A Stochastic Mixed Integer Linear Programming Framework for Open Stope Mine Production Scheduling Optimization Considering Grade Uncertainty and Stockpiling Strategy

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ABSTRACT

The use of traditional geostatistical estimation methods such as ordinary kriging for mine planning have been identified to be biased and introduce smoothing effect which results in unrealistic mine production schedules. The introduction and development of simulation methods where equiprobable orebody realizations are used as input in the mine planning process greatly helps in measuring and managing the associated risk due to grade uncertainty. Often, the outcome of the uncertainty related to orebody modelling is crucial and affects the resultant performance of stope designs and stope production. This is directly linked to the failure to meet set production targets and expected project financial outcomes during the mine-life. Uncertainty-based solutions in underground mine production scheduling optimization helps to minimize the variations between planned and actual production key performance indicators. To incorporate grade uncertainty into the strategic mine plan, a stochastic mixed integer linear programming (SMILP) framework was developed to optimize open stope underground mining production schedules. The objective of the model is to maximize the Net Present Value (NPV) of the mining project while minimizing the associated risk of deviation from the set production targets based on simulated orebody realizations.

1. Introduction

There are two approaches of conducting mine planning, namely, deterministic approach, and stochastic approach. Deterministic mine planning uses historical data collected from previous mine production activities and assumes that this trend will continue in future processes. Conversely, stochastic mine planning can be described as a premeditated course of action based on data collected from historical mine production figures incorporating uncertainty into the modelling processes. Deterministic approaches are known to ignore variations; thus, stochastic mine planning is promoted because it incorporates variations considering that mine production activities are erratic in nature and not static. Stochastic mine planning is a complex scheduling process due to its ability to incorporate uncertainties such as geological, technical, and economic, inherent in mining operations [27; 37]. Researchers like Ramazan and Dimitrakopoulos [42]; Grieco and Dimitrakopoulos [14]; Leite and Dimitrakopoulos [24]; Carpentier et al. [4]; Malaki [28]; MacNeil and Dimitrakopoulos [25]; Huang et al. [15] have investigated, evaluated, and analyzed risks associated with geological and grade uncertainty to considerably help the decision makers in better understanding various cases and conditions associated with the optimization process. The authors concluded that uncertain attributes other than grade and geology should be added to the optimization problem. Hence, subsequent models should be extended to take stochastic variables into account during optimization.

ISBN: 978-1-55195-503-2

The application of stochastic mine planning in production scheduling has been reported by various researchers in open pit mining taking into consideration grade uncertainty and associated risk [6; 9; 23], orebody uncertainty, in situ grade and geological uncertainty [9; 8; 10; 12; 47], geological and market uncertainty [46] and NPV [43] to mention a few. Conceptual developments in open pit mining have gradually led to the application of stochastic mine planning in underground mine planning notwithstanding the intricate nature of underground mine planning. The uncertainty related to orebody is a critical aspect affecting the forecasted performance of designs and is linked to the failure of meeting production targets and project financial expectations in mine planning [14; 38; 37].

Grieco and Dimatrakopoulos [14] developed and explored a new probabilistic Mixed Integer Programming (MIP) model to optimize stope designs, including size, location, and number of stopes under consideration of grade uncertainty and predefined levels of acceptable risk. The model was applied on data from Kidd Creek Mine, Ontario, Canada to demonstrate its practicality. The application exhibited aspects including risk quantification for contained ore tons, grade, and economic potential. The authors reported that unlike any conventional stope optimization approach, the stope designs generated based on the concept of acceptable risk gives the mine planner control over the final stope layout and its potential future performance while considering grade uncertainty. The application of the proposed approach is based on the ability to stochastically simulate equally probable representations of the deposit.

Martinez [30] gives a classic presentation on why accounting for uncertainty and risk leads to an improvement in decision making so far as mine evaluation is concerned. He introduced a new mine evaluation framework; the Integrated Valuation Optimization Framework (IVOF). He presented this as an alternative tool for mine project evaluation where uncertainty and risk are incorporated in the evaluation process. His paper highlights two main objectives; i) what problems can arise when single estimated values are substituted for a distribution of values when evaluating a mine project in the face of uncertainty and ii) how the ability to deal with uncertainty and risk in mine project evaluation can have a significant impact on the owners' and stakeholders' investment decision-making. The author asserts that the complexity of mine projects makes it a business that requires constant assessment of risk because the value of a mine project is typically influenced by many underlying economic and physical uncertainties, such as metal prices, metal grades, costs, schedules, quantities, and environmental issues, among others, which are not known with absolute certainty. He identifies the main sources of uncertainty that arise at the commencement of a mine project evaluation to be uncertainty in orebody modelling, uncertainty in metal prices and costs, and uncertainty and risk in mine planning and design.

