Open Stope Mine Production Scheduling Optimization with Stochastic Mixed Integer Linear Programming: Considering Grade Uncertainty and Stockpiling Strategy

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ABSTRACT

Conventional geostatistical techniques, like ordinary kriging, can produce biased and overly smoothed estimates, leading to unrealistic production schedules. In contrast, simulation-based methods offer a more effective approach by generating multiple equiprobable models of the orebody, allowing for better assessment and management of grade uncertainty risks in mine planning. The study presents a Stochastic Mixed Integer Linear Programming (SMILP) model, which is an advanced version of an initial Mixed Integer Linear Program (MILP). This enhanced model encompasses a variety of mining operations, including primary (decline) development, ventilation development, operational development, ore pass development, stope design and extraction, as well as backfilling and stockpile management, all aimed at maximizing the net present value (NPV). Six comprehensive case studies were conducted. These case studies aimed to analyze and compare the economic and operational outcomes when employing the SMILP model. Among the case studies presented, the SMILP model with stockpile management (Case 4) exhibited superior performance, reporting the highest NPV when compared to the other five cases. Case 4achieved a 25-year mine life production schedule with a generated NPV amounting to $8,077.86M, while the MILP model with stockpile management (Case 2) yielded $7,801.20 M, indicating a4% increment in financial gain by the SMILP model. These findings underscore the effectiveness and practical applicability of the developed model in real-world mining scenarios.

1. Introduction

Mine planning can be conducted using two distinct approaches: deterministic and stochastic. The deterministic approach relies on historical data from past mining activities, if future trends will mirror previous patterns [1-3] . In contrast, the stochastic approach acknowledges the uncertainty inherent in mining processes by incorporating variability into the planning model, based on historical data. Unlike deterministic methods, which overlook fluctuations, stochastic mine planning recognizes the inherent unpredictability of mining activities, making it a more comprehensive and realistic approach [1-6].

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1.1. Stochastic Integrated Mine Planning Approaches in Underground Mining

Stochastic mine planning is a complex scheduling process due to its ability to incorporate uncertainties such as geological, technical, and economic, inherent in mining operations [7, 8]. Researchers have analyzed risks associated with geological and grade uncertainty to considerably help the decision makers in better understanding various cases and conditions associated with the optimization process [2, 9, 10]. The authors [11-13] concluded that uncertain attributes other than grade and geology should be added to the optimization problem. Hence, subsequent models should be extended to take stochastic variables into account during optimization.

The application of stochastic mine planning in production scheduling has been reported by various researchers in open pit mining taking into consideration grade uncertainty and associated risk [3, 4, 14] , orebody uncertainty, in situ grade and geological uncertainty [3, 5, 15-17] , geological and market uncertainty [18] and NPV [19] to mention a few. Conceptual developments in open pit mining have gradually led to the application of stochastic mine planning in underground mine planning notwithstanding the intricate nature of underground mine planning. The uncertainty related to orebody is a critical aspect affecting the forecasted performance of designs and is linked to the failure of meeting production targets and project financial expectations in mine planning [8,13, 20] .

The essence of accounting for uncertainty and risk which leads to an improvement in decision making was introduced in mine evaluation though the Integrated Valuation Optimization Framework (IVOF) [21] . The author asserts that the complexity of mine projects makes it a business that requires constant assessment of risk because the value of a mine project is typically influenced by many underlying economic and physical uncertainties, including metal prices, metal grades, costs, schedules, quantities, and environmental issues, among others, which are not known with absolute certainty. In underground mining, the use of stochastic variables like grade uncertainty and predefined levels of acceptable risk [13] were developed and explored in a probabilistic Mixed Integer Programming (MIP) model to optimize stope designs, including size, location, and number of stopes. The application of the proposed approach was based on the ability to stochastically simulate equally probable representations of the deposit. This approach forms the basis of the model presented in this paper.

The advent of Stochastic Integer Programming (SIP) [22] adopted risk-based concepts developed in open pit mining to the underground stoping environment and showcased how conventional technologies cannot quantify risk since they are unable to foresee a significant upside potential/or downside risk for the conventionally produced designs. The work quantified risk in terms of the uncertainty a conventional stope design has in expected contained ore tones, grade, and economic potential. Evidently, the model could not achieve the set goals and recommendations from the authors led to the development of a new model which incorporates uncertainties from both the geological and economic factors while minimizing cost [23] . The model integrated two elements: stochastic simulation and stochastic optimization. These elements provided an extended mathematical framework that allows modelling and direct integration of orebody uncertainty to mine design, production planning, and evaluation of mining projects and operations. This stochastic framework increases the value of production schedules by 25%. Further improvements of SIP models [2, 9, 24, 25] indicated that stochastic solution maximizes the economic value of a project and minimizes deviations from production targets in the presence of ore/metal uncertainty because SIP models account and manage risk in ore supply despite the difficulties surrounding creating such planning process [7] .

As the need to incorporate multiple components of the mining value chain increased several methods were developed over the past decades. Efforts have been made in new models to incorporate more decisions and flexibility to the mining optimization of a mining complex. However, they either ignored such realities needed to be considered in the presence of uncertainties associated with the underground mining process or consider decisions taken before optimization commenced with simultaneously optimizing mining, blending, processing, and transportation decision variables while accounting for geological uncertainty [1]. The expansion of SIP led to the introduction and use of Stochastic Mixed integer Programming (SMIP) which permitted the use of both continuous and integer variables for multiple constraints including primary development, ventilation, and operational development [11]. The SMIP model allowed the optimization of mining complexes in open-pit, underground operations, and processing destinations [1].

1.2. Stockpiling in Underground Optimization

The adoption of stockpiling in mining operations is an approach used in situations where ore materials mined exceeds the plant requirement. In this regard, the best grades are allowed to be processed directly while lower grades are stockpiled for a future date. It is possible to use one or more grade stockpiles, were there could be a low grade and a medium-low grade stockpile [26]. In many cases, such stockpiles may not get processed for years, possibly until: a) the mine is depleted b) the mined grades are lower than those in the stockpile. Such stockpiles can grow to enormous size if accumulated over many years. In some instances, oxidation and processability may be a concern for reactive materials with long term stockpiles [26, 27]. In long-term open-pit production planning (LTOPP), non-linear stockpiling constraints were incorporated for a problem-specific solution method [28]. The formulation helped to track material flow from aggregate to stockpile and plant. This concept [28] forms the foundation of the stockpiling strategy adopted in the stochastic underground formulation approach in this paper.

A goal programming model for a surface iron ore processing plant blending requirement was presented [29]. The model illustrated blending material from two high grade and a low-grade stockpile. The proposed model provided an optimal reclamation schedule by dividing the stockpiles into blocks and subsequently assigning grade values to each block. However, the proposed model did not include decisions on material flow from the mine and stockpile but solely focused on reclamation decisions. A proposed production scheduling model with uncertain supply that includes stockpiling was also presented [30]. The model used a predetermined constant grade for reclaiming material from the stockpile and allow blocks into the stockpile based on the probability of block grade being within the acceptable range for the stockpile. However, the authors do not compare the actual grade of material in the stockpile to the predefined grade.

A Mixed Integer Programming (MIP) for medium-term production planning with stockpiling was presented [31]. The authors divided ore into different categories based on low, high grade and recovery of the main two elements and defined a stockpile for rehandling low-grade ore when needed. However, they avoid nonlinearity by not keeping track of elements grades going to and reclaimed from the stockpile. The cost of uncertainty in a long term mine production plan presented for a Mixed Integer Linear Programming (MILP) model [32] which considers stockpiles. The model finds the mining sequence of blocks from a predefined pit shell and their respective destinations, with two objectives: to maximize the net present value of the operation and to minimize the cost of uncertainty. The presence of stockpile in the model allowed the optimization to extract extra ore at early stages of the mine-life which helped to reduce the chance of short falls at later years.

A multi-step approach to long-term open-pit production planning that determined pushbacks based on a hybrid binary programming-heuristic method were presented [33, 34]. The authors introduced stockpiling to the model with non-linear objective function and later illustrated the benefit of developing a linear objective function and constraints. The linearized model improved reducing the error as the number of stockpiles increased. However, it became evident that the number of stockpiles and grade ranges will depend heavily on the characteristics of the deposit and operations due to homogenous reclamation and other important stockpiling assumptions with respect to multiple element grades.

Subsequent mathematical programming models [35-37] also considered stockpiling with a predetermined fixed grade. They compared their results against solutions obtained, showed how close to the optimum solution their solutions were, and showed that the observed element head grades are within the required boundaries [36]. However, they did not report the errors caused by assuming a fixed reclamation grade for the stockpile

1.3. Summary of Gaps in Literature

Majority of studies so far illustrate the benefits of incorporating grade uncertainty in open-pit production scheduling optimization. Notwithstanding the complexities associated with the development and implementation of underground mining methods, some efforts have been made in incorporating grade uncertainty in underground mining production scheduling optimization [2, 11, 13, 38]. However, the application of stochastic optimization is relatively recent in underground mining and the methods used in the literature remains to be verified when applied to different types of deposits and underground mining methods.

Generally, stockpiles can be considered as buffers of material for future use for controlling capacity and blending requirements. Again, the presence of a stockpile allows optimization to extract extra ore at early stages of the mine-life and the extra ore reduce the chances of short falls at later years [32, 34, 39]. The use of stockpiling in the optimization process has been extensively used in open pit mining but less considered in underground mining [28, 30, 33-35, 37]. The proposed optimization approach is to consider grade uncertainty for underground open stope extraction with primary development, ventilation development, operational development, ore pass development, backfilling, and stockpiling management [40].

This research, therefore, seeks to develop a risk-based optimization framework using stochastic mixed integer linear programming (SMILP) that effectively integrates grade uncertainty into the optimization of long-term production scheduling in open stope underground mining, by developing a stochastically simulated equally probable representations of a deposit for underground production scheduling optimization. The objective function of the proposed SMILP model is to maximize the net present value and balance the risk associated with grade uncertainty simultaneously, while meeting all technical and operational requirements.

2. Methodology

An assessment of the proposed approach and results will be illustrated through a case study corresponding to a gold deposit. Figure 1 is a schematic representation of the proposed approach.

Studies on the effect of grade uncertainty on NPV in long-term production plans have been reported for underground mining projects. It has been identified that there are differences between the actual and expected production targets especially in the early years of production [9, 10, 13]. To address the issue of grade uncertainty, instead of using a single estimated block model for production scheduling, 50 simulated realizations, that are representative of ore grade variability will be used as input to the SMILP model. A conventional model which was based on Ordinary Kriging (OK) estimation was considered as the base case model. Both models in addition to the E-type model were then compared to illustrate the impact of grade uncertainty on the integrated mine development and production scheduling problem.

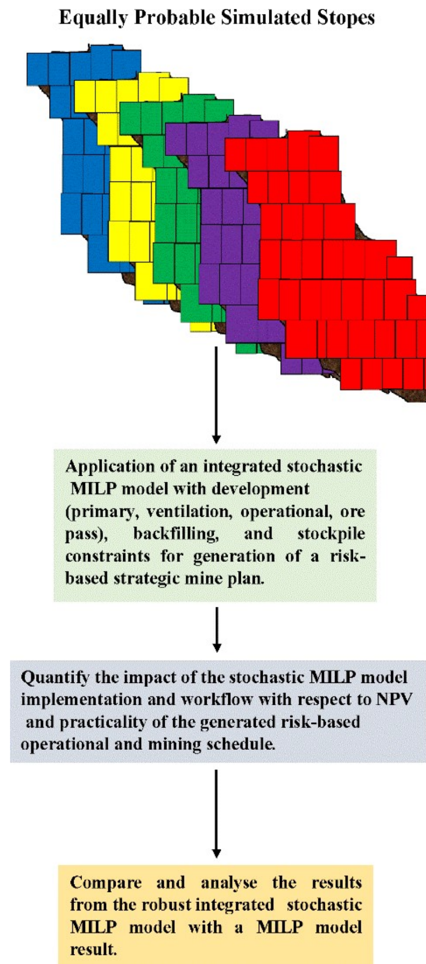

Figure 1. Illustration of proposed approach.

The proposed workflow used in this study to generate production schedule under grade uncertainty from the SMILP framework are as follows:

a. Design stopes from an economic gold block model using Promine AutoCAD [41]. The gold grades in the block model are estimated using OK and serves as the base case model.
b. Implement geostatistical modelling using Sequential Gaussian Simulation (SGS) algorithm to map out gold ore grade uncertainty in the designed stopes, utilizing. Stanford Geostatistical Modeling Software (SGeMS) [42].
c. Select all the stopes for all the realizations, E-type, and Ordinary Kriging. Save the selected stopes in ASCI file format.
d. For each case study, define the input scheduling parameters in MATLAB to formulate the problem.
e. Implement the developed mathematical programming formulations in MATLAB. TOMLAB/CPLEX [43] is used as the solver for the defined optimization problem.
f. Perform production scheduling optimization and comparative analysis based on the generated results from the OK, E-type and SGS block models.

3. Statistical analysis of gold data

To start geostatistical modelling, it is necessary to perform preliminary statistical analysis including compositing, recognizing outliers, identifying trends, and data transformation. This is necessary because data collection practices in general focuses on portions of the study area that are most important. For this reason, the element of interest is gold grades since its variability in estimation creates uncertainty which potentially impacts the overall net present value of the mining project [11]. Presented in Figure 2 is a 3D map of the 120 stopes with gold grades.

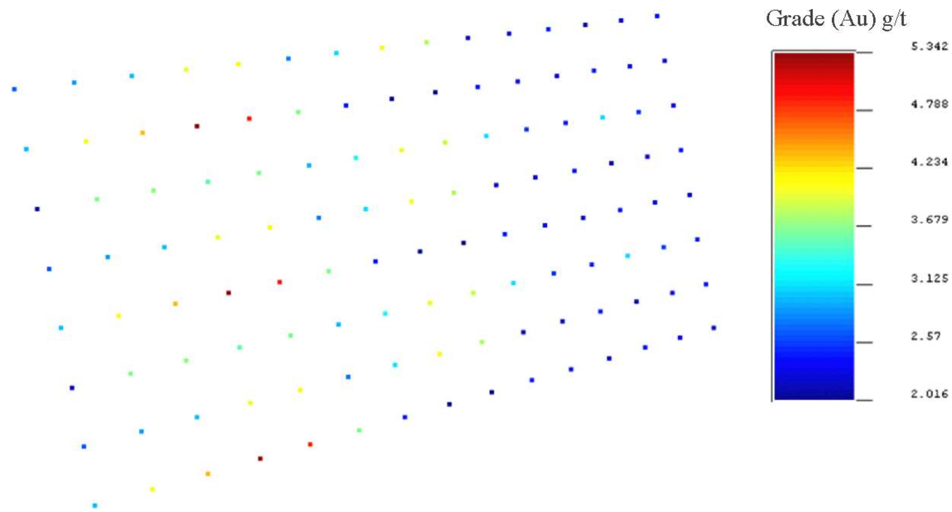

Figure 2. Location of stopes with gold grade distribution.

3.1. Spatial Correlation Analysis Using Variogram

The measurement of spatial continuity was employed to understand the correlation between the observations of the univariate sample at different locations. This analysis is useful to detect the presence of general trends in the data. Geostatistical techniques were used to analyze spatial variability and distribution of sample data to estimate parameters at unsampled locations in three main steps [44]:

1. Assumption of stationarity,
2. Spatial modelling of sample data, and
3. Estimation of variable value at unsampled location.

The analysis of spatial correlation can be undertaken using Stanford Geostatistical Modeling Software, (SGeMS) [42].

The original data set containing gold grades for 120 stopes were transformed to a gaussian space using standard normal score transformation applied in geostatistical analyses. Transformation of data to normal score distribution satisfies the assumption of stationarity of data. The transformed normal score data is also useful as input data in the stochastic gaussian simulation technique [45] as shown in Figure 3 for the gold grades.

