An Optimization Approach for Ultimate Pit and Push-Back Design in an Open-Pit Mine Planning

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ABSTRACT

Designing an open pit mine plan is a specialized problem focused on optimizing the economic value of a mining project before implementation. Two primary operations in an open-pit mine planning are ultimate pit and push-back design. This paper develops an ultimate pit and push-back optimization by incorporating maximum flow network problem while considering the precedence relationships between all blocks in the block model. Using GEOVIA SURPAC, a block model comprising 26,162 blocks was developed, each characterized by volume, gold grade percentage, rock code, rock type, and density. Then, the block data was prepared for use in Python to solve a Mixed-Integer Model for ultimate pit and push-back optimization. For push-back design, a mixed-integer model was used to constrain the capacity of each push-back. In this way, each block is assigned to a specific push-back and evaluated to determine its inclusion within the ultimate pit. The designed block model was applied to generate pushbacks and implement a production schedule using Whittle software, in which the Milawa Balanced algorithm obtained a more robust production schedule. Comparative analysis indicated that solving the push-back optimization problem using Mixed-Integer Programming yielded a higher Net Present Value compared to implementing in Whittle.

1. Introduction and Literature Review

Open-pit mines are surface excavations designed to extract ore located near the surface by creating a series of benches, or horizontal layers, that are mined sequentially from top to bottom. The process begins by exposing a sufficient area and then progressing to deeper levels until the pit limit is reached. Significant amounts of waste material are removed in this process, known as stripping, and the extracted ore is sent to processing plants while waste is disposed of in designated areas [1].

The mining region is represented as a 3D grid of blocks, each containing geological and economic information. Effective mine production requires practical pushback designs, which connect regions with sufficient width and access via haulage ramps. These pushbacks maximize the financial return of the mine and form the ultimate pit. However, models used to define pushbacks often lack operational constraints, leading to manual adjustments by engineers. Previous attempts to model operational conditions include heuristic methodologies and optimization techniques, aiming to create optimal semi-practical pushback designs that ensure connectivity and minimum mining width [2].

A hybrid methodology for long-term open-pit production planning was proposed by Tabesh [3], focusing on three key components: optimal phase design, selective mining-unit characterization, and long-term scheduling optimization. The approach integrates integer programming with a local search heuristic for phase design and employs hierarchical clustering to aggregate blocks into minable

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polygons within pushback boundaries. A mixed-integer linear programming model then generates near-optimal life-of-mine schedules, allowing planners to optimize large-scale multi-pit and multi-process scenarios, including cut-off grade optimization. Two case studies, involving small and large iron ore deposits, demonstrate the methodology's effectiveness. The first case study uses a small pushback from a block model for exact design, while the second employs heuristic procedures for larger pushbacks. The results show that the pushback designs are practical and uniform in tonnage, improving over traditional parameterized revenue methods. The MILP model ensures optimal production schedules and dynamic cut-off grade management, proving the robustness and efficiency of the proposed multi-step approach.

In open pit mining, achieving stable pit slopes is crucial for both safety and economic efficiency. However, many mines are designed with insufficient geotechnical investigations, leading to inaccurate slope angle estimations. Conservative slopes increase operating costs due to higher stripping ratios, while overly steep slopes risk failures and potential mine closure. This study highlights the importance of geotechnical assessments for optimizing pit slopes. Using Pickstone Peerless mine as a case study, geotechnical data was collected through core logging and face mapping, then analyzed using empirical, kinematic, limit equilibrium, and numerical modeling methods with Rocscience software. The study of Nyamande [4] used the current 52° slope angle as steepened by 1° without compromising stability, potentially saving over $12 million in stripping costs. However, further evaluations are necessary for pit extensions to ensure stability. The analysis also indicated that the pit could be deepened by 5 meters without requiring a pushback.

The problem of open-pit mining aims to maximize the net present value (NPV) by defining an optimal extraction sequence under various constraints and uncertainties. Jelvez [5; 6] presented a multi-stage methodology that integrates mathematical optimization and conditional simulation to address geological uncertainty at all stages of planning: computing the ultimate pit, optimizing pushbacks, and scheduling production. Unlike previous models that treat each stage independently, this approach evaluates the impact of uncertainty across the entire process. Using a real case study, the methodology demonstrates that incorporating uncertainty can improve solution quality and reduce the risk of failing to meet production goals, though its effects vary across different planning stages. Their study emphasizes the need for robust models and further experiments to refine understanding and management of geological uncertainty in mine planning.

