Mining Optimization Laboratory

Report Three – 2010/2011

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Executive Summary

On behalf of all the researchers at the MOL, I would like to thank our sponsors:

1. Newmont Mining Corporation
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Your sponsorship of our research team shows the strategic vision at your organization. We appreciate your sponsorship and look forward to your ongoing and growing support of the MOL in the future. As promised, industry-driven research has been conducted and documented in two main research areas: mine planning and design and simulation optimization of mining systems. This report is a deliverable that would rationalize sponsorship of the *MOL* research.

We look forward to active collaboration with the MOL sponsors and new members in order to:

1. Increase real-world experiences that are gained by graduate students while working closely with industry,
2. Establish internships and fellowships for the researchers at the MOL,
3. Increase the networking opportunities to the MOL students, and
4. Establish a Thesis Partners program, where students are paired with sponsoring companies to work on innovative and challenging mine planning / simulation research projects directly related to the companies' problems.

This year, we have prepared a hardcopy report including 17 papers, a CD-ROM containing all the papers in PDF format, PowerPoint presentations, and software source code. The prototype software code is documented and is available through HTML links in the appendix section of the papers online. We continue to update all the research results on the MOL webpage www.ualberta.ca/mol on the members section. Sponsors have access to current and past research results, publications, prototype software, and source code. Limited information is available to non-members, hopefully just enough for them to seriously consider membership. Let's review the contributions in the MOL Report Three (2010/2011) by considering some of the main contributors.

Mohammad has continued working on development, implementation, and verification of clustering algorithms for block aggregation (paper 101). The main algorithms tested are horizontal hierarchal clustering, vertical hierarchal clustering, k-means, and tabu search. The algorithms aggregate blocks into selective mining units based on a similarity index which is defined based on rock-types, ore grades and distances between blocks. ***Mohammad*** also has proposed a multi-destination MILP model, which handles various destinations and multiple rock-types. The model incorporates various processing capacities as well as cumulative waste dump capacities. Three variable reduction techniques are proposed to pre-determine some variables that will not have an impact on the solution to decrease the time and resources required to solve the mathematical programming problem. These techniques do not affect the optimality of the problem. Having the methodology implemented on a number of sample datasets, on average, 70% of the original variables were removed from the formulation.

Hesam has been working on developing models for open pit mine production planning and scheduling. He has developed a hierarchical long-term and medium-term open pit mine production scheduling model (paper 102). Two mixed integer linear programming (MILP) models are applied for long-term and medium-term mine production scheduling in an iron ore mine. The mixed integer linear programming model of the long-term mine production scheduling maximizes the net present value of the mining operation subject to a number of technical constraints. The MILP formulation, developed for medium-term mine production scheduling, models open pit mines with multiple destinations, multiple elements, multiple stockpiles, and multiple ramps/routes. The objective of the medium-term planning is to minimize the total operational costs subject to technical constraints and logical constraints such as tracking the long-term plan. The proposed hierarchical model is then verified in an iron ore open pit mine using TOMLAB/CPLEX environment. Hesam has also developed a stochastic long-term mine production scheduling model to deal with the grade uncertainty present in production planning (paper 103).

Behrang has been working on long-term mine planning in the presence of grade uncertainty. The optimality of an open pit production scheduling problem is affected dramatically by grade uncertainty. A mathematical programming formulation is presented to find a sequence in which ore and waste blocks should be removed from a predefined open pit outline and their respective destinations, over the life of mine, so that the net present value of the operation is maximized and the deviations from the annual target ore production is minimized in the presence of grade uncertainty. The results of the optimization model show that the extraction of high uncertain blocks is deferred to the later years of production. A penalty value is applied for any probable over and under production. The main idea is that uncertainty costs money and should be deferred. The challenging question is the quantification of the cost of uncertainty. In paper 104, a methodology is presented to quantify the cost of uncertainty.

Yashar has worked on long-term block cave mine production scheduling using mixed integer linear programming (MILP). He has carried out a mathematical programming formulation for block cave production scheduling (paper 105). The production scheduler aims at maximizing the net present value of the mining operation while the mine planner has control over the: development rate, vertical mining rate (production rate per drawpoint), lateral mining rate (rate of opening new drawpoints), dilution entry, mining capacity, maximum number of active drawpoints, cave draw strategies and advancement direction, and draw rate. The production scheduler defines: the opening and closing time of each drawpoint, the draw rate from each drawpoint, the number of new drawpoints that need to be constructed, and the sequence of extraction from the drawpoints to support a given production target.

At the Mining Optimization Laboratory (MOL), the MILP models are implemented in MATLAB and TOMLAB/CPLEX environment. We are always dealing with MILP formulations with many variables and many constraints, consequently, the branch and cut tree becomes so large that insufficient memory remains to solve an LP sub-problem. On the other hand, sometimes we need to run several problems at the same time. **Yashar** has prepared a document (paper 401) on how to run and compile TOMLAB projects on the University cluster server to be able to use parallel computing.

Elmira has been working on the truck-and-shovel dispatching system in presence of uncertainty. She has developed a discrete event simulation model in Arena to study the truck-shovel operations in an open pit mine (paper 302). The objective of her study is to determine the number of trucks and shovels based on the optimal mine schedule, and assess the effect of uncertainties on the key performance indicators in the system. **Elmira** also, has presented a mixed integer linear programming model to optimize the allocation of trucks and shovels in an open pit mine (paper 106). The objective of the presented MILP model is to minimize the operating costs associated with trucks and shovels while considering technological, operational and financial constraints. Her final goal is to build a interactive simulation-optimization model that links the optimal short-term schedule to the truck-and-shovel operations.

Eugene has been carrying out research on oil sands long term production planning and waste management. His current work is on in-pit and external oil sands dyke construction scheduling using goal programming (paper 201). In oil sands mining, timely provision of ore and low-footprint tailings containment are crucial for profitability and sustainability. The recent Alberta Energy Resources Conservation Board Directive 074 requires oil sands waste disposal planning to be an integral part of mine planning. This requires an integrated strategy of directional mining and tailings dyke construction for in-pit and ex-pit tailings management. The objective of his work is to: 1) determine the order and time of ore, dyke material, and waste removal from a predefined final pit limit over the mine life that maximizes the net present value of the operation; and 2) determine the destination of dyke material that minimizes construction cost while meeting construction requirements as per stipulated designs.

Mohammad Mahdi has been working towards integration of mine and tailings plans in oil sands industry. Tailings is considered to be the main by-product of oil sands processing. Due to the noticeable amount of fresh and recycled water used in the process of bitumen extraction, huge volume of slurry is produced at the end point of the process. The amount of tailings produced is also important from environmental point of view. The ultimate goal of this research is to develop a multi-objective mine planning framework that will maximize the net present value of the mining operations while minimizing the environmental impacts. The short-term goal of the work is to first calculate the amount of tailings produced as the result of extraction of each block and secondly, revise the MILP mine planning models in a way to consider the constraint of tailings pond capacity that holds the tailings. The tailings calculation formula is retrieved from Suncor's process flow sheet. The derived formulation is verified by applying on a real mining production plan. Then, a sub-gradient algorithm is developed to solve the MILP model with Lagrangian relaxation method. The results are reported and the future steps of research are mentioned at the end.

Mohammad **and** ***Ebrahim*** have aimed at using Arena discrete event simulation software to study a processing plant. A magnetic iron separation plant is considered and modeled in paper 303. A processing plant usually works in a continuous manner. However, it is possible to simulate the system and study parameters of importance in a discrete event time steps with modifications in the simulation approach. They have used the batching approach for achieving this goal. In this technique, batches of material are considered as entities and carry important properties of the material through the simulation model. The simulation model captures uncertainties in feed, machine performances, downtimes and the effects on the global system performance.

Hooman Askari
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Hooman Askari

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Following are researchers and students affiliated with Mining Optimization Laboratory in September 2011.

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Clustering and Multi-Destination Production Scheduling

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Abstract

Mine production planning is the process of deciding on the order of extraction of blocks from the block model. This can be done in various ways, but a known and reliable approach is through mathematical programming. The goal of this paper is to propose a mathematical model which can handle multiple destinations with different capacities, profits, mining and haulage costs, and processing costs. In order to make the problem tractable, two clustering techniques are proposed and compared based on their accuracy and resource consumption. Three procedures for removing unnecessary variable are also proposed, which together eliminate around 70% of variables in a test dataset.

1. Introduction[1]

Mining is a complex industry with large capital investment requirements. Critical decisions are made at different stages in the exploration, design, extraction and reclamation of mines; these deal with the movement of millions of tonnes of materials and cash flows in the order of billions of dollars. Therefore, operations research has been broadly used to assist with decision making processes in the mining industry.

Open pit mine planning is commonly defined as the process of deciding on the sequence of extracting blocks from a mine in order to gain the highest net present value (NPV) subject to a set of constraints. This process is usually undertaken based on two main approaches. The first approach is in two stages: 1) decide which blocks to extract (finding final pit limits); and 2) find the appropriate sequence of extraction (scheduling). In the second approach both problems are solved simultaneously. Although the second approach seems to result in a better solution with respect to maximization of the objective function, Caccetta and Hill (2003) have proven that the optimum pit found using the simultaneous approach will fall inside the pit outline found using pit optimization techniques. Therefore, the first approach is taken for this paper.

Open pit production scheduling is a known problem which has attracted many researchers. This problem has been studied in various levels of detail and various time horizons. Production scheduling models are usually categorized into long-term, mid-term, and short-term time horizons. One of the main issues with various mathematical models proposed for open pit production scheduling in the literature is the size of the problem. A typical midsize open pit mine block model consists of hundreds of thousands of blocks at least. The mathematical formulation of the production scheduling of such a mine is beyond the ability of today's commercial software and

[1] Some parts of introduction and literature review are direct excerpts from (Tabesh and Askari-Nasab, 2011)

hardware to solve the optimization problem in a reasonable time or to find a solution to the problem at all. Attempts have been made to overcome this curse of dimensionality, but one of the main obstacles of using exact optimization methods in open pit mine production scheduling is still the intractability of such formulations.

The first objective of this paper is to develop a mathematical formulation for multi-destination production scheduling of a mine. The proposed formulation is a mixed-integer linear programming model which uses continuous variables for determining the portion of block to each destination and binary variables for controlling the order of extraction of blocks and slope constraints. This means that for every block in the model there are T binary and $T \times D$ continuous variables, where T and D represent the number of planning periods and destinations respectively. For a real size block model this can lead to a mathematical formulation with tens of millions of variables – a situation which is not tractable using currently available hardware and software.

Clustering algorithms are then proposed in order to make the problem tractable and also to have more practical production plans from the mining point of view. The clustering algorithms aggregate blocks into selective mining units based on rock types, ore grades and distances between blocks. The aggregated blocks are referred to as mining-cuts, which are consequently used in the aforementioned mathematical formulation. Two known clustering algorithms are developed and compared based on the variability of grades and rock types inside cuts, the NPV that results when the algorithm is used with the multi-destination model, and the shape of the created clusters. The first algorithm is a hierarchical clustering technique proposed in Tabesh and Askari-Nasab (2011) which defines a similarity index between blocks and aggregates blocks together until it reaches a predefined number of clusters. The next clustering algorithm is an implementation of the k-means algorithm based on kernel functions. The term *k-means*, first used in MacQueen (1967), also covers a large series of heuristics proposed in subsequent literature, all of which partition data points by selecting mean points for clusters and assigning each data point to the nearest mean.

The next part of this paper presents some efforts made to reduce the number of variables and consequently the size of the problem. In many mathematical formulations, paying attention to the structure of the problem can lead to pre-determining some variable values and removing them from the model. Three variable reductions are proposed in this paper which together can remove 73% of variables on a test problem.

The rest of the paper is organized as follows. A review on the long-term open pit mine production planning (LTOPP) literature is followed by a review of the relevant clustering techniques and implementations in the next section. The third section explains the mathematical model and the clustering algorithm proposed. The algorithm is then implemented on a small subset of a real dataset and the results are presented in section four. The paper is concluded in the fifth section.

2. Review of literature

2.1. Long-term open pit mine production planning

Various models for LTOPP have been proposed in the literature. The traditional approach was to define production levels (in tonnages) of ore and waste (Gershon, 1983). Other models of interest consist of zero-one decision variables indicating whether a block is going to be extracted in a period, subject to some constraints on mining and processing capacities, grade blending, and slope constraints. Usually the objective function aims at maximizing the NPV in order to take the time value of money into account. For a complete review of the mathematical programming models used in mine production scheduling, see Osanloo et al. (2008) and Newman et al. (Newman et al., 2010). In this literature survey we will focus on block aggregation and clustering techniques.

The first attempts to cluster blocks and reduce the size of problem were made by (Busnach et al. (1985) and Klingman and Phillips (1988); the authors of these studies decided to aggregate all the blocks on the same bench without considering any of the block properties. Gershon and Murphy (1989) then tried to form layers of material labeled as ore or waste. Samanta et al. (2005) used the same approach but solved the problem using a genetic algorithm. Another interesting approach for grouping blocks and reducing the number of binary variables is taken in Ramazan (2001). The author of this thesis creates a tree based on the blocks in the model and their extraction sequence constraints. Ramazan (2001) then extracts the fundamental trees and solves the problem using these trees. The same author has refined the approach in Ramazan (2007). Although this can lead to optimal solution of the problem, finding the fundamental trees and solving the problem using them is still NP-Hard. Zhang (2006) uses a genetic algorithm to create mining cuts and solve the problem in cut level.

2.2. Clustering

Clustering is defined as the process of grouping similar entities together in a way that maximum intra cluster similarity and inter cluster dissimilarity is achieved. This can be modeled and solved as a mathematical programming problem, but the difficulty lies in the amount of resources and time required to solve the problem. The clustering problem has been proven to be NP-Hard (Gonzalez, 1982). Therefore, a wide range of non-exact algorithms has been developed in the literature. These algorithms are usually performed by defining a measure of similarity or dissimilarity between the objects. These techniques can be categorized into two major groups: hierarchical and partitional clustering. As its name implies, hierarchical clustering is performed by creating a hierarchy of clusters. On the other hand, partitional clustering is performed by partitioning data objects into a number of groups. Hierarchical clustering is known to result in better clusters than partitional algorithms, but by taking more CPU time (Feng et al., 2010). A famous example of partitional algorithms is the k-means, which attempts to find cluster means and assign data points to the closest mean. K-means is a partitioning technique which tries to find a good partitioning scheme by iteratively modifying the partitions. This has also proven to be an NP-Hard problem (Mahajan et al., 2010), but it can still be used to find good partitions on the data. Some recent modified versions of k-means clustering can be found in Chung and Lin (2006), Bagirov (2008) and Zalik (2008). This algorithm has also been combined with other methods to form hybrid faster or more accurate techniques (Chang et al., 2009), (Niknam and Amiri, 2010) and (Niknam et al., 2010). A more complete review of the algorithm and its extensions can be found in Jain (2010). One extension to k-means clustering which is relevant to this project is the kernel k-means which is developed to be able to partition data points which are not linearly separable by mapping them into a kernel space (Dhillon et al., 2004).

3. Mathematical formulation

The mixed integer linear programming model for multi destination mine planning is explained in this section. The model consists of continuous variables for extracting material and sending it to various destinations as well as binary decision variables for controlling the sequence of extractions and slope stabilities. Decision variable $x_k^{d,t} \in [0,1]$ is used in determining the portion of block/cut k in period t and sending the material to destination d. Therefore, $x_k^{d,t}$ has to be bounded by the rock tonnage in block/cut k that can be sent to the corresponding destination. This is done through defining the set R_d and constraints (6) and (7). Binary decision variables are also used in the model in order to control the order of extraction of blocks/cuts. b_k^t is set to one, if extraction of block/cut k is allowed in period t; i.e., all of the predecessor blocks/cuts are completely removed. This is guaranteed through constraint (8). Parameter L in this constraint is the number of members in set $C_k(L)$ which holds all the predecessor blocks/cuts of the block/cut k .When b_k^t gets a value of one, it remains at one until the end of the mine life. On the other hand, mining is prevented through constraint (10) as long as b_k^t is equal to zero.

The objective function of the model is the net present value (NPV) of extracting material from the mine and sending them to the corresponding destinations. The block value earned from sending one tonne of rock from each block/cut is calculated and discounted to the present value and is called $v_k^{d,t}$. This parameter holds all the profits and costs such as mining, haulage, dumping and processing of one ton of rock from the corresponding block/cut. Since haulage and dumping costs are considered, this value can also be different for different waste destinations.

Constraint set (2) is responsible for controlling the minimum and maximum mining capacity in each year. There is also a set of constraints for controlling the capacity of each processing destination in each period, as in constraint (3). Waste dump capacities are controlled through constraint (4). The limitation on waste dumps is usually the available lease area or the area prepared for dumping material. Therefore, having these as cumulative capacity constraints is more reasonable than having a yearly upper and lower bound. Average head grades of elements and deleterious material sent to each processing destination is also controlled by constraint (5). Finally, constraint (11) makes sure that all the material in the pit is extracted and sent to an appropriate destination.

The rest of parameters and sets used in the mathematical formulation are defined as follows.

3.1. Sets

D_P	Set of indices of the processing destinations
D_W	Set of indices of the waste dumps
$C_k(L)$	For each mining-cut k, there is a set $C_k(L)$ defining the immediate predecessor mining-cuts that must be extracted prior to extracting mining-cut k, where L is an integer number presenting the total number of cuts in the set $C_k(L)$.
R_d	For each destination d, R_d holds the rock types that are allowed to be sent to this destination if the there is enough capacity.

3.2. Indices

$k \in \{1,2,...,K\}$	Index for the mining cuts
$t \in \{1,2,...,T\}$	Index for the planning periods
$d \in \{1,2,...,D\}$	Index for the destinations
$e \in \{1,2,...,E\}$	Index for the elements of interest

3.3. Parameters

$v_k^{d,t}$	The discounted value (Net Present Value) of extracting one tonne of cut k in period t and sending it to destination d. If $d \in D_P$ the value is expected to be positive; otherwise, it is expected to be negative.
t_k	The total tonnage of material in mining-cut k
t_k^r	The tonnage of rock type r in mining-cut k
$\underline{m}^t$	The lower bound of mining capacity in period t
$\overline{m}^t$	The upper bound of mining capacity in period t

$\underline{p}^{d,t}$ The lower bound of processing capacity of destination $d \in D_p$ in period t

$\overline{p}^{d,t}$ The upper bound of processing capacity of destination $d \in D_p$ in period t

$\underline{w}^{d,t}$ The lower bound on waste dump cumulative capacity for destination $d \in D_W$ from the first period to period t

$\overline{w}^{d,t}$ The upper bound on waste dump cumulative capacity for destination $d \in D_W$ from the first period to period t t

g_k^e The grade of element e in mining-cut k

$\underline{g}^{e,d,t}$ The lower bound of the average head grade of element e for destination $d \in D_p$ in period t

$\overline{g}^{e,d,t}$ The upper bound of the average head grade of element e for destination $d \in D_p$ in period t

3.4. Decision Variables

$x_k^{d,t} \in [0,1]$ Continuous variable, representing the portion of mining-cut k extracted and sent to processing destination $d \in D_p$, in period t

$b_k^t \in \{0,1\}$ Zero-one variable for controlling the extraction precedence. It is equal to one if it is allowed to extract mining-cut k in period t. If this variable gets the value of one it will remain at one until the end of the mining period.

3.5. Objective Function

$$\max \sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{k=1}^{K} v_k^{d,t} \times t_k \times x_k^{d,t} \tag{1}$$

3.6. Constraints

$$\underline{m}^t \le \sum_{k=1}^{K}\left(t_k \times \sum_{d=1}^{D} x_k^{d,t}\right) \le \overline{m}^t \qquad \forall t \tag{2}$$

$$\underline{p}^{d,t} \le \sum_{k=1}^{K} t_k \times x_k^{d,t} \le \overline{p}^{d,t} \qquad \forall d \in D_P, t \tag{3}$$

$$\underline{w}^{d,t} \le \sum_{t'=1}^{t}\sum_{k=1}^{K} t_k \times x_k^{d,t'} \le \overline{w}^{d,t} \qquad \forall d \in D_W, t \tag{4}$$

$$\underline{g}^{e,d,t} \le \frac{\sum_{k=1}^{K} t_k \times g_k^e \times x_k^{d,t}}{\sum_{k=1}^{K} t_k \times x_k^{d,t}} \le \overline{g}^{e,d,t} \qquad \forall e, d \in D_P, t \tag{5}$$

$$\sum_{t=1}^{T} x_k^{d,t} \le \frac{\sum_{r \in R_d} t_k^r}{t_k} \qquad \forall k, d \in D_P \tag{6}$$

$$\sum_{t=1}^{T} x_k^{d,t} + \sum_{t=1}^{T} x_k^{d',t} \le \frac{\sum_{r \in R_D \cup R_{D'}} t_k^r}{t_k} \qquad \forall k,d,d' \tag{7}$$

$$b_k^t \le \frac{1}{L} \sum_{d=1}^{D} \sum_{l \in C_k} x_k^{d,t} \qquad \forall k,t \tag{8}$$

$$b_k^t \le b_k^{t+1} \qquad \forall k,t \tag{9}$$

$$\sum_{d=1}^{D} x_k^{d,t} \le b_k^t \qquad \forall k,t \tag{10}$$

$$\sum_{t=1}^{T} \sum_{d=1}^{D} x_k^{d,t} = 1 \qquad \forall k,t \tag{11}$$

4. Clustering

The first proposed clustering algorithm in this paper is based on a hierarchical approach presented in Tabesh and Askari-Nasab (2011). In this approach, there is a need for a similarity index to be defined in order to define similarities between objects. It is defined based on grade difference, distance, rock type and the beneath bench clustering scheme. Then the clusters are formed in a hierarchical manner, as the name implies. Each block is considered as one cluster at the beginning of the algorithm. In each step, the two most similar clusters are merged together and form a new cluster. Then the similarities between the newly created cluster and all others are updated based on one of these techniques: single-link, complete-link or average-link. The single-link approach takes the similarity between the two most similar objects from each cluster as the similarity between the two clusters. In contrast, the complete-link approach looks for the two most dissimilar objects. The average-link method, as its name implies, averages all of the pair similarities. In this paper, the update is done based on the complete-link approach. The procedure continues until a predefined number of clusters are achieved. The main contributions by Tabesh and Askari-Nasab (2011) are the definition of the similarity index and calibration of parameter weights, forcing the algorithm to respect maximum cluster size and to prevent non-adjacent clusters from merging together. For more details readers are referred to Tabesh and Askari-Nasab (2011).

The next clustering technique used is an implementation of the k-means algorithm based on gradient descent search. In this approach, K initial cluster centers are randomly selected at each replication. Then the objects are assigned to the nearest center. Afterwards, based on the gradient descent search technique, the centers are manipulated in such a way that the summation of distances between the objects and the means is locally minimized. Another replication is then started with a new random set of means and the process continues for a limited number of replications.

The first step for this algorithm is to form the feature matrix, which holds all the important properties of all objects. To be consistent with the hierarchical clustering technique, the same parameters are used with the same weighting approach. Then the matrix has to be kernelized in order to get better results. When objects are not linearly separable in their initial space, kernel functions are used to map data points from the initial space to the kernelized space and do the clustering in there. Then the same map is used in returning to the initial space with all the objects labeled as belonging to various clusters. Having tested various kernel functions and parameters, a polynomial kernel function with $d = 1$ is used in this implementation. Afterwards, K initial cluster centers are randomly selected in the kernelized space and objects are assigned to the closest mean. Then the objective function, which is a summation of Euclidean distances between all objects and cluster means, is calculated. Cluster means are then manipulated in an iterative manner based on gradient descent until a local minimum is found. This is stored as a solution to the clustering problem and a new replication starts with another random definition of cluster means. Finally, all of the replications are compared, and the one with the lowest objective function is selected as the solution to the clustering problem on that bench.

5. Variable reduction techniques

5.1. Clustering

As mentioned earlier, clustering brings two advantages to the problem. The first is that it reduces the number of variables in the model, since only one variable is defined for each group of blocks referred to as mining-cuts. The second advantage of clustering lies in having bigger units for planning, which makes the resulted long-term production plan more practical from the mining point of view.

5.2. Removing unnecessary variables

Since the model is built to deal with multi-destination production planning, a variable is used for sending a portion of each cut to some (but not all) destinations. As mentioned above, clustering seeks to have cuts with homogenous rock types; i.e., most of the blocks grouped together have the same rock type. Therefore, it can be determined based on R_D (the possible rock type destination combinations) which cuts cannot be sent to which destinations. The corresponding variables can be removed without making any change to the optimality of the final solution. The only exception is for a waste destination called general waste dump which accepts all types of material. For example, ore rock types can be sent to this destination if the processing capacity is insufficient to let them into processing destinations.

5.3. Predecessor cone

Another technique used is borrowed from Bley et al. (2010), who remove extraction decision variables for certain blocks based on their availability. To do so, the whole tonnage of rock in the predecessor cone of the block is calculated. This is the cone which has to be extracted before one can get access to the block based on slope constraints. The cone tonnage is then compared against the cumulative mining capacity from the first period up to period t. If the cone tonnage exceeds this cumulative mining capacity, the binary and consequently continuous variable for extracting that cut in that period is removed from the problem, because one can already be sure the variables cannot have values other than zero. The idea can be expanded to processing capacities too, i.e. by comparing the ore tonnage in the cone to the cumulative processing capacity. However, this makes the decision a bit tricky since there are multiple destinations and rocks that can be sent to different destinations if it results in higher objective function value. Therefore, this approach is not used in this paper.

5.4. Successor cone

Since there is an assumption that all the material has to be extracted from the mine, the same idea can be applied to the cone below each block. This is the set of blocks which can only be extracted after the block is completely extracted. In this case, if the total tonnage in the cone below the block exceeds the cumulative mining capacity from period t to the end of mine life, the block has to be totally extracted prior to period t. The two-cones concept is also used in a meta-heuristic approach based on simulated annealing in Kumral and Dowd (2005). The predecessor and successor cones are shown in Fig 1 with green dashed and blue dotted filling patterns respectively.

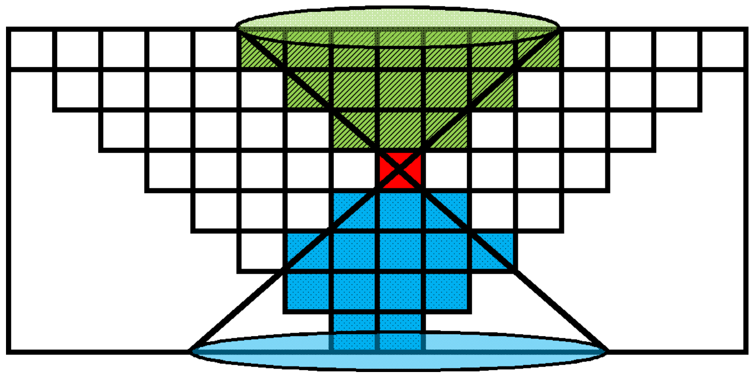

Fig 1.Predecessor and Successor Cones

6. Results

Three different experiments are undertaken to verify the ideas proposed in this paper. The first experiment is designed in order to compare the two clustering techniques based on the homogeneity of the created clusters as well as runtimes. This is done on an iron ore block model with more than 19,000 blocks. The goal of the second experiment is to study the effects of the variable reduction techniques. The same dataset is clustered with hierarchical clustering and variable reduction techniques are applied after forming mathematical programming matrices. In the third experiment, the clustering techniques are used to form mining-cuts as the input to the multi-destination mathematical model, and the resulting NPVs and required CPU times are compared for different techniques as well as different cluster sizes. Since running the multi-destination mathematical model is time- and resource-consuming, a small subset of the model with 2413 blocks from the bottom of the pit is selected and the optimal production plan for 11 periods is determined.

6.1. Hierarchical versus K-Means

The test dataset is an iron ore block model with 19,561 blocks which are distributed in 12 benches. Three elements are tracked in the model: iron, which is tracked as magnetic weight recovery (MWT), and sulfur and phosphor, which are tracked as deleterious material. The properties of the block model are summarized in Table 1. The most significant fact from this pre-processing is the high correlation between Sulfur and Phosphor grades and the MWT grade. Hence, Sulfur and Phosphor grades are removed from the feature vector.

Table 1.Block model summary

	MWT Grade	S Grade	P Grade
MIN	0	0	0
Max	0.83	0.0380	0.0039
Mean	0.11	0.0023	0.0002
STDev	0.27	0.0056	0.0005
Range	0.83	0.0380	0.0039
Correlation with MWT grade		0.96	0.97

After running both clustering algorithms on the dataset, the techniques are compared based on the following measures. As mentioned earlier, it is required to have mining-cuts homogenous in grade and rock types. Consequently, the resulting mining-cut scheme is evaluated using two separate measures for grade and rock type, which are numerical and categorical attributes respectively. It is common to evaluate numerical attributes by the use of squared error criterion (Hsu et al., 2007). This is the square distance of each data point from the cluster mean. Since there is only one numerical attribute having an effect on the quality of the results, this distance is the same as the standard deviation of that attribute. Therefore, the first criterion is the average coefficient of variation (CV) of block grades in each mining-cut, which is a normalized version of the standard deviation. Mining-cuts with lower grade variation are more effective in practice. The same thing applies to the rock type distribution. Since it is a categorical variable, neither standard deviation nor CV can be used. One measure used for categorical attributes is the categorical utility which is the probability of having two objects from same category in one cluster (Hsu et al., 2007). Another measure is the relative frequency of categories in clusters. This measure puts more weight on rare categories if considered as the objective of clustering (Huang, 1997). Based on the structure of the problem and existing criteria for categorical variables, a new index is defined as the percentage of blocks in a mining-cut belonging to the most dominant rock type in that mining-cut[2]. This is called the rock unity and is depicted in Table 2 along with other comparison measures. The hierarchical clustering technique takes more time to run but provides more accurate results, as can be seen in Table 2. The shape of the created clusters is also of importance. Therefore, the clustering schemes of one bench in the middle of the block model are provided for comparison in Fig 2 and Fig 3.

Table 2.Comparing clustering techniques

Technique	CPU Time (s)	Number of Clusters	Average Rock Unity (%)	Average MWT CV (%)	Average S CV (%)	Average P CV (%)
Hierarchical	119	402	94.6	0.3	0.008	0.0008
K-Means	67	399	67.4	0.4	1.8	1.2

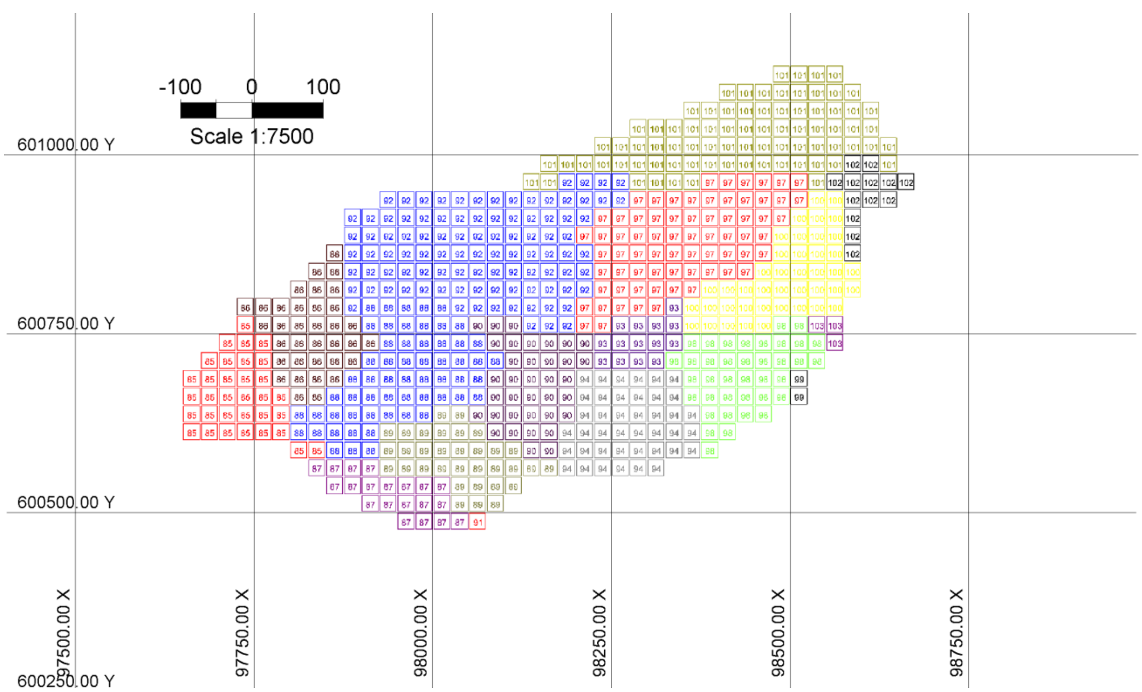

Fig 2.Hierarchical Clustering

[2] This part is a direct excerpt from (Tabesh and Askari-Nasab, 2011)

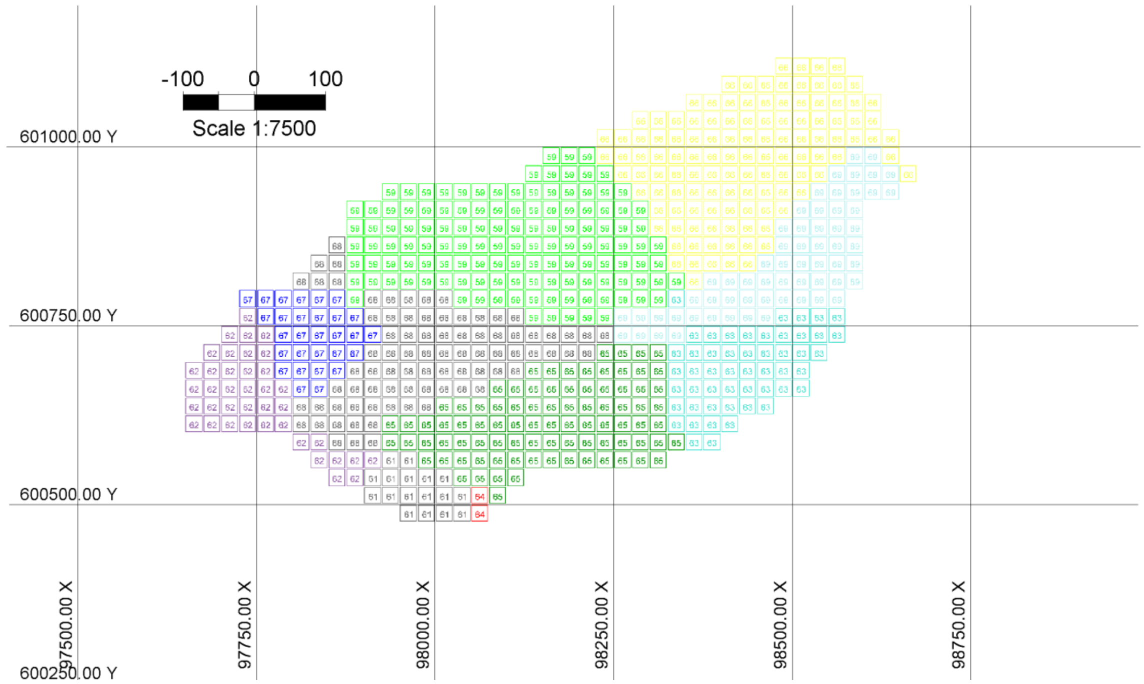

Fig 3.K-Means Clustering

6.2. Variable reduction

The same iron ore data set clustered with hierarchical clustering is used to test the effectiveness of the variable reduction techniques. A more accurate study on the effects of predecessor cone and some other techniques can be found in Bley et al. (2010). There are 19,561 blocks in the model and 402 mining-cuts. The number of binary and continuous variables for a model with 3 destinations over 21 periods is presented in Table 3.

Table 3.Effectiveness of variable reduction techniques

	Number of variables	**Number of continuous variables**	**Number of binary variables**
Original block model	1,643,124	1,232,343	410,781
Clustered model	33,768	25,326	8,442
Clustered model after variable reduction	9,147	4,863	4,284

6.3. Mathematical formulation

The multi-destination mathematical formulation is also tested on the same dataset. However, solving the MILP with available hardware and software is a resource- and time-consuming operation. Therefore, a small subset from the bottom of the pit with 2413 blocks on 6 benches is extracted and clustered with hierarchical clustering with various cluster sizes. The optimal schedule for each cluster size is obtained and compared in Table 4. The scheduling is to be done for 11 periods. The total rock tonnage in the dataset is 81 million tonnes, which is made up of 56 million tonnes of rock type 101 and 1.5 million tonnes of rock type 301, and the rest of which is waste. Lower and upper bounds of 5 and 8 million tonnes per year are considered for mining capacity. Two processing destinations are considered, the first of which is the main processing plant. The second destination is a backup process which has lower capacity and lower efficiency but can be used if the first process is working at full capacity. Rock type 101 is the main ore and can be processed in either of the destinations, unlike rock type 301, which can only be economically processed in the second destination. The processing capacities are considered to be 5 and 1 million tons per year, respectively, for the two processing destinations. For simplification, cumulative capacity constraints are not considered for the only waste dump in the model. The production schedule and the stripping ratio for the four settings are presented in Fig 4 to Fig 7. Blue bars represent the production schedule at the main processing plant, while green bars represent that of the second processing plant. Material sent to the waste dump is shown in brown. Generated schedules for a sample bench are also presented in Fig 8 to Fig 11. It can be seen that by having bigger cuts, the CPU time required for finding the optimal solution and the NPV drop. However, by increasing blocks per cut from 25 to 50, the CPU time is reduced by 99% while the NPV decreases by only 0.4%. On the other hand, the practicality of the generated schedule is one of the most important factors for judging on various clustering options. A higher number of drop-cuts, as can be seen in Fig 8, is a usual drawback of using smaller planning units. On the other hand, higher resolution clustering results in smoother schedule, as shown in Fig 4.

Table 4.Mathematical Modeling Summary

#	Average Blocks per Cut	Number of Cuts	Coefficient Matrix Size	Number of Binaries	CPU Time (s)	NPV (million dollars)
1	25	115	3345*2806	881	41,372	2501.8
2	50	62	1741*1312	407	42.26	2490.7
3	75	43	1195*816	245	6.64	2479.0
4	100	33	931*573	170	3.13	2461.6

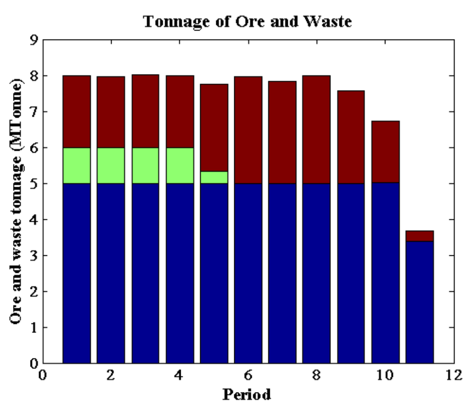

Fig 4.Production Schedule #1

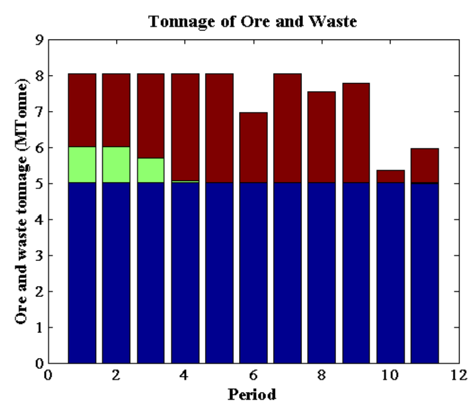

Fig 5.Production Schedule #2

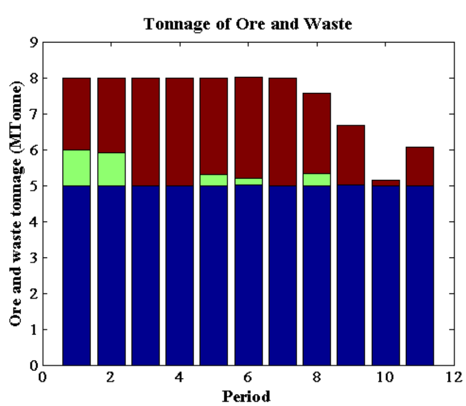

Fig 6.Production Schedule #3

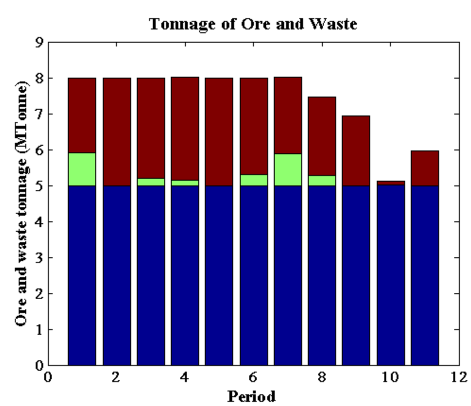

Fig 7.Production Schedule #4

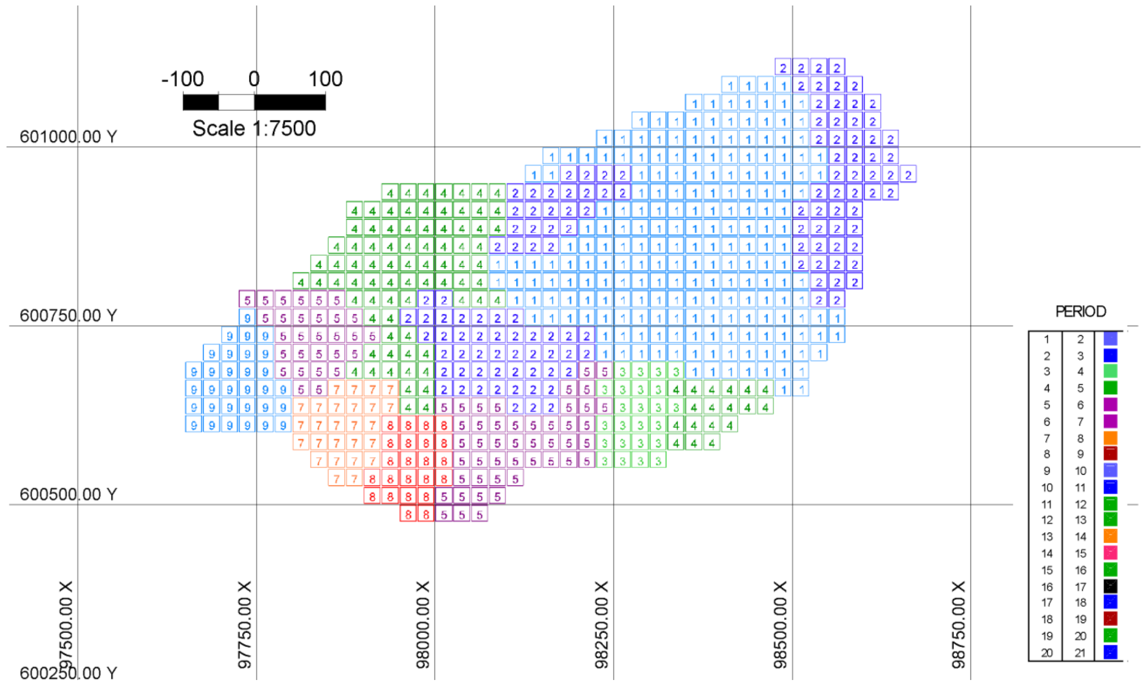

Fig 8.Sample Plan View of Schedule #1

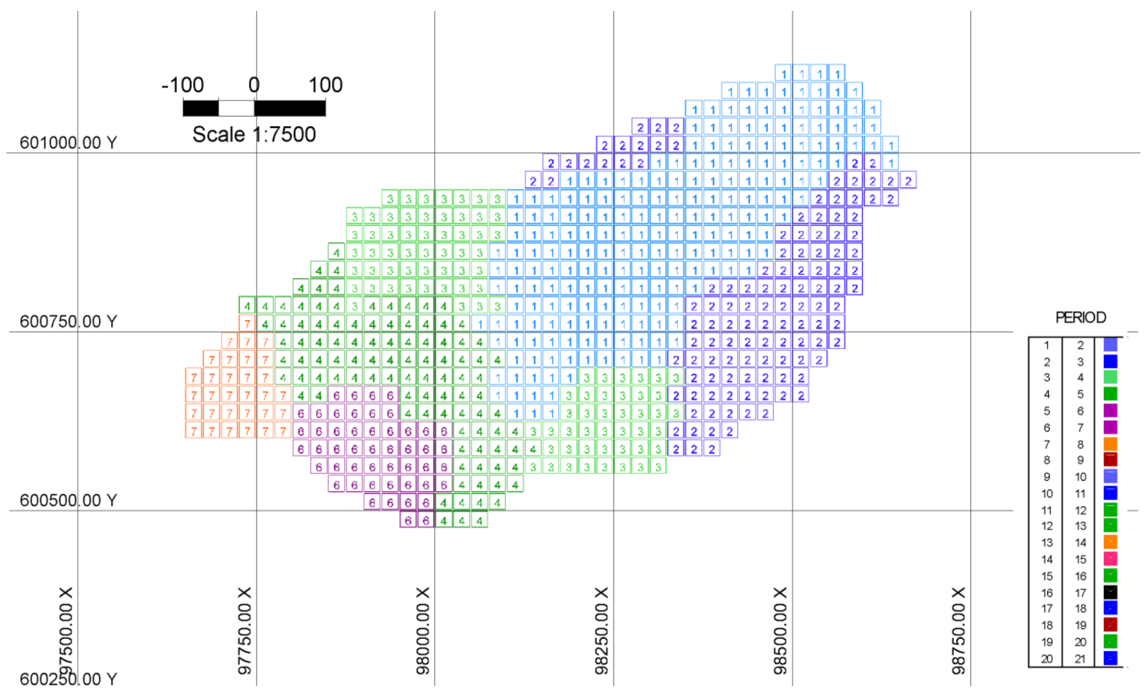

Fig 9.Sample Plan View of Schedule #2

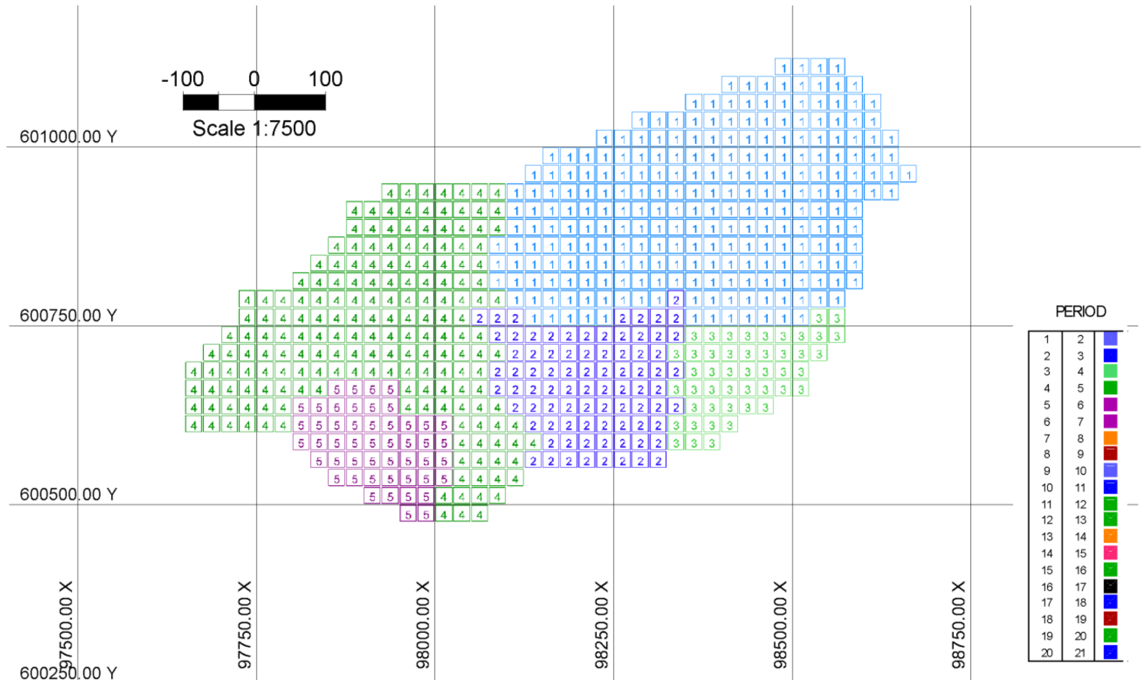

Fig 10.Sample Plan View of Schedule #3

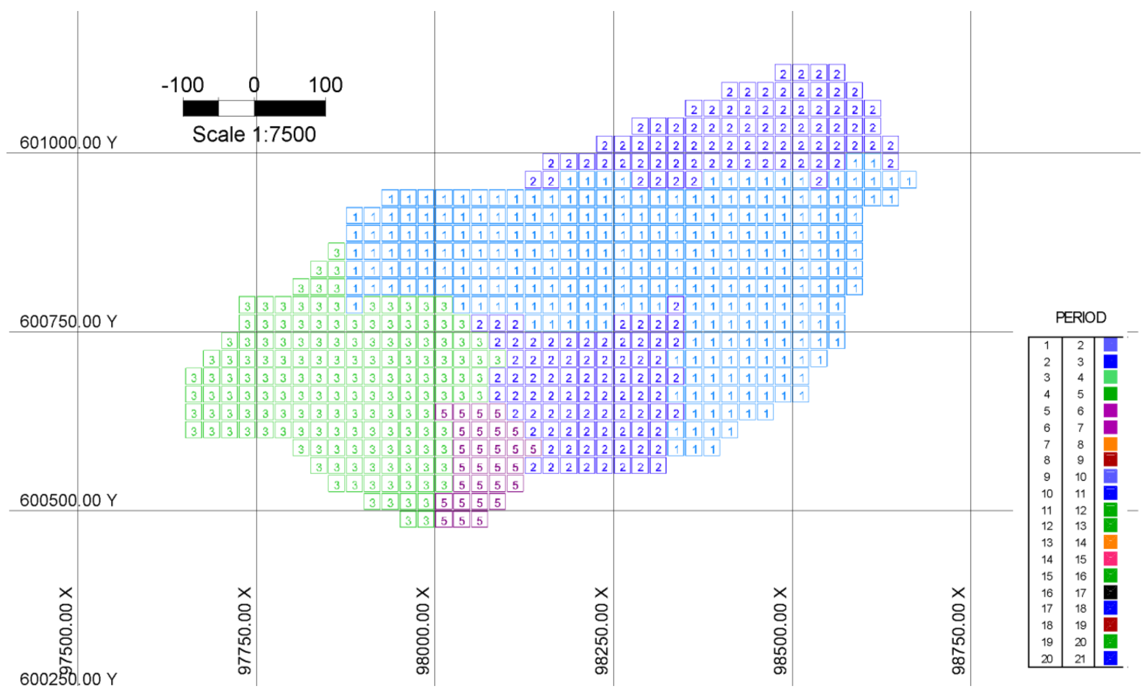

Fig 11.Sample Plan View of Schedule #4

7. Conclusions

The term "open pit long-term production planning" has received significant attention from researchers and practitioners. Various models have been introduced in literature, and solution procedures have been proposed to address the tremendous amount of computational resources required to solve the problem. One well-known approach is to cluster blocks into larger sets in order to reduce the size of the problem[3]. Two clustering algorithms are developed and compared in this paper. The hierarchical clustering technique results in more homogenous clusters than k-means, but it takes more processing time. Hierarchical clustering is then used to provide input to the proposed mathematical formulation for studying the effectiveness of the variable reduction techniques and the schedules generated using the multi-destination MILP. There is an obvious trend in the results suggesting that having smaller cuts leads to higher NPV but consumes more resources for divergence. However, the two factors do not change at the same rate, as a significant drop can be seen in CPU time while decreasing the number of cuts from 115 to 62, which is dramatically larger than the drop in NPV. The practicality of the generated schedules and the number of drop-cuts also change when the resolution of the model changes.

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9. Appendix

MATLAB and CPLEX Code Documentation

Alignment of Optimal Medium-Term Plans with Long-Term Plans using Mixed Integer Linear Programming

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Abstract

Open pit mine production planning and scheduling is one of the most important steps of any mining project. In this step, how to extract the ore as the valuable asset of mine is defined. Thus, the open pit mine production planning must be concerned well to build a good schedule of extraction. In a hierarchical categorizing, the open pit mine production planning and scheduling is divided into two main groups: long-term, medium-term and short-term planning. The long-term planning provides a strategic plan of the mining project while maximizing the expected profit of the mining project. On the other hand, medium- and short-term mine production scheduling provides operational scheme for extraction while tracking the strategic plan. Primarily, the operational costs are minimized in medium- and short-term mine production scheduling. In this paper, a dataset on an iron ore mine is used to apply long-term and medium-term open pit mine production scheduling. Both models are mixed integer linear programming models for the open pit mine. By applying the long-term and medium-term open pit mine production planning models, it is tried to show the applicability of models in satisfying technical constraints.

1. Introduction

Simply, mining is defined as the process of recovering the valuable materials from the earth crust in the following five steps (Newman et al., 2010).

- Prospecting,
- Exploration,
- Development,
- Exploitation,
- Reclamation.

Prospecting and exploration steps include studies carried out by geologists in order to identify and measure the earth's properties, the value of deposit, mineral concentration and its variability throughout the orebody. The development phase primarily includes detailed engineering design of mine, determination of mining method, and estimating production capacity and invested mining capital. Ore is recovered from the mine in the exploitation step by open pit (surface) and/or underground mining techniques. In other words, mine production is performed in the exploitation stage. Extracted ore is hauled to different destinations such as processing plant, stockpiles, and waste dumps through designed ramps. Reclamation stage consists of restoring the area in which mining operations are done to its natural state to the possible extent (Newman et al., 2010).

Like any business, mining industry seeks the maximum profit. Thus, each of mining stages are planned and carried out in order to guarantee the maximum profit. In each step, data obtained from the previous steps are used for planning of the current stage. Thus, weak planning and performing any phase will create troubles in planning of the next stages which threaten the profitability of the whole mining project. As mentioned, the mine production is done in the exploitation phase. In this stage, the valuable asset of mine, raw ore, is extracted. Fundamentally, mine production planning and scheduling aims to decrease mining costs, maximize the production while considering the quality and operation requirements like mining and processing capacity and truck/shovel allocation (Fioroni et al., 2008). In other words, mine production planning and scheduling optimizes the way to extract the ore reserve as the most valuable asset of any mining operations (Franklin, 1985). Thus, careful planning and scheduling of mine production is a vital part in the chain of mining operations.

The focus of the current study is on open pit mine production planning and scheduling. A mine production schedule has to present a mining sequence scheme that meets physical limitations and aims to produce ore to reply to demand throughout the life of mine (Kuchta et al., 2004). Also, the mine production scheduling can be defined as a decision making process to determine the sequence of extraction of materials (block sequencing) and that how much ore and waste are sent to their corresponding destinations, e.g. processes, stockpiles, and waste dumps, over a time horizon.

Depending on the time horizon of scheduling, the mine production planning and scheduling is classified as long-term, medium-term and short-term planning that each category involves with certain objectives and issues (Chanda, 1992).

The long-term mine production planning mainly focuses on ore reserves, stripping ratio and major yearly investment plans (Fytas et al., 1993). The long-term mine production planning and scheduling is provided while looking at the mine operations in long-range time horizon (e.g. 20 years) of life of mine with specifically aiming to increase the profitability (usually net present value (NPV) maximization) (Chanda, 1992). There are numerous research results available in the area of long term open pit production scheduling using mathematical programming. A number of noticeable recent contributions in this area are carried out by Bienstock and Zuckerberg (2010), Bley et al. (2010), and Askari-Nasab et al. (2010). Mines use long-term schedule as a strategic tool to determine the starting time of mining a production region and as a guideline for medium-term and short-term mine production planning (Kuchta et al., 2004). In fact, the long-term mine production plan presents a general and long-term financial status of mine. However, satisfying blending constraints, head grade constraint, and other technical constraints which are imposed on the long-term planning does not ensure their fulfillment in the medium- and short-term plans (Kumral and Dowd, 2002). In fact, a long-term mine production plan gives a conceptual framework for mine activities while medium-term and short-term mine production plans translate the long-term concept into operational level by presenting effective and doable functions (Franklin, 1985).

Unlike the long-term mine planning, in the medium-term mine production planning some aspects of mine production such as haulage roads, mining sequence, and equipment investment are involved with more detail (Chanda, 1992). The medium- and short-term mine production planning is concerned with determination of sequence of ore and waste removal in medium-term periods (e.g. month, week, etc.). The medium- and short-term mine production scheduling should match with constraints applied by long-term plan, plant capacity, inventory restriction, equipment availability, haul and road. In short-term mine production scheduling, usually more detailed data on ore grade distribution are available (Muge et al., 1992). Unlike long-term mine production planning which tries to maximize profit, often medium-term mine production planning's purposes include minimizing the operational cost (Kumral and Dowd, 2002). The medium- and short-term mine production plan must obey the long-term plan to guarantee the expected profitability of mining project (Chanda, 1992). Thus, ensuring the profitability in the medium-term and short-term planning, some operational costs involved in the mining process are going to be taken into account. Then, minimizing the cost is the main objective of medium-term and short-term plans.

The rest of the paper is organized as follows: in section 2, objectives of the paper are explained. Section 3 indicates the theoretical framework of the research. Different steps of research including both long-term and medium-term open pit mine production scheduling models are briefly described. In section 4, a dataset regarding an iron ore open pit mine is used to apply long-term and medium-term open pit mine production scheduling formulations. The long-term mine production planning is performed for 21 years. For medium-term mine production scheduling, the corresponding MILP model is applied for two years 10 and 16, separately. The results of implementation of mine production scheduling are elaborated and discussed. Finally, conclusion and future research directions are stated in section 5.

2. Objectives of research

The main objective of the current research is to apply long-term and medium-term open pit mine production scheduling to an iron ore mine case study. Also, the present study aims to provide both long-term and medium-term mine production schedules. The schedules and different plan views and issues of mining are plotted to show the results of mine production planning.

3. Theoretical framework

The general framework of the present study is as follows:

- Applying a long-term open pit mine production scheduling model to a dataset related to an iron ore open pit mine;
- Selecting a few long-term periods (years) for applying medium-term mine production planning;
- Applying a medium-term open pit mine production scheduling in the selected years;
- Interpreting the results of mine production planning models in form of plan view and resulted schedules.

The long-term open pit mine production scheduling model is the formulation # 2 developed by Askari-Nasab and Awuah-Offei (2009). The long-term model as a mixed integer linear program (MILP) maximizes the discounted cash flow of mining project in the life of mine subject to a number of constraints as follows:

- Mining capacity constraints;
- Processing capacity constraints;
- Head grade constraints;
- Precedence or slope constraints;
- Complete extraction of whole pit.

On the other hand, the medium-term open pit mine production scheduling model of this research is the formulation developed by Eivazy and Askari-Nasab (2010). The medium-term open pit mine production scheduling model considers a number of stockpiles, processes, and waste dumps that are named as the destinations of extracted material. Generally, the medium-term open pit mine production scheduling model decides on followings:

- Sequence of block extraction including amount of extraction of each block in each period through the time horizon,
- Amount of total material that must be extracted from the mine in each period,
- Destinations that the mined materials are sent to,

- Amount of ore reclaimed from the stockpiles to the processes in each period.

Briefly, the assumptions of the medium-term open pit mine production scheduling model are as follows:

- Each destination can receive material with a specific rock-type or a combination of rock-types,
- Each element has an acceptable grade range at different processes and stockpiles. Generally, stockpiles are separated by rock-type and grade range of ore.
- There are numerous routes to send extracted materials from the mine to different destinations. Mined material of each block could be hauled to corresponding destination through some routes.
- Material within each stockpile is homogeneous and the ore reclaimed from each stockpile has a certain grade equivalent to the average grade of stockpiled material.

The medium-term open pit mine production scheduling formulation minimizes total operational costs including total processing cost, total waste rehabilitation cost, total rehandling cost, and total haulage cost. The minimization of total costs is performed under a number of constraints as follows:

- Complete extraction of all blocks within the considered year of long-term plan,
- Assigning one destination for the extracted ore and waste in each period,
- Minimum extraction of blocks in each period,
- Mining capacity constraints,
- Processing capacity constraints,
- Stockpile capacity constraints,
- Head grade constraints,
- Vertical precedence in extraction of blocks (slope constraint),
- Route selection constraints,

4. Case study and discussion of results

Here, a dataset regarding an iron ore open pit mine is used to apply the long-term and medium-term mine production scheduling models. The mixed integer linear program developed for the long-term mine production scheduling, is applied for 21 years. In addition, to apply medium-term mine production scheduling model, two year 10 and 16 are selected. Figs. 1 and 2 show the 3D and 2D views of whole pit, respectively. Totally, 33 ramps have been designed for the iron ore mine. The ramps can be seen in Fig. 2. As there are large number of blocks, solving the problem in this size could take much time. Thus, blocks are aggregated by fuzzy C-means method into 1217 cuts. Fig. 3 to Fig. 13 show the resulting long-term schedule in terms of plan view plots for levels with specific z-values. These specific levels are selected in which their blocks are extracted in years 10 and 16. The numbers inside the plan view plots shown in Fig. 3 to Fig. 13 indicate the year that the maximum portion of corresponding block is extracted.

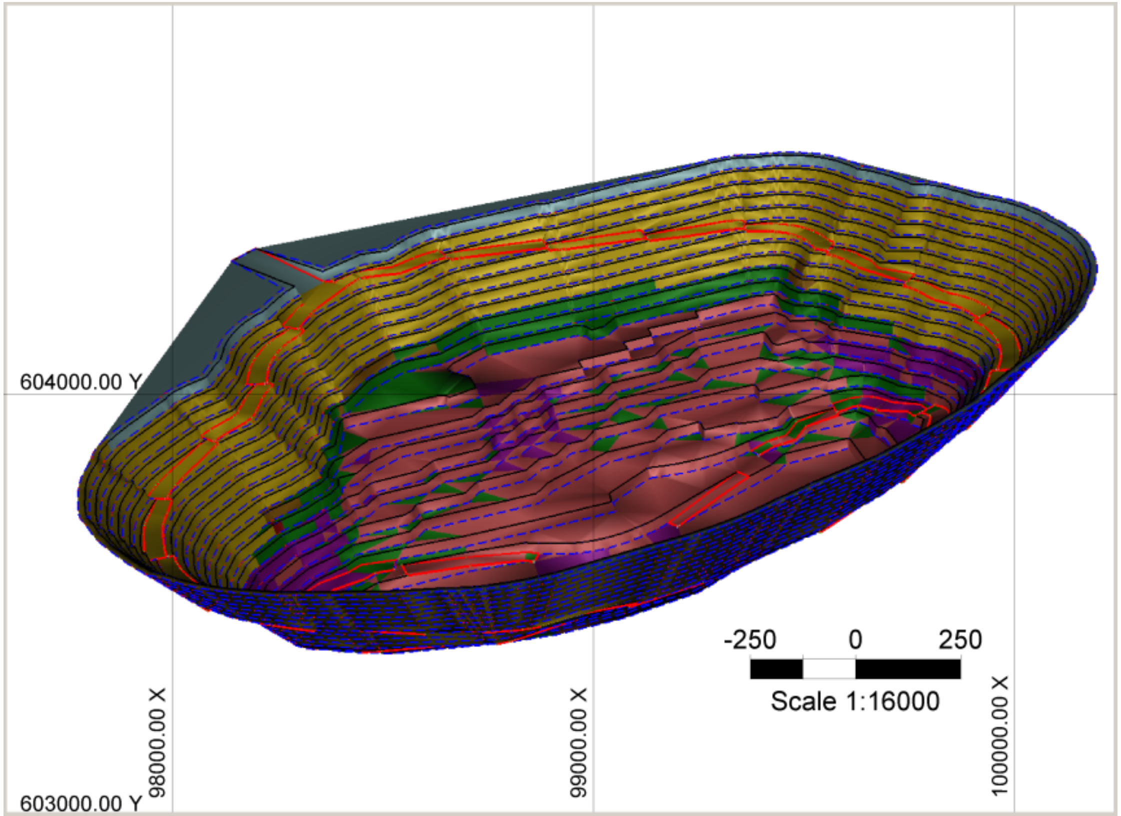

Fig. 1 3D view of iron ore mine

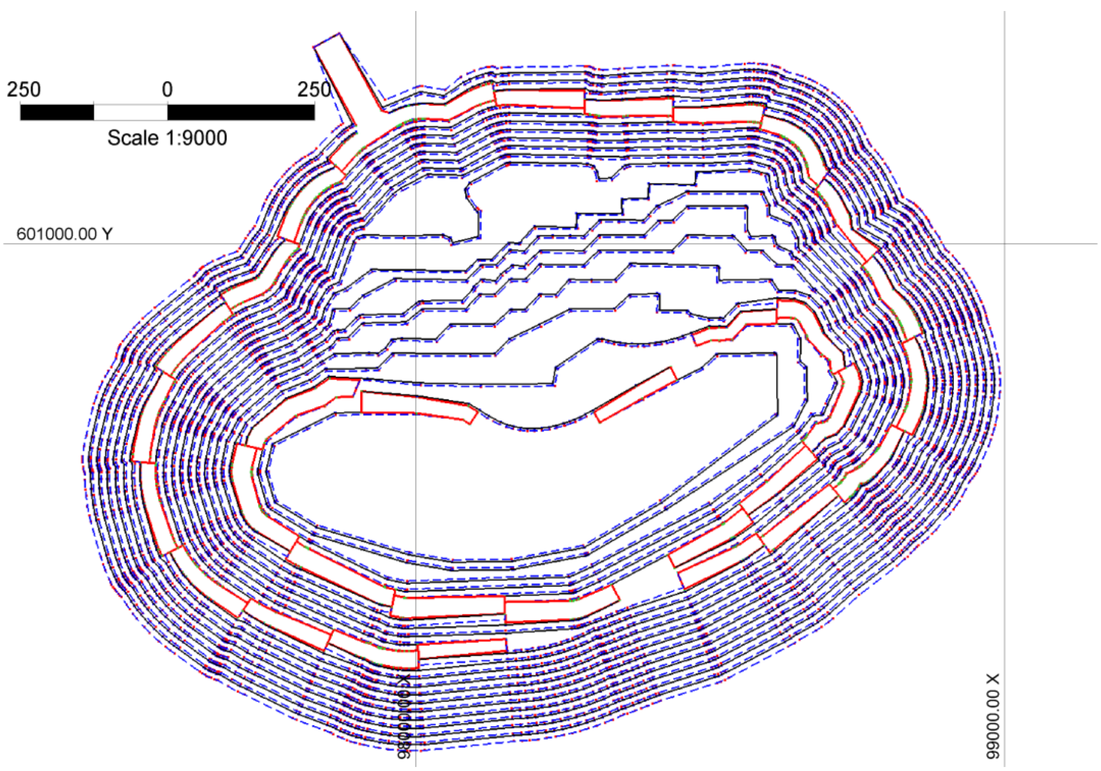

Fig. 2 2D view of pit with designed ramps

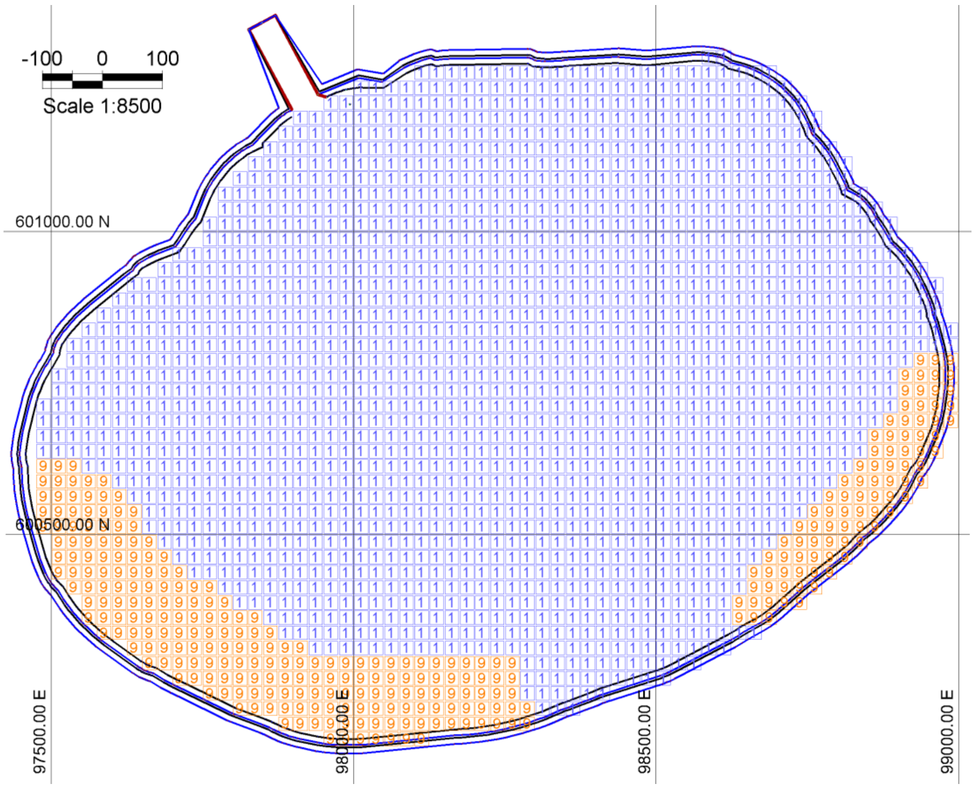

Fig. 3 Plan view plot of long-term schedule-Z=1725 m.

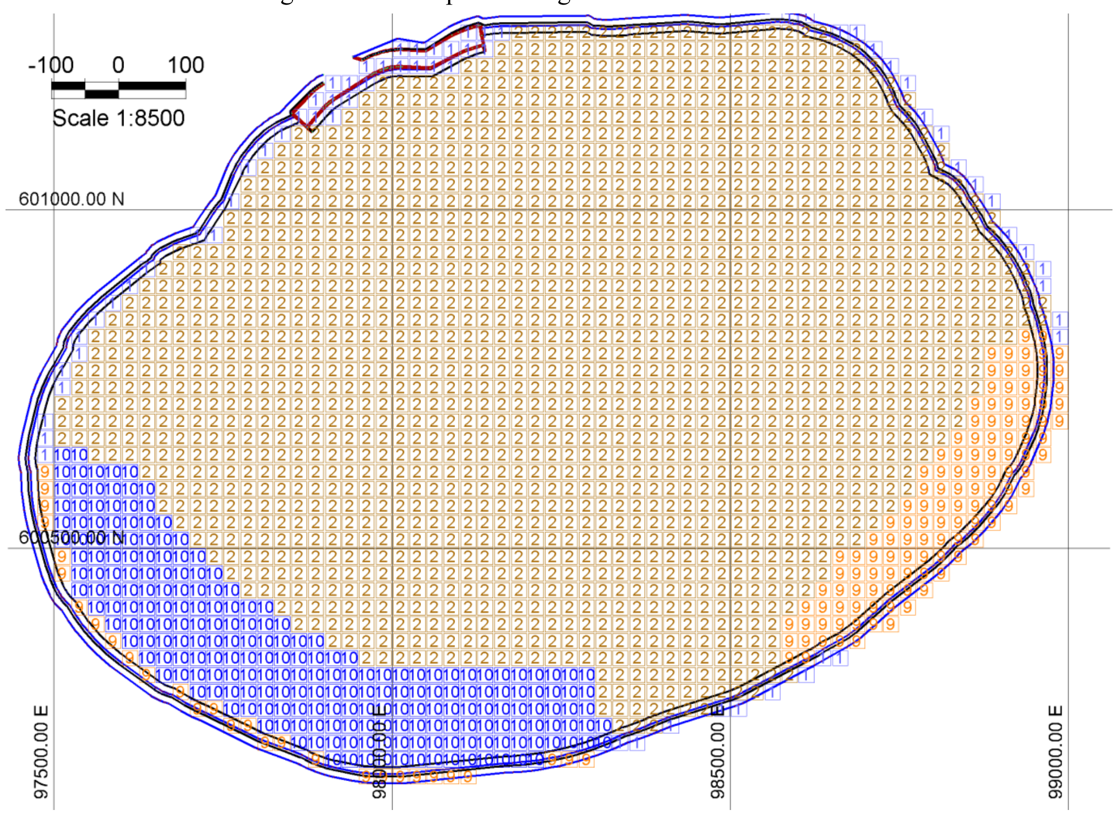

Fig. 4 Plan view plot of long-term schedule-Z=1710m.

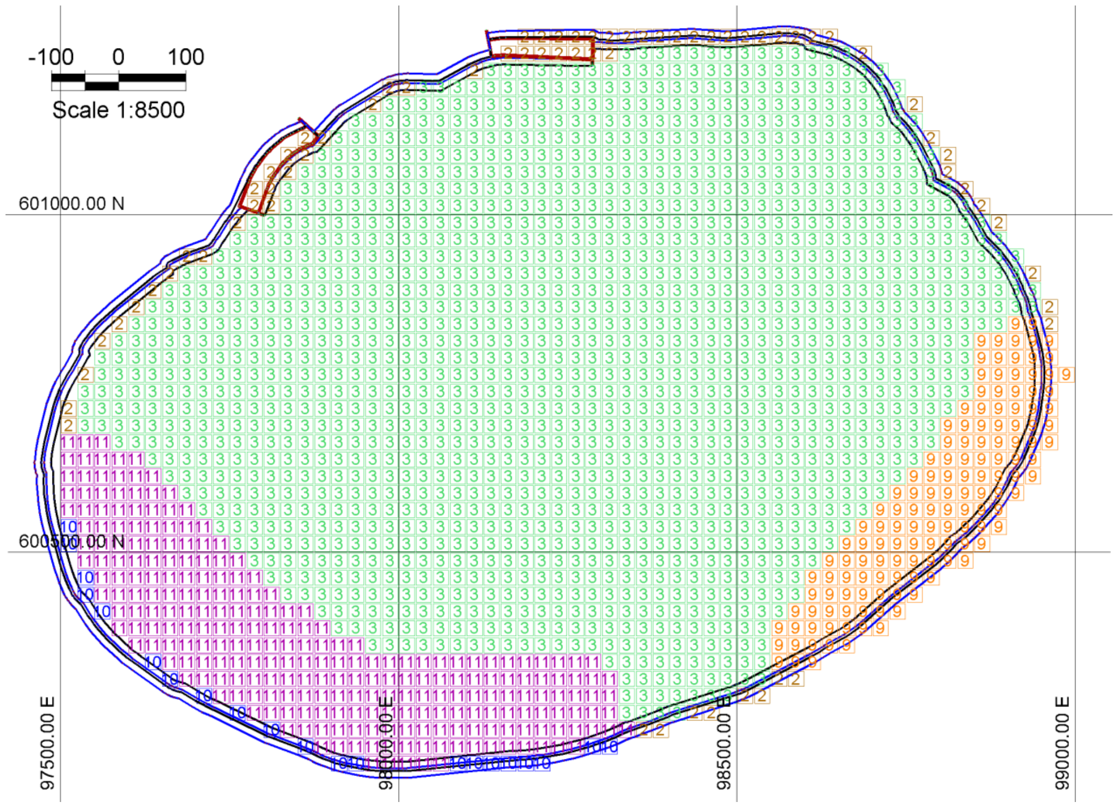

Fig. 5 Plan view plot of long-term schedule-Z=1695m.

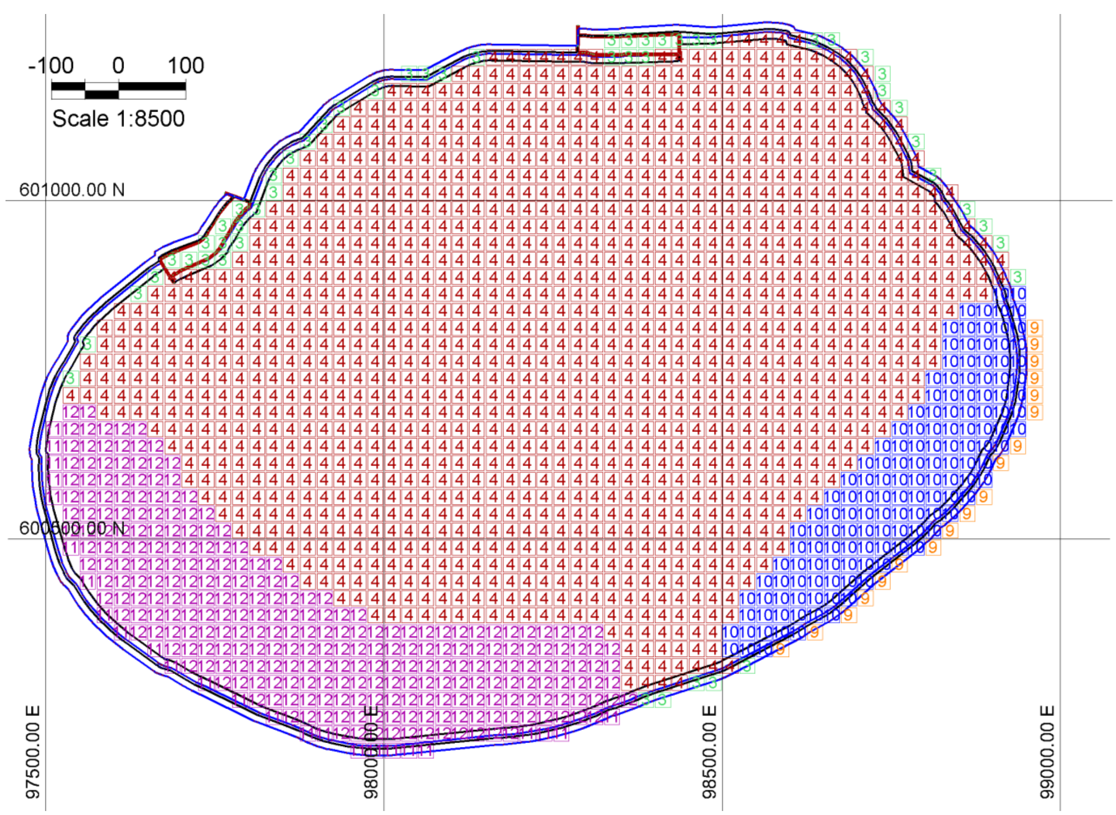

Fig. 6 Plan view plot of long-term schedule-Z=1680m.

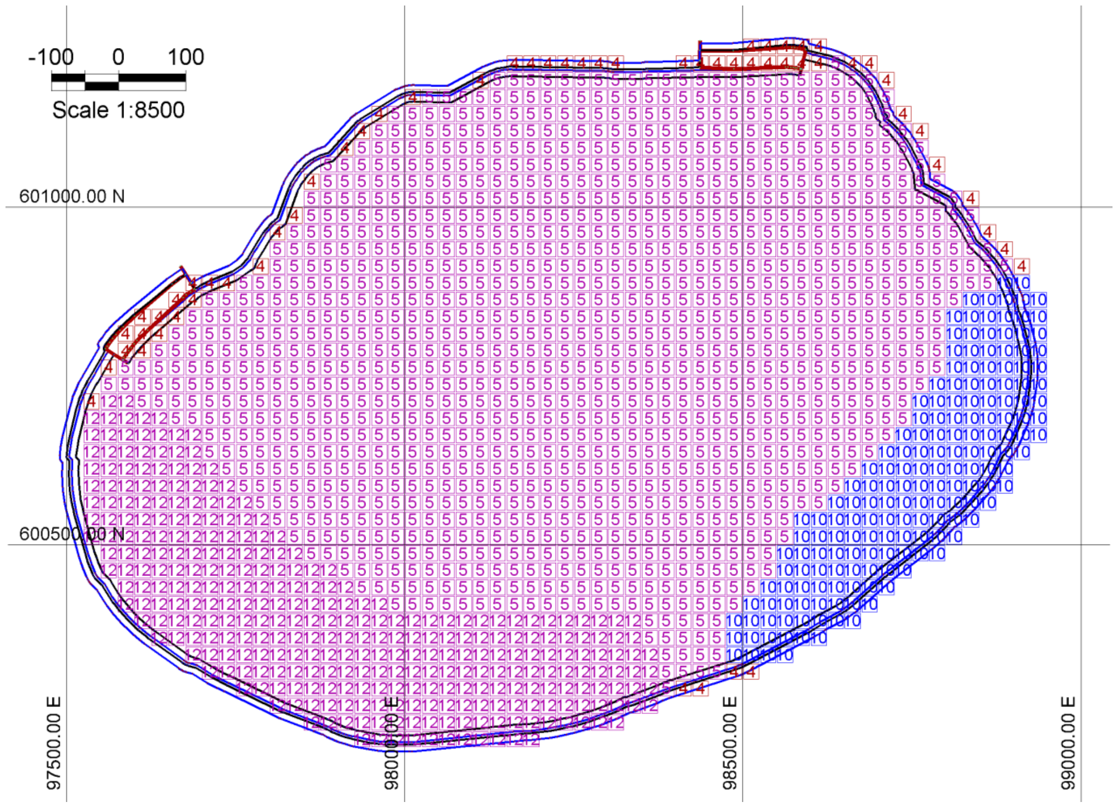

Fig. 7 Plan view plot of long-term schedule-Z=1665m.

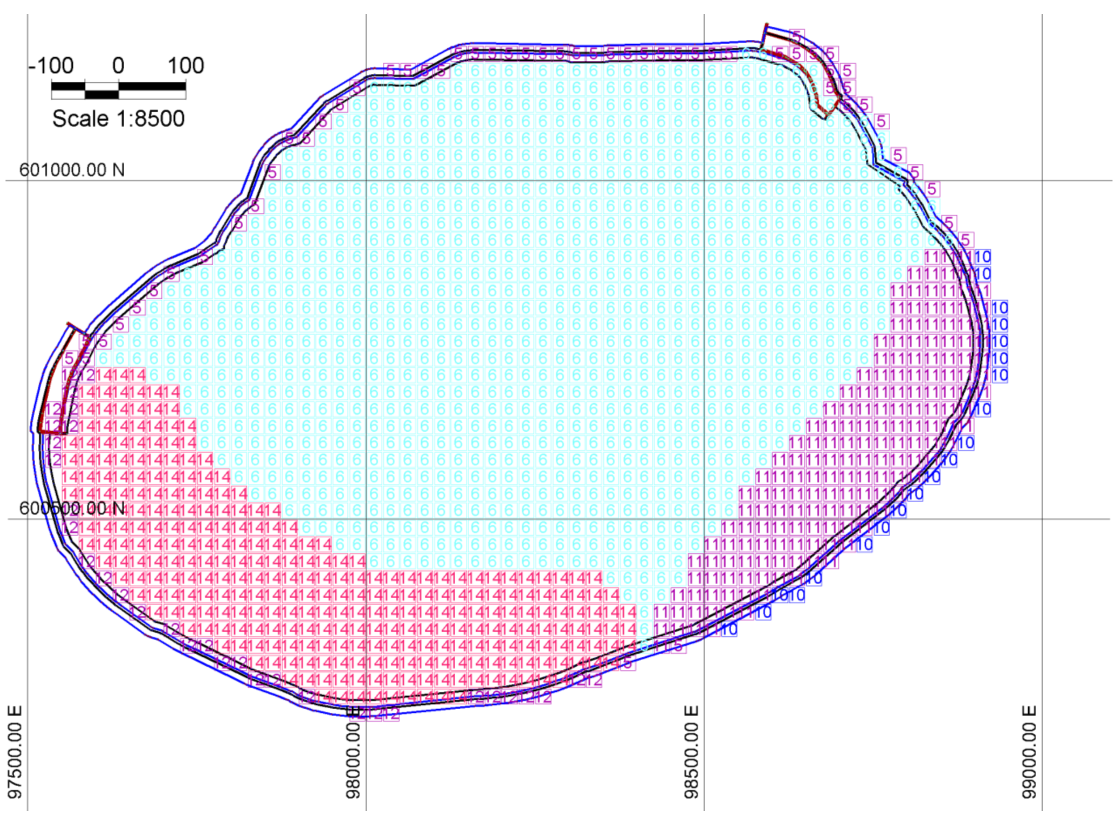

Fig. 8 Plan view plot of long-term schedule-Z=1650m.

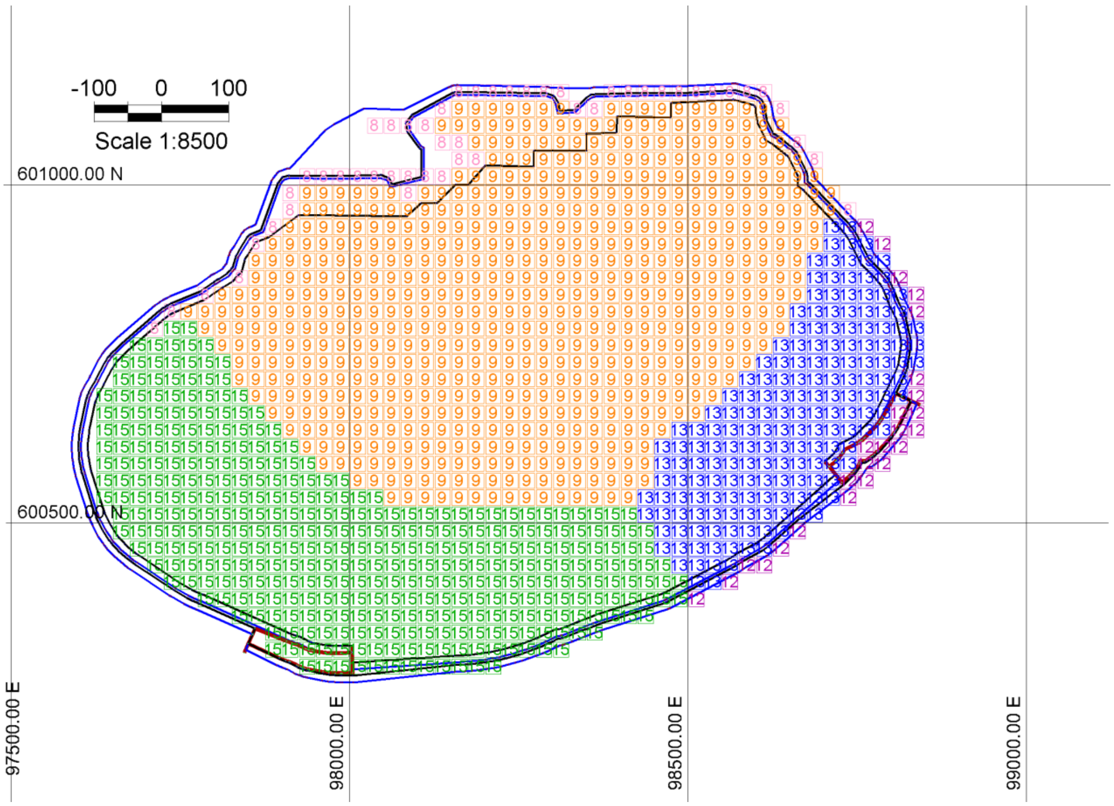

Fig. 9 Plan view plot of long-term schedule-Z=1590m.

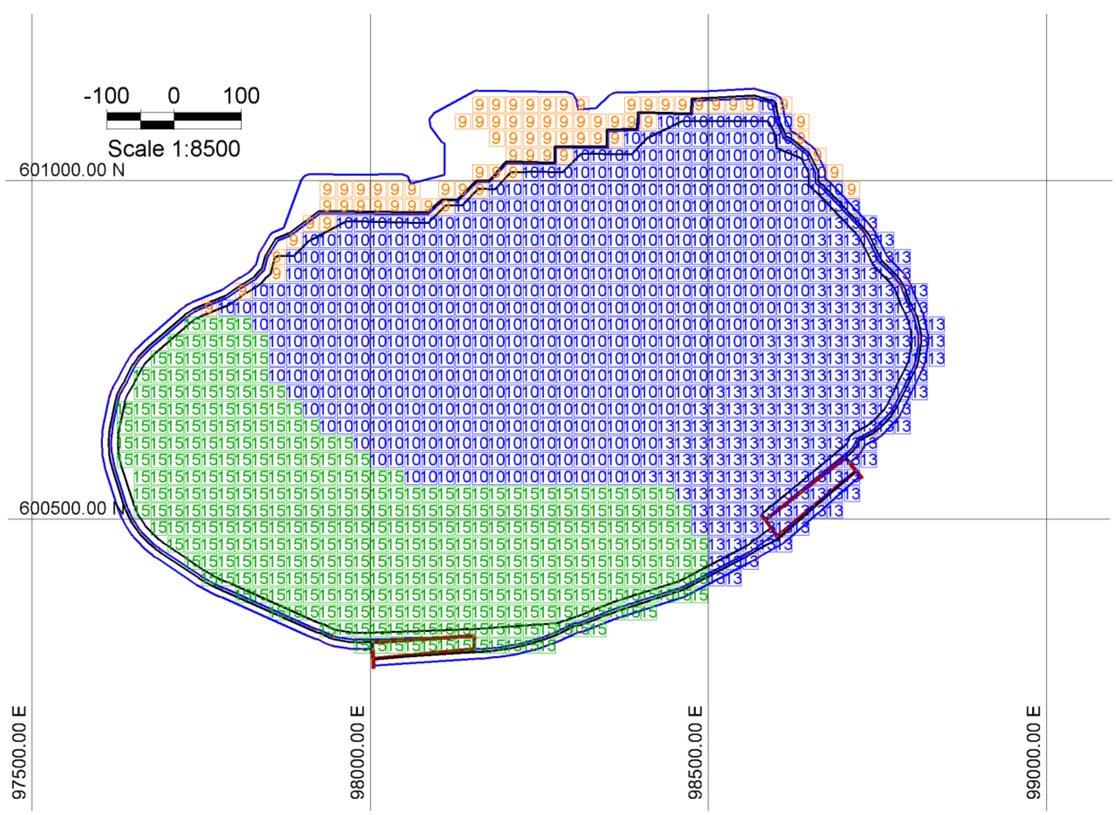

Fig. 10 Plan view plot of long-term schedule-Z=1575m.

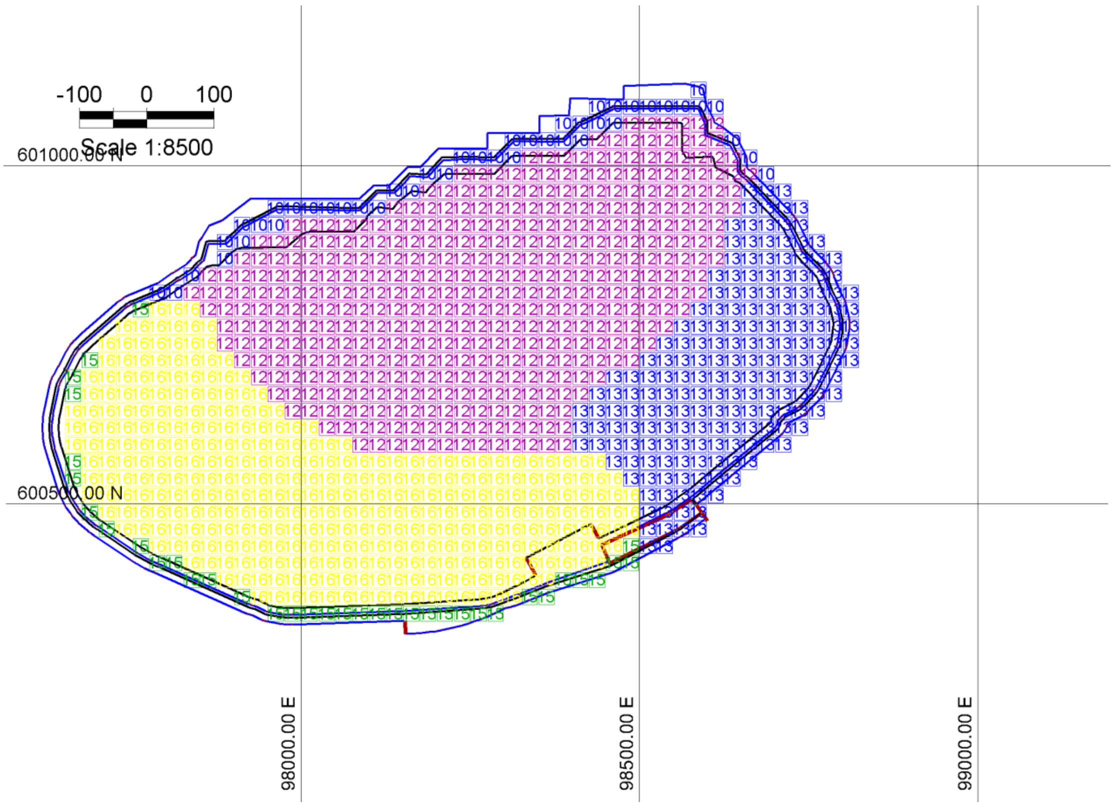

Fig. 11 Plan view plot of long-term schedule-Z=1560m.

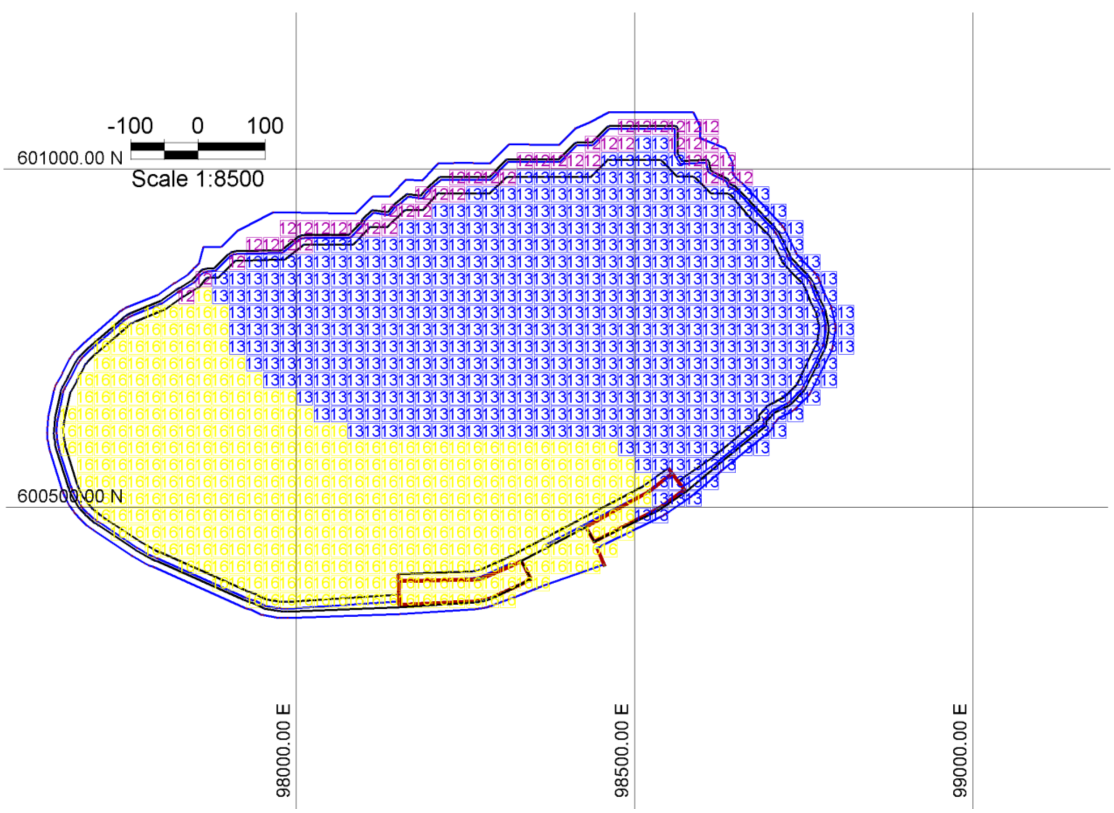

Fig. 12 Plan view plot of long-term schedule-Z=1545m.

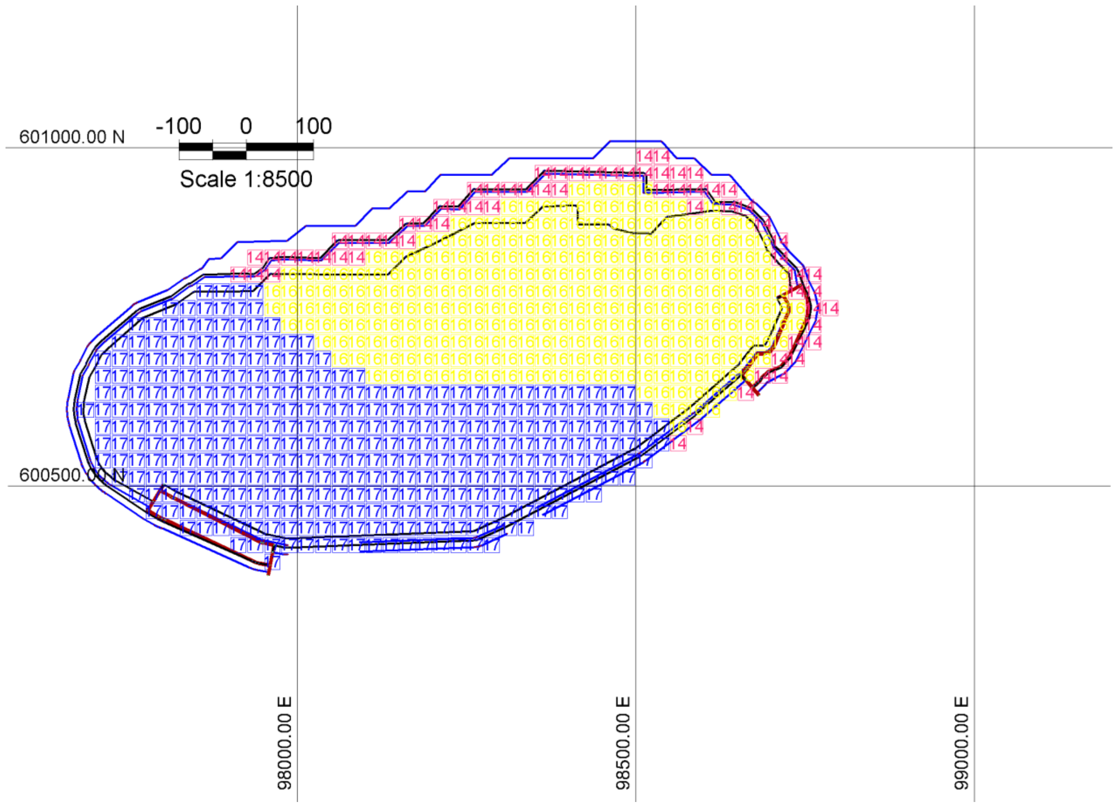

Fig. 13 Plan view plot of long-term schedule-Z=1515m.

Table 1 and 2 indicate the general information about the domain of medium-term scheduling in years 10 and 16, respectively. In addition, Table 3 shows the values of unit costs (waste rehabilitation, processing, mining, rehandling, and haulage) considered in the medium-term scheduling model. In Table 3, SP and P indicate stockpile and process, respectively.

Table 1 General specifications of the domain considered in year 10

Number of periods	12 months
Total number of blocks	2464
Total number of cuts	142
Block size	25×25×15 (m^3)
Total rock tonnage	25 million tonnes
Total mineral tonnage	8 million tonnes

Table 2 General specifications of domain considered in year 16

Number of periods	12 months
Total number of blocks	1296
Total number of cuts	72
Block size	25×25×15 (m^3)
Total rock tonnage	25 million tonnes
Total mineral tonnage	8 million tonnes

Table 3 Values of different unit costs

Unit Processing Cost ($.tonne^{-1})		Unit Rehandling Cost ($.tonne^{-1})		Unit Waste Rehabilitation Cost ($.tonne^{-1})		Unit Mining Cost ($.tonne^{-1})	Unit Haulage Cost ($.tonne^{-1}.m^{-1})
P1	P2	SP1to p1	SP2 to p2	W1	W2	1	0.001
5.5	5.75	0.5	0.25	1.75	2		

Tables 4 and 5 show process 1 and 2's features that are involved in the medium-term mine production scheduling model in years 10 and 16, respectively. These features include the values of lower and upper bound on ore tonnage and grade fed into them. In addition, Table 5 and 6 indicate different specifications of stockpiles such as the values of lower and upper bound on ore tonnage and grade fed into them and the values of their output grade in years 10 and 16, respectively.

Table 4 Specifications of processes for medium-term planning in year 10

Process	**Lower Grade (%)**			**Upper Grade (%)**			**Capacity (Million Tonnes)**	
	MWT	S	P	MWT	S	P	Min	Max
Process 1	73	0	0	83	2	0.18	0.31	0.35
Process 2	60	0	0	73	1.8	0.2	0.32	0.35

Table 5 Specifications of processes for medium-term planning in year 16

Process	**Lower Grade (%)**			**Upper Grade (%)**			**Capacity (Million Tonnes)**	
	MWT	S	P	MWT	S	P	Min	Max
Process 1	73	0	0	83	2	0.18	0.35	0.35
Process 2	60	0	0	73	1.8	0.2	0.33	0.35

Table 6 Specifications of stockpiles for medium-term planning in year 10

Stockpile	**Lower Grade (%)**			**Upper Grade (%)**			**Output Grade (%)**			**Initial Inventory (Million Tonnes)**
	MWT	S	P	MWT	S	P	MWT	S	P	
Stockpile 1	75	0	0	85	2	0.16	80	1.5	0.15	0.1
Stockpile 2	64	0	0	78	2.5	0.18	69	1	0.17	0

Table 7 Specifications of stockpiles for medium-term planning in year 16

Stockpile	**Lower Grade (%)**			**Upper Grade (%)**			**Output Grade (%)**			**Initial Inventory (Million Tonnes)**
	MWT	S	P	MWT	S	P	MWT	S	P	
Stockpile 1	75	0	0	80	2	0.16	80	1.5	0.15	0.2
Stockpile 2	64	0	0	75	2.5	0.18	69	1	0.17	0.1

Figs. 14 to 21 show the plan view plots of the resulting medium-term schedule in year 10. In addition, Figs. 22 to 25 show the plan view plots of medium-term schedule for year 16. The numbers inside these figures represents the month that the maximum portion of the block is extracted within year 10. For example, in Fig. 14, value of 1 indicates that green blocks are at most extracted in the first month of year 10. Also, numbers 2 in Fig. 22 present that blue blocks are at most extracted in the second month of year 16. Fig. 26 and Fig. 27 show the medium-term mine production schedules in years 10 and 16, respectively. These figures present following information about the production:

- Amount of rock tonnage which is extracted in each month of the considered year
- Amount of waste tonnage which is fed into waste dump in each month of the considered year
- Amount of ore tonnage which is fed into processes in each month of the considered year
- Amount of ore tonnage which is fed into stockpiles in each month of the considered year

As well as above information, the optimal values of operational costs resulted from medium-term scheduling are as follows:

- 137 million dollars for year 10
- 157 million dollars for year 16

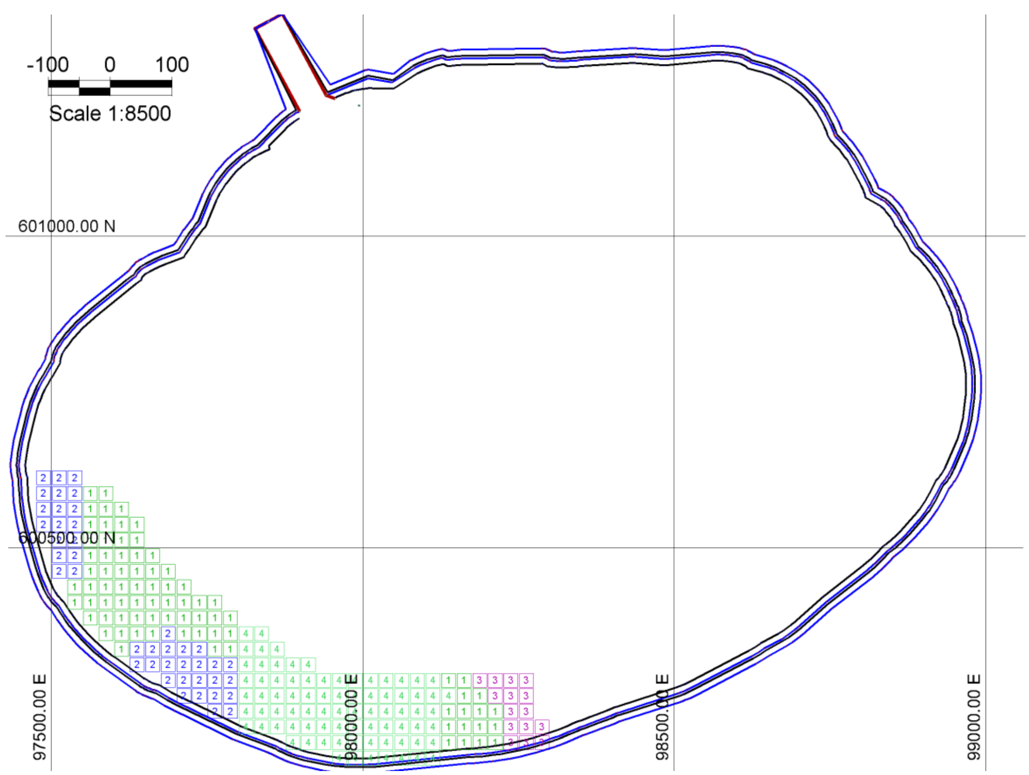

Fig. 14 Plan view plot of medium-term schedule-Year 10 (Z=1725m)

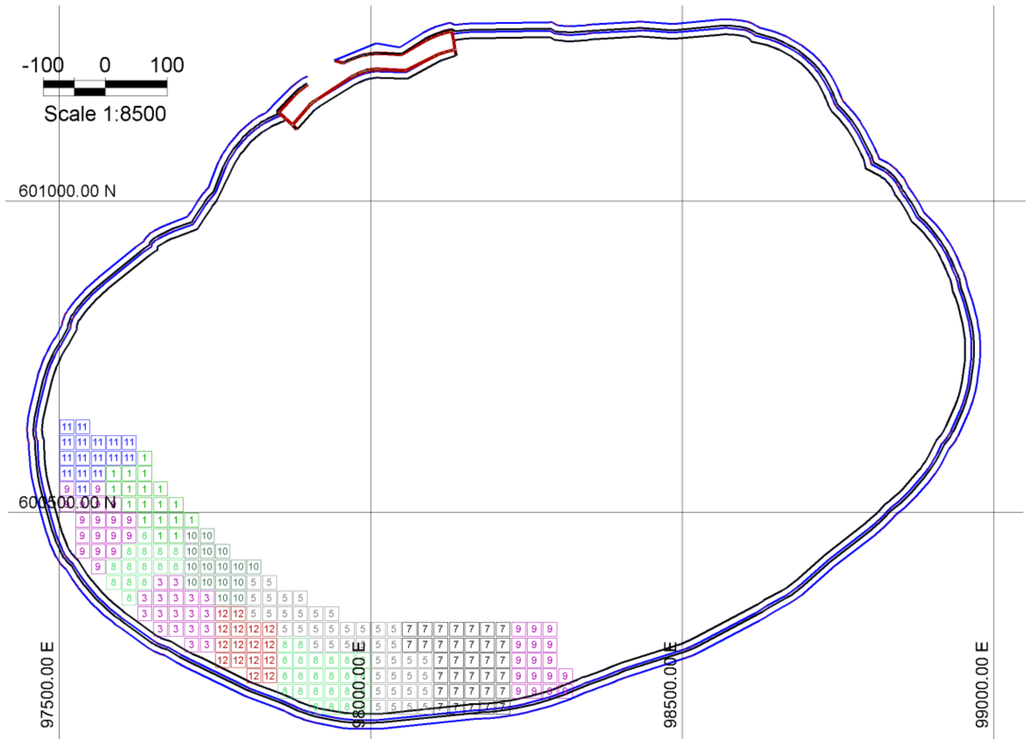

Fig. 15 Plan view plot of medium-term schedule-Year 10 (Z=1710m)

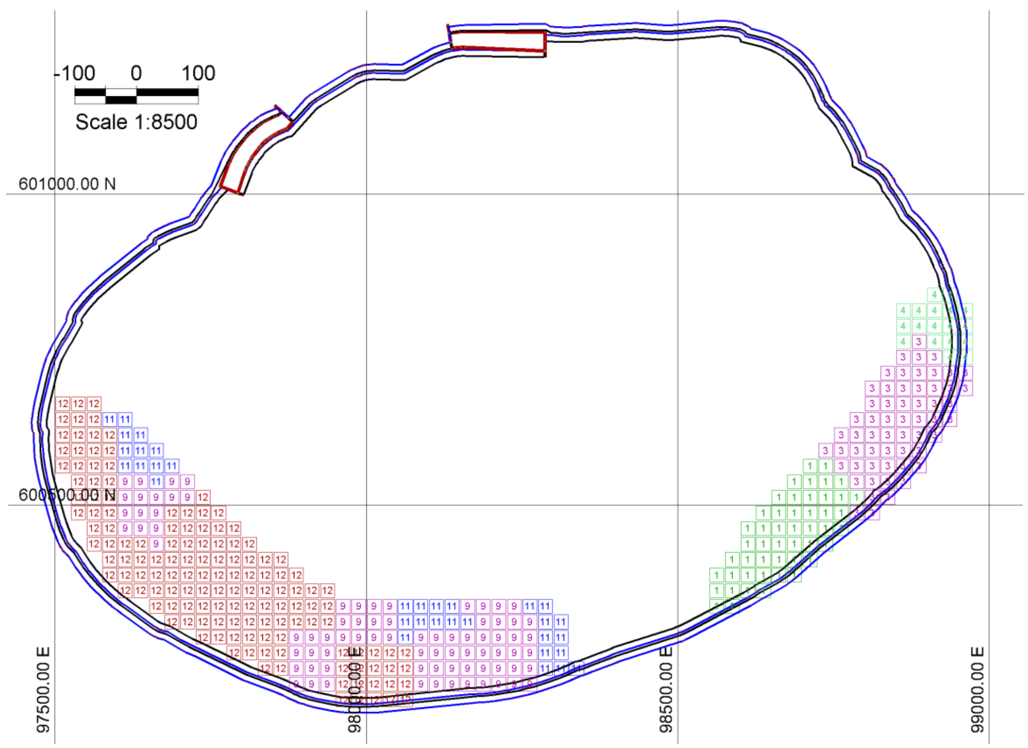

Fig. 16 Plan view plot of medium-term schedule-Year 10 (Z=1695m).

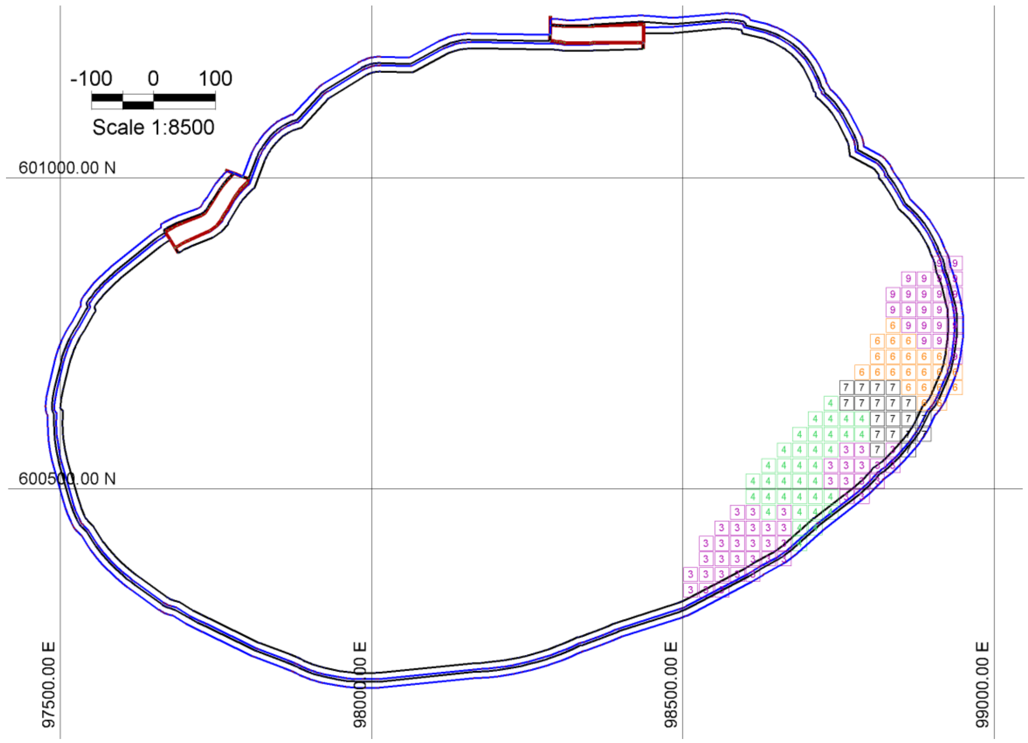

Fig. 17 Plan view plot of medium-term schedule-Year 10 (Z=1680m)

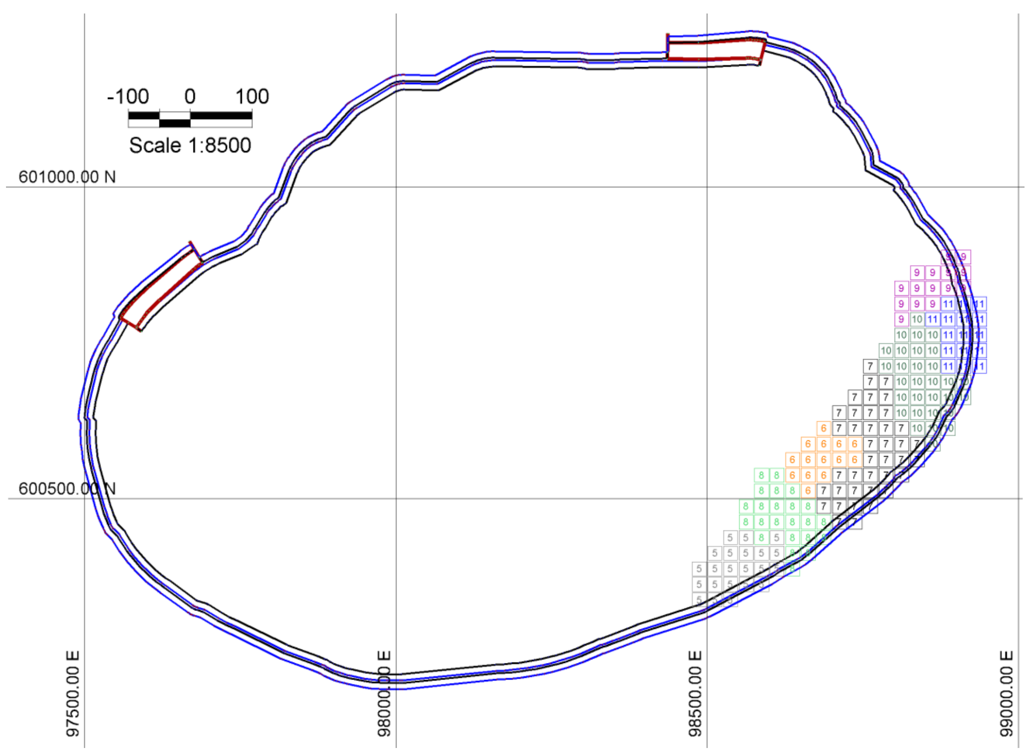

Fig. 18 Plan view plot of medium-term schedule-Year 10 (Z=1665m)

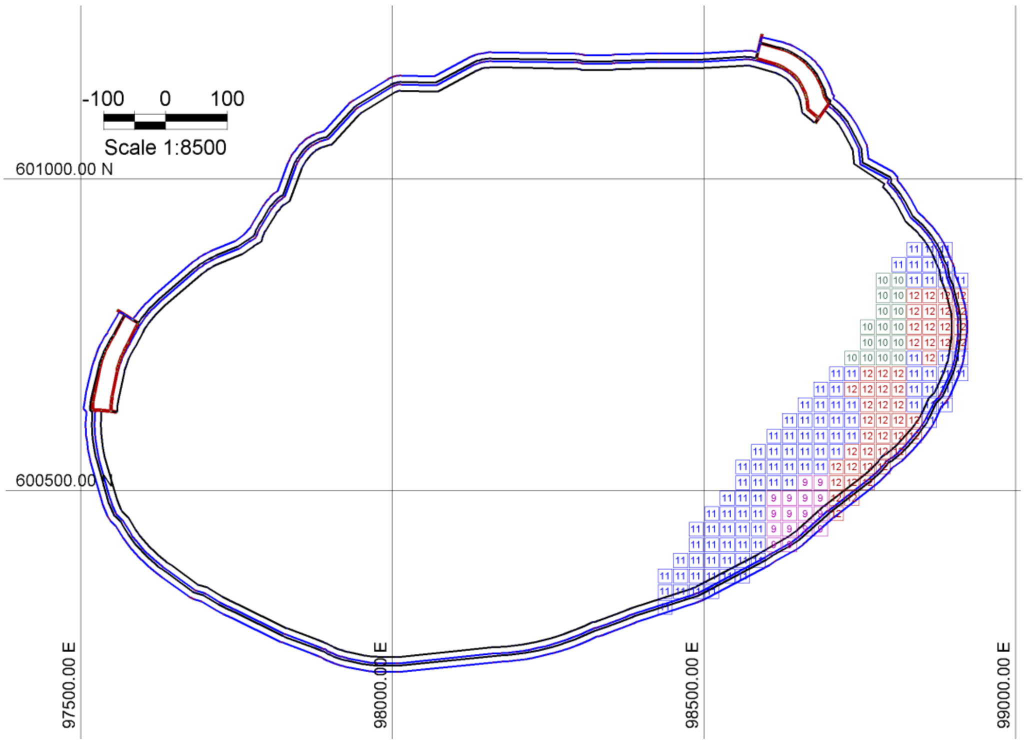

Fig. 19 Plan view plot of medium-term schedule-year 10 (Z=1650m)

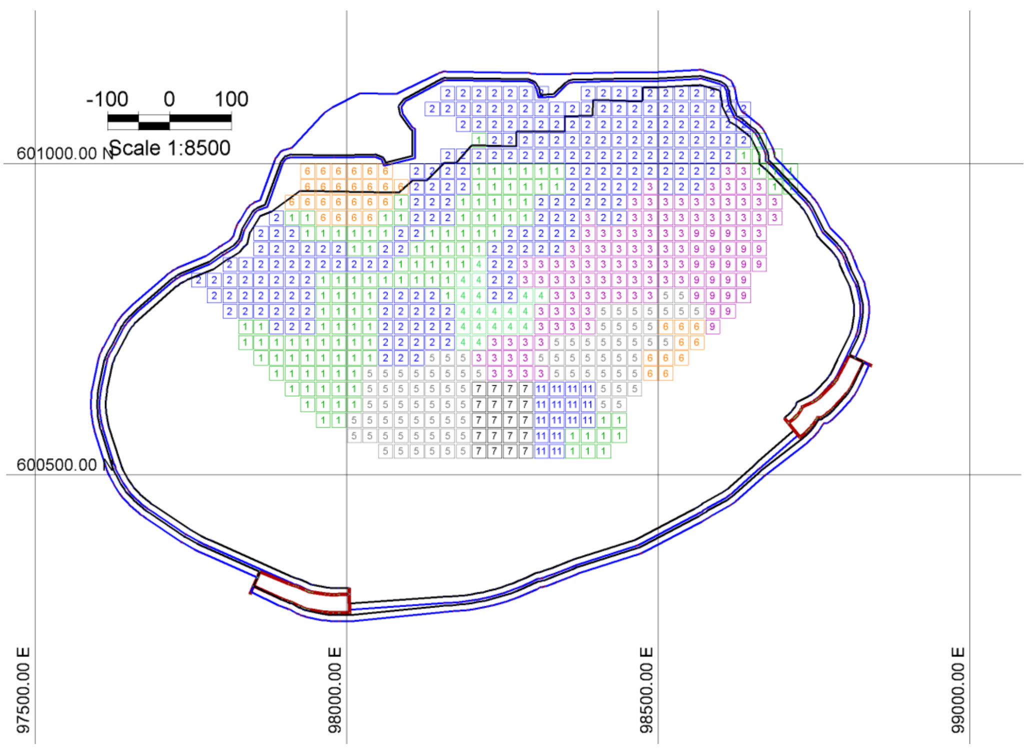

Fig. 20 Plan view plot of medium-term schedule-Year 10 (Z=1590m)

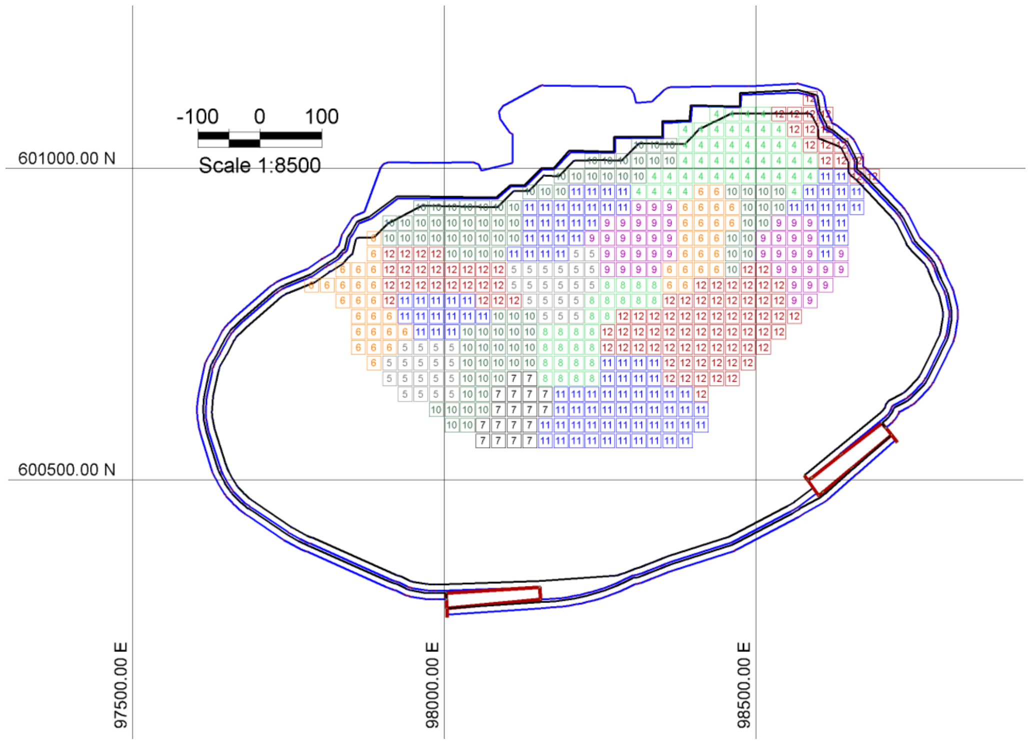

Fig. 21 Plan view plot of medium-term schedule-Year 10 (Z=1575m)

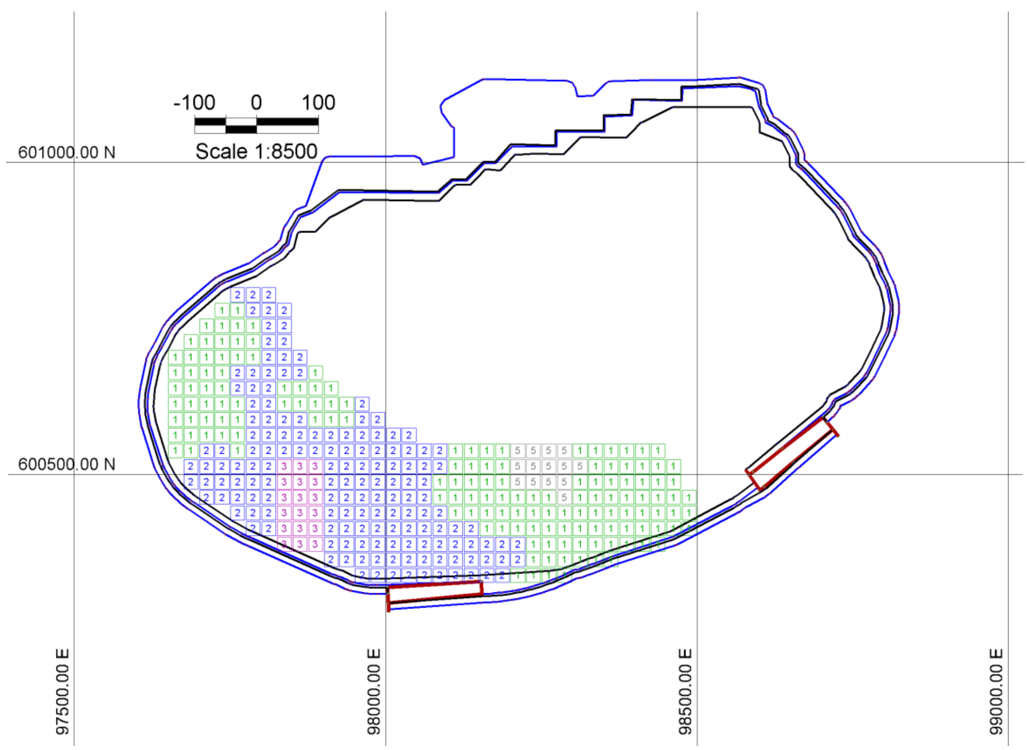

Fig. 22 Plan view plot of medium-term schedule-year 16 (Z=1575m)

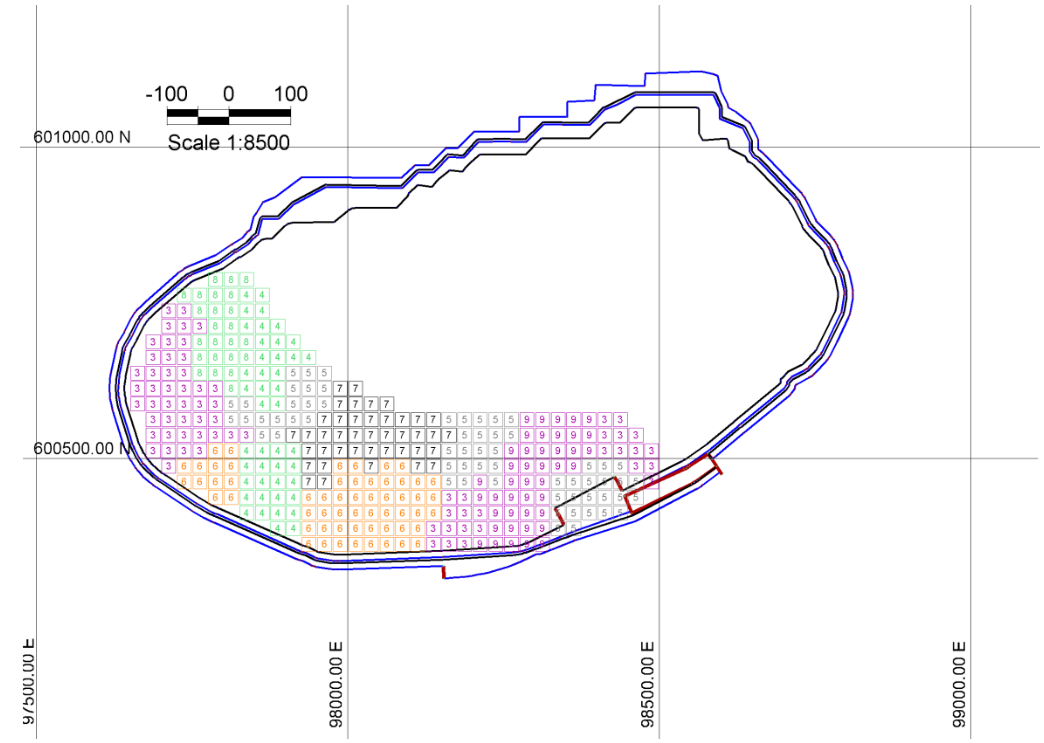

Fig. 23 Plan view plot of medium-term schedule-Year 16 (Z=1560m)

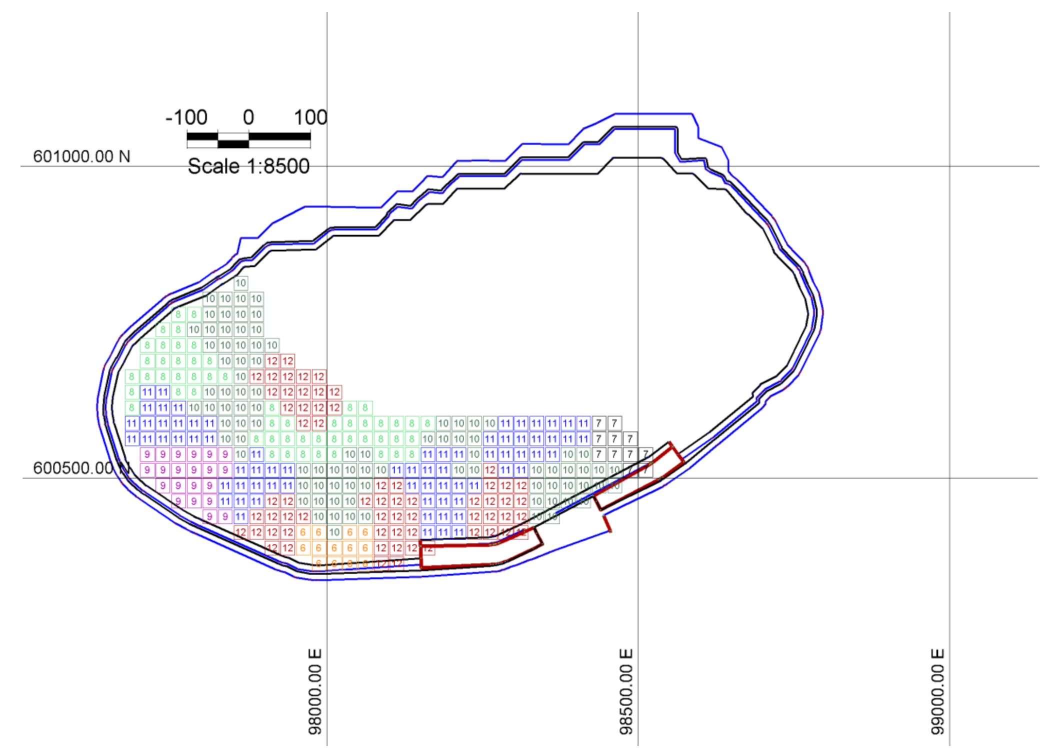

Fig. 24 Plan view plot of medium-term schedule-Year 16 (Z=1545m)

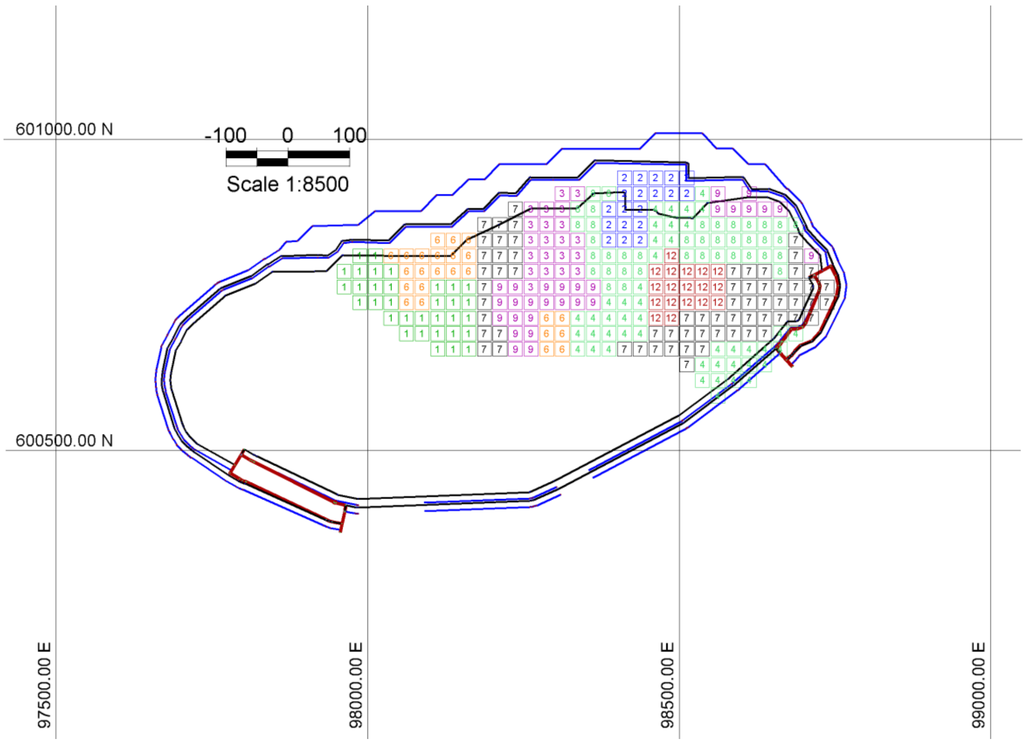

Fig. 25 Plan view plot of medium-term schedule-Year 16 (Z=1515m)

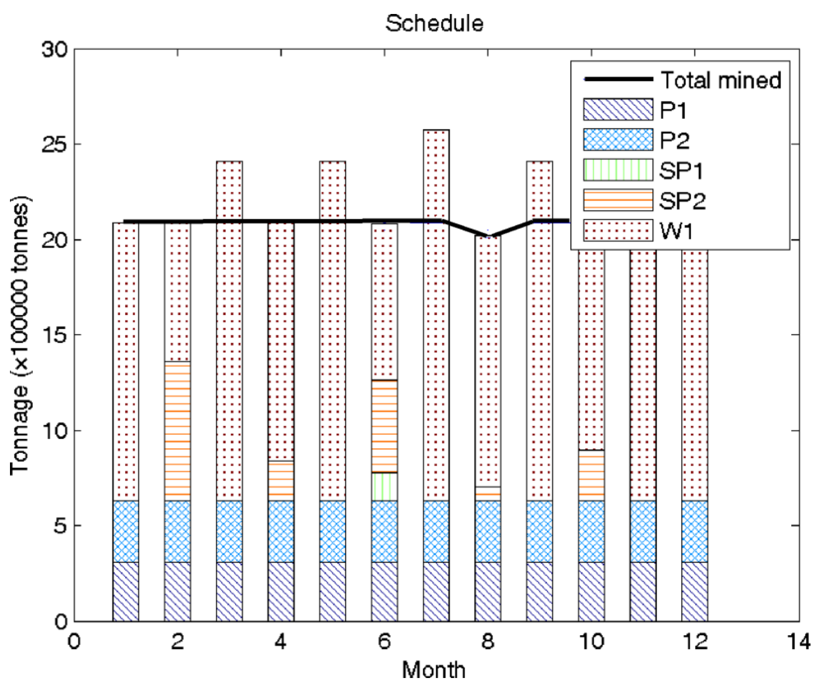

Fig. 26 Medium-term mine production schedule-Year 10

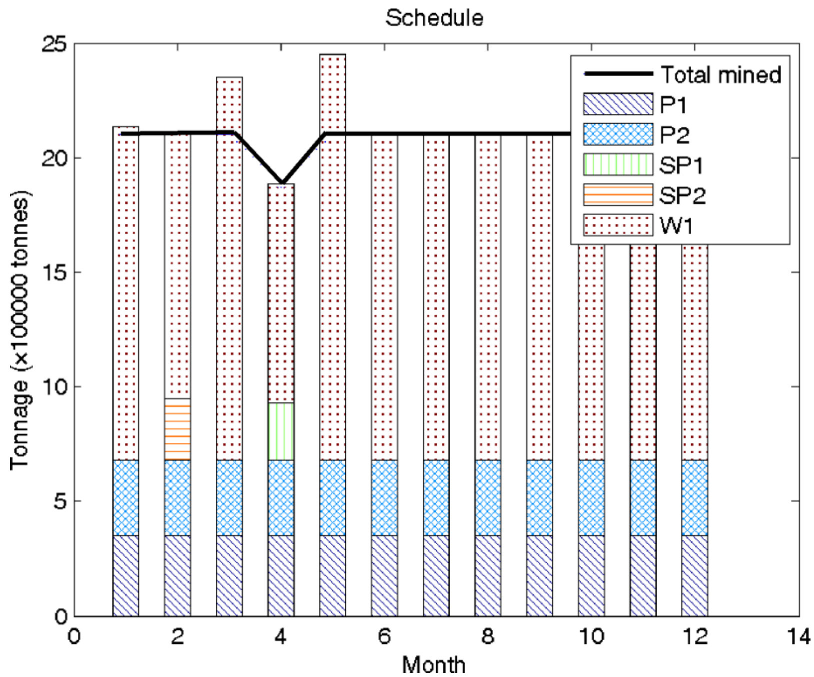

Fig. 27 Medium-term mine production schedule-Year 16

5. Conclusions

In this paper, an open pit mine production scheduling model including long-term and medium-term was applied to a dataset on an iron ore mine. Both of the production schedule models were mixed integer linear programming. The long-term scheduling model tried to maximize the discounted cash flow in the life of mine while the medium-term production plan minimized the total operational costs. Since there were many blocks, solving the models in this large size could take so long time. Thus, blocks were aggregated into a number of cuts and then solved by TOMLAB/CPLEX. After applying the long-term mine production scheduling model, two years 10 and 16 were selected to apply the medium-term mine production scheduling. For each year, the medium-term mine production scheduling model was applied. The results of scheduling were presented in terms of plan view plots of schedule and mine production schedule plots. The plan view plots of schedule showed the plan view plots of the considered domain of planning in which the month that each bock is extracted was shown. The medium-term mine production schedule indicated that in each month of the considered year, what amount of material is extracted and sent to different destinations (stockpiles, processes, and waste dump).