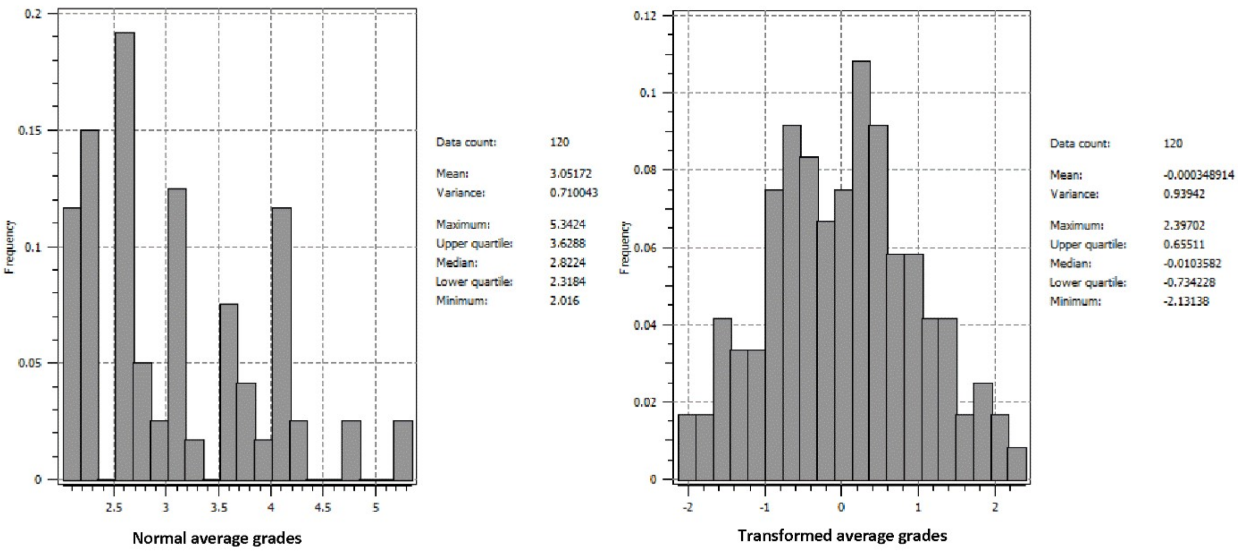

Figure 3. Histogram and transformed normal score for gold grades.

Variogram analysis, which allows for examination of data correlation with respect to distance, was done for gold grades. Omnidirectional variogram for the grades was first computed to identify the sill while vertical variograms were used to identify the nugget effect. Primary variogram maps were calculated to determine the orientation of the major axis in the presence of anisotropy. Directional experimental variograms were calculated and theoretical variogram models were fitted to the experimental variograms. The parameters used to model the experimental variogram are presented in Table 1.

Table 1. Parameters used for variogram modelling.

Direction	Azimuth (°)	Variogram model	Sill contribution	h_{min} (m)	h_{max} (m)	Nugget
Vertical	0.0	Gaussian	0.5	3.5	5.9	0.5
Minor	112.5	Gaussian	0.5	3.5	5.9	0.5
Major	67.5	Gaussian	0.5	3.5	5.9	0.5

A gaussian variogram model was fitted to the experimental variograms in the horizontal major direction and the horizontal minor direction. Figure 4 shows the experimental and fitted variogram models in the major and minor directions, and in the vertical direction. The general equation for gaussian variogram model is as shown in Eq. (1).

$$\gamma(h) = C.\left(1-\exp\left(\frac{-3h^2}{a^2}\right)\right) \qquad (1)$$

Where *C* is the structure variance and *a* is the effective range.

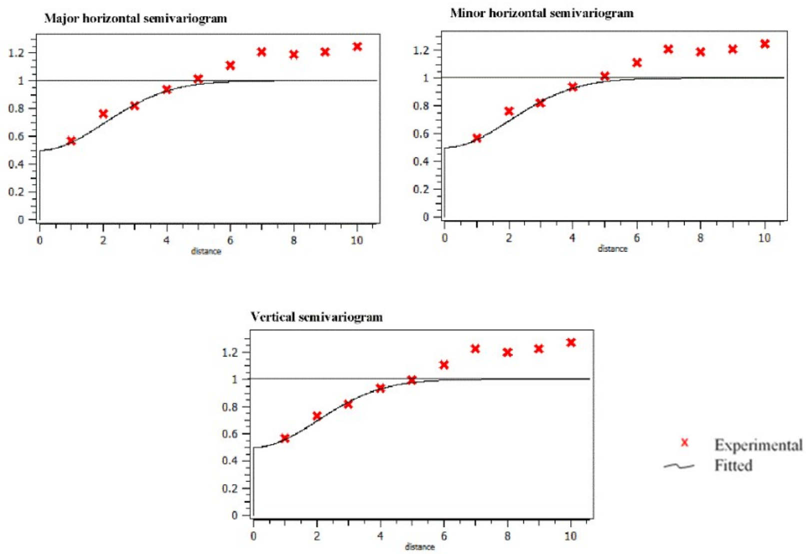

Figure 4. Experimental directional variogram and fitted variogram models for gold grades (distance in meters).

4. Assumptions

It is assumed that the mineral reserve will be extracted using underground mining techniques with a predetermined classical optimum design for stope limits. The optimum stope outline represents reserve discretization units that will maximize the profit from the project. For the open stope underground mining method in this research, ore extraction is achieved by an advancement method with a top-down approach. In addition:

1. The ore pass bin is considered in the mining system as the primary point from where all material mined (ore and waste) exit the mine.
2. Materials from the ore pass bin are managed on a first-in first-out basis with no mixing.
3. Material from the ore pass bin can be sent directly to the processing plant or stockpiled at the surface ore stockpile.
4. The surface ore stockpile is adopted to store ore that exceeds the current processing plant capacity and is reclaimed in the future.
5. There are no physical or chemical modifications should ore stay longer in the surface stockpile.

It is also assumed that grade uncertainty will be the only stochastic variable considered during the geologic block modelling. It is also assumed that the future cost and price data used for the economic block models are constant. This assumption implies that as cost and price changes in the future, there is a need for re-optimization of the production schedules based on prevailing economic conditions and current industry practice. Ore extraction is achieved by open stoping and includes either retreating or advancement methods. Caving mining methods will not be evaluated.

Activities involved in collecting drilling and sampling data for developing the geologic block model will not be considered. Other geotechnical properties of the deposit will not be modeled or considered during optimization aside stope sequencing. No stability analysis of the development and stope designs will be undertaken. Figure 5 is a representation of the material flow from the mine to the processing plant through the ore pass bin and/or surface ore stockpile. As illustrated in Figure 5, material extracted, from stope y_k^t can be sent directly to processing plant x_k^t through the ore pass. Alternatively, material extracted from stope $y_{k,}^t$ can be sent to the stockpile $u_{k,si}^t$ and material from the stockpile $u_{k,so}^t$ can be reclaimed to the processing plant.

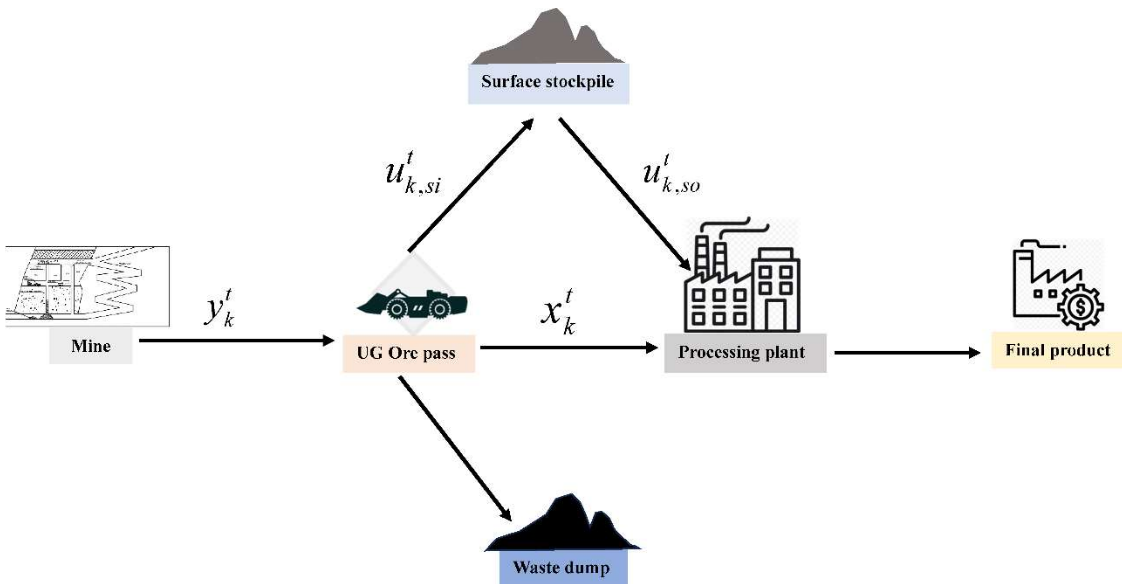

Figure 5. Material flow diagram from mine to final product

4.1. Notation

We present a SMILP formulation for the open stope underground mining production scheduling problem. The notation for indices, decision variables, sets and parameters are as follows:

t	index for schedule time periods: t = 1, 2, 3, …, T, where T is the schedule duration.
k	index for stope identification: k = 1, 2, 3, …, K, where K is the total number stopes.
l	index for level identification: l = 1, 2, 3, …, L, where L is the total number of levels.
s	index for realization identification: s = 1, 2, 3, …, S, where S is the total number of realizations.

4.2. Sets

$K = \{1,...,K\}$	set of all stopes in the model
$K_l = \{1,...,K_l\}$	set of all stopes on level l in the model
$S = \{1,...,S\}$	set of all equally probable stope realizations in the model
$C = \{1,...,C\}$	set of all primary development in the model
$C_l = \{1,...,C_l\}$	set of all primary development on level l in the model

$A = \{1,...,A\}$	set of all operational development in the model
$A_l = \{1,...,A_l\}$	set of all operational development on level *l* in the model
$V = \{1,...,V\}$	set of all ventilation development in the model
$V_l = \{1,...,V_l\}$	set of all ventilation development on level *l* in the model
$P = \{1,...,P\}$	set of all ore pass development in the model
$P_l = \{1,...,P_l\}$	set of all ore pass development on level *l* in the model
$A_k(J)$	for each stope *k*, there is a set $A_k(J) \subset K$ defining the adjacent stopes that cannot be mined simultaneously with the extraction or backfilling of stope *k*; where *J* is the total number of stopes in the set $A_k(J)$.
$G_k(S)$	for each stope *k*, there is a set $G_k(S) \subset K$, defining the immediate predecessor stopes that must be extracted prior to stope *k*; where *S* is the total number of stopes in the set $G_k(S)$.
$D_a(L)$	for each level *l*, there is a set $D_a(L) \subset OD_l$, defining the number of operational development on that level that must be completed before a stope and an ore pass can be extracted; where *L* is the total number of operational development on that level in the set $D_a(L)$.
$D_c(L)$	for each level *l*, there is a set $D_c(L) \subset CD_l$, defining the number of primary development that must be completed before operational development on that level can be started; where *L* is the total number of primary development on that level in the set $D_c(L)$.
$D_v(L)$	for each level *l*, there is a set $D_v(L) \subset VD_l$, defining the number of ventilation development that must be completed before operational development on that level can be started; where *L* is the total number of ventilation development on that level in the set $D_v(L)$.
$L_l(M)$	for each level l, there is a set $L_l(M)$ defining all stopes on this level; where M is the total number of stopes in the set $L_l(M)$.
$L_l(D)$	for each level l, there is a set $L_l(D)$ defining all operational development on this level; where D is the total number of operational development activities in the set $L_l(D)$.

4.3. Parameters

$R^t_{k,p,s}$	revenue generated by selling the final mineral commodity in stope k of realization s in period t from the ore pass.
$R^t_{k,so,s}$	revenue generated by selling the final mineral commodity in stope k of realization s in period t from the stockpile.
i	discount rate.
o_k	ore tonnage in stope k.
$o_{k,so}$	ore tonnage in stope k sent from stockpile to plant.
$o_{k,so,s}$	ore tonnage in stope k for realization s sent from stockpile to plant.
$o_{k,s}$	ore tonnage in stope k of realization s.
w_k	waste tonnage in stope k.
$w_{k,s}$	waste tonnage in stope k of realization s.
$g_{k,s}$	average grade of mineral in ore portion of stope k for realization s.
$g^t_{k,so}$	average grade of mineral in ore portion of stope k sent from stockpile to plant.
$g^t_{k,so,s}$	average grade of mineral in ore portion of stope k for realization s sent from stockpile to plant.
r^t	processing recovery: the portion of mineral recovered in stope in period t.
sp^t	selling price in present value terms obtainable per unit of mineral commodity.
sc^t	selling cost in present value terms per unit of mineral commodity.
mp^t_k	extra cost in present value terms per tonne of ore for mining and processing of stope k in period t.
rc^t	mining recovery of stope in period t.
$di_{k,s}$	mining dilution of stope k of realization s.
C^t_k	total cost in present value terms of primary development for stope k in period t.
c^t_k	variable cost in present value terms per length of primary development for stope k in period t.

H_k^t	total cost in present value terms of ventilation development for stope *k* in period *t*.
h_k^t	variable cost in present value terms per length of ventilation development for stope *k* in period *t*.
E_k^t	total cost in present value terms of operational development for stope *k* in period *t*
e_k^t	variable cost in present value terms per length of operational development for stope *k* in period *t*.
Q_k^t	total cost in present value terms of mining from stope *k* in period *t*.
q_k^t	variable cost in present value terms per tonne of mining from stope *k* in period *t*.
F_k^t	total cost in present value terms of backfilling stope *k* in period *t*.
cf_k^t	variable cost in present value terms per unit volume of backfilling stope *k* in period *t*.
Z_k^t	total cost in present value terms of ore pass development for stope *k* in period *t*.
z_k^t	variable cost in present value terms per length of ore pass development for stope *k* in period *t*.
cs_k^t	variable cost in present value terms per tonne for stockpiling ore from stope *k* in period *t*.
$L_{c,lb}^t$	lower bound on primary development length (decline) in period *t*.
$L_{c,ub}^t$	upper bound on primary development length (decline) in period *t*.
d_{cl}	primary development length cl.
$L_{v,lb}^t$	lower bound on ventilation development length in period *t*.
$L_{v,ub}^t$	upper bound on ventilation development length in period *t*
d_{vl}	ventilation development length vl.
$L_{a,lb}^t$	lower bound on operational development length for period *t*.
$L_{a,ub}^t$	upper bound on operational development length for period *t*.

d_a	operational development length a.
$L^t_{p,lb}$	lower bound on ore pass development length in period *t*.
$L^t_{p,ub}$	upper bound on ore pass development length in period *t*.
d_p	ore pass development length p.
$T^t_{m,lb}$	lower bound on available mining capacity in period *t* (tonnes).
$T^t_{m,ub}$	upper bound on available mining capacity in period *t* (tonnes).
$T^t_{pr,lb}$	lower bound on ore processing capacity in period *t* (tonnes).
$T^t_{pr,ub}$	upper bound on ore processing capacity in period *t* (tonnes).
$T^t_{si,ub}$	upper bound on stockpile capacity in period *t* (tonnes).
$gr^t_{pr,lb}$	lower bound on acceptable average grade of ore at the processing plant in period *t*.
$gr^t_{pr,ub}$	upper bound on acceptable average grade of ore at the processing plant in period *t*.
$gr^t_{si,lb}$	lower bound on acceptable average grade of ore at the stockpile in period *t*.
$gr^t_{si,ub}$	upper bound on acceptable average grade of ore at the stockpile in period *t*.
$V^t_{f,lb}$	lower bound on backfilling volume in period *t*.
$V^t_{f,ub}$	upper bound on backfilling volume in period *t*.
d_{fv}	backfilling volume fv.
N^t_{lc}	the maximum number of active levels available for operational development in period *t*.
N^t_{le}	the maximum number of active levels available for stope extraction in period *t*.
N_{xd}	the maximum length of extraction duration for stopes.
GDR	geological discount rate.

$pntCT^{t}_{o,+}$ penalty cost in terms of the upper ore tonnage target deviation in period *t*.

$pntCT^{t}_{o,-}$ penalty cost in terms of the lower ore tonnage target deviation in period *t*.

$pntCT^{t}_{g,+}$ penalty cost in terms of the upper ore grade target deviation in period *t*.

$pntCT^{t}_{g,-}$ penalty cost in terms of the lower ore grade target deviation in period *t*.

$SPpntCT^{t}_{g,+}$ penalty cost in terms of the stockpile upper grade target deviation in period *t*.

$SPpntCT^{t}_{g,-}$ penalty cost in terms of the stockpile lower grade target deviation in period *t*.

4.4. Decision variables

$x^{t}_{k} \in [0,1]$ continuous variable representing the portion of stope k to be extracted as ore and processed in period t

$y^{t}_{k} \in [0,1]$ continuous variable representing the portion of the stope k to be mined in period t; fraction of y characterizes both ore and waste in the stope

$f^{t}_{k} \in [0,1]$ continuous variable representing the portion of stope k to be backfilled in period t.

