economic stope outlines over T different periods of extraction – COPOS mining. The schedule should maximize the NPV of the operation subject to a variety of physical, technical and economic constraints. Fig. 1 shows a schematic diagram of the problem definition. A MILP formulation was developed for this strategic mining options optimization study.

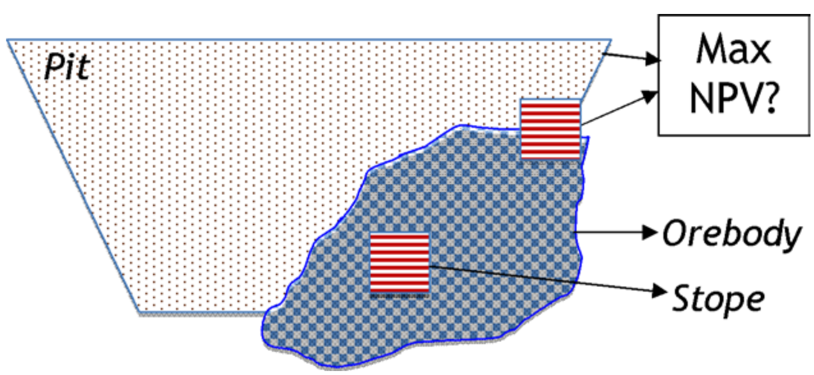

Fig. 1. Schematic representation of the problem definition showing the mining options (Ben-Awuah et al., 2015)

3. Integrated MILP model for OP, OS and COPOS mining

The basic problem of concern can be simplified as finding the time and sequence of extraction of ore and waste blocks to be removed from the predefined open pit and/or open stopes outlines and their respective destinations over the mine life so that the NPV of the operation is maximized. The production schedule is subject to a variety of technical, physical and economic constraints which enforce the mining extraction sequence, mining and processing capacities, and blending requirements. The notations used in the formulation have been classified as sets, indices, subscripts, superscripts, parameters and decision variables.

3.1. Economic block value modeling

The objective function of the MILP model is to maximize the net present value of the mining operation. This requires that economic block values are defined based on ore parcels which could be mined selectively. The profit generated from mining a block depends on the value of that block and the costs incurred in mining and processing the block. The cost of mining a block is a function of its spatial location. When a mined block is sent to the stockpile prior to processing, then an extra cost of re-handling is applied.

The discounted economic block value for block *k* is equal to the discounted revenue generated by selling the final product contained in block *k* minus all the discounted costs involved in extracting block *k* and processing it. The discounted economic block value is computed separately for when the block is extracted by open pit mining and when it is extracted by open stope mining. This can be summarized by Eqs. (1) to (6).

Discounted economic block value = discounted revenue - discounted costs

$$d_k^{op,t} = v_k^t - q_k^{op,t} - l_k^{op,t} \tag{1}$$

$$d_k^{os,t} = v_k^t - q_k^{os,t} \tag{2}$$

The variables in Eqs. (1) and (2) can be decomposed into Eqs. (3) to (6).

$$v_k^t = \sum_{e=1}^{E} o_k \times g_k^e \times r^{e,t} \times \left(p^{e,t} - cs^{e,t}\right) - \sum_{e=1}^{E} o_k \times cp^{e,t} \tag{3}$$

$$q_k^{op,t} = \left(o_k + w_k\right) \times cm^{op,t} \tag{4}$$

$$l_k^{op,t} = o_k \times rh^{op,t} \tag{5}$$

$$q_k^{os,t} = \left(o_k + w_k\right) \times cm^{os,t} \tag{6}$$

Where

$t \in \{1,......,T\}$ index for scheduling periods

$k \in \{1,.....,K\}$ index for blocks

$e \in \{1,.....,E\}$ index for element of interest in each block

$j \in \{1,.....,J\}$ index for phases (pushback)

$d_k^{op,t}$ the open pit discounted economic block value generated by extracting block *k* in period *t*

$d_k^{os,t}$ the open stope discounted economic block value generated by extracting block k in period t

v_k^t the discounted revenue generated by selling the final product within block k in period t minus the extra discounted cost of mining all the material in block k as ore and processing it

$q_k^{op,t}$ the open pit discounted cost of mining all the material in block k in period t as waste

$l_k^{op,t}$ the open pit discounted cost of re-handling for all material in block k in period t processed from the stockpile

$q_k^{os,t}$ the open stope discounted cost of mining all the material in block k in period t as waste

o_k the ore tonnage in block k

w_k the waste tonnage in block k

g_k^e the average grade of element e in ore portion of block k

$r^{e,t}$ the processing recovery factor for element e

$p^{e,t}$ the price of element e in present value terms per unit of product

$cs^{e,t}$ the selling cost of element e in present value terms per unit of product

$cp^{e,t}$ the extra cost in present value terms per tonne of ore for mining and processing

$cm^{op,t}$ the open pit cost in present value terms of mining a tonne of waste in period t

$rh^{op,t}$ the open pit cost in present value terms of re-handling a tonne of ore in period t

$cm^{os,t}$ the open stope cost in present value terms of mining a tonne of waste in period t

3.2. Integrated MILP model objective function

In the proposed integrated MILP model, the formulation is cast to ensure that material can be extracted only once by either of the mining options. The MILP model objective function can be formulated as: 1) maximizing the NPV of the open pit mining operation and 2) maximizing the NPV of the open stope mining operation. This is represented by Eq. (7). We used the concepts presented in Askari-Nasab et al. (2012) as the starting point of our development. The amount of ore processed is controlled by the continuous decision variables $x_k^{op,t}$ and $x_k^{os,t}$ for open pit and open stope mining respectively. The amount of material mined is controlled by the continuous decision variables $y_k^{op,t}$ and $y_k^{os,t}$ for open pit and open stope mining respectively. The amount of material re-handled through the open pit stockpile is controlled by the decision variable $h_k^{op,t}$. The continuous variables enable fractional extraction of blocks in different periods.

$$Max\left[\sum_{t=1}^{T}\left(\sum_{j=1}^{J}\sum_{k\in B_j}\left(v_k^t\times x_k^{op,t}-q_k^{op,t}\times y_k^{op,t}-l_k^{op,t}\times h_k^{op,t}\right)+\sum_{p=1}^{P}\sum_{k\in B_p}\left(v_k^t\times x_k^{os,t}-q_k^{os,t}\times y_k^{os,t}\right)\right)\right] \quad (7)$$

Where

$x_k^{op,t},\ x_k^{os,t}\in\{0,1\}$ are continuous decision variables representing the portion of block k extracted as ore and processed in period t for open pit and open stope mining respectively.

$y_k^{op,t},\ y_k^{os,t} \in \{0,1\}$	are continuous decision variables representing the portion of block *k* mined in period *t* for open pit and open stope mining respectively. Fractions of *y* characterize both ore and waste material in the block.
$h_k^{op,t} \in \{0,1\}$	is a continuous decision variable representing the portion of block *k* re-handled in period *t* from the open pit mining stockpile
B_j	represents the set of blocks within the mining phase *j*
B_p	represents the set of blocks within the mining stope *p*

3.3. Integrated MILP model mining and processing constraints

The MILP model constraints are used in controlling the mining, processing, stockpiling and plant head grade targets in the mining options. They are defined in the form of upper and lower bounds which limit the amount of resource allocated to the associated activity. These constraints are represented by Eqs. (8) to (17).

$$T_{m,lb}^{op,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(o_k + w_k\right)\times y_k^{op,t}\right) \le T_{m,ub}^{op,t} \tag{8}$$

$$T_{m,lb}^{os,t} \le \sum_{p=1}^{P}\left(\sum_{k\in B_p}\left(o_k + w_k\right)\times y_k^{os,t}\right) \le T_{m,ub}^{os,t} \tag{9}$$

$$\left(\left(o_k + w_k\right)\times y_k^{op,t}\right) + \left(\left(o_k + w_k\right)\times y_k^{os,t}\right) = o_k + w_k \tag{10}$$

$$T_{pr,lb}^{op,t} \le \sum_{k=1}^{K}\left(o_k \times x_k^{op,t}\right) \le T_{pr,ub}^{op,t} \tag{11}$$

$$T_{pr,lb}^{os,t} \le \sum_{k=1}^{K}\left(o_k \times x_k^{os,t}\right) \le T_{pr,ub}^{os,t} \tag{12}$$

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\left(\left(o_k \times m_k^{op,t}\right) - \left(o_k \times h_k^{op,t}\right)\right) = 0 \tag{13}$$

$$\sum_{k=1}^{K}\left(g_k^e \times o_k \times x_k^{op,t}\right) - \overline{g}^{op,t,e}\sum_{k=1}^{K}\left(o_k \times x_k^{op,t}\right) \le 0 \tag{14}$$

$$\sum_{k=1}^{K}\left(g_k^e \times o_k \times x_k^{op,t}\right) - \underline{g}^{op,t,e}\sum_{k=1}^{K}\left(o_k \times x_k^{op,t}\right) \ge 0 \tag{15}$$

$$\sum_{k=1}^{K}\left(g_k^e \times o_k \times x_k^{os,t}\right) - \overline{g}^{os,t,e}\sum_{k=1}^{K}\left(o_k \times x_k^{os,t}\right) \le 0 \tag{16}$$

$$\sum_{k=1}^{K}\left(g_k^e \times o_k \times x_k^{os,t}\right) - \underline{g}^{os,t,e}\sum_{k=1}^{K}\left(o_k \times x_k^{os,t}\right) \ge 0 \tag{17}$$

Where

$m_k^{op,t} \in \{0,1\}$	is a continuous decision variable representing the portion of block *k* mined in period *t* and sent to the open pit stockpile
$T_{m,ub}^{op,t},\ T_{m,lb}^{op,t}$	are the upper and lower bounds on the available open pit mining capacity in period *t*
$T_{m,ub}^{os,t},\ T_{m,lb}^{os,t}$	are the upper and lower bounds on the available open stope mining capacity in period *t*
$T_{pr,ub}^{op,t},\ T_{pr,lb}^{op,t}$	are the upper and lower bounds on processing capacity of ore from open pit mining in period *t*
$T_{pr,ub}^{os,t},\ T_{pr,lb}^{os,t}$	are the upper and lower bounds on processing capacity of ore from open stope mining in period *t*
$\overline{g}^{op,t,e},\ \underline{g}^{op,t,e}$	are the upper and lower bounds on required average head grade of element *e* in period *t* in open pit mining
$\overline{g}^{os,t,e},\ \underline{g}^{os,t,e}$	are the upper and lower bounds on required average head grade of element *e* in period *t* in open stope mining

Eqs. (8) and (9) are the mining capacity constraints for open pit and open stope mining respectively. These constraints are controlled by the continuous decision variables $y_k^{op,t}$ and $y_k^{os,t}$. These inequalities ensure that the total tonnage of material mined (ore and waste) in each period is within the range of the total available equipment capacity in that period. Eqs. (8) and (9) are used in manipulating the stripping ratio over the mine life. The set mining capacity is a function of the ore reserve, targeted mine-life, designed processing capacity, overall stripping ratio and the available capital for mining fleet acquisition.

Eq. (10) represents the concurrent mining constraint that controls the mining options. This constraint is controlled by the continuous decision variables $y_k^{op,t}$ and $y_k^{os,t}$. This inequality manages the relationship of block extraction in the open pit mining and in the open stope mining. It ensures that all blocks are extracted once either through the open pit and/or through the open stope mine. This constraint can be extended to represent mining bins which are made up of accumulation of blocks.

Eqs. (11) and (12) represent the processing capacity functions which control the mill feed quantities for open pit and open stope mining respectively. These constraints are controlled by the continuous decision variables $x_k^{op,t}$ and $x_k^{os,t}$. These inequalities help the mine planner to provide a uniform feed throughout the mine life resulting in an effectively integrated mine-to-mill operation. Depending on the ore grade distribution of the orebody, the processing target may not be achieved in some periods. In such cases, pre-stripping and stockpiling could be explored to provide a uniform mill feed. This amounts to forcing the optimizer to mine waste in the early periods, or mining more ore than needed when available and feeding the mill with the stockpiled ore when required.

Eq. (13) represents the stockpile management constraint which controls the level of the stockpile during the mine life of the open pit mining operation. The constraint is controlled by the continuous decision variables $m_k^{op,t}$ and $h_k^{op,t}$. This equation ensures that all the material stockpiled in the open pit operation is re-handled back to the processing plant by the end of mine life. Thus, limits the stockpile material to only the ore that has potential positive cash flow in the future prior to the end of the mine life. Stockpiling is not recommended for open stope mining due to the relatively high cost of production.

The grade blending constraints are represented by Eqs. (14) to (17). These constraints monitor the mill feed quality and are controlled by the continuous decision variables $x_k^{op,t}$ and $x_k^{os,t}$. Eqs. (14) and (15) specify the limiting grade requirements for material from the open pit operation for processing whilst Eqs. (16) and (17) specify the limiting grade requirements for material from the open stope operation for processing. The objective of blending in production scheduling is to mine in a way that the run-of-mine materials meet the quality and quantity specification of the processing plant. The mill head grade is a function of the ore grade distribution, processing plant design and mine cash flow requirements.

3.4. Integrated MILP model general constraints

The general constraints that apply to the MILP model discussed relate to the mining precedence and the logics of the variables during optimization. These have been documented in Ben-Awuah et al. (2012) and Ben-Awuah and Askari-Nasab (2013). These constraints include:

a) Open pit vertical mining precedence: all the immediate predecessor mining blocks above the current mining block should be extracted prior to extracting the current mining block.

b) Open stope vertical mining precedence: all the immediate predecessor mining development decline above the current development level should be extracted prior to extracting the current development decline.

c) Open pit horizontal mining precedence: all the immediate predecessor mining phases preceding the current mining phase in the horizontal mining direction are extracted before or together with the current mining phase. These are referred to as absolute and concurrent precedences respectively.

d) Open stope horizontal mining precedence: all the immediate predecessor mining stope and development drive preceding the current mining stope and development drive in the horizontal mining direction are extracted before or together with the current mining stope and development drive. These are referred to as absolute and concurrent precedences respectively.

e) Variables logic control: the logic of the mining, processing and stockpiling variables with regards to their limits and definitions are within acceptable ranges.

4. Modeling the mining options problem

The mining options problem is modeled as a multiple mine and destination optimization problem in Evaluator (Snowden Mining Industry Consultants, 2013); an optimization modelling platform. The modelled problem is solved with a commercial optimization solver Gurobi (Gurobi Optimization, 2013). Fig. 2 shows a schematic material flow network diagram of the scheduling project. The COPOS problem is set up to ensure that during concurrent mining all material is mined once by either of the mining options. This is enforced using Eq. (10). The COPOS problem was modeled with two mining nodes namely OP and OS. The OP node holds all the mining blocks data relating to open pit mining and the OS node holds all the access development and mining stopes data relating to open stope mining.

Material from the open pit operation can be sent to the open pit processing plant, open pit stockpile or open pit waste dump based on the material type and mine economics. Material sent to the open pit processing plant results in a product that generates revenue for the open pit mining project. Material sent to the open pit stockpile is later re-handled back to the processing plant. The open pit stockpile management constraint (Eq. (13)) ensures that by the end of mine life no material is left on the stockpile. This equation indirectly constrains the stockpile material to only those that can still generate positive cash flow in the future. Material that does not qualify for processing or stockpiling is sent to the open pit waste dump. The constraints set up to control the open pit mining operation are mainly the mining capacity, processing limits, stockpiling control and ore quality requirements throughout the mine life. The vertical and horizontal mining sequences for the open pit mining blocks and mining phases which include both absolute and concurrent precedences are defined as well.

Material from the open stope operation can be sent to the open stope processing plant or access development waste dump based on the material type and mine economics. The access development includes both decline and drive developments. Material sent to the processing plant results in a product that generates revenue for the open stope mining operation. The decline and drive development material mined prior to stope developments are sent to the access development waste dump. No other waste or stockpile materials are extracted due to the high cost of production associated with underground mining. The constraints set up to control the stope mining are mainly mining capacity, processing limits and the ore quality requirements throughout the mine life. The vertical and horizontal mining sequences for the mining stopes, development declines and development drives which include both absolute and concurrent precedences are applied as well.

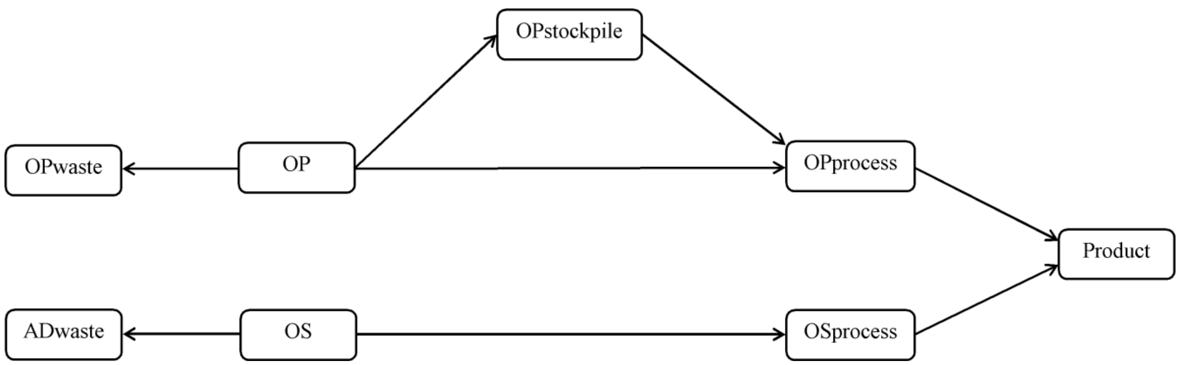

Fig. 2. Schematic material flow network diagram (Ben-Awuah et al., 2015)

5. Computational experiment

The MILP model for the mining options problem was implemented on a gold-silver-copper orebody which has a potential for both open pit mining and underground mining. The objective was to use the MILP model to investigate a potential mining strategy that maximizes the overall profit for this deposit. The performance of the proposed model was assessed based on net present value and smoothness of the generated schedules. The MILP model was setup for OP mining and OS mining to compete for the same material during optimization subject to each method's respective mining and economic parameters.

An initial analysis of the orebody for OP mining resulted in a final pit shell being generated. Based on the incremental revenue factors, four pit stages were identified and designed as the main stages suitable for the open pit mining operation. Similarly, initial analysis of the orebody for selective underground mining option resulted in the decision to use long hole open stope mining with blocks above a cut-off grade of $80/t net smelter return (NSR). This had the potential for an economically viable underground mining operation. Table 1 summarizes the mineable inventory for OP and OS mining. It should be noted that the total ore tonnes for OP mining is 143 Mt; for OS mining is 147 Mt; and for COPOS mining is 147 Mt (Table 1). Since this study includes a combined option of simultaneous open pit and open stope mining, the block model had to be re-blocked to a common size that would serve both OP and OS requirements. Each block in the economic block model therefore carries an economic block value, $d_k^{op,t}$ when it is extracted by OP mining and $d_k^{os,t}$ when it is extracted by OS mining. In this case, the decision to mine a block during optimization is based mainly on the mining economics. All scenarios were solved to within 1% optimality gap. In the COPOS mining option, no geotechnical assessment of the interaction of the mining systems was done.

Table 1. Total mineable inventory

Mining phases	Ore tonnes (Mt)	Au (g/t)	NSR ($/t)
OP_1, OP_2, OP_3, OP_4	143	1.13	50.45
OS	4	2.50	115.95

6. Results and discussions

The optimization study was based on three scenarios namely: i) open pit mining only (OP); ii) open stope mining only (OS); and iii) concurrent open pit and open stope mining (COPOS). All scenarios were based on a high pre-production capital with corresponding lower operating costs. The mineable inventory for OP and OS combined is 147 Mt of ore with recoverable metal of 4.34 Moz Au. A production schedule was generated with the MILP model using the mining and processing constraints summarized in Table 2.

Table 2. Summary of mining and processing constraints

Scheduling constraints	Mining option scenario		
	OP	OS	COPOS
Mining limit, OP (Mtpa)	80	-	80
Mining limit, OS (Mtpa)	-	3	3
Processing limit, OP (Mtpa)	11	-	11
Processing limit, OS (Mtpa)	-	3	3
Vertical mining rate limit, OP (m/yr)	96	-	96
Vertical mining rate limit, OS (m/yr)	-	50	50

After investigating the scenarios at a 5% annual discount rate, the results of the production schedule optimization have been summarized in Table 3. The results show that for OP mining option, the total ore processed was 83 Mt generating an NPV of $2103 M over nine years mine life. For the OS mining option, the total ore processed was 18 Mt generating an NPV of $822 M over nine years mine life. For the COPOS mining option, the total ore processed by the open pit processing plant was 83 Mt and the total ore processed by the open stope processing plant was 3 Mt generating a total of $2154 M over nine years mine life.

Table 3. Production scheduling optimization results

Mining option scenario	Total ore processed OP/OS (Mt)	Mine life OP/OS (yrs)	NPV ($M)
OP mining	83/0	9/0	2103
OS mining	0/18	0/9	822
COPOS mining	83/3	9/9	2154

Fig. 3 to Fig. 10 show the production schedule profile over the mine life for each of the mining options. Fig. 3 and Fig. 4 show that during the OP mining, all the material extracted come from the OP mining phases alone. Mining starts from OP phase 1 (OP_1) and phase 2 (OP_2) and progresses to phases 3 and 4 maintaining a relatively uniform stripping ratio from year 2 to the end of mine life. Year 1 was mainly used for pre-stripping and ore processing starts in year 2. The processing plant operated at it maximum capacity throughout the mine life. Material from the stockpile was used in supporting the processing plant from year 6 (Fig. 5).

Fig. 6 and Fig. 7 show that during the OS mining, material is extracted from both the open pit phases and the open stope phase. Mining starts from open pit phases 2, 3 and 4 (OP_2, OP_3 and OP_4) and progresses to the open stope phase (OS) and open pit phase 1 (OP_1). Year 1 is mainly for underground access development. Processing starts in year 2 and ramps up to full capacity in year 3. All material mined are processed since it falls above the cut-off grade.

Fig. 8 and Fig. 9 show that during the COPOS mining option, material is extracted from both the open pit phases and the open stope phases. Mining starts from open pit phases 1 and 2 and open stope access developments (AD). Mining progresses to the open stope phase and open pit phases 3 and 4 maintaining a relatively uniform stripping ratio from year 2. Year 1 is used mainly for pre-stripping for the open pit mining and access development for the open stope mining. Ore processing starts in year 2 at full plant capacity and remains steady for the rest of the mine life. Material from the stockpile was used in supporting the processing plant from year 6 (Fig. 10). All material processed for the open stope mine comes from the open stope phase.

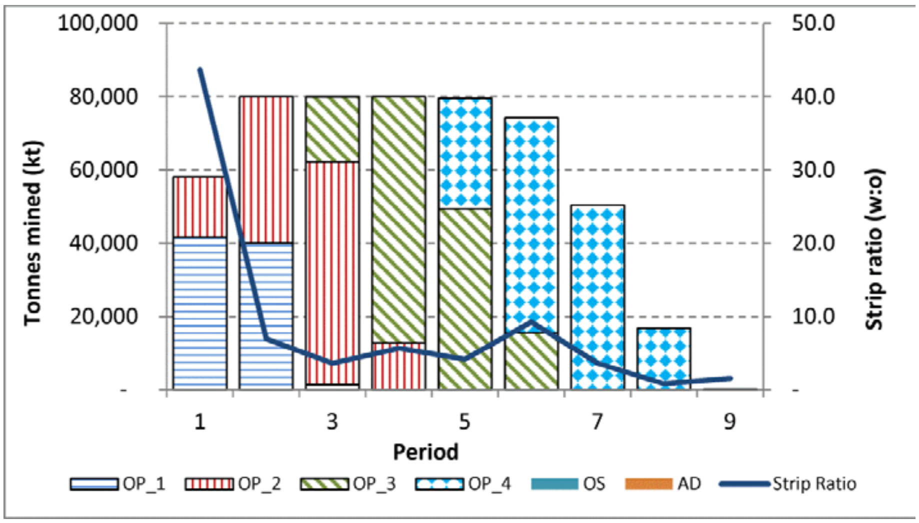

Fig. 3. OP mining – mining by phases

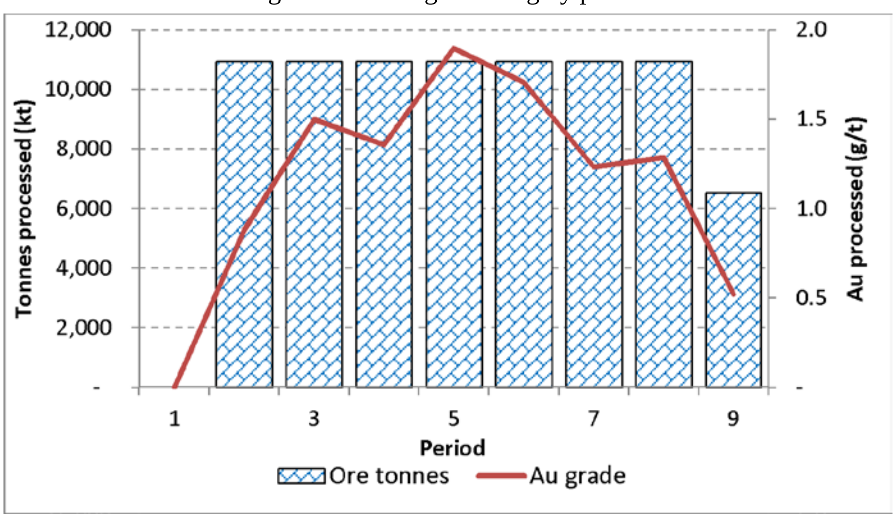

Fig. 4. OP mining – ore processed

Fig. 5. OP mining – stockpile variations

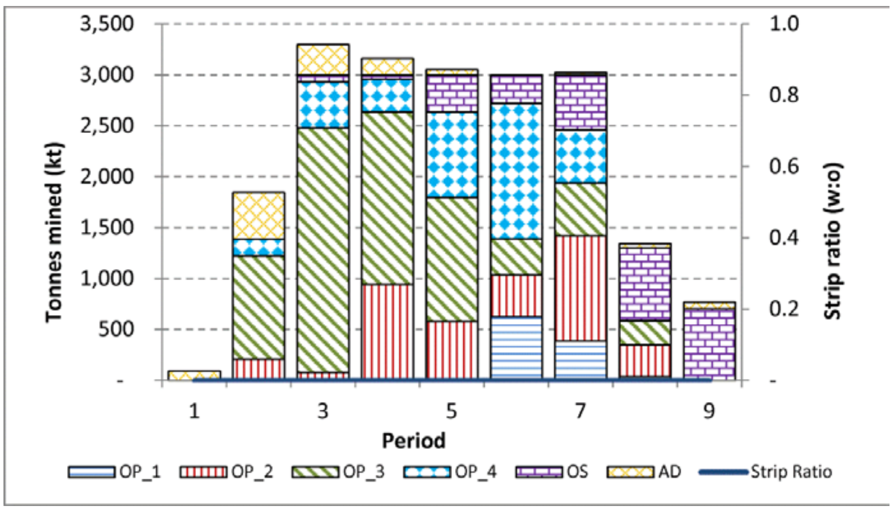

Fig. 6. OS mining – mining by phases

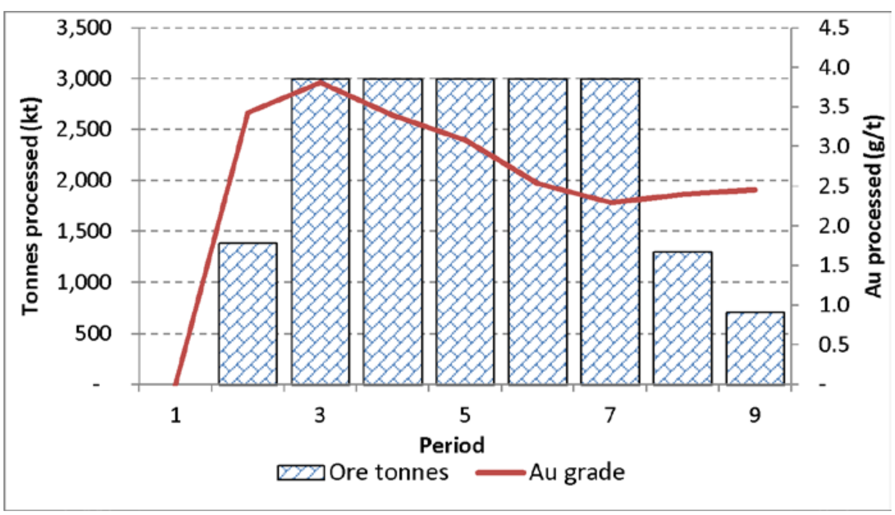

Fig. 7. OS mining – ore processed

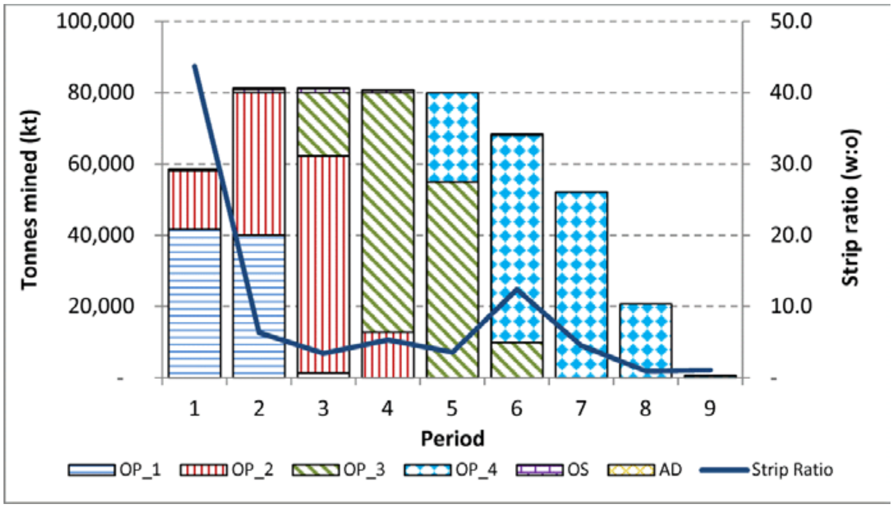

Fig. 8. COPOS mining – mining by phases

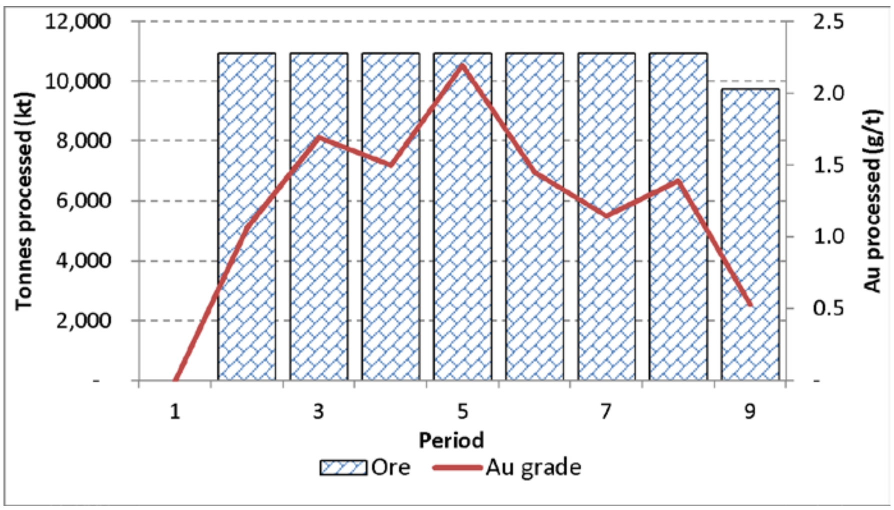

Fig. 9. COPOS mining – ore processed

Fig. 10. COPOS mining – stockpile variations

7. Conclusions

We have developed, implemented and verified a MILP formulation and methodology which seeks to evaluate the value of an orebody using different mining options with complex production requirements. The MILP optimization framework has proved to be robust in providing a global optimization solution when assessing different mining options. It can also be extended to determine the change-over point between an open pit mining operation and an underground mining operation. The different mining options were evaluated based on the assumption of a high pre-production capital investment with low operating cost. The NPV generated at a 5% discount rate when the orebody is extracted with i) OP mining option is $2103 M, ii) OS mining option is $822 M and iii) COPOS mining option is $2154 M. A summary of the conclusions drawn after the mining options optimization study with the MILP model are:

1. In the COPOS mining scenario, the optimizer prefers extracting blocks using OP mining. This is due to the fact that although the OS mine could access ore sooner, the mining cost differential for OP mining is more than compensated for by the discounting benefits associated with earlier OS mining.
2. The COPOS mining option generates a higher relative NPV compared to the individual mining cases. However, it requires a higher capital expenditure (CAPEX) outlay. The addition of the OS material to the OP material only adds a marginal increase of 2% in NPV which cannot offset the CAPEX required to develop an OS mining operation.
3. Comparatively, the OP mining option generates a higher NPV than the OS mining option. However considering the investment required for these mining options, the OS mining option generates a better return on investment than the OP mining option.
4. Sensitivity analysis for the COPOS mining option shows that as the discount rate or mine life significantly increases some OP material become attractive for OS extraction in the early years of mine life.

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Determination of Development Precedence for Drawpoints in Block-Cave Mining

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Abstract

High rate of production, low operational cost, and automated systems have made block caving attractive to mine owners and mining engineers. Two of the key steps in block-caving operation scheduling are the development direction and the drawpoints' precedence determination. An optimum direction for cave development and precedence for drawpoint extraction can add remarkably to the net present value of the mining project. In this paper, a methodology is introduced in order to find the best direction for development and the best precedence of extraction based on the development direction. In the first step, drawpoints are evaluated using the adjacent concept to find the best direction of development; the selection is based on the draw economic value (DEV). In the next step, the best precedence for all drawpoints is determined. Results show that this methodology can lead the decision makers to more realistic and near-optimum options in short- and long-term scheduling.

1. Introduction

As the mineral resources near the surface are being exploited, the mining operation goes deeper into the ground, waste removal rates increase, capital and operation costs become higher, and environmental impacts are more evident. In such a situation, underground mining with lower waste removal and less environmental impact are becoming more attractive. Usually the main problem with underground mining is the low rate of production and high operation costs that make the undertaking less practical. Among underground methods, block-cave mining with its high rate of production, low operational cost, and automated systems can be one of the best choices instead of surface mining or block-cave mining can be considered as part of production after surface mining during the life of mine.

In block-cave mining, gravity plays the main role for production. After creating drawpoints (production level) and developing an empty space as the undercut level (the gap between the rock mass and drawpoints), the rock is fractured because of the weight of rock mass above it and then the crushed rock is exploited using drawpoints. The whole procedure seems to be easy, but there are actually many factors involved with the operations. In particular, production scheduling in block-cave mining is a multi-criteria decision making problem with related constraints. In this paper, we

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model one of the most important steps in the production scheduling of block caving in order to improve the profitability of the mining project.

2. Production scheduling in block-cave mining

Production scheduling in block-cave mining is the determination of the amount of ore extraction from each drawpoint in each period of production in order to achieve the maximum net present value (NPV) regarding the project's constraints. Production scheduling is one of the most challenging problems in both open-pit and underground mining. Many researchers have focused on solving production scheduling problems using different methods of mathematical programing, such as Linear Programming (LP), Mixed-Integer Linear Programming (MILP), and Quadratic Programming (QP). Some mathematical programming models have been proposed to optimize the production scheduling for a block-cave mining operation: LP (Winkler 1996, Guest, Van Hout et al. 2000, Hannweg and Van Hout 2001), MILP (Song 1989, Chanda 1990, Winkler 1996, Guest, Van Hout et al. 2000, Rubio 2002, Rahal, Smith et al. 2003, Rubio and Diering 2004, Rahal 2008, Rahal, Dudley et al. 2008, Weintraub, Pereira et al. 2008, Smoljanovic, Rubio et al. 2011, Epstein, Goic et al. 2012, Parkinson 2012, Pourrahimian 2013, Alonso-Ayuso, Carvallo et al. 2014, Khodayari and Pourrahimian 2014), and QP (Rubio and Diering 2004, Diering 2012).

The first step of the production scheduling is the determination of the mining advancement direction. The second step is finding the precedence of extraction based on the defined direction. Reviewing the literature, we see that the proposed production scheduling models are based on a manual direction selection. In the other words, the mining direction is selected manually and then the production schedule is generated for the defined direction at the first step. Since the first step of the optimization is manual, there is no guarantee that the resulting production schedules are optimal. Also, some researchers have applied the optimization for different directions (Pourrahimian 2013). Our research, however, proposes a methodology to find the best mining direction in block-caving operations using an automated mathematical method.

3. Mining direction determination

In block-cave mining, the production starts from one part of the ore-body and then continues to the other side(s) of the ore-body. Fig 1 indicates that the mining started at the north-east side of the layout (A to B). Mining direction is determined according to different factors, such as the geotechnical parameters of the ore-body and overburden, grade distribution in different parts of the ore-body, commodity price, and available equipment.

The direction can be defined as a straight line(s), curve(s), or triangles. In this paper, at the first step, adjacent drawpoints are defined using the distance between drawpoints so that the combination of each drawpoint with its adjacent drawpoints are called production block (PB). Based on the locations of drawpoints in the production layout, each drawpoint can appear in several production blocks with its different sets of adjacent drawpoints. Therefore, in a layout with "n" drawpoints, there are "n" production blocks.