6. References

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7. Appendix

MATLAB and CPLEX Code Documentation

Stochastic Long-Term Mine Production Scheduling

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Abstract

Mining is one of capital-intensive industries with high exploration, exploitation, and processing operating costs in order of hundreds of millions of dollars. Thus, mining projects should be planned with care to avoid possible losses. Generally, mine production planning of open pit mines cannot lead to a perfect plan in terms of mine management's goals if it does not consider uncertainties present in the mining operations. In this paper, a long-term mine production scheduling model is proposed to deal with the grade uncertainty present in production planning. the proposed model is applied regarding a number of realizations and generates a final schedule and profit as the most probable schedule and profit, respectively.

1. Introduction

Long-term planning of mining projects is a critically important part of open pit mining ventures and deals with the efficient management of cash flows in the order of hundreds of millions of dollars (Leite and Dimitrakopoulos, 2007). The main traditional objective of long-term mine scheduling is the maximization of cash flow or the net present value (NPV). However, various factors of risk in mining operations create uncertainties in the schedule and may lead to its failure. Two major concerns of investors and shareholders are the payback period and the risk associated with it. Uncertainties will endanger the mine schedule and finally the forecasted cash flows. Thus, consideration of risk in the schedule will show a more realistic plan to the mine's managers and investors. Uncertainties in the mining projects often originate from sources such as price volatility, geological uncertainty, economical instability, and different uncertainties in availability and utilization of mining and processing facilities. Each factor of risk can influence positively or negatively on the cash flow of a mining project. To protect the cash flow from the fluctuations, incorporating uncertainties into mine planning can be an effective approach (Leite and Dimitrakopoulos, 2007). Primarily, two types of risks are considered in the problem: economical and geological. Economical uncertainties are related to prediction of commodities' prices. On the other hand, geological uncertainties mainly include uncertainty in estimation of grade and ore tonnage. Controlling these types of uncertainty could decrease the risk of mining projects.

In this paper, an approach is proposed for long-term open pit mine production planning in the presence of uncertainties. The main source of uncertainty is the uncertainty in the estimation of grade of elements. Therefore, how to sequence the extraction of mining blocks inside an open pit mine to maximize the cash flow while minimizing the risk and cash flow fluctuation is the main problem.

The rest of this paper is organized as follows: section 2 reviews the literature regarding the problem of long-term planning with uncertainty issues. In section 3, objectives of research are briefly presented. Also, the theoretical framework of proposed research is reflected in section 4. This section includes the mathematical model and the theoretical framework. A case study is considered in section 5 and how the proposed framework is applied is highlighted in this section. Results are discussed with tables and figures to show the practicality of the proposed framework better. Finally, the future work that could be performed related the proposed framework is explained in section 6.

2. Literature review

Mine production scheduling in the presence of uncertainty has been the focus of attention from researchers and practitioners in recent years. Leite and Dimitrakopoulos (2007) provide a framework for life of mine (LOM) production scheduling based on simulated annealing and stochastic orebody representations. The authors take into account geological uncertainty on the tonnage of ore deposit and grade in their proposed mine production scheduling framework. Their suggested mine production scheduling algorithm produces a final schedule in the presence of geological uncertainties to minimize the risk of not meeting production targets and maximize the net present value (NPV). This algorithm includes the following steps:

- Designing the pit limit and mining rates: to define pit limits, conventional methods such as Lerchs-Grossman algorithm are used. Also, mining rates are defined by Milawa scheduler or by geometric constraints and mill demands. Mining rates and pit limit set in this step will be used in the next steps.
- Generating a number of schedules respecting to pit limit: a set of schedules is generated regarding the pit limit and production targets (mining rates) by conventional schedulers and simulated ore bodies. These schedules are produced based on distinct equally probable spatial distribution of grade of ore body within the deposit while honoring different constraints such as mining precedence and production target. The produced set of schedules is used for forming a cumulative distribution indicating the probability that a typical block should be mined within the time horizon (mine's life).
- Selecting an optimal schedule by implementing simulated annealing: simulated annealing approach is used to combine different schedules from step 2 to get an optimal schedule regarding the corresponding objectives. The objective of applied simulated annealing is the minimization of deviation from the production target during the life of mine over all available representations of the deposit.

The algorithm was implemented on a disseminated copper deposit. The final schedule obtained by the framework shows 26% improvement in the NPV and low chance of significant deviations from the production targets comparing to the conventional models of mine production scheduling. The proposed algorithm considers predefined pit limit, pushback, and cut-off grades. Thus, involvement of dynamic cut-off grade and uncertainties in the pit limit and pushback design would be of high value.

Halatchev and Lever (2005) propose a risk model for long-term production planning of an open pit Gold mining. Most of works performed on the uncertainty and risk's factors of mining project have just taken into consideration the geological and price uncertainty. However, they consider four types of risk as economic, technological, technical, and geological parameters. The risk model is based on the cash flow of mining project through the mine's life and tries to formulate the effect of different risk factors on the cash flow by implementing the Monte Carlo simulation approach. The focus of the proposed risk model is on two well-known indicators: payback period and NPV. The proposed risk model minimizes the risk of deviation of cash flow from the planned cash flow.

Dimitrakopoulos and Ramazan (2008) work on providing a stochastic integer programming for optimization of long-term mine production schedule. The authors propose a stochastic integer programming model considering grade and quality of ore uncertainties. The authors review three main stochastic integer programming algorithms (anticipative models, adaptive models, and anticipation/adaptive-resource models). Then, authors explained their proposed model to maximize the NPV and minimize the mining project risk profile. In their model, a number of equal probable representations of ore body are considered and the model aims to minimize the deviation from the production targets. It is noteworthy that the proposed model is primarily on the basis of anticipation/adaptive-resource models. Dimitrakopoulos and Ramazan (2008) implemented their model on a copper-gold deposit. The results show better NPV and less deviation from the target production. The model does not take into account other sources of uncertainties such as mining equipment availability. Also, one of the main assumptions in their model is that pit limit is known. Integration of pit limit design and mine scheduling would be interesting.

As one of the first papers on considering geological uncertainty into long-term mine planning, Dimitrakopoulos et al. (2002) present a simulation based optimization model to generate a number of realizations of geological uncertainties such as grade and tonnage. The authors firstly apply conditional Gaussian simulation to produce realizations. Then, based on each realization, scheduling of block extraction is performed to reach the maximum NPV. A distribution for NPV is obtained which with a confidence interval for value of cash back can be presented to mine managers and investors. The approach is applied on a disseminated low-grade, epithermal, quartz breccia-type gold deposit with 50 realizations. The proposed approach does not determine what could be the best schedule. Only it indicates the distribution of NPV.

Dimitrakopoulos and Abdel Sabour (2007) introduce a framework to show the advantages of real option valuation (ROV) comparing to NPV. The authors propose a simulation-oriented real option valuation model to consider three types of uncertainties: geological uncertainty, foreign exchange uncertainty, and variability of real options (price uncertainty). The authors use geometric Brownian motion (GBM) method (Dixit and Pindyck, 1994) and mean-reverting process (MRP) proposed by Schwartz (1997) to model the variability of market and economic variables like foreign exchange and prices. To model the grade uncertainty, the authors apply sequential Gaussian simulation (SGS) method (Dimitrakopoulos and Luo, 2004). The authors use historical data and regression to estimate the parameters of the above models. A number of equal likely realizations of grade distribution by using above models are produced. The optimization process is performed on each of the realizations. In each period, managers can decide if to close the mining project or to continue. A switch cost is applied when the project is shut down. Thus, optimization is done by taking into account this switch cost and other operating cost. Dimitrakopoulos and Abdel Sabour (2007) apply the simulation-based ROV and NPV method on an Australian gold mine with 3 years. 12 mine designs are considered as the realizations. The mining valuations obtained by these methods are the expected values (average values got from applying the methods on the 12 mine designs). It should be noted, as the major element is a precious metal, GBM method is applied for model of market variables. The results of comparisons show the high variability (variance) in valuation by NPV than the proposed ROV method. Also, cash flow of simulation based ROV through 3 years is much higher than NPV. Thus, implementing simulation based ROV can cause better investment return than traditional methods like NPV. The main reason is that in simulation real option valuation some uncertainties are modeled and absorbed. However, the authors point out that the overcome of ROV on NPV is just by one case study and the generalization requires implementation of models on more case studies. It seems that the optimization process is performed without considering cut-off grade role. In fact, the proposed optimization model presented in the paper does not involve decision making on the values of cut-off. The values of cut-off influence on the economic block values. In other words, as in each realization price pattern of gold is known, consideration of cut-off variable could be of value to adjust the schedule.

Zhang et al. (2007) present a reactive approach for mining project assessment. The proposed reactive model attempts to model the decision making about the mining schedule in each period when moving forward in time, while incorporating new information. The authors first introduce an

MILP model which seeks to maximize the discounted cash flow while optimizing the cut-off grade. Then, they model the price uncertainty by use of log-normal mean reverting model to generate price patterns through time. As time passes by, the pattern of price is updated by use of log-normal mean reverting model in each period. In the reactive model, the model calculates the expected value of NPV. Also, a model of optimization based on the conditional expectation of future price is presented. This conditional expectation does not update the price pattern. In fact, it uses only a static pattern based on the historical data which are on-hand at the first period. In the end, the authors present a model based on perfect knowledge of future price. The authors show that the NPV obtained by perfect knowledge is more than the NPV by reactive method and conditional model. The reason is clear because in the perfect knowledge model the future price is completely known. Thus, the scheduling is carried out based on certain information while in the reactive and conditional based model, the pattern of price are estimated. The authors implement three models on a case study with 16000 blocks with 25 realizations (patterns). The results show the superiority of reactive approach proposed by in the paper on conditional-based model of future price.

Ramazan and Dimitrakopoulos (2007) introduce a stochastic integer programming formulation to tackle production scheduling problem with the ore body uncertainty. The mathematical formulation aims at maximizing the NPV while minimizing the deviations from the production target (in ore tones, grade, and quality). The authors aggregated these goals as an objective function to maximize the net profit in term of the total expected NPV minus the cost of missing the production targets. To impose the grade uncertainty, a number of equally probable realizations of ore body are generated. The authors implement their formulation on a synthetic two-dimensional data set. To quantify the uncertainty within each schedule, three measures are used:

- Average deviations from the production targets
- Average of non-zero deviation from the production targets
- Probability to deviate from the production targets

Based on the objective function values and the three above-mentioned measures, a schedule is picked up as the best. The paper does not show how to select the best schedule. It seems that applying different realizations of geology uncertainty, a distribution for objective function is obtained. Also, as we do not know what realization is the real case, selecting based on the introduced measures appears to be impractical.

Boland et al (2008) present a multistage stochastic programming model for the problem of long-term mine production scheduling. The authors consider geology uncertainty, including rock type and grade, as the main stochastic nature of the problem. The proposed mixed integer multistage model involves multiple estimates of geology and tries to adopt the mining and processing capacities in later periods regarding the decisions made in the earlier. The authors first re-introduce their proposed deterministic MILP model in their previous work. Then, they apply stochastic approach in two steps: 1) scenario-dependent processing decision and 2) scenario dependent mining and processing decisions, where scenarios are different realizations of uncertainty.

3. Objectives of research

As mentioned, the problem, which is considered in this research is the long-term planning of open pit mines with accounting for grade uncertainties. Grade uncertainty is one of the main sources of not meeting the production targets in mining operations. In this paper, a long-term mine production scheduling formulation in the presence of grade uncertainty is proposed. Thus, in addition to a practical production schedule, cut-off grade values are decided by the model. The following steps are completed throughout the paper:

- Grade uncertainty modeling: to model the uncertainty in estimation of grade distribution, grade values of each block is considered to have Normal distribution with mean and standard deviation obtained by sequential Gaussian simulation method.
- Calculation of economic block value (EBV) regarding the considered grade distribution.
- Mathematical modeling with involving cut-off grade value optimization to handle both cut-off and scheduling to obtain the maximum value of expected profit.

4. Theoretical framework

Following is the proposed theoretical framework:

- Modeling and generating a number of grade distribution realizations.
- Synchronization of EBV estimation and each realization of grade distribution.
- Presenting a mathematical formulation for the corresponding problem.
- Solving the mathematical formulation with each realization of grade distribution.
- Analysis of schedules obtained in step 4 and distribution of objective function (NPV).

The first three of above items are elaborated in the current section. Other two items are explained with the case study in section 5. The proposed framework has the following improvements over the previous research:

- Consideration of both schedule and cut-off optimization.
- Estimation of EBV related to each realization of grade distribution. Since, EBV primarily depends on the value of grade of elements, each realization of grade distribution leads to a different EBV. Thus, estimation of EBV itself is a problem that should be performed with long-term scheduling.
- Producing a number of schedules which can be used for estimation of NPV distribution. NPV distribution can be used as an indicator for investors. Investors can expect profit with their desired confidence by seeing the NPV distribution.

4.1. Grade distribution modeling

To take into account the grade uncertainty, a number of simulation realizations are generated; each of these realizations represents a certain distributions for grade of elements. In this research framework, SGS is implemented to produce realizations. Based on SGS, in each realization, a value for grade of elements is assigned which obeys from the Gaussian distribution with a mean and a standard deviation. In this research, a case study is used in that grade of elements is reproduced by mean and standard deviation obtained by kriging (Journel and Huijbregts, 1978).

4.2. Synchronization of EBV and grade distribution

Since EBV depends on the value of grade of elements, determination of EBV related to each realization of grade is a problem. In other words, EBV corresponding to each realization of grade distribution has to be calculated. In this paper, to estimate the values of EBV related to each realization, an artificial intelligence method called support vector machine (SVM) (Schuurman, 2010) is applied. In this method, three features are considered for estimation of EBV. These features are as follows:

- Ore tonnage
- Mining cost
- Grade

Above features directly influence the block economic value. The values of these features in each block are used to estimate the best value of EBV for the corresponding block in each realization.

4.3. Mathematical formulation

A mixed integer programming formulation is presented to model the multiple-period extraction of blocks in the long-term. The proposed model, presented by Zhang et al. (2007) model.

$$Max\ Z = \sum_{i=1}^{N}\sum_{j=1}^{cg}\sum_{t=1}^{T} EBV_{i,j,t} * u_{i,j,t} / (1+df)^t \tag{1}$$

Subject to:

$$\sum_i \sum_j R_i * u_{i,j,t} <= MU_t \qquad \forall t = 1...T \tag{2}$$

$$\sum_i \sum_j R_i * u_{i,j,t} <= MU_t \qquad \forall t = 1...T \tag{3}$$

$$\sum_i \sum_j O_{i,j} * u_{i,j,t} <= PU_t \qquad \forall t = 1...T \tag{4}$$

$$N_{PB(i)} \cdot b_{i,t} \le \sum_{\tau=1}^{t} \sum_{k \in PB(i)} \sum_{j}^{cg} u_{k,j,\tau},\ \forall i = 1...N, \forall t = 1...T \tag{5}$$

$$\sum_j u_{i,j,t} \le b_{i,t},\ \forall i = 1...N, \forall t = 1...T \tag{6}$$

$$\sum_j c_{j,t} = 1, \quad \forall t = 1...T \tag{7}$$

$$u_{i,j,t} \le c_{j,t},\ \forall i = 1...N, \forall t = 1...T, \forall j = 1...cg \tag{8}$$

$$u_{i,j,t} >= 0 \quad b_{i,t} = 0/1, \forall i = 1...N, \forall t = 1...T \tag{9}$$

Where

t : period of scheduling (t=1,…,T)

N : number of blocks

MU^t : upper bound of mining capacity used in period t

PU_i^t : upper bound of plant capacity i in period t

$O_{i,j}$: mineral zone tonnage of block I, if it is extracted by cut-off grade j

R_i : rock tonnage of block i

cg: all of cut-off grade values

df: discounting factor

$PB(i)$: set of precedent blocks of block i

$N_{PB(i)}$: number of blocks in set PB (i)

$u_{i,j,t}$: fraction of block i extracted in period t with cut-off grade j

$b_{i,t}$: binary variable, if block i is extracted in period t it gets value of 1 otherwise 0

$c_{j,t}$: binary variable, if cut-off grade j is selected in period t it gets value of 1 otherwise 0

Eq. (1) indicates the maximization objective function. This function is the summation of discounted revenue by extraction of blocks with different cut-off grade values (maximization of profit). Eq. (2) indicates that in each period, total amount of extraction has to be less than the maximum capacity for mining. Also, Eq. (3) conveys the same concept of Eq. (2) for the processing capacity. Eq. (4) implies the precedence or slope constraint in the extraction process. Based on this constraint, the extraction of each block can be started when all its above blocks have been mined completely. Eq. (5) defines the meaning of binary variable $b_{i,t}$. Eq. (6) indicates that in each period only one cut-off grade can be imposed. Eq. (7) states the definition of binary variable for cut-off grade choosing in each period. Based on this constraint, whenever a cut-off grade value is chosen for a period, no fraction of blocks can be extracted by that specific cut-off grade in that period. At last, Eq. (8) indicates the sign constraint.

5. Case study and discussion of results

To show the practicality of the proposed framework, model #2 of Askari-Nasab and Awuah-Offei (2009) is used as the mathematical model. The mathematical formulation is run in TOMLAB/CPLEX environment (Holmstrom, 2009) with realizations of grade distributions. In this section, an illustrative example of long-term planning of Gol-E-Gohar iron ore mine in south of Iran is presented to validate the proposed framework. The main element of interest in the deposit is Iron (Fe). The process employs magnetic separators; therefore, the main criterion in selecting ore to be sent to the concentrator is the magnetic weight recovery (percent MWT) of iron ore. The contaminants present are phosphor and sulfur that are considered as secondary elements to be controlled. The open pit has 20 benches which only blocks of benches 14, 15, 16, and 17 with 3089 blocks are used for the purpose of long-term planning over 10 years. Blocks are clustered into 150 mining-cuts using fuzzy C-means method. For more information on clustering algorithms for block aggregation see Askari-Nasab (2010).

Fig. 1 shows the plan view of four benches, 14 to 17. Table 1 represents the general information of the problem. Based on this table, tonnage of rock and ore inside the four benches are around 94 and 47 million tonnes which should be extracted and processed within 10 years. The number of cuts in the benches 14 to 17 are 45, 40, 35, and 30.

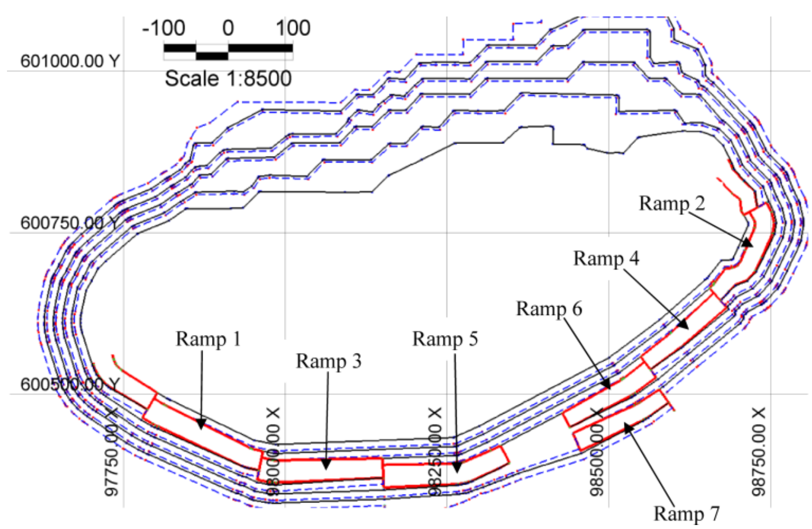

Fig. 1. Plan view of benches scheduled over a 10-year time horizon (Eivazy and Askari-Nasab, 2010)

For simplicity, only one process (process #1) is considered for processing the ore extracted from the mine. Table 2 indicates the acceptable grade range for different elements and the processing capacity. The acceptable grade ranges and processing capacity are the same over the 10 years of scheduling.

Table 1 General information of problem

	BENCH NUMBER			
	17	16	15	14
Number of blocks	614	726	820	929
Number of Cuts	30	35	40	45
Number of periods	10 years			
Total number of blocks	3089			
Total number of cuts	150			
Block size	25×25×15 (m^3)			
Total rock tonnage	94 million tonnes			
Total mineral tonnage	47 million tonnes			

Table 2 Processes' main features.

PROCESS	LOWER GRADE (%)			UPPER GRADE (%)			CAPACITY (MILLION TONNES)	
	MWT	S	P	MWT	S	P	Min	Max
Process #1	0	0	0	90	0.02	0.01	0	4.8

Fig. 2 sketches the rock code of blocks in bench 17. Mineralized zone is indicated by rock code 101. All other rock codes are waste.

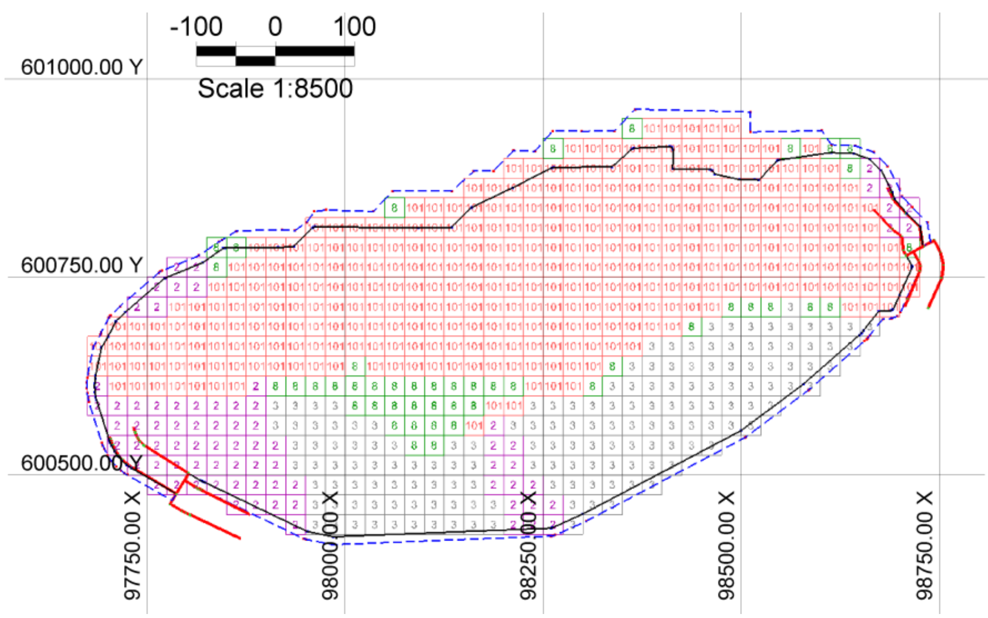

Fig. 2. Plan view of bench 17 showing waste and mineralized zone (Eivazy and Askari-Nasab, 2010)

After producing 100 realizations for grade distributions, the mathematical model is solved with each realization and the best schedule is obtained. Fig. 3, Fig. 5, Fig. 7, and Fig. 9 show the plan view of levels 14 to 17 based on the optimal production schedule with the first realization. Also, Fig. 4, Fig. 6, Fig. 8, and Fig. 10 illustrate the plan view of levels 14 to 17 based on the optimal production schedule with the realization #100. The numbers inside each block represent the year that maximum portion is extracted from that block. For instance, 10 placed in a block refers to that most of rock tonnage of block is extracted in year 10. It can be seen that each realization of grade distribution would result in an optimal schedule. Thus, with 100 realizations, we have 100 best production schedules. In each of schedules, it is determined that each block should be extracted at most in what period. For example, block # 1 can be extracted in years 8, 9, and 10 in different realizations. To aggregate the 100 optimal schedules, a rough schedule could be inferred from these 100 schedules. Fig. 11 to Fig. 14 show the rough schedule obtained by average of 100 schedules for benches 14 to 17. Each number inside each block in these figures shows the year that maximum fraction of that block is going to be extracted in that year in most of years. In other words, based on 100 schedules we know that one block at most is extracted in what periods. Thus, the most repeated period is selected as the year that block is better to be extracted.

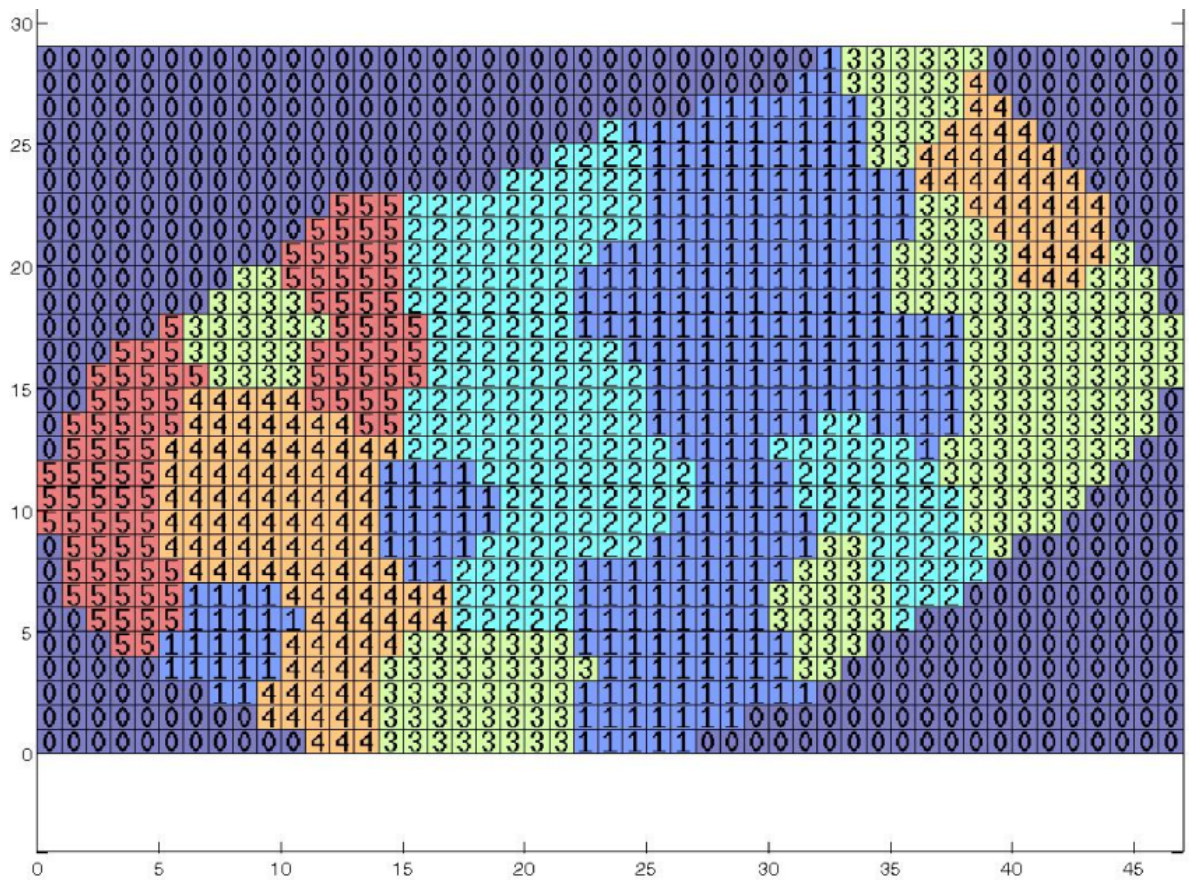

Fig. 3. Plan view f bench 14-realziation #1 (each block: 25m×25m)

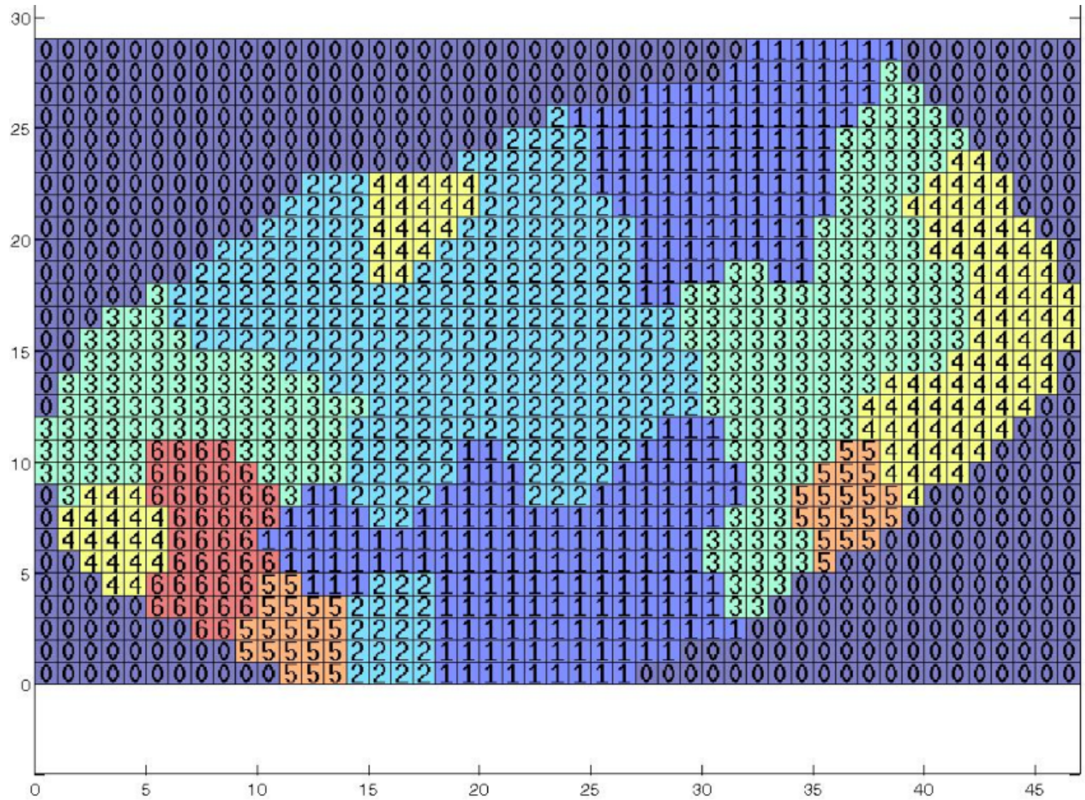

Fig. 4. Plan view of bench 14-realization #100 (each block: 25m×25m)

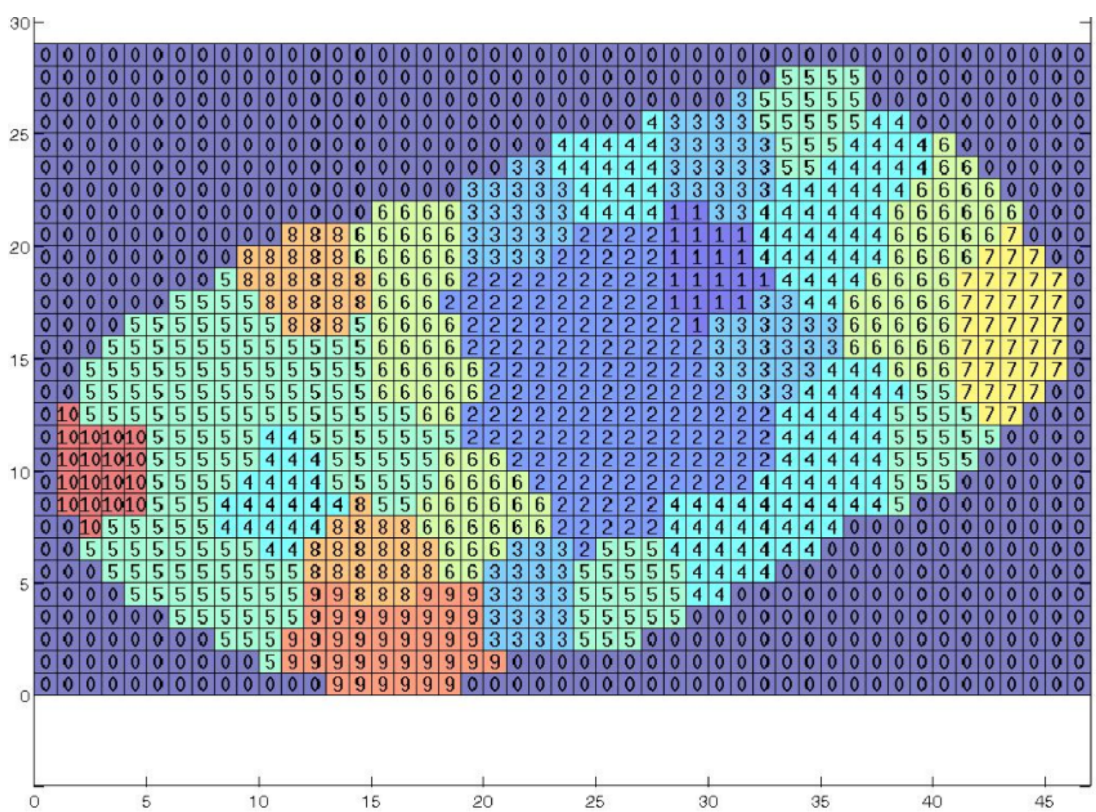

Fig. 5. Plan view of bench 15-realization #1 (each block: 25m×25m)

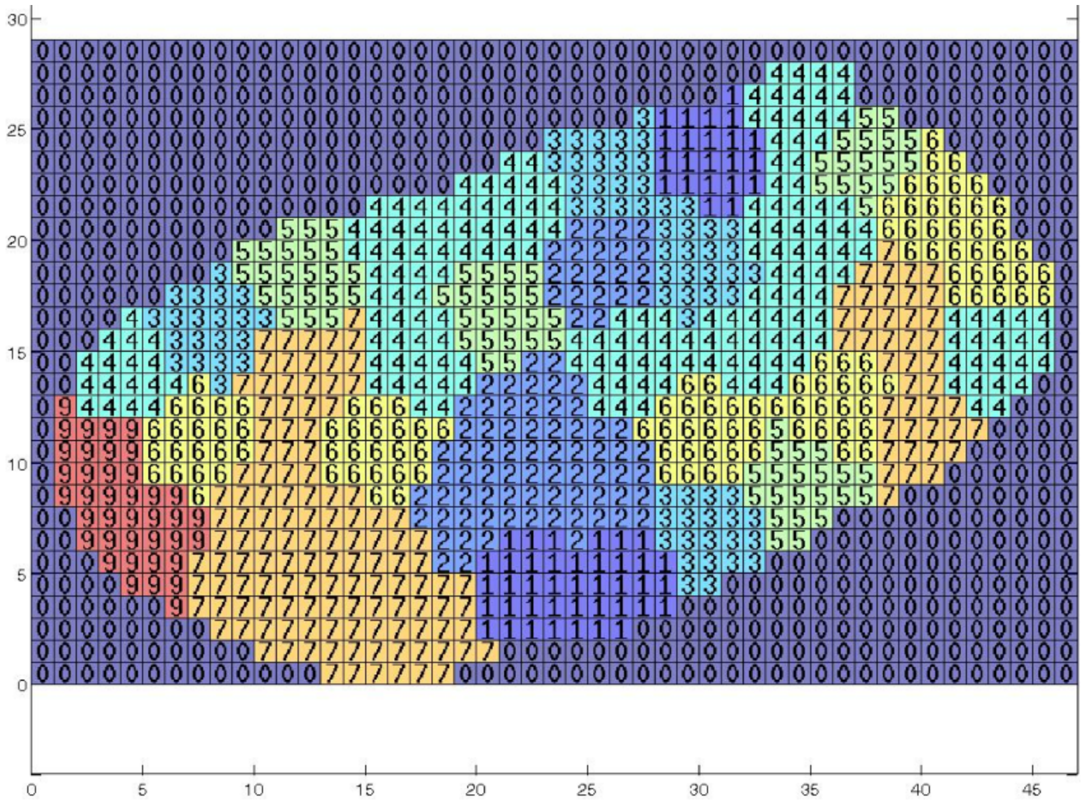

Fig. 6. Plan view of bench 15-realization #100 (each block: 25m×25m)

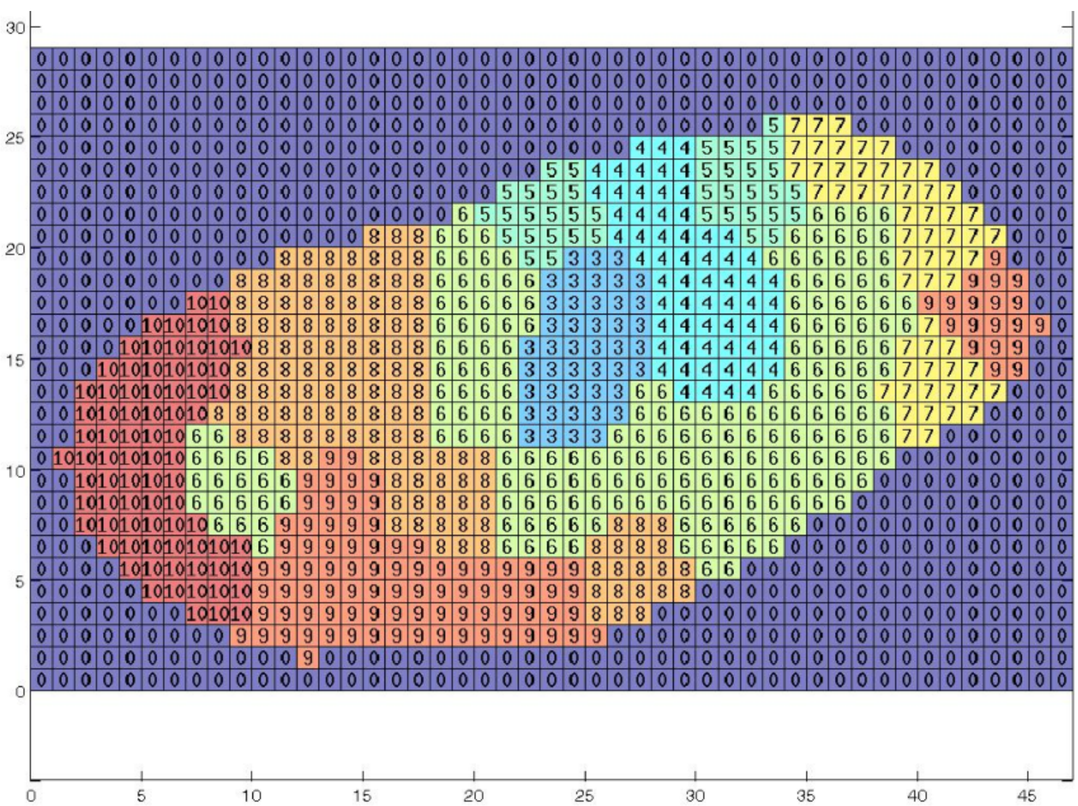

Fig. 7. Plan view of bench 16-realization #1 (each block: 25m×25m)

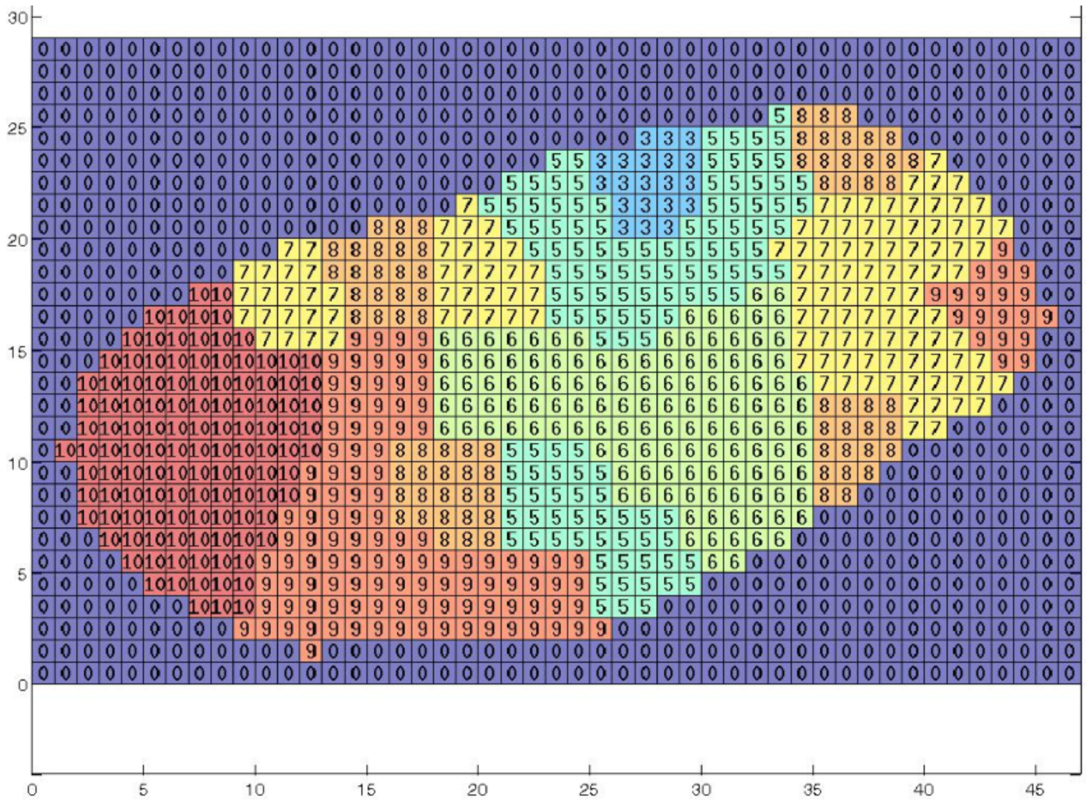

Fig. 8. Plan view of bench 16-realization #100 (each block: 25m×25m)

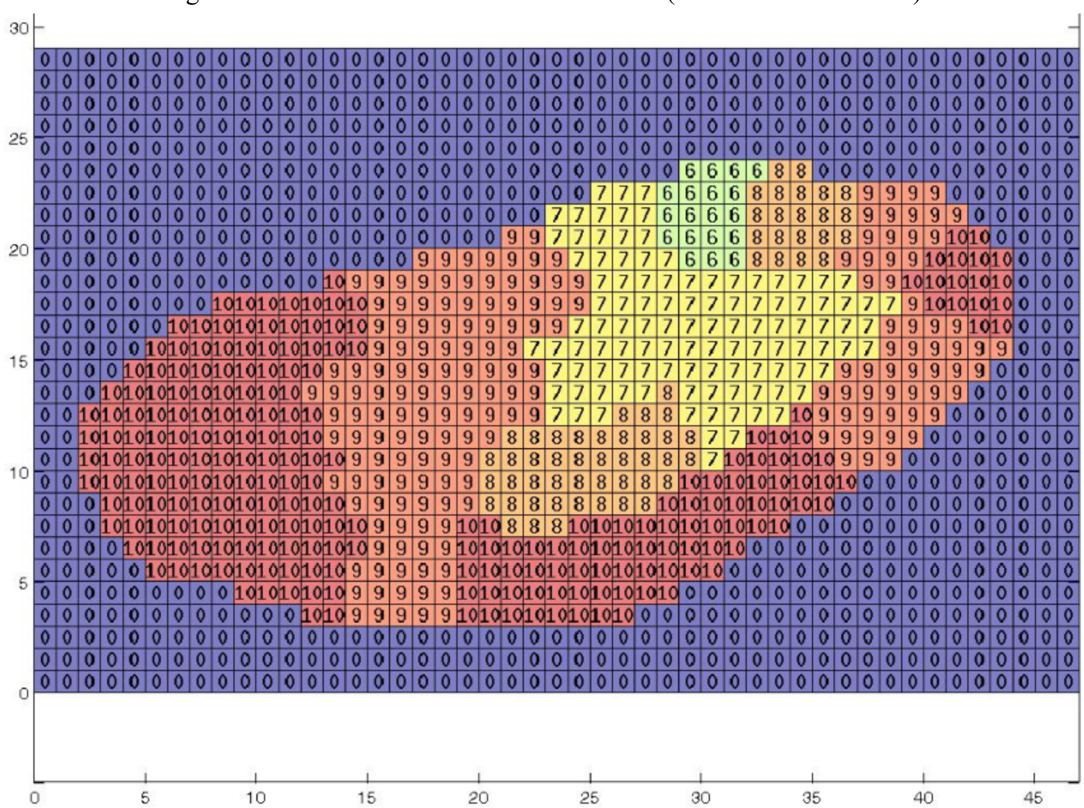

Fig. 9. Plan view of bench 17-realization #1 (each block: 25m×25m)

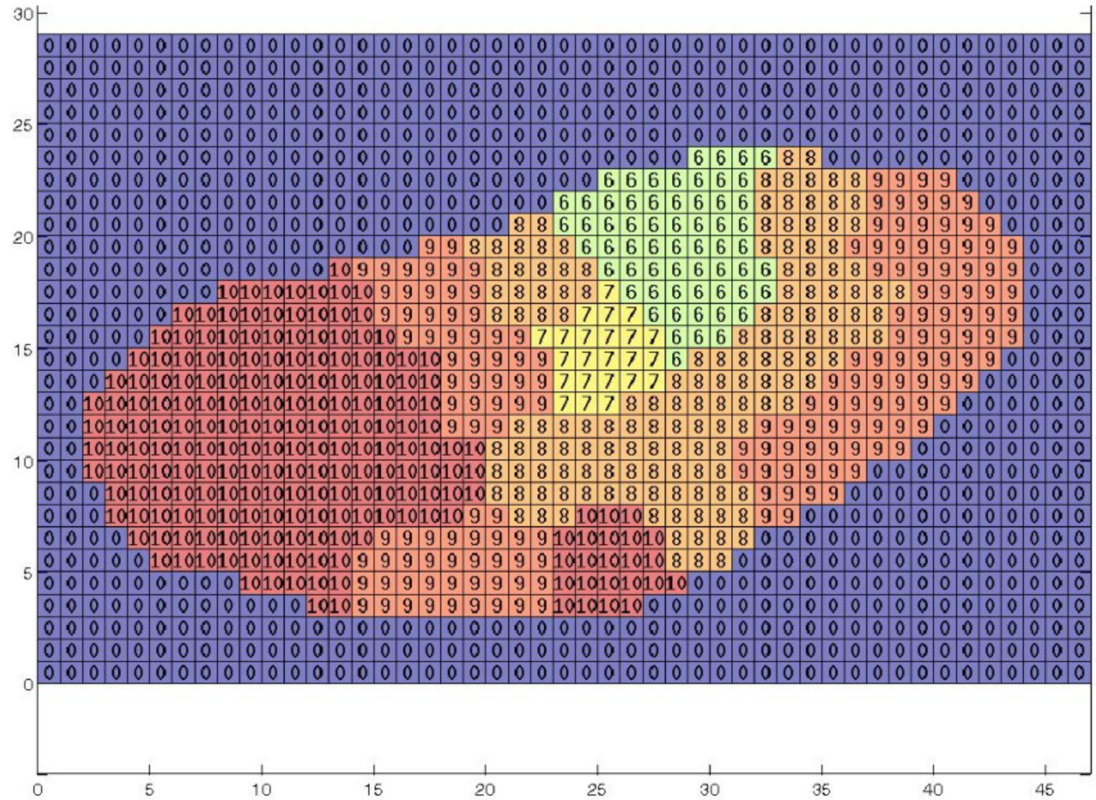

Fig. 10. Plan view of bench 16-realization #100 (each block: 25m×25m)

Fig. 15, Fig. 17, and Fig. 19 show the fluctuations of total rock tonnage mined in different 100 schedules based on 100 realizations in years 1, 8, and 9, respectively. Also, Fig. 16, Fig. 18, and Fig. 20 indicate the total ore processed in the process in different schedules in years 1, 8, and 9. As can be seen, fluctuation of total extracted rock tonnage is higher than total extracted ore in these years. Fig. 21 shows the trend of profit (objective function-net present value) in 100 realizations' schedules. Also, Fig. 22 presents the histogram plot of profit. Based on this plot, mean and standard deviation of profit are 21,648 and 6.7523 million dollars. Thus, no matter of what the grade distribution is, it is expected to earn 21,648 million dollars.

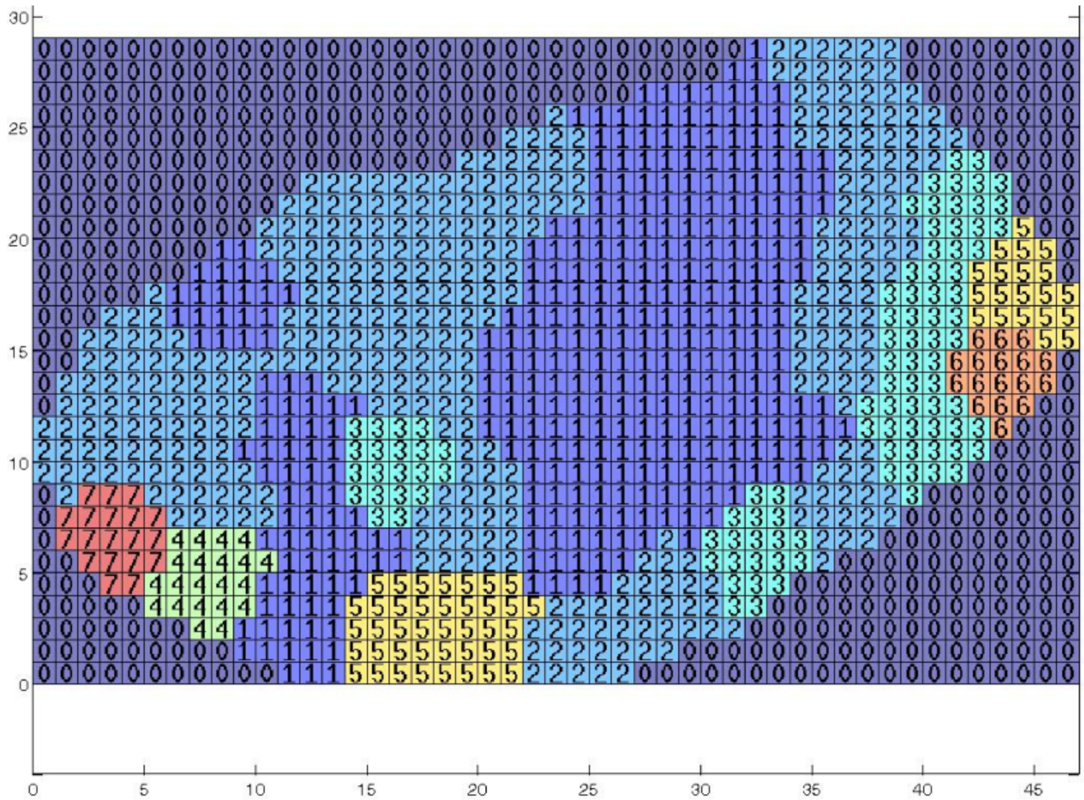

Fig. 11. Bench 14-average of realizations (each block: 25m×25m)

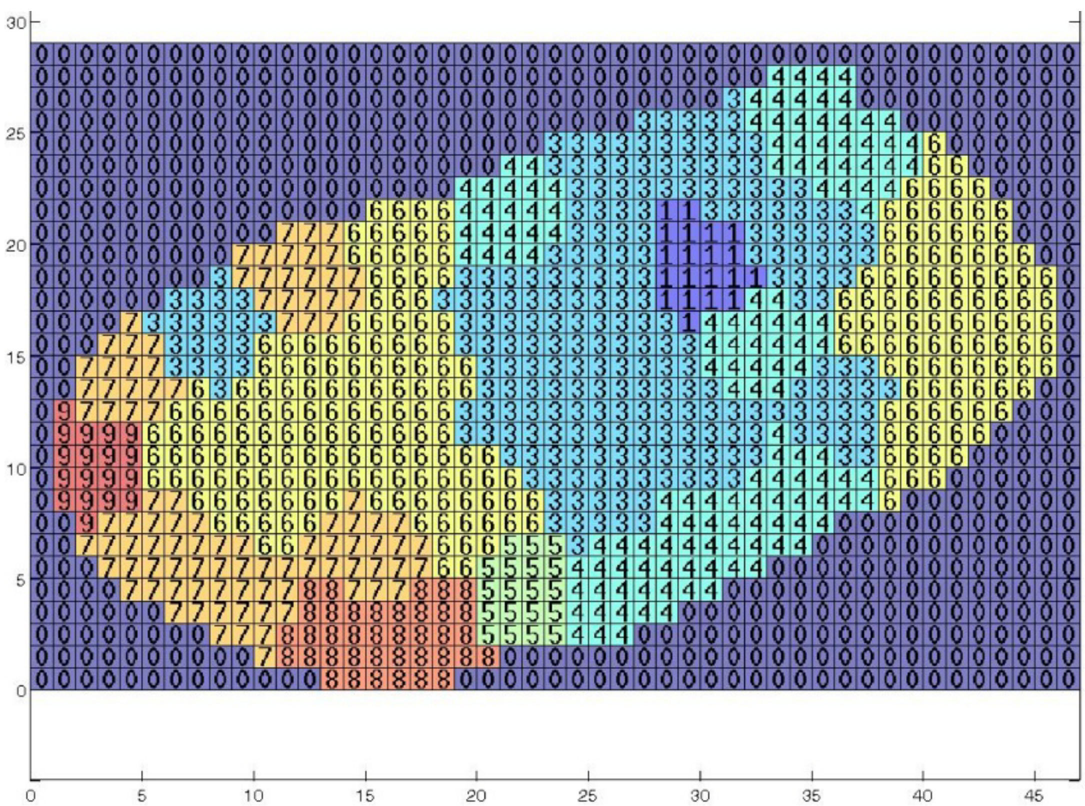

Fig. 12. Bench 15-average of realizations (each block: 25m×25m)

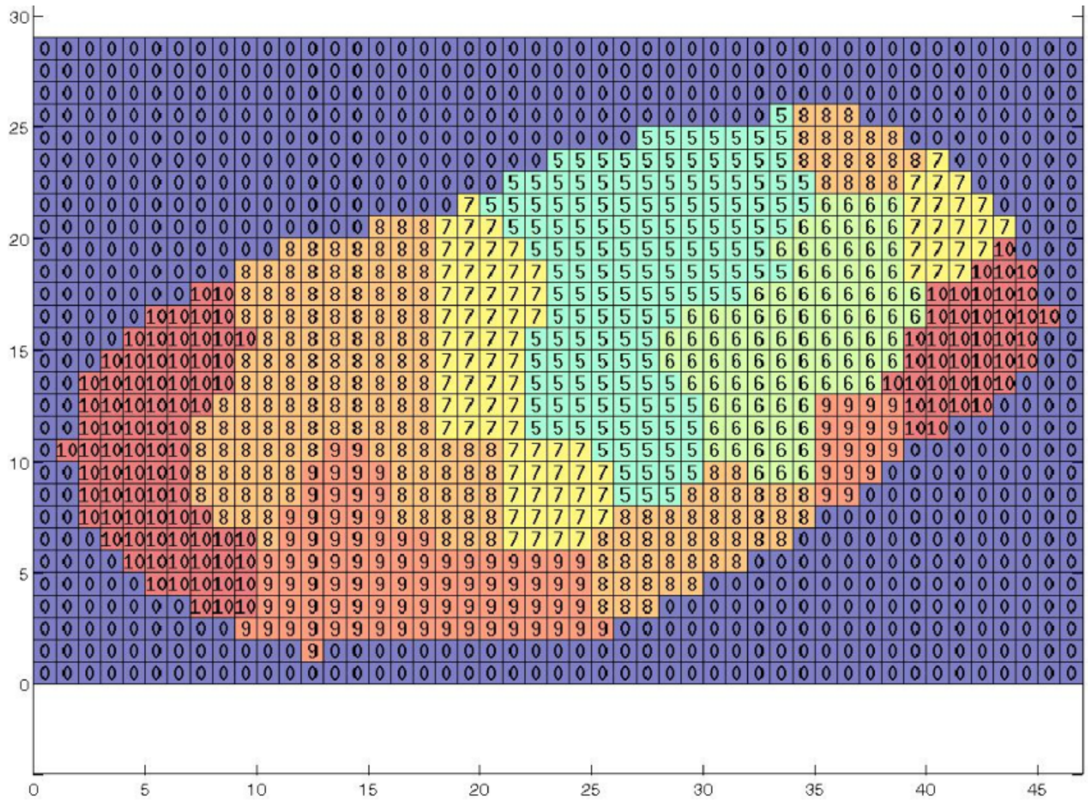

Fig. 13. Bench 16-average of realizations (each block: 25m×25m)

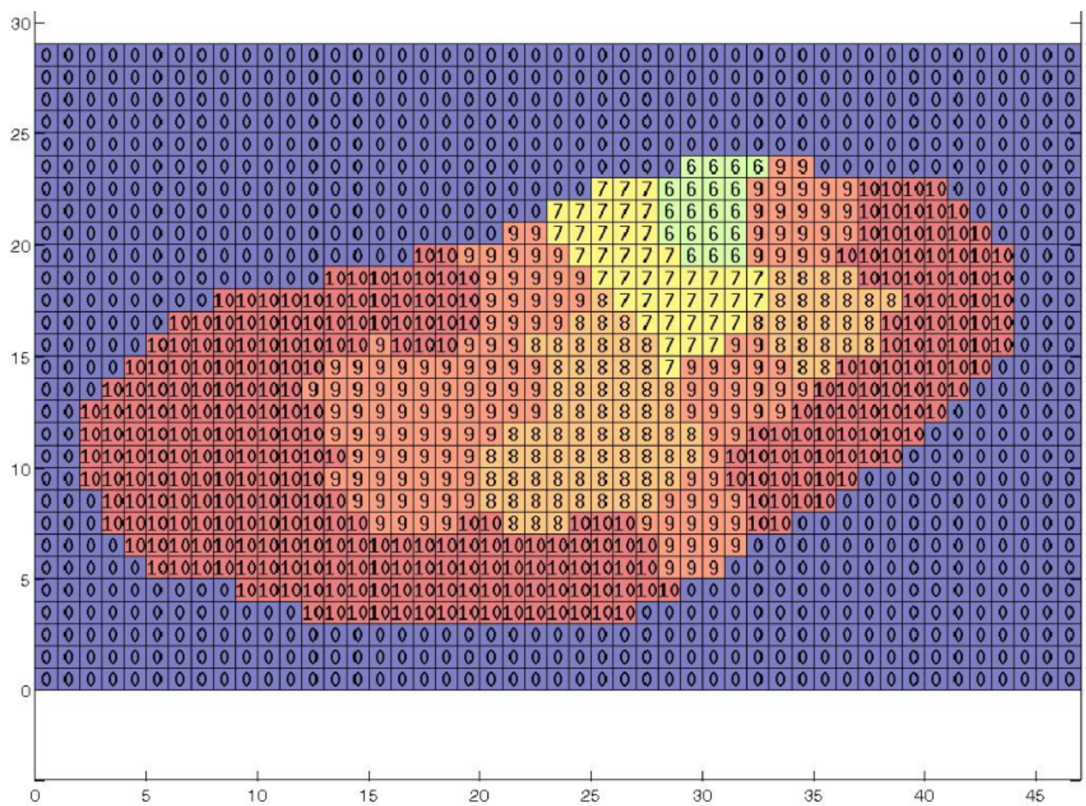

Fig. 14. Bench 17-average of realizations (each block: 25m×25m)

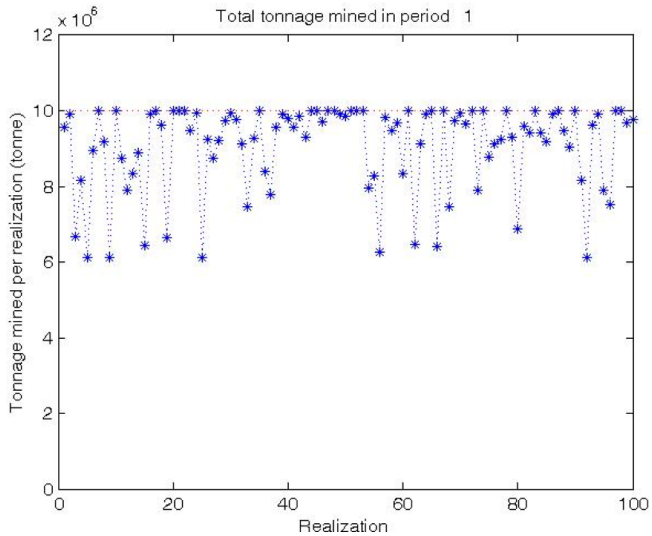

Fig. 15. Total tonnage mined in year 1 per realizations

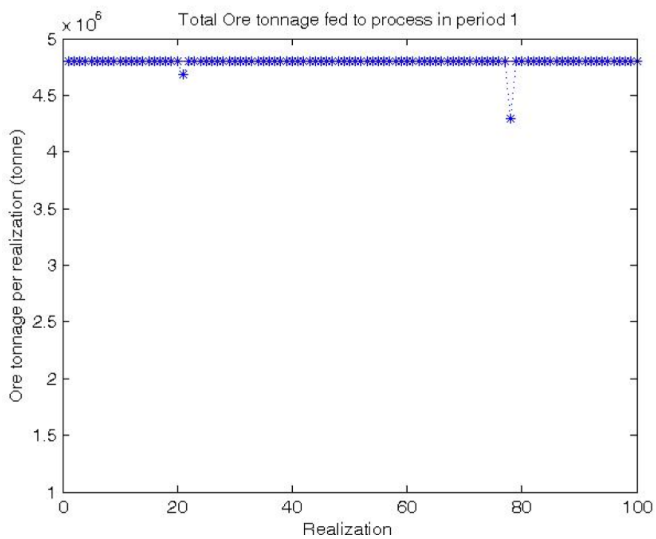

Fig. 16. Total Ore processed in year 1 per realizations

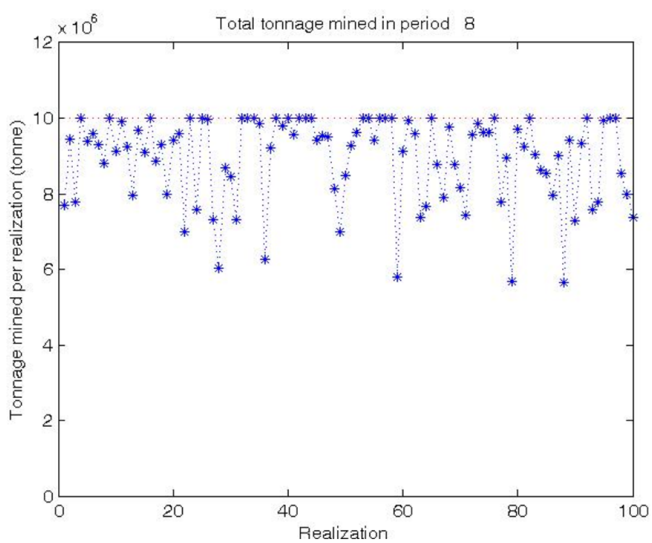

Fig. 17. Total tonnage mined in year 8 per realizations

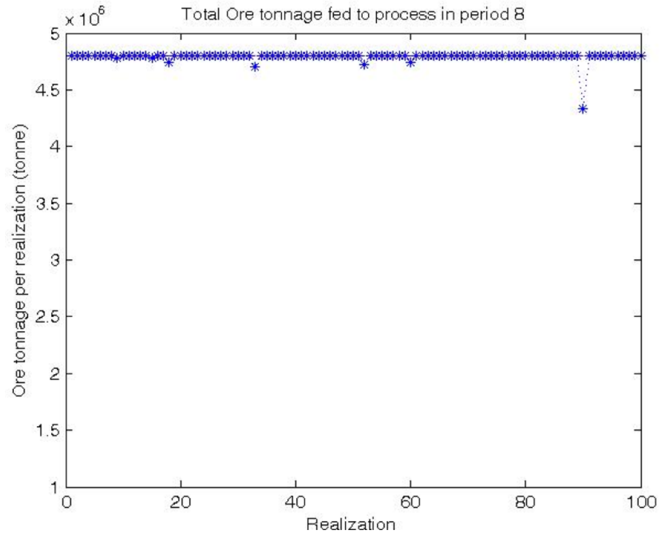

Fig. 18. Total ore processed in year 8 per realizations

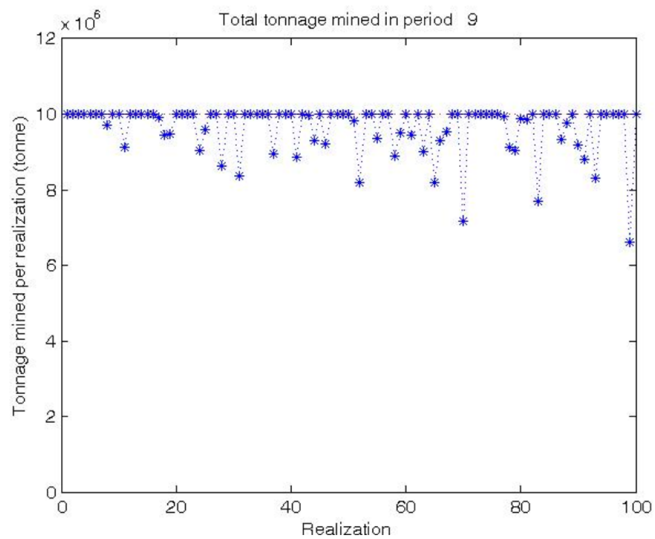

Fig. 19. Total tonnage mined in year 9 per realizations

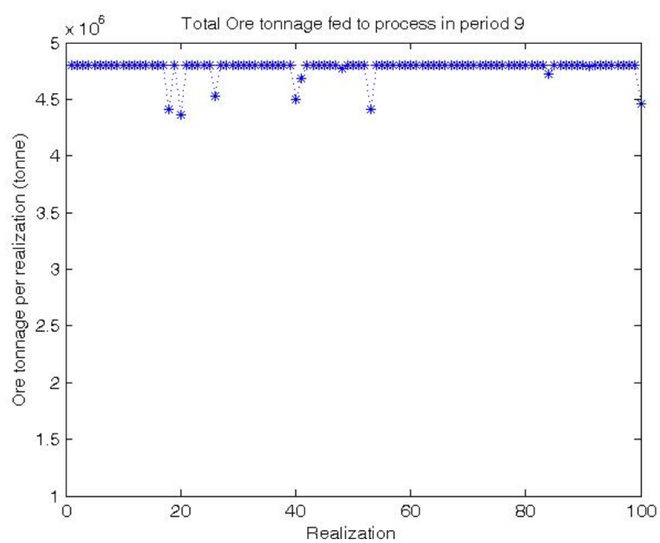

Fig. 20. Total ore processed in year 9 per realizations

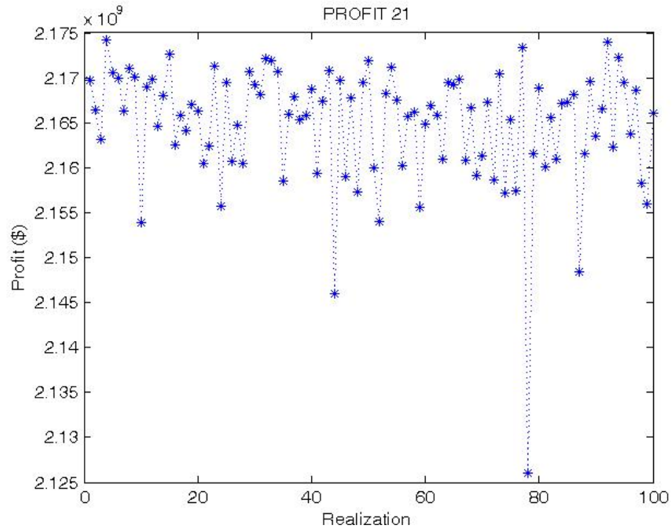

Fig. 21. Expected profit per realizations

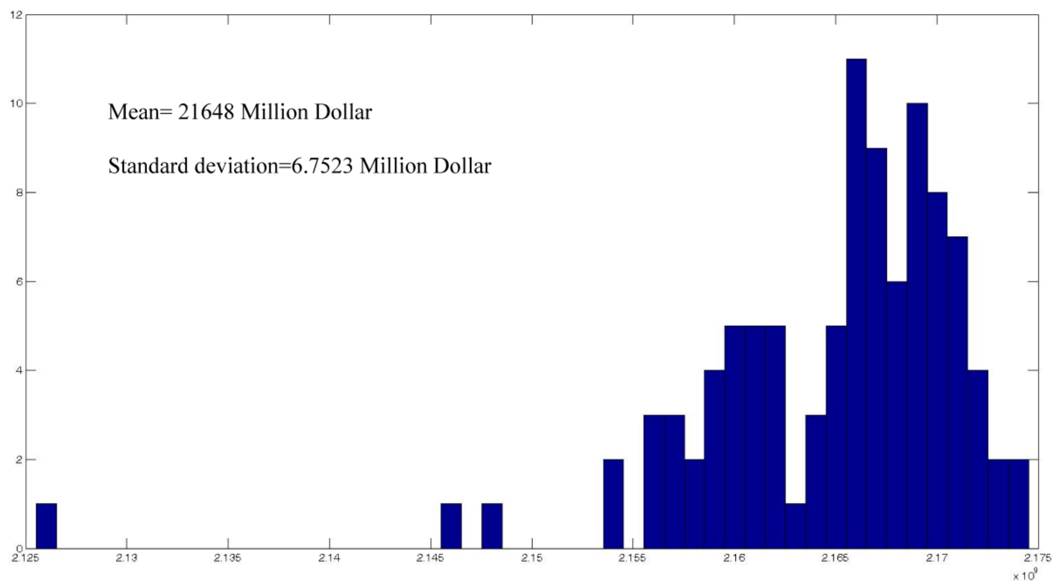

Fig. 22. Histogram plot of profit

6. Conclusions and future work

The problem of long-term mine production planning plays an important role in profitability of mining projects. Presence of uncertainty sources such as uncertainty in estimation of grade, shape of mineralized zone, and price could be potential perils for application of the long-term mine schedules. Therefore, how to involve these uncertainties in the mine scheduling stage is a challenging issue. In this paper, a mixed integer linear programming model (MILP) was developed to optimize the cash flow and cut-off grade with uncertainty approach. As the proposed model was complex, a simpler version of long-term mine production planning developed by Askari-Nasab and Awuah-Offei was implemented on a case study with generating 100 realizations of grade distribution. Hundred schedules were obtained showing how the profit changes and the period that each block most expectedly is going to be extracted. For future work, applying the proposed MILP with the grade uncertainty approach on larger models and improving the performance could be a valuable research theme. Also, integrating the price uncertainty with grade uncertainty in order to have better understanding of potential profitability of mining projects would be another noticeable work.

7. References

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Quantifying the Cost of Grade Uncertainty in Mine Plans

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Abstract

Uncertainty is present because of a lack of information. Conditional simulation algorithms are widely used to model grade uncertainty. This uncertainty has some negative effects on the planning process and it is necessary to transfer and measure risk in the long-term production schedule. In this paper, the effect of grade uncertainty in processing and on the economic value of extracted blocks is assessed and a quantitative method is presented to calculate the discounted cost of uncertainty in a production schedule. An oil sand deposit is used to demonstrate the presented methods.

1. Introduction

Grade uncertainty is modeled by generating equal probable realizations using geostatistical conditional simulation techniques. For each block, a local distribution of grade can be generated with simulated values, which show the local uncertainty. Usually, the average grade of a block is used to determine whether a block should be processed or not. The average grade may be the arithmetic mean of all simulated values, or may simply be the estimated grade value of the block using Kriging (Journel and Huijbregts, 1981). The cut-off grade is a critical threshold; any block with a grade above this limit is considered as ore and has an economical value, and any material bellow the cut-off grade is treated as waste with no economic value. Lane (1988) presented the fundamentals of cut-off grade calculation.

There are four different situations that may occur depending on the cut-off grade and local distribution of a block generated by n conditional simulations.

1. All n simulation values are below the cut-off grade (Fig. 1)
2. All n simulation values are above the cut-off grade (Fig. 2)
3. Not all simulation values are above or below the cut-off, but the average grade is below the cut-off grade (Fig. 3)
4. Not all simulation values are above or below the cut-off, but the average grade is above the cut-off grade (Fig. 4)

It is assumed that the number of realizations is sufficient to capture grade uncertainty with a reasonable level of statistical confidence. A synthetic case is assumed to demonstrate all four situations with lognormal distributions for grade of blocks. Four different mean and variances and a cut-grade of 2% have been chosen. Fig. 1 to Fig. 4 show the Probability Density Function (PDF) at left and Cumulative Density Function (CDF) at right for all four situations.

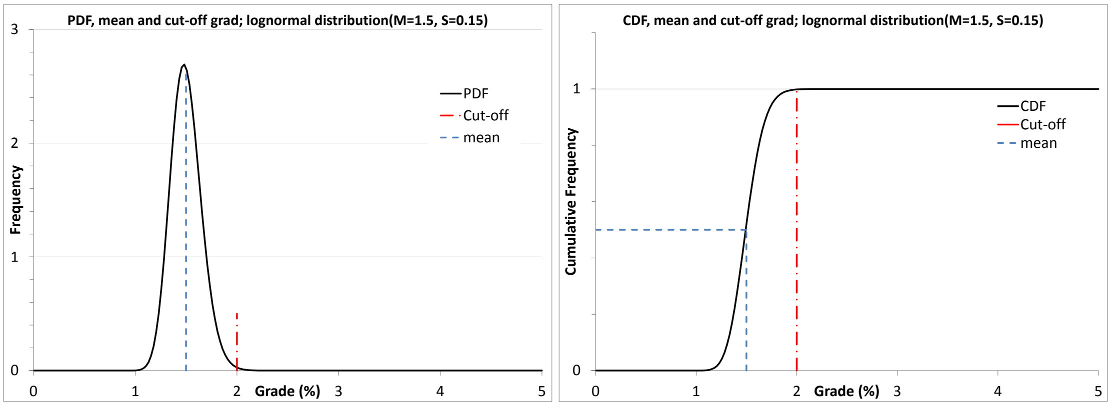

Fig. 1. PDF (left) and CDF (right) for case 1, all n realizations and mean (dashed blue line) are less than cut-off grade (dashed red line).

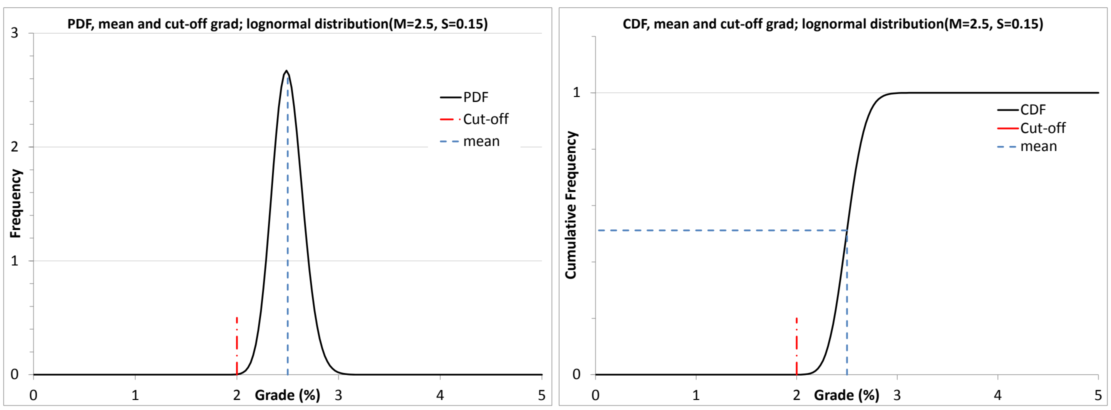

Fig. 2. PDF (left) and CDF (right) for case 2, all n realizations and mean (dashed blue line) are higher than cut-off grade (dashed red line).

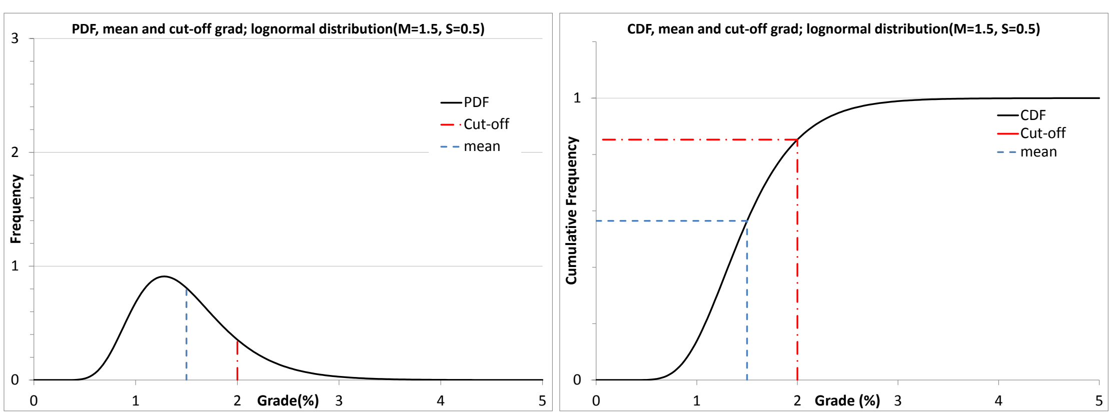

Fig. 3. PDF (left) and CDF (right) for case 3, not all n realizations and mean (dashed blue line) are less than cut-off grade (dashed red line).

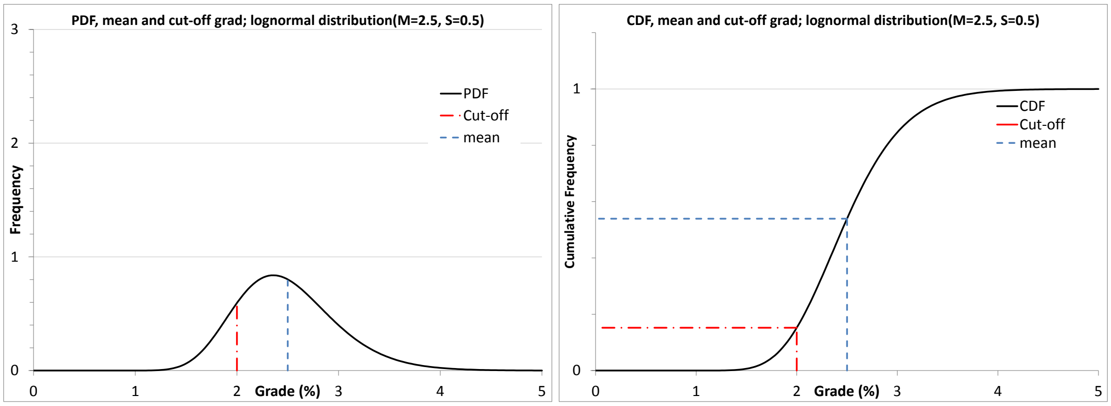

Fig. 4. PDF (left) and CDF (right) for case 3, not all n realizations and mean (dashed blue line) are higher than cut-off grade (dashed red line).

For cases 1 and 3, because the average grade of the blocks is below the cut-off, these cases are considered waste blocks (Fig. 1 and Fig. 3). The average grades of cases 2 and 4 are above the cut-off; hence these blocks are treated and sent to the processing plant (Fig. 2 and Fig. 4). Based on n simulation values, there is a very small risk involved in decisions that are made for cases 1 and 2 (Fig. 1 and Fig. 2). This risk increases in blocks with distributions described in case 3 and 4. In case 3 the block is chosen as waste because average grade is lower than the cut-off grade. But there is 14.7 percent chance of obtaining a higher value than the cut-off grade. This means that there is a 14.7 percent chance that this block should be considered as ore, but it will be treated as waste. Let's assume an extraction schedule that is generated according to the average grade of blocks. In such a case, this block, with probability of 14.7 percent, may generate over-production.

The same situation would occur in case 4 (Fig. 4). This block is classified as ore block because its average grade is higher than the cut-off grade. There is a 15.2 percent chance that the grade of this block is below the cut-off grade. This means that with a block extraction schedule generated based on average grade, this block with a probability of 15.2 percent may be considered as waste. This will cause under production following the designed schedule.

2. Grade uncertainty and economic block value

The Economic Block Value (EBV) is presented by Eq. (1).

$$EBV_{\mathrm{n}}^{\mathrm{t}}=\begin{cases}\mathrm{o_n}\times\left(\overline{\mathrm{g}}_{\mathrm{n}}\times \mathrm{p}_r\times \mathrm{price}^{\mathrm{t}}-p_c^t\right)-\left(\mathrm{o_n}+\mathrm{w_n}\right)\times \mathrm{m}_c^t & if\ \overline{\mathrm{g}}_{\mathrm{n}}\geq g_{cut}\\ -\left(\mathrm{o_n}+\mathrm{w_n}\right)\times \mathrm{m}_c^t & if\ \overline{\mathrm{g}}_{\mathrm{n}}< g_{cut}\end{cases} \tag{1}$$

where $\mathrm{o_n}$ is the tonnage of ore, $\mathrm{w_n}$ is tonnage of waste, $\overline{\mathrm{g}}_{\mathrm{n}}$ average grade of block n, p_r processing recovery, p_c^t processing cost, m_c^t mining cost, $\mathrm{price}^{\mathrm{t}}$ is the present selling price of final product, g_{cut} is cut-off grade.

EBV is a positive value when the average grade of the block is above the cut-off grade and equal to a negative mining cost for waste blocks (Fig. 5).

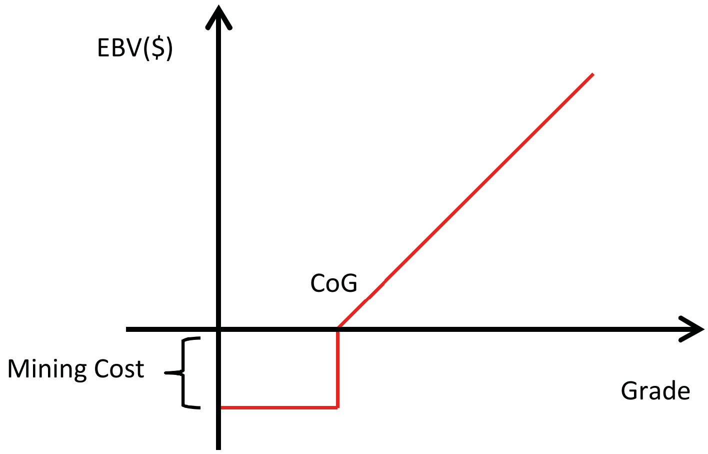

Fig. 5. Cut-off grade and EBV.

To show the effect of grade uncertainty on EBV, a synthetic case is assumed. A block with a log normal distribution of grade with a mean and standard deviation of 2.2% and 0.5% respectively is simulated 10,000 times (Fig. 6). The cut-off grade is assumed to be 2%. Therefore, this block is considered as an ore block because the average grade is above the cut-off grade.

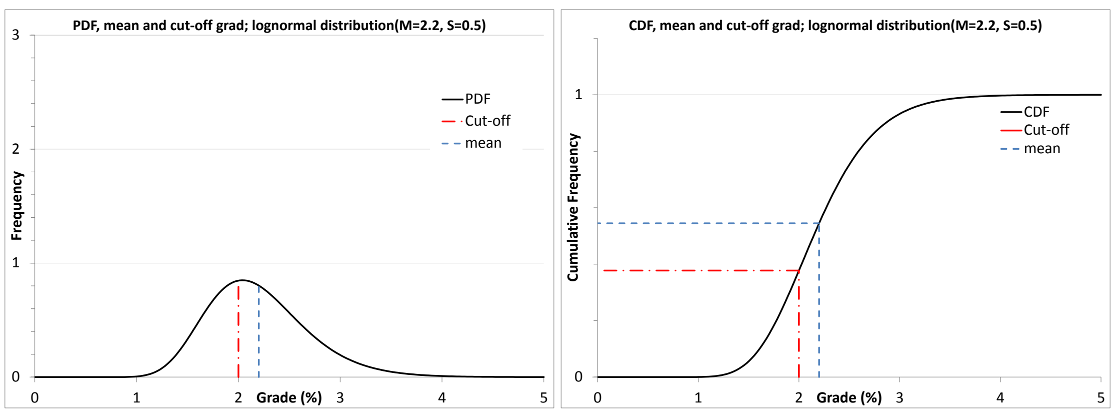

Fig. 6. PDF (left) and CDF (right) for a synthetic case to calculate expected value of EBV.

EBV of this block is calculated for all 10,000 realizations where $p_r = 100\%$, $\text{Price} = 1\$$, $p_c = 0.5\$$ and $m_c = 1.5\$$. The tonnage of this block is assumed to be 1 tonne. Histogram of EBV is shown in Fig. 7. 3744 times (37.44%) over 10,000 generated values, the block is assumed to be waste because the simulated grade is less than the cut-off grade. This is revealed by the trimmed black column in Fig. 7. 62.56% of the times, the simulated grade is above the cut off-grade and the block is assumed to be ore (gray columns at the Fig. 7). The average of EBV is -0.26 \$ and less than zero. This means that even for a block with an average grade above the cut-off grade, the average EBV of simulations may be less than zero and it is not economical to be processed.

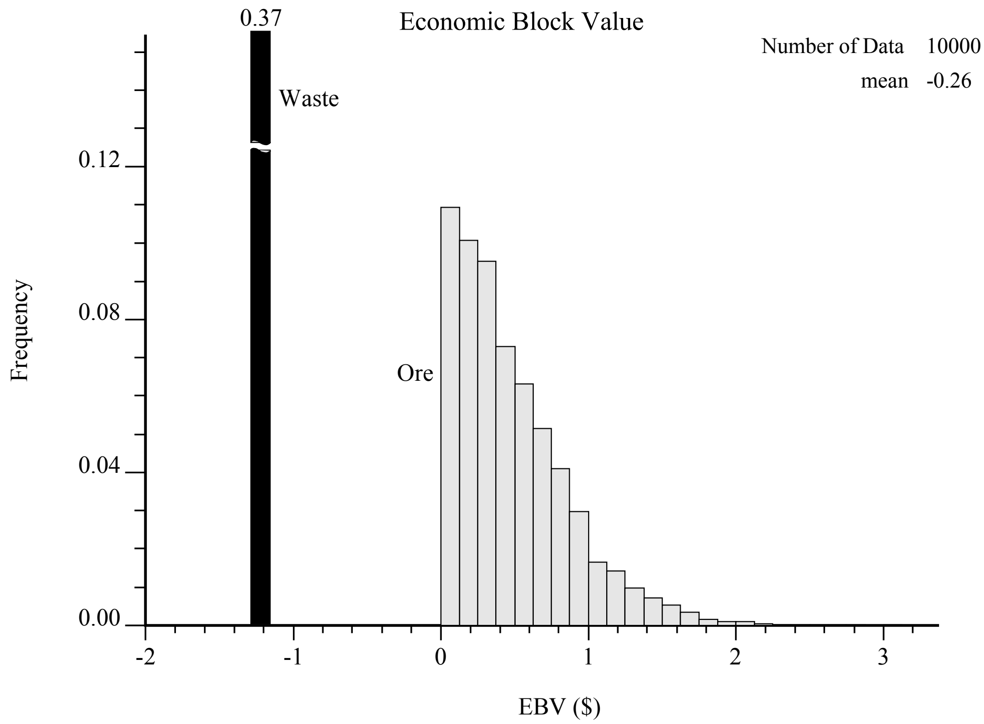

Fig. 7. Histogram of EBV for a block with lognormal distribution, mean=2.2 and Std.dev=0.5.

3. Cost of uncertainty

The main objective of long-term mine planning is usually to maximize the net present economic value of a project subject to technical and other (e.g. environmental) constrains. Such an objective is usually modeled using optimization techniques. The goal is usually to find the sequence of extraction of blocks or mining-cuts to reach the maximum achievable net present value of the project. The input data used in production scheduling optimization, such as geological block model, grades, costs, prices, recoveries, and practical mining constraints are usually based on the best point estimates available at the time of optimization. Traditional mine planning techniques do not consider grade uncertainty and only one estimate of the block grade is used.

As noted in the previous section, grade uncertainty might cause shortfalls from and surpluses over the designed target production at the processing plant. This is because decisions are made only based on an estimated grade value for a block. Different estimation techniques, such as different Kriging methods or the E-type mean method, are used to determine the average grade of a block. If the average grade of a block is less than the cut-off grade, then the block is classified as waste and vice versa. Therefore, uncertainty in the grade can result in misclassification; a block that has been classified as waste can be an ore block and vice versa. This misclassification is not considered in traditional mine planning methods, which use only a smooth block model, in most cases, an average grade estimation such as Kriging or E-type is used.

The secondary objective seen increasingly in the literature is to take uncertainty into account. The fact that the input variables into the optimization model are uncertain, affects the optimization process. Recently some authors, such as Dimitrakopoulos and Ramazan (2008), have presented optimization algorithms which aim to maximize expected value of the target function, which is net present value (NPV), and to minimize the negative effect of uncertainty, which is called risk. These methods try to maximize NPV and minimize the risk of grade uncertainty by deferring the extraction of more uncertain blocks into the future so that the effect of grade uncertainty is reduced by gathering new information during mine life. The main idea is that uncertainty somehow costs money and should be deferred. The challenging question is how to quantify of the cost of uncertainty. The cost of uncertainty has two main reasons:

1. For any shortfalls that may happen at processing plant, there is a missing profit. This profit could be achieved by a feeding plant with full capacity.
2. On the other hand, assume that the processing plant is working at full capacity and there is enough ore to feed the plant for a while. At this moment, trucks and shovels are working to remove waste blocks as planned. Because of grade uncertainty and misclassification, a block that has been classified as waste is extracted and has a grade above cut-off grade. In real life, this block would not be extracted and the schedule would be changed or in most cases, there would be a stockpile where this extra ore would be sent to be used in the future. Not having a stockpile is very unlikely in real life. We may, however, assume a hypothetical case in which, there are no stockpiles to store extra ore and in which the block is being extracted to follow exactly the same schedule. In such a case, this ore block will be sent to the waste dump and its revenue will be lost. This lost profit is also part of the cost of uncertainty.

To quantify these two costs, a cost of not meeting the target production is presented in Eq. (2)

$$C_t = \left| P_t - \text{Target}_t \right| \times \left(\overline{g}_t \times P_r \times \text{Price - } P_c \right) \tag{2}$$

Where C_t is the cost on not meeting the target production in period t, P_i is the input ore to the mill at period t, Target_t is target production for period t, $\overline{g}_t$ average grade of input ore at period t, P_r processing recovery, Price is the selling price of final product and P_c is the processing cost. By averaging C_t over all realization and discounting it over all the planning periods, the discounted cost of uncertainty is defined by Eq. (3)

$$C_u = \frac{1}{L} \sum_{t=1}^{T-1} \sum_{l=1}^{L} \left(\frac{C_t^l}{(1+i)^t} \right) \tag{3}$$

Where L is the number of realizations, T is the number of periods and i is the discounting rate. The cost of uncertainty is calculated over all period except final period. Because any ore that is left for final period will be processed and will not exceed the target production, any shortfall in the final period does not occur because of grade uncertainty.

4. Case study

An oil sands deposit in Fort McMurray, Alberta, Canada was used for this case study. The location of boreholes and the histogram of data are presented at Fig. 8 and Fig. 9 respectively.

GSLIB (Deutsch and Journel, 1998) programs were used in this case study. Directional experimental variograms were calculated and fit using `gamv` and `vmodel` programs. The azimuths of major and minor directions were 50 and 140 degrees. Fig. 10 shows the experimental and the fitted variogram models in major, minor and vertical directions.

`KT3d` was employed to estimate the bitumen grade (with no normal score transform) at each block location using Ordinary Kriging (OK). Multiple realizations of the bitumen grade were generated using Sequential Gaussian Simulation (SGS) (Isaaks and Srivastava, 1989) at a very high resolution three-dimensional grid in the point scale. To determine the average grade of a block, simple arithmetic averaging was done between all simulated points inside the block. This step is called up-scaling, and a program called `blkavg` was used at this stage. The E-type mean was calculated using the `postsim` program. Fig. 11 illustrates the map of the bitumen grade for the Kriged and the E-type methods, and of realization no. 26 at block scale. It is well-known that

Kriging is conditionally biased (Isaaks, 2005) and that, on the other hand, "there is no conditional bias of simulation when the simulation results are used correctly" (McLennan and Deutsch, 2004). The conditional biasness of Kriging can be reduced by tuning estimation parameters but it cannot be eliminated (Isaaks, 2005). The Grade-Tonnage curve is a good tool for use in checking the impact of Kriging biasness.

Fig. 12 shows the grade tonnage curve of simulation realization (dashed lines), Kriging (bold solid line) and E-type (bold dashed line). Although an effort was made to minimize the conditional biasness of Kriging, there are still differences between the Kriging and simulation results because SGS uses Simple Kriging(SK). Also, E-type is slightly different than Kriging; theoretically, the E-type model is identical with the SK result in Gaussian space (Journel and Huijbregts, 1981).

Histogram and variogram reproduction were checked using `gamsim` and `histpltsim` respectively. Fig. 13 shows the histogram reproduction, and Fig. 14 shows the variogram reproduction in major and minor horizontal and vertical directions. Generally, SGS should reproduce the histogram and variograms of the original data if it is carefully implemented. In this case, the reproduction of the histogram and variograms was acceptable.

The ultimate pit limit design was carried out based on the Syncrude's costs in CAN$/bbl of sweet blend for the third quarter of 2008 (Jaremko 2009). The price of oil was assumed to be US$45, with an exchange rate of 1.25:1 equal to CAN $56.25/bbl SSB in the same time period. We assumed that every two tonnes of oil sands with an average grade of 10% mass would produce one barrel of sweet blend, which is approximately 200 kg. We also assumed a density of 2.16 tonne/m^3 of oil sands, and a density of 2.1 tonne/m^3 of waste material, including clay and sand.

Table 1 shows the pit design and production scheduling input parameters. The mining cost of of $4.6/tonne and processing cost of $0.5025/tonne was applied. Thirty-three pit shells were generated using 49 fixed revenue factors ranging from 0.1 to 2.5, based on the Kriged block model. The number of pit shells was reduced to 14 after applying the minimum mining width of 150 meters for the final pit and the intermediate pits. Table 2 summarizes the information related to the final pit limit based on Kriged block model at 6% bitumen cut-off grade. The minimum slope error, the average slope error and the maximum slope error respectively are: 0.0 degrees, 0.2 degrees, and 0.4 degrees. The final pit limit was designed for the E-type model and all fifty simulation realizations with the exact same input variables.

The final pit, based on the Kriging block model was used at this stage. There are 14607 blocks inside the final pit. Using the MATLAB (MathWorks Inc., 2007) c-mean clustering function, 1834 mining cuts were generated by aggregating blocks at the same level with similar grades. The Kriging block model was used at LP optimization(Askari-Nasab and Awuah-Offeri, 2009). Two years of pre-stripping were assumed to provide enough operating space and ore availability. No stockpile was defined and the target production was set to 36 million tonnes of ore per year with a mining capacity of 135 million tonnes per year. The interest rate is 10%. The mine life was 10 years. There were 653.61 million tonnes of material inside the final pit, 282.44 million tonnes of which were ore. The strip ratio was 1.31; in addition, there were 37.4 million tonnes of ore with an average grade less than the cut-off grade. These were assumed as waste blocks.

The mixed integer programming was solved using TOMLAB CIPLEX (Holmström, 1989-2009) with a gap of 1%. Fig. 15 shows the schedule generated by MILP and using Kriging block model. Gray and yellow bars present removed waste and ore materials respectively. The plan view and two cross sections of blocks and their extraction periods are shown in Fig. 16. To capture the effect of grade uncertainty, the Kriged value of blocks were replaced by simulated values and the same extraction schedule was followed. Any blocks that had less simulated grade than cut-off grade were sent to waste dump even if they were considered to be ore using the Kriging block model and vice-versa. Following the same schedule, some realizations are generated over-produced ore, This revenue was counted in the NPV. This NPV is incorrect, because is it impossible to process over-produced ore where there is no stockpile. Therefore the revenue of these surplus ores must be removed. For this reason, there are two versions of results. One is the row results, surplus ore is not removed and revenue is counted. In the second version the removed over-produced ore is removed and called "Cleaned Version". From Fig. 17 to Fig. 20, row results are shown on the left and the cleaned version is at right. Fig. 17 to Fig. 19 show the effect of grade uncertainty on the generated plan in terms of cumulative NPV, head grade and feed respectively. The bold black line is Kriging, the bold dashed blue line is Etype and the dashed red lines are 50 conditional simulation results. Fig. 17 demonstrates that the NPV of the cleaned realizations is less than of the row version. For example in the row version there is a realization that generates more NPV than the Kriging block model, but in the cleaned one there is not any realization to exceed the NPV of Kriging. Fig. 20 illustrates the box plot of input ore into the plant calculated with using simulation realizations. Yellow bars show the deviation from target production. As it is clear from this graph and Fig. 19, surplus ore production and under-production are probable in periods 2 and 4 respectively, while shortfall may only happen in period 3.

Table 3 shows the summary statistics for the generated schedule for the two versions. The NPV of Kriging is $2461 million. The expected NPV is calculated from all 50 realizations. In the first table, the statistics show the row results and in the second table, the results are for removed surplus ores. The average NPV for row and cleaned version respectively are 2335.4 and 2317.5, and are less than the Kriged NPV. It is reasonable to have a lower expected NPV, because the whole MILP algorithm was solved to maximize NPV of the Kriging and the solution is optimized for the Kriged block model. Table 4 shows the summary of statistics for cumulative discounted case flow at different periods. In the first two periods, the cumulative discounted case flow is negative because of pre-striping at these periods and because there is no extraction of ore.

The discounted cost of uncertainty was calculated based row results. Therefore the discounted cost of uncertainty based on periods 3 to 9 was $178.9 Million.

For this case study, 861 of 1834 cuts were considered to be ore based on comparison of the Kriging average grade to the cut-off grade. 84 mining cuts of 861 ore cuts had less expected EBV than the minimum acceptable EBV, which is calculated based on cut-off grade. Therefore, if the average EBV is the criterion for choosing whether a block will be processed or not, then not all of these 84 mining cuts should be considered as ore cuts. This shows the effect of grade uncertainty on misclassification even with an average grade above the cut-off grade. The difference summation of EBVs of these 86 mining cuts is $64 million.

5. Conclusions

In this paper the effect of grade uncertainty on four synthetic cases was illustrated. Four possible situations may occur, depending upon on the estimated mean and a cut-off grade. Two cases may cause shortfalls or extra ore production in the generated schedule using only an estimated value. This is because of grade uncertainty.

It has also been shown that the cut-off grade and one average value for a block are not good criteria to choose a block as ore or waste. There are some situations in which the average grade of a block is above cut-off grade, but the expected dollar value of block is less than minimum threshold. Therefore the average EBV of a block over realizations may be a better criterion for classifying a block as either ore or waste.

The cost of not meeting the target production and cost of uncertainty were presented. Using all simulation realizations, the cost of uncertainty can be calculated. It is a good criterion for use in comparing two different schedules. The Cost of Uncertainty is an average dollar value that, by following a schedule over all realizations, is imposed on the plant because of probability of not meeting the target production.

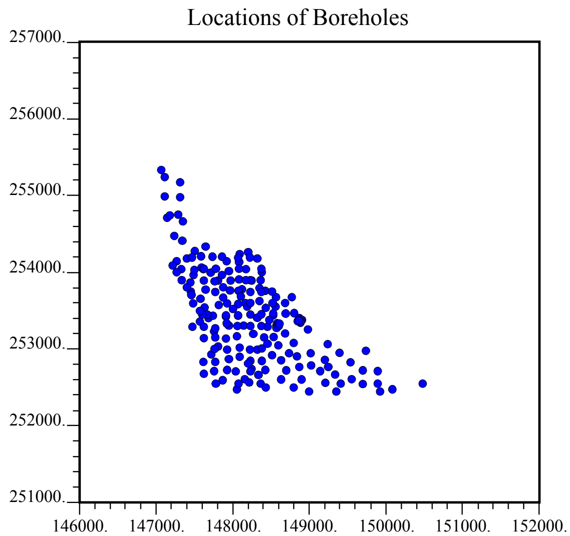

Fig. 8. Location map of boreholes.

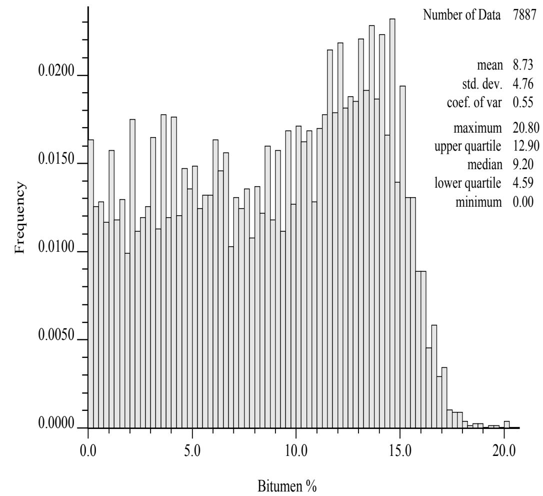

Fig. 9. Histogram of Bitumen.

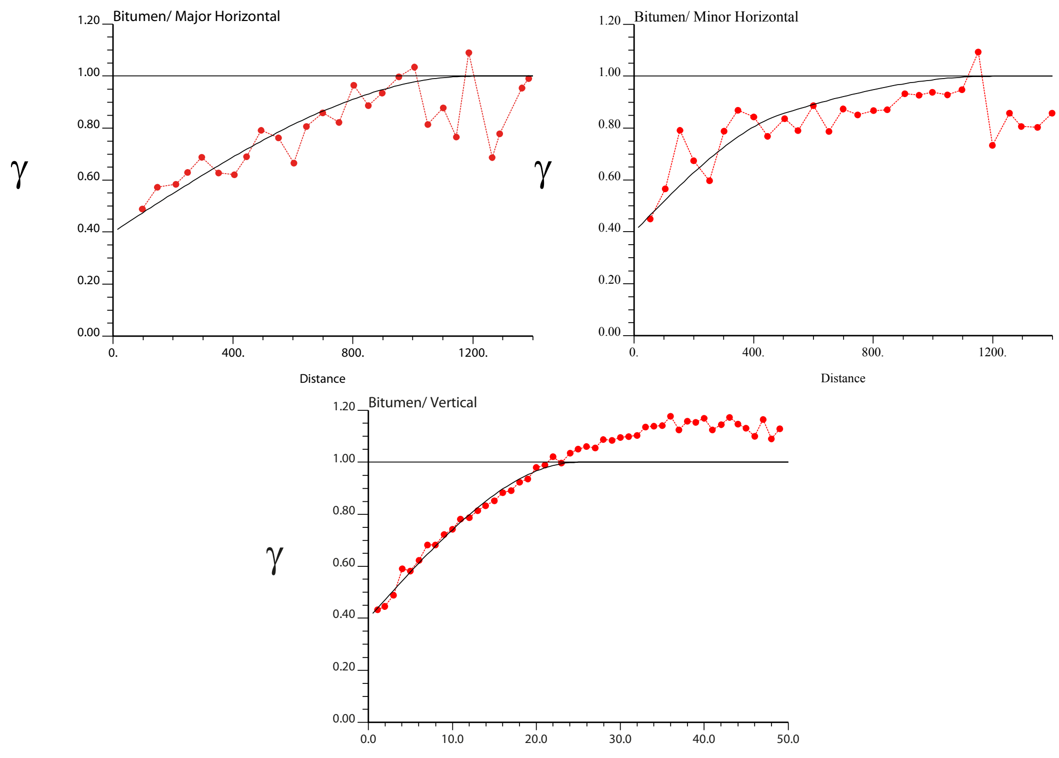

Fig. 10. Experimental directional variograms (dots) and the fitted variogram models (solid lines)

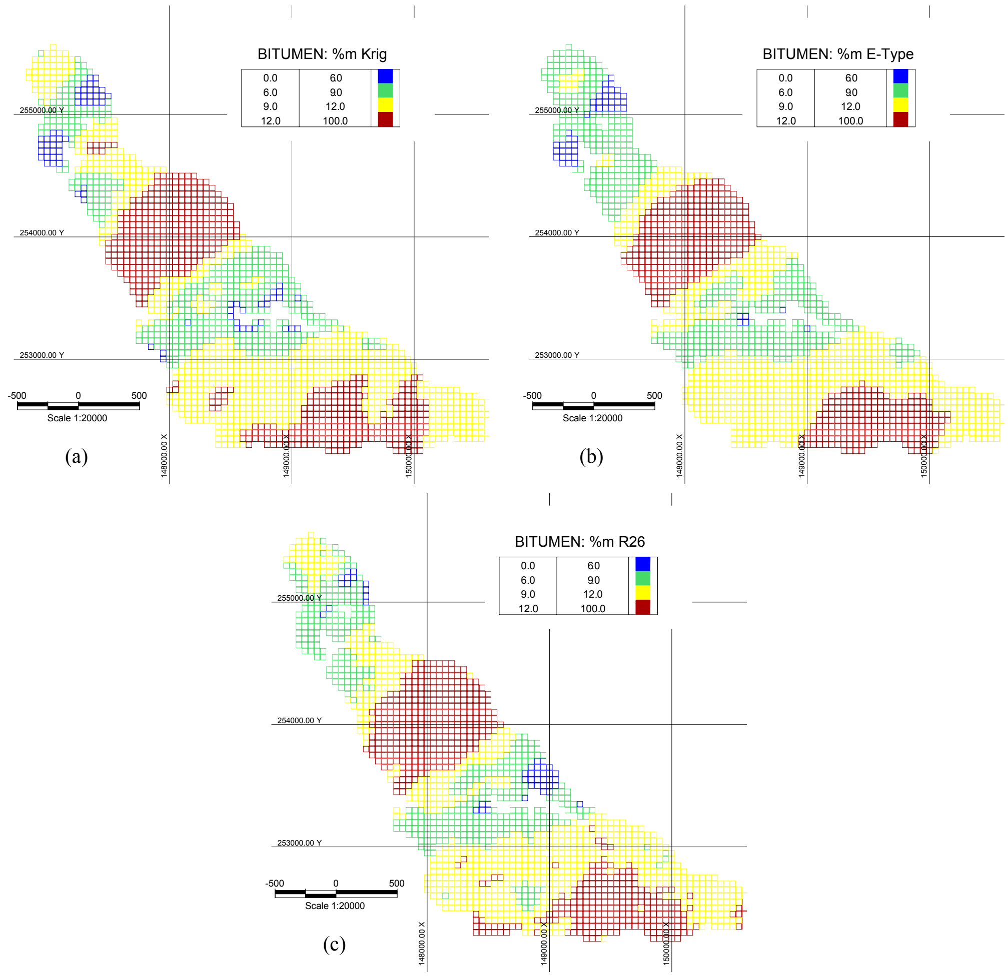

Fig. 11. Plan view at 260m; (a) Kriged model, (b) E-type model, (c) realization 26.

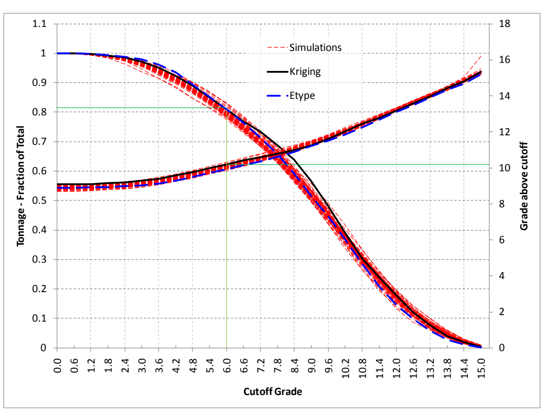

Fig. 12. Grade tonnage curve of simulation realizations, kriged, and Etype block models

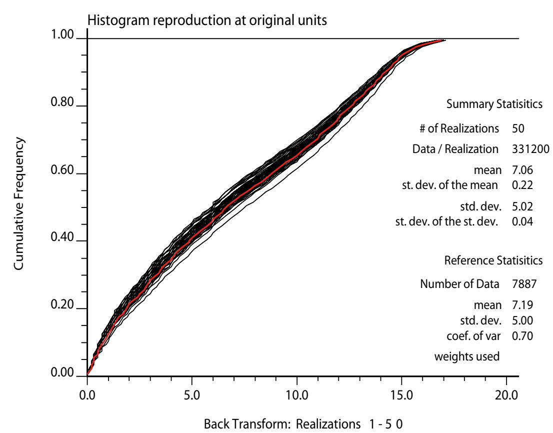

Fig. 13. Histogram reproduction of simulation realizations (dashed lines) and histogram of original data (bold line)

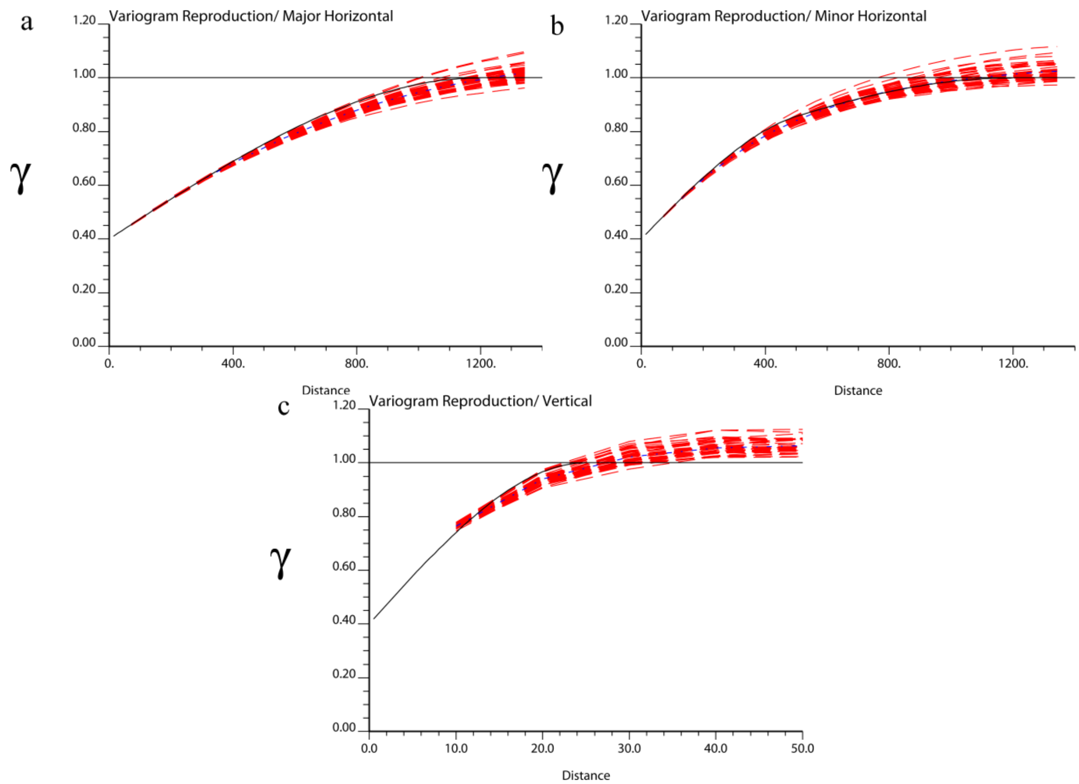

Fig. 14. Variogram reproduction of simulation realizations (red dash lines) and reference variogram model (black line).

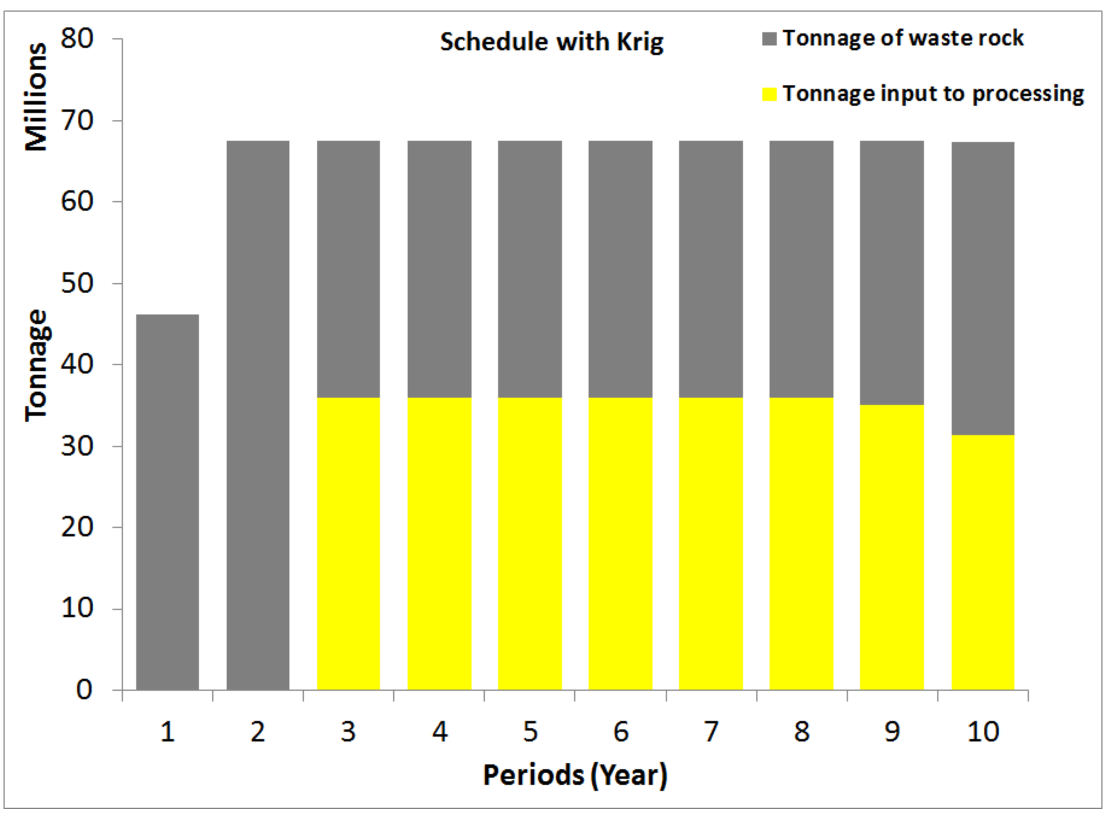

Fig. 15. Schedual generated using krig model

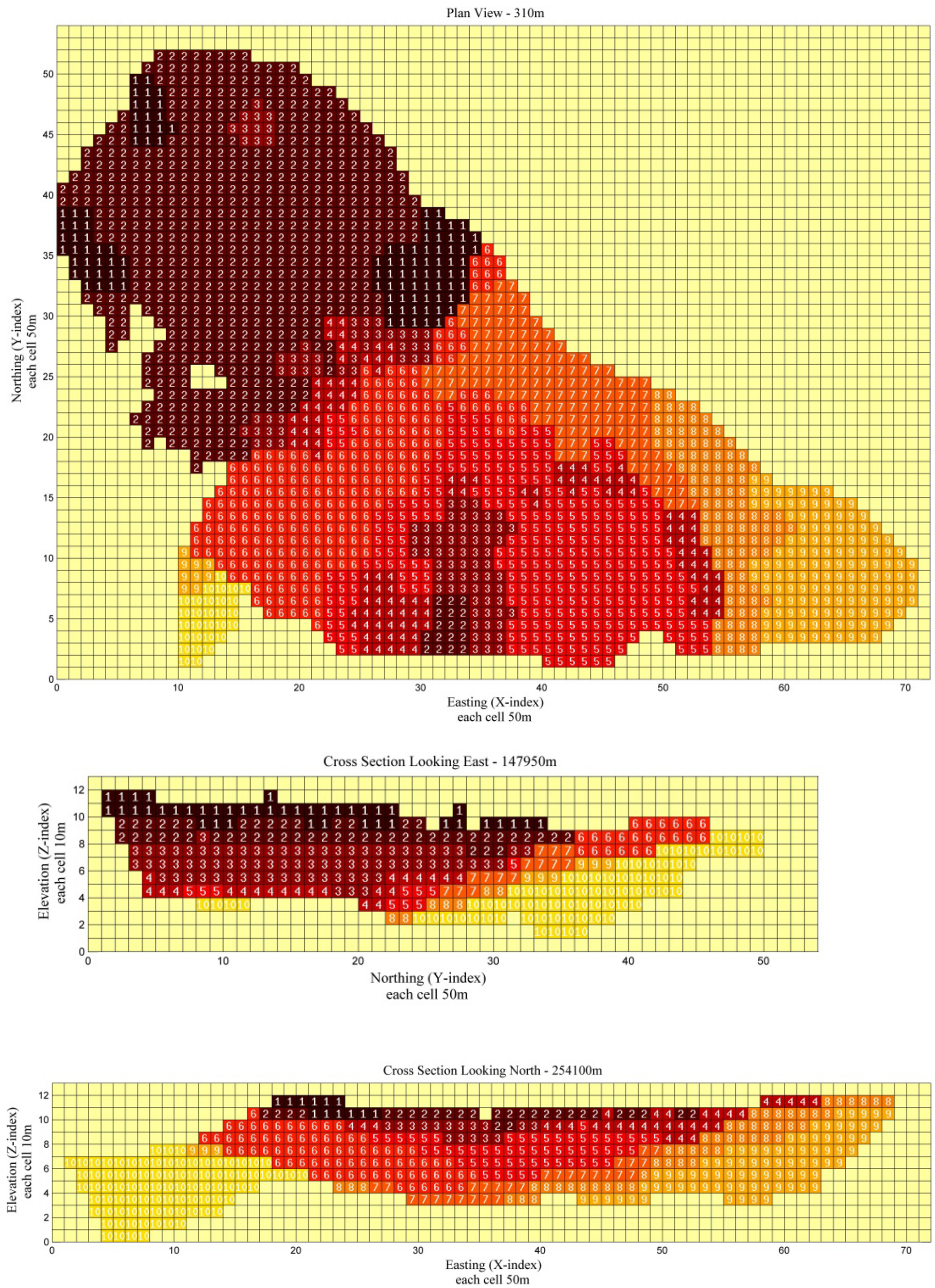

Fig. 16. Plan view, cross section looking east and north for generated schedual

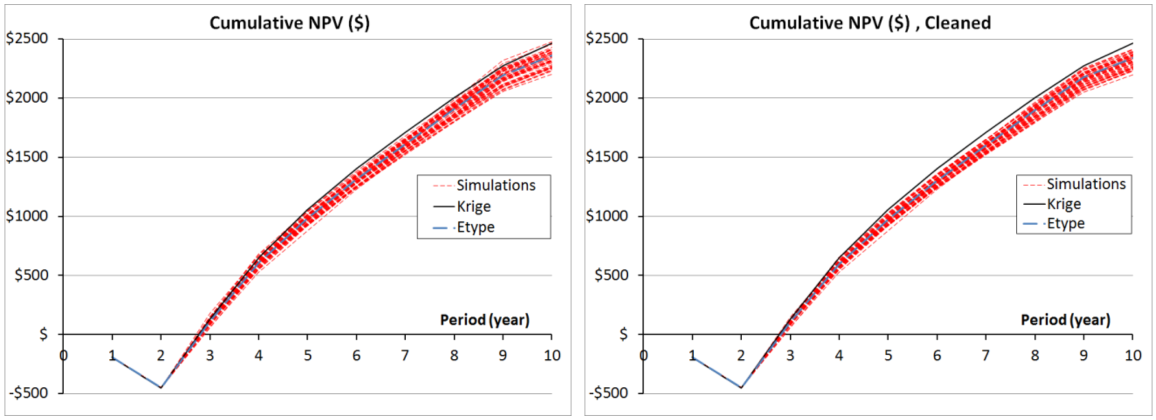

Fig. 17. Cumulated NPV over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

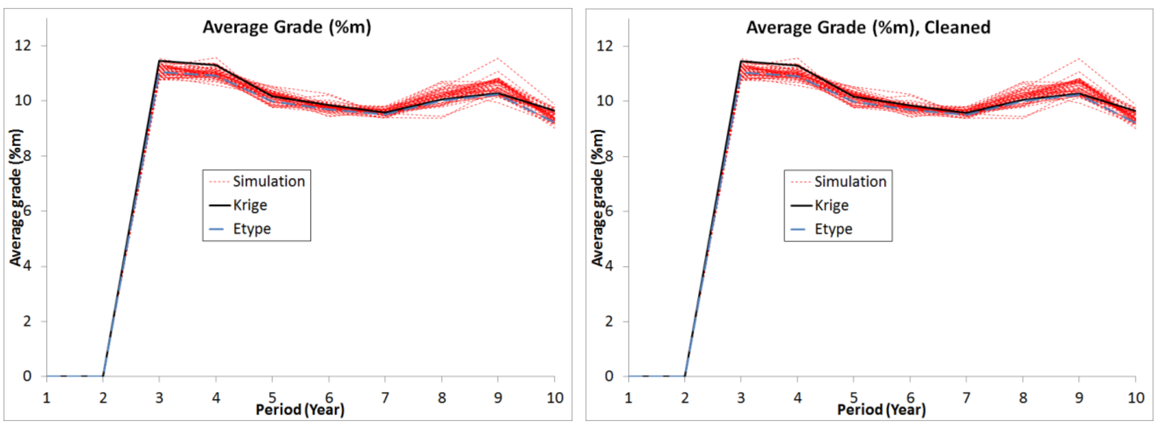

Fig. 18. Input head grade to the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

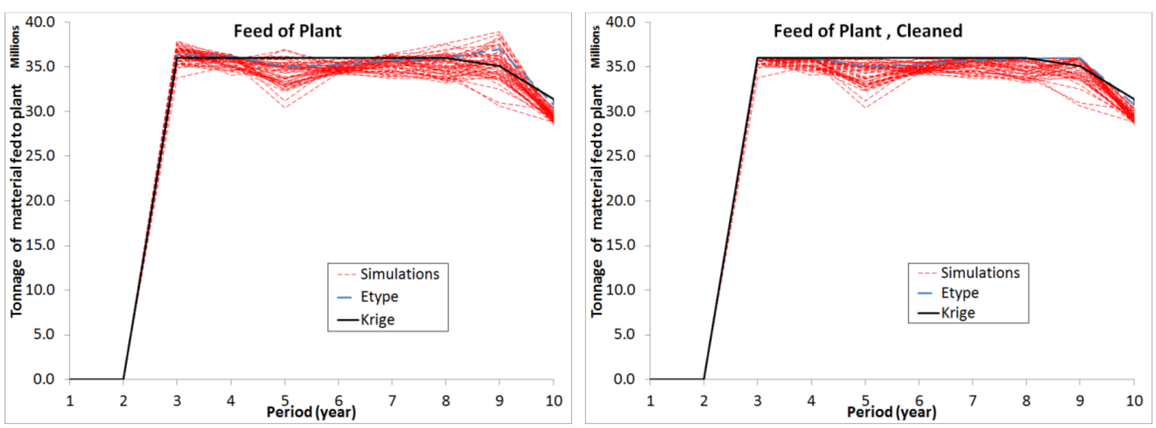

Fig. 19. Feed of the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), surplus ore not removed at left and cleaned version at right

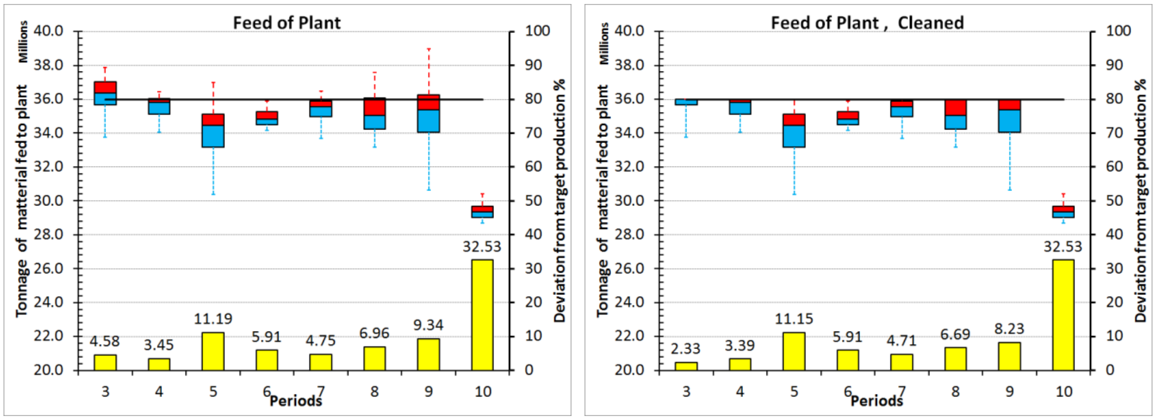

Fig. 20. Boxplot and deviation from target production (yellow bars), calculated using simualtion values, surplus ore not removed at left and cleaned version at right

Table 1. Final pit limit and mine planning input parameters.