$u^{t}_{k,si} \in [0,1]$ continuous variable representing the portion of stope k sent to stockpile in period t.

$u^{t}_{k,so} \in [0,1]$ continuous variable representing the portion of stope k sent from stockpile to the plant in period t.

$d^{t}_{c} \in [0,1]$ continuous variable representing the portion of primary development length c to be completed in period t.

$d^{t}_{a} \in [0,1]$ continuous variable representing the portion of operational development length a to be completed in period t.

$d^{t}_{v} \in [0,1]$ continuous variable representing the portion of ventilation development v to be completed in period t.

$d^{t}_{p} \in [0,1]$ continuous variable representing the portion of ore pass development p to be completed in period t.

$d^{t}_{a,l} \in [0,1]$ continuous variable representing the set of operational development activities a to be completed on level l in period t.

$odev_{s,+}^{t} \in [0,1]$ continuous variable representing the excess from the ore tonnage upper bound in period *t* for realization *s*.

$odev_{s,-}^{t} \in [0,1]$ continuous variable representing the shortage to the ore tonnage lower bound in period *t* for realization *s*.

$gdev_{s,+}^{t} \in [0,1]$ continuous variable representing the excess from the grade upper bound in period *t* for realization *s*.

$gdev_{s,-}^{t} \in [0,1]$ continuous variable representing the shortage to the grade lower bound in period *t* for realization *s*.

$jgdev_{s,+}^{t} \in [0,1]$ continuous variable representing the excess from the stockpile grade upper bound in period *t* for realization *s*.

$jgdev_{s,-}^{t} \in [0,1]$ continuous variable representing the shortage to the stockpile grade lower bound in period *t* for realization *s*.

$b_{k}^{t} \in \{0,1\}$ b_{k}^{t} is a binary integer variable controlling the precedence of extraction of mining stope *k;* b_{k}^{t} is equal to one if extraction of stope *k* has started in period *t,* otherwise it is zero

$bf_{k}^{t} \in \{0,1\}$ bf_{k}^{t} is a binary integer variable controlling the precedence of backfilling of stope *k;* bf_{k}^{t} is equal to one if backfilling of stope *k* has started in period *t,* otherwise it is zero

$b_{c,l}^{t} \in \{0,1\}$ $b_{c,l}^{t}$ is a binary integer variable controlling the precedence of operational developments $b_{c,l}^{t}$ is equal to one if primary development c on level l has started by or in period t, otherwise it is zero.

$b_{a,l}^{t} \in \{0,1\}$ binary integer variable controlling the precedence of operational developments. $b_{a,l}^{t}$ is equal to one if operational development a on level l has started by or in period t, otherwise it is zero.

$b_{p,l}^{t} \in \{0,1\}$ binary integer variable controlling the precedence of ore pass development. $b_{p,l}^{t}$ is equal to one if ore pass development p on level l has started by or in period t, otherwise it is zero.

$b_{v,l}^{t} \in \{0,1\}$ binary integer variable controlling the precedence of ventilation development (installations). $b_{v,l}^{t}$ is equal to one if ventilation development (installation) v on level l has started by or in period t, otherwise it is zero.

$lc_l^t \in \{0,1\}$	binary integer variable: when it is equal to one, it implies operational development activities are in progress on level l in period t.
$le_l^t \in \{0,1\}$	binary integer variable: when it is equal to one, it implies stope extraction activities are in progress on level l in period t.

5. Stochastic Mixed Integer Linear Programming (SMILP) Model Formulation

The SMILP optimization framework for generating long-term production schedule in the presence of grade uncertainty was modeled using multiple realizations from Sequential Gaussian Simulation (SGS) of the orebody. Revenue is calculated for each stope in each realization. The revenue of a stope is generated by selling the final product less all the costs involved in developing, extracting, and processing the stope. The mining cost per stope is a function of the distance between its location and its destination (processing plant and stockpile). Since the long-term production plan isa multi-period optimization problem and stopes are extracted in different periods, a discount rate is applied to calculate the present value of the revenue and the costs. Grade uncertainty causes shortfall from target production levels.

The application of geological discount rate referred to as GDR [14] in this work helps to control the grade and ore tonnage risk distribution over time and ensures the extraction of lower-risk and higher-grade stopes in the early periods, leaving higher-risk stopes for later periods when more information becomes available from mining. It is essential for the operation to mine less risky parts of the deposits in the early stages with limited geological information. As more geological information becomes available in the form of operational data, new schedules can be produced for later periods.

Penalty cost parameters are financial consequences for deviations from desired outcomes, that make simulation exercises more realistic and reveals the impact of decisions made. By incorporating penalty costs, simulations can effectively evaluate different strategies, manage risks, and promote optimal performance. Therefore, the penalty cost can be altered to prioritize the most critical operational targets according to the optimization requirements in the mining complex.

Penalty cost parameters $pntCT_{o,+}^t$, $pntCT_{o,-}^t$, $pntCT_{g,+}^t$, $pntCT_{g,-}^t$, $SPpntCT_{g,+}^t$ and $SPpntCT_{g,-}^t$ are defined and used for minimizing deviations from ore tonnage and ore grade production targets.

The objective function of the SMILP model in Eq. (2) is to maximize the expected NPV and minimize deviations from production targets. The first part of the objective function comprises of the total discounted cashflow from ore material from direct mining and stockpile to the processing plant for all realizations. The second part of the objective function minimizes the costs associated with deviating from the processing plant ore tonnage operating targets for all realizations. The third and fourth part minimizes the total costs associated with deviating from the processing plant and stockpile ore grade operating targets for all realizations respectively. Thus, the second, third, and fourth parts aim to control grade uncertainty by minimizing the variability from the ore tonnage and ore grade targets. Deviations are discounted in terms of geological discount rate (GDR) to implement a time dependent risk profile [14]. The concept presented in Appianing et al. [46] was used as the starting point of this work.

The objective function expressed in Eq. (2) is composed of the discounted revenues considering ore recovery rc^t and mining dilution $di_{k,s}$ factors in Eqs. (3) and (4), discounted cost of stope extraction

Eq. (5), discounted cost of primary development Eq. (6), discounted cost of ventilation development Eq. (7), discounted cost of operational development Eq. (8), discounted cost of ore pass development Eq. (9) and discounted cost of backfilling Eq. (10). Therefore, to obtain an optimum schedule, NPV must be maximized and the deviation from target productions must be minimized simultaneously among all simulation realizations. Therefore, the revenue for the stochastic model is calculated using Eqs. (3) and (4). By this uncertainty integrated objective function, the SMILP model can reduce the risk of not meeting the planned production targets and provide a feasible schedule.

$$Max\frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{k=1}^{K}\left[\frac{\begin{array}{l}(R_{k,p,s}^{t}\times x_{k}^{t})+(R_{k,so,s}^{t}\times u_{k,so}^{t})-(Q_{k}^{t}\times y_{k}^{t})-(C_{k}^{t}\times d_{c}^{t})\\-(H_{k}^{t}\times d_{v}^{t})-(E_{k}^{t}\times d_{a}^{t})-(Z_{k}^{t}\times d_{p}^{t})-(F_{k}^{t}\times f_{k}^{t})\end{array}}{(1+i)^{t}}\right]$$

$$-\left[\begin{array}{l}\frac{1}{s}\sum_{s=1}^{S}\sum_{t=1}^{T}\left[\frac{(pntCT_{o,+}^{t}\times odev_{s,+}^{t})+(pntCT_{o,-}^{t}\times odev_{s,-}^{t})}{(1+GDR)^{t}}\right]\\+\frac{1}{s}\sum_{s=1}^{S}\sum_{t=1}^{T}\left[\frac{(pntCT_{g,+}^{t}\times gdev_{s,+}^{t})+(pntCT_{g,-}^{t}\times gdev_{s,-}^{t})}{(1+GDR)^{t}}\right]\\+\frac{1}{s}\sum_{s=1}^{S}\sum_{t=1}^{T}\left[\frac{(SPpntCT_{g,+}^{t}\times jgdev_{s,+}^{t})+(SPpntCT_{g,-}^{t}\times jgdev_{s,-}^{t})}{(1+GDR)^{t}}\right]\end{array}\right] \tag{2}$$

$$R_{k,p,s}^{t}=(g_{k,s}\times o_{k,s}\times r_{k}\times rc^{t}\times(sp^{t}-sc^{t}))-(o_{k,s}\times(1+di_{k,s})\times(mp_{k}^{t}+cstp_{k}^{t})) \tag{3}$$

$$R_{k,so,s}^{t}=(g_{k,s}\times o_{k,s}\times r_{k}\times rc^{t}\times(sp^{t}-sc^{t}))-(o_{k,s}\times(1+di_{k,s})\times mp_{k}^{t}) \tag{4}$$

$$Q_{k}^{t}=q_{k}^{t}\times(o_{k}+w_{k}) \tag{5}$$

$$C_{k}^{t}=c_{k}^{t}\times d_{c} \tag{6}$$

$$H_{k}^{t}=h_{k}^{t}\times d_{v} \tag{7}$$

$$E_{k}^{t}=e_{k}^{t}\times d_{a} \tag{8}$$

$$Z_{k}^{t}=z_{k}^{t}\times d_{p} \tag{9}$$

$$F_{k}^{t}=cf_{k}^{t}\times d_{fv} \tag{10}$$

5.1. Constraints

Constraints considered for the formulation follow the underground mining sequence of operation. Thus, for a typical underground mining system, capital development precedes ventilation development before operational development. Extraction then precedes operational development and finally backfilling. Illustrated in Figure 6 are the constraint dependencies for the open stope mining production schedule formalism. In summary, the primary contribution of this research is the integration of mine development, grade uncertainty, stockpiling and production schedules in a stochastic optimization framework that generates a practical mine-plan.

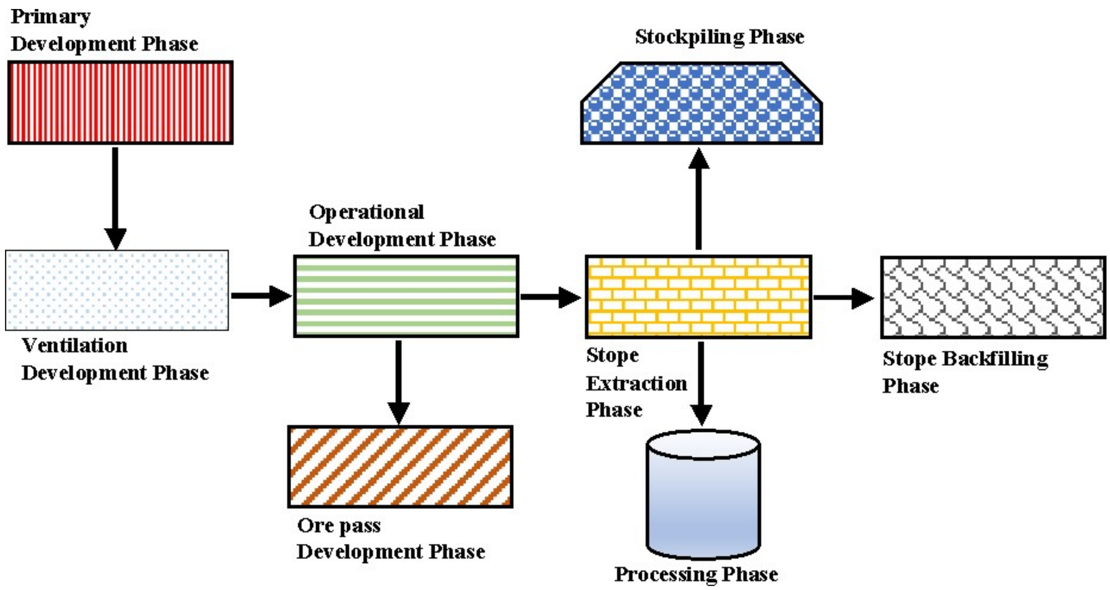

Figure 6. Constraints dependencies.

5.1.1. Primary Development Constraints

Eqs. (11) and (12) define the primary development upper and lower limits capacity constraints for the mine. These inequalities ensure that the total length of primary development required in a period is within the stated lower and upper limits of the total available equipment capacity for developing the mine. Eqs. (13) to (16) control the precedence relations between sections of primary development leading to each mining level and the ventilation development on each level.

Specifically, Eqs. (13) to (15) ensure that the set of primary development representing a section above a level must be completed before the primary development $d_{c,l}^{t}$ of the level commences. Eq. (16) ensures that the ventilation development $b_{v,l}^{t}$ on a level linking the primary development can only commence after completion of a set of required primary development $\sum_{t=1}^{t} d_{c,l}^{t}$ above and on that level.

$$\sum_{c=1}^{C}\left(d_{c,l} \times d_{c}^{t}\right) \leq L_{c,ub}^{t}, \ \forall t \in \{1,...,T\}; \quad (11)$$

$$\sum_{c=1}^{C}\left(d_{c,l} \times d_{c}^{t}\right) \geq L_{c,lb}^{t}, \ \forall t \in \{1,...,T\}; \quad (12)$$

$$b_{c,l}^{t} - \sum_{t=1}^{t} d_{s,l}^{t} \leq 0, \ \forall l \in (1,...,L), s \in D_{c}(L); \quad (13)$$

$$\sum_{t=1}^{t} d_{c,l}^{t} - b_{c,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, c \in \{1,...,C\}; \quad (14)$$

$$b_{c,l}^{t} - b_{c,l}^{t+1} \leq 0, \quad \forall l \in \{1,...,L\}, t \in \{1,...,T-1\}, c \ \in \{1,...,C\}; \quad (15)$$

$$b_{v,l}^{t} - \sum_{t=1}^{t} d_{c,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, c \in D_{c},(L), v \in D_{v}(L); \quad (16)$$

5.1.2. Ventilation Development Constraints

Ventilation is required to provide airflow into development and workings for the comfort of workers and functioning of equipment. Eqs. (17) to (22) present the set of constraints defining the ventilation requirements to enhance stope extraction. Eqs. (17) and (18) define the ventilation development upper and lower limits capacity constraints for the mine. These inequalities ensure that the total length of ventilation development required in a period is within the stated lower and upper limits of the total available equipment capacity for ventilation development. Eqs. (19) to (22) control the precedence relationships between the sections of ventilation development leading to each mining level and the operational development on each level.

Specifically, Eqs. (19) to (21) ensure that the set of ventilation development representing a section above a level must be completed before the ventilation development $d_{v,l}^{t}$ of the level commences. Eq. (22) ensures that the operational development $d_{a,l}^{t}$ on a level linking the ventilation development $b_{v,l}^{t}$ can only commence after completion of a set of required ventilation development $\sum_{t=1}^{t} d_{v,l}^{t}$ above and on that level.

$$\sum_{v=1}^{V}\left(d_{vl} \times d_{v}^{t}\right) \leq L_{v,ub}^{t}, \quad \forall t \in \{1,...,T\}; \tag{17}$$

$$\sum_{v=1}^{V}\left(d_{vl} \times d_{v}^{t}\right) \geq L_{v,lb}^{t}, \quad \forall t \in \{1,...,T\}; \tag{18}$$

$$b_{v,l}^{t} - \sum_{t=1}^{t} d_{s,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, s \in D_{v}(L); \tag{19}$$

$$\sum_{t=1}^{t} d_{v,l}^{t} - b_{v,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, v \in \{1,...,V\}; \tag{20}$$

$$b_{v,l}^{t} - b_{v,l}^{t+1} \leq 0, \quad \forall l \in \{1,...,L\}, t \in \{1,...,T-1\}, v \in \{1,...,V\}; \tag{21}$$

$$b_{a,l}^{t} - \sum_{t=1}^{t} d_{v,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, a \in D_{a}(L), v \in D_{v}(L); \tag{22}$$

5.1.3. Operational development constraints

If a stope is scheduled to be mined in a period, a set of operational development must be completed ahead or in that period. Eqs. (23) to (28) present the set of constraints defining the operational development and lateral precedence relations for stope extraction sequence. This includes the type and length of each operational development (level drives and crosscuts). Eqs. (23) and (24) define the operational development upper and lower limits capacity constraints for the mine. These equations ensure that the total length of operational development required in each period is within the defined lower and upper limits of the total available equipment capacity for developing the mine. Eqs. (25) to (27) control the lateral precedence relation of the operational development required for each stope. Eq. (28) ensures that for each stope, there is a set of operational development $\sum_{t=1}^{t} d_{a,l}^{t}$ that must be completed before mining the stope $b_{k,l}^{t}$.

$$\sum_{a=1}^{A}\left(d_a \times d_a^t\right) \le L_{a,ub}^t, \quad \forall t \in \{1,...,T\}; \tag{23}$$

$$\sum_{a=1}^{A}\left(d_a \times d_a^t\right) \ge L_{a,lb}^t, \quad \forall t \in \{1,...,T\}; \tag{24}$$

$$b_{a,l}^t - \sum_{t=1}^{t} d_{s,l}^t \le 0, \quad \forall l \in \{1,...,L\}, s \in D_a(L); \tag{25}$$

$$\sum_{t=1}^{t} d_{a,l}^t - b_{a,l}^t \le 0, \quad \forall l \in \{1,...,L\}, a \in \{1,...,A\}; \tag{26}$$

$$b_{a,l}^t - b_{a,l}^{t+1} \le 0, \quad \forall l \in \{1,...,L\}, t \in \{1,...,T-1\}, a \in \{1,...,A\}; \tag{27}$$

$$b_{k,l}^t - \sum_{t=1}^{t} d_{a,l}^t \le 0, \quad \forall l \in \{1,...,L\}, k \in \{1,...K_l\}, a \in D_a(L); \tag{28}$$

5.1.4. Ore Pass Development Constraints

Eqs. (29) and (30) define the ore pass development upper and lower limits capacity constraints for the mine. These inequalities ensure that the total length of ore pass developed in a period is within the stated lower and upper limits of the total available equipment capacity for developing the ore pass for the mine. Eqs. (31) to (34) control the precedence relations between the sections of ore pass development leading to each mining level and the operational development on the level.