Fig 2 shows the schematic view of the method. For the considered drawpoint (hatched block), the adjacent drawpoints are determined using the defined adjacent radius of R. Depending on the geometry of the production layout, there might be some production blocks with a smaller number of the drawpoints compared to the other production blocks (this situation happens in the boundaries of the layout). The adjacent radius as an input parameter is imported to the model and then the adjacent drawpoints for each drawpoint is determined. The adjacent radius depends on different factors, such as geotechnical parameters of the ore-body and its host rock(s), mining equipment, and operational constraints.

In the input model, the draw economic value (DEV) for each draw column has been calculated based on the ore tonnage of the draw column, which is the summation of the tonnage resulting from the block model according to the best height of draw (BHOD), grade distribution in the draw column, the ore price, mining cost, processing cost, and selling cost. The BHOD is the height that produces the best economic value and it is usually not discounted with time.

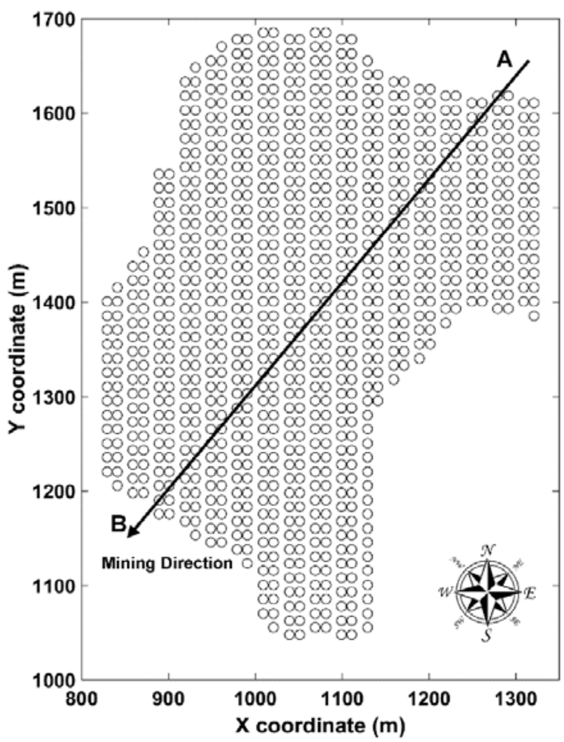

Fig 1. Mining direction for a block-cave mining layout (A to B)

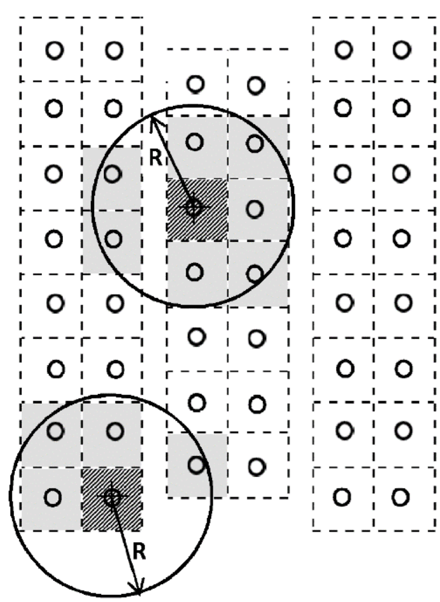

Fig 2. Adjacent drawpoints for the considered drawpoint with the adjacent radius of R (small circles represent the drawpoints)

Using draw economic values, the summation of DEV for each production block is calculated, and then the production block economic value (PBEV) profile is created.

$$PBEV_i = \sum_{j=1}^{J} DEV_j = \sum_{j=1}^{J}\sum_{n=1}^{N}[(p-s)*g_n*\rho*]-[c_m+c_p] \quad (1)$$

Where:

$PBEV_i$ is the i[th] production block's economic value ($)

DEV_i is the draw economic value for drawpoint j associated with draw column 'j' ($)

p is the ore price per metric ton of product ($/t)

s is the selling costs per metric ton of product ($/t)

g_n is the grade for block 'n' in the block model (%)

ρ is the processing recovery of ore in the processing plant (%)

c_m is the mining cost per metric ton of ore ($/t)

c_p is the processing cost per metric ton of ore ($/t)

J is the number of defined production blocks in the block-caving layout using the adjacent concept

N is the number of blocks (in the original block model) in draw column j according to the best height of draw (BHOD)

Using MATLAB (The MathWorks Inc. 2014), the adjacent drawpoints for each drawpoint are determined based on X and Y coordinates of the drawpoints and then the production block economic values are calculated. In the next step, the production block with the highest economic value is selected as the starting area for block-cave mining production. The production is started from the maximum economic value and then will continue to the area with lower economic value. During the production periods, the economic value of the current production block is equal or less than the previous one and greater or equal than the next one.

The methodology has been applied in two real case block-cave mining projects that will be discussed in this paper. The first case is a designed layout with 941 drawpoints (Fig 1). In total, 941 production blocks are defined for the production layout based on the 25 meter adjacent radius. The results show that the central area of the production layout is the best choice for starting the caving operation. Based on the proposed methodology, two major directions are suggested as the mining direction to move from the central part of the ore-body to the boundaries. The major mining direction is from the center to the east and the minor direction is to the south of the layout (Fig 3).

The second case is a block-cave mining layout with 1546 drawpoints (Fig 4). In this case, the adjacent radius of 30 meters and then the same methodology has been applied to find the best mining direction. Using the adjacent concept, 1546 production blocks were defined for this block-cave mine layout.

Results show that the best starting point is north-west and then the caving operation continues to the south-east of the deposit. The suggested direction is a triangle shape that is shown in Fig 5.

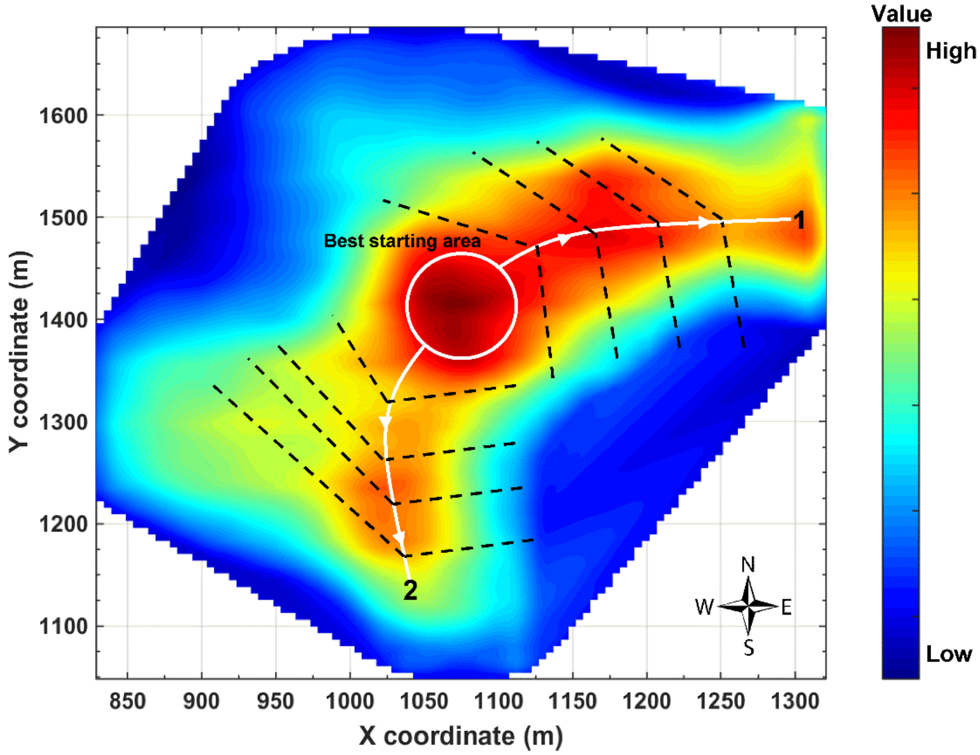

Fig 3. Mining direction determination for Case 1 based on the PBEV concept (1: major advancement direction, 2: minor advancement direction)

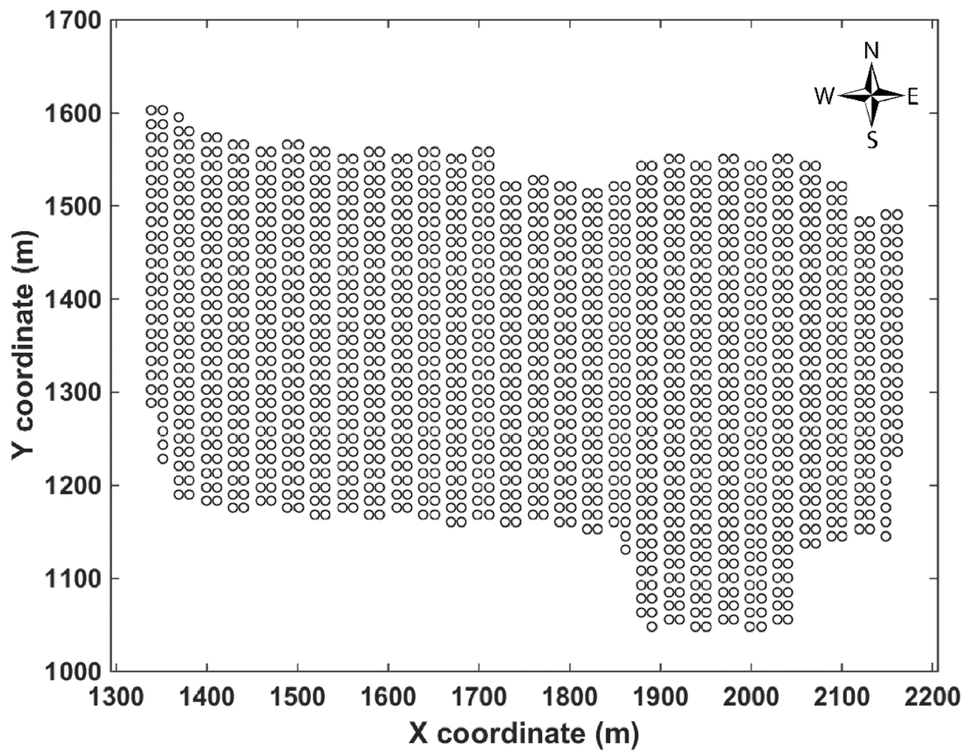

Fig 4. Production layout of the drawpoints for Case 2 (circles represent the drawpoints)

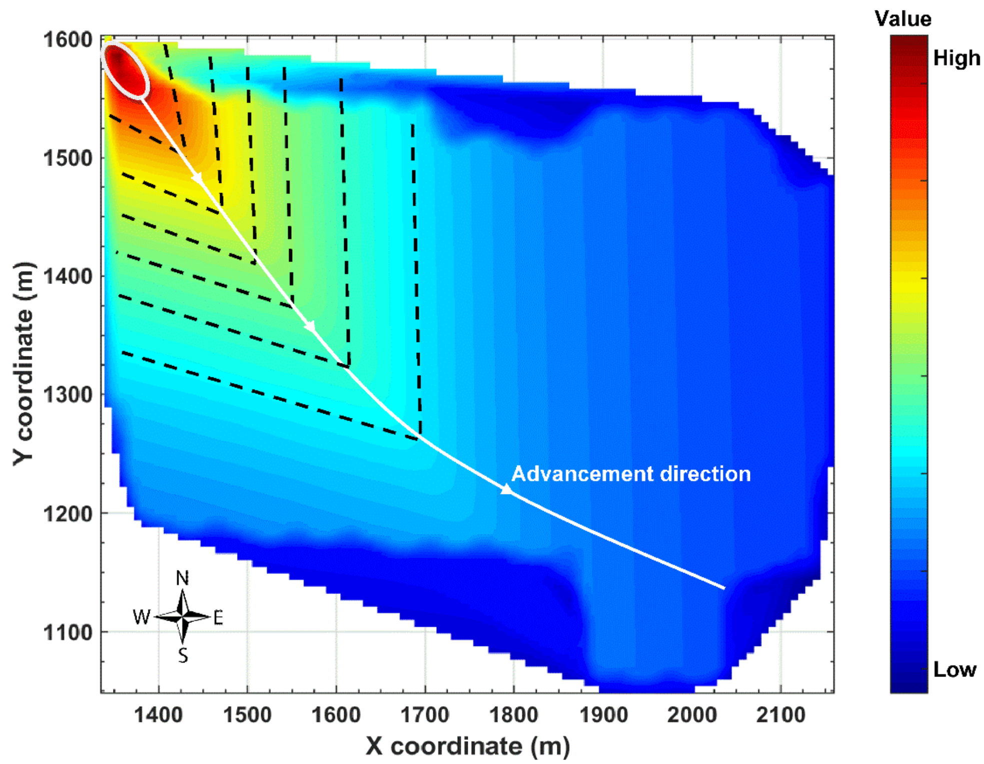

Fig 5. Mining direction determination for Case 2 based on the PBEV concept

4. Precedence determination of the drawpoints

Using the determined mining direction in the previous step, the appropriate precedence for each drawpoint can be defined. The precedence of drawpoints is considered as a production constraint in order to optimize the production scheduling of the block-cave mining operation. According to the advancement direction, for each drawpoint there is a set which defines the predecessor drawpoints among adjacent drawpoints that must be started before the considered drawpoint is extracted.

The proposed mining direction in the previous section (Fig 5) has been applied to find the best mining precedence for drawpoints in Case 2. In Fig 6, it can be seen that the production priority starts from the north-west and it continues to the south-east of the deposit, with drawpoints obtaining their precedence numbers based on the defined direction. For instance, the arrows show that extraction form drawpoint 47 must be started before drawpoints 45, 48, and 46. On the other hand, all four drawpoints 45, 46, 47, and 48 must be opened before drawpoints 41, 42, 43, 44, 97, 98, 99, 100, 101, and 103.

5. Conclusion

The first step of production scheduling in block caving is the determination of the mining direction in which the caving operation starts from an area of the ore-body and then expands to the other parts. For a block-caving layout, there are infinite numbers of directions in which the ore-body can be mined but not all of them will generate an optimal production schedule. Different methodologies have been proposed for production scheduling optimization in block-cave mining operation, but one of the shortcomings is the mining direction, which has been determined manually. In this paper, we have proposed a methodology in order to find the best mining direction and production precedence based on the economic value of the draw columns. The input block model of the ore-body was used for calculating the draw economic value of draw columns based on the tonnage, grade, price, operational costs (mining, processing and selling), processing recovery, and the best height of draw (BHOD). Then the production blocks were defined using the adjacent concept. Afterwards, the best starting area and mining direction among all possible directions were determined based on the PBEV. In the second step, the best production precedence was generated based on the best mining direction proposed in step 1. The application of an automated procedure for mining direction determination can generate a more accurate input compared to the manual methods for production scheduling models in block-cave mining.

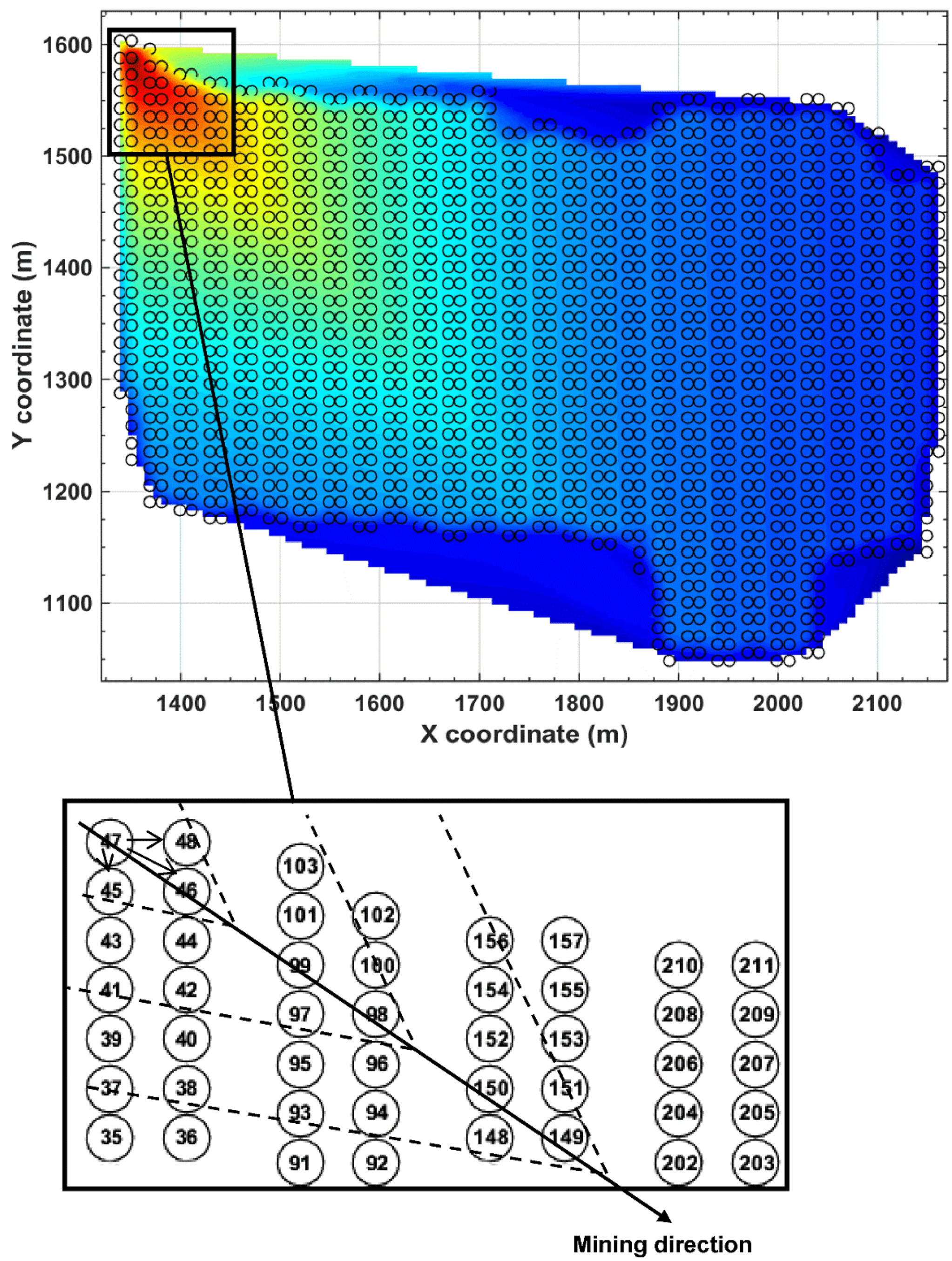

Fig 6. Production precedence for drawpoints in Case 2 based on the determined mining direction

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Footprint Calculation for Block-Cave Mining under Grade Uncertainty

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Abstract

The initial evaluations of a range of levels for starting the extraction of block-cave mining is an important issue which needs to consider a variety of parameters such as extraction rate, block height, discount rate, block profit, cost of mining and processing and revenue factors. This paper deals with finding the best level to place a block cave extraction level based on the maximum discounted ore profit.

Grade uncertainty has profound impact on the total ore profit of each level and as a result on the placement of extraction level. A set of simulations of the mineral grade is modelled and for each realization the best level for starting extraction is specified.

The maximum NPV is determined using mathematical programming approach (MP) after determining the best extraction level. The model is created in MATLAB and CPLEX as a solver is implemented to solve the optimization problem. The purpose of this paper is to present a methodology to find the best extraction level under grade uncertainty. The extraction level is used to maximize the NPV; given some constraints such as mining capacity, grade of production, extraction rate and precedence.

1. Introduction

Among the underground mining methods available, caving methods are favored because of their low-cost and high-production rates. Production scheduling in block caving, because of its significant impact on the project's value, has been considered a key issue to be improved, so researchers have applied different methods to model production scheduling in block caving.

Grade uncertainty can lead to significant differences between actual production and planning expectations and as a result the NPV of the project (Osanloo et al., 2008; Koushavand and Askari-Nasab, 2009). Various researchers have considered the effects of grade uncertainty in open-pit mines and introduced different methodologies (Dimitrakopoulos, 2011). Dowd (1994) presented a risk-based algorithm for surface mine planning. For different variables such as commodity price, processing cost, mining cost, investment required, grade and tonnages a predefined distribution function is implemented. Several types of schedules are generated for a number of realizations of the grades. The result of this methodology is various schedules that accounts for grade uncertainty. Simulated ore-bodies are utilized by Ravenscroft (1992) and Koushavand and Askari-Nasab (2009) to show the influence of the grade uncertainty on production scheduling. Ramazan and Dimitrakopoulos (2004) used a mixed-integer linear programming

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(MILP) model to maximize NPV for each realization. Then, the probability of extraction of a block at each period is calculated. These probabilities are the input of a second stage of the optimization to have one schedule at the end. Dimitrakopoulos and Ramazan (2008) presented a linear integer programming (LIP) model for generating optimal production schedules. This model considers multiple realizations of the block model and defines a penalty function that is the cost of deviation from the target production. This cost is calculated based on the geological risk discount rate which is the discounted unit cost of deviation from target production. A linear programming is used to maximize a new function that is NPV less penalty costs.

Otherwise, a few numbers of authors have considered geological uncertainty in underground mining. Vargas et al. (2014) developed a tool that took geological uncertainty into account by using a set of conditional simulations of the mineral grades and defining the economic envelope in a massive underground mine. One of the main steps involved in the optimization of underground mines is determining a mining outline and inventory. The open-pit corollary to this is open-pit optimization, which is completed with algorithms such as Lerchs and Grossmann (1965). For optimization of block-caving scheduling, most of researchers have used mathematical programming; Linear Programming (LP), Mixed-Integer Linear Programming (MILP), and Quadratic programming (QP). LP is the simplest one in modelling and solving. Table 1shows some of the applied mathematical methodologies in block-caving production scheduling.

This paper will introduce a method in which the best level for initializing extraction according to the maximum discounted ore profit and grade uncertainty is found. Several realizations are modelled by using geostatistical studies to consider the grade uncertainty. The production schedule is generated for the given advancement direction and in presence of some constraints at the chosen level.

2. Methodology, assumption and notations

A geological block model represents the ore-body. Numerical data are used to represent each block's attributes, such as tonnage, density, grade, rock type, elevation, and profit data. Fig. 1 shows the summary of the methodology.

2.1. Geological Uncertainty

GSLIB (Deutsch and Journel, 1998) is used for geostatistical modeling in this paper. The first step for a geostatistical study is defining different rock types based on the drillhole data. In this study with assuming stationary domain within each rock type, the geostatistical modeling is performed for each of the rock types separately. The following steps are implemented for generating geological model:

1. Declustering
2. Multivariate statistical analysis
3. Determining the principle direction of continuity
4. Transforming data to Gaussian units
5. Calculating the variogram
6. Simulation

First, declustering algorithm is used to adjust the variable distributions for each rock type to decrease the weight of clustered samples (declus program). Then, the correlation of the multivariate data is determined (scatplt program). To determine the principle directions of continuity, global kriging is performed using arbitrary variograms with high range. In the case of rock-type modeling, indicator kriging (ik3d program) and for the rock-property modeling simple kriging (kt3dn program) is used. Afterwards, the data is transformed to Gaussian units; in the case of univariate data, nscore and for multivariate data, ppmt programs are used to removes the correlation between the variables in each rock type (Barnett, 2015).

Table 1. Summary of applied mathematical methodologies in block-caving production scheduling (Khodayari and Pourrahimian, 2014)

Author	Methodology	Model's objective(s)	Features
Song (1989)	MILP	Minimization of total mining cost	**LP:** This method has been used most extensively and it can provide a mathematically provable optimum schedule. But straight LP lacks the flexibility to directly model complex underground operations which require integer decision variables. **MILP:** MILP could be used to provide a series of schedules which are marginally inferior to a provable optimum. Computational ease in solving an integer programming problem is dependent upon the formulation structure. It can provide a mathematically provable optimum schedule. The advantage that MILP has over simulation when used to generate sub-optimal schedules is that the gap between the MILP feasible solution and the relaxed LP solution provides a measure of solution quality. The drawback in using MILP is that it is often difficult to optimize large production systems by the branch-and-bound search method. **QP:** Block caving process is non-linear, so it would not be appropriate to use linear programming for production scheduling in block caving. But solving of this kind of problems could be a challenge because we must change them to LP and then solve them, so we have conversion errors.
Chanda (1990)	Simulation and MIP	Minimization of the deviation in the average production grade between operating shifts	
Guest et al. (2000)	LP	Maximization of NPV	
Rubio (2002)	MIP	Two models (a) maximization of NPV and (b) optimization of the mine life	
Diering (2004)	NLP	Maximizing NPV for M periods and minimization of the deviation between a current draw profile and a defined target	
Rubio and Diering (2004)	LP, IP, QP	Maximization of NPV, optimization of draw profile, and minimization of the gap between long and short term planning	
Rahal et al. (2008)	MILGP	Minimizing deviation from the ideal draw profile while achieving a production target	
Weintraub et al. (2008)	MIP	Maximization of profit	
Smoljanovic et al. (2011)	MILP	Optimization of NPV and mining material handling system	
Parkinson (2012)	IP	Finding an optimal opening sequence in an automated manner	
Epstein et al. (2012)	LP, IP	Maximization of NPV	
Diering (2012)	QP	Objective tonnage (to optimize the shape of the cave)	
Pourrahimian et al. (2013)	MILP	Maximization of NPV	
Alonso-Ayuso et al. (2014)	MILP	Maximization of NPV with considering uncertainty in copper price	
Pourrahimian and Askari-Nasab (2014)	MILP	Maximization of NPV, Determining the BHOD based on the optimization	

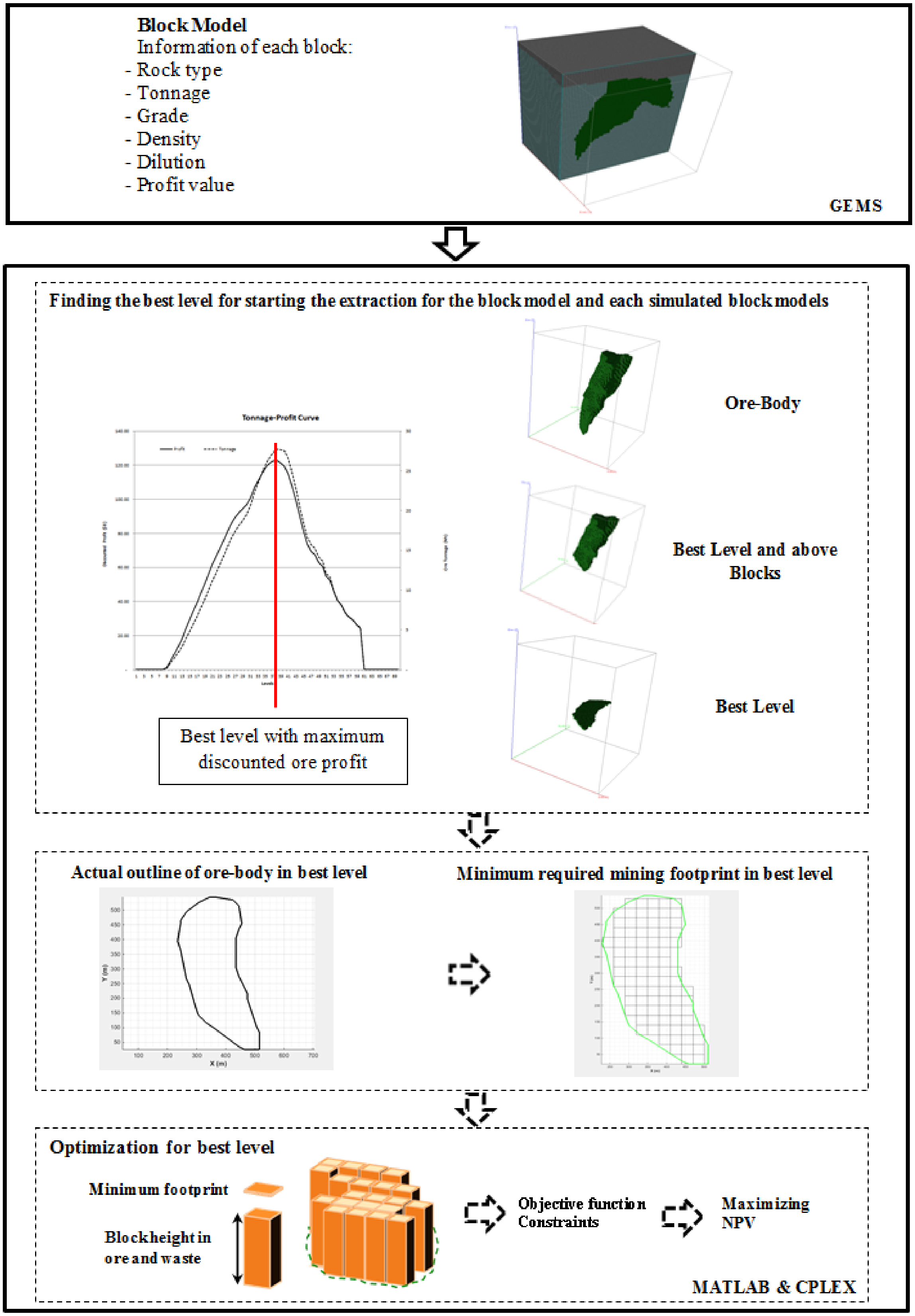

Fig. 1. Schematic representation of the methodology

The experimental variograms are calculated by using the determined directions of continuity in the previous step and a model is fitted to these variograms in different directions. Varcalc, varmodel and varplot programs are utilized for this purpose respectively. In the case of rock-type modeling, indicator variogram and for the rock-property modeling traditional variogram is used.

Rock-type model is generated for the chosen grid definition by using Sequential Indicator Simulation algorithm (SIS). Blocksis program is used for this purpose that generates multiple realizations of the rock-types.

Rock-property model such as grade for each rock-type is generated by using sgsim program which is based on Sequential Gaussian Simulation algorithm (SGS). Then the data is back-transformed to original units by implementing backtr program or ppmt-b program for univariate and multivariate data respectively. In this step histplotsim program can be used for plotting cumulative distribution function (CDF) of each realization and reference distribution in one graph.

Finally, the rock-property model is matched with the rock-type model by using mergemod program for each realization.

At the next stage the best level described in determined for each of the simulated models.

2.2. Placement of extraction level

After importing the block model into the MATLAB (Math Works Inc, 2014), the ore tonnage and discounted profit for each level are calculated. Discounted profit of each ore block (Diering et al., 2008) and the total discounted profit of each level are calculated through Eqs. (1) and (2).

$$Dis\ \mathrm{P}_{blL} = \frac{\mathrm{P}}{(1+i)^{d/ER}} \tag{1}$$

$$Dis\ \mathrm{P}_{L} = \sum_{bl=1}^{n} Dis\ \mathrm{P}_{blL} \tag{2}$$

Where $Dis\ \mathrm{P}_{blL}$ is the discounted profit of ore block bl in level L; $Dis\ \mathrm{P}_{L}$ is the total discounted profit of level L; P is the profit of ore block bl and its above ore blocks; i is the discount rate; d is the distance between the center points of ore block bl in level L and its above blocks; ER is the extraction rate per period. The profit of each block is calculated by the following equations:

$$TR = G \times Ton \times R \times (P - SC) \tag{3}$$

$$TC = Ton \times (MC + PC) \tag{4}$$

$$P = TR - TC \tag{5}$$

Where TR is the total revenue; R is the processing plant recovery; P is the price per tonne of the product; SC is the selling cost per tonne of material; G is the element grade; TC is the total cost; PC is the processing plant cost and MC is the cost of mining per tonne of material.

Then the tonnage-profit curve is plotted and the level with the highest profit is selected for starting the extraction.

2.3. Making decision

Two methods are introduced to select the best level among all the realizations:

Method I)

If the numbers of realizations are n, there are n block models with separate grade numbers for each of them. In this method just one block model is considered instead of n block models in which the average grade of all the block models for each cell is calculated. At the next step the best level of extraction is found for the created block model according to section 2.2. Fig. 2 shows the summary of the method.

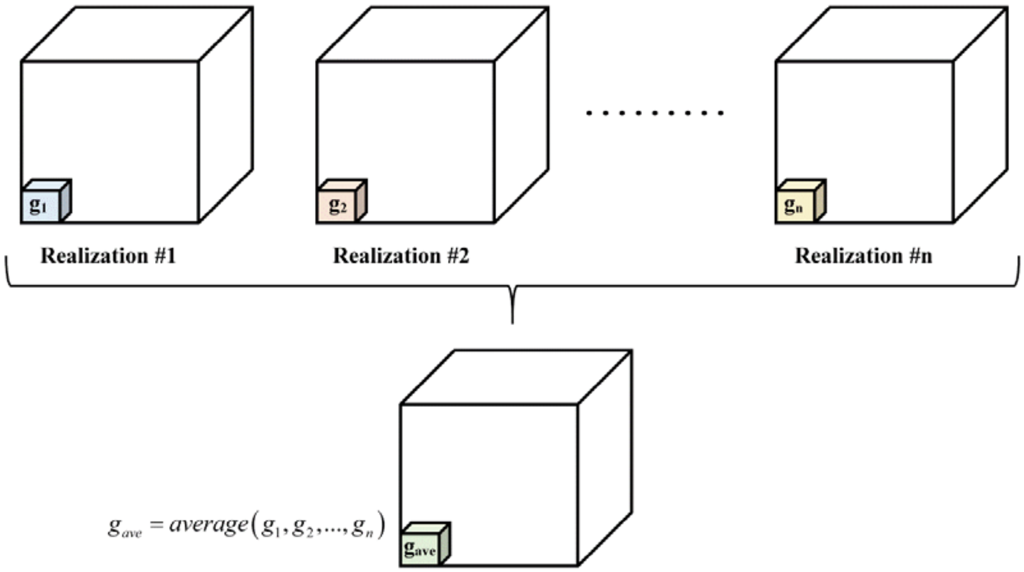

Fig. 2. Schematic representation of method I

Method II)

The best level for each realization is found according to section 2.2. The next step is calculating the probability of each selected levels based on the number of repetition. The level with the probability higher than p , is defined by planner, will be chosen for starting the extraction.

2.4. Production Scheduling

The actual outline of the ore-body for the best elevation is determined. Then the inside of the ore-body outline is divided into rectangles based on the given information about the minimum mining footprint. The minimum mining footprint (plan view) represents the minimum sized shape that will induce and sustain caving. This is similar to the hydraulic radius in a caving operation. It could be a rectangle, a circle, or any other shape. Finally the NPV through the objective function and subjected to a set of constraints is maximized.

The notation of sets, indices and decision variables are as follows:

2.4.1. Notation

Indices

$t \in \{1,....,T\}$	Index for scheduling periods
$bl \in \{1,...,BL\}$	Index for big blocks

Set

S^{bl}	For each block, bl , there is a set S^{bl} defining the predecessor blocks that must be started prior to extraction of block bl .

Decision Variables

$B_{bl,t} \in \{0,1\}$	Binary variable controlling the precedence of the extraction of blocks. It is equal to one if the extraction of block bl has started by or in period t; otherwise it is zero.

$x_{bl,t} \in [0,1]$	Continuous variable, representing the portion of block bl to be extracted in period t.

Parameters

$MCL\,(Mt)$	Lower bound of mining capacity
$MCU\,(Mt)$	Upper bound of mining capacity
g_{bl}	The average grade of considered element in the ore portion of block bl
$GL\,(\%)$	Lower bound of the acceptable average head grade of considered element
$GU\,(\%)$	Upper bound of the acceptable average head grade of considered element
$ExtU\,(Mt)$	Maximum possible extraction rate from each big block
$i\ (\%)$	Discount rate
$R\,(\%)$	Processing plant recovery
$P\,(\$/tonne)$	Price per ton of the product
$MC(\$/tonne)$	Cost of mining per ton of material
$PC(\$/tonne)$	Cost of processing per ton of material
$SC(\$/tonne)$	Selling Cost per ton of material
L	An arbitrary big number
T	Maximum number of scheduling periods
BL	The number of big blocks in the model

2.4.2. Objective function and constraints

The objective function of the MILP formulation is to maximize the net present value (NPV) of the mining operation which depends on the value of the big blocks (based on distances between drawpoints and footprint size, the blocks are placed into bigger blocks.). The objective function, Eq. (6). is composed of the block profit value, discount rate, and a continuous decision variable that indicates the portion of a block, which is extracted in each period. The most profitable blocks will be chosen to be part of the production in order to maximize the NPV.

$$Max \quad \sum_{t=1}^{T}\sum_{bl=1}^{BL}\frac{Profit_{bl}\times x_{bl,t}}{(1+i)^{t}} \tag{6}$$

Where $Profit_{bl}$ is the profit value of the block bl which is equal to the summation of the blocks profit within the big blocks; and i the discount rate.

The objective function is subject to the following constraints:

Mining capacity:

$$MCL_t \le \sum_{bl=1}^{BL} Ton_{bl}\times x_{bl,t} \le MCU_t\,, \quad \forall t \in \{1,....,T\} \tag{7}$$

These constraints ensure that the total tonnage of material extracted from big blocks in each period is within the acceptable range. The constraints are controlled by the continuous variable $x_{bl,t}$.

Grade Blending:

$$GL_t \le \frac{\sum_{bl=1}^{BL} g_{bl} \times Ton_{bl} \times x_{\mathrm{bl},t}}{\sum_{bl=1}^{BL} Ton_{bl} \times x_{\mathrm{bl},t}} \le GU_t, \quad \forall t \in \{1,...,T\} \tag{8}$$

These constraints control the production's average grade.

Block Extraction Rate:

$$Ton_{bl} \times x_{bl,t} \le ExtU_t, \quad \forall bl \in \{1,...,\mathrm{BL}\}, \mathrm{t} \in \{1,...,\mathrm{T}\} \tag{9}$$

These constraints ensure that the extraction rate from each big block is less than or equal to the given maximum value.