Description	Value	Description	Value
Mining Cost ($/tonne)	4.6	Processing Cost ($/tonne)	0.5025
Cutoff grade (%mass bitumen)	6	Processing limit (M tonne/year)	36
Mining recovery fraction	0.88	Mining limit (M tonne/year)	67.5
Processing recovery factor	0.95	Overall slope (degrees)	20
Minimum mining width (m)	150	Pre-stripping (years)	2

Table 2. Material in the final pit using the kriged block model.

Description	Value
Total tonnage of material (M tonne)	653.61
Tonnage of ore (M tonne)	282.44
Tonnage of material below cutoff (M tonne)	37.4
Tonnage of waste (M tonne)	371.166
Bitumen recovered (M tonne)	23.29
Stripping ratio (waste:ore)	1.31

Table 3. Summary statistic of realization simulations when generated schedule with kriging is followed, surplus ore not removed at top and cleaned version at bottom

Row Version	Ore(MT)	STRO	Input Bitumen (MT)	Average %	NPV (M$)
Mean	276.2	1.4	28.3	10.2	2335.4
Std. dev	3.9	0.0	0.5	0.1	64.2
Min	269.2	1.3	27.3	10.0	2201.9
Quartile 1	273.3	1.3	27.9	10.2	2285.1
Median	276.4	1.4	28.3	10.2	2332.2
Quartile 2	278.7	1.4	28.6	10.3	2385.8
Max	287.3	1.4	29.4	10.5	2473.8
Krig	282.4	1.3	29.1	10.3	2461.0
Etype	282.2	1.3	28.5	10.1	2360.8

Cleaned Version	Ore(MT)	STRO	Input Bitumen (MT)	Average %	NPV (M$)
Mean	275.0	1.4	28.1	10.2	2317.5
Std. dev	3.1	0.0	0.4	0.1	56.0
Min	268.0	1.3	27.2	10.0	2195.8
Quartile 1	272.7	1.4	27.8	10.2	2267.7
Median	275.2	1.4	28.2	10.2	2322.7
Quartile 2	277.0	1.4	28.4	10.3	2363.0
Max	280.5	1.4	28.8	10.5	2415.6
Krig	282.4	1.3	29.1	10.3	2461.0
Etype	280.7	1.3	28.3	10.1	2341.5

Table 4. Summery statistics of Cumulative NPV at each period, surplus ore not removed at top and cleaned version at bottom.

Period	1	2	3	4	5	6	7	8	9	10
Mean	-193.2	-449.8	112.4	603.3	975.7	1,303.9	1,600.6	1,893.0	2,175.6	2,335.4
Std. dev	0.0	0.0	24.6	32.7	36.1	38.9	42.6	51.7	62.9	64.2
Min	-193.2	-449.8	58.8	525.9	879.5	1,228.8	1,522.8	1,794.7	2,055.0	2,201.9
Quartile 1	-193.2	-449.8	95.4	578.6	947.2	1,280.9	1,565.7	1,849.1	2,124.9	2,285.1
Median	-193.2	-449.8	110.6	604.1	975.3	1,297.5	1,602.6	1,904.7	2,169.6	2,332.2
Quartile 2	-193.2	-449.8	129.9	620.1	1,002.2	1,337.7	1,638.8	1,935.7	2,230.3	2,385.8
Max	-193.2	-449.8	180.6	679.3	1,049.9	1,378.2	1,678.9	1,987.6	2,319.8	2,473.8
Krig	-193.2	-449.8	132.0	650.6	1,054.0	1,403.1	1,707.8	2,005.4	2,274.7	2,461.0
Etype	-193.2	-449.8	110.5	606.3	982.7	1,313.2	1,611.0	1,905.4	2,194.3	2,360.8

Period	1	2	3	4	5	6	7	8	9	10
Mean	-193.2	-449.8	102.2	592.2	964.0	1,292.2	1,588.4	1,879.1	2,157.7	2,317.5
Std. dev	0.0	0.0	17.8	27.4	32.1	35.0	38.7	46.4	54.8	56.0
Min	-193.2	-449.8	58.8	525.8	879.5	1,228.8	1,520.7	1,792.7	2,048.9	2,195.8
Quartile 1	-193.2	-449.8	89.4	574.5	941.0	1,266.4	1,554.6	1,840.6	2,112.6	2,267.7
Median	-193.2	-449.8	102.1	593.6	962.3	1,290.7	1,585.8	1,889.9	2,161.6	2,322.7
Quartile 2	-193.2	-449.8	116.8	608.6	984.1	1,320.6	1,618.2	1,917.1	2,202.5	2,363.0
Max	-193.2	-449.8	144.5	654.0	1,030.6	1,360.8	1,655.3	1,964.5	2,255.1	2,415.6
Krig	-193.2	-449.8	132.0	650.6	1,054.0	1,403.1	1,707.8	2,005.4	2,274.6	2,461.0
Etype	-193.2	-449.8	103.4	596.2	972.5	1,303.0	1,600.8	1,895.3	2,175.0	2,341.5

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Block Cave Production Scheduling Optimization using Mixed Integer Linear Programming

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Abstract

Planning of caving operations poses complexities in different areas such as safety, environment, ground control, and production scheduling. As the mining industry is faced with more marginal resources, it is becoming essential to generate production schedules which will provide optimal operating strategies while meeting practical, technical, and environmental constraints.

Production scheduling of any mining operation has an enormous effect on the economics of the venture. The scheduling problems are complex due to the nature and variety of the constraints acting upon the system. Relying only on manual planning methods or computer software based on heuristic algorithms will lead to mine schedules that are not the global optimal solution.

The objective of this paper is to develop a practical optimization framework for production scheduling of caving operations. A mixed integer linear programming (MILP) formulation is developed, implemented and verified in TOMLAB/CPLEX environment. The production scheduler aims at maximizing the net present value (NPV) of the mining operation while the mine planner has control over the development rate, vertical mining rate (production rate per drawpoint), lateral mining rate (rate of opening new drawpoints), dilution entry, mining capacity, maximum number of active drawpoints, cave draw strategies and advancement direction, and draw rate. The production scheduler defines the opening and closing time of each drawpoint, the draw rate from each drawpoint, the number of new drawpoints that need to be constructed, and the sequence of extraction from the drawpoints to support a given production target. The successful application of the model for production scheduling of a real mine data is also presented.

1. Introduction

Long-term mine production scheduling is one of the optimization problems. A production schedule must provide a mining sequence that takes into account the physical limitations of the mine and, to the extent possible, meets the demanded quantities of each raw ore type in each period throughout the mine life.

Underground mining is more complex in nature than surface mining (Kuchta et al., 2004). Underground mining is less flexible than surface mining due to the geotechnical, equipment, and space constraints (Topal, 2008).

A number of optimization techniques have been used in the past; many include significant simplifications or fail to produce acceptable results within the required timeframe. In spite of the difficulties associated with the application of mathematical programming to production scheduling in underground mines, authors have attempted to develop methodologies to optimize production schedules. Mathematical programming is a generic term for a variety of optimization algorithms developed to solve different mathematical formulations. All share the combination of variables, constraints, and an objective function. The algorithms used to solve the variables all treat the problem as a multidimensional solution space. It also reduces complexity and uncertainty to a level that is manageable, providing a quantifiable basis for mine design and planning.

Williams et al. (1972) planned sublevel stoping operations for an underground copper mine over one year using a linear programming approximation model. Jawed (1993) formulated a linear goal programming model for production planning in an underground room and pillar coal mine. Tang et al. (1993) integrated linear programming with simulation to address scheduling decisions, as did Winkler (1998). Trout (1995) used the MIP method to schedule the optimal extraction sequence for underground sublevel stoping. Ovanic (1998) used mixed integer programming of type two special ordered sets to identify a layout of optimal stopes. Carlyle et al. (2001) presented a model that maximized revenue from Stillwater's platinum and palladium mine. Topal et al. (2003) generated a long-term production scheduling MIP model for a sub-level caving operation and successfully applied it to Kiruna Mine. Sarin et al. (2005) scheduled a coal mining operation with the objective of net present value maximization. Ataee-Pour (2005) critically evaluated some optimization algorithms according to their capabilities, restrictions and application for use in underground mining. McIssac (2005) formulated the scheduling of underground mining of a narrow veined polymetallic deposit utilizing MIP.

Scheduling of underground mining operations is primarily characterized by discrete decisions to mine blocks of ore, along with complex sequencing relationships between blocks. Since linear programming (LP) models cannot capture the discrete decisions required for scheduling, MIPs are generally the appropriate mathematical programming approach to scheduling.The methods currently used to compute production schedule in block cave mines can be classified in two main categories: (a) heuristic methods and (b) exact optimization methods.

Heuristic methods are particularly used to rapidly come to a solution that is hoped to be close to the best possible answer, or optimal solution. These methods are used when there is no known method to find an optimal solution under the given constraints.

The original heuristic methods were the manual draw charts used at the beginning of block caving. These methods evolved through use at Henderson mine where a way to avoid early dilution entry was described by constraining the draw profile to an angle of draw of 45 degrees (Dewolf, 1981). Heslop et al. (1981) described a volumetric algorithm to simulate the mixing along the draw cone. Carew (1992) described the use of a commercial package called PC-BC to compute production schedules at Cassiar mine. Diering (2000) showed the principles behind the commercial tool PC-BC to compute production schedules, providing several case studies where different draw methods have been applied depending on the ore body geometry and rock mass behavior.

The application of operation research methods to the planning of block cave mines was first described by Riddle (1976). This development intended to compute mining reserves and define the economic extent of the footprint. The final algorithm did not reflect the operational constraints of block caving described above since it worked with the block model directly instead of defining the concept of draw cone as an individual entity of the optimization process.

The first attempt to use mathematical programming in block cave scheduling was made by Chanda (1990) who implemented an algorithm to write daily orders. This algorithm was developed to minimize the variance of the milling feed in a horizon of three days. Guest et al. (2000) made another application of mathematical programming in block cave long term scheduling. In this case, the objective function was explicitly defined to maximize draw control behavior. However, the author stated that the implicit objective was to optimize NPV. There are two problems with this approach. The first one is that maximizing tonnage or mining reserves will not necessarily lead to maximum NPV. The second problem is the fact that draw control is a planning constraint and not an objective function. The objective function in this case would be to maximize tonnage, minimize dilution or maximize mine life.

Rubio (2002) developed a methodology that would enable mine planners to compute production schedules in block cave mining. He proposed new production process integration and formulated two main planning concepts as potential goals to optimize the long term planning process, thereby maximizing NPV and mine life.

Rahal et al. (2003) used a dual objective mixed integer linear programming algorithm to minimize the deviation between the actual state of extraction (height of draw) and a set of surfaces that tend towards a defined draw strategy. This algorithm assumes that the optimal draw strategy is known. Nevertheless, it is postulated that by minimizing the deviation to the draw target, the disturbances produced by uneven draw can be mitigated.

Diering (2004) presented a non-linear optimization method to minimize the deviation between a current draw profile and the target defined by the mine planner. He emphasized that this algorithm could also be used to link the short-term plan with the long-term plan. The long-term plan is represented by a set of surfaces that are used as a target to be achieved based on the current extraction profile when running the short-term plans. Rubio et al. (2004a) presented an integer programming algorithm and an iterative algorithm to optimize long-term schedules in block caving integrating the fluctuation of metal prices in time.

We critically reviewed the MILP formulations of the block cave production scheduling problem. We modeled the problem considering different possible. We divided the major decision variables into two categories, continuous variables representing the portion of a slice that is going to be extracted in each period and binary integer variables controlling the order of extraction of drawpoints and the number of active drawpoints in each period. We implemented the optimization formulation in TOMLAB/CPLEX (Holmstrom, 1989-2009) environment. A scheduling case study with real mine data was carried out over fifteen periods to verify the MILP model.

The next section of the paper covers the assumptions, problem definition, and the notations of variables. Section 3 presents mixed integer linear programming formulation of the problem, while section 4 presents the numerical modeling techniques. Section 5 presents an example, conclusions and future work followed by the list of references in the next section.

2. Assumptions, problem definition, and notation

We assume that a geological block model represents the orebody, which is a three-dimensional array of rectangular or cubical blocks used to model orebodies and other sub-surface structures.

The column of rock above each drawpoint, draw cone, is simulated and stored in a slice file. The draw cones, which are vertical, are created based on the block model and the total column is divided into slices which match the vertical spacing of the geological block model. Numerical data are used to represent each attribute of the orebody such as tonnage, densities, grade of elements, elevations, percentage of dilution, and economic data for each slice. Five basic drawpoint layouts which are being used at caving operations include continuous trough, herringbone, offset herringbone, Henderson or Z design, and the El Teniente or parallelogram (Brown, 2003). This research assumed that the physical layout of the production level is offset herringbone. There is the assumption of selective mining, meaning that based on the existing conditions either all the material in the draw cone or some part of it can be extracted.

Fig. 1 shows different steps from creation of the initial block model to creation of the slices. All stages before scheduling are done using GEMS and PCBC (GEMCOMSoftwareInternational, 2011). First of all, using GEMS a block model is created to provide a quantitative description of the rockmass including and surrounding the cave zone. Then drawpoint locations are defined and block model data is converted into drawpoint based data using PCBC. Afterwards, the slices are constructed for each drawpoint. These slices represent the draw column above each drawpoint before any extraction begins. The best height of draw (BHOD) for each draw cone is estimated.

The BHOD is the height which produces the best economic value and it is usually not discounted with time. The number of possibilities for finding the optimal height is equal to the number of slices above each drawpoint. A simple comparison of the dollar value for each combination (slice 1, then slices 1,2, then slices 1,2,3) allows the best height to be found. This process or technique is shown schematically in Fig. 2. The relevant dollar value of each number in the horizontal axis is equal to the summation of dollar values of slices 1 to that number. The maximum value, in this case, is obtained for slice number 33. If the height of each slice is h (m), the best height of draw for this drawpoint is 33h (m).

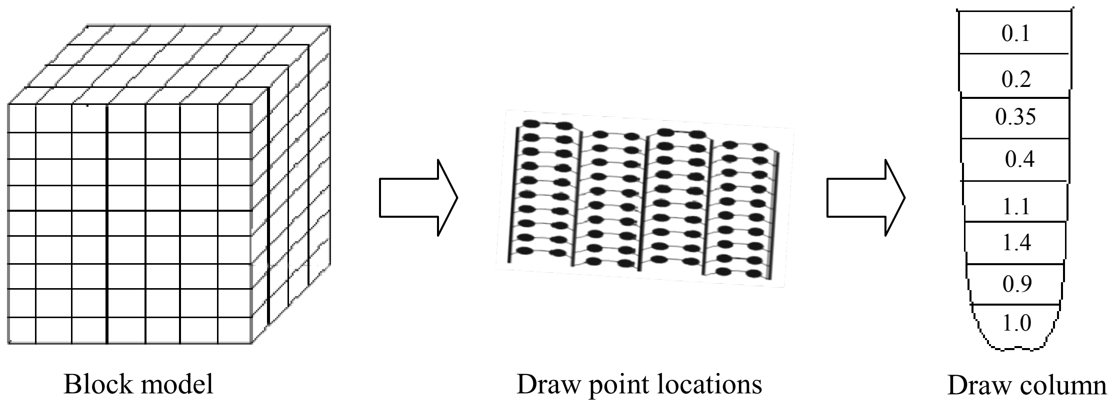


Fig. 1. Flow chart from initial block model to draw column.

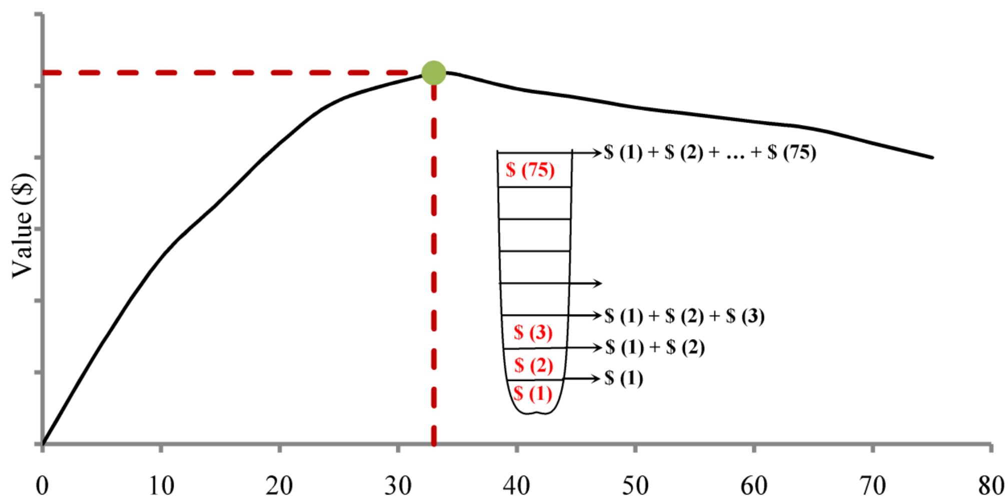


Fig. 2. Determination of the BHOD

After applying the BHOD, the final height of draw is obtained. Afterwards the production schedule of a block cave mine can be optimized using the MILP formulation. The problem is maximizing the net present value of the mining operation while the mine planner has control over the development rate, vertical mining rate (production rate per drawpoint), lateral mining rate (rate of opening new drawpoints), dilution entry, mining capacity, maximum number of active drawpoints, cave draw strategies and advancement direction, and draw rate. To solve the problem, four decision variables are employed, one continuous decision variable and three binary integer variables. Two of them are used in controlling slice level and two in controlling drawpoint level. The continuous decision variable indicates the portion of extraction from each slice in each period and three binary integer variables control the number of active drawpoints, precedence of extraction between slices and drawpoints, the opening and closing time of each drawpoint, the draw rate from each drawpoint, and the number of new drawpoints that need to be constructed in each period. This formulation is implemented for eight advancement directions to maximize the NPV. Fig. 3 illustrates these advancement directions.

According to the advancement direction for each drawpoint, *d*, there is a set S^d which defines the predecessor drawpoints among adjacent drawpoints that must be started prior to extraction of drawpoint *d*. Based on the search direction, eight different predecessor data sets can be defined for each drawpoint. Fig. 4 shows that in the offset herringbone layout, each drawpoint is surrounded by a maximum of seven drawpoints. The members of set S^d in each direction are determined using an imaginary line prependicular to the desired advancement direction at the location of the considered drawpoint. All located adjacent drawpoints behind the imaginary line are defined as members of set S^d in the considered advancement direction. For instance, Fig. 5a shows that adjacent drawpoints for drawpoint d4 include d1, d2, d3, d5, d6, d7, and d8. In advancement direction of North to South (NS), extraction of drawpoints d1, d2, and d3 have to be started prior to drawpoint d4. Fig. 5b shows the advancement direction of South West to North East (SW to NE), extraction of drawpoints d1, d6, and d7 have to be started prior to drawpoint d4.

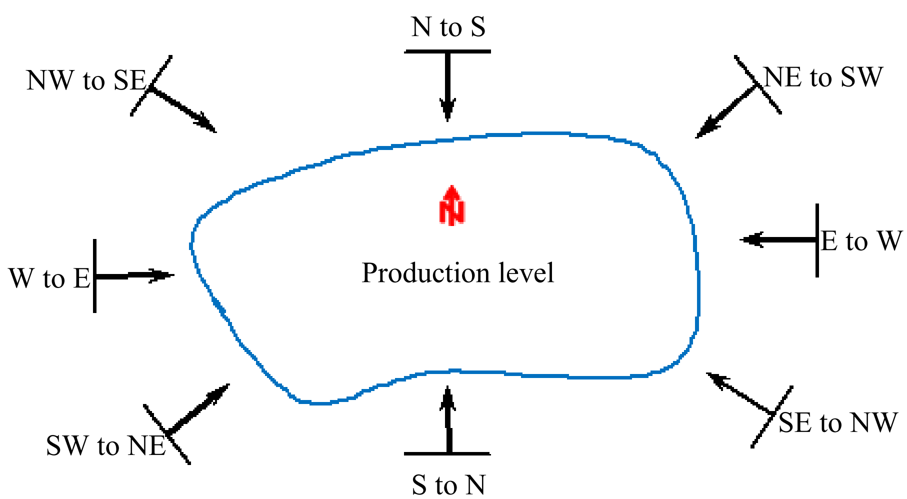


Fig. 3. Alternative cave advancement directions (Pourrahimian and Askari-Nasab, 2010)

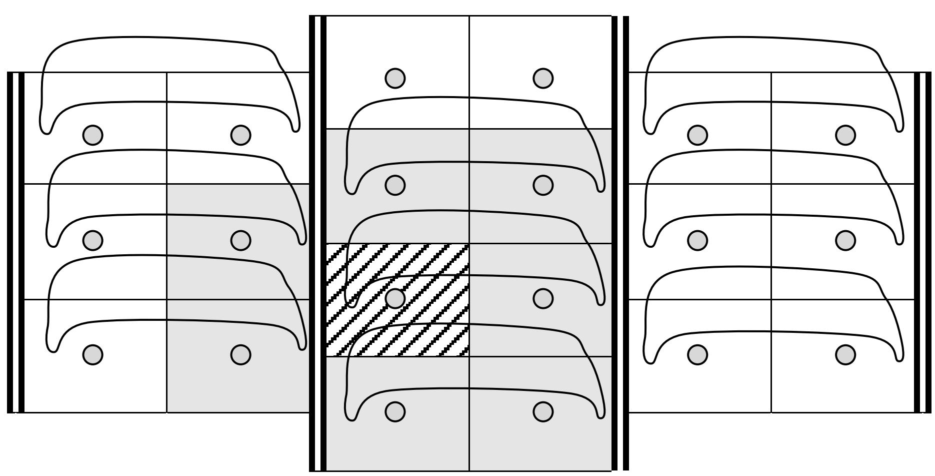


Fig. 4. Offset herringbone extraction level layout (after Brown, 2003)

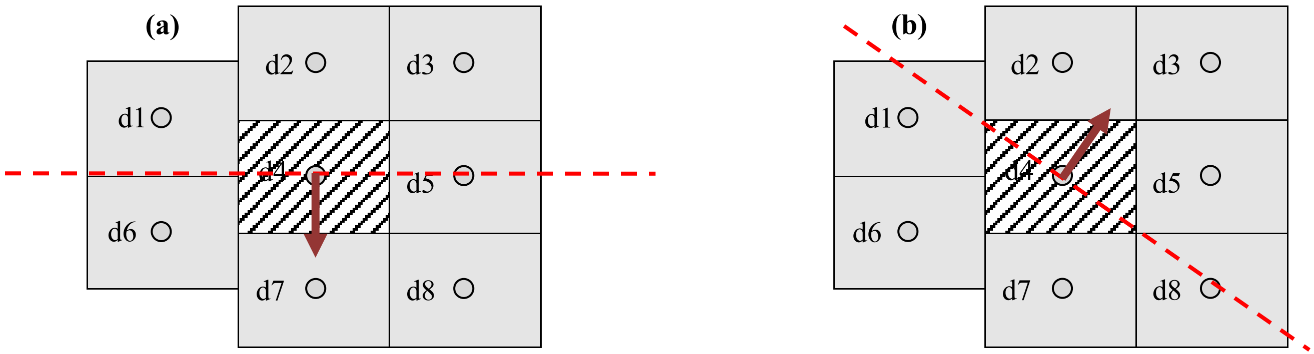


Fig. 5. Determination method of members for set S^d in different directions.

2.1. Notation

The notation of decision variables, parameters, sets, and constraints are as follows:

2.1.1. Sets

S^d	For each drawpoint, *d*, there is a set S^d defining the predecessor drawpoints that must be started prior to extraction of drawpoint *d*.
S^{ds}	For each drawpoint, *d*, there is a set S^{ds} defining the slices in draw cone associated with drawpoint *d*.
S^s	For each slice, *s*, there is a set S^s defining the predecessor slice that must be extracted prior to extraction of slice *s*.
S^{dls}	For each drawpoint, *d*, there is a set S^{dls} defining the lowest slice within the draw cone associated with drawpoint *d*.

2.1.2. Indices

$t \in \{1,....,T\}$	Index for scheduling periods.
$d \in \{1,...,D\}$	Index for drawpoints.
$s \in \{1,...,S\}$	Index for slices.
$e \in \{1,...,E\}$	Index for elements of interest in each slice.
m	Index for a slice belonging to one of the sets S^s or S^{dls}
j	Index for a drawpoint belonging to one of the sets S^d or S^{ds}

2.1.3. Parameters

SEV_s	Economic value of slice *s*.
i	The discount rate.
N_d	Number of drawpoints.
Ns_d	Number of slices within the draw cone associated with drawpoint *d*.
DC_d	Development and construction cost of drawpoint *d*.
SC_{ds}	Development and construction cost of slice s in the draw cone associated with drawpoint *d*.
N_{at}	Maximum number of active drawpoints in period *t*.
$\overline{N_{nt}}$	Upper limit of number of new drawpoints in period *t*.

$\underline{N_{nt}}$	Lower limit of number of new drawpoints in period t.
O_s	Total tonnage of ore in slice s.
W_s	Total tonnage of waste in slice s.
$O_s + W_s$	Total tonnage of material in slice s.
TD_d	Total tonnage of material in draw cone associated with drawpoint d.
G_{es}	Average grade of element e in ore portion of slice s.
$\overline{G_{et}}$	Upper limit of acceptable average head grade of element e in period t.
$\underline{G_{et}}$	Lower limit of acceptable average head grade of element e in period t.
$\overline{M_t}$	Upper limit of mining capacity in period t.
$\underline{M_t}$	Lower limit of mining capacity in period t.
DR_{dt}	Draw rate of drawpoint d in period t.
$\underline{DR_{dt}}$	Minimum possible draw rate of drawpoint d in period t.
$\overline{DR_{dt}}$	Maximum possible draw rate of drawpoint d in period t.
γ_d	Density of material in drawpoint d.

2.1.4. Decision variables

$X_{st} \in [0,1]$	Continuous variable, representing the portion of slice s to be extracted in period t.
$E_{dt} \in \{0,1\}$	Binary integer variable controlling the starting period of drawpoints and the precedence of extraction of drawpoints. E_{dt} is equal to one if extraction of drawpoint d has started by or in period t, otherwise it is zero.
$C_{dt} \in \{0,1\}$	Binary integer variable controlling the closing period of drawpoints. C_{dt} is equal to one if extraction of drawpoint d has finished by or in period t, otherwise it is zero.
$B_{st} \in \{0,1\}$	Binary integer variable controlling the precedence of extraction of slices. It is equal to one if extraction of slice s has started by or in period t, otherwise it is zero.

3. Mathematical model

A mixed integer linear programming (MILP) problem contains both integer and continuous variables and there are no quadratic terms in the objective function.

3.1. Objective function

The objective function of the MILP formulation is to maximize the net present value of the mining operation. The profit from mining a drawpoint depends on the value of the slices and the costs incurred in mining.

The objective function, Eq.(1), is composed of the slice economic value (SEV), discount rate, slice cost, and a continuous decision variable that indicates the portion of a slice which is extracted in each period. The most profitable slices will be chosen to be part of the production call in order to optimize the NPV.

In Eq.(1), construction cost of drawpoint *d* is divided among the slices in the draw cone associated with drawpoint *d*. For example, if there are 15 slices in the draw cone associated with drawpoint number 20 and drawpoint development and construction cost of this drawpoint is \$ DC_{20}, the cost of each slice within the relevant draw cone is given by Eq.(2).

$$\text{Maximize} \quad \sum_{t=1}^{T}\sum_{s=1}^{S}\left(\frac{SEV_s}{(1+i)^t} - SC_s\right)\times X_{st} \tag{1}$$

$$SC_{20s} = \frac{\$DC_{20}}{15} \qquad s \in S^{20s} \tag{2}$$

3.2. Constraints

3.2.1. Mining capacity

This constraint forces mining system to achieve desired mining capacity. It is applied using inequalities in Eq.(3), which ensures that the total tonnage of material extracted from drawpoints in each period is within the acceptable range that allows flexibility for potential operational variations.

$$\underline{M_t} \le \sum_{s=1}^{S}(O_s + W_s)\times X_{st} \le \overline{M_t} \qquad \forall t \in \{1,...,T\} \tag{3}$$

3.2.2. Grade blending

This constraint forces the mining system to achieve the desired grade. The average grade of the element of interest has to be greater than or equal to a certain value, $\underline{G_{et}}$, and less than or equal to a certain value, $\overline{G_{et}}$, for each period *t*. It is applied using inequalities in Eq.(4), which ensure that the average grade of production is within the desired range in each period.

$$\underline{G_{et}} \le \frac{\sum_{s=1}^{S} G_{es}\times(O_s + W_s)\times X_{st}}{\sum_{s=1}^{S}(O_s + W_s)\times X_{st}} \le \overline{G_{et}} \qquad \forall t \in \{1,...,T\}, e \in \{1,...,E\} \tag{4}$$

3.2.3. Maximum number of active drawpoints and continuous extraction from draw cone

During the mine life, each drawpoint can be in three different situations: open, active, and close. Fig. 6 illustrates how the situations change. In each period, we need to know the number of active drawpoints because this number must not exceed the allowable number. This constraint controls the maximum number of active drawpoints at any given period of the schedule.

As an example, Fig. 7 shows that the draw cone associated with drawpoint *d* contains four slices. The starting period of extraction of drawpoint *d* can be controlled by the lowest slice. This means, extraction of drawpoint *d* is started by extraction of the relevant lowest slice. When extraction of the last portion of a slice is finished in period *t*, extraction of the above slice can be started in the period *t* or *t+1*. In other words, extraction of a slice can be started if the below slice is totally extracted. If extraction of a slice is not started after finishing the extraction of the below slice in period *t* or *t+1*, the relevant drawpoint must be closed. Fig. 8 shows values of variables when extraction of a slice is finished. The mentioned concept is applied using inequalities in Eqs.(5), (6), (7), and (8). Eq.(7) ensures that when drawpoint *d* is open, at least a portion of one of the slices within the draw cone associated with drawpoint *d* is extracted otherwise the drawpoint must be closed. This means extraction must be continuous otherwise the drawpoint will be closed. Parameter *L* in Eq.(7) must be a big enough number. Eqs.(6) and (8) ensure that when variables

E_{dt} and C_{dt} change to one, they remain one until the end of the mine life. This helps us to recognize the periods when the drawpoint is active. Fig. 9 shows the relationship between opening time, closing time, and active time. Eq. (9) controls the maximum number of active drawpoints in each period. N_{at} should be given as an input to the algorithm.

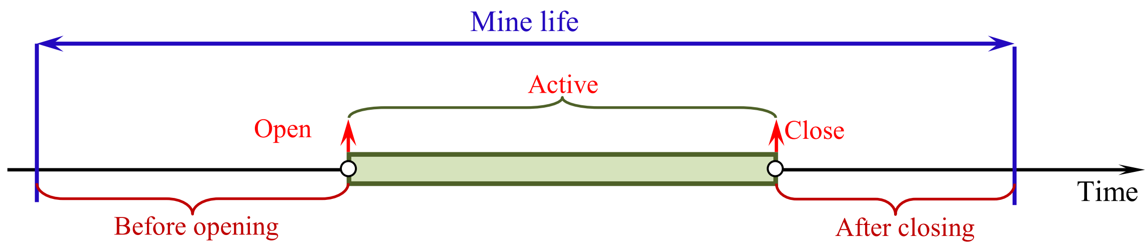


Fig. 6. Changes of drawpoint situation during the mine life

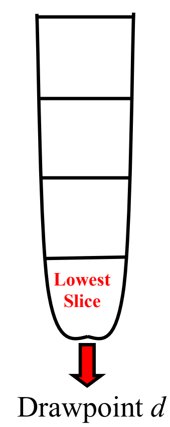


Fig. 7. Extraction time of the lowest slice is equivalent to drawpoint starting period.

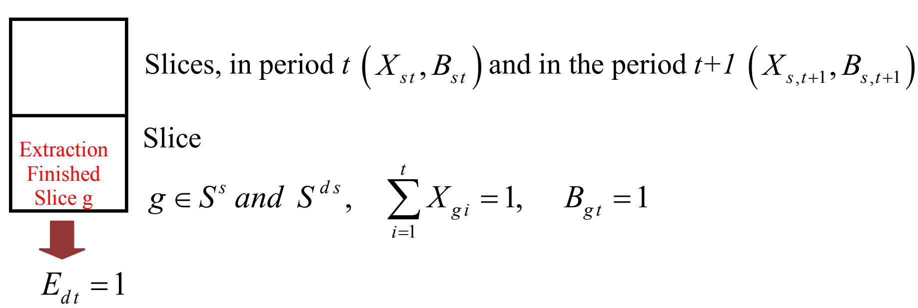


Fig. 8. Value of variables when extraction of a slice is finished.

$$X_{st} - E_{dt} \le 0 \qquad \forall t \in \{1,...,T\},\ d \in \{1,...,D\},\ s \in S^{dls} \tag{5}$$

$$E_{dt} - E_{d(t+1)} \le 0 \qquad \forall t \in \{1,...,T\},\quad d \in \{1,...,D\} \tag{6}$$

$$E_{dt} - C_{dt} \le L \times \sum X_{mt} \qquad \forall t \in \{1,...,T\}, d \in \{1,...,D\},\ m \in S^{ds} \tag{7}$$

$$C_{dt} - C_{d(t+1)} \le 0 \qquad \forall t \in \{1,...,T\},\quad d \in \{1,...,D\} \tag{8}$$

$$\sum_{d=1}^{D}(E_{dt}-C_{dt})\le N_{at} \qquad \forall t\in\{1,...,T\} \qquad (9)$$

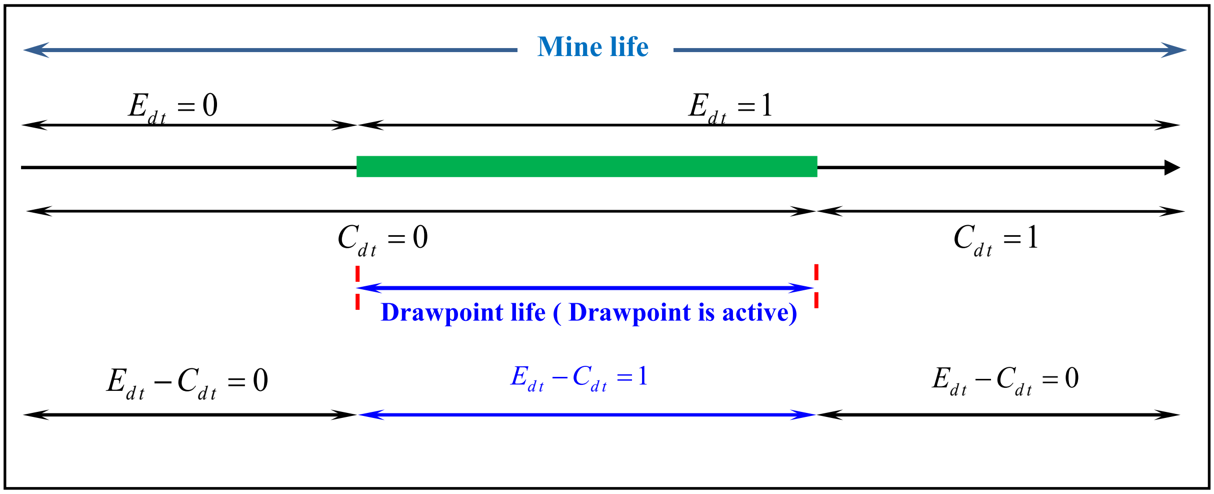


Fig. 9. Drawpoint activity duration based on opening and closing periods

3.2.4. Precedence

- Drawpoints

These constraints control the extraction precedence of drawpoints. Eq. (10) ensures that all drawpoints belonging to the relevant set, S^d, have been started prior to extraction of drawpoint *d*. This set is defined based on the selected mining advancement direction. This set can be empty, which means the considered drawpoint can be extracted in any time period in the schedule. Eq.(10) ensures that only the set of immediate predecessor drawpoints need to be started prior to starting the drawpoint under consideration.

$$E_{jt}-E_{dt}\le 0 \qquad \forall d\in\{1,...,D\}, t\in\{1,...,T\},\ j\in S^d \qquad (10)$$

- Slices

Extraction of slice, *s*, can be started if the slice below it has been extracted totally. Fig. 10 shows that for each slice except the lowest, there is a set S^s defining the predecessor slice that must be extracted prior to extraction of slice *s*. The extraction precedence of slice within each draw cone is controlled by Eqs.(11), (12) and (13). Eqs. (11) and (12) ensure that extraction of slice belonging to the relevant set, S^s, has been finished prior to extraction of slice *s*. Eq.(14) ensures that slice *s* is extracted when the relevant drawpoint is active.

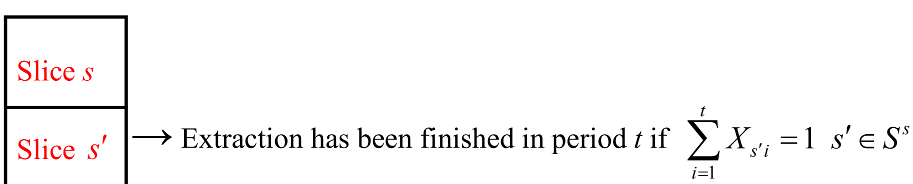


Fig. 10. Sequence of extraction between slices

$$B_{st} - \sum_{i=1}^{t} X_{mi} \le 0 \qquad \forall s \in \{1,...,S\}, t \in \{1,...,T\}, m \in S^{s} \tag{11}$$

$$\sum_{i=1}^{t} X_{si} - B_{st} \le 0 \qquad \forall s \in \{1,...,S\}, t \in \{1,...,T\} \tag{12}$$

$$B_{st} - B_{s(t+1)} \le 0 \qquad \forall s \in \{1,...,S\}, t \in \{1,...,T\} \tag{13}$$

$$\frac{\sum X_{mt}}{Ns_{d}} \le E_{dt} - C_{dt} \qquad \forall d \in \{1,...,D\}, t \in \{1,...,T\}, m \in S^{ds} \tag{14}$$

3.2.5. Number of new drawpoints (Development rate)

This constraint defines the maximum feasible number of drawpoints to be opened at any given time within the scheduled horizon. This constraint is usually based on the footprint geometry, the geotechnical behavior of the rock mass and the existing infrastructure of the mine, which will typically define available mining faces.

The drawpoint opening is controlled by the variable E_{dt}, which takes a value of one from the opening period to end of the mine life. From period two to the end of the mine life, the difference between the summation of opened drawpoints until and including period *t*, $\sum_{d=1}^{D} E_{dt} \;\; t \in \{2,...,T\}$, and the summation of opened drawpoints until and including previous period *t-1*, $\sum_{d=1}^{D} E_{d(t-1)} \;\; t \in \{2,...,T\}$, indicates the number of new drawpoints. Eq.(15) ensures that the number of new drawpoints which are opening in each period except period one is within the acceptable range. Eq. (16) ensures that in period one the number of new drawpoints is equal to the number of active drawpoints.

$$\underline{N_{nt}} \le \sum_{d=1}^{D} E_{dt} - \sum_{d=1}^{D} E_{d(t-1)} \le \overline{N_{nt}} \qquad \forall t \in \{2,...,T\} \tag{15}$$

$$\sum_{d=1}^{D} E_{d1} \le N_{a1} \tag{16}$$

3.2.6. Reserves

Eq.(17) ensures that there is selective mining for the slices, and thereby based on the existing conditions either all the material in the draw cone or some part of that can be extracted.

$$\sum_{t=1}^{T} X_{st} \le 1 \qquad \forall s \in \{1,...,S\} \tag{17}$$

3.2.7. Draw rate

This constraint controls the maximum and minimum rate of draw and is a function of fragmentation and capability. This rate should be fast enough to avoid compaction and slow enough to avoid air gaps. The maximum limit to the draw rate is usually determined by the fragmentation process since time is required to achieve good fragmentation. However, sometimes, the maximum rate may be determined by the LHD productivity. Inequalities in Eq. (18) ensure that the draw rate from each drawpoint is within the desired range in each period.

$$\left(E_{dt}-C_{dt}\right).\underline{DR_{dt}} \leq \sum\left(O_m+W_m\right).X_{mt} \leq \overline{DR_{dt}} \qquad \forall d \in \{1,...,D\}, t \in \{1,...,T\}, m \in S^{ds} \qquad (18)$$

4. Numerical modeling

To solve linear programming problems in which the variables of the objective function are continuous in the mathematical sense, with no gaps between real values, ILOG CPLEX implements optimizers based on simplex algorithms (Winston, 1995) (both primal and dual simplex) as well as primal-dual logarithmic barrier algorithms.

The branch-and-cut method is an efficient way for solving combinatorial optimization problems that are formulated as mixed integer linear programming problems. It is an exact algorithm which combines cutting plane and the branch-and-bound algorithms. It works by solving a sequence of linear programming relaxations of the IP problem. The cutting plane improves the relaxation of the problem to a closer approximation (Horst and Hoang, 1996).

In this study we used TOMLAB/CPLEX version 12.1.0 (Holmström, 1989-2009) as the MILP solver. TOMLAB/CPLEX efficiently integrates the solver package CPLEX (ILOGInc, 2007) with a MATLAB environment(MathWorksInc, 2007).

Table 1 represents the required number of decision variables for the proposed MILP formulation as a function of the number of drawpoints, N_d, number of slices, N_s, and number of scheduling periods, T. Thus, the number of continuous and binary decision variables for each problem are $\left(N_s \times T\right)$ and $\left[T \times \left(2N_d + N_s\right)\right]$, respectively.

Table 1. Number of decision variables in the presented formulation

Variable	Type	Relevant level	Number of variables
X_{st}	continuous	Slice	$N_s \times T$
B_{st}	binary	Slice	$N_s \times T$
E_{dt}	binary	Drawpoint	$N_d \times T$
C_{dt}	binary	Drawpoint	$N_d \times T$

5. Illustrative example

The presented model has been implemented and tested in TOMLAB/CPLEX environment. It was verified based on a real data set containing 20 drawpoints. There were 607 slices in the primary slice data file which was reduced to 324 after calculating the BHOD. Fig. 11 illustrates tonnage and grade distribution of the 324 slices. The grade of copper varies between 0.7and 1.5 percent and the majority of slices are more than 6000 tonnes. The total tonnage of material within drawpoints is almost 2.08 Mt. Fig. 12 shows a plan view of the drawpoints and tunnels based on relevant coordinates. Fig. 13 illustrates a 3D view of draw columns. As can be seen, southern draw columns are taller than northern ones.

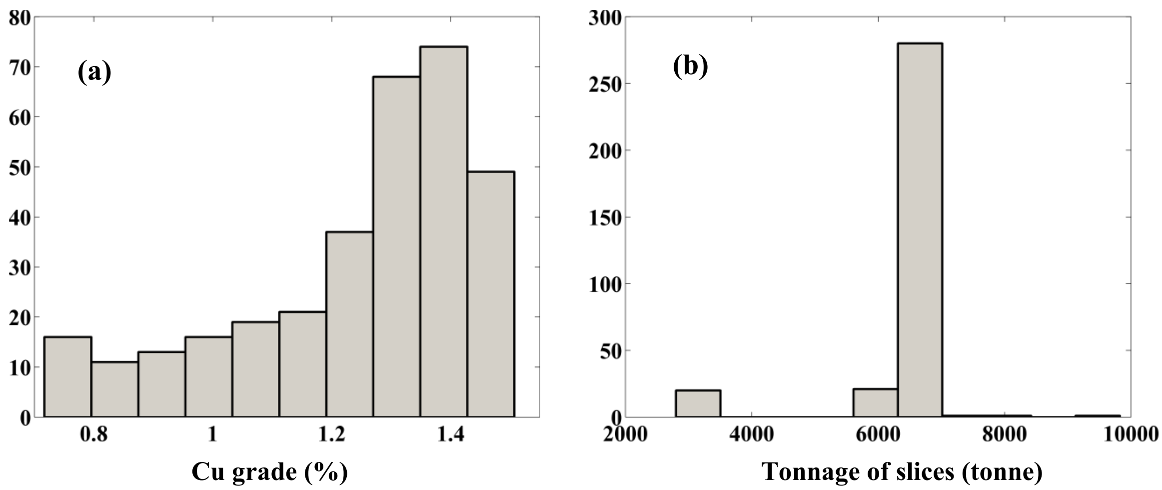


Fig. 11. Tonnage and grade distribution of slices

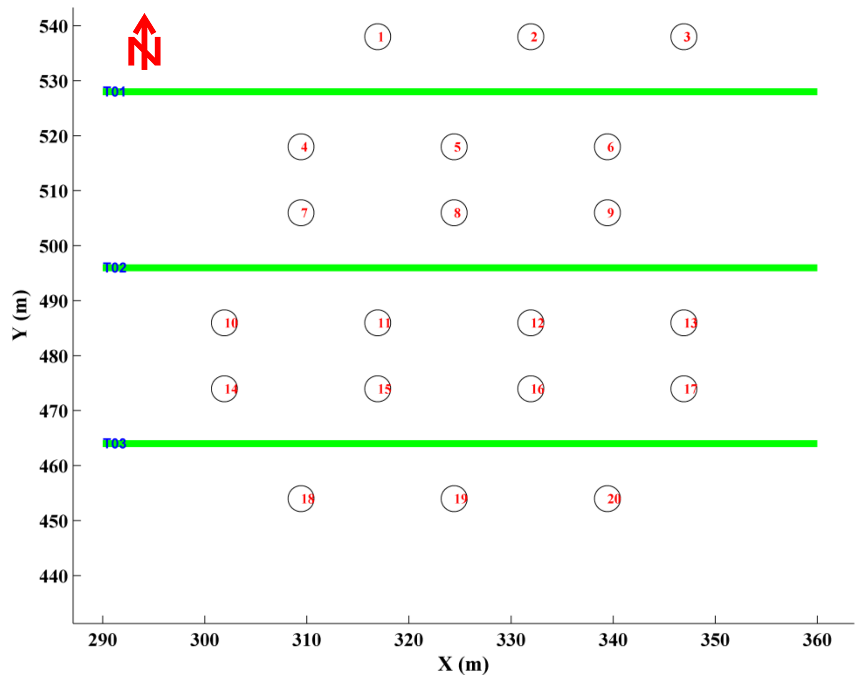


Fig. 12. Plan view of drawpoints and tunnels

6. Results and discussion

The presented MILP formulation was implemented for all mining advancement directions to find the optimum production schedule. Table 2 shows CPU time and size of the problem to test the MILP model for 20 drawpoints over 15 periods of extraction along the different advancement directions. The same scheduling parameters were used as input to solve the problem in the different advancement directions (see Table 3)

Table 3 shows scheduling parameters and obtained results for each advancement direction. Fig. 14 shows that difference between the highest and the lowest NPVs is more than five percent. The maximum NPV is obtained in the South to North direction. Fig. 15 to Fig. 22 show that all assumed constraints have been satisfied. Fig. 15 illustrates average grade of production for each period along the different advancement directions.

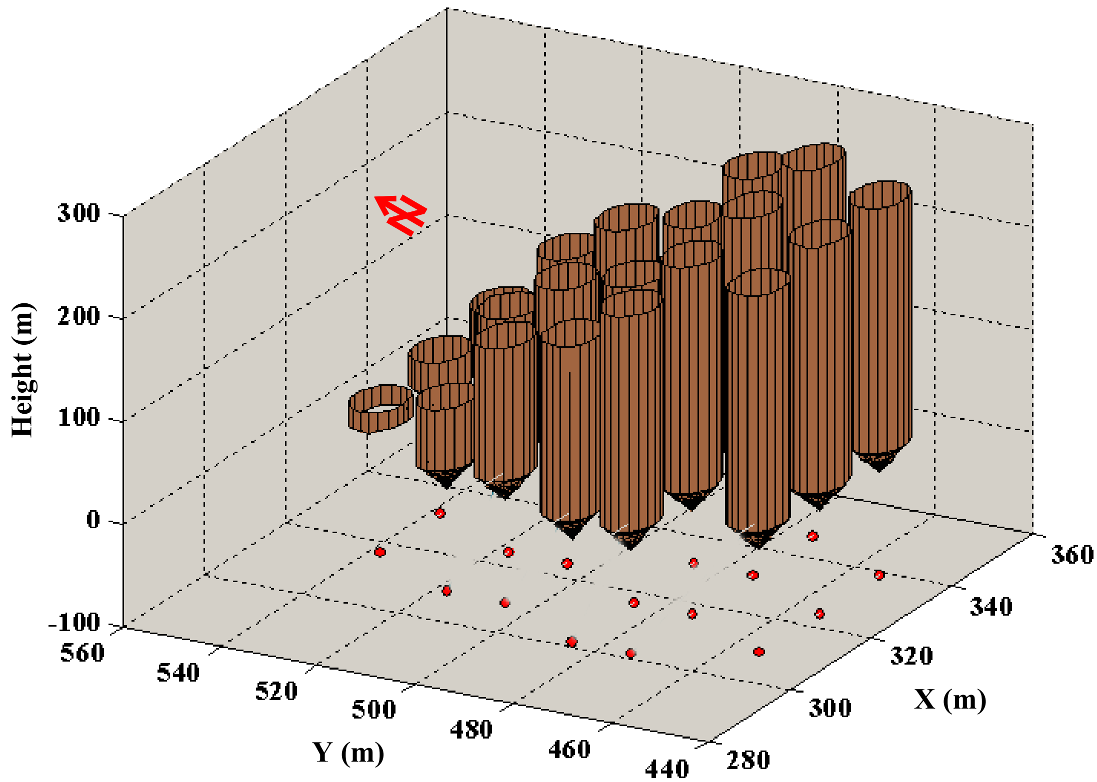


Fig. 13. 3D view of draw columns

Table 2. Numerical results for the solution time and problem size

Direction	CPU time (s)	Size of matrix A (row×col)	Continuous variables	Binary variables
West to East	208	17030 × 10320	4860	5460
East to West	1343.7	17030 × 10320	4860	5460
North to South	2644.7	16925 × 10320	4860	5460
South to North	2188.1	16925 × 10320	4860	5460
South East to North West	2380.3	16925 × 10320	4860	5460
North West to South East	95.3	16925 × 10320	4860	5460
South West to North East	308.27	16940 × 10320	4860	5460
North East to South West	403.45	16940 × 10320	4860	5460

In South to North direction in Fig. 15, it can be seen that the formulation tries to extract high grade slices earlier than others. Fig. 16 shows that tonnage of extraction in South to North direction during the first 11 years is equal to the upper bound of production that has been set up as a scheduling parameter.

Table 3. Scheduling parameters and obtained net present value

Advancement Direction	$\underline{G_{et}} / \overline{G_{et}}$ (%)	$\underline{M_t} / \overline{M_t}$ (t)×10^3	$\underline{DR_{dt}} / \overline{DR_{dt}}$ (mm/day)	N_{at}	$\underline{N_{nt}} / \overline{N_{nt}}$	NPV ($M)
WE	0.5 / 1.9	100/139	50 / 300	7	0 / 3	2.8074
EW	0.5 / 1.9	100/139	50 / 300	7	0 / 3	2.7649
NS	0.5 / 1.9	100/139	50 / 300	7	0 / 3	2.6936
SN	0.5 / 1.9	100/139	50 / 300	7	0 / 3	2.8497
SE-NW	0.5 / 1.9	100/139	50 / 300	7	0 / 3	2.8019
NW-SE	0.5 / 1.9	100/139	50 / 300	7	0 / 3	2.8033
SW-NE	0.5 / 1.9	100/139	50 / 300	7	0 / 3	2.8280
NE-SW	0.5 / 1.9	100/139	50 / 300	7	0 / 3	2.7210

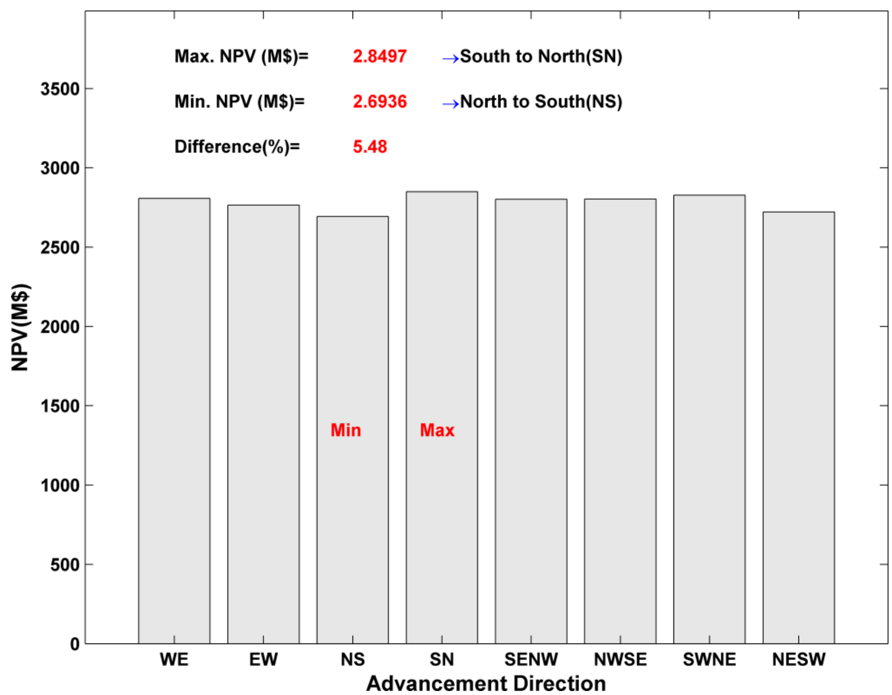


Fig. 14. Amount of NPV for different directions over 15 years scheduling horizon

Fig. 17 illustrates the maximum number of active drawpoints in each period for the different advancement directions. It can be seen that this constraint has been satisfied for all directions. In South to North direction, the mine works with the maximum allowable number of active drawpoints except for years one, six and fourteen. Fig. 18 illustrates the number of new drawpoints that are opened in each period for all advancement directions.

According toIn South to North direction in Fig. 15, it can be seen that the formulation tries to extract high grade slices earlier than others. Fig. 16 shows that tonnage of extraction in South to North direction during the first 11 years is equal to the upper bound of production that has been set up as a scheduling parameter.

Table 3, the number of new drawpoints that can be opened in each period varies between 0 and 3. As can be seen in Fig. 18, the upper bound is equal to 3 for all periods except period one. In period one, the number of new drawpoints that can be opened is equal to the maximum number of active drawpoints because at the beginning, we need more flexibility to reach the target grade and production level.

Fig. 18 shows a striking difference in the number of new drawpoints in the different advancement directions. In the South to North direction, extraction starts with six new drawpoints in the first period then the number of new drawpoints falls sharply to two in the second period and it remains almost constant until the end of the mine life. Extraction of all drawpoints is started before period 15 and there is no new drawpoint in this period.

Fig. 19 shows the production schedule for South to North direction. It illustrates the starting and finishing periods of extraction and the drawpoints life during the mine life. The ID number of active drawpoints and new drawpoints can be obtained by comparing Fig. 19 and the South to North direction of Fig. 17 and Fig. 18. For instance, according to the South to North direction of Fig. 17 and Fig. 18, there are seven active drawpoints and one new drawpoint in period five. In Fig. 19, it can be seen that the ID number of active drawpoints include DP7, DP10, DP11, DP15, DP16, DP19 and DP20 and the new drawpoint which has to be opened is DP7.

Fig. 20 shows the draw rate for drawpoints DP8 and DP11 in the South to North direction. It can be seen that the defined upper and lower bounds for both selected drawpoints have been satisfied. Fig. 20a shows the fluctuation in the draw rate of DP8; in contrast, Fig. 20b shows that DP11 has an almost uniform draw rate. This situation happened because of constant numbers which have been defined as upper and lower bounds. The formulation tries to extract material from drawpoints with a draw rate within the acceptable range without considering a specific shape.

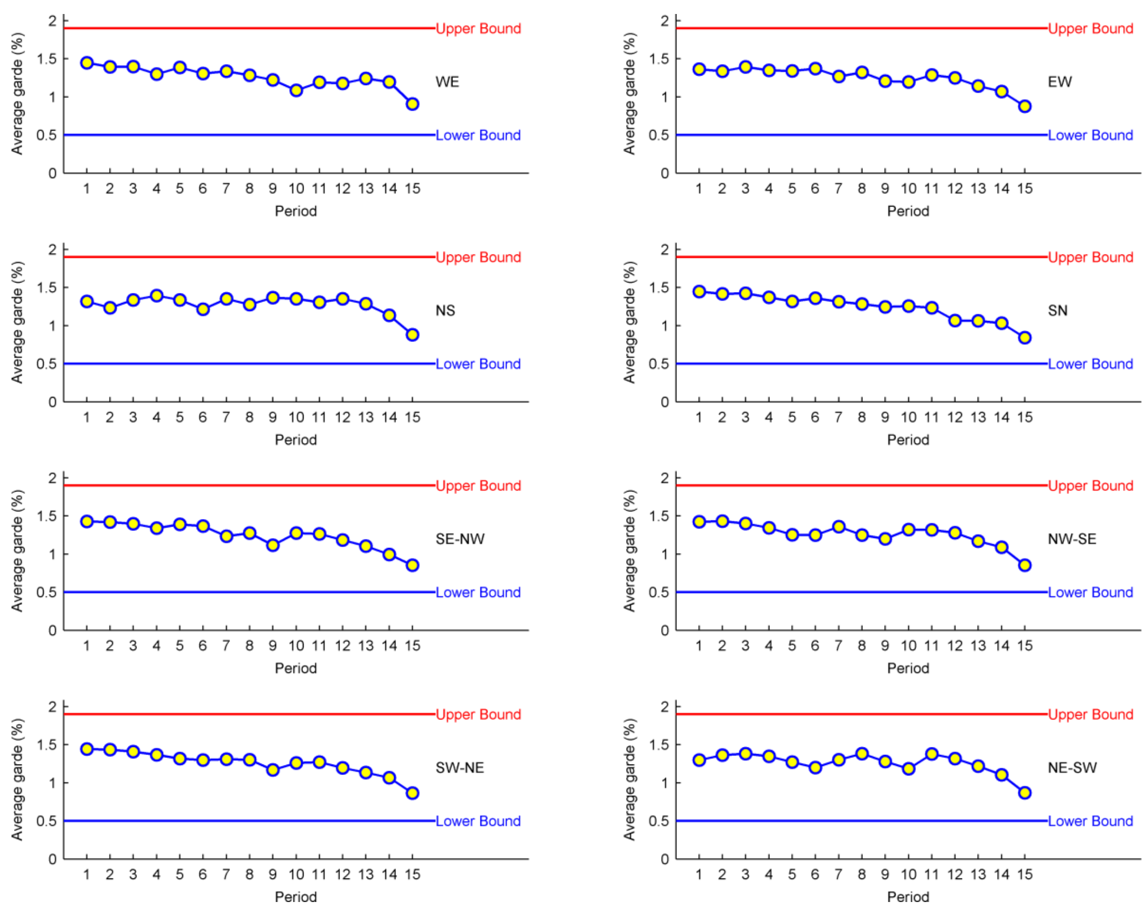


Fig. 15. Average grade of production for different advancement directions over 15 periods

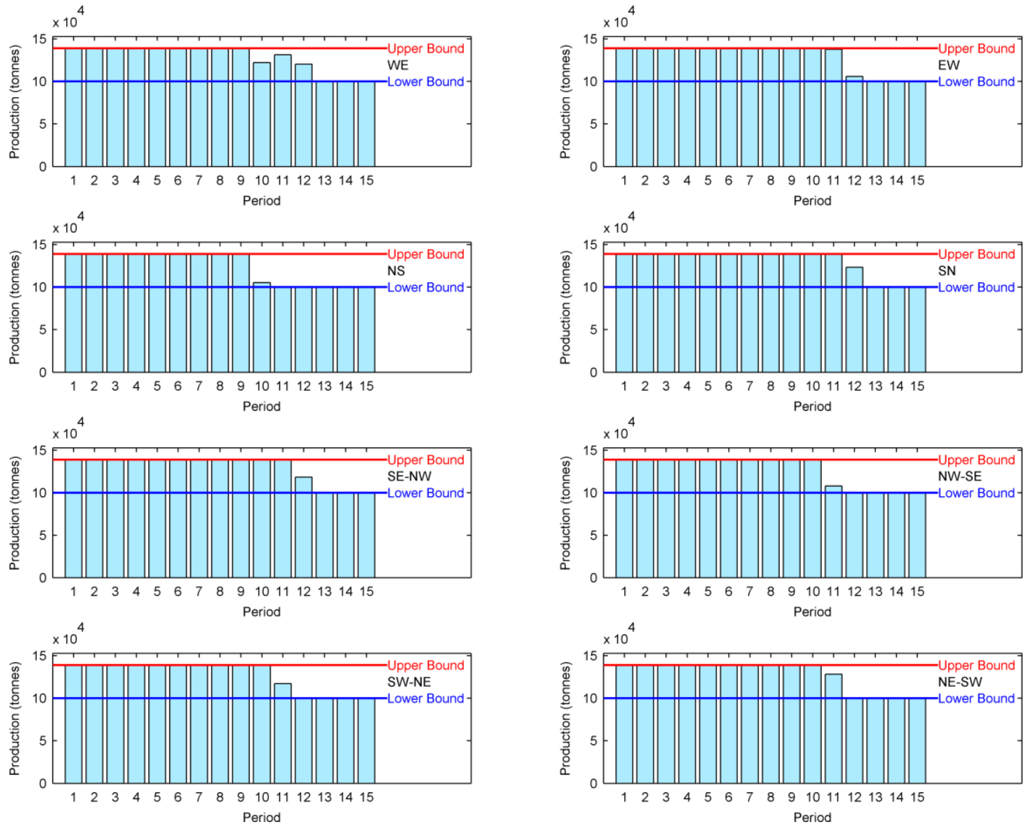


Fig. 16. Tonnage of production for different advancement directions over 15 periods

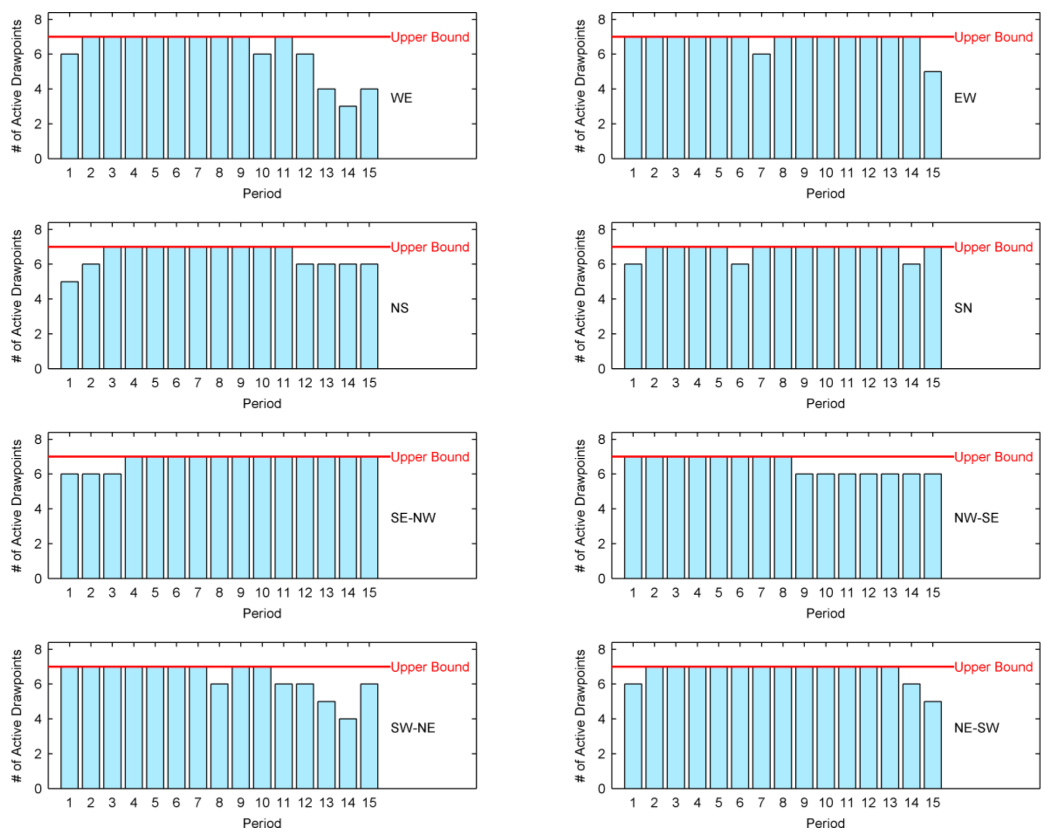


Fig. 17. Maximum number of active drawpoints for different advancement directions over 15 periods

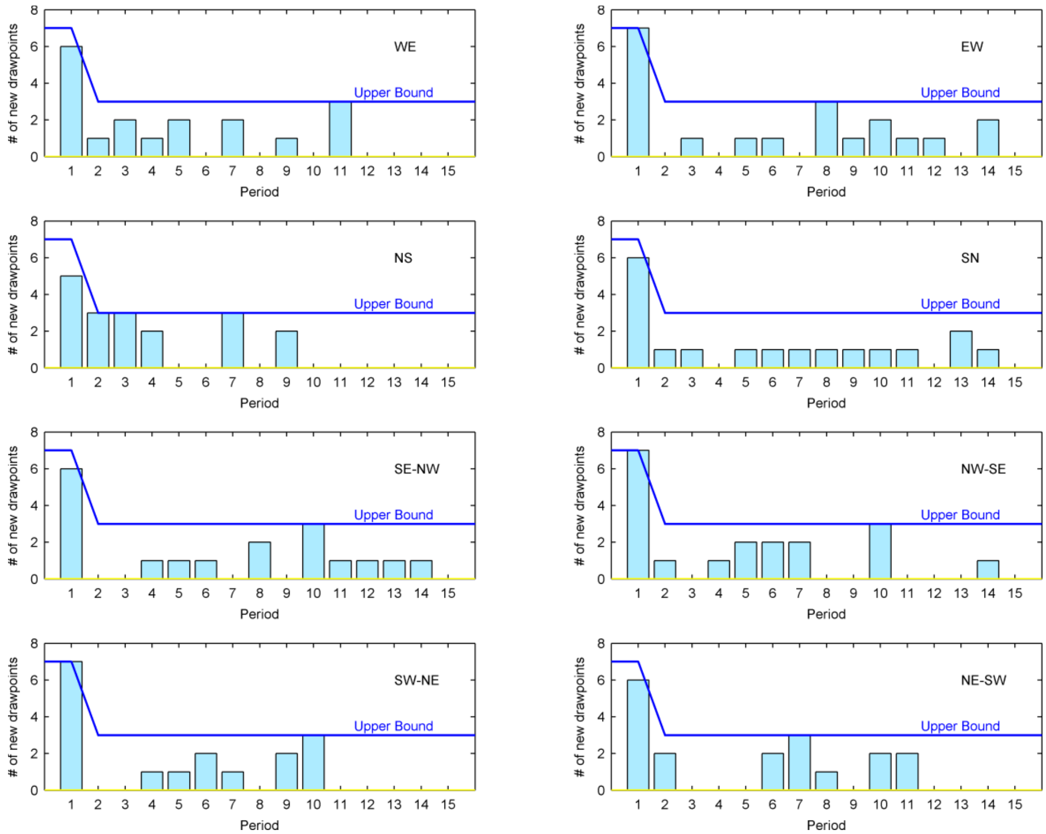


Fig. 18. Development rate for different advancement directions over 15 periods

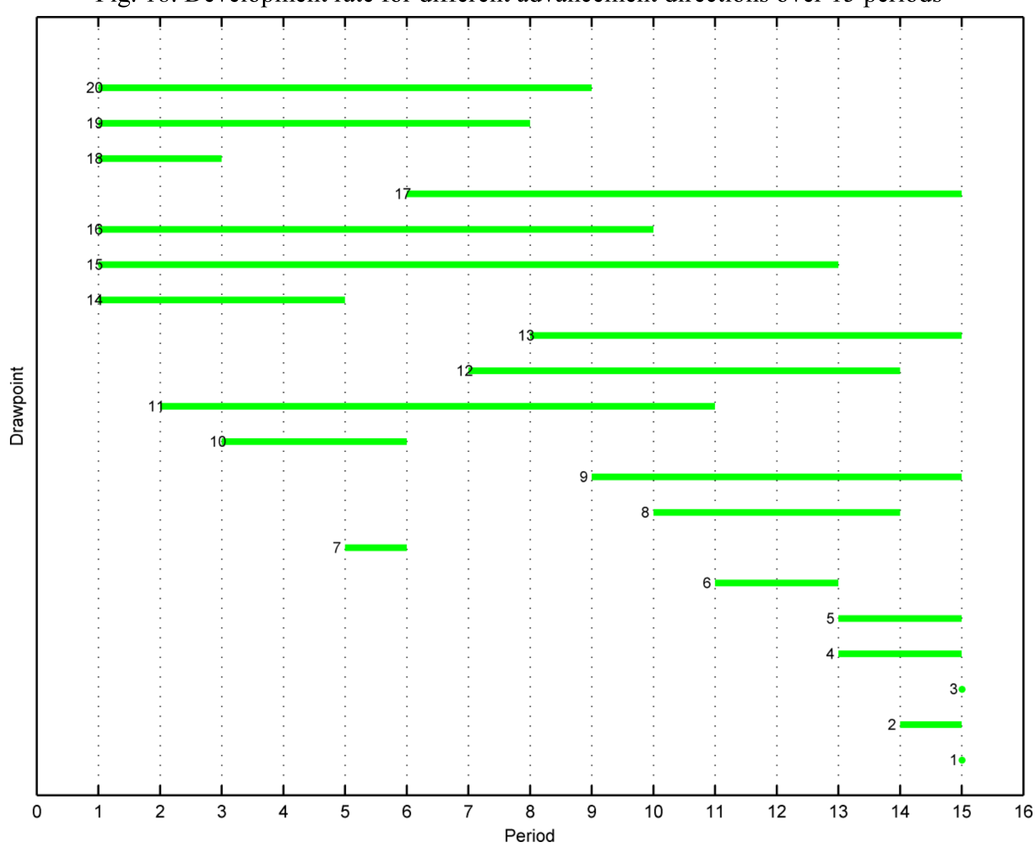
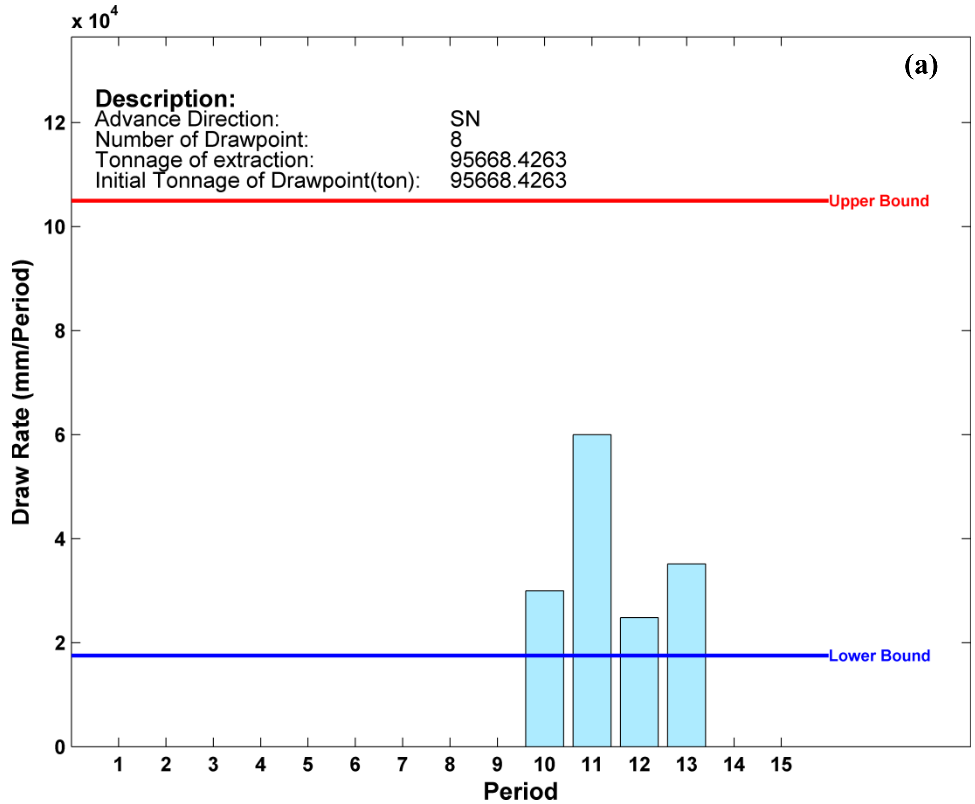


Fig. 19. Sequence of extraction and the drawpoints life for South to North advancement direction



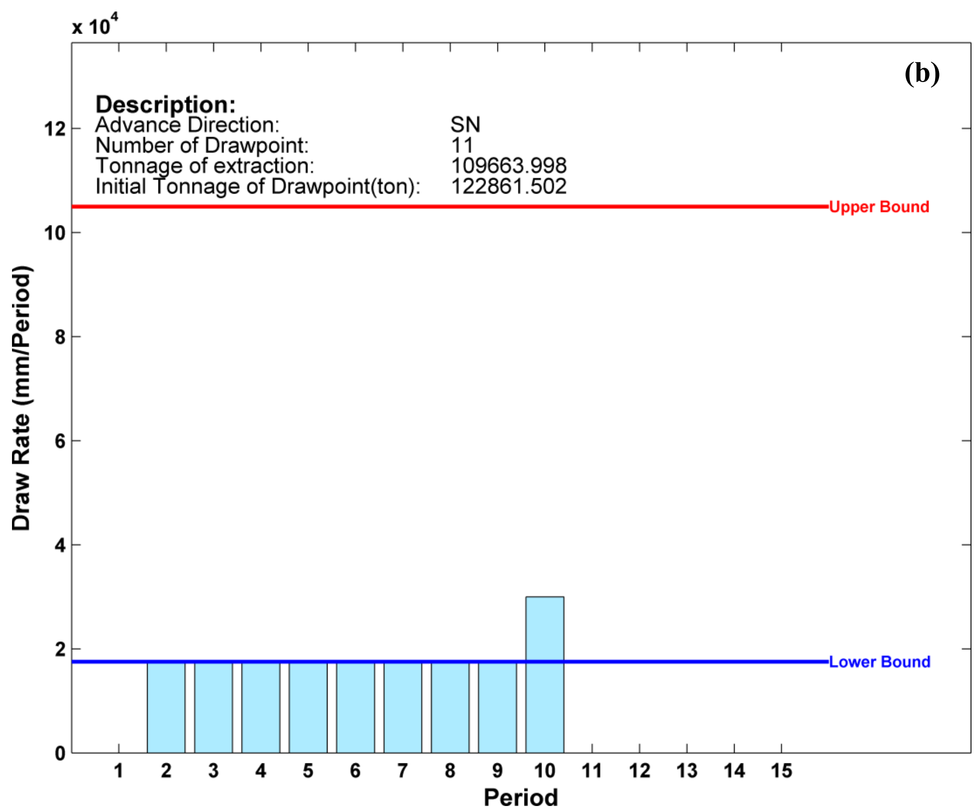


Fig. 20. Draw rate of drawpoints DP8 and DP11 in the South to North direction.

Fig. 21 shows cumulative tonnage extracted from DP8 and DP11 and percentage of extraction from slices within draw columns associated with drawpoints DP8 and DP11. The Y-axis of Fig. 21a and Fig. 21c represents the ID number of slices within the draw cone associated with the considered drawpoints DP8 and DP11, respectively. The smallest and the biggest numbers in these graphs indicate the lowest and the topmost slices in the draw cone associated with the considered drawpoints. For example, Fig. 21a shows that the lowest and the topmost slices within draw cone associated with drawpoint DP8 are slices 53 and 67, respectively. The blue numbers in these graphs are the percentage of extraction from each slice in the related period. It can be clearly seen that extraction from each slice is started after finishing the extraction of the slice below. Fig. 21b shows that all the material within the draw cone associated with drawpoint DP8 is extracted, while according to Fig. 21d, almost 90 percent of the materials within the draw cone associated with drawpoint DP11 are extracted.

Fig. 22a shows defined precedence among drawpoints for South to North direction based on the mentioned advancement direction concept in Fig. 5 . For example, extraction from drawpoint DP11 can be started if extraction from drawpoints DP14, DP15, and DP16 has been started. Fig. 22b shows obtained start and end periods from the MILP formulation for South to North direction. It can be seen that all defined precedence among drawpoints has been observed. According to Fig. 22a, extraction of drawpoints DP14, DP15, and DP16 can be started after starting the extraction of drawpoints DP18, DP19, and DP20, but Fig. 22b shows that extraction of drawpoints DP14, DP15, DP16, DP18, DP19, and DP20 starts at the same period. This happened because when a small portion of the lowest slices of drawpoints DP18, DP19, and DP20 is extracted that means extraction has been started and extraction of drawpoints DP14, DP15, and DP16 can be started in the same period.

7. Conclusions and future work

The economics of today's mining industry are such that the major mining companies are increasing the use of massive mining methods. Of the methods available, caving mines are favored because of their low cost and high production rates.

Improvement in both computer processing power and optimization solution algorithms have caused increased ability to find an optimum schedule. These advances have increased the importance of MILP for production scheduling because it can provide a mathematically provable optimum schedule.

This paper presented an MILP formulation for block cave mines production scheduling. The formulation maximizes the NPV subject to several constraints such as development rate, vertical mining rate (production rate per drawpoint), lateral mining rate (rate of opening new drawpoints), mining capacity, ore production target, maximum number of active drawpoints, cave draw strategies and advancement direction. The production scheduler defines the following: the opening and closing time of each drawpoint, the draw rate from each drawpoint, the number of new drawpoints that need to be constructed, and the sequence of extraction from the drawpoints to support a given production target.

Further focused research is underway to add new capabilities to the model. In the presented model, constant draw rates were used as upper and lower bounds. One method of managing drawpoint production is by establishing a production rate curve, which limits production based on the amount of material that has been drawn previously. This means that production depends on the cumulative tonnes mined from a drawpoint. So in the new model production rate curve (PRC) will be used instead of the constant upper and lower boundaries. Sometimes extraction from drawpoints is started from two or more different areas of the mine; hence we need to have a schedule which considers all mining areas at the same time. For this reason, some new constraint will be added to the MILP formulation for handling multiple-lift and multiple-mine scenarios.

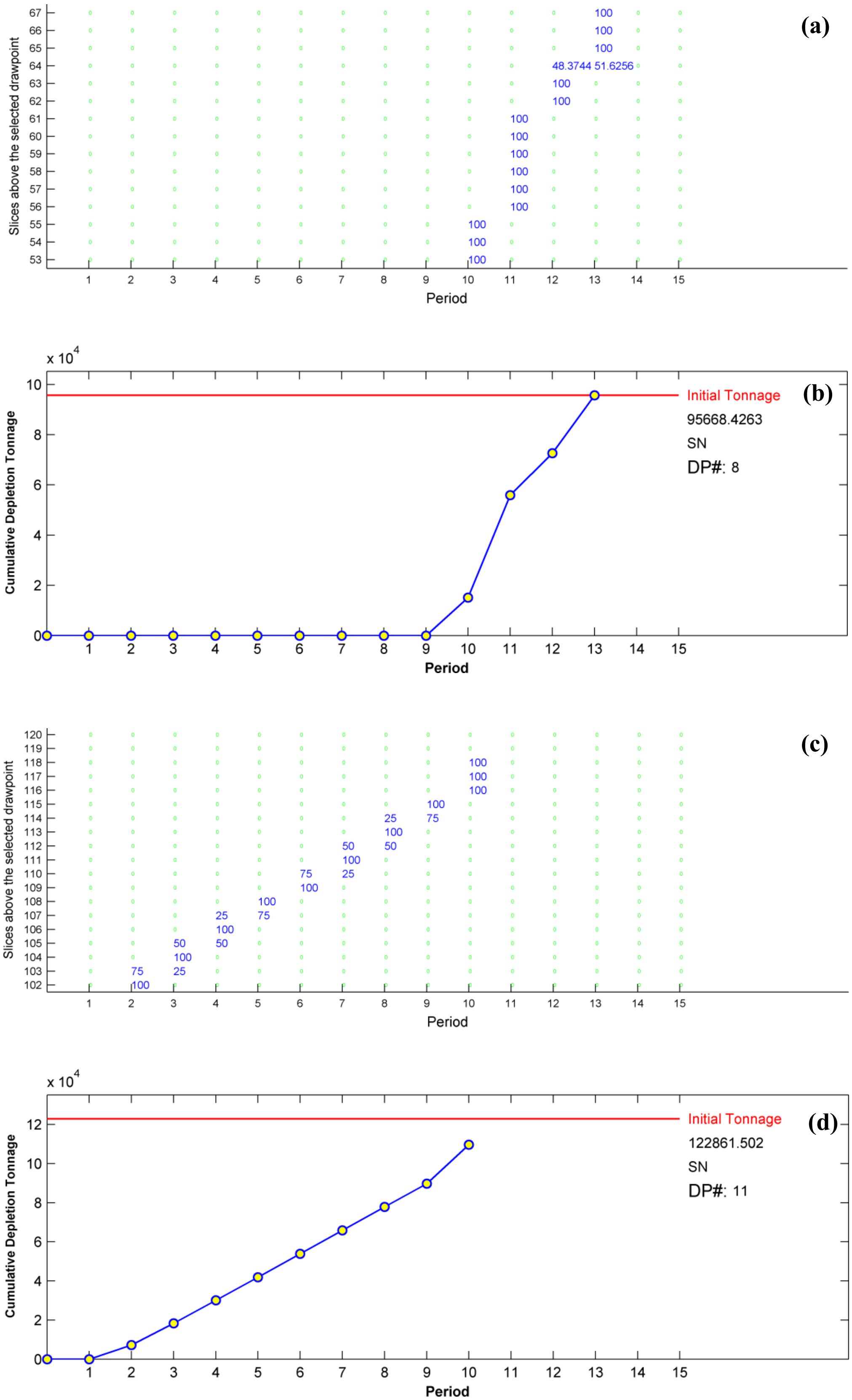


Fig. 21. Cumulative depletion tonnage and extraction percent from slices of DP8 and DP11 in South to North direction

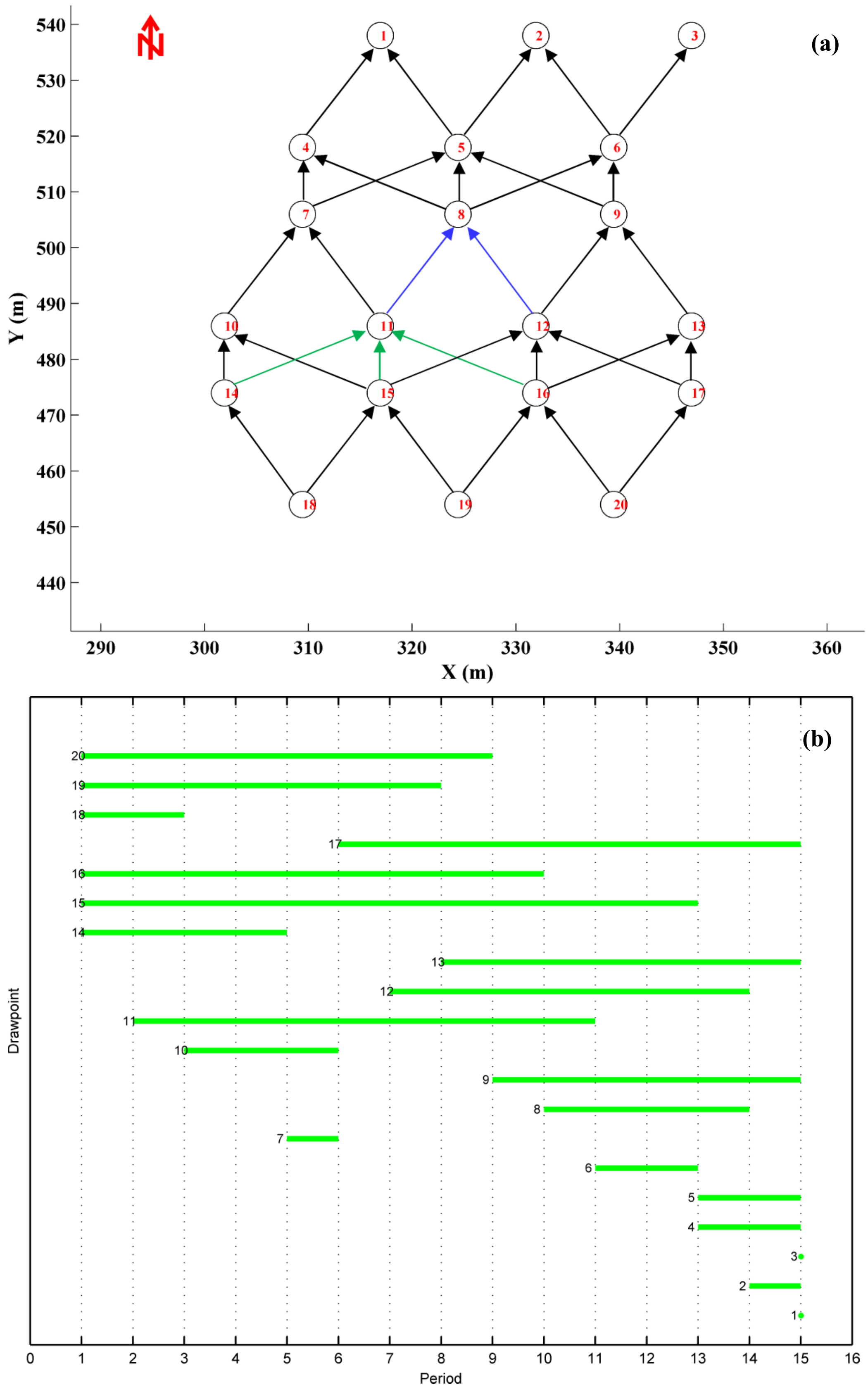


Fig. 22. Precedence of extraction between drawpoints

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9. Appendix

MATLAB and TOMLAB/CPLEX documentation for block cave mines scheduling

A Mathematical Programming Model for Optimal Truck-Shovel Allocation

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Abstract

The fundamental objective of any mine plan is to maximize the mine profit by extracting ore at the lowest possible cost over the mine life. Since the costs associated with the operation of trucks and shovels as resources are significant, the optimum allocation and dispatching of these resources is an essential issue. This paper presents a Mixed Integer Linear Programming (MILP) model to determine the optimum number of shovels and trucks required to meet the short-term plan's goals. Also, the model takes into account the allocation of trucks and shovels to mining faces. This model minimizes operational costs, while attempting to meet the production demand and consider technological constraints.

1. Introduction and literature review

Mine planning consists of two the planning level and the operational level. The fundamental objective of any mine plan is to maximize the mine profit by extracting ore at the lowest possible cost over the mine life. Geological, operational, technological and financial requirements constrain this objective. Mining equipment is one of the most expensive necessities of a mine. At the operational level, the goal is to use the trucks and shovels efficiently, minimizing the resources required which results in reduction in hauling, operating and maintenance costs, while meeting production targets.

In this paper resource allocation refers to the allocation of trucks and shovels to mining faces over a shift. Shovels are used to extract the ore and trucks are used to haul the ore to various destinations for further processing. Since the costs associated with the operation of trucks and shovels are significant, the allocation of these resources is an essential issue. Many mining companies try to allocate the trucks and shovels to produce an optimal schedule in a way that the operating costs are minimized and the utilization of resources is maximized through the planning horizon. Increasing the efficiency of the trucks and shovels results in savings.

Allocation of resources in the mining context is a complex and important process. The main factor that makes the allocation problem complex is the uncertainties in the operation of trucks and shovels such as truck cycle time, load tonnage, and truck and shovel reliabilities. These factors affect the production of a mine. Ignoring such uncertainties in the operation of a mine could result in deviations from the optimal plans. Any deviation from the production plan because of operational uncertainties increases the overall cost. One of the solutions to prevent the risk of not meeting the production demand is to provide extra trucks and shovels which again imposes extra costs to the system.

In many of the open pit mining systems, dispatching is considered a two level process. The first level is to allocate the shovels and the trucks at the beginning of the period and the second level is to implement the solution for real time operations. Most of the studies develop a mathematical programming model to solve the allocation problem. They usually aim to minimize the overall operating costs or maximize the profit, while meeting the target production. Other studies apply heuristic rules or stochastic approaches to solve the allocation and dispatching problems.

Li (1990) proposed a new dispatching methodology called intertruck-time deviation to keep truck flows as close to the optimum as possible. This methodology is based on material transportation rather than operational costs and can be used in real time open pit mining operations. Czaplicki (1999) proposed a procedure based on the queuing theory to assess the optimum number of operating trucks and reserve trucks in a mine. Two types of truck-and-shovel systems are considered: (1) one shovel and a certain number of trucks and (2) a certain number of shovels and trucks. This study considers many important technical and stochastic properties of the system.

A mixed integer programming (MIP) model was proposed by Topal and Ramazan (2010) to produce an optimum schedule for a fixed fleet of trucks over a year. This model minimizes the maintenance costs, while trying to achieve the target production. Yuriy and Vayenas (2008) developed a reliability assessment model based on genetic algorithm to evaluate and generate the time between truck failures. The output of the model is used as an input to a discrete event simulation model to analyze the impact of failures on production. Two different simulation software are used to compare the merits.

Temeng et al. (1997) developed a transportation algorithm for real time dispatching system. This algorithm evaluates the criteria called cumulative production ratio and minimizes the deviation of this criteria from the mean for each shovel route. Yan et al. (2008) and Yan and Lai (2007) developed an integrated mixed integer model to study the production scheduling and truck dispatching problems in the same framework. The methodology was applied to a ready mixed concrete (RMC) case in Taiwan. Fioroni et al. (2008) presented a two stage method; firstly a mathematical programming model is used to allocate the shovels and the trucks; secondly simulation is used to assess the results in real time operations.

Muduli and Yegulalp (1996) modeled the dispatching system as a closed queuing network considering different classes of trucks with various attributes. Mean value analysis (MVA) is used to evaluate performance measures. Erselebi and Bascetin (2009) presented a two stage procedure to optimize the truck and shovel system. In the first stage a model based on the closed queuing network theory is used to determine the optimal number of operating trucks. At the second stage a linear programming (LP) model is used to specify the dispatching sequence of trucks to shovels.

This paper presents a mixed integer linear programming (MILP) model to determine the number of shovels and trucks and solve the allocation problem to the mining faces available at any given shift. The model minimizes operational costs, while trying to meet the production demand and considers technological constraints. The next section introduces the problem and defines the assumptions considered in this study. Section 3 presents the mathematical formulation of the problem. The last section includes the conclusion and future work.

2. Problem definition

In the mining industry, trucks and shovels are used as resources to extract ore and haul it to various destinations for further processing or dumping as waste. Shovels are used to extract the material and load them to the trucks. Trucks operate continuously to haul the material to other locations and to feed the processing plant.

The number and the type of trucks and shovels are important elements in optimal designing of open pit hauling systems. The truck-and-shovel allocation problem involves determining the number and size of trucks and shovels, and the matching between them. Availability, useful economic life, spare parts availability, maintenance, and operating costs are factors affecting the type of trucks and shovels to be chosen for hauling.

The following assumptions are the basis of the truck-shovel allocation mathematical model. This paper studies a mine consisting of different mining faces. A number of shovels and trucks of different types are available. Each type of truck has a specific size and hauls a different volume of material. Due to the failures and predicted maintenance the number of available trucks of each type and available shovels may vary for each period.

At the beginning of each period the decision is made about assigning trucks and shovels to the mining faces which are ready to be extracted. The type of the material of each mining face specifies each truck's destination. If the material type is ore, assigned trucks go to the mill and if it is waste, they go to the waste dump. The number of trips of each type of truck to different destinations is another variable to be decided in the model. This assignment must be in a manner that the loading and haulage costs are minimized.

The grade of different minerals and metals directly affects the mining costs. The grades of materials in the ore faces are considered in the model. Shovels and trucks are allocated to the mining faces in order to meet the blending constraints at the mill or stockpiles. Any deviation from the target production at the mill results in extra system costs as a penalty. Other assumptions of the problem are as follow:

- Specific types of trucks can work with specific types of shovels;
- The number of available trucks of each type is known at the beginning of the period;
- The number of available shovels is known at the beginning of the period;
- Maximum and minimum production capacity of shovels and load capacity of trucks are known;
- Only one shovel operates at each mining face at a time;
- Each shovel can operate at only one mining face at a time;
- The time horizon for the model is an 8-hour shift.

3. Mathematical formulation

In this section the MILP model of the problem is presented.

3.1. Sets

$I = \text{set of mining faces}$

$J = \text{set of shovels}$

$K = \text{set of types of trucks}$

3.2. Indices

$i \in I =$ Index for mining faces

$j \in J =$ Index for shovels

$k \in K =$ Index for types of trucks

3.3. Parameters

$$MAT_i = \begin{cases} 1 & \text{if current material type of mining face } i \text{ is ore} \\ 0 & \text{otherwise} \end{cases}$$

ORE_i = remaining ore tonnage at mining face i (ton)

$WASTE_i$ = remaining waste tonnage at mining face i (ton)

$$AVL_i^{face} = \begin{cases} 1 & \text{if mining face } i \text{ is available} \\ 0 & \text{otherwise} \end{cases}$$

$$AVL_j^{shovel} = \begin{cases} 1 & \text{if shovel } j \text{ is available} \\ 0 & \text{otherwise} \end{cases}$$

$SHCAP_j^{\max}$ = maximum production capacity of shovel j (ton/hour)

$SHCAP_j^{\min}$ = minimum production capacity of shovel j (ton/hour)

NUM_k = number of available trucks of type k

$$COMP_{jk} = \begin{cases} 1 & \text{if truck } k \text{ is compatible with shovel } j \\ 0 & \text{otherwise} \end{cases}$$

CT_{ik}^{ore} = cycle time of truck type k transferring ore from mining face i to the mill (second)

CT_{ik}^{waste} = cycle time of truck type k transferring waste from mining face i to the waste dump (second)

CAP_k^{ore} = capacity of truck type k transferring ore (ton)

CAP_k^{waste} = capacity of truck type k transferring waste (ton)

GR_{il} = grade of variable l at mining face i (%)

UB_l = upper bound of grade blending for variable l (%)

LB_l = lower bound of grade blending for variable l (%)

$PMAX$ = maximum processing capacity of the mill (ton)

$PMIN$ = minimum processing capacity of the mill (ton)

MC_{ij} = moving cost of shovel j from its current location to minign face i (%)

TRC_{ik}^{ore} = trip cost of truck type k from mining face i to the mill ($)

TRC_{ik}^{waste} = trip cost of truck type k from mining face i to the waste dump ($)

DC = cost of deviation from target production ($/ton)

T = planning time duration (hour)

3.4. Decision variables

$$a_{ij} = \begin{cases} 1 & \text{if shovel } j \text{ is assigned to mining face } i \\ 0 & \text{otherwise} \end{cases}$$

n_{ik}^{ore} = number of trips of truck type k from mining face i to the mill

n_{ik}^{waste} = number of trips of truck type k from mining face i to the waste dump

x_i = extracted tonnage from mining face i (ton)

3.5. Objective Function

MIN Z =

$$\sum_{i\in I}\sum_{j\in J} MC_{ij}\cdot a_{ij} + \sum_{i\in I}\sum_{k\in K}(TRC_{ik}^{ore}\cdot n_{ik}^{ore} + TRC_{ik}^{waste}\cdot n_{ik}^{waste}) + DC\cdot(PMAX - \sum_{i\in I} MAT_i\cdot x_i) \quad (1)$$

3.6. Constraints

$$\sum_{j\in J} a_{ij} \le AVL_i^{face} \qquad \forall i\in I \quad (2)$$

$$\sum_{i\in I} a_{ij} \le AVL_j^{shovel} \qquad \forall j\in J \quad (3)$$

$$CT_{ik}^{ore}\cdot n_{ik}^{ore} \le 3600\cdot T\cdot NUM_k\cdot MAT_i \qquad \forall i\in I, k\in K \quad (4)$$

$$CT_{ik}^{waste}\cdot n_{ik}^{waste} \le 3600\cdot T\cdot NUM_k\cdot(1-MAT_i) \qquad \forall i\in I, k\in K \quad (5)$$

$$n_{ik}^{ore} \le \sum_{j\in J} a_{ij}\cdot COMP_{jk} \qquad \forall i\in I, k\in K \quad (6)$$

$$n_{ik}^{waste} \le \sum_{j\in J} a_{ij}\cdot COMP_{jk} \qquad \forall i\in I, k\in K \quad (7)$$

$$\sum_{i\in I} n_{ik}^{ore}\cdot CT_{ik}^{ore} + \sum_{i\in I} n_{ik}^{waste}\cdot CT_{ik}^{waste} \le 3600\cdot T\cdot NUM_k \qquad \forall k\in K \quad (8)$$

$$x_i \le \sum_{j\in J} T\cdot SHCAP_j^{\max}.a_{ij} \qquad \forall i\in I \quad (9)$$

$$x_i \ge \sum_{j\in J} T\cdot SHCAP_j^{\min}.a_{ij} \qquad \forall i\in I \quad (10)$$

$$\sum_{i\in i} x_i\cdot MAT_i \le PMAX \quad (11)$$

$$\sum_{i\in i} x_i\cdot MAT_i \ge PMIN \quad (12)$$

$$x_i\cdot MAT_i \le ORE_i \qquad \forall i\in I \quad (13)$$

$$x_i\cdot(1-MAT_i) \le WASTE_i \qquad \forall i\in I \quad (14)$$

$$x_i = \sum_{k\in K} CAP_{ik}^{ore}\cdot n_{ik}^{ore} + \sum_{k\in K} CAP_{ik}^{waste}\cdot n_{ik}^{waste} \qquad \forall i\in I \quad (15)$$

$$\sum_{i\in I} GR_{il}\cdot x_i \le \sum_{i\in I} UB_l\cdot x_i \qquad \forall l\in L \quad (16)$$

$$\sum_{i\in I} GR_{il}\cdot x_i \ge \sum_{i\in I} LB_l\cdot x_i \qquad \forall l\in L \quad (17)$$

$$a_{ij} \in \{0,1\} \quad \forall i \in I, j \in J \tag{18}$$

$$n_{ik}^{ore}, n_{ik}^{waste} \in Z \quad \forall i \in I, k \in K \tag{19}$$

$$x_i \geq 0 \quad \forall i \in I \tag{20}$$

Objective function seeks to minimize the operational costs associated with the mine. The first term in Eq. (1) is the total cost of moving shovels to new faces, the second term is the total transportation cost of trucks moving to the waste dump or to the mill, and the last term is the cost of negative deviation from the production target at the mill. Constraint Eq. (2) indicates that at each available mining face only one shovel can operate, and if a face is not available, no shovel should be assigned to that face. Constraint Eq. (3) assures that each available shovel can operate at only one face. Eq. (4)limits the number of trips for a fleet of trucks travelling from each mining face to the mill. Eq. (5)restricts the number of trips for a fleet of trucks travelling from each mining face to the waste dump. Constraints Eq. (6) and Eq. (7) guarantee that a truck could travel to a mining face only if a shovel is assigned to that face and the shovel is compatible with that truck type. Equation Eq. (8) denotes that the total number of trips of each truck type travelling to the mill or the waste dump is less than the maximum possible trips of that truck type. Constraints Eq. (9) and Eq. (10) ensure that the production of each mining face is between minimum and maximum possible production of the shovel assigned to that face. Eq. (11) and Eq. (12) aim to meet the limits of processing capacity of the mill. Constraints Eq. (13) and Eq. (14) force the production of each mining face to be smaller than the maximum amount of available material. Eq. (15) defines the production of each mining face based on the number of trips of each fleet of trucks. Eq. (16) and Eq. (17) ensure that the grade blending at the mill is between specified upper and lower limits. Eqs. (18), (19), and (20) define types of decision variables.

4. Conclusions and future work

This paper presented a mixed integer programming model to determine the number and the optimum allocation of trucks and shovels in an open pit mine. The model is proposed for the problem under certain assumptions. The next stage in this study is to code, solve, and verify the model using optimization tools. The approach should be applied to a real case to validate the model and to study the efficiency of the model and the solution algorithm.

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A Mathematical Programming Model for Mine Planning in the Presence of Grade Uncertainty

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Abstract

The optimality of an open pit production scheduling problem is affected dramatically by grade uncertainty. Recent research initiatives have attempted to consider the effect of grade uncertainty on production schedules. These methods are aimed either at minimizing the risk using grade uncertainty or at maximizing the net present value (NPV) without taking into account grade uncertainty explicitly. Another major problem in open pit production scheduling is the size of the optimization problem. The mathematical programming formulation of real size long-term open pit production schedules is beyond the capacity of current hardware and optimization software. In this paper a mathematical programming formulism is presented to find a sequence in which ore and waste blocks should be removed from a predefined open pit outline and their respective destinations, over the life of mine, so that the net present value of the operation is maximized and the deviations from the annual target ore production is minimized in the presence of grade uncertainty. Two main methods are presented: (1) without a stockpile and (2) with a stockpile. The new parameters that are controlling the uncertainty part of the optimization are studied. At the end, an oil sand deposit in northern Alberta is used to generate an optimum schedule.

1. Introduction

Mine planning is the process of finding a feasible block extraction schedule that maximizes net present value (NPV) and is one of the critical processes in mining engineering. It entails consideration of some technical, financial and environmental constraints. In the case of open pit mines, as Whittle (1989) notes, planning involves: "Specifying the sequence of blocks extraction from the mine to give the highest NPV, subject to variety of production, grade blending and pit slope constraints".

In this context, the uncertainty of ore grade may cause some shortfalls in the designed production and discrepancies between planning expectations and actual production (Koushavand and Askari-Nasab, 2009; Osanloo et al., 2008; Vallee, 2000). Therefore using only one block model would not be optimal. Various authors present methodologies to employ grade uncertainty, and show its impact, in mine planning.

Dowd (1994) propose a risk-based algorithm for surface mine planning. Ravenscroft (1992) and Koushavand and Askari-Nasab (2009) uses conditional simulated orebodies to show the impact of grade uncertainty on production scheduling. Dowd (1994) and Ravenscroft (Ravenscroft, 1992) use stochastic orebody models sequentially in traditional optimization methods. However the sequential process cannot produce an optimal schedule which takes uncertainty into account.

Godoy and Dimitrakopoulos (2003) and Leite and Dimitrakopoulos (2007) present a new risk-inclusive Long Term Production Plan(LTPP) approach based on simulated annealing. A multistage heuristic framework is presented to generate a final schedule, which considers geological uncertainty so as to minimize the risk of deviations from production targets. A basic input to this framework is a set of equally probable scenarios of the orebody, generated by conditional simulation. The authors report a significant improvement in NPV in the presence of uncertainty. Heuristic methods do not guarantee the optimality of the results. Also these techniques are sometimes very complex, and many parameters need to be chosen carefully in order to get reasonable results.

Dimitrakopoulos and Ramazan (2004) propose a probabilistic method for long-term mine planning based on linear programming. This method uses probabilities of being above or below a cut-off to deal with uncertainty. The LP model is used to minimize the deviation from target production. This method does not directly and explicitly account for grade uncertainty and also does not maximize the NPV.

Leite and Dimitrakopoulos (2007) present a technique that generates an optimal schedule for each realization. Simulated annealing is subsequently used to generate a single schedule based on all schedules, such that the deviation from target production is minimized. For each conditional simulation, an optimum schedule is generated. Using simulated annealing, a single schedule is generated based on all schedules such that deviation from target production is minimized. The main drawback of this method is that it does not necessarily find the optimum solution.

Dimitrakopoulos and Ramazan (2008) present a linear integer programming (LIP) model to generate the optimal production schedule. Equally probable simulated block models are used as input. This model has a penalty function that is the cost of deviation from the target production and is calculated based on the geological risk discount rate (GDR), which is the discounted unit cost of deviation from a target production. They use linear programming to maximize a new function that is NPV minus penalty costs. With this method, it is not clear how to define the GDR parameter. For different GDR values there are different optimal solutions. In the presented model, mixed integer programming has been used. A variable is defined for each block. Adding constraints increases the complexity and CPU time to solve the optimization. This method is not tractable with a real case study. In addition, there is no stockpile defined in this method.

2. Background

Geological characteristics of each point (grade) are assigned using available estimation techniques. Kriging (Deutsch and Journel, 1998a; Goovaerts, 1997) is the most common estimation method used in industry; however, Kriging results do not capture uncertainty and may leading to conditional biased reserve estimates (Isaaks, 2005). Also, mine plans that are generated based on one input block model fail to quantify the geological uncertainty and its impact on the future cash flows and production targets.

Geostatistical simulation algorithms are widely used to quantify and assess uncertainty. The generated realizations are equally probable and represent possible outcomes (Deutsch and Journel, 1998b; Goovaerts, 1997; Journel and Huijbregts, 1981). Choosing one of these realizations will not be objective to fair uncertainty assessment. Also, generated final pit limit and production schedule based on one block model would not necessarily be the optimum one. Therefore to get robust and optimum long-term production planning (LTPP), a sufficient number of realizations should be used simultaneously.

Mine planning is a process that defines a sequence of extraction of blocks with the objective of net present value (NPV) maximization. The mine production scheduling can be formulated as an optimization problem. NPV is the discounted revenue that discounted cost has been deducted from it:

$$\text{discountedprofit} = \text{discounted revenue} - \text{discounted costs} \tag{1}$$

Askari-Nasab and Awuah-Offei (2009) have presented the objective functions of the LP formulations that maximize the net present value of the mining operation. It is important to define a clear concept of economic block value based on ore parcels which could be mined selectively. The profit from mining a block depends on the value of the block and the costs incurred in mining and processing. The cost of mining a block is a function of its spatial location, which characterizes how deep the block is located relative to the surface and how far it is relative to its final dump. The spatial factor can be applied as a mining cost adjustment factor for each block according to its location on the surface. The discounted profit from a block is equal to the discounted revenue generated by selling the final product contained in block n minus all the discounted costs involved in extracting the block; this is presented at Eq.(1) The discounted cost can rewrite as Eq. (2) and Eq. (3):

$$v_n^t = o_n \times \left(g_n \times r^t \times \mathrm{P}^t - cp^t \right) \tag{2}$$

$$q_n^t = \left(o_n + w_n \right) \times cm^t \tag{3}$$

Where n is the id number of the block, and v_n^t and q_n^t are discounted revenue and cost of extraction from block n at period t respectively. o_n and w_n are the tonnage of ore and waste for block n, and cp^t and cm^t are cost of processing and mining at period t per ton respectively. r^t is processing recovery; P^t is the price of the final product. If there is more than one valuable element in the final product, the revenue of the block will be added up for each element. In addition, if there are contaminants that are to be processed and eliminated from final product, the cost of processing will be deducted from the revenue of that block.

The objective function is to maximize the summation profit (Eq. (1)) of all blocks at all periods with two separate decision variables for each block in each period. First, q_n^t is the portion of the block n to be extracted at period t, and second, z_n^t, is the portion of block n to be processed (if it is ore) at period t. Therefore the mathematical form of the optimal mining schedule is presented in Eq. (4):

$$Max \quad \sum_{t=1}^{T} \sum_{n=1}^{N} \left(v_n^t \times z_n^t - q_n^t \times y_n^t \right) \tag{4}$$

Subject to:

$$gl^t \le \frac{\sum_{n=1}^{N} g_n \times o_n \times z_n^t}{\sum_{n=1}^{N} o_n \times z_n^t} \le gu^t \qquad \forall\, t = 1,2,\ldots,T \tag{5}$$

$$pl^t \le \sum_{n=1}^{N} o_n \times z_n^t \le pu^t \qquad \forall\, t = 1,2,\ldots,T \tag{6}$$

$$ml^t \le \sum_{n=1}^{N} \left(o_n + w_n \right) \times z_n^t \le mu^t \qquad \forall\, t = 1,2,\ldots,T \tag{7}$$

$$z_n^t \le y_n^t \qquad \forall\, t = 1,2,\ldots,T \;,\; n = 1,2,\ldots,N \tag{8}$$

$$a_n^t - \sum_{i=1}^{t} y_l^i \le 0 \qquad \forall t = 1,2,\ldots,T \ , n = 1,2,\ldots,N \ , \ l = 1,2,\ldots,C(L) \tag{9}$$

$$\sum_{i=1}^{t} y_n^i - a_n^t \le 0 \qquad \forall t = 1,2,\ldots,T \ , n = 1,2,\ldots,N \tag{10}$$

$$a_n^t - a_n^{t+1} \le 0 \qquad \forall t = 1,2,\ldots,T-1 \ , \ n = 1,2,\ldots,N \tag{11}$$

Where Eq. (5) is grade blending constraints; these inequalities ensure that the head grade is within the desired range in each period. g_n is the estimated grade of block n, gl^t and gu^t are the allowable lower limit and upper limit of the input grade at period t. There will be separate constraints for each element of interest and any contaminants in each period. There are two equations (upper bound and lower bound) per element per scheduling period in Eq.(5). Eq. (6) is the processing capacity constraints, where pl^t and pu^t are the lower limit and upper limit (target production) for the designed processing plan; these inequalities ensure that the total ore processed in each period is within the acceptable range of the processing plant capacity. There are two equations (upper bound and lower) per period per ore type. Eq. (7) is the mining constraints where ml^t and mu^t are lower and upper limit for mining limits; these inequalities ensure that the total tonnage of material mined (ore, waste, overburden, and undefined waste) in each period is within the acceptable range of mining equipment capacity in that period. There are two equations (upper bound and lower bound) per period. Eq. (8) represents inequalities that ensure that the amount of ore of any block which is processed in any given period is less than or equal to the amount of rock extracted in the considered time period.

Eqs. (9) to (11) control the relationship of block extraction precedence by binary integer variables a_n^t, which is equal to one if the extraction of block n has started by or in period t (otherwise it is zero), and i, which is the index for set of the blocks, C(L), that need to be extracted prior to the extraction of block n. This model only requires the set of immediate predecessors' blocks on top of each block to model the order of block extraction. This is presented by set C(L) in Eq. (9).

The amount of ore processed and amount of material mined are controlled by two separate continuous variables rather than by binary integer variables. In this model, there is T (number of periods) multiplied by N (number of blocks) integer variables.

The estimated block model is usually used to maximize NPV; therefore, NPV_{es} is calculated from Eq. (12):

$$NPV_{es} = \sum_{t=1}^{T}\sum_{n=1}^{N}\left(v_n^t \times z_n^t - q_n^t \times y_n^t\right) \tag{12}$$

In this model, the number of decision variables equals two times of the number of blocks multiplied by the number of periods. Therefore, it would be a time-consuming process to solve this using linear programming. Boland et.al. (2009) and Askari-Nasab and Awuah-Offei (2009) tried to solve this problem by clustering the blocks in order to reduce the number of variables. Using some grade aggregation methodology and based on lithological information, similar blocks were summarized to a group and are dealt with as one variable which will be extracted in the same period. Each group of blocks is called a mining cut. Grouping the blocks into mining cuts is done without sacrificing the accuracy of the estimated (or simulated) values and to model a more realistic equipment movement strategy. The mining-cut clustering algorithm developed uses fuzzy logic clustering (Kaufman and Rousseeuw, 1990). The coordinates of each mining-cut are represented by the center of the cut and its spatial location.

The proposed linear programming was formulated in a MATLAB environment (MathWorks Inc., 2007). TOMLAB/CPLEX (Holmström, 1989-2009) was used as the Linear programming Solver. TOMLAB/CPLEX efficiently integrates the solver package CPLEX (ILOG Inc, 2007) with MATLAB.

3. MILP formulation based on grade uncertainty without stockpile

A Mixed Integer Linear Programming (MILP) model for optimizing long term production scheduling in open pit mines is developed with an objective function that maximizes the total NPV of the project under a managed grade risk profile. Grade uncertainty causes shortfalls from target productions. Therefore, to obtain an optimal solution, NPV must be maximized and deviation from target production must be minimized simultaneously among all simulation realizations:

$$\begin{cases} \text{Max. NPV} \\ \text{Min. Deviationfrom target production} \end{cases}$$

Two main assumptions are made to model this optimization problem:

1. There is no stockpile to store any possible overproduced ore. Most of the time not having a stockpile is very unlikely in real life. However the assumption made here is a hypothetical case.
2. Long term scheduling is a dynamic process. This means that it changes during mine life. There are many situations that may occur at the operational level that management needs to change the extraction schedule such as misclassification of ore and waste, so called as information effect, failures at equipment, and price changes at final production. In addition, during every period, the generated schedule is updated with new information such as blasthole data and new exploration drill holes. Therefore no long term schedule is followed from the first year until the end of the mine life. However the goal here is to find a robust long term schedule using all useful information. In the case of violation from target production in operational level, the optimization should run again with new information to find the new optimum schedule.

The proposed method tries to postpone the extraction of uncertain blocks to later years when there will be new information and less uncertainty. The main idea is to minimize the risk of not meeting the target production, because the uncertainty may impose costs to the project. The cost of uncertainty is considered in paper 104. The schedule generated using the method proposed here poses less risk in the early years of production.

The objective function has two components. As in Eq.(4), only one block model is used to maximize NPV; this is usually an estimated block model such as Kriging. The generated schedule is such that in all periods except the last one, the plant is fully fed by this block model and there is no deviation from target production. The second objective is applied to realizations. There is a probability that any schedule may not meet the target production because of grade uncertainty. These probabilities can be calculated using simulation values. The method presented here tries to minimize these probabilities in the early years of mine production. Two new variables, op_l^t and up_l^t, represent the amount of over-production and under production for realization l at period t. Each of these variables is multiplied by the discounted cost for over and under production, called $c_{op}{}^t$ and $c_{up}{}^t$. These two parameters are chosen by the user. They are the discounted penalty dollar values per ton for probable over and under production, and they are discounted based on the defined discount rate.

It is also important to note that in most cases, there is not enough ore to feed the plant during the final year of mine life. Therefore, as discussed in paper number 104, the cost of underproduction for final year is assumed to be equal to zero: ${c_{up}}^{T}=0$.

Therefore, the mathematical form of the optimal mining schedule in the presence of grade uncertainty is shown in Eq.(13).

$$Max\sum_{t=1}^{T}\left\{\underbrace{\sum_{n=1}^{N}\left(v_n^t\times z_n^t-q_n^t\times y_n^t\right)}_{first\ part:MAX\ NPV}-\underbrace{\frac{1}{L}\sum_{l=1}^{L}\left({c_{op}}^{t}\times op_l^t+{c_{up}}^{t}\times up_l^t\right)}_{second\ part:MIN\ Risk.}\right\} \tag{13}$$

The first part of this model maximizes the NPV using one block model which usually is estimate values, and the second part tries to minimize risk by deferring high uncertain blocks to the later years.

There are two more constraints for each period and each realization. These constraints control two new variables: op_l^t and up_l^t. The other constraints of this model are the same as those in Eq. (5) to (11).

Two new constraints are defined by Eq.(14) and (15):

$$\sum_{n=1}^{N}\left(o_{n,l}\times z_n^t-op_l^t\right)\le P_u^t \qquad \forall\, t=1,2,\ldots,T\ ,\ l=1,2,\ldots,L \tag{14}$$

$$\sum_{n=1}^{N}\left(-o_{n,l}\times z_n^t-up_l^t\right)\le -P_l^t \qquad \forall\, t=1,2,\ldots,T\ ,\ l=1,2,\ldots,L \tag{15}$$

Where $o_{n,l}$ is the tonnage of ore at block n in realization l. The number of decision variables is: $2\times N\times T+2\times L\times T$. The Numbers of binary variable is the same as in Eq. (4). This model was implemented in MATLAB (MathWorks Inc., 2007) and solved by the CPLEX TOMLAB (Holmström, 1989-2009) library. Because the size of the problem is too large to handle with current hardware and software, the fuzzy clustering technique (Askari-Nasab and Awuah-Offeri, 2009) was used to aggregate similar blocks within a group called mining cut.

4. MILP formulation based on grade uncertainty with stockpile

The assumption that a plant does not have a stockpile is a very severe assumption. In most plants, there is a stockpile for storing over produced ore when there is enough material to feed the plant. These surplus ores are used when there is some problem with feeding the plant, such as failure of the extraction and hauling system, or when there is a grade blending problem with input material to the mill. Therefore any possible over produced ore would be processed in later years, and the penalty value defined in Eq.(13) for over production would be less than it would be in the presence of stockpile. This means that any plausible over production based on a realization will be kept in a stockpile and will be used in the next period of extraction.

The costs of over production are:

- The cost of re-handling materials from a stockpile
- The loss of discounted value of ore transferred ore to the next period

In Eq.(13), the cost of over production for each period is deducted from the cost of over production in the next period, which means that for each period the penalty value is only the loss of the discounted value of ore that is transferred to the next period. Meanwhile, any re-handling costs are added to the cost of over production in each period. Therefore, a new optimization model for long term mine planning in the presence of grade uncertainty and of a stockpile is presented at Eq. (16):

$$Max \sum_{t=1}^{T} \left\{ \underbrace{\sum_{n=1}^{N} \left(v_n^t \times z_n^t - q_n^t \times y_n^t \right)}_{first\ part: MAX\ NPV} - \underbrace{\frac{1}{L} \sum_{l=1}^{L} \left[\left(c_{op}{}^{t} - c_{op}{}^{t+1} \right) \times op_l^t + c_{up}{}^{t} \times up_l^t \right]}_{second\ part: MIN\ Risk.} \right\} \tag{16}$$

The only difference between Eqs. (13) and (16) is in the cost of over production. Having a stockpile reduces the cost of possible over production.

It is very unlikely that in the last period of mine life, one realization will create over production, because in most cases there is not enough ore to feed the plant in the final year of mine life. But if, in a hypothetical case, a realization does generate over production in the final year, the ore will not be processed and the cost of over production will be the same as in the previous model. This means that there will be no deduction from over production costs for the final year. This can be imposed to mathematical form as: $c_{op}{}^{T+1} = 0$. There are some modifications to be made in the constraints which control the two variables op_l^t and up_l^t. Because any possible over produced ore will be used in the next period, this should be considered by two constraints as shown in Eqs. (14) and (15). Modified versions of these two constrains are shown in Eqs. (17) and (18):

$$\sum_{n=1}^{N} \left[o_{n,l} \times z_n^t - \left(op_l^{t-1} + op_l^t \right) \right] \le P_u^t \qquad \forall\, t = 1,2,\ldots,T \ ,\ l = 1,2,\ldots,L \tag{17}$$

$$\sum_{n=1}^{N} \left[-o_{n,l} \times z_n^t - \left(op_l^{t-1} + up_l^t \right) \right] \le -P_l^t \qquad \forall\, t = 1,2,\ldots,T \ ,\ l = 1,2,\ldots,L \tag{18}$$

Note that there is no overproduction in period 0: $op_l^0 = 0$.

The number of decision variables and binary variables are the same as in the previous model. Matlab and Tom lab are used to solve this optimization problem. Clustering method is used as before to reduce number of variables.

5. Discussion

In the early stages of production, the cost of uncertainty is higher than in later years because of the new information and the reduced uncertainty that later emerges. It is a reasonable decision to postpone the extraction of a highly uncertain block to later years, which will reduce the probability of obtaining deviations from target production. Higher penalty values mean that in the early years of production, less uncertain blocks are preferred. The optimizer generates a schedule that maximizes the NPV using the Kriging block model and minimizes the average penalty value calculated from L realizations. There is trade off in choosing high grade and low uncertainty blocks in the early years of production. Conversely, a higher $c_{op}{}^{t}$ and $c_{up}{}^{t}$ result in a lower NPV generated from the first part of Eqs. (13) and (16). Therefore it is very important to find the optimum values for these parameters. In order to do so, two techniques are proposed.

a) Numerical method: In this method, different $c_{op}{}^{t}$ and $c_{up}{}^{t}$ are used. In addition, over production and under production costs are considered to be equal and are called c. The optimization is run with different cost values. For each number, NPV_{es}^{c}, which is the NPV derived from the first part of Eqs. (13) and (16), is calculated. The cost of uncertainty is calculated (based on notation in paper 301), as are the differences:

$$Delta = NPV_{es}^{c} - Unc.Cost \tag{19}$$

Fig. 1 shows the expected results for this method. As is shown in this graph, NPV_{es}^{c} decreases slowly with higher c factors, while the cost of uncertainty decreases rapidly. The optimum c value is chosen where the delta value exceeds to its maximum value.