Specifically, Eqs. (31) to (33) ensure that the set of ore pass development representing a section above a level must be completed before the ore pass $d_{p,l}^t$ of the level commences. Eq. (34) ensures that the development of the ore pass $d_{p,l}^t$ linking the operational level can only commence after completion of a set of required operational development $\sum_{t=1}^{t} d_{a,l}^t$ above and on that level.

$$\sum_{p=1}^{P}\left(d_p \times d_p^t\right) \le L_{p,ub}^t, \quad \forall t \in \left\{1,...,T\right\}; \tag{29}$$

$$\sum_{p=1}^{P}\left(d_p \times d_p^t\right) \ge L_{p,lb}^t, \quad \forall t \in \left\{1,...,T\right\}; \tag{30}$$

$$b_{p,l}^t - \sum_{t=1}^{t} d_{s,l}^t \le 0, \quad \forall l \in \left\{1,...,L\right\}, s \in D_p(L); \tag{31}$$

$$\sum_{t=1}^{t} d_{p,l}^t - b_{p,l}^t \le 0, \quad \forall l \in (1,...,L), p \in \left\{1,...,P\right\}; \tag{32}$$

$$b_{p,l}^t - b_{p,l}^{t+1} \le 0, \quad \forall l \in (1,...,L), t \in \left\{1,...,T-1\right\}, p \in \left\{1,...,P\right\}; \tag{33}$$

$$b_{p,l}^t - \sum_{t=1}^{t} d_{a,l}^t \le 0, \quad \forall p \in \left\{1,...,P\right\}, a \in D_a(L), l \in (1,...,L); \tag{34}$$

5.1.5. Mining Capacity Constraints

The total material mined is expressed as the sum of ore tonnage, and waste tonnage. Though the total material may not be extracted all together in a specific period, the fractional extraction of material is controlled using the continuous decision variable y_k^t. The mining capacity constraints are expressed in Eqs. (35) and (36).

$$\sum_{k=1}^{K}\left[(o_{k,s}+w_{k,s})\times y_k^t\right]\leq T_{m,ub}^t,\ \forall t\in\{1,...,T\},\forall s\in\{1,...,S\}; \tag{35}$$

$$\sum_{k=1}^{K}\left[(o_{k,s}+w_{k,s})\times y_k^t\right]\geq T_{m,lb}^t,\ \forall t\in\{1,...,T\},\forall s\in\{1,...,S\}; \tag{36}$$

5.1.6. Processing Capacity Constraints

To complete the SMILP formulation, Eqs. (37) and (38) are the processing capacity constraints that control the quantity of mill feed for the mine in each period for all realizations. We introduce continuous variables $odev_{s,+}^t$ and $odev_{s,-}^t$ which serve as buffers to allow for deviations from the processing targets. They are however penalized in the objective function. Penalizing such deviations ensure that the tonnage of ore processed at the plant is close to the required targets as much as is feasible.

$$\sum_{k=1}^{K}\left[((o_{k,s}\times x_{k,\mathrm{m}}^t)+\sum_{so=1}^{SO}(o_{k,so,s}\times u_{k,so}^t)-odev_{s,+}^t\right]\leq T_{pr,ub}^t,\forall t\in\{1,...,T\},\forall s\in\{1,...,S\}; \tag{37}$$

$$\sum_{k=1}^{K}\left[((o_{k,s}\times x_{k,\mathrm{m}}^t)+\sum_{so=1}^{SO}(o_{k,so,s}\times u_{k,so}^t)+odev_{s,-}^t\right]\geq T_{pr,lb}^t,\forall t\in\{1,...,T\},\forall s\in\{1,...,S\}; \tag{38}$$

5.1.7. Grade Blending Constraints

Eqs. (39) and (40) define the upper and lower limit of the grade to be sent to the plant in each period. The deviation decision variables $gdev_{s,+}^t$ and $gdev_{s,-}^t$ are introduced to serve as buffers to the ore grade targets. Eqs. (41) and (42) also define the upper and lower limit of grade delivered to the stockpile. The deviation decision variables $jdev_{s,+}^t$ and $jdev_{s,-}^t$ are introduced to serve as buffers to the stockpile ore grade targets. Similar to the processing quantity constraints, the excess and shortage of the desired grade range in each period across all geological realizations are penalized as indicated in the objective function(Eq. (2)). These constraints are controlled by the continuous decision variables $x_{k,m}^t, u_{k,si}^t, u_{k,so}^t, gdev_{s,+}^t, gdev_{s,-}^t, jgdev_{s,+}^t$ *and* $jgdev_{s,-}^t$ in each period.

$$\sum_{k=1}^{K}\left[(g_{k,s}-gr_{pr,ub}^t)\times(o_{k,s}\times x_{k,m}^t)\ +\ \sum_{so=1}^{SO}(g_{k,so,s}^t-gr_{pr,ub}^t)\times(o_{k,so,s}\times u_{k,so}^t)-gdev_{s,+}^t\right]\leq 0,$$
$$\forall t\in\{1,...,T\},\forall k\in\{1,...,K\},\ \forall s\in\{1,...,S\}; \tag{39}$$

$$\sum_{k=1}^{K}\left[(gr_{pr,lb}^t-g_{k,s})\times(o_{k,s}\times x_{k,m}^t)\ +\ \sum_{so=1}^{SO}(gr_{pr,lb}^t-g_{k,so,s}^t)\times(o_{k,so,s}\times u_{k,so}^t)+gdev_{s,-}^t\right]\leq 0,$$
$$\forall t\in\{1,...,T\},\forall k\in\{1,...,K\},\ \forall s\in\{1,...,S\} \tag{40}$$

$$\sum_{k=1}^{K}((g_{k,s}-gr_{si,ub}^t)\times(o_{k,s}\times u_{k,si}^t)-jgdev_{s,+}^t)\leq 0,$$
$$\forall t\in\{1,...,T\},\forall k\in\{1,...,K\},\ \forall s\in\{1,...,S\} \tag{41}$$

$$\sum_{k=1}^{K}((gr_{si,lb}^{t} - g_{k,s}) \times (o_{k,s} \times u_{k,si}^{t}) + jgdev_{s,-}^{t}) \leq 0, \quad (42)$$
$$\forall t \in \{1,...,T\}, \forall k \in \{1,...,K\}, \forall s \in \{1,...,S\};$$

5.1.8. Stope Extraction Precedence Constraints

Eqs. (43) to (45) control the lateral precedence relations of stope extraction on each level of the mining operation. The stoping sequence for the deposit is implemented in an advancing and top-down approach. Thus, mining generally starts from the upper level to the lower level exploiting the deposit from the center of the mine towards to the periphery of the mine.

$$b_k^t - \sum_{t=1}^{t} y_s^t \leq 0, \quad \forall s \in G_k(S), k \in \{1,...,K\}; \quad (43)$$

$$\sum_{t=1}^{t} y_k^t - b_k^t \leq 0, \quad \forall k \in \{1,...,K\}; \quad (44)$$

$$b_k^t - b_k^{t+1} \leq 0, \quad \forall t \in \{1,...T-1\}, k \in \{1,...,K\}; \quad (45)$$

5.1.9. Backfilling Management Constraints

The backfilling constraints include backfilling capacity and sequencing constraints, and stope status constraints. Eqs. (46) and (47) present the upper and lower limit backfilling capacity constraints. The total volume of material backfilled in each period cannot exceed the capacity of the backfill plant and equipment in the period.

The stope status constraints are defined to monitor geotechnical and ground quality conditions. These constraints control where to start operational development and extraction for a level and progress sequentially. Eq. (48) ensures that each active stope can only be in either extraction or backfilling phase during any specific period. The two activities cannot be in progress at the same time for a stope. Eq. (49) ensures that stope ore extraction precedes backfilling. Eq. (50) ensures that each stope is in either the extraction or backfilling phase immediately after a non-zero stope extraction period. The stope status and backfilling sequencing constraints work together to ensure each stope progresses through discrete mining activities sequentially from operational development to extraction and then backfilling. These constraints indirectly limit the exposure time of the void area after mining to reduce potential excessive geotechnical stresses prior to backfilling.

$$\sum_{k=1}^{K}\left(d_{fv} \times f_k^t\right) \leq V_{f,ub}^t, \quad \forall t \in \{1,...,T\}; \quad (46)$$

$$\sum_{k=1}^{K}\left(d_{fv} \times f_k^t\right) \geq V_{f,lb}^t, \quad \forall t \in \{1,...,T\}; \quad (47)$$

$$b_k^t + f_k^t \leq 1, \quad \forall t \in \{1,...,T\}, k \in \{1,...,K\}; \quad (48)$$

$$f_k^t - \sum_{t=1}^{t}(x_{k,\mathrm{m}}^t + u_{k,si}^t) \leq 0, \quad \forall t \in \{1,...,T\}, k \in \{1,...,K\}; \quad (49)$$

$$b_k^t \leq b_k^{t+1} + bf_k^{t+1}, \quad \forall t \in \{1,...,T-1\}, k \in \{1,...,K\}; \quad (50)$$

5.1.10. Non-Adjacent Stope Extraction Constraints

These are geotechnical constraints that focus on limiting the size of unsupported void areas. To demonstrate the functioning of these constraints, the nine adjacent stopes depicting the two kinds of spatial constraints required to facilitate geotechnical stability of the stoping area in extraction and backfilling scenarios can be seen in Figure 7. When a stope is being extracted or backfilled, the adjacent stopes cannot be active. As illustrated in Figure 7, if the central stope Sp5 is in the extraction state, all operational activities are forbidden in the near adjacent stopes (Sp2, Sp4, Sp6, Sp8). Similarly, when a stope is being backfilled, the adjacent stopes cannot be exploited. Eqs. (51) and (52) ensure that all forms of adjacent stope extraction or backfilling activities are avoided to prevent excessively large unsupported voids.

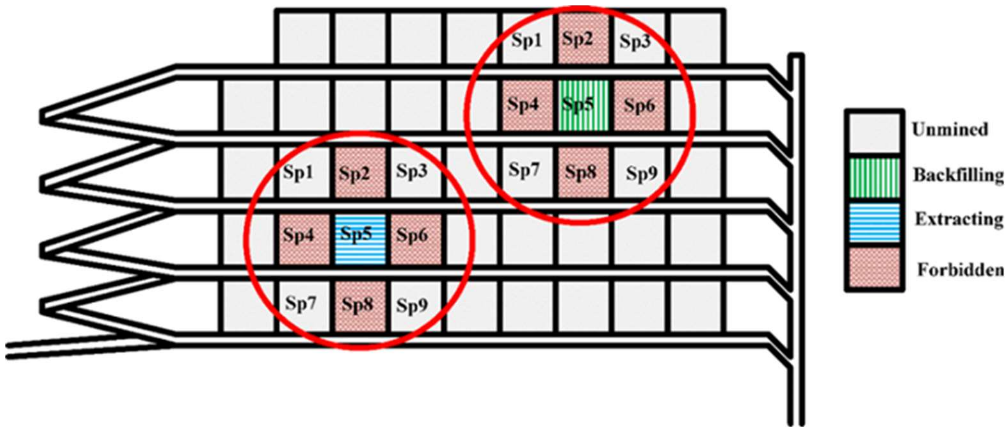

Figure 7. A schematic illustration of non-adjacent stope extraction and backfilling activities modified after Huang et al. [11].

$$b_k^t + b_j^t \leq 1, \ \forall t \in \{1,...,T\}, k \in \{1,...,K\}, j \in A_k(J); \tag{51}$$

$$bf_k^t + b_j^t \leq 1, \ \forall t \in \{1,...,T\}, k \in \{1,...,K\}, j \in A_k(J); \tag{52}$$

5.1.11. Active Levels Control Constraints

Many active levels in a mine might lead to serious operational problems. A controlled maximum number of simultaneous active levels allow mine planners to adjust extraction to different application scenarios depending on available infrastructure, production rates, equipment capacities, and the stability of the surrounding rocks. The constraints expressed in Eqs. (53) to (56) control the maximum number of simultaneous active levels in each period. In Eqs. (53) and (54), when operational development activity is ongoing, or a portion of a stope is being extracted in period *t,* on level l, the relevant binary variable lc_l^t and le_l^t must be equal to one. In Eqs. (55) and (56), N_{lc}^t and N_{le}^t are provided as allowable input in the formulation for which the maximum number of simultaneous active levels must not exceed in each period. The active levels control constraints guide the model to generate a more operationally feasible mining pattern, thus ensuring a practical maximum NPV.

$$\sum_{t=1}^{t}\left(\frac{1}{D}\sum_{d=1}^{D} d_{a,l}^t - lc_l^t\right) \leq 0, \ \ \forall t \in \{1,...,T\}, l \in \{1,...,L\}, \ \mathrm{a} \in L_l(D); \tag{53}$$

$$\sum_{t=1}^{t}\left(\frac{1}{M}\sum_{m=1}^{M} x_m^t - le_l^t\right) \leq 0, \ \ \forall t \in \{1,...,T\}, l \in \{1,...,L\}, \ \mathrm{m} \in L_l(M); \tag{54}$$

$$\sum_{l=1}^{L} lc_l^t \leq N_{lc}^t, \ \ \forall t \ \in \{1,...,T\}; \tag{55}$$

$$\sum_{l=1}^{L} le_l^t \leq N_{le}^t, \quad \forall t \in \{1,...,T\}; \tag{56}$$

5.1.12. Stope Extraction Duration Constraints

The length of stope extraction depends on notably, stope size, rate of production, and mining plan. Eq. (57) ensures that the period of extraction of a stope cannot exceed a pre-defined number of periods. This inequality enables the model to implement continuous stope extraction, mirror the mining environment, and regulate the allowed extraction period. This adds practicality and strengthens the functionality of the formulation.

$$\sum_{t=1}^{T} y_k^t \leq N_{xd}, \quad \forall t \in \{1,...,T\}; \tag{57}$$

5.1.13. Stockpile Management Constraints

The stockpile capacity constraint that control the quantity of ore material sent to the stockpile is presented in Eq. (58). This constraint also ensure that the capacity of stockpile does exceed predefined limit. To control ore sent to the stockpile, the decision variable is used. Since the stockpile is primarily grade controlled, there is no additional requirement to introduce a deviational variable to minimize ore tonnage deviation from the stockpile capacity target. An additional stockpile ore tonnage deviation variable adds to the complexity of the optimization problem and extends the solution time. Eq. (59) ensures that the cumulative quantity of ore sent to the stockpile exceeds the cumulative quantity of ore sent out of the stockpile to the plant. Thus, at the end of the mine-life, there could be some ore left in the stockpile. Eq. (60) accumulates the metal content of ore sent to the stockpile and ensures that the cumulative metal content must not exceed the cumulative metal content sent out from the stockpile to the plant.