Binary Constraints:

$$B_{bl,t} \le L \times x_{bl,t}, \quad \forall bl \in \{1,...,BL\}, \ t \in \{1,...,T\} \tag{10}$$

$$\mathrm{x}_{bl,t} \le B_{bl,t}, \quad \forall bl \in \{1,...,BL\}, \ t \in \{1,...,T\} \tag{11}$$

$$B_{bl,t} - B_{bl,t+1} \le 0, \quad \forall bl \in \{1,...,BL\}, \ t \in \{1,...,T\} \tag{12}$$

Where $B_{bl,t}$ is a Binary variable controlling the precedence of the extraction of blocks. It is equal to one if the extraction of block bl has started by or in period t ; otherwise it is zero and L is an arbitrary big number. In these constraints every block has two variables (continuous and binary) in which constraints (10) and (11) ensure if the extraction of one block is started its binary variable should be one and otherwise it should be zero. Also Eq. (12) controls the fact that if the extraction of one block in period t has been started, so for the next period the extraction of that block is started. The results of these constraints is been used for the precedence constraint that the maximum number of active blocks is needed.

Precedence Constraints:

$$n \times Bl_{bl,t} \le \sum_{k=1}^{n} B_{k,t}, \quad k \in S^{bl}, \quad \forall bl \in \{1,...,BL\}, \ t \in \{1,...,T\} \tag{13}$$

These constraints ensure that all the predecessor blocks of a given block bl have been started prior to the extraction of this block.

For applying this constraint at first the adjacent blocks of each block is determined then by considering the extraction direction and determining the perpendicular line to it which crosses the center point of a given block, the adjacent blocks under this line have been specified which are the blocks that should be extracted before that given block. The formulas that have been used in finding the predecessor blocks of each block are described in the following equations:

$$Y_{new} - y_{bl} = -m(X_{new} - x_{bl}) \tag{14}$$

$$D = (x_{adj} - x_{bl})(Y_{new} - y_{bl}) - (y_{adj} - y_{bl})(X_{new} - x_{bl}) \tag{15}$$

Where $-m$ is the slope of the perpendicular line to the mining direction; y_{bl} and x_{bl} are the coordinates of each big block in the extraction level; X_{new} is an arbitrary coordinate and as a result

Y_{new} is calculated by Eq. (14). (perpendicular line crosses both the center point of each big block and a point with coordinates of (X_{new}, Y_{new})). In Eq. (15). x_{adj} and y_{adj} are the coordinates of the adjacent blocks of each big block. By calculating D, the blocks with $D < 0$ are below the perpendicular line and considered as the predecessors of a given big block.

Reserve Constraints:

$$\sum_{t=1}^{T} x_{bl,t} = 1, \quad \forall bl \in \{1,...,BL\} \tag{16}$$

In this formulation, all material inside of the big blocks should be extracted which is controlled by Eq. (16).

3. Case study

3.1. Grade Uncertainty

Geostatistical study based on the drillholes data and according to what mentioned in section 2.2. is performed. The data is belonged to cupper grade, so it is a univariate data and multivariate statistical analysis to find the correlation between the variables is not needed. The initial inspection of drillholes locations demonstrated equally-spaced drillholes and as a result declustering algorithm is not implemented.

The next step is grid definition for simulation. Distance between the grid nodes in each direction, the number of grid nodes in each direction and the coordinates of first grid node are important parameters for defining a grid. By considering all of these parameters the size of the grid is chosen according to

Table 2.

Table 2. Grid Definition for Geostatistical Study

Direction	Number of nodes	Center coordinates of first node	Grid Spacing
Easting	45	145	10
Northing	60	25	10
Elevation	70	395	10

The modeling has two parts: rock-type modeling and rock-property modeling. There are two types of blocks in this model: ore blocks (18%) and waste blocks (82%). Rock-property modeling should be implemented for both rock-types (ore and waste) separately.

3.1.1. Rock-type Modeling

The principal directions of continuity are found using indicator kriging based on two categories; 0 (waste) and 1 (ore). The azimuths of major and minor directions were chosen to be 0 and 90 degrees (Fig. 3) that are used for variogram calculation at the next step.

Afterwards indicator variograms are calculated and a theoretical variogram model is fitted with three structures, nugget effect of 0 and sill of 0.14 (Fig. 4).

At the next step, 20 realizations for rock-types are generated using Sequential Indicator Simulation (SIS) algorithm. Fig. 5 illustrates the results for the first realization.

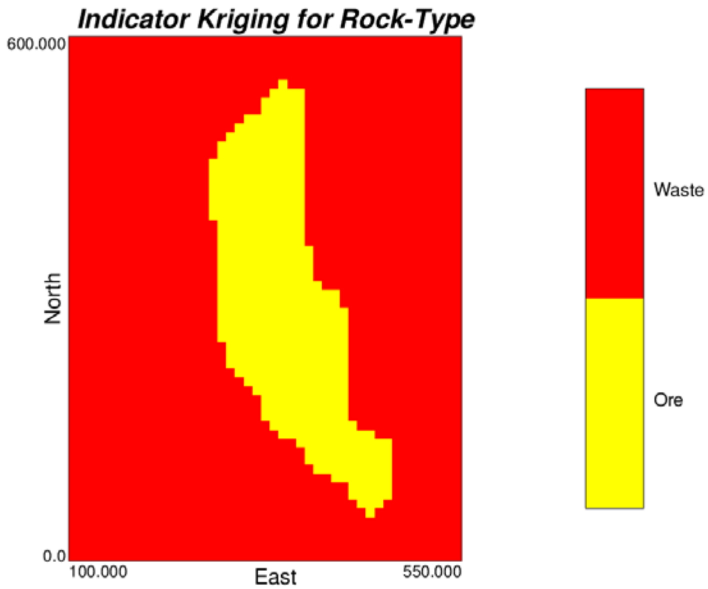

Fig. 3. Plan view of maximum direction of continuity for rock-types at elevation 40

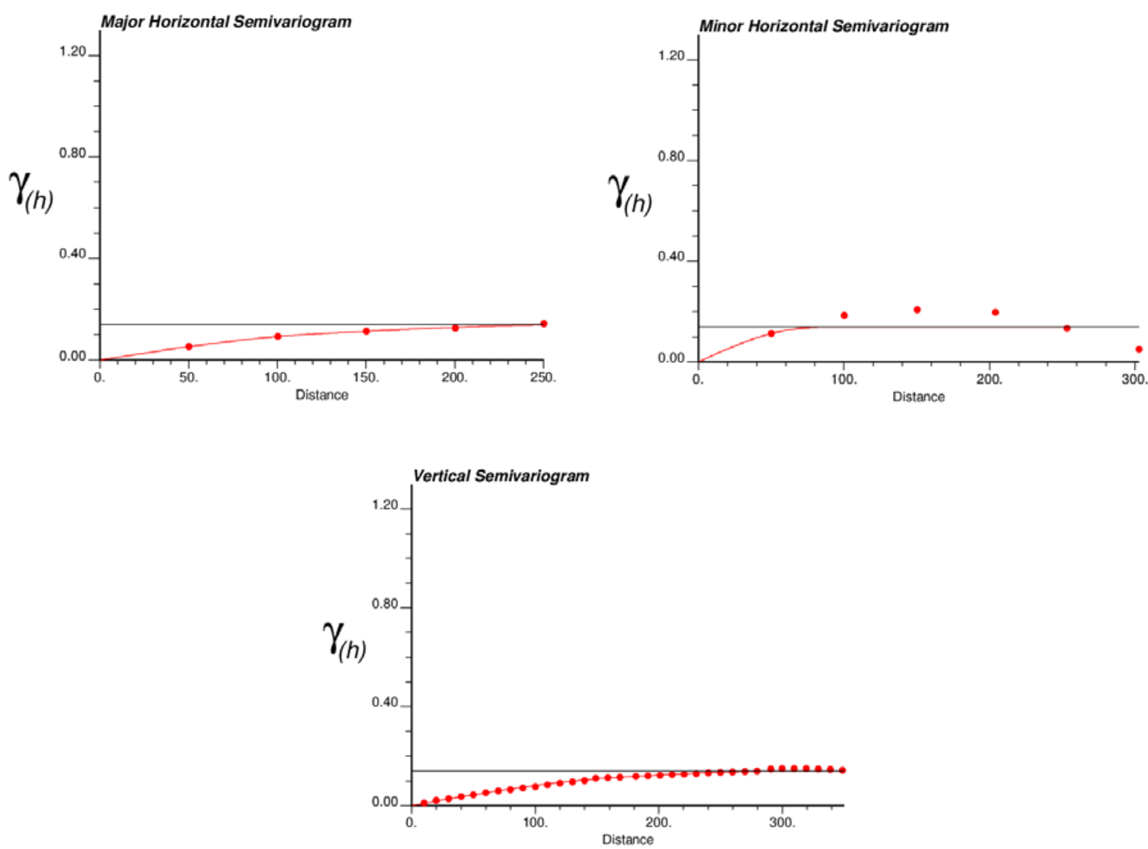

Fig. 4. Experimental directional variograms (dots) and the fitted variogram models (solid lines) for rock-type, distance units in meter

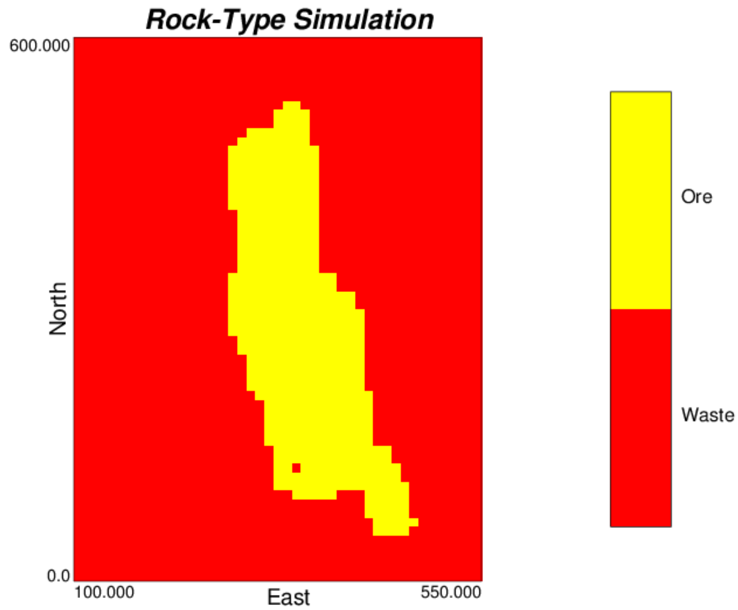

Fig. 5. Plan view of rock-type simulation for first realization at elevation 40

3.1.2. Rock-Property Modeling

As in this case all the cupper grades related to waste blocks were zero, rock-property modeling is unnecessary to be performed for the waste and just zero values equal to grid-size are considered for waste modeling.

For ore modeling, the principal directions of continuity are extracted by doing simple kriging with the help of arbitrary variograms. As it can be seen in Fig. 6 the azimuth of 90° (major) and zero (minor) in horizontal direction are selected for the following variogram calculation.

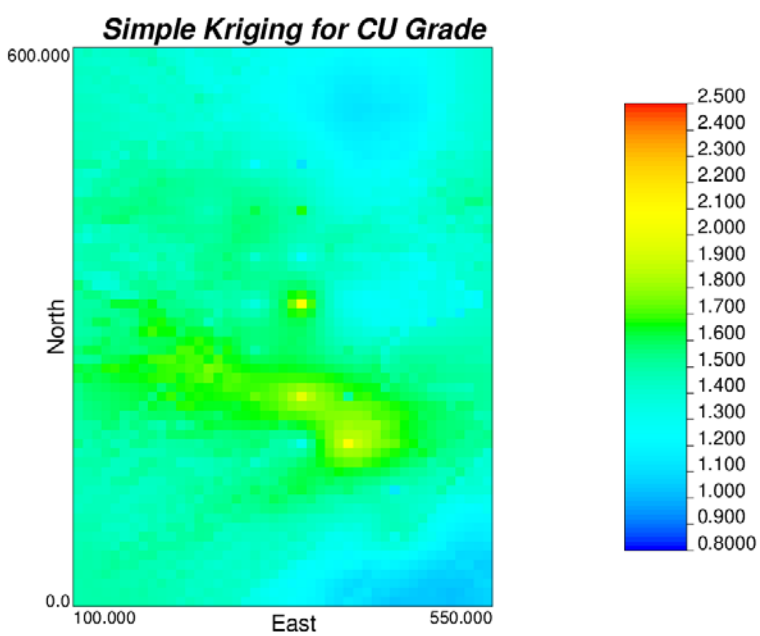

Fig. 6. Plan view of maximum direction of continuity for cupper grade at elevation 40

At the next step the cupper grades should be transformed to Gaussian space. Then traditional variogram calculation and modeling with three structure and nugget effect of 0.1 are done for the cupper grade and the results are demonstrated in the Fig. 7.

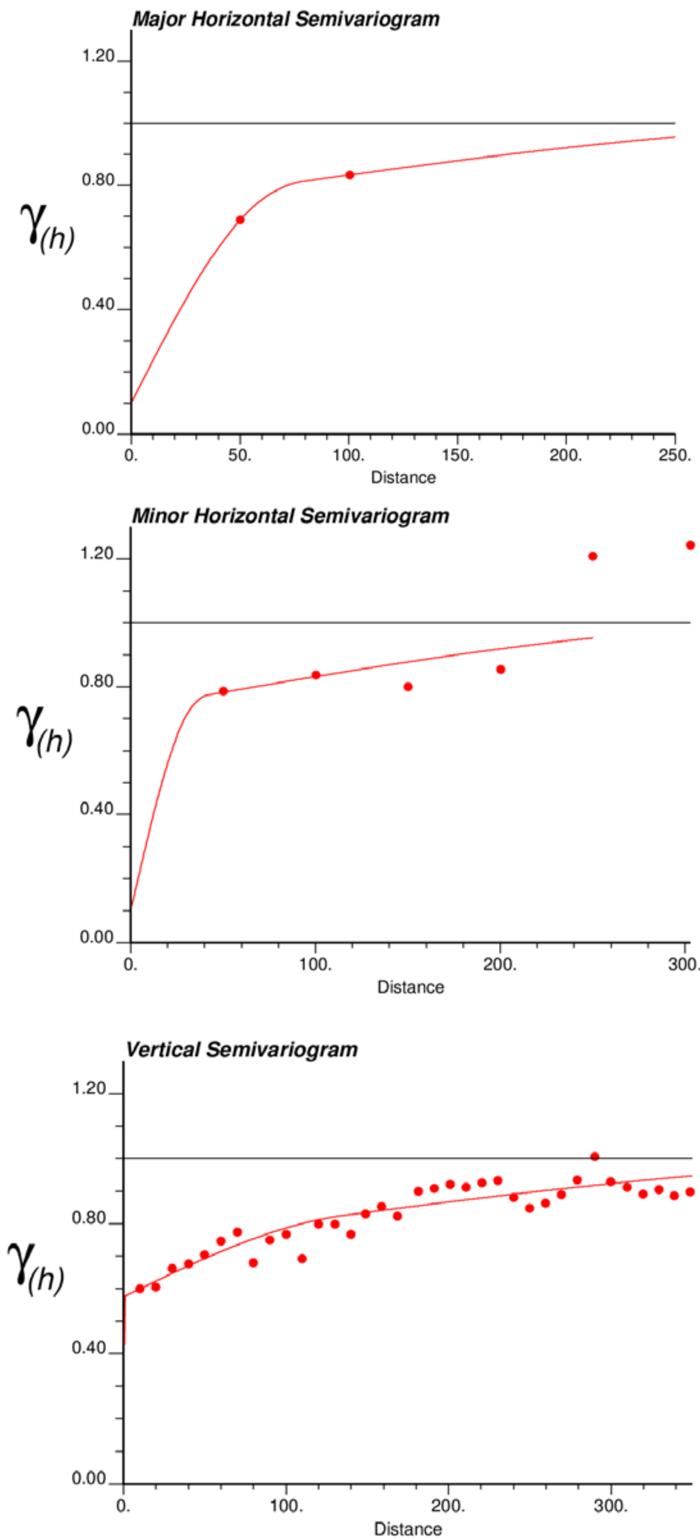

Fig. 7. Experimental directional variograms (dots) and the fitted variogram models (solid lines) for Cu grade of ore blocks, distance units in meter

At the next step, 20 realizations for cupper grade are generated using Sequential Gaussian Simulation (SGS) algorithms. The SGS needs a back-transformation to original units that the results are demonstrated in Fig. 8.

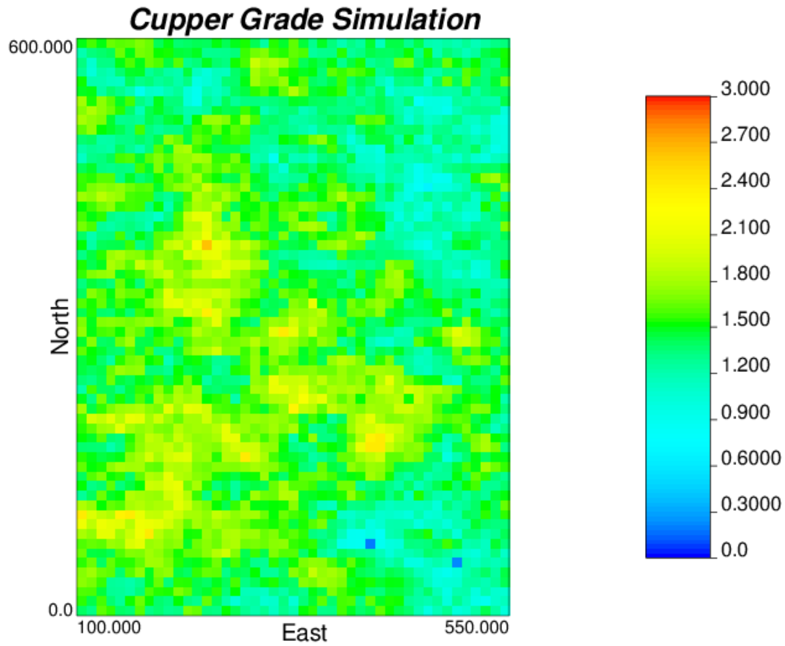

Fig. 8. Plan view of Cu grade simulation for first realization at elevation 40

3.1.3. Merging rock-type and rock-property models

The next step is merging rock-type model with rock-property model. The plan view of final simulation for first realization is shown in Fig. 9.

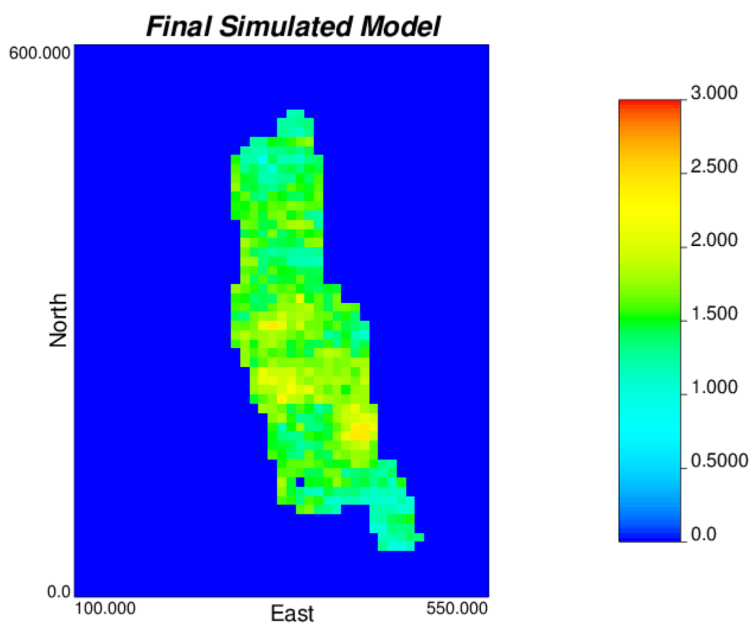

Fig. 9. Final simulation at elevation 40 and realization 1

Fig. 10 shows the variogram reproduction of rock-property (ore) simulation (left) and rock-type simulation (right) in three major, minor and vertical directions. Since the variogram is reproduced quite reasonably, the generated realizations are considered representative of the grade uncertainty.

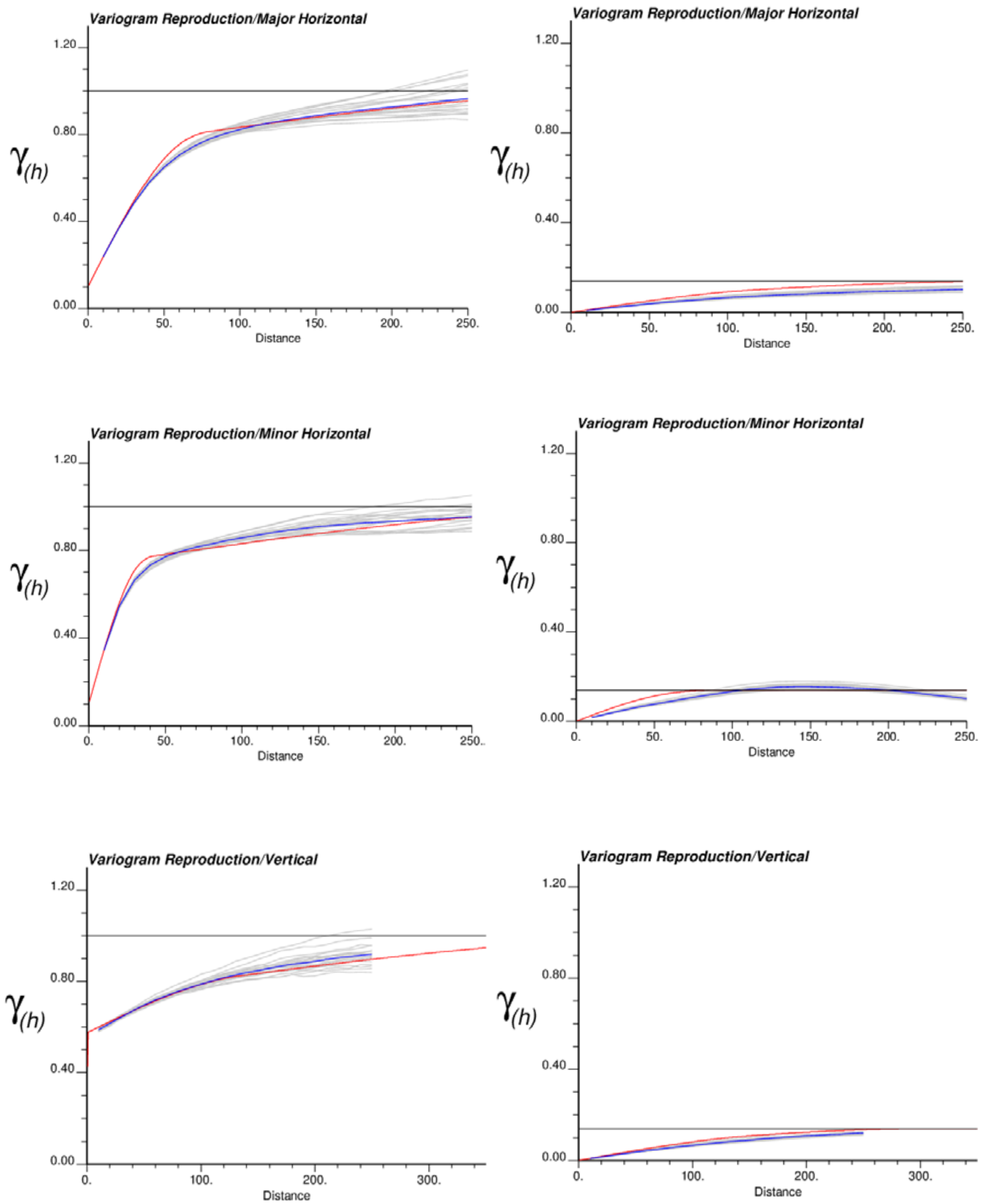

Fig. 10. Variogram reproduction at Gaussian units of cupper grade (left) and rock-type (right) realizations (gray lines), the reference variogram model (red line) and the average variogram from realizations (blue line) in three directions.

3.2. Placement of extraction level

For each block model, at the first step the discounted profit and tonnage of the ore blocks above each ore block in each level is calculated and the profit-tonnage curve is plotted which leads to selecting the best level for starting extraction based on the maximum profit (Fig. 11).

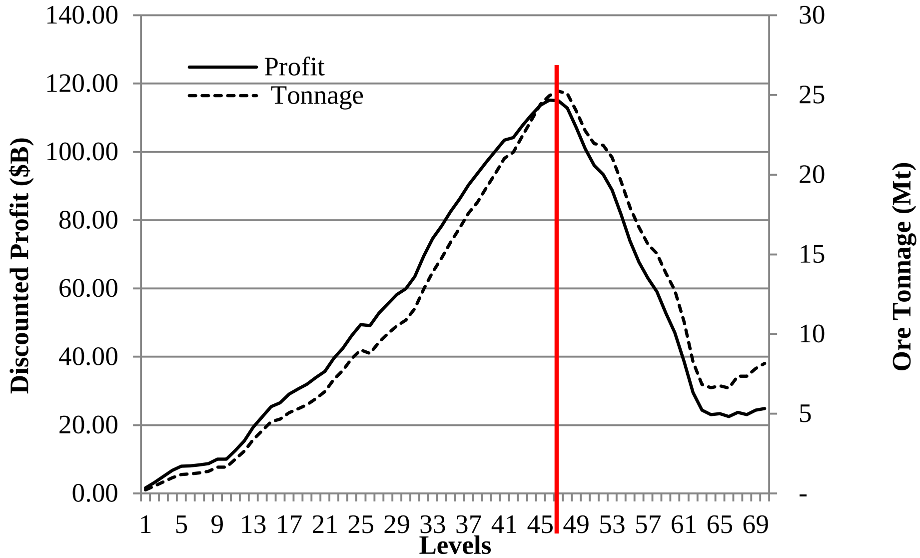

Fig. 11. Tonnage-Profit curve against the levels for one of the simulated block models

This method has been implemented for all the realizations (Table 3). The average grade of all realizations is 1.4794 that is quite near the average grade of original data which is 1.4759.

Table 3. Best level for a number of realizations

Realization	Average grade	Best level	Ore Tonnage (Mt)	Ore discounted profit ($B)
1	1.4784	48	28.884	123.907
2	1.4873	46	24.108	109.964
3	1.4845	46	24.964	115.114
4	1.4648	49	25.015	110.174
5	1.4649	47	25.300	112.752
6	1.4629	48	26.910	116.964
7	1.4607	45	25.188	109.274
8	1.4986	47	24.284	108.451
9	1.4901	46	23.892	108.673
10	1.4831	46	25.452	113.599
11	1.4779	47	25.930	116.430
12	1.4822	46	25.130	110.634
13	1.4921	49	26.630	116.611
14	1.4817	47	24.012	109.150
15	1.4747	47	22.920	101.623
16	1.4798	46	25.636	117.686
17	1.4942	46	26.642	120.052
18	1.4872	46	23.746	110.302
19	1.4767	47	24.276	107.577
20	1.4664	47	26.210	114.670

3.3. Making Decision

Method II is implemented to select the best level. The probability of each level from Table 3 is calculated. Level 46 with the highest probability is selected for the placement of extraction level and for production scheduling at the next step.

3.4. Production Scheduling

The proposed mathematical model will be used in further studies to generate the production schedule for the level 46 to maximize the NPV.

4. Conclusion

Geological uncertainty has been utilized recently in open-pit mining, but less studied in underground mining especially in block caving in which revision in production plan is not easy after caving (Vargas et al., 2014). This methodology is able to find the best extraction horizon placement under grade uncertainty. Also it will be able to implement mathematical programming through defining an optimization model in MATLAB and running it by CPLEX in the future studies.

The results related to the profit of each level have been compared with each other thorough the profit-tonnage curve and as a consequence, the best level is recommended for initializing the extraction from all the realizations. Then by optimizing the proposed model maximum NPV will be obtained for the selected level.

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Block-Cave Production Scheduling Using Mathematical Programming

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Abstract

As the mineral resources near the surface are being exploited, the mining operation goes deeper into the ground, waste removal rates increase, capital and operation costs become higher, and environmental impacts are more evident. In such a situation, underground mining with lower waste removal and less environmental impact are becoming more attractive. Among underground methods, block-cave mining with its high rate of production, low operational cost, and automated systems can be one of the best choices instead of surface mining or block-cave mining can be considered as part of production after surface mining during the life of mine. Production scheduling is one of the critical steps in the block-caving design process so that an optimum production scheduling could add significant value to a mining project. Block-cave mining operations can be complicated and behave as a non-linear phenomenon. So, production scheduling for this kind of operations with lots of involving dynamic parameters could be a big size problem in non-linear environment. This research uses mining background and its parameters, with the help of mathematical programming and computer science, to model the production scheduling in block-cave mining to maximize the net present value of the project using MILP and also implement MIQP as non-linear tool to minimize the difference between the objective and the target tonnage of the mining project considering the related constraints of the operations.

1. Introduction

In block cave mining, the gravity of the material is simply used for extraction. It means that compare to other underground methods, extraction is easier and cheaper. Theoretically it is simple but practically it is complicated because many constraints are involved. Omitting any of them can result in inefficiency or even failure in operations. For each drawpoint, its draw rate can affect the draw rate and even grade of other drawpoints which are located in its neighborhood. This nonlinear relationship makes the production scheduling complicated. An ununiformed shape of extraction from drawpoints can result in a high amount of mixing between the draw columns which are located in its neighborhood. This research proposes a methodology to model a block cave operation using mathematical programming. The aim is to minimize the gap between certain amount of production (as an initial expectation) and the results from the optimization. Quadratic programming as a strong tool is used to model this non-linear relationship in the block-cave mining. In this research, two methodologies with different objective functions are proposed to model the block

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cave operations: mixed integer linear programming (MILP) and mixed integer quadratic programming (MIQP). The proposed models are tested for a real case block cave mine.

2. Summary of literature review

Production scheduling in caving means determining how much of material should be extracted from each drawpoint in each period during the life of the mine. Having an optimum extraction of drawpoints can add a significant value to the mining project. There are many constraints limiting the production: geotechnical, economic, environmental, and operational. Mathematical programming is a useful tool to model such a problem in order to find the best solutions for catching the goals while considering the related constraints. Like open pit mining, many researchers have already worked on production scheduling for block cave mining. They have mostly used LP (Guest, Van Hout, & Von Johannides, 2000; Hannweg & Van Hout, 2001; Winkler, 1996), MILP (Alonso-Ayuso, et al., 2014; Chanda, 1990; Epstein, et al., 2012; Guest, et al., 2000; Parkinson, 2012; Pourrahimian, 2013; Rahal, 2008; Rahal, Dudley, & Hout, 2008; Rahal, Smith, Van Hout, & Von Johannides, 2003; Rubio, 2002; Rubio & Diering, 2004; Smoljanovic, Rubio, & Morales, 2011; Song, 1989; Weintraub, Pereira, & Schultz, 2008; Winkler, 1996), and QP (Diering, 2012; Rubio & Diering, 2004). A detailed literature review can be find in (Khodayari & Pourrahimian, 2015b).

3. Methodology

We model the production scheduling of a block-cave mining operations using two different types of mathematical programming: mixed-integer linear programming (MILP) and mixed-integer quadratic programming (MIQP). The models carry same constraints with different objective functions. Models, the related indices, variables, and parameters are discussed in this section.

3.1. Notation

Indices

$t \in \{1,...,T\}$	Index for scheduling periods
$n \in \{1,...,N\}$	Index for drawpoints
g_n	Average grade of draw column associated with drawpoint n
$tonnage_n$	Ore tonnage of draw column associated with drawpoint n

Variables

tarton_n^t	Target tonnage of extraction for the drawpoint n at period t based on the solution of the production scheduling problem (the optimum tonnages that we are looking for, considering the problem's constraints)
$obj\ \text{ton}_n^t$	Objective tonnage of extraction for the drawpoint n at period t based on the production goals
$X_n^t \in [0,1]$	Continues decision variable that represents the portion of draw column n which is extracted in period t
$(Y1)_n^t \in [0,1]$	Binary variable which determines whether drawpoint "n" in period "t" is active [$(Y1)_n^t = 1$] or not [$(Y1)_n^t = 0$]
$(Y2)_n^t \in [0,1]$	Binary variable which determines whether drawpoint "n" till period "t" (periods 1, 2,..,t) has started extraction [$(Y2)_n^t = 1$] or not [$(Y2)_n^t = 0$]

$(Y3)_n^t \in [0,1]$ Binary variable which determines whether the Depletion Percentage (DP) of drawpoint "n" in period "t" is less than Draw Control Factor (DCF) or not (based on the draw rate curve):

$$if\ \ DP \le DCF \ \ \rightarrow \ \ (Y3)_n^t = 0$$

$$if\ \ DP \ge DCF \ \ \rightarrow \ \ (Y3)_n^t = 1$$

$(Y4)_n^t \in [0,1]$ Binary variable which determines whether the material which has remained from drawpoint "n" in period "t" is greater than maximum draw rate (DRMax) based on the draw rate curve [$(Y4)_n^t = 1$] or not [$(Y4)_n^t = 0$]

Parameters

price	Metal price
ir	Interest rate of return
rec	Metal recovery in the processing plant
cost	Operating cost per ton of ore (including mining and processing)
DRMin	Minimum production rate based on the draw rate curve
DRMax	Maximum production rate based on the draw rate curve
M_{min}	Minimum mining capacity base on the capacity of mining equipment
M_{max}	Maximum mining capacity base on the capacity of mining equipment
G_{min}	Minimum production grade
G_{max}	Maximum production grade
ActMin	Minimum number of active drawpoints in each period
ActMax	Maximum number of active drawpoints in each period
$DP_n^t \in [0,1]$	Depletion Percentage which is the portion of draw column "n" which has been extracted till period "t"
$(DP4)_n^t$	Depletion Percentage which is the portion of draw column "n" which has been extracted till period "t-1"
$DCF \in [0,1]$	Draw Control Factor which is the turning point at the draw rate curve
M	An arbitrary big number

3.2. MILP objective function

The MILP objective function is going to maximize the net present value (NPV) of the mining project during the life of mine:

$$Maximize \quad NPV = \sum_{t=1}^{T}\sum_{n=1}^{N}\frac{DEV_n^t}{(1+i)^t}\times X_n^t = \sum_{t=1}^{T}\sum_{n=1}^{N}\frac{[(price\times g\times ton\times rec)-\cos t]_n^t}{(1+ir)^t}\times X_n^t \tag{1}$$

3.3. MIQP objective function

Production goals determine the required tonnage of extraction in a mining project. But there are always some constraints that control the goals. In this research, the optimization problem is looking for the best solution to reduce the gap between the expected production and the practical production considering the related constraints. The objective function is going to minimize the difference between the objective and the target tonnage:

$$Minimize \sum_{t=1}^{T}\sum_{n=1}^{N}(\text{tarton}_n^t - objton_n^t)^2 \tag{2}$$

$$= \sum_{t=1}^{T}\sum_{n=1}^{N}(\text{tarton}_n^t)^2 - (2*\text{objton}_n^t)*\text{tarton}_n^t$$

$$= \sum_{t=1}^{T}\sum_{n=1}^{N}(tonnage_n * X_n^t)^2 - (2*\text{objton}_n^t*tonnage_n)*X_n^t$$

Extraction from drawpoints while having a uniform extraction surface is one of the most important concerns in block cave mining, to minimize the dilution, which can be improved by solving this optimization problem.

3.4. Constraints

There are lots of geotechnical, operational, and economical constraints related to mining projects which limit the whole system in achieving the operational and strategic plans. This research will try to make sure that related constraints are considered so that the model's results can be applicable in real case block-cave mining.