Mining companies need advanced mathematical tools and techniques for effective decision-making, leading to better returns on investment and benefits for local communities. Despite a trend towards mining larger, lower-grade deposits and an increasing preference for underground mining methods like block caving, surface mining remains popular due to its lower costs. Operations research plays a vital role in mine planning, providing mathematical models to address complex engineering challenges. Deutsch [7] studied these tools to address under-represented operational constraints and improve long-range open pit mine planning. Accurate models are essential, but increased accuracy often leads to greater complexity and longer computational times. Thus, the focus is on developing computationally efficient techniques applicable to real mining datasets. The dissertation aims to create high-quality, flexible, and efficient models for long-range open pit mine planning, detailed through a structured review of literature, algorithmic improvements, and computational comparisons. In the other study [8], the floating cone optimization algorithm was presented, replacing complex programming loops with efficient SQL queries, significantly speeding up calculations. A case study showed a 35% reduction in computation time compared to traditional methods. The approach demonstrated SQL's potential in mining applications, offering faster and more efficient data management.

In open pit mine design, pushbacks are subregions mined in distinct phases. Practical pushbacks must be connected, maintain a minimum width for equipment, and include a haulage ramp. Current models often overlook these conditions, requiring significant adjustments by engineers. Yarmuch [9] presented an integer programming model to generate high-value, practical pushbacks. It introduces a compactness factor, weighting blocks by their distance from the ramp, and a closeness factor to measure design similarity, optimizing the balance between value and mine-ability. The solutions showed that the model produces more practical and valuable pushbacks than traditional methods. To handle larger instances, re-blocking and sequential generation of pushbacks were used. The model achieved better mineability compared to traditional methods, improving the net present value. An integer linear programming model was proposed in a research work to optimize pushback designs [10]. The model ensures adequate space for large equipment and maximizes profit. The model generated practical solutions in minutes, improving design quality and computational efficiency compared to traditional methods. The optimized pushbacks achieve near-optimal profits with better geometric properties and faster computation times.

Developing a mine is a complex task requiring significant initial investment and efficient mine planning to ensure economic success. The planning process has evolved due to advancements in extraction and processing technologies, the decline in average ore grades, and the increasing complexity of permitting and environmental policies. High-quality mine development plans are essential for securing stakeholder support and financial backing. Push-back design using ultimate pit parameterization often encounters the gap problem, characterized by large jumps in push-back size. Mieth [11] proposed an alternative approach, iteratively solving an optimization model to create pushbacks of predefined sizes in terms of overall and ore tonnage. This method, which falls under precedence constrained knapsack problems, employs linear relaxation and two heuristics to overcome long runtimes. The developed program was tested on three deposits (copper, iron ore, and gold), showing it can mitigate the gap problem. However, it trades off between runtime and solution quality, with heuristics failing to always find the optimal solution.

Optimizing open pit mine design involves managing geological uncertainties, particularly ore grade. A minimum cut algorithm is able to determine ultimate pit limits and pushback designs, integrating economic values and geospatial constraints [12]. The push–relabel algorithm efficiently processes block values under multiple scenarios. Applied to an Indian iron ore mine, the new method increased net present value compared to traditional models, despite greater complexity and computation time. The proposed approach enhances mine planning by incorporating uncertainties, yielding higher cumulative metal production and more reliable production schedules, ultimately maximizing profitability and reducing risks. Designing pushbacks in open-pit mines is essential for optimizing economic value and operational feasibility. An advanced Geometrically Constrained Production Scheduling Problem using a parallel genetic algorithm (PGA) is able to generate efficient pushbacks while considering geometric constraints [13]. The PGA, implemented on a network of computers, evaluates potential solutions, ensuring operational space and optimal resource utilization. Applied to test cases, the algorithm showed consistent ore grades and realistic mine geometries. The study highlights the PGA's robustness and effectiveness in handling real-world constraints, offering a significant improvement over traditional methods in terms of profitability and computational efficiency.