$$\sum_{k=1}^{K}\left[(o_{k,s} \times u_{k,si}^t)\right] \leq T_{si,ub}^t, \forall t \in \{1,...,T\}, \forall s \in \{1,...,S\}; \tag{58}$$

$$\sum_{i=1}^{t}\sum_{k=1}^{K}(o_{k,so,s} \times u_{k,so}^i) \leq \sum_{i=1}^{t-1}\sum_{k=1}^{K}(o_{k,s} \times u_{k,si}^i), \forall t \in \{1,...,T\}, \forall s \in \{1,...,S\}; \tag{59}$$

$$\sum_{i=1}^{t}\sum_{k=1}^{K}(o_{k,so,s} \times g_{k,so,s}^t \times u_{k,so}^i) \leq \sum_{i=1}^{t-1}\sum_{k=1}^{K}(o_{k,s} \times g_{k,s} \times u_{k,si}^i), \forall t \in \{1,...,T\}, \forall s \in \{1,...,S\}; \tag{60}$$

5.1.14. Variables Control Constraints

Eq. (61) defines the relations between mining, ore extraction, and stockpiling variables controlling the stope mining, ore stockpiling, and waste decisions. Thus, the continuous variables x_k^t and $u_{k,so}^t$ representing ore extraction and stockpiling plus amount of the waste dilution should always be smaller than or equal to the continuous variable y_k^t representing stope mining in each period. Eq. (62) show the relation between stockpiling variables controlling material that goes to stockpile and out of stockpile to processing plant. Thus, the sum of the continuous variables must be smaller than or equal to 1. Eqs. (63) and (64) ensure that the total fractions of stopes and backfilling volumes over the scheduling periods sum up to one. Eqs. (65) to (66) ensure that the total fractions of ore sent to stockpile from the mine and sent from stockpile to processing plant from stockpile over the scheduling periods sum up to one. Thus, ore in each circumstance may be scheduled once. Eqs. (67) to (70) ensure that all developments (primary, ventilation, operational, and ore pass) are going to be constructed once in the life of the mine. The variable control constraints define the logics and inter-relations of the binary and continuous variables that define extraction, backfilling, and primary, ventilation, operational, and ore pass development to ensure they are within desired ranges.

$$x_k^t + u_{k,so}^t \le y_k^t, \forall t \in \{1,...,T\}, k \in \{1,...,K\}; \quad (61)$$

$$u_{k,si}^t + u_{k,so}^t \le 1, \forall t \in \{1,...,T\}, k \in \{1,...,K\}; \quad (62)$$

$$\sum_{t=1}^{T} f_k^t \le 1, \forall k \in \{1,...,K\}; \quad (63)$$

$$\sum_{t=1}^{T} x_k^t \le 1, \forall k \in \{1,...,K\}; \quad (64)$$

$$\sum_{t=1}^{T} u_{k,si}^t \le 1, \forall k \in \{1,...,K\}; \quad (65)$$

$$\sum_{t=1}^{T} u_{k,so}^t \le 1, \forall k \in \{1,...,K\}; \quad (66)$$

$$\sum_{t=1}^{T} d_c^t \le 1, \forall c \in \{1,...,C\}; \quad (67)$$

$$\sum_{t=1}^{T} d_v^t \le 1, \forall v \in \{1,...,V\}; \quad (68)$$

$$\sum_{t=1}^{T} d_a^t \le 1, \forall a \in \{1,...,A\}; \quad (69)$$

$$\sum_{t=1}^{T} d_p^t \le 1, \forall p \in \{1,...,P\}; \quad (70)$$

5.1.15. Non-Negativity Constraints

Eq. (71) ensures that all continuous decision variables are non-negative. Eq. (72) defines all binary variables as non-negative and integers. Eq. (73) defines the additional non-negativity constraints for deviational decision variables for processing tonnes and ore grade targets. These constraints enforce that none of these variables can take on negative values during the optimization process.

$$x_k^t, u_{k,si}^t, u_{k,so}^t, y_k^t, f_k^t, d_c^t, d_v^t, d_a^t, d_p \ge 0 \quad (71)$$

$$b_k^t, b_{c,l}^t, b_{v,l}^t, b_{a,l}^t, b_{p,l}^t \ge 0 \text{ and integers} \quad (72)$$

$$odev_{s,+}^t, odev_{s,-}^t, gdev_{s,+}^t, gdev_{s,-}^t, jgdev_{s,+}^t, jgdev_{s,-}^t \ge 0, \forall_t \in \{1,...T\}, \forall_s \in \{1,...S\}; \quad (73)$$

6. Results and discussions of Case studies

This section presents the implementation of the SMILP optimization models for a gold deposit. Six Case studies are presented. For Cases 1 and 2, a single orebody model which was based on Ordinary Kriging (OK) was used as the input block model. Cases 3 and 4 were implemented based on stochastic block models represented by 50 SGS realizations. The SGS realizations generate a set of equally probable block models used to capture and assess uncertainty in the deposit and production schedule. Cases 5 and 6 were based on the E-type block model which is the average simulated block model generated by post-processing of the SGS realizations. The E-type estimates showed slight differences from the Ordinary Kriging model. However, in theory, the E-type model is identical to the kriging results in Gaussian space [47, 48] . For Case 1, the MILP model is implemented without the grade controlled stockpile management strategy and for Case 2, the MILP model is implemented with the grade controlled stockpile management strategy. For Case 3, the integrated SMILP model is also implemented without the grade-controlled stockpile management strategy and finally for Case 4, the SMILP model is implemented with the grade-controlled stockpile management strategy. Similarly, for Case 5 was implemented without the grade-controlled stockpile management and Case 6 implemented with the grade-controlled stockpile management. Grade uncertainty is incorporated into the mine planning by using multiple simulated orebody realizations as inputs for the SMILP model. The SMILP model optimizes on a block-by-block basis to account for the consistent variation in ore grades across different blocks in each realization. Thus, this chapter documents the implementation of the uncertainty-based SMIL model, which integrates stockpiling management into production planning optimization under conditions of grade uncertainty and further presents the comparison between the MILP and SMIL models' performance, and the identified indicators needed to achieve desired production levels. The effects of the identified indicators like risk factors and deviational parameters on the production schedules are also presented through sensitivity analysis. Additionally, the advantage of incorporating a grade-controlled stockpile management strategy is presented. This chapter also documented.

Table 2 provides details about the gold deposit's characteristics and the designed stopes. Figure 8 displays a cross-section of the block model, illustrating the distribution of gold grades within the deposit. Figure 9 shows the section of the block model, some designed stopes, and development layout created using Promine software [41]. IBM CPLEX Optimization Studio V12.6.3 [43] was integrated within a MATLAB 2023a [49] environment to define the modelled framework and solve the optimization problem at a gap tolerance of 1%. The model was tested on an Intel (R) Core^{TM} i7-6500U CPU at 2.50 GHz with 64 GB of RAM.

Table 2. Characteristics of gold deposit and designed stopes.

Description	Value
Total mineralized material (Mt)	2.88
Maximum grade value of Au (g/t)	5.34
Minimum grade value of Au (g/t)	1.40
Average grade value of Au (g/t)	3.05
Number of levels	8
Stope height (m)	30
Stope length (m)	25
Stope width (m)	10-12

The orebody exhibits scattered high-grade mineralization throughout the deposit, while certain areas in the upper region contain prominent low-grade sections. The surrounding waste rock is composed of medium to low strength rock, making open stoping with backfilling the preferred mining method. 120 stopes were designed from the block model with 8 operational levels. Each level had 15 stopes (numbering from left to right) and continue on the level below following same pattern from left to right. Thus, the decending and advancing method of extracton was adopted where stope extraction starts from the first level continues downwards while exploiting stopes sequentially on each level towards the end of each level before moving to the next level downwards.

Table 3 contains the economic and technical criteria used to assess the case study. On the basis of the total amount of material to be extracted and the proposed plant capacity, respectively, annual capacities for mining and processing were established. Cost projections in Canadian dollars were gathered from industry professionals and technical mining company reports [50-52].

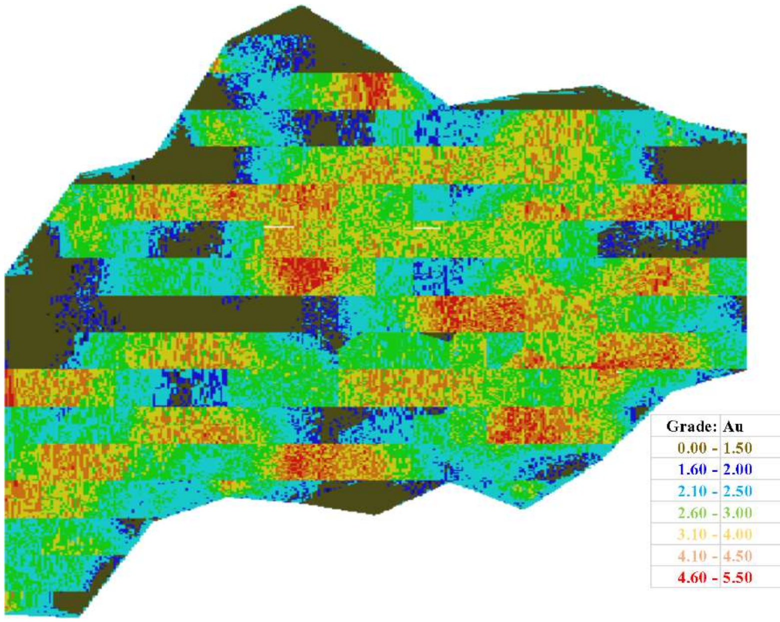

Figure 8. Cross-section of the block model depicting the distribution of gold grades (Not drawn to scale).

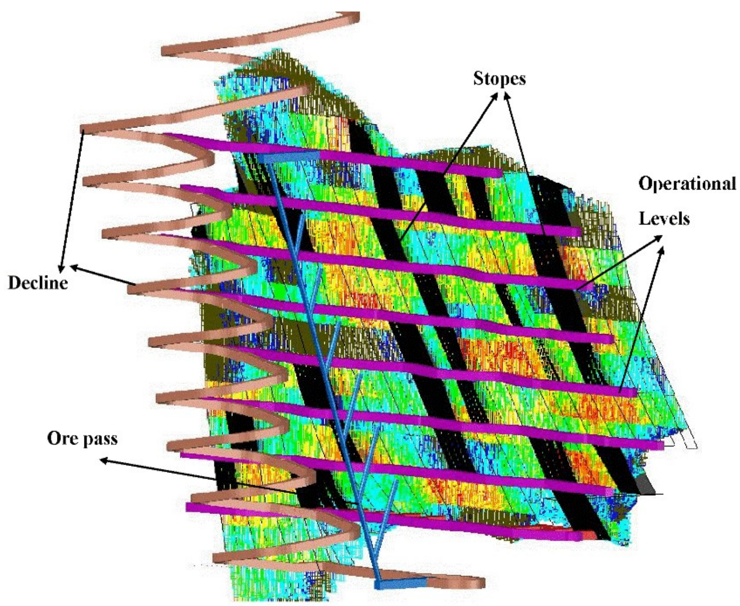

Figure 9. A pictorial illustration of the block model with some designed stopes and development.

Table 3. Technical data used.

Parameter	Value
UG mining cost ($/t)	96.04
Processing cost ($/t)	15.21
Backfilling cost ($/t)	15.00
Operational development cost ($/m)	4,500
Capital development cost ($/m)	6,500
Ventilation cost ($/m)	3,000
Selling price of gold ($/oz)	2,000
Discount rate (%)	8.00
Mining recovery (%)	93.00
Variable Mining dilution* (%)	5.00 / 10.00
Maximum mining capacity (Kt/period)	140.0
Maximum processing capacity (Kt/period)	110.0
Maximum backfilling capacity (m^3/period)	50,000
Maximum operational development (m/period)	200
Maximum capital development (m/period)	500
Maximum ventilation development (m/period)	500
Maximum stope extraction duration (Period)	2
Maximum ore pass development (m/period)	143
Maximum active levels	7

*Mining dilution of 10% was applied to stopes at the orebody's periphery, while mining dilution of 5% was applied to the remaining stopes. For our case studies, all the lower boundaries of the mining productivity data were set to zero. The optimizer was allowed to operate on multiple active mining levels due to the incorporation of stope backfilling, which improves ground control and increases operational flexibility.

The risk factors indicated in Table 4 were designed largely for Cases 3 and 4 to reduce the risk associated with processing plant and stockpile ore tonnage and ore grade deviations from predetermined targets during production scheduling. These risk parameter values were used for Cases 3 and 4 optimization runs, and were adopted from Huang, Li [11] to assess their impact on the SMILP model solution. In Table 5 are the parameters used for all stockpile implementation cases.

Table 4: Risk parameters for SMILP case studies.

Parameter	Cases 3 and 4
Number of realizations (#)	50
Geological risk discount rate (%)	20
Cost of shortage in ore production ($/tonne)	1500
Cost of excess in ore production ($/tonne)	1500
Cost of shortage in grade ($/g)	500
Cost of excess in grade ($/g)	500
Cost of shortage in stockpile grade ($/g)	500
Cost of excess in stockpile grade ($/g)	500

Table 5: Stockpile parameters.

Stockpile	Value
Maximum grade value of Au (g/t)	1.40
Minimum grade value of Au (g/t)	0.10
Rehandling cost ($/t)	0.5

6.1. Summary Results of Cases 1 and 2

Cases 1 and 2 which involves the MILP model formulation without stockpiling and with stockpiling respectfully benefited from optimized schedules for primary, ventilation, operating, and ore pass development, as well as mining, processing, and backfilling production schedules for a 25-year mine-life. Presented in Table 6 presents the summary of results from Cases 1 (MILP without stockpiling management) and 2 (MILP with stockpiling management). From Table 6, Case 2 achieved more ore tonnage even though it recoded a lower average grade. The high ore tonnage enabled Case 2 to record a higher metal content. These results made Case 2 record a higher metal content which subsequently caused Case 2 to record the highest NPV which is approximately 13% higher than that of Case 1.

Table 6: Summary of cases 1 and 2 results.

Case	Ore tonnage (Mt)	Average grade (g/t)	Au metal (t)	NPV (M$)
Case 1	2.19	2.98	6.53	7344.00
Case 2	2.52	2.87	7.23	7801.20

6.2. Case 3 – SMILP without Stockpile Management

In Figure 10, the primary, ventilation, and ore pass development schedules are presented. The NPV of the SMILP model was $7,601.30 M. Throughout the mine life, 2.64 Mt of material was mined, and 2.37 Mt of material was processed. Aside ore pass development which had the last schedule occurring in Year 23, primary and ventilation development occurred within the first 5 periods. Figure 11 shows the operational development schedule with varying capacities from Year 1 to Year 25 per production requirements.

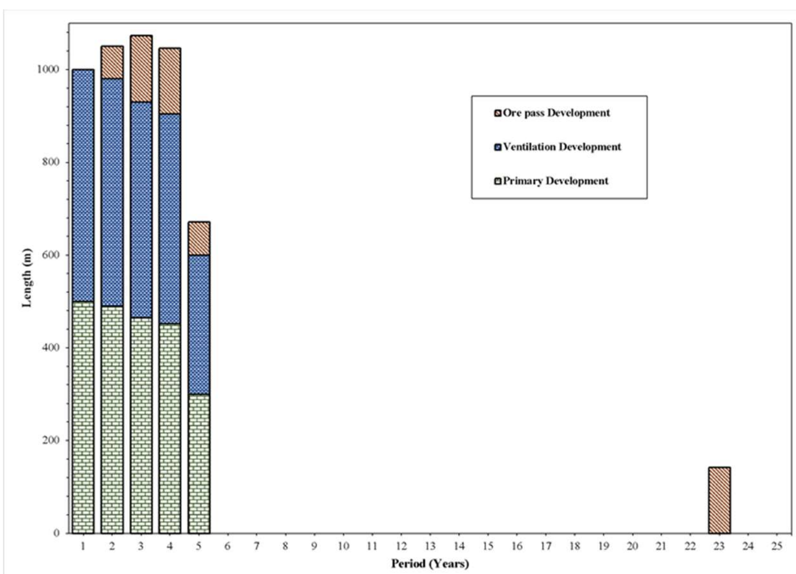

Figure 10. Primary, ventilation, and ore pass development schedule for Case 3.