3.4.1. Binary variables

These sets of constrains define the required binary variables. Totally 4 sets of binary variables are defined in order to be able to apply the related constraints for the:

Set 1 $\left((Y1)_n^t \in [0,1], \begin{Bmatrix} n \in N \\ t \in T \end{Bmatrix} \right)$:

This set contains N*T variables, it means for each drawpoint there is one variable per each period. Variables (N*T)+1 to (2*N*T) in the model are allocated to this set. This set determines whether drawpoint "n" is active in period "t" or not; if any extraction from drawpoint "n" at period "t" occurs it means the drawpoint is active (x>0) then Y1=1 and if there is no any extraction (x=0) it means it is not active then Y1=0. The mathematical formulation of this set of constraint includes two parts of equations:

$$Y1 - Mx \le 0 \tag{3}$$

$$x - Y1 \le 0 \tag{4}$$

Set 2 $\left((Y2)_n^t \in [0,1], \begin{Bmatrix} n \in N \\ t \in T \end{Bmatrix} \right)$:

This set contains variables N*T variables, it means for each drawpoint there is one variable per each period. Variables (2*N*T)+1 to 3*N*T in the model are allocated to this set. This set determines whether the depletion percentage of drawpoint "n" in period "t" is 0 or not. Depletion percentage (DP) is the summation of the x values for drawpoint "n" from period "1" till period "t" based on the draw rate curve.

$$DP = \sum_{t=1}^{t} x_n^t \tag{5}$$

If the depletion percentage is 0 (DP=0) then Y2=0 and if depletion percentage is greater than 0 (DP>0) then Y2=1. Two equations are defined for this set:

$$DP - Y2 \le 0 \tag{6}$$

$$Y2 - M * DP \leq 0 \tag{7}$$

Set 3 $((Y3)_n^t \in [0,1], \left\{\begin{matrix} n \in N \\ t \in T \end{matrix}\right\})$:

This set contains variables N*T variables, it means for each drawpoint there is one variable per each period. Variables (3*N*T)+1 to 4*N*T in the model are allocated to this set. This set determines whether the depletion percentage (DP) is in the second area of the depletion curve or it is in the third area of the depletion curve. If the depletion percentage (DP) is in the second area of the depletion curve (DP < DCF) then Y3=0 and if it is in the third area of the depletion curve (DP > DCF) then Y3=1. DCF is Draw Control Factor in the draw rate curve (DCF϶[0,1]). This set contains 2 equations:

$$DP - Y3 \leq DCF \tag{8}$$

$$Y3 - DP \leq 1 - DCF \tag{9}$$

Set 4 $((Y4)_n^t \in [0,1], \left\{\begin{matrix} n \in N \\ t \in T \end{matrix}\right\})$:

This set contains variables N*T variables, it means for each drawpoint there is one variable per each period. Variables (4*N*T)+1 to 5*N*T in the model are allocated to this set. This set determines whether the remained material in draw column “n” at period “t” is less than maximum allowable draw rate (DRMax) or not. If the remained material in draw column “n” at period “t” is less than DRMax then Y4=0 if it is greater than DRMax then Y4=1. Two equations in the constraints define this set:

$$(ton - DP4 * ton) - DRMax \leq M * Y4 \rightarrow -(DP4 * ton) - M * Y4 \leq DRMax - ton \tag{10}$$

$$DRMax - (ton - DP4 * ton) \leq M * Y4 \rightarrow (DP4 * ton) + M * Y4 \leq M + ton - DRMax \tag{11}$$

3.4.2. Mining capacity

This constraint defines the whole production from all drawpoints for each period of time. It can be determined based on the whole operations system capacity. It helps to make sure that the system is working optimally.

$$\forall t \in T \rightarrow M_{\min} \leq \sum_{n=1}^{N} ton_n \times X_n^t \leq M_{\max} \tag{12}$$

3.4.3. Average grade of production

The average grade of the extracted material should be in an acceptable range. This constraint helps to have a uniform extraction of the ore during the mine life and can be determined based on processing plant requirements.

$$\forall t \in T \rightarrow G_{\min} \times (\sum_{n=1}^{N} ton_n \times X_n^t) \leq \sum_{n=1}^{N} g_n \times ton_n \times X_n^t \tag{13}$$

$$\forall t \in T \rightarrow \sum_{n=1}^{N} g_n \times ton_n \times X_n^t \leq G_{\max} \times (\sum_{n=1}^{N} ton_n \times X_n^t) \tag{14}$$

3.4.4. Reserve

The BHOD is calculated before applying the mathematical model. This constraint controls the amount of resource that is going to be extracted during the life of mine (based on the BHOD).

$$\forall n \in N \rightarrow \sum_{t=1}^{T} X_n^t \leq 1$$
$$\text{or } \sum_{t=1}^{T} X_n^t = 1 \quad (15)$$

3.4.5. Number of allowable active drawpoints at each period of time

This constraint controls maximum and minimum number of active drawpoints at each period of time.

$$\forall t \in T \rightarrow \sum_{n=1}^{N} (\mathrm{Y}1)_n^t \leq ActMax \quad (16)$$

$$\forall t \in T \rightarrow \sum_{n=1}^{N} (\mathrm{Y}1)_n^t \geq ActMin \quad (17)$$

3.4.6. Development direction and mining precedence

Extraction of each drawpoint can be started if the previous drawpoints (based on the defined precedence) have been started before. Two of the key steps in block-caving operation scheduling are development direction and drawpoints' precedence determination. More details about this constraint can be found in (Khodayari & Pourrahimian, 2015a). The priorities for development direction and precedence of extraction of the case study of this research is shown layout in Fig. 1.

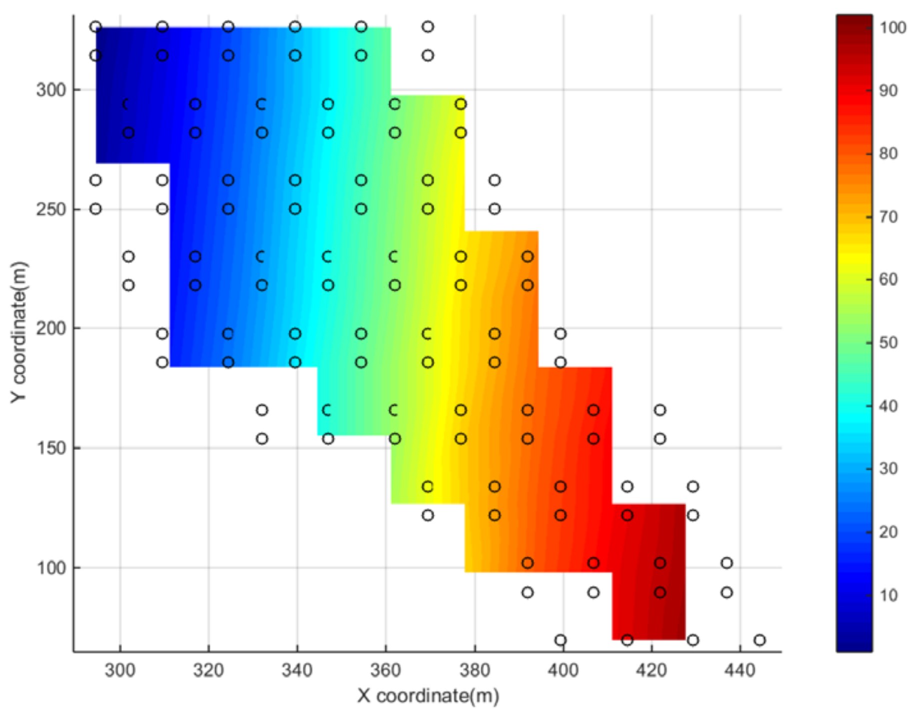

Fig. 1. Mining direction determination for block-cave layout

The precedent constraint is defined by the following function:

$$\forall n \in N \,\&\, t \in T \rightarrow (Y2)_n^t \le (Y2)_{n-1}^t \quad (18)$$

Y2 is the second set of binary variables (section 3.4.1). It means that i^{th} drawpoint can start its production if only if drawpoint i-1 has already started its production in previous periods or they can be started at the same period. i represents the precedence of that specific drawpoint.

3.4.7. Continuous mining

This constraint makes sure that if extraction from a drawpoint starts in a period, then the extraction will continue till end of its life. It means that there is no gap between extractions for drawpoints in their lives of production.

$$\forall n \in N \,\&\, t \in T \rightarrow (Y1)_n^t + (DP4)_n^t \ge (Y1)_n^{t-1} \quad (19)$$

Y1 is the first set of binary variable (section 3.4.1). DP4 represents the total of draw percentage for periods 1,…,t-1 which doesn't include the current period (t).

3.4.8. Draw rate

This constraint controls the production rate for each drawpoint based on the draw rate curve. Draw rate curve is a function of the material that has been already extracted during the previous periods of production in a drawpoint. It makes sure that the neighboring ratio is considered to have an uniform extraction profile during the production which results in low mixing and dilution. The draw rate curve is divided to different area based on the amount of extraction (depletion percentage). Fig. 2 Presents draw rate curve and its different area.

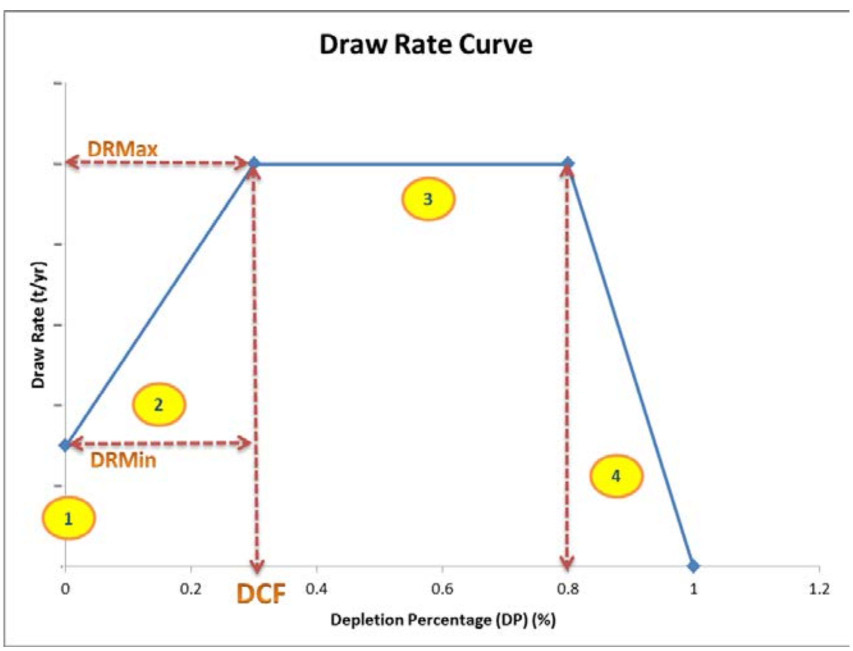

Fig. 2. Dividing draw rate curve to model it using binary variables

For each area (1, 2, 3, 4) in the draw rate curve, some formulations will be added to the model to have the production based on the draw rate curve. The current model doesn't include this constraint and we are still working on that.

4. Implementation of the models

A real case data for a block-cave mining operation is implemented for testing both the MILP and MIQP models. The resource estimation shows that the main element of the ore body is Copper. Mine development has been finished and the life of mine is 5 years. The mine has been designed and the production is going to be based on 102 drawpoints. Fig. 3 shows the designed lay out.

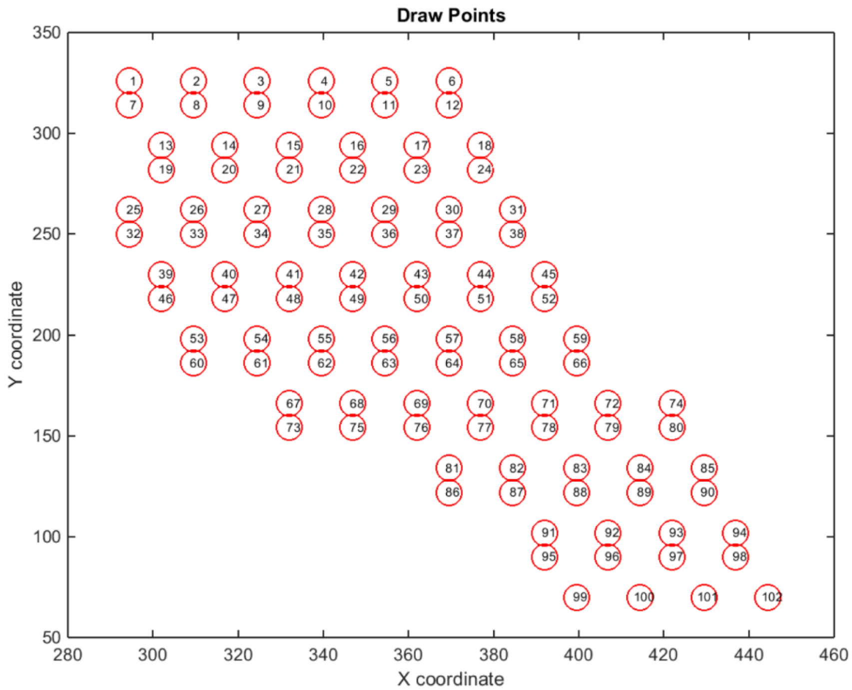

Fig. 3. The layout of drawpoints (the numbers inside drawpoints are just initial record numbers)

Based on the reserve estimation, total tonnage is 13.4 million tonne with the average weighted grade of %1.33 of Cu (the grade range is %0.5 to %1.61). The constraints are the same for both models. The scheduling parameters are presented in table 1.

Table 1. The scheduling parameters

Parameter	Value	unit
G_{min}	0.9	%
G_{max}	1.6	%
M_{min}	2.5	Mt
M_{max}	3	Mt
ActMin	50	-
ActMax	90	-
Price	4,318	USD/tonne
Cost	18	USD/tonne
Recovery	85	%
Interest rate	10	%
EGap	5	%

4.1. MILP model

We tested the MILP model for the case study, the model contains 5*N*T decision variables in which the first N*T variables are continuous and the rest are binary variables. As it was mentioned, the objective function is going to maximize net present value of the project considering the related constraints. The resulted production during the life of mine is shown in Fig. 4.

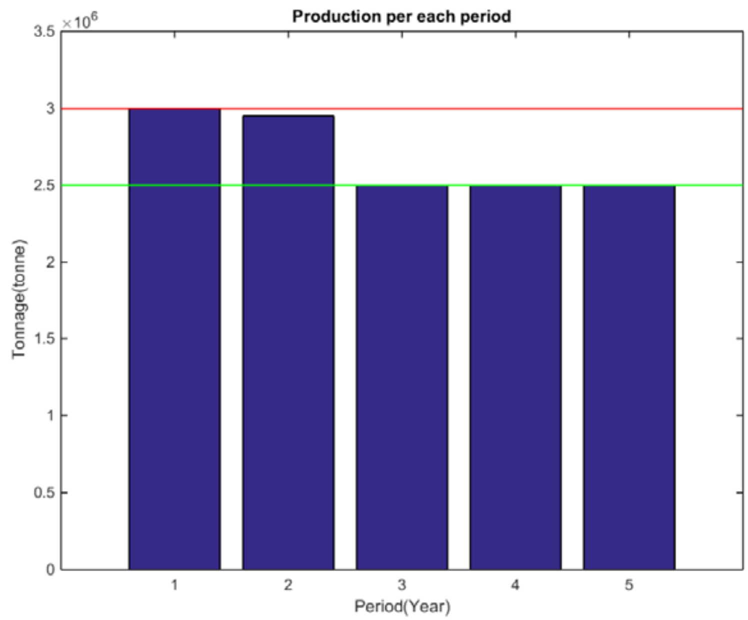

Fig. 4. Total production during the life of mine (the green and red lines are the minimum and maximum mining capacity respectively) (MILP results)

It can be seen that the MILP model tries to produce more during the first years of production to maximize the NPV while satisfying the mining capacity constraint. The average production grade is presented in Fig. 5.

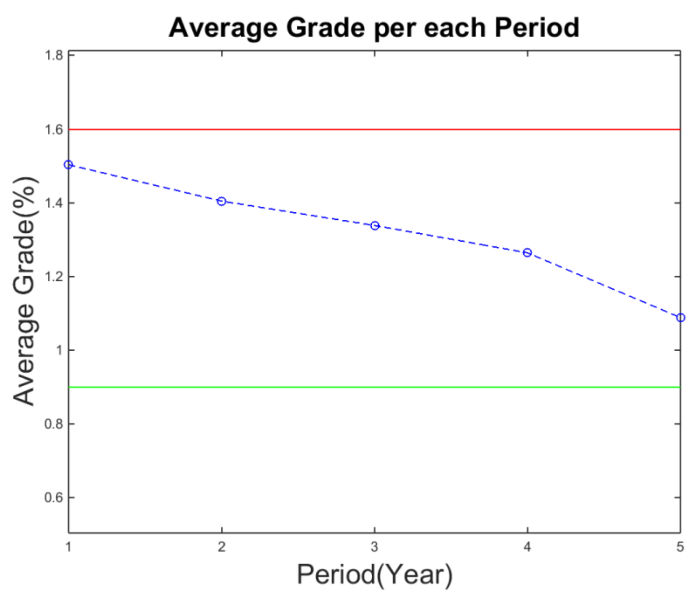

Fig. 5. Average production grade (%Cu)–the green and red lines show the acceptable range of grade for production (MILP results)

The MILP model tries to extract higher grades first while satisfying the grade constraint. The precedence of the drawpoints is determined based on the direction on Fig. 1. The considered precedence is presented in Fig. 6.

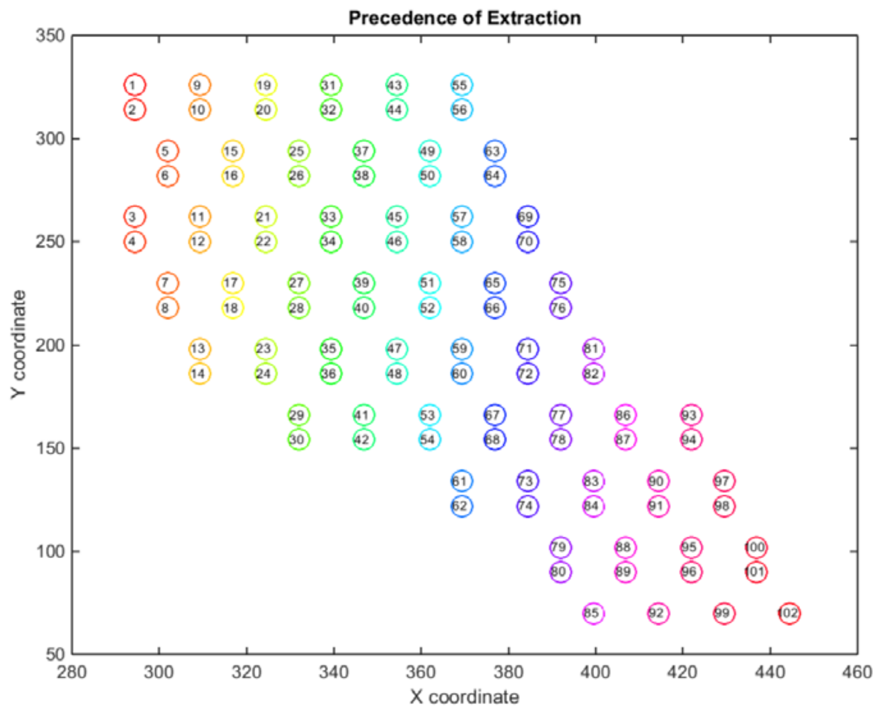

Fig. 6. Precedence of the drawpoints based on the defined direction

Considering the precedence, the extraction of drawpoints starts from number 1 to 102. Fig. 7 shows the starting period for drawpoints during the life of mine.

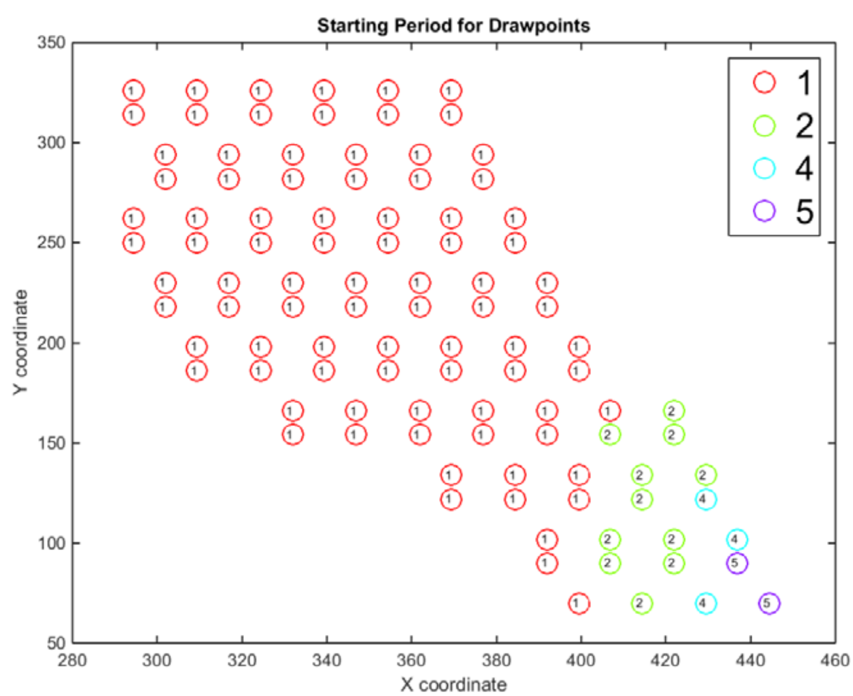

Fig. 7. Starting period for drawpoints during the life of mine (MILP results)

It can be seen that the results follow the defined precedence. Draw rates of the drawpoints during their production life doesn't follow a specific order or trend (Fig. 8). It needs more research to add the draw rate constraint to the optimization model.

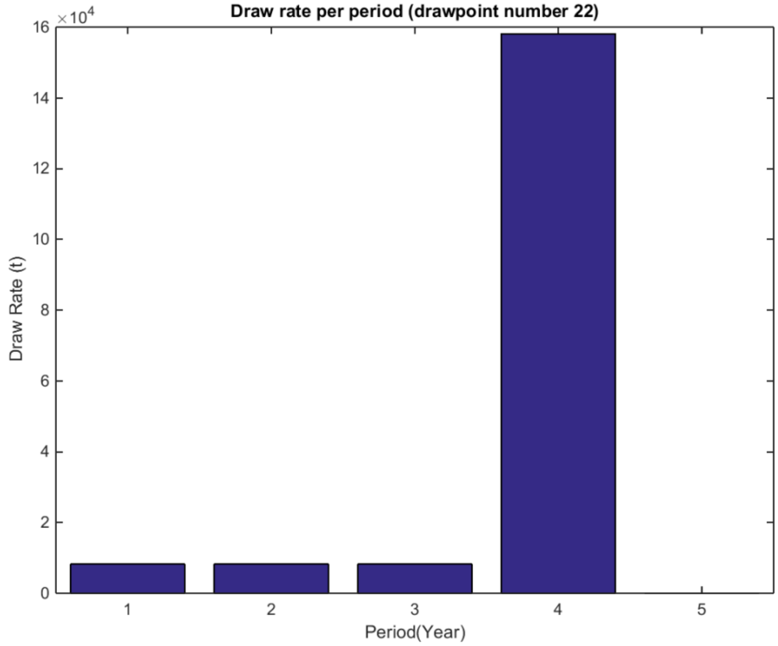

Fig. 8. Draw rate for drawpoint number 22 during the life of mine (MILP results)

The cave surface or the profile of extraction resulted from MILP model shows high fluctuations between drawpoints and their neighbors (Fig. 9). This extraction increases the probability of dilution in the production.

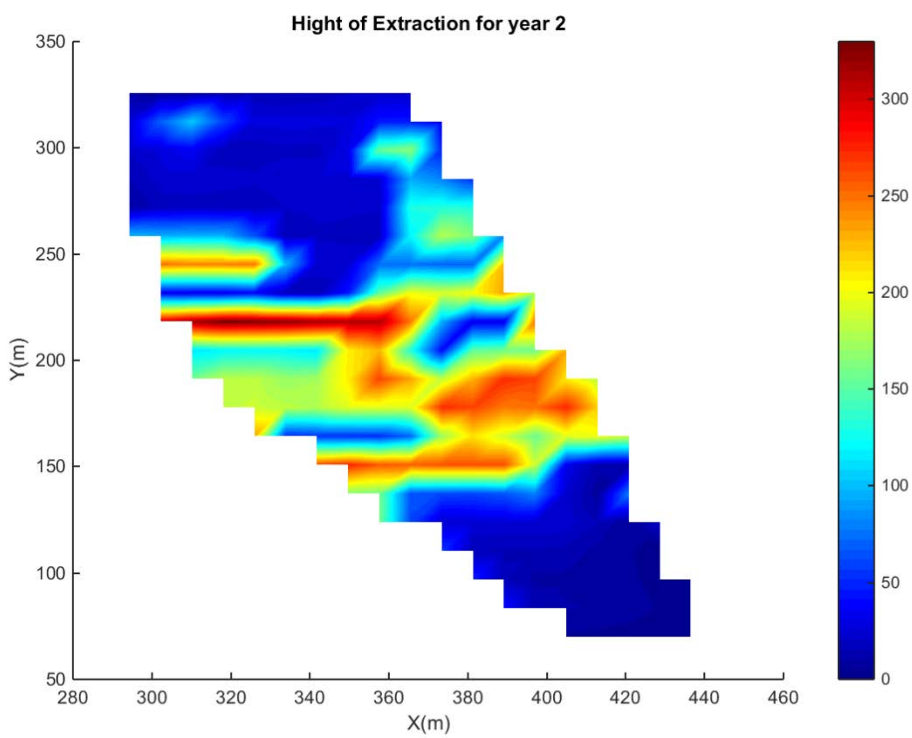

Fig. 9. Extraction profile (MILP results)

For better visualization, the 3-D plot of the extraction surface is presented in Fig. 10. It can be seen that the sharp edges are so common in the MILP model.

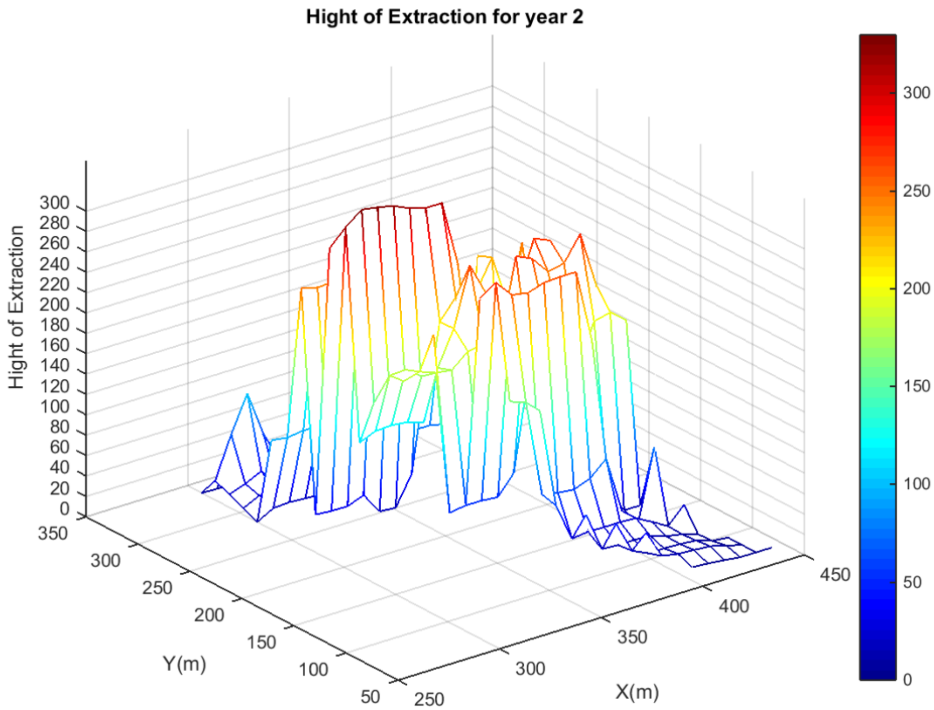

Fig. 10. Extraction profile surface (MILP results)

4.2. MIQP model

We used the same case study for testing the MIQP model. The model contains 5*N*T variables in which the first N*T variables are continuous and the rest are binary variables. The objective function is going to minimize the difference between an initial tonnage of extraction and the tonnage of extraction which is based on the production scheduling. The resulted production during the life of mine is shown in Fig. 11.

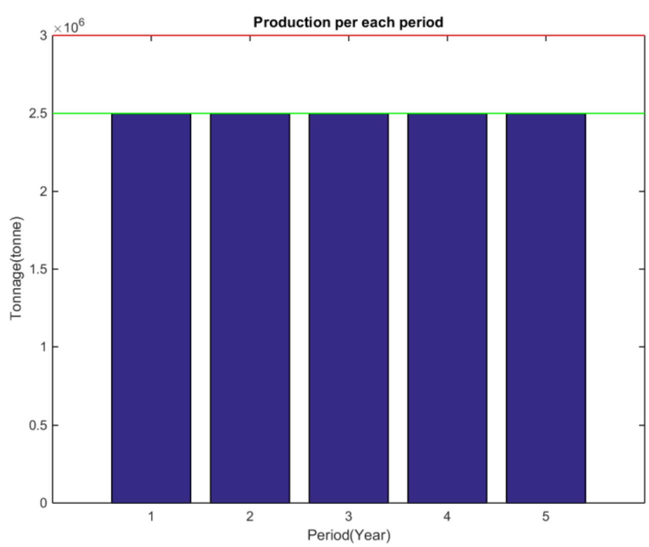

Fig. 11. Total production during the life of mine (the green and red lines are the minimum and maximum mining capacity respectively) (MIQP results)

It can be seen that the MIQP model tries to produce an even amount during the life of the mine while satisfying the mining capacity constraint. The average production grade is presented in Fig. 12.

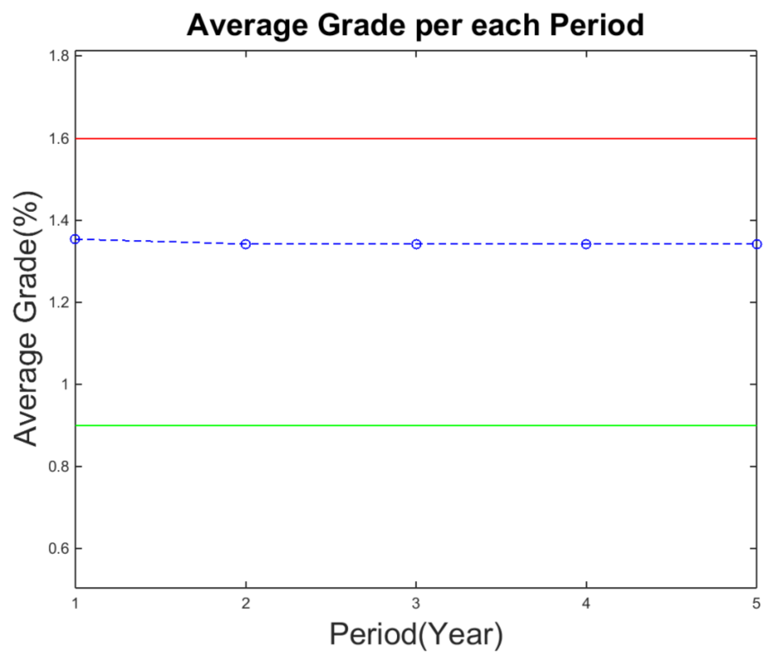

Fig. 12. Average production grade (%Cu)–the green and red lines show the acceptable range of grade for production (MIQP results)

The MIQP model produce an almost fix grade for production. Considering the precedence, the extraction of drawpoints starts from number 1 to 102. Fig. 13 shows the starting period for drawpoints during the life of mine.

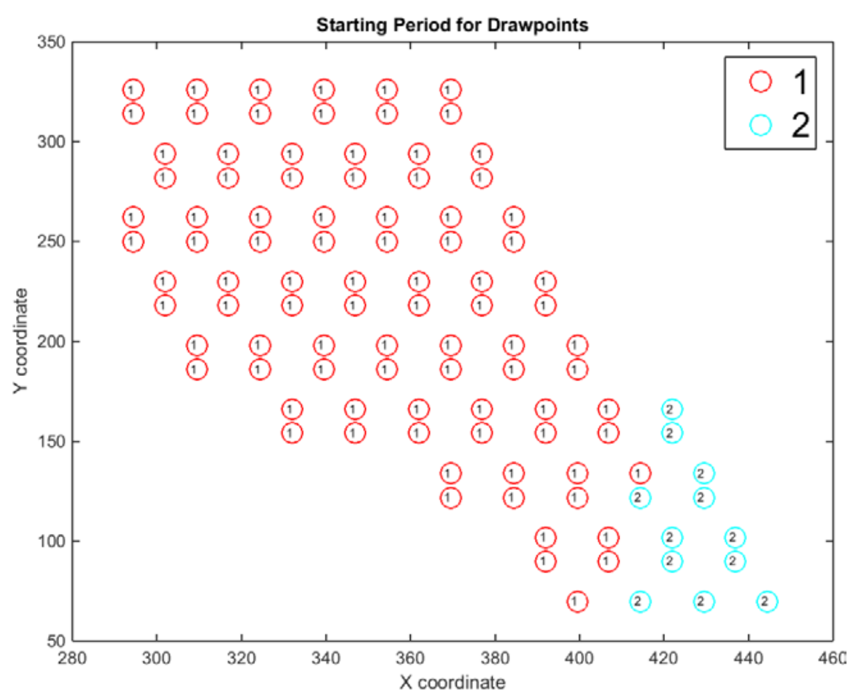

Fig. 13. Starting period for drawpoints during the life of mine (MIQP results)

It can be seen that the results follow the defined precedence. Draw rates of the drawpoints during their production life resulted from MIQP model is reasonable even without adding the draw rate constraint (Fig. 14).

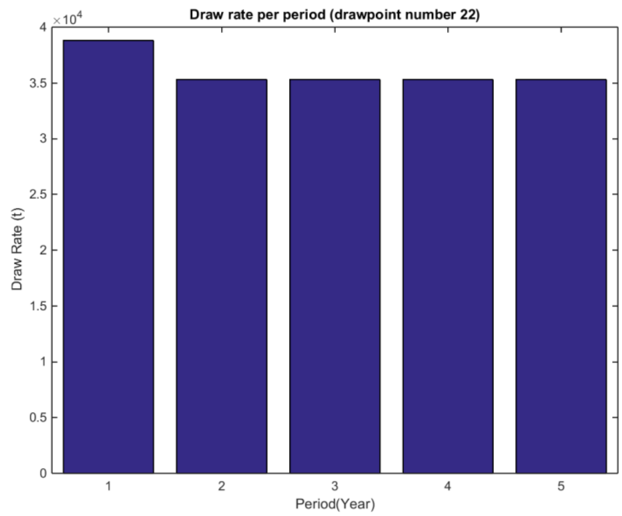

Fig. 14. Draw rate for drawpoint number 22 during the life of mine (MIQP results)

The profile of extraction resulted from MIQP model is more uniformed compare to the MILP model, as we expected (Fig. 15).

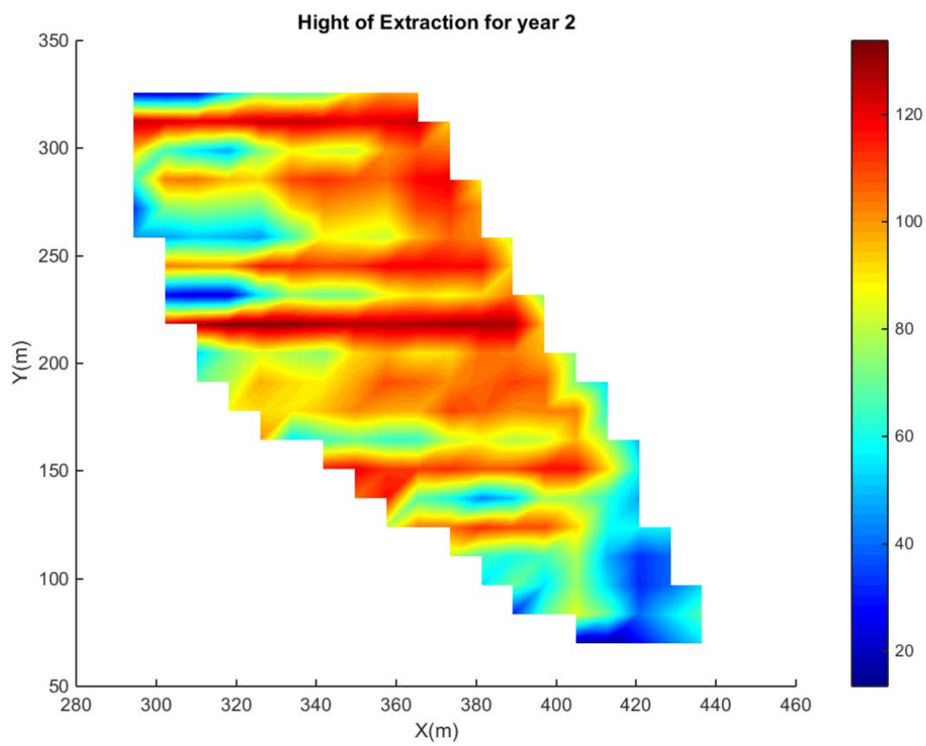

Fig. 15. Extraction profile (MIQP results)

For better visualization, the 3-D plot for extraction profile resulted from MIQP model is presented in Fig. 16. It shows that the MIQP model can generate a more practical profile compare to MILP model.

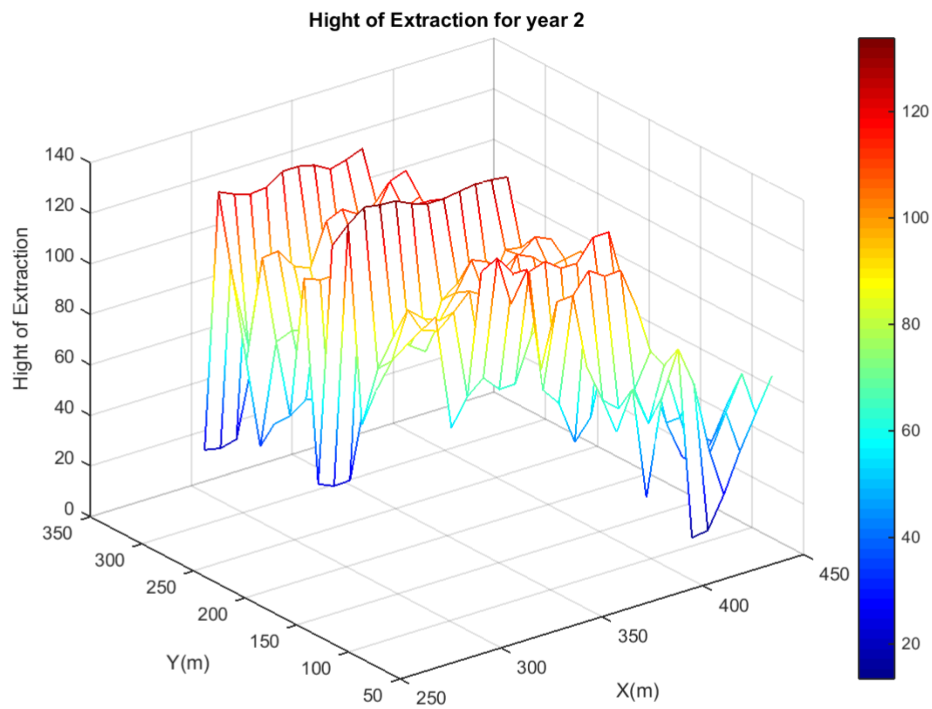

Fig. 16. Extraction profile surface (MIQP results)

5. Conclusion

In this research, we tested both MILP and MIQP models for production scheduling in block-cave mining operation. The MILP model aims to maximize NPV of the project while the MIQP model tries to minimize the tonnage deviation for achieving a uniform extraction profile. We implemented both models with same constraints for one case study. Results show that the MILP model tries to produce more in first years with higher grades. This will result in ununiformed extraction profile with high probability of dilution. The MIQP model extracts from the drawpoints smoothly with a very low fluctuation of tonnage and grade during the life of mine. This will generate uniform extraction profile with low probability of dilution.