Determining the ultimate pit limit is vital in open-pit mining and is usually calculated using profit-maximizing algorithms like Lerchs-Grossman. Saleki [14] proposed a mathematical model that integrates ultimate pit limit determination with long-term production planning to maximize profit. To manage the model's nonlinearity, it is broken down into two linear sub-problems, thereby simplifying the solution process and reducing the number of decision variables, even though it remains NP-Hard. The Dynamic Pit Tracker heuristic algorithm, developed for this purpose, uses economic block models to guide the optimization. By comparing economic values and positional weights of blocks, the algorithm identifies the optimal blocks at each step. A comparative analysis with a two-dimensional block model demonstrates that the ultimate pit limits generated by the new model, LG, and Latorre-Golosinski algorithms are consistent. When applied to a 3D block model, the Dynamic Pit Tracker achieved a final pit profit that was 97.95% of the LG ultimate pit limit profit.

In the other study on the ultimate pit design [15], a large-scale open-pit mine production planning model was designed which is NP-hard, requiring efficient solutions. The research compares the genetic algorithm (GA) with Linear Programming (LP) and the Floating Cone (FC) algorithm for designing the ultimate pit limit. Using the Marvin mine block model, the study found that LP provides the highest UPL value, but GA offers a near-optimal solution much faster, crucial for large-scale planning. Sensitivity analysis improved GA's UPL value to just 8% less than LP's. When optimizing an ultimate pit limit, ecological costs into open pit mine planning can also be modeled [16]. By calculating carbon emissions, ecological service value loss, and reclamation costs, an iterative optimization algorithm adjusts the ultimate pit design. A case study shows that incorporating ecological costs reduces land damage, carbon emissions, and overall ecological costs, while economic profits decrease. Despite lower profits, the comprehensive benefit of the ecological pit is higher than the traditional pit, achieving a better balance between environmental and economic outcomes.

Open-pit mining, while profitable, significantly harms the environment. Traditional mine design focuses on economic benefits, often neglecting ecological costs. The study of Xu [17] aims to integrate ecological costs into the mine design process for more sustainable development. Ecological costs, including ecological service value loss, reclamation costs, and carbon emissions, are calculated using specific equations. An iterative optimization algorithm for pit design, considering these costs, was applied to a coal deposit in Northwest China. The case study revealed that incorporating ecological costs reduced the mine's environmental impact: the ecological optimum pit had 31.08% less damaged area, 36.17% lower carbon emissions, and 34.63% reduced overall ecological costs compared to the economic optimum pit. Despite a 19.01% decrease in economic profit, the ecological optimum pit provided a 2% greater comprehensive benefit. This approach demonstrates the potential for balancing economic and ecological benefits in open-pit mining.

For defining final pit boundaries in open-pit mining, using the maximum flow Pseudo flow method is useful which can be solved by programming software such as Python and SGeMS [18]. The analysis of 480,000 blocks (10x10x10 meters) generated 20 pits with varying revenue factors. Pit 20 emerged as optimal, with an estimated NPV of 17,855 MUSD, extracting 212 million tons of ore and 58 million tons of waste, resulting in a favorable stripping ratio and high economic feasibility. The ultimate pit problem in open pit mine planning is crucial but computationally intensive. An open-source C++ library was designed to solve this problem efficiently using the pseudo flow algorithm which is Mine-Flow [19]. This method processes large block models significantly faster than commercial alternatives by employing implicit precedence schemes. In a test, it handled 16 million blocks in nine seconds, compared to over two minutes for the fastest commercial solver. Mine-Flow, available under the MIT license, supports extensive models with millions of blocks and constraints. The tool is designed for academic and commercial use, facilitating quicker and more efficient mine planning and optimization.

Despite significant advances in ultimate pit design and pushback optimization through the application of mixed-integer programming (MIP) models, the following critical gaps remain unaddressed in the current literature:1) The integration of maximum flow network and minimum cut problems into the ultimate pit design, considering precedence relationships among blocks, presents a complex challenge that has yet to be fully explored; 2)Existing models, such as those implemented in Whittle software, typically simplify these relationships, potentially leading to suboptimal solutions. While the use of MIP has demonstrated a higher net present value (NPV) compared to traditional methods, there is a need for more sophisticated models that can accurately represent the spatial and economic attributes of the block model, including coordinates, block size, volume, gold grade, rock type, and density; and 3) Current methodologies often lack robust techniques for dynamically adjusting pushback designs based on real-time data and evolving geological conditions. Further research is required to develop integrated frameworks that can seamlessly transition from data preparation in mining software such as Surpac to advanced optimization in Python, ensuring that each block is optimally assigned to pushbacks while maximizing overall NPV. Addressing these gaps will enhance the accuracy, efficiency, and economic outcomes of open pit mine planning.