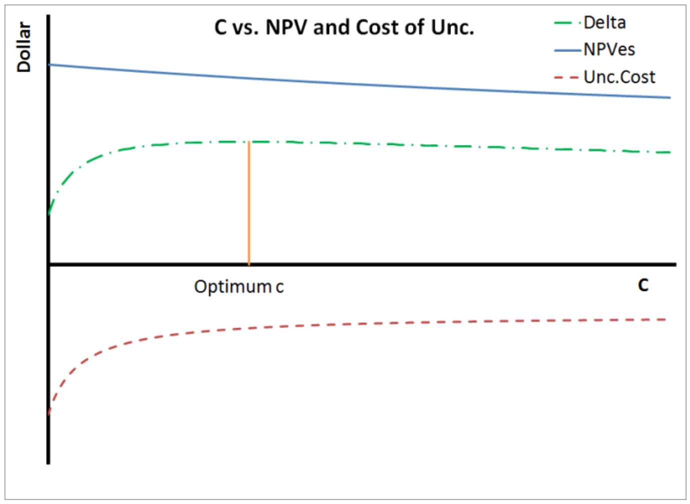

Fig. 1. Expected graph for optimum C factor based on numerical method.

b) The second method is based on the cost of uncertainty that is presented in paper 104. With equal $c_{op}{}^{t}$ and $c_{up}{}^{t}$ values, the second part of Eq. (13) is changed to the similar notation that was presented in paper 104.

$$\begin{aligned}&\frac{1}{L}\sum_{l=1}^{L}\left[c^t \times op_l^t + c^t \times up_l^t\right] = \frac{1}{L}\sum_{l=1}^{L}\left[c^t \times \left(op_l^t + up_l^t\right)\right] \\ &= \frac{1}{L}\sum_{l=1}^{L}\left[c^t \times \left(op_l^t + up_l^t\right)\right] = \frac{1}{L}\sum_{l=1}^{L}\left[c^t \times \left|P_l^t - \text{Target}_l^t\right|\right]\end{aligned} \tag{20}$$

On the other hand the cost of uncertainty is:

$$CoU = \frac{1}{L}\sum_{l=1}^{L}\left[\left(\overline{g}^t \times P_r^t \times \text{Price}^t \text{ - } P_c^t\right) \times \left|P_l^t - \text{Target}_l^t\right|\right] \tag{21}$$

By comparing Eqs. (20) and (21), $\overline{C}^t$, the average penalty cost for over and under production can be calculated from Eq. (22):

$$\Rightarrow \overline{C}^t = \frac{c^t}{L} = \frac{\overline{g}^t \times P_r^t \times \text{Price}^t \text{ - } P_c^t}{L} \tag{22}$$

Where P^t is the input ore to the mill in period t, Target^t is the target production for period t, $\overline{g}^t$ is the average grade of input ore in period t, P_r^t is the processing recovery in period t, Price^t is the selling price of the final product in each period, P_c^t is the processing cost and L is the number of realizations.

In Eq.(13), there is no stockpile. Therefore any possible over produced ore is not processed and will be considered waste. On the other hand modeling the stockpile is a very difficult problem, because it is necessary to have one extra variable to control the average grade of the stockpile and this extra variable is multiplied by the decision variables that control the portion of extraction and processing; this causes the model to be a nonlinear optimization problem. One way to solve this problem is to have the average grade of stockpile as an input. Eq. (16) uses this value as an input parameter. The penalty function that is applied in both Eqs. (13) and (16) is deferent in an over-production situation. Fig. 2 shows the penalty values at different periods; they are discounted for not having any stockpile (at left) and with stockpile (on the right). In each graph, the vertical axis shows the penalty value per tonne of over- and under-production. The horizontal axis shows the under (left side) and over (right side) tonnage of material that is sent to the mill to be processed. The slope of the lines is reduced over time. This means that the penalty value in period 1 for over- and under-production is less than that in period 2. In addition, having a stockpile reduces the penalty value for over production of ore. This cost is related to the re-handling of material at the stockpile and the revenue loss of resulting from the transfer of ore materials to the later years.

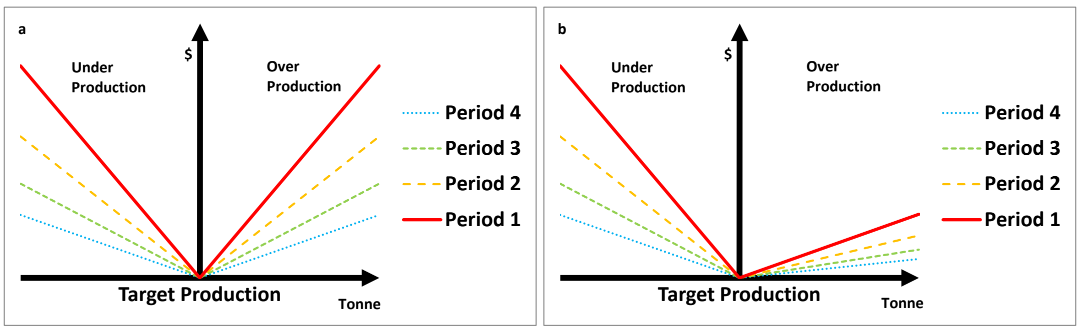

Fig. 2. Penalty function for over and under production at different periods based on a discounting factor, a: no stockpile and b: with stockpile

6. Case study

The same oil sand deposit that is cited in paper 104 is examined in this section. GSLIB (Deutsch and Journel, 1998) programs were used to generate an ordinary Kriging block model and 50 conditional realizations. The Kriging block model was used in the first part of both models to maximize NPV. The realizations were used in the second part to minimize deviation from target production. In this case study a 0.5$ per tonne penalty value was consider for both over-production and under-production ($c_{op}{}^{t}$ and $c_{up}{}^{t}$). Using an average grade 9.5 mass percent bitumen in all periods, a processing recovery of 95 percent, a selling price of \$2.8125 per tonne and a processing cost of \$0.5025 per tonne in Eq. (22), the c factor is calculated with 50 realizations as $\overline{C}^{t}=0.5$. The gap of 1% was used in CPLEX optimizer. The final gaps for LP without stockpile and with stockpile respectively are 0.98% and 0.68%.

Fig. 3 shows the schedules generated by two methods. In both methods, the optimizer did quite well in feeding the plant over 7 years of production, and the only shortfall, in the last period, resulted from the fact that there was less ore remaining. Fig. 4 shows the cumulative cash flow over periods for Kriging, Etype and simulation realizations for both methods. Note that the generated schedule was followed for each of realizations. Average input grade to the mill in each period is presented in Fig. 5. Having a stockpile relaxes the optimizer that it is able to extract and stock high grade ore in period three, and this increases the average grade in period four. Fig. 6 illustrates input tonnage to the mill using the Kriging, E-type and realization block models for both methods. It is clear that having a stockpile reduced the probability deviation from target production in the early years of production (period 3 and 4). This fact is clear in Fig. 7 which shows the box plot of realizations and deviations from target production. Deviations from target production in period 3 and 4 using the first method (without stockpile) are 2.5 and 5.28 percent respectively. These values for second method (with stockpile) are 0.39 and 3.32. This shows that a stockpile reduce the risk of not meeting the target production.

Table 1 summarizes the results of the two methods when each realization follows the two schedules. The expected NPVs for first and second methods are 2319.18 and 2322.60.

Table 2 shows the statistics for realizations and cumulative cash flow over the periods. The first two periods have a negative cash flow because of pre-stripping. The summary results of the three methods are shown in Table 3. The first method is Kriging without uncertainty and without a stockpile. These results derived from paper 104. The second and third methods involve the optimization of NPV with Kriging and minimization for deviation from target production using simulation realization with and without a stockpile. The NPV_{es}^{c}, which is calculated based on a Kriging block model, is maximized in the first method because there are fewer constraints and more flexibility for optimization: 2461 versus 2449.3 and 2453.8. On the other hand, the cost of uncertainty is higher in this method than in the next two methods: 178.9, 151.6 and 141.9 respectively. The stockpile also reduces the cost of uncertainty. The delta value, which is defined in Eq. (19), is calculated in final column. It is clear that the delta value for the method with a stockpile is higher than two other methods. In addition using realizations creates a higher delta value than the schedule creates using only the Kriging block model.

7. Conclusions

In this paper two methods were presented to generate long term production schedules using a linear programming technique. The net present value was maximized based on the estimate block model, which is usually created using Kriging methods. The second objective was to minimize the deviation from target production or minimize the cost of uncertainty. In both methods, highly uncertain blocks are extracted in later years when more information is provided by new drill holes. This is controlled by two factors, called the cost of over and under production in each period. Having high values for each of these parameters means that less uncertain blocks are preferred in the early years of production. High values for these parameters also reduce the NPV that is calculated with Kriging. Two methods were presented to calculate the optimum values for these parameters.

There is not over or under production for the Kriging block model. The probability of deviations from target production in each period is calculated using simulation realizations. The generated schedules are more robust because the probability of not meeting the target production is lower in the early years of production.

There are two types of variables in the proposed optimization models: binary variables that control the precedence of block extraction and decisions variables that indicate the portion of the block that is going to be extracted and sent to the mill in each period. The number of variables for both of these methods is too large to be handled by current commercial software in a real case study. For example, there are 800,000 decision variables for a project with 20,000 blocks and 20 years of mine life. To solve this problem, a clustering technique is used. Blocks in the same level with similar grades are aggregated into mining cuts and the number of variables is reduced. Recently some studies have sought to find better techniques and more complex criteria.

The future work for this study is to use pushbacks to reduce the size of the problem. Pushbacks can also be used to define the large scale strategy of mine development and are used widely in industry. First Lerchs-Grossman algorithm (1965) is used to find the nested pits then the block at each pushbacks are determined. The new constrains are added to the LP model such that the blocks in a certain pushback are extracted before starting to remove the blocks in the next at pushback.

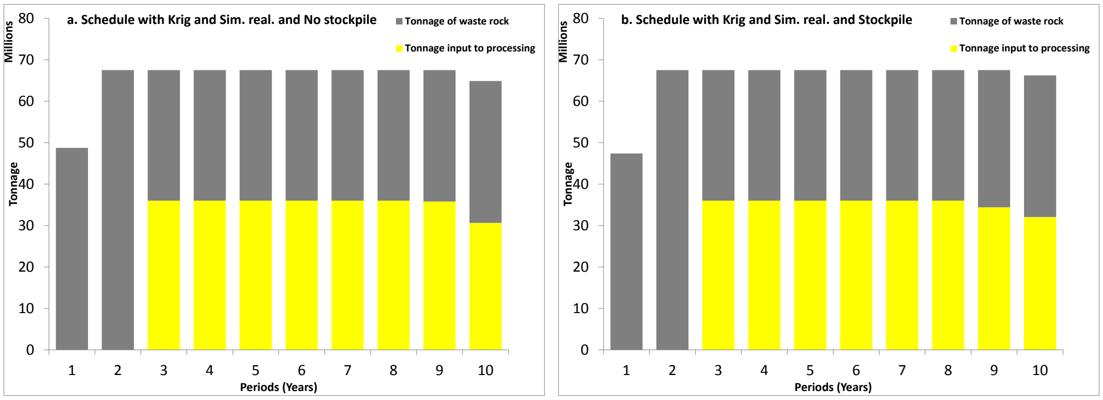

Fig. 3. Schedules generated using krig model and simulation realizations a: without stockpile and b: with stockpile.

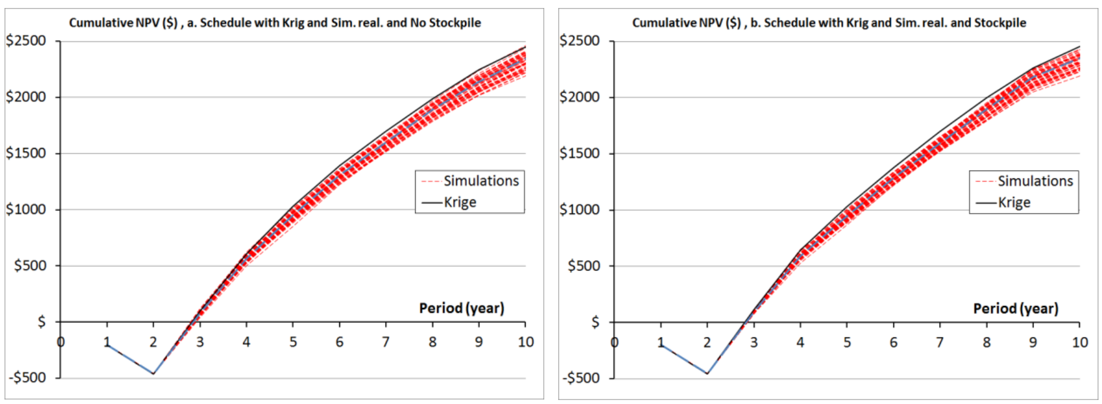

Fig. 4. Comulative NPV over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), a: without stockpile and b: with stockpile.

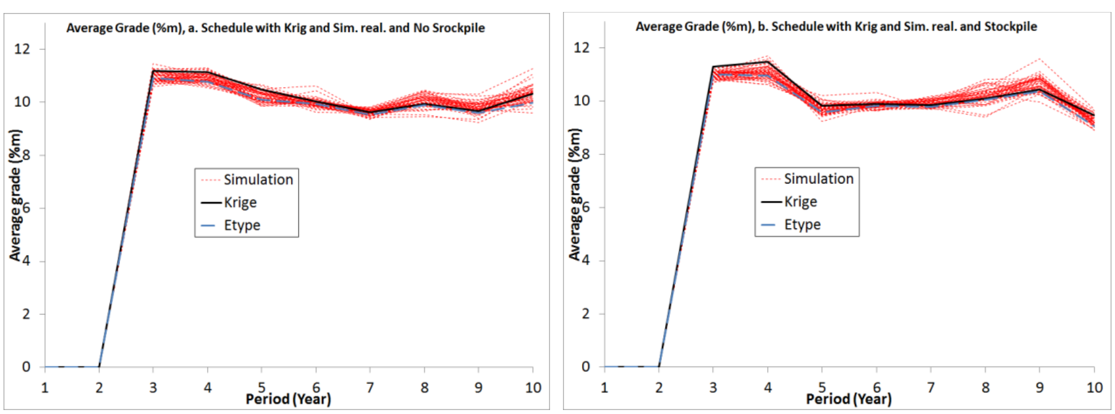

Fig. 5. Input head grade to the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), a: without stockpile and b: with stockpile.

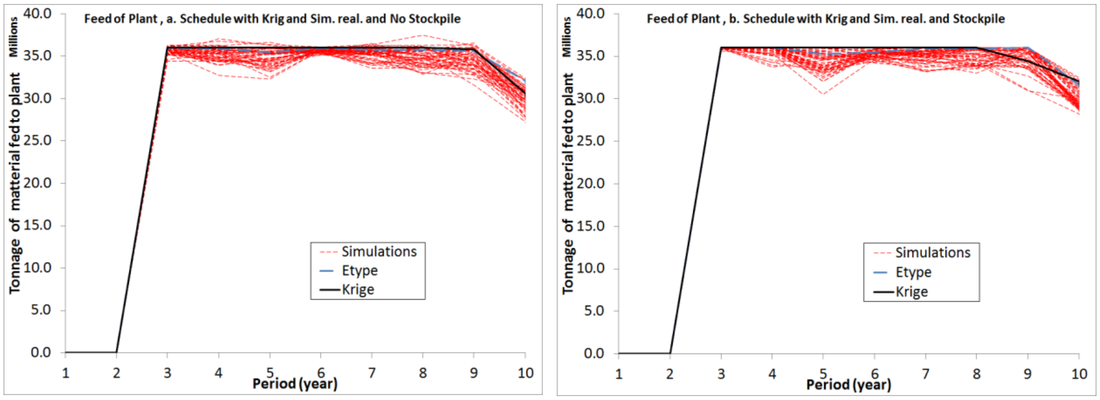

Fig. 6. Feed of the plant over periods for kriging (back line), etype (dashed blue line) and simulations (dash red line), a: without stockpile and b: with stockpile.

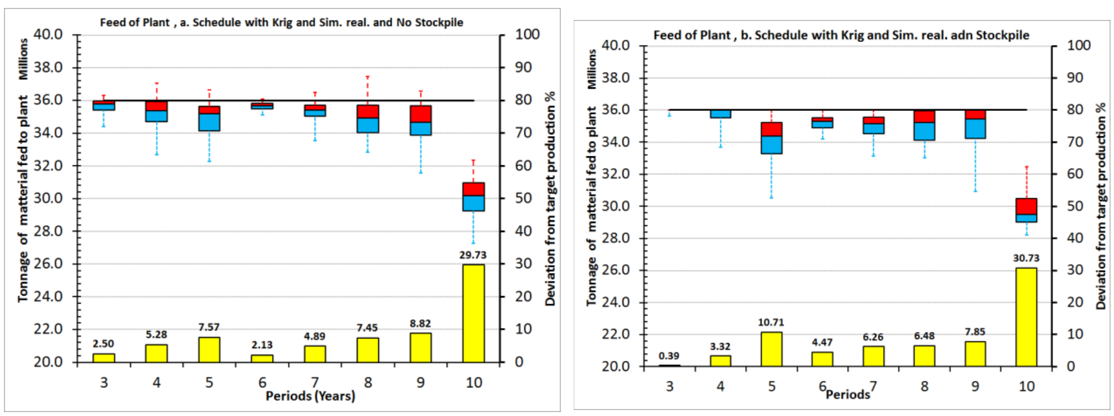

Fig. 7. Boxplot and deviation from target production (yellow bars), calculated using simualtion values, a: without stockpile and b: with stockpile.

Table 1. Summary statistic of realizations when generated schedule with Kriging is followed, at above without stockpile and bottom with stockpile.

a: LP With Krig & Sim. Realizations Without Stockpile	Ore Millions Tonnes	STRO	Input Bitumen Millions Tonnes	Average %	NPV Millions Dollars
Mean	276.14	1.37	28.27	10.24	2319.18
Std. dev	3.60	0.03	0.44	0.09	60.74
Min	269.28	1.29	27.28	10.02	2189.42
Quartile 1	273.31	1.34	27.89	10.19	2269.00
Median	276.50	1.36	28.25	10.24	2316.67
Quartile 2	278.83	1.39	28.61	10.29	2367.12
Max	284.06	1.43	29.12	10.52	2428.65
Krig	282.44	1.31	29.11	10.31	2449.44
Etype	282.22	1.32	28.49	10.10	2346.25

b: LP With Krig & Sim. Realizations With Stockpile	Ore Millions Tonnes	STRO	Input Bitumen Millions Tonnes	Average %	NPV Millions Dollars
Mean	275.92	1.37	28.24	10.24	2322.60
Std. dev	3.61	0.03	0.44	0.09	59.42
Min	269.29	1.29	27.22	10.02	2191.25
Quartile 1	272.97	1.35	27.84	10.19	2269.99
Median	276.37	1.36	28.23	10.24	2318.70
Quartile 2	278.39	1.39	28.56	10.29	2372.09
Max	284.47	1.43	29.10	10.52	2430.31
Krig	282.44	1.31	29.11	10.31	2453.85
Etype	282.07	1.32	28.48	10.10	2349.99

Table 2 Summery statistics of cumulative cash flow at each period, at above without stockpile and bottom with stockpile.

Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Mean	-203.8	-460.4	77.7	555.5	943.5	1,294.5	1,590.8	1,875.0	2,119.8	2,319.2
Std. dev	0.0	0.0	16.9	26.1	35.2	37.4	41.0	47.7	53.7	60.7
Min	-203.8	-460.4	42.8	501.9	851.8	1,219.6	1,516.5	1,784.7	2,017.3	2,189.4
Quartile 1	-203.8	-460.4	66.3	537.5	918.5	1,273.6	1,560.1	1,837.7	2,075.1	2,269.0
Median	-203.8	-460.4	76.6	549.7	943.9	1,289.8	1,588.3	1,883.2	2,125.4	2,316.7
Quartile 2	-203.8	-460.4	90.9	578.2	971.6	1,328.1	1,627.5	1,911.1	2,163.3	2,367.1
Max	-203.8	-460.4	120.5	620.4	1,015.7	1,360.7	1,669.2	1,954.8	2,202.1	2,428.6
Krig	-203.8	-460.4	101.6	609.3	1,032.0	1,391.6	1,698.3	1,991.2	2,244.6	2,449.4
Etype	-203.8	-460.4	80.0	561.5	951.4	1,304.4	1,602.7	1,890.4	2,138.0	2,346.3

Period	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Mean	-198.1	-454.8	93.2	590.7	939.0	1,277.4	1,585.1	1,878.4	2,162.2	2,322.6
Std. dev	0.0	0.0	9.2	25.6	31.3	32.9	37.7	46.2	54.1	59.4
Min	-198.1	-454.8	76.9	525.7	870.9	1,214.9	1,520.8	1,792.2	2,049.0	2,191.2
Quartile 1	-198.1	-454.8	86.9	573.0	916.6	1,254.4	1,557.1	1,841.2	2,119.9	2,270.0
Median	-198.1	-454.8	93.3	594.0	939.9	1,272.8	1,579.4	1,886.8	2,165.2	2,318.7
Quartile 2	-198.1	-454.8	100.6	607.2	962.5	1,305.2	1,616.9	1,915.1	2,202.6	2,372.1
Max	-198.1	-454.8	114.9	649.2	1,008.3	1,337.5	1,654.8	1,957.1	2,255.8	2,430.3
Krig	-198.1	-454.8	115.1	644.7	1,028.1	1,379.8	1,697.3	1,997.4	2,264.9	2,453.8
Etype	-198.1	-454.8	94.6	591.0	946.4	1,285.8	1,598.3	1,895.0	2,179.9	2,350.0

Table 3. Summary of NPV and Cost of uncertainty at different methods.

Method	NPV_{es}^{c}	**Cost of Unc.**	**Delta**
Kriging Without Unc. No Stockpile	2461.0	178.9	2282.1
Kriging With Sim. No Stockpile	2449.3	151.6	2297.7
Kriging With Sim. and Stockpile	2453.8	141.9	2311.9

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In-Pit and External Oil Sands Dyke Construction Scheduling using Goal Programming

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Abstract

Historical analysis of mineral resource evaluations has demonstrated the sensitivity of project's profitability to decisions based on long-term mine production planning. In oil sands mining, providing processable ore and tailings containment with less environmental footprints at the right time are the main drivers for profitability and sustainability. Recent environmental and regulatory requirements makes waste management an integral part of mine planning in the oil sands industry (Directive 074). This requires the development of a well integrated strategy of directional mining and tailings dyke construction for in-pit and ex-pit tailings storage management systems. The objective of this paper is to: 1) determine the order and time of extraction of ore, dyke material and waste to be removed from a predefined final pit limit over the mine life that maximizes the net present value of the operation; and 2) determine the destination of dyke material that minimizes construction cost depending on the construction requirements of the various dykes as per their designs. We have developed, implemented, and verified a theoretical optimization framework based on mixed integer linear goal programming (MILGP) to address this objective. The research introduced a MILGP mine planning model for multiple material types and destinations. This study also presents an integration of mixed integer linear programming and goal programming in solving large scale mine planning optimization problems using clustering and pushback techniques. Application of the MILGP model was presented with an oil sands mining case. The MILGP model generated a smooth and uniform mining schedule that generates value and provides a robust framework for effective waste disposal planning.

1. Introduction

Open-pit mining involves the process of extracting blocks of earth from the surface to retrieve the ore contained in them. This mining process causes the surface of the land to be continuously excavated causing an increasingly deeper pit to be formed until the end of the mine life (Hochbaum and Chen, 2000; Newman et al., 2010). Prior to the mining operation, the complex strategy of displacement of ore, waste, overburden, and tailings over the mine life need to be decided and this is known as mine planning. Open-pit mine planning can be defined as the process of finding a feasible block extraction sequence that generates the highest net present value (NPV) subject to operational and technical constraints (Whittle, 1989). Mine planning is done for different time horizons and these include short-term, medium-term, and long-term production scheduling plans. This paper focuses on the long-term production scheduling optimization process which is the backbone of the entire mining operation. In mining projects, deviations from optimal mine plans will result in significant financial losses, future financial liabilities, delayed reclamation, and resource sterilization.

The objective of this study is to develop a theoretical framework that maximizes the NPV of an oil sands mining operation and minimizes dyke construction cost for tailings containment using a mixed integer linear goal programming (MILGP) model. The MILGP model incorporates multiple material types with multiple elements for multiple destinations in long-term production scheduling. Though operation research methods have been applied in mine production scheduling, very little work has been done in terms of oil sands mine planning which has a unique scenario in terms of waste management. Oil sands mining profitability depends on a carefully planned and integrated mine planning and waste management strategy that generates value and sustainability by maximizing NPV and creating timely tailings storage areas with less environmental footprints. Recent mining regulations by Alberta Energy Resources and Conservation Board (Directive 074) (McFadyen, 2008) requires that oil sands mining companies develop an integrated mine planning and waste management strategy for their in-pit and external tailings facilities. This requires a new and more systematic approach in looking at the planning of oil sands mining operations.

The next section of this paper presents the problem definition and section 3 is on our conceptual mining model. Section 4 covers a literature review on goal programming (GP), mixed integer programming (MIP) and mixed integer linear programming (MILP). The application of MILGP to the long-term production planning (LTPP) problem is formulated in section 5. The formulation is applied to an oil sands mine planning and waste management case with an example and the results discussed in sections 6 and 7 respectively. Section 8 outlines the conclusions and future research direction.

2. Problem definition

Mine management is always faced with the problem of achieving multiple goals with the available limited resources. In oil sands mining, due to the limitation of lease area, the pit phase advancement is carried out simultaneously with the construction of tailings dykes in the mined out areas of the pit and designated areas outside the pit. These dykes are constructed to hold tailings that are produced during the processing of the oil sands. Dykes with different configurations are required during the construction. Most of the materials used in constructing these dykes come from the oil sands mining operation. The dyke materials are comprised of overburden and interburden (OI) dyke material and tailings coarse sand (TCS) dyke material. It is assumed that the material sent to the processing plant (ore) must have a specified amount of bitumen and percentage fines as well as the material sent for dyke construction (dyke material). Any other material that does not meet the requirements of ore or dyke material is sent to the waste dump.

The main problem here has been categorized in two parts: 1) determining the order and time of extraction of ore, dyke material and waste to be removed from a predefined ultimate pit limit over the mine life that maximizes the net present value of the operation; 2) determining the destination of dyke material that minimizes construction cost depending on the construction requirements of the various dykes as per their designs.

Fig. 1 illustrates the scheduling of an oil sands ultimate pit block model containing K mining-cuts. Mining-cuts are clusters of blocks within the same level or mining bench that are grouped based on the attributes; location, rocktype and grade distribution (Askari-Nasab and Awuah-Offei, 2009; Ben-Awuah and Askari-Nasab, 2011). Each mining-cut k, is made up of ore o_k, OI dyke material d_k, and waste w_k. The material in each mining-cut is to be scheduled over T periods depending on the goals and constraints associated with the mining operation. The OI dyke material scheduled, d_k^T and the TCS dyke material from the processed ore, l_k^T must further be assigned to the dyke construction sites based on the construction requirements. For period t_1, the dyke construction material required by site 1 is $dyke_1$, the dyke construction material required by site 2 is $dyke_2$ and the dyke construction material required by site i is $dyke_i$.

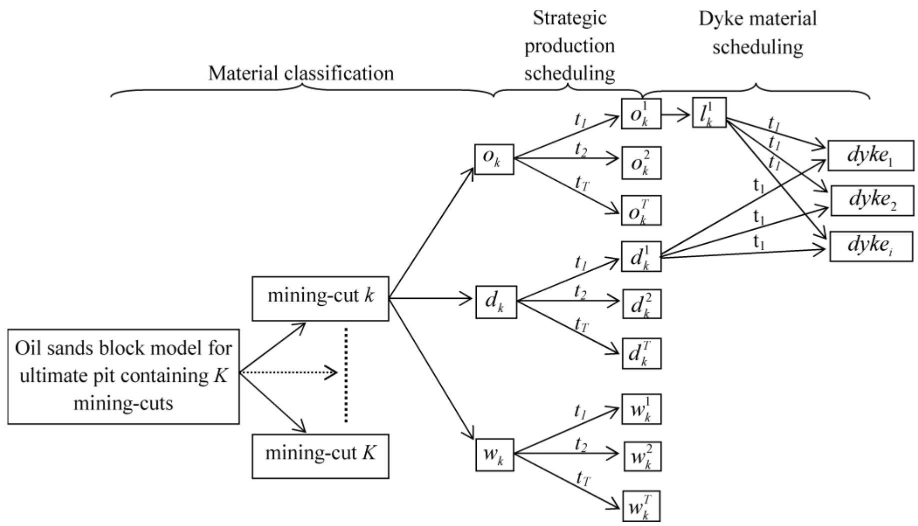

Fig. 1. Schematic representation of the problem definition showing strategic and dyke material production scheduling modified after Ben-Awuah and Askari-Nasab (2011)

The strategic and dyke material production schedules to be developed are subject to a variety of economic, technical, and physical constraints. The constraints control the mining extraction sequence, ore and dyke material blending requirements and mining, processing, and dyke material goals. The mining, processing, and dyke material goals specify the quantities of material allowed for the mining operation, processing plant, and dyke construction respectively.

The strategic and dyke material production schedules are the main drivers for the profitability of oil sands mining operation. The schedules control the NPV of the operation and enable a robust waste management planning strategy. Improper waste management planning can lead to environmental issues causing immediate mine closure by regulatory agencies and major financial liabilities.

3. Conceptual mining model

The key drivers for oil sands mine planning are the provision of a processable blend of ore at the required grade and the provision of tailings containment at the right time. A conceptual mining model that is consistent with practical oil sands mining and waste management was set up to illustrate how the MILGP model can be used to generate a strategic and dyke material production schedules. As shown in Fig. 2, the mining model is made up of an oil sands deposit area which is to be mined and simultaneously used as an in-pit tailings storage area as mining progresses in a specified direction and the in-pit tailings dyke footprints are released. Each oil sands mining-cut is made up of ore, OI dyke material and waste. After processing the ore to extract bitumen, two main types of tailings are produced; fine and coarse tailings. The coarse tailings also referred to as TCS dyke material and OI dyke material are used in the construction of dykes for tailings facilities. The fine tailings form the slurry which needs to be contained in the tailings facilities.

3.1. Tailings storage management strategy

Each tonne of ore is made up of bitumen, fines, sand, and water. Using the oil sands extraction process volume changes on the path from ore to waste as outlined in a report for Alberta Energy Research Institute (Devenny, 2009), the volume of tailings to be produced can be calculated and an appropriate storage management strategy planned. In the conceptual mining model, using the tailings storage volume required and the total in-pit tailings facilities volume available, an external tailings facility (ETF) volume required to support the mining operation can be calculated.

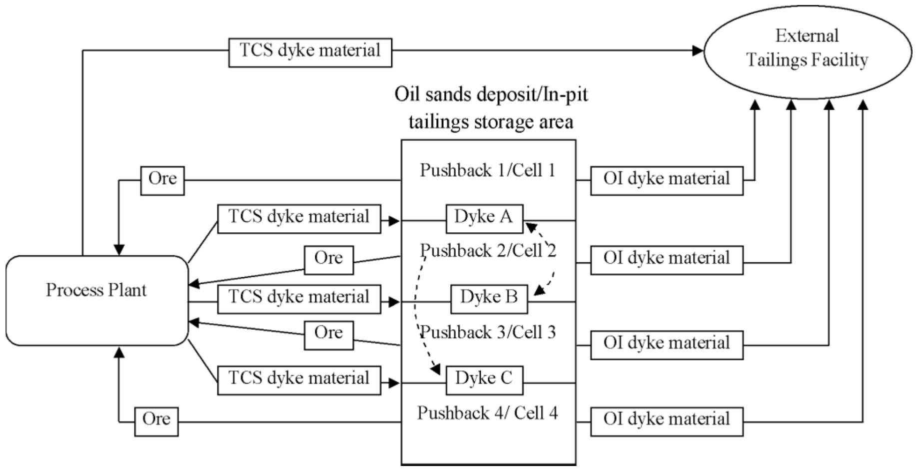

Fig. 2. Conceptual mining model showing mining and waste management strategy (Askari-Nasab and Ben-Awuah, 2011)

The oil sands deposit area was divided into pushbacks which coincide with the areas required by tailings dam engineers to set up in-pit tailings facility cells. In the case of our illustrative example in Fig. 2, the deposit covers an area of 8 km x 4 km with an average height of 75 m. Based on literature on oil sands mining operations with regards to standard sizes of ex-pit and in-pit tailings facility cells (Fort Hills Energy Corporation, 2009; Jackpine Mine, 2009; Kearl Oil Sands Project, 2009; Muskeg River Mine, 2009; Suncor Energy Incorporated Oil Sands, 2009; Syncrude Aurora North, 2009; Syncrude Aurora South, 2009; Syncrude Mildred Lake, 2009), it was decided to divide the mining area into four pushbacks which will result in four in-pit cells as shown in Fig. 2. Each cell will have approximate dimensions of 2 km x 4 km x 75 m except cells 1 and 4. The mining operation will stay ahead of dyke construction by about 100 m resulting in cell 1 having a size of 1.9 km x 4 km x 75 m and cell 4 having a size of 2.1 km x 4 km x 75 m. It is assumed that mining will start in pushback 1 and progress in a north-south direction. During the mining of pushback 1, all IO and TCS dyke material will be sent to the ETF for the construction of the ETF dyke. Fluid fine tailings produced from pushback 1 will be sent to the ETF after the key trench and starter dyke construction is completed. Once mining of pushback 1 is completed, the dyke ‘A’ footprint required to construct cell 1 becomes available. OI and TCS dyke material from pushback 2 will be used for the construction of dyke ‘A’ to enable in-pit tailings storage to start in cell 1.

As mining progresses to pushbacks 3 and 4, the OI and TCS dyke material produced can be used to construct dykes ‘B’ and ‘C’ to make available cells 2 and 3 respectively for tailings storage. Any excess OI and TCS dyke material can be used for other purposes like shelling dumps, road construction, sand capping, and fines trapping as in non-segregating tailings. It is assumed that cell 4 will not be available for tailings storage until the end of the mine life; therefore it was not used for the volume balance calculations in the tailings storage management strategy. Table 1 shows estimates from the balancing of tailings storage requirements for the conceptual mining model. From the in-pit cell volumes generated for cells 1, 2, and 3, the required capacity of the ETF can be calculated and designed. The ETF was designed to cover an area of 16 Mm^2 with a height of 60 m resulting in a 13% excess containment capacity. The freeboard used for the designs is 5 m.

This tailings storage management strategy is based on the assumption that, all the available ore will be mined and processed. After the optimization of the production schedule, the actual mined ore tonnes can be used to reassess the tailings storage management strategy and appropriate modifications made. Further analysis of the conceptual mining model was done by starting the mining operation in pushback 4 and progressing in a south-north direction.

Table 1. Estimates for tailings storage requirements for the conceptual mining model

Material type	Oil sands deposit (Mtonnes)	Available dyke material (Mm^3)	Tailings/Waste produced (Mm^3)	Cells/ETF designed capacity (Mm^3)
Ore	2792.5	-	2251.1	Cell 1: 532
OI dyke material	1697.8	797.6	-	Cell 2: 560
TCS dyke material	2110.0	975.0	-	Cell 3: 560
Waste	375.9	-	179.0	ETF: 880

3.2. Conceptual dykes' designs

Simplified conceptual dykes' designs were made for all the dykes and used as the basis for OI and TCS dyke material scheduling in all pushbacks. It was assumed that each dyke is made up of a key trench, a starter dyke and the main dyke as shown in Fig. 3. The key trench and starter dyke will be constructed using OI dyke material and the main dyke will be constructed using TCS dyke material. Once construction of the key trench and starter dyke is complete, the tailings facility can be used whiles construction of the main dyke progresses. In line with the geology of the McMurray formation, it was assumed that the ETF dyke will be constructed possibly on a weak foundation and the in-pit cell dykes will be constructed on a good foundation, thus requiring different side slopes. Table 2 shows the designed material requirements for the main dyke, starter dyke, and key trench at various destinations. The estimates are the minimum material required at the various destinations for dyke construction and any excess material can be used for other purposes.

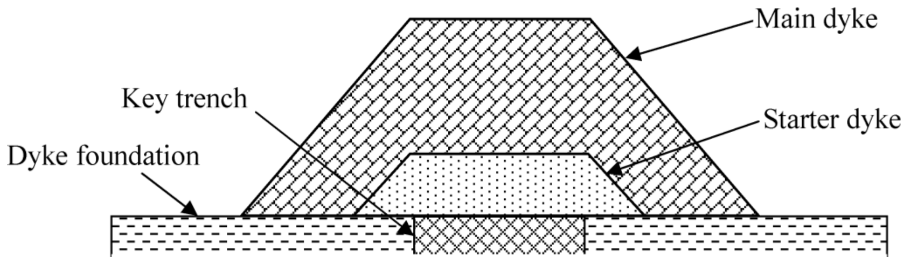

Fig. 3. Schematic diagram showing cross section of a dyke (Askari-Nasab and Ben-Awuah, 2011)

Table 2. Material requirements for dykes at different locations

Dyke location	OI and TCS dyke material required (Mm^3)		
	Key trench	Starter dyke	Main dyke
ETF dyke	1.96	20.58	507.63
Dykes A+B+C	1.38	10.80	304.95

4. Literature review

Mining is the process of extracting a beneficial natural resource from the earth (Newman et al., 2010) and historical analysis of mineral resource evaluations has demonstrated the sensitivity of project's profitability to decisions based on long-term mine production schedules. LTPP problems have been a major research area for some time now and though major improvements have been made, the current dynamic mining environment brings about new and complex problems. Effective LTPP can increase the profitability and life of mine considerably.

Using mathematical programming models with exact optimization methods to solve the LTPP problem have proved to be robust. Mathematical programming models including GP, LP, MIP, and MILP have the capability of considering multiple material types, elements, and destinations. Solving them with exact optimization methods result in solutions with known extent of optimality. As the solution gets closer to optimality, it leads to production schedules that generate higher NPV than those obtained from heuristic optimization methods. GP allows for flexible formulation, specification of priorities among goals, and some level of interaction between the decision maker and the optimization process (Zeleny, 1980; Hannan, 1985). Zhang et al. (1993) applied GP to a production scheduling problem using multiple criteria decision-making formulation. This formulation was developed for one ore type process and multiple goals were considered based on their priorities. The model was implemented for a surface coal mine production schedule. A 0-1 non-linear GP model was used by Esfandiri et al. (2004) in defining a mineral dressing criterion for an iron ore mine. The GP model was defined based on multiple criteria decision making and the deviations from the goals for economics, mining, and mineral dressing functions were minimized. The model was found to have limitations and constraints that were numerous for practical applications. The scheduling of multiple maintenance projects for a mineral processing equipment at a copper mine was developed by Chen (1994) using a 0-1 GP model. Chen used 0-1 binary integer decision variables and multiple scheduling periods, to schedule 4 projects, 40 jobs and 9 resource types. Comparing the results to a heuristic method that was used by the mine, the GP model reduced the total project cost, project duration, and overall workload. Some mine production related problems have been tackled using modified forms of GP. Chanda and Dagdelen (1995) used GP and an interactive graphics system for optimal blending in a coal mine production. A fuzzy GP model was developed by Oraee and Asi (2004) for optimizing a haulage system in an open pit mine. Other industrial production planning and project selection decision-making problems that have been solved making use of the advantages of GP formulations includes the works of Jääskeläinen (1969), Mukherjee and Bera (1995), Leung et al. (2003), and Lee et al. (2010). These GP applications can be considered as small-scale optimization problems in comparison with mine production scheduling optimization problems which involve a large number of decision variables and constraints.

Ramazan and Dimitrakopoulos (2004) developed MIP formulations where they attempt to decrease the number of binary variables and solution times by setting some variables as binary and others as continuous. The results showed partial mining of blocks with the same ore value, thus affecting the NPV generated. LP and MIP models that were subsequently developed by Akaike and Dagdelen (1999), Caccetta and Hill (2003), Ramazan et al. (2005), Ramazan (2007), and Boland et al. (2009) were either not able to generate a global optimum solution for large-scale LTPP problems or there were not enough information to assess the practicality of the generated schedules from mining operation point of view. Recent applications of MILP models to the LTPP problem by Askari-Nasab et al. (2010) has lead to the development of models that use block clustering techniques to reduce the number of decision variables. The formulation was implemented for an iron ore mine case study where long-term production scheduling was done for 21 periods. This model does not consider multiple material types or destinations.

In summary, these GP, LP, MIP, and MILP applications lack the framework that can be used in solving the oil sands mine production planning and waste management problem. They are limited to either single ore, element, or destination, small-scale optimization problems or no consideration for directional mining, and integration of mine production and waste disposal planning. Some efforts have been made to combine GP, MIP, and MILP models to solve some industrial problems because of the advantages of such hybrids. This model referred to as MILGP, has been successfully applied to scheduling and budgeting problems in nursing, business administration, and manufacturing industries (Selen and Hott, 1986; Ferland et al., 2001; Liang and Lawrence, 2007; Nja and Udofia, 2009). The application of MILGP to the oil sands mine production planning and waste management problem as outlined in this paper has been setup in an optimization framework that integrates multiple material types, elements, and destinations. It includes large-scale optimization, directional mining, and integration of mine production planning and waste management. The practical implementation of the MILGP model and the generated production schedules are also discussed.

5. MILGP model for open pit production scheduling

The long-term mine production scheduling problem will be formulated using a combination of mixed integer and goal programming. Using goal programming is appropriate in this context because the structure enables the optimization solution to try achieving a set of goals where some goals can be traded off against one another depending on their priority. Hard constraints can also be converted to soft constraints which otherwise could lead to infeasible solutions. In simple terms, goal programming allows for flexible formulation and the specification of priorities among goals (Liang and Lawrence, 2007). The formulated model for the strategic production and dyke material scheduling problem has an objective function, goal functions and constraints. The goal objectives are mining, processing and dyke construction (Ferland et al., 2001; Esfandiri et al., 2004; Liang and Lawrence, 2007).

5.1. Notations

The notations used in the formulation of the oil sands strategic and dyke material production scheduling problem has been classified as sets, indices, subscripts, superscripts, parameters, and decision variables. Details of these notations can be found in Appendix 1. In general, the MILGP formulation is for multiple material types and destinations as well as pushbacks which ties into the waste management strategy. The MILGP formulation framework was developed based on mining-cuts. This MILGP model is an extension of the oil sands mine planning formulation by Ben-Awuah and Askari-Nasab (2011).

5.2. Modeling of economic mining-cut value

The objective function of the MILGP model for LTPP is to maximize the net present value of the mining operation and minimize the dyke construction cost and deviations from the mining goal, processing goal, OI dyke material goal, and TCS dyke material goal for all destinations. The concept of economic mining-cut value is based on ore parcels within mining-cuts which could be mined selectively. The profit from mining a mining-cut is a function of the value of the mining-cut based on the processing destination and the costs incurred in mining, processing, and dyke construction at a specified destination. The cost of dyke construction is also a function of the location of the tailings facility being constructed and the type and quantity of dyke material needed. The discounted profit from mining-cut k is equal to the discounted revenue obtained by selling the final product contained in mining-cut k minus the discounted cost involved in mining mining-cut k as waste minus the extra discounted cost of mining OI dyke material minus the extra discounted cost of mining TCS dyke material. This has been simplified into Eqs. (1) to (5).

$$d_k^{u,t} = v_k^{u,t} - q_k^{u,t} - p_k^{u,t} - h_k^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\},\ k \in \{1,...,K\} \tag{1}$$

Where:

$$v_k^{u,t} = \sum_{e=1}^{E} o_k \times g_k^e \times r^{u,e} \times \left(p^{e,t} - cs^{e,t}\right) - \sum_{e=1}^{E} o_k \times cp^{u,e,t} \quad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{2}$$

$$q_k^{u,t} = \left(o_k + d_k + w_k\right) \times cm^{u,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{3}$$

$$p_k^{u,t} = d_k \times ck^{u,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{4}$$

$$h_k^{u,t} = l_k \times ct^{u,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{5}$$

5.3. The MILGP model

Using multiple criteria decision making analysis, the objective functions of the MILGP model for strategic and dyke material LTPP as applied in oil sands mining, can be formulated as: i) maximizing the NPV, ii) minimizing the dyke construction cost, and iii) minimizing deviations from the goals. These are represented by Eqs. (6), (7), and (8) respectively.

$$\sum_{j=1}^{J}\left(Max\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{k\in B_j}\left(v_k^{u,t}\times x_k^{u,t}-q_k^{u,t}\times y_k^{u,t}\right)\right)\right) \tag{6}$$

$$\sum_{j=1}^{J}\left(Min\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{k\in B_j}\left(p_k^{u,t}\times z_k^{u,t}+h_k^{u,t}\times s_k^{u,t}\right)\right)\right) \tag{7}$$

$$\sum_{j=1}^{J}\left(Min\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{k\in B_j}\begin{bmatrix}P_1\left(a_1d_1^{-,u,t}\right)+P_2\left(a_2d_2^{-,u,t}\right)+\\ P_3\left(a_3d_3^{-,u,t}+a_3d_3^{+,u,t}\right)+P_4\left(a_4d_4^{-,u,t}+a_4d_4^{+,u,t}\right)\end{bmatrix}\right)\right) \tag{8}$$

Eqs. (6) to (8) can be combined as a single objective function formulated as in Eq. (9).

$$\sum_{j=1}^{J}\left(Max\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{k\in B_j}\begin{bmatrix}\left(v_k^{u,t}\times x_k^{u,t}-q_k^{u,t}\times y_k^{u,t}\right)-\left(p_k^{u,t}\times z_k^{u,t}+h_k^{u,t}\times s_k^{u,t}\right)-\\ \begin{pmatrix}P_1\left(a_1d_1^{-,u,t}\right)+P_2\left(a_2d_2^{-,u,t}\right)+\\ P_3\left(a_3d_3^{-,u,t}+a_3d_3^{+,u,t}\right)+P_4\left(a_4d_4^{-,u,t}+a_4d_4^{+,u,t}\right)\end{pmatrix}\end{bmatrix}\right)\right) \tag{9}$$

The complete MILGP model comprising of the objective function, goal functions and constraints can be formulated as;

Objective function:

$$\sum_{j=1}^{J}\left(Max\sum_{u=1}^{U}\sum_{t=1}^{T}\left(\sum_{k\in B_j}\begin{bmatrix}\left(v_k^{u,t}\times x_k^{u,t}-q_k^{u,t}\times y_k^{u,t}\right)-\left(p_k^{u,t}\times z_k^{u,t}+h_k^{u,t}\times s_k^{u,t}\right)-\\ \begin{pmatrix}P_1\left(a_1d_1^{-,u,t}\right)+P_2\left(a_2d_2^{-,u,t}\right)+\\ P_3\left(a_3d_3^{-,u,t}+a_3d_3^{+,u,t}\right)+P_4\left(a_4d_4^{-,u,t}+a_4d_4^{+,u,t}\right)\end{pmatrix}\end{bmatrix}\right)\right) \tag{10}$$

Goal functions:

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(o_k+w_k+d_k\right)\times y_k^{u,t}\right)+d_1^{-,u,t}=T_m^{u,t} \qquad \forall t\in\{1,...,T\},\ u\in\{1,...,U\} \tag{11}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(o_k\times x_k^{u,t}\right)\right)+d_2^{-,u,t}=T_p^{u,t} \qquad \forall t\in\{1,...,T\},\ u\in\{1,...,U\} \tag{12}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(d_k\times z_k^{u,t}\right)\right)+d_3^{-,u,t}-d_3^{+,u,t}=T_d^{u,t} \qquad \forall t\in\{1,...,T\},\ u\in\{1,...,U\} \tag{13}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(l_k\times s_k^{u,t}\right)\right)+d_4^{-,u,t}-d_4^{+,u,t}=T_l^{u,t} \qquad \forall t\in\{1,...,T\},\ u\in\{1,...,U\} \tag{14}$$

Constraints:

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} g_k^e \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \le \overline{g}^{u,t,e} \qquad \forall t\in\{1,..,T\},\ u\in\{1,..,U\}, e\in\{1,..,E\} \tag{15}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} g_k^e \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \ge \underline{g}^{u,t,e} \qquad \forall t\in\{1,..,T\},\ u\in\{1,..,U\}, e\in\{1,..,E\} \tag{16}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^e \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \le \overline{f}^{u,t,e} \qquad \forall t\in\{1,..,T\}, u\in\{1,..,U\}, e\in\{1,..,E\} \tag{17}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^e \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \ge \underline{f}^{u,t,e} \qquad \forall t\in\{1,..,T\}, u\in\{1,..,U\}, e\in\{1,..,E\} \tag{18}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^d \times d_k \times z_k^{u,t} \Big/ \sum_{k\in B_j} d_k \times z_k^{u,t}\right) \le \overline{f}^{u,t,d} \qquad \forall t\in\{1,..,T\}, u\in\{1,..,U\}, e\in\{1,..,E\} \tag{19}$$

$$\sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^d \times d_k \times z_k^{u,t} \Big/ \sum_{k\in B_j} d_k \times z_k^{u,t}\right) \ge \underline{f}^{u,t,d} \qquad \forall t\in\{1,..,T\}, u\in\{1,..,U\}, e\in\{1,..,E\} \tag{20}$$

$$\sum_{u=1}^{U}\left(x_k^{u,t} + z_k^{u,t}\right) \le \sum_{u=1}^{U} y_k^{u,t} \qquad \forall t\in\{1,..,T\},\ k\in\{1,..,K\} \tag{21}$$

$$\sum_{u=1}^{U} s_k^{u,t} \le \sum_{u=1}^{U} x_k^{u,t} \qquad \forall t\in\{1,..,T\},\ k\in\{1,..,K\} \tag{22}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} x_k^{u,t} \le 1 \qquad \forall k\in\{1,..,K\} \tag{23}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} z_k^{u,t} \le 1 \qquad \forall k\in\{1,..,K\} \tag{24}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} s_k^{u,t} \le 1 \qquad \forall k\in\{1,..,K\} \tag{25}$$

$$b_k^t - \sum_{u=1}^{U}\sum_{i=1}^{t} y_s^{u,i} \le 0 \qquad \forall t\in\{1,...,T\},\ k\in\{1,...,K\},\ s\in C_k(L) \tag{26}$$

$$b_k^t - \sum_{u=1}^{U}\sum_{i=1}^{t} y_r^{u,i} \le 0 \qquad \forall t\in\{1,...,T\},\ k\in\{1,...,K\},\ r\in M_k(P) \tag{27}$$

$$\sum_{u=1}^{U}\sum_{i=1}^{t} y_k^{u,i} - b_k^t \le 0 \qquad \forall t\in\{1,...,T\},\ k\in\{1,...,K\} \tag{28}$$

$$b_k^t - b_k^{t+1} \le 0 \qquad \forall t\in\{1,...,T-1\},\quad k\in\{1,...,K\} \tag{29}$$

$$c_j^t - \sum_{u=1}^{U}\sum_{i=1}^{t} y_h^{u,i} \le 0 \qquad \forall t \in \{1,...,T\},\ j \in \{1,...,J\},\ h \in B_j(H) \tag{30}$$

$$\sum_{u=1}^{U}\sum_{i=1}^{t} y_h^{u,i} - c_j^t \le 0 \qquad \forall t \in \{1,...,T\},\ j \in \{1,...,J\},\ h \in B_{j+1}(H) \tag{31}$$

$$c_j^t - c_j^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\},\ \ j \in \{1,...,J\} \tag{32}$$

$$d_1^{-,t},\ d_2^{-,u,t},\ d_3^{-,u,t},\ d_3^{+,u,t},\ d_4^{-,u,t},\ d_4^{+,u,t} \ge 0 \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{33}$$

$$P_1 > P_2 > P_3 > P_4 \tag{34}$$

Eq. (10) is the objective function of the formulation which seeks to i) maximize the NPV, ii) minimize the dyke construction cost, and iii) minimize deviations from the goals. Eqs. (11), (12), (13), and (14) are the goal functions which define the mining, processing, OI dyke material, and TCS dyke material goals that are required for all destinations. Eqs. (15) to (20) specify the limiting grade requirements for ore bitumen, ore fines, and OI dyke material fines for all destinations. Eq. (21) ensures that the total material mined in each period for all destinations does not exceed the sum of the ore and OI dyke material mined. Eq. (22) states that the fraction of TCS dyke material mined in each period should be less or equal to the fraction of ore material mined for all destinations. Eqs. (23), (24), and (25) ensures that the total fractions of mining-cut k sent to all destinations in all periods is less or equal to one. Eqs. (26), (27), (28), and (29) check the set of immediate predecessor mining-cuts that must be mined prior to mining mining-cut k for all periods and destinations. These equations control the vertical and horizontal block extraction sequence. They ensure that mining proceeds in the specified mining direction as the mine goes deeper. Eqs. (30), (31), and (32) check the set of immediate predecessor pit phase that must be mined prior to mining phase j in all periods for all destinations. Eq. (33) ensures that the negative and positive deviations from the targeted mining, processing, OI dyke material, and TCS dyke material goals are always positive for all periods and destinations. Eq. (34) states the order of prioritization associated with achieving the goals. The model assumes that there exists a pre-emptive priority structure among the goals and this can be changed depending on the mining operation and aim of optimization.

Using mathematical models like the MILGP formulation for mine optimization usually results in large-scale optimization problems. A commercial optimization solver capable of handling such problems is ILOG CPLEX (ILOG Inc., 2007). This optimization solver uses branch and cut algorithm and makes the solving of large-scale problems possible for the MILGP model. Branch and cut is a method of combinatorial optimization for solving integer programming problems. This algorithm is a hybrid of branch-and-bound and cutting plane methods (Horst and Hoang, 1996; Wolsey, 1998).

The MILGP model solver in this research is TOMLAB/CPLEX (Holmström, 2009). The user sets an optimization termination criterion in CPLEX known as the gap tolerance (EPGAP). The EPGAP which is a measure of optimality sets an absolute tolerance on the gap between the best integer objective and the objective of the best node remaining in the branch and cut algorithm. It instructs CPLEX to terminate once a feasible integer solution which is within the set EPGAP has been found (ILOG Inc., 2007).

6. Implementing the MILGP model for production scheduling and waste disposal planning

The MILGP model for open pit strategic and dyke material production scheduling has the objective of maximizing the NPV of the mining operation, minimizing the dyke construction cost for the tailings management plan, and minimizing the deviations from the set goals. The goals are the mining, processing, OI dyke material, and TCS dyke material targets in tonnes. The size of the mining-cuts used for production scheduling must be carefully selected to ensure that it is comparable to the selective mining units of the operation in practice. The proposed MILGP model uses continuous decision variables, $y_k^{u,t}$, $x_k^{u,t}$, $z_k^{u,t}$, and $s_k^{u,t}$ to model mining, processing, OI dyke material and TCS dyke material requirements respectively for all destinations. Binary integer decision variables, b_k^t and c_j^t are used to control precedence of mining-cuts and pushback extraction. Continuous deviational variables, $d_1^{-,t}$, $d_2^{-,u,t}$, $d_3^{-,u,t}$, $d_3^{+,u,t}$, $d_4^{-,u,t}$, and $d_4^{+,u,t}$ have been defined to support the goal functions that control mining, processing, OI and TCS dyke material for all destinations. The deviational variables make available a continuous range of units (tonnes) that the optimizer chooses from to satisfy the set goals and these deviational variables are minimized in the objective function. The objective function also contains deviational penalty cost and priority parameters which are important aspects of this formulation. The deviational penalty cost parameters defined by a_1, a_2, a_3, and a_4 penalizes the NPV for any deviation from the set goals. This parameter forces the optimizer to meet the set goals to avoid penalizing the NPV. The priority parameters P_1, P_2, P_3, and P_4 are used to place emphasis on the goals that are more important. This parameter is also set up to penalize the NPV more if the most important set goal is not met.

In setting up these parameters, the modeler needs to monitor how smooth the mining proceeds from one period to another and the uniformity of tonnages mined per period; as well as the corresponding NPV generated in other to keep track of the impact of any parameter change on these key performance indicators. In some cases, the extent of setting the priority or penalty cost depends on the extent to which the modeler wants to trade off NPV to meet the set goals. A higher priority or penalty may enforce a goal to be met whilst reducing the NPV of the operation. A case showing this trend has been analyzed.

7. Results and discussions

The developed MILGP model was implemented and tested in TOMLAB/CPLEX environment (Holmström, 2009). The performance of the proposed model was analyzed based on NPV, mining production goals, smoothness and practicality of the generated schedules and the availability of tailings containment areas at the required time. The proposed formulation was verified by numerical experiments on a synthetic and an oil sands data set. The model was implemented on a Dell Precision T3500 computer at 2.4 GHz, with 3GB of RAM.

Further implementation of the MILGP model was done for a large scale oil sands deposit covering an area of 8 km x 4 km which is similar to that used in the conceptual mining model. 864 drillholes with an average depth of 82 m were sampled in this area. The drillhole data were used in developing the rock types and grade models for the oil sands deposit using Gemcom GEMS software (Gemcom Software International, 2008). The modeled rock types are made up of the Pleistocene, Clearwater, Upper McMurray, Middle McMurray and Lower McMurray formations. Whittle software and Gemcom GEMS (Gemcom Software International, 2008) were used in determining and designing the final pit which contains 61490 blocks over five 15 m mining benches ranging from 265 m to 325 m. Each block represents a volume of rock equal to 50 m x 50 m x 15 m. The model contains 4866.2 million tonnes of material with 2792.5 million tonnes of ore, 1697.8 million tonnes of OI dyke material, 2110.0 million tonnes of TCS dyke material and 375.9 million tonnes of waste. The deposit is to be scheduled over 20 periods.

The designed final pit block model was divided into 4 pushbacks that are consistent with the conceptual mining model. The sizes of the pushbacks are determined in consultation with tailings dam engineers and are based on the required cell capacities and the timeliness required in making the cell areas available for tailings containment. The blocks within each pushback are clustered into mining-cuts using fuzzy logic clustering algorithm (Kaufman and Rousseeuw, 1990) to reduce the number of decision variables required in the MILGP model. Clustering of blocks into mining-cuts ensures the MILGP scheduler generates a mining schedule at a selective mining unit that is practical from mining operation point of view. The material in the designed final pit is to be scheduled for the processing plant and four dyke construction destinations with the objective of maximizing the NPV of the mining operation and minimizing the dyke construction cost. An EPGAP of 2% was set for the optimization of all pushbacks. A summary of the details for each pushback to be used for production scheduling are shown in Table 3.

For processing plant feed and dyke construction, bitumen grade and fines percent need to be controlled within an acceptable range for all pushbacks and destinations. It is required to keep an average processing plant head grade with bitumen content between 7 and 16% and fines content less than 30%. The OI dyke material is required to have bitumen content less than 7% and fines content less than 50%. Mining will proceed in a north-south direction starting from pushback 1 to 4. When mining of pushback 1 starts, the OI and TCS dyke material will be used in constructing the key trench, starter dyke, and main dyke of the ETF where the initial fluid fine tailings will be stored. When pushback 1 is completely mined, cell 1 area becomes available and OI and TCS dyke material from pushback 2 can be used in constructing dyke ‘A’ about 100 m from the mine face to create cell 1 for in-pit tailings containment to start. This mining and tailings storage management strategy similar to the conceptual mining model will be utilized until all pushbacks are mined.

Table 3. Details for each pushback to be used for production scheduling

Description	**Value**			
	Pushback 1	Pushback 2	Pushback 3	Pushback 4
Number of blocks	14,535	16,433	16,559	13,963
Number of mining-cuts	971	970	977	999
Tonnage of rock (Mt)	1,144.6	1,303.9	1313.2	1104.5
Ore tonnage (Mt)	631.1	758.7	775.7	627.0
OI dyke material tonnage (Mt)	432.4	434.2	435.6	395.7
TCS dyke material tonnage (Mt)	479.4	568.0	587.0	475.5
Average ore bitumen grade (%)	11.7	11.5	11.6	11.6
Average ore fines (%)	8.6	9.7	8.9	8.7
Average OI dyke material fines (%)	4.1	5.8	5.1	4.6

Our objectives are to generate a uniform schedule and a smooth mining sequence based on the availability of material, the plant processing capacity, and dyke construction requirements. The dyke construction material scheduled should meet the minimum requirements of material for the specified destination with any excess material being available for other purposes. Further to this, to ensure that the mining equipment capacity is well utilized throughout the mine life, we intend to keep a uniform stripping ratio when the mining of ore starts. Table 4 shows the input mining, processing and dyke material goals for the MILGP model for 20 periods. Table 5 shows the input grade limits for ore and OI dyke material for the MILGP model for 20 periods.

Table 4. Mining, processing, OI and TCS dyke material goals for the MILGP model for 20 periods

Mining goal (Mt)	**Processing goal (Mt)**	**OI dyke material goal (Mt)**	**TCS dyke material goal (Mt)**
244	140	70	106

Table 5. Ore and OI dyke material grades for the MILGP model for 20 periods

Ore bitumen grade (wt%)		Ore fines (wt%)		OI dyke material fines (wt%)	
$\underline{g}^{u,t,e}$	$\overline{g}^{u,t,e}$	$\underline{f}^{u,t,e}$	$\overline{f}^{u,t,e}$	$\underline{f}^{u,t,d}$	$\overline{f}^{u,t,d}$
7	16	0	30	0	50

Some of the important features that make this MILGP formulation a robust and flexible platform for mine planning are that apart from the NPV maximization and dyke construction cost minimization, the planner has control over the setting of goals and their deviational variables and the upper and lower limits of grades in each period for all pushbacks and destinations. The planner can also decide on tradeoffs between NPV maximization or dyke construction cost minimization and goals achievement using the penalty and priority functions. The penalty cost and priority parameters used in the MILGP model for this optimization were: 0 for mining; 20 for processing; 30 for OI dyke material; and 30 for TCS dyke material. These generated the required tonnages at the various production destinations. Table 6 summarizes the results from the MILGP model in terms of the NPV and dyke construction cost generated after optimization. The four pushbacks were optimized separately over a total of 20 periods. The overall NPV generated including the dyke construction cost for all pushbacks and destinations is $14,237M.

Table 6. Results from the MILGP model in terms of the NPV and dyke construction cost for all pushbacks and destinations

Pushback #	NPV ($M)	Dyke construction cost ($M)	EPGAP (%)
Pushback 1	6,493.77	714.44	2.0
Pushback 2	4,695.34	524.20	2.0
Pushback 3	3,184.72	312.74	1.7
Pushback 4	1,588.65	174.39	1.1

Figs. 4a, 4b, 4c, and 4d show the mining sequence at level 295m for all pushbacks with a north-south mining direction. The MILGP model generated a practical mining sequence that is smooth and consistent with the mining of oil sands. Mining proceeds in the specified direction to ensure least mobility and increased utilization of loading equipment. This is very important in the case of oil sands mining where large cable shovels are used. The size of the mining-cuts in each period enables good equipment maneuverability and the number and size of active bench phases in each period also reduces the number of loading equipments required as well as providing alternative loading points if needed. Another strategic aspect of mining in the specified direction within each pushback is to ensure that the dyke footprints are released on time as the mining proceeds to enable in-pit dyke construction for tailings containment to start. This is an important integral part of the waste management strategy for oil sands mining operations, and a key driver for profitability and sustainable operations. This also reduces the environmental footprints of the ETF.

The results from Fig. 5 shows a uniform mining, processing, OI and TCS dyke material schedules which ensures effective utilization of mining fleet and processing plant throughout the mine life. The schedule ensures that apart from meeting the processing plant requirements to maximize NPV, the required quality and quantity of dyke material needed to build the dykes of the ETF, cells ‘A’, ‘B’, and ‘C’ are provided in a timely manner at a minimum cost for tailings containment. The schedule basically ensures that the minimum dyke material requirements of each dyke construction destination as per the conceptual dykes’ designs are met so that any excess material can be used for other purposes.

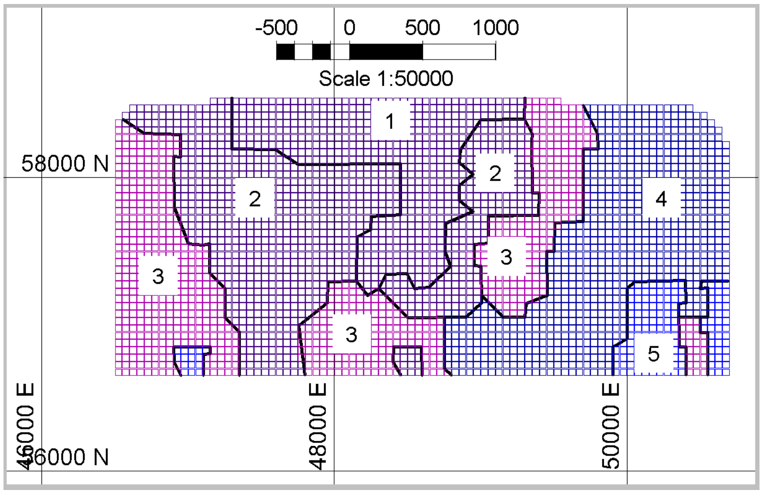

Fig. 4a. Pushback 1 mining sequence at level 295 m

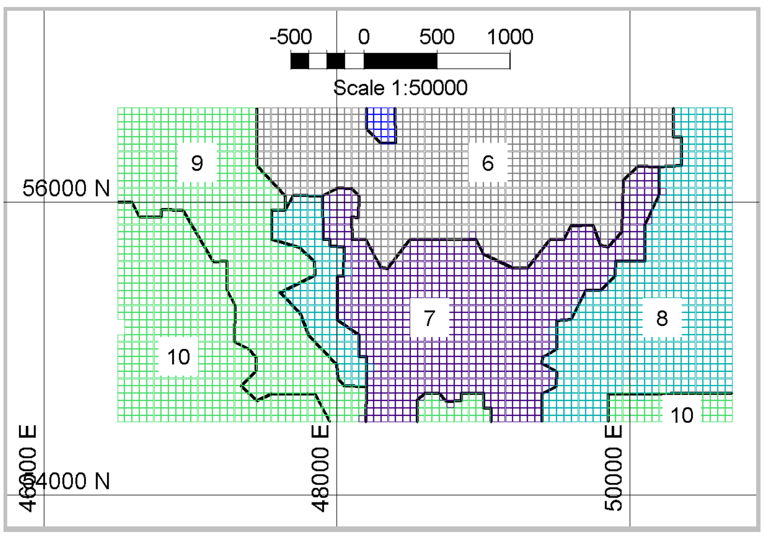

Fig. 4b. Pushback 2 mining sequence at level 295 m

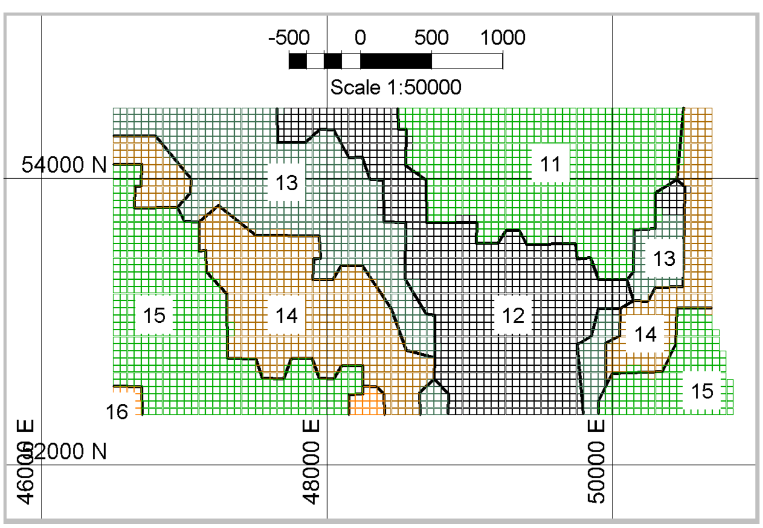

Fig. 4c. Pushback 3 mining sequence at level 295 m

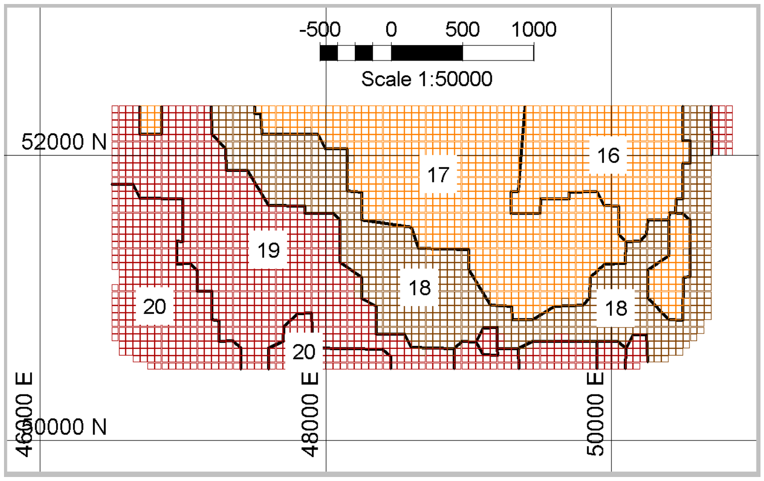

Fig. 4d. Pushback 4 mining sequence at level 295 m

During the first year, due to the requirements of the ETF dyke construction material, less ore is mined and more OI dyke material is mined to facilitate the construction of the key trench and starter dyke and then subsequently, TCS dyke material can be used to continue constructing the main dyke as planned in the conceptual dyke design. This ensures that tailings containment area is created in time for the storage of fluid fine tailings. Ore becomes available at full processing plant capacity from year 2 until the end of the mine life and subsequently TCS dyke material. The OI dyke material supply was also maintained at a uniform rate throughout the mine life. Fig. 5 shows the schedules for ore, OI and TCS dyke material, and waste tonnages generated for 20 periods. Fig. 6 shows the material mined and TCS dyke material tonnage produced in each pushback for 20 periods. Fig. 7 shows the dyke material tonnage sent to the various dyke construction destinations for 20 periods and Fig. 8 shows the OI and TCS dyke material volume scheduled for 20 periods. It can be seen from Table 2 that 23Mm^3 of OI dyke material is required for the ETF key trench and starter dyke construction and this material requirement has been adequately catered for by scheduling 40Mm^3 of OI dyke material in period 1 as shown in Fig. 8.

The total material mined was 4866.2Mt. This is made up of 2720.4Mt of ore and 1386.7Mt of OI dyke material whilst 2055.2Mt of TCS dyke material was generated. A total of 1602.1Mm^3 of dyke material was scheduled. The schedules give the planner good control over dyke material and provides a robust platform for effective dyke construction planning and tailings storage management.

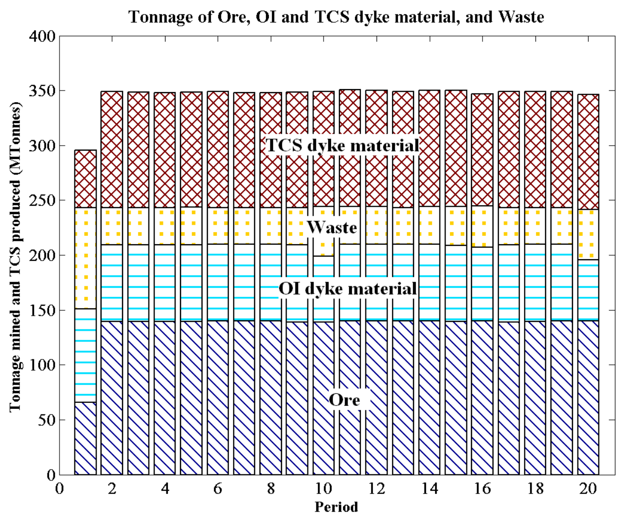

Fig. 5. Schedules for ore, OI and TCS dyke material, and waste tonnages produced over 20 periods

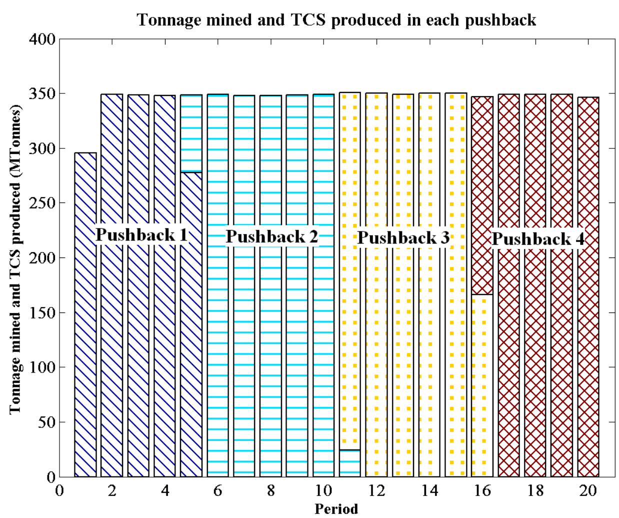

Fig. 6. Material mined and TCS dyke material tonnage produced in each pushback for 20 periods

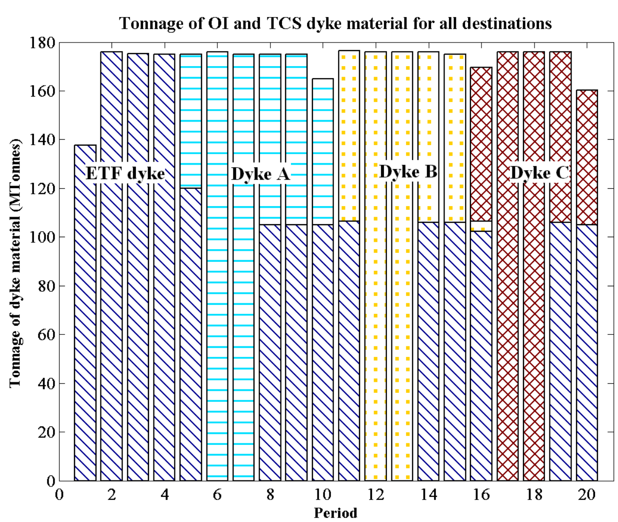

Fig. 7. Dyke material tonnage sent to the various dyke construction destinations for 20 periods

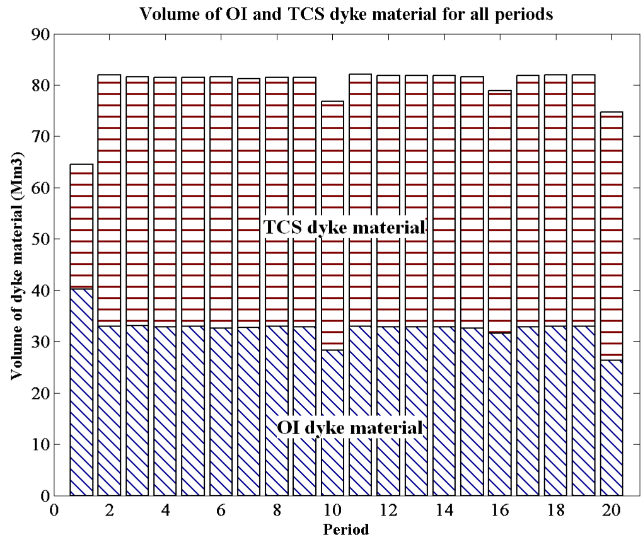

Fig. 8. OI and TCS dyke material volume scheduled for 20 periods

There is also an inherent task of blending the run-of-mine materials to meet the quality and quantity specifications of the processing plant and dyke construction. The blending problem becomes more prominent as more detailed planning is done in the medium to short term. The processing plant head grade and OI dyke material grade that was set were successfully achieved in all periods for all destinations. With the exception of period 1, the scheduled average ore bitumen grade was between 10.9 and 12.2%. The average ore bitumen grade for period 1 was 10.3% basically due to the emphasis placed on mining OI dyke material for the ETF key trench and starter dyke construction. This was required to construct the initial tailings containment when ore processing starts. The average ore and OI dyke material fines percent were between 14 and 30%, and 10 and 23% respectively. Figs. 9 and 10 show the average ore bitumen grade and ore fines percent for all pushbacks respectively. Fig. 11 shows the average OI dyke material fines percent for all pushbacks.

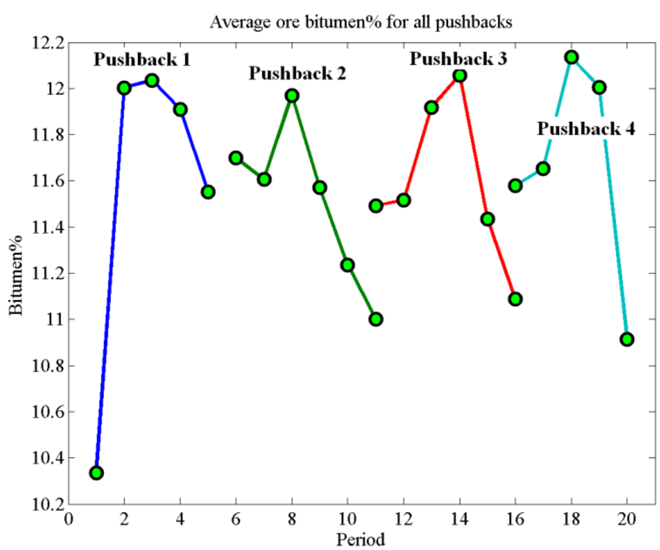

Fig. 9. Average ore bitumen grade for all pushbacks

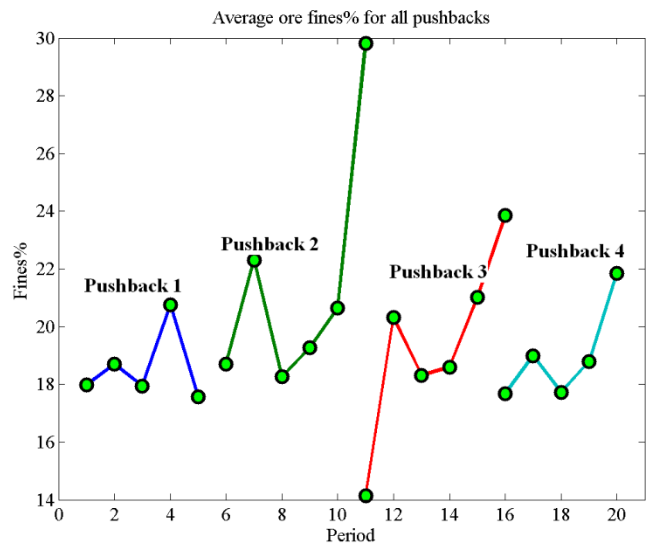

Fig. 10. Average ore fines percent for all pushbacks

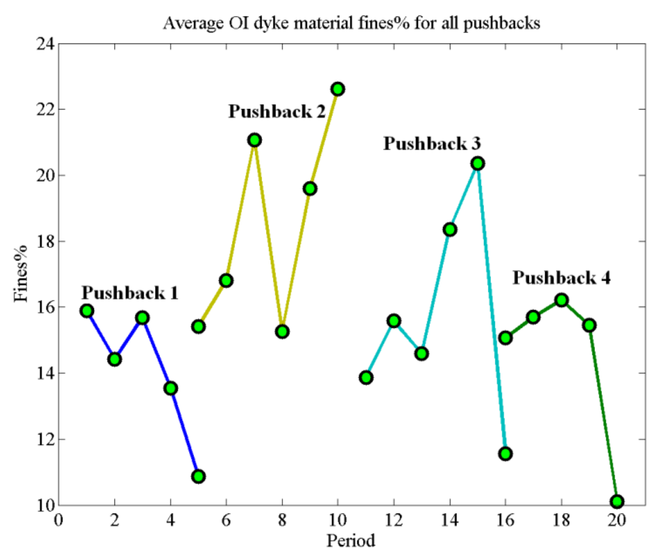

Fig. 11. Average OI dyke material fines percent for all pushbacks

7.1. Supplementary experiments

The data shown in Table 7 represents the summary of results for other optimization experiments that were conducted prior to selecting the illustration presented in this paper. The illustration corresponds to run 3 on the table. The initial optimization experiment conducted was run 1 which schedules for a north-south mining direction. Further work was done by optimizing with a south-north mining direction (run 2) which yielded a lower NPV and a lower dyke material tonnage. The lower NPV results from mining pushbacks with lower economic block values in the early years. Less ore was mined and a less uniform schedule was produced due to the mining direction.

Further investigations were conducted by increasing the number of mining cuts as in run 3. This resulted in an increase in NPV resulting from an increase in the resolution of the optimization problem. The increased resolution increases the flexibility of the problem as well as the number of decision variables thereby increasing the optimization runtime. A smooth and uniform schedule was generated. Another experiment (run 4) was done to test the MILGP model in terms of placing a higher penalty cost and priority (PP) value on one goal as compared to the others. The increased PP value for OI dyke material further constrains the optimization problem decreasing the ore to dyke material ratio and causing a decrease in the overall NPV which includes dyke construction cost. The dyke material tonnes increases and hence the dyke construction cost. As illustrated in Fig. 12, in general within the set mining constraints, as the PP values for dyke material increases, the NPV decreases as a result of a reduction in ore tonnes and/or an increase in dyke material tonnes. This approach is useful when more dyke material is required for tailings containment construction to enable a sustainable mining operation.

Comparing these experiments, run 3 was selected because it generates the best overall NPV as well as a good schedule and the required dyke material tonnage.

Table 7. Results for supplementary experiments showing that run 3 generates the highest NPV and best schedule

Run #	Total Cuts	Mining dxn	P_1a_1	P_2a_2	P_3a_3	P_4a_4	Runtime (minutes)	Overall NPV ($M)	Dyke material (Mt)	Schedule uniformity & smoothness ranking
1	1977	NS	0	20	30	30	105	13,810	3315	3
2	1977	SN	0	20	30	30	17	10,713	3012	4
3	3917	NS	0	20	30	30	288	14,237	3442	1
4	3917	NS	0	20	60	30	59	14,121	3460	2

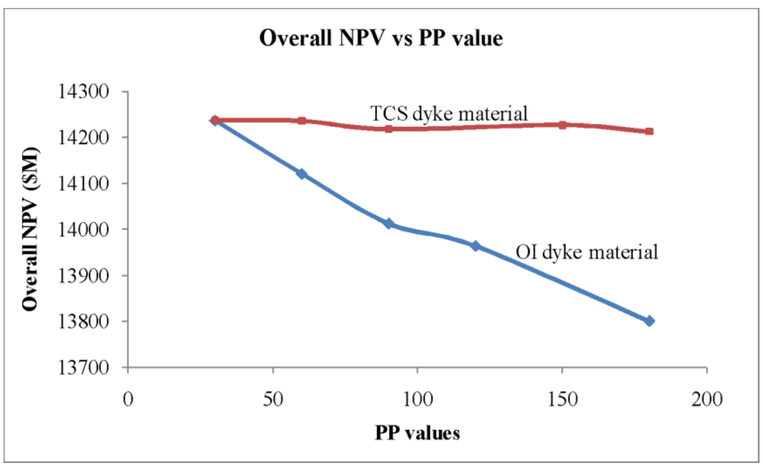

Fig. 12. General trend of overall NPV with PP values of dyke material

8. Conclusions and future work

This paper discussed some of the shortcomings and applications of MIP, MILP and GP to the open pit production scheduling problem. Further to this, the use of MILGP formulations in solving industrial problems due to the advantages derived from this hybrid formulation was reviewed. In this paper, the authors have developed, implemented, and verified a MILGP theoretical framework for open pit production scheduling of multiple material types with multiple elements for multiple destinations. The developed model proved to be able to handle the integration of large-scale mine production planning and waste management problems in the oil sands mining industry.

Oil sands mining requires a carefully planned and integrated mine planning and waste management strategy that generates value and sustainability. This requires that production schedules are generated for ore, dyke material and waste to ensure that whilst ore is fed to the processing plant, there is enough dyke material available for dyke construction for both the ex-pit and in-pit tailings facilities. This ensures there is adequate storage space for the tailings throughout the mine life whilst reducing the size of the disturbed landscape by making the best use of in-pit tailings facilities and reducing the size of the external tailings facility. The MILGP formulation uses binary integer variables to control mining precedence and continuous variables to control mining of ore and dyke material. There are also goal deviational variables and penalty costs and priorities that must be set up by the planner. The optimization model was implemented in TOMLAB/CPLEX environment.

The developed model was able to create value and a sustainable operation by generating a practical, smooth and uniform schedule for ore and dyke material using mining-cuts from block clustering techniques. The schedule gives the planner good control over dyke material and provides a robust platform for effective dyke construction and waste disposal planning. The schedule ensures that the key drivers for oil sands profitability and sustainability which is maximizing NPV whilst creating timely tailings storage areas are satisfied within an optimization framework. This is in accordance with recent regulatory requirements by Alberta Energy Resources and Conservation Board (Directive 074) that requires oil sands mining companies to develop an integrated life of mine plans and tailings disposal strategies for in-pit and external tailings disposal systems (McFadyen, 2008). The planner also has the flexibility of choosing goal deviational variables, penalty costs and priorities to achieve a uniform schedule and improved NPV. Similarly, tradeoffs between achieving goals and maximizing NPV or minimizing dyke construction cost can be made.

Future research will focus on developing more efficient mathematical formulation techniques for the MILGP model that will reduce the solution time for large-scale open pit production scheduling and waste management problems.

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10. Appendix 1

10.1. Notations

10.1.1. Sets

$\boldsymbol{K} = \{1,.....,K\}$ set of all the mining-cuts in the model.

$\boldsymbol{J} = \{1,......,J\}$ set of all the phases (push-backs) in the model.

$\boldsymbol{U} = \{1,.....,U\}$ set of all the possible destinations for materials in the model.

$C_k(L)$ for each mining-cut k, there is a set $C_k(L) \subset \boldsymbol{K}$ defining the immediate predecessor mining-cuts above mining-cut k that must be extracted prior to extraction of mining-cut k, where L is the total number of mining-cuts in the set $C_k(L)$.

$M_k(P)$ for each mining-cut k, there is a set $M_k(P) \subset \boldsymbol{K}$ defining the immediate predecessor mining-cuts in a specified horizontal mining direction that must be extracted prior to extraction of mining-cut k at the specified level, where P is the total number of mining-cuts in the set $M_k(P)$.

$B_j(H)$ for each phase j, there is a set $B_j(H) \subset \boldsymbol{K}$ defining the mining-cuts within the immediate predecessor pit phases (push-backs) that must be extracted prior to extracting phase j, where H is an integer number representing the total number of mining-cuts in the set $B_j(H)$.

10.1.2. Indices, subscripts and superscripts

A parameter, f, can take indices, subscripts, and superscripts in the format $f_{k,j}^{u,e,t}$. Where:

$t \in \{1,......,T\}$ index for scheduling periods.

$k \in \{1,.....,K\}$ index for mining-cuts.

$e \in \{1,.....,E\}$ index for element of interest in each mining-cut.

$j \in \{1,.....,J\}$ index for phases.

$u \in \{1,.....,U\}$ index for possible destinations for materials.

d,l,m,p subscripts and superscripts for overburden and interburden dyke material, tailings coarse sand dyke material, mining and processing respectively.

10.1.3. Parameters

$d_k^{u,t}$ the discounted profit obtained by extracting mining-cut k and sending it to destination u in period t.

$v_k^{u,t}$	the discounted revenue obtained by selling the final products within mining-cut k in period t if it is sent to destination u, minus the extra discounted cost of mining all the material in mining-cut k as ore and processing at destination u.
$p_k^{u,t}$	the extra discounted cost of mining all the material in mining-cut k in period t as overburden and interburden dyke material for construction at destination u.
$h_k^{u,t}$	the extra discounted cost of mining all the material in mining-cut k in period t as tailings coarse sand dyke material for construction at destination u.
$q_k^{u,t}$	the discounted cost of mining all the material in mining-cut k in period t as waste and sending it to destination u.
g_k^e	the average grade of element e in ore portion of mining-cut k.
$\underline{g}^{u,t,e}$	the lower bound on the required average head grade of element e in period t at processing destination u.
$\overline{g}^{u,t,e}$	the upper bound on the required average head grade of element e in period t at processing destination u.
f_k^e	the average percent of fines in ore portion of mining-cut k.
$\underline{f}^{u,t,e}$	the lower bound on the required average fines percent of ore in period t at processing destination u.
$\overline{f}^{u,t,e}$	the upper bound on the required average fines percent of ore in period t at processing destination u.
f_k^d	the average percent of fines in overburden and interburden dyke material portion of mining-cut k.
$\underline{f}^{u,t,d}$	the lower bound on the required average fines percent of overburden and interburden dyke material in period t at dyke construction destination u.
$\overline{f}^{u,t,d}$	the upper bound on the required average fines percent of overburden and interburden dyke material in period t at dyke construction destination u.
o_k	the ore tonnage in mining-cut k.
w_k	the waste tonnage in mining-cut k.
d_k	the overburden and interburden dyke material tonnage in mining-cut k.
l_k	the tailings coarse sand dyke material tonnage in mining-cut k.
$T_m^{u,t}$	the mining goal (tonnes) in period t at destination u.
$d_1^{-,u,t}$	the negative deviation from the mining goal (tonnes) in period t at destination u.
$d_1^{+,u,t}$	the positive deviation from the mining goal (tonnes) in period t at destination u.
$T_p^{u,t}$	the processing goal in period t at destination u (tonnes).

$d_2^{-,u,t}$	the negative deviation from the processing goal in period t at destination u (tonnes).
$d_2^{+,u,t}$	the positive deviation from the processing goal in period t at destination u (tonnes).
$T_d^{u,t}$	the overburden and interburden dyke material goal in period t at destination u (tonnes).
$d_3^{-,u,t}$	the negative deviation from the overburden and interburden dyke material goal in period t at destination u (tonnes).
$d_3^{+,u,t}$	the positive deviation from the overburden and interburden dyke material goal in period t at destination u (tonnes).
$T_l^{u,t}$	the tailings coarse sand dyke material goal in period t at destination u (tonnes).
$d_4^{-,u,t}$	the negative deviation from the tailings coarse sand dyke material goal in period t at destination u (tonnes).
$d_4^{+,u,t}$	the positive deviation from the tailings coarse sand dyke material goal in period t at destination u (tonnes).
$r^{u,e}$	the proportion of element e recovered (processing recovery) if it is processed at destination u.
$p^{e,t}$	the price of element e in present value terms per unit of product.
$cs^{e,t}$	the selling cost of element e in present value terms per unit of product.
$cp^{u,e,t}$	the extra cost in present value terms per tonne of ore for mining and processing at destination u.
$ck^{u,t}$	the cost in present value terms per tonne of overburden and interburden dyke material for dyke construction at destination u.
$ct^{u,t}$	the cost in present value terms per tonne of tailings coarse sand dyke material for dyke construction at destination u.
$cm^{u,t}$	the cost in present value terms of mining a tonne of waste in period t and sending it to destination u.
P_1	the priority level associated with minimizing the deviations from the mining goal.
P_2	the priority level associated with minimizing the deviations from the processing goal.
P_3	the priority level associated with minimizing the deviations from the overburden and interburden dyke material goal.
P_4	the priority level associated with minimizing the deviations from the tailings coarse sand dyke material goal.
a_1	the penalty paid per tonne in deviating from the mining goal.
a_2	the penalty paid per tonne in deviating from the processing goal.
a_3	the penalty paid per tonne in deviating from the overburden and interburden dyke material goal.

a_4 the penalty paid per tonne in deviating from the tailings coarse sand dyke material goal.

10.1.4. Decision variables

$x_k^{u,t} \in [0,1]$ a continuous variable representing the portion of mining-cut k to be extracted as ore and processed at destination u in period t.

$z_k^{u,t} \in [0,1]$ a continuous variable representing the portion of mining-cut k to be extracted as overburden and interburden dyke material and used for dyke construction at destination u in period t.

$s_k^{u,t} \in [0,1]$ a continuous variable representing the portion of mining-cut k to be extracted as tailings coarse sand dyke material and used for dyke construction at destination u in period t.

$y_k^{u,t} \in [0,1]$ a continuous variable representing the portion of mining-cut k to be mined in period t and sent to destination u, which includes both ore, overburden and interburden dyke material and waste.

$b_k^t \in [0,1]$ a binary integer variable controlling the precedence of extraction of mining-cuts. b_k^t is equal to one if the extraction of mining-cut k has started by or in period t, otherwise it is zero.

$c_j^t \in [0,1]$ a binary integer variable controlling the precedence of mining phases. c_j^t is equal to one if the extraction of phase j has started by or in period t, otherwise it is zero.

11. Appendix 2

HTML documentation of MATLAB code

Towards Integration of Oil Sands Mine and Tailings Plans

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Abstract

Tailings is considered to be the main by-product of oil sands processing. Due to the noticeable amount of fresh and recycled water used in the process of bitumen extraction, huge volume of slurry is produced at the end point of the process. The amount of tailings produced is also important from environmental point of view. By regulations, the oil sands companies are required to monitor and control the tailings ponds conditions and minimize the footprints of their operations when closing the mine. Tailings ponds are the most important footprint left from the mining operations. On the other hand, the available facilities for construction of tailings ponds to hold the slurry is limited and restricted to the lease areas. Therefore, the volume of tailings produced downstream is a key operational factor that affects both operation planning and environmental costs of decommissioning. In the literature, several production scheduling formulations using mixed integer liner programs (MILPs) are developed to maximize the net present value (NPV) as the main objective function. These formulations are subject to different operational constraints such as mining capacity, processing capacity, and extraction precedence. The objective of this paper is first, to calculate the amount of tailings produced as a result of extraction of each block and secondly, to revise the MILP in a way to consider the constraint of tailings pond capacity. The tailings calculation formula is retrieved from Suncor's process flow sheet. The derived formulation is verified by applying on a real mining production plan. Then, a sub-gradient algorithm is developed to solve the MILP model by Lagrangian relaxation method. Some future steps of research are mentioned at the end.

1. Introduction

The economic value that the business generates is the most important driver in the mining industry. The net present value (NPV) is well introduced to measure the economic value of the production over the active mine's life. However, apart from the economic aspects of the business, related social and environmental impacts must be considered in mine plans as well. Particularly in mining industry, limited natural resources that contain minerals are the main source that brings money into companies. Mining companies are now required to minimize the land disturbance and exploit these resources in a responsible manner. In addition, many of the mines are in remote areas, where either there is no urban population or in some cases, there are just some small rural societies in the area. As a result, large number of work force and facilities stream into the site after the start of production. It may alter the social and demographic patterns of the region. Therefore, the industry needs to be aware of its responsibility to consider and also minimize the social impacts of any development. Such social and environmental concerns have made the companies pay more attention to long term consequences of their production.

The oil sands industry is one of the fastest growing industries in North America. Most of the bitumen resources of the world are located in northern Alberta boreal forests. Oil sands deposit is a mixture of bitumen and water in sands and clay. It is a thick, sticky, heavy and viscous material and needs rigorous extraction treatment. According to Government of Alberta (2011), the proven oil reserves of Alberta are 171.3 billion barrels (more than 95% of Canadian oil reserves), making Alberta oil sands the third-largest proven crude oil reserve in the world, next to Saudi Arabia and Venezuela. Based on the depth of the resource, there are two extraction methods for oil sands' bitumen; surface mining and steam-assisted gravity drainage (SAGD) technology. Surface mining is used for near-surface reserves, requiring an open-pit mine operation. Oil sands are dug up with shovels and moved by trucks to processing facilities where the recoverable oil is separated from sand by means of hot water. On the other hand, more than 80 percent of Alberta's bitumen is located deeper in sub-surface and needs to be extracted with in-situ method. SAGD technology is used in the majority of in-situ operations. This involves pumping steam underground through the first horizontal well beneath the bitumen formation to decrease the viscosity of bitumen and then pumping the liquefied bitumen up to the surface through a second well.

Different lists of environmental impacts and their significance corresponding to mining projects are addressed in literature. Singh (2008) reviews some general environmental issues of mining projects as the land use, socio-economic impacts, public health and safety, noise and vibrations, impacts on water quality, air and dust and the impacts on environment ecology. More specific lists of impacts corresponding to the oil sands industry are presented in the literature as well. Woynillowicz et al. (2005) and Rodriguez (2007) consider the following environmental issues for open-pit mining and in-situ operations regarding oil sands industry. The impacts are classified in three categories as water related, land related and air-quality related impacts.

1. Water-related impacts

   1.1. Withdrawal from surface fresh waters:

   Between two to five barrels of fresh water are withdrawn from the nearby Athabasca River to extract one barrel of bitumen from Alberta oil sands. Due to added chemicals from extraction process, more than 90% of this amount is not returned to the river anymore. Reduction in the flow of water could reduce the amount of available habitat for fish.

   1.2. Tailings:

   The slurry of water, bitumen, sand, silt and fine clay that is produced from the extraction process is called tailings and is pumped to tailings ponds. There are a number of environmental issues associated with the tailings ponds due to the bitumen remained in the tailings. The pollutants could migrate into the groundwater system and also leak into the surface water and surrounding soil. The tailings water is also toxic to the aquatic life, nearby plants and migratory birds landing on tailings ponds.

   1.3. Freshwater aquifers:

   Both SAGD and oil sands mining operations could decrease the water pressure in mining pit or horizontal wells region. Then it may cause "leaking down" the water from aquifers closer to the subsurface to operation regions. Therefore, the groundwater is discharged to lakes, wetlands and streams and the level of water table goes down as a result. This phenomenon is called "the drawdown effect".

   1.4. Water treatment waste:

   A considerable portion of water used in bitumen extraction, whether in surface mining or SADG operations, is the saline water or produced water that is recycled. Treatment of such water results in significant amount of solid waste. This waste is injected into disposal wells or dumped in landfills and in both cases, many other environmental issues are raised regarding proliferation of waste disposal facilities.

2. Land-related impacts

2.1. Effect on the boreal forests:

Canada's boreal forests cover about 30% of the country's land. This piece of land contains 35% of world's wetlands and provides habitat for many important wildlife species. Most of the Alberta oil sands deposits are found in these forests. Oil sands operations have disturbed the landscape and also groundwater drastically. In surface mining, large land-clearings, in addition to the noise and presence of human have resulted in less presence of wildlife in the area. In in-situ operations, despite the thought that the impact on the landscape is less, the dense network of roads, power line corridors, pipelines and seismic lines have fragmented the habitat and changed it to smaller patches. According to AXYS (2005) the most recently filed environmental impact assessment (EIA) shows that the currently planned oil sands development in Alberta will result in cumulative disturbance of more than 2000 square kilometers, which is a very fast growing footprint.

2.2. Reclaimed landscapes:

The lands affected by oil sands development are required to be reclaimed to an "equivalent land capability" to be returned to the Province of Alberta. However, the reclaimed landscapes currently proposed by the industry are very different from the original nature of boreal forests and wetlands. In fact, it is impossible to re-create the ecological diversity of the boreal ecosystem and the inter-relationships of ecosystem components.

2.3. Erosion:

In most cases, it is required to move the vegetation and thin fertile surface soil to get access to the minerals. In surface mining activities, stripping happens in large scales, resulting in large clearings in natural landscapes. In addition to the pit, construction of access roads also requires wiping the vegetation out. Since the plants' roots protect the soil from erosion, absence of vegetation increases the rate of erosion.

3. Air quality-related impacts

3.1. Emission from purification processes:

Alberta has been number one for air releases from industrial sources among Canadian provinces in 2003 (Pollution-watch, 2003). The main source for Alberta industrial emissions is the oil sands industry. Criteria Air Contaminants (CACs) including sulfur dioxide (SO_2), nitrogen oxides (NOx), particular matter (PM) and volatile organic compounds (VOC) are the most common air pollutants. CACs are released by burning fossil fuels in processes such as oil sands and conventional oil production. Due to the fact that many more steps are involved in producing synthetic crude oil from oil sands comparing to conventional oil, pollutant emissions are much higher in bitumen recovery processes.

3.2. Dust and emission from mining operations:

Apart from the bitumen extraction process that release large amounts of pollutants into the air, other production operations in open pit mining also trigger some issues related to air quality. Drilling and blasting activities generate dust and noise and spreads dust into the air. In addition, construction of the access roads in early stages of mine life, loading and hauling activities and dried tailings are other sources of dust generation. Emissions from mining machinery such as trucks are the other source of air pollution in oil sands mining.

Environmental impacts and mine planning

In response to such environmental concerns, some new concepts in mine planning and optimization are developed. S*ustainable mining* represents this new line of thinking which takes the environmental concerns of any mining project into account. Now the question is, what environmental issues should be considered in decision making and how should they be embedded in the problem. In fact, there are two related categories in which the environmental issues should be considered in; mine design and mine planning.

Mine design refers to the group of techniques that are applied to determine what the overall view of the mine will be at the end of its life. Particularly in open pit mining, the final pit limit is determined in a way that the most possible amount of the ore body can be extracted, based on the estimations of ore value and also extraction and processing costs over the mine-life. A number of environmental issues could be considered in mine design phase by defining a new cost as "environmental cost". The new term may cover estimations of environmental costs corresponding to different stages in mine life such as exploration, excavation and reclamation (Rodriguez, 2007).
On the other hand in mine planning, the objective is to find the optimal production plan to extract all the material out of the optimal pit. A typical mine plan maximizes the NPV over the mine life subject to some technical constraints such as production and processing capacities. For sustainable mine planning, the impacts with ties to the block model should be considered in planning as well. In any block model, there are some attributes such as ore grade, ore tonnage, waste tonnage, rock type and block coordinates. If a valid relation between any of the environmental issues and corresponding blocks can be defined, then that issue could be considered at the mine planning stage. For example, extraction of each block results in generating some amount of waste, either as solid (waste) or wet (as tailings). This waste should be dumped or pumped into an appropriate waste facility and it results in landscape disturbance. Now if based on the specifications of each block, a value called disturbance factor could be assigned to that block, then the disturbance can be measured and considered in planning phase. As a result, the solution to the model maximizes the NPV and at the same time, minimizes the disturbance to the landscape.

From the presented list of environmental impacts, two important impacts are considered to have significant ties to the block model; the tailings and the land disturbance. Thus, these two environmental issues are considered in this paper. It is first required to establish a relationship between the environmental impacts of land disturbance and the block model and then revise the traditional mine planning models in a way to consider these impacts in finding the optimal solution to the problem. The new model makes it possible to take into account both NPV as the financial driver of the industry and environmental issues as the public concerns at the same time.

The rest of the paper is organized as follows: the problem definition is discussed in section 2. Section 3 covers the review of the related literature. The theoretical framework is discussed in section 4. The formulation to calculate the amount of tailings produced out of each block is presented and verified in section 5. The mathematical model for the problem is presented in section 6. Finally, the conclusions and further steps for the research are discussed in section 7.

2. Problem definition

By reviewing the literature, it turns out that already there are many works addressing the maximization of profit in mine planning. Also, there are some models that have considered different environmental costs in finding the optimal pit limits for the mine. However, the missed critical aspect in mine planning is the merger between two areas; profit maximization and environmental cost minimization.

For a better understanding of reclamation process and also the missing part in mine planning chain, revision of Shell's plan in fulfillment of Directive 074 (ERCB, 2009) is helpful. Shell Canada has considered some dedicated disposal areas (DDA) for its JackPine Mine (JPM) and Muskeg River Mine (MRM) sites in Athabasca river region, Alberta, Canada. The two sites are different in terms of tailings facilities; JPM has in-pit tailings facilities while MRM has external tailings facilities. However, the concept of reclamation is almost the same. In both cases, the tailings facilities are constructed with multiple cells adjacent to each other. The thickened tailing (TT) is discharged into the cells consecutively, meaning that the cells receive TT in the order of their location. The cell that receives the discharge earlier is considered as the first DDA, meaning that after a certain period of time, it changes into a dried and reclaimed landscape and reclamation continues in the next cell. The drainage system is designed in a way to discharge any flow of surface water to the adjacent cell. The layout for JPM is illustrated in Fig. 1.

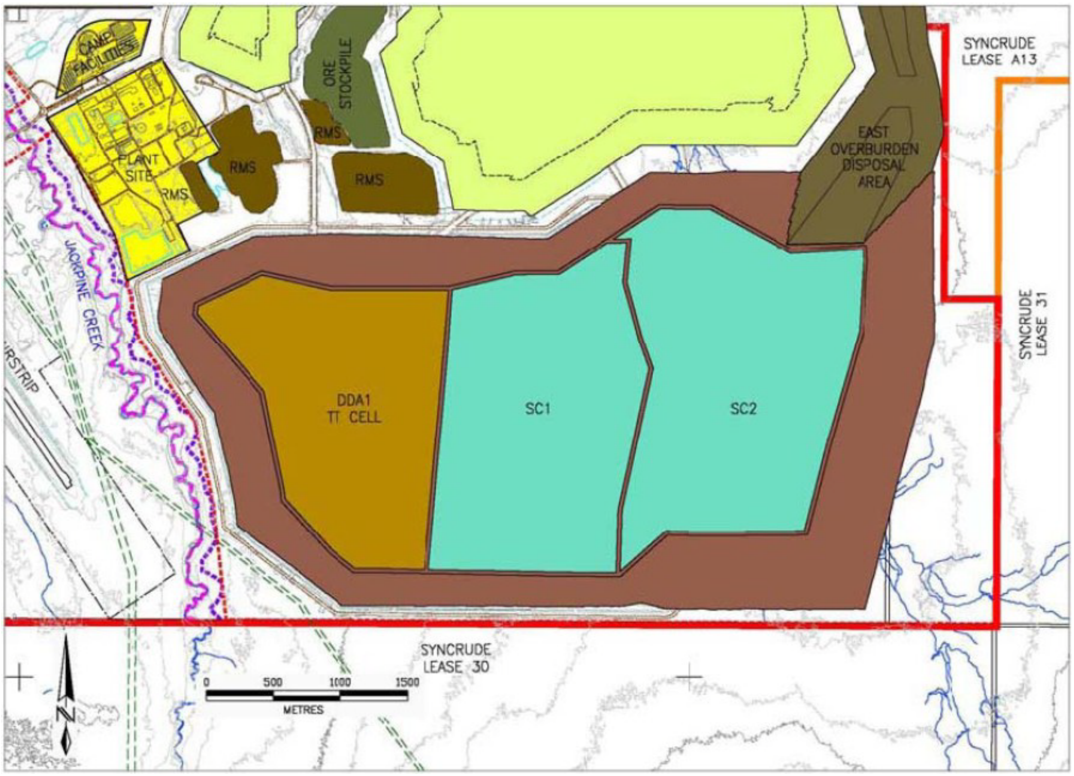

Fig. 1. Layout of dedicated disposal area 1 within the JPM external tailings facility (Shell-Canada, 2011).

Shell Canada considers three main categories in its plan for decommissioning of the external tailings facility as the DDA; (1) construction, (2) operations and (3) closure. Among these three, the construction and operations have ties to the extraction plan of the mine. The amount of waste material that is produced in extraction operations is used for the preparation of starter dyke, external dyke walls and upstream dyke. Also the thickened tailings (TT), centrifuge cake manufacturing and coarse sand tailings (CST) are the by-products of extraction and processing operations. Thus, any change to the production schedule has some impacts on the required material for decommissioning. Now, the question is "*what is the optimal extraction plan that simultaneously maximizes the NPV and minimizes the disturbance on the landscape by considering the decommissioning costs of the mine site?*" in other words, not only the production plan should maximize the NPV with respect to the technical constraints, but also providing required material for decommissioning must be considered as well. Any delay in providing the material required for decommissioning or any additional shipment of material from other stockpiles to the DDA causes extra costs. Therefore, it is important to take into account the destinations for different extracted materials so as to properly manage the stream of materials for decommissioning. As the first step towards considering decommissioning costs in the mine plan, the amount of downstream tailings that is produced from processing oil sands is calculated (section 5) and considered as a new constraint in the mathematical model (section 6). Table 1 presents the steps involved in the decommissioning of an external tailings facility in Jackpine Mine site (Shell-Canada, 2011).

Table 1. Summary of time line for decommissioning of
an external tailings facility by Shell Canada Energy in JMP site.

	Start date	End date
Construction		
Preparation of Starter Dyke	2008	2010
Preparation of External Dyke Walls (centerline)	2015	2029
Preparation of Upstream Dyke	2010	2011
Operations		
TT deposition – initial filling period(1)	2010	2027
Centrifuge Cake Manufacture / Deposition	2014	2027
TT deposition – in-pit tailings CST capping activities(2)	2035	2036
TT deposition – in-pit tailings CST capping activities	2049	2050
TT deposition – in-pit tailings CST capping activities	2054	2055
TFT transfer to SC1	2010	2055
Closure, Capping and Final Landform Design		
Completion of TT deposition	n/a	2055
Trafficable tailings surface	2055	2057
Overburden capping and drainage contouring	2057	2059
Reclamation cover soil placement	2060	2061
Nurse crop coverage and cap settlement	2060	2062
Re-vegetation	2062	2063
Monitoring	2063	TBD
Completion of TT deposition	n/a	2055

Already, the mathematical model that maximizes the NPV corresponding to the mining and processing constraints is well proposed as MILP models in the literature. The overview of such models is as follows:

Maximize (NPV)
Subject to:
Processing plant constraints
Mining capacity constraints
Extraction precedence

In this new model, the costs and constraints corresponding to decommissioning operations is considered as well. Decision variables are revised in a way that facilitates sending of each extracted block or its fractions to different destinations. New objective function terms and constraints are added to the original MILP model to quantify the costs regarding decommissioning. The overview of the revised MILP is as follows:

Maximize (NPV – decommissioning costs)
Subject to:
Processing plant constraints
Mining capacity constraints

Extraction precedence
Capacity constraints for each destination
Capacity constraints for tailings facilities
Providing required material for decommissioning purposes

3. Literature review

During the recent decades, many papers have been published around different aspects of environmental impacts in the mining industry and sustainable mining. In the literature, two groups of tools are used to evaluate sustainability in the mining industry, the descriptive tools and the quantitative ones. Descriptive approaches mostly are based on some reports, for example about the environmental conditions and concerns in mining projects. These reports are either mandatory, obliged by governments or regional authorities, or voluntarily with which the company aims to show its differences from others in the market in terms of environmentally clean practices. On the other hand, quantitative approaches try to quantify the qualitative measures and provide quantitative assessment results for mining operations.

As a descriptive work, Sinding (1999) reviews the environmental management and communication tools for mining industry and discusses the specifications for some of them such as environmental impact assessment, environmental management systems, environmental accounting and life cycle assessment. Sinding (1999) focuses on different stages in a typical mining production as (1) mineral exploration, (2) mine development decision-making, (3) production phase, and (4) mine closure and decommissioning. Then the proper tools for each stage are suggested. Some general recommendations about sustainable mining practices are found in this group of descriptive papers. For instance, the mining companies should consider full range of environmental management and documentation for their activities. Also, it is necessary to establish a global environmental reporting mechanism for the mining industry so that in general, different mining products can be comparable and "the cleanest" can be determined. Furthermore, there should be more emphasis on the effective environmental management in new projects and for effective environmental assessment, increased monitoring and post audit reviews are essential. As a more practical step towards the implementation of these remedies, some have recommended the environmental assessment of projects to be considered as per ISO 14001.

Descriptive works have discussed relatively a complete list of environmental issues tied to mining activities. A comprehensive list of environmental management tools that are in-hand and essential to be implemented are available (Sinding, 1999). However, most of descriptive tools are very general and provide qualitative suggestions rather than explicit quantitative and practical solutions for the problem.

Some other works take a step towards more practical recommendations on environmental aspects of mining projects. Manteiga and Sunyer (2000) modify the recently developed environmental evaluation methodologies in order to make them more practical. A simplified three-step methodology for environmental evaluation assessment is proposed in their work, consisting of; (1) establishment of an assessment framework, (2) assessment of the environmental situation and (3) environmental assessment. Then, these steps are elaborated in details and some indicators to quantify the final results of each step are defined. However, greater efforts are required to achieve the operational implementation of these indicators, both by the environmental authorities who define the indicators and by the mining sector who implement the projects and is responsible for recording precise data corresponding to the indicators.

Quantitative approaches can be classified as the second category of sustainable mining works. Several quantitative approaches are used to take into account the social and environmental impacts of mining industry. In some cases, the environmental impacts are quantified in the designing phase of a mining project. Rodriguez (2007) develops a heuristic algorithm that considers the environmental cost and adds it up to the other mining costs to find the pit limit for an open pit mining problem. The focus in this work is on the technical issues regarding environmental impacts, rather than social impacts. A new term known as environmental cost (EC) is defined that covers a variety of costs regarding environmental issues such as land clearing, construction of access roads, drilling and blasting, pit excavation, waste rock dumping, tailings disposal and decommissioning. EC is deducted from the economic block value (EBV) and the revised EBV is used to find the pit limit in an iterative algorithm.

Odell (2004) uses the sustainability primer methodology proposed by the association of professional engineers and geoscientists of British Columbia (APEGBC) to integrate sustainability into the mine design process. The basis for APEGBC process is the multi criteria analysis (MCA), also known as multiple accounts evaluation (MAE) and multiple attribute analysis (MAA). MCA consists of a number of distinct approaches, but the basis for all of them is to define different options (scenarios) and assess each option with respect to series of explicit criteria, which is typically done through MCA tables. Some important factors in selecting the proper MCA are (1) availability of time and financial resources, (2) availability and the amount of supporting data, (3) analytical expertise of the project team, (4) administrative culture of decision-making body and (5) the number of decision options (finite or infinite). Odell (2004) applies the methodology for an open pit copper deposit in Peru. Since the decision context in the case study shows a high complexity and a wide range of interested stakeholder groups, the MCA seems to be the proper choice for the problem. It turns out that with MCA approach, it is strongly possible to take into account the different aspects of mining projects, including social and environmental issues, in order to assess different scenarios. New packages of holistic mine design tools should be refined and prepared so as to consider the social and environmental impacts of the mining projects in advance, not just mitigating the environmental consequences afterward.

Odell (2004) shows that MCA is a strong tool that can be used in the development of new packages for assessment of mining projects. However, some pre-determined options (scenarios) should be defined to be used in MCA matrices. In other words, the MCA approach requires two major building blocks to form the MCA matrices; these are some scenarios (as matrix columns) and a variety of indicators representing different engineering, economical, social and environmental criteria (as matrix rows). Therefore, MCA is more suitable for general decision-making in feasibility study stages when several scenarios can be defined and there are some pre-estimations for different indicators under each scenario. MCA is considered as a powerful tool for mine design, but is not that handy when it is intended to optimize production scheduling problem, because optimization requires more detail and numerical values to be used in mathematical programming. At that stage, the idea of defining scenarios fails. Therefore, MCA does not work for mine planning purpose.

Fuzzy logic is the other tool that can be used in quantification of descriptive and qualitative values. Many of environmental impacts are either described qualitatively or there are some quantitative indicators but still the judgment of an expert is required to assess them. Thus, fuzzy sets and fuzzy logic are strong tools to capture the uncertainty and fuzzy nature of environmental variables. Shepard (2005) makes an introduction to the fuzzy logic and discusses its implementation in quantification of environmental impact assessment. The traditional approach in environmental impact assessment is reviewed and fuzzy logic is introduced as the modern approach in the field.

The focus in the current literature is mostly on qualitative approaches for assessment of environmental impacts. There are few works that have considered the impacts quantitatively. However, the scope of the current quantitative approaches is mine design, not mine planning. Therefore, this paper aims at integrating the mine and decommissioning long term plans for the case of oil sands.

4. Theoretical framework

In a typical mine planning problem, the number of variables is directly related to the number of blocks and the number of time periods considered in planning. Since in real world problems, usually hundreds of thousands to millions of blocks are considered in the optimum pit and for long-term planning, multiple periods are taken into account, a typical mine planning problem has millions of variables, among them some integers and others continuous. This makes the problem NP-hard and non solvable with current software, or solvable with a very long solution time. In a brief review, the literature regarding the methodology used in finding the solution can be classified into two main categories; (1) those based on the exact methods, mainly relying on linear programming (LP) to find the exact optimal solution with a long CPU time, and (2) those based on the approximation of optimal solution by applying heuristic algorithms. Applying the heuristic approach may result in quality solution in reasonable time, but the solution may not be necessarily optimal.

In many of the works focusing on exact methods, the binary nature of integer variables is relaxed. For instance, Tan and Romani (1992) consider the equipment capacity constraints and find the optimal extraction schedule over multiple periods, using both linear programming (LP) and dynamic programming (DP). Since the integer nature of decision variables is relaxed, the block sequencing constraints are not satisfied. Fytas et al. (1993) use different approaches for long-term and short-term decision making problems. As the first phase, the simulation is used to find those blocks that should be extracted in long-term. At this phase, the precedence constraints, the minimum and maximum production and processing capacity constraints and also the bounds on the grade of entering material to the processing plant are considered. Then for the second phase, the LP is used for the short-term planning, subject to the same constraints of long-term, but with the assumption that the partial extraction of blocks is permitted. Finally, an iterative approach is proposed to deliver a practical mining sequence.

There are some techniques to reduce the problem size and make large problems solvable using exact methods. Block aggregation is one of these techniques. The idea is to merge the blocks to create "*mining-cuts*" and hence, reduce the number of MILP variables (Askari-Nasab et al., 2010). However in the 1980s, a new approach emerged, aiming to reduce the problem size, not by reducing the number of variables, but by relaxing some of the constraints. The Lagrangian relaxation is used to relax some of the constraints and help to find the exact solution of the relaxed problem. The main idea in the Lagrangian relaxation approach is to relax some of the constraints and instead, add penalty terms into the objective function. The constraints of the MILP can be classified into two categories: (1) hard constraints, including those that define the precedence of blocks extraction and (2) soft constraints, including those constraints that are defined to satisfy the limited production and processing capacities and grade bounds for the material entering the processing plant. In most of the papers, the soft constraints are relaxed and corresponding terms are added into the objective function with a penalty multiplier. This relaxation triggers another question: to what extent the objective function should be penalized if the corresponding constraint is not satisfied? In other words, what are the proper values for Lagrangian multipliers? Some algorithms are developed to find the answer to such questions. Among them, one of non-heuristic algorithms is the sub-gradient method.

Dagdelen and Johnson (1986) use the Lagrangian relaxation to relax constraints on the maximum production in each period. The sub-gradient method is applied to update the multipliers in some small examples. This is one of the first papers in the field of Lagrangian relaxation, using the sub-gradient method. Akaike and Dagdelen (1999) extend the previous work by changing the value of the Lagrangian multipliers in an iterative procedure. The procedure continues until the relaxed problem reaches the optimal solution that is feasible for the original problem.

Kawahata (2006) expands the idea of Dagdelen and Johnson (1986) by defining a variable cut-off grade that specifies the destination of extracted material, i.e. whether to go to the processing plant, to be stockpiled or to be dumped as waste. Then two Lagrangian relaxation sub problems are used to find the bounds of the original problem; one for the most conservative case of mine sequencing and the other for the most aggressive case. Since the solution for the relaxed problem is not necessarily feasible for the original problem, some bounds are adjusted on capacities to guarantee the feasibility of Lagrangian solution for the original problem.

To develop the theoretical concept of Lagrangian relaxation, it is assumed that there is a maximization problem with some constraints. This original problem is called the *primal* problem. In addition, it is assumed that the constraints have made the primal problem so complicated that it takes a long run-time to find the optimal solution. One practical way to tackle the complexity of the problem is to relax some of the constraints and consider the relaxed version of the primal problem as the *dual* problem. It is proved that the optimal solution to the dual problem always equals, or is greater than the optimal solution to the primal problem (Fisher, 2004). In other words, the dual optimal solution is always considered as an upper bound to the primal optimal solution.

On the other hand, we assume that just a feasible solution to the primal problem can be found. Since by definition, the optimal solution to any maximization problem is greater than, or equal to any point in the feasible space of the problem, any feasible solution is considered as a lower bound for the optimal solution of the primal problem. The idea of upper bound and lower bound works for any minimization problem as well, just in reverse order for lower and upper bounds. The concept of lower and upper bounds in a maximization problem is illustrated in Fig. 2.

To avoid the violations from relaxed constraints, proper penalty terms are added to the objective function to penalize any violation from the corresponding constraints. For any penalty term, a penalty multiplier called the Lagrangian multiplier, is considered and multiplied to the penalty term. One of the most well known and typical approaches used to find the multiplier values is the sub-gradient method.

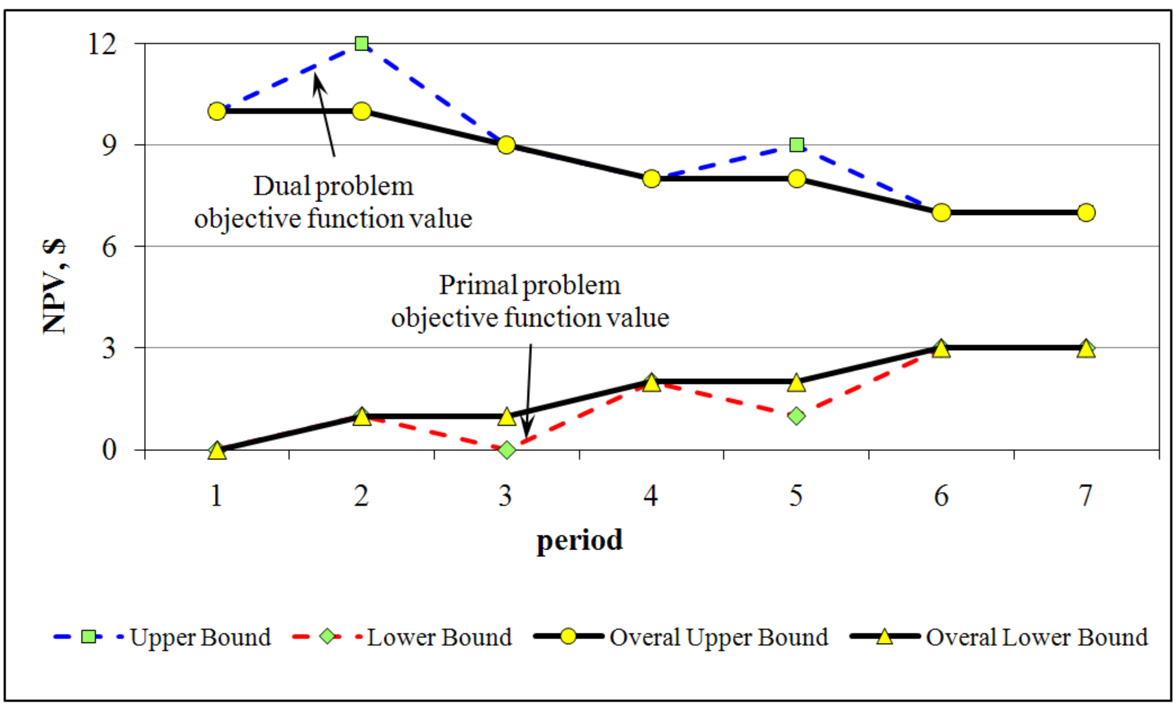

Fig. 2. Upper and lower bounds for a typical maximization problem.

The sub-gradient method

Fisher (1985) provides an application oriented guide to Lagrangian relaxation and presents the following formulation for typical primal and dual problems and the sub-gradient method.

The integer programming problem, called the primal is formulated as Eqs. (1) and (2).

$$Z = \max cx \tag{1}$$

Subject to:

$$\begin{aligned} &Ax \le b \\ &Dx \le e \\ &x \ge 0, \text{int} \end{aligned} \tag{2}$$

Where x is $n \times 1$, b is $m \times 1$, e is $k \times 1$ and all other matrices have conformable dimensions. In this formulation, the constraints are partitioned into two sets, $Ax \le b$ and $Dx \le e$. It is assumed that it is easy to solve the primal problem, if the set of constraints $Ax \le b$ are relaxed. This relaxation produces the dual problem LR_u^k, using an m vector of non-negative multipliers u. (Eqs. (3) and (4))

$$Z_D(u) = \max cx + u(b - Ax) \tag{3}$$

Subject to:

$$\begin{aligned} &Dx \le e \\ &x \ge 0, \text{int} \end{aligned} \tag{4}$$

Since the dual problem provides an upper bound for the primal maximization problem, ideally the vector u should be found in a way that $Z_D(u)$ be minimized. This is formulated with Eq. (5).

$$Z_D = \min Z_D(u) \tag{5}$$

Eq. (5) is considered as the basis of the sub-gradient method. The goal is to find the proper set of Lagrangian multipliers that minimizes $Z_D(u)$. Multiplier values are updated, considering the initial value of u^0 and according to Eq. (6).

$$u^{k+1} = \max\{0, u^k - t_k(b - Ax^k)\} \tag{6}$$

Where x^k is the optimal solution to LR_u^k, the Lagrangian problem with dual variables set to u^k, and t_k is a scalar step size value. According to Fisher (1985), a formula for t_k that has been proven to be effective in practice is given by Eq. (7).

$$t_k = \frac{\lambda_k (Z_D(u^k) - Z^*)}{\sum_{i=1}^{m} (b_i - \sum_{j=1}^{n} a_{ij} x_j{}^k)^2} \tag{7}$$

In this formula, Z^* is the objective value of the best known feasible solution to (P) and λ_k is a scalar chosen between 0 and 2. Frequently, the sequence λ_k is determined by starting with $\lambda_k = 2$ and reducing it by a factor of two whenever $Z_D(u^k)$ has failed to decrease in a specific number of iterations.

5. Tailings calculation

The volume and tonnage of tailings that is produced as a result of oil sands processing is required in surface mine planning. In this paper, Suncor's flow sheet is used to find the mass-balance relationship between ore feed and tonnage of the total pond slurry tailings (Suncor, 2009). Suncor has applied some assumptions in its flow sheet. These assumptions are based on Suncor's operational factors and are implied here as well. A schematic view of a related part from Suncor's oil sand processing flow diagram is presented in Fig. 3.

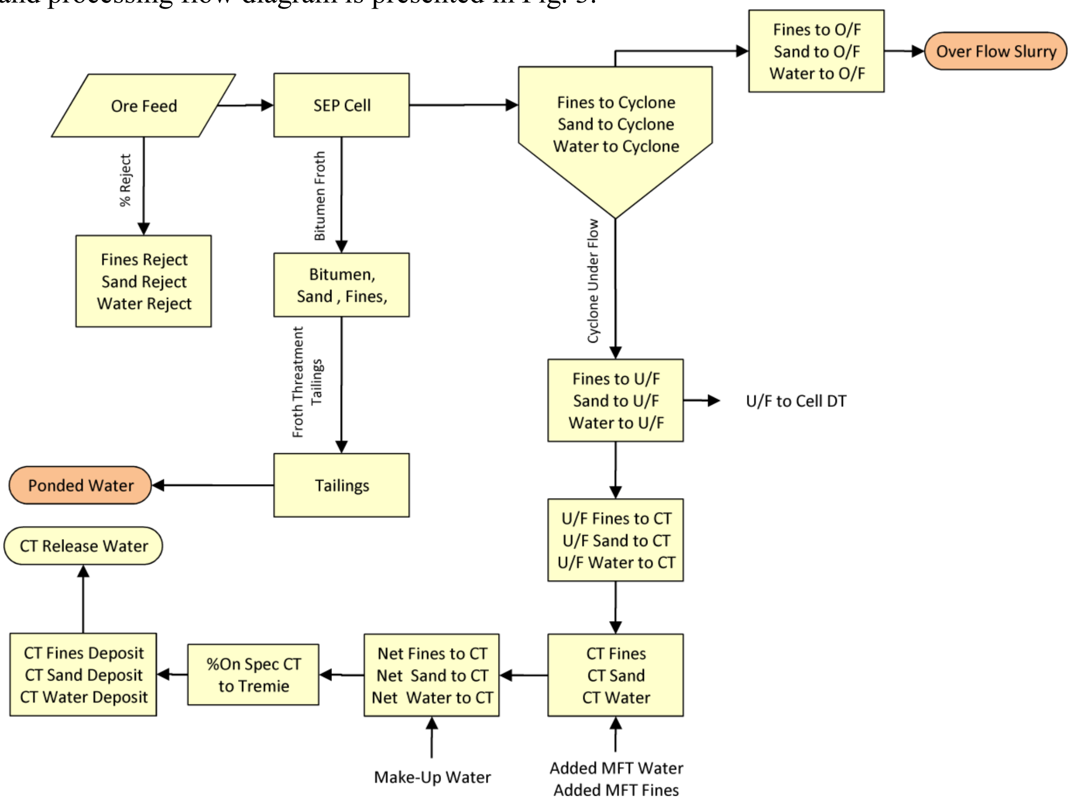

Fig. 3. Suncor processing flow diagram.

In this paper, the focus is on two main streams that make slurry and water, ending in the tailings pond. The first one feeding the greatest portion of the tailings material is the over flow slurry. The second one is the pond water from bitumen froth treatment. These two parts are highlighted in Fig. 3. In addition to these streams, there are two other streams in the process. These two streams (that result in producing CT and MFT) come from cyclone under flow. Since it is assumed that these two products are held in different cells, they are not considered in calculating the total amount of ponded tailings.