6. References

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A Guideline to Block-Cave Production Scheduling Graphical user Interface

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Abstract

This paper introduces the open-source software application with a graphical user interface called drawpoint scheduling in block-caving (DSBC). This appendix explains how the sequence of extraction in a block cave mine can be optimized using the mixed-integer linear programming (MILP) formulations in the DSBC. This appendix presents the workflow and documentation on how to use DSBC software. The prototype software helps transfer knowledge and optimization technology developed in this research to practitioners and end-users in the field of block-cave production scheduling.

1. Introduction

The developed prototype software helps transfer knowledge and optimization technology developed in our research to practitioners and end-users in the field of block-cave production scheduling.

In this paper, an open-source software application with a graphical user interface called drawpoint scheduling in block-caving (DSBC) is introduced. The paper also explains how the sequence of extraction in a block-cave mine can be optimized using the MILP formulations in the DSBC.

All stages before scheduling, from creating a block model to converting a slice file, are done using GEMS and PCBC (Geovia, 2014). These stages include:

1. Creating a block model using GEMS.
2. Importing drawpoints data such as coordinates, dip, and azimuth.
3. Creating a slice file using PCBC.
4. Calculating the best height of draw (BHOD).

After creating the slice file, all the clustering and optimization steps are done using the DSBC. These steps are as follows:

1. Importing the slice file, the BHOD file, and coordinates of drawpoints into DSBC.
2. Creating all required databases and sets to use in the developed MILP models.
3. Clustering the draw columns based on the similarity of the draw column’s tonnage, average grade, and physical location.

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4. Defining the scheduling parameters.
5. Creating the objective function and constraints for three levels of resolution: cluster level, drawpoint level, and drawpoint-and-slice level.
6. Solving the problem using one of the methods: either single-step or multi-step.
7. Plotting the results.

2. Required Software to Run the DSBC

To run the DSBC, at the beginning MATLAB and TOMLAB/CPLEX must be installed on the computer. MATLAB is a high-level language and interactive environment for numerical computation, visualization, and programming. Using MATLAB, the user can analyze data, develop algorithms, and create models and applications. The language, tools, and built-in math functions enable users to explore multiple approaches and reach a solution faster than with spreadsheets or traditional programming languages, such as C/C++ or Java. TOMLAB is a general-purpose development and modeling environment in MATLAB for research, teaching, and finding practical solutions to optimization problems. TOMLAB/CPLEX integrates the solver package CPLEX with MATLAB and TOMLAB.

3. Experiments Framework for the MILP Models

The methodology applied to the block-cave production scheduling problem in the MILP framework includes a solution scheme that is based on the branch-and-cut optimization algorithm (Horst and Hoang, 1996) implemented in TOMLAB/CPLEX (Holmstrom, 2011). To be able to obtain reliable experimental results, the solution scheme employed in solving the problem should be able to capture the complete definition of the block-cave production scheduling problem. The assumptions are based on the framework for applying operations research methods in mining. Fig 1 shows the general workflow of database creation in the DSBC. After creating the database, based on the selected level of resolution, the problem is solved. These levels include the cluster level, drawpoint level, and drawpoint-and-slice level.

The three MILP formulations for three levels of problem resolution -- cluster level, drawpoint level, and drawpoint-and-slice level -- can be used in two ways: (i) single-step, in which each formulations is used independently; and (ii) multi-step, in which each step's solution is used to reduce the number of variables in the next level and consequently generate a practical block-cave schedule in a reasonable CPU runtime for large-scale problems. Fig 2 and Fig 3 show the general workflow for single-step and multi-step methods, respectively. It should be mentioned that the MILP formulations use a solver developed based on exact solution methods for optimization. In this solver, an optimization termination criterion is set up to stop the algorithm when an integer feasible solution has been proved to be within a specific percentage of optimality, subject to the practical and technical mining constraints.

4. Input Data

To solve the problem we use the PCBC's slice file. Three Excel files with the following information have to be prepared:

- *Coordinates*: This file contains two sheets titled "Drawpoints" and "Tunnels." Fig 4 shows the order of these Excel sheets. The order of columns in the sheet titled Drawpoints is record, drawpoint's name, X, Y, and depth. In the sheet titled Tunnels, the coordinates of the start point and endpoint of each tunnel are defined.

- *Slice info:* This file contains information about slices within each draw column. These data include dilution, density, tonnage, dollar value, and percentage of the elements within each slice. Fig 5 shows the order of parameters in this file.
- *BHOD:* This file contains the BHOD information for draw columns and other economic information. Fig 6 shows the order of this file.

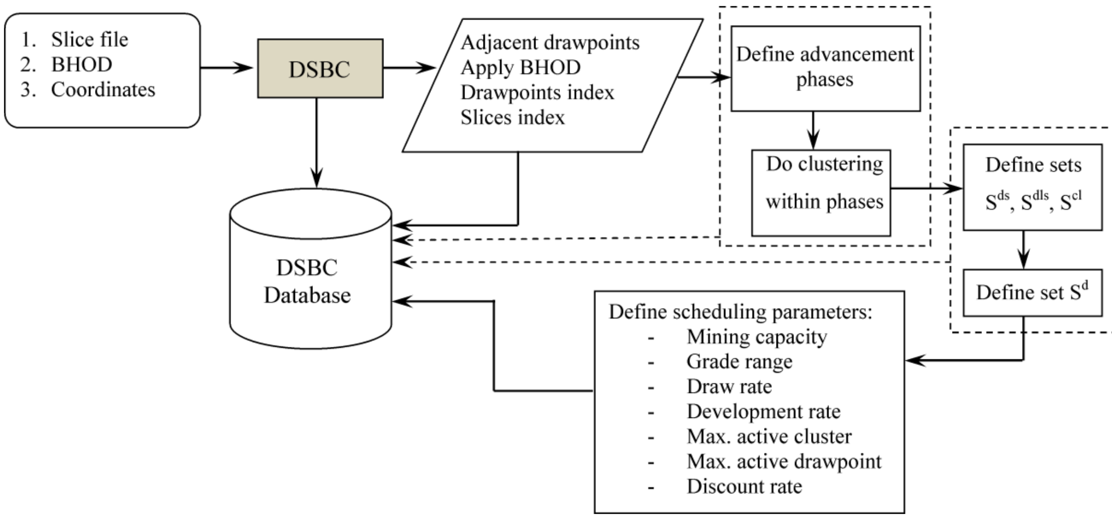

Fig 1. Database creation in the DSBC

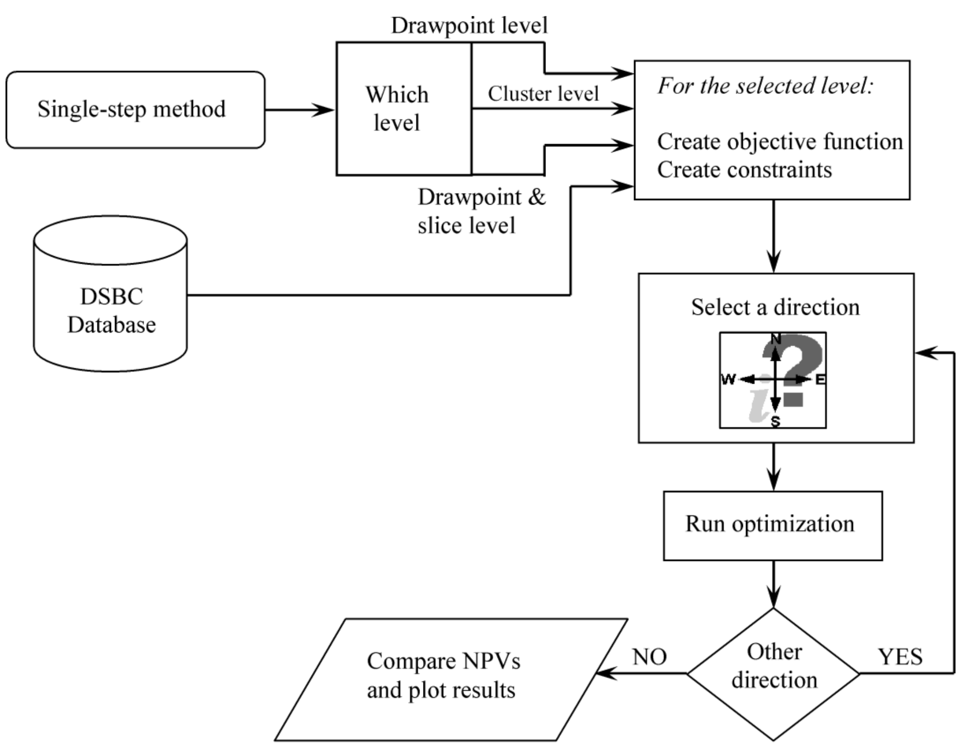

Fig 2. General workflow for single-step method

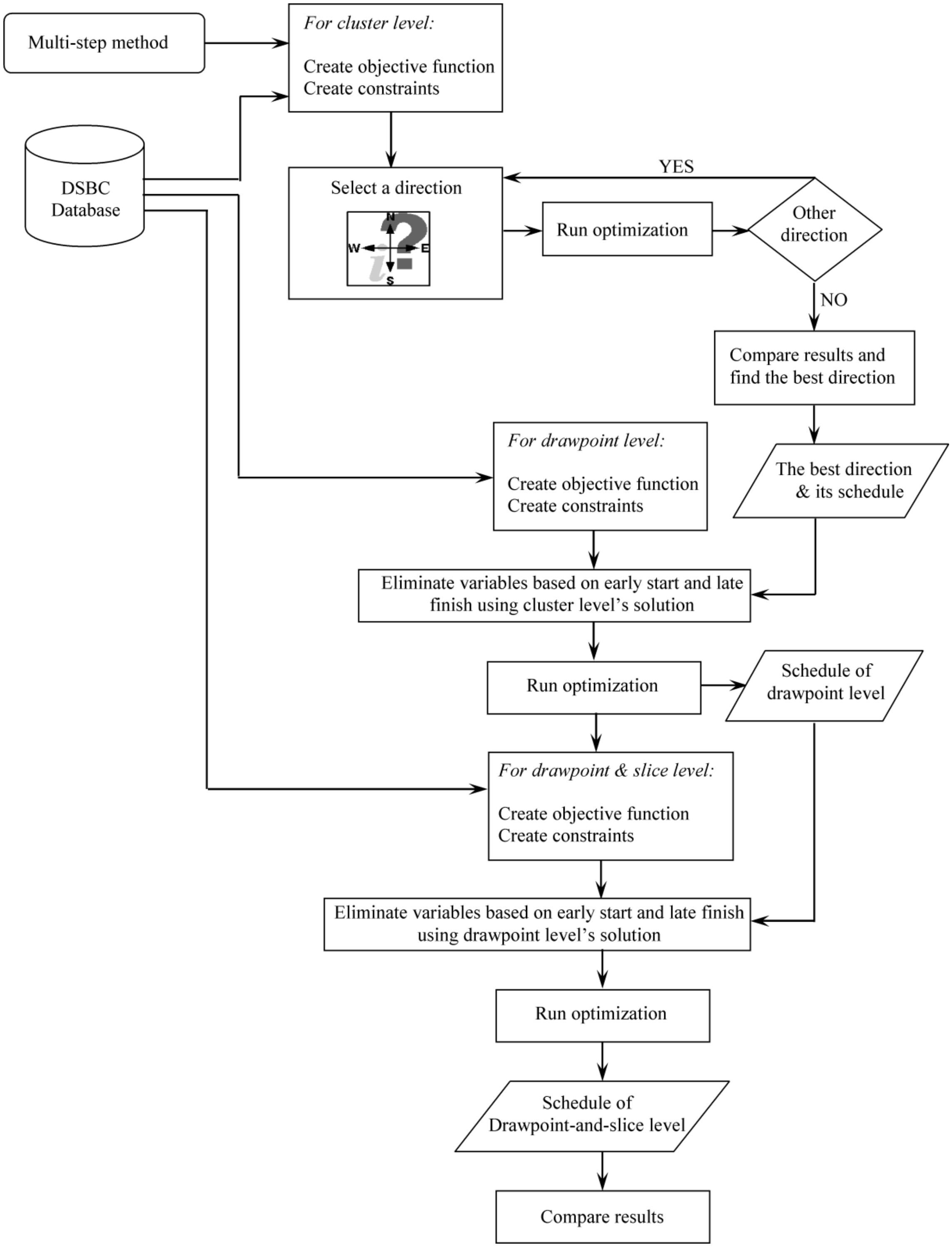

Fig 3. General workflow for multi-step method

	A	B	C	D	E
1	Record	Drawpoint	x	Y	Depth
2	1	P01N06	301.92	538	590
3	2	P01N07	316.92	538	590
4	3	P01N08	331.92	538	590
5	4	P01N09	346.92	538	590
6	5	P01S04	279.42	518	590
7	6	P01S05	294.42	518	590
8	7	P01S06	309.42	518	590
9	8	P01S07	324.42	518	590
10	9	P01S08	339.42	518	590
11	10	P01S09	354.42	518	590
12	11	P01S10	369.42	518	590
13	12	P02N03	264.42	506	590
14	13	P02N04	279.42	506	590
15	14	P02N05	294.42	506	590
16	15	P02N06	309.42	506	590
17	16	P02N07	324.42	506	590
18	17	P02N08	339.42	506	590
19	18	P02N09	354.42	506	590
20	19	P02N10	369.42	506	590
21	20	P02S02	241.92	486	590
22	21	P02S03	256.92	486	590

Drawpoint sheet

	A	B	C	D	E	F
1	Tunnel Record	Tunnel Name	Xstart	Ystart	Xend	Yend
2	1	T01	226	528	460	528
3	2	T02	226	496	460	496
4	3	T03	226	464	460	464
5	4	T04	226	432	460	432
6	5	T05	226	400	460	400
7	6	T06	226	368	460	368
8	7	T07	226	336	460	336
9	8	T08	226	304	460	304
10	9	T09	226	272	460	272
11	10	T10	226	240	460	240
12	11	T11	226	208	460	208
13	12	T12	226	176	460	176

Tunnels sheet

Fig 4. Structure of drawpoints and tunnels sheets in the Excel file

	A	B	C	D	E	F	G	H	I	J
1	Record	Slice Name	Descriptic	Height	Dil %	Density	Tons	$val	CU	AU
2	1	P07S03	... Vmix10	10.00	0	2.80	3,007.59	25.95	1.48	0.50
3				20.00	1	2.80	5,790.37	24.73	1.47	0.43
4				30.00	4	2.80	6,438.14	22.97	1.42	0.41
5				40.00	8	2.79	6,702.38	20.89	1.36	0.38
6				50.00	13	2.79	6,676.88	18.16	1.27	0.35
7				60.00	20	2.78	6,675.91	14.94	1.17	0.32
8				70.00	26	2.77	6,639.00	11.39	1.06	0.28
9				80.00	33	2.77	6,696.08	7.68	0.94	0.25
10				90.00	41	2.76	6,731.95	3.84	0.82	0.22
11				100.00	49	2.75	6,605.59	(0.09)	0.70	0.18
12				110.00	57	2.74	6,491.40	(3.86)	0.58	0.15
13				120.00	65	2.73	6,414.18	(7.30)	0.47	0.12
14				130.00	72	2.73	6,396.01	(10.30)	0.37	0.10
15				140.00	78	2.72	6,417.30	(12.85)	0.29	0.08
16				150.00	83	2.72	6,406.64	(15.00)	0.22	0.06
17				160.00	87	2.71	6,390.34	(16.77)	0.17	0.04
18				170.00	91	2.71	6,390.87	(18.18)	0.12	0.03
19				180.00	94	2.71	6,409.45	(19.27)	0.09	0.02
20				190.00	96	2.70	6,380.00	(20.10)	0.06	0.02
21				200.00	97	2.70	6,428.43	(20.70)	0.04	0.01
22				210.00	98	2.70	6,493.94	(21.12)	0.03	0.01
23				220.00	99	2.69	6,513.13	(21.41)	0.02	0.00
24				230.00	99	2.68	6,472.15	(21.60)	0.01	0.00
25				240.00	99	2.67	6,478.63	(21.72)	0.01	0.00
26				250.00	100	2.66	6,507.08	(21.79)	0.01	0.00
27				260.00	100	2.65	6,520.76	(21.83)	0.01	0.00
28				270.00	100	2.63	6,482.01	(21.84)	0.01	0.00
29				280.00	100	2.61	6,505.49	(21.84)	0.01	0.00
30				290.00	100	2.59	6,062.47	(21.83)	0.01	0.00
31				300.00	100	2.57	6,069.73	(21.81)	0.01	0.00
32				310.00	100	2.55	6,071.42	(21.79)	0.01	0.00
33				320.00	100	2.54	6,071.42	(21.77)	0.01	0.00
34				330.00	100	2.54	6,071.42	(21.77)	0.01	0.00
35				334.50	100	2.50	2,439.55	(21.86)	0.00	0.00
36										
37	2	P08N03	... Vmix10	10.00	1	2.80	3,007.59	27.37	1.57	0.42
38				20.00	3	2.80	5,790.37	24.71	1.48	0.40

Fig 5. Structure of the slice file

	A	B	C	D	E	F	G	H	I	J
1	Record	Draw Point	OK?	Best HOD	Ave_Dol	Net_Dol	Tot_Dol	Tonnage	CU	AU
2	1	P07S03	OK	90	15.95049	882.9919	882.9919	55358.29	1.198847	0.337136
3	2	P08N03	OK	80	14.90038	723.4717	723.4717	48553.92	1.173203	0.311591
4	3	P07S04	OK	160	19.85667	2037.188	2037.188	102594.6	1.3276	0.360297
5	4	P08N04	OK	140	19.59518	1744.636	1744.636	89033.96	1.323794	0.348382
6	5	P07S05	OK	220	21.16943	3000.261	3000.261	141726.1	1.372936	0.363632
7	6	P08N05	OK	210	20.68158	2789.119	2789.119	134860.1	1.358006	0.358263
8	7	P07S09	OK	280	20.46782	3713.835	3713.835	181447.5	1.344116	0.371735
9	8	P08N09	OK	280	21.31608	3871.473	3871.473	181622.1	1.378115	0.363758
10	9	P07S10	OK	280	17.35229	3178.504	3178.504	183174.9	1.244121	0.347454
11	10	P08N10	OK	280	19.56856	3588.879	3588.879	183400.3	1.320371	0.353706
12	11	P07S11	OK	270	12.15976	2116.752	2116.752	174078.4	1.073345	0.315853
13	12	P08N11	OK	280	15.41513	2786.104	2786.104	180738.3	1.184623	0.326591
14	13	P08S04	OK	100	14.25443	879.1486	879.1486	61675.44	1.151218	0.309256

Fig 6. Structure of the file containing the BHODs

5. Guideline on Running DSBC

The DSBC folder contains sub-folders, .tif and .m files (see Table 1). Within the folder DSBC, right click on the *DSBC_Login.m*, and then open it in MATLAB. In MATLAB, run the opened text file. If you have been asked to change the directory, accept it and change the directory. In the opened login window, enter User name and Password. Then press **Login** (see Fig 7). The main window of DSBC comes up (see Fig 8).

Table 1. Existing sub-folders, photos and MATLAB files within the DSBC's folder

Folders	.tif files	.fig files	.m files
8. FUNCTION	16.BlockCave1	19.DSBC_Login	20.DSBC_Login
9. IMPORT	17.BlockCave2		
10.Input Data	18.BlockCave3		
11.MODELS			
12.REPORTS			
13.RESULTS			
14.TOMLAB Input			
15.TOMLAB Output			

Fig 7. DSBC's login window

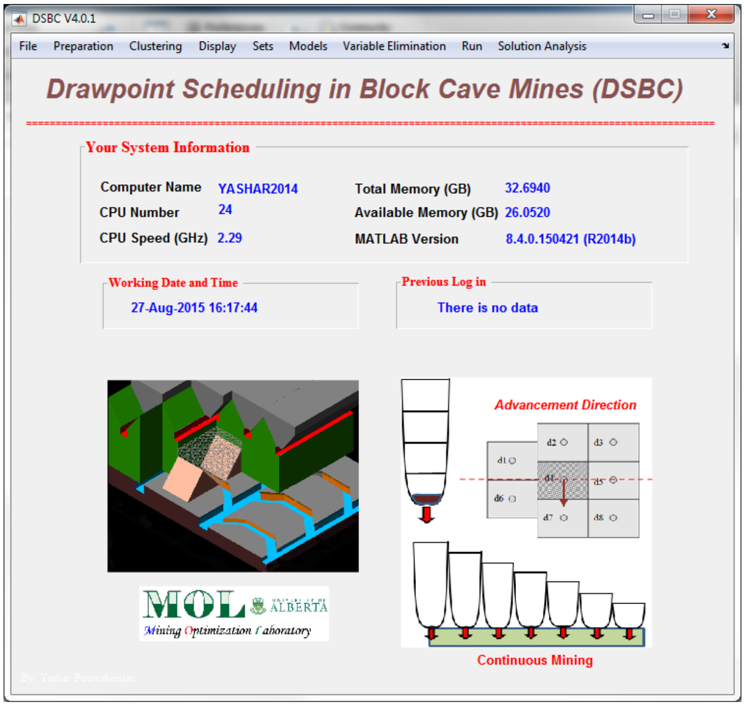

Fig 8. Main window of the DSBC

The components of the main window are menu bar, system information area, and date and time information area. The menu bar contains File, Preparation, Clustering, Display, Sets, Models, Variable Elimination, Run, and Solution Analysis.

In the system information area, useful information about the computer which the DSBC is running on that and version of the MATLAB are presented. Under the system information area, the date and time of the current and previous login is displayed.

5.1. Database Preparation

After executing the DSBC, to create a new project, go to:

File > Set New Problem

If there is another project, this option makes a backup from that project and then creates a new project. To import the three main Excel files into the DSBC, go to **File > import .xls to DSBC**. In the opened window, import the Excel files one by one. Then press **OK** (see Fig 9). After the Excel files are imported, they have to be converted to the MATLAB format. For this purpose, go to **File > Convert To .mat**.

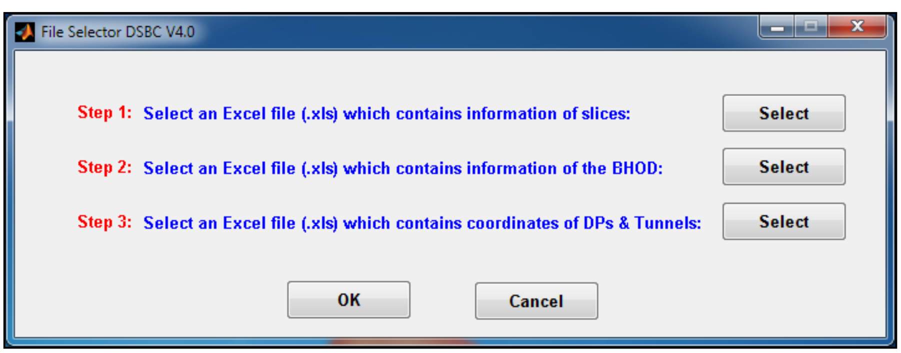

Fig 9. Import window

A window titled **Excel 2 MATLAB** comes up (see Fig 10). In this window you have to follow the steps. After each step the window is updated at the left bottom corner, and green lines appear.

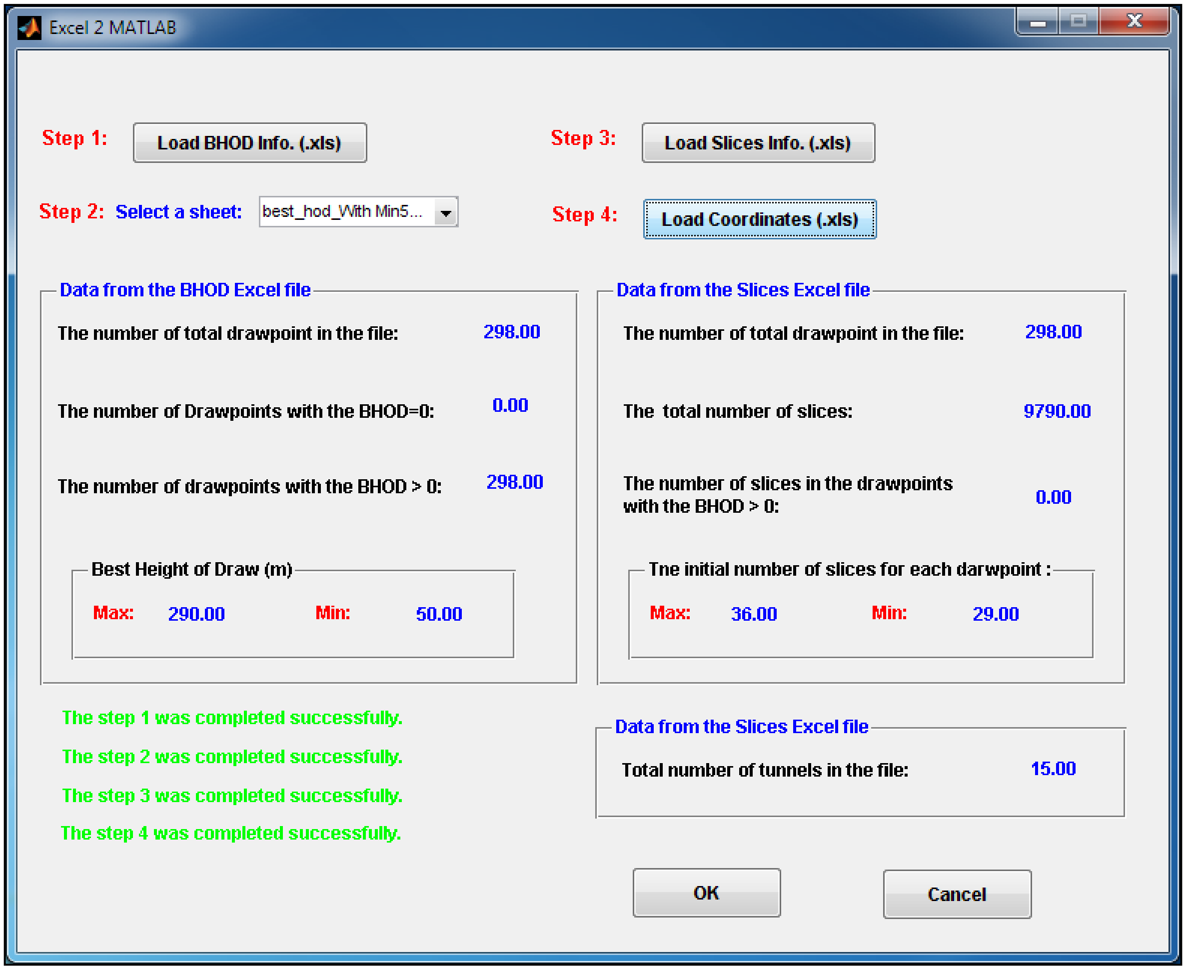

Fig 10. Excel 2 MATLAB window to convert .xls files to .mat

In the Excel 2 MATLAB window, Press **Load BHOD Info. (.xls)**. Then, from the pop-up menu in front of Step 2, select the sheet which contains the BHODs (see Fig 11).

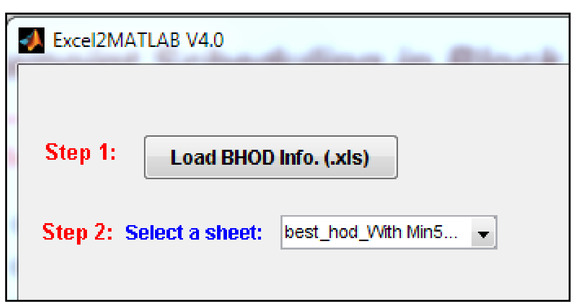

Fig 11. Selection of the sheet which contains the BHOD information

Press **Load Slice Info. (.xls)** to convert the slices info, and then Press **Load Coordinates (.xls)**. Check the data. Then press **OK**.

As the next step, the adjacent drawpoints and the drawpoints located between tunnels are recognized. Also, for each drawpoint-and-slice, an index is defined and the BHODs are applied on the initial draw columns. All this is done using the **Preparation** menu. Under this menu, there are five options. Select these options in the following order. After each step a dialog box is opened and displays useful information about the step.

1. **Preparation > Find Adjacent Drawpoints**
2. **Preparation > Create Index for Drawpoints**

In the opened window, enter the height of the slices.

3. **Preparation > Find Drawpoints Between Tunnels**
4. **Preparation > Apply BHOD on Slice file**
5. **Preparation > Create Index for Slices**

The **Display** menu allows a user to show the plan view of drawpoints, 3D view of draw columns, initial height of draw columns, and height of draw columns after applying the BHOD. To show these views, go to the **Display** menu and select the related option. If there is a draw column with BHOD = 0 in the data, it can be displayed using the last option under the **Display** menu.

5.2. Clustering and Creating the Sets

The next step is clustering. Before clustering, the advancement directions must be defined. For this purpose, go to **Clustering > Advancement Areas**. In the opened window, select a direction and press **APPLY**. This will display the plan view of the drawpoints and tunnels. Now, based on the advancement direction, the boundary of phases must be created in the following order:

- WE or EW: the lines should be created from left to right.
- NS or SN: the lines should be created from bottom to top.
- SWNE or NESW: the lines should be created from the left bottom corner to the right top corner.
- NWSE or SENW: the lines should be created from the right bottom corner to the left top corner.

Each line has two points. These points must be out of the black dash-line boundary. To pick the start and end points of the lines, use the LEFT click on the mouse. However, for the last line, the last point MUST be selected by the RIGHT click on the mouse. Fig 12 shows the advancement lines for the west to east (WE) and east to west (EW) directions. There are seven lines for the WE and EW directions. To define the phases boundaries for other directions, press **CLEAR** and then select another direction and press **APPLY**. Then, repeat the steps described in the previous sentence. Finally, press **OK**. The next step is the clustering of the draw columns within each phase. However, before clustering, the data must be prepared. For this purpose, go to **Clustering > Preparation**.

To cluster the draw columns within each phase using the hierarchical clustering method, go to **Clustering > Method > DPs Clustering Hierarchical**. Then, select a direction for clustering and press **Plot**.

Afterwards, the clustering parameters input window comes up; in this window type the required numbers (see Fig 13). If you press **OK**, the clusters in the selected direction appear. To find the best weights that create practical clusters, you have to repeat the clustering with different weights.

After clustering, you can analyze the clusters or display them. For this purpose, go to **Clustering > Display**.

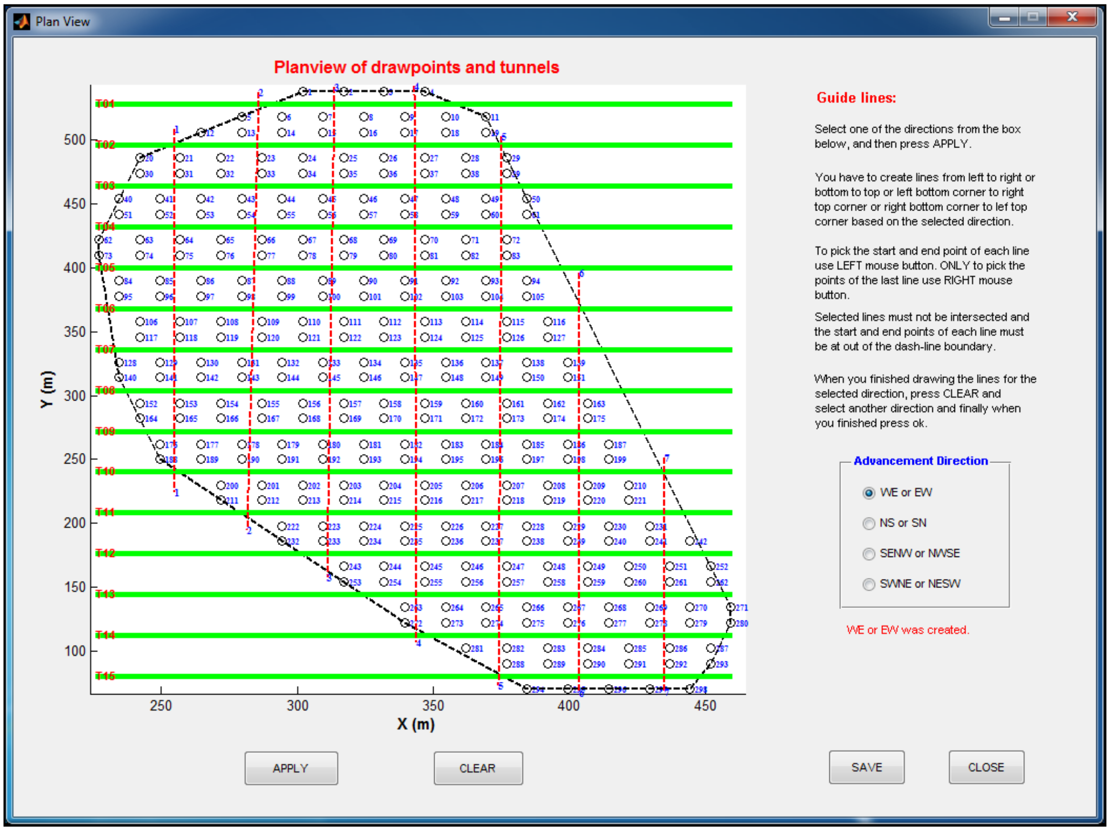

Fig 12. The window for advancement line selection with the phases boundaries for the WE or EW directions

Two options are available (see Fig 14). Using **DP Clusters**, you can display the clusters in the selected direction. Using **DP Clusters Analysts**, you can obtain useful information about the clusters and drawpoints within each cluster. This information includes tonnage, average grade, economic value, and number of drawpoints within each cluster. Go to **Clustering > Display > DP Clusters Analysts**.

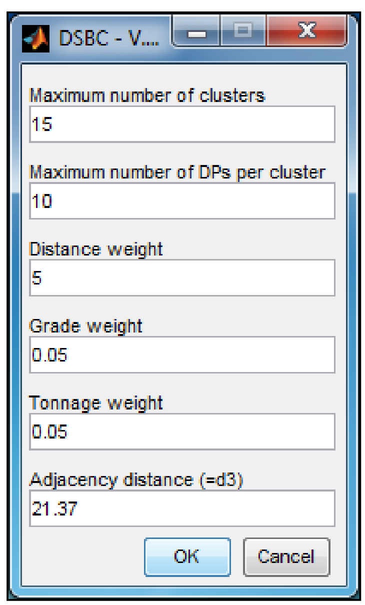

Fig 13. The clustering parameters input window

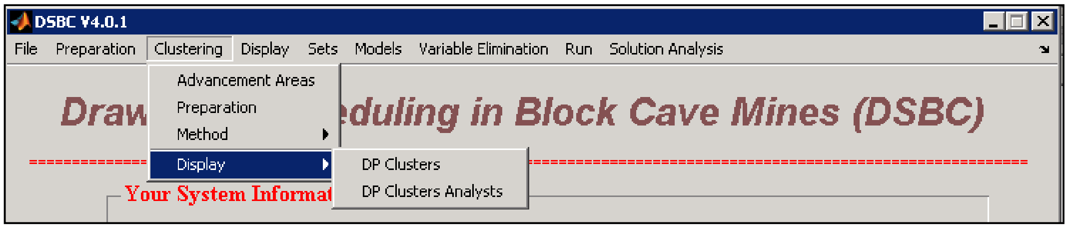

Fig 14. Available options for cluster displaying and analyzing in the DSBC

The DP Cluster Analysts window comes up. Select one of the existing directions and then press **PLOT**. The window on the left will display clusters from the selected direction. The window on the right will display tonnage, average grade, economic value, and number of draw columns for each cluster. At the bottom of the main window, each cluster can be analyzed based on tonnage, grade, and the economic value of the draw columns within the cluster. Fig 15 illustrates the DP Cluster Analysts window.