In the rest of this paper, we present the model for ultimate pit limit and push back optimization, then the methodology which is applied for the problem. Then, the results and outcome are discussed, and the potential developments for the future are proposed.

2. Methodology

This section explores the complexities of mine planning, focusing on ultimate pit limit and pushback design in open-pit mining. The primary objective is to address irregular increments in nested pits result in inefficient pushbacks. We use an alternative method using operations research and linear programming to create pushbacks of specific overall and ore tonnage, developed by Mieth [20]. This method iteratively solves an optimization model, incorporating techniques such as linear relaxation and heuristics to manage the complexity and computational demands. The developed approach was implemented in a computer program and tested on a gold deposit. The expected results should show that the program effectively creates practical pushbacks and improves conventional parameterization methods. The developed method is driven by advancements in extraction and processing technology, depletion of high-grade deposits, stricter environmental policies, and rising commodity prices. Efficient mine planning is crucial for the economic success of mining projects, and pushback design plays a central role in long-term production scheduling.

The methodology is divided into two main steps of ultimate pit limit and pushback optimization.

1. **Ultimate Pit Calculation:** The ultimate pit is calculated using the maximum flow algorithm, where blocks outside the ultimate pit are discarded as they are not included within the economic extraction boundaries of the pit. Simultaneously, this will reduce the problem size.

The general maximum flow network model is as follows.

$$\text{Min} \sum_i \sum_j c_{ij} x_{ij} \tag{1}$$

$$\sum_j x_{ij} - \sum_k x_{ki} = b_i \quad \text{for } i = 1, \ldots, n \tag{2}$$

$$l_{ij} \leq x_{ij} \leq u_{ij} \tag{3}$$

$$x_{ij} \geq 0 \tag{4}$$

Where the parameters and variables are as below.

c_{ij} The cost of each arc.

b_i The supply for each node.

x_{ij} The flow from node i to node j.

l_{ij} The lower bound on the flow amount from node i to node j.

u_{ij} The upper bound on the flow amount from node i to node j.

Based on the block data obtained from mining software, precedence graph, Picard's graph and the max flow network model we can model ultimate pit limit programming model in a programming language (Python or Matlab) to get the optimized ultimate pit limit. This final pit limit is used in the next step, pushback optimization.

The Picard graph is a concept used in open-pit mining optimization, particularly in the context of maximum flow problems. Introduced by J. C. Picard [21], it involves creating a network flow model to solve the open-pit mine design problem. The model utilizes nodes and edges to represent blocks and their precedence relationships, respectively. The maximum flow algorithm is applied to determine the optimal sequence of mining blocks while satisfying spatial and temporal constraints. This approach helps identify the ultimate pit limit and subsequent pushback phases efficiently [22].

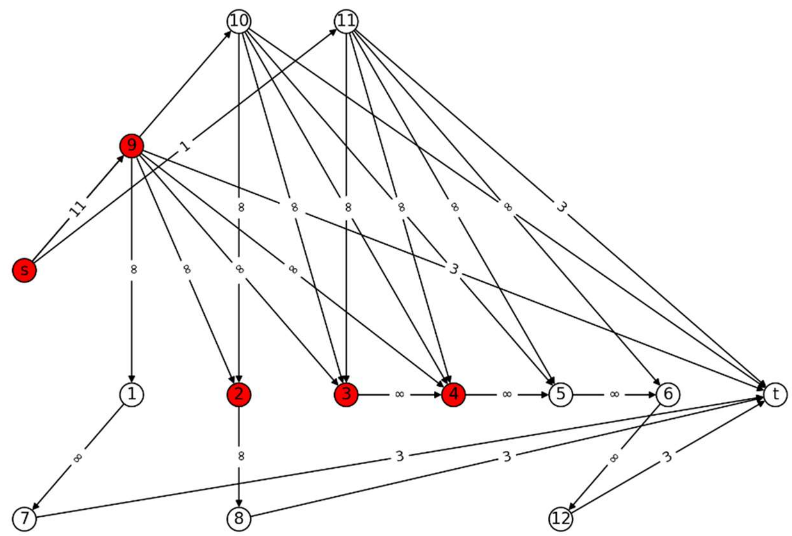

Figure 1. Visual indication of Picard's graph.