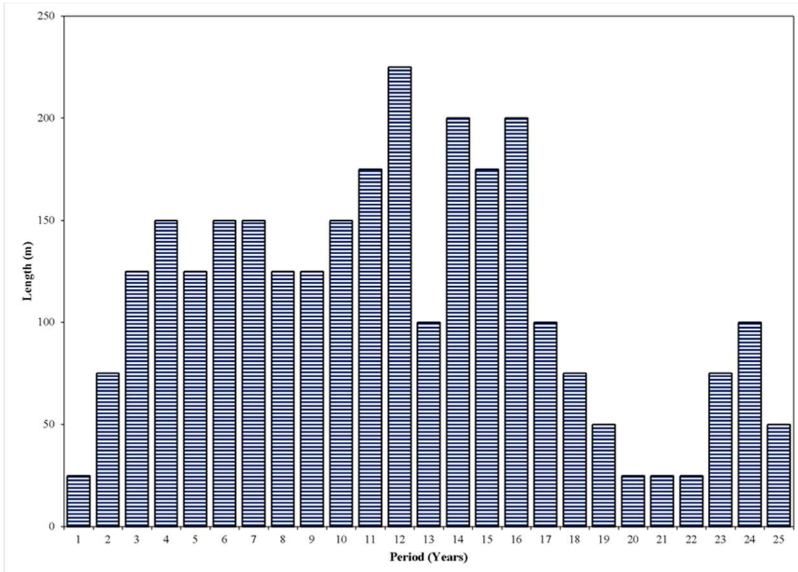

Figure 11. Operational development schedule for Case 3.

The material mined schedule presented in Figure 12 includes waste dilution. The mining operation commenced in Year 1 ramping up following the completion of primary, ventilation, and some initial operational development. The mining output gradually increases from Year 1 to a maximum capacity of 140 Kt in Year 9 and gradually decreases from Year 14 to the end of mine life in Year 25.

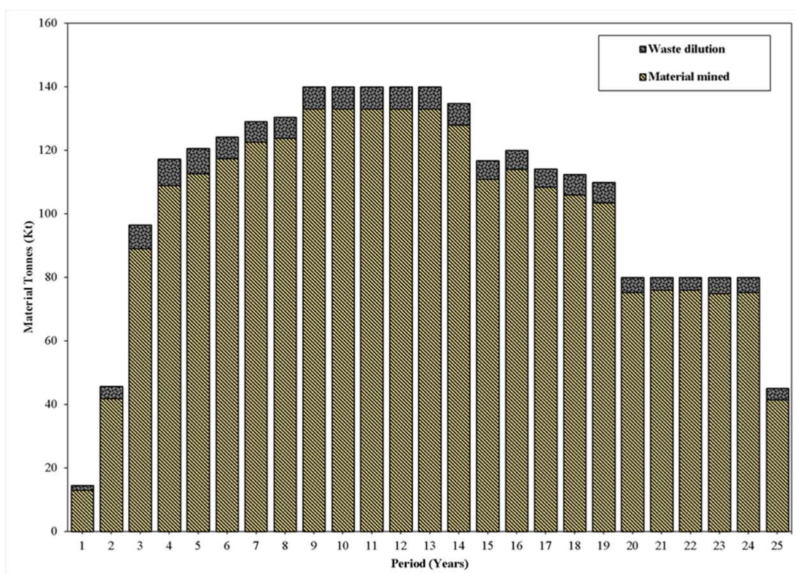

Figure 12. Mined material with estimated waste dilution for Case 3.

Figure 13 shows the material processing schedule and average grade curve from the SMILP model without stockpile management consideration. The plant's operation started in Year 1 ramping up to full plant capacity of 110 Kt in Year 4, and gradually ramping down to 80 Kt from Year 20 to Year 24. The SMILP model emphasizes targeting high-grade locations during the early years of mining based on the average grade profile to optimize the project's Net Present Value (NPV). However, due to development constraints, only a limited number of stopes are available for extraction during the first three years. As a result, the mining system is limited to lower grades that are instantly accessible on the upper levels. As development advances and more stopes become accessible for extraction, the processing plant's head grade improves from Year 1 to Year 7. In general, medium to high-grade ores are largely extracted during the first ten years of mining, from Year 1 to Year 10. As the higher-grade areas become depleted, the mining operation turns to extracting medium to lower -grade ores from Year 11 to Year 25. This facilitates strategic resource management and effective extraction of higher-grade material when it contributes most to the project's overall financial performance.

The schedule illustrated in Figure 14 shows the order in which each of the three underground mining production phases is completed, ensuring a systematic mining process.

- Operational Development: This phase entails the development of drives and crosscuts within the mine to provide access to the various sections of the orebody. Before proceeding to stope extraction, operational development is prioritized and completed to facilitate ventilation.
- Stope Extraction: After the operational development phase is completed, the stope extraction phase begins. During this phase, accessing and removing of valuable ore material takes place while maintaining the structural integrity of the surrounding rock is mined from the stope
- Backfilling: This phase constitutes the structural integrity of stope extraction. It entails filling the voids left after ore extraction with prescribed material. Backfilling helps to maintain ground stability, reduce surface subsidence and managing of waste material

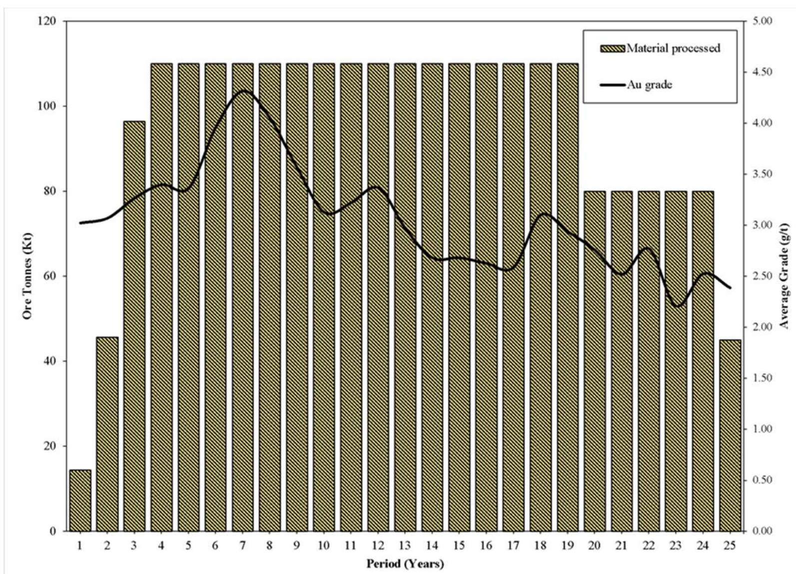

Figure 13. Material processed with average grade curve for Case 3.

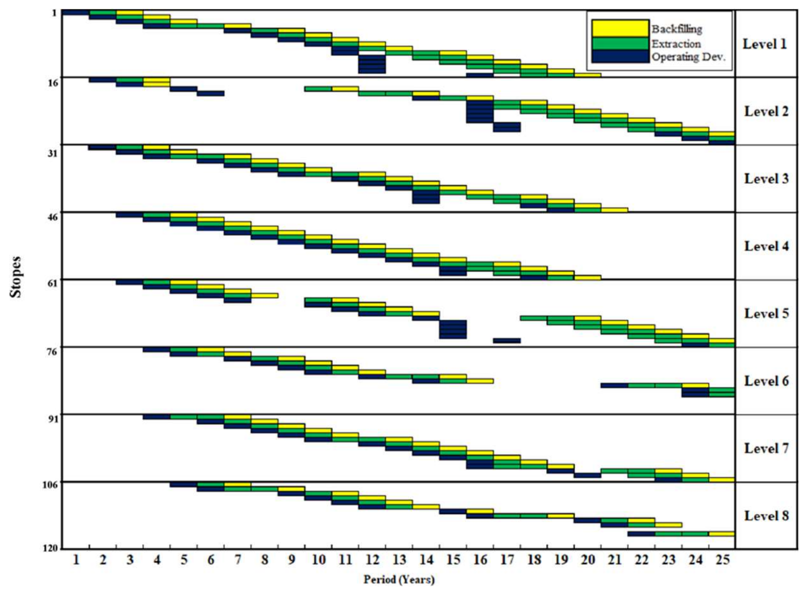

Figure 14. Annual operational development, stope extraction and backfilling schedule for Case 3.

There were certain unique instances in the mining operation where some levels become inactive after mining has started on them due to the availability of higher-grade stopes elsewhere. These extraction variations can be seen on Level 2 from Period 7 to Period 9 and on Level 6 from Period 17 to Period 20.

On Level 2, after the operational development in Periods 6 and 7 for Stopes 18 and 19, there were no immediate extraction and backfilling activities. On Level 6, no immediate operational development, extraction, and backfilling activities occurred from Period 17 to Period 20 before operational development of Stope 84 started. As mentioned earlier, these mining variations were caused by the presence of higher-grade stopes on Level 7, which were prioritized for extraction.

On Levels 6 and 8, some stopes were not extracted by the end of mine life. Specifically, Stopes 87 to 90 on Level 6, and Stopes 116, 118, and 119 on Level 8 respectively. The optimizer made these decisions based on the availability of varied ore grades in the deposit, with the goal of maximizing the project's NPV while maintaining efficient resource governance and safety considerations.

6.3. Case 4 – SMILP with Stockpile Management

The optimized primary, ventilation, operational and ore pass development schedules, and mining, stockpiling, processing, and backfilling schedules allowed for efficient resource utilization and a resulting NPV of $8,077.86 M, which is approximately 10% higher than MILP Case. Figure 15 indicates that primary and ventilation development follow the same timeline as in Case Studies 1, 2 and 3. However, ore pass development, which directly connects to ore extraction and stockpiling, occurred in three separate times; in Years 2 to 4, in Year 16, and in Years 18 and 21.

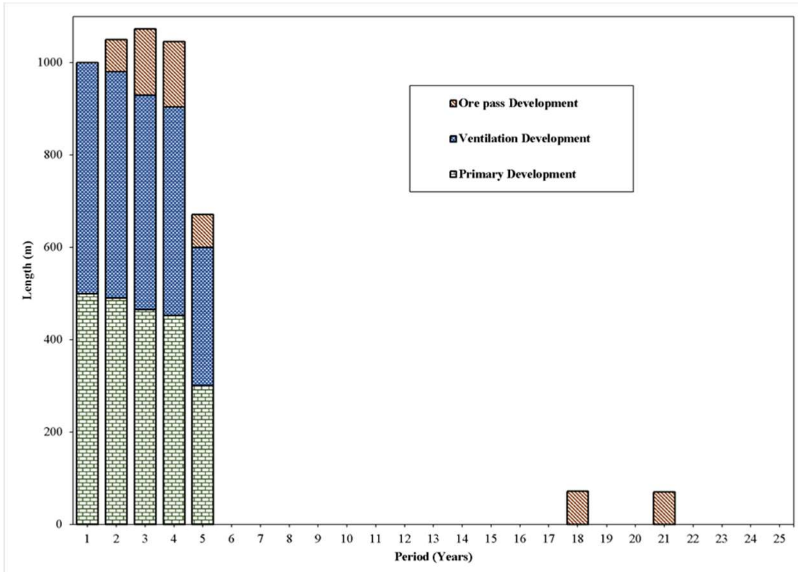

Figure 15: Primary, ventilation and ore pass development schedules for Case 4.

Except for Year 24, operational development occurred at variable rates based on production requirements throughout the mine life as presented in Figure 16.

Figure 17 shows the schedule for material mined with estimated waste dilution. During the first twelve years of the mine life, the planned maximum mining capacity of 140 Kt was achieved from Year 5 to Year 12. The mining operation started ramping down from Year 13 to the end of mine life. This Case resulted in the extraction of 2.5 Mt of material from the open stopes.

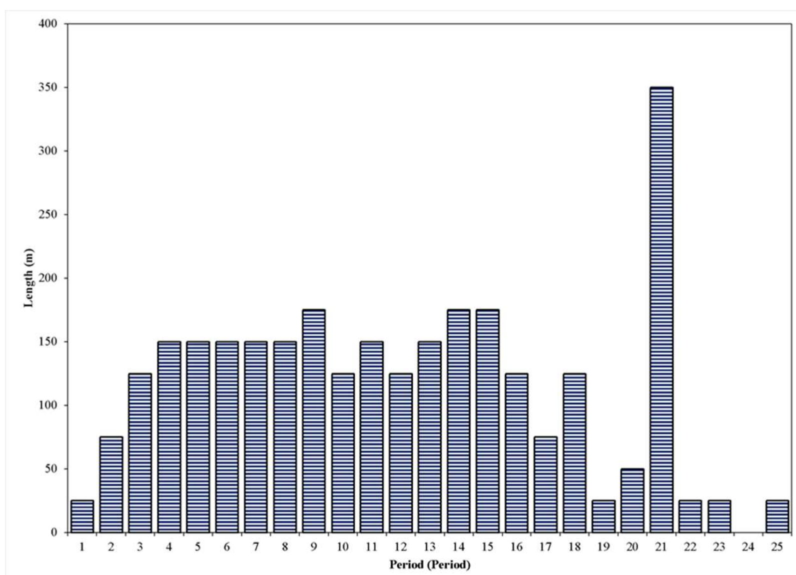

Figure 16: Operational development schedule for Case 4.

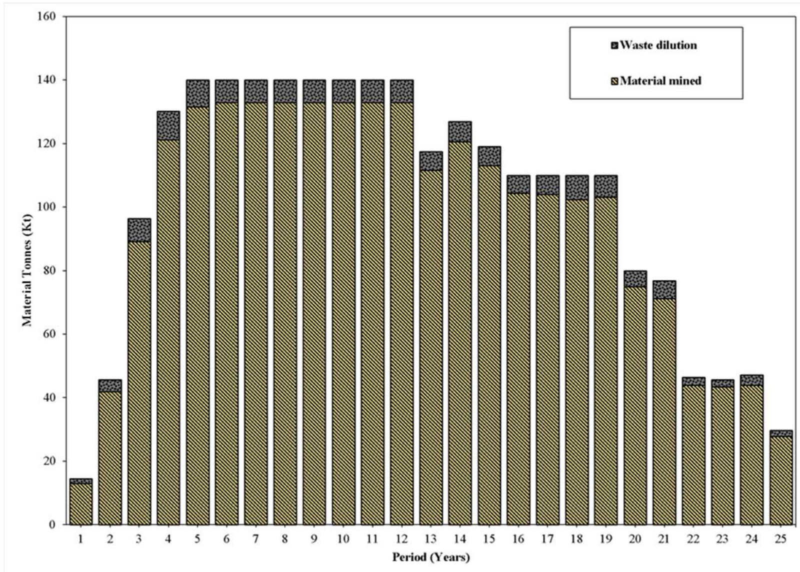

Figure 17: Material mined schedule with estimated waste dilution for Case 4.

Figure 18 depicts a detailed schedule that includes both material processed directly from the mine and material processed from the stockpile. The application of the grade controlled stockpiling strategy, and selective material blending guarantees a steady supply of high-quality ore to the processing plant in the initial stages of the mining operation, optimizing the overall financial returns of the project. Generally, the stockpile inventory profile indicates how the optimizer effectively controlled material storage, strategically utilizing the stockpile to ensure steady plant operations and enhance total material processed. By carefully regulating the stockpile and using it to supplement the direct ore extraction, the project adapts to varying ore grades.

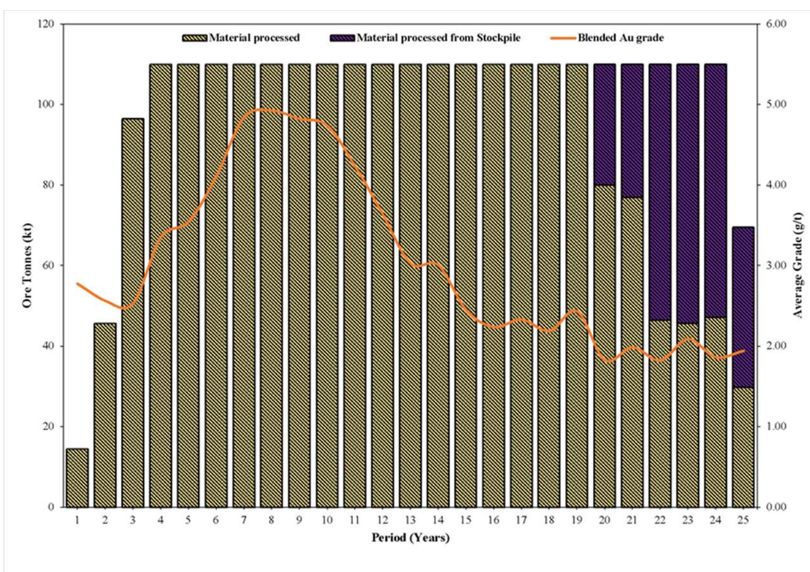

Figure 18: Processed material schedule with blended grade curve for Case 4.