The following notations are used in the tailings calculation:

Parameters

$Sd\%_{UF}$: Sand content of the underflow

$Sl_{solid\%}$: Slurry solid percent sent to cyclone

$UF_{Sd\%}$: Sand percent in cyclone underflow

$UF_{F\%}$: Fine percent in cyclone underflow

$UF_{W\%}$: Water percent in cyclone underflow

R : SET recovery percent

$B\%_{SET}$: SET bitumen percent

$F\%_{SET}$: SET fines percent

$Sd\%_{SET}$: SET sand percent

$W\%_{SET}$: SET water percent

$Rj\%$: Reject percent

$Rj\%_F$: Fines reject percent

$Rj\%_{Sd}$: Sand reject percent

$Rj\%_W$: Water reject percent

$Rj\%_B$: Bitumen reject percent

HPW : HPW

SG_f : Fines specific gravity

SG_s : Sand specific gravity

$F\%_{Beach}$: Fines content in beach solids (%)

BDD : Beach dry density

$S\%_{MFT}$: MFT solid content (%)

Input variables

M^O_{Feed} : Mass of ore in the feed

B_{Feed} : Bitumen content of the feed (%)

F_{Feed} : Fines content of the feed (%)

W_{Feed} : Water content of the feed (%)

Outputs

$M_{Ponded}^{Overflow}$ Mass of total overflow ponded material

M_{Ponded}^{W} Mass of total ponded water from froth treatment

M_{Ponded}^{Slurry} Mass of total ponded material

The total tonnage of ponded slurry is calculated as in Eq. (8) and consists of two parts, the overflow slurry and the ponded water as the downstream product from bitumen froth treatment.

$$M_{Ponded}^{Slurry} = M_{Ponded}^{Overflow} + M_{Ponded}^{W} \tag{8}$$

The overflow slurry, $M_{Pounded}^{Overflow}$, is the summation of total fines, sand and water that is the overflow material produced by the cyclone and is calculated as Eq. (9).

$$M_{Ponded}^{Overflow} = M_{Feed}^{O} \times \left(\begin{array}{l} \left(1 - B_{Feed} - W_{Feed}\right) \times \left(\begin{array}{l} -\dfrac{1 - F_{Feed}}{UF_{Sd\%}} \times \left(UF_{F\%} + UF_{W\%}\right) \\ \times Sd\%_{UF} \\ -\left(1 - F_{Feed}\right) \times Sd\%_{UF} \\ +\dfrac{1}{Sl_{solid\%}} \end{array} \right) \\ +\left(B_{Feed} - Rj\% \times Rj\%_{B}\right) \times \dfrac{R}{B\%_{SET}} \\ \times \left(\begin{array}{l} \dfrac{Sd\%_{SET} \times Sd\%_{UF}}{UF_{Sd\%}} \times \left(UF_{F\%} + UF_{W\%}\right) \\ +Sd\%_{SET} \times Sd\%_{UF} - \dfrac{F\%_{SET}}{Sl_{solid\%}} - \dfrac{Sd\%_{SET}}{Sl_{solid\%}} \end{array} \right) \\ +Rj\% \times \left(\begin{array}{l} \dfrac{Rj\%_{Sd} \times Sd\%_{UF}}{UF_{Sd\%}} \times \left(UF_{F\%} + UF_{W\%}\right) \\ +Rj\%_{Sd} \times Sd\%_{UF} - \dfrac{Rj\%_{F}}{Sl_{solid\%}} - \dfrac{Rj\%_{Sd}}{Sl_{solid\%}} \end{array} \right) \end{array} \right) \tag{9}$$

Finally, the ponded water from bitumen froth treatment is calculated through Eq. (10).

$$M_{Ponded}^{W} = \left(\frac{\left(B_{Feed} - Rj\% \times Rj\%_{B}\right) \times M_{Feed}^{O} \times R}{B\%_{SET}} \right) \times \left(\begin{array}{l} W\%_{SET} - \dfrac{Sd\%_{SET}}{\left(1 - F\%_{Beach}\right) \times BDD} \\ +\dfrac{Sd\%_{SET}}{SG_{s}} + \dfrac{Sd\%_{SET} \times F\%_{Beach}}{\left(1 - F\%_{Beach}\right) \times SG_{f}} \\ +\left(\dfrac{S\%_{MFT} - 1}{S\%_{MFT}} \right) \\ \times \left(\dfrac{F\%_{SET} \times \left(1 - F\%_{Beach}\right) - Sd\%_{SET} \times F\%_{Beach}}{\left(1 - F\%_{Beach}\right)} \right) \end{array} \right) \tag{10}$$

Validation of the formula

In order to check the results from the formulations, the tailings tonnage is calculated based on the derived formulation for an optimized long term mining production case. The final pit limit for the case contains 61490 blocks of 50 by 50 by 15 meters and the production is planned for 19 periods. In order to reduce the number of variables and make the selective mining units more practical, the blocks are aggregated into 302 mining cuts. According to the presented formulation and notation, four main input variables are required to calculate the tailings amount; (1) percentage of bitumen content of the block, (2) percentage of fines content of the block, (3) tonnage of ore in extracted portion of each block and (4) percentage of water content of the block. The first two inputs already exists in the block model for the case study. The ore tonnage of the block is multiplied by the portion of the block that is extracted as ore in each period to calculate the ore tonnage of the processing plant feed. Eq. (11) is used to calculate the water content (Masliyah, 2010) of the processing plant feed.

$$W_{Feed} = 0.75 \times F_{Feed} + 2.3 \tag{11}$$

The amount of mined material, processing material sent to the mill and the produced tailings sent to the tailings pond for 19 periods is illustrated in Fig. 4.

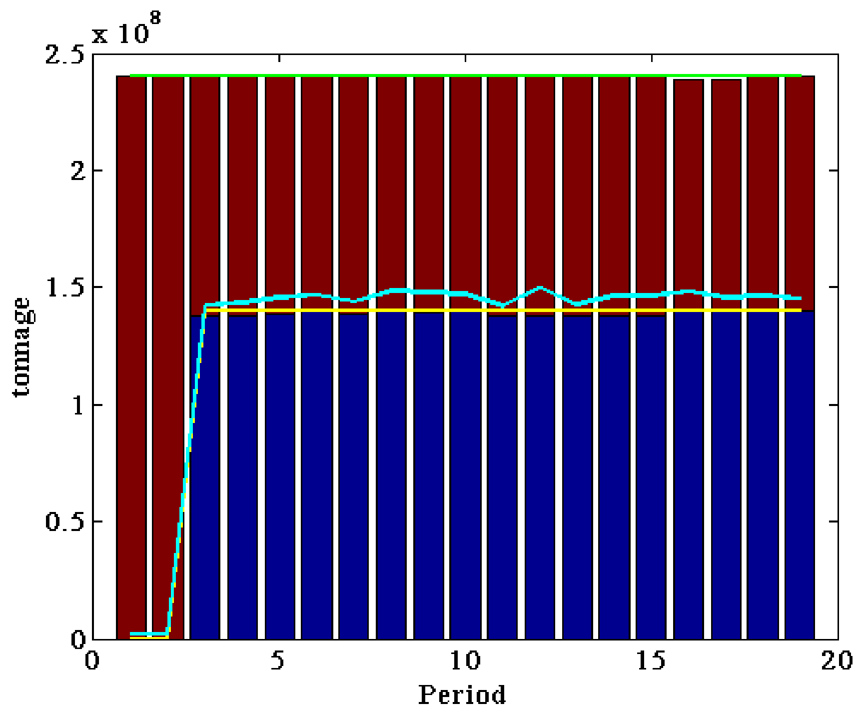

Fig. 4. Mining, processing and tailings amount per period.

The horizontal lines in Fig. 4 represent the processing and mining capacities. Based on the optimal mine production schedule, all the extracted material in the first two periods are waste and sent to the waste dump (two years of pre-stripping). As a result, the amount of processing material is zero in periods one and two. The bright curve represents the amount of tailings that is produced in each period. The presented formulation is used to figure out the total tonnage of tailings in each period. Initially the tailings amount corresponding to the processed portion of each block in a period is calculated. Then, the calculated tailings tonnages are aggregated to build the total amount of tailings in the period.

To double check the result of the formulation, the tailings tonnage is also calculated in another method. In the second method, the tailings tonnage is calculated for a block with the ore tonnage of 1000 tonne. For each period, the average values for bitumen content, fines content and water content of the blocks that are going to be extracted in the period are considered in calculations. Then the result for tailings tonnage in each period is multiplied by the total tonnage of the material that is processed in that period. The minimum, maximum and average differences between the amounts of tailings resulting from the two methods are 1.25%, 1.40% and 1.32%, respectively. It shows that the two methods result in almost the same amounts of tailings and the derived formulation works well. The differences between the results from the first method and the second one for the 17 periods with non-zero values for the tailings are compared in Fig. 5.

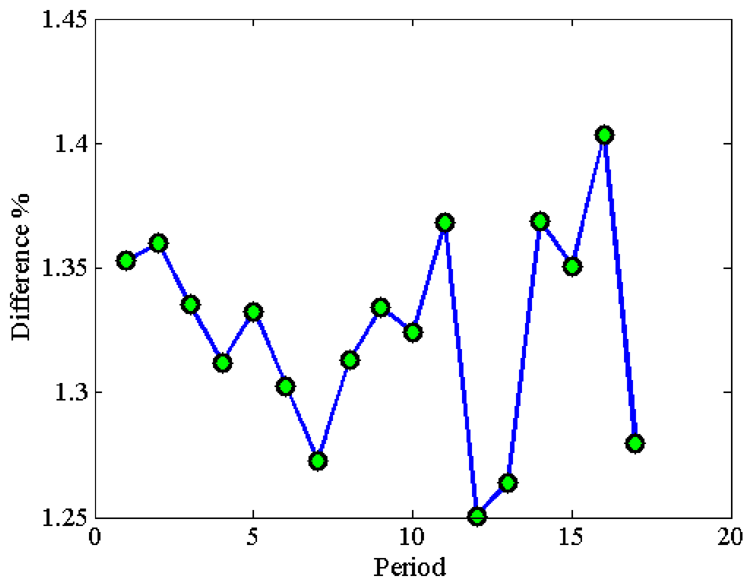

Fig. 5. Percent of difference between two methods for tailings calculation.

6. Mathematical model

The long-term mine production scheduling problem is formulated using mixed integer linear programming. The formulated model for the strategic production and operational decommissioning (capping) material scheduling problem has an objective function and number of constraints. The material used for capping purposes in oil sands surface mining, which are overburden, interburden and coarse sands tailings, are all from the block model. However, the costs regarding to each portion are different. In reality, due to the different activities associated with dumping, reloading and hauling of each type of material, the costs are different. Thus, different decision variables and cost coefficients are defined in the mathematical model to differentiate between different portions of each block.

The notation used in the formulation of the problem has been classified as sets, indices, subscripts, superscripts, parameters, and decision variables. Multiple material types and destinations are taken into account in the MILP formulation. Finally, the MILP formulation framework is developed based on mining-cuts.

Sets

$\boldsymbol{K} = \{1, ..., K\}$ set of all the mining-cuts in the model.

$\boldsymbol{J} = \{1,......,J\}$ set of all the phases (push-backs) in the model.

$\boldsymbol{U} = \{1,.....,U\}$ set of all possible destinations for materials in the model.

$C_k(L)$ For each mining-cut k, there is a set $C_k(L) \subset \boldsymbol{K}$ defining the immediate predecessor mining-cuts above mining-cut k that must be extracted prior to extraction of mining-cut k, where L is the total number of mining-cuts in the set $C_k(L)$.

$M_k(P)$ For each mining-cut k, there is a set $M_k(P) \subset \boldsymbol{K}$ defining the immediate predecessor mining-cuts in a specified horizontal mining direction that must be extracted prior to extraction of mining-cut k at the specified level, where P is the total number of mining-cuts in the set $M_k(P)$.

$B_j(H)$ For each phase j, there is a set $B_j(H) \subset K$ defining the mining-cuts within the immediate predecessor pit phases (push-backs) that must be extracted prior to extracting phase j, where H is an integer number representing the total number of mining-cuts in the set $B_j(H)$.

Indices, subscripts and superscript

A parameter, f, can take indices, subscripts, and superscripts in the format $f_{k,j}^{u,e,t}$. Where:

$t \in \{1,......,T\}$ index for scheduling periods.

$k \in \{1,.....,K\}$ index for mining-cuts.

$e \in \{1,.....,E\}$ index for element of interest in each mining-cut.

$j \in \{1,.....,J\}$ index for phases.

$u \in \{1,.....,U\}$ index for possible destinations for materials.

D, S, M, P subscripts and superscripts for overburden and interburden material, tailings sand, mining and processing respectively.

Parameters

$d_k^{u,t}$ the discounted profit obtained by extracting mining-cut k and sending it to destination u in period t.

$r_k^{u,t}$ the discounted revenue obtained by selling the final products within mining-cut k in period t if it is sent to destination u, minus the extra discounted cost of mining all the material in mining-cut k as ore and processing at destination u.

$n_k^{u,t}$ the extra discounted cost of mining all the material in mining-cut k in period t as overburden and interburden material for capping at destination u.

$m_k^{u,t}$ the extra discounted cost of mining all the material in mining-cut k in period t as tailings sand material for capping at destination u.

$q_k^{u,t}$	the discounted cost of mining all the material in mining-cut k in period t as waste and sending it to destination u.
g_k^e	the average grade of element e in ore portion of mining-cut k.
$\underline{g}^{u,t,e}$	the lower bound on the required average head grade of element e in period t at processing destination u.
$\overline{g}^{u,t,e}$	the upper bound on the required average head grade of element e in period t at processing destination u.
f_k^o	the average percent of fines in ore portion of mining-cut k.
$\underline{f}^{u,t,o}$	the lower bound on the required average fines percent of ore in period t at processing destination u.
$\overline{f}^{u,t,o}$	the upper bound on the required average fines percent of ore in period t at processing destination u.
f_k^c	the average percent of fines in overburden and interburden capping material portion of mining-cut k.
$\underline{f}^{u,t,c}$	the lower bound on the required average fines percent of overburden and interburden capping material in period t at capping destination u.
$\overline{f}^{u,t,c}$	the upper bound on the required average fines percent of overburden and interburden capping material in period t at capping destination u.
o_k	the ore tonnage in mining-cut k.
w_k	the waste tonnage in mining-cut k.
d_k	the overburden and interburden material tonnage in mining-cut k.
l_k	the tailings sand material tonnage in mining-cut k.
t_k	the tailings tonnage produced downstream from extraction of ore from mining-cut k.
$T_{Mu}^{m,t}$	the upper bound on mining capacity (tonnes) in period t.
$T_{Ml}^{m,t}$	the lower bound on mining capacity (tonnes) in period t.
$T_{Pu}^{u,t}$	the upper bound on processing capacity (tonnes) in period t at destination u.
$T_{Pl}^{u,t}$	the lower bound on processing capacity (tonnes) in period t at destination u.
$T_{Cu}^{u,t}$	the upper bound on overburden and interburden capping material requirement (tonnes) in period t at destination u.
$T_{Cl}^{u,t}$	the lower bound on overburden and interburden capping material requirement (tonnes) in period t at destination u.
$T_{Nu}^{u,t}$	the upper bound on tailings sand capping material requirement (tones) in period t at destination u.

$T_{Nl}^{u,t}$	the lower bound on tailings sand capping material requirement (tones) in period t at destination u.
$T_{Tu}^{u,t}$	the upper bound on capacity of tailings pond (tones) in period t at destination u.
$T_{Tl}^{u,t}$	the lower bound on capacity of tailings pond (tones) in period t at destination u.
$r^{u,e}$	the proportion of element e recovered (processing recovery) if it is processed at destination u.
$p^{e,t}$	the price of element e in present value terms per unit of product.
$cs^{e,t}$	the selling cost of element e in present value terms per unit of product.
$cp^{u,e,t}$	the extra cost in present value terms per tonne of ore for mining and processing at destination u.
$cl^{u,t}$	the cost in present value terms per tonne of overburden and interburden dyke material for capping at destination u.
$cu^{u,t}$	the cost in present value terms per tonne of tailings sand dyke material for capping at destination u.
$cm^{u,t}$	the cost in present value terms of mining a tonne of waste in period t and sending it to destination u.

Decision variables

$x_k^{u,t} \in [0,1]$	a continuous variable representing the portion of ore from mining-cut k to be extracted and processed at destination u in period t.
$w_k^{u,t} \in [0,1]$	a continuous variable representing the portion of overburden and interburden material from mining-cut k to be extracted and used for capping purposes at destination u in period t.
$v_k^{u,t} \in [0,1]$	a continuous variable representing the portion of tailings sand material from mining-cut k to be extracted and used for capping purposes at destination u in period t.
$y_k^t \in [0,1]$	a continuous variable representing the portion of mining-cut k to be mined in period t, which includes ore, overburden and interburden capping material, tailings sand capping material and waste.
$b_k^t \in [0,1]$	a binary integer variable controlling the precedence of extraction of mining-cuts. b_k^t is equal to one if the extraction of mining-cut k has started by or in period t, otherwise it is zero.
$c_j^t \in [0,1]$	a binary integer variable controlling the precedence of mining phases. c_j^t is equal to one if the extraction of phase j has started by or in period t, otherwise it is zero.

Modeling of economic mining-cut value

The objective function of the MILP model is to maximize the net present value of the mining operations, including operation-related portion of the decommissioning costs. The concept of economic mining-cut value is based on ore parcels within mining-cuts which could be mined selectively. The profit from mining a mining-cut is a function of the value of the mining-cut based on the processing destination and the costs incurred in mining, processing and cell decommissioning at a specified destination. The cost of cell decommissioning is also a function of the location of the tailings facility being constructed and the type and quantity of used dyke and tailings sand material. The discounted profit from mining-cut k is equal to the discounted revenue obtained by selling the final product contained in mining-cut k minus the discounted cost involved in mining mining-cut k as waste (Askari-Nasab and Awuah-offei, 2009). In this paper, in addition to the previous terms, two new terms are considered in calculation of economic mining cut value; the extra discounted cost of mining overburden/interburden (OI) and tailings sand (TS) material for capping purposes. This has been simplified into Eqs. (12) to (16).

$$d_k^{u,t} = r_k^{u,t} - q_k^{u,t} - n_k^{u,t} - m_k^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\},\ k \in \{1,...,K\} \tag{12}$$

Where:

$$r_k^{u,t} = \sum_{e=1}^{E} o_k \times g_k^e \times r^{u,e} \times \left(p^{e,t} - cs^{e,t}\right) - \sum_{e=1}^{E} o_k \times cp^{u,e,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{13}$$

$$q_k^{u,t} = \left(o_k + d_k + w_k\right) \times cm^{u,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{14}$$

$$n_k^{u,t} = d_k \times cl^{u,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{15}$$

$$m_k^{u,t} = l_k \times cu^{u,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,..,K\} \tag{16}$$

The mixed integer linear programming model

The objective functions of the MILP model for strategic and operational production plan for oil sands mining can be formulated as: i) maximizing the NPV and ii) minimizing the decommissioning (capping) cost. These are represented by Eqs. (17) and (18), respectively.

$$Max \sum_{u=1}^{U} \sum_{t=1}^{T} \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(r_k^{u,t} \times x_k^{u,t} - q_k^{u,t} \times y_k^t \right) \right) \tag{17}$$

$$Min \sum_{u=1}^{U} \sum_{t=1}^{T} \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(n_k^{u,t} \times w_k^{u,t} + m_k^{u,t} \times v_k^{u,t} \right) \right) \tag{18}$$

Eqs. (17) and (18) can be combined as a single objective function, formulated as in Eq. (19).

$$Max \sum_{u=1}^{U} \sum_{t=1}^{T} \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left[\left(r_k^{u,t} \times x_k^{u,t} - q_k^{u,t} \times y_k^t \right) - (n_k^{u,t} \times w_k^{u,t} + m_k^{u,t} \times v_k^{u,t}) \right] \right) \tag{19}$$

The complete MILP model comprising of the combined objective function and constraints can be formulated as;

Objective function:

$$Max \sum_{u=1}^{U} \sum_{t=1}^{T} \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left[\left(r_k^{u,t} \times x_k^{u,t} - q_k^{u,t} \times y_k^t \right) - (n_k^{u,t} \times w_k^{u,t} + m_k^{u,t} \times v_k^{u,t}) \right] \right) \tag{20}$$

Constraints:

$$T_{Ml}^{t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(o_k + w_k + d_k\right)\times y_k^t\right) \le T_{Mu}^{t} \qquad \forall t \in \{1,...,T\} \tag{21}$$

$$T_{Pl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(o_k \times x_k^{u,t}\right)\right) \le T_{Pu}^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{22}$$

$$T_{Cl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(d_k \times w_k^{u,t}\right)\right) \le T_{Cu}^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{23}$$

$$T_{Nl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(l_k \times v_k^{u,t}\right)\right) \le T_{Nu}^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{24}$$

$$\underline{g}^{u,t,e} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j} g_k^e \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \le \overline{g}^{u,t,e} \qquad \forall t \in \{1,..,T\},\ u \in \{1,..,U\}, e \in \{1,..,E\} \tag{25}$$

$$\underline{f}^{u,t,o} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^o \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \le \overline{f}^{u,t,o} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\} \tag{26}$$

$$\underline{f}^{u,t,c} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^c \times d_k \times w_k^{u,t} \Big/ \sum_{k\in B_j} d_k \times w_k^{u,t}\right) \le \overline{f}^{u,t} \qquad \forall t \in \{1,..,T\}, u \in \{1,..,U\} \tag{27}$$

$$T_{Tl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(t_k \times x_k^{u,t}\right)\right) \le T_{Tu}^{u,t} \qquad \forall t \in \{1,...,T\},\ u \in \{1,...,U\} \tag{28}$$

$$\sum_{u=1}^{U}\left(o_k \times x_k^{u,t} + d_k \times w_k^{u,t}\right) \le \left(o_k + d_k\right)\times y_k^t \qquad \forall t \in \{1,..,T\},\ k \in \{1,..,K\} \tag{29}$$

$$\sum_{u=1}^{U}\left(l_k \times v_k^{u,t}\right) \le \sum_{u=1}^{U}\left(o_k \times x_k^{u,t}\right) \qquad \forall t \in \{1,..,T\},\ k \in \{1,..,K\} \tag{30}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} x_k^{u,t} \le 1 \qquad \forall k \in \{1,..,K\} \tag{31}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} w_k^{u,t} \le 1 \qquad \forall k \in \{1,..,K\} \tag{32}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} v_k^{u,t} \le 1 \qquad \forall k \in \{1,..,K\} \tag{33}$$

$$b_k^t - \sum_{i=1}^{t} y_s^i \le 0 \qquad \forall t \in \{1,...,T\},\ k \in \{1,...,K\},\ s \in C_k(L) \tag{34}$$

$$b_k^t - \sum_{i=1}^{t} y_r^i \le 0 \qquad \forall t \in \{1,...,T\},\ k \in \{1,...,K\},\ r \in M_k(P) \tag{35}$$

$$\sum_{i=1}^{t} y_k^i - b_k^t \le 0 \qquad \forall t \in \{1,...,T\},\ k \in \{1,...,K\} \tag{36}$$

$$b_k^t - b_k^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\},\quad k \in \{1,...,K\} \tag{37}$$

$$c_j^t - \sum_{i=1}^{t} y_h^i \le 0 \qquad \forall t \in \{1,...,T\},\ j \in \{1,...,J\},\ h \in B_j(H) \tag{38}$$

$$\sum_{i=1}^{t} y_h^i - H \times c_j^t \le 0 \qquad \forall t \in \{1,...,T\},\ j \in \{1,...,J\},\ h \in B_{j+1}(H) \tag{39}$$

$$c_j^t - c_j^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\},\ j \in \{1,...,J\} \tag{40}$$

$$\sum_{t=1}^{T} y_k^t = 1 \qquad \forall k \in \{1,...,K\} \tag{41}$$

Eq. (20) is the objective function of the formulation which seeks to i) maximize the NPV and ii) minimize capping costs. Eq. (21) is the total mining capacity constraint. Eqs. (22), (23) and (24) are the capacity constraints for processing, OI and TS for capping requirements, respectively. Eqs. (25) , (26) and (27) specify the limiting requirements for bitumen in ore, fines in ore and fines in OI capping material for all destinations. Eq. (28) represents the upper and lower bounds on the capacity of each tailings facility in each period. Eq. (29) ensures that the total material that is mined in each period for all destinations does not exceed the sum of the ore and OI material that is mined. Eq. (30) states that the tonnage of TS that is mined for capping in each period should be less than or equal to the tonnage of ore material that is mined for all destinations. Any unscheduled TS material becomes available for preparation of mature fine tailings (MFT). Eqs. (31), (32) and (33) ensure that the total fractions of mining-cut *k* sent to all destinations in all periods are less than or equal to one. Eqs. (34), (35), (36) and (37) check the set of immediate predecessor mining-cuts that must be mined prior to mining mining-cut *k* for all periods and destinations. Eqs. (38), (39) and (40) check the set of immediate predecessor pit phase that must be mined prior to mining phase *j* in all periods for all destinations. Eq. (41) ensures that the whole blocks in the optimized pit are completely extracted.

7. Conclusions and future work

Processing of oil sands produces huge volume of tailings, pumped to the tailings ponds and being kept there for long periods of time. Keeping the tailings in their conventional form in tailings ponds results in many sever environmental issues. Thus, mining companies are required to take care of their tailings ponds and take the responsibility for site decommissioning before leaving the mine site. For decades in oil sands industry, the mine plans have been scheduled independently from tailings plans. Production planning directly affects the amount of produced tailings and also the availability of material that is required for decommissioning. In this paper, a new MILP model is developed that maximizes the NPV and at the same time considers the decommissioning costs as the new term in its objective function. In addition, the result from the proposed MILP model ensures that the required material for site decommissioning is available. Suncor processing flow sheet is used to capture the mass balance relation in oil sands processing. The derived formulation to calculate the amount of tailings is verified by testing the formulation on a real data from an oil sands surface mining case. Some steps toward solving the MILP model with Lagrangian relaxation method are passed. As the future work, it is required to develop an efficient Lagrangian relaxation method to solve the proposed MILP model and validate the results. In addition to decommissioning operations, dyke construction can also be added to the model. Finally, the model can be upgraded by considering multiple pits and finding the mine production, dyke construction and decommissioning schedules accordingly for multiple pits.

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9. Appendix

HTML documentation of the MATLAB code for tailings formulation

Modeling Truck-Shovel Energy Efficiency under Uncertainty

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Abstract

The US coal mining industry consumes approximately 142 billion kWh per year of energy. The US Department of Energy estimates that the industry's annual energy consumption can be reduced by 49% (24.6 billion kWh/year by using currently available best practices and a further 44.8 billion kWh/year with more research). This constitutes nearly $3.7 billion of potential savings on coal production costs at 5.3¢/kWh of energy. Additionally, with climate change regulation on the horizon, any benefits from energy savings in the near future are compounded by associated reductions in CO_2 *emissions.*

The goal of this work was to apply stochastic process simulation to model the energy efficiency of a typical truck and shovel mining system and use the model to evaluate production strategies to improve energy efficiency. The research team conducted energy audits of truck-and-shovel overburden removal and highwall miner operations. This information was used to develop regression models describing truck and shovel fuel consumption. The research team then built a stochastic simulation model of the truck-and-shovel overburden removal operation and used it to assess a variety of improvement measures by simulation experimentation.

Valid fuel consumption models for shovel loading and truck haulage have been formulated based on the energy audit results. Valid stochastic process models of truck-and-shovel operations have been formulated to study energy efficiency. The following strategies, in decreasing order of impact, provide the most energy savings for truck-and-shovel overburden removal at the mine: (1) shorten haul roads; (2) increase shovel capacity; and (3) increase shovel utilization through optimal truck matching. Additional data will be required to adequately describe operator effects on energy efficiency.

1. Introduction

The US mining industry consumes approximately 365 billion kWh of energy annually to produce vital products to support the US economy. Of this, coal mining accounts for approximately 142 billion kWh per year. The US Department of Energy (DOE) estimates that energy consumption can be reduced by 24.6 billion kWh/year by using current best practice and a further 44.8 billion kWh/year with more research to make coal mining more energy efficient (US Department of Energy, 2007). This translates into almost 49% decrease in energy consumption or nearly $3.7 billion of potential savings on coal production costs at 5.3¢/kWh of energy. With climate change regulation on the horizon, the benefits of energy savings in any production endeavor will be compounded in the near future. According to US Department of Energy (2007), the most promising

[1] Paper submitted to Transactions of the Society of Mining, Metallurgy, and Exploration.

processes for energy efficiency improvement are grinding and materials handling, including loading and hauling.

Current energy-saving strategies in coal mining tend to involve improvements in technology (e.g. improving engine performance). Energy consumption monitoring and reporting emphasizes system performance without regard to the operating conditions. However, there is evidence that operator practices and mine operating conditions significantly affect the energy consumption. For instance, simulation experiments conducted by Awuah-Offei (2009) suggest that an electric shovel operator who operates near optimal with a 58 yd^3 bucket can save over $114,000/year in electricity costs for the digging cycle alone, when compared to an average operator. Other research shows that equipment utilization and loading, for instance, are key factors in the energy efficiency of mining operations (Kecojevic and Komljenovic, 2010). Fig 1 shows, more comprehensively, the factors that affect energy consumption of mine equipment.

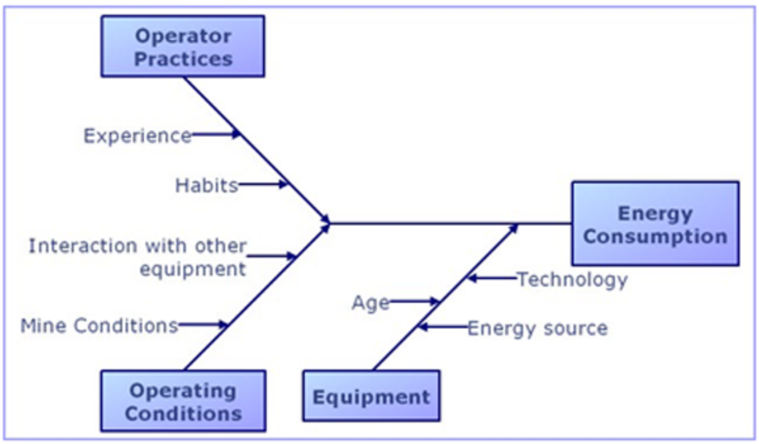

Fig 1. Factors affecting coal mine equipment energy consumption.

In order to understand the impact of operating conditions on energy efficiency, there is a need to conduct process specific, as opposed to system-wide, energy audits which account for operating conditions (Bogunovic, et al., 2009a). The knowledge can then be used to increase energy efficiency of mining operations and processes. Modeling and simulation is a cost effective and reliable way to assess the impact of different operating conditions on energy efficiency. Stochastic process simulation based on Monte Carlo simulation is capable of modeling process interactions and quantifying the uncertainty surrounding model outputs (Kelton, et al., 2003).

The objective of this research was to (i) apply stochastic process simulation to model the energy efficiency of a typical truck and shovel mining system; and (ii) evaluate production strategies to improve energy efficiency. The researchers conducted energy audits of the truck and shovel overburden removal operations at a surface coal mine. Following data analysis, they modeled the truck and shovel system in Arena® (Rockwell, 2010). The model was then validated with the field data and used to evaluate three improvement strategies.

Managing energy efficiency is an important goal in reducing process cost, which has become even more important due to concerns over energy availability and supply. Energy efficiency is defined as the ratio of effective energy to the total energy (Zhu and Yin, 2008). For loading and hauling, the amount of material loaded and hauled and the fuel consumed in the process are used as proxies for effective and total energies, respectively.

A fraction of the energy generated by an internal combustion engine is available for useful work as brake output due to heat, engine friction and pumping losses. In shovel loading, this output is used to overcome digging resistance and inertia, swing and spot, travel, and provide energy for accessories (Awuah-Offei, 2009). In trucking, the output is used to overcome aerodynamic drag, rolling resistance, drive train friction, inertial forces and accessory loads (Ang-Olson and Schroeer, 2002). For trucking, the contribution of each of these resistances to energy losses, and hence efficiency drops, depend on driving speed, truck weight, terrain, driver behavior, wind speed and direction, and road conditions (Ang-Olson and Schroeer, 2002). Researchers have evaluated the effect of reduced idling, speed reduction, driver training, and reduced empty runs through optimal truck scheduling on energy efficiency (Ang-Olson and Schroeer, 2002; Bates, et al., 2001; Hubbard, 2003; Leonardi and Baumgartner, 2004).

Loading and hauling in surface mines is different from freight trucking because loading is a significant aspect of the energy demand due to significantly shorter distances. Consequently, the inefficiencies due to shovel-truck interactions (under-matched or over-matched shovels) are also significant. Bogunovic, et al. (2009b) show variability in energy consumption of trucks and shovels due to operating conditions (e.g. which coal seam is being mined) and operators.

Kecojevic and Komljenovic (2010) show that just 10% reduction in engine load factor can result in fuel savings ranging from $40,000 to $267,000 per year depending on the size of truck, based on original equipment manufacturer (OEM) data and existing literature. Various studies have also shown that shovel depth of cut, which is a function of operator experience and preferences, significantly affects shovel energy consumption (Awuah-Offei and Frimpong, 2007; Patnayak, et al., 2008). However, what is missing in the literature is predictive modeling of energy efficiency as a function of operating conditions under uncertainty to facilitate evaluation of improvement strategies. As far as the authors know, there has been no work that characterizes uncertainty in a model and also attempts to determine statistically significant correlations. This can be achieved with stochastic process simulation with adequate data for uncertainty characterization.

An important aspect of this study was the use of stochastic process simulation to evaluate energy saving production strategies, before implementation, and characterize uncertainty. Stochastic simulation is a well known technique that has been used to study several mining systems (Awuah-Offei, et al., 2003; Raj, et al., 2009). Several special-purpose simulation languages like GPSS, Simscript, SLAM and SIMAN exist for modeling continuous, discrete and mixed continuous-discrete event systems. An important advantage of these packages is the use of Monte Carlo simulation to handle uncertainty modeling. In this work, we used Arena® (Rockwell, 2010), which is based on the SIMAN simulation language, to model the energy consumption of the truck-shovel system of the mine (Kelton, et al., 2003).

This work represents a novel attempt to model truck and shovel energy efficiency with uncertainty. Existing OEM software are deterministic and do not account for the inherent uncertainty. This approach to energy improvement planning will allow inexpensive experimentation prior to implementation so that the strategies that are implemented are likely to succeed and the risks are properly understood.

2. Experimental procedures

2.1. Study site

The study site is a strip mine in the Illinois basin and recovers coal mainly from the Murphysboro seam, with some coal mined from the Mount Rorah seam. The mine produces about 600,000 tons of coal annually at an average stripping ratio of 17:1. The overburden is made up of grey, well laminated non-marine shales, overlain with up to 40 feet of glacial outwash clays and sand channels. The overburden is fragmented through blasting prior to removal. Overburden removal is mainly by carry dozers. However, the final overburden is removed by a Hitachi EX1900 hydraulic shovel (14.4 yd^3 dipper) and Caterpillar 785C (150-ton), rigid frame, haul trucks. Both the shovel and trucks had on-board data logging systems that were used to collect data on engine load factor and fuel consumption, respectively.

2.2. Shovel Energy Audit

Hitachi's onboard shovel data logging system, Machine Information Center (MIC) logs, among others, engine running time, the operating time of the front end[1], travel time, and engine load factor. MIC data from January 1 to July 12, 2010 was downloaded from the shovel for this study. After careful review, we used the shift averages of the engine running time, the operating time of the front end, travel time, and engine load factor for data analysis. Since MIC does not log fuel consumed, Hitachi data on fuel consumption and load factors was used to establish the relationships in Eq. (1), which relates engine load factor to fuel consumption for the two shovel models analyzed in this study.

$$\begin{aligned} &\text{EX1900 fuel consumption [gals/hr]} = 52.971 \times \text{Load factor} + 0.0133 \\ &\text{EX2500 fuel consumption [gals/hr]} = 71.304 \times \text{Load factor} + 0.0059 \end{aligned} \quad (1)$$

Additionally, researchers conducted time and motion studies of the shovel loading operation to obtain cycle times. Productivity of the shovel was obtained by correlating the time stamps on the data with the truck OEM data logging system (discussed in the next section).

2.3. Truck energy audit

Caterpillar's Vital Information Management System (VIMS) logs payload, empty stopped time, empty travel time, empty travel distance, loading time, loaded stopped time, loaded travel time, loaded travel distance, total cycle distance, total cycle time, and fuel used for each cycle. The team downloaded data from May 3 to July 2, 2010. The summary performance was based on all this data. However, given the variability in haul distance, and haul road profile and conditions, the team only used the data from the experimental period (June 28-July 2) for detailed analysis. This was because the haul distances, profile and conditions were similar. The haul profile was surveyed with Topcon Hyperlite GPS units for real-time kinematic (RTK) surveying. Even though VIMS logged the cycle times, the team conducted manual time and motion studies of the trucks as well to confirm the VIMS data. The OEM data was proven to be reliable and better than the manual data and therefore all the analysis was based on that.

3. Data analysis and model input

3.1. Shovel data analysis and modeling

Statistical correlation analysis, at 95% confidence, was used to examine correlation between the load factor (the proxy for fuel consumption) and the engine running time, the operating time of the front end, front-end utilization (ratio of time the front-end was active in the shift), travel time and ratio of time the shovel traveled in the shift. The decision to use the time ratios in correlation analysis was to enable extension of the results to different shift lengths. Regression analysis was then used to determine the relationship between load factor and key independent variables.

Figs. 2-4 show plots of load factor against engine running time, front end operating time and travel time. As shown by Table 1, there is positive linear correlation between load factor and each of the variables except ratio of travel time (p-value is greater than α). The correlation between load factor and engine running time is due to the fact that short shifts (less than five hours) are usually for non-production related work and do not result in significant loading of the engine.

[1]This time is cumulatively logged so long as any of the hydraulic pumps controlling cylinders on the shovel front end is active. The time logged is, thus, always more than the loading time of the shovel.

Table 1 Shovel fuel consumption correlation analysis.

Independent variable	Pearson correlation coefficient	p-value ($\alpha = 0.05$)
Engine running time	0.66979	0.0000
Front end operating time	0.76525	0.0000
Front end utilization	0.77948	0.0000
Travel time	0.51037	0.0000
Ratio of travel time	-0.11818	0.1392

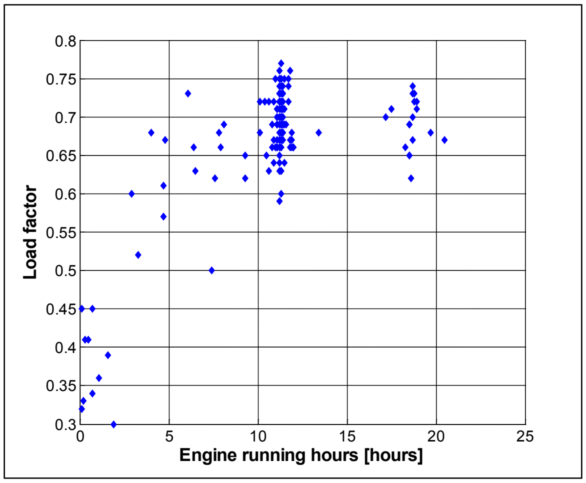

Fig 2. Plot of load factor against engine running hours for a shift.

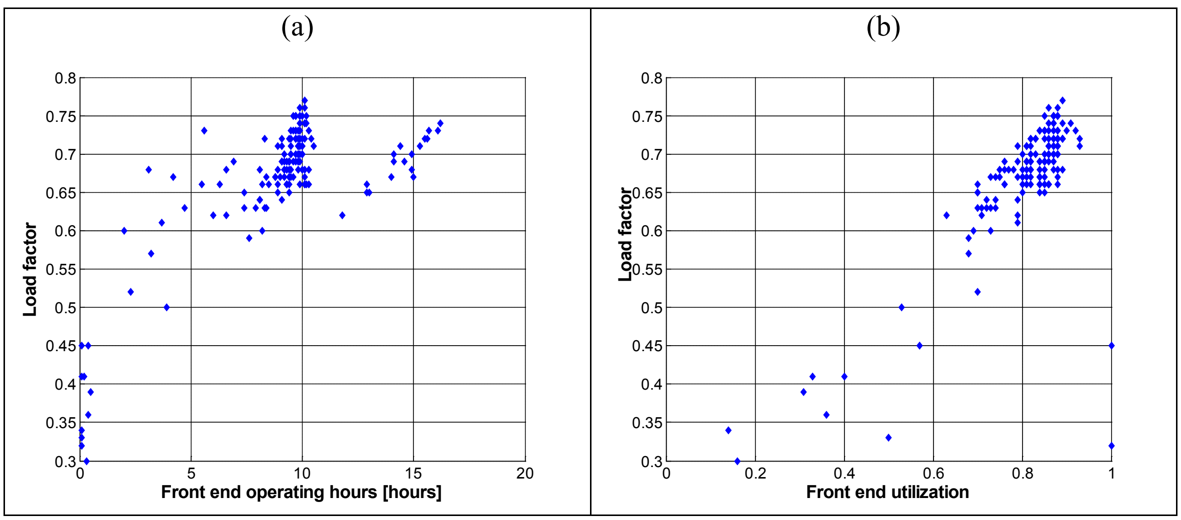

Fig 3. Plot of load factor against (a) front end operating time; (b) front end utilization.

Front end utilization allows one to extend the model to different shift times. Eq. (2) is the resulting regression model. Fig 5 shows the residuals of the model compared to the actual data. The mean residual for the 158 data points is 3.0918×10^{-17}%. Fig 5 shows only 3 out of 158 data points could not be predicted with confidence. The R^2, the F statistic and its p-value, and the error variance are 0.6062, 240.1482, 0, and 0.0030, respectively.

$$\text{Shovel load factor} = 0.2391 + 0.5337(\text{front end utilization}) \tag{2}$$

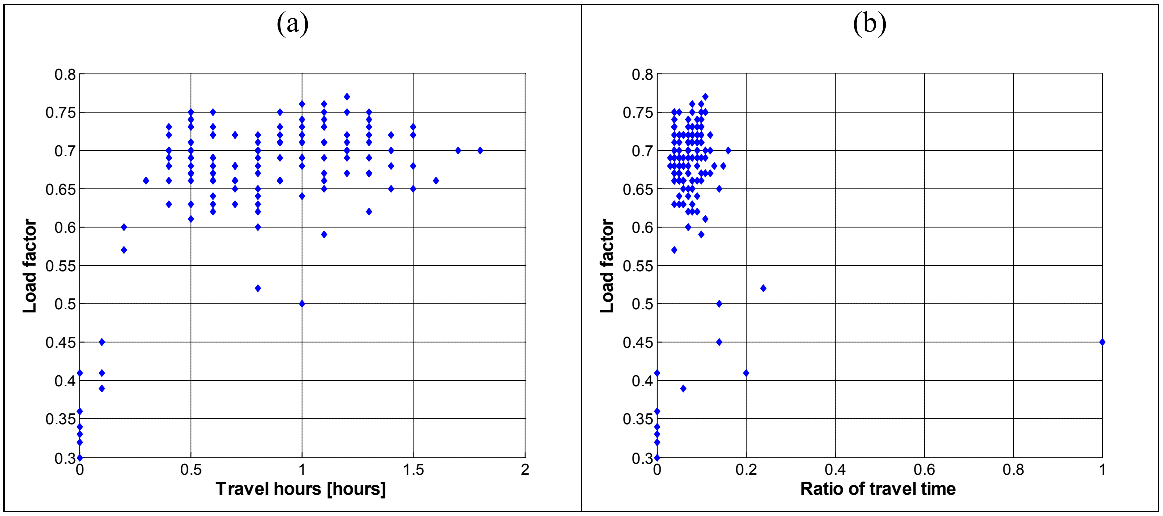

Fig 4. Plot of load factor against: (a) travel time; (b) ratio of travel time in a shift.

Fig 5. Residuals and 95% confidence intervals of residuals for Equation (2) model. Red data points are residual intervals that do not include zero.

3.2. Truck data analysis and modeling

First, we used statistical hypothesis testing at 95% confidence to determine if the different operators and trucks had any impact on the fuel consumption and total cycle time. We used two sample, unequal variances, t-test hypothesis testing (NIST/SEMATECH, 2010). The data was collected by switching operators to ensure that different operators drove the different trucks. Tables 2 and 3 show the summary of the input and output from the t-test. The null hypothesis was accepted for both cycle time and fuel when comparing the two trucks (Table 2).

Hence, we conclude that there is not enough evidence at 95% confidence to reject the notion that the means of the cycle times and fuel consumption for both trucks are the same, given the available data.

The null hypothesis was accepted in the test to compare the means of cycle times of the two operators (Table 3). However, when comparing the fuel consumptions with the null hypothesis, $H_0 : \mu_A = \mu_B$, the hypothesis was rejected. The team then proceeded to test the hypothesis that operator A was consuming more fuel/cycle than operator B (null hypothesis and corresponding alternate hypothesis shown in parenthesis in Table 3).

Table 2. t-test summary for comparing trucks.

	Cycle time [mins]		Fuel/cycle [gals]	
	Truck 1	*Truck 2*	*Truck 1*	*Truck 2*
No. of samples	115	113	115	113
Mean	11.66	11.44	4.36	4.22
Standard deviation	5.05	4.15	0.60	0.51
Degrees of freedom	226		226	
Pooled standard deviation	4.62		0.56	
t-statistic	0.3591		1.8615	
H_0	$\mu_1 = \mu_2$		$\mu_1 = \mu_2$	
H_1	$\mu_1 > \mu_2$		$\mu_1 > \mu_2$	

Table 3. t-test summary for comparing operators.

	Cycle time [mins]		Fuel/cycle [gals]	
	Operator A	*Operator B*	*Operator A*	*Operator B*
No. of samples	116	112	116	112
Mean	11.29	11.83	4.36	4.21
Standard deviation	3.98	5.20	0.55	0.56
Degrees of freedom	226		226	
Pooled standard deviation	4.62		0.56	
t-statistic	0.3591		2.0944	
H_0	$\mu_A = \mu_B$		$\mu_A = \mu_B$ ($\mu_A > \mu_B$)	
H_1	$\mu_A < \mu_B$		$\mu_A > \mu_B$ ($\mu_A \le \mu_B$)	

Again, there was enough evidence to reject the null hypothesis. One would have to conclude based on the t-tests at 95% confidence, that: (i) the means of the cycle times of the two operators are equal; (ii) the means of the fuel/cycle for the two operators are not equal; and (ii) the mean fuel/cycle of operator A is not greater than the mean of operator B. This leads to an inconclusive overall conclusion. On one hand, cycle times of the two operators are similar but there are indications that the fuel/cycle is not the same. Yet, one cannot definitively say, that the fuel consumption of operator A is higher than that of operator B. More data over a longer period, and possibly involving more operators, is needed to better characterize the impact of operators on fuel consumption. Given the foregoing, the team concluded that the different trucks and operators made no significant difference and, hence, all the data will be treated as one population.

We then proceeded to conduct linear correlation analysis to determine the correlation between fuel/cycle and the cycle time components and payload. Table 4 and Figs 6-9 show the correlation coefficients with their corresponding p-values and the scatter plots, respectively. Surprisingly, there was no statistically significant correlation between the payloads for the experimental period and the fuel/cycle as indicated by the p-value of 0.1801 (greater than $\alpha = 0.05$). This was contrary to expectation and hence the correlation between payload for the entire available data set (May 3 to July2) and fuel/cycle was also analyzed. This yielded a statistically significant correlation (p-value of 0). Modeling fuel/cycle per ton is desirable so that the model can be extended to different truck payloads. In fact, it is expected that fuel consumption should be correlated to amount of material carried since more work is done. Hence, we proceeded to test the correlations between the cycle time components in Table 4 and fuel/cycle/ton. There was, statistically significant, positive correlation between the cycle time components and the fuel/cycle/ton. Based on this, the regression model in Eq. (3) was formulated. t_i is cycle time in minutes for component i. The subscripts *es, et, l, ls,* and *lt* mean empty stopped, empty travel, loading, loaded stopped, and loaded travel.

Table 4. Truck fuel correlation analysis.

Independent variable	**Pearson correlation coefficient**	**p-value (α = 0.05)**
Payload (June 28-July 2)	0.0891[1]	0.1801
Payload (May 3-July 2)	0.1518	0.0000
Loading time	0.1861	0.0049
Empty stopped time	0.3951	0.0000
Empty travel time	0.5206	0.0000
Loaded stopped time	0.1861	0.0049
Loaded travel time	0.3511	0.0000

[1] Correlation is between the independent variable and fuel/cycle.

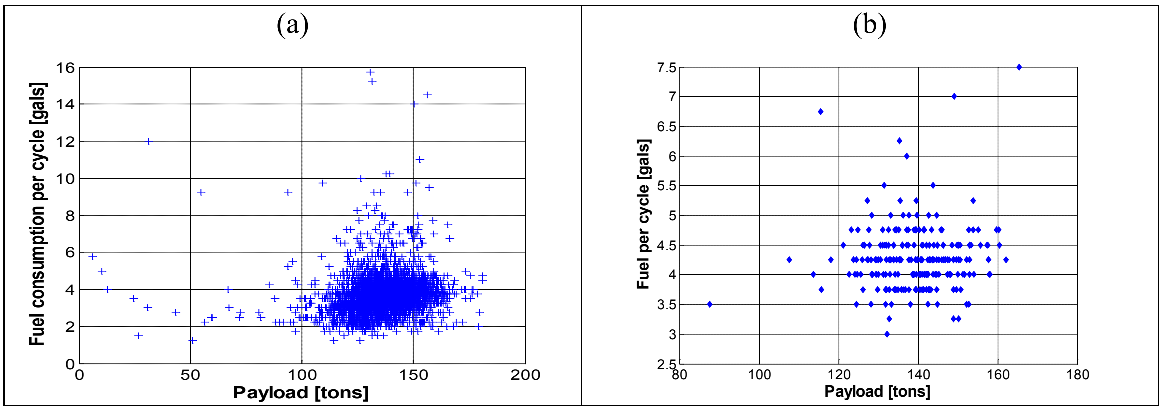

Fig 6. Fuel/cycle against: (a) payload over the period 6/28-7/2; (b) payload over the period 6/28-7/2.

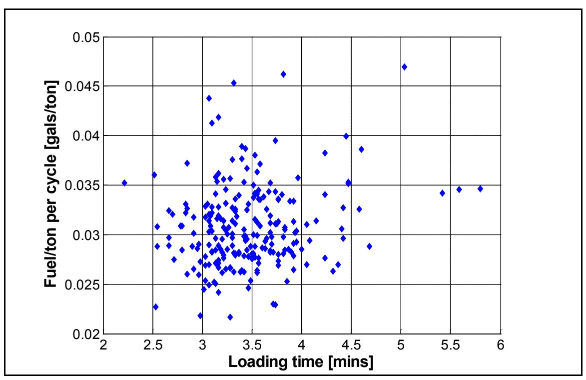

Fig 7. Fuel/cycle/ton against loading time.

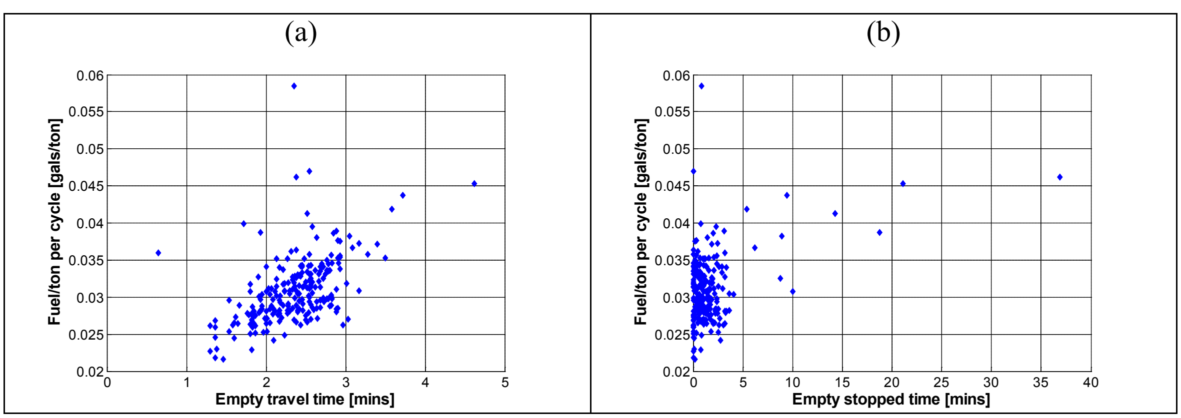

Fig 8. Fuel/cycle/ton against: (a) empty travel time; (b) empty stopped time.

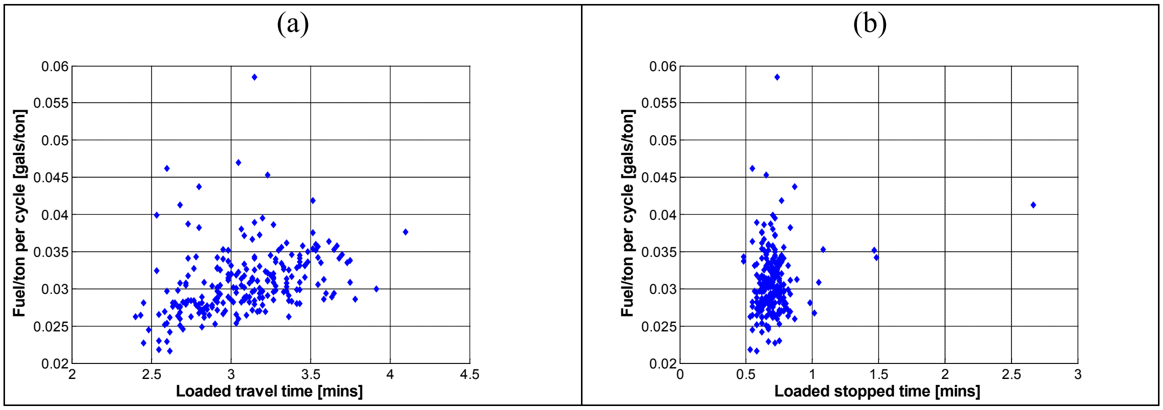

Fig 9. Fuel/cycle/ton against: (a) loaded travel time; (b) loaded stopped time.

$$\text{Fuel/cycle/ton} = 0.0037 + 0.0005t_{es} + 0.0035t_{et} + 0.0008t_l + 0.0031t_{ls} + 0.0043t_{lt} \tag{3}$$

Fig 10 shows the residuals of the model compared to the actual data. The mean residual for the 158 data points is -8.6857 × 10^{-18}%. Fig 10 shows only 6 out of 153 data points compared could not be predicted with confidence. The R^2 statistic, the F statistic and its p-value, and the error variance are 0.8356, 139.2678, 0 and 0.0519, respectively.

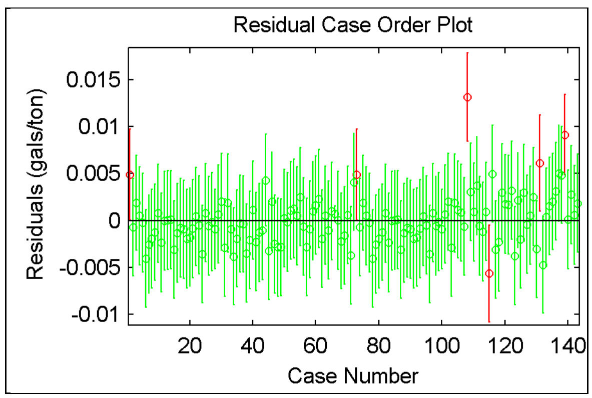

Fig 10. Residuals and 95% confidence intervals of residuals for Equation (3) model. Red data points are residual intervals that do not include zero.

3.3. Simulation model input

Eq. (3) was used, in the simulation model, to predict the amount of fuel consumed by a truck in each cycle. Theoretical statistical distributions were fitted to data to describe the stochastic processes in the simulation model. Chi-squared goodness-of-fit tests were used to fit these distributions to the processes that needed to be modeled this way using the Input Analyzer module in Arena®. Table 5 shows the results of distribution fitting using the data from the time and motion studies and the OEM onboard data logger. The expressions in Table 5 were used to describe the activity times. The average haul distance for the experimental period was surveyed to be 1,207 m (3,960 ft).

Table 5a. Shovel distribution fitting results.

Process	Distribution	Expression	Square Error
Dumping time (mins)	Lognormal	LOGN(0.0349, 0.0156)	0.093891
Return time (mins)	Lognormal	LOGN(0.173, 0.0969)	0.019817
Loading time (mins)	Gamma	GAMM(0.0464, 3.05)	0.027245
Spotting time (mins)	Lognormal	LOGN(0.155, 0.109)	0.047969

Table 5b. Truck distribution fitting results.

Process	Distribution	Expression	Square Error
Payload (tons)	Normal	NORM(139, 10.8)	0.001313
Empty stopped time (mins)	Beta	37 × BETA(0.171, 2.31)	0.011708
Empty travel time (mins)	Normal	NORM(2.3, 0.471)	0.006764
Loaded stopped time (mins)	Erlang	ERLA(0.458, 2)	0.000268
Loaded travel time (mins)	Beta	2.26 + 1.66 × BETA(3.3, 4.06)	0.003836

4. Truck-shovel simulation model

Discrete systems, such as the truck-and-shovel system, are modeled in Arena® using the process orientation approach usually referred to as object-oriented simulation. In this type of model, the modeler identifies the system's entities, processes, and resources. The system is then conceptualized by letting entities go through static processes in a logical way. At each process, entities wait their turn to use up required resources to go through the process (Awuah-Offei, et al., 2003). In Arena®, the modeler can create different entities, which can be given characteristics by specifying attributes. The software provides numerous modules for model construction (Kelton, et al., 2003). A model is therefore an appropriate assembly of blocks to mimic reality as closely as possible.

In modeling energy efficiency of the mine's truck-and-shovel system, drivers/operators were identified as entities. Component cycle times (empty stopped, empty travel, loading, loaded stopped and loaded travel times), arrival and departure time at a station, and payload were defined as attributes which were changed for each cycle by sampling from the distributions in Table 5b or assigning current simulation time. Two stations were defined in the model and transporters (trucks) used to move entities between these stations. The logic at each station is shown in Fig 11. The shovel was defined as a resource, which was needed for an entity to go through the loading process. The shovel schedule was used to enforce the 30-minute break during an 11-hour shift. At the end of each cycle, fuel consumed in that cycle is calculated using Eq. (3). In order to ensure accurate fuel consumption data, the model was set-up to write the fuel consumed by trucks in each cycle to a comma separated text file for processing at the end of the simulation. Appropriate data, including shovel utilization, were collected and reported at the end of the simulation. Time consuming and instantaneous processes are defined as those that affect the simulation clock and those that do not, respectively.

The main simulation output was energy efficiency, which (in this model) is a function of shovel utilization, truck fuel consumption, and productivity per shift. We, therefore, selected the number of replications to ensure that the half-widths[2] of production, truck fuel consumption per shift, shovel utilization, and energy efficiency were less than 1% of the mean in the base case simulation. It was determined that 100 replications achieved the desired half-widths for these key outputs. Hence, all simulation experiments were set up to run for 100 replications of eleven hours each (equivalent to 100 shifts). The simulation run until it was terminated after eleven hours. All trucks are allowed to dump the last load at the end of the shift.

[2] Half-width, h, is defined, using central limit theorem, as $h = s_x \times t_{v,\left(1-\alpha/2\right)}$, where s_x is the standard deviation of x, and $t_{v,\left(1-\alpha/2\right)}$ is the t-statistic for v degrees of freedom and (1-α) confidence level.

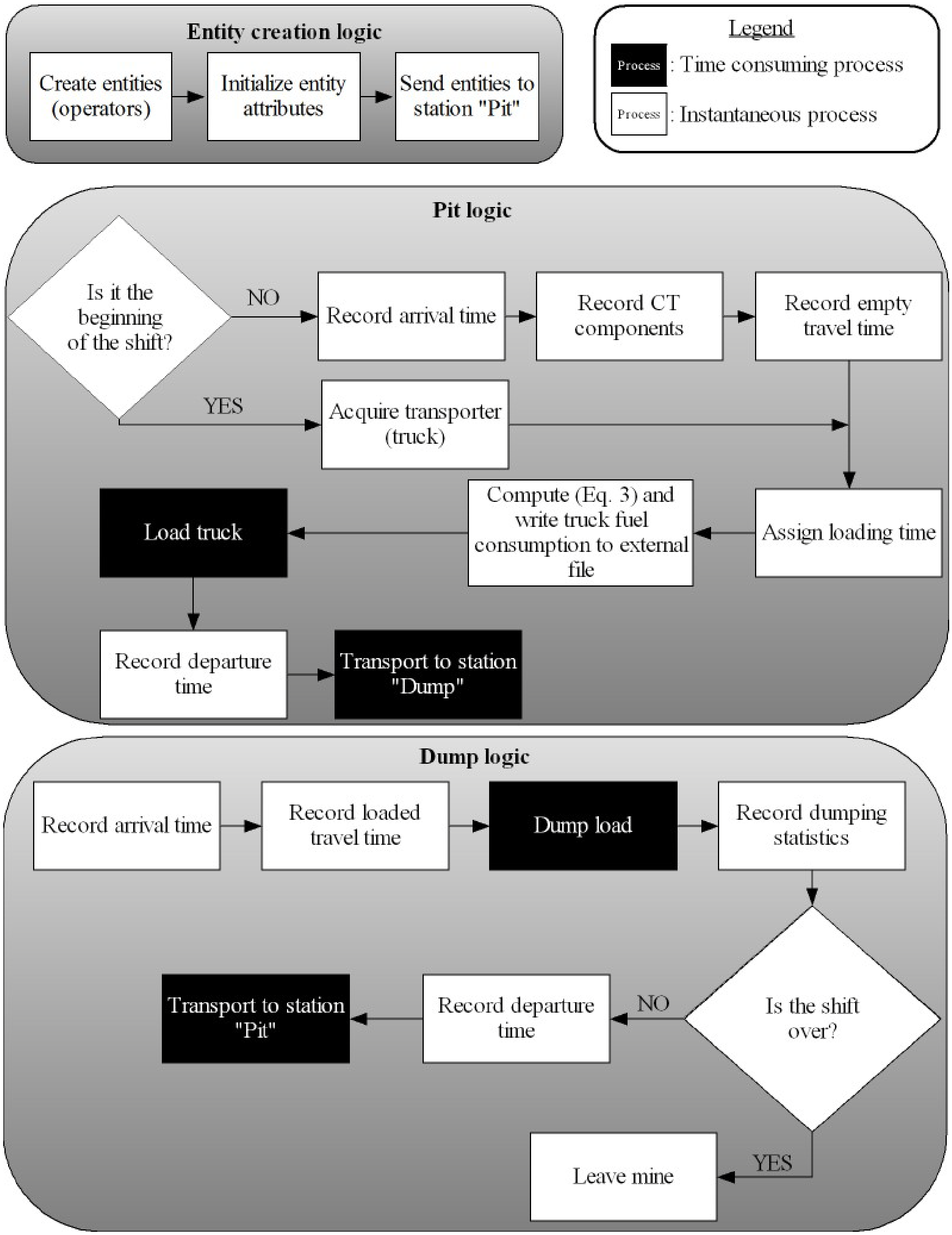

Fig 11. Flowchart of Arena® model logic.

The model was verified and validated with animation and comparing with the field data, respectively. Table 6 shows comparison of the actual truck data for the experimental period and the average values after the 100 replications. The model was validated based on the truck data because the VIMS data was more detailed and useful for validation. Average shovel shift front end utilization and load factor from the MIC data is 80.31% and 66.78%, respectively. Since the cycle time data did not capture any action of the front end apart from loading activities, it was not possible to predict front end utilization from the simulation. The simulation model, however, predicts shift shovel utilization to be 67.43% (less than the front end utilization, as expected). Given, the similarity between the shovel utilization and the load factor, it was assumed that the shovel utilization is a good predictor of the engine load factor for subsequent analysis. On the basis of the truck predictions, the model was deemed validated as it predicts fuel efficiency and fuel consumed per cycle to within 5%.

Table 6. Simulation model validation.

	Actual	**Simulated**		**Error**
		Mean	*Half-width*	
Production [tons]	15,887	16,590	57	4%
Number of loads	114	120	0.4	5%
Total fuel consumption [gals]	488.87	502.60	1.54	3%
Average fuel consumption per cycle [gals]	4.24	4.27	0.01	1%
Overall fuel efficiency [tons/gal]	17.81[1]	18.51	0.03	4%

[1] Based on calculated shovel fuel efficiency of 39.29 tons/gal from average engine load factor of 66.78% and hourly production from truck production data.

The half-width is a good measure of the uncertainty surrounding each of the estimates from the model. At (1-α) confidence, the estimate (predicted by the expected value, $\overline{x}$) will be between $\overline{x}-h$ and $\overline{x}+h$. All half-widths in Table 6 are less than 1% of the mean, which indicates the 100 replications are adequate. The ability to quantify that uncertainty is a key advantage of stochastic process simulation over deterministic methods such as those found in OEM haulage software.

5. Evaluating energy-saving strategies

The validated simulation model was then used to evaluate the energy saving improvement strategies. The following strategies were evaluated:

- **Strategy 1:** Increase shovel utilization through optimal truck matching. This scenario involved increasing the number of trucks in the system in order to identify the optimal truck-shovel matching. In an alternate scenario, additional Caterpillar 777 (100-ton) trucks were added to the fleet since the mine has two of 777 trucks available already. The mine is more likely to add these 777 trucks than purchase new 785 trucks. It is estimated that the EX1900 shovel is able to load the 777 trucks in five passes.
- **Strategy 2:** Increase shovel capacity. This scenario involved simulating the use of an EX2500 (20.4 yd^3 dipper), which is the next size up in Hitachi's fleet, instead of the EX1900 shovel currently in use. In order to do this, it was assumed (after consultation with Hitachi dealer staff) that the cycle times for the EX2500 shovel were the same as those for the EX1900 shovel. This shovel will load the 785 trucks in 5 passes.
- **Strategy 3:** Shorten haul roads. This can be achieved by reducing the size of the pit. This involved varying the haul distance from 0.2 to 1.0 miles in steps of 0.2 miles while keeping everything else constant.

6. Results and discussions

6.1. Strategy 1: increase shovel utilization through optimal truck matching

Fig 12 shows there are potential gains in production and shovel utilization from adding trucks as expected (for each box, the central mark is the median, the notches represent the 95% confidence interval, the edges of the box are the 25th and 75th percentiles, the whiskers extend to the most extreme data points the MATLAB boxplot algorithm does not consider to be outliers, and the outliers are plotted individually in red). The largest gain comes when the number of trucks is increased from two to three – mean production increases by 4,400 and 5,700 tons/shift (Fig 12a and Fig 13a), and shovel utilization increases by 19.53 and 23.26% (Fig 12b and Fig 13b) for adding 777 (100-ton) and 785 (150-ton) trucks, respectively. However, increasing the number of trucks increases queue lengths or time spent waiting at the shovel (Fig 12c and Fig 13c) and longer queue lengths causes total fuel consumed by trucks in a shift to increase, even when shovel fuel consumption is constant (Fig 12b & d and Fig 13b & d). The overall effect is that fuel efficiency declines in spite of the increased productivity. Adding one 777 or 785 truck decreases fuel efficiency by 1.06 and 0.89%, respectively. The reduction is significant at 95% confidence, as shown by the non-overlapping notches in Fig 12e and Fig 13e.

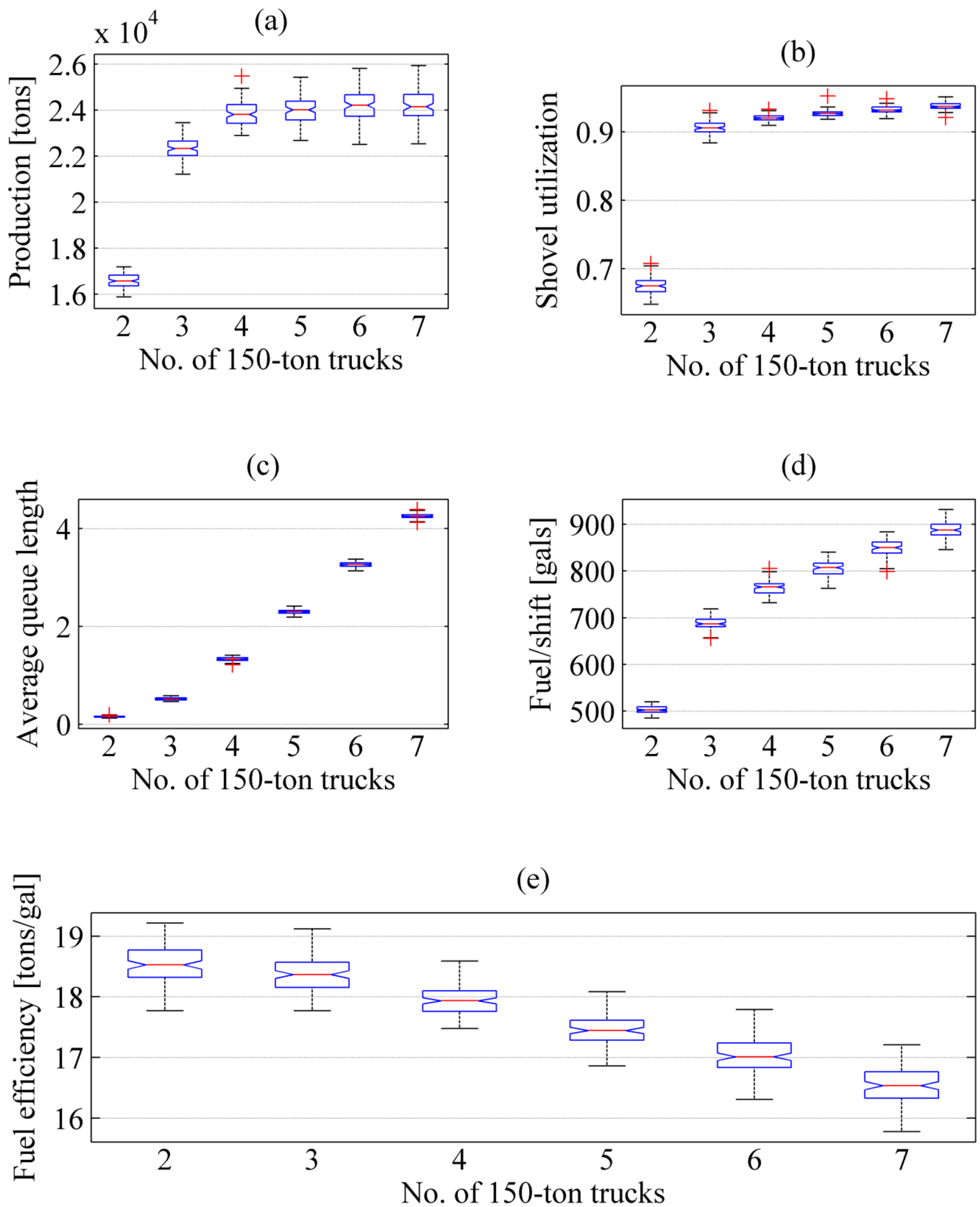

Fig 12. Simulation Results for Strategy 1 – Adding 150-ton Trucks.

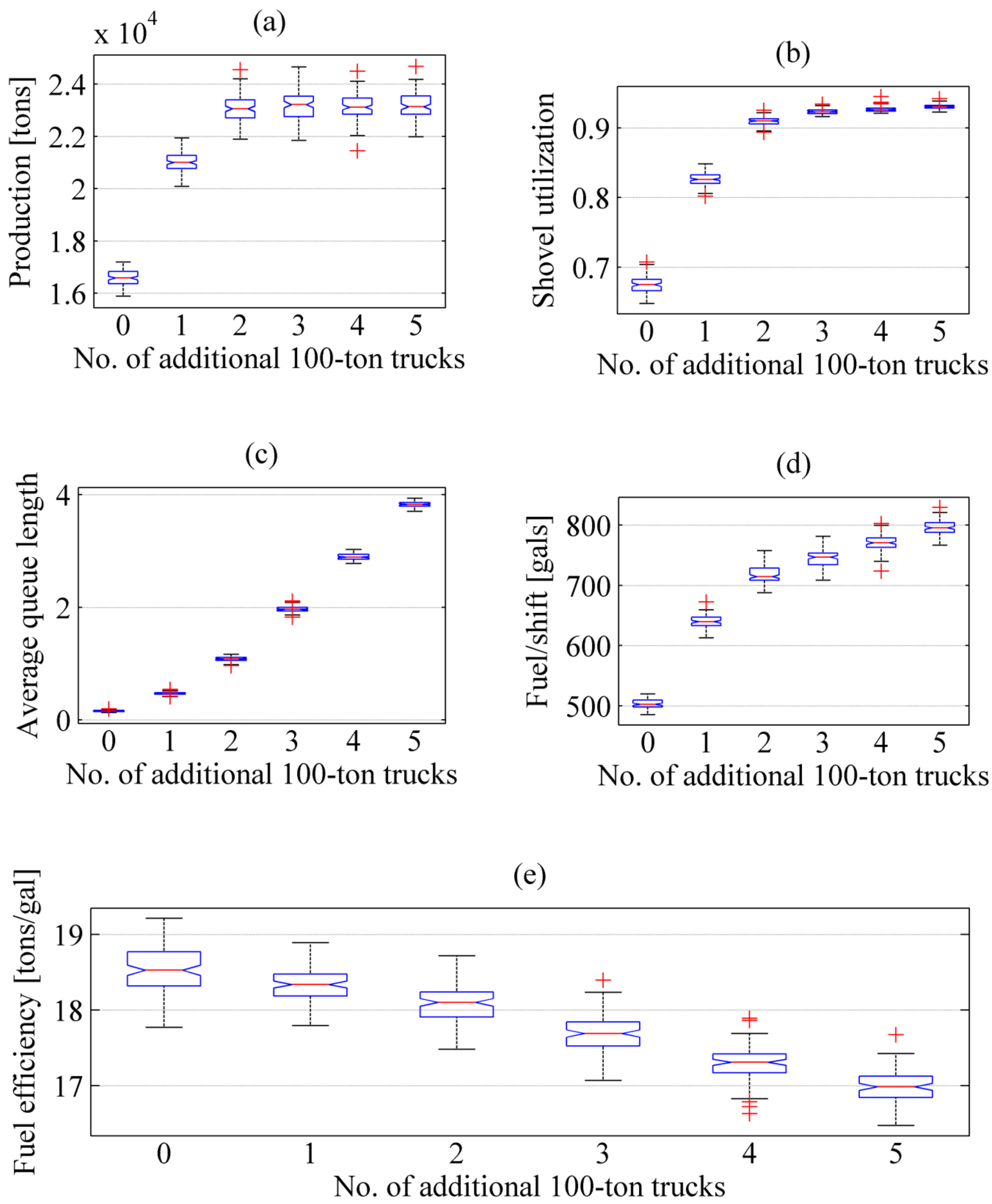

Fig 13. Simulation Results For Strategy 1 – Adding 100-ton Trucks.

This phenomenon is described more clearly in Fig 14, which shows that the only cycle time component that varies as trucks are added to the system is empty stopped time. The increase in empty stopped time is smallest when a third truck is added to the system, but it rises sharply as additional trucks are added and the resulting inefficiencies outweigh any gains in productivity and shovel utilization. The conclusion is that for this mine, having more than three trucks in the system is sub-optimal.

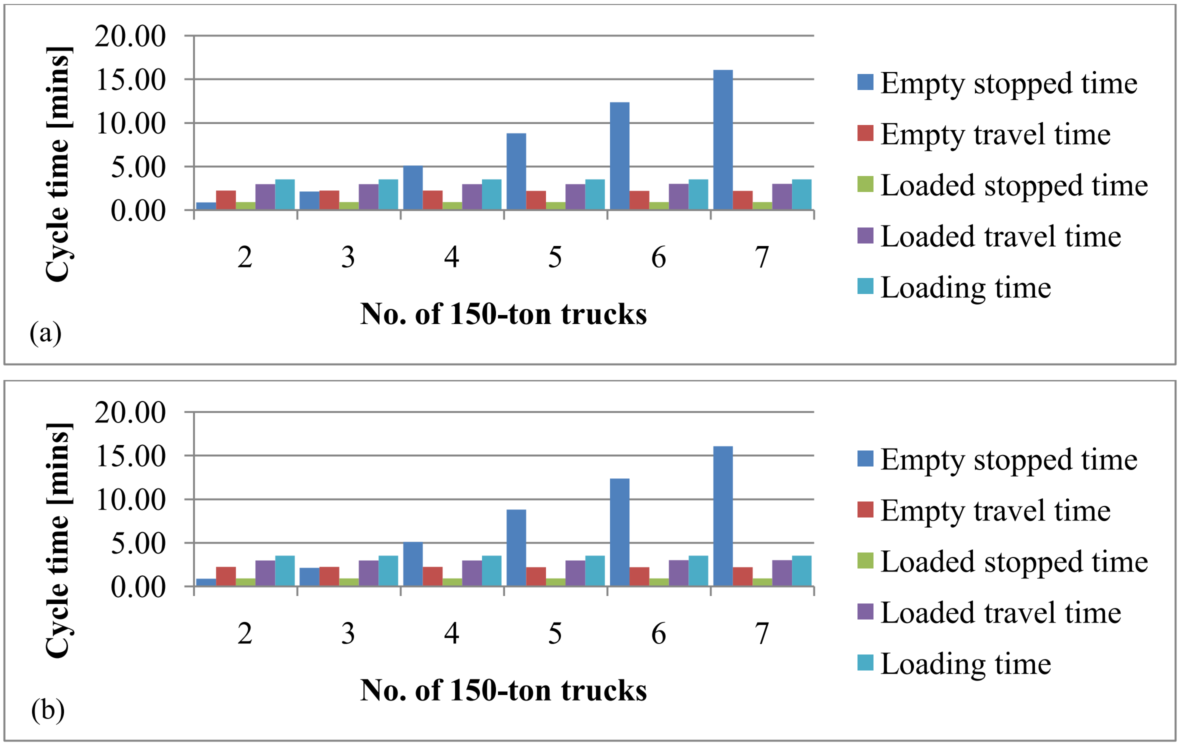

Fig 14. Simulated Cycle Time Components for Trucks: (a) 100-ton; (b) 150-ton.

The uncertainty surrounding the estimates in Figs 12 and 13 are relatively low, which builds confidence in the estimates and the conclusions drawn from them.

6.2. Strategy 2: increase shovel capacity

Fig 15 gives results when use of the larger EX2500 shovel, instead of the currently used EX1900, was evaluated. It shows a statistically significant increase in production and fuel consumed by the trucks (Fig 15a & d) and decreases in shovel utilization (Fig 15b) and average queue length (Fig 15c), which lead to a 3.3% increase in fuel efficiency (Fig 15e). Even though shovel utilization is lower for the larger shovel, fuel consumption is 38.8 gals/hr compared to 35.4 gals/hr for the smaller shovel. This is due to a higher consumption rate for the larger engine. While the increase in production more than compensates for the increase in fuel consumption rate, lower utilization of the larger shovel is economically undesirable given its higher ownership costs.

Fig 16 shows average cycle time components for trucks working with the two shovels. Average travel time and loaded stopped time (dumping time) remain the same. Loading time and empty stopped time (waiting on shovel) decrease with use of the larger shovel. Consequently, truck fuel consumption is reduced from 4.27 to 4.14 gals/cycle. The result is increased fuel efficiency when using the EX2500 shovel.

6.3. Strategy 3: shorten haul roads

The variation in truck fuel consumption and average haul distance over a shift is evident in the VIMS ('Actual') data shown in Fig 17. However, the observed variation cannot be, solely, attributed to changes in haul distance since other factors (e.g. haul road conditions and profiles) were not kept constant. This explains the different fuel consumptions for the same truck (Truck 2) at 0.3 miles observed on June 19 and 20. Consequently, simulation experiments were conducted to quantify the relation between fuel consumption and haul distance when only the haul road distance is varied in a controlled experiment ('Simulated' data points and line in Fig 17).

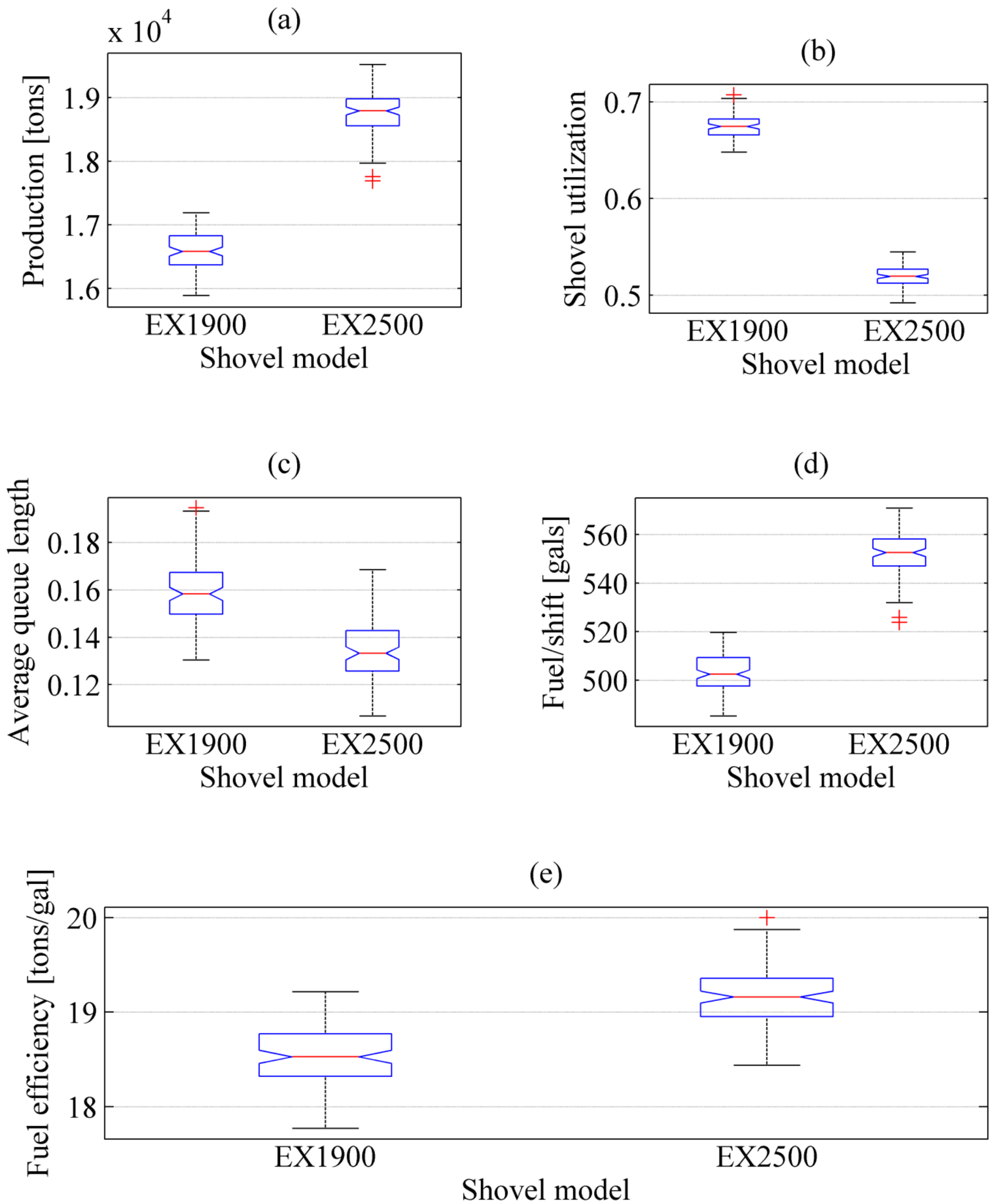

Fig 15. Simulation Results for Strategy 2 – Larger Shovel.

Fig 18 shows that production, shovel utilization, queue length, and fuel efficiency decrease with increasing haul distance. The only one of these that is an efficiency gain from increasing haul distance is the reduction in queuing or truck waiting. This decrease in empty stopped time diminishes with increasing haul distance, as shown in Fig 19, such that beyond approximately 0.8 miles, empty stopped time is not dependent on haul distance. Fig 19 shows both travel times increasing with longer haul distances. This predictably increases the overall cycle time.

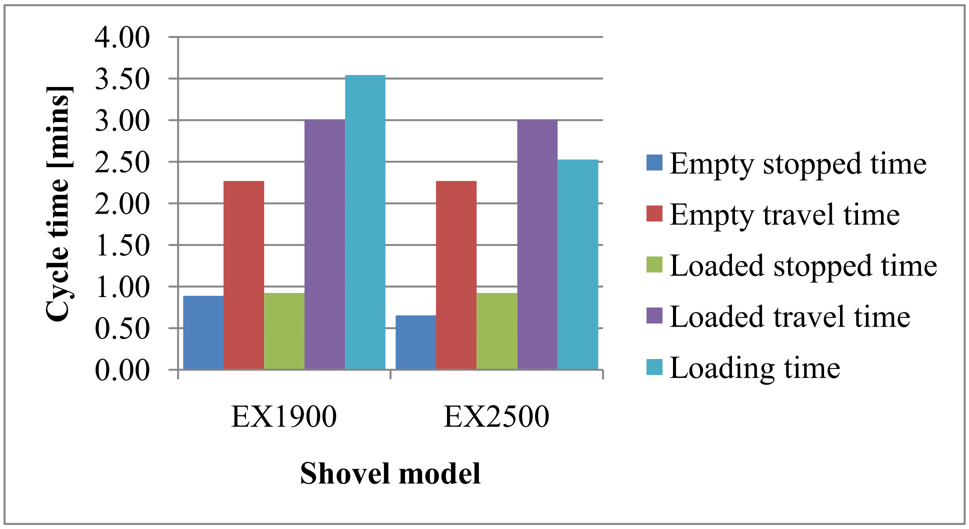

Fig 16. Truck Cycle Time Components When Different Shovels Are Used.

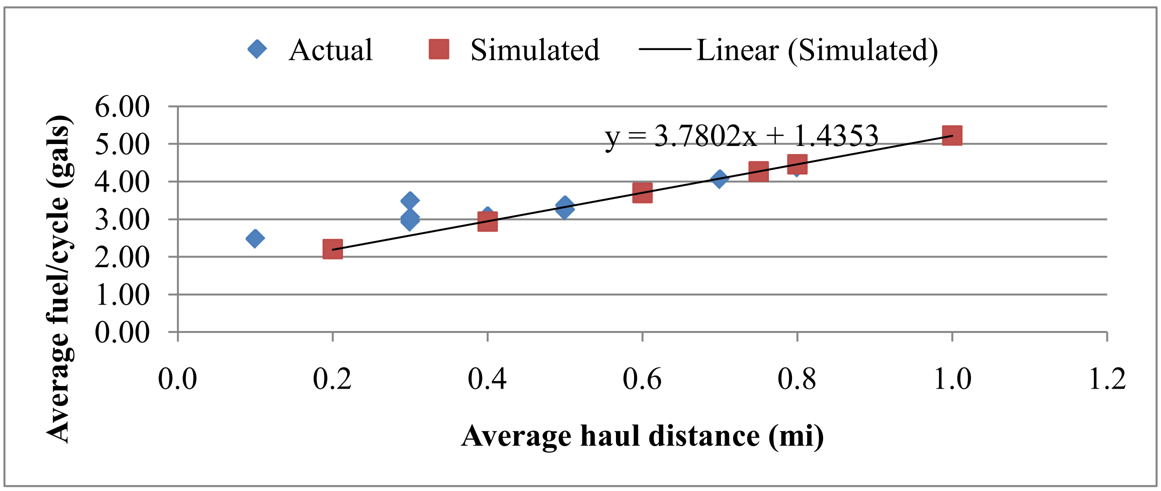

Fig 17. Variation in Truck Fuel/cycle with Average Haul Distance.

While significant gains can be achieved by shortening haul distances, a systems approach should be taken in implementing this strategy. In reducing haul road length, the mine operator must be careful not to significantly increase either the haul road grade or the dozer push distance. A limitation of this analysis is that it did not include dozer fuel consumption.

7. Conclusions

The objective of this work was to apply stochastic process simulation to model the energy efficiency of a typical truck and shovel mining system and use the model to evaluate production strategies to improve energy efficiency. The following conclusions are drawn from the results and discussion presented:

- Process specific energy audits provide insights into improving operations in a way that is not possible with global energy consumption figures. This was illustrated by the fuel consumption analysis of the truck-and-shovel system.

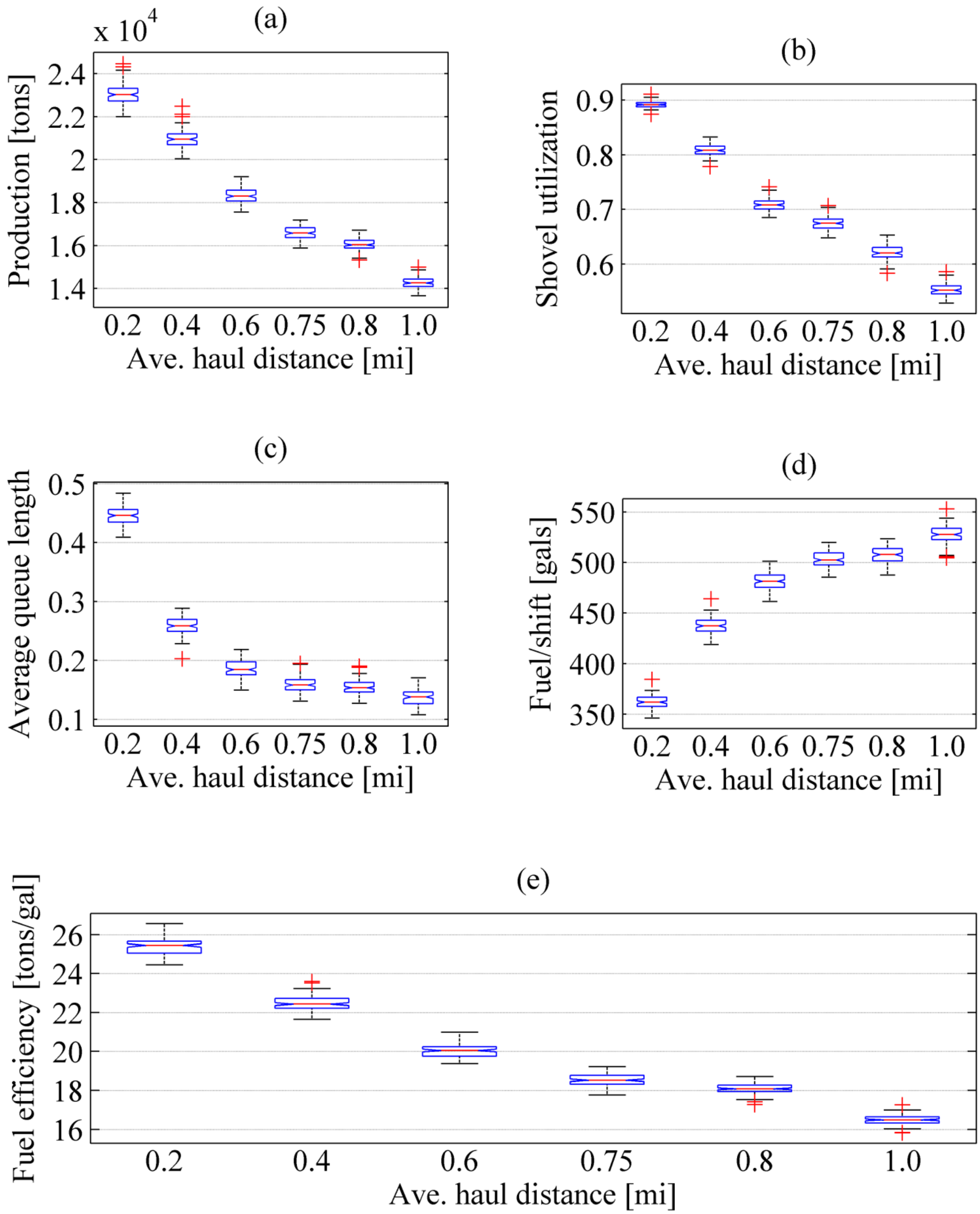

Fig 18. Simulation Results for Strategy 3 – Shorten Haul Distance.

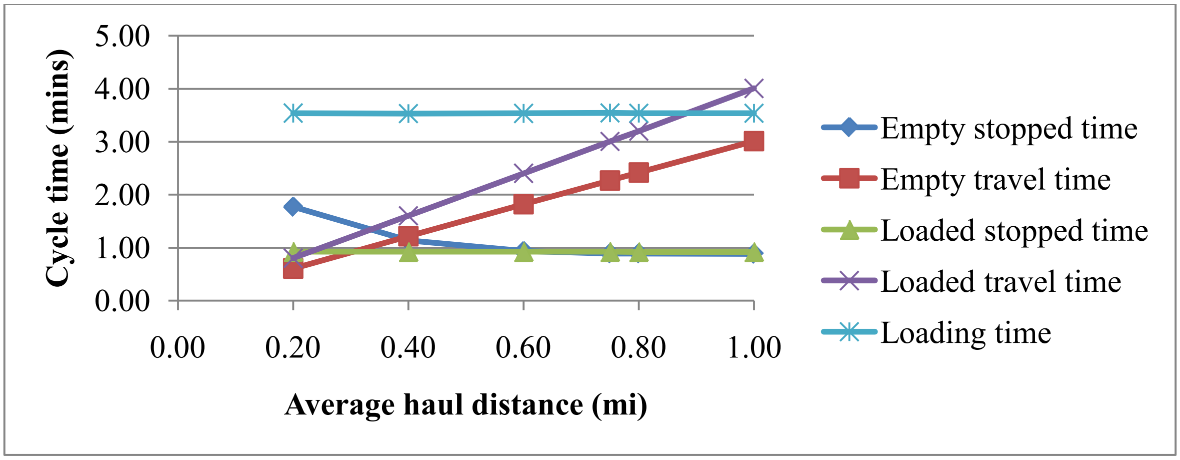

Fig 19. Variation in Truck Cycle Time Components with Haul Distance

- Eqs. (2) and (3) are valid fuel consumption models for shovel loading and truck haulage, respectively.
- Valid stochastic process models of truck-and-shovel operations have been formulated to study fuel efficiency.
- For the study mine, the following strategies, in decreasing order of impact, provide the most improvement in energy efficiency for truck-and-shovel overburden removal: (i) shorten haul road lengths while maintaining similar haul road grades and dozer push distances; (ii) increase shovel capacity by using next size model (Hitachi EX2500); and (iii) increase shovel utilization by adding one more truck. While adding one more truck actually results in 1.5 and 1.3% decreases in fuel efficiency, for 777 and 785 trucks, respectively, this is compensated for by 4,400 and 5,700 tons/shift increases in production, and by 19.53 and 23.26% increases in shovel utilization.
- The effect of operators cannot be adequately described without additional data.

8. Acknowledgements

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Alignment of Short-Term and Operational Plans using Discrete Event Simulation

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Abstract

Optimal resource allocation is an important aspect in many industries. In the mining industry, resource allocation usually refers to the truck-and-shovel allocation. Since truck-and-shovel technology has been widely used in open-pit mining operations, the efficiency of trucks-and-shovel mining systems is an essential issue in mining operations. Truck and shovel operations are main contributors to mine operating costs. Therefore, determining the optimum number of trucks and shovels is an important and complex process, which can result in reducing the overall mining cost. This paper analyzes the operations of trucks and shovels in an open pit mine using ARENA simulation software.

1. Introduction

Open pit mining is widely used for large deposits. Because of the size of open pit mines, the revenue and the costs of such large projects are enormous. Most of the time, revenue cannot be controlled due to unpredictable behavior of the final price of the product, which in metal mining is usually a metal concentrate. On the other hand, one of the major costs for an open pit mine is the number of trucks and shovels and the reallocation strategy of the mining fleet. A small percentage reduction in costs involved with trucks and shovels would have a significant impact on the profit of a project.

Yuriy and Vayenas (2008) developed a reliability assessment model based on a genetic algorithm to evaluate and generate the time between trucks' failures. The output of the model is used as an input to a discrete event simulation model to analyze the impact of the failures on the production. Two different simulation software packages are used to compare the merits of them. Fioroni et al. (2008) presented a two stage method in which the first step is a mathematical programming model applied to allocate the shovels and the trucks. In the second stage, simulation is used to assess the results in real time operations.

In this paper, a discrete event simulation methodology is proposed and Arena Software is used for modeling (Rockwell Automation Technologies, 2009). In the first stage, various scenarios with different number of trucks and shovels are examined to determine the appropriate number of each resource. Then, the uncertainty in velocity of empty and full trucks, movement speed of shovel, loading time, and dumping time are taken into account. The results show the effect of such uncertainties on the extracted tonnage of different material types and grades of elements of interest.

2. Problem definition

In the first stage, this paper presents the problem of determining the number of shovels and trucks to be employed at the beginning of the mine plan. The only constraint considered in this part is meeting the extraction schedule in each period. The decision on the number of trucks and shovels is made based on criteria such as shovel utilization and truck utilization in different scenarios.

Using the results from the previous stage, the effect of uncertainties on trucks and shovels operations are evaluated in the second stage. These uncertainties consist of full truck movement speed, empty truck movement speed, shovel movement speed, loading time, dump time, and load tonnage. Variables of interest which are assessed for multiple realizations includes: total ore tonnage dumped at the mill, total ore tonnage dumped at the stockpile, total waste tonnage, P grade, S grade, MWT grade at the mill and stockpile, and resource utilizations.

The extraction plan considered consists of 12 time periods. The extraction schedule provides information about the number of blocks to be extracted and their sequence of extraction, block type, tonnage, coordinates, extraction fraction, and grade of different materials. The type of each block specifies its destination either as process area or waste dump. Distances between blocks and different destinations are also provided. Based on the information of each period, a shovel travels to the first block which is at the top of the sequence. It takes some time for the shovel to travel from its current location to the block's location. Simultaneously, a truck travels from its current location to the block's location where the shovel is going. The shovel starts its work to extract a portion of the block and load it into the truck. The tonnage which the shovel can extract at each time is not constant and is determined based on a random distribution. The truck stays by the working shovel until it is fully loaded. Knowing the type the block which is extracted, the truck travels to either the process area or the waste dump. At the same time, another truck travels to the shovel to be loaded. The shovel moves to the next block right after the current block is completely depleted.

On the other side of the mine, trucks unload their material at the waste dump or the processing plant. At the processing plant, based on the MWT grade, trucks go to the mill or the stockpile. There is no limit on the number of trucks which can unload at one destination, either at the mill or the waste dump, but there is a limitation on the number of trucks that can dump their material into the crusher simultaneously. There is space for only one truck at the crusher to dump at a time. Fig. 1 shows a schematic view of the problem. The next section introduces the logic of the simulation model which is modeled in ARENA.

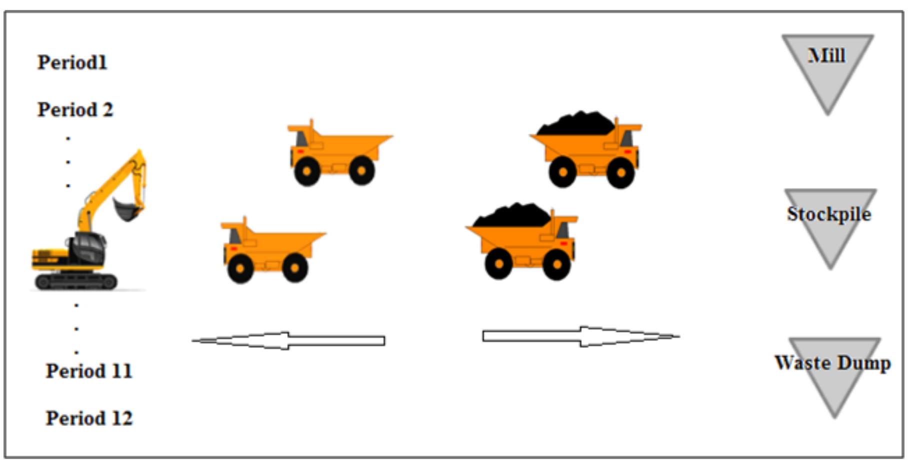

Fig. 1. Schematic view of the problem

3. Methodology

For modeling a problem in ARENA simulation software, the first step is to define appropriate entities. In the model built in this study, entities are block portions for each period. For simplification, each block portion is considered as an entity. At the beginning of the model, entities are introduced to the system at small time intervals. As an entity arrives to the system its attributes are assigned, consisting of the entity's number, coordinates of x, y and z, block tonnage, fraction, destination, distance to destination, and grades. Each entity's tonnage is calculated using Eq. (1).

$$entity\,tonnage = origin\,block\,tonnage \cdot fraction \tag{1}$$

At this stage, variables such as total waste tonnage and ore tonnage entering the system and the initial coordinates of the shovels' locations are calculated and assigned. Also, coordinates of the mill, stockpile, and waste dump are defined. Blocks entering the system wait in a queue to seize a shovel and a truck as resources. Once they seize a shovel and a truck, the shovel and the truck travel from their current locations to the block's location. Speeds of the shovel and the truck follow a Normal distribution with a small deviation (Eq. (2) and Eq. (3)). The time that it takes for the shovel and the truck to reach the block is the maximum of Eq. (4) and Eq. (5).

$$speed_{shovel} \approx Normal(100,50) \;\; (m/min) \tag{2}$$

$$speed_{empty\,truck} \approx Normal(500,20) \;\; (m/min) \tag{3}$$

$$time_{shovel\,travels\,to\,the\,block} = \frac{distance_{shovel-block}}{speed_{shovel}} \;\; (min) \tag{4}$$

$$time_{truck\,travels\,to\,the\,block} = \frac{distance_{truck-block}}{speed_{empty\,truck}} \;\; (min) \tag{5}$$

Once both truck and shovel reach the block, the extraction begins and it takes a time of Eq. (6) to load the truck each time. The truck's nominal capacity is 200 tonnes and is fully loaded by 2 loads.

$$time_{loading} \approx Triangular(0.67, 0.80, 0.91) \;\; (min) \tag{6}$$

The tonnage that the shovel can load follows Triangular distribution (80, 90, 100) tonnes. If the remaining tonnage of the entity is less than 80 tonnes, the remaining amount will be loaded to the truck. After each load, three variables are updated: (1) the remaining entity tonnage, (2) the number of loads on the current truck, and (3) the tonnage loaded to the truck. In this part of the model a decision must be made according to three situations as follows:

- If the block is completely extracted, the shovel working at this block is released and the truck working with the shovel travels to its destination. When a shovel is released it moves from its current location, which is the extracted block's location, to the next block's location and new coordinates are assigned to the shovel.
- If the entity is not completely extracted but the truck is fully loaded, the model duplicates the entity to 2 entities: block and load. The block entity which is the original entity seizes a new truck and the load entity representing the full truck will go to its destination.
- If the entity is not completely extracted and the truck is not fully loaded, the shovel and the truck remain at their current locations and continue the extraction and loading process.

When a truck decides to leave the block, a decision must be made about its destination. If a truck's load is ore, it goes to the processing plant and if it is waste, it goes to the waste dump. Trucks going to the processing plant can dump at either the mill or the stockpile. If the average MWT grade of the coming truck is less than 70%, it dumps at the low grade stockpile, otherwise it dumps at the mill. There is no limit on the number of trucks that can dump at the waste dump or the stockpile at the same time. The traveling time of the truck is based on its speed when it is loaded (Eq. (7)) and the distance between the block and its destination. Distance between each block and all possible destinations (mill or stockpile or waste dump) are provided as inputs to the model. Eq. (8), Eq. (9) and Eq. (10) indicate the traveling time of the full truck based on its destination.

$$speed_{full\,truck} \approx Normal(300,100)\ (m/min) \tag{7}$$

$$time_{truck\,travels\,to\,the\,destination} = \frac{distance_{block-destination}}{speed_{full\,truck}} \quad (min) \tag{8}$$

$$time_{truck\,travels\,to\,the\,stockpile} = \frac{distance_{block-stockpile}}{speed_{full\,truck}} \quad (min) \tag{9}$$

$$time_{truck\,travels\,to\,the\,waste\,dump} = \frac{distance_{block-waste\,dump}}{speed_{full\,truck}} \quad (min) \tag{10}$$

After dumping, the truck returns to the shovel and they wait to be seized by the next entity. The returning time is calculated using Eq. (8) to Eq. (10), with the only difference that the truck is empty, so Eq. (3) is used as the denominator. After returning, the truck is released and is ready to be seized by the next coming block.

The model keeps track of some variables of interest during the run. These variables consist of ore tonnage at the mill, ore tonnage at the stockpile, waste tonnage at the waste dump, average P grade at the mill, average P grade at the stockpile, average S grade at the mill, average S grade at the stockpile, average MWT grade at the mill, average MWT grade at the stockpile, average shovel utilization, and average truck utilization.

At the end of the simulation time the model creates a dummy entity to record these variables and write them to an output EXCEL file. Fig. 2 shows the flowchart of the model.

4. Implementation and results

As explained in the previous sections, each fraction of a block is an entity. We used the output of WITTLE software as an input data for our model. The model reads the production schedule data of an iron ore mine. The input data include block tonnage, block fraction extracted, destination, coordinates, P grade, S grade, MWT grade, distance to the mill, distance to the waste dump, and random distributions from the excel file for each period. The entire time horizon consists of 13 years. The yearly production schedule of the 13 periods is shown in Fig. 3.

As mentioned before, in the first stage the problem is to determine the optimal number of trucks and shovels. In the model proposed in this paper, shovels and trucks are modeled as resources. Different scenarios are produced with different number of trucks and shovels, with year 7 selected to examine different scenarios. ARENA Process Analyzer is used to run the scenarios and the results are demonstrated in Table 1.

Starting with 1 shovel and 3 trucks, in some periods there are some blocks which are not extracted completely. By increasing the number of trucks, the remaining tonnage in each period decreases but then stabilizes (at that level). After this point, further increasing the number of trucks reduces utilization of resources and does not affect the extracted tonnage.

It is inferred that the number of shovels must increase. Scenario number 12 with 4 shovels and 10 trucks seems to be the best scenario. In this scenario all waste and ore are completely extracted and utilization of resources is reasonable. After this point, increasing the number of trucks or shovels is not going to change the level of production (see Fig. 4).

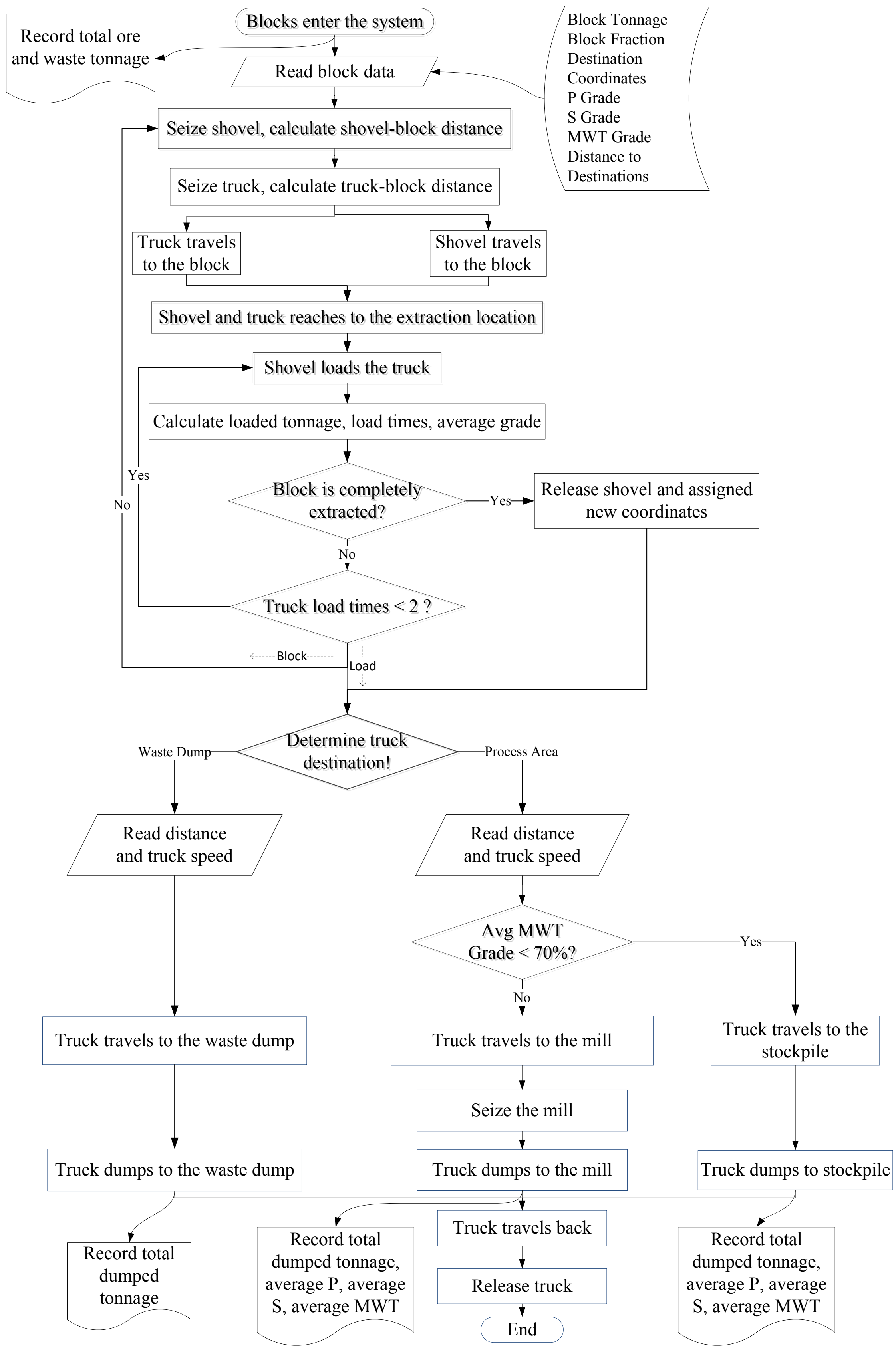

Fig. 2. Flowchart of the model

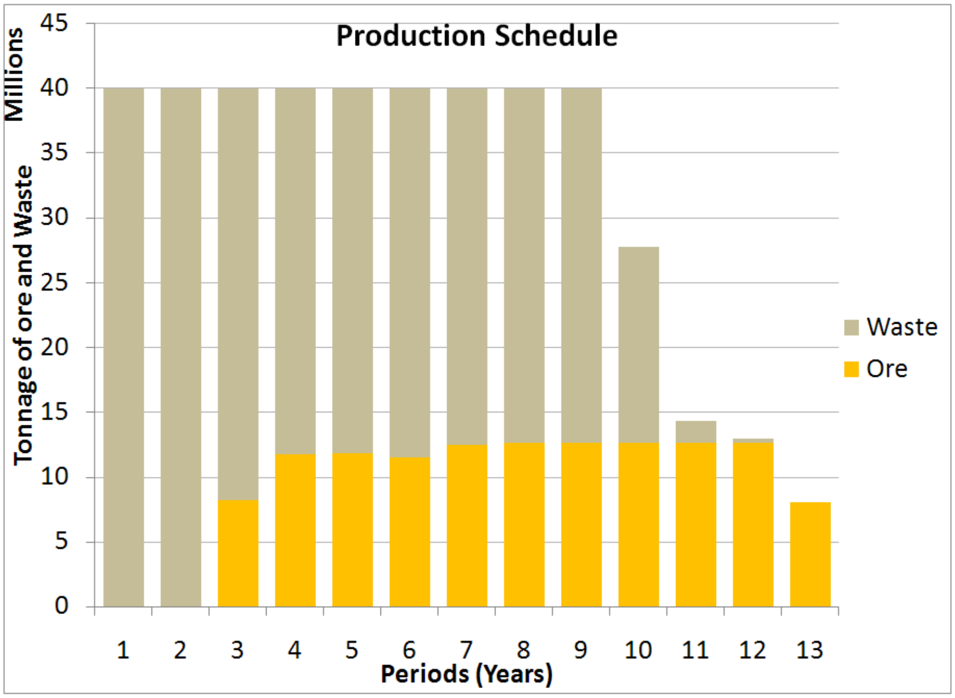

Fig. 3. Production schedule as input data

Table 1. Results of different scenarios for year 7

Scenario ID	Shovel #	Trucks #	Extracted ore (tonnes)	Extracted Waste (tonnes)	Shovel Utl.	Truck Utl.
1	1	3	10147861	2656537	0.903	0.7431
2	1	6	10460710	3753151	0.9997	0.4123
3	2	4	12487312	6656762	0.7721	0.9049
4	2	6	12487312	6584494	0.9311	0.7752
5	3	3	12483208	4897333	0.4455	1
6	3	6	12487312	18068258	0.8486	0.8953
7	3	9	12487312	23475918	0.9753	0.7017
8	3	11	12487312	24642616	0.9952	0.5955
9	3	13	12487312	25637419	0.9974	0.5771
10	4	6	12445212	15059571	0.7872	0.9567
11	4	8	12487312	23715803	0.8119	0.9446
12	4	10	12487312	27512677	0.8616	0.7306
13	4	16	12487312	27497624	0.9161	0.5318
14	4	19	12453528	24971572	1	0.4102
15	5	8	12452663	25489468	0.8024	0.9613
16	5	10	12460822	27512677	0.8808	0.8235
17	5	13	12483268	27512677	0.9019	0.6362
18	5	16	12487312	27512677	0.7252	0.4709

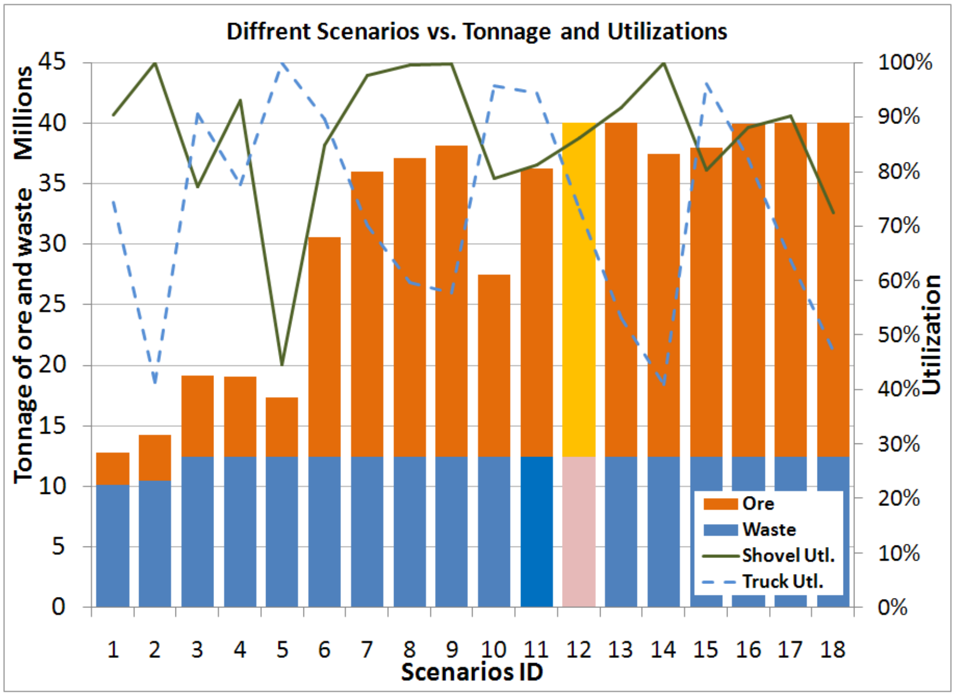

Fig. 4. Illustrative results of different scenarios

Using the fixed number of resources determined in the previous stage, the effect of uncertainties is studied during the second stage. These uncertainties include the variations in speed of loaded truck, speed of unloaded truck, speed of shovel, loading time, dump time, and extraction tonnage. These uncertain parameters result in variations in outputs. The model is run for 100 328-day replications.

Fig. 5 shows extracted ore tonnage going to the mill or to the stockpile, and extracted waste tonnage. For each period, box plots are used to show the variations in the waste tonnage going to the waste dump (Fig. 5), average shovel utilization (Fig. 6), and average truck utilization (Fig. 7) over 100 replications. The top and the bottom of the box represent the upper quartile (75th percentile) and the lower quartile (25th percentile) respectively. The line in the middle of the box shows the median (50th percentile). Ends of the upper and lower whiskers indicate the maximum and minimum values respectively. Summary of the related statistics are given in Table 2, Table 3 and Table 4. As an instance, the histogram of average truck utilization for 100 realizations is sketched in period 7 (Fig. 8).

Fig. 9 through Fig. 14, respectively, show the following results: average P grade at the mill, average S grade at the mill, average MWT grade at the mill, average P grade at the stockpile, average S grade at the stockpile, and average MWT grade at the stockpile.

5. Conclusions and future work

This paper presented a simulation model to assess the operational plans in the mine by taking into account the uncertainties associated with the operation of trucks and shovels. In further research the failure of trucks, shovels, mill, and their regular service times should be considered in long term planning. In order to achieve a more precise model, different number of stockpiles with various grade limitations should be used. To extend the model much further, some complex processes after extraction can be added to the model. Results from mathematical programming models for allocating the trucks and shovels can be used as an input to the simulation model. Consequently, an interactive approach between the simulation model and the mathematical programming model can be studied.

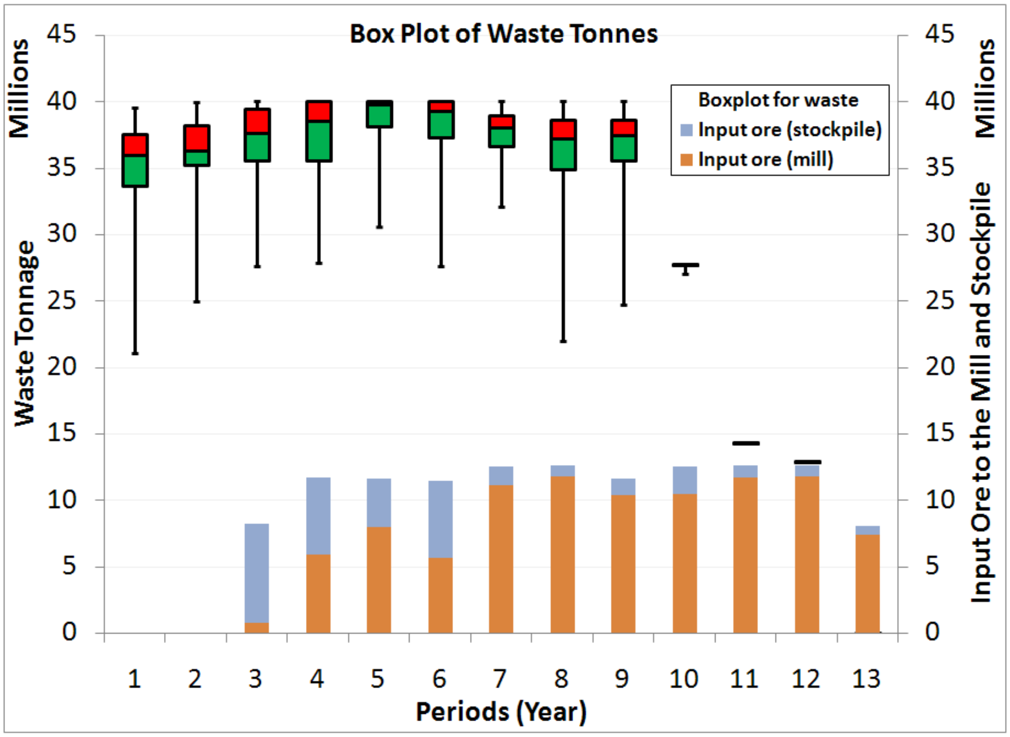

Fig. 5. Material tonnage going to the mill, stockpile and waste dump

[1]Table 2. Summary statistics of waste tonnage going to the waste dump (over 100 realizations)

Period	1	2	3	4	5	6	7	8	9	10	11	12	13
Minimum	21.1	25.0	19.4	16.2	18.7	16.1	19.6	9.4	12.1	14.4	1.7	0.3	0.0
Quartile 1	33.7	35.2	27.4	23.9	26.3	25.8	24.1	22.3	23.0	15.2	1.7	0.3	0.0
Meadian	35.9	36.3	29.4	26.9	27.9	27.8	25.6	24.6	24.8	15.2	1.7	0.3	0.0
Quartile 2	37.5	38.2	31.2	28.3	28.2	28.5	26.5	26.0	26.0	15.2	1.7	0.3	0.0
Maximum	39.6	39.9	31.8	28.3	28.2	28.5	27.5	27.4	27.4	15.2	1.7	0.3	0.0
Average	35.4	36.1	28.8	25.8	26.9	26.6	25.1	23.8	24.3	15.1	1.7	0.3	0.0
Standard deviation	2.8	2.6	3.0	2.9	2.0	2.6	2.0	3.1	2.5	0.1	0.0	0.0	0.0

[1] Values in Table 2 are in millions.