Dimitrakopoulos and Grieco [7] adopted risk-based concepts developed in open pit mining to the underground stoping environment and shows examples employing data from Kidd Creek Mine. This example illustrates how conventional technologies cannot quantify risk since they are unable to foresee a significant upside potential and/or downside risk for the conventionally produced designs. The work quantified risk in terms of the uncertainty a conventional stope design has in expected contained ore tones, grade, and economic potential. They further outlined a new probabilistic mathematical formulation that optimizes the size, location, and number of stopes in the presence of grade uncertainty with an additional constraint introduced as the minimum acceptable risk allowed in a design. The model was applied to demonstrate the advantages of a user-defined level of acceptable risk. It is worth mentioning, that the authors recommended that further developments of the work could be to include: i) the formulation of a stope optimization formulation that replaces the probability of grades above cutoff with the direct use of all available simulated orebodies, integrating more geological information; ii) consider sequencing and thus accommodating risk management and/or geological risk discounting as part of the stope design process; and iii) extend to integrate geotechnical uncertainties starting from over-breaking and under-breaking.

Dimitrakopoulos [5] proposed a risk-based Stochastic Integer Programming (SIP) optimization model which incorporates uncertainties from both the geological and economic factors while minimizing cost. The model integrates two elements: stochastic simulation and stochastic optimization. These elements provide an extended mathematical framework that allows modelling and direct integration of orebody uncertainty to mine design, production planning, and evaluation of mining projects and operations. This stochastic framework increases the value of production schedules by 25%. Case studies also show that stochastic optimal designs i) can be about 15% larger in terms of total tonnage when compared to the conventional design, while ii) adding about 10% of NPV comparing to the traditional scheduling using a determined averaging orebody model design. Results suggest a potential new contribution to the sustainable utilization of natural resources. However, Magagula [27] reports that the author indicated the difficulties surrounding creating such planning process.

To integrate ore/metal uncertainty into the optimization of mine production scheduling, a SIP formulation was proposed and tested at a copper deposit by Leite and Dimitrakopoulos [24]. The stochastic solution maximizes the economic value of a project and minimizes deviations from production targets in the presence of ore/metal uncertainty. Unlike the conventional approach, the SIP model accounts and manages risk in ore supply, leading to a mine production schedule with a 29% higher NPV than the schedule obtained from the conventional, industry-standard optimization approach, thus contributing to improving the management and sustainable utilization of mineral resources.

Macneil [26] used stochastic mine planning methods to identify the optimal open pit to underground mining transition depth by identifying a series of candidate scenarios where it is feasible to make an OP-UG transition. The author evaluated the economic viability of each member of the set of candidate transition depths by producing uncertainty-based life-of-mine production plans that were used to outline expected yearly cash flows. The benefits of using stochastic mine planning to provide well-informed long-term strategic decision-making criteria are observed. Results of the application of the stochastic approach produced operational schedules with an increase NPV compared to the corresponding deterministic framework. An application of the proposed method at Geita gold mine in Eastern Africa indicate that future ore production is forecasted to fall well below the mill's capacity, and to supplement this deficiency a transition from open pit to underground mining was considered. Interestingly, results of the analysis from the proposed stochastic framework reflect that the most profitable decision favored continuing production through solely open pit mining for the foreseeable future. Valuable insights towards the risk associated with the proposed mine design are gained through stochastic risk analysis.

As the need to incorporate multiple components of the mining value chain increased several methods were developed over the past decades. Efforts have been made by these new methods to incorporate more decisions and flexibility to the mining optimization of a mining complex. However, they either ignore uncertainties associated with the mining project or consider decisions taken before optimization [34]. A method that optimizes mining complexes comprised of multiple open-pits, underground operations and processing destinations was presented by Montiel et al., [34]. The proposed method simultaneously optimizes mining, blending, processing, and transportation decision variables while accounting for geological uncertainty. The method employs a simulated annealing model at different decision levels to generate a stochastic-based extraction sequence and processing policies. An application based on a case study shows the method's ability to generate a higher NPV while facing a reduced amount of risk when compared to traditional optimization methods.

Malaki [28] employs the application of grade uncertainty in block cave mining. The author presents a methodology to find the best extraction level and the optimum sequence of extraction for that level under grade uncertainty. The work uses stochastic sequential simulation to address this problem by modelling a set of simulated realizations of the average mineral grade. The Mixed Integer Linear Programming (MILP) model was formulated to obtain the maximum NPV given some constraints such as mining capacity, grade, extraction rate and precedence. Finally, risks associated with grade uncertainty are investigated and analyzed, considerably helping the decision makers in better understanding of various cases and conditions. The author concludes among other things that more uncertain attributes other than grade should be added to the optimization problem. Hence, the MILP model should be extended to take stochastic variables into account during optimization.