The schedule of material sent to the stockpile and from the stockpile to the processing facility is shown in Figure 19. This schedule demonstrates the execution of the grade control technique. Low-grade material was transported to the stockpile at certain times; from Years 4 to 15. This enabled the extraction of higher-grade material while accumulating lower-grade material for future use. Grade-blended stockpiled material was reclaimed from the stockpile to the processing plant as needed from Year 20 until the end of mine life. This demonstrates how the stockpile was effectively used to deliver a continual supply of ore to the processing plant, ensuring efficient processing and resource management in the later stages of the mining operation. Approximately 294 Kt of material was stored and reclaimed, adding to the project's overall strategic planning and grade control.

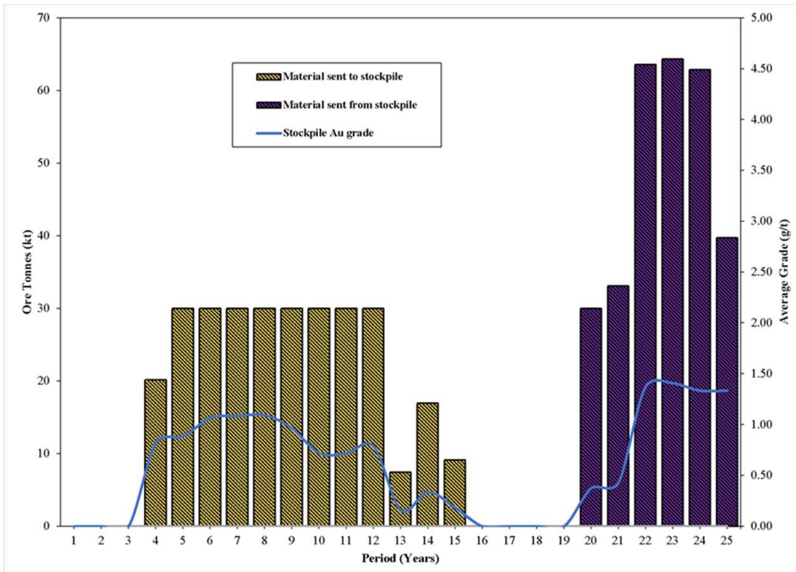

Figure 19: Stockpiled material schedule and grade curve for Case 4.

The stockpile inventory profile is shown in Figure 20 demonstrating material buildup and depletion over the mine life. The stockpile inventory gradually builds up from Year 4 to Year 15. Strategically, lower-grade material is preferentially delivered to the stockpile during these times, while higher-grade material is processed directly from the mine. This stockpiling strategy guarantees a stable supply of material to the processing facility, allowing for a consistent output rate. The stockpile inventory decreases from Year 20 to the end of mine life.

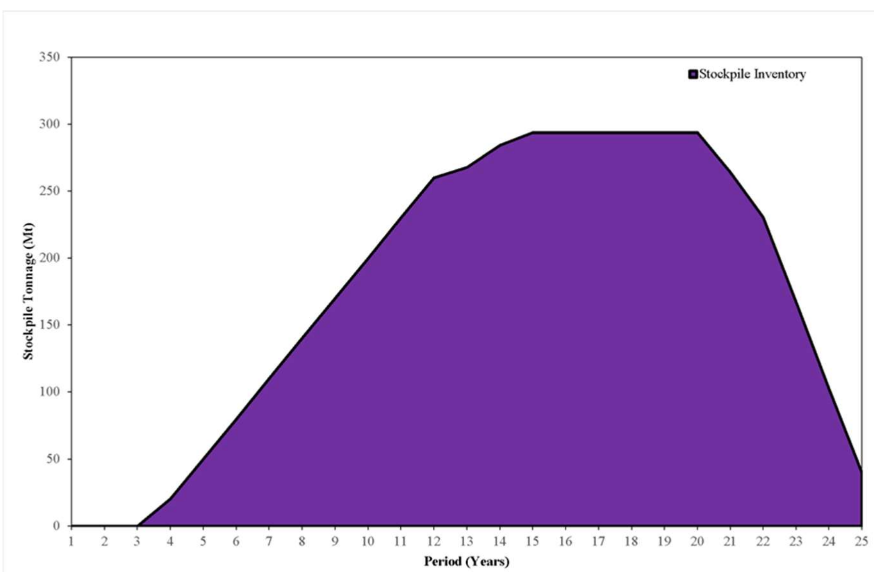

Figure 20: Stockpile inventory profile for Case 4.

Figure 21 depicts the stope production steps. On Levels 6, 7, and 8 during Period 24, there was no operational development work due to extraction and backfilling activities already taking place on these levels. In Period 21 on Level 8, a series of operational advancements and extraction occurred for Stopes 110 to 119. However, the optimizer did not initiate backfilling activity since these stopes constitute the last set of stopes on this level prior to the end of mine life. Subsequently, Stopes 110 to 119 on Level 7 were backfilled from Periods 22 to 25.

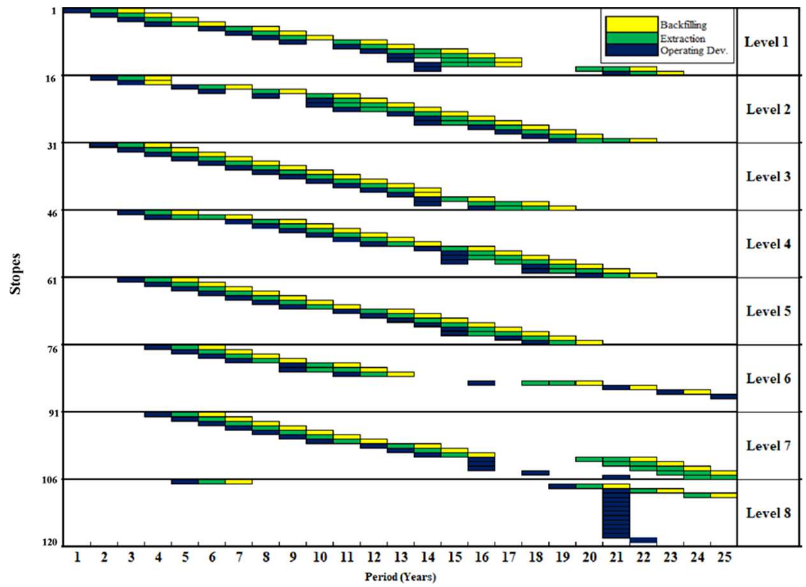

Figure 21: Annual operational development, stope extraction and backfilling schedules for Case 4.

6.4. Case 5 – E-type without Stockpile Management

The E-type block model is the average block model of all generated realizations. This is the expected block model representing a conservative risk profile. Similarly, this experiment was designed to create a consistent processing feed over the course of the mine's existence. 2.29 Mt of material was extracted and total NPV realized was $7,036.10 M. Figure 22 indicate the yearly development schedules for primary, ventilation and ore pass and Figure 23 shows the operational development schedule.

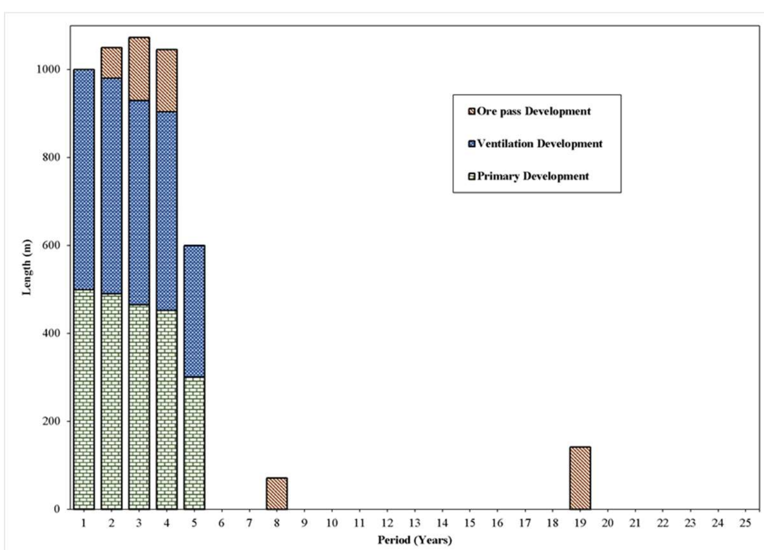

Figure 22: Primary, ventilation and ore pass development schedules for Case 5.

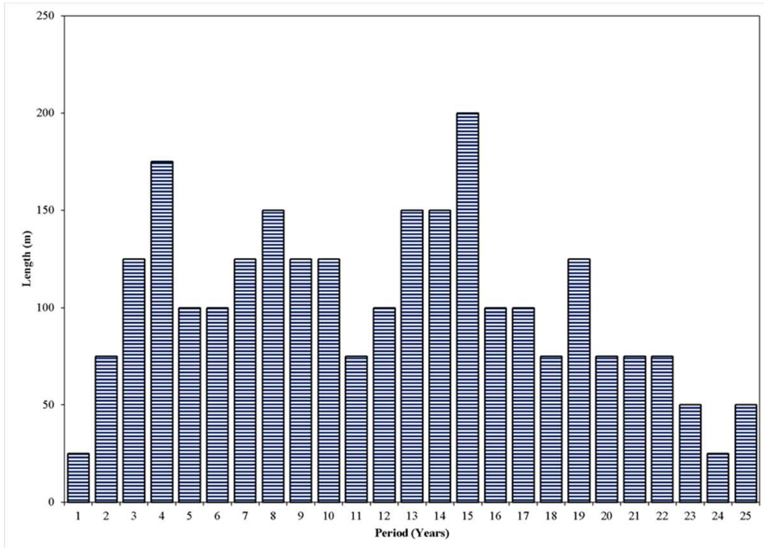

Figure 23: Operational development schedule for Case 5.

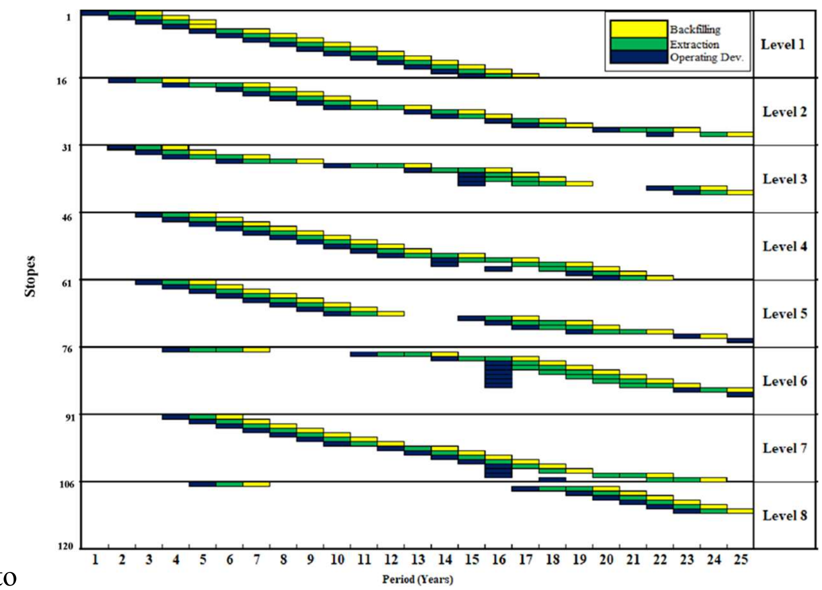

Figure 24 to

Figure 26 presents the mining schedule, processing schedule with grade curve, and operational phases schedules. Case 5 equally met all operating restrictions, and material quality and quantity requirements. It can be seen from Figure 24 that due to development activities, little access was gained to the ore during the first 3 years of the production plan. Gradually as more development were completed, stopes become accessible in subsequent years and the processing plant operate at maximum capacity from Period 4 until Period 16 (Figure 25) when the mine begins a ramping down phase due to limited available stopes. The annual operational development, stope extraction and backfilling schedules are presented in

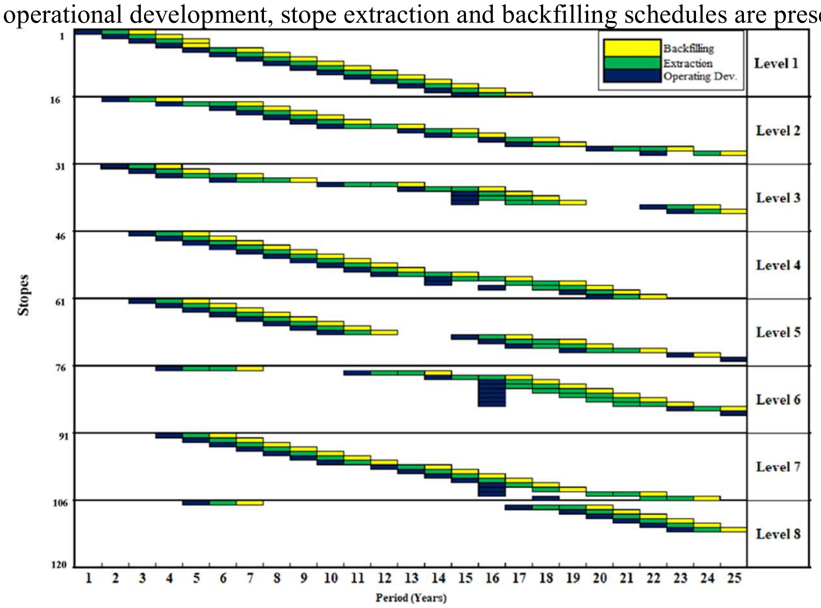

Figure 26.

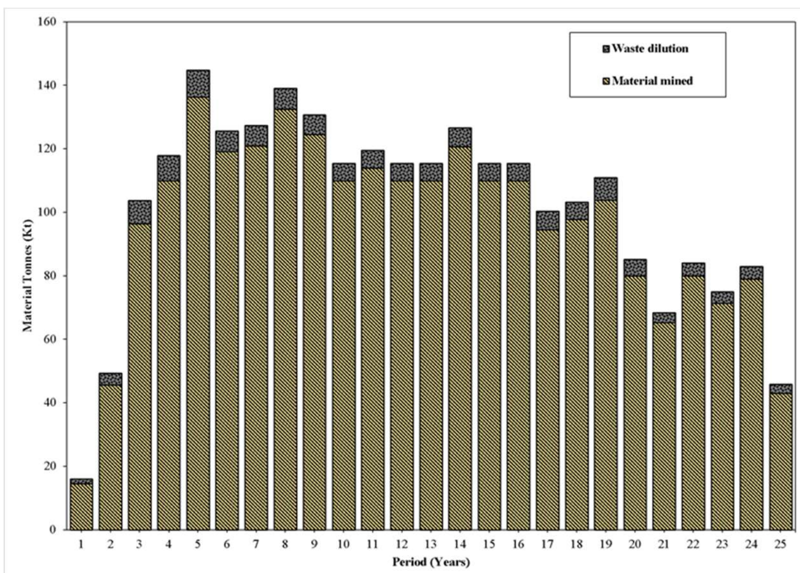

Figure 24. Material mined schedule with estimated waste dilution for Case 5.

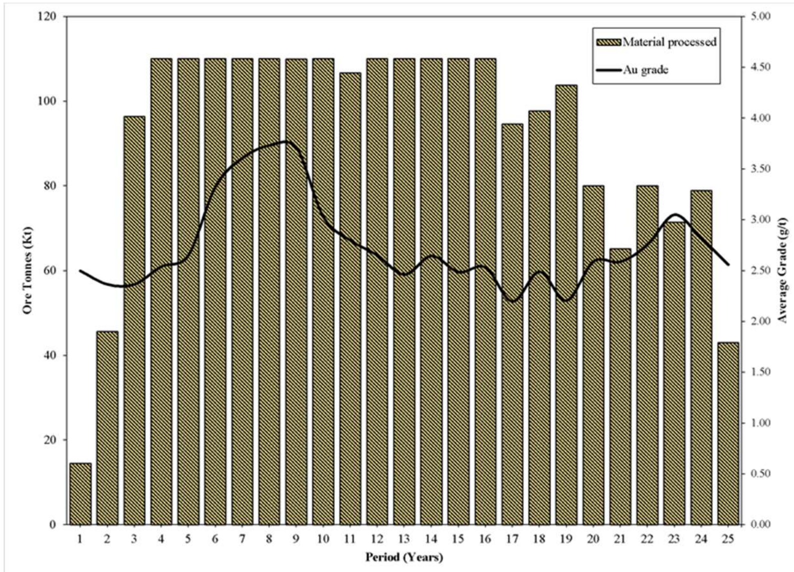

Figure 25. Material processed schedule with average grade curve for Case 5.