This Picard graph represents a maximum flow problem applied to an open pit mining scenario, including these components.

- The starting point for the flow of network, which is a source node.
- The endpoint for the flow of network, which is a sink node.
- Other nodes are blocks within the mining pit and included in precedence graph.
- A flow capacity between nodes on the arcs.
- A constraint-free flow, indicating no limit on the flow capacity for those connections.
- The specific limits on the flow capacity between nodes.

The graph helps in determining the maximal closure, which is a subset of nodes (blocks) where the sum of the values is maximal, and all predecessors of each node are included. The edges and their capacities represent the precedence constraints in mining, ensuring that certain blocks must be mined before others. By solving the maximum flow problem on this graph, we can determine the optimal sequence of blocks to mine to maximize economic returns while adhering to operational constraints.

1. **Push-Backs Optimization:** The following three steps are followed here:

- Mixed integer linear programming: Pushbacks are sequential sub-regions of an open pit mine designed to be mined in phases, guiding the spatial advancement of mining activities. Traditional methods for creating pushbacks involve parameterizing the ultimate pit limits, resulting in nested pits that often suffer from the irregular jumps in pushback sizes, leading to inefficiencies in production scheduling. In this way, we used Operations Research (OR) and Linear Programming (LP), where OR offers mathematical and computational tools to optimize complex decision-making processes, and LP, a subset of OR, is used to model and solve optimization problems where relations are linear. The applied approach leverages OR techniques to address the pushback design problem, formulating it as a precedence constrained knapsack problem.

- Model constraints: The model includes constraints for mining and processing capacities and ensures slope stability through block precedence. Due to the NP-hard nature of the problem, exact solutions are computationally intensive. Therefore, the model employs linear relaxation and heuristics to approximate solutions efficiently.
- Implementation and Testing: Implement the model in a computer program, and test on the case study of a gold deposit. The expected output is the efficient and practical pushbacks with reduced deviations from capacities.

The following mathematical model optimizes the pushback decision.

$$\max \sum_{i \in \mathcal{N}} p_i x_i \tag{5}$$

$$x_i - x_j \leq 0 \quad \forall (i,j) \in E \tag{6}$$

$$\sum_{i \in \mathcal{N}} t_i x_i \leq mc \tag{7}$$

$$\sum_{i \in \mathcal{N}} o_i x_i \leq pc \tag{8}$$

$$x_i \in \{0,1\} \quad \forall i \in \mathcal{N} \tag{9}$$

Where the parameters, variables and indices are as the following.

$\mathcal{N}$	Set of all nodes in the precedence graph representing all blocks in the block model.
E	Set of all edges in the precedence graph.
p_i	Economic block value of block i.
t_i	Overall tonnage of block i.
o_i	Ore tonnage of block i.

mc Push-back overall tonnage.

pc Push-back ore tonnage.

$x_i \in \{0,1\}$ Binary integer variable representing block extraction. x_i is 1 if block i is extracted, otherwise 0.

3. Results and Discussion

The developed approach is implemented in Python and tested on a gold deposit. The problem is expected to manage pushback sizes, reducing deviations from mining and processing capacities, and achieving more uniform and practical pushbacks compared to traditional parameterization methods.

One sample result is expected to be same as the below which is from Clemens [11]. The chart demonstrates a well-managed pushback strategy that maximizes net value while adhering to tonnage and ore tonnage restrictions, but also highlights the natural decline in economic returns as mining progresses.

- The consistent alignment of the total tonnage and ore tonnage with their respective restrictions for the first 15 pushbacks indicates an efficient mining strategy that maximizes resource extraction while adhering to capacity limits.
- The decreasing trend in net value suggests that the economic returns from mining diminish over successive pushbacks. This could be due to the depletion of high-value ore blocks or increasing extraction costs.
- The significant drop in both tonnage and net value in the last pushback suggests the end of economically viable mining operations within the given constraints.