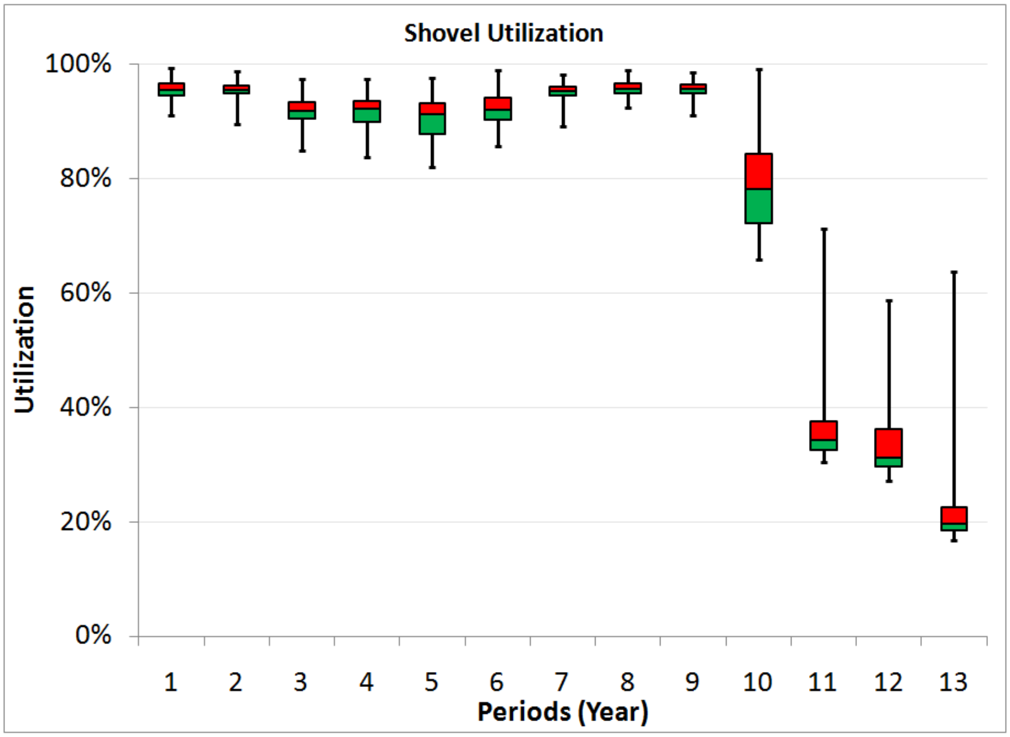

Fig. 6. Shovel utilization

Table 3. Summary statistics of average shovel utilization (over 100 realizations)

Period	1	2	3	4	5	6	7	8	9	10	11	12	13
Minimum	0.91	0.90	0.85	0.84	0.82	0.86	0.89	0.92	0.91	0.66	0.30	0.27	0.17
Quartile 1	0.94	0.95	0.90	0.90	0.88	0.90	0.95	0.95	0.95	0.72	0.33	0.30	0.19
Meadian	0.96	0.95	0.92	0.92	0.91	0.92	0.95	0.96	0.96	0.78	0.34	0.31	0.20
Quartile 2	0.97	0.96	0.93	0.94	0.93	0.94	0.96	0.97	0.96	0.84	0.37	0.36	0.23
Maximum	0.99	0.99	0.97	0.97	0.98	0.99	0.98	0.99	0.99	0.99	0.71	0.59	0.64
Average	0.95	0.95	0.92	0.92	0.90	0.92	0.95	0.96	0.96	0.79	0.36	0.34	0.22
Standard deviation	0.02	0.02	0.03	0.03	0.04	0.03	0.01	0.01	0.01	0.09	0.07	0.07	0.07

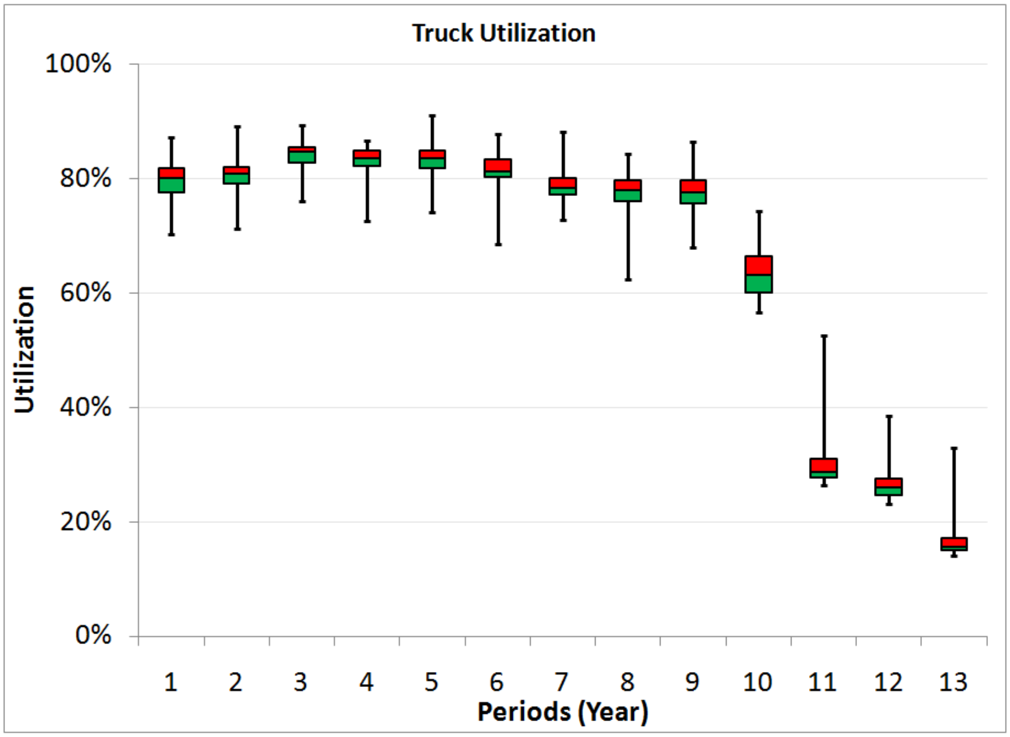

Fig. 7. Truck utilization

Table 4. Summary statistics of average truck utilization (over 100 realizations)

Period	1	2	3	4	5	6	7	8	9	10	11	12	13
Minimum	0.70	0.71	0.76	0.73	0.74	0.69	0.73	0.62	0.68	0.57	0.26	0.23	0.14
Quartile 1	0.78	0.79	0.83	0.82	0.82	0.80	0.77	0.76	0.76	0.60	0.28	0.25	0.15
Meadian	0.80	0.81	0.85	0.84	0.84	0.81	0.78	0.78	0.78	0.63	0.29	0.26	0.16
Quartile 2	0.82	0.82	0.86	0.85	0.85	0.83	0.80	0.80	0.80	0.66	0.31	0.28	0.17
Maximum	0.87	0.89	0.89	0.87	0.91	0.88	0.88	0.84	0.86	0.74	0.53	0.39	0.33
Average	0.80	0.81	0.84	0.83	0.83	0.81	0.79	0.78	0.78	0.64	0.30	0.27	0.17
Standard deviation	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.03	0.03

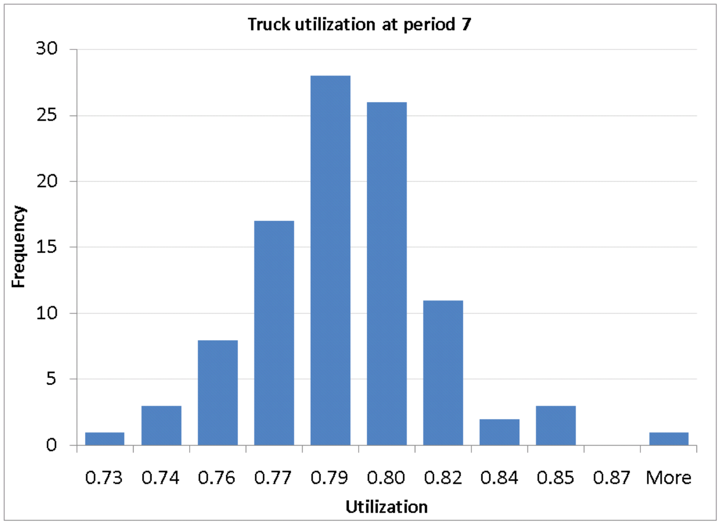

Fig. 8. Histogram of average truck utilization at period 7 over 100 replications

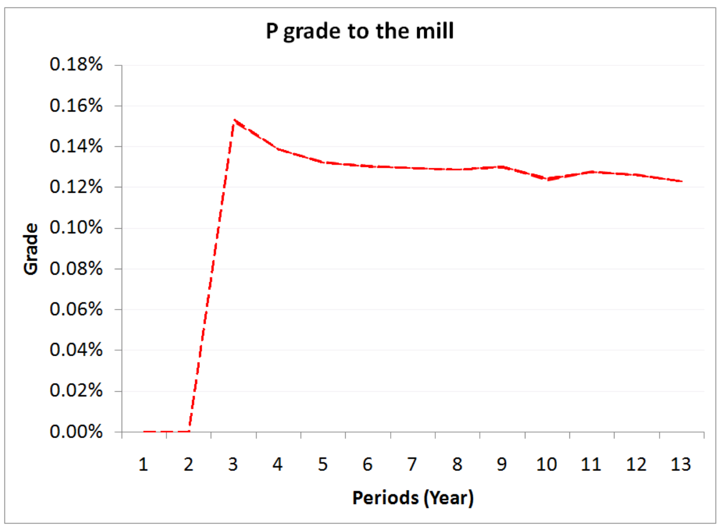

Fig. 9. Average P grade of material going to the mill

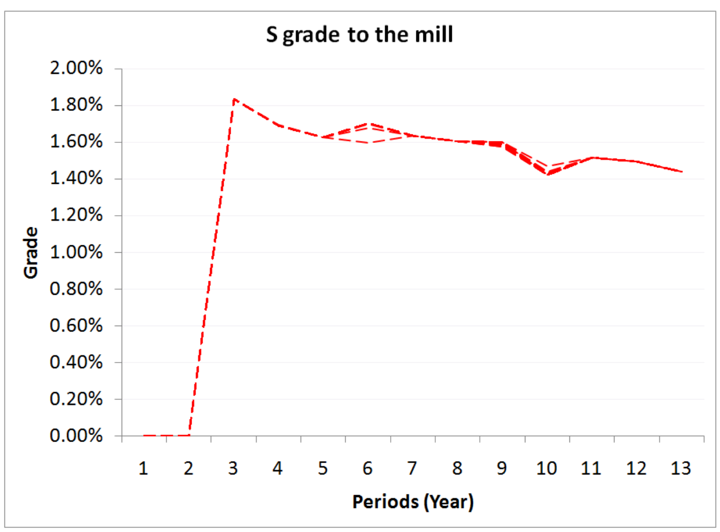

Fig. 10. Average S grade of material going to the mill

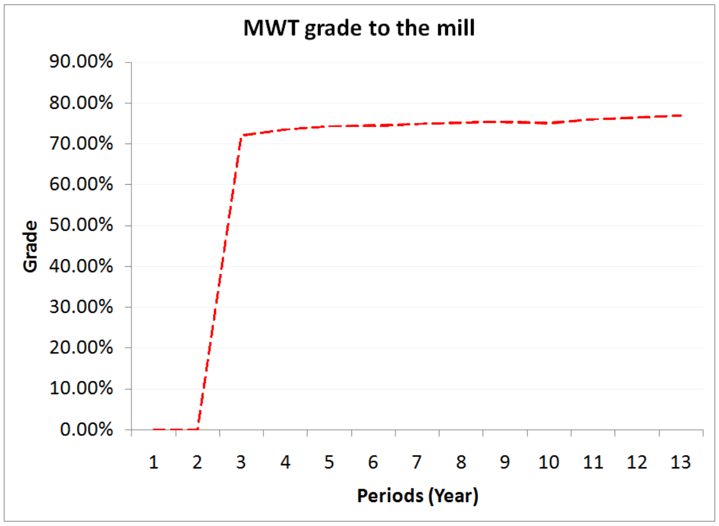

Fig. 11. Average MWT grade of material going to the mill

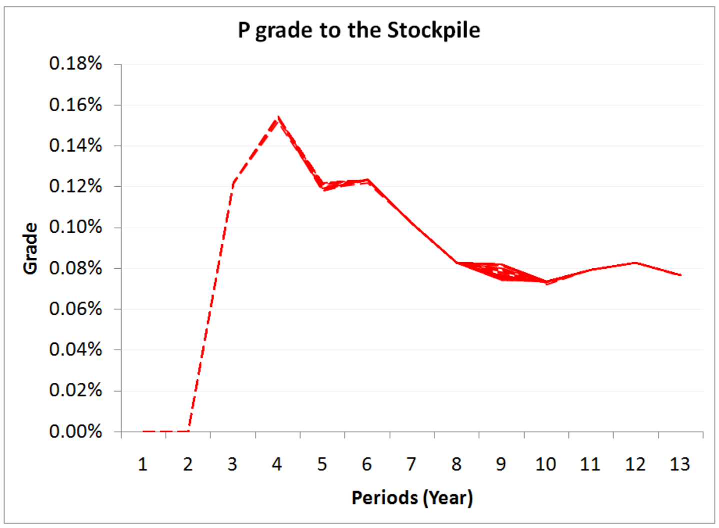

Fig. 12. Average P grade of material going to the stockpile

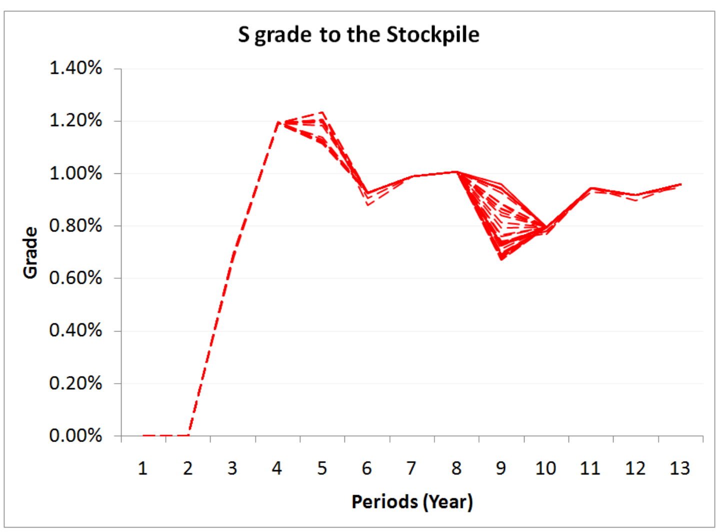

Fig. 13. Average S grade of material going to the stockpile

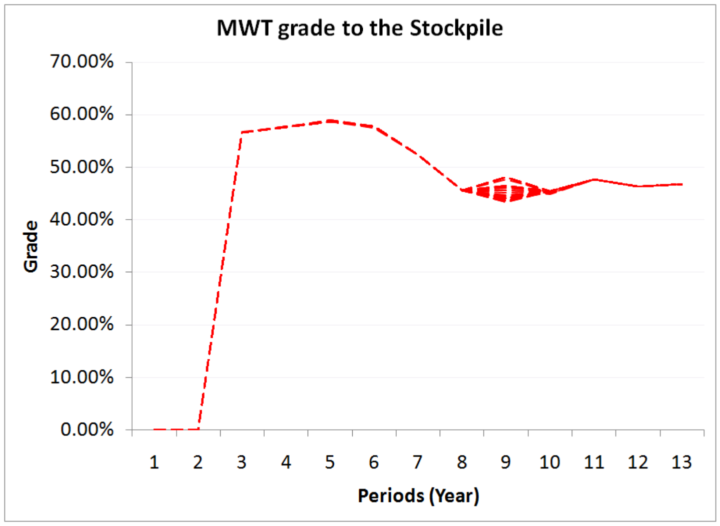

Fig. 14. Average MWT grade of material going to the stockpile

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[2] Rockwell Automation Technologies, Inc. (2009). Arena (Version 13.00.00000 - CPR 9).

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Discrete Event Simulation of an Iron Ore Milling Process

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Abstract

One of the most common techniques for studying a system's behaviour, predicting its outputs and anticipating challenges along the way is simulation. This is a very powerful technique, especially when uncertainty and time-varying parameters are involved. Numerous simulation studies have been conducted on the material processing plants. Simulation and modelling of mineral processing systems focuses on design and optimization of circuits and machine performances. The focus of this project is on simulating the interactions between interior components of a plant using a discrete event approach. An ordinary iron ore processing plant, with several comminution and separation stages, is considered for simulation. The system here involves a continuous process, but it is supposed to be modeled as discrete events. One way is to use the batching approach and consider one hour's worth of material fed into the plant (or that of any other time period) as an entity flowing through the model. The model is developed in Arena simulation software. The goal is to get an understanding on the quality of the output and the effects of the uncertainty in input parameters and operations on it. Therefore, recoveries, processing times, capacities and etc. are selected based on the authors' experience, in an effort to mimic a real plant while avoiding complications.

1. Introduction

Mining is the process of extracting raw materials from the ground in a profitable manner. It usually consists of five major steps: prospecting and exploration, development, exploitation, mineral processing and reclamation. The oldest known mine dates back to 43,000 years ago in southern Africa, where pre-historic humans started to extract iron from a near surface mine (Newman et al., 2010). Since then people have been looking for mines and extracting material from them. A wide range of extraction procedures such as truck-shovel systems, underground mining, block caving and continuous bucket wheel excavations have been developed and implemented all over the world. The most common and best-known technique is the truck-shovel operation used in open pit mines. This technique is also used for extracting iron ore; shovels dig into the ground and dump the material onto the back of the trucks. Rocks are then delivered to plants where further processing activities such as grinding, classification, purification and concentration are done and the material becomes ready to be sold in the market. The purified product is then shipped to various destinations but mostly used for making steel.

The main goal of any mining operation is to maximize profits. The processing plant is the site where the mineral dressing or ore dressing, which is the process of separating commercially valuable minerals from the ore, is completed. Customers who wish to buy the product have a set of requirements for its properties, such as minimum metal grade, maximum deleterious material and even the tonnage of concentrate needed to be delivered on time. On the other hand, increasing the concentrate metal grade and decreasing deleterious content leads to larger amounts of tailings and lost metal, as well as to higher costs. Finding the trade-off, which enables the company to make the highest level of benefit and managing the mineral processing plant such that the desired output and costs are achieved, is a complex and challenging activity. Therefore, studying the plant is of great value. Another property of a processing plant is the variety of machines involved in concentrating the product. All of these machines impose uncertainty because of their operations characteristics as well as their failures. The uncertainty in characteristics of the rock delivered to the plant also increases the process uncertainties and makes things even more complicated.

One of the common techniques for studying a system's behaviour and predicting its outputs and challenges on the way is simulation. This is a very powerful technique, especially when uncertainty and time-varying parameters are involved. There are some fairly comprehensive software packages, such as MODSIM (Mineral Technologies International), USIM PAC (BRGM) and JKSimMet (JKTech), which nowadays are used to simulate mineral processing plants. These software packages are based on predefined models, which predict a plant's outputs for the specific input data range. The models need to be calibrated on a case-by-case basis. The outputs need verification and validation because of the complex nature of the ore and some unique characteristics of each mine.

This project aims at modeling and simulating the behaviour of an iron ore magnetic separation plant using discrete event simulation. The goal is to gain an understanding of the quality of the output and the effects of the uncertainty in input parameters on the output variables. A brief review of the previous work done on mineral processing simulation is presented in the next section. The third section defines the system of interest for modeling. The proposed approach and the modeling procedure are described in fourth and fifth sections respectively. The verification process is then presented and the output results of the model on synthetic data are presented. The conclusion and future work are discussed in the final section.

2. Literature review

Several simulation studies have been conducted on mineral processing plants. Simulation and modeling of mineral processing systems focuses on the design and optimization of circuits and machine performance (Lynch and Morrison, 1999). The first studies were intended to propose equations representing relations between various parameters of the system. The concepts of modeling and simulation for mineral processing were introduced after the 1960s. Developments in the capabilities of computers in the 1980s helped researchers conduct vast studies on the models and observe deficiencies of the models, through the use of computer programs and simulations (Lynch and Morrison, 1999). These studies can be categorized into two main groups: standalone machine simulations and plant simulations. Most of the simulation studies belong to the first group, in which researchers try to mimic the behaviour of a specific machine based on experiments and computer simulations. The idea behind the second group of studies is to analyze the plant as a single system consisting of various machines and investigate the interactions among components of the system. However, one may only be interested in studying the whole plant as a system and investigating the relationships between inputs, outputs and operating conditions of the plant instead of a machine by machine study. The focus of this project is on simulating the interactions between interior components of a plant.

Standalone machine studies have been conducted widely on grinding machines, separators, classifiers, etc. Austin et al. (2007) simulate wet ball milling of iron ore using laboratory scale tests. Another study conducted by Wang et al. (2009) investigates the grinding process within vertical roller mills. Pothina et al. (2007) propose a model to relate impact parameters to energy consumption in gyratory crushers. Dlamini et al. (2005) simulate the hydrocyclone to obtain physically realistic velocity and pressure profiles. Morrell and Man (1997) use computer simulation as well as existing plant data to design full-scale ball mill circuits. Sosa-Blanco et al. (2000) develop a simulation model for tuning a grinding circuit with the objective of optimizing a flotation plant. Another simulation of a grinding plant is conducted in Duarte et al. (2002), in which the authors use simulation to compare five control strategies in a copper grinding plant. A simulation study on the control parameters of flotation columns can be found in Bergh and Yianatos (1995). For a complete review of models for column flotation, the reader can consult the study by Bouchard et al. (2009).

One of the first simulations of the ore processing plant was done by Ford and King (1984). De Andrade, Lima and Hodouin (2006) performed another simulation study, which falls into the second category. The authors of this paper simulate cyanide distribution in a gold leaching circuit. A simulation design which treats the processing plant as a whole and suggests an approach for measuring and managing variations in a mineral processing plant is proposed in Robinson (2003). Fourie (2007) also proposes a modeling approach for studying any metal separation circuit (flotation, magnetic separation or electrostatic separation). Delgadillo et al. (2008) integrate the grinding machines along with the classifiers and magnetic separators and simulate the combination in a magnetite plant.

3. System definition

In this study, a whole processing plant simulation modeling approach is followed. The study focuses on modeling an iron ore magnetic separation processing plant in Arena discrete event simulation software (Rockwell Automation). A typical iron ore processing plant, with several comminution and separation stages, is considered for simulation. The flow sheet of the process is illustrated in Fig 1.

The plant receives run-of-mine (ROM) as trucks dump loads with specific tonnage and known metal grade into the primary crusher (in this case a Gyratory crusher); the crushed materials are carried to the stockpile through feeders and some conveyer belts. The plant's main stockpile serves as ore feed storage for all process stages.

In the next stage, ore with a known tonnage rate per hour is fed to the size reduction section. The final output of this section is two streams of ore with restricted particle size distributions (one in the range of 20 to +10 mm and the other in the range of -10 mm) and a tailing stream with particles finer than 3 mm (which is supposed to have a lower iron content and higher sulfur and phosphor grades). An Auto Genius mill and a secondary crusher are considered in this area.

Ore coming from the size reduction section is fed to the dry low magnetic separator (LMS) and dry high magnetic separator (HMS) in the order depicted in Fig 1. The final output of the dry separation section is a mixed concentrate, which is fed to the wet plant, and a tail material which is sent to the dry tailings dump.

At the beginning of the wet separation area (mixing upgraded ore with water), a closed circuit grinding mill with hydrocyclone is included to grind the material down to -2.5 mm in order to achieve higher degree of freedom of materials. The ore is divided into a higher grade material and a wet tail in wet HMS, which is disposed of in the wet tail dam. The final concentrate of the process line is obtained after meeting one more size reduction stage in a ball mill (minus 1mm) and a wet LMS machine. Both the tail and the final concentrate may need to go through the thickener, the filter, the dryer and the pumping station before settling in their final points.

The approach of this study is to simulate and trace the material characteristics through the different stages of the processing plant, from the point that material is delivered to the plant from the mine (with trucks feeding the Gyratory crusher) to the four exit points of the process defined here. The main ore feed tonnage, its rock type and its respective three grades (Fe, S and P) can be defined as inputs of the system. The main parameters of importance in output streams of materials are the recoverable tonnage in each of four system exit points and the iron, sulfur and phosphor grade in each stream.

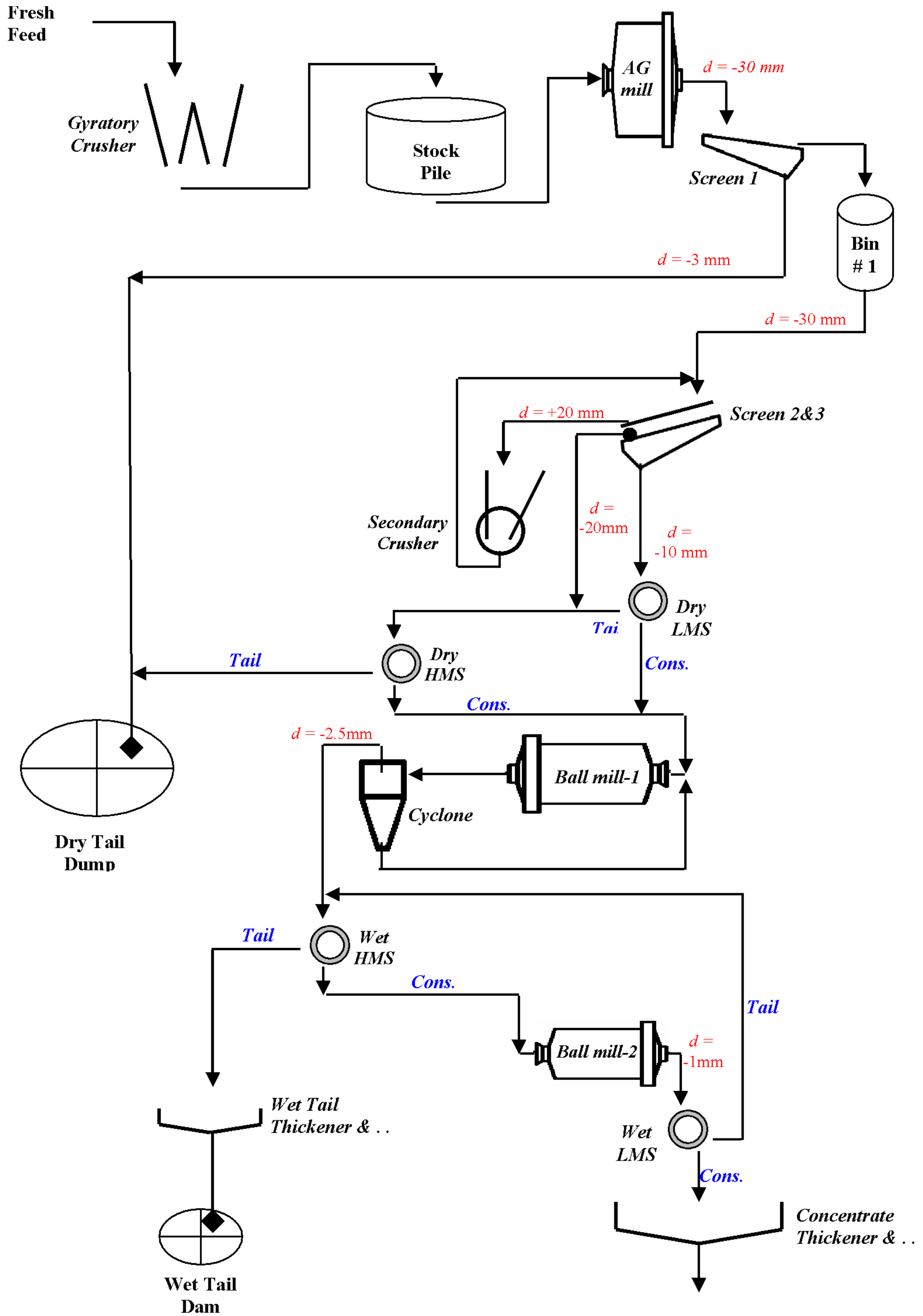

Fig 1.Hypothetical magnetic iron ore separation process flow sheet

4. Modeling approach

Among all the machines used in the processing plant, the ones which have direct effect on the recoveries and performance of the system are considered to be system modules in the simulation. The other facilities do not affect ore characteristics, but still have an effect on plant operations. So whenever it is possible, their positive or negative effects (such as failures, capacity restrictions, etc.) are added to the specifications of the corresponding main machine (e.g. conveyer belts).

The system here is a continuous process, but this project seeks to model it as discrete events. One way to do so is to use the batching approach as introduced in Lu et al. (2007), considering one hour's worth of material fed into the plant (or that of any other time period) as an entity flowing through the model. In order to make the simulation more realistic, the production of each part is stopped if any of the machines in that division fails, if the bin which feeds the plants gets empty, or if the bin at the end of the plant fills up.

Considering the process flow sheet and parameters of interest, facilities can be categorized into four main groups:

1. Storage bins and piles
2. Comminution machines
3. Classifiers
4. Separators

In the next part, the specifications of each group which are important from the simulation point of view are discussed.

4.1. Storage bins and piles

In all mineral processing plants there is a need to consider some storage areas as bins or stockpiles to keep 4-5 days of plant feed. These stockpiles/storage bins are used to store material between different stages of the processing plant in order to avoid unexpected shutdowns of the whole plant. Also, the presence of stockpiles/storage bins assure continuation of material flow in the downstream processes, when for any reason the upstream is shutdown for a short period of time. These storage bins can be considered to be shock absorbers of the processes.

Throughout this study, bins and stockpiles are modeled by tank module, to store ore material tonnage and grade and extract from it whenever process line needs feed material as representative entities. In order to keep track of the material's grades in each bin, the average weighting method is used, i.e. the material is blended in each bin and input batches are not recognizable among the outputs. The material content (level) of the tank (tonnage) is calculated based on the receiving entities tonnage (adding) and the exiting entities tonnage (subtracting). The tank grade (as a weighted average) of each species is defined by the following equation:

$$\left(AverageTankGrade\right)_i = \frac{\left(TankLevel \times TankGrade_i + EntityTonnage \times EntityGrade_i\right)}{\left(TankLevel + EntityTonnage\right)} \tag{1}$$

where i can be Fe, S or P. It is also possible to consider limited capacities for each bin, when defining them as tank modules in Arena to avoid the accumulation of material in the plant when the next processing step is out of order because of a fault. On the other hand, if a bin runs out of material, the next part of the plant has to be stopped until a certain level (tonnage) of material accumulates in the bin.

4.2. Comminution machines

Comminution machines play an important role in mineral processing plants. In fact, it is not possible to recover any mineral or metal from ore without comminuting it down to a proper size (reaching the acceptable degree of freedom). Two types of comminution facilities are considered in this flow sheet (Fig 1). First there are some crushers which are designed to deal with coarse particles of ore. The second type of size reduction facilities considered here are mills. Mills usually deal with finer feed size in comparison with crushers, and they also grind the material to much finer particle sizes.

Regardless of all designing and operational conditions, it is important to have an idea about two main parameters of size reduction machines in order to have a correct model in the Arena simulator and to achieve a reality-mimicking model. First, one should consider when and how frequently they are out of order. This can be used as the failure schedule in the model. The second important point related to these facilities is the particle size distribution (PSD) of the discharge material. The minerals' degree of freedom, which defines the recovery of metal and grade of the concentrate, is strongly related to the PSD of feed stream to a separator.

The discharge PSD of the materials is affected by various parameters in a size reduction machine. Some of the main affecting parameters are: rock hardness, mineral size and type, dominant comminution mechanism, machine operational condition and ball content, the ratio of ore to ball and the amount of water in the mill.

Various functions, models and procedures have been developed to describe the discharge PSD for any specific type of crusher or mill; they can be categorized into two main groups. The first category contains those which are determined based on experiments and correlations of the results to a logically proper model. The second group of models is proposed based on empirical functions. Here, too, the discharge PSD of several rock types fed to a special kind of size reduction machine is examined to determine some constant coefficients. None of these methods can predict the exact PSD of the comminution machine, but they can still obtain a reasonable PSD of the discharge material in each case.

In this study, as no experiment is performed, two famous predicting functions for discharge PSD of size reduction machines are considered: the Gaudin function and the Rosin-Rammler function. The Gaudin function is used for crushers, as in Eq. (2).

$$w = 100\left(\frac{d}{n}\right)^{m} \tag{2}$$

Where w is the weight cumulative percent of the particles with diameter of d or smaller size and n and m are model constants effecting the PSD range. For each machine, different n and m are defined according to the desired coarse particle size-.

Rosin-Rammler function is used for any of the mills discharge PSD. a and b are model constants. Considering the desired coarse particle size, different n and m are defined for each machine individually.

$$100 - w = 100 \times \exp\left(-\left(\frac{d}{a}\right)^{b}\right) \tag{3}$$

4.3. Classifiers

Classifiers are used to separate particles based on physical properties such as size, density, shape. Two types of classifiers, screens and hydrocyclones, are placed in the aforementioned plant flow sheet (Fig 1). Some of them merely separate or dispose a portion of materials, while some are placed so as to create a close grinding or crushing circuit. There are some advantages to using such a circuit, including lower energy consumption per ton of fragmented ore down to a specific size and lower fine particle (over ground particles) production in comparison with the equivalent open circuit fragmentation. Design quality also affects their industrial (actual) performance to a great extent. Some of the important parameters in classifier designing procedure are: dry or wet operation, feeding rate, shape and density of particles, proportion of open area (in screens) and slurry feed pressure (hydrocyclones).

From the simulation point of view, one needs to trace particle size range in feed, over flow, and under flow. Tonnage and grades of over flow and under flow streams should be traced as well. The mean time between failures (MTBF) and mean time between repairs (MTBR) for the classifiers must be taken into account in the simulation modeling based on the historical data. The MTBF and MTBR should be considered for the classifier itself, and for supplementary instruments such as belt conveyors, pumps, feeders and slurry tanks.

In the simulation model, output tonnages are determined based on feed tonnage and particle size distribution. Also, the PSD of the outputs of classifiers are determined based on the input feed PSD and linear interpolations. It is assumed that the grade of the ore in over flow and under flow of the classifier is not the same as the input feed. Therefore, using the R_g concept (to be discussed later) and considering each classifier as a separator of the metal, the metal content of each stream is determined.

4.4. Separators

Separators in mineral processing plants play a significant role in the mineral processing flow sheet design. For any kind of separator, the machine recovery and concentrate stream grade are two of the most important parameters that managers in plants are interested in knowing and controlling. In the assumed iron ore separation flow sheet (Fig 1), two types of magnetic separators in both operational conditions (wet and dry) are considered. High intensity magnetic separators (HMS) and low magnetic separators (LMS) are classified under the category of physical separators. As the category name suggests, in such separators we deal with inherent physical properties of minerals (magnetic property of iron minerals) much more than their chemical properties.

There are many parameters affecting magnetic separators recovery. The metal recovery of a magnetic separator (either low or high intensity) can be affected by various parameters such as particle size of material fed to the machine; mineral's degree of freedom; metal carrying (ore) type; magnetic intensity of the machine; physical and operational characteristics of the machine; bed thickness of material fed to machine in dry magnetic separators and solid content of slurry fed to machine in wet separators.

For modeling purposes it is necessary to define a function for metal recovery, and a function for determining what weight percentage of the materials should go to concentrate stream or tail stream. Logically, these functions should be defined based on grade, mineral type and particle size distribution of the feed. But since there are no experimental data available, an acceptable constant metal recovery (R_g) and concentration ratio (CR) for each separation machine is defined. At this step of simulation, it is possible to calculate all mass balance related parameters for each machine output stream.

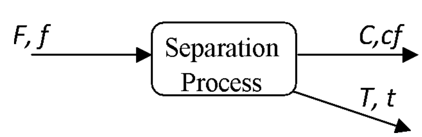

Fig 2.A schematic separator

Considering Fig 2, if F, C and T are defined as feed concentrate and tail tonnage, and f, c and t as feed concentrate and tail metal grades respectively, R_g and CR can be defined as in Eq. (4) and Eq. (5).

$$R_{g(i)} = \frac{Cc}{Ff} \tag{4}$$

$$CR = \frac{F}{C} \tag{5}$$

i stands for iron, sulfur or phosphor. Having $R_{g(i)}$ and CR, concentrate and tail parameters for all separators can be calculated using Eq. (6) to Eq. (9).

Concentrate tonnage: $C = \frac{F}{CR}$ (6)

Tail tonnage: $T = F - C$ (7)

Concentrate grade of species: $c_i = R_{g(i)} \times CR \times f_i$ (8)

Tail grade of species: $t_i = \frac{F \times \left(1 - R_{g(i)}\right) \times f_i}{T}$ (9)

5. Modeling

The model is developed in Arena simulation software (Rockwell Automation), version 13.00. The plant is separated into 4 divisions which are shown schematically in Fig 3 to Fig 6. Recoveries, processing times, capacities etc. are selected based on the authors' experience and are presented in Tables 1 to 4. All of the machines on the processing line are assumed to have the same capacity, which means that the processing time for each batch on each machine is calculated as the tonnage of the batch divided by the hourly production capacity of the line.

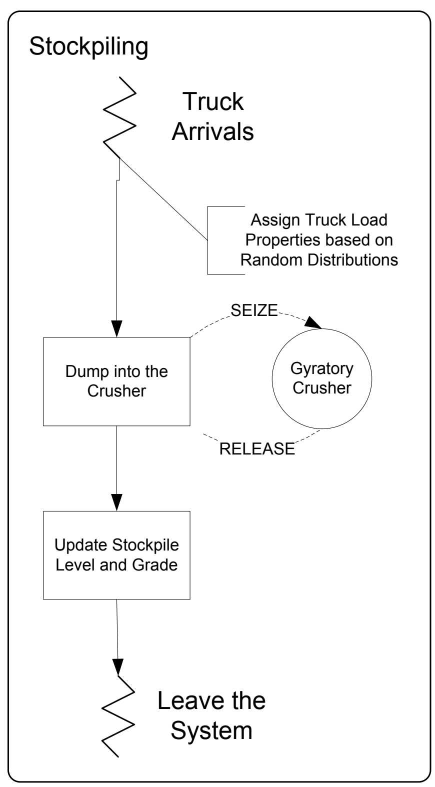

Fig 3.Truck Arrival Section

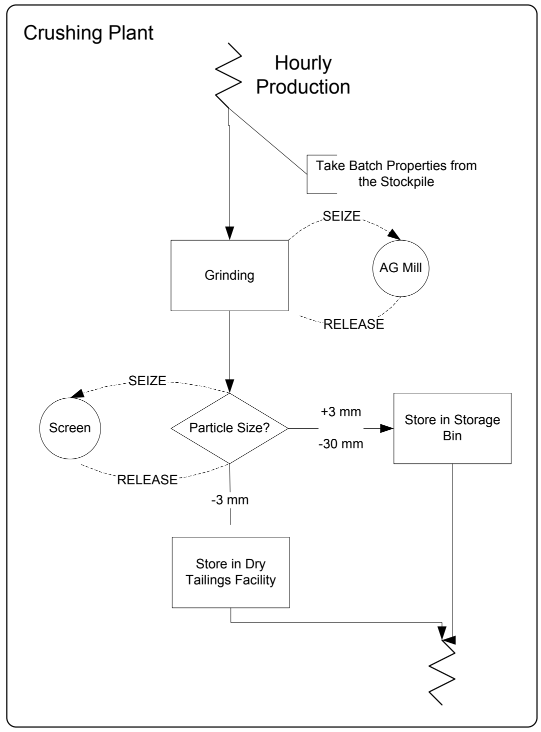

Fig 4.Crushing Plant

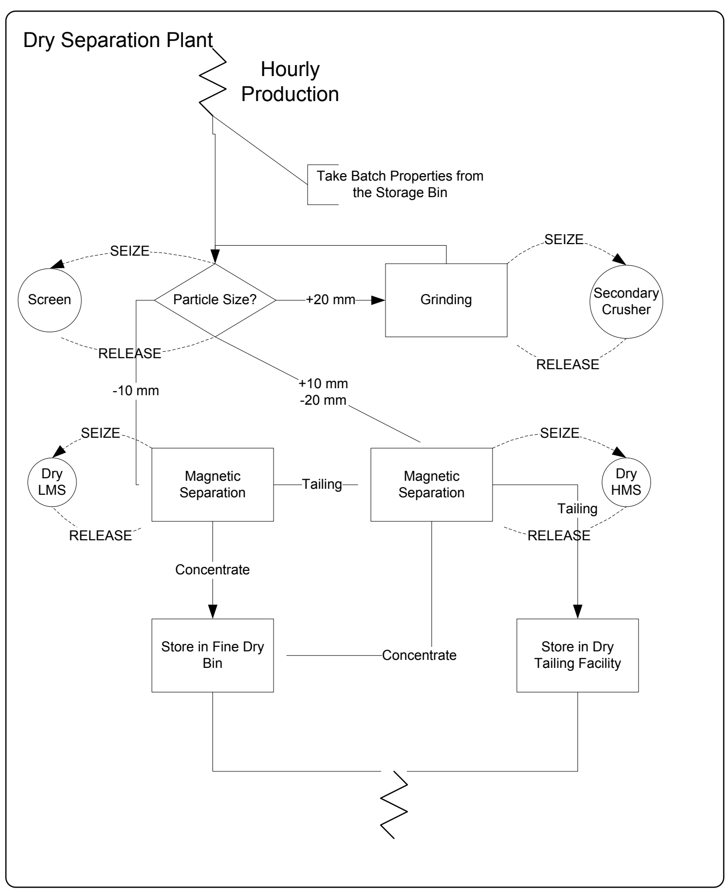

Fig 5.Dry Separation Plant Part 1

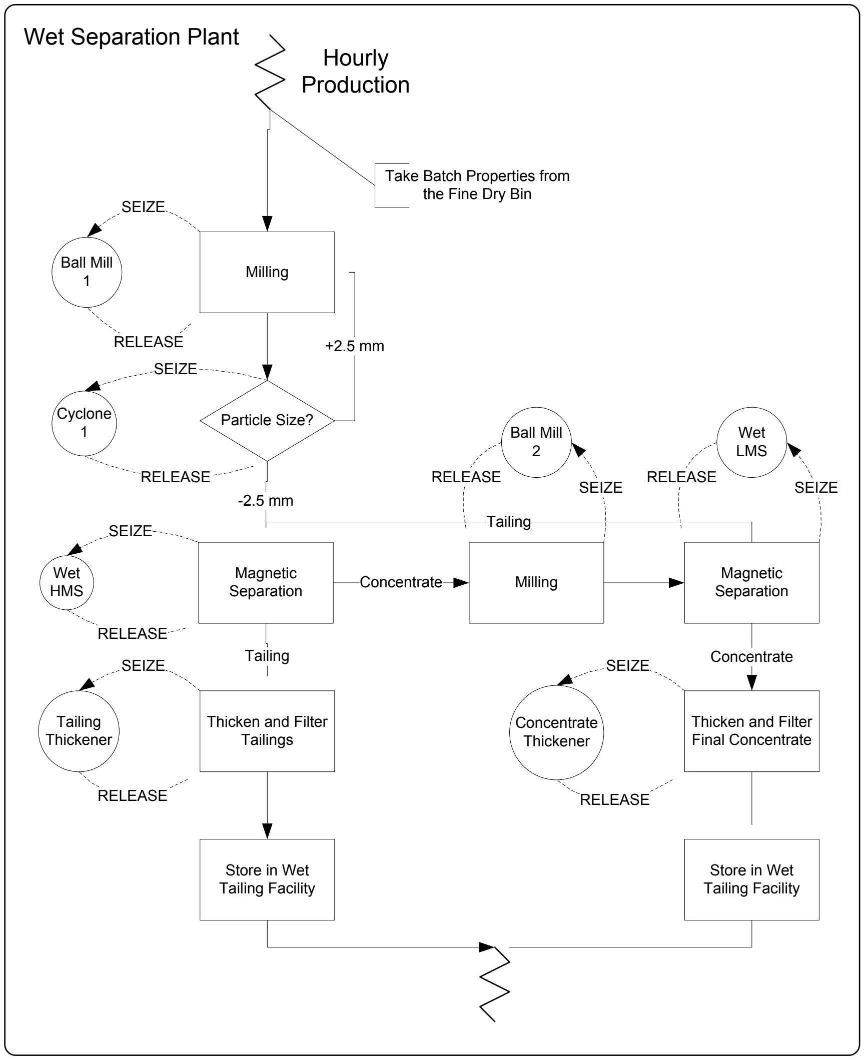

Fig 6.Wet Separation Plant Part 2

Table 1.Production Rates

Trucks Inter Arrival Time	**Truck Load**	**Crushing Plant Production Rate**	**Dry Plant Production Rate**	**Wet Plant Production Rate**
9 mins	NORM(200,10) tonnes	TRIA(950,1000,1050) tonnes	TRIA(830,860,900) tonnes	TRIA(670,710,750) tonnes

Table 2.Machine Failures

Failure	**Resource**	**Uptime (days)**	**Downtime (days)**
fOverhaul	All	355	10
fGyratoryCrusher	rGyratoryCrusher	59	1
fSAGMill	rSAGMill	9.6	0.3
fSAGMill2	rSAGMill2	175	7
fScreen1	rScreen1	59.6	0.3
fScreen2	rScreen2	9.6	0.3

fCrusher	rCrusher	89	1
fLMS	rLMS	29.8	0.2
fHMS	rHMS	19.8	0.2
fBallMill1	rBallMill1	89.6	0.3
fCyclone1	rCyclone1	29.6	0.3
fBallMill2	rBallMill2	179.6	0.3

Table 3.Recoveries

Machine	CR	Rg Fe	Rg P	Rg S
Screen1	-	TRIA(0.97,0.98,0.99)	TRIA(0.50,0.60,0.70)	TRIA(0.60,0.70,0.80)
Screen2	-	TRIA(0.35,0.40,0.45)	TRIA(0.60,0.70,0.80)	TRIA(0.50,0.60,0.70)
Dry LMS	TRIA(1.60,1.70,1.80)	TRIA(0.90,0.92,0.94)	TRIA(0.25,0.30,0.35)	TRIA(0.30,0.35,0.40)
Dry HMS	TRIA(1.30,1.35,1.40)	TRIA(0.92,0.94,0.96)	TRIA(0.25,0.30,0.35)	TRIA(0.30,0.35,0.40)
Wet HMS	TRIA(1.30,1.35,1.40)	TRIA(0.90,0.92,0.94)	TRIA(0.18,0.20,0.22)	TRIA(0.10,0.15,0.20)
Wet LMS	TRIA(1.30,1.40,1.50)	TRIA(0.80,0.82,0.84)	TRIA(0.09,0.10,0.11)	TRIA(0.09,0.10,0.11)

Table 4.Bin Capacities

	Stockpile	Storage Bin 1	Fine Dry Bin	Concentrate Bin
Capacity	5 days of plant operation feed (120,000 tonnes)	1 days of downstream operation feed (24,000 tonnes)	Unlimited (10,000,000 tonnes)	Unlimited (100,000,000 tonnes)

6. Model verification

The developed model is run for a pilot evaluation and verified using synthetic data. Since the objective of the study is to track the grades and tonnages of material, a limited amount of rock with constant grades is fed into the plant and the changes in its grades and tonnages are studied. In order to verify the model and make sure that all of the tonnage and metal content of the feed is retrieved at either of the system outputs, a 24-hour run is considered. In this case, the trucks are scheduled to arrive in constant 2 hour periods with 200 tonnes of rock. In order to make sure that no material is inside the system when the simulation terminates, the number of created trucks is limited to 10, which leaves 2 hours free of input, during which time the system can process existing entities. All material carried by the trucks are assumed to have a constant metal grade of 40 percent and 0.2 and 1.5 percent for phosphor and sulfur respectively. The verification results are summarized in Table 5.

Table 5.Model verification results

	Feed	Screen1 Tailing	Storage Bin	D-HMS Tailing	Fine Dry Bin	W-HMS Tailing	Final Concentrate
Rock (tonnes)	2000	242	1758	293	1465	473	966
Metal Grade (%)	40	7		7		15	71.92
Metal Content (tonnes)	800	16		21		72	682
Phosphor Grade (%)	0.1	0.331		0.262		0.086	0.001
Phosphor Content	2	0.80		0.77		0.41	0.01
Sulfur Grade (%)	1.5	3.719		2.877		1.539	0.014
Sulfur Content	30	9.00		8.43		7.28	0.13

There is no change in grades or tonnages in the first part of the model (stockpile). Before the material is sent to the grinder from the stockpile, a screen separates very fine particles and sends them to the dry tailing dump. 242 tonnes of material are sent to the dump with the properties presented in Table 5 . Two other tailings can be identified at dry and wet HMS machines, which hold 293 and 473 tonnes of material respectively. The total amount of tailings and final concentrate is almost equal to the tonnage of feed to the plant. However, 26 tonnes of material are lost in the system. The explanation lies in the cycling load of the system. In order to avoid having an unlimited number of entities in the system, batches smaller than 0.2 tonnes are removed from cycles without having recorded anywhere. These two holes are responsible for the losses in the metal, as well as in the phosphor and sulfur contents.

7. Results

After verification, the model is run for 365 days with the failures and maintenance plans applied to it. The first day of the run is considered as warm-up and is not used in calculating statistics. In addition, uncertainty is added to grade, recovery and production tonnages. In order to have a more balanced production line, different batch sizes are selected for different parts of the model. The crushing plant is run with average production of 1000 tonnes/hr where the dry and wet lines are set to run with average rates of 880 and 735 tonnes/hr based on the tail and concentrate produced in each part. Approximately 8.1 million tonnes of rock are fed into the processing plant and 1.6 million tonnes of concentrate are recovered. This concentrate has an average metal grade of 70.15 percent. This means the overall recovery of the plant is approximately 86.6 percent. The results obtained are presented in Table 6 and Table 7.

Table 6.Tonnages and grades

Plant Divisions	Stockpile	Crushing Plant	Dry Plant	Wet Plant
Total Ore (million tonnes)	7.99	6.92	5.75	3.72
Average Metal Grade (%)	39.68	44.21	52.28	70.44
Average Phosphor Grade (%)	0.1183	0.0807	0.0361	0.0013
Average Sulfur Grade (%)	1.4670	1.1650	0.4852	0.0132
Average Working Time (%)	68.35	89.74	91.12	92.44
Average Residence Time (mins)	-	0.9	0.9	2.76
Total Waste (million tonnes)	-	0.95	1.15	1.82
Metal Lost (1000 tonnes)	-	62	82	277

Table 7.Bin Levels

	Stockpile	Storage Bin	Fine Dry Bin	Concentrate Bin
Capacity (tonnes)	120,000	24,000	2,000	10,000,000
Average (tonnes)	2183	765	194	0
Maximum (tonnes)	4338	949	548	90

8. Conclusions and future work

The study shows that we can simulate a mineral processing plant, which is a continuous process, as a combination of continuous and discrete events by assuming batches of feed as entities through the system. The batching strategy seems to be the best method of dealing with continuous systems, but it is important to be careful about batch size definitions. During this step, using some simple assumptions for separation mechanisms, we could trace all materials along the process line; and in the end, we came to the same sum of material introduced into the plant at four exit points. The experience shows that it is difficult to use many equations in Arena models (Assign modules). In such cases we can integrate Arena with some other powerful mathematical software.

The most important step for future work is to develop the model based on real processing plant data and to use mine plans for scheduling the material delivered to the plant. The next step is to study a real plant and come up with better failure and uptime distributions. Additionally, it is possible to balance the processing line and calculate appropriate processing times based on machine capacities.

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Bulk Material Terminal Simulation Modeling

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Abstract

Bulk terminals are designed to facilitate the material flow from mines to plants in a network of mining facilities and receiver plants. Therefore, such terminals are considered to be one of the most important nodes in mining logistic networks. Optimal design of such terminals has a dominant effect on the success rate of mining material supply chain. In the literature, a number of mathematical models are developed to decide on the optimal design and schedule of such terminals. However, due to the presence of high uncertainty in real terminal operations, such deterministic models fail to deliver reliable solutions. Simulation is a useful technique to capture the uncertainty existing in the arriving material contents and terminal operations such as unloading, sampling, sorting, stacking, reclaiming and loading operations. In this paper, a simulation model is developed for a typical iron ore bulk terminal. The terminal is consisted of three main parts as; train unloading section, stockyard area and vessel loading section. An Arena model is developed to capture the performance measures of these three terminal sections, specifically the mean and maximum values for the inventory level of cells and total waiting time for any single train to be unloaded and vessel to be loaded. The number of replications is adjusted in a way to increase the accuracy of the estimations for the desired performance indicators. The proposed simulation model is capable of testing different cell allocations in the stock yard area and also more complex scenarios about stacking/reclaiming functions. The model is verified and the results are presented and discussed. As the further step, the proposed model should be validated, based on the real parameters and input data from a bulk material terminal.

1. Introduction

Designing a proper logistic network that facilitates the shipment and storage of the mined material is considered to be an important task in mining supply chain management. This network consists of two main elements; nodes and arcs. The nodes represent all of the facilities in the network such as mining and processing plants (the supply side), the production plants as a final destination of the material (the demand side) and finally, some transshipment points such as bulk material terminals that facilitate the transfer of the mined/processed material to the destinations. In fact, the terminals balance the demand and supply flows through the network. The arcs represent the routes that interconnect different nodes in the network together. Some sample views from the network are illustrated in Fig. 1. One critical decision making problem for the network design is to determine the number, location, size and design of terminals. Designing such terminals is considered to be important from two points of view; balancing the material flow in the network and also cost reduction aspects. The design and capacity of terminals mostly depends on the input rate (from suppliers) and the output rate of commodity (from demand points). In reality, due to the uncertainty existing in schedules of transportation facilities such as trains and vessels, both input rate to the terminal and output rate from it may follow stochastic patterns.

In the literature, one can find this issue under the category of logistics management and more specifically, the inventory management. Different approaches are developed to evaluate the performance of bulk terminals. In many of related works, the focus is on the coal terminal which represents other bulk solid material terminals as well. In some of them, the focus is on development of an exact mathematical model for terminal operations, while others concern more about finding a reliable solution through heuristic approaches to solve such large scale mathematical models.

Fig. 1. Iron ore supply network.

Abdekhodaee et al. (2004) investigate a typical coal terminal problem and divide the whole terminal system into two sub-systems; railing sub-system and stockyard sub-system. For each of these sub-systems, an integer programming (IP) formulation is developed that works properly for small scale problems. The railing IP model minimizes the summation of penalties that are applied to any violation from trains' schedule. The stockyard IP model minimizes the summation of conflicts that happen during stacking and reclaiming operations in the stockyard (section 3). As a result, the proposed IP model for stockyard system allows physical infeasibilities to be scheduled, but minimizes them. In order to solve the model for large scale problems in a reasonable time, a greedy heuristic algorithm is developed to find the solution. Simulation results are used in proposed heuristic.

In presence of uncertainty, one powerful technique that can be applied to investigate performance of the system is discrete event simulation. The first step in building a simulation model is to determine different components of the system, which in this case is a typical bulk solid material terminal. All bulk terminal activities can be categorized in four groups as; (1) receiving, (2) storing, (3) blending and (4) shipping activities.

Weiss et al. (1999) discuss the implementation of simulation techniques for solid bulk material terminals and review different simulation components that are required to model all four different activities of a terminal in more details as the followings:

1) *Stockpiling area*: occupies most of the terminal area and is partitioned in order to avoid any probable dilution and contamination due to mixing different material.
2) *Rail facility*: provides required area and facility for arriving trains which have brought the bulk load to the terminal.

3) *Ship loading facility*: consisted of berths and docks to accommodate both relatively small ships and ocean-going vessels.
4) *Stacking – reclaiming*: installed between stockpiles and are used to stack the trains load into the stockpiles or reclaim the material from stockpiles and send it to the quays.
5) *Other process facilities*: crushing and sizing small plants, drying plants, power plants and other required logistical facilities in the terminal.
6) *Conveyor system*: complex belt system which connects other facilities of the terminal.

In this paper, some of initial steps to simulate a solid bulk material terminal are presented and an Arena simulation model (Arena, 2009) is developed for the purpose. The paper has the following sections: Problem definition is presented in section 2. A sample mathematical model is presented in section 3. The proposed simulation model, in details with its components is discussed in section 4. Model verification is presented in section 5 and the results are discussed in section 6. Finally, future steps in validating the terminal model and testing more scenarios are discussed in section 7.

2. Problem definition

In an overall point of view, an iron ore terminal receives ore with different qualities from one or a number of mines. Then, the ore is stocked in proper stockpiles and sent to a number of quays to be loaded into the vessels. More specifically, following operations and activities take place in the iron ore terminal:

The ore is shipped from different mines by trains. It is assumed that each train has a capacity of 3000 tonnes and trains arrive into the terminal in three shifts (Table 1). The trains' load is then unloaded, using a number of unloaders, and sent for sampling to determine the specifications of the load and its type. Sampling time follows a Normal distribution with mean of 1 hour and standard deviation of 30 minutes. According to the sampling results, the proper cell for each load in the terminal is determined. It is assumed that each arriving load has an equal chance to be either of types A, B, C or D (each with chance of 25%). The load is sent to the proper stockyard cell via a network of conveyors. The stockyard is partitioned into 12 separated cells to avoid any dilution and mixture of different quality ore types. Four different ore types can be stocked in these cells (three cells for each kind of iron ore). Stackers are used to dump the material into proper cell. A schematic view of the terminal, including the train unloading section, stockyard cells and ships loading section, is illustrated in Fig. 2.

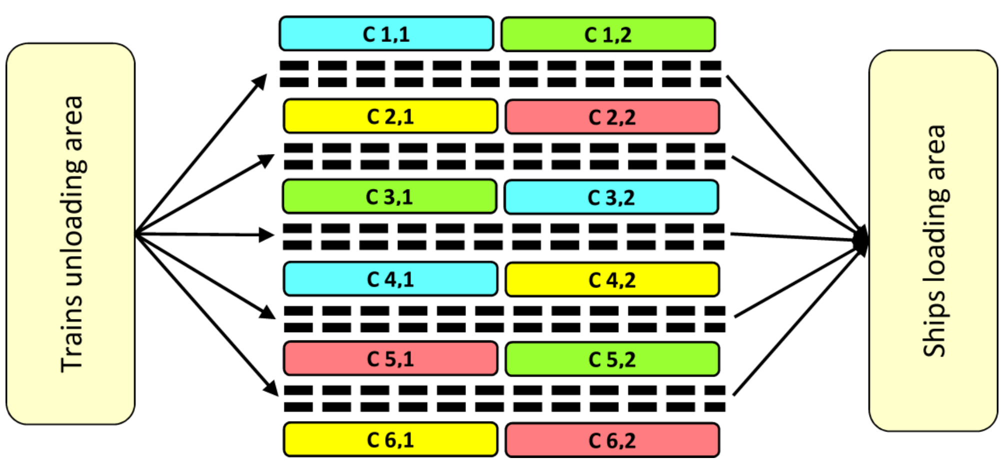

Fig. 2. Schematic view of a typical iron ore terminal.

On the other hand, the loading process is triggered by receiving a load order from customer. Orders are received by the terminal 24 hours a day and the time between order arrivals is eight hours. Same to the arriving loads, the orders have an equal chance to be for ore types A, B, C and D (each with chance of 25%).

Table 1. Trains' first arrival times.

	First arrival time
Shift 1:	Triangular (12:00 AM , 04:00 AM , 08:00 AM)
Shift 2:	Triangular (08:00 AM , 12:00 PM , 04:00 PM)
Shift 3:	Triangular (04:00 PM , 08:00 PM , 12:00 AM)

Based on the records and due to real unstable conditions of the ocean, ships arrive into the terminal, not exactly when the orders are received. Since the loading to the ships can only be initiated when the order and corresponding ship are present in the terminal, the order is held until the corresponding ship arrives. The inter arrival time for ships follows a triangular distribution as (7.5 , 8.0 , 8.5) hours. In the simulation model, it is assumed that the first order and the first ship are arrived randomly with a triangular distribution between 3:00 AM to 7:00 AM, with the most chance for 5:00 AM. There are three quays in the terminal. 30% of ships are assigned to quay 1, 40% to quay 2 and 30% to quay 3. Four ships can be loaded at the same time at each quay. When the ship with predetermined order berths, the reclaimers load the dumped ore from proper cell into the conveyor belt. The belts are designed in a way to load the coal into the ships that are berthed in quay. As a result, the ore is loaded into the ship based on the specified amount and quality of the order. The tonnage for each order is assumed to be 6000 tonnes.

When loading of the ships finishes, the ship decouples from the quay and it takes a time with Normal distribution with a mean of one hour and standard deviation of 15 minutes. To prevent any unexpected shut downs of the terminal flow, it is decided to stop the flow of material to the cells when the level of each cell reaches to 5% of its capacity and restart it when it reaches to 10% of capacity. On the other hand, to avoid any overflow of the cells, it is assumed that the flow to the cells becomes zero when the level of each cell reaches to 95% of its capacity and restarts when it reaches to 85% of capacity. The complete data about the capacity of terminal cells, the input rate, output rate and the initial level of ore in each cell is presented in Table 2.

Table 2. Specification of cells.

Capacity (tonnes):	10,000	Min level settings (% of capacity)	Stop flow:	5
Initial level (tonnes):	3,000		Restart flow:	10
Input rate (tonne/h):	1,000	Max level settings (% of capacity)	Stop flow:	95
Output rate (tonne/h):	8,000		Restart flow:	85

The aim is to develop a simulation model for the terminal which makes testing different scenarios possible. Specifically, the key performance indicators (KPIs) considered to evaluate the performance of the terminal are as follows:

- Utilization of quays,
- Total waiting time for ships and trains in the terminal,
- Average number of ships and trains being served in the terminal,
- Minimum, maximum and average of ore inventory in each cell.

Since the bulk material in terminal is carried by conveyor belts smoothly and in a very constant rate, the system can be assumed to be continuous and the cells can be modeled as tanks.

3. Mathematical formulation

Some mathematical programming models are developed to find the exact and optimal design for the terminal. As an example, Abdekhodaee et al. (2004) consider the supply network as a combination of two sub systems, the railway system and the coal terminal system. Two integer programming formulations are developed to optimize performance of each sub system. Although the model is developed for a coal terminal, the same concept works for almost every solid bulk material. The integer programming objective is to minimize the total number of conflicts that may occur during stacking and reclaiming activities. The basic inputs in this model are the different types of products, opening and closing times for stockpiles, stackers and reclaimers. Pairs of (*r,m*) are used to identify the row number and dimension (in meters) of each stockpile. Three types of opening and closing time intervals are considered as following:

$[to^1{}_i, tc^1{}_i)$: an interval beginning at the time a stockpile becomes ready to receive coal for storage, and ending at the time that this stockpile is available to receive a new product.

$[to^2{}_i, tc^2{}_i)$: an interval over which a stacking operation occurs.

$[to^3{}_i, tc^3{}_i)$: an interval over which a reclaiming operation occurs.

For instance, for the list of products P = {1, 2, 3, 2}, There will be a corresponding list of stockpile opening-closing times $[to^1{}_i, tc^1{}_i)$ given by $T_{OC}{}^1 = \{(1,15),(3,18),(9,21),(12,21)\}$. It is assumed that pre-designated stockpile locations are assigned to different products in the terminal. So, a particular product may be permitted for instance in stockpiles at locations $(r,m) \in \{(1,11),(1,7),(3,7),(2,10)\}$. "S" is defined as the union of these sets of permissible stockpile locations.

S_{P_i} : the set of possible stockpiles for a product P_i.

$K_{r,m}$: the set of machines that can access stockpile (*r,m*).

K_j : the set of machines allocated to row (bund) *j*.

$d_{k,m}$: the distance of a stockpile from the end of its row, in the direction of machine k (there are generally two stacker/reclaimer machines in each row).

Three zero-one decision variables are defined as:

$Y_{r,m,i,t}$: Is one, if *i*-th item in product list is allocated to position (*r,m*) in the stockyard at time *t* and zero, otherwise.

$X_{k,m,i,t}$: Is one, if *i*-th opening-closing for stacking machines is allocated and zero, otherwise.

$Z_{k,i,t}$: Is one, if *i*-th opening-closing for reclaiming machines is allocated and zero, otherwise.

Finally, Decision variable C_t^i is defined as the number of conflicts occurs in different terminal sections in each period. The proposed formulation allows scheduling of physical infeasibilities, but tries to minimize them.

$$Min \sum (C_t^1 + C_t^2 + C_t^3) \quad (1)$$

$$\sum_r \sum_m Y_{r,m,i,to_i^1} = 1 \qquad \forall i \quad and \quad (r,m) \in S_{P_i} \quad (2)$$

$$Y_{r,m,i,t} = Y_{r,m,i,to_i^1} \qquad \forall r,m,i \quad and \quad t \in [to_i^1, tc_i^1) \quad (3)$$

$$\sum_k \sum_m X_{k,m,i,t} = Y_{r,m,i,to_i^1} \qquad \forall i, t \in [to_i^2, tc_i^2) \quad and \quad k \in K_{r,m} \tag{4}$$

$$\sum_k \sum_m Z_{k,m,i,t} = Y_{r,m,i,to_i^1} \qquad \forall i, t \in [to_i^3, tc_i^3) \quad and \quad k \in K_{r,m} \tag{5}$$

$$\sum_i Y_{r,m,i,t} \le 1 + C_t^1 \qquad \forall r, m, t \tag{6}$$

$$\sum_m \sum_i (X_{k,m,i,t} + Z_{k,m,i,t}) \le 1 + C_t^2 \qquad \forall k, t \tag{7}$$

$$\sum_{k \in K_j} \sum_m \sum_i (X_{k,m,i,t} + Z_{k,m,i,t}) + MS \le BL + C_{t,j}^3 \qquad \forall t \quad and \quad j = 1,...,|r| \tag{8}$$

Eq. (1) defines the objective function of the model that is to minimize the total number of conflicts in the system. Eqs. (2) and (3) allocate a product (parcel) to a location (*r,m*) and retain the allocation throughout the opening and closing of that parcel. Eq. (4) allocates a stacking operation to a machine, while Eq. (5) allocates a reclaiming operation to a machine. The next three constraints deal with the number of possible conflicts. Eq. (6) measures the possible number of conflicts in assigning different overlapping parcels to a single stockpile. Eq. (7) calculates the number of conflicts in assigning overlapping reclaiming and stacking tasks to a single machine. Finally, Eq. (8) determines the amount of conflict in machines-on-the-same-bund movement (where *MS* is the minimum separation between machines and *BL* is the bund length). By introducing different weight coefficients, it is possible to change the relative importance for different types of conflicts.

Since the proposed integer programming formulation is not practical for large scale problems, the model is used for just small size problems. To solve the large scale problems, a heuristic algorithm is developed which uses simulation results in its iterations.

4. Simulation model

Arena (2009) is a powerful simulation package and is used in this paper for building the simulation model to study the performance of the terminal in presence of uncertainty. Stockyard cells and conveyor belts are modeled with the flow process modules of Arena, which is a powerful tool for simulating the continuous behavior of input and output rates. The model is consisted of three main parts as; (1) train unloading section, (2) ship loading section and (3) stockyard area.

Dumping of arriving loads to the stockyard cells is modeled in the train unloading section. The entity in this section is the train load, which has four attributes; load size, iron ore grade, phosphor grade and sulfur grade. During each 8 hour shift, one train arrives in the railway station, but the first arrival time is not always the same and follows a probability distribution function. A sampling process is done for each arriving load to verify the grades. Then according to the grade distributions, the load is directed to the proper stockyard cells through conveyor belts. There are three cells corresponding to each grade quality. These cells are selected in a random order for the loads with that specific grade range. The train load entity is disposed when the load is conveyed completely to the cells. The Arena model for train unloading section is illustrated in Fig. 3.

On the other hand, the ship loading section covers different processes that are triggered when a load order arrives. It is assumed that the orders are received almost always earlier than corresponding ship arrival time. That is because of the nature of oceanic transportation. As a result, the terminal freezes each order until its corresponding ship arrives and then, lets the order/ship wait to be loaded. Same to the train load entities, corresponding attributes are defined for each ship load entity as order size, order quality (grade) and the quay number which the ship is going to be loaded at. According to the quay assignment, ships line up in corresponding queue to be loaded. Each quay can load four ships at the time. When loading finishes, the ships are decoupled from the quay and leave the loading dock. Ship load entity is disposed when the ship leaves the terminal. The Arena model for ship loading section is illustrated in Fig. 4.

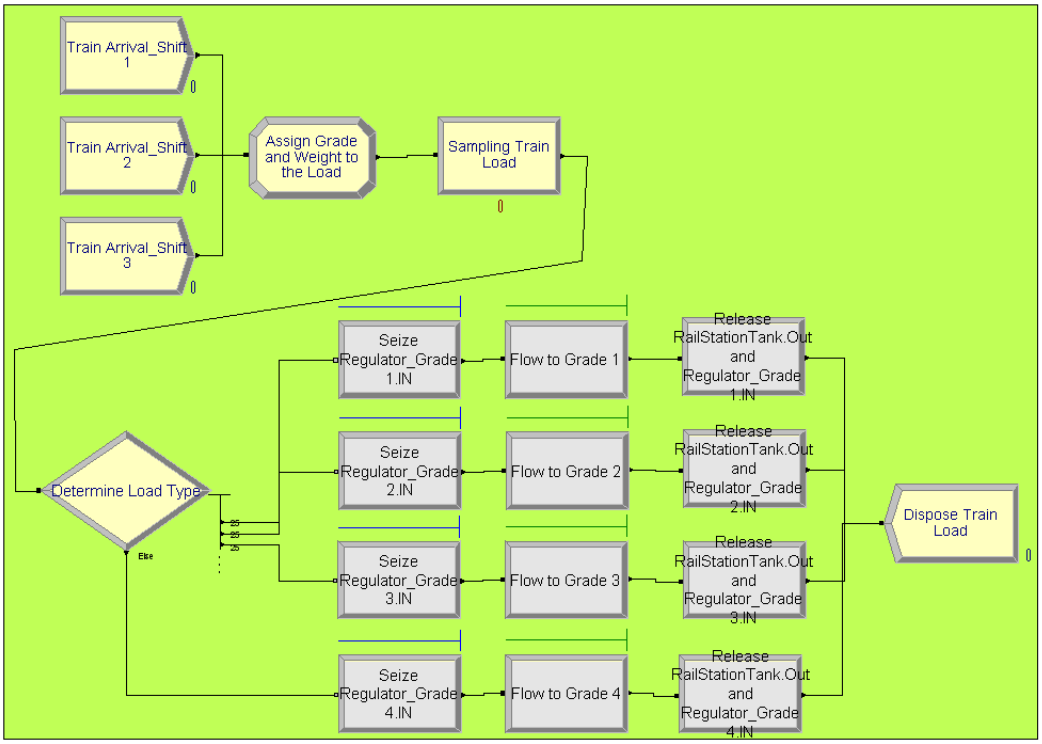

Fig. 3. Arena simulation model for train unloading section.

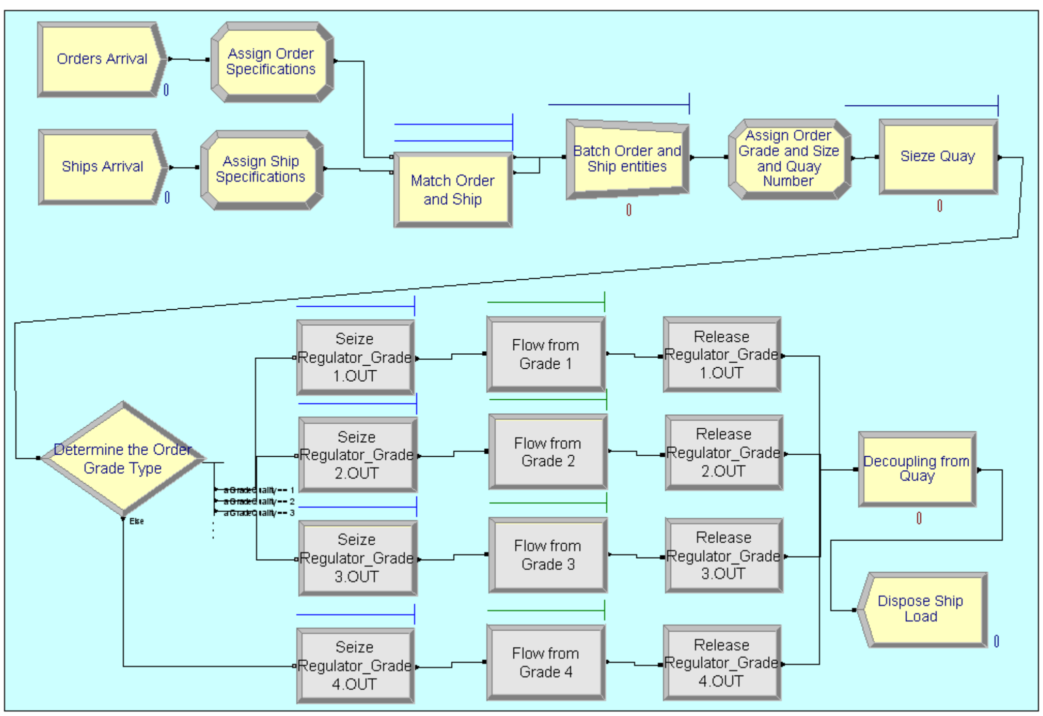

Fig. 4. Arena simulation model for ship loading section.

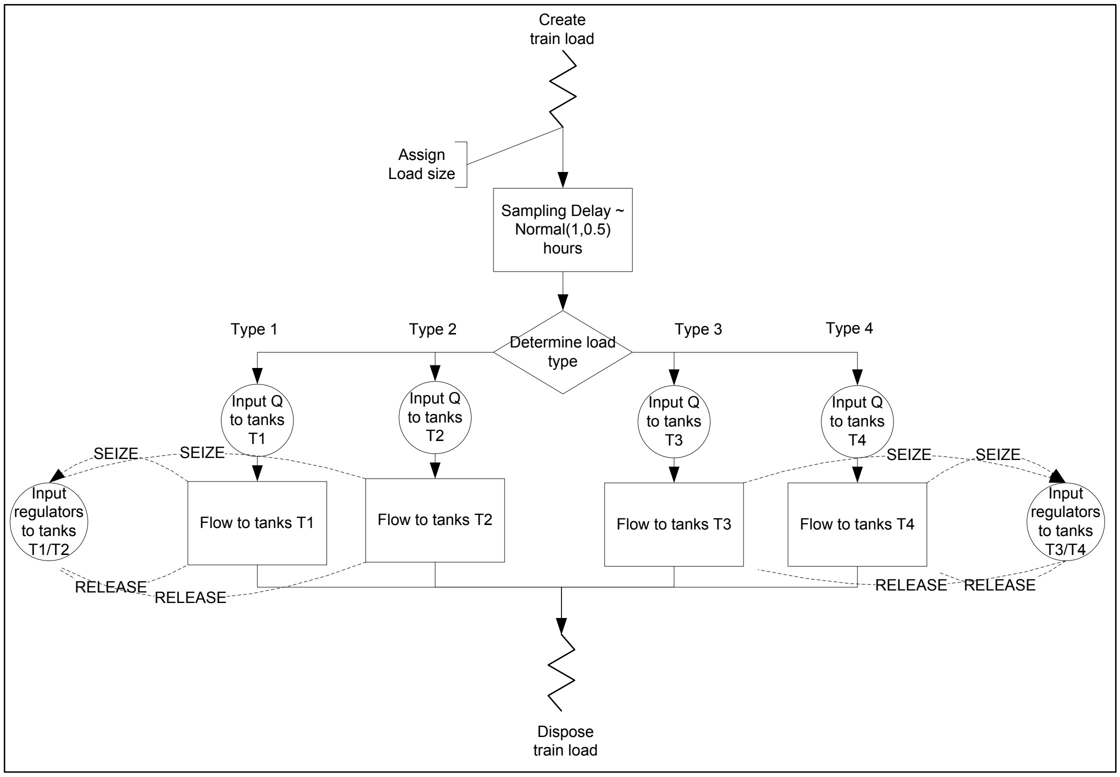

Fig. 5. Logic flow chart for train unloading section.

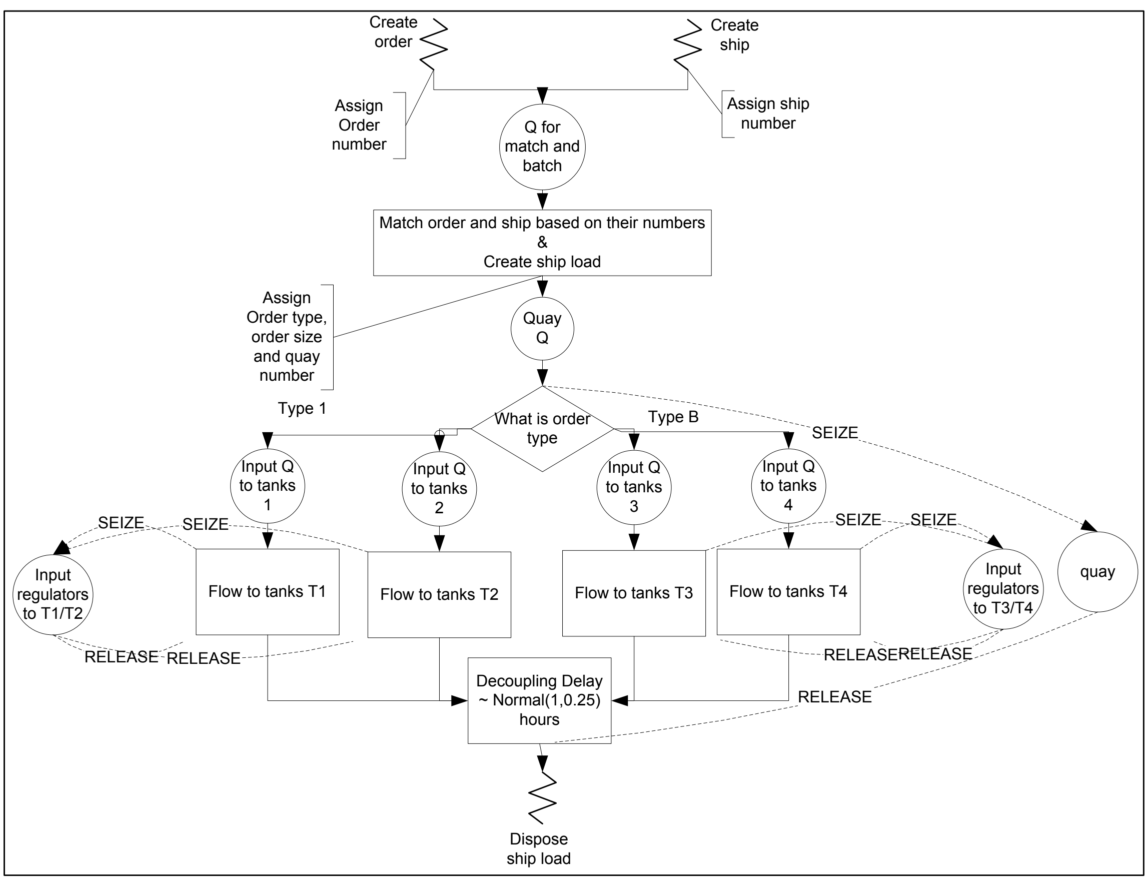

Fig. 6. Logic flow chart for ship loading section.

Logic flow charts for train-unloading and ship-loading sections are presented in Figs. 5 and 6, respectively. For more details on the steps of creating the model, see paper 402. Add a sentence referring the reader to paper 402 for detailed modeling. The stockyard section is consisted of 12 separated cells. Each three of these cells are assigned to a specific iron ore quality. The train load is conveyed into the proper cell and stacked in it by the means of stackers. On the other hand, when an order/ship arrives, using reclaimers, the ore is reclaimed from the cell, convoyed to the proper quay and loaded into the ship. It is assumed that each cell had its own stacker and reclaimer. To simulate stacking and reclaiming processes in the cells, each cell is considered as a tank which has an input regulator (conveyor and stacker) and an output regulator (reclaimer and conveyor). All of the tanks have the same capacity and almost one third of their capacity is full when the simulation initiates. There are two rules for stacking to and reclaiming from each cell. In order to avoid any dilution or mix between neighbor cells, the input flow to the cell should be stopped when 95% of cells capacity is full and can be restarted when its inventory reaches to 85% of the capacity. On the other hand, the output flow from the cell should be stopped when 5% of cells capacity is remaining and can be restarted again when its inventory reaches to 10% of capacity. The Arena model for stockyard section is illustrated in Fig. 7. Different colors represent different grade qualities that are contained in the cells.

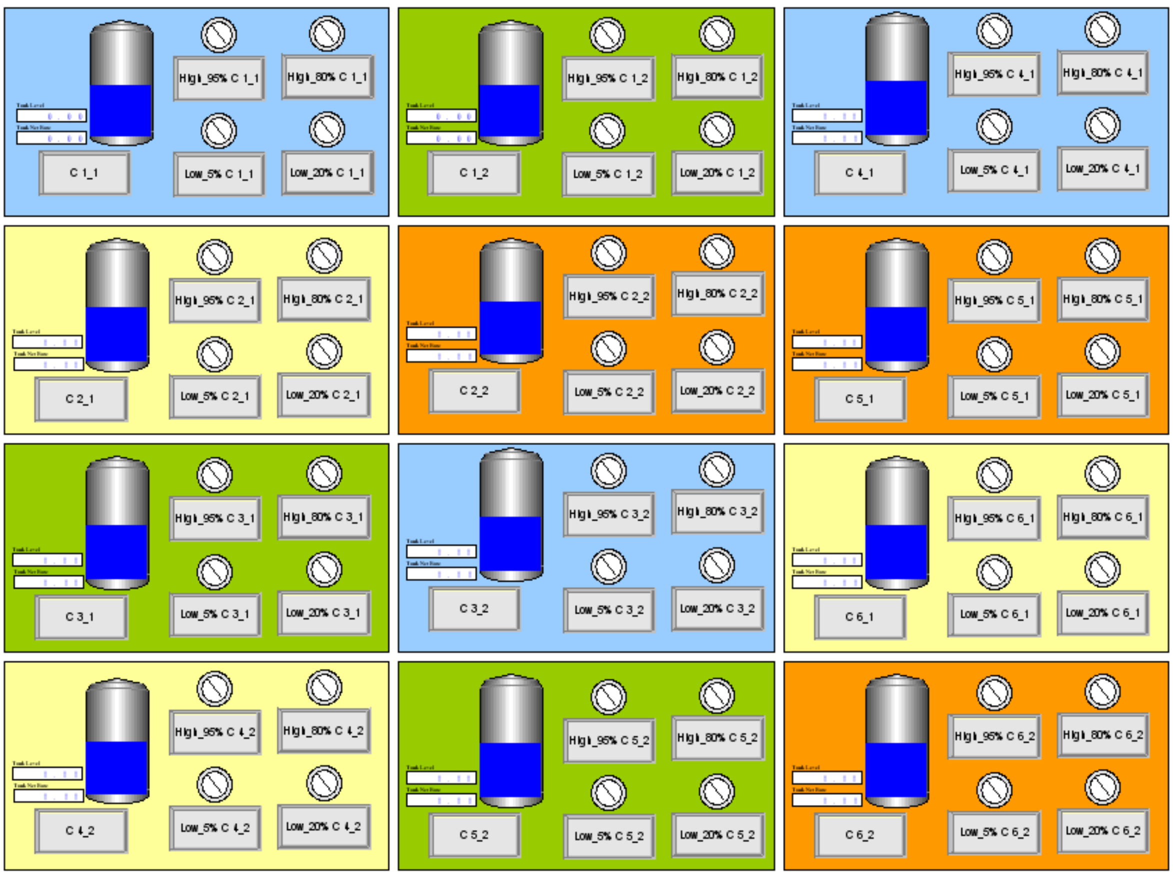

Fig. 7. Arena simulation model for stockyard section.

The following Arena modules are used in building the terminal model:

- *Create*: to create train load, order and ship entities.
- *Assign*: to define required variables and attributes for the entities through the model.
- *Process*: to introduce processes take place in the model, including sampling train loads and decoupling of ships from the quay.

- *Decide*: to distinguish between different load and order qualities in train unloading and ship loading sections.
- *Seize regulator*: to seize cells' input regulators in train unloading section and output regulators in ship loading section when the flow starts.
- *Flow*: to make the flow to and from the cells in train unloading and ship loading sections.
- *Release regulator*: to release cells' input regulators in train unloading section and output regulators in ship loading section when the flow stops.
- *Match*: to hold ship and order entities until they match together and let them move in the model.
- *Batch*: to batch ship and order quantities when they are match together and make a new entity (ship load).
- *Tank*: to represent cells capacity and filling/emptying behavior.
- *Sensor*: to capture stop/restart levels in the cells (tanks).
- *Dispose*: to dispose the train load and ship load entities.

5. Model verification

Prior to run the model, it is first required to make sure that the mass of material that is flowed in the terminal is balanced. Eq. (9) defines the material mass balance in the terminal.

$$M_{in}^{i} + L_{I}^{i} = M_{out}^{i} + M_{W}^{i} + L_{F}^{i} \tag{9}$$

Where;

M_{in}^{i} is the mass of ore type *i* that enters into the terminal (tonne),

M_{out}^{i} is the mass of ore type *i* that leaves the terminal (tonne),

M_{W}^{i} is the mass of ore type *i* that has entered into the terminal, but is waiting to be unloaded (tonne),

L_{I}^{i} is the initial level of tanks for ore type *i* when the simulation starts (tonne),

L_{F}^{i} is the final level of tanks for ore type *i* when the simulation ends (tonne).

The model is run with different values for train load sizes and ship order sizes. Eq. (9) is the checked for each pair of train and ship load sizes. The results for two cases are reported in Table 3.

The results show that the material in the model is completely balance, meaning that the total amount of input plus the initial tank levels adds up with the total amount of output, plus the final tank levels and waiting loads in the terminal. These values confirm that the model is performing right in terms of material mass balance. The results are also illustrated in Figs. 8 and 9.

Table 3. Verification results.

	Load size: 2,500 Order size: 50 (All values are in tonnes)					Load size: 3,000 Order size: 100 (All values are in tonnes)				
	M_{in}^{i}	L_{I}^{i}	M_{out}^{i}	M_{W}^{i}	L_{F}^{i}	M_{in}^{i}	L_{I}^{i}	M_{out}^{i}	M_{W}^{i}	L_{F}^{i}
Type 1	0	9,000	1,800	0	7,200	0	9,000	3,600	0	5,400
Type 2	2,500	9,000	2,050	0	9,450	3,000	9,000	4,100	0	7,900
Type 3	0	9,000	1,900	0	7,100	0	9,000	3,800	0	5,200
Type 4	5,000	9,000	1,750	0	12,250	6,000	9,000	3,500	0	11,500

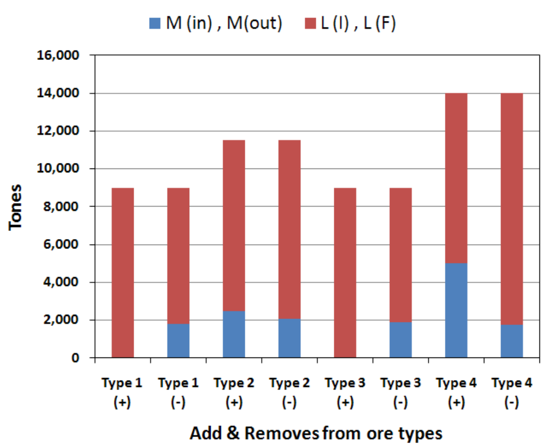

Fig. 8. Verification results (load size: 2500 , order size: 50).

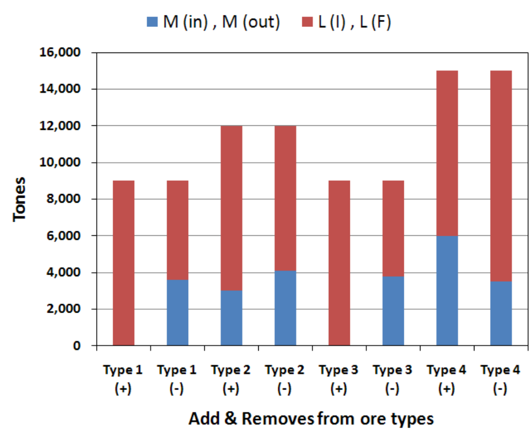

Fig. 9. Verification results (load size: 3000 , order size: 100).

6. Results and discussion

Number of replications:

The terminal model is run for 13 days and simulation base time unit is in hours. In order to decide on the proper number of replications, two important KPIs of the terminal are taken into account; (1) the total time that it takes to unload a train completely and (2) the total time that it takes to load a ship. Since these two are the most important measures in assessing the terminal performance, reduction of the uncertainty in estimation of these KPIs is a goal. In other words, the half-widths for these two KPIs should be considered in determining the number of replications. Eq. (10) is used to find the proper number of replications:

$$n = n_0 \times (\frac{h_0}{h})^2 \tag{10}$$

Where n and n_0 are the proper and test numbers of replications, respectively and h and h_0 are the corresponding half widths. In order to reach into the half widths of 3 minutes for the trains total unloading time and 30 minutes for the ships loading time, Eq. (10) results in following numbers of replications:

$$n^{train} = n_0{}^{train} \times (\frac{h_0{}^{train}}{h^{train}})^2 \qquad n^{train} = 10 \times (\frac{0.08}{0.05})^2 = 25.6 \simeq 26$$

$$n^{ship} = n_0{}^{ship} \times (\frac{h_0{}^{ship}}{h^{ship}})^2 \qquad n^{ship} = 10 \times (\frac{0.96}{0.50})^2 = 36.86 \simeq 37$$

After running the model for 37 replications, it turns out that few more replications are required in order to reach to 30 minutes half width for ship loading time. Therefore, 39 replications are considered for this model. The entity-related results from 39 replications are presented in Table 4.

According to these results, on average, total number of trains in the terminal are much less than the total number of ships. However, the average total time that a train spends in the terminal to be fully unloaded is four hours on average, which is almost twice the same value for the ships. It can be inferred from these numbers that any improvement in unloading facilities may result in smaller waiting time of the trains. In addition, in some cases there exist some considerations which do not let the filling and emptying of the cells take place simultaneously. In such cases, any upgrade in either sides of cells input or output flows may improve another as well. In general, higher input rate to the cells will result in smaller trains waiting time and number of trains hold in the terminal. The same statement is true for the ship loading section as well.

The utilization of quays is reported in Table 5. All three quays have the same capacity of four loading docks. According to these values, the average utilization of second quay has been more than two others. That is because of the quays assignments pattern to the ships. Since the first and third quays are planned to have some maintenance activities, 60 percent of ships are assigned equally to the first and third quays (30 percent to each one), while 40 percent of ships are sent to the second quay. The total number of times that each of three quays are seized are almost equal.

Table 4. Simulation results (based on entities).

	Total waiting time (hour)			Total number in system		
	Ave.	Max.	H. width	Ave.	Max.	H. width
Ships	2.12	4.74	0.50	18.58	19.44	0.20
Trains	4.00	4.20	0.03	2.13	4.76	0.24

Table 5. Simulation results (resources utilizations).

	Utilization (%)		Total number seized	
	Min.	Ave.	Ave.	Max.
Quay 1	53.12	79.23	4.36	7
Quay 2	56.88	84.68	4.31	6
Quay 3	16.70	74.66	4.51	7

Table 6. Simulation results (cells' inventories / tank levels).

Cell (tank) name	inventory (ktonne)			Cell (tank) name	inventory (ktonne)		
	Min.	Ave.	Max.		Min.	Ave.	Max.
C 1_1	997.13	4696.14	8667.20	C 4_1	1295.69	5263.42	8270.20
C 1_2	998.17	4970.76	8623.00	C 4_2	1073.70	5299.47	9051.16
C 2_1	940.30	5099.61	8837.48	C 5_1	813.81	4114.66	7074.27
C 2_2	856.64	4587.93	8291.86	C 5_2	681.34	5176.91	8852.94
C 3_1	1471.19	4585.67	8465.26	C 6_1	923.10	5325.42	8490.64
C 3_2	1146.08	4435.35	8627.87	C 6_2	1025.60	4433.25	8758.50

The inventories of the cells (tank levels) are reported in Table 6 and illustrated in Fig. 10. According to the illustrated results, average tank levels are deviating between their initial level (3,000 tonnes) and their capacities (10,000 tonnes). Since in this scenario, filling and emptying of the cells could happen at the same time, on average, the maximum level of tanks has not reached to the tanks' capacities.

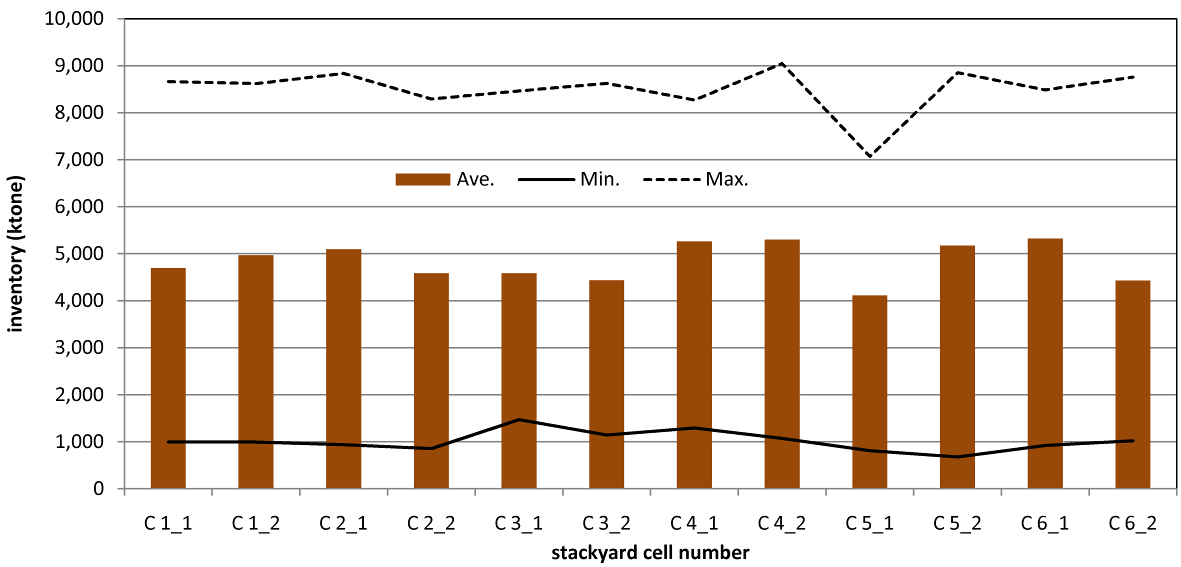

Fig. 10. Simulation results for cells' average inventory (tank levels).

7. Conclusions and future work

The design of terminals that receive bulk loads from suppliers, store them and send them to the demand points is an important issue in mining supply chain management. In this paper, the focus is on an iron ore terminal case and the first steps toward building a complete simulation model for bulk material terminal are presented. The model covers main operations over three areas in any typical terminal as train-unloading section, ship-loading section and the stockyard area. The number of simulation replications is determined in a way to increase the preciseness of the estimations for the selected KPIs. Arena continuous flow modules are used to model the continuous nature of iron ore bulk loads in the system. Then, the constructed Arena model is verified through running the model under different patterns of arriving loads and checking the results. The model is capable of testing different scenarios regarding input and output rates of material to the terminal and number of different variations for stockyard layout and operation options. More realistic assumptions about terminal operations should be considered in the next steps. Furthermore, the model needs to be validated, using real data sets. In addition, more scenarios regarding the different layouts for the terminal stockyard and different types of stackers and reclaimers can be tested and compared to figure out the best layout and allocations for a real case.

8. References

[1] Abdekhodaee, A., Dunstall, S., T. Ernest, A., & Lam, L. (2004). *Integration of stockyard and rail network: A scheduling case study.* Paper presented at the Proceedings of the Fifth Asia Pacific Industrial Engineering and Management Conference 2004, 25.25.21 - 25.25.17.

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9. Appendix

Arena file for the simulation model

Guidelines for using TOMLAB on Linux Server

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Abstract

This is a document on how to run and compile TOMLAB projects on the University cluster server. The servers are referred to as the AICT numerical and cluster servers.

1. Introduction

AICT operates and maintains a general purpose Linux cluster for research use by faculty, staff, and students at the University of Alberta. It offers a 64-bit environment with excellent performance for serial programs written in C/C++ and Fortran 77/90. MPI and OpenMP parallel programs are also supported. Users share the available resources by running jobs in batch mode.

The cluster is made up of one head node (login node) and 29 execution nodes, all running Linux. They are interconnected by standard Gigabit Ethernet (for NFS) as well as high speed Infiniband (for MPI). The head node contains two dual-core AMD Opteron 275 processors (four cores total) at 2.2GHz and 4GB of RAM installed. The execution nodes provide a total of 188 cores in the following configuration (AICT, 2011).

Access to the cluster and numerical servers are not automatic. Your supervisor must send an email to idadmin@ualberta.ca, Cc to research.support@ualberta.ca, requesting access on your behalf.

The remainder of this document outlines basic use of the cluster. It assumes you have prior experience working in a UNIX environment.

2. Required software

A basic Windows installation does not come with an ssh client program. Therefore, you should download and install PuTTY (terminal program) and WinSCP (file transfer program). You will also need to install an X display server program such as Xming-mesa.

In addition to the above general software, you need TOMLAB software for Unix-like systems and MATLAB compiler. You can download these from MOL homepage. Copy these files in a folder titled “Reqsoftware”.

You also need TOMLAB Linux license to run TOMLAB on the server.

3. Software installation on the cluster server

1. Run Xming
2. Login to the cluster server cluster.srv.ualberta.ca through WinSCP
3. Copy the folder (Reqsoftware) containing the tomlab-lnx64-setup.bin and MCRInstaller.bin, to the server. Drag the folder and put in WinSCP page. This folder is appeared in the following directory

→ /state/partition1/home/pourrahi

4. Run putty. Host Name: cluster.srv.ualberta.ca , In the putty configuration panel select tunnels under X11 from connection category then check "Enable X11 forwarding" and type "localhost:0" in Xdisplay location.
5. To see your folder in the cluster server use the command 'ls' as follows:

```
[pourrahi@cluster-login ~]$ ls
```

There is only one folder titled Reqsoftware.

6. Change directory to the folder:

```
[pourrahi@cluster-login ~]$ cd Reqsoftware
```

7. If you type ls, you will see there are two files, tomlab-lnx64-setup.bin and MCRInstaller.bin. We can only run executable files. You can convert files to the executable format as follows:

```
chmod +x file name
```

For example: chmod +x tomlab-lnx64-setup.bin

8. Type ls, it can be seen that the color of tomlab-lnx64-setup.bin file has been changed. This shows that this file is executable.
9. Execute the installer binary, tomlab-lnx64-setup.bin, as follows:

```
[pourrahi@cluster-login ~/Reqsoftware]$ ./tomlab-lnx64-setup.bin
```

If you are running the installer as a regular user, a warning may be displayed. Currently this warning can be ignored since a manual step. Press forward for the rest and at the final page; simply press the Finish button to exit the installer.

10. A new folder titled tomlab will have to appear in the following path:

→ /state/partition1/home/pourrahi

11. Copy the TOMLAB server license (tomlab.lic) from your computer to the tomlab folder through WinSCP.
12. Create a new folder titled MCR in the cluster server through WinSCP
13. Repeat Steps 7 to 8 for the installer binary, MCRInstaller.bin.
14. Execute the installer binary, MCRInstaller.bin, as follows:

```
[pourrahi@cluster-login ~/Reqsoftware]$ ./ MCRInstaller.bin
```

Directory name: /state/partition1/home/pourrahi/MCR

15. A new folder titled InstallShield will have to appear in the following path:

→ /state/partition1/home/pourrahi

4. Compiling TOMLAB/CPLEX files on the numerical server

MATLAB compiler can be used to convert your MATLAB code to a stand-alone application. To compile, we need a copy of the TOMLAB/CPLEX required files. This is a list of files that you get when you run TOMLAB/CPLEX code, in order to learn how to access the files, study paper 301 in the first report of MOL (Askari-Nasab, 2009).

Table 1, shows an example of the files required by TOMLAB/CPLEX to compile the MILP on the server for a specific project. Table 1 should be used just as a guideline only.

Table 1. Files required by TOMLAB/CPLEX to compile the MILP on the Linux cluster

tomRun	PrintResult	optParamDef	checkAssign	LineParamDef
cplexTL	ProbCheck	DefPar	SOLGet	tomFiles
cplex	optParamSet	endSolveMini	defblbu	mkbound
cplexmex	mipAssign	checkType	cpx2retvec	iniSolveMini
cpxBuildConflict	tomlablic	ProbDef	cplexStatus	ResultDef
tomlabVersion	StateDef	Str2num	etime	getfield
blanks	xprint			

1. Copy all the required files and TOMLAB code in a folder. For example, the TOMLAB code is Run_BlockCave.m, we copy this file, and all required files in a folder titled Reqfunctions.
2. Login to the numerical server num.srv.ualberta.ca through WinSCP
3. Copy the folder, Reqfunctions, to the numerical server. Drag the folder and put in WinSCP page.
4. Run putty. Host Name: num.srv.ualberta.ca , In the putty configuration panel select X11 under SSH from connection category then check “Enable X11 forwarding” and type “localhost:0” in Xdisplay location.
5. Change directory to the folder:

```
o/pourrahi> cd Reqfunctions
```

6. To compile your TOMLAB code use the command 'mcc-matlab' as follows:

```
Pourrahi\ Reqfunctions> mcc-matlab –m –v Run_BlockCave.m
```

7. After compiling, new files are generated within the Reqfunctions folder. The name of one of them is similar to the TOMLAB code function, Run_BlockCave.m, but without extension. Copy this file on your computer desktop through WinSCP.
8. The TOMLAB code loads coefficient, upper bound, and lower bound matrices. You may use this code to load other matrices, too. Create a folder and copy all these matrices and the compiled file, which does not have extension in this folder, for example BC.

Table 2, shows the files which were copied in the BC folder for a block caving problem.

Table 2. List of files in BC folder

File	Description
A.mat	Coefficient matrix
b_L.mat	Lower bound matrix
b_U.mat	Upper bound matrix
c.mat	Matrix of objective function
Input.mat	A MATLAB file which is contain required input data
zarib.mat	A MATLAB file which is contain required data
Run_BlockCave	Compiled file without extension

9. Login to the cluster server through WinSCP
10. Copy the folder, BC, which described in previous step to the server. Drag the folder and put in WinSCP page.

5. Test your program

Before submitting the job on the cluster, run your code interactively and make sure it works.

1. Login to the cluster server through putty
2. Change directory to the folder (BC).

[pourrahi@cluster-login ~]$ cd BC

3. Convert the file without extension to the executable format as follows:

chmod +x Run_BlockCave

4. Execute the file without extension, as follows:

[pourrahi@cluster-login ~/BC]$ ./ Run_BlockCave &

6. Batch jobs

Once you have successfully tested your program, you can submit the job on the cluster. A job is simply a shell script that invokes the program you want to execute.

Contention among jobs for the finite resources on a given node can lead to poor performance (or even a node crash). To avoid this, you should accurately specify your job's needs so that Portable Batch System (PBS) can reserve the necessary resources.

A convenient alternative to specifying resources on the command line is to introduce a PBS directive into your job script. A PBS directive is a shell script comment that begins with the string PBS. It must appear before the first executable statement.

To write a .pbs script file on the server use gedit command as follows:

[pourrahi@cluster-login ~]$ gedit

Write the following lines in the opened window and then save the script with a proper name, for example, Run in the created folder (BC). Table 3 shows description of each line (AICT, 2011).

```
 #!/bin/bash -l
#PBS -N BC_NESW
#PBS -S /bin/bash
#PBS -l pvmem=2gb
#PBS -l nodes=1:ppn=8
#PBS -l walltime=168:00:00
#PBS -m bea
#PBS -M pourrahi@ualberta.ca

LD_LIBRARY_PATH=/home/pourrahi/tomlab;
LD_LIBRARY_PATH=${LD_LIBRARY_PATH}:/home/pourrahi/tomlab/shared;
LD_LIBRARY_PATH=${LD_LIBRARY_PATH}:/home/pourrahi/MCR/v711/runtime/glnxa64;

TOMLAB_LICENSE_FILE=/home/pourrahi/tomlab/tomlab.lic

cd /home/pourrahi/BC

./Run_BlockCave
```

Table 3. Description of commands in the script file

Line	Description
`#PBS -N BC_NESW`	Gives your job BC_NESW name
`#PBS -S /bin/bash`	This job is configured to use the recommended bash-shell login environment
`#PBS -l pvmem=2gb`	Specifies virtual memory of 2GB per-process
`#PBS -l nodes=1:ppn=8`	Is read as: one node multiplied by one processor per node equals eight CPUs required.
`#PBS -l walltime=168:00:00`	Specifies an elapsed time of 168 hours
`#PBS -m bea`	An email notification will be sent when the job (b)egins and (e)nds or is (a)borted
`#PBS -M pourrahi@ualberta.ca`	Your email address
`LD_LIBRARY_PATH= /home/pourrahi/tomlab;` `LD_LIBRARY_PATH= ${LD_LIBRARY_PATH}:/home/ pourrahi/tomlab/shared;` `LD_LIBRARY_PATH= ${LD_LIBRARY_PATH}:/home/ pourrahi/MCR/v711/runtime/glnxa64;`	Define the library paths for TOMLAB, MATLAB Compiler runtime(MCR)
`TOMLAB_LICENSE_FILE=/ =/home/pourrahi/tomlab/tomlab.lic`	Defines the directory of TOMLAB license
`cd /home/pourrahi/BC`	Changes the directory to the folder where your compiled file is
`./Run_BlockCave`	Runs your compiled file

Overestimating any of these parameters could potentially waste resources and cause jobs to be delayed, *including yours*, particularly when the cluster is busy. Therefore, specify the **minimum** requirements that will allow your program to run successfully.

If you omit a resource in the specification, a default value will be applied from the following list.

```
pvmem=512mb
nodes=1:ppn=1
walltime=24:00:00
```

The maximum resources available are

```
pvmem=126gb
nodes=32:ppn=4
nodes=3:ppn=16
walltime=168:00:00
```

7. Job submission

To run a job on the cluster, you must submit the created script for batch processing, thus

1. Change directory to the folder (BC).

   [pourrahi@cluster-login ~]$ cd BC

2. Submit the job (the script file titled Run)

   [pourrahi@cluster-login ~/BC]$ qsub Run

   This will return a unique job id, and your job will take its place in a queue that includes all jobs from all the other users. It will wait there until the necessary resources become available.

Use the **qstat** command to view the status of jobs in the queue. Also try **qstatx**. Job status will be either running (R) or queued (Q). To remove a running or queued job that you previously submitted, use the **qdel** command with the numerical portion of the job id as the argument.

As an example, here is what to expect when you submit a job script called Run.

```
[pourrahi@cluster-login ~/BC]$ qsub Run
5480026.opteron-cluster.nic.ualberta.ca
[pourrahi@cluster-login ~/BC]$ qstatx –a
Mon jan 31, 2011 11:51:02
                    --Requested-------------- --Used (mb)---------------------
Job Id   S Username Nodes    PVMem  Walltime  Mem    VMem   Walltime  Cputime
-------- - -------- -------- ------ --------- ------ ------ --------- --------
.
.
.
5480020  R jes6     1:ppn=4    10gb 168:00:00   2459   2512  21:20:20  83:48:20
5480021  R jes6     1:ppn=4    10gb 168:00:00   3656   3715   2:36:54   9:47:59
5480022  Q jes6     1:ppn=4    10gb  48:00:00
5480023  Q jes6     1:ppn=4    10gb 168:00:00
5480024  Q jes6     1:ppn=1     4gb  48:00:00
5480025  Q jes6     1:ppn=1     4gb  48:00:00
5480026  Q pourrahi 1:ppn=1     2gb 168:00:00
                    ----------------------------------------------------------
                    103 cpus used
```

8. References

[1] AICT. (2011). AICT General Purpose Linux Cluster Retrieved 08/08/2011, 2011, from http://www.ualberta.ca/CNS/RESEARCH/LinuxClusters/

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Guidelines for Building a Bulk Material Terminal Simulation Model in Arena

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1. Introduction

Designing of a proper logistics network that facilitates the shipment and storage of the mined materials is considered to be an important issue in mining supply chain management. This network is consisted of two main elements; the nodes and the arcs. The nodes represent all of the facilities in the network such as mining and processing plants (the supply side), the production plants as the final destination of the materials (the demand side) and finally, some transshipment points such as bulk material terminals that facilitate the transfer of the mined/processed material to the destinations. The arcs represent the routes that interconnect network nodes. One of the critical decisions in the design of the network is the number, location, size and the design of terminals.

2. Learning Objectives

This guideline aims to learn how to:

- Use resource set and regulator set in a model
- Use a combination of continuous and discrete modules in a model
- Define resources with variable capacities
- use animation for entities
- Combine multiple entities and create a new entity

3. Concepts and Terminology

The following modules are repeated from Arena Help (Rockwell, 2010).

3.1. Create module

This module is intended as the starting point for entities in a simulation model. Entities are created using a schedule or based on a time between arrivals. Entities then leave the module to begin processing through the system. The entity type is specified in this module.

3.2. Decide module

This module allows for decision-making processes in the system. It includes options to make decisions based on one or more conditions (e.g., if entity type is Gold Card) or based on one or more probabilities (e.g., 75% true; 25% false). Conditions can be based on attribute values (e.g., Priority), variable values (e.g., Number Denied), the entity type, or an expression (e.g., NQ (ProcessA.Queue)).

3.3. Assign module

This module is used for assigning new values to variables, entity attributes, entity types, entity pictures, or other system variables. Multiple assignments can be made with a single Assign module.

3.4. Process module

This module is intended as the main processing method in the simulation. Options for seizing and releasing resource constraints are available. Additionally, there is the option to use a "sub-model" and specify hierarchical user-defined logic. The process time is allocated to the entity and may be considered to be value added, non-value added, transfer, wait or other. The associated cost will be added to the appropriate category.

3.5. Match module

The Match module brings together a specified number of entities waiting in different queues. The match may be accomplished when there is at least one entity in each of the desired queues. Additionally, an attribute may be specified such that the entities waiting in the queues must have the same attribute values before the match is initiated. When an entity arrives at the Match module, it is placed in one of up to five associated queues, based on the entry point to which it is connected. Entities will remain in their respective queues until a match exists. Once a match exists, one entity from each queue is released to be matched. The matched entities are then synchronized to depart from the module.

3.6. Batch module

This module is intended as the grouping mechanism within the simulation model. Batches of entities can be permanently or temporarily grouped. Temporary batches must later be split using the Separate module. Batches may be made with any specified number of entering entities or may be matched together based on an attribute. Entities arriving at the Batch module are placed in a queue until the required number of entities has accumulated. Once accumulated, a new representative entity is created. The entity type of the outgoing entity may be changed by specifying a representative entity type.

3.7. Tank module

The Tank module defines a location where material is held or stored. The Capacity is the maximum quantity that may be stored in the tank. The Initial Level is the quantity in the tank at the beginning of the simulation or when the system is cleared. The Regulators repeat group specifies a list of devices that may be used to add or remove material from the tank (e.g., valves or pumps) at a specified rate. Semi-continuous flow operations may then be executed by discrete entities using the regulators via the Flow module. If a system contains both semi-continuous batch processes as well as high-speed packaging or filling operations, then it may be desirable to use the Flow Process panel with Arena Packaging Template. Enable the Packaging Input Connection and/or Packaging Output Connection options to graphically connect a Flow Process Tank to a Valve or Machine module in the Packaging panel. When an Arena model (i.e., .doe file) is run, the simulation results are stored in a Microsoft Access database (.mdb) file by the same name (e.g., the simulation results for Banking.doe are stored in Banking.mdb). If statistics collection for all tanks is enabled in the Project Parameters dialog AND Report Statistics is checked for an individual tank module, then Arena will automatically create, collect, and report statistics for that tank.

3.8. Sensor module

The Sensor module defines a detection device that monitors the level of material in a tank (Tank module). A sensor’s location is specified using the Tank Name, Location Type, and Level/Percentage prompts. A sensor is activated (triggered) when its location is crossed by the tank level in the Crossing Direction AND the sensor is enabled. When the sensor is activated, one or more Actions may be executed. Additionally, the Create Discrete Entity option may be used to create a discrete entity and send it to custom logic. The Initial State field specifies whether the sensor is enabled or disabled at the beginning of the simulation run. If a sensor is disabled it will be ignored and never activate. A sensor may be dynamically enabled/disabled at any point during the simulation run by assigning the SensorState(Sensor ID) variable.

3.9. Flow module

The Flow module is used in conjunction with the Tank module to model semi-continuous flow operations such as adding material to a tank, removing material from a tank, or transferring material between two tanks. When an entity enters the Flow module, a flow operation of the specified Type is initiated (i.e., Add, Transfer, or Remove). The tanks affected by the flow are determined by the regulators specified as the source and/or destination. A regulator is a device (e.g., a valve) that is used to add or remove material from a tank. A tank's regulators are defined in the Tank module. The entity is held in the Flow module until its flow operation is completed. The flow operation is completed when the first of three possible conditions is satisfied:

When the specified Quantity has been transferred.

When the specified Time has elapsed.

When the specified Signal Value is received via a Signal module.

The flow rate of the entity is regulated according to the maximum rate(s) of the source regulator and/or destination regulator. Additionally, the flow rate may be further constrained due to starvation or blocking from empty or full tanks. The initial maximum rate of a regulator is specified in the Tank module. A regulator's maximum rate may be changed during a run using the Regulate module. The RegulatorMaxRate(Regulator ID) function returns the current maximum rate of a regulator. The RegulatorRate(Regulator ID) function returns the current rate of a regulator. The actual quantity transferred by an entity in the Flow module may be stored in the Quantity Save Attribute when the entity exits the module.

3.10. Seize regulator module

A tank regulator may be used for only one flow operation at any given time. The Seize Regulator module may be used to control ownership of regulators and thus avoid situations of multiple entities trying to simultaneously use the same regulator in a Flow module. This module is also useful for choosing from a set of alternative regulators using a selection rule. The Seize Regulator module allocates one or more regulators to an entity. When an entity enters this module, it waits in a queue until all specified regulators are available simultaneously. An allocated regulator is released by an entity using the Release Regulator module.

3.11. Release regulator module

The Release Regulator module is used to release tank regulators that have been allocated to an entity using the Seize Regulator module. This makes those regulators available to other entities waiting to seize the regulator(s). When the entity enters the Release Regulator module, it gives up control of the specified regulator(s). Any entities waiting in queues for those regulators will gain control of the regulators immediately.

3.12. Regulator set module

The Regulator Set module defines a set of regulators to select from and use in other Flow Process modules

3.13. Dispose module

This module is intended as the ending point for entities in a simulation model. Entity statistics may be recorded before the entity is disposed.

4. Required capacity of a terminal for bulk material

Although there are some deterministic exact solutions for network problems, but in presence of uncertainty, one of the most powerful techniques that can be applied to investigate the model performance is discrete event simulation. To make a simulation model, first it is essential to distinguish between different components of a typical bulk solid material terminal. A complete list of terminal activities is separable into four minor activities as: 1) receiving, 2) storing, 3) blending and 4) Shipping activities.

4.1. Problem definition

An iron ore terminal receives the ore from different mines by trains. It is assumed that the time between trains' arrivals follows a triangular distribution as (5 , 9 , 13) hours. The trains load is then unloaded, using a number of unloaders, and sent for sampling to measure the specifications of the load and determine its type. Sampling time follows a Normal distribution with mean of 1 hour and st.dev of 30 minutes. The tonnage of each train load is 3000 tones and it is assumed that 60% of loads are of type A and the rest are type B. After sampling, the load is sent to the proper stockyard cell via a network of conveyors. The stockyard is partitioned into 4 separated cells in order to avoid any mixture of different quality ore types. Two different ore types can be stocked in these cells (two cells for each kind of ore). The material is dumped into the proper cell by the means of stackers. A schematic view of the terminal, including the train unloading section, stockyard cells and ships loading section, is illustrated in Fig. 1.

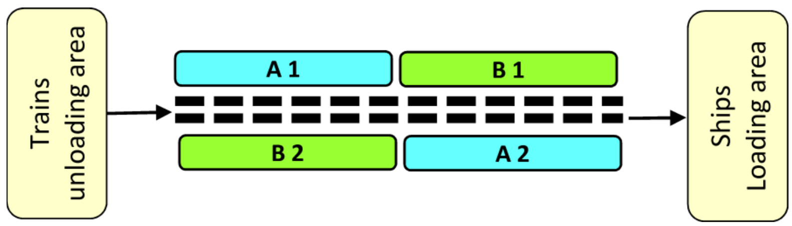

Fig. 1. A schematic view of the bulk terminal problem.

On the other hand, the loading process is triggered by receiving a load order from customer. The terminal receives the orders in a regular daily rate and the time between order arrivals follows a Uniform distribution of (9.5 , 11.5) hours. 65% of orders are for type A and 35% for type B. According to the past records, the orders and ships will not arrive into the quays at the same time and the loading can be initiated only when the order and corresponding ship are present in the quay. The inter arrival time for ships is Uniform (7 , 15) hours. There are three quays in the terminal and 30% of ships are assigned to quay 1, 40% to quay 2 and 30% to quay 3. Capacity schedule for quays is presented in Table 1.

Table 1. Capacity of each quay in 24 hours.

	from	to	from	to	from	to
	8:00 AM	4:00 PM	4:00 PM	12:00 AM	12:00 AM	8:00 AM
Quay 1	3		2		2	
Quay 2	2		1		3	
Quay 3	1		1		3	

When the ship with predetermined order berths, the reclaimers load the dumped ore from proper cell into the conveyor belt. The belts are designed in a way to load the material into the ships that are berthed in nearby quay. As the result, the ore is loaded into the ship based on the specified amount and quality of the order. It is assumed that the tonnages and specifications of orders are known as Table 2.

Table 2. Order specifications.

Order tonnage	6000
Request for ore tone A	65 %
Request for ore tone B	35 %

When loading of the ships finishes, the ship decouples from the quay and it takes a time with Normal distribution with a mean of 1 hour and st.dev of 15 minutes. In order to avoid any unexpected shot downs of the terminal flow, terminal manager had decided to stop the flow of material to the cells when the level of each cell reaches to 5% of its capacity and restart it again when it reaches to 10% of capacity. On the other hand, to avoid any overflow of the cells, it is assumed that the flow to the cells becomes zero when the level of each cell reaches to 95% of its capacity and restarts again when it reaches to 85% of capacity. The complete data about the capacity of terminal cells, the input rate, output rate and the initial level of ore in each cell is presented in Table 3.

Table 3. Specification of cells.

	capacity	Initial level	Rates (tone/h)		Min level settings (% of capacity)		Max level settings (% of capacity)	
			Input	Output	Stop flow	Restart flow	Stop flow	Restart flow
Cell A1	10,000	3,000	2,000	8,000	5	10	95	85
Cell A2	8,000	2,000	1,000	6,000				
Cell B1	8,000	2,000	1,000	6,000				
Cell B2	10,000	3,000	2,000	8,000				

The aim is to develop a simulation model for the solid bulk material terminal which makes testing different scenarios possible. Specifically, the key performance indicators (KPIs) considered to evaluate the performance of the terminal are as follows:

- Utilization of each quay,
- Total waiting time for ships and trains in the terminal,
- Average number of ships and trains in the terminal,
- Minimum, maximum and average of ore inventory in each cell

Since the bulk material in terminal is carried by conveyor belts smoothly and in a very constant rate, the system is assumed to be a continuous and the cells can be modeled as tanks.

Use blue balls to animate the train loads, red page to animate orders and boat to animate ships. Also animate the quays.

Run the simulation for 30 replications, each including 13 working days of 24 hours.

4.2. Flow diagram

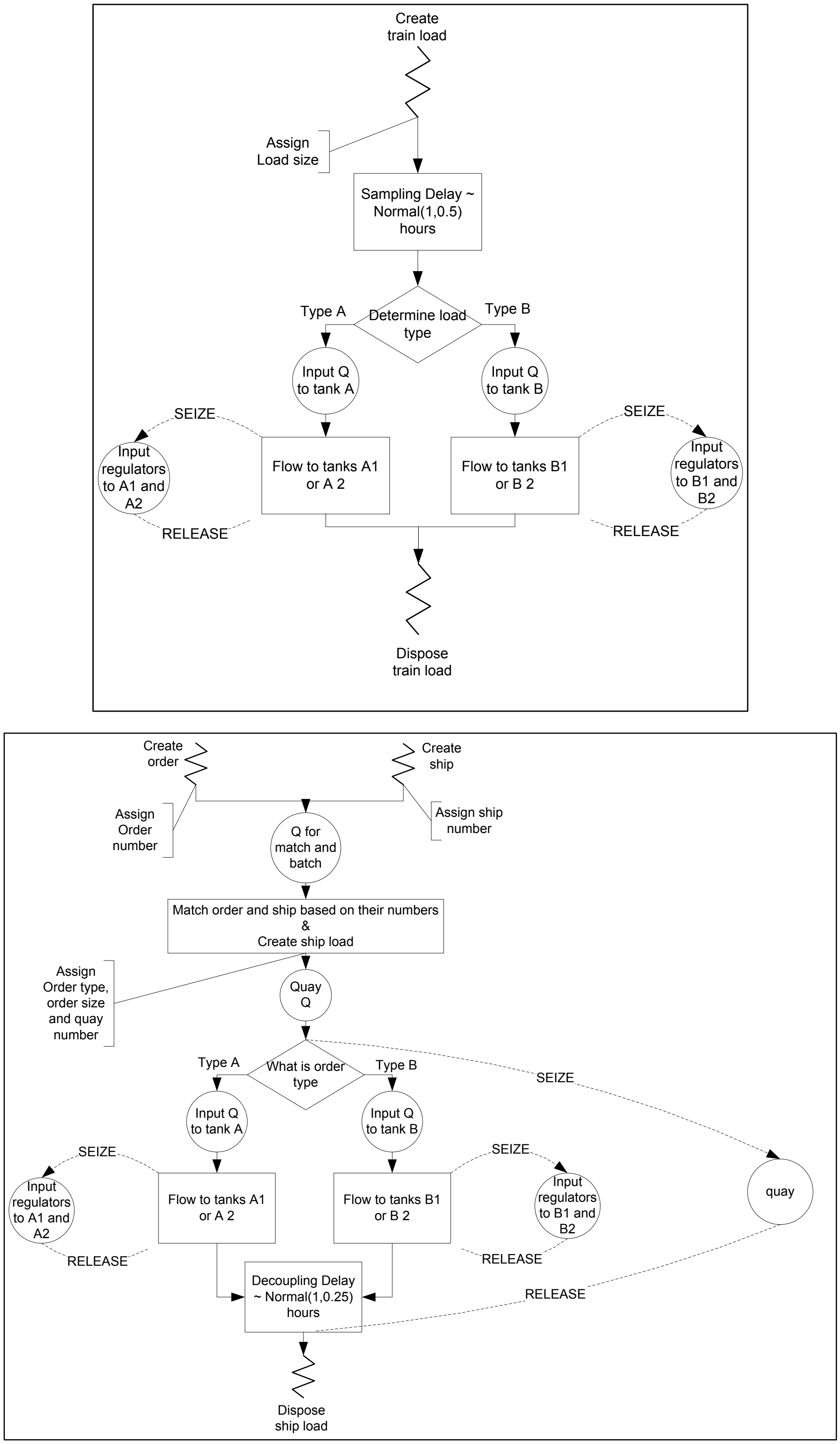

Fig 1. Flow Diagram.

4.3. Definition of terminology

4.3.1. *Entities*

- entShip: represents arriving ships to the terminal
- entOrder: represents arriving orders to the terminal
- entShipLoad: represents loads that are going to be loaded into the ships (combination result from entShip and entLoad entities)
- entTrainLoad: represents arriving train loads to the terminal

4.3.2. *Variables*

- varOrderNumber: variable, used to hold the serial number of arriving orders temporarily.
- varShipNumber: variable, used to hold the serial number of arriving ships temporarily.

4.3.3. *Attributes*

- atrGradeQuality: Used to assign a value as the quality of the arriving load to each train load entity.
- atrLoadSize: Used to assign a value as the load size to each arriving train load entity.
- atrOrderNumber: Used to assign a serial number to each arriving order entity.
- atrOrderSize: Used to assign a value as the order size to each arriving order entity.
- atrQuayNumber: Used to assign a value to determine the quay number that each ship should be loaded at.
- atrSeizedInputRegulator: Used to save the ID of the input regulator in the set of input regulators that is seized to stream the flow into the cells.
- atrSeizedOutputRegulator: Used to save the ID of the output regulator in the set of output regulators that is seized to stream the flow out of the cells.

4.3.4. *Resources*

- resQuay1,2 & 3: Represents each quay as a resource with specific scheduled capacity.

4.3.5. *Schedules*

- schQuay_1,2 & 3_Capacity: Represents the capacity schedule for each quay resource.

4.3.6. *Sets*

- setQuays: Set of all three quays.

4.3.7. *Tanks*

- A1, A2, B1 & B2: Tanks representing four separated cells for ore types A & B in the terminal.

4.3.8. *Regulator set*

- setInputRegulators_A & B: Set of input regulators to tanks A & B.
- setOutputRegulators_A & B: Set of output regulators from tanks A & B.

4.4. Modeling approach and Pseudo code

The simulation model has three main sub-models as “train unloading part”, “ship loading part” and stock yard area. Each sub-model is constructed separately as follows:

4.4.1. *Train unloading sub-model:*

- Trains arrival into the terminal: Use create module to create a train load entities that arrive into the terminal.
- Assign the train's load size: Use assign module to assign load size to each arriving train load.
- Sampling the train's load: Use process module to make a delay time as the sampling to determine the quality of each train load.
- Determine load type: Use decide module to differentiate between different load types (A or B).

For each ore type A or B:

- Seize input regulator A or B: Use seize regulator module to seize a regulator from the set of proper input regulators.
- Flow to A or B: Use flow module to let the flow to cells of ore type A or B.
- Release input regulator A or B: Use release regulator module to release the previously seized regulator from the set of input regulators.
- Dispose train load: Use dispose module to dispose the train load entities.

4.4.2. *Ship loading sub-model:*

- Orders arrival into the terminal: Use create module to create order entities that arrive into the terminal.
- Assign order number: Use assign module to assign serial number to each arriving order.
- Ships arrival into the terminal: Use create module to create ship entities that arrive into the terminal.
- Assign ship number: Use assign module to assign serial number to each arriving ship.
- Match order and ship: Use match module to check the similarity of arriving orders and ships according to their serial number.
- Batch order and ship entities: Use batch module to batch similar orders and ships and create new ship load entity.
- Assign Order type and size and quay number: Use assign module to assign order type (A or B), order size and also the quay number to each newly created ship load entity.
- Seize quay: Use process module to seize the assigned quay from the set of quays.
- Determine the order type: Use decide module to differentiate between different order types (A or B).

For each order type A or B:

- Seize output regulator A or B: Use seize regulator module to seize a regulator from the set of proper output regulators.
- Flow from A or B: Use flow module to let the flow from cells of ore type A or B.
- Release output regulator A or B: Use release regulator module to release the previously seized regulator from the set of output regulators.
- Decoupling from quay: Use process module to make a delay for decoupling of ship from quay.
- Dispose ship load: Use dispose module to dispose the ship load entities.

4.4.3. *Stock yard area:*

For each of four cells in the stock yard area:

- Create the cell: Use tank module to create the cell.

- Top level sensors: use two sensor modules to control the flow when the tank level is close to its maximum level.
- Low level sensors: use two sensor modules to control the flow when the tank level is close to its Minimum level.

4.5. A snapshot of the completed Arena model

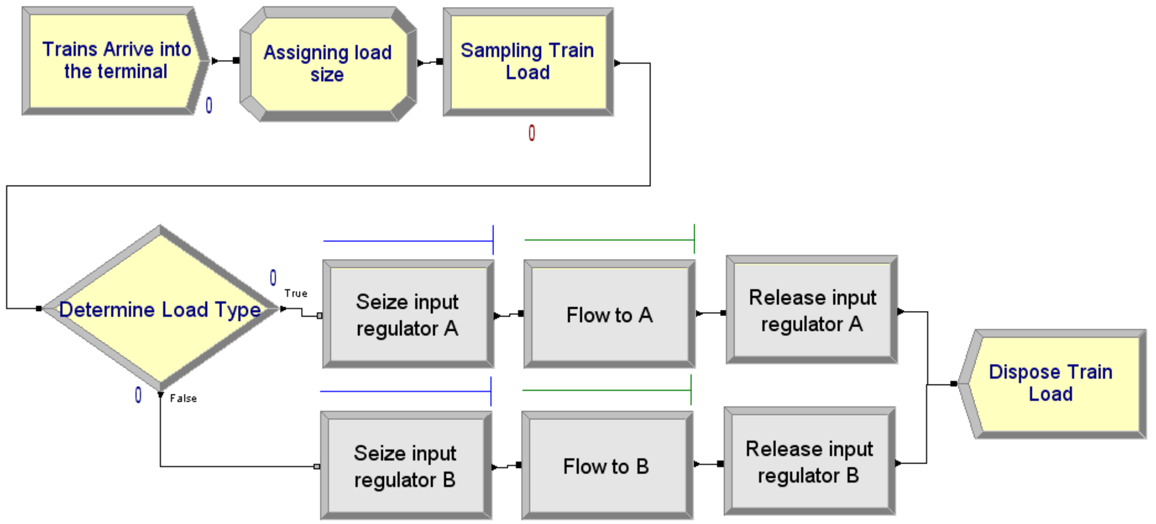

Fig. 2.Snapshot of the sub-model “train unloading part”.

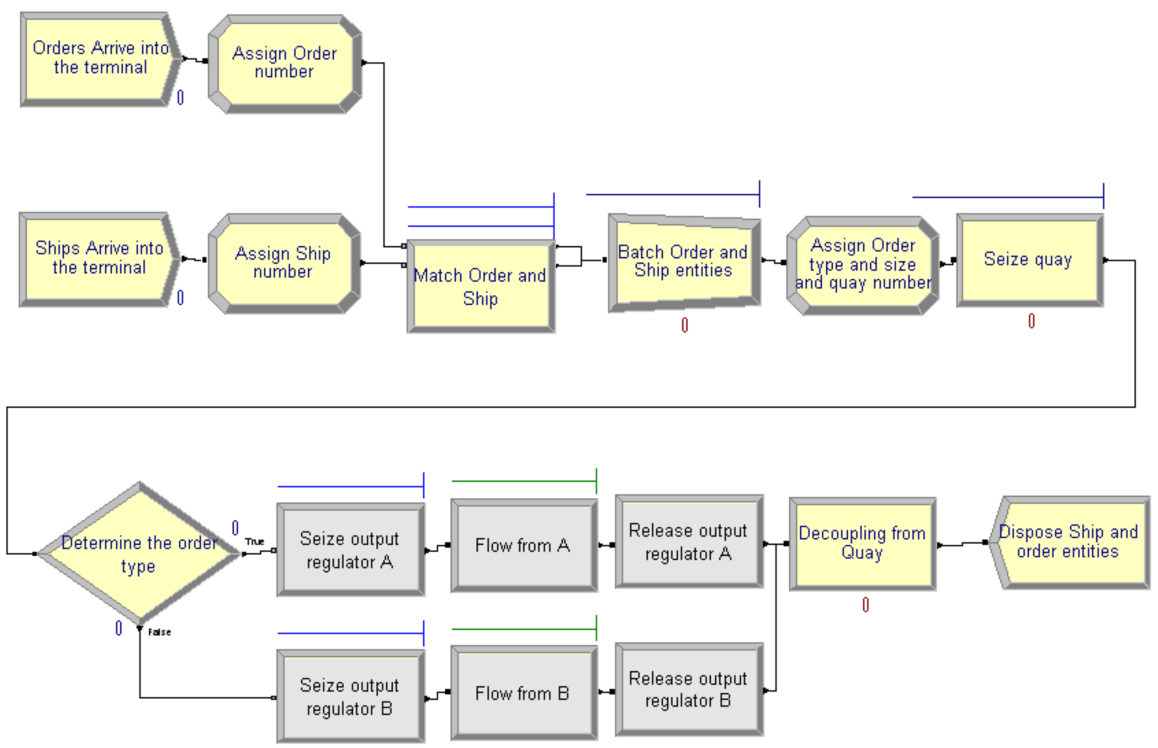

Fig. 3. Snapshot of the sub-model “ship loading part”.

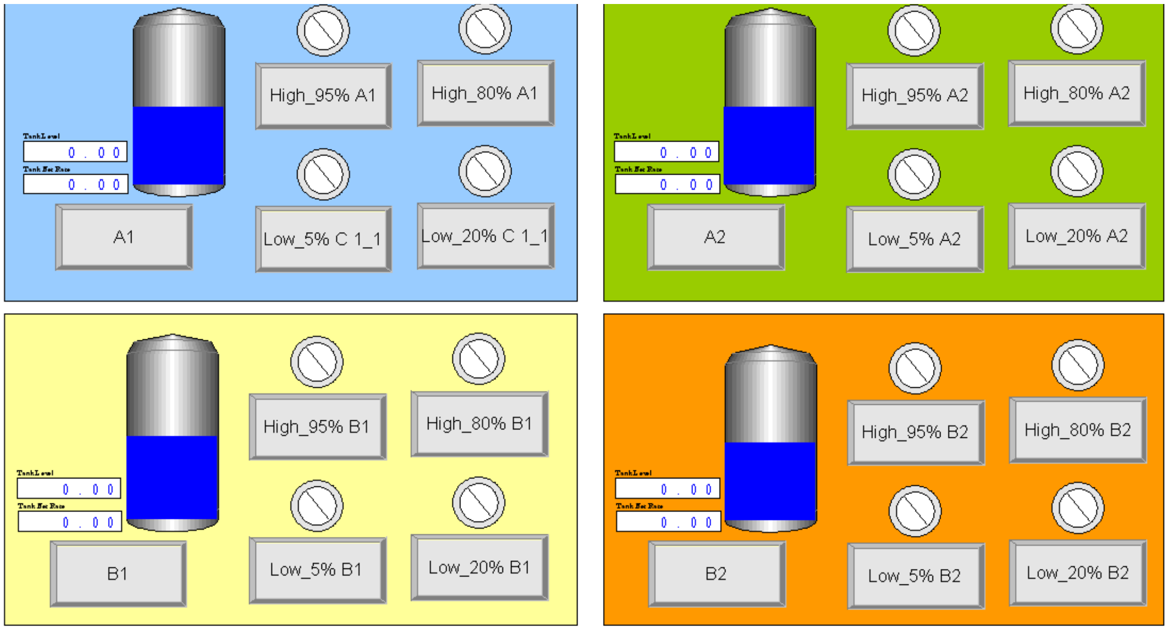

Fig. 4. Snapshot of the “stock yard area”.