All the required sets explained in Chapter 3 are created through the **Sets** menu. To create sets S^{ds}, S^{dls}, S^{adj}, S^{CL}, and S^{d}, go to the menu called **Sets** (see Fig 16) and then select the options in the following order:

Sets > Create Sds, Sdls, Sadj

Sets > Create SCL

Sets > Create Sd

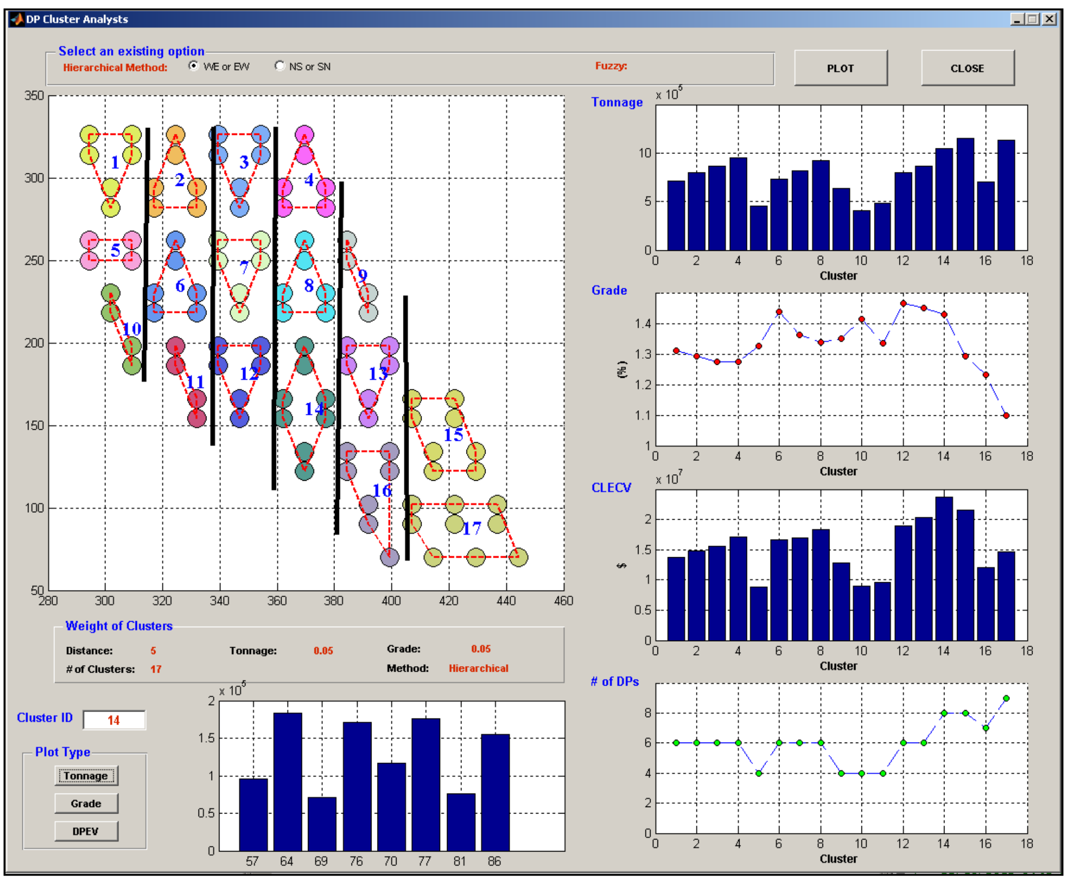

Fig 15. Cluster analysts window

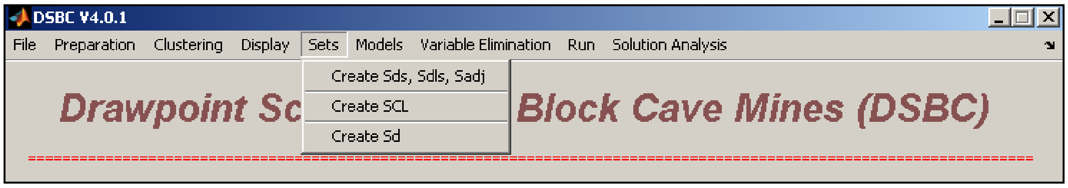

Fig 16. Sets menu to create all the required sets

5.3. MILP Models Preparation

To create the MILP models, the first step is to define of all the scheduling parameters based on the considered model. For this purpose there are two options. To define the scheduling parameters for the clustering-level model, go to

Models > Scheduling Parameters > Model With Cluster

In the opened window, select a direction and then press the **Schedule** button. A window will come up in which to enter the scheduling parameters for the clustering-level model (see Fig 17).

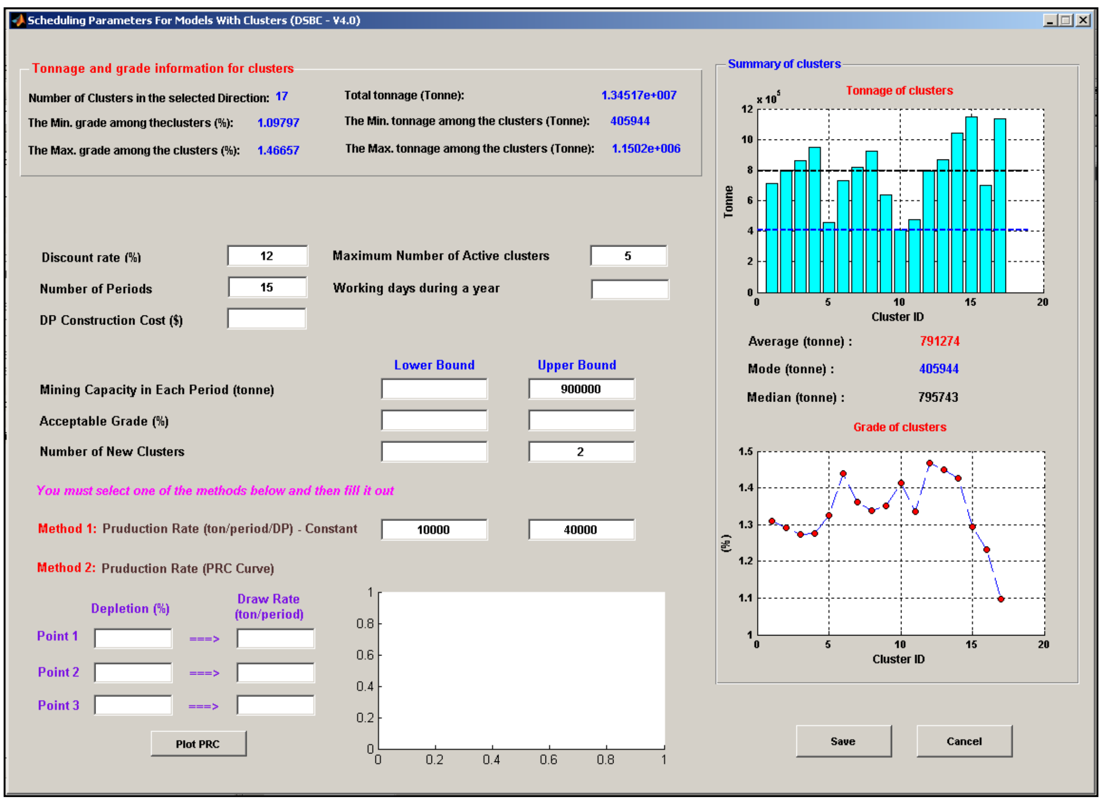

Fig 17. Scheduling parameters definition window for the cluster-level model

At the top part of this window, there is a summary about the minimum and maximum grade and tonnage among the clusters. Displayed on the right side of the window are the average grade of each cluster and its tonnage with the average, mode, and median lines. Fill out the blank boxes based on the project requirements. At the cluster-level model, user does not need to define the acceptable grade. For the draw rate, two different methods can be applied: (i) constant range, and (ii) production rate curve (PRC). Enter the drawpoint production rate in method 1. It will be automatically calculated for each cluster. Finally press the **SAVE** button. The summary of the entered data will appear; review the summary. If it is correct, press **OK**.

To define the scheduling parameters for the other models, go to

Models > Scheduling Parameters > Model Without Cluster

At the top of the open window, a summary of the useful data about the drawpoints and slices is presented. The right-hand side of the window displays the histograms related to the mentioned data (see Fig 18).

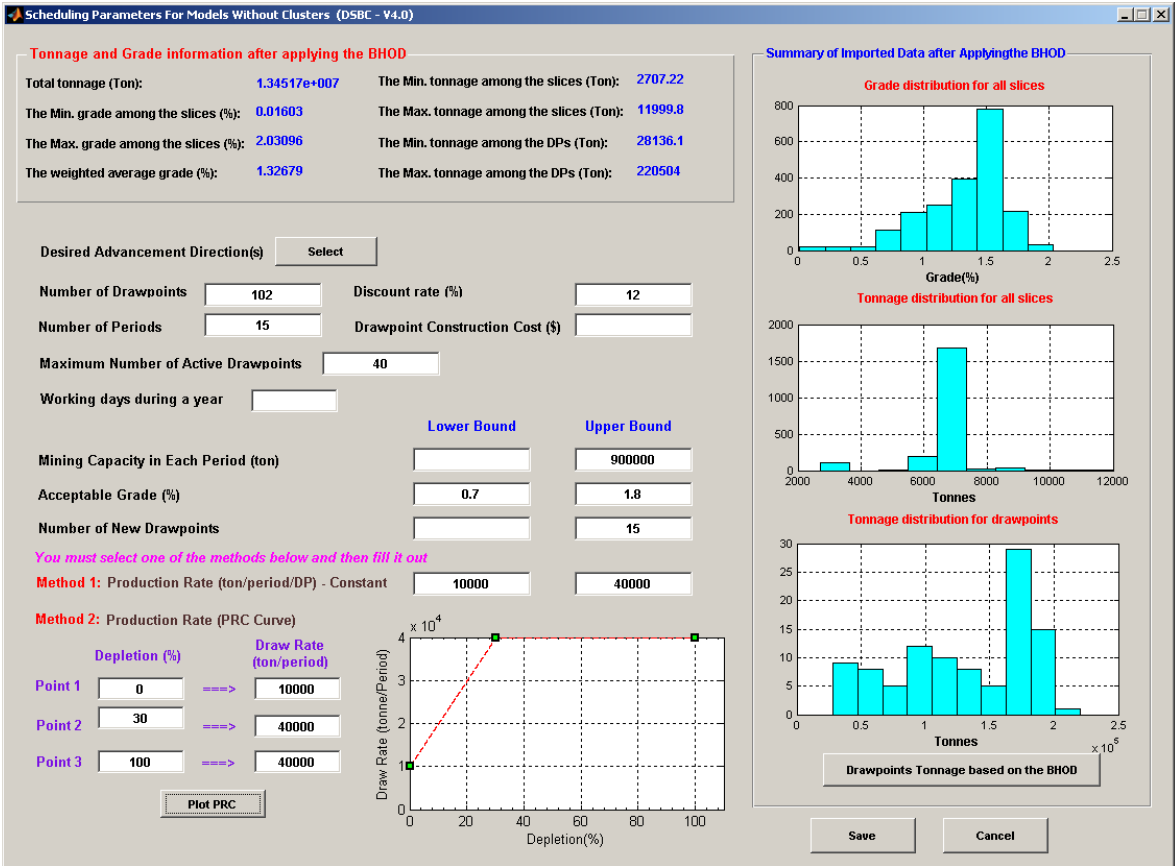

Fig 18. Scheduling parameters definition window for the drawpoint level and drawpoint-and-slice level models

Press the **Select** button in front of Desired Advancement Direction(s) and then select the scheduling directions. Then, press **OK** (see Fig 19). Fill out the blank boxes according to the project requirements. This time the acceptable grade must be defined. Finally, press the **SAVE** button. The summary of the entered data will appear. Check for accuracy. If everything is correct, press **OK**.

After defining the scheduling parameters and in order to create the models go to **Models > Create** and select the level of resolution needed to solve the problem. For example, to solve the problem at the cluster level, go to **Models > Create > Clustered DPs**. A mathematical model creator window will appear (see Fig 20). To see the formulation in detail, press the **Display Model** button. Create objective function and constraint in the order that has been appeared.

To create the draw rate constraint, select the **Lower and Upper Bounds option**. For the mining precedence, at the cluster level select **Set_SCL.mat** from the folder **SetSCL**. At the drawpoint level, select **Set_Sd.mat** from the folder **SetSd**. Finally, press **OK**.

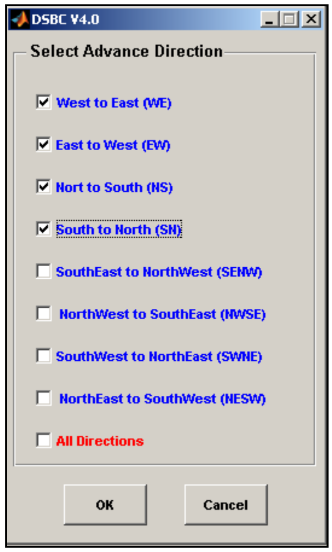

Fig 19. Direction selection window

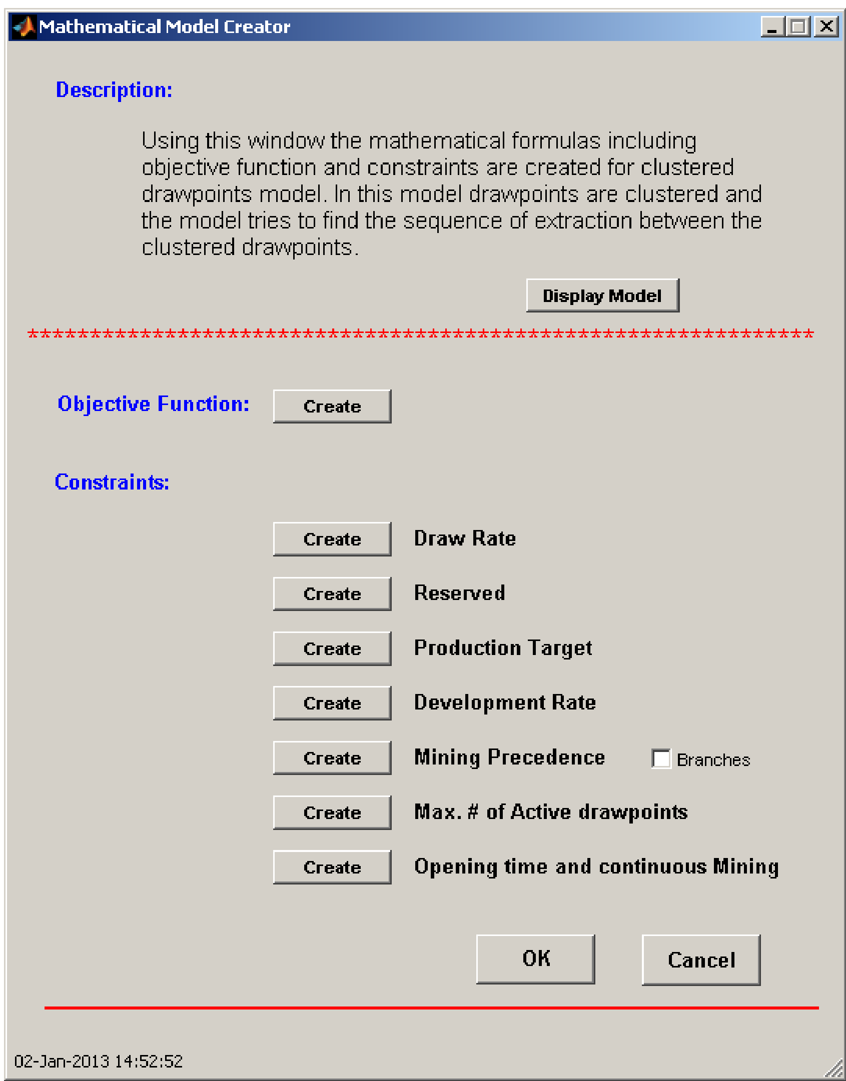

Fig 20. Mathematical model creator window for the cluster-level model

5.4. Scheduling Optimization

The MILP formulations presented for three levels of problem resolution -- cluster level, drawpoint level, and drawpoint-and-slice level -- can be used in two ways: (i) a single-step method in which each of the formulations is used independently, and (ii) a multi-step method in which each step's solution is used to reduce the number of variables in the next level and consequently generate a practical block-cave schedule in a reasonable CPU runtime for large-scale problems.

- **Single-Step Method**

After the model for each level of resolution has been created, it can be solved using the **Run** menu. For this purpose, go to **Run > Export to TOMLAB**. Select the model that you have created, and press the **Export** button. Then go to **Run > Solve the Model**.

TOMLAB/CPLEX is executed automatically. Then, a window comes up (see Fig 21). In this window, select the model and the advancement direction for optimization. Then enter the relative tolerance on the gap between the best integer objective and the objective of the best node remaining. For example, for the EPGAP of 1%, type 1 in the box. Finally, based on the number of available CPUs on your computer, enter the number of required CPUs to solve the selected model and press **Solve**.

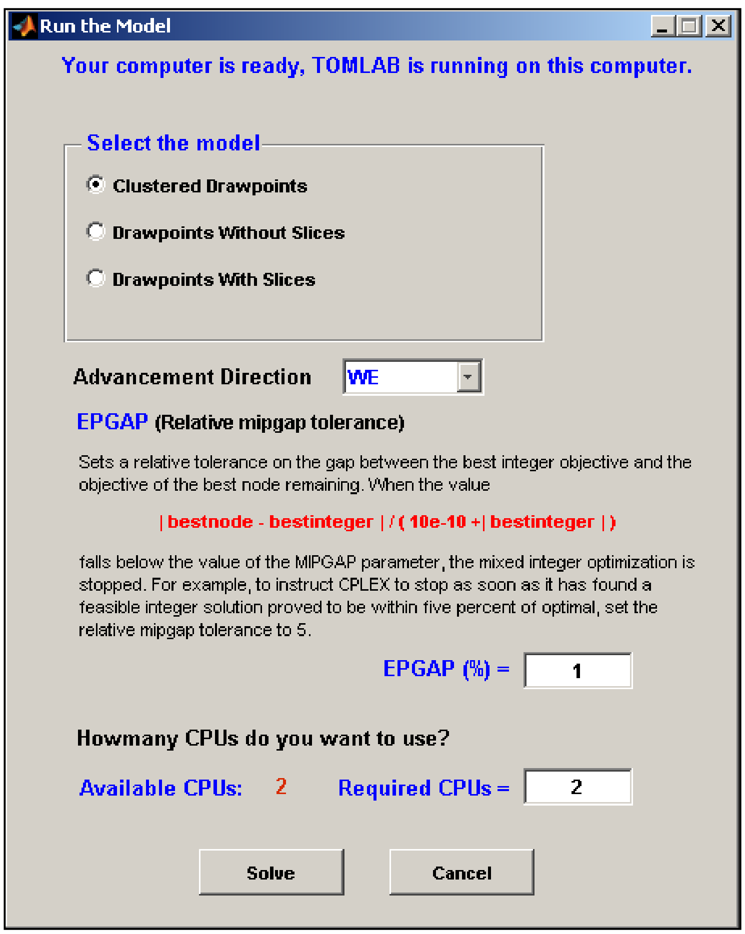

Fig 21. The window for defining the optimization criteria

After solving the problem in the selected advancement direction, to display the obtained results, go to **Solution Analysis > Results Preparation** and then select the related model and prepare it. To plot the results, go to **Solution Analysis > Plot Results**.

- **Multi-Step Method**

To solve the problem using the multi-step method, create all models. After creating the models, go to **Run > Export to TOMLAB**.

Select the cluster level model and export it. Then go to **Run > Solve the Model**. Select **Clustered Drawpoints** as a model and a direction to run the optimization. Enter the proper numbers for the EPGAP and the number of required CPUs. When the problem was solved, select another direction and solve the problem in that direction. Using **Plot Wizard**, recognize the best advancement direction for the obtained solutions in the selected directions. For this purpose, go to **Solution Analysis > Results Preparation** and then select the cluster-level model and prepare it. Afterwards, go to **Solution Analysis > Plot Results** and analyze the results to find the best advancement direction.

Then, go to **Variable Elimination > Cluster to Drawpoint Level**. Afterwards, go to **Run > Export to TOMLAB**. Select **Drawpoint Without Slices** and export it. Then, go to **Run > Solve the Model**. Select **Drawpoint Without Slices** as the model. The advancement direction must be the direction which had the maximum net present (NPV) value at the cluster level. Enter the proper numbers for EPGAP and the number of required CPUs and then solve the problem. After solving the problem at the drawpoint level, in order to solve it at the drawpoint-and-slice level, go to **Solution Analysis > Results Preparation** and select **Drawpoint Without Slices** and prepare it. Afterwards, go to **Variable Elimination > Drawpoint to Slice Level**. Select the solution of the drawpoint level as the starting period. Then, go to **Run > Export to TOMLAB**. Select **Drawpoint With Slices** and export it. Solve the problem at the drawpoint-and-slice level. The advancement direction must be that direction which had the maximum NPV at the cluster level. Enter the proper numbers for EPGAP and the number of required CPUs and then solve the problem.

After solving the problem, using the **Plot Wizard** you can analyze the results and compare different models.

5.5. Result Analysis

After solving the problem, results must be prepared for display. For this purpose, go to **Solution Analysis > Results Preparation** and select the related model and prepare it. Next, to plot the results, go to **Solution Analysis > Plot Results**. The **Plot Wizard** window will appear (see Fig 22). Select a model and then plot the results based on available options.

Fig 23 to Fig 28 illustrate the example of plots generate through DSBC's plot wizard.

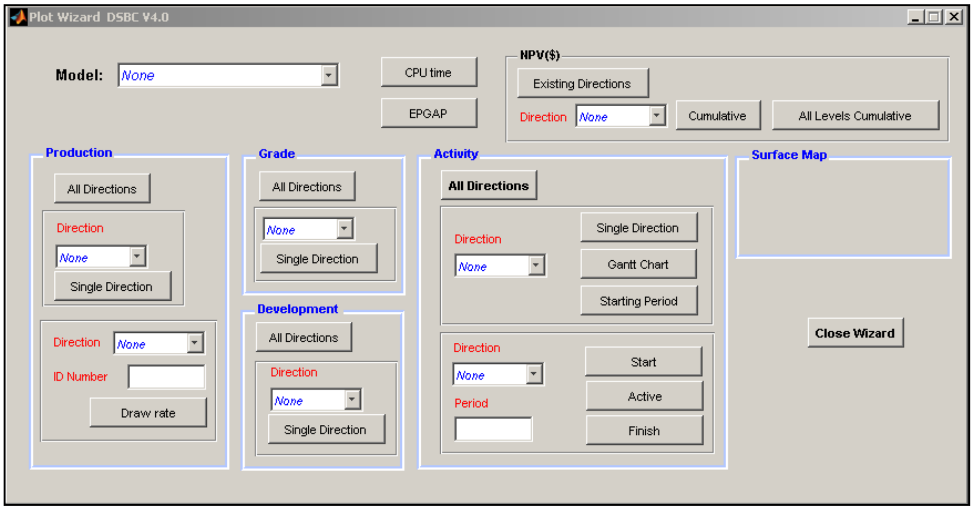

Fig 22. Plot wizard window

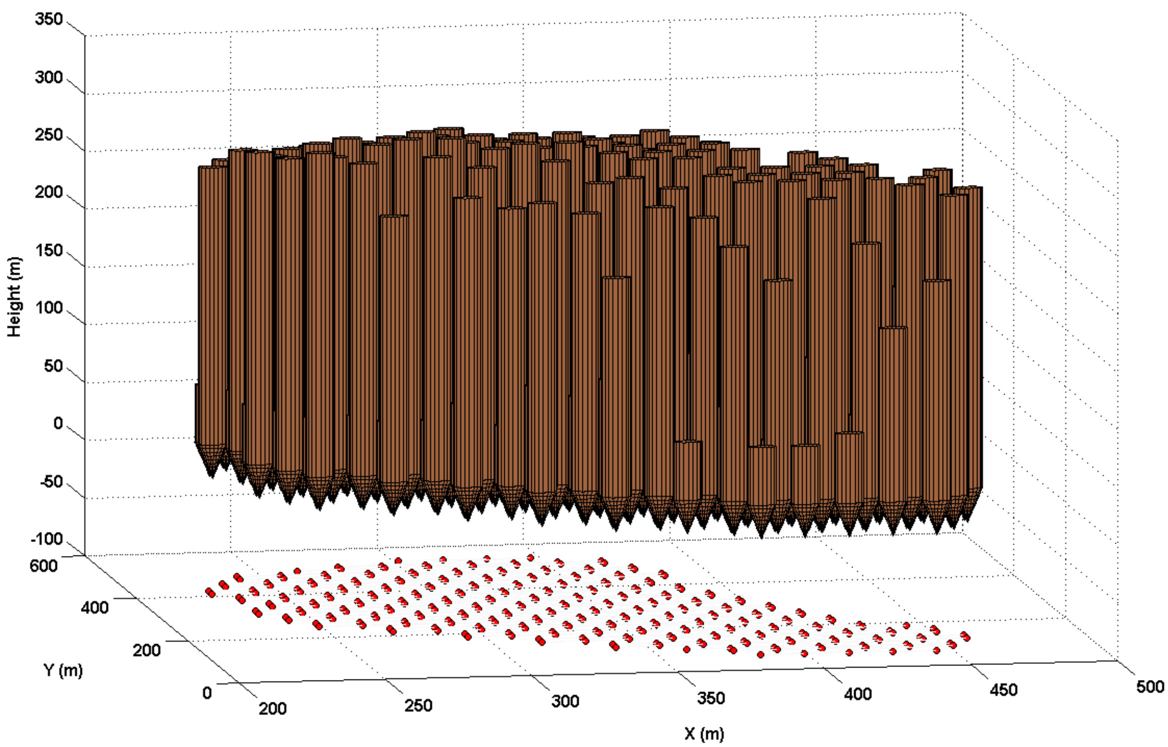

Fig 23. Example of plot for 3D view of the draw columns

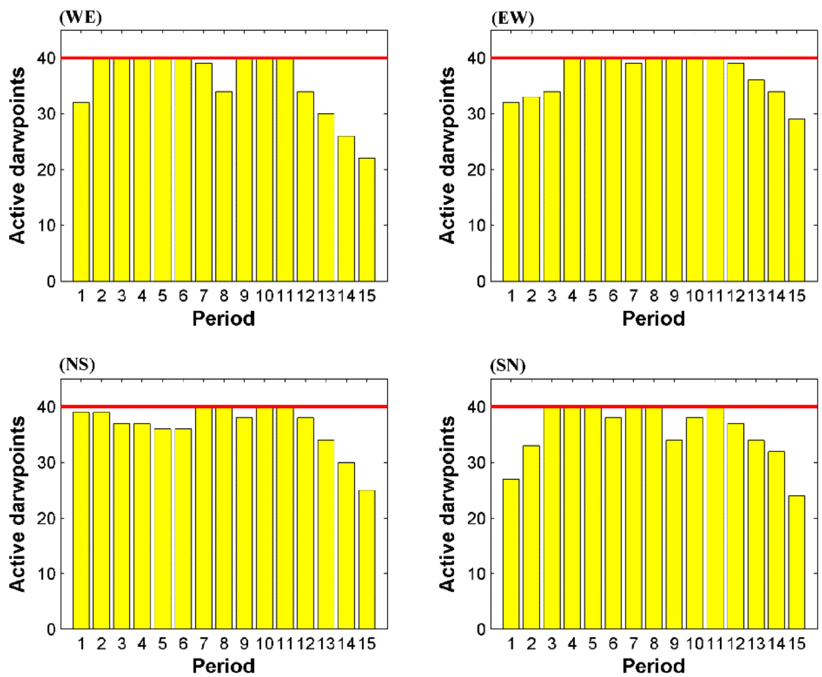

Fig 24. Example of plot for number of active drawpoints for different directions at the drawpoint level over the mine life

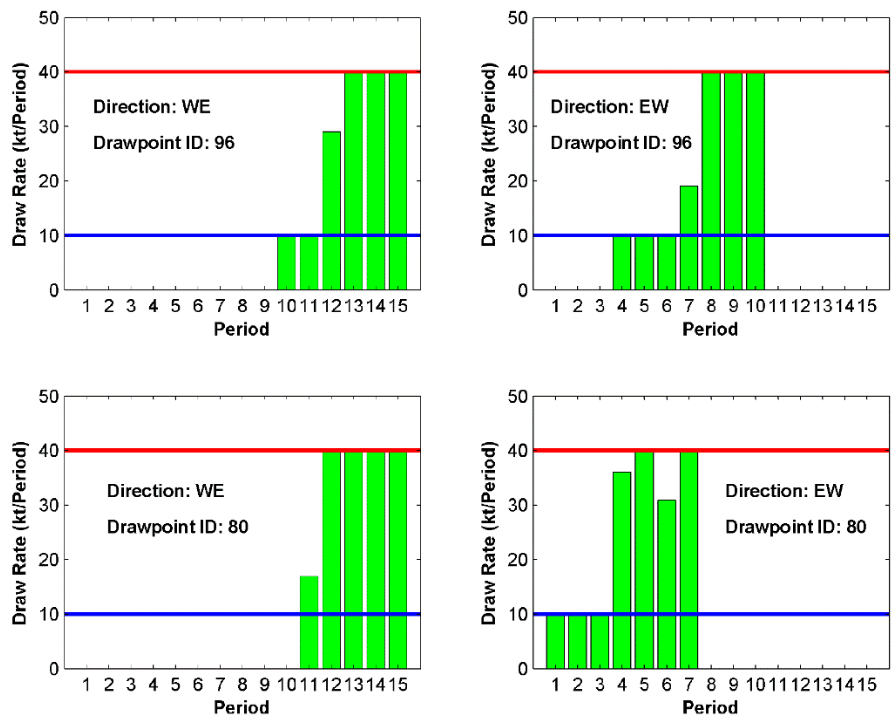

Fig 25. Example of plots for amount of depletion from drawpoints in different directions

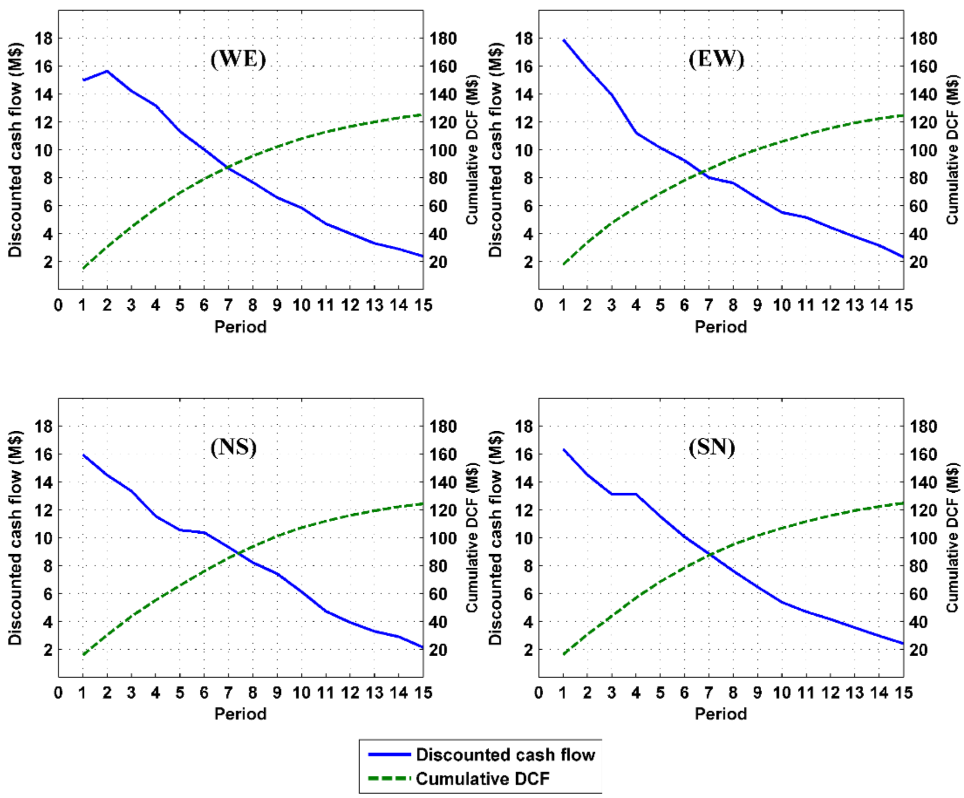

Fig 26. Example of discounted cash flow and cumulative DCF for different directions at the drawpoint level over the mine life

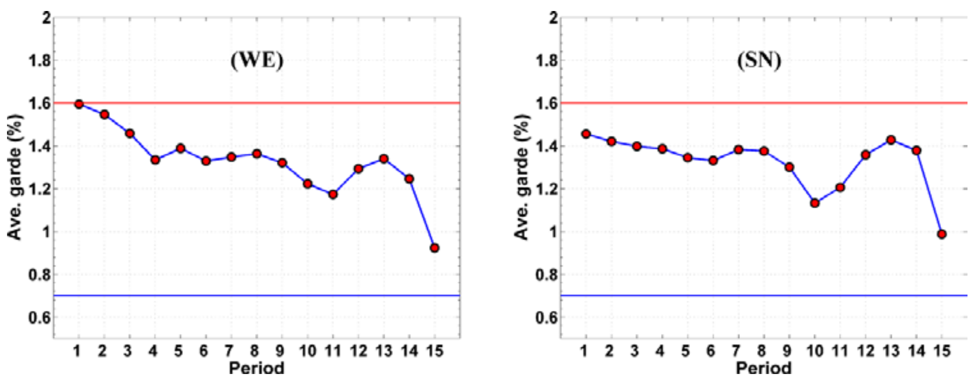

Fig 27. Example of plot for average grade of production in the different advancement directions at the drawpoint- and-slice level over the mine life

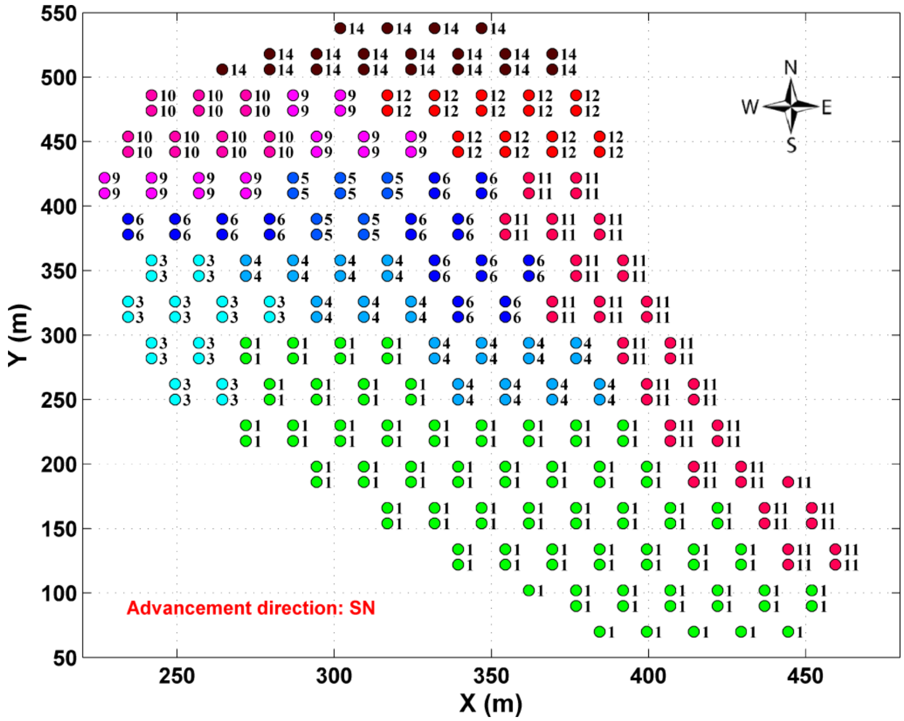

Fig 28. Example of plot for opening pattern using the cluster level formulation in the selected direction

6. References

[1] Geovia (2014). GEMS and PCBC. Ver. 6.5, Vancouver, BC, Canada.

[2] Holmstrom, K. (2011). TOMLAB/CPLEX, ver. 11.2. Ver. Pullman, WA, USA: Tomlab Optimization

[3] Horst, R. and Hoang, T. (1996). *Global optimization : deterministic approaches.* Springer, New York, 3rd ed,xviii, 727 p.

Automated Mine Polygons & Production Scheduling Interface

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Abstract

We have developed a Matlab application to implement our clustering algorithms and mathematical models for long-term open-pit production planning. The application is designed to handle all the steps of importing blocks data from various sources, clustering blocks with two different techniques, setup MILP parameters and interpret and plot results. The application has a graphical user interface for easy use. In this document, we explain the details of programming and guidelines on how to use the application. Moreover, we review the implementations and outcomes by studying a small case study. The case study includes multiple scenarios for clustering the blocks for long-term and short-term planning and using the GUI to complete the tasks.

1. Step 1: Prepare ".dat" Block Model

In order to start working with the clustering application, we need a text file containing the block information. This file is a tab-separated text file with 3 sections: the header, column headers and the block data. It can be exported from any mine planning software and formatted to adhere to the following features. It has to be saved with a ".dat" extension. You can find a sample block model file in "2014_MILP/InputData/Test/Blocks.dat". Open the file in a text editor to have an example of formatting the block model file.

1.1. The Header

The header consists of 17 lines , see Table 1, ("-…" at the end of each line is a comment and is not imported into Matlab).

1.2. The Column Headers

Line number 18 in the file contains the column headers. The number of columns and their headers changes based on the block model properties such as number of elements. The columns should be formatted in the following order (Table 2).

1.3. The Block Model

The block data is listed under the header according the aforementioned columns. Bear in mind that no field is allowed to be left empty. Therefore, you have to use NA and 0 for unavailable string and number fields respectively. The block data is allowed to have multiple rows per block where a block consists of multiple rock types or it can be sent to multiple destinations. It is also allowed to have multiple rows per block when a block is divided between two periods or pushbacks. In this

Pourrahimian, Y., Ben-Awuah, E., and Askari-Nasab, H. (2015), Mining Optimization Laboratory (MOL), Report Six, © MOL, University of Alberta, Edmonton, Canada, Pages 250 , ISBN: 978-1-55195-356-4, pp. 225-240.

case, the import function will add up tonnages from the two rows. However, make sure that multiple rows describing a single block have to have a common block ID.