A new SIP model incorporating geological uncertainty to optimize long-term scheduling of an underground project extension was introduced by Carpentier et al., [4]. To represent a deposit integrating the uncertainty, they stochastically generated a set of simulations and placed them in the optimization model. The results show that the schedule generated has a higher expected value when considering and managing grade risk. They also demonstrated the benefits of risk control, allowed by the approach.

MacNeil and Dimitrakopoulos [25] provided an approach to determine an optimal depth at which a mine should transition from open pit to underground mining, based on managing technical risk. The proposed approach is tested on a gold deposit. This work aims to improve on previous attempts to solve this problem by jointly considering geological uncertainty and effectively describing the optimal transition depth in 3-D. Results show the benefits of managing geological uncertainty in long-term strategic decision-making frameworks. The stochastic result produces a 9% increase in NPV over a similar deterministic formulation. The risk-managing stochastic framework also produces operational schedules that reduce a mining project's susceptibility to geological risk. The authors direct future studies to aim at improving the method by considering more aspects of financial uncertainty such as inflation and mining costs.

Huang et al. [15] presented a Stochastic Mixed integer Programming (SMIP) framework to evaluate the effect of grade uncertainty for a gold deposit and to integrate consideration of grade uncertainty in the optimization of the long-term production schedule for underground cut-and-fill mining. The authors reported that conventional mine planning approaches use an estimated orebody model as input to generate optimal production schedules. The smoothing effect of some geostatistical estimation methods causes most of the mine plans and production forecasts to be unrealistic and incomplete. With the development of simulation methods, the risks from grade uncertainty in ore reserves can be measured and managed through a set of equally probable orebody realizations. Thus, a set of equally probable orebody realizations were used as input in the strategic mine plan and to generate a more profitable and risk-based optimized production schedule. The SMIP model showed the ability to minimize risk and improve financial profitability. The model was able to manage geological risk more effectively compared with the conventional method by directly incorporating grade uncertainty into the integrated model formalism. Results from the implemented case study demonstrated a 2-9% improvement in the expected NPV by the SMIP model, and the ability to control the geological risk when compared to the conventional approach.

1.1. Stockpiling in Underground Optimization

The adoption of stockpiling in mining operations is an approach used in situations where ore materials mined exceeds the plant requirement. In this regard, the best grades are allowed to be processed directly while lower grades are stockpiled for a future date. It is possible to use one or more grade stockpiles, were there could be a low grade and a medium-low grade stockpile. In many cases, such stockpiles may not get processed for years, possibly until: a) the mine is depleted b) the mined grades are lower than those in the stockpile. Such stockpiles can grow to enormous size if accumulated over many years. In some instances, oxidation and processability may be a concern for reactive materials with long term stockpiles [21; 18]. Bley et al. [3] modelled the Long-term open-pit production planning (LTOPP) incorporating stockpiling. The authors added non-linear constraints and proposed a problem-specific solution method. The model presented a formulation which helps to track material flow from aggregate to stockpile and plant.

Gholamnejad and Kasmaee [11] presented a goal programming model for an Iron ore processing plant blending requirement. The model illustrated blending material from two high grade and a low-grade stockpile. The proposed model provided an optimal reclamation schedule by dividing the stockpiles into blocks and subsequently assigning grade values to each block. However, their proposed model did not include decisions on material flow from the mine and stockpile but solely focused on reclamation decisions. Ramazan and Dimitrakopoulos [44] proposed a production scheduling model with uncertain supply that includes stockpiling. Their model uses a predetermined constant grade for reclaiming material from the stockpile and allow blocks into the stockpile based on the probability of block grade being within the acceptable range for the stockpile. However, the authors do not compare the actual grade of material in the stockpile to the predefined grade.

Smith and Wicks [48] presented a MIP for medium-term production planning with stockpiling in a copper mine. The authors divided ore into different categories based on low, high grade and recovery of the main two elements and defined a stockpile for rehandling low-grade ore when needed. However, they avoid nonlinearity by not keeping track of elements grades going to and reclaimed from the stockpile. Koushavand et al. [20] introduced the cost of uncertainty in a long term mine production plan of a mixed integer linear programming model which considers stockpiles. The model finds the mining sequence of blocks from a predefined pit shell and their respective destinations, with two objectives: to maximize the net present value of the operation and to minimize the cost of uncertainty. However, the presence of a stockpile in the model allows optimization to extract extra ore at early stages of the mine-life which helps reduce the chance of short falls at later years.

Tabesh et al, [50] presented a multi-step approach to long-term open-pit production planning by using different resolutions for making mining and processing decisions. They further determined the pushbacks based on a hybrid binary programming-heuristic method and used the intersections of pushbacks and mining benches as mining units. Afterwards, they divided the bench-phases into smaller units with similar rock type and grade using an agglomerative hierarchical clustering algorithm. These units were then used as processing units. They presented a mathematical model to solve the LTOPP problem with the aggregated units and finally, introduced stockpiling to the model with non-linear and linear objective functions and constraints. The idea of piecewise linearization was used to