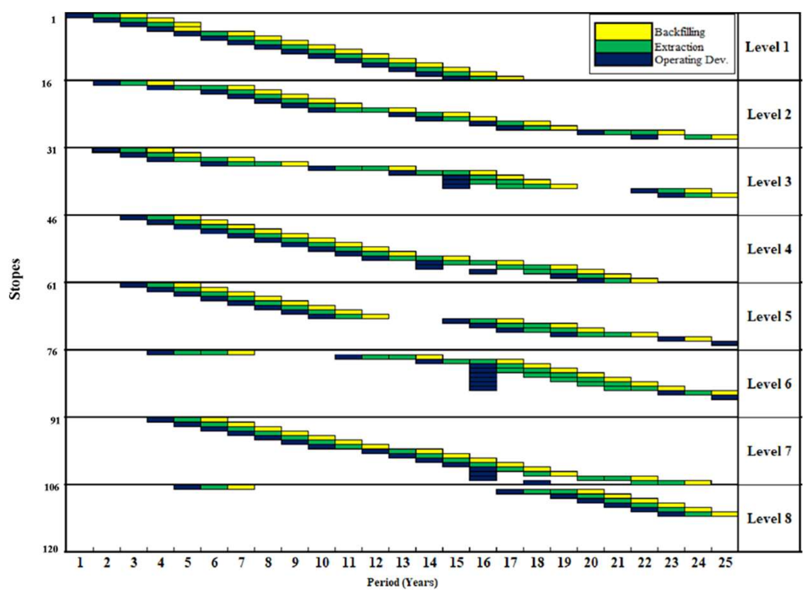

Figure 26. Annual operational development, stope extraction and backfilling schedules for Case 5.

6.5. Case 6 – E-type with Stockpile Management

The optimal primary, ventilation, operational, and ore pass development schedules, and mining, stockpiling, processing, and backfilling schedules allowed for efficient resource utilization generating a NPV of $7,360.52 M, approximately 8% lesser than Case 4 because of the conservative grades in the E-type block model. Figure 27 shows the yearly development schedules for primary, ventilation, and ore pass, whereas Figure 28 highlights the operational development schedule.

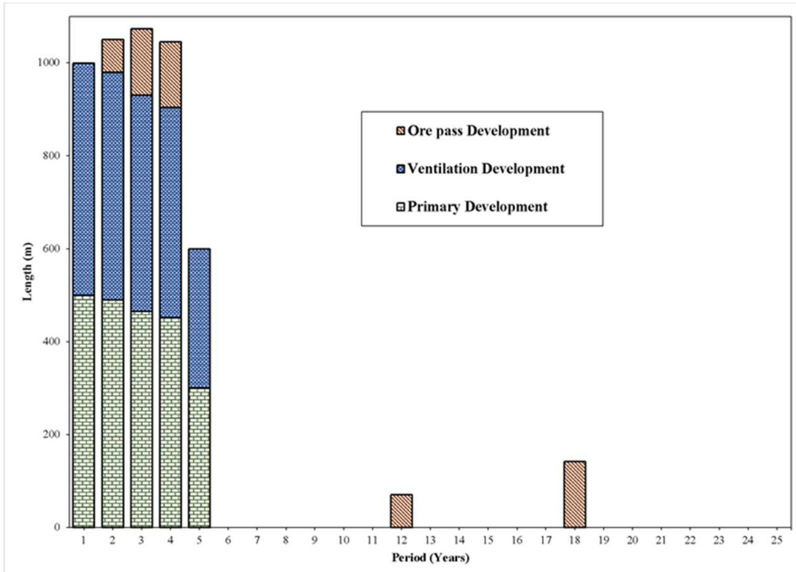

Figure 27. Primary, ventilation and ore pass development schedules for Case 6.

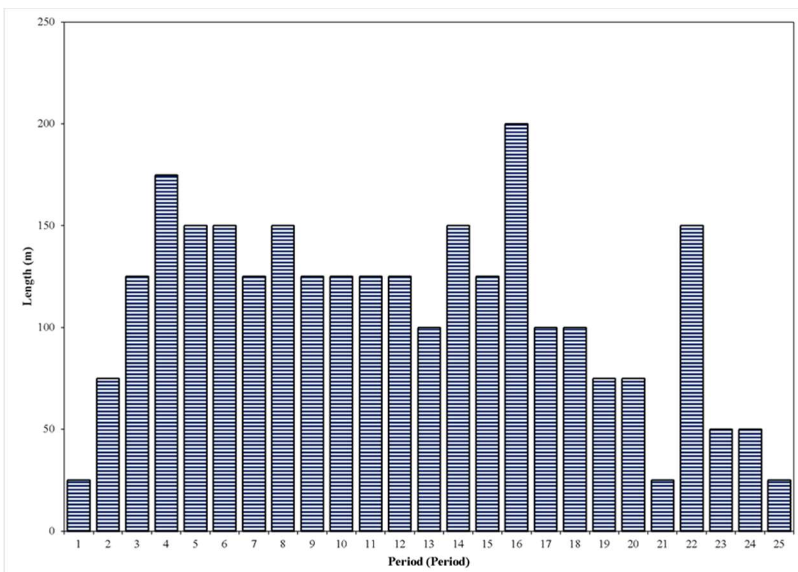

Figure 28. Operational development schedule for Case 6.

Figure 29 presents a graphical representation of the mining material schedule. This schedule outlines the tonnage of material to be mined over the life of the mining operation, including stockpiling. By controlling the capacity of the stockpile, it can be seen in Figure 30 that the mining operation was able to maintain control over ore accumulation and reclamation for processing. Thus, the management of stockpiling (Figure 31) was critical in maintaining control over ore accumulation and reclamation resulting in 2.4 Mt of material being processed from open stopes during the mine life.

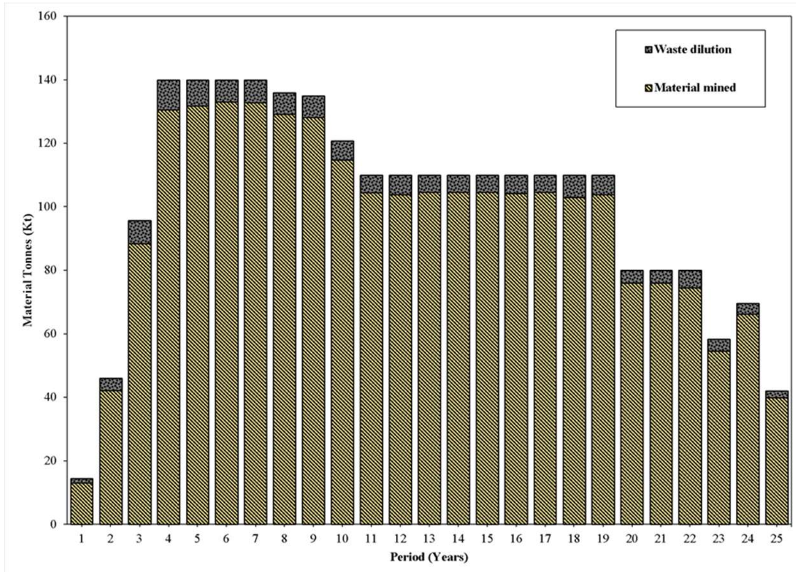

Figure 29: Material mined schedule with estimated waste dilution for Case 6.

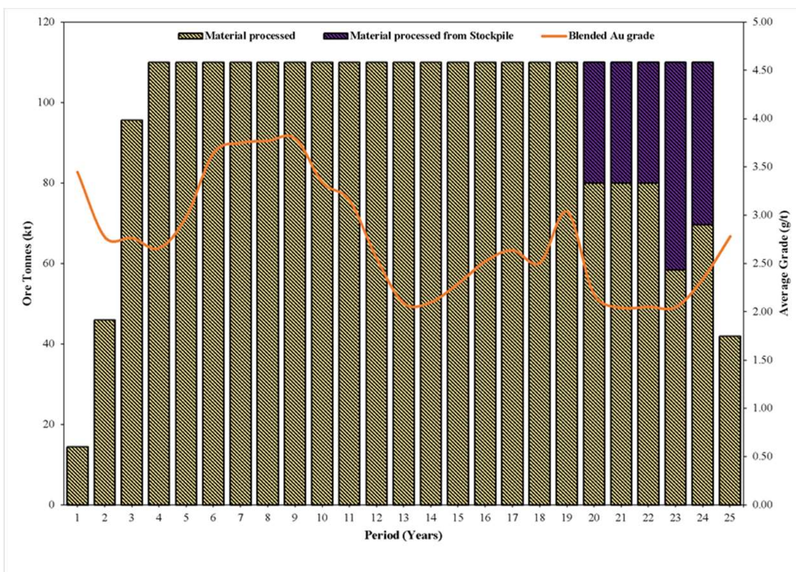

Figure 30: Processed material schedule with blended grade curve for Case 6.

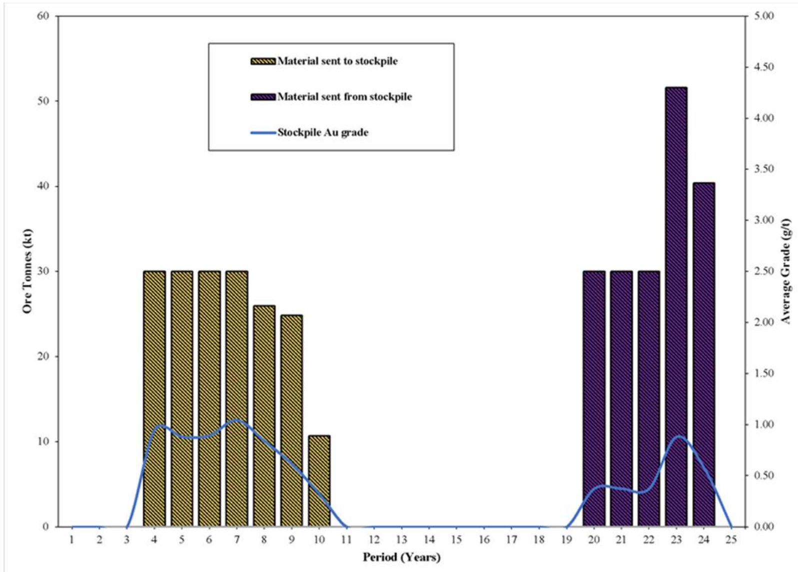

Figure 31: Stockpiled material schedule and grade curve for Case 6.

The stockpile inventory profile is presented in Figure 32. Illustrated in Figure 33 is the progression of stope production phases. During Periods 11 to 13 on Level 5, Periods 8 to 11 on Level 6, and Periods 9 to 12 on Level 8, there was inactivity on these levels due to higher grades in other areas including Levels 3, 4 and 7.

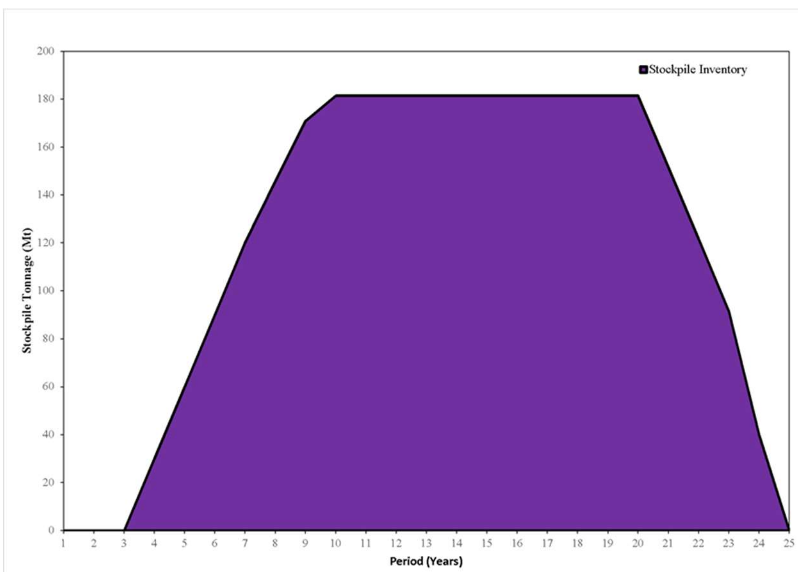

Figure 32: Stockpile inventory profile for Case 6.

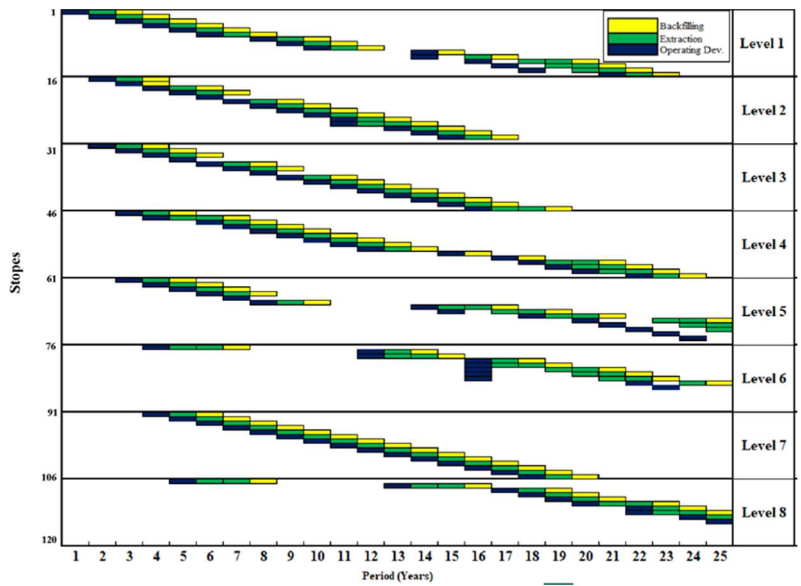

Figure 33: Annual operational development, stope extraction and backfilling schedules for Case 6.

6.6. Summary of results and findings on case studies

The study used the same set of constraints and parameters to compare the results and test the robustness of the model under different scenarios. This criterion of analysis was adopted to ensure consistency while examining the impact of data and constraint variations. It enables direct comparison of results when the same input data and restrictions are utilized for different scenarios, making it easier to identify the consequences of specific changes or improvements. Presented in Table 7 is the summary of results produced from the implementation of the MILP and SMIL formulations. In Table 7, Case 4 which incorporates the SMILP with a stockpile management strategy stands out as the superior performer among the five other case studies. It processed more ore tonnage at a higher grade and achieved the best NPV. Therefore, Case 4, which incorporates stockpiling in a SMILP framework to generate a risk-based production schedule provides the best NPV in the presence of grade uncertainty.

Table 7: Summary of case study results.

Case	Ore tonnage (Mt)	Average grade (g/t)	Au metal (t)	NPV (M$)
Case 1	2.19	2.98	6.53	7344.00
Case 2	2.52	2.87	7.23	7801.20
Case 3	2.33	3.05	7.11	7601.30
Case 4	2.54	3.02	7.67	8077.86
Case 5	2.29	2.77	6.34	7036.10
Case 6	2.51	2.93	7.35	7360.52

7. Conclusions

A comprehensive literature review revealed limitations that exist in the current body of knowledge in underground mining production scheduling optimization. Despite numerous works in underground production scheduling optimization, the literature review exposed that there has never been any previous attempt to formulate a stochastic underground mining mathematical programming framework with integrated stockpile management and grade uncertainty considerations. In effect, such limitations affect the practicality and optimality of the generated underground mine plan. This research therefore pioneers the effort to employ a stochastic mathematical programming framework to contribute to the body of knowledge and provide a novel understanding in uncertainty-based integrated mine planning optimization for underground mining operations.

An integrated SMILP theoretical framework has been developed. The research objectives have been successfully achieved within the research scope. The following conclusions were drawn from the research:

1. The multi-objective SMILP optimization framework with stockpile management maximizes the NPV of the mining project while generating underground development, extraction and processing schedules in the presence of grade uncertainty.
2. The SMILP model minimizes the geological risk cost by placing higher penalties for ore grade and ore tonnage deviations from production targets in the early years of mine life and deferring production deviations to later years when more geological information becomes available to update the block model and mine plan.
3. The SMILP optimization framework with SGS block model realizations performed better in terms of NPV and total ore tonnage processed compared to a similar MILP optimization framework implementation with Kriged and E-type block models.
4. Grade uncertainty influences the ore tonnage and input ore grade sent to both the stockpile and plant at different periods. The implementation of the SMILP model contributes significantly to decrease the effect of grade uncertainty during mining. The uncertainty-based integrated production schedule with stockpile management generates better profitability with a higher chance of success during implementation.
5. The proposed SMILP framework was verified in terms of both feasibility and risk assessment using six case studies while providing a systematic workflow towards promoting robust risk-based strategic underground mine planning with stockpile management.

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