5. Step by step modeling

The steps that are required to build the simulation model for three separated sub models of the problem are as follows:

TRAIN UNLOADING PART

5.1. Trains arrive into the terminal

Use create module

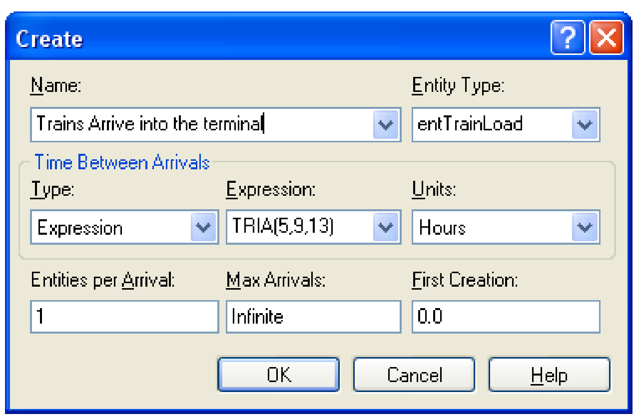

Fig. 5. Trains arrive into the terminal.

Name	Trains arrive into the terminal
Entity Type	ent TrainLoad
Type	Expression
Expression	TRIA(5,9,13)
Units	Hours
Entities per arrival	1
Max Arrivals	Infinite
First Creation	0.0

5.2. Assigning load size

Use assign module

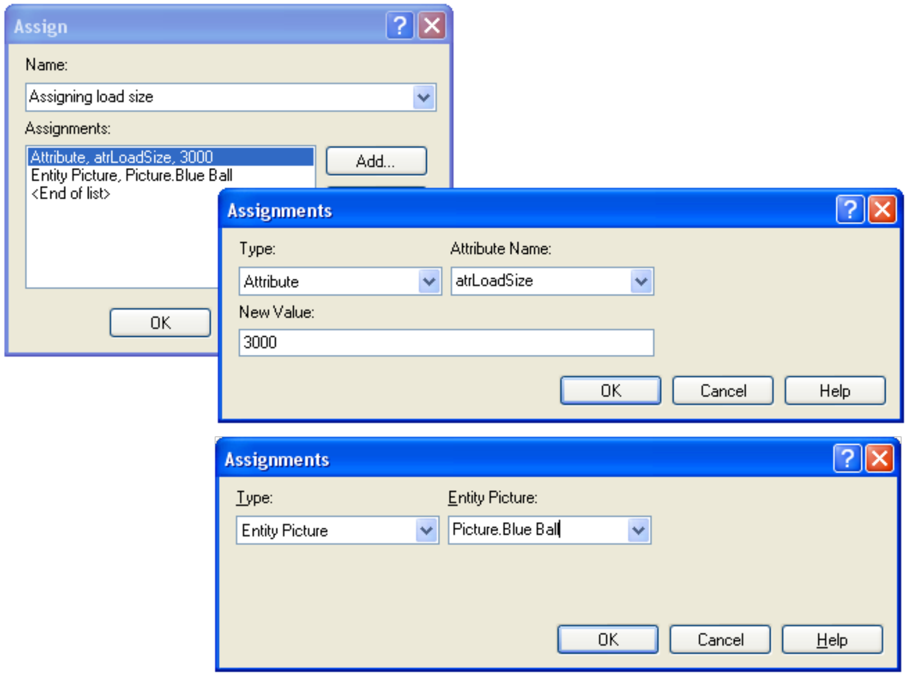

Fig. 6. Assign load size.

Name		Assigning load size
Assignments	Type	Attribute
	Attribute Name	atrLoadSize
	New Value	3000
	Type	Entity Picture
	Entity Picture	Picture.Blue Ball

5.3. Sampling train load

Use process module

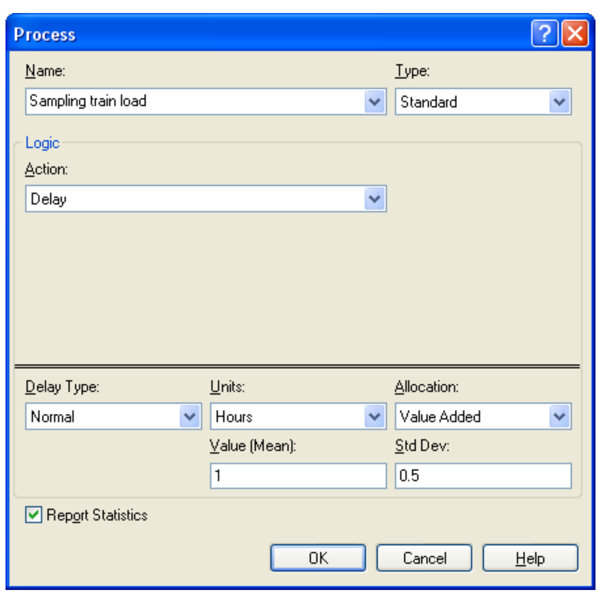

Fig. 7. Sampling train load.

Name	Sampling train load
Type	standard
Action	Delay
Delay Type	Normal
Units	Hours
Allocation	Value added
Value (Mean)	1
Std Dev	0.5

5.4. Define tanks and regulators

Use tank module from flow process panel

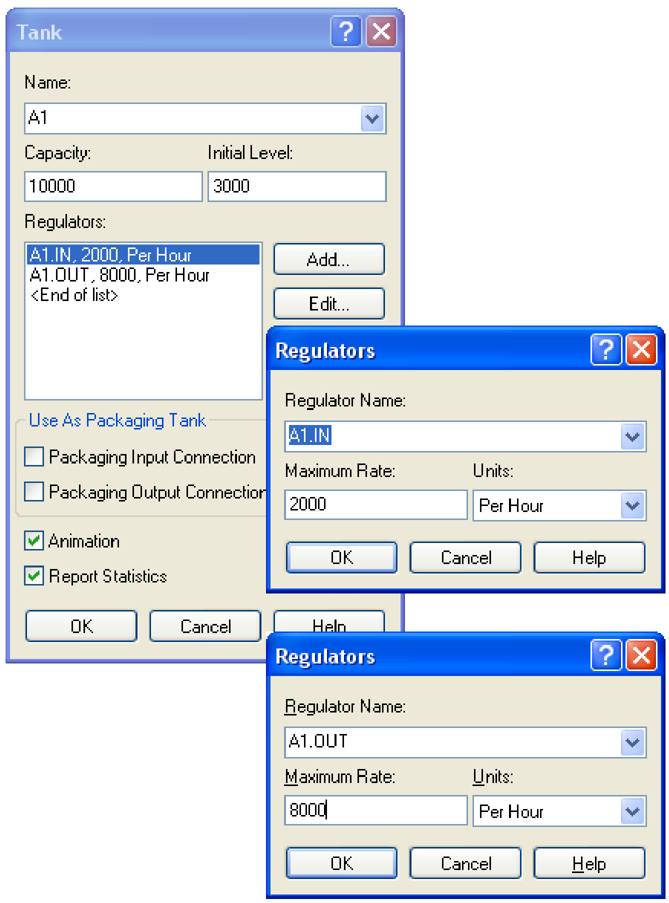

Fig. 8. Defining tanks and regulators.

Name	A1	
Capacity	10,000	
Initial Level	3,000	
regulators	Regulator Name	A1.IN
	Maximum Rate	2,000
	Units	Per Hour
	Regulator Name	A1.OUT
	Maximum Rate	8,000
	Units	Per Hour

Do the same for other three tanks A2, B1 and B2 (note the different capacities and regulator rates):

Name	A2		
Capacity	8,000		
Initial Level	2,000		
regulators	Regulator Name	A2.IN	
	Maximum Rate	1,000	
	Units	Per Hour	
	Regulator Name	A2.OUT	
	Maximum Rate	6,000	
	Units	Per Hour	

Name	B1	
Capacity	8,000	
Initial Level	2,000	
regulators	Regulator Name	B1.IN
	Maximum Rate	1,000
	Units	Per Hour
	Regulator Name	B1.OUT
	Maximum Rate	6,000
	Units	Per Hour

Name	B2	
Capacity	10,000	
Initial Level	3,000	
regulators	Regulator Name	B2.IN
	Maximum Rate	2,000
	Units	Per Hour
	Regulator Name	B2.OUT
	Maximum Rate	8,000
	Units	Per Hour

5.5. Define regulator sets

Use regulator set data module from flow process module

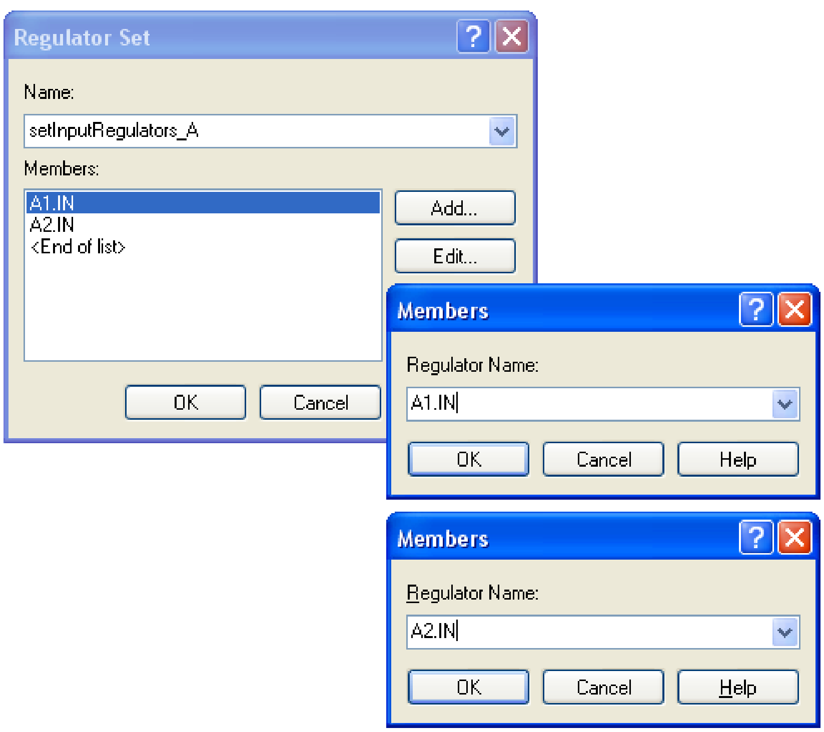

Fig. 9. Define regulator sets.

Name	setInputRegulators_A	
Members	Regulator Name	A1.IN
	Regulator Name	A2.IN

Do the same for other three sets for setOutputRegulators_A, setInputRegulators_B, setOutputRegulators_B:

Name	setOutputRegulators_A	
Members	Regulator Name	A1.OUT
	Regulator Name	A2.OUT

Name	setInputRegulators_B	
Members	Regulator Name	B1.IN
	Regulator Name	B2.IN

Name	setOutputRegulators_B	
Members	Regulator Name	B1.OUT
	Regulator Name	B2.OUT

5.6. Determine load type

Use decide module

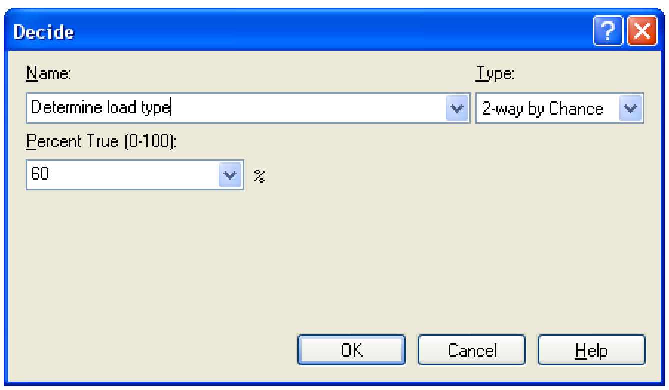

Fig. 10. Determine load type.

Name	Determine load type
Type	2-way by Chance
Percent True (0-100)	60

If the load was of type “A”:

5.7. Seize input regulator A

Use seize regulator module

Name	Seize input regulator A	
Priority	Medium(2)	
Regulators	Regulator Type	Regulator Set
	Regukator Set Name	setInputRegulators_A
	Selection Rule	Random
	Save Attribute	atrSeizedInputRegulator

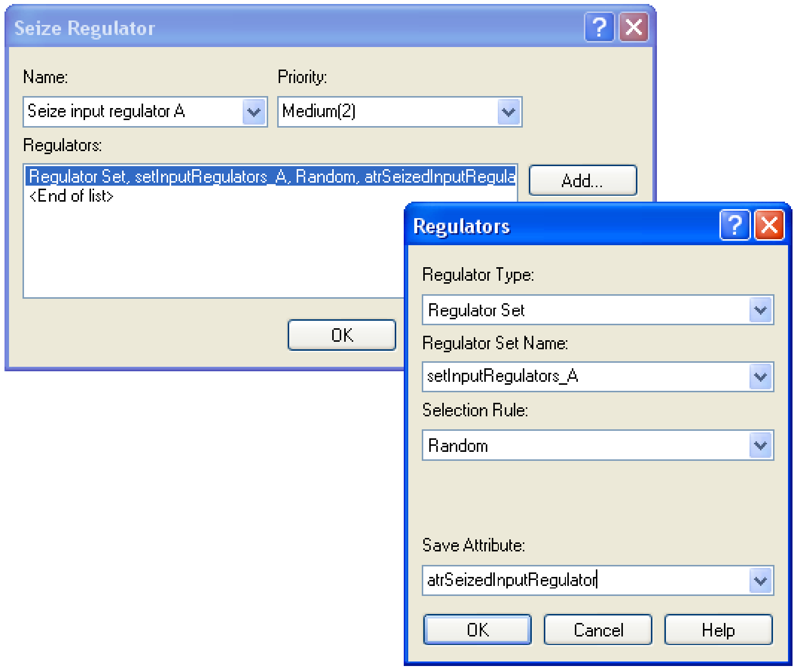

Fig. 11. Seize input regulator A.

5.8. Flow to A

Use flow module

Name	Flow to A
Type	Add
Destination Regulator Type	Regulator Set
Regukator Set Name	setInputRegulators_A
Set Index	atrSeizedInputRegulator
Stop Flow After Quantity	MN(atrLoadSize ,TankCapacity(A2))
Priority	Medium(2)
Allocation	Value Added

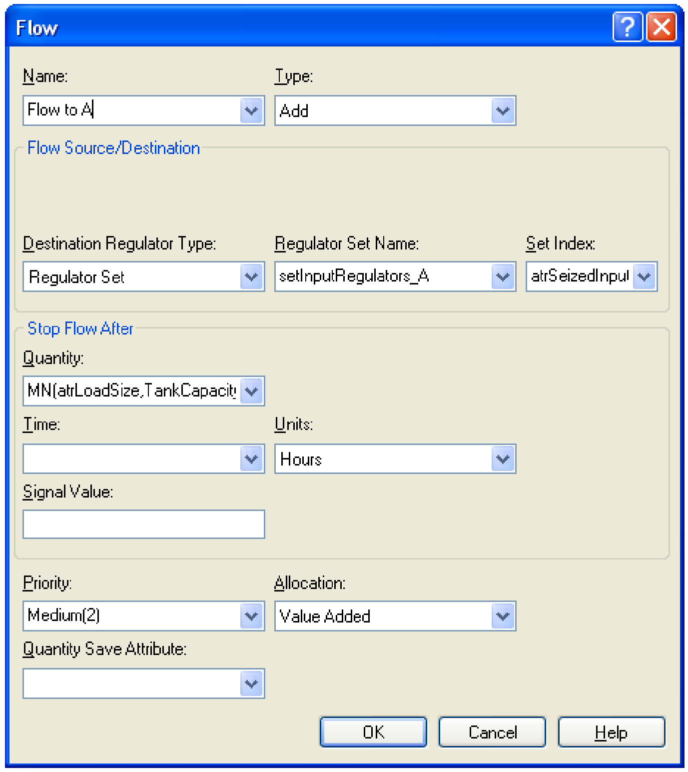

Fig. 12. Flow to A.

5.9. Release input regulator A

Use release regulator module

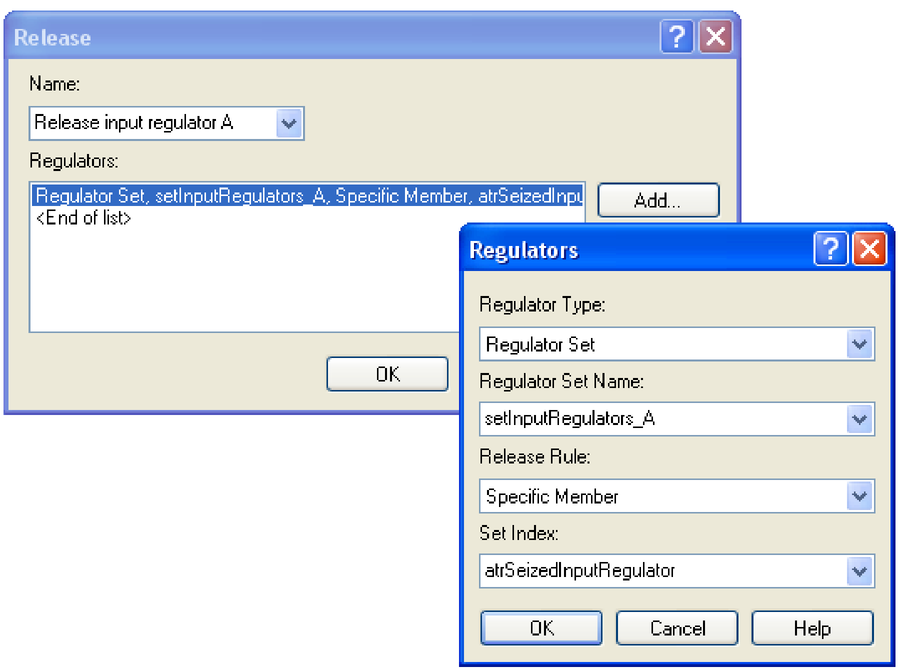

Fig. 13. Release input regulator A.

Name	Release input regulator A	
Regulators	Regulator Type	Regulator Set
	Regukator Set Name	setInputRegulators_A
	Selection Rule	Specific Member
	Save Attribute	atrSeizedInputRegulator

If the load was of type "B":

5.10. Seize input regulator B

Use seize regulator module

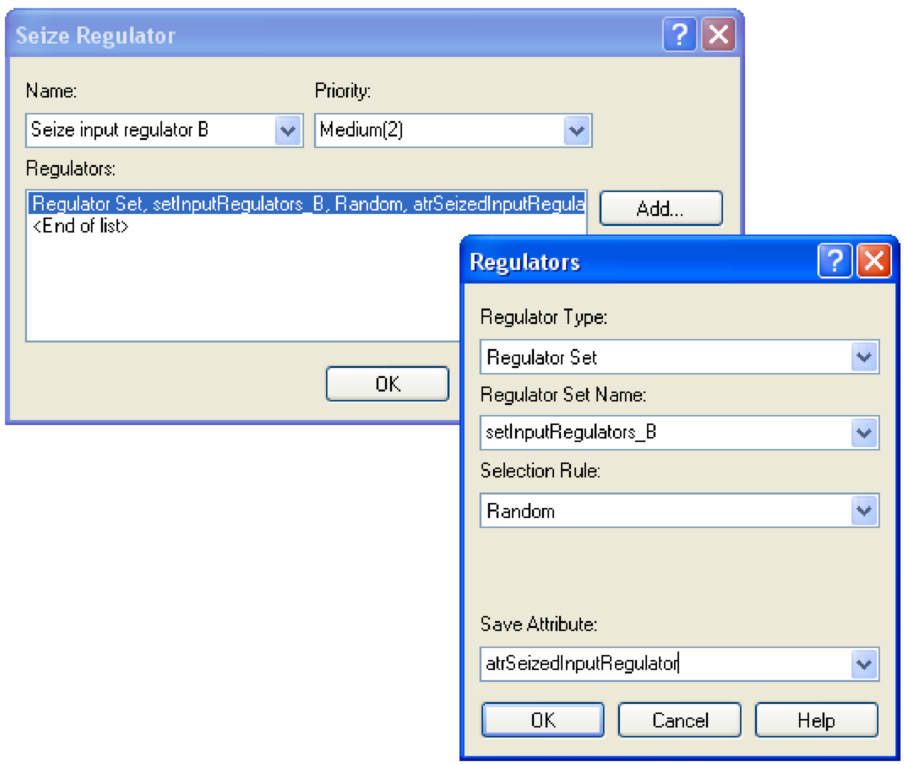

Fig. 14. Seize input regulator B.

Name	Seize input regulator B	
Priority	Medium(2)	
Regulators	Regulator Type	Regulator Set
	Regukator Set Name	setInputRegulators_B
	Selection Rule	Random
	Save Attribute	atrSeizedInputRegulator

5.11. Flow to B

Use flow module

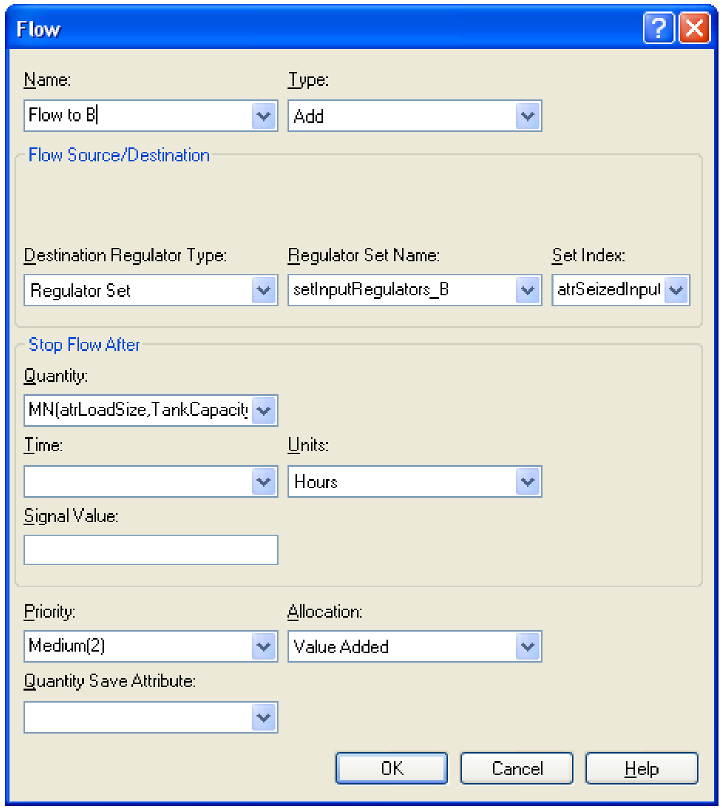

Fig. 15. Flow to B.

Name	Flow to B	
Type	Add	
Destination Regulator Type		Regulator Set
Regukator Set Name		setInputRegulators_B
Set Index		atrSeizedInputRegulator
Stop Flow After Quantity		MN(atrLoadSize,TankCapacity(B1))
Priority		Medium(2)
Allocation		Value Added

5.12. Release input regulator B

Use release regulator module

Name	Release input regulator B	
Regulators	Regulator Type	Regulator Set
	Regukator Set Name	setInputRegulators_B
	Selection Rule	Specific Member
	Save Attribute	atrSeizedInputRegulator

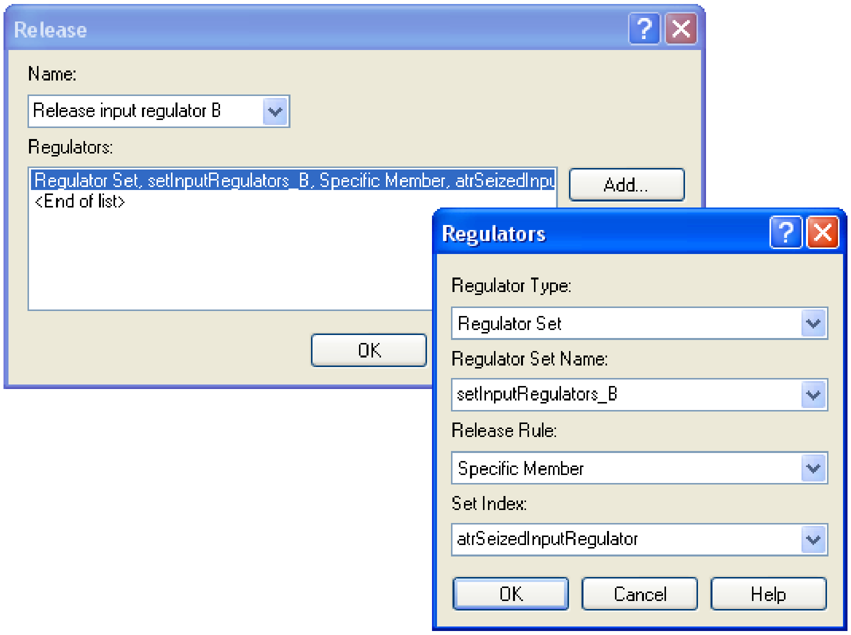

Fig. 16. Release input regulator B.

5.13. Dispose train load

Use dispose module

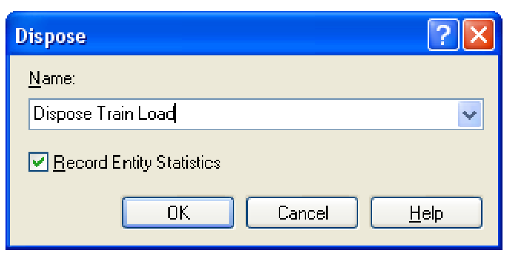

Fig. 17. Dispose train load.

Name	Dispose train load
Record Entity Statistics	checked

SHIP LOADING PART

5.14. Orders arrive into the terminal

Use create module

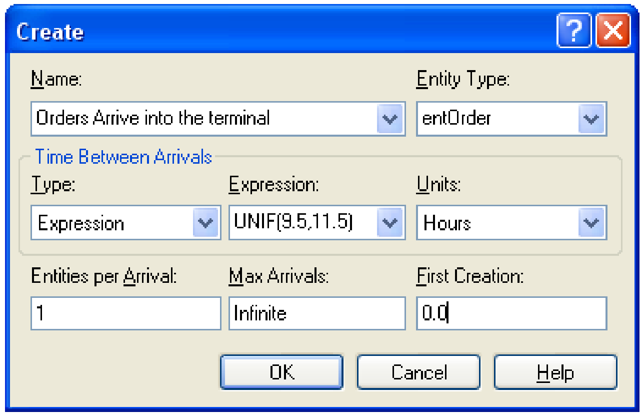

Fig. 18. Orders arrive into the terminal.

Name	Orders arrive into the terminal
Entity Type	ent Order
Type	Expression
Expression	UNIF(9.5,11.5)
Units	Hours
Entities per arrival	1
Max Arrivals	Infinite
First Creation	0.0

5.15. Assigning order number

Use assign module

<table>
<tr><td colspan="2">Name</td><td>Assigning order number</td></tr>
<tr><td rowspan="8">Assignments</td><td>Type</td><td>Variable</td></tr>
<tr><td>Attribute Name</td><td>varOrderNumber</td></tr>
<tr><td>New Value</td><td>varOrderNumber + 1</td></tr>
<tr><td>Type</td><td>Attribute</td></tr>
<tr><td>Attribute Name</td><td>atrOrderNumber</td></tr>
<tr><td>New Value</td><td>varOrderNumber</td></tr>
<tr><td>Type</td><td>Entity Picture</td></tr>
<tr><td>Entity Picture</td><td>Picture.Red Page</td></tr>
</table>

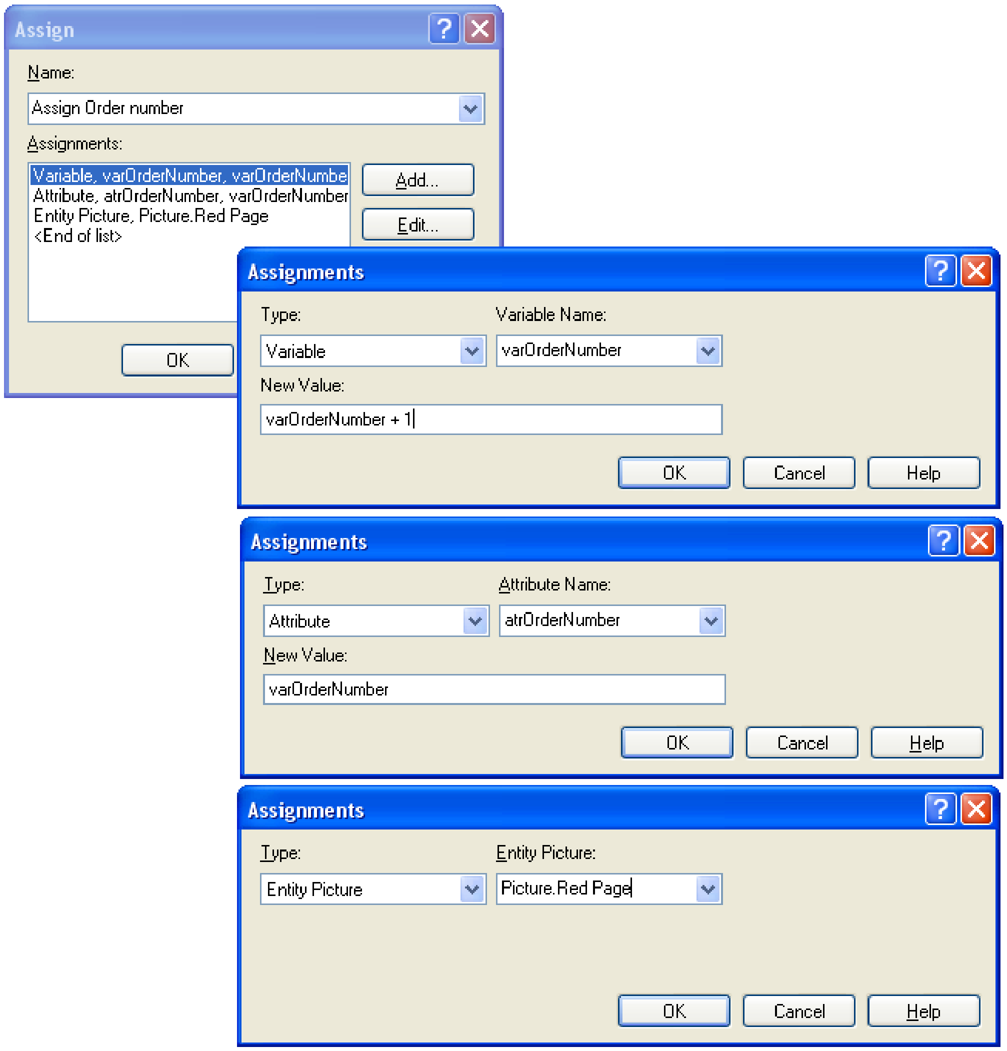

Fig. 19. Assign order number.

5.16. Ships arrive into the terminal

Use create module

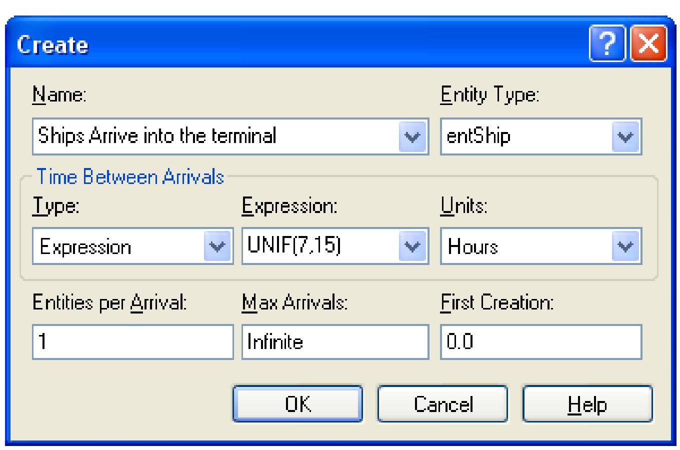

Fig. 20. Ships arrive into the terminal.

Name	Ships arrive into the terminal
Entity Type	ent Ship
Type	Expression
Expression	UNIF(7,15)
Units	Hours
Entities per arrival	1
Max Arrivals	Infinite
First Creation	0.0

5.17. Assigning ship number

Use assign module

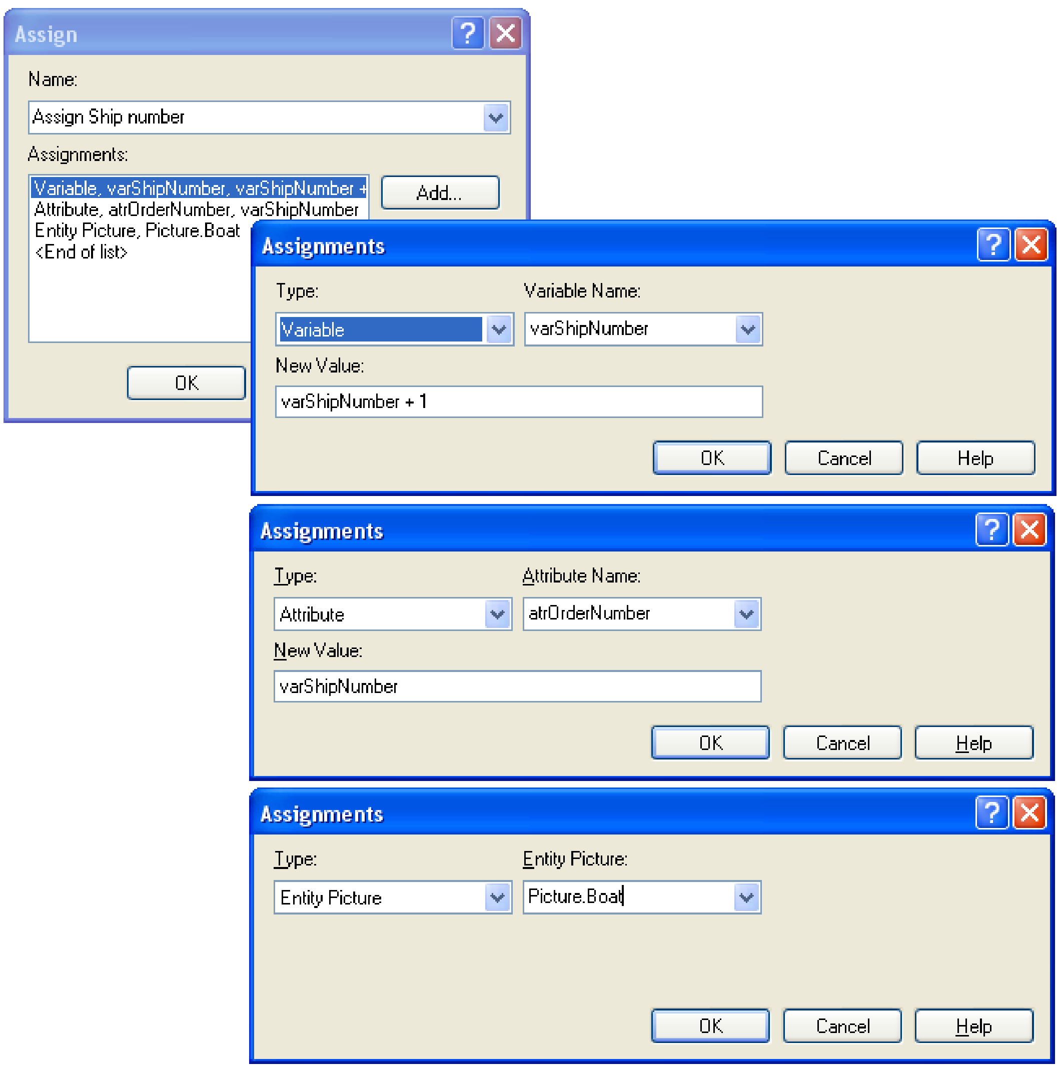

Fig. 21. Assign ship number.

Name		Assigning ship number
Assignments	Type	Variable
	Attribute Name	varShipNumber
	New Value	varShipNumber + 1
	Type	Attribute
	Attribute Name	atrOrderNumber
	New Value	varShipNumber
	Type	Entity Picture
	Entity Picture	Picture.Boat

5.18. Match order and ship

Use match module

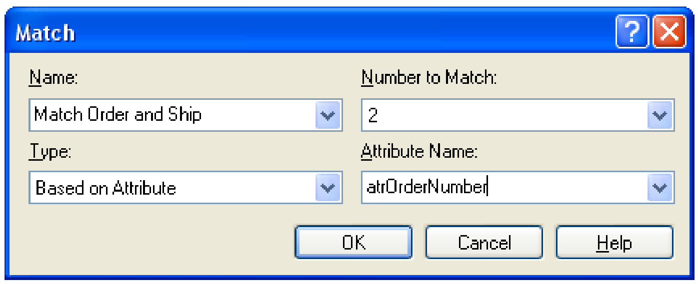

Fig. 22. Match order and ship.

Name	Match order and ship
Number to Match	2
Type	Based on Attribute
Attribute Name	atrOrderNumber

5.19. Batch order and ship entities

Use batch module

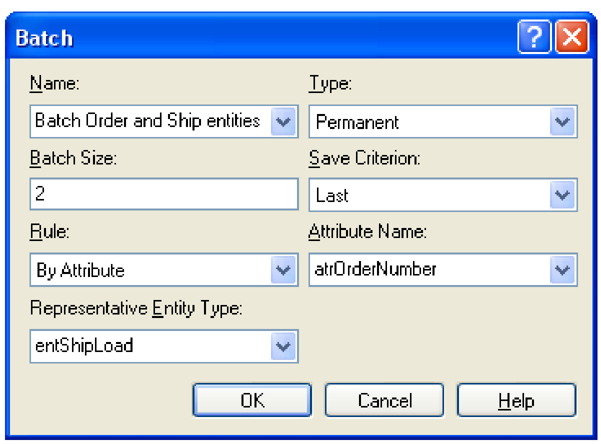

Fig. 23. Match order and ship.

Name	Batch order and ship entities
Type	Permanent
Batch Size	2
Save Criterion	By Attribute
Attribute Name	atrOrderNumber
Representative Entity Type	entShipLOad

5.20. Assigning order type and size and quay number

Use assign module

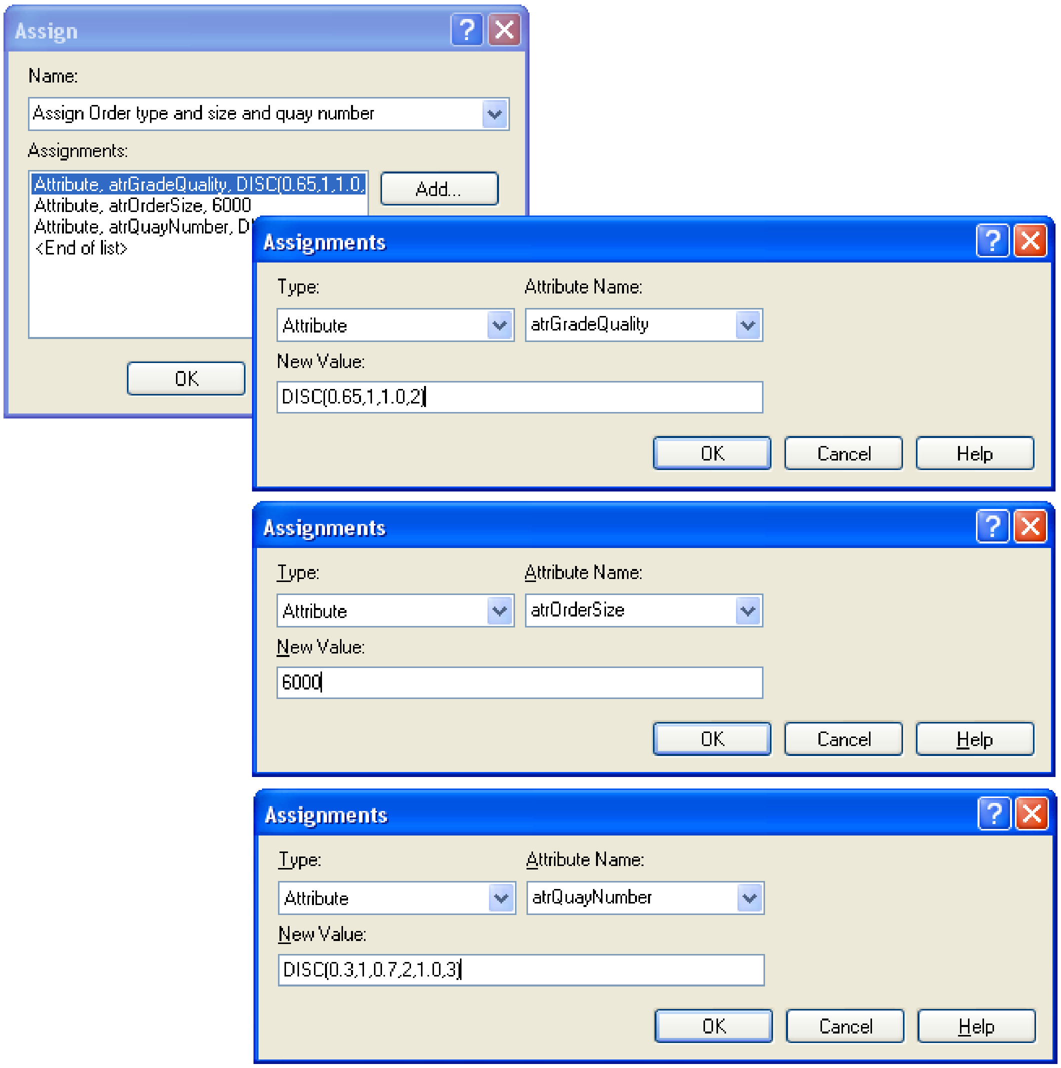

Fig. 24. Assign order type and size and quay number.

Name		Assigning order type and size and quay number
Assignments	Type	Attribute
	Attribute Name	atrGradeQuality
	New Value	DISC(0.65 , 1 , 1.0 , 2)
	Type	Attribute
	Attribute Name	atrOrderSize
	New Value	6000
	Type	Attribute
	Attribute Name	atrQuayNumber
	New Value	DISC(0.3 , 1 , 0.7 , 2 , 1.0 , 3)

5.21. Define schedule for quays

Use schedule data module

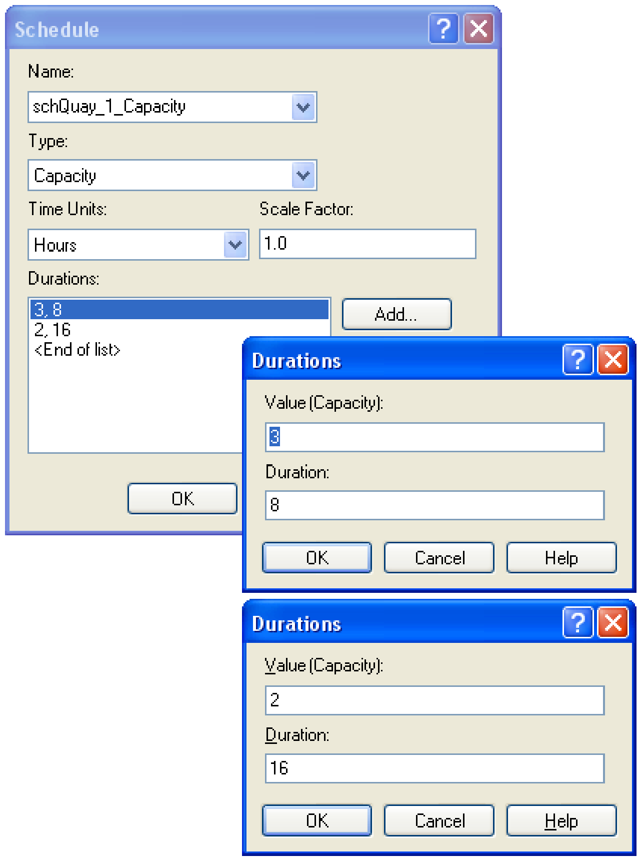

Fig. 25. Define schedule for quay 1.

Name		schQuay_1_Capacity
Type		Capacity
Time Units		Hours
Scale Factor		1
Durations	Value (Capacity)	3
	Duration	8
	Value (Capacity)	2
	Duration	16

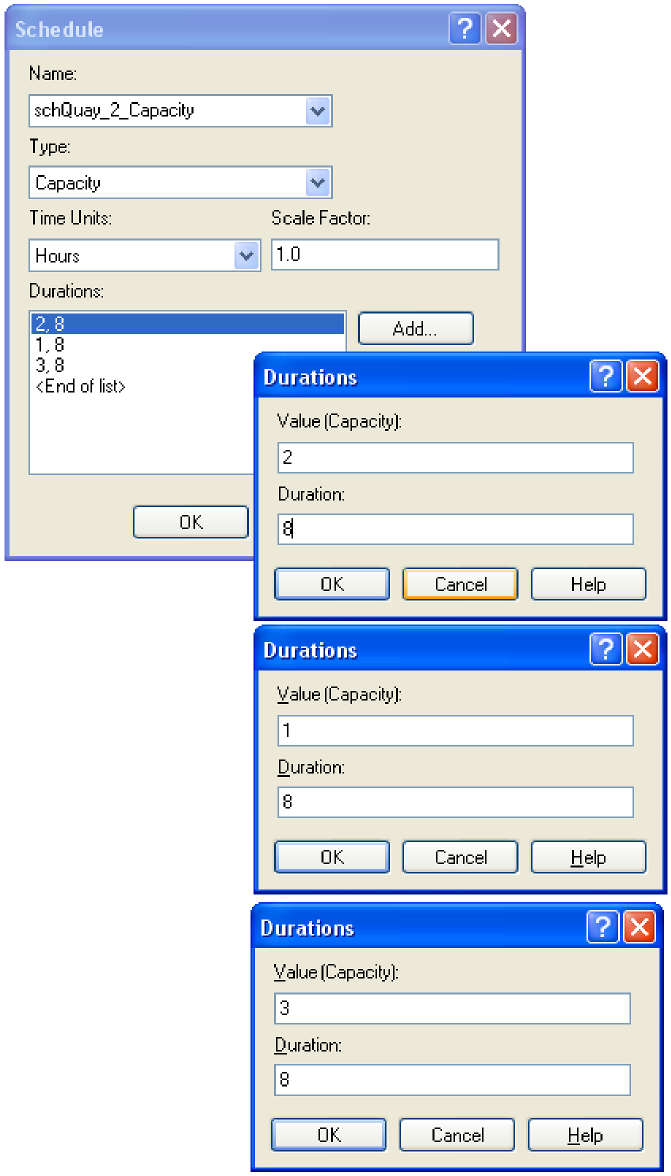

Fig. 26. Define schedule for quay 2.

Name		schQuay_2_Capacity
Type		Capacity
Time Units		Hours
Scale Factor		1
Durations	Value (Capacity)	2
	Duration	8
	Value (Capacity)	1
	Duration	8
	Value (Capacity)	3
	Duration	8

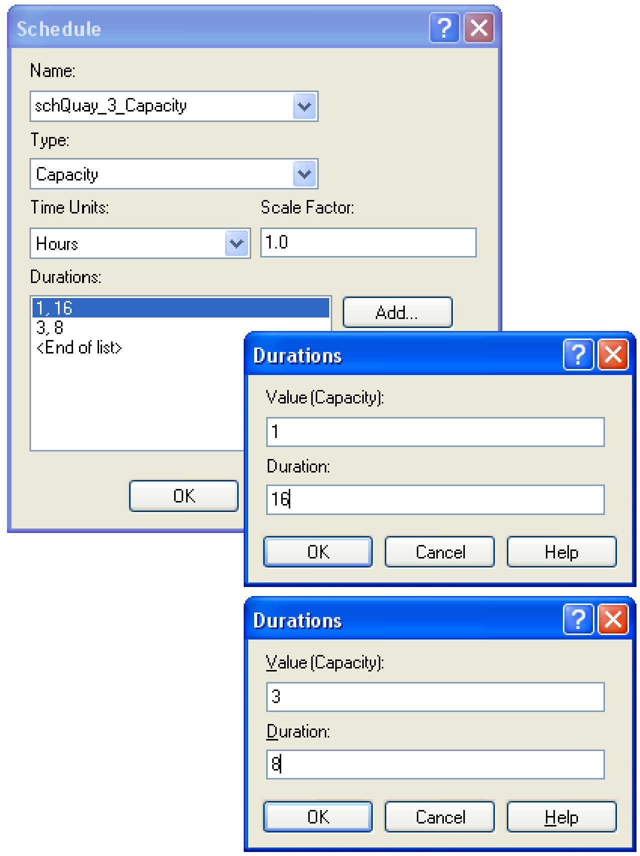

Fig. 27. Define schedule for quay 3.

Name		schQuay_3_Capacity
Type		Capacity
Time Units		Hours
Scale Factor		1
Durations	Value (Capacity)	1
	Duration	16
	Value (Capacity)	3
	Duration	8

5.22. Define quays as resources

Use resource data module

Name	resQuay1
Type	Based on Schedule
Schedule Name	schQuay_1_Capacity
Schedule Rule	Wait
Costs	0.0
Failure	---

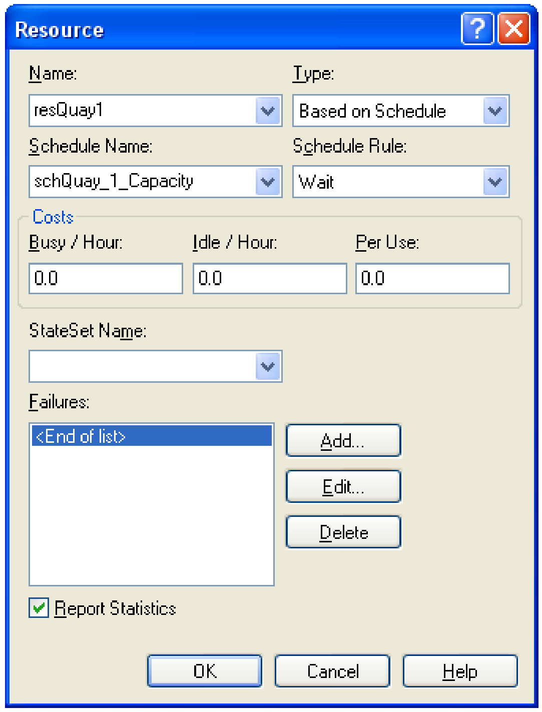

Fig. 28. Define quay 1 as resource.

Name	resQuay2
Type	Based on Schedule
Schedule Name	schQuay_2_Capacity
Schedule Rule	Wait
Costs	0.0
Failure	---

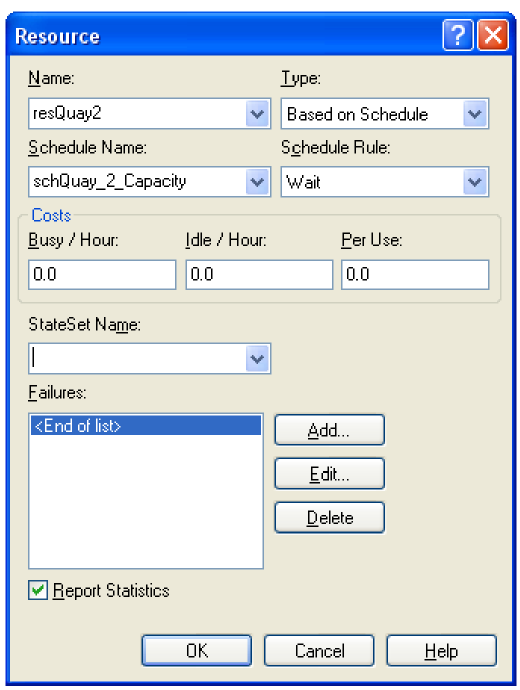

Fig. 29. Define quay 2 as resource.

Name	resQuay3
Type	Based on Schedule
Schedule Name	schQuay_3_Capacity
Schedule Rule	Wait
Costs	0.0
Failure	---

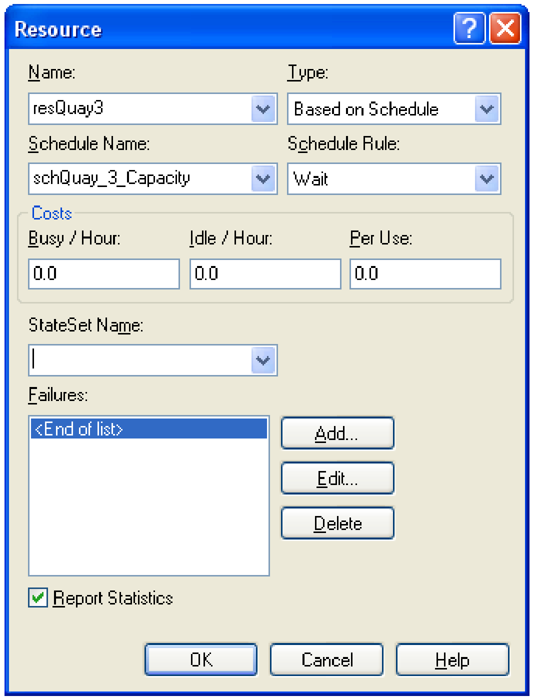

Fig. 30. Define quay 3 as resource.

5.23. Define resource set for quays

Use set data module

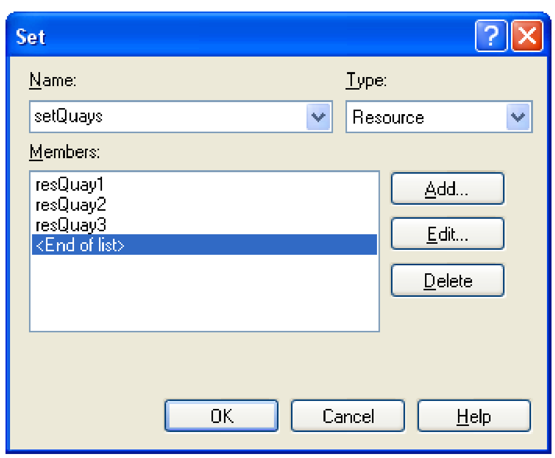

Fig. 31. Define resource set for quays.

Name	setQuays
Type	Resource
Members	resQuay1
	resQuay2
	resQuay3

5.24. Seize quay

Use process module

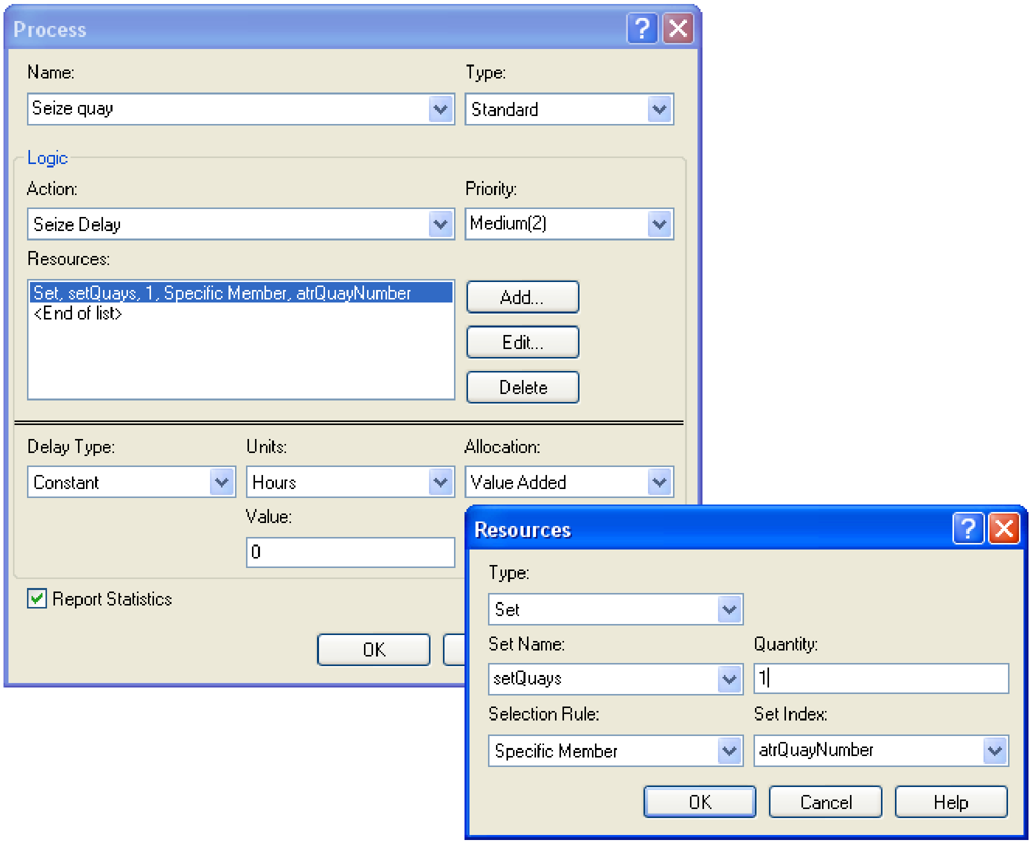

Fig. 32. Seize quay.

Name	Seize quay	
Type	Standard	
Action	Seize Delay	
Priority	Medium(2)	
Resources	Type	Set
	Set Name	setQuays
	Quantity	1
	Selection Rule	Specific Member
	Set Index	atrQuayNumber
Delay Type	Constant	
Units	Hours	
Allocation	Value Added	
Value	0.0	
Report Statistics	Checked	

5.25. Determine the order type

Use decide module

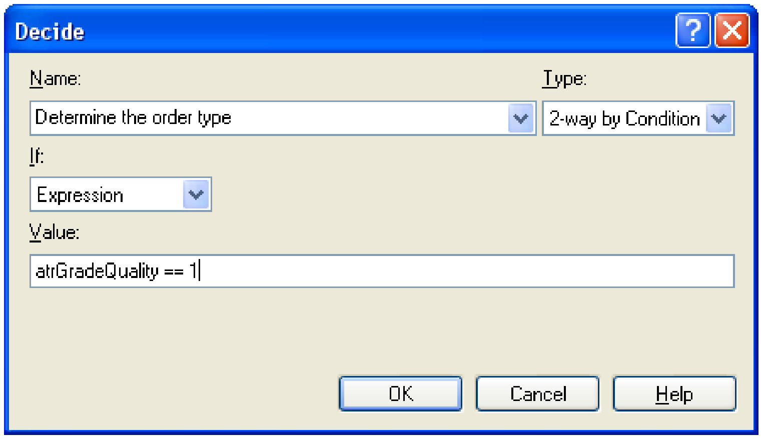

Fig. 33. Determine the order type.

Name	Determine the order type
Type	2-way by Condition
If	Expression
Value	atrGradeQuality == 1

If the order was of type “A”:

5.26. Seize output regulator A

Use seize regulator module

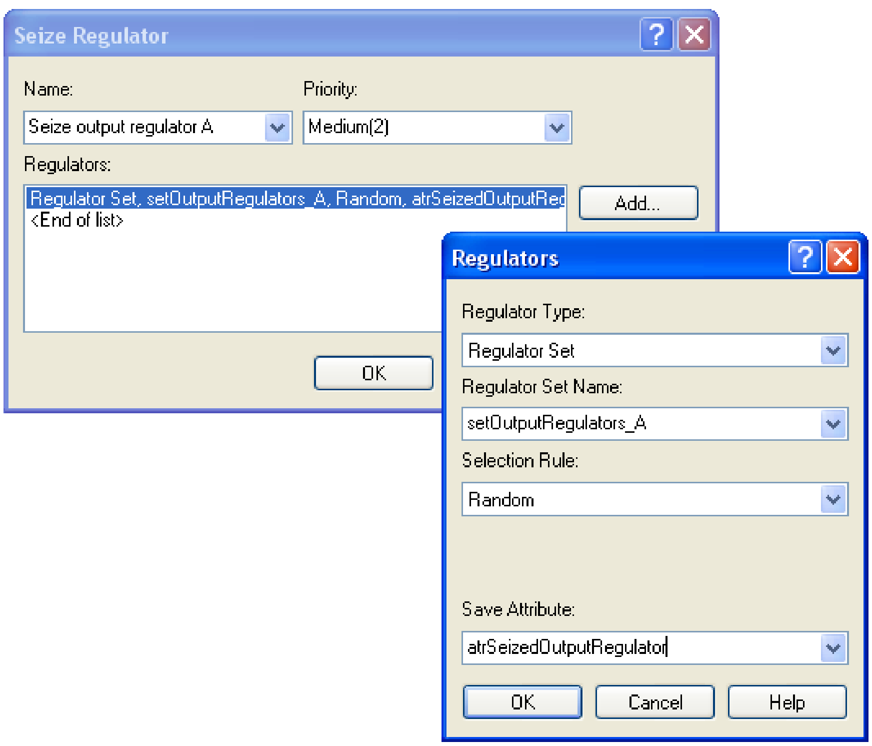

Fig. 34. Seize output regulator A.

Name	Seize output regulator A	
Priority	Medium(2)	
Regulators	Regulator Type	Regulator Set
	Regukator Set Name	setOutputRegulators_A
	Selection Rule	Random
	Save Attribute	atrSeizedOutputRegulator

5.27. Flow from A

Use flow module

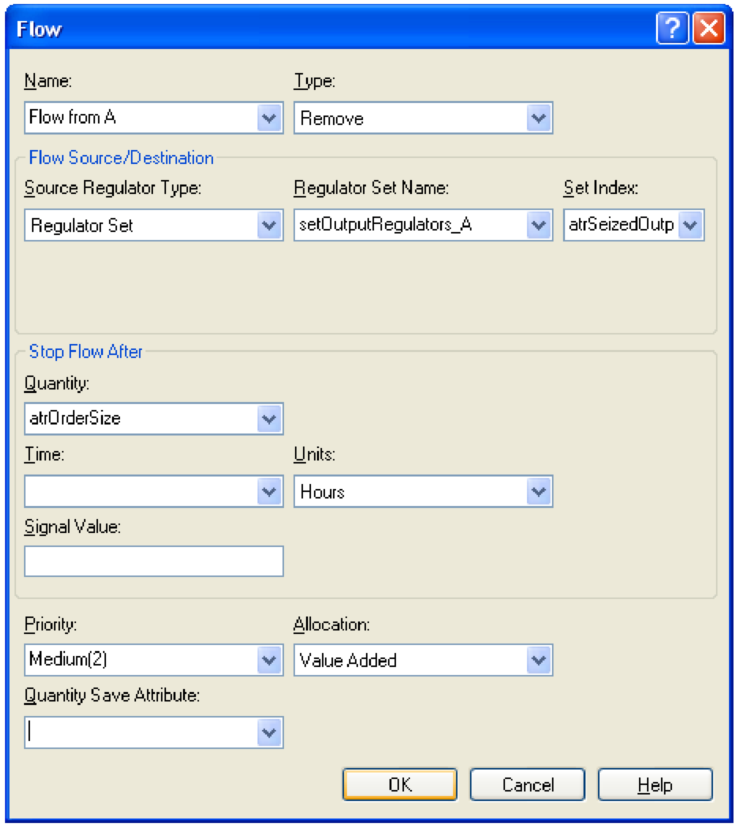

Fig. 35. Flow from A.

Name	Flow from A
Type	remove
Destination Regulator Type	Regulator Set
Regukator Set Name	setOutputRegulators_A
Set Index	atrSeizedOutputRegulator
Stop Flow After Quantity	atrOrderSize
Priority	Medium(2)
Allocation	Value Added

5.28. Release output regulator A

Use release regulator module

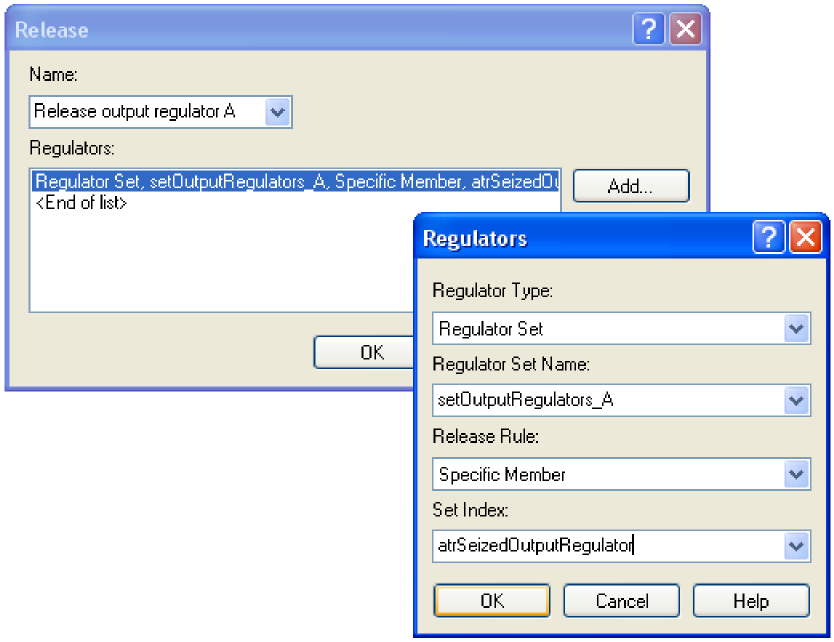

Fig. 36. Release output regulator A.

Name	Release output regulator A	
Regulators	Regulator Type	Regulator Set
	Regukator Set Name	setOUputRegulators_A
	Selection Rule	Specific Member
	Save Attribute	atrSeizedOUputRegulator

If the load was of type "B":

5.29. Seize output regulator B

Use seize regulator module

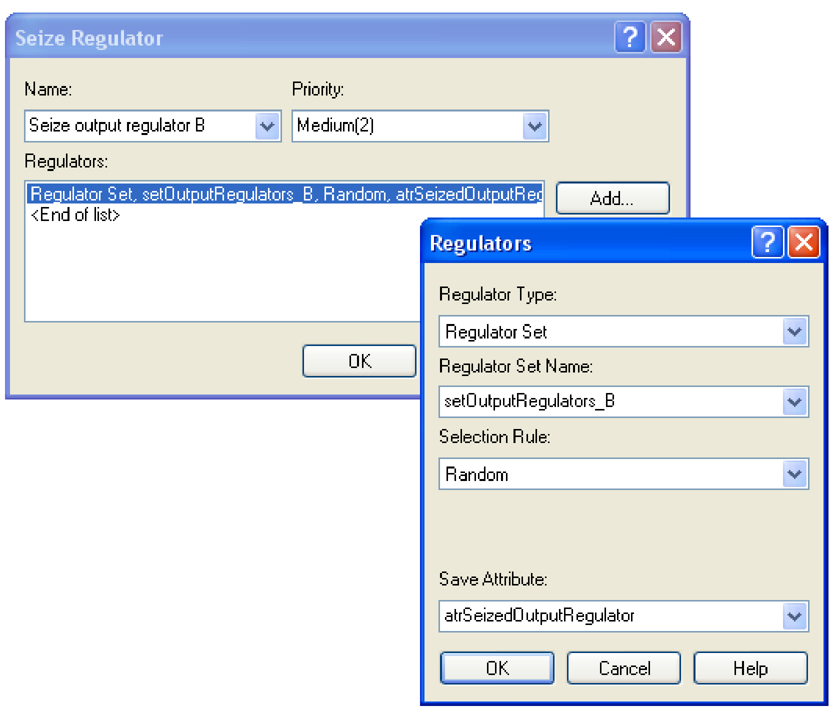

Fig. 37. Seize output regulator B.

Name	Seize output regulator B	
Priority	Medium(2)	
Regulators	Regulator Type	Regulator Set
	Regukator Set Name	setOutputRegulators_B
	Selection Rule	Random
	Save Attribute	atrSeizedOutputRegulator

5.30. Flow from B

Use flow module

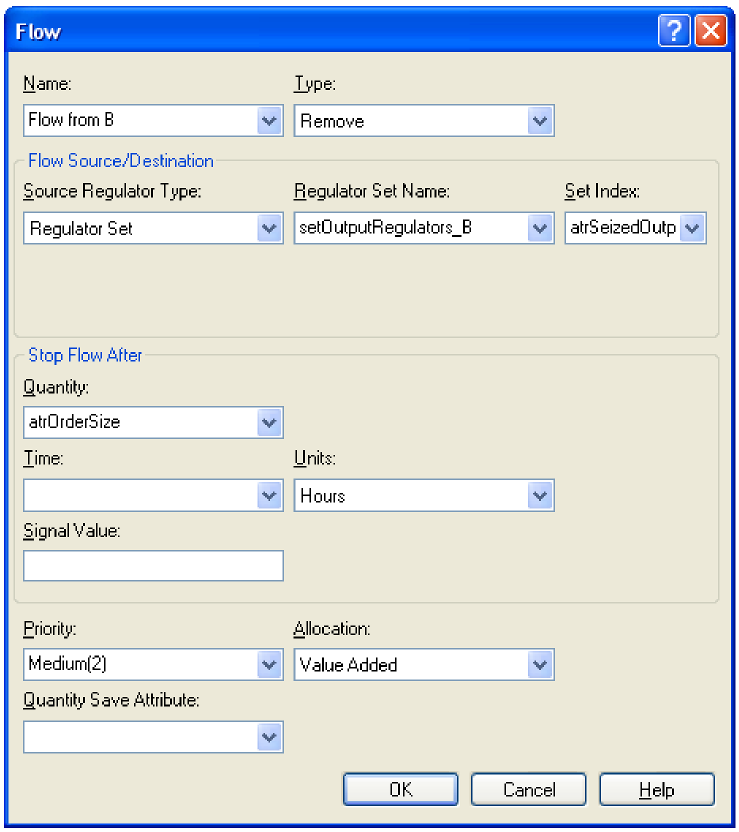

Fig. 38. Flow from B.

Name	Flow from B
Type	remove
Destination Regulator Type	Regulator Set
Regukator Set Name	setOutputRegulators_B
Set Index	atrSeizedOutputRegulator
Stop Flow After Quantity	atrOrderSize
Priority	Medium(2)
Allocation	Value Added

5.31. Release input regulator B

Use release regulator module

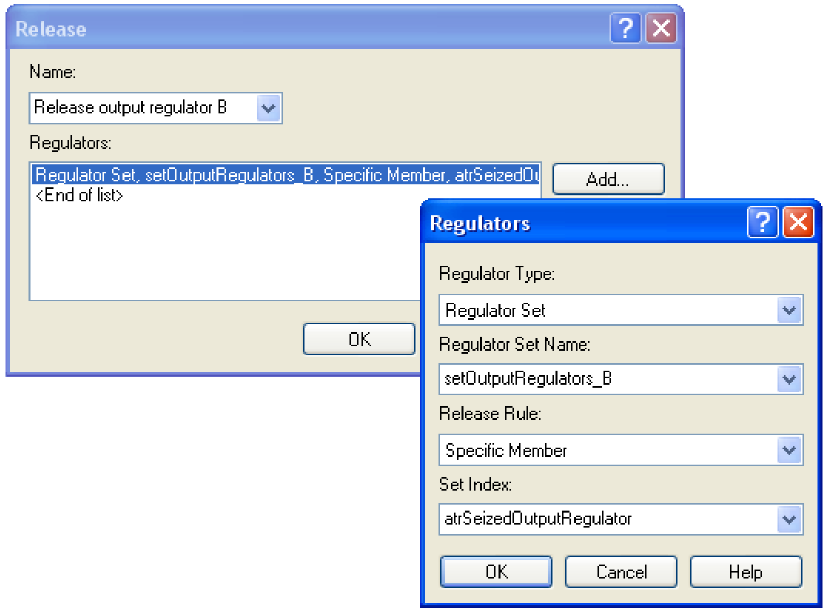

Fig. 39. Release output regulator B.

Name	Release output regulator B	
Regulators	Regulator Type	Regulator Set
	Regukator Set Name	setOUputRegulators_B
	Selection Rule	Specific Member
	Save Attribute	atrSeizedOUputRegulator

5.32. Decoupling from quay

Use process module

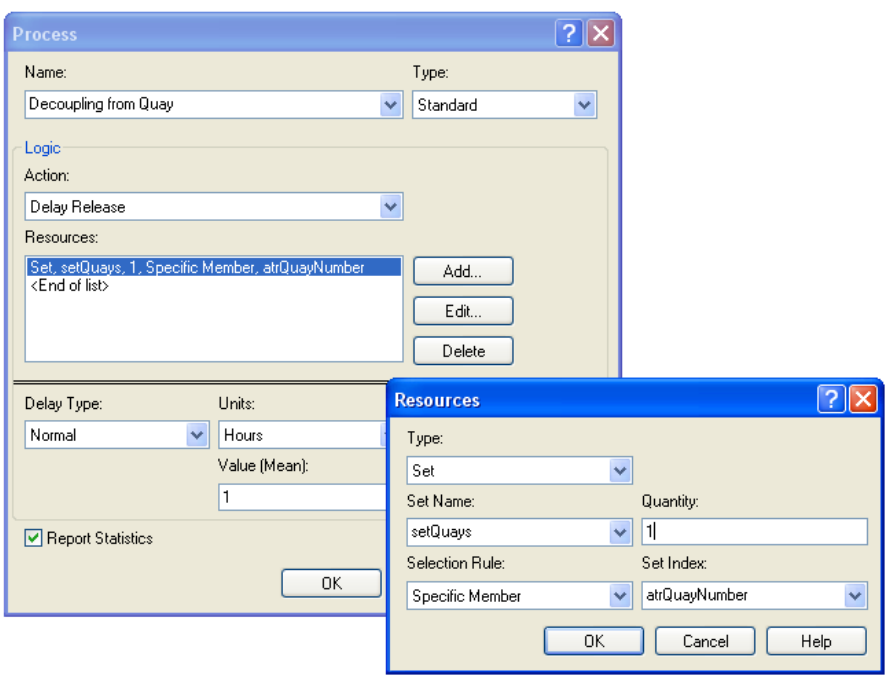

Fig. 40. Decoupling from quay.

Name	Decoupling from quay	
type	standard	
Action	Delay Release	
Resources	Type	Set
	Set Name	setQuays
	Quantity	1
	Selection Rule	Specific Member
	Set Index	atrQuayNumber
Delay Type	Normal	
Units	Hours	
Allocation	Value Added	
Value (Mean)	1	
Std Dev	0.25	
Report Statistics	Checked	

5.33. Dispose ship and order entities

Use dispose module

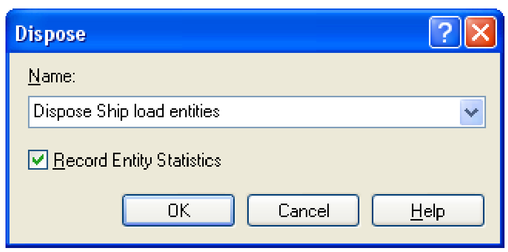

Fig. 41. Dispose ship load entities.

Name	Dispose ship load entities
Record Entity Statistics	checked

STOCK YARD AREA

Tanks and regulators are already created. The only modules that are left are sensors.

Use sensor module from flow process panel

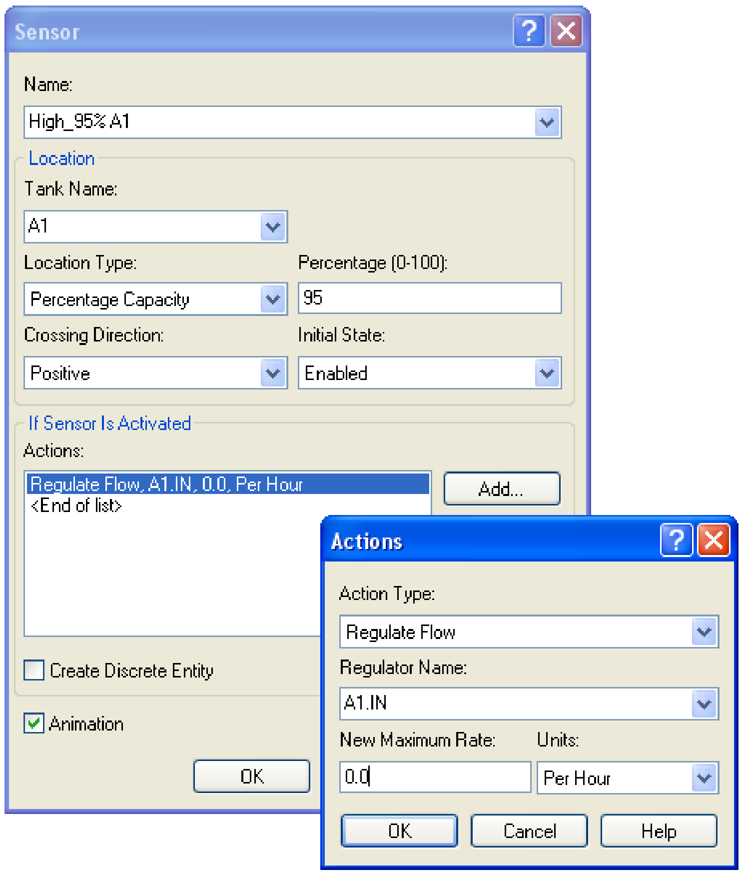

Fig. 42. Define sensors.

Name	High_95%A1	
Tank Name	A1	
Location Type	Percentage Capacity	
Percentage (0-100)	95	
Crossing Direction	Positive	
Initial State	Enabled	
Actions	Action Type	Regulate Flow
	Regulator Name	A1.IN
	New Maximum Rate	0.0
	Units	Per Hour
Create discrete event	---	
Animation	Checked	

Do the same for other three sensors of tank A1 as:

Name	High_85%A1	
Tank Name	A1	
Location Type	Percentage Capacity	
Percentage (0-100)	85	
Crossing Direction	Negative	
Initial State	Enabled	
Actions	Action Type	Regulate Flow
	Regulator Name	A1.IN
	New Maximum Rate	RegulatorMaxRate(A1.IN)
	Units	Per Hour
Create discrete event	---	
Animation	Checked	

Name	Low_5%A1	
Tank Name	A1	
Location Type	Percentage Capacity	
Percentage (0-100)	5	
Crossing Direction	Negative	
Initial State	Enabled	
Actions	Action Type	Regulate Flow
	Regulator Name	A1.OUT
	New Maximum Rate	0.0
	Units	Per Hour
Create discrete event	---	
Animation	Checked	

Name	Low_10%A1	
Tank Name	A1	
Location Type	Percentage Capacity	
Percentage (0-100)	10	
Crossing Direction	Positive	
Initial State	Enabled	
Actions	Action Type	Regulate Flow
	Regulator Name	A1.OUT
	New Maximum Rate	RegulatorMaxRate(A1.OUT)
	Units	Per Hour
Create discrete event	---	
Animation	Checked	

Exactly the same to the four sensors for tank A1, create four sensors for each of tanks A2, B1 and B2.

5.34. Define replication parameters

Use Run / setup

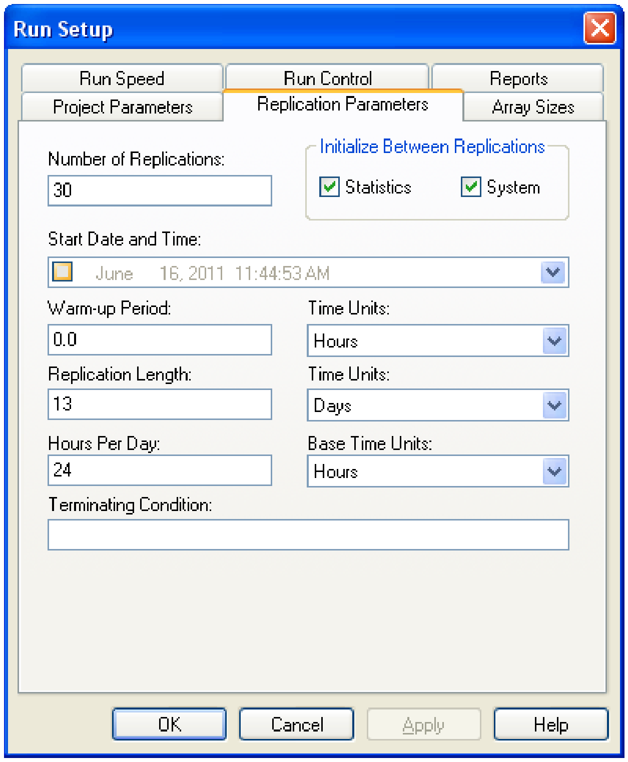

Fig. 43. Define replication parameters.

Number of replications	30
Warm-up Period	0.0
Time Units	Hours
Replication Length	13
Time Units	Days
Hours Per Day	24
Base Time Units	Hours
Terminating Condition	-

6. References

[1] Rockwell. (2010). Arena Simulation Softwre. 13.0 ed. Pittsburgh: Rockwell Automation.

Step by Step Guidelines for Modeling Coal Storage Bin Design using Arena

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1. Introduction

Simulation models become handy when an engineer is looking forward to modifying the system in pursuit of a specific goal. It can also be useful in finding the bottlenecks of the system to investigate what can be done to resolve the issues with the bottle neck. In this example a design engineer is trying to identify the system bottleneck, propose solutions and test the effects of the solutions on the system. This document serves as a step by step guideline to create the model in Arena Simulation Software (Rockwell, 2010). The problem definition is based on example number 10 from the book "Mine design: examples using simulation" by Sturgul (2000).

2. Learning Objectives

By the end of this lab you should be able to:

- Model storage bins/stockpiles
- Use continuous simulation modules such as tank, signal, regulators etc.
- Combine discrete and continuous models
- Illustrate variable values in a plot
- Modify inter-arrival times during the simulation
- Use n-way conditions
- Hold entities until a specific condition is met

3. Concepts and Terminology

Some new modules introduced in this lab are explained briefly in this section. The explanations are brought from the Arena help file.

3.1. Tank Module

The Tank module defines a location where material is held or stored.

The Capacity is the maximum quantity that may be stored in the tank. The Initial Level is the quantity in the tank at the beginning of the simulation or when the system is cleared.

The Regulators repeat group specifies a list of devices that may be used to add or remove material from the tank (e.g., valves or pumps) at a specified rate. A regulator is a device (e.g., a valve) that is used to add or remove material from a tank. A tank's regulators are defined in the Tank module. Semi-continuous flow operations may then be executed by discrete entities using the regulators via the Flow module.

3.2. Seize Regulator Module

A tank regulator may be used for only one flow operation at any given time. The Seize Regulator module may be used to control ownership of regulators and thus avoid situations of multiple entities trying to simultaneously use the same regulator in a Flow module. This module is also useful for choosing from a set of alternative regulators using a selection rule.

The Seize Regulator module allocates one or more regulators to an entity. When an entity enters this module, it waits in a queue until all specified regulators are available simultaneously. An allocated regulator is released by an entity using the Release Regulator module

3.3. Flow Module

The Flow module is used in conjunction with the Tank module to model semi-continuous flow operations such as adding material to a tank, removing material from a tank, or transferring material between two tanks.

When an entity enters the Flow module, a flow operation of the specified Type is initiated (i.e., Add, Transfer, or Remove). The tanks affected by the flow are determined by the regulators specified as the source and/or destination.

The entity is held in the Flow module until its flow operation is completed. The flow operation is completed when the first of three possible conditions is satisfied:

- When the specified Quantity has been transferred.
- When the specified Time has elapsed.
- When the specified Signal Value is received via a Signal module.

The flow rate of the entity is regulated according to the maximum rate(s) of the source regulator and/or destination regulator. Additionally, the flow rate may be further constrained due to starvation or blocking from empty or full tanks. The initial maximum rate of a regulator is specified in the Tank module.

3.4. Release Regulator Module

The Release Regulator module is used to release tank regulators that have been allocated to an entity using the Seize Regulator module. This makes those regulators available to other entities waiting to seize the regulator(s).

When the entity enters the Release Regulator module, it gives up control of the specified regulator(s). Any entities waiting in queues for those regulators will gain control of the regulators immediately.

3.5. Sensor Module

The Sensor module defines a detection device that monitors the level of material in a tank (Tank module). A sensor's location is specified using the Tank Name, Location Type, and Level/Percentage prompts.

A sensor is activated (triggered) when its location is crossed by the tank level in the Crossing Direction AND the sensor is enabled. When the sensor is activated, one or more Actions may be executed. Additionally, the Create Discrete Entity option may be used to create a discrete entity and send it to custom logic.

The Initial State field specifies whether the sensor is enabled or disabled at the beginning of the simulation run. If a sensor is disabled it will be ignored and never activate. A sensor may be dynamically enabled/disabled at any point during the simulation run by assigning the SensorState(Sensor ID) variable.

4. Coal Storage Bin Design Problem

The system studied in this lab is the crushing plant of a coal mine where loaded coal trucks arrive from the mine to dump their loads. The ore dumped into the crusher is then carried to a storage bin via conveyer belts. For the purpose of this study, the storage bin always has enough capacity. Other capacities and properties of the mine are brought in more details in the next chapter. The goal of this simulation study is to find out the best way for increasing the production level of the mine.

4.1. Problem definition

Example number 10 of the “Mine design: examples using simulation” book by Sturgul (2000).

Trucks arrive at the dumping area with exponential inter-arrival times. Studies have shown that the rate of truck arrivals varies based on the number of trucks currently in the dumping area due to the limited number of trucks available. This means the more trucks in the dumping area the more it takes for the next truck to arrive. The inter-arrival times based on the number of trucks waiting in the crusher queue or dumping material are presented in Table 1. Inter-arrival times are assumed to have exponential distribution.

Table 1.Inter-Arrival Times

Trucks in the dumping area	0	1	2	3	4	5	6	7 or more
Means inter-arrival rate (Minutes)	2	2.5	4	6	8	10	15	20

Trucks are the same in size but the ore they’re hauling can vary from 250 to 290 tons. Only one truck can dump at a time. Additionally, if a truck arrives and the crusher bin does not have capacity for its load it will wait until the crusher frees enough room. In order to make sure that the crusher does not overflow, dumping will be prevented until room for 290 tons of ore becomes available. When a truck gets permission to dump its load, it takes sometime uniformly distributed between 1 and 2 minutes to be spotted. The dumping itself takes 0.35 minutes exactly. A schematic representation of the system is shown in Fig 1.

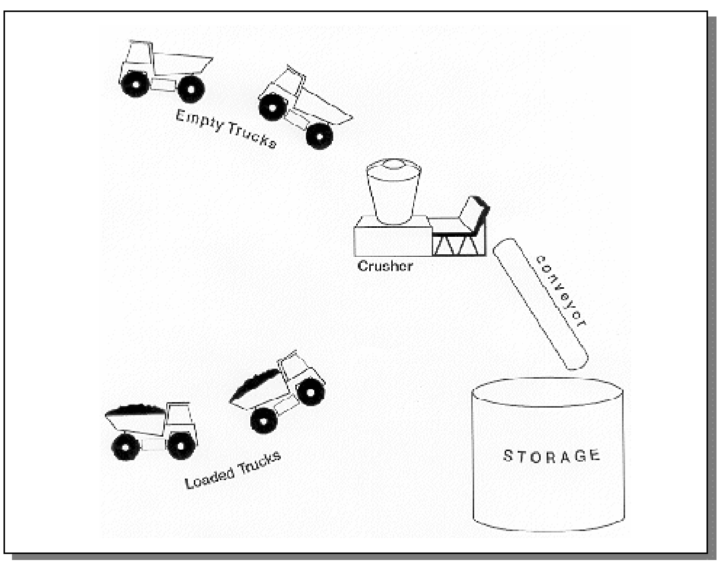

Fig 1.Coal Storage Bin – borrowed from Sturgul (2000).

The crusher has a bin with 350 tons capacity and a rating of 8,000 tons per hour. The crusher capacity can be increased if needed. The conveyer belt has the ability to haul 10,000 tons of coal in each hour but it can be slowed down to save money. The conveyer belt will only work if there is at least 50 tons of coal in the crusher bin. The objective of this example is to study the effects of changing the crusher bin size, crushing rate and the conveyer's haulage rate.

Every workday at the processing plant consists of one 8 hour shift. The actual working time is considered 400 minutes in each shift. Simulate the plant for one shift and judge on the bottlenecks of the system and propose modifications which can increase production capacity.

4.2. Flow diagram

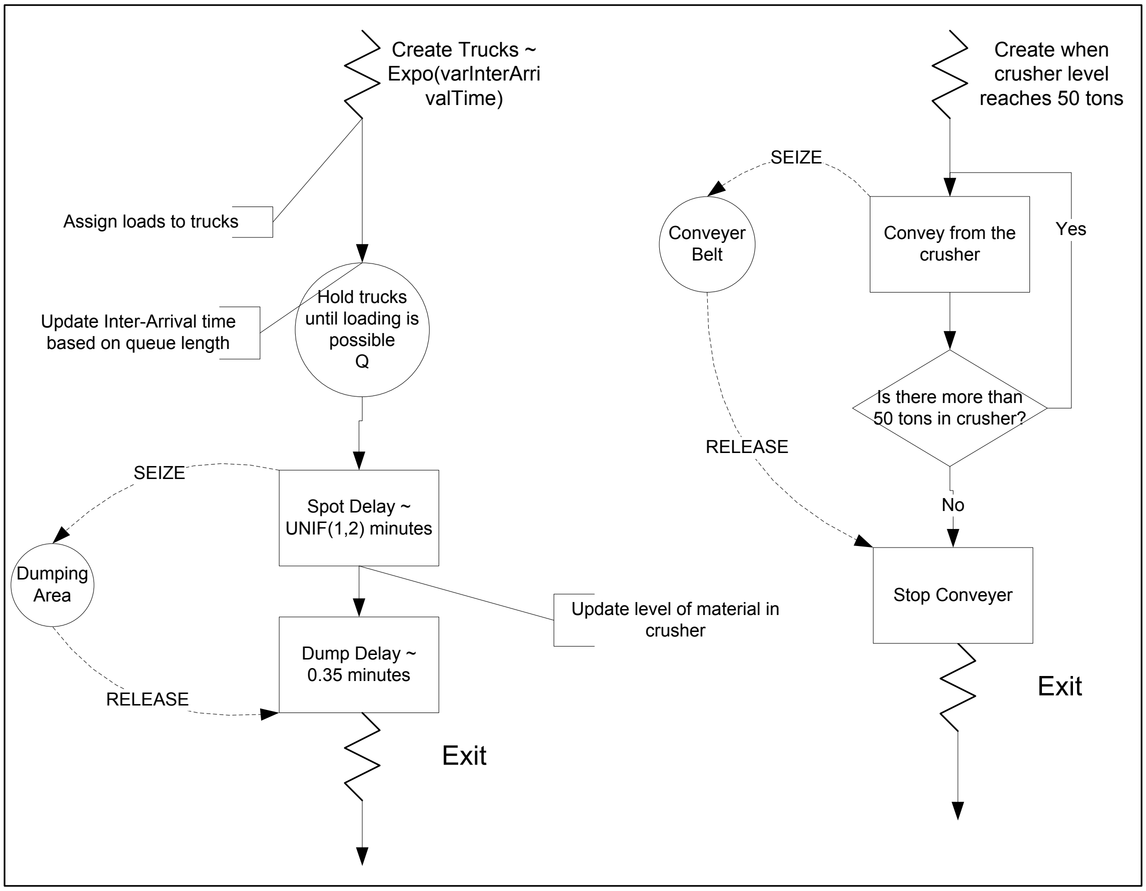

Fig 2. Flow diagram of the model.

4.3. Definition of terminology

4.3.1. Entities

- entTruck: the entity representing trucks
- entDummy: the dummy entity used for having smooth animation

4.3.2. Variables

- varInterArrivalTime: used for counting the number of loads from area C dumped at the crusher

4.3.3. Attributes

- atrLoadTonnage: the tonnage of load each truck is hauling

4.3.4. Resources

- resDumpArea: the area in which the trucks dump the material

4.4. Modeling approach and Pseudo code

The model consists of two parts: discrete and continuous

4.4.1. Discrete model:

- ***Create Trucks with Specified Inter-Arrival Rate:*** use create module with variable mean inter-arrival time to create truck entities
- ***Set Load Tonnages:*** assign a tonnage value to each truck from the uniform distribution
- ***Modify Inter-Arrival Time:*** set the inter-arrival time based on the number of trucks in the dumping area (the queue and the one dumping if exist)
- ***Hold the Trucks Until Dumping is Allowed:*** use a hold module to hold entities until the dumping area is free and there is enough room in the crusher bin
- ***Seize Dumping Area and be Spotted:*** seize the dumping area and delay for the spotting time
- ***Update the Level of Ore in the Crusher:*** add the amount of ore in the truck to the crusher level
- ***Delay for Dumping and Release the Dumping Area:*** delay for the dumping time and release the dumping area for the next truck

4.4.2. Continuous Model

- ***Crusher bin:*** place a tank module for the crusher bin
- ***Add Outflow Regulator to the Crusher Bin:*** add an outflow regulator to the crusher bin with 8,000 tons/hour rate
- ***Storage Bin:*** place a tank module for the final storage bin
- ***Add Inflow Regulator to the Storage Bin:*** add an inflow regulator to the storage bin with 10,000 tons/hour rate (equal to the conveyer belt)
- ***Initiate the Flow:*** use a sensor module to start the flow when the level of material in the crusher bins passes 50 tons. The sensor has to create discrete entities for the flow management
- ***Stop the Flow:*** use a sensor module to stop the flow when the level of material in the crusher bin falls below 50 tons. The sensor will send a signal value of 1 if the level of material falls below 50 tons.
- ***Seize Regulators:*** seize the outflow regulator on the crusher bin and the inflow regulator on the storage bin
- ***Flow:*** start the flow from the crusher bin to the storage bin until a signal value of 1 is received
- ***Release Regulators:*** release the regulator when the flow is stopped

4.5. A snapshot of the completed Arena model

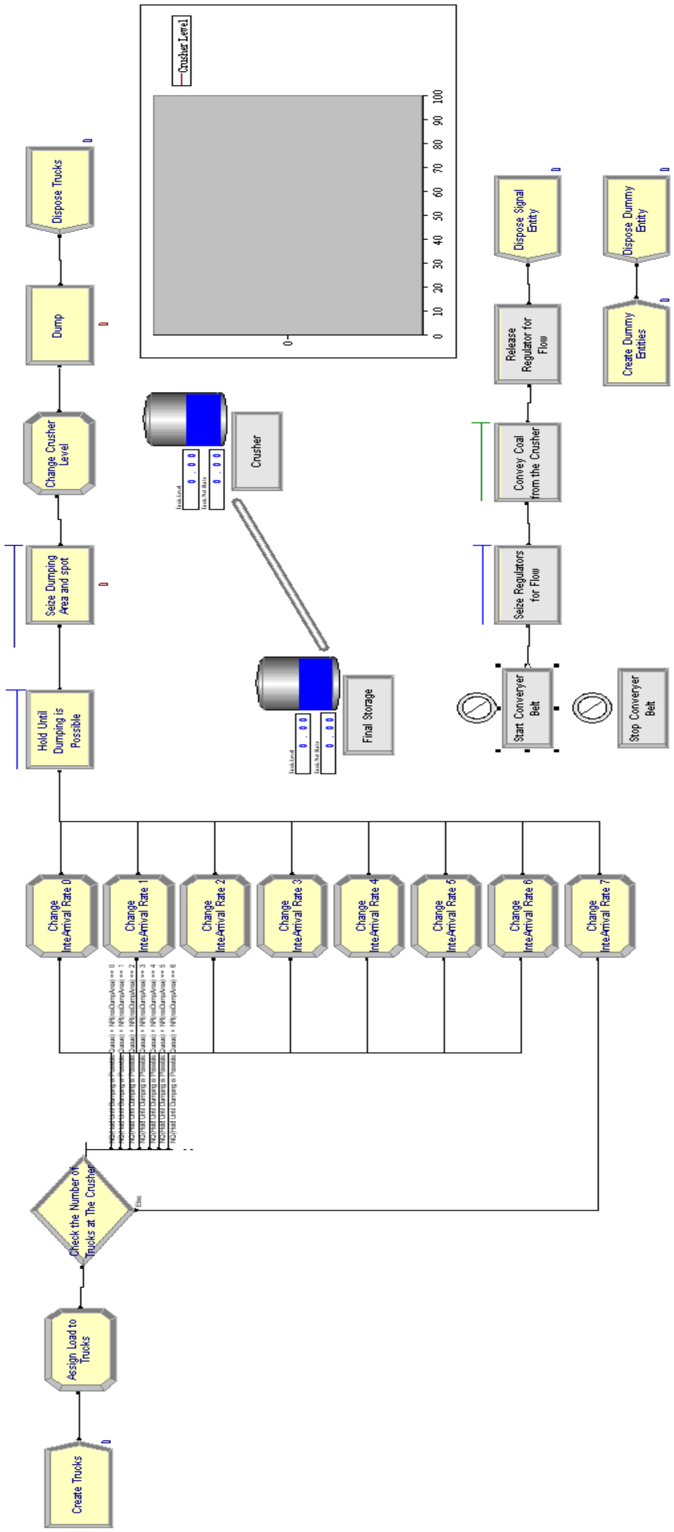

Fig 3.Model Snapshot.

4.6. Step by step modeling

4.6.1. Define Replication Parameters— Use Run > Setup

The model is going to be run for one shift of 400 minutes. The hours per day field is set to 8 and the base time unit is set to minutes. In order to have the Tank statistics in the reports you have to check the corresponding box in the run setup form as in Fig 3.

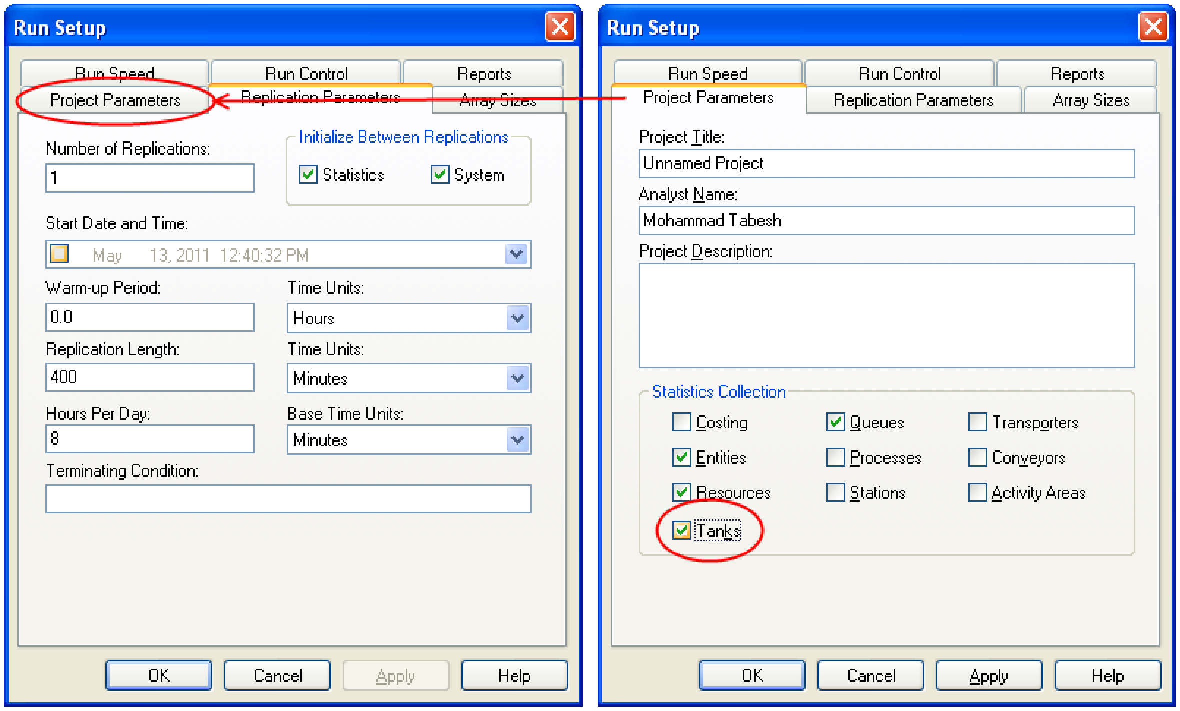

Fig 4.Replication Parameters

Number of Replications	1
Warm-up Period	0 hours
Replication Length	400 minutes
Hours per Day	8
Base Time Units	Minutes

4.6.2. **Define Inter-Arrival Variable** — Use the Variable Data Module

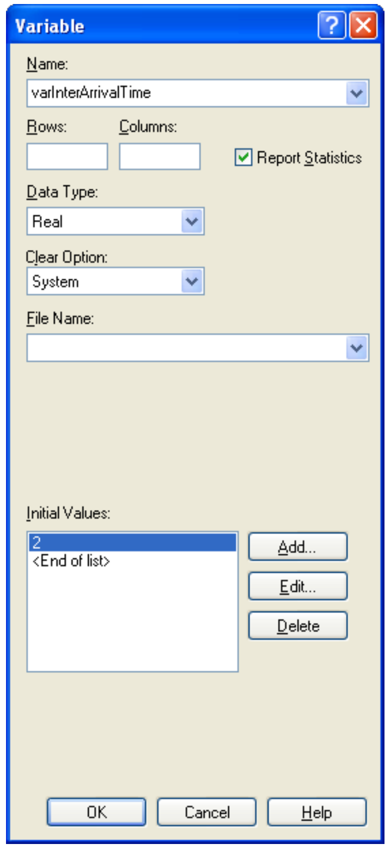

Fig 5.Define Inter-Arrival Variable

Name	varInterArrivalTime
Initial Value	2

4.6.3. **Truck arrivals** — Use the Create Module

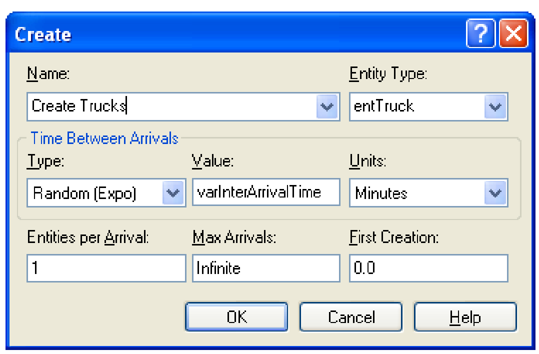

Fig 6.Create Module for Trucks from Area A

Name		Create Trucks
Entity Type		entTruck
Time Between Arrivals	Type	Random (Expo)
	Value	varInterArrivalTime
	Units	Minutes

4.6.4. **Assign Load to Trucks** — Use the Assign Module

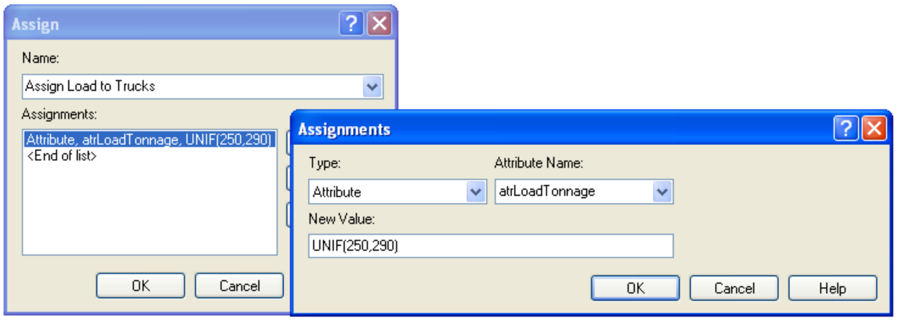

Fig 7.Assign Load to Trucks

Name		Assign Load to Trucks
Assignment	Type	Attribute
	Attribute	atrLoadTonnage
	New Value	UNIF(250,290)

4.6.5. **Add Crusher Bin —** Use the Tank Module from Flow Process panel

You first need to attach the "Flow Process" panel as shown in Fig 7**.**

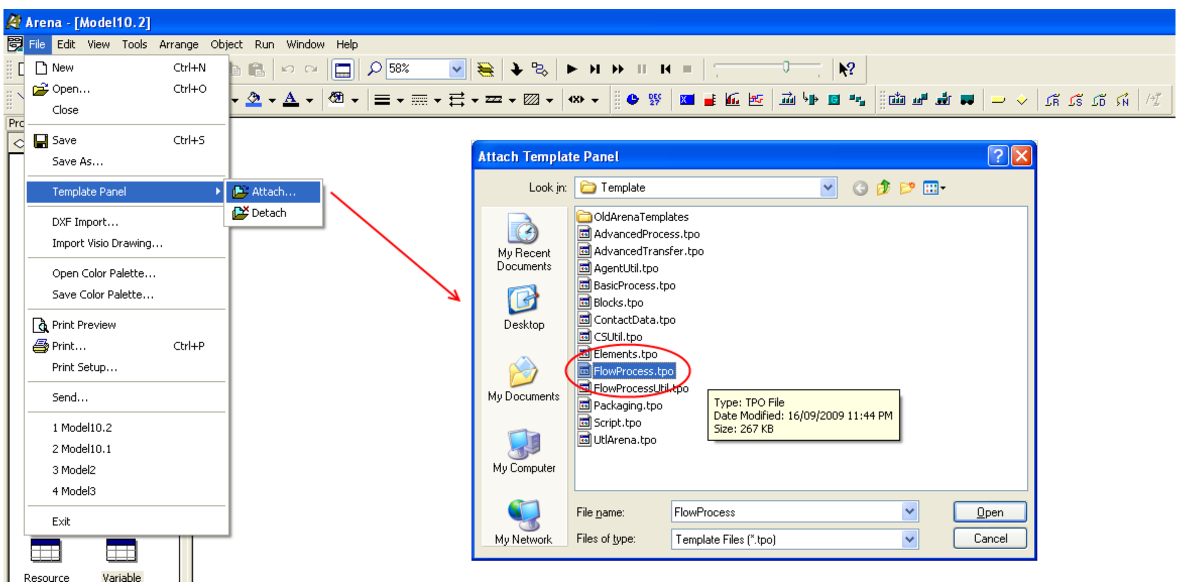

Fig 8.Attach Flow Process Panel

Now add the Tank module from the flow process panel.

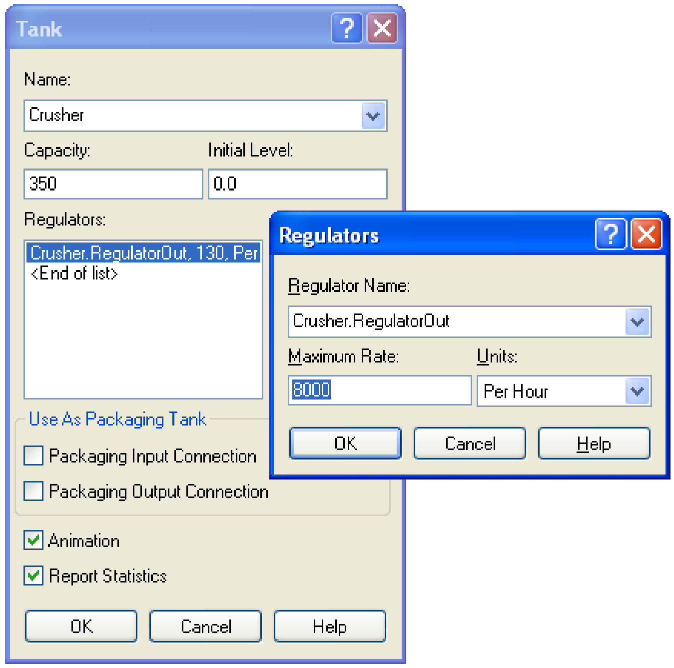

Fig 9.Crusher Tank

Name		Crusher
Capacity		350
Initial Level		0
Regulators	Regulator Name	Crusher.RegulatorOut
	Maximum Rate	8000
	Units	Per Hour

4.6.6. **Hold until Dumping is Possible** — Use the Hold Modules

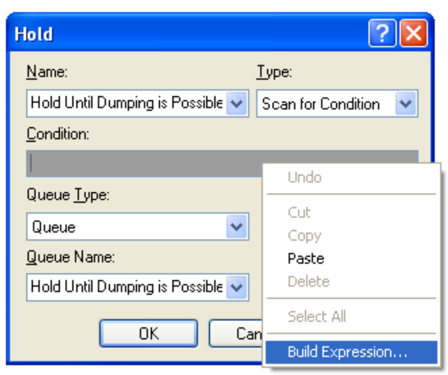

Fig 10.Hold until Dumping is Possible

Name	Hold Until Dumping is Possible
Type	Scan for Condition
Condition	TankLevel(Crusher) <= (TankCapacity(Crusher)-290) && STATE(resDumpArea) == IDLE_RES

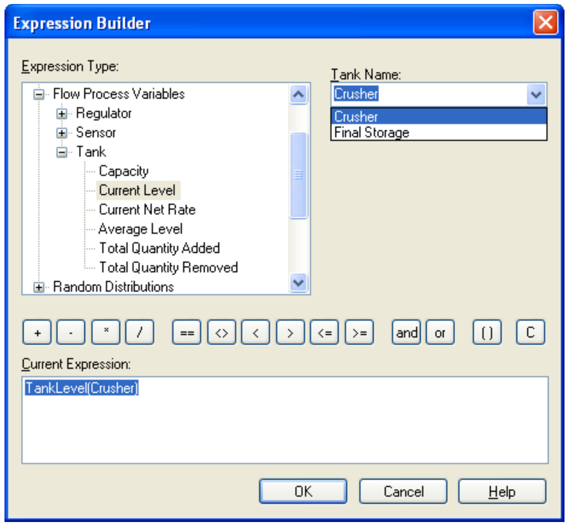

Fig 11.Select Tank Level Expression

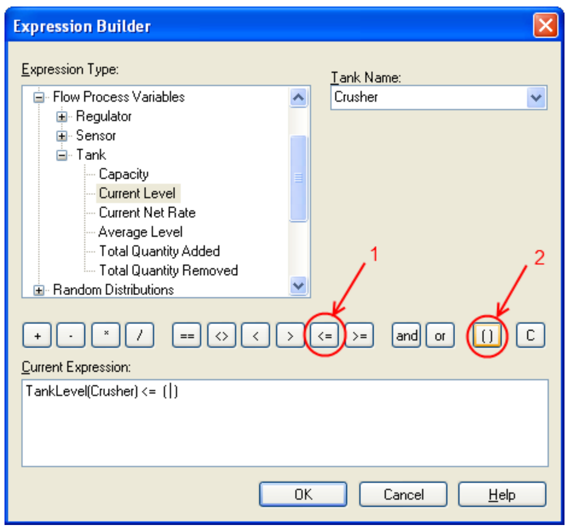

Fig 12.Use Smaller than or Equal to Operator

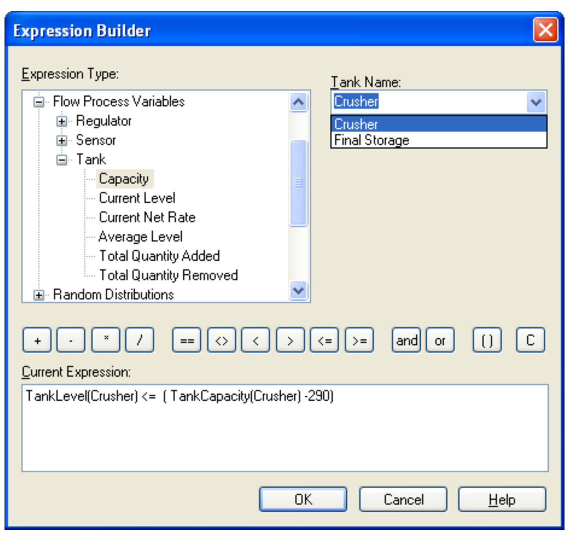

Fig 13.Use Tank Capacity Expression

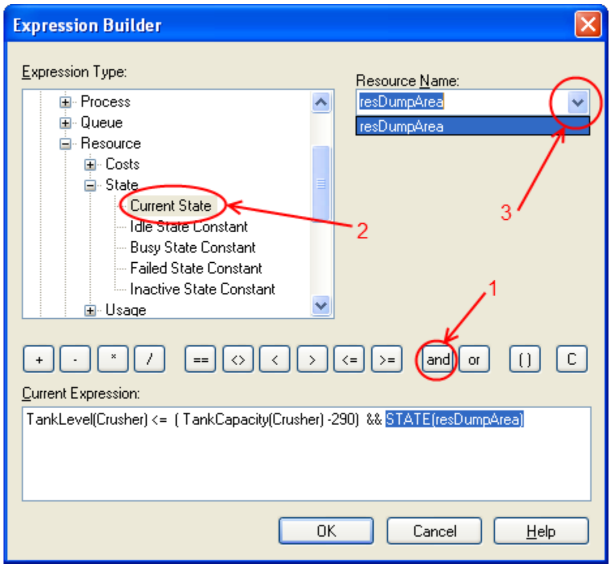

Fig 14.Use Resource State Expression

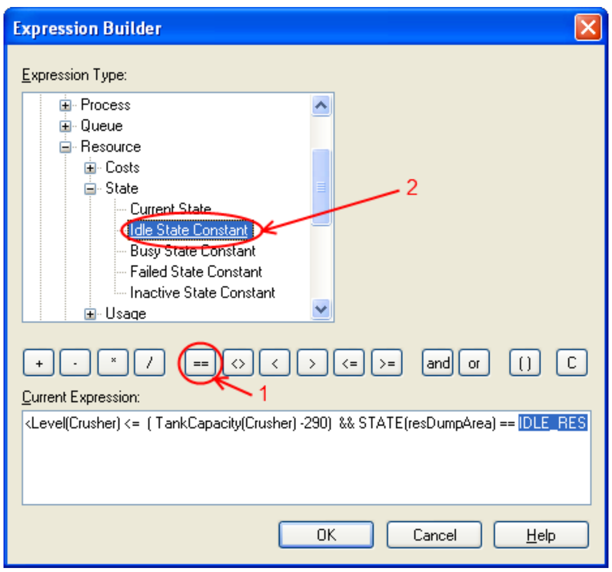

Fig 15.Check versus Idle State

4.6.7. **Seize Dumping Area and Be Spotted** — Use the Process Module

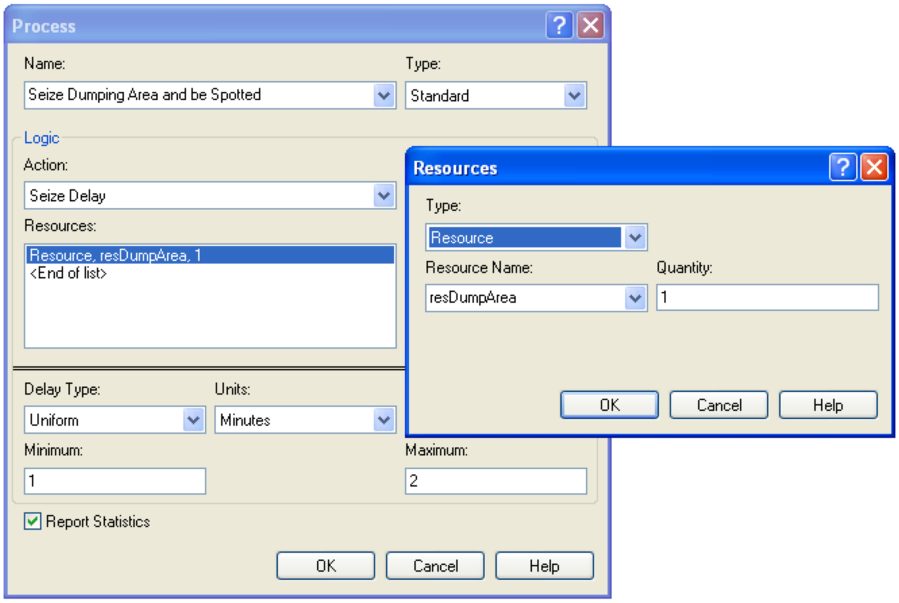

Fig 16.Seize Dumping Area and be Spotted

Name		Seize Dumping Area and be Spotted
Action		Seize Delay
Resources	Type	Resource
	Set Name	resDumpArea
	Quantity	1
Delay Time	Delay Type	Uniform
	Units	Minutes
	Minimum	1
	Maximum	2

4.6.8. **Change Crusher Bin Level** — Use the Assign Module

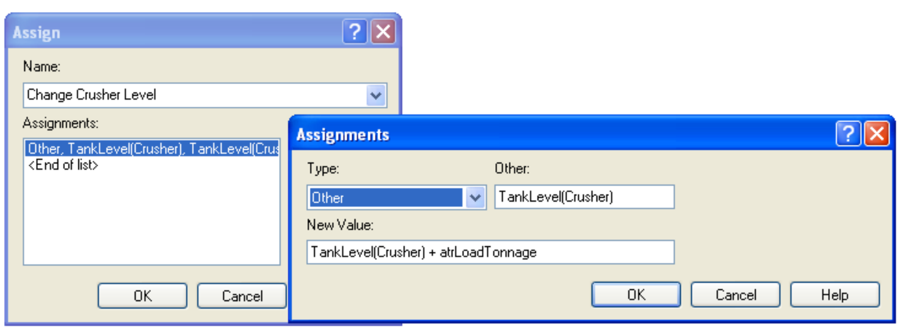

Fig 17.Change Crusher Level

Name		Change Crusher Level
Assignment	Type	Other
	Other	TankLevel(Crusher)
	New Value	TankLevel(Crusher) + atrLoadTonnage

4.6.9. **Release the Dumping Area** — Use the Process Module

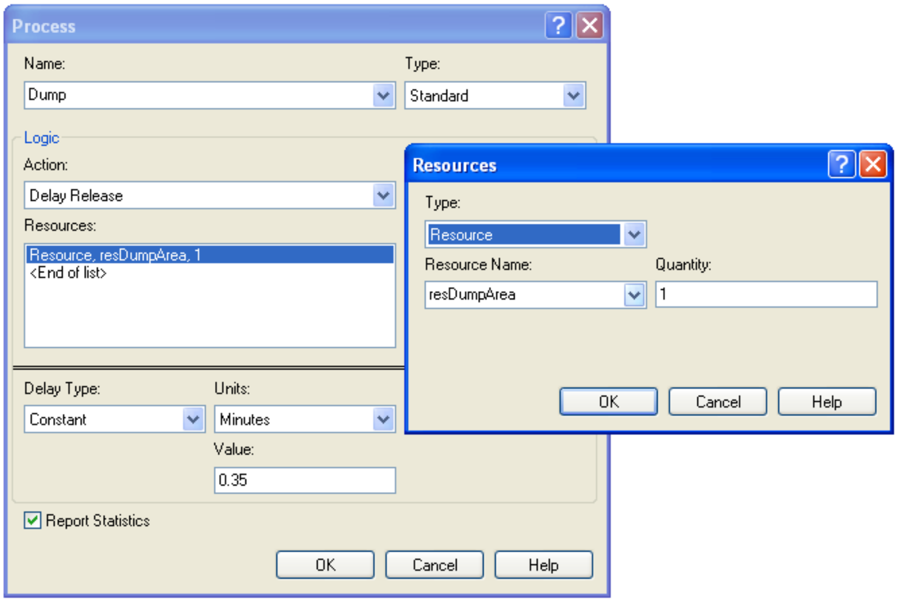

Fig 18.Dump

Name		Dump
Action		Delay Release
Resources	Type	Resource
	Set Name	resDumpArea
	Quantity	1
Delay Time	Delay Type	Constant
	Units	Minutes
	Value	0.35

4.6.10. Dispose Truck Entities — Use the Dispose Module

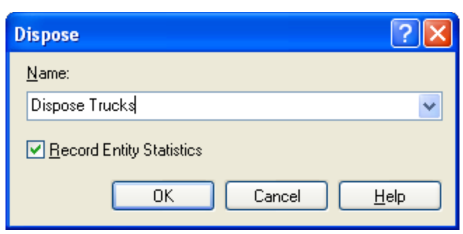

Fig 19.Dispose Trucks

4.6.11. **Add Final Storage Bin** — Use the Tank Module

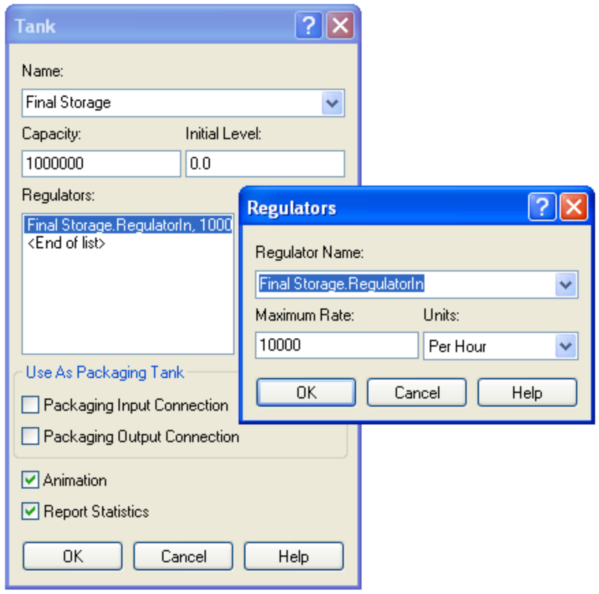

Fig 20.Final Storage Bin

Name		Final Storage
Capacity		1,000,000
Initial Level		0
Regulators	Regulator Name	Final Storage.RegulatorIn
	Maximum Rate	10,000
	Units	Per Hour

4.6.12. **Start Conveyer Belt** — Use the Sensor Module

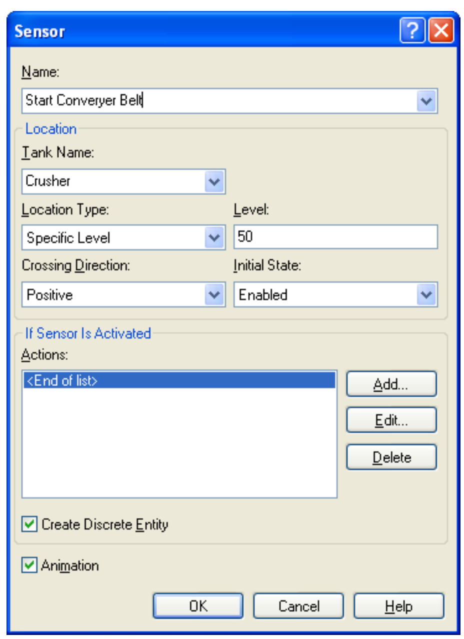

Fig 21.Start Conveyer Belt

Name	Start Conveyer Belt
Tank Name	Crusher
Location Type	Specific Level
Level	50
Crossing Direction	Positive
If Sensor is Activated	Create Discrete Entity

4.6.13. **Seize Regulators for Flow** — Use the Seize Regulator Module

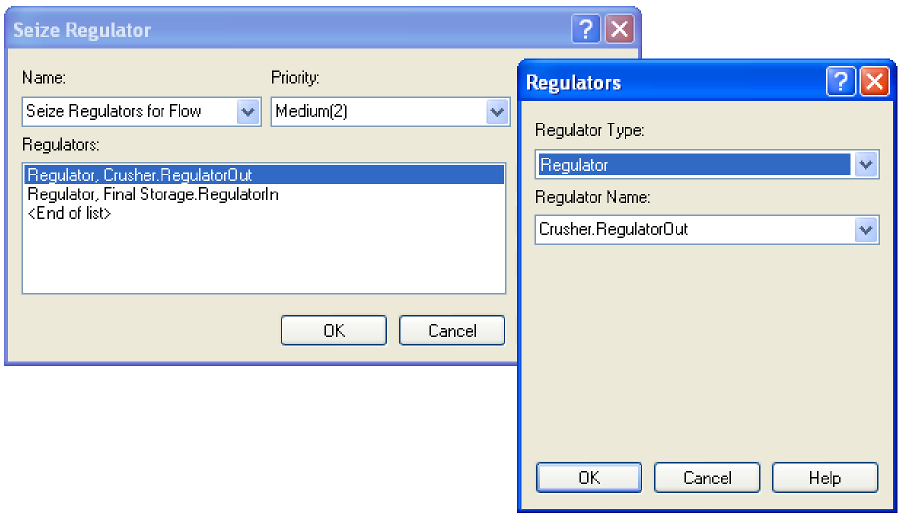

Fig 22.Seize Regulators

Name		Seize Regulators for Flow
Regulators	Regulator Type	Regulator
	Regulator Name	Crusher.RegulatorOut
Regulators	Regulator Type	Regulator
	Regulator Name	Final Storage.RegulatorIn

4.6.14. **Convey Coal from the Crusher** — Use the Flow Module

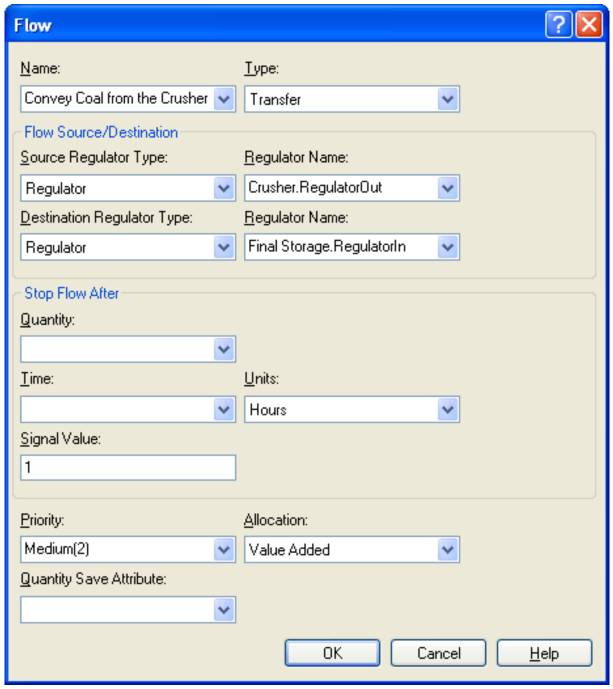

Fig 23.Convey Coal from the Crusher

Name	Convey Coal from the Crusher
Type	Transfer
Source Regulator Type	Regulator
Regulator	Crusher.RegulatorOut
Destination Regulator Type	Regulator
Regulator	Final Storage.RegulatorIn
Signal Value	1

4.6.15. Release Regulators for Flow — Use the Release Regulator Module

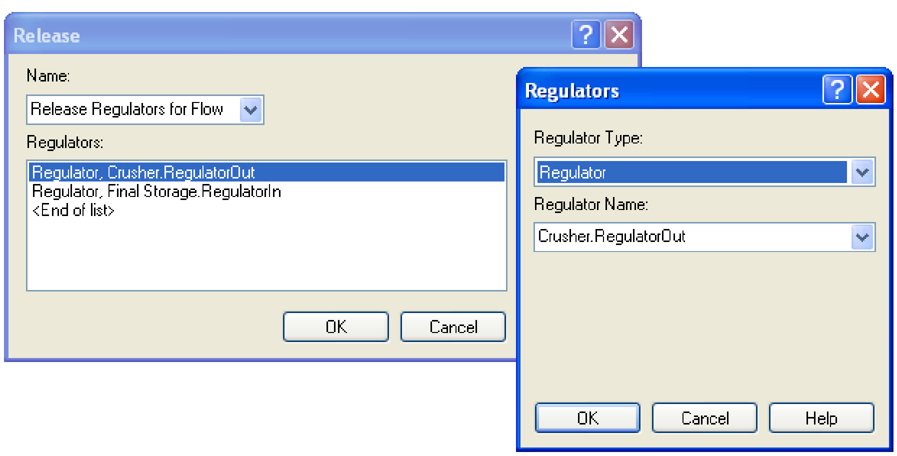

Fig 24.Release Regulators

Name		Release Regulators for Flow
Regulators	Regulator Type	Regulator
	Regulator Name	Crusher.RegulatorOut
Regulators	Regulator Type	Regulator
	Regulator Name	Final Storage.RegulatorIn

4.6.16. **Dispose Sensor Entities** — Use the Dispose Module

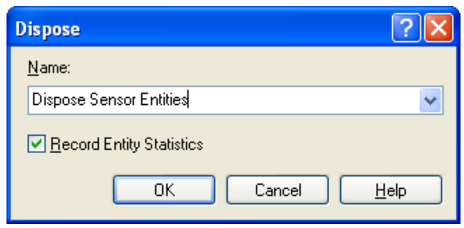

Fig 25.Dispose Sensor Entities

4.6.17. **Stop Conveyer Belt** — Use the Sensor Module

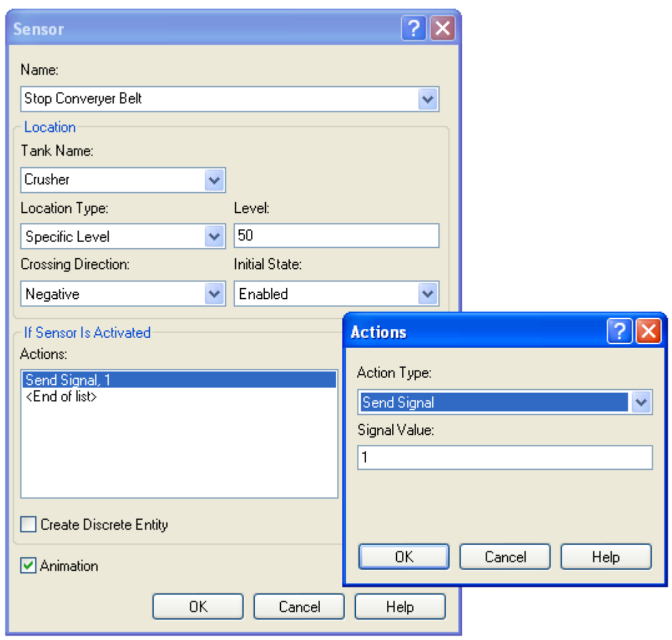

Fig 26.Stop Conveyer Belt

Name		Stop Conveyer Belt
Tank Name		Crusher
Location Type		Specific Level
Level		50
Crossing Direction		Negative
If Sensor is Activated	Action Type	Send Signal
	Signal Value	1

4.6.18. **Animate Flow** — Use Level Button

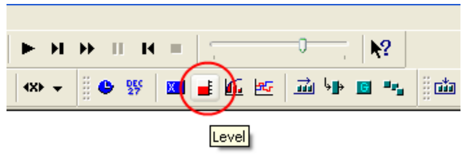

Fig 27.Add Level Animation

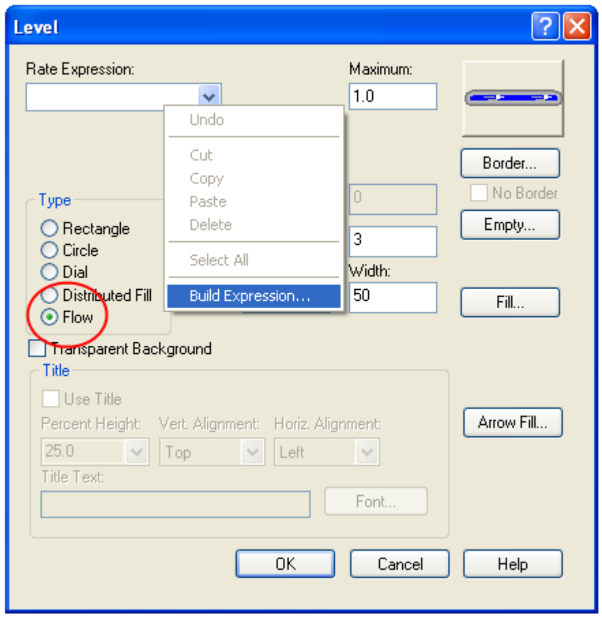

Fig 28.Select Flow

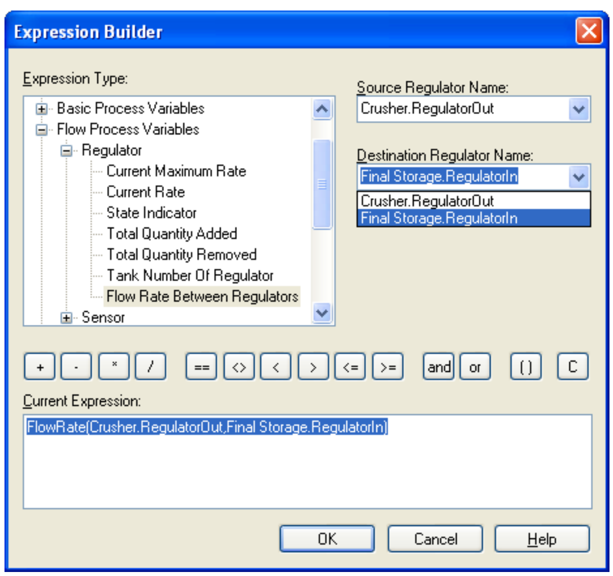

Fig 29.Build Flow Rate Expression

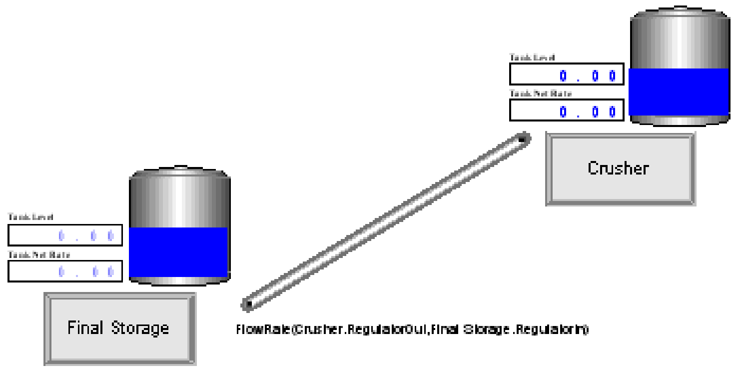

Fig 30.Draw Object in the Appropriate Place

4.6.19. **Plot Tank Level** — Use Plot Button

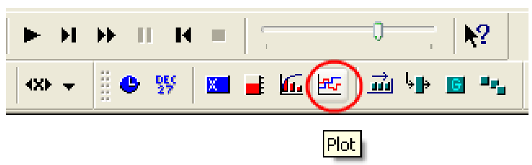

Fig 31.Plot Button

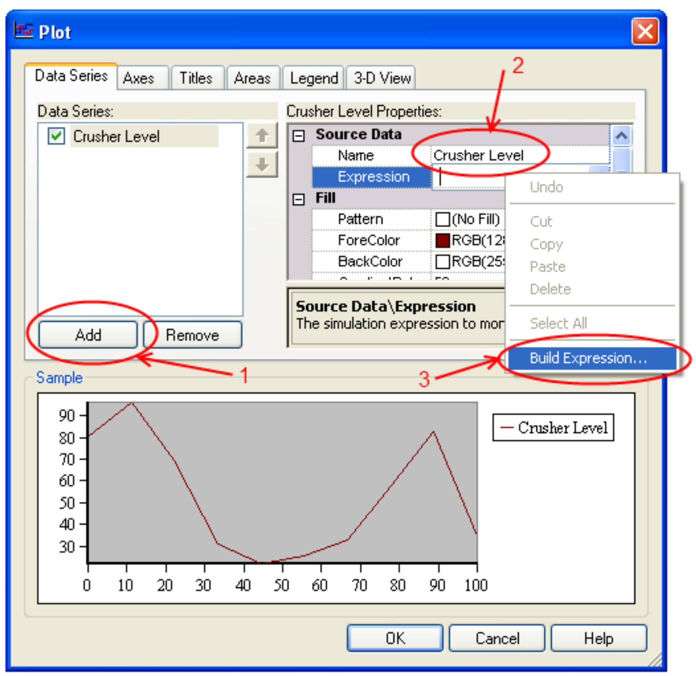

Fig 32.Plot Dialogue

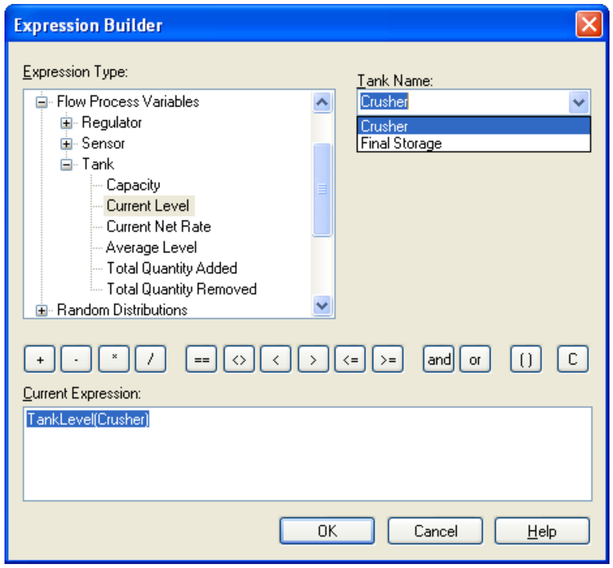

Fig 33.Tank Level Expression

Source Data	Name	Crusher Level
	Expression	TankLevel(Crusher)

4.6.20. **Change Truck Entity Picture** — Use Entity Data Modules

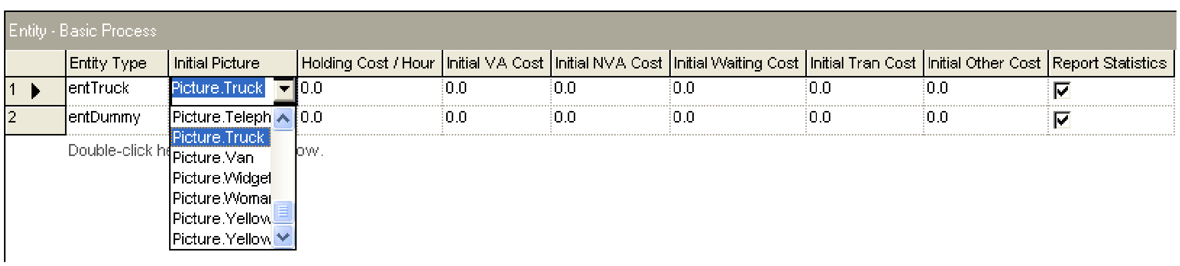

Fig 34.Change Truck Entity Picture

4.6.21. **Change Inter-Arrival Time** — Use the Decide and Assign Modules

Delete the link between "Assign Load to Trucks" and "Hold Until Dumping is Possible" modules and add one decide module and 8 assign modules. Set the values based on the following figures and draw links as shown in Fig 20.