Table 1. Header Lines

Line #	Description	Example	Notes
1	Number of Destinations	4{tab}-Number of Destinations	
2	Destination Names	MILL{tab}Process{tab}W005{tab}-np-{tab}-Destination Names	The destination names should match the number of destinations in line 1 (e.g. –np- refers to not processed material and defined as a waste destination)
3	Number of Processes	2{tab}-Number of Processes	The first 2 destinations are assumed to be processes
4	Number of Waste Dumps	2{tab}-Number of Waste Dumps	The last 2 destinations are assumed to be waste dumps
5	Number of Stockpiles	0{tab}-Number of Stockpiles	Considered for later extensions
6	Number of Rock Types	7{tab}-Number of Rock Types	Number of rock types in the block model
7	Rock Types	2{tab}3{tab}5{tab}8{tab}101{tab}201{tab}301{tab}-Rock Types	The rock codes should match the number of rock types in line 6
8	Number of Element	3{tab}-Number of Elements	Number of elements in the block model
9	Elements	P{tab}S{tab}MWT{tab}-Elements	The elements should match the number of elements in line 8
10	Number of Elements Processed	1{tab}-Number of Elements Processed	Number of elements processed (not contaminants)
11	Number of Rock Types Processed	3{tab}-Number of Rock Types Processed	Number of rock types processed (mineralized rock types)
12	The origin of Indices	41{tab}68{tab}22{tab}-XI,YI,ZI Origin	Only saved for later references and not used for clustering operations (can be left 0,0,0)
13	The block dimensions	25{tab}25{tab}15{tab}-X,Y,Z Dimension	
14	The origin of coordinates	97525{tab}600200{tab}1440{tab}-X,Y,Z Origin	
15	Number of Blocks	19561{tab}-Number of Blocks	The number of blocks in the block model/The number of unique rows
16	Number of Rows	42117{tab}-Number of Rows	The number of rows in the file excluding the header rows
17	Number of Production Periods	20{tab}-Number of Periods	The number of production periods currently determined using mine planning tools

Table 2. Block Data Columns

Column #	Header	Type	Description
1	BlockID	Integer	Unique block incremental ID number
2	IX	Integer	Block column index
3	IY	Integer	Block row index
4	IZ	Integer	Block elevation index
5	X	Float	Block X coordinate
6	Y	Float	Block Y coordinate
7	Z	Float	Block Z coordinate
8	PitID	Integer	Pit number (in case there are multiple pits in one model)
9	PhaseID	Integer	Push back number
10	BenchPhaseID	Integer	Bench-phase/panel number
11	numRockTypesInBlock	Integer	Number of rock types in the block
12	RockType	String	The rock type e.g. 101, HYPO
13	RockCode	Integer	The rock type order (defined in the header e.g. 5 for 101)
14	NumDestination	Integer	Number of possible destinations for the block
15	Destination	String	Block destination name e.g. MILL
16	DestinationCode	Integer	Block destination order e.g. 1 for MILL
17	BlockTonnage	Float	Total block tonnage
18	BlockValue	Float	Total block value if sent to this destination
19	Revenue	Float	Total revenue if sent to this destination
20	ProcessingCosts	Float	Total processing cost at this destination
21	MiningCostAndHaulage	Float	Total cost of mining and hauling to this destination
22	BlockReferenceMiningCost	Float	Total cost of mining and hauling to this destination
23	MineralizedTonnage	Float	Total extractable mineralized tonnage from this rock type (considering dilution e.g. 0.95*block tonnage)
24	OreTonnage	Float	Total Ore tonnage (mineralized tonnage above cut-off) from this rock type
25	WasteTonnage	Float	Total waste rock in the block
26	TotalWasteTonnage	Float	Total waste tonnage (waste rock + undefined waste + dilution)
27	ProcessThroughPut	Float	Process throughput at this destination for this block
28	BlockFraction	Float	Fraction of block extracted in this period
29	Period	Integer	Period number
30	{element}_Quantity	Float	Element quantity in the block (tonnage)
31	{element}_Grade	Float	Element grade in the block (%mass)
32	{element}_Recovery	Float	Element recovery at this destination (0-1)
33	{element}_Revenue	Float	Element revenue (in addition to block processing revenue)
34	{element}_ProcessingCost	Float	Element processing cost (in addition to block processing cost)
…	Repeat 30 to 34 for each element		
	DestinationFraction	Float	Fraction of block sent to this destination in this period

The clustering algorithm works based on a single element in most cases and it is easier to have one element imported into Matlab. However, it is possible to define multiple elements, import them into Matlab and try different combinations when using the clustering procedure. Many of the columns introduced earlier are defined to account for MILP formulations and are not needed for clustering. Therefore, they can be left as NA or 0 based on their data type.

2. Step 2: Import to Matlab

The next step is to import the block model into Matlab with the specified format.

1. Open Matlab
2. Change the active directory to "2014_MILP"
3. Right-click on the script file "OpenMainForm.m" and click on "Run" as in Fig 1. It will open the GUI developed for running the clustering algorithm.

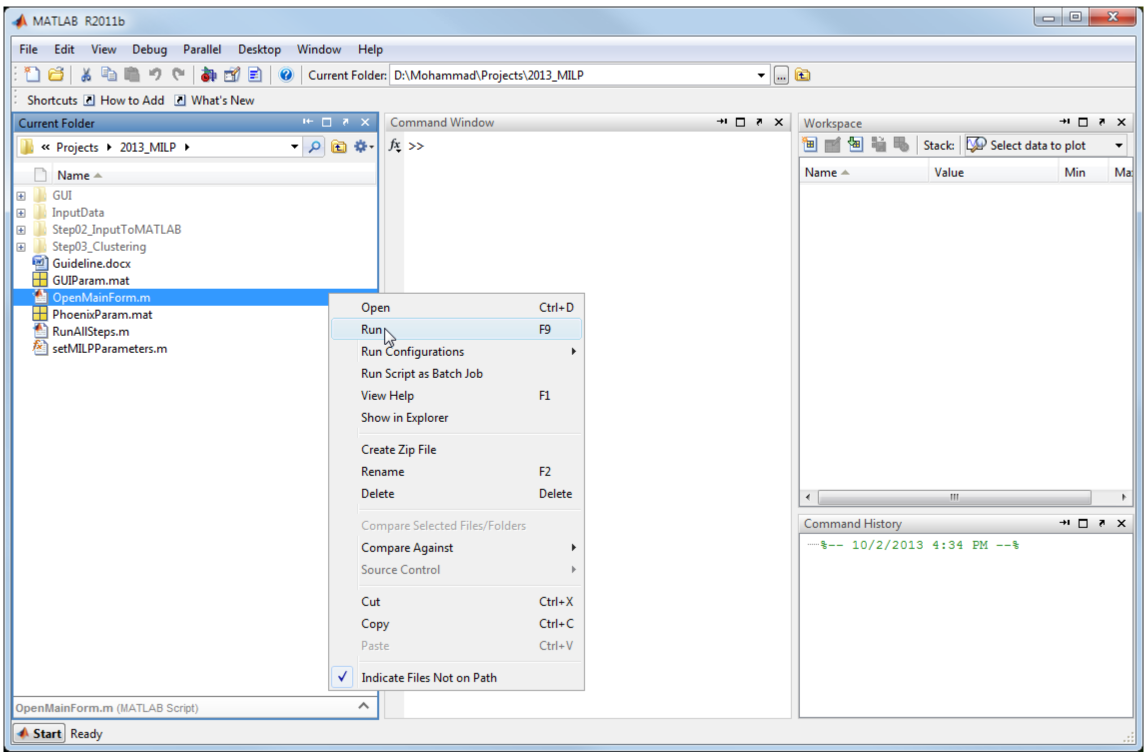

Fig 1. Opening GUI

The main GUI has three sections to be explained separately: the menus, the active directory and memory usage option. The menus are used to navigate through different parts of the application. However, using the application starts by setting the working directory. Results of every step are loaded from and saved into the active directory. There are two options for memory usage: "Save Blocks Variable" and "Keep Everything in memory". The first option takes more time since it loads blocks variable, performs the operation, saves the results and clears memory every time. This helps the application consume less memory. In contrast, the second option keeps the blocks variable in the memory and the user has to save it if needed. There is also a read-only table on the main GUI that checks which files exist in the active directory.

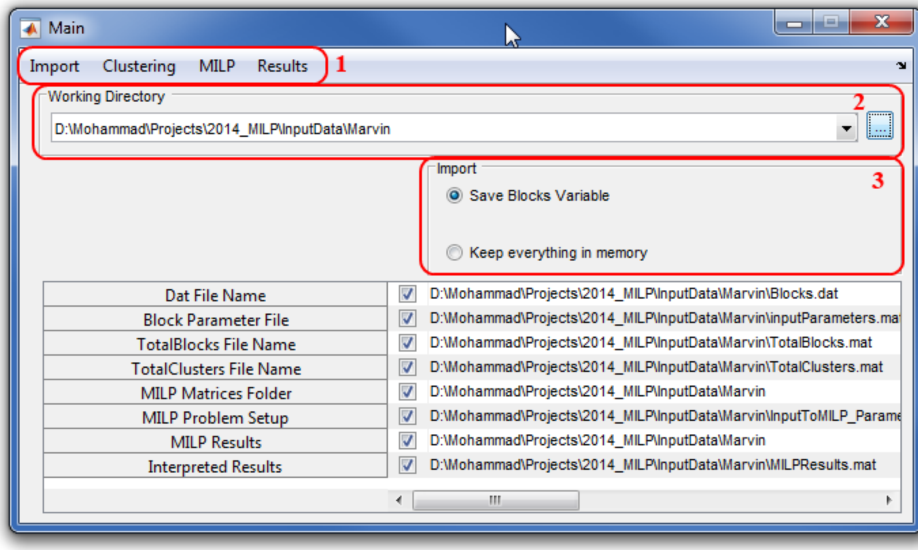

Fig 2. Main GUI

The first step is to create a directory and set it as the active directory.

1. Click on the "…" button in section 2 of the GUI
2. Browse to the desired directory (e.g. 2014_MILP/InputData)
3. Press the "New Folder" button and give it a name

Afterwards, you have to import the block file into Matlab. Choose the "Import DAT File" command from the "Import" menu and browse to the tab-separated block model file. It will create a copy of the file in the active directory and import the block data into Matlab structured variable. It will then save the TotalBlocks variable to hard disk or keep it in memory based on the option chosen. The "Load Existing TotalBlocks" and "Save TotalBlocks" menus can be used to manually load and save the blocks variable when the keep in memory option is chosen.

3. Step 3: Clustering

Two clustering methods are currently available in the software: horizontal hierarchical and k-means. They can be called from the clustering menu on the upper left corner of the main GUI.

- If the "Save Blocks Variable" option is chosen:

The clustering algorithms automatically load the TotalBlocks variable. However, if the program is not able to find the TotalBlocks.mat file in the active directory the user has to use the "Load Existing TotalBlocks" menu to locate the TotalBlocks variable which will be copied to the active directory then and loaded when the clustering algorithm starts.

- If the "Keep in Memory" option is chosen:

The user has to load the TotalBlocks variable if it is not already loaded in the memory by using "Load Existing TotalBlocks" menu and browsing to the TotalBlocks.mat file.

3.1. Horizontal Hierarchical Clustering

The two algorithms and their control parameters are explained in the following sections. The clustering GUI (Fig 3) contains five textboxes to enter the weight and penalty parameters as explained in Appendix I.

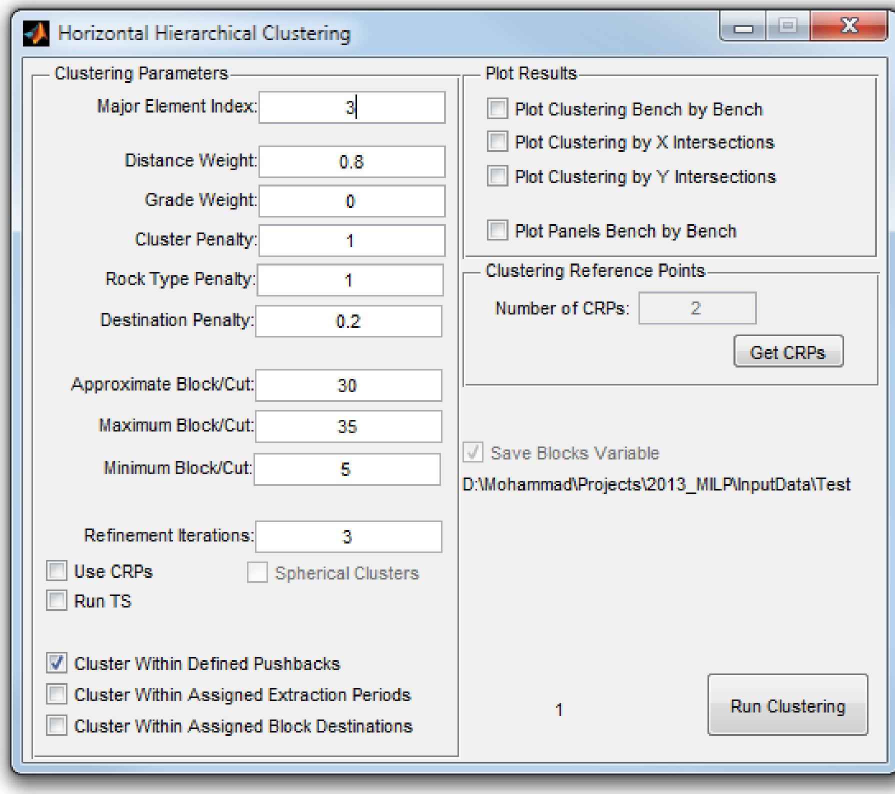

Fig 3. Hierarchical Clustering GUI

- Major Element Index: Since the data structure is able to handle multiple elements you need to indicate which element grade to use for calculating the grade difference between the blocks. This textbox is considered to input the order index of the element of interest in the input file (3 represents MWT in our example).

- Distance Weight: The weight put on distance and direction measure in calculating the similarities (W_D)

- Grade Weight: The weight put on the major element grade difference measure in calculating the similarities (W_G)

- Cluster Penalty: The penalty value for the blocks located above different clusters in calculating the similarities (C_{ij})

- Rock Type Penalty: The penalty value for the blocks from different rock types in calculating the similarities (R_{ij})

- Destination Penalty: The penalty value for the blocks determined to be sent to different destinations in calculating the similarities (N_{ij})

- There are also 3 textboxes considered for average, maximum and minimum blocks per cut (cluster) that control the cluster sizes.

- Refinement Iterations: The number of shape refinement iterations to be performed after the clustering

- Use CRPs: If checked the user has to provide CRPs for directional clustering, otherwise the M_{ij} will be set to 1.

- Spherical Clusters: This checkbox will be enabled if “Use CRPs” is checked. If this is checked, the mining direction factor will be calculated.

- Run TS: The Tabu search (TS) procedure will be called, if this checkbox is checked, in order to reduce the number of arcs between formed clusters in each bench and the ones in the bench below. This helps planning stage get to the bottom of the pit faster. However, experiments of various datasets have shown that reducing the precedence arcs by manipulating the cluster shapes decreases the homogeneity of the clusters and the practicality of the production plans.

- There are 3 checkboxes in the lower left side of the GUI for clustering within boundaries. If the phase (pushback) IDs, production periods or destinations are assigned for the blocks in the input file they can be used to apply strict boundaries on the clustering. This means blocks from different regions cannot be merged to form a cluster.

- There are 4 checkboxes in the “Plot Results” panel that can be used to plot the clustering results after the clustering is performed. They will produce plots of clustering bench by bench, X intersections and Y intersection. However, for a real-size dataset there will be many figures generated and it is not recommended to plot them at the same time in this GUI. Another GUI is designed for plotting the results in plan views and cross sections. The last checkbox “Plot Panels Bench by Bench” plots the panels/bench phases to be able to compare against the results when “Clustering within Defined Pushbacks” is desired.

- The number of CRP points is currently fixed to 2 but the textbox is considered for future expansions on the algorithm. The GUI has a button to add the CRPs to each bench. It starts from the lowest bench and continues to the higher benches. The red squares represent blocks on each bench and the user can use the cross hair to specify the start and end point of the mining direction vector. The same way, the user can specify 2 points for what we call the spherical clustering.

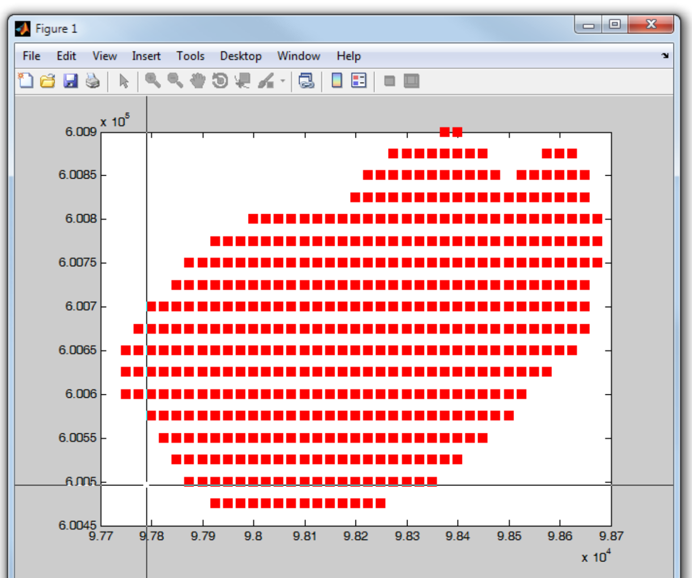

Fig 4. Get CRPs Plot

There are instances that the user prefers to enter exact coordinates for CRPs; for example, ramp entrance coordinates can be used as CRPs for spherical clusters. The CRPs are saved in a variable called “TotalCRPs.mat” in the active directory. If the user needs to change the values or input them manually, the file can be opened in Matlab. The TotalCRPs variable is a cell array with a cell per bench starting from the lowest bench. Each cell contains a 2×2 matrix with the coordinates of the two reference points as in Fig 5 and Fig 6.

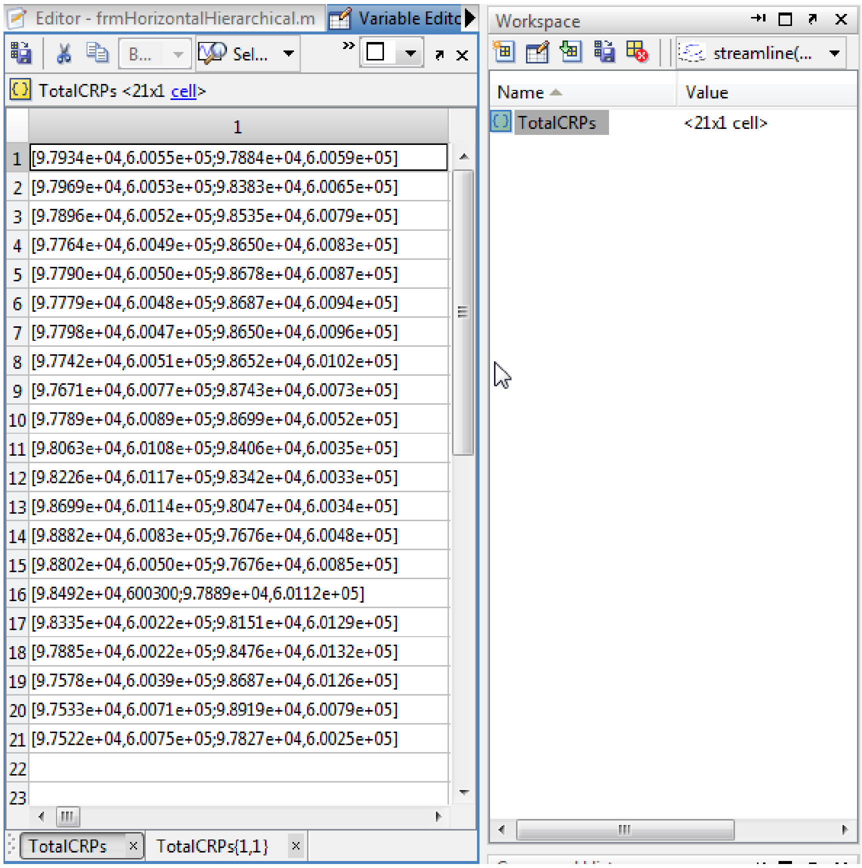

Fig 5. TotalCRPs Variable with 21 Benches

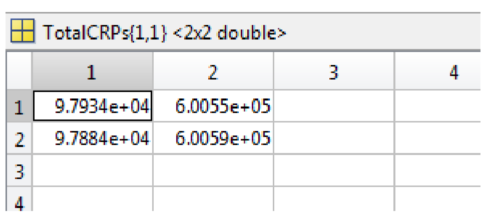

Fig 6. CRP Coordinates for the First Bench (X,Y)

After running the clustering algorithm the user may want to “Save the Clustering Results” if working with “Keep in Memory” option and then use the “Plot and Export” GUI to plot the results or export them into a text file.

3.2. K-Means Clustering

A famous example of partitional algorithms is the k-means, which attempts to find cluster means and assign data points to the closest mean. K-means is a partitioning technique which tries to find a good partitioning scheme by iteratively modifying the partitions. One extension to k-means clustering which is relevant to this project is the kernel k-means which is developed to be able to partition data points which are not linearly separable by mapping them into a kernel space. The clustering technique used is an implementation of the kernel k-means algorithm based on gradient descent search. In this approach, K initial cluster centers are randomly selected at each replication. Then the objects are assigned to the nearest center. Afterwards, based on the gradient descent search technique, the centers are manipulated in such a way that the summation of distances between the objects and the means is locally minimized. Another replication is then started with a new random set of means and the process continues for a limited number of replications.

The first step for this algorithm is to form the feature matrix, which holds all the important properties of all objects. To be consistent with the hierarchical clustering technique, the same parameters are used with the same weighting approach. Then the matrix has to be kernelized in order to get better results. When objects are not linearly separable in their initial space, kernel functions are used to map data points from the initial space to the kernelized space and do the clustering in there. Then the same map is used in returning to the initial space with all the objects labeled as belonging to various clusters. Having tested various kernel functions and parameters, a polynomial kernel function with $d = 1$ is used in this implementation. Afterwards, K initial cluster centers are randomly selected in the kernelized space and objects are assigned to the closest mean. Then the objective function, which is a summation of Euclidean distances between all objects and cluster means, is calculated. Cluster means are then manipulated in an iterative manner based on gradient descent until a local minimum is found. This is stored as a solution to the clustering problem and a new replication starts with another random definition of cluster means. Finally, all of the replications are compared, and the one with the lowest objective function is selected as the solution to the clustering problem on that bench [3]. The problem with the k-means approach is that there is no size control. In addition, it is possible to generate fragmented clusters that are problematic if used in later planning stages.

The k-means algorithm GUI (Fig 7) can be called from the GUI, clustering menu. Instead of penalty values, there are only weights in k-means GUI. The X, Y, grade and rock type are put in a matrix and each are powered to their weights. User can also input the average blocks per cut that determines the number of center points in the k-means algorithm. The last parameter is the number of replications. More replications usually results in better results but by talking more time. The rest of the k-means GUI works the same as the horizontal hierarchical GUI.

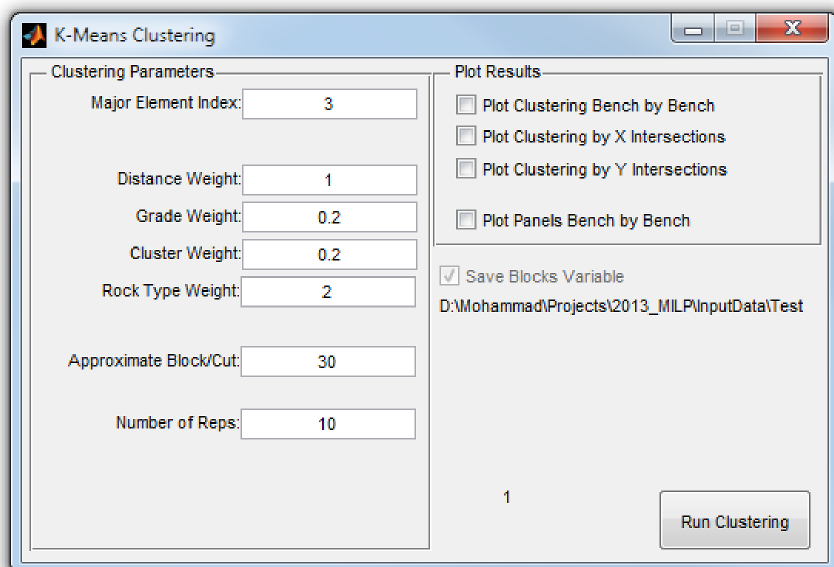

Fig 7. K-Means Clustering GUI

3.3. Plot and Export

After performing the clustering with hierarchical or k-means algorithms, the user may need to evaluate the clustering results by looking at plan views or cross sections, or export the results to a text file to import into other applications. A GUI, which can be called from the clustering menu, is developed for this purpose (Fig 8). The GUI consists of three main panels: Clustering Stats, Plot and Export.

The first panel is designed to provide a summary of the clustering measures to be able to quickly evaluate clustering scheme and change the parameters to get better results. The clustering statistics table is updated by pressing the "Update Stat" button on the upper left corner. Once updated, it will provide the user with the following information:

- Cut Tonnage: Average and the standard deviation of the tonnage of material in the cuts is the first evaluation criterion in the table. This can be used to check if the differences in the tonnage of material in the generated cuts are reasonable.

- Rock Unity: Based on the structure of the problem and existing criteria for categorical variables, a new index is defined as the percentage of rocks in a mining-cut belonging to the most dominant rock type in that mining-cut. This is called the rock unity and is depicted in second row of the table.

- DDF: Destination Dilution Factor is defined in the same way as the rock unity but by considering the predetermined block destination as the homogeneity factor.

- Element Variation: The coefficient of variation (CV) is defined as the standard deviation of a variable divided by its mean. The average and standard deviation of the CV values for each element is presented in the following rows. Average CV can represent the variations in the grade values among the blocks grouped together. This can be helpful when creating mining polygons for estimating the head grade to the processing plant.

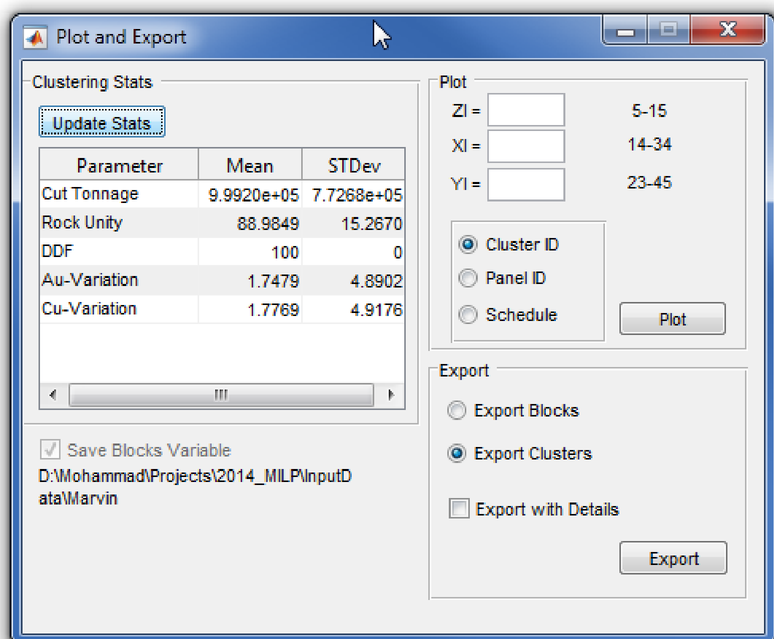

Fig 8. Plot and Export GUI

The “Plot” panel can be used to plot the clustering results in plan views, cross sections and in 3D. The minimum and maximum index values for the blocks are presented in front of each textbox. The user can choose to filter the blocks to be plotted by entering the index value in each textbox. The application will create a 3D plot of the dataset if all textboxes are left empty. Otherwise, a plan view (if ZI is specified) or cross section (if XI or YI is specified) will be created. Note that the user cannot enter a range in the filter textboxes but only integer numbers. There are three options to plot clusters, panels and the schedule from the input file.

The “Export” panel is developed to export the clustering results into tab-delimited text files to be used as input to other software. The “Export Blocks” option creates a file with a row for each block with indices, coordinates, cluster IDs etc. The columns in the text file are presented in Table 3 The “Export Clusters” option provides a file with the same format but with one row for each cluster. The indices and coordinates presented for clusters are the average values of the blocks in that cluster.

Table 3. Blocks Brief Output File

Column Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Value	XI	YI	ZI	X	Y	Z	Panel ID	Cluster ID	Cluster ID in Bench	Period	Destination

There is also a checkbox labeled “Export with Details” that adds extra columns to the original format and can be mostly useful for cluster exports. This option can be used to get detailed information about the quality of the generated clusters such as their rock unity, destination dilution factor and element grade variations. It will also provide the precedence arcs between the generated clusters in different benches. This can be especially useful when evaluating the Tabu Search results. The extra columns are presented in Table 4 . After column 20, there are 2 columns for each element in the dataset: mean and variance of grades.

Table 4. Block Detailed Output File

Column Number	**1**	**…**	**13**	**14**	**15**	**16**
Value	ID	…	Phase ID	Rock Unity	Destination Dilution Factor	Total Tonnage
Column Number	17	18	19	20	…	
Value	Downward Relation Count	Upward Relation Count	ID in Bench	Bench ID	Average Grade	Grade Variance

4. Prepare Input to MILP

We have to prepare the matrices for the MILP formulation based on four different resolutions: cluster-panel, cluster-cluster (CC), block-cluster (BC) and block-block (BB). This can be done through the menu items under MILP menu as shown in Fig 9.

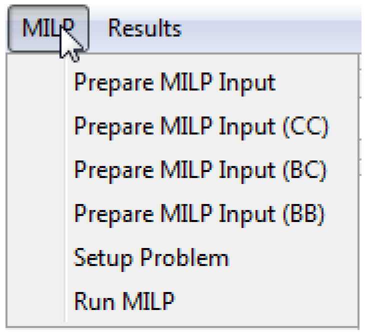

Fig 9. MILP Menu

5. Setup MILP Parameters

We have developed a GUI for setting up the MILP by changing the parameters in two user-friendly forms. The first form is called the setup form and includes any parameters related to the MILP except than the stockpiles. The stockpile settings are in a separate form that can be opened from the main setup form. The setup form can be opened by the “Setup Problem” item under the MILP menu.

5.1. Initial Setup

The first step for setting up the MILP parameters is to define the number of periods and the default values for mining and processing constraints. The initial setup panel on the top-left corner of the form (Fig 10) is where the initial settings have to be defined. We start by defining the number of periods in the textbox marked with number 1 in Fig 10. Two textboxes, market with number 2 in Fig 10, are designed to input the default values for lower and upper limits on the mining capacity. The table marked with number 3 in Fig 10 presents the list of destinations defined in the input file. We can check the Process checkbox for processing destinations (the ones that have a limited capacity) and assign lower and upper bounds for them. Note that the lower and upper bound values are in millions. The table marked with number 4 in Fig 10 presents the elements in the block model and provides the option to assign default lower and upper values for average grade constraints for each element. Capacities and average grade bounds can be edited for each period in the next step. The interest rate is a fixed number for the mine life and can be provided in the textbox market with number 5. If the checkbox “Extract All” is checked the model will be forced to extract everything within the final pit instead of determining what to mine and what to leave in ground based on NPV. The checkbox “Use Predetermined Periods” will limit the extraction variables to predetermined periods for each panel and cut in cases where a multi-step solution is required. This will be explained in more details later. The “Data Stats” panel on the top-right corner is the summary of the block model to be used for determining the proper capacities.

5.2. Period by Period Setup

Pressing the “Initial Setup” button will replicate the default values for lower and upper limits for the number of periods and make the corresponding tables visible. Now we can change the constraints for different periods in the three tables marked with 1 to 3 in Fig 11. The numbers in these three tables are not in millions in contrast to the default values.

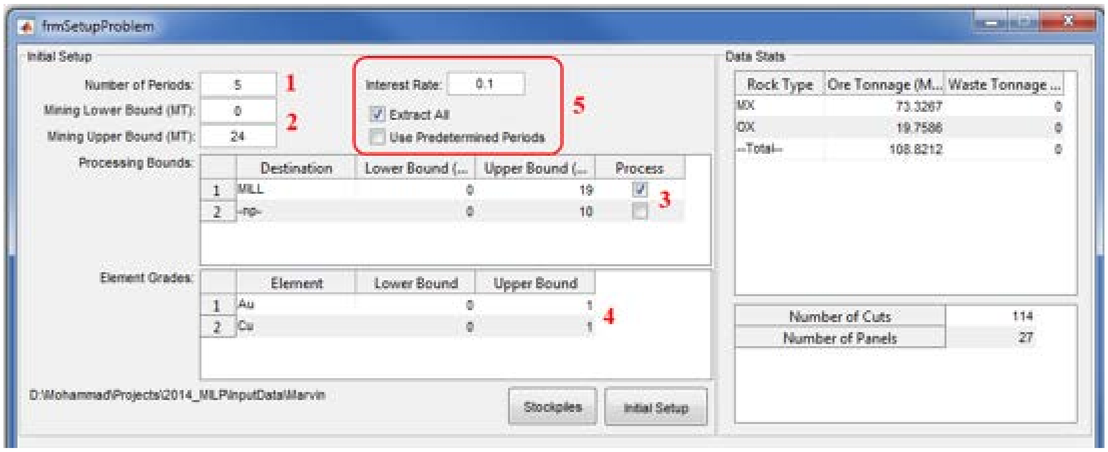

Fig 10. Initial Setup Panel

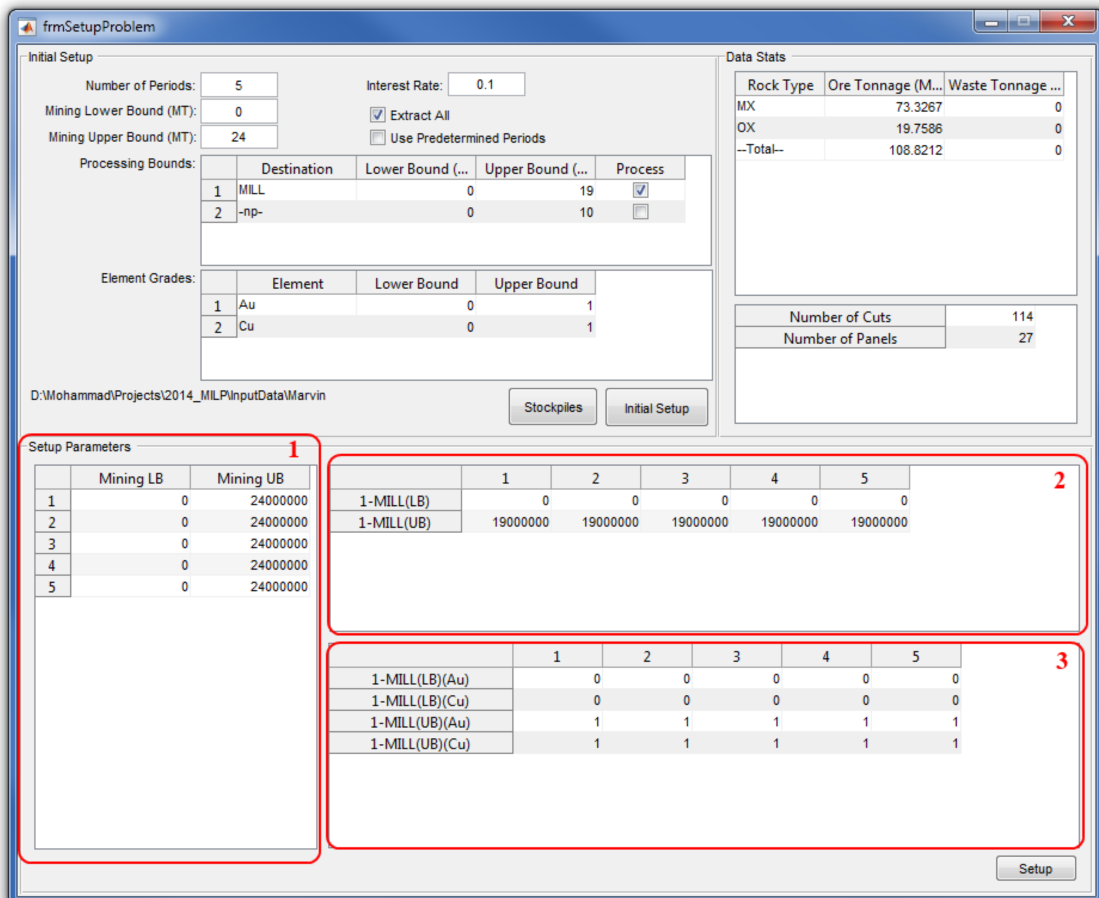

Fig 11. Period by Period Setup

5.3. Stockpile Setup

Following the same design as the main setup form, the stockpile setup requires an initial setup with the number of stockpiles and default values for lower and upper bounds on average grades as well as the reclamation revenues (Fig 12). After setting the number of stockpiles and pressing the initial setup, we can limit stockpiles to rock types and assign re-handling cost per ton of material reclaimed in the top-right table market with number 2. Moreover, we can limit the tonnage sent to the stockpile in each period through the table marked with number 3. The lower and upper bounds on the average grade of material sent to the stockpile and the reclamation grade for each stockpile and element can be set in table 4. The revenue made from processing elements in the stockpile can be set in table 5.