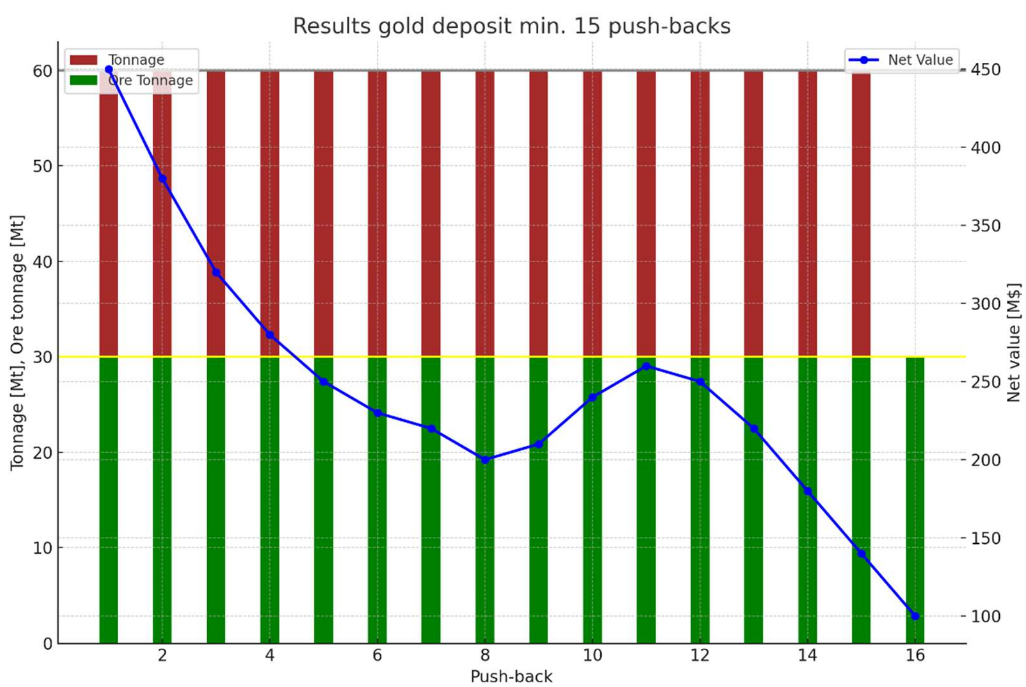

Figure 2. The sample result from solving ultimate pit limit and push back optimization problem.

In addition to Mixed-Integer Programming (MIP), the pushback optimization problem was also solved using the Whittle software. The designed block model was applied to generate pushbacks and implement a production schedule using Whittle software. Whittle uses the Milawa Balanced algorithm, known for its robustness in creating balanced production schedules over the mine life. The Milawa Balanced algorithm focuses on balancing the stripping ratio and ore grades, ensuring that each pushback phase contributes to a stable production output. This approach aims to optimize operational efficiency and economic returns within each mining phase.

Comparative analysis between the two methods revealed that the MIP approach obtained a higher Net Present Value compared to the implementation in Whittle, which indicates a more profitable mining operation. While Whittle's Milawa Balanced algorithm provided a robust and balanced production schedule, the flexibility and precision of MIP allowed for better optimization of the mining sequence. The MIP approach could also tailor the pushbacks more closely to the specific economic and operational constraints of the mine, leading to enhanced financial outcomes.

4. Conclusion and Future Directions

The integration of advanced optimization techniques, particularly Mixed-Integer Linear Programming, in mining operations has demonstrated significant advantages in terms of economic returns and operational efficiency. While traditional software such as Whittle provides a robust foundation for production scheduling through algorithms such as Milawa Balanced, incorporating MILP allows for more detailed and precise optimization, thereby unlocking higher profitability. The comparative analysis revealed that solving the pushback optimization problem using MILP yielded a higher NPV compared to the implementation in Whittle, highlighting the superior financial outcomes achievable with MILP.

The proposed MILP model and heuristic algorithms have provided a robust solution to the pushback design problem, effectively addressing the gap problem and improving mine planning efficiency. This research contributes to the field of mine planning by developing a MILP model for direct pushback creation without the need for nested pits and by utilizing maximum flow network, linear relaxation and heuristics to solve large-scale optimization problems efficiently. The implementation of this model in a computer program, tested on real-world deposit data, has demonstrated its practical applicability and effectiveness.

Future research directions should focus on incorporating additional constraints such as greenhouse gas emissions considerations to enhance the sustainability of the problem. Incorporating minimum mining width considerations into the model will further improve its practicality by ensuring the designed pushbacks adhere to operational constraints and safety standards. Further improvements in algorithmic efficiency can be achieved by applying advanced algorithms such as simulated annealing and Tabu search for the implementation of problem. This would enable the handling of even larger datasets and more complex mining scenarios.

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