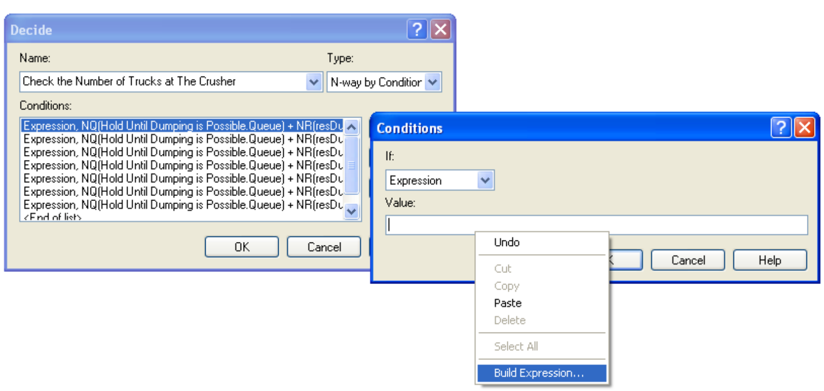

Fig 35.N-Way by Condition Decide Module

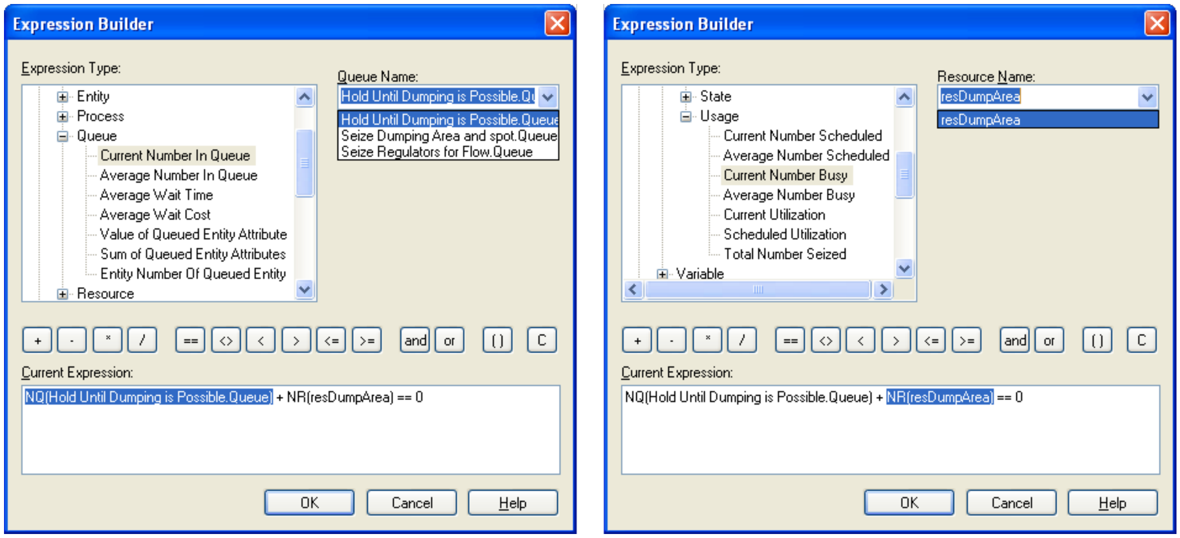

Fig 36.Build Conditions

Name		Check the Number of Trucks at The Crusher
Type		N-way by Condition
Condition	Type	Expression
	Value	NQ(Hold Until Dumping is Possible.Queue) + NR(resDumpArea) == 0
Condition	Type	Expression
	Value	NQ(Hold Until Dumping is Possible.Queue) + NR(resDumpArea) == 1
Condition	Type	Expression
	Value	NQ(Hold Until Dumping is Possible.Queue) + NR(resDumpArea) == 2
Condition	Type	Expression
	Value	NQ(Hold Until Dumping is Possible.Queue) + NR(resDumpArea) == 3
Condition	Type	Expression
	Value	NQ(Hold Until Dumping is Possible.Queue) + NR(resDumpArea) == 4
Condition	Type	Expression
	Value	NQ(Hold Until Dumping is Possible.Queue) + NR(resDumpArea) == 5
Condition	Type	Expression
	Value	NQ(Hold Until Dumping is Possible.Queue) + NR(resDumpArea) == 6
Condition	Type	Expression
	Value	NQ(Hold Until Dumping is Possible.Queue) + NR(resDumpArea) >= 7

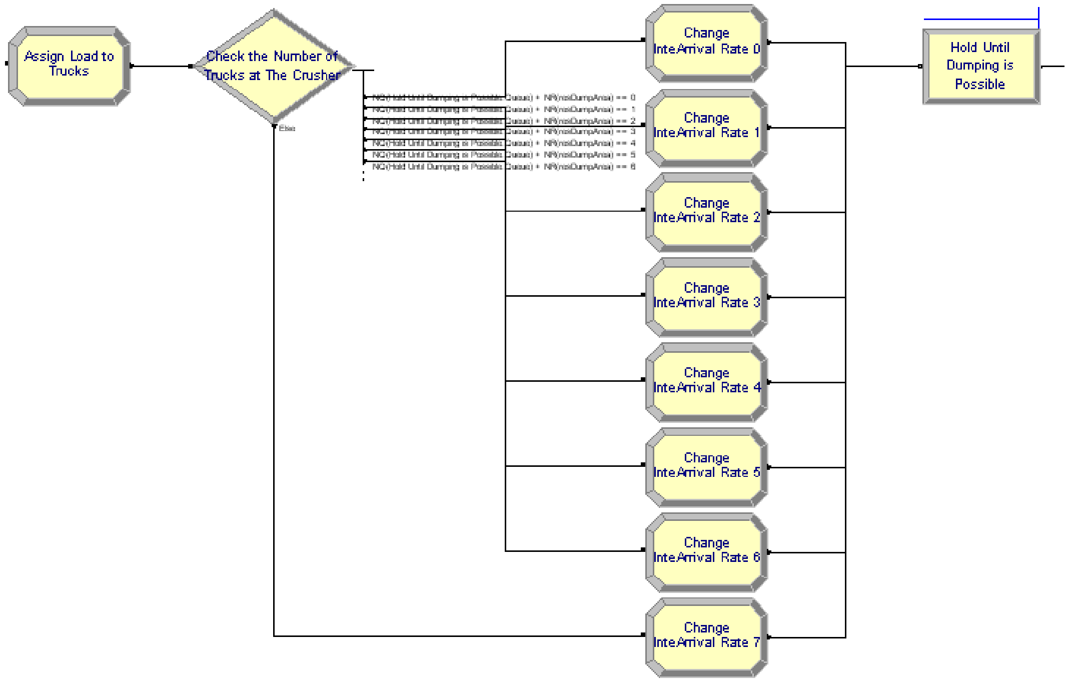

Fig 37.How to Draw Links

5. References

[1] Rockwell. (2010). Arena Simulation Softwre (Version 13.0). Pittsburgh: Rockwell Automation.

[2] Sturgul, John R. (2000). *Mine design : examples using simulation*. Littleton, CO: Society for Mining, Metallurgy, and Exploration.

Guidelines for Creating Block Models in Gems

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1. Introduction

This guideline was written using Gemcom Gems – Version. 6.3 (Gemcom Software International, 2011). Gemcom Gems Resource Modelling Manual - Version 6.1 (Gemcom Software International, 2007) from Gemcom Training Course series was used as the main source in writing and compiling this guideline.

GEMS is a 2D (two-dimensional) and 3D (three-dimensional) graphical environment within which you can display, edit, and model data from a variety of sources. GEMS provides various practical tools that make all aspects of geological modelling, as well as open pit and underground mine planning and design, faster, easier to understand, and more flexible.

GEMS is designed to help you with:

- ***Geological modeling** – You can interactively query drillhole data, use the data to interpret lithology zones or ore zones on plans or sections, and then use the data to construct accurate solids models.*
- ***Block modeling** – You can create numerical models to characterise your orebody or seam deposit. Powerful interpolation tools allow you to create detailed grade models, which you can then use to determine economic feasibility.*
- ***Ore reserves** – Solid models, surfaces, and block models can all contribute to ore reserve calculations.*

GEMS offers two distinct but interlinked ways to model reserves: numerical or block modelling and solids modelling. Typically, you will need to construct both types of models. Block models can be used to update solids, and solids can be used to update and filter block model data. Both types of model data can contribute to reserves reports. Block models are three-dimensional arrays of cubical blocks used to model orebodies and other sub-surface structures. Each block cell is usually used to represent a homogeneous volume of material and each cell is assigned a series of attributes that describe the characteristics of the material in a block. These include but are not limited to rock types, grades, density and dollar value. These attributes are stored in individual models, which have the same number of rows, columns and levels. The models can be used in reserve estimation and in mine planning. A typical project will have a group of models organized in folders with a single geometry (Fig 1). You may create multiple block model projects within the same GEMS session with different geometries. Once you create block model projects, you access them through the Project View Area.

2. Objectives

- Single block model setup
- Multiple folder block model setup
- Block model interpolation using inverse distance squared
- Block model interpolation using ordinary kriging
- Cross validation, and

- Resource classification into measured, indicated, and inferred.

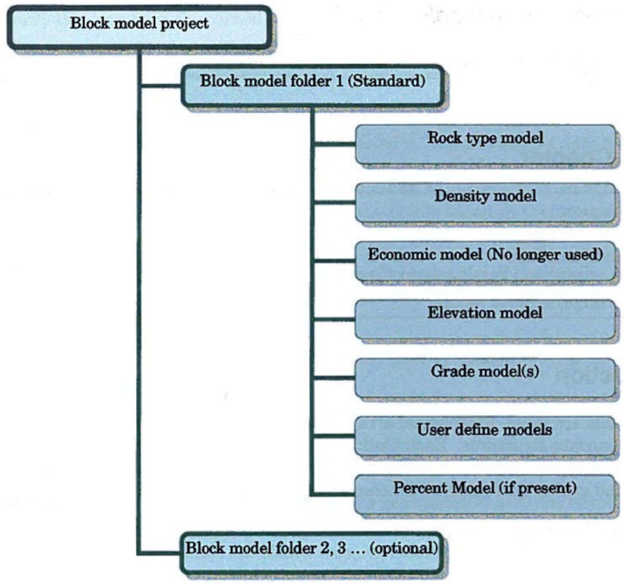


Fig 1. Block model project format.

3. General Procedures

The following list provides a summary of the main activities involved in a block modeling exercise using GEMS. Most of the items listed are procedures. Fig 2 illustrates this workflow in Gems.

- Define colour profiles
- Define rock codes
- Define grade names
- Create geological plans or sections
- Create the geological domain or constraint
- Create a topographic surface and other surfaces such as pit bottom etc.
- Prepare extracted data into point area workspace for grades
- Create the block model Project. You must define the geometry here, and you should also create folders, models, and model type mappings
- Create additional block folder if needed
- Create additional model if needed
- Check mapping for grade elements
- Initialize block models to background values
- Code the rock-type or domain model
- Create density models (sometime calculated)
- Prepare interpolation profile:
  → Trace blocks
  → Search ellipsoid
  → Semi-variogram (if interpolating using kriging)
- Calculate grade models
- Use simple manipulation to calculate other models if necessary such as category model
- Use Volumetric menu commands to produce in-situ reserve reports.

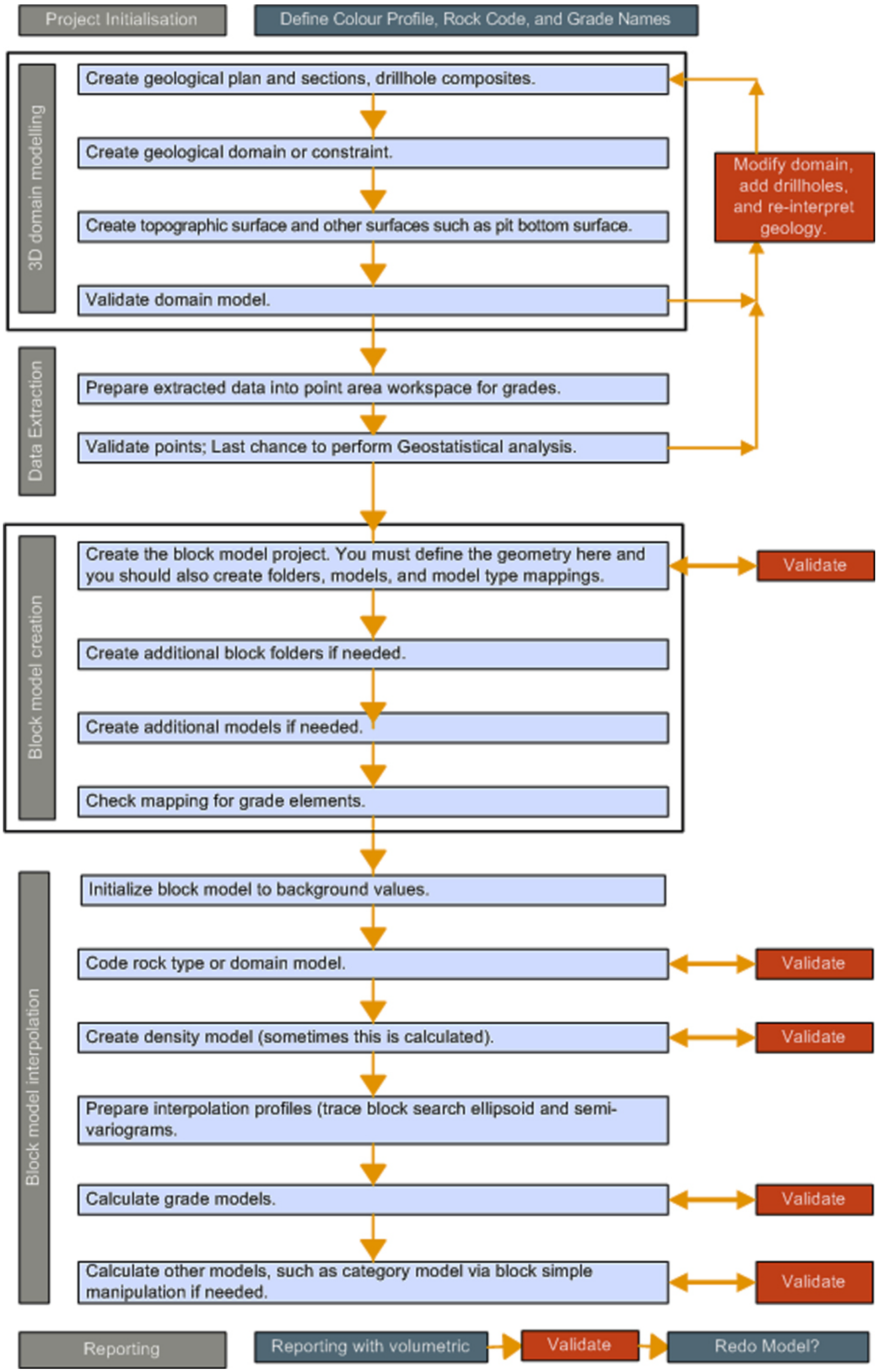


Fig 2. Workflow of block modeling in Gems.

4. Steps required for block model project initiation

This section provides the steps on activities that you must complete when you want to start a new block model project. These activities include the following:

- Define colour profiles
- Define rock codes
- Define grade names

4.1. Colour profiles

When creating block models in GEMS, it is useful to separate colour profiles for the block model display. For example, a typical gold colour profile may be set as follows in the left table, and the block model is set similar to the right table. Note the "From" starts in the block model profile at 0.001 thus avoiding GEMS to display an all zero grade block (Table 1).

Table 1. Colour profile setup.

From	To	Colour	From	To	Colour
0.00	1.0	Blue	0.001	1.0	Blue
1.0	1.5	Yellow	1.0	1.5	Yellow
1.5	2.0	Orange	1.5	2.0	Orange
2.0	5.0	Red	2.0	5.0	Red
5.0	999	Magenta	5.0	999	Magenta

These can always be edited or added to later if they are not displaying the information you are looking for. These colour profiles may have been created before, otherwise create them now. Typical colour profiles that you need include the following:

1. **Rock code or domain colour profile -** This colour profile is of type "value" and is based on the integer value of the rock code.
2. **Grade name colour profiles** - This colour profile is of type "value" and is based on the grade ranges appropriate for the deposit.
3. **Density colour profiles** - This colour profile is of type "value" and is based on the density ranges appropriate for the deposit.

Tip: Prefix all block model colour profiles with "BM_" (i.e. BM_ROCKS, BM_DEN etc.), so that you can easily identify them.

To define colour profiles, follow these steps.

1. **Choose Format > Other Profiles > Colour** to open the Colour dialog.
2. Click **New Profile**, type a name for the profile, and click **OK**.
3. Enter a **Description** for the profile.
4. Select the **Type** of profile you want to create.
5. Complete the colour profile as needed, and click Apply.
6. Repeat steps 2 to 5 to add more profiles.
7. When you are finished, click **OK**.

4.2. Rock Codes

When you create rock codes for block models, you are creating the profiles for the domain codes. Take note that for each domain code, you need to create one rock code.

To create a rock code, follow these steps.

1. Choose **Format > Other profiles >Rock Codes** to open the Rock Codes dialog.
2. Click **New Profile**, type a name for the profile, and click **OK**.
3. In the General tab, enter the following information:

a. Comment: Enter a comment/description for this rock code.

b. **Rock type:** Select **Ore, Waste**, or **Air**. This distinction is used for solid reserves reporting. Air and waste rock types are not used for grade calculations.

c. **In-situ Density**: Enter the average density for the domain expressed in tonnages per unit volumes (tonnes per cubic meter or tons per cubic feet). This density can be used to code the density model during the block model interpolation phase in lieu of calculated (or interpolated) densities.

d. **Colour**: Define the colour for the rock code.

e. Optional details:

- **Block-model code:** Enter a unique integer (1 to 9999) to represent the current rock code within block models. This model code needs to be unique for a particular project.
- **Block-model folder:** Select the block model folder to contain the lithology model using this rock code. For a standard block model, choose *Default Folder*. For a partial block model, you use this option to map the rock code profile to a specific block model folder.
- **Radius:** Enter the radius for reserves in the units of your project. This is only needed for polygonal resource estimates.

4. Use the **Pit Design** tab for defining variable pit slopes in the GEMS pit design application. You can complete the following optional settings:

a. **Slope angle:** Enter the angle (between 0 and 90 degrees) measured from the toe of one bench to the toe of the next bench in the current rock code domain.

b. **Batter angle:** Enter the individual face angle (between 0 and 90 degrees) for a bench in the current rock code domain. The angle is measured from the toe to the crest of the bench.

c. **Berm width:** enter the width measured between the toe and crest line at the same elevation for the current rock-code domain. The berm width is the bench width.

d. **SEG slope angles:** Enter an angle (between 0 and 85 degrees) for each direction. These angles are used when generating a pit design from a surface-elevation grid.

5. Use the **Economic** tab only for Whittle 3D. It is no longer needed with more recent Whittle products. You can complete the following optional settings:

a. **Mining services cost, Drilling cost, and Blasting cost**: For each type of cost listed, enter the cost in monetary units per unit volume (for example, dollars per cubic meter).

b. **Primary mineral:** Enter the following:

- **Primary grade:** Select the index that represents the primary grade element in the rock. GEMS lists grade element indexes under Recovery factors.
- **Ore stockpile cut-off grade:** Enter a threshold that the primary mineral grade must exceed before a block with this rock code is defined as ore.
- **In Waste stock cut-off grade:** Enter a threshold that the primary mineral grade must exceed before a block with this rock code is defined as stockpile.

6. **Recovery factors:** For each grade name, enter a value that represents the amount of grade element recoverable as a percentage of grade element present.
7. Use the **Block-caving** tab only if you are using the rock codes within the PCBC application.
8. Click **Apply**.
9. Add more profiles if required.
10. When you are finished, click **OK**.

4.3. Grade names

Generally, you create the grade names when you set up the project. Prior to creating the block model matrix, review the grade names you created to ensure you haven't missed anything.

To create a grade name, follow these steps.

1. Choose **Format > Other profiles > Grade Names** to open the Grade Names dialog.
2. Review the list and add any elements.
3. If you add grade names, click in the table, and press **INSERT** to create a new row.

   Note: When you insert a new row, GEMS will add the new row above the row you clicked. To add a row to the bottom of the list, click in the last Decimals cell, and press TAB.
4. Enter the following details:

   a. **Grade Name:** Enter the name of the grade element.

   b. **Decimals:** Enter the number of decimal places to use when reporting concentrations of the specified grade element.
5. To add more rows, place the mouse cursor in the last cell on the right-hand column, and press **Tab.**

   Tip: If you are planning to report your resources categorized by a level of confidence (inferred, indicated, and proven), you will need to add a CAT grade element to the list.

5. Types of block models

This section describes the block models that are available in Gems. Fig 3 lists each block model type and the icon that represents it in the project view area. Block model names appear with the generic icon if they have not been otherwise mapped.

Table 2 summarizes different types of block model in gems and a description of their usage.

Icon	Block Model Type	Example	Typical Units
	Rock type	Standard rock type Ore rock type	None
	Percent	Percent ore Percent mined	%
	Density	Standard density Ore density	tons/m^3
	Elevation	Elevation	m
	Grade	Grade 1 through 10 or more	g/ton
	Generic	Variance 1 through 10	
	Economic	Economic	dollars

Fig 3. Block model types.

Table 2. Types of block models in Gems.

Model Type	Description
Rock Type Model	This model represents the geological domains of the deposit. Each block contains a numerical code indicating its assigned rock type. Domain (Rock type) codes are indexed to the rock code profile that contains details about the domain, such as slope angles, economics, primary minerals, recovery factors, and cut–off grades. Each folder should contain one rock type model.
Density Model	This model defines the density of the rock represented by the rock type model stored in the same folder. Although each rock type profile specifies a default density, you may wish to control accuracy further, for example, by linking density to grade. Each folder containing a rock type model should contain a corresponding density model.
Elevation Model (Very rarely used)	Multiple topographic and geological surfaces are represented by surface elevation grid (SEG) data contained in this model. SEG cells have the same locations and dimensions as blocks in the other models and contain average elevation data.
Grade Models	You can create up to 10 (or more) separate grade models. Each grade or assay element has an individual grade model associated with it.
User defined or Generic models	You may create as many additional block model files as you wish to fully characterise your reserves. Typically, these might be used for special purpose manipulation, backing up models, or storing results of IK or MIK interpolating runs. Another typical user-defined model is the percent mined model.
Percent Model (Used Only in Partial Models)	A percent model can represent the portion of a block occupied or unoccupied by any solid(s) or extruded polygon(s). The standard percent model, however, represents the portion of the block volume occupied by the rock type represented in the rock type model in the same folder. The remaining volume of the block is associated with rock types found in rock type models stored in other folders. If you have n types of rock, each in a separate folder, you need n-1 percent models to account for 100% of the deposit.
Economic Model (Use Only for older version of Whittle 3D)	The economic model is constructed after all other models have been built. This model contains an economic factor for each block, calculated from block-mining costs and block revenues as follows: 1. Determine a cost penalty for the block volumetric mining cost that is a function of the block volume, its rock type, and its associated rock type-dependent mining costs. 2. Determine a further cost penalty if the block can be classified as ore, which is a function of block grades, cut–off grades, or primary minerals. This cost penalty consists of processing costs and administrative overhead and is calculated per ton of ore. 3. Determine a final cost penalty to take into account variable haulage distances. This cost is determined as a

function of block location and haulage costs.
4. Assign to each block the block revenue that is dependent on grades, product revenues, and recovery factors.
5. The mining cost penalty (a negative value) and the block revenue (a positive value) combine to form the economic factor. If the result is positive, then the block is payable; if it is negative, the block is not economical.

6. Single folder block model setup

Block models can be used to numerically model many characteristics of your ore deposit. The size of the block model defined has a great influence on the speed and ease of working with it (the machine speed, hard disk and graphics card being used are also important to the speed of manipulation). A block model 50 x 50 x 40 yields a model with 100,000 cells. Displaying the entire model on screen as filled outlines can be SLOW so caution should be used in setting display parameters. In general, the dimensions for a block model must be consistent throughout the model. That is the X, Y and Z dimensions although they may be of different dimensions do not change from one level or area to the next. Some packages allow for variable block dimensions, via sub-blocking regions (Surpac), however these blocks will not work properly with pit optimization software (Whittle). We will follow the steps explained above to setup a block model for Gol-E-Gohar data set.

6.1. Color display profiles and setup

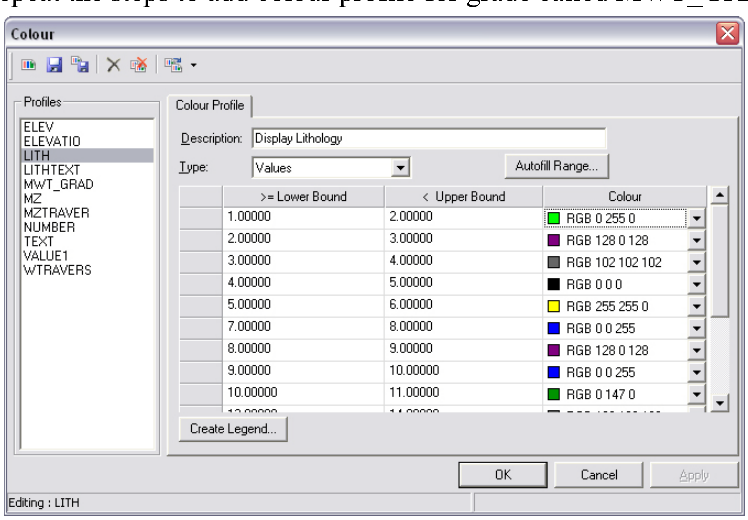
To define colour profiles:

8. Choose **Format > Other Profiles > Colour** to open the colour dialog
9. Click New Profile
   → Name: Lith
   → Description: Display Lithology
   → Type: Values

   Complete the colour profile as needed and click Apply (see Fig 4)

   → Repeat the steps to add colour profile for grade called MWT_GRAD

(a)



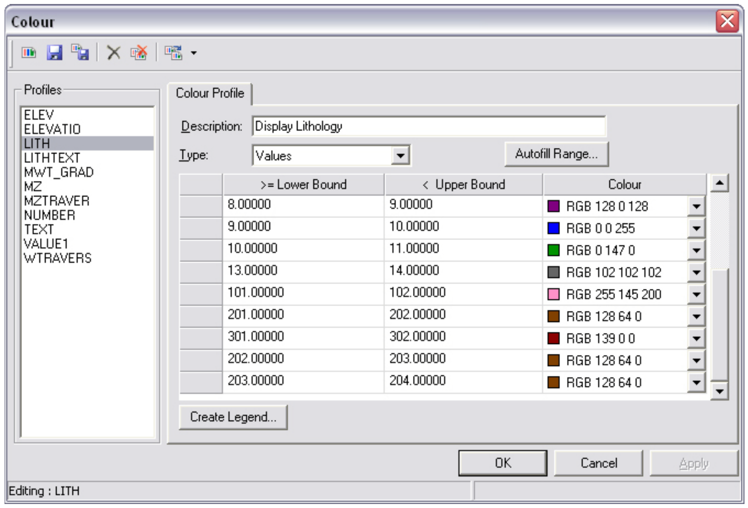
(b) 

Fig 4. Lithology profile.

6.1.1. Define grade elements

By defining the grade elements prior to setting up the block model, the grade models will automatically be created. Make sure this is already done; if not go to **Format > Other Profiles > Grade Names**, and add in MWT, P, S, and FE and specify 2 decimals of iron elements and 5 decimal places for S and P (see Fig 5).

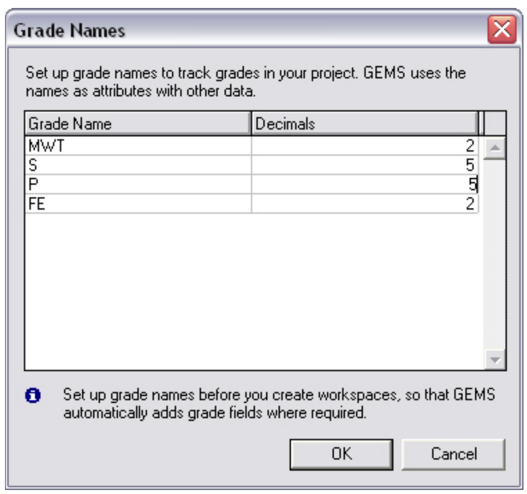


Fig 5. Grade names.

6.2. Block model geometry and workspace

The geometric parameters of a block model project (including the location, orientation, number of blocks, and block dimensions) are called the geometry of the block model matrix Specific column, row, and level indices uniquely identify a given cell (i, j, k) in the geometry (see Fig 8). This cell encompasses a specific volume of material. Any kind of block model data assigned to a cell (i, j, k) refers to the material contained in this cell. In block model geometry, row width and column width are always uniform and all individual blocks are orthogonal.

Caution: It is recommended to define the block model geometry large enough to cover potential pit perimeters at surface in order to facilitate long term planning activities in the future.

To create a block model project and to define its geometry:

1. Go to **Block > Create > Workspace**

2. Enter the name of the Block Model (Fig 6):
   a. **Name:** SingleID (single folder inverse distance)
   b. **Description:** Single folder block model using inverse distance estimation
3. Block model geometry description
   a. **Origin and orientation**: Enter the **LOWER left** corner of the matrix X, Y and the **TOP Z** coordinate (not the block centroid). In GEMS, the block index (1,1,1) is located at the upper, left corner of the block model matrix (see Fig 7).
   b. **Rotation:** You can rotate the block model matrix. The angle of rotation is counter clockwise from the project north.
   c. **Block sizes**: Enter the block size in project coordinate where:
      - Column = block size in the X direction.
      - Row = block size in Y direction.
      - Level = block size in Z direction.
   d. **Number of blocks:** Enter the total number of blocks for each axis where
      - Column = number of blocks in the X direction.
      - Row = number of blocks in Y direction.
      - Level = number of blocks in Z direction.

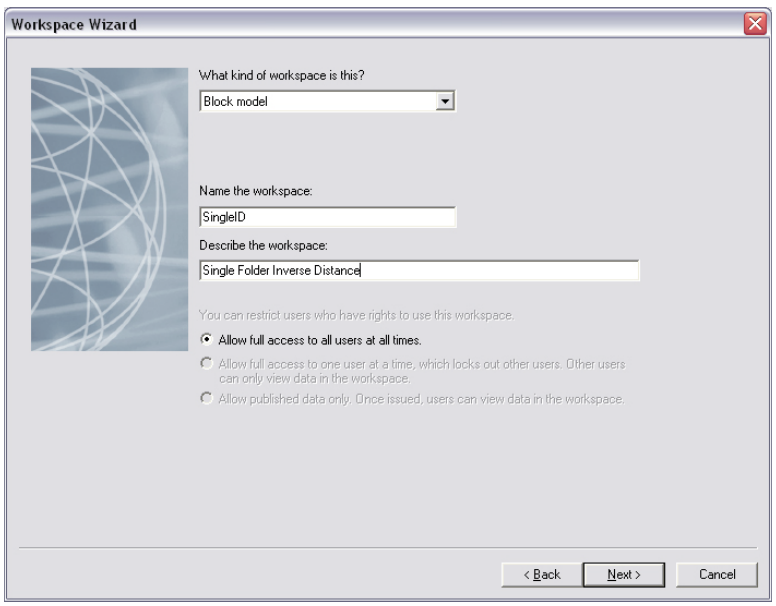


Fig 6. Block Model workspace.

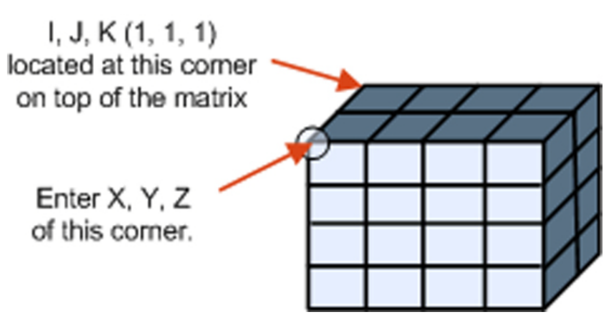


Fig 7. Block model origin.

Tip: If possible, it is a good idea to have a block model lineup with the drill sections. The block model is easier to validate afterwards.

Define the size and location of the model (see Fig 8):

- Origin: X=100200; Y=599500; Z=1760
- Block Size: C=20; R=10; L=5
- Number of Blocks: C=130; R=150; L=76
- Rotation: 0

This will create a standard block model with 1,482,000 blocks. The create block folder page shows the information that GEMS will use in creating the block model. Select **OK** and finish the Setup.

4. Right click on block model in the Project View Area (left window) and add workspace.
5. Right click on block model workspace (SingleID) in the left window and open. This opens a screen showing the information GEMS used in creating the block model (Fig 9). Change the decimals for S and P to 5.

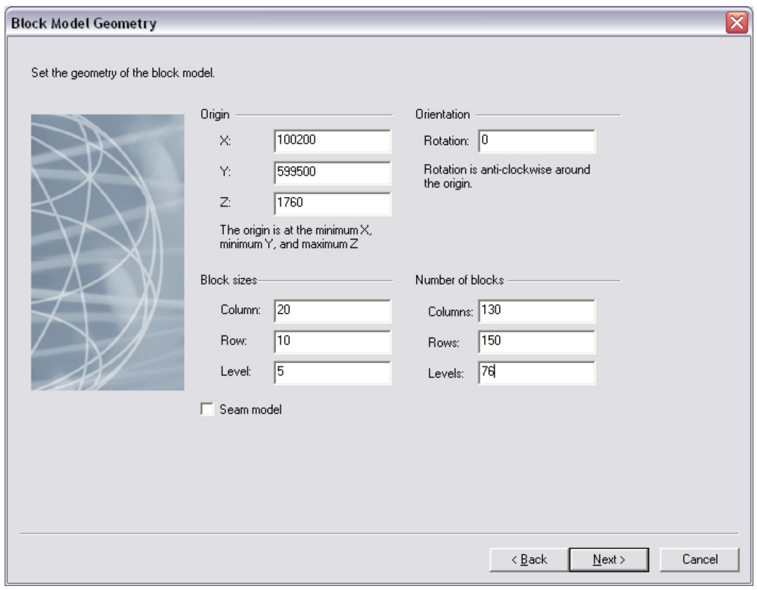


Fig 8. Block model geometry.

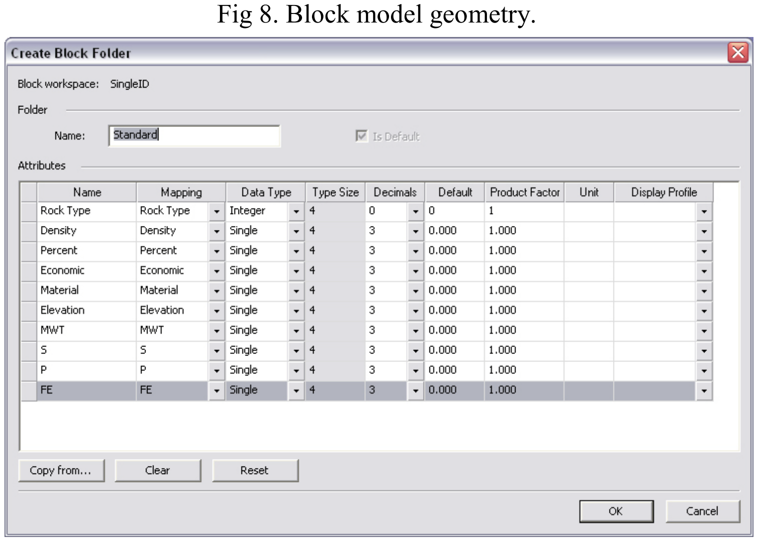


Fig 9. Block folder attributes.

6. Go to **Block > Edit > Workspace properties.** Check the summary of all the levels that was created in the block model along with their respective top and bottom elevations (Fig 10). This dialog allows you to add more levels if changes are required. No changes are required here.

7. In the project view area, you will see that under Block models, there is a folder SingleID that is now red with an asterisk at the end. You should right click on it and save the created block model. Under the block model project there is a standard folder that branches to RockType, Density, Percent, Economic, Material, Elevation, MWT, S, P, and FE. These are the properties associated to the block model.

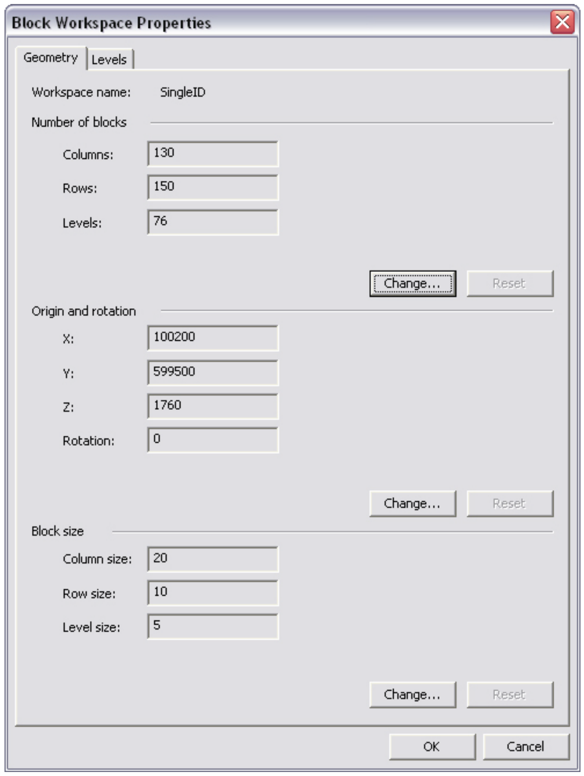


Fig 10. Block workspace properties.

Each block model created within a folder can be mapped to a block model type. After you create block models, you can choose **Block > Create > Folders > Edit Block Folder** to re-map model types at any time.

In each folder, you may create as many models as you wish. Note that at any one time, you can have only one for each type (e.g. one model mapped as the rock type model, one as the density model, etc.) for that folder. You can alter the mapping to experiment with different scenarios, but if you wish to make two models of the same type active, you must assign them to separate projects. For example, if you want two rock type models active, they must be in separate folders.

6.3. Create additional attributes

To create additional block model attributes that was not part of the initial standard folder:

1. Select the "Standard" folder under SingleID, then
2. Go **Block > Create > Attribute** (see Fig 11)
   - Name: Class
   - Mapping: Generic
   - Data Type: Integer

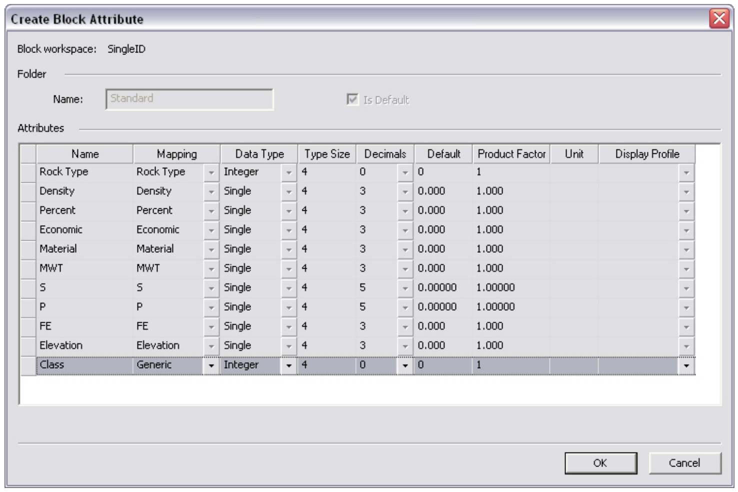


Fig 11. Creating additional block model attributes.

We will use this attribute later on for resource classification into measured, indicated, and inferred.

6.4. Create cell display profiles for block model

Click on the block model you just created in the Project View Area to make sure it is open (there should be a little circle with a white check mark to the right of the folder icon).

1. **Format > Blocks > Block Display Setting** will show up
2. Add a profile called ROCKS (see Fig 12)
   - Show all block attributes as Outlines; cubes would be more visually appealing but can be very slow when the block selection is large and rendering turned on
   - Since information we want to display is rock type, choose the color profile: LITH
   - Accept the default shrinkage factor. This can be used to scale the sizes of the boxes.
3. Add another cell display profile called MWT and choose MWT_GRAD as color profile.
4. Click **Apply** and the **OK** to save the profile and exit out of this screen.
5. Back in the Project View area, right click the Rock Type folder and select Properties
   - Display profile: ROCKS
   - Click OK and save the changes to the SingleID block model

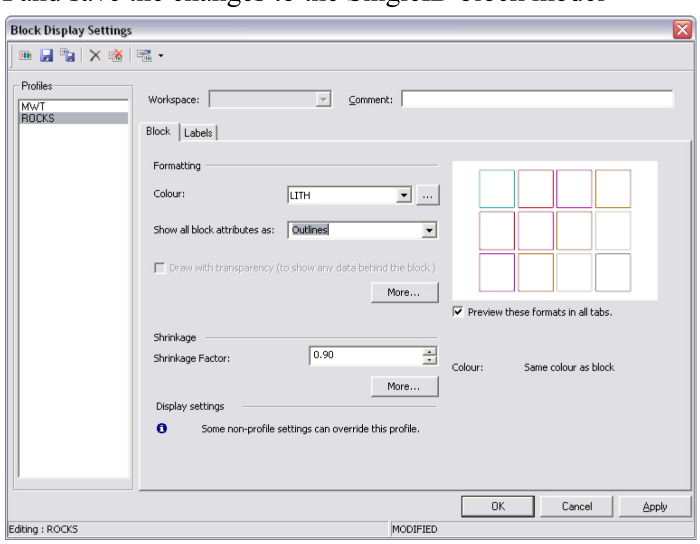


Fig 12. Block display settings.

6.5. Block model interpolation

6.5.1. Assignment of rock codes

When you assign rock codes to blocks from geological solids or polygons, GEMS performs the following tasks:

- Determines the rock codes contained within each block
- Determines the predominant rock code
- Assigns the corresponding rock code to each block

Typical process is shown in Fig 13.

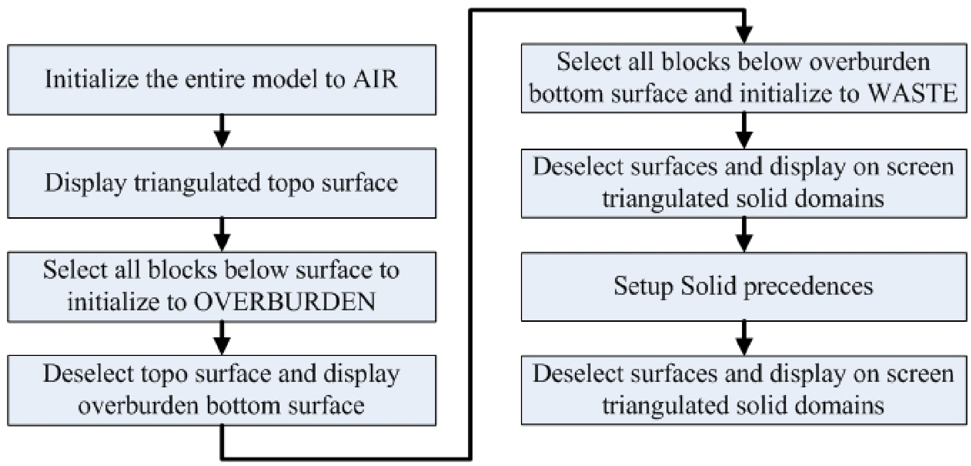


Fig 13. Process of updating blocks from surfaces and solids.

To initialize the block model rock codes you can follow these steps as an alternative to the process represented in Fig 13:

1. In the Project View Area, right click the block model project, and select initialize.
2. To initialize a single model, in the Project View area, expand a block model project to view all models, right click the model and select initialize. You can initialize all blocks in the model, only the blocks that are currently selected, or blocks not currently selected.

To initialize the entire rock type model to a default background value, which is usually unspecified waste follow these steps:

- Right click the Rock Type folder under the block model project and select initialize. Enter the rock code of the unspecified waste 8, as the Initial Value and check, **All blocks in the model**. This effectively assigns all the blocks as belonging to the undefined waste (see Fig 14).

Before anything is displayed, the blocks to be displayed must be selected. Various methods are possible and will be looked at as data is added to the model. This is one way to select Level 1 of the block model coloured by undefined waste material:

- Block > Select > By Column/Row/Level Range
- Level: from 1 to 1
- Row: from 1 to 150
- Column: 1 to 130

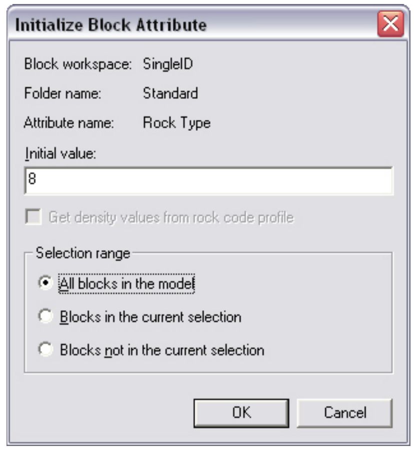


Fig 14. Initializing the Rock Type folder

To verify that all the blocks were initialized to the undefined waste material with Rock code 8 you can use one the methods below:

- **Block > Edit > Edit Block Data**; you can go through some columns, rows and levels and they should all show a Rock Type of 8 (see Fig 15).

Now the entire model is considered to be the undefined waste material, but we know that any blocks above the surface topography should be set as air.

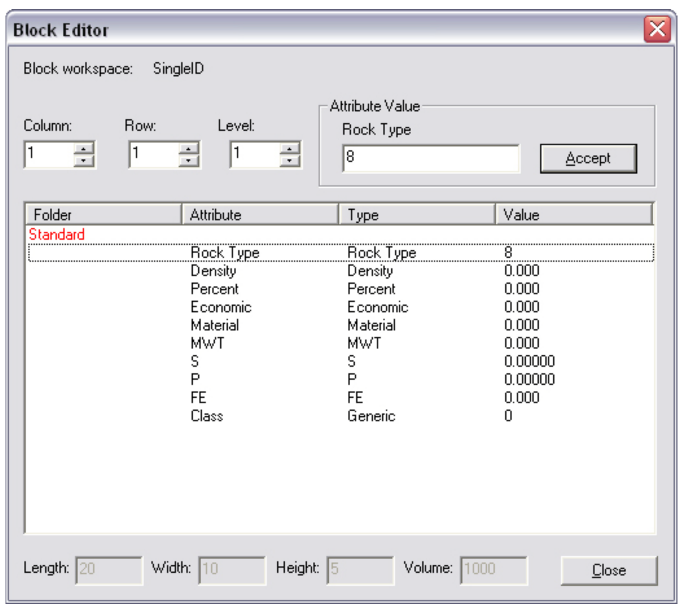


Fig 15. Block Editor.

Also, we are interested in setting any blocks between topography and the surface representing the bottom of the overburden material as overburden. Ordinarily this information could come from several places – DXF files, point data, or digitizing topographical maps. If the topographic data is not available, the drillhole collar elevations could also be used to create the topographical surface. We have modeled the topography and a surface representing bottom of the overburden in the Surfaces section. The topography model is a Surface object named Topo and the surface representing the bottom of the overburden is named “Bottom OB”.

3. Before moving ahead, make sure that you have added **Air** to the Rock codes. To check go **Format > Other Profiles > Rock Codes**. If there is no Air profile, create one:
   a. Rock type: Air
   b. In-situ density: 0
   c. Block rock code 999

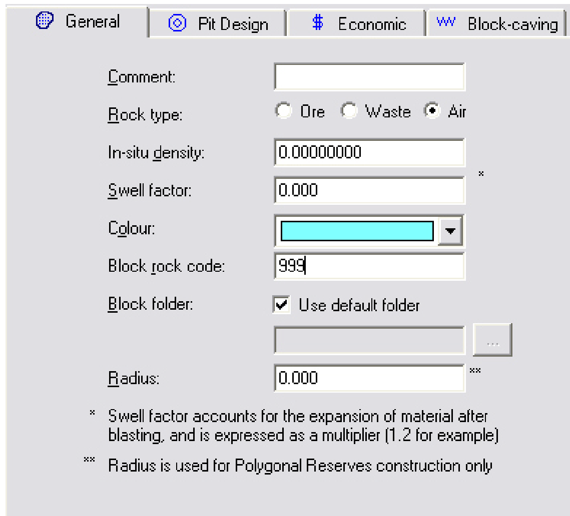


Fig 16. Create Air rock code.

4. Load the Bottom OB surface by right clicking on the Surfaces workspace and opening up the surface.
5. To change any blocks above the Bottom OB surface to Overburden, they must first be selected.
   - Create a selection from the block model using the surface by going to **Block > Select > Above Surface**. Now select the surface with your mouse.
   - Set the integration level to 4, and the minimum percent of block above the surface to 50. Leave all other defaults. If at least 50% of the block is above the surface, it will be selected. The integration level refers to the number of 'needles' which are passed through each block. An integration level of 4 means 16 needles/block is used (Fig 17).
   - All the blocks, which are above the surface, are selected. Do not check the option to show selected blocks – this can be time consuming.

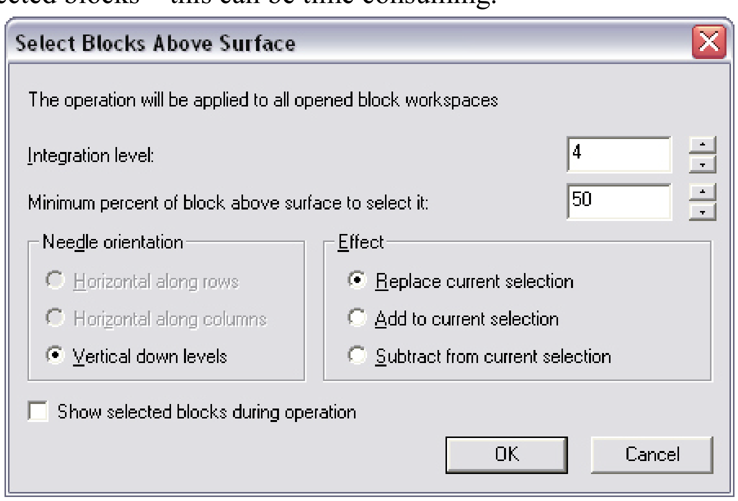


Fig 17. Select blocks above the overburden bottom surface.

6. Now that the blocks above the surface have been selected, they can be initialized to the Overburden with rock code 5:

- Right click Rock Type in the Project view area, and select Initialize
- Initial value: 5, select 'Blocks in current selection' (Fig 18).

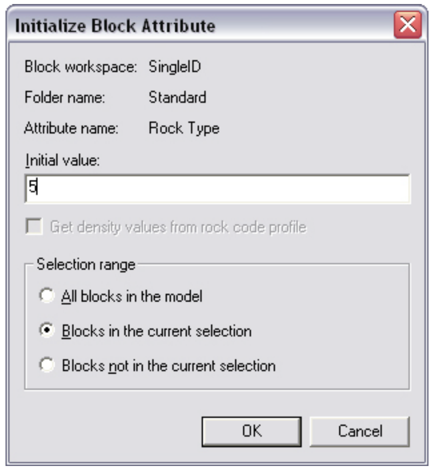


Fig 18. Set block rock type to overburden.

All the blocks above the overburden bottom surface are now tagged with Rock Type 5. We will do the same procedure to code the blocks above the topography as Air blocks.

7. Load the topography surface by right clicking on the Surfaces workspace and opening up the Topo surface.
8. To change any blocks above the Topo surface to Air, they must first be selected.
9. Create a selection from the block model using the surface by going to **Block > Select > Above Surface**. Now select the Topo surface with your mouse.
   - Set the integration level to 4, and the minimum percent of block above the surface to 50. Leave all other defaults. If at least 50% of the block is above the surface, it will be selected. The integration level refers to the number of 'needles' which are passed through each block. An integration level of 4 means 16 needles/block is used.
   - All the blocks, which are above the Topo surface, are selected. Do not check the option to show selected blocks – this can be time consuming.
   - Now that the blocks above the surface have been selected, they can be initialized to the Air with rock code 999:

     → Right click Rock Type in the Project view area, and select Initialize

     → Initial value: 999, select 'Blocks in current selection'.
   - To display air blocks and undefined waste, you have to add air to the LITH color:

     → **Format > Other Profiles > Colour**

     → Profile colour (RGB 192 255 255)
10. Go to **Block > Select > By Column/Row/Level Range** and select All blocks in rows, columns, and levels in the left window, under standard folder right click on Rock Type and select show. You should see a block model rock type similar to Fig 19.

Fig 19. Rock type model containing default background value undefined waste, overburden, and air blocks.

6.5.2. Update Rock types from Solids

Now that we have the entire block model set as either air, undefined waste, or overburden we can further update these rock codes for the mineralized zones and other waste types.

A block model can be updated using one of the following methods:

- Pre-select the blocks then initialize the selected blocks to a specific value.
- Update block model from points, polygons, or solids. This method is accomplished by a technique called needling.

We will use the second method to update the rock type model. Updating a block model with solids or polygons requires a numerical integration technique called "needling" to determine a value to assign to each cell. With this method, GEMS creates long, horizontal, or vertical "needles" to intersect with block models and any active solids or extruded polygons. The needles are similar to drillholes, having a starting point, a direction, and a length.

Block model updating employs a simpler needling approach – needles are always oriented along the block model columns, rows, or levels in a regular pattern. GEMS generates the needles from a "grid plane", the dimensions of which are taken directly from the block model (the size and number of levels and rows, and the co-ordinates of the upper left-hand corner). All needles have equal weighting. The fixed needle orientation and regular spacing reduce both the complexity of the application and the required processing time. By checking the entrance and exit points of each needle through the solids, GEMS calculates what proportion of each block is inside each geological or excavation solid or extruded polygon (see the following image). GEMS then reassigns the block values based both on this amount and the type of updating (attribute or percentage), see Fig 20.

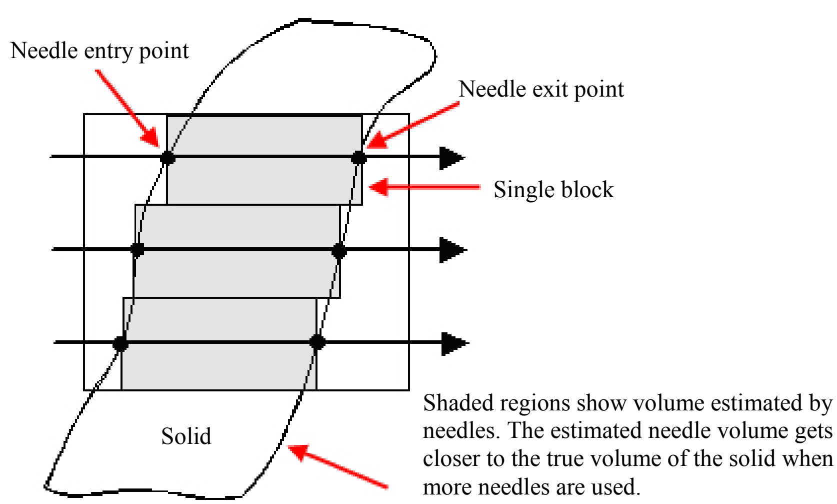


Fig 20. Three needles penetrating one block of a block model

The density of the needles (the integration level) can be varied for block model updating according to the desired level of accuracy. The integration levels (1 to 99) determine the number of needles used in each cell of the block model. The number of needles equals the integration level squared (that is, level 3 means that nine needles will penetrate each cell; level 8 means 64 needles will be used).

Higher integration levels will yield more precise results. However, as the integration level increases, so does the time require for processing. As is the case with virtually all statistical methods, a larger number of samples (in this case, needles) does not necessarily give proportionally more precise results. For example, using level 20 (400 needles per cell) may provide results within .01% of using level 5 (25 needles per cell), and the report would take 16 times as long to generate. In this example, the additional degree of report accuracy would probably not justify the extra processing time.

When you assign rock codes to blocks from geological solids, GEMS performs the following tasks:

- Determines the rock codes contained within each block
- Determines the predominant rock code
- Assigns the corresponding rock code to each block

In GEMS, you can use solids to update block models in the following ways:

1. Each block can be assigned a value based on one or more geology class solids and their assigned attributes:
    - → Rock type
    - → Density
    - → Grade (from list of defined grade elements)
    - → Percent of block inside (or outside) solids
2. Each block of a percent ore or waste model can be updated with percentages using one or more geology class solids
3. Each block of a percent-mined model (a common user-defined model) can be updated with percentages using one or more excavation class solids (either as-mined or planned pits or stopes)

You can activate varying solid types depending on the type of model you wish to update, see Table 3. If there is any chance of overlapping solids, you must activate all solids so that they may be used during the needling process to resolve overlaps. For example, even if you are only updating an ore rock type model, you must select all ore and waste solids.

Table 3. Type of solids which should be activated for different model types

For this model type	Activate these solids
Standard rock types	All ore solids, All waste solids
Ore rock type, Percent ore	All ore solids
Waste rock type	All waste solids
Percent mined	All excavation solids

There will be occasions where solids used for the block model updating procedure will overlap. In some cases, the overlap may result from errors made when creating or selecting the solids. However, in other cases you may produce this overlap intentionally (for example, to simplify solid creation). In either case, GEMS has some simple rules to handle solid overlaps, so areas where solids overlap are never accumulated twice.

- If solids overlap, precedence is determined by the solid precedence. This can be explicitly set for each solid using the **Volumetrics > Solid and Surface Precedence** command.

- If no solid precedence has been explicitly set, the geology solid with the rock code occurring higher in the alphanumeric rock code list will take precedence. For example, if one solid with rock code 10 and another with rock code 20 overlap, 10 will be listed first and will take precedence. Unless you set precedence explicitly, the rock codes that represent the most recent geological formations should be at the top of the list so as to override the older formations. This list may be edited at any time.
- If excavation class solids overlap and no solid precedence has been explicitly set, precedence is given to the first excavation solid in the solid listing. You may view (but not edit) the list with the **Solid > Data > Select Solids from List** command.

Overlapping solids can be used to greatly simplify the solid creation process if your goal is to update a rock type block model. For example, you may have a property where the host rock contains dykes that were formed more recently. Instead of creating separate solids for all the parts (which can be quite time consuming), it may be more convenient to simply create one solid for the host rock type, and let the application handle the overlaps for you. If there is any chance of overlapping solids, you must activate all solids so that they may be used during the needling process to resolve overlaps. For example, even if you are only updating an ore rock type model, you must select all ore and waste solids.

We will update the block model for the mineralized zones and wastes in the following precedence order from left to right, can you explain why? **101 → 201 → 301→ 1 → 9 → 2 → 13**

To set up the precedence of solids:

1. Load all the solids representing mineralized zone and waste areas. This includes: SolidMZ101, SolidMZ201, SolidMZ301, Waste02, W9Solid, and W13Solid.

   a. Right click on **Triangulations** in the **Project View Area,**

   b. **Add Workspace,**

   c. Right Click on the Workspace > Open > then open the desired Solid,

   d. Repeat steps **a** to **c** for all the solids.

2. On the **Volumetrics** menu, click **Solid and Surface Precedence**. In the dialog box that appears, under Order Value, assign an order to the solids that overlap (Fig 21).

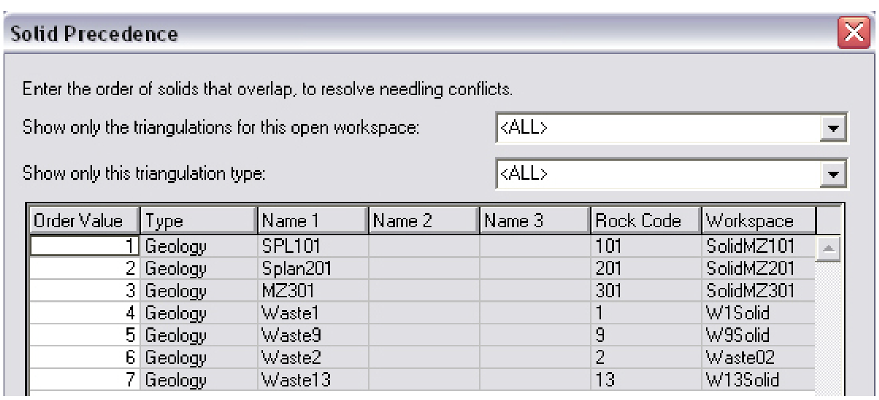


Fig 21. Solid Precedence.

3. In the **Project View Area** and select **Update from Solids**; or from the main menu, select **Block > Edit > Update Block Model from Solid**. In the dialogue box which opens (Fig 22)

   - Solid attribute to update form: Rock Type,
   - Integration Level 4 – 16 needles per block,
   - Use: All Materials,

- Minimum percent required to reassign blocks: 55,
- leave all blocks in model checked,
- Needle Orientation – Vertical down levels,
- ***UNCHECK*** *Initialize Blocks to Default Values*

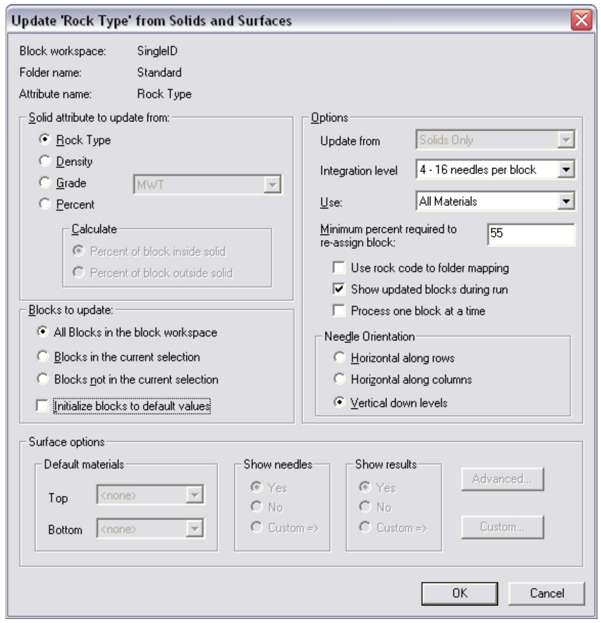


Fig 22. Update Rock Type dialog.

- Click **OK**
- During the run blocks are going to be highlighted.
- Close all the Solid triangulations.
- Save the block model after the completion of run.

4. To verify that the rock type model is properly updated, you can visually check the block model.

   - **Select Block > Select > By Column/Row/Level Range**
   - Select different cross sections to check the rock types and the order that the blocks have been updated. Fig 23and Fig 24 illustrate the block model at two sections after rock type updates.
   - To visually check for a specific rock type:

     → **Block > Select > By Block Attribute Value Range.**
     → Select the Rock Type and specify 100 and 302 as the lower and upper thresholds, respectively, this will select all the mineralized zones. Make sure that "Replace Current Selection" is checked.
     → In the left window, right click on the Rock Type under standard folder and select show, you will see blocks representing three mineralized zones (Fig 25 and Fig 26). You can experiment with these blocks – try changing the display profile from outline to cubes.

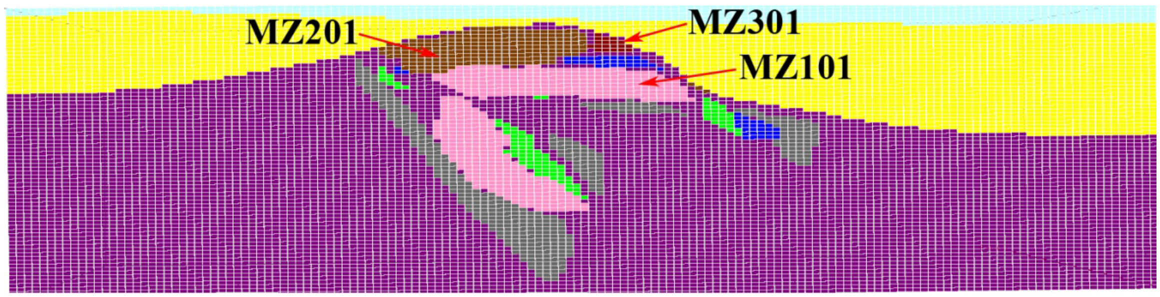


Fig 23. Rock types in column 52 looking east.

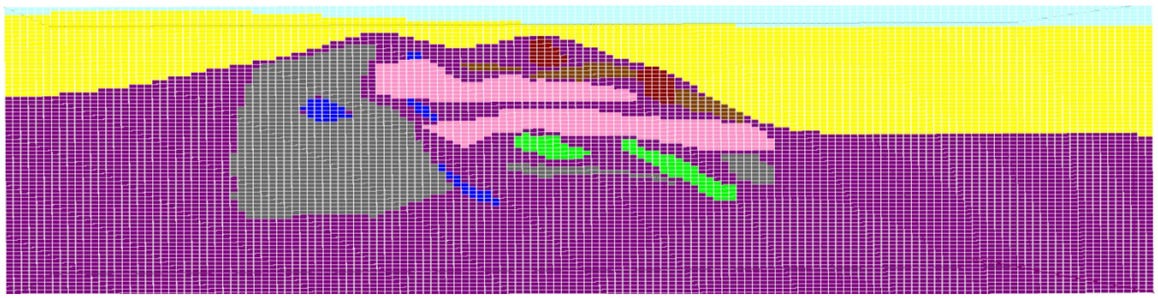


Fig 24. Rock types in column 72 looking east.

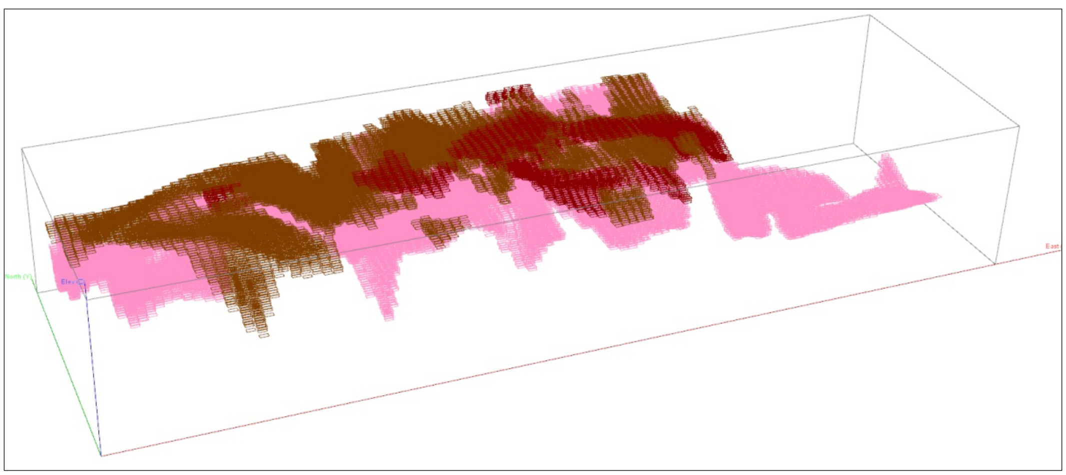


Fig 25. Mineralized zone rock types- block outline view.

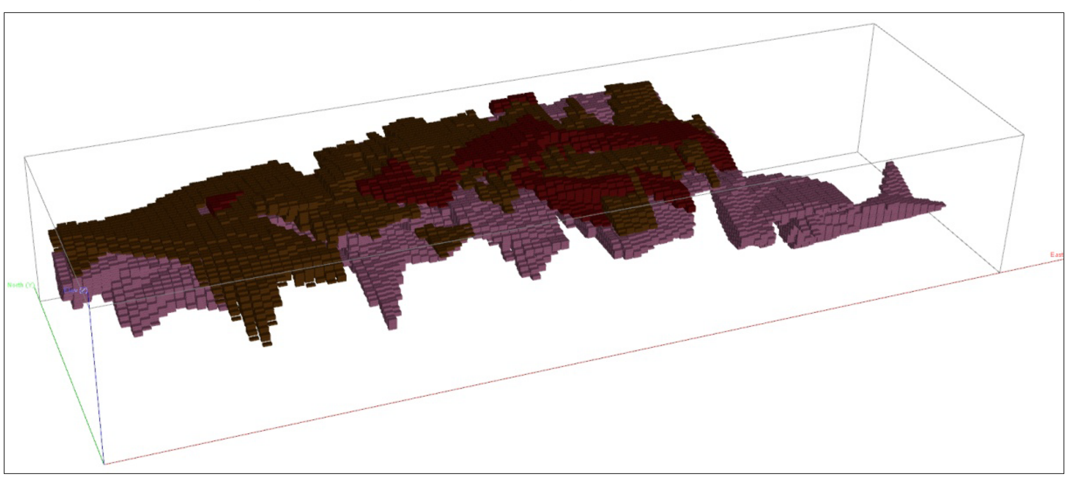


Fig 26. Mineralized zone rock types- cubes view.

6.5.3. Updating density

The rock densities are very easy to enter into the model – they come directly from the rock profiles and the rock codes already in the block model.

1. In the project view area, right click on the Density folder, and select initialize. Set the initial value to 0.0 and make sure that the selection range includes all blocks in the model. Also, check the Get Density values from Rock code profile box. **Click OK**.

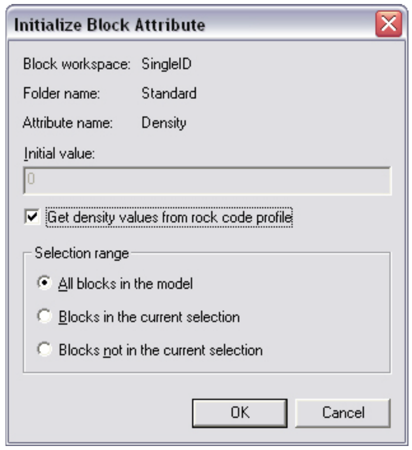


Fig 27. Updating densities from rock profiles.

2. To check that you now have densities, use the block editor to check the values in some of the blocks: **Block > Edit > Edit Block Data** and select a block near the solids region.

6.5.4. Grade estimation using Inverse Distance Squared (IDS)

You have extracted the drillhole data points for each mineralized zone based on the rock types into PAMZ101, PAMZ201, and PAMZ301 in the previous labs. We are going to use these Point Area Workspaces and the Variogram profiles and the Search Ellipse profiles associated with each mineralized zone in the interpolation using inverse distance squared.

When using an inverse distance method, you must set the ID power to specify the rate at which influence falls off with distance (1/D, 1/D2, etc.). The distance (true or anisotropic) from sample to block is used twice. The first instance is during the search for eligible samples. The second is for the actual computation of sample weights. The distances used for sample searching and selection are anisotropic if an anisotropic search volume is defined.

a. **Inverse Distance (True)**: If you select this method, true distances from block to sample will be used for computation of sample weights.

b. **Inverse Distance (Anisotropic)**: If you select this method, anisotropic distances will be used for computation of sample weights. Anisotropic scaling occurs when something (distance) is scaled by different amounts (axis length) in different directions (axis rotation). Fig 28 illustrates the difference between the true and anisotropic distances.

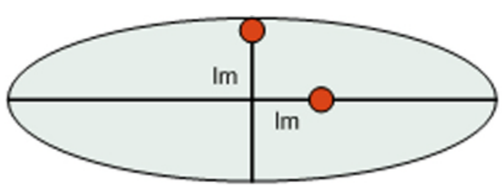


Fig 28. True distance- both samples will have the same weight. Anisotropic distance- sample on the long axis will have different weight than the sample on the short axis.

Go to **Format > Geostatistics > Interpolation**. As usual, we need to create a profile, in this case an interpolation profile:

- Click **New Profile**, type in the **Profile** name, and call it MWT-ID (MWT inverse distance squared), click **OK**.
- **Interpolation Tab** (Fig 29**)**:
  - Calculation Method: Inverse Distance (true)
  - ID Power: 2
  - Number of samples to use: is a powerful tool when you want to control the number of samples that will affect the interpolation results.

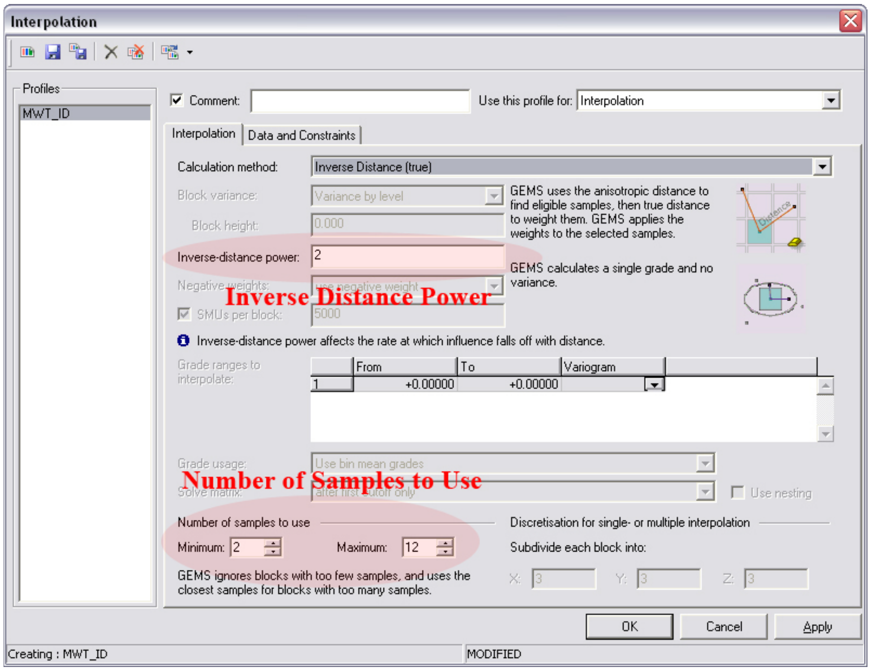


Fig 29. Interpolation tab.

b. **Data and Constraints Tab**:

- Click on “Block Model” in the Tree View Area to select the block model
- Use this block workspace: SingleID (Fig 30).

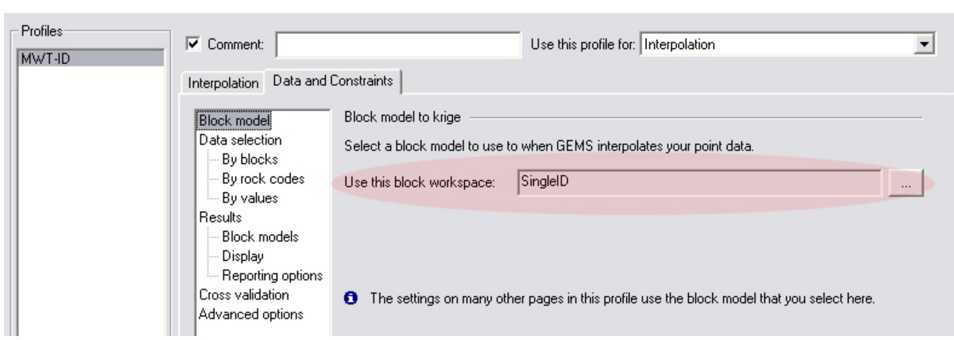


Fig 30. Block model selection.

- **Data selection – By blocks**

  → These options let you perform calculations on a sub set of or the entire block model. Table 4 summarizes the options available.

Table 4. By blocks

If you select …	GEMS will …
Use all blocks in the block-model layout	look at and calculate every block in the model.
Use the blocks that you select before interpolation starts	only calculate the selected blocks.
Exclude the blocks that you select before interpolation starts	calculate all other blocks with the exception of the blocks that are already selected.
Always use these specific blocks	calculate only on the specified sets of blocks.

→ Under "By Blocks", select "Use all blocks in the block-model layout" (Fig 31).

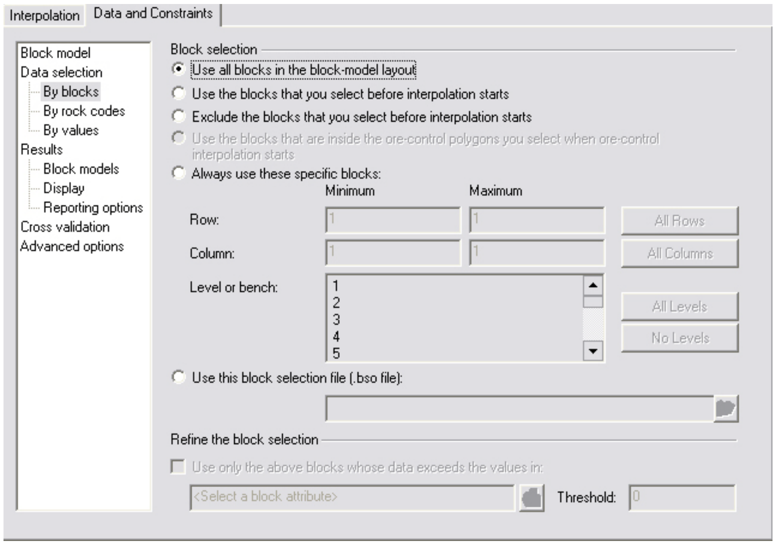


Fig 31. Data selection – by blocks.

- **Data selection – By rock codes**

Data selection by rock codes is the most important portion of the profile.

→ **Target rock code:** Enter the target rock code in the list. Only blocks that have this code(s) in the rock-type model will be considered for interpolation. The extracted data in the point area are typically structures using one or two methods (Fig 32).

(a) One point area per geological domain or rock code.

(b) One big point area for all domains where an integer field controls the domain affiliation.

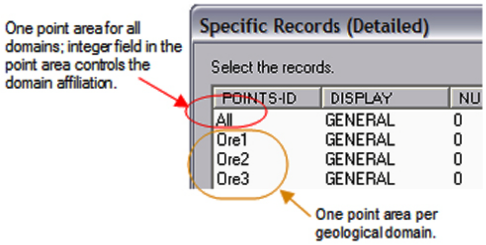


Fig 32. An example showing one point area per geological domain or one point area for all the domains.

For each rock code in the list on the left of the dialog, you need to select the source of the data that will be used for the interpolation.

→ **Target rock codes:** Get the rock codes from your rock model (Fig 33).

→ Select 101.

(a) **Point-area workspace:** Select the point area workspace.

→ Select PAMZ101.

(b) **Point area:** Select the point area. This list is initialised based on the selection of Point-area workspace.

→ Select MZ101.

Tip: If your extracted data are in one point area per domain then the point area will be different for each rock code in the list.

(c) **Grade (real) field:** Select the grade field or the input point values to be used from the point area.

→ Select MWT_CUT

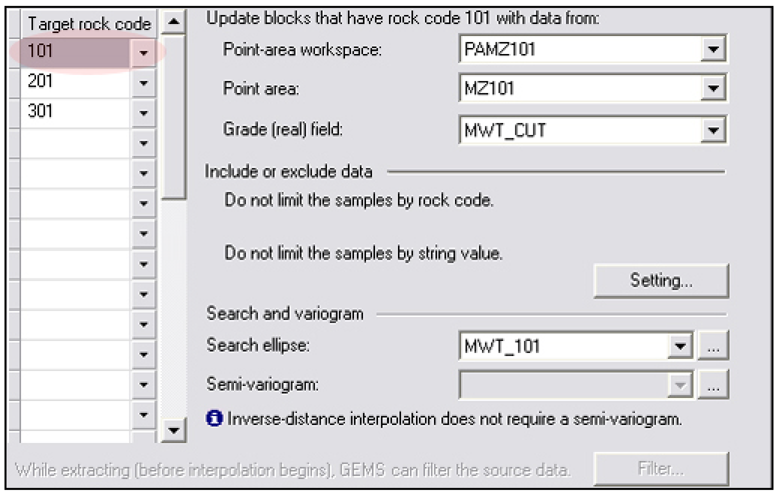


Fig 33. Target rock code 101.

→ **Include or exclude data:** This is the rock-code (integer) field. To access this field, click **Setting**. This setting is optional, with the function that each point in the point area can carry an integer code that tells GEMS to which geological domain they belong. If you choose this option, you will need to update the fields in the **By limiting the sample frequency** section.

→ You can limit the sample frequency based on a string value in the sample dataset. This option is a de-clustering tool typically used to cap the number of samples per drillholes. The drillhole ID is usually stored in a field called Hole-id in the point area workspace. If more than the maximum number of occurrences is reached for one drillhole, GEMS will look for other points, even though the closest point may still be from the drillholes (Fig 34 and Fig 35).

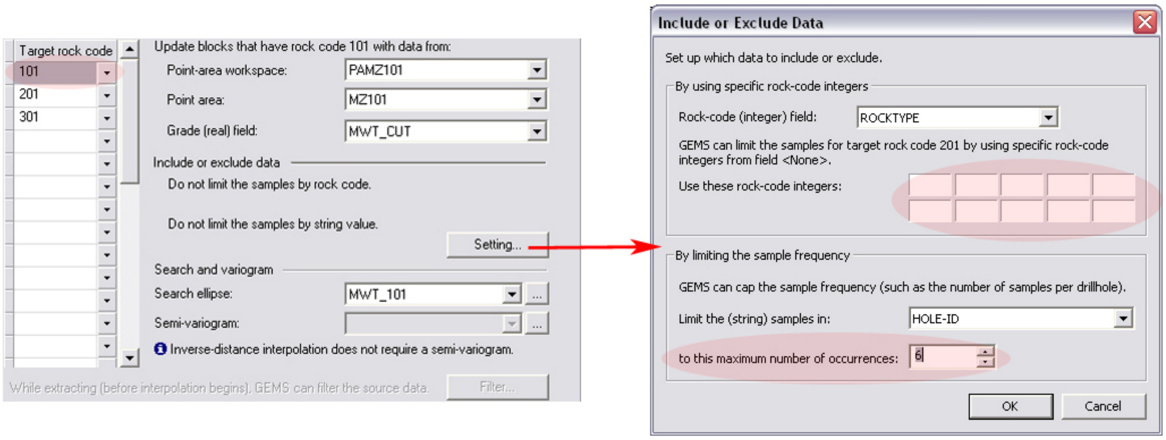


Fig 34. Include or exclude data with the rock-code (integer) field.

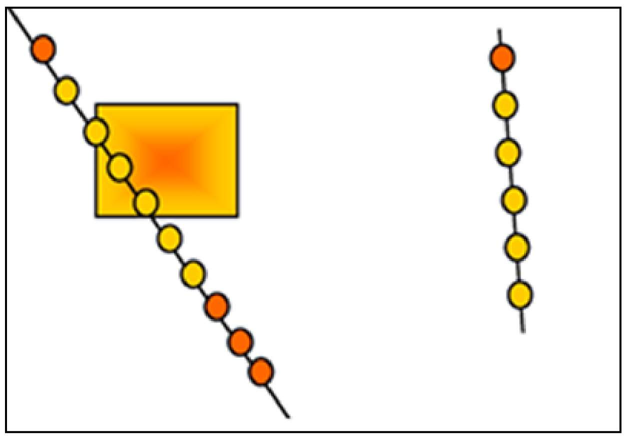


Fig 35. 2 samples minimum, 12 maximum, Max per hole limited to 6.
Only yellow samples will be selected.

→ Search and variogram: click on the "…" button next to Search ellipse. This will open another window where you can create a profile for the search ellipse specification. You have created this before based on the Variogram modeling in the previous labs. Select **MWT_101** for Search ellipse (Fig 33).

→ You are using Inverse Distance Squared so the semi-variogram option will not be active.

→ Call this profile "MZ101". In this case, no rotation will be selected; however, restrictions will be placed on the searching, so change only the ranges (on the left) to 150 for all three directions.

- Now go back to the target rock code on the left and in the second row, select 201. You will notice that all the information you selected on the left is reset (think of this as setting up a different set of specifications for a zone). Similar to above, select the point area workspace and all appropriate fields for grade and rock type information (Fig 36a).
- Go back to the target rock code on the left and in the second row, and this time select 301. Similar to above, select the point area workspace and all appropriate fields for grade and rock type information (Fig 36b).

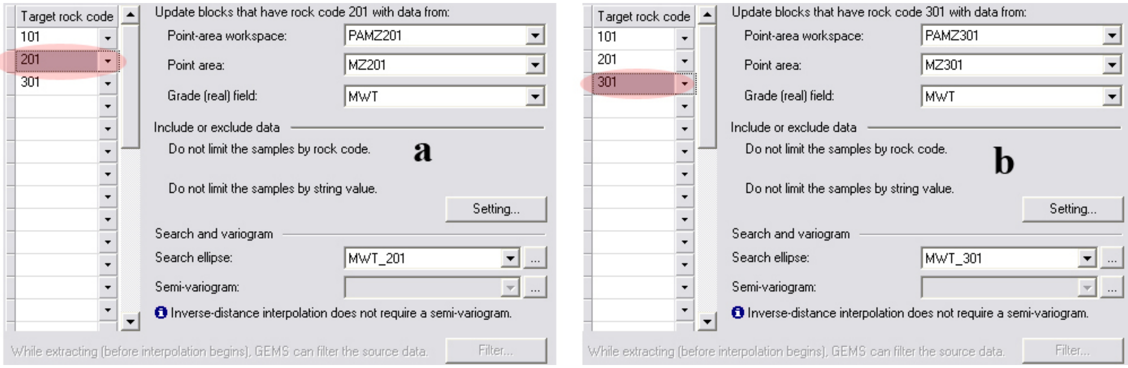


Fig 36. Updating rock codes 201 and 301.

- **Data selection – By values (**Fig 37**)**

  This setting lets you limit the interpolation to the values within a specific range. If you haven't already limited the values of the raw samples within a specific range, the high-grades limit (which is commonly referred to as "Top Cut") can truncate the input data to a certain value. If you have duplicated data points or points that are at the same location, you can set **To resolve** the conflict by:

  → Do nothing

  → Detect and report only

  → Detect and attempt to fix by moving

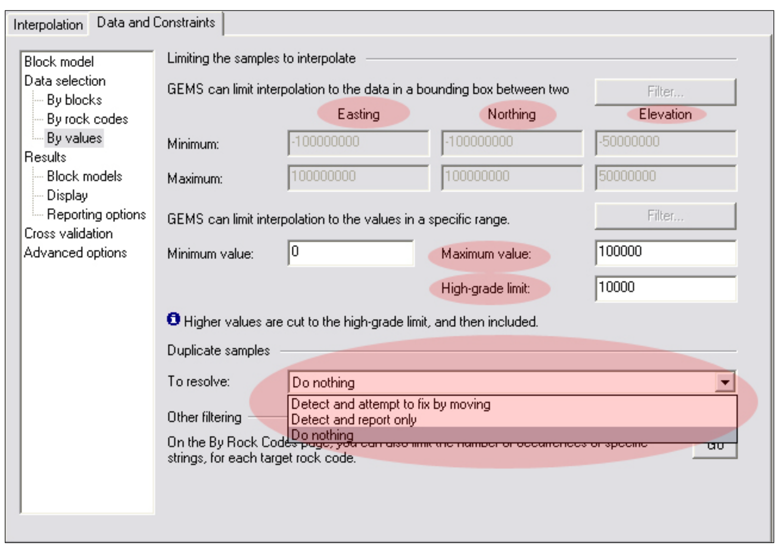


Fig 37. Data selection – By values.

  → Under "By Values", leave the defaults.

- **Results – Block models**

  This setting lets you indicate the target model where the results of the interpolation will be written to. You can also restrict the update to all blocks, only blocks that have selected rock codes, or update blocks that have zero grades. You can also save any special kriging results to a specific model in the block model project.

  For "Block Models", allow GEMS to overwrite completely. Choose to "Save grade results in Grade attributes", and select the **Standard\MWT (**Fig 38**)**.

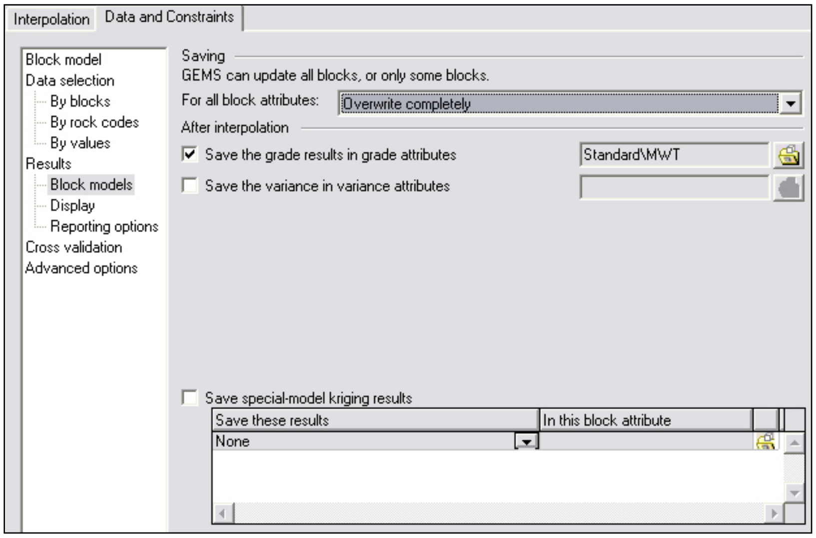


Fig 38. Results – Block Models.

- **Results – Display**

  This setting lets you display the data point, and the blocks interpolated on screen during the interpolation run. Note that these options consume processing, and thus, will slow down your interpolation run.

  → Colour profile for blocks and points: **MWT_GRAD**.

  → Leave all other defaults.

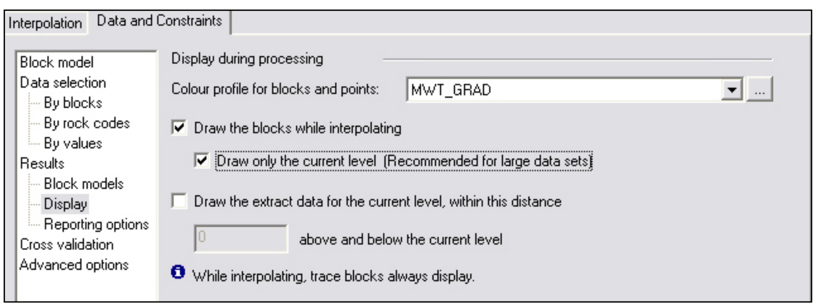


Fig 39. Results – display.

- **Results – Reporting options**

  Use these settings to define the reporting options.

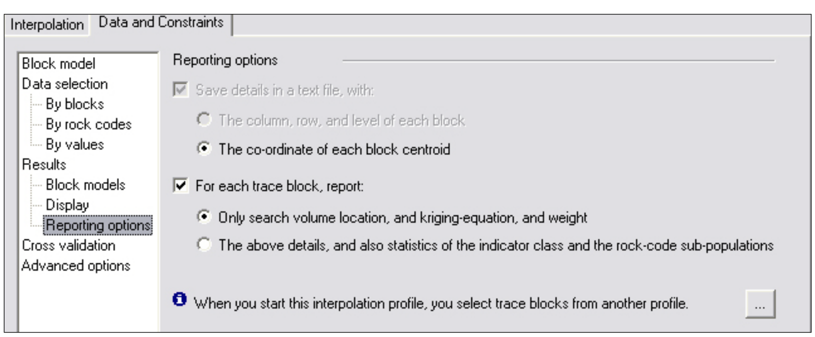


Fig 40. Reporting options.

- **Cross Validation**

Cross validation or Jack-knifing is a method where a given set of sample points are replaced by a pseudo block of a given size and a value is kriged in this block. The result of the calculation is documented in the report**.** The results can be compared against the real grades at the point to validate the model (Fig 41). You can leave the default for now. You will use this later on when we estimate grades by kriging method.

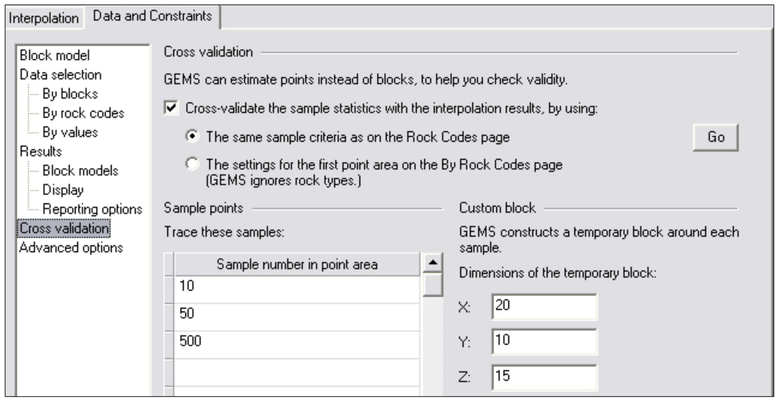


Fig 41. Cross validation.

- Press **Apply**.
- Save the profile and press **OK**.

- **Calculate Grade Model**: Once the interpolation profile has been entered the actual interpolation may be carried out. Before you perform an interpolation run, close or hide all images, models, etc. in the graphics area. To perform an interpolation run, follow these steps.
- Go to **Tools > Geostatistics > Interpolation > Interpolate and Report…**, and select your interpolation profiles under "Interpolation Profile", (see Fig 42).
- Press **OK** and you should see blocks being interpolated on-screen (they will flash on-screen as an indication of progress).
- In **Use data from**, select and add all the interpolation profiles you want to run with their matching **Trace Block Profiles (**we don't have any at this stage).
- Click **OK**. GEMS will run the interpolation.
- When completed, GEMS displays **the Interpolation Reports dialog (**Fig 43). Click **Summary**, **View Detail Trace**, or **View Report File** to open any of the reports.
- Look at the reports, which are produced after the grade estimation has been carried out. The report contains useful information. The trace file contain detailed information on the estimation of the trace blocks – since we are not using any trace blocks, it will be empty. We will cover this later on. You can specify certain blocks to trace as a check on the interpolation results. To look at the results
- Click Summary a text file summary will open up (Fig 44).
- Select Interpolation Profile then press View Report File, a text file will open (Fig 45).
- When you are finished, click **Close**.

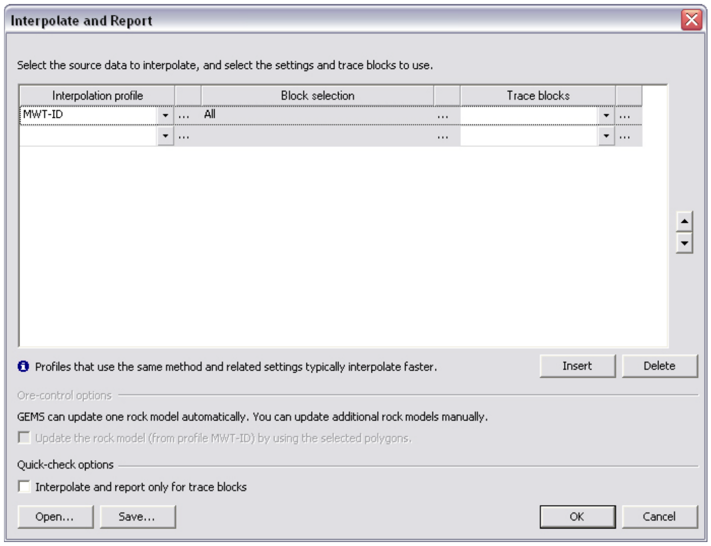


Fig 42. Interpolate and report.

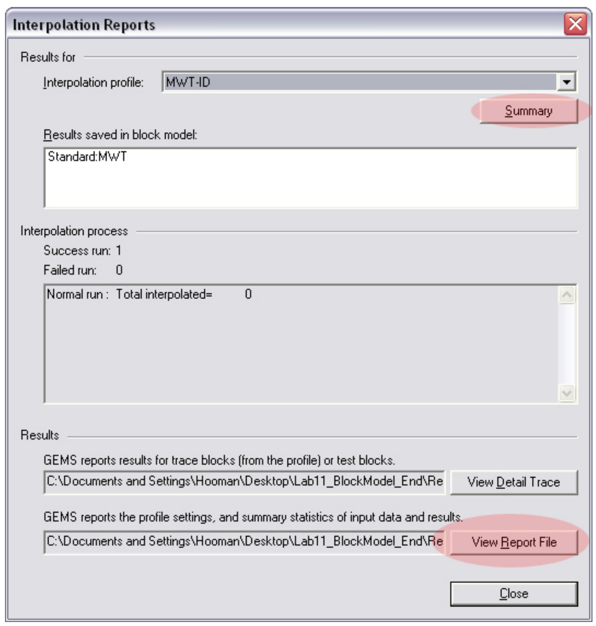
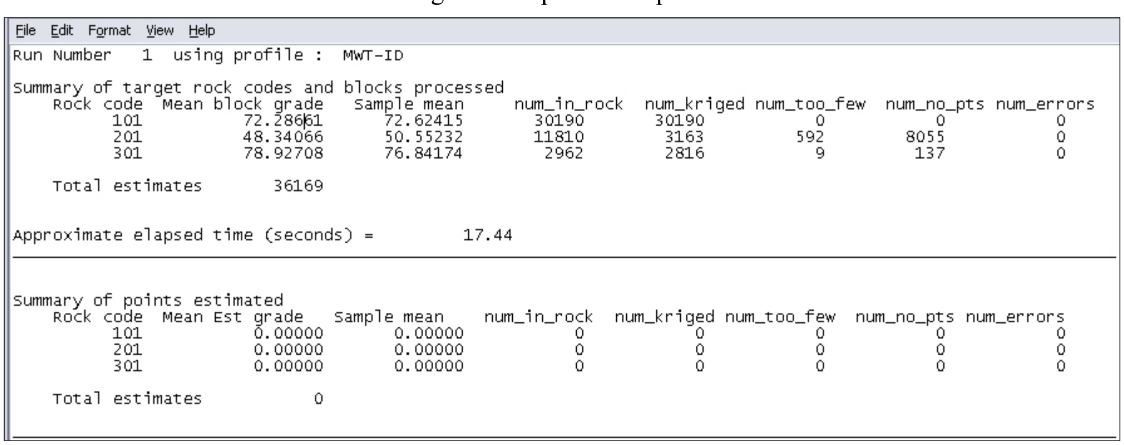
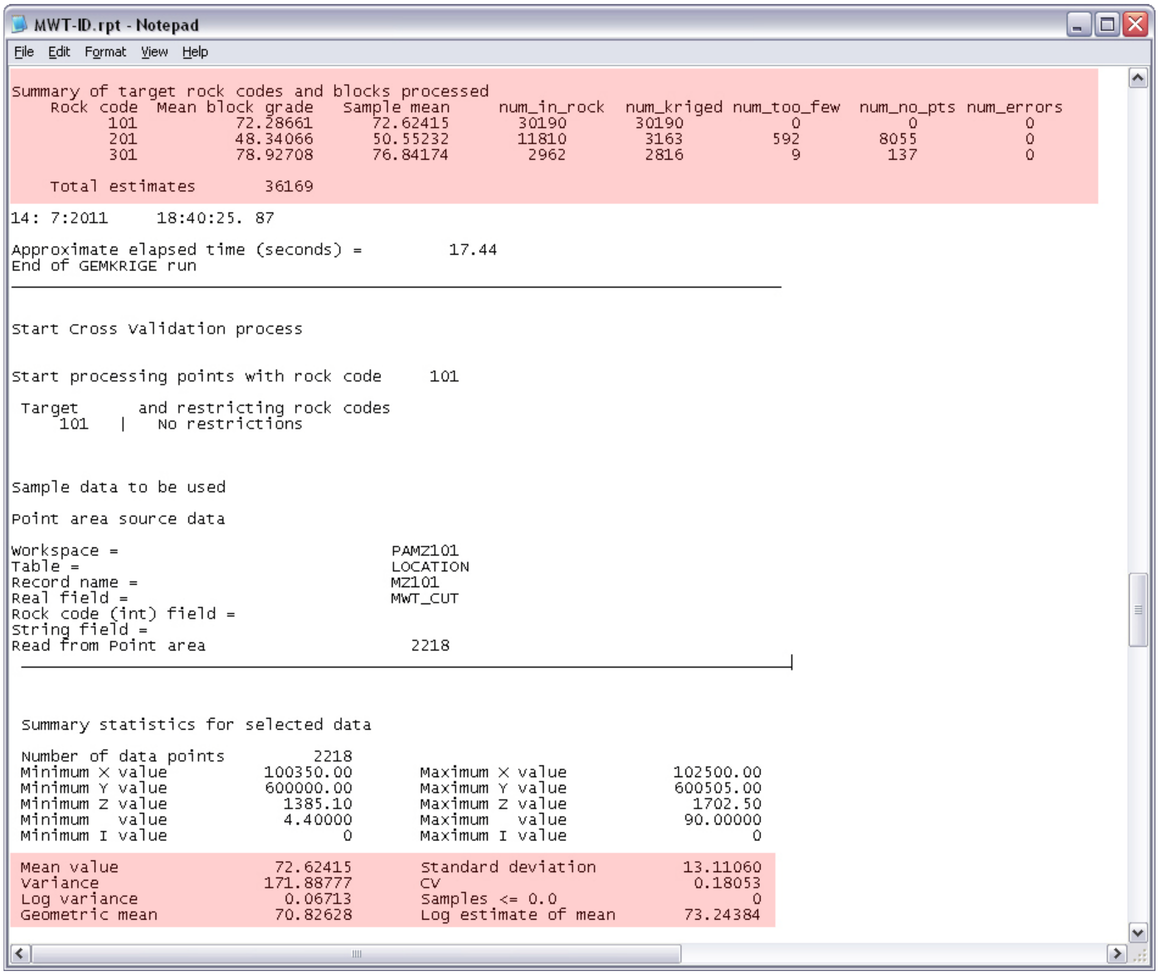


Fig 43. Interpolation reports.


File Edit Format View Help

```
Run Number   1  using profile :  MWT-ID

Summary of target rock codes and blocks processed
    Rock code  Mean block grade   Sample mean     num_in_rock   num_kriged num_too_few  num_no_pts num_errors
          101          72.28661      72.62415       30190        30190          0           0          0
          201          48.34066      50.55232       11810         3163        592        8055          0
          301          78.92708      76.84174        2962         2816          9         137          0

    Total estimates        36169

Approximate elapsed time (seconds) =        17.44

Summary of points estimated
    Rock code  Mean Est grade   Sample mean    num_in_rock   num_kriged num_too_few  num_no_pts num_errors
          101          0.00000      0.00000           0            0          0           0          0
          201          0.00000      0.00000           0            0          0           0          0
          301          0.00000      0.00000           0            0          0           0          0

    Total estimates            0
```

Fig 44. Summary report.



MWT-ID.rpt - Notepad

File Edit Format View Help

```
Summary of target rock codes and blocks processed
    Rock code  Mean block grade   Sample mean     num_in_rock   num_kriged num_too_few   num_no_pts  num_errors
          101         72.28661     72.62415        30190         30190           0            0           0
          201         48.34066     50.55232        11810          3163         592         8055           0
          301         78.92708     76.84174         2962          2816           9          137           0

    Total estimates       36169

14: 7:2011     18:40:25. 87

Approximate elapsed time (seconds) =        17.44
End of GEMKRIGE run


Start Cross Validation process


Start processing points with rock code     101

 Target      and restricting rock codes
     101    |    No restrictions


Sample data to be used

Point area source data

Workspace =                             PAMZ101
Table =                                 LOCATION
Record name =                           MZ101
Real field =                            MWT_CUT
Rock code (int) field =
String field =
Read from Point area                      2218


 Summary statistics for selected data

 Number of data points          2218
 Minimum X value           100350.00      Maximum X value          102500.00
 Minimum Y value           600000.00      Maximum Y value          600505.00
 Minimum Z value             1385.10      Maximum Z value             1702.50
 Minimum   value             4.40000      Maximum   value            90.00000
 Minimum I value                   0      Maximum I value                   0

 Mean value                 72.62415      Standard deviation         13.11060
 Variance                  171.88777      CV                          0.18053
 Log variance                0.06713      Samples <= 0.0                    0
 Geometric mean             70.82628      Log estimate of mean       73.24384
```

Fig 45. View Report File

6.6. Block display

Choose **Block > Display** commands to access options for viewing your block models. You can display data from multiple block models by associating each desired block model with a cell display profile, in which you can define the following display parameters:

- Block colour using a standard colour profile
- Display type
- Text height for block value display
- Number of decimal places for block value display
- Block scale factor, so you can shrink blocks (either uniformly or in proportion to the values they contain) to clarify their display.

You can display blocks in conjunction with associated SEGs or a surface. By selecting different view modes (such as 2D or 3D) or view planes (vertical sections, inclined sections, or plan views), you can have greater flexibility in displaying block models.

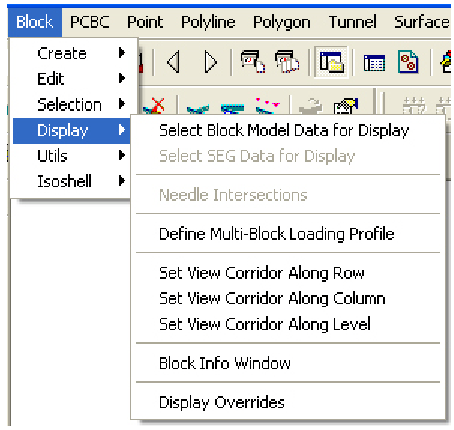


Fig 46. Block display menu.

Selecting Blocks for Display: In order to see blocks displayed, you must load, create, or define a selection.

6.6.1. Display Modes

Use one of the following methods for displaying block models:

- Block Outlines (Fig 47)
- Block Values
- Block Outlines and Block Values
- Cubes (Fig 48)

6.6.2. Text Height

When displaying block values, you can define the height of the text in scaleable units using the project measurement system.

Tip: Text height should be in proportion to the block size. If the block model is 5x5x5 then the text size should be about 2.5 – 3 if you display with no decimals points, smaller with decimals.

6.6.3. Decimal Places for Value String

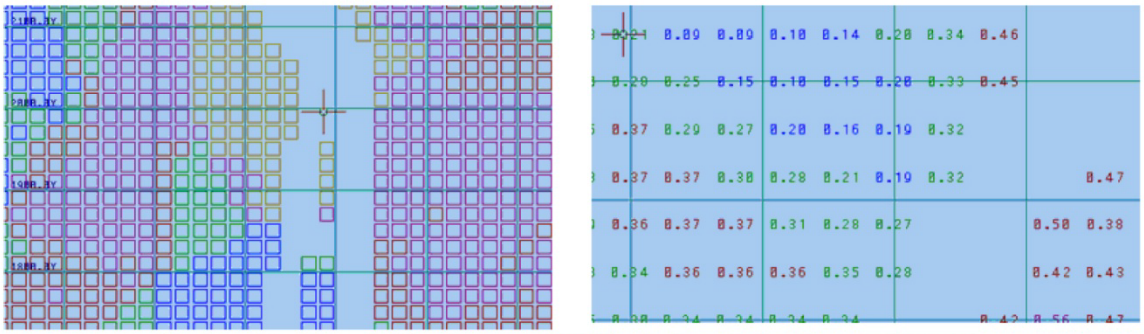
You can specify how many decimal places will appear when block values are displayed. For example, if you set this parameter to 2, the value 100 will be shown as 100.00.

6.6.4. Scaling

When blocks are displayed as cubes or outlines, the blocks are drawn with dimensions obtained from the block model definition. You can reduce the displayed size of the blocks by specifying a shrinkage factor less than one. For example, if the blocks are actually cubes with edges 20 feet long, setting the shrink factor to 0.5 will result in the blocks being displayed as cubes with edges 10 feet long.

You may specify a default shrink factor, or scale blocks so a block's size represents the value it contains. You can turn scaling on or off. Also, you can display or hide blocks with values outside the scaling limits.

Method	Description
Block outlines	This method displays 2D outlines of the block model (see the following images). No data values are displayed when you select this option.
Block values	This method displays the numerical data values of the block model (see the following images). You will be required to define the text height and the number of decimal places.

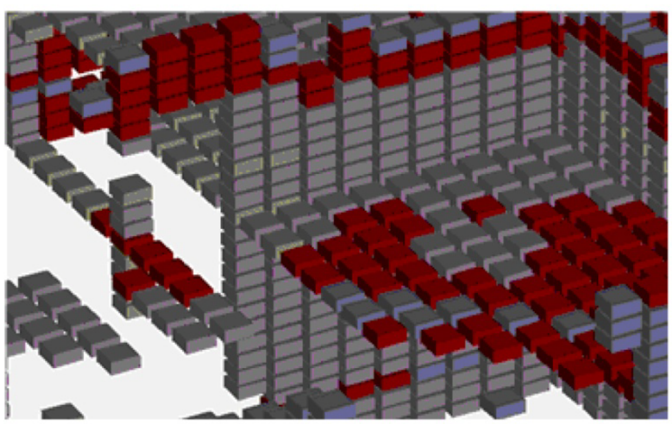


Block model outlines display, shown in 2D (left) and values display, shown in 2D (right).

Block outlines and block values	This display method combines the previous two methods.

Fig 47. Blocks displayed as outlines and values.

Block outlines and block values	This display method combines the previous two methods.
Cubes	This method displays blocks as 3D cubes or rectangular prisms (see the following image). This display method is generally preferred when viewing in 3D mode, since it provides superior visualization of solid blocks in relation to drillholes and solids.



Block model cubes display (rendered in 3D)

Fig 48. Block model displayed as cubes.

6.6.5. Choosing Block Models for Display

Use one of the following methods to choose block models for display.

- In the Project View Area, click on the desired block model. A check mark will appear alongside a displayed model.
- Define a multi–block loading profile, and load it using **Block > Display > Select Block Model Data for Display**

Note: Gems only displays selected blocks. You may display from multiple block models in 2D mode.

The grade values can now be displayed.

- Go **Block > Select > By Block Attribute Value Range**, select MWT and 0.1 and 100 as Lower threshold and Upper threshold values, respectively.
- Right click on MWT and select show and choose the MWT display profile (Fig 49).
- Change to display to cubes in the block display profile (Fig 50).

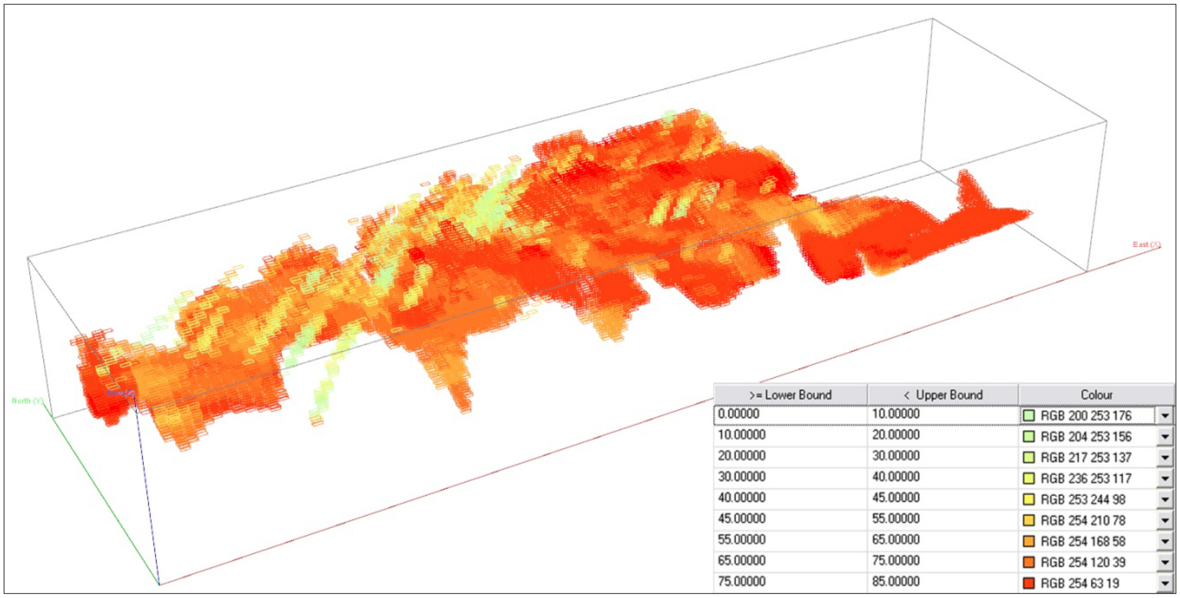


Fig 49. Estimated grades for blocks showing block outline.

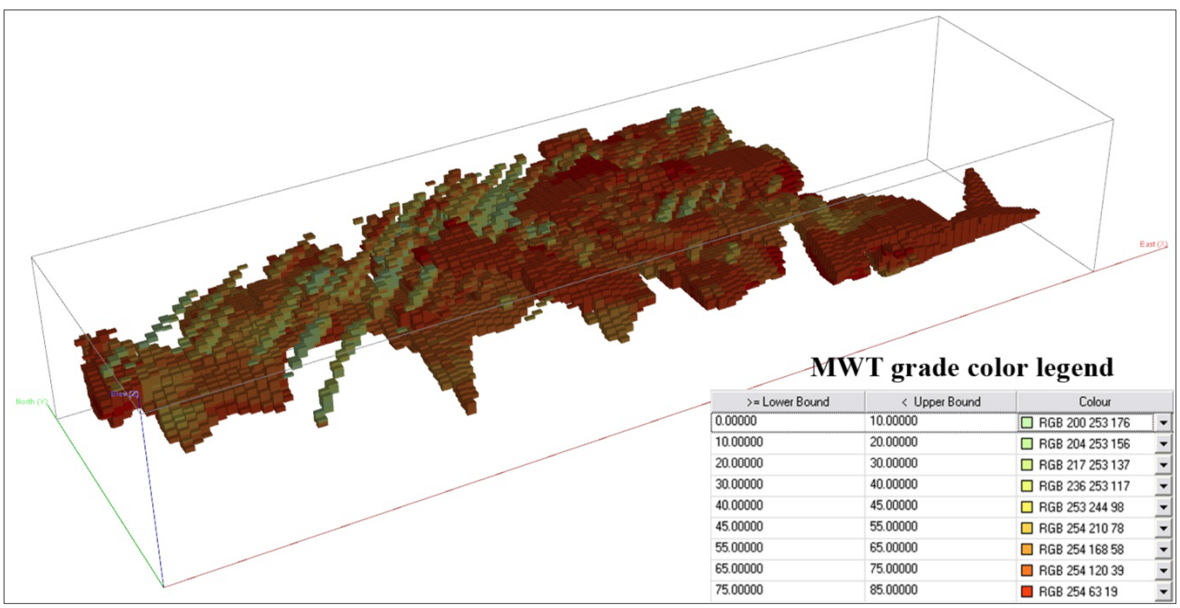


Fig 50. Estimated grades for blocks showing block cubes.

6.6.6. Hiding Block Models

You can inhibit display of block models in one of the following ways.

- You can globally suppress the display by selecting **Block > Display > Display Overrides** and turning off the option **Show block-model data**.

- You can terminate the display of an individual model by right-clicking the block model folder in the Project View Area and choosing **Hide**.
- You can clear the display by choosing a new multi-block loading profile.

6.6.7. Block Display in 2D

Outlines displayed in 2D will occupy a scaled portion of a share of the cell (for example, if outlines of two block model quantities are displayed, neither can occupy more than 50% of the cell.) There are, however, two ways to draw the outline of the entire cell, as shown in the following image:

To modify the display settings, follow these steps:

- Choose Block > Display > Display Overrides to open the View Settings dialog.
- In the Block-model display overrides section, click on or off the following options:
  - → Draw first outline as block outline: Select this option to expand the first value outline to encompass the entire cell. Remaining outlines will be scaled as defined in the cell display profile.
  - → Draw outline grid: Select this option to draw a grid to show cell boundaries. This is most useful when not drawing the first outline as block outline.
  - → Click **OK**.

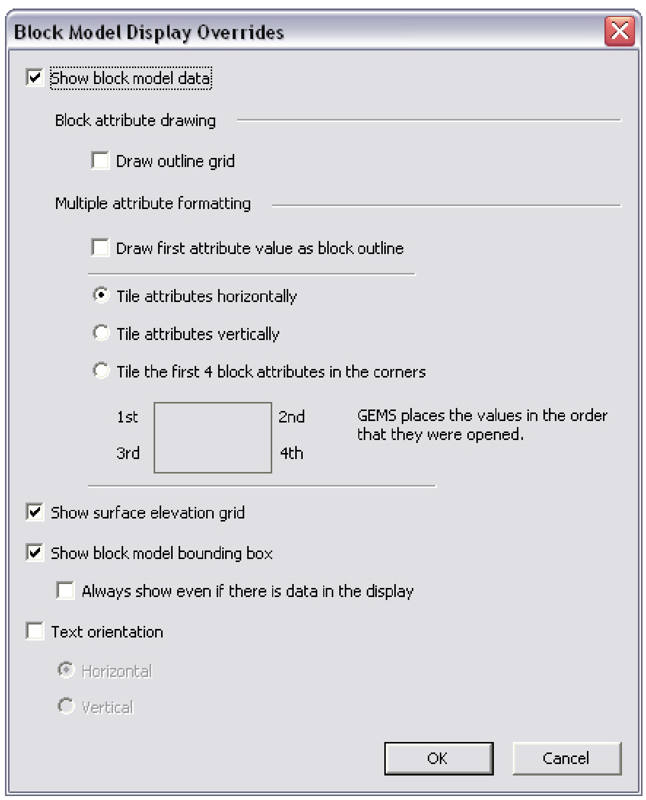


Fig 51. Block Model Display Overrides.

7. References

[1] Gemcom Software International (2007). *Resource Modelling Manual - Version 6.1, June 2007*.Unpublished manuscript, Vancouver, BC Canada.

[2] Gemcom Software International (2011). Gemcom Gems (Version 6.3). Vancouver, BC Canada: Gemcom Software International.