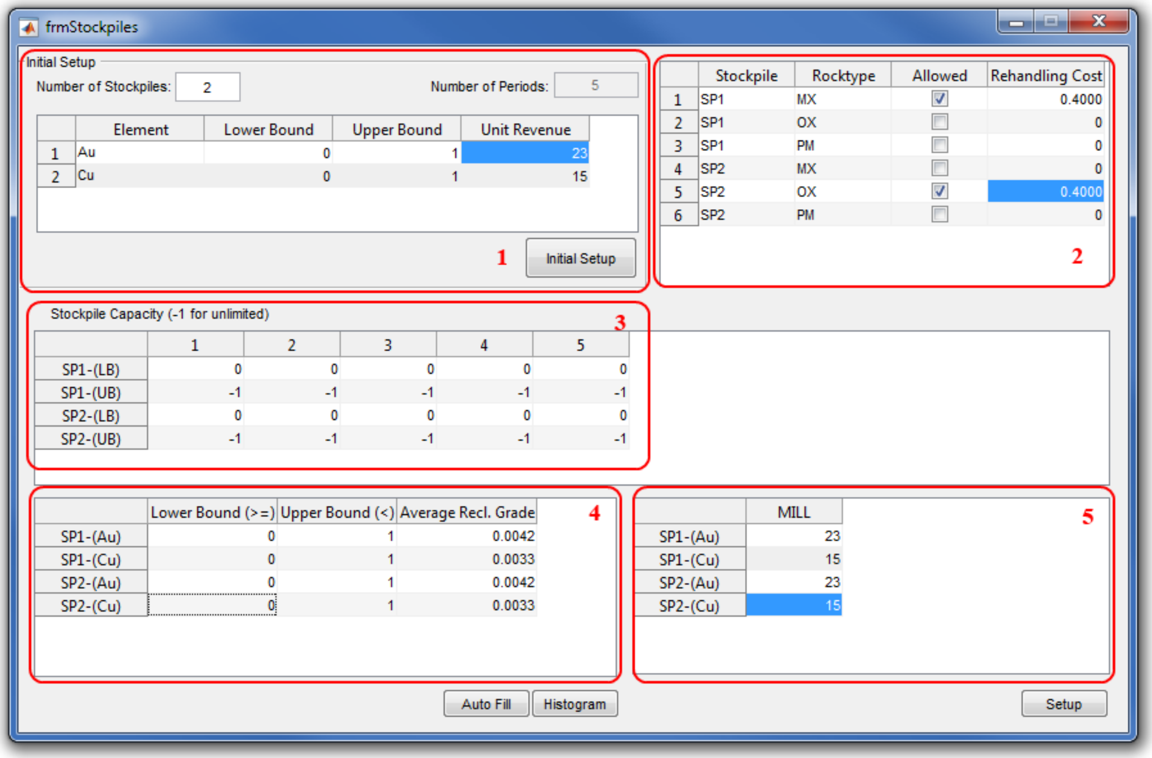

Fig 12. Stockpile Initial Setup

We have added two buttons to help find out the proper reclamation grades for stockpiles. The first button is the "Auto Fill" that will calculate the average grade of element by filtering clusters based on the lower and upper bounds provided. The "Histogram" button plots the histograms of the grades based on the provided lower and upper bounds.

6. Run MILP Solver

After setting up the MILP parameters, we run the solver to the determined gaps. We developed a GUI for starting up Tomlab, setting the gaps and calling the solver. The GUI is presented in Fig 13.

7. Interpret Results

The next step, after solving the MILP, is to interpret the results (Fig 14). For this step, we have developed a GUI that provides various options for the user to interpret, plot and export the results in different formats. There are four options for the different resolutions that were used in creating the MILP matrices as well as two options for updating cluster schedules based on panel schedules. Next, we have the plot functions available for initial evaluation of the output. We usually use the saved variables to create the plots with better looks in Excel. The first four plots are stripping ratio, destination (production), stockpiling (re-handling) and stockpile inventory. For the rest of the plots, the user has to choose an element from the drop-down list before calling the plot function to plot the head grade, cut-off grade (lowest grade of each element sent to process in each period), real stockpile grade and head grade with stockpile.

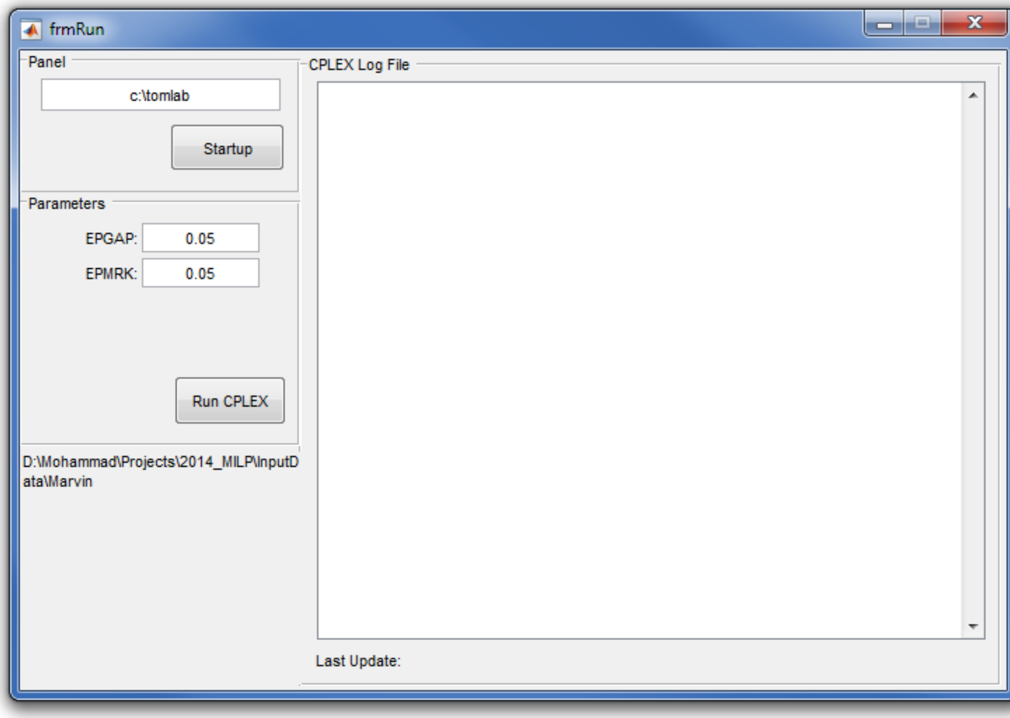

Fig 13. Run MILP

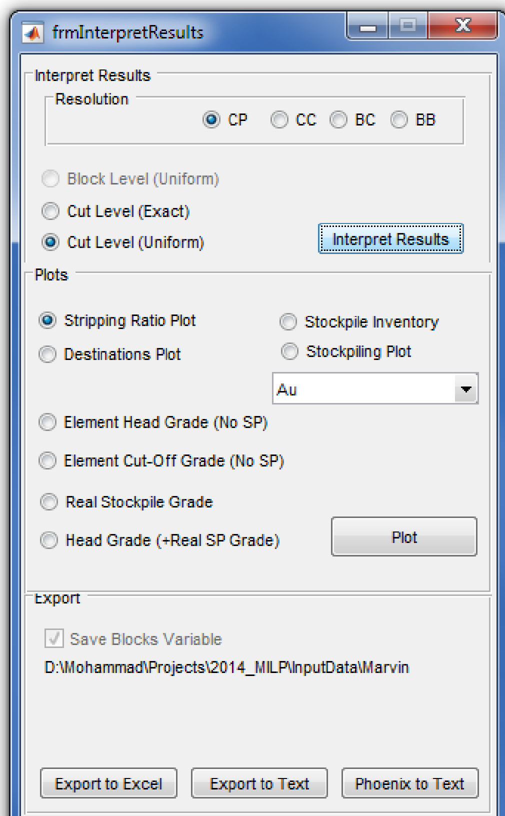

Fig 14. Interpret Result GUI

The export to Excel button exports the matrices into an Excel file where the Export to Text feature creates a text file from all the blocks with their assigned extraction period and destination.

8. Update Possible Periods

As mentioned we sometimes solve the MILP model and use the solution to limit the decision variables and resolve the model. For example, if the mine life is 15 years, we can initially solve the model for 5 periods of three years. Then, by using a multiplier of 3 and a tolerance of 1, a panel that is initially scheduled to be extracted in period 2 will have extraction variables for years 3 to 7. We call these the possible periods of that panel and save it as an attribute for the each panel. We have developed a simple GUI to update the possible periods as shown in Fig 15. After updating the possible periods, the user has to go back to the MILP setup, set the number of periods to 15 and check the "Use Possible Periods" checkbox to use this information to solve the model in shorter time.

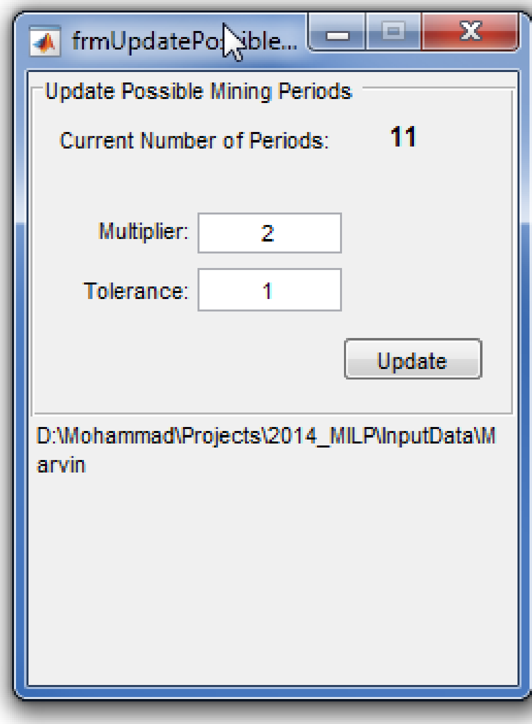

Fig 15. Update Possible Period

Mine Road Network and Rimpull Curve Configuration Interface

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University of Alberta, Edmonton, Canada

1. Introduction

Simulation models have proved to be an effective tool for understanding the behavior of the open pit mining systems and for “what if...?” analysis. Truck haulage, having a significant portion of the operating cost and a bottle neck in production for most operations, is required to be modeled precisely for an efficient simulation model of the production operations in open pit mines. Most of the existing simulation packages used for haulage simulation in mining industry are based on Macroscopic Simulation, which are incapable of capturing the interactions and platoon formations of trucks on the haul roads. A Microscopic modeling of truck movements is therefore necessary to capture these interactions and model the truck travel times with a relatively more precision, so as to have a better representative simulation model of the actual mining operation. Apart from taking into consideration the accelerations and decelerations of trucks based on gradient, turning angle and road condition, this approach takes into account the interactions of trucks on the haul roads, which may cause platoon formations, and affect significantly the total travel time of trucks.

To develop a simulation model, exercising control on the truck movements and capturing interactions, we need to model the haul road network and its characteristics within the simulation model and move the trucks through it based on truck characteristics and the haul road gradient, rolling resistance, turning angle and interactions with other trucks. Thus we will first describe how we can generate the haul road network within Arena using a MATLAB application with GUI. In the next section we will discuss a MATLAB application to generate the speeds of trucks on haul roads based on rimpull characteristics of the trucks and various haul road gradients. Finally we will model the trucks in Arena and discuss how to incorporate the desired characteristics into the simulation model.

2. Learning Objectives

By the end of this lab you should be able to:

- Use the RoadNetwork GUI to build haulage road network for Arena simulation
- Generate speeds of trucks based on rimpull curve characteristics and haul road gradients
- Model haulage system in Arena simulation capturing truck interactions and platoon formations on haul roads

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3. Concepts and Terminology (Rockwell Automation, 2010)

3.1. Transporters

Transporters are one type of device that moves entities through the system. They can be used to represent material-handling or transfer devices such as fork trucks or delivery vehicles. Transporters can also be used to model personnel whose movement is important to modeling a system, such as a nurse or a food server. When transporters are used, you provide information defining the transporter's speed and the travel distances between stations served by the transporter.

3.2. Free-path Transporters

Free-path transporters move freely between stations and are not influenced by other transporter traffic.

3.3. Guided Transporters

Guided transporters are restricted to run on fixed paths such as tracks or rails. Movement may be affected by traffic congestion from other vehicles.

3.4. Distance

The Distance module defines the distance between two stations in the distance set of a free-path transporter device. The beginning station, ending station, and distance are used to create the appropriate distance set, which is used during the simulation run by the transporter moving between the specified stations.

3.5. Network

The Network module defines a system map that a set of guided transporters will follow. A network encompasses the set of links specified in its Network Links repeat group. The parameters of a network link (e.g., length, intersections, directions), are defined in the Network Link module.

3.6. Network Link

The Network Link module defines the characteristics of a guided transporter path between an intersection pair Beginning Intersection Name and Ending Intersection Name. The Network module then references a set of network links to define a network that guides transporters follow for movement. Each link is composed of a Beginning Intersection ID, an Ending Intersection ID, and one or more Number of Zones -- each Length of Each Zone units long.

3.7. The Link Type

The link type — Unidirectional, Bidirectional, or Spur — dictates whether the simulation will allow transporters to move from Ending Intersection Name toward Beginning Intersection Name. When traveling on a unidirectional link, a guided transporter may only move from Beginning Intersection Name toward Ending Intersection Name. A bidirectional link allows vehicles to move either from Beginning Intersection Name toward Ending Intersection Name or from Ending Intersection Name toward Beginning Intersection Name. In the case of spurs, the Ending Intersection Name must be a "dead end" — not connected to the network by any links other than the spur. When the transporter arrives at the Ending Intersection Name of a spur, it does not give up control of the link zones between Ending Intersection Name and Beginning Intersection Name; instead, the transporter keeps control of the entire link to ensure that it can return to the main path intersection (Beginning Intersection Name).

3.8. Beginning and Ending Directions

The fields Beginning Direction and Ending Direction are used to define the direction of the link (in degrees) as it leaves the beginning intersection and as it enters the ending intersection; the direction entering the ending intersection defaults to the direction leaving the link's beginning intersection.

The value entered should be an integer value between 0 and 360 representing direction of travel in degrees (0 and 360 represent right or east). These directions are used in conjunction with the turning velocity of a vehicle to slow down a transporter as it turns a corner.

3.9. Number of Zones

The Number of Zones operand is used by SIMAN to determine how to move transporters through the links. If the number of zones is one, then SIMAN simply moves the transporter through the link as a single event. If the number of zones is greater than one, then SIMAN moves the transporter through the link zone by zone. The entity controlling the transporter seizes the first zone in the link before commencing movement. When it gets the first zone, it moves through the new zone, and depending upon the vehicle size and zone control policy, it releases trailing zones as they are no longer needed. When the transporter arrives at the end of the zone, it waits to seize the next zone on the link. When it gets this next zone, it commences movement through the new zone. This process repeats until the entity arrives at the end of the link.

3.10. Length of zones

The Length of Each Zone is the same for all zones in a link. The product of Number of Zones and Length of Each Zones is the total length of the link. The units used to measure Length of Zone should be consistent with all other measurements for the transporters using this link.

3.11. Velocity change factor

The Velocity Change Factor specifies a multiplier that is to be applied to the current velocity of any vehicle moving through the link only during travel through the link. The value entered is multiplied by the transporter unit’s current velocity to determine the travel velocity through the link.

4. MATLAB Application for Generating Haul Road Networks in Arena

To generate the haul road network in Arena, a text file containing the road information is required. We can export the road information from within a dxf file representing the road network of the mine as polylines. Before exporting the polylines, it is necessary that:

- Polylines are connected.
- Polylines start and end at junctions or end points, i.e. any polyline must not extend beyond a junction.

The polyline data is then exported as ASCII file. The exported ASCII file must be formatted as Line Number, Line Type, Y, X, Z, Tag. It should be noted that this format is essential for the text file so that we may use it with MATLAB application, RoadNetwork.m GUI, to create road network in Arena. Line Type and Tag fields in the exported output can take any value as it is discarded during reading the file.

Create an excel file named “Config.xlsx”. Create two sheets “DumpLocations” and “Schedule” within the excel file. Provide all the dump locations, their coordinates as X, Y, Z and each dump locations capacities i.e. how many simultaneous dumps are possible at each dump location. Name the ranges as “DumpCoordinates” and “DumpCapacities”. Provide all the scheduled mining polygons and respective coordinates as X, Y, Z in the “Schedule” sheet. Name the coordinate range as “PolygonLocations”. Also provide the scheduled shovels to mine respective polygons. Name this range as “ShovelScheduled”.

Run the “RoadNetwork.m” GUI. Browse the ASCII file containing the road network information that we exported as polylines. Provide the other required data in the GUI and click on run to generate three sheets “Nodes”, “Links” and “FailureNetwork” in the Config.xlsx file. The data from these sheets will be read into Arena to generate the road network, which will be discussed in the last section.

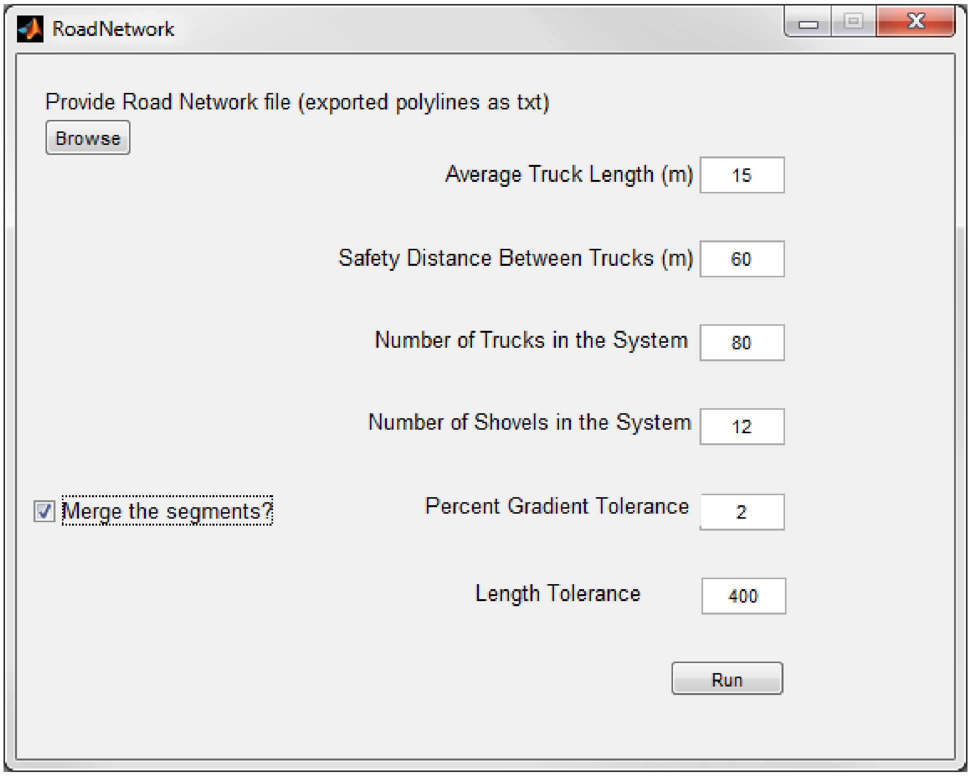

Fig. 1. RoadNetwork GUI to generate road network data for Arena Simulation

Average truck length and safety distance between trucks is required to model the zone lengths for individual network links in Arena. Zone length provides the length of the part of a road segment that a truck (transporter unit) seizes when it is stationary. A truck unit moves through the network zone by zone by seizing the next zone and releasing the previous zone for the trailing transporters. The zone control rule thus can be defined as 'Start', i.e. a truck releases a zone, to be seized by a trailing truck, as soon as it is given the next required zone. This setting enables trucks not to overtake and maintain a safe following distance on the haul roads.

Check the "Merge the segments?" checkbox if the road network is very large. This will enable merging the road segments based on the length and gradient tolerance. If the difference between the gradients of two consecutive road segments is less than percent gradient tolerance or total length of the combined segment is less than length tolerance, the segments will be merged together to form a single segment.

5. MATLAB Application for Modeling Truck Speeds

In normal case, when there is no interaction between vehicles, they try to move freely on their normal driving speed. According to Bonates (1996), maximum obtainable speed by any truck can be determined by the rimpull curves generally provided by the manufacturers. He describes the rimpull as the force exerted on ground by the drive wheels to get the truck in motion. This force is generated by the torque that the engine develops and it is a function of the gear ratios.

To model the velocities of trucks in simulation, a table is created which provides a speed factor for each truck type on different total resistance haul roads. Total resistance of haul roads is the summation of haul road gradient resistance and rolling resistance. Based on the total resistance of each haul road segment, which we created in previous section, and referring to this table, we can determine the maximum possible speed of each truck type while traveling on that haul road segment.

To generate the speed factor table, a MATLAB application is created which reads in truck payloads and rimpull characteristics from a separate excel workbook and writes down the speed factors table in “Config.xlsx” file. To run this application:

- Create an excel file “RimpullRetardCurves.xlsx”.
- Create a sheet “TruckSpecification”.
- Write down the names of all truck types employed followed by their gross empty and full vehicle weights. Name the range as “Specifications”

Name of the truck type	Truck Type 1	Truck Type 2
Gross Empty Vehicle Weight	283495	278690
Gross Loaded Vehicle Weight	610082	623690

- Create two worksheets for each truck type. Name the worksheets as “Rimpull-” and “Retard-” followed by name of the truck type given in truck specifications.
- Provide the rimpull and retard curve data as Point, Rimpull/Retard Force (Kg. force), Speed (Km/h).
- Save the excel file and run the MATLAB application TruckSpeedsByRimpull.m”.

The MATLAB application will create a table in “Config.xlsx”, sheet “RimpullSpeeds”.

6. Haulage Modeling in Arena

A flowchart of the haulage simulation model, incorporating the interaction between trucks on haul roads and intersections, is given in Fig. 2. This chapter focuses only on the haulage simulation model, and a detailed modeling of the entire simulation model is not presented.

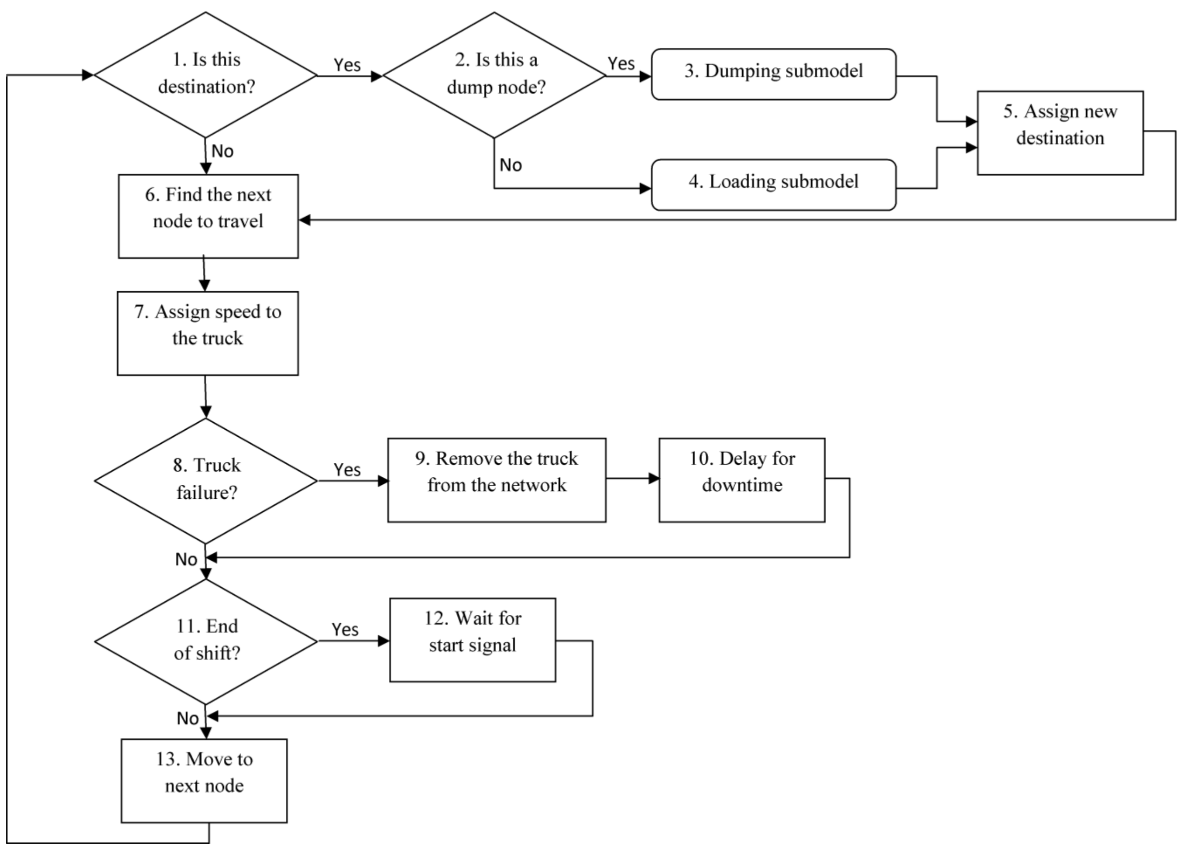

Fig. 2. Flow chart of the haulage simulation model

Prior to building the simulation model a VBA code is written which uses the data in "Nodes" sheet of "Config.xlsx" file to create stations and intersections. A Network, along with Network Links, is created between the intersections using the data in "Links" and "FailureNetwork" sheet. All the stations coupled with their corresponding intersections are written in a station set module in sequential order. This station set module is the module where entity enters back into the model after being transported to next node at step 13 in Fig. 2.

To model the truck interactions on haul roads, guided path transporters will be used which move from zone to zone by seizing next zone and releasing the occupied zone on the network link. This characteristic prohibits overtaking of trucks and models the truck interactions precisely. Moving the trucks from node to node on haul roads also provides opportunity to control the speeds of trucks based on the haul road gradient and rolling resistance.

6.1. Definitions of terminology

6.1.1. VBA Macro

- BuildModelFromExcel: A macro is written in the VBA editor of Arena to read the data in "Config.xlsx" file and build the sets, expressions, variables, transporters, network, network links and stations in the model to avoid manual work to enter each data, create haulage road network and formulate expressions in the model.

6.1.2. Entities

- entLoad: load entities associated, each associated with a transporter (truck).

6.1.3. Attributes

- atrTruckID: unique ID associated with each transporter (truck).
- atrTruckType: Truck type represented by the transporter
- atrTruckInitSpeed: Sampled speed of truck on a flat haul road (m/hr)
- atrVelocity: Velocity of the transporter on a Network Link (road segment)
- atrIsLoaded: 0 or 1 if travelling empty or loaded
- atrCurrentStation: Station name of the current node of the transporter.
- atrNextStation: Station name of the next node to travel
- atrNextLinkNum: Network Link sequence number of the next network link (road segment) to travel
- atrDumpStation: Station name of the dump station node
- atrDigStation: Station name of the node at which shovel loads the trucks

6.1.4. Variables

- v2DNetworkLinks: Variable containing information about each Network Link (road segment), i.e. start node, end node, Angle of segment from the east direction, zone lengths, Number of zones, Gradient, rolling resistance and total resistance of the road segment.

6.1.5. Station

- setStnNodes: A set of all the nodes in the haulage road network including the shovel reach node and dump nodes is created as stations associated with their corresponding intersections. It is created by the user macro "BuildModelFromExcel" written in VBA.

6.1.6. Network Link

- Network Links are created by the user macro written in VBA. Individual network links are named as "Network Link 1", "Network Link 2" and so on. Each network link is created as unidirectional in nature between a pair of intersections, with a beginning direction, number of zones, zone length and the default velocity change factor; which are taken directly from the "Config.xlsx" file by the VBA macro "BuildModelFromExcel".

6.1.7. Network

- Network: It contains the names of all the links which were created in Network Link. Arena provides flexibility to create separate Networks using different Network Links.

6.1.8. Transporter

- Transporter: Created by the user macro written in VBA "BuildModelFromExcel". All the trucks in the system are modeled as transporters. Number of units equals number of trucks in the system. Network Name is "Network" which is the haulage road network used by trucks. A default velocity is given as 40,000 per hour. The basic distance unit is considered as meter, so the velocity is in meter per hour. Rest is left as default. Initial position of transporters is assigned sequentially on haulage road intersections.

6.1.9. Files

- filConfig: Input file "Config.xlsx" containing information about number of trucks, types, rimpull speed factors, flat haul velocities, haulage road intersections (nodes), links (segments) and haul road characteristics.

6.1.10. Advanced Sets

- setIntersections: Set containing names of all intersections in the model in sequential order
- setLinks: Set containing names of all the network links (haul road segments) in sequential order
- setTruckSpeed: Set of expressions as e2DTruckSpeed1, e2DTruckSpeed2 and so on for each truck type in sequential order
- setSpeedFactors: Set of expressions as e2DSpeedFactor1, e2DSpeedFactor2 and so on for each truck type

6.1.11. Expressions

- e2DTruckSpeed: Expressions e2DTruckSpeed1, e2DTruckSpeed2 and so on, refer to the recordsets in filConfig representing flat haul empty and loaded speeds of each truck type.
- e2DSpeedFactor: Expressions e2DSpeedFactor1, e2DSpeedFactor2 and so on, refer to the recordsets in filConfig representing empty and loaded speed factors for each truck type on various total resistances of haul roads.
- expVelocityTransporter: Expression to determine the speed of a truck on a haul road segment:

  *atrTruckInitSpeed * 1000 * EXPR (MEMBER (setSpeedFactors, atrTruckType), atrIsLoaded + 1, AINT(v2DNetworkLinks (atrNextLinkNum, 8)) + 11)*
- expNextStation: Expression to determine the next station to travel to reach the destination:

  MEMIDX(setIntersections,NEXTX(Network,INXNUM(setStnNodes(atrCurrentStation)),INXNUM(setStnNodes(atrDestStation))))

6.2. Pseudo code

- ***Check current station*** if it is not the destination station
- ***Assign*** the next station and velocity of the transporter on the next road segment
- ***Transport*** the truck through the next network link
- ***Station:*** Receive the transporter (truck) on the next station and update the current station

6.3. Step by step haulage simulation modeling

6.3.1. Station set module: setStnNodes

Create a station module and go to its properties and change the tag as “NodeStations”. When the “BuildModelFromExcel” macro will be run, all the nodes in the haul road network will be added here as stations forming part of the set “setStnNodes”.

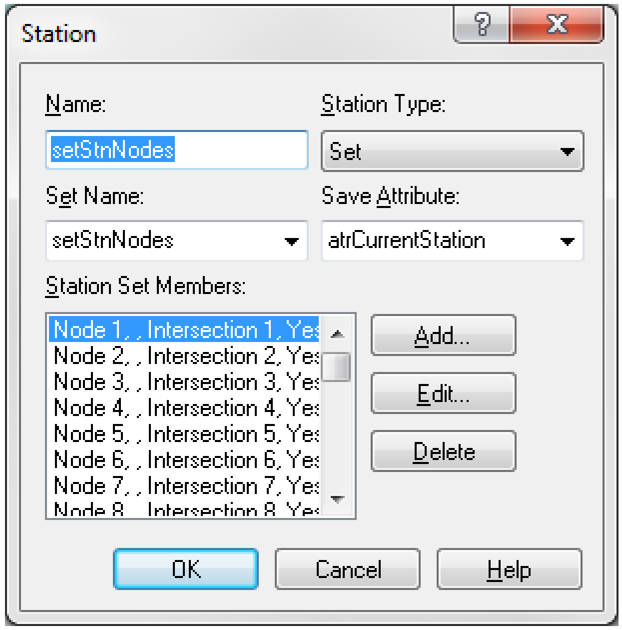

Fig. 3. Station module containing all the stations and associated intersections in the model

6.3.2. Condition module

Create a condition module to check if the current station of the transporter is its destination station. If it is the destination station of the transporter, the entity is sent into the loading or dumping sub-model which we are not discussing in this chapter. Otherwise, the entity is sent to the assign module.

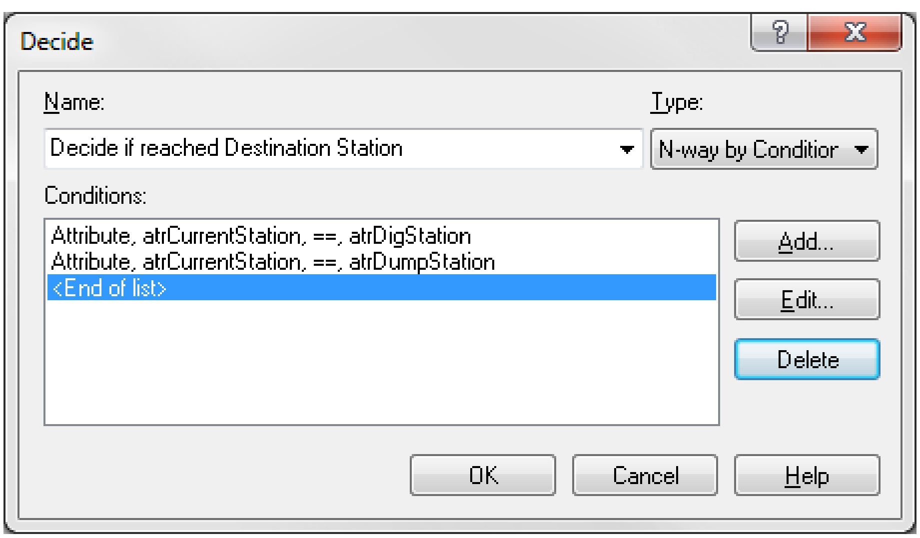

Fig. 4. Condition module to check if the current station is the dig station or the dump station (destination)

6.3.3. Assign Module: Assign Next Station

The transporter is assigned the next node station on its path to its destination and the travelling speed.

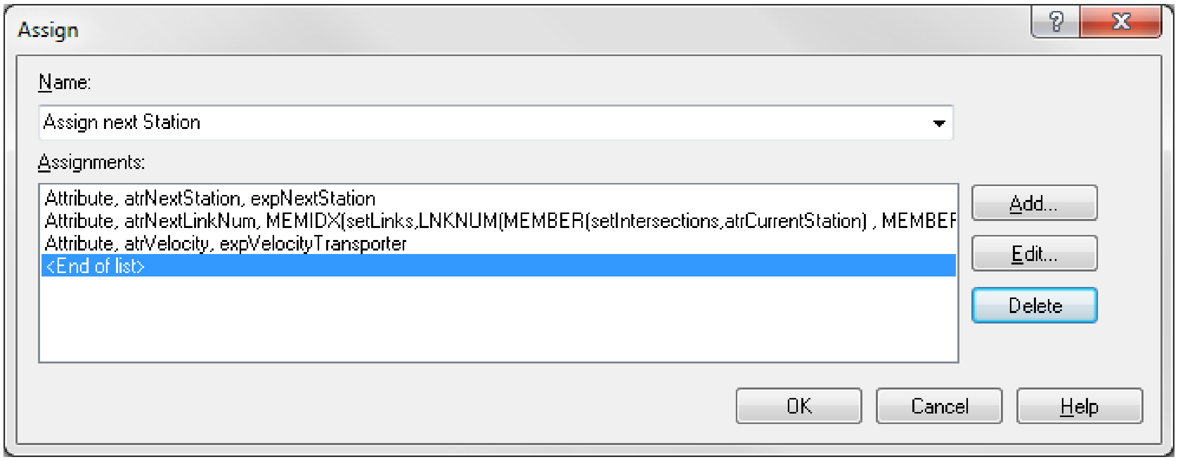

Fig. 5. Assign Module to assign next station and speed of travel to the transporter

Name		**Assign next Station**
Assignments	Type	Attribute
	Attribute	atrNextStation
	New Value	expNextStation
Assignments	Type	Attribute
	Attribute	atrNextLinkNum
	New Value	MEMIDX(setLinks,LNKNUM(MEMBER(setIntersections,atrCurrentStation) , MEMBER(setIntersections,atrNextStation)))
Assignments	Type	Attribute
	Attribute	atrVelocity
	New Value	expVelocityTransporter

6.3.4. Transport

The transporter is sent to the next station node travelling through the next network link (haul road segment) using the transport module.

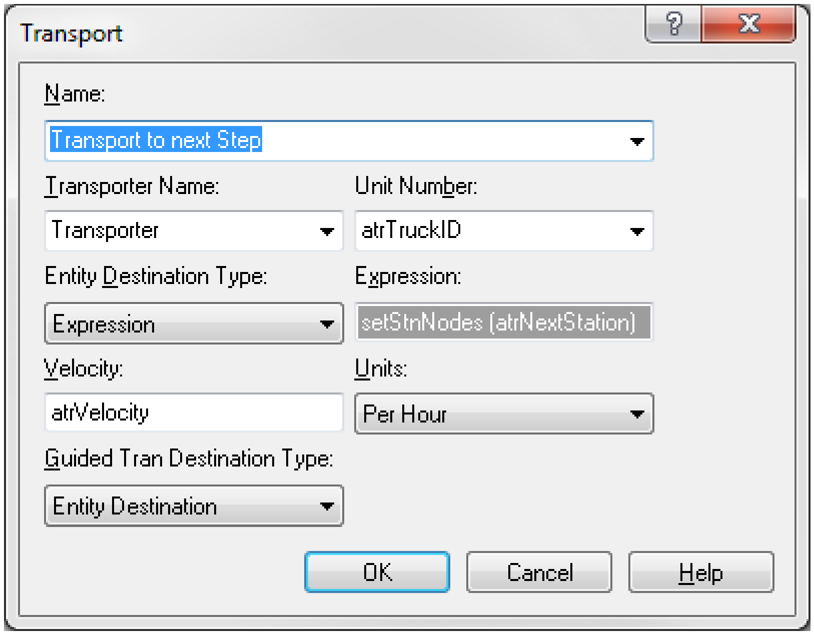

Fig. 6. Transport the load entity through the transporter to the next node station on the path to its destination.

Name	**Transport to next Step**
Transporter Name	Transporter
UnitNumber	atrTruckID
Entity Destination Type	Expression
Expression	setStnNodes(atrNextStation)
Velocity	atrVelocity
Units	Per Hour
Guided Tran Destination Type	Entity Destination

7. References

1. Bonates, E. (1996). Interactive truck haulage simulation program. *Mine Planning and Equipment Selection*, 51-57.

2. Rockwell Automation, I. (2010). Arena (Version 14.70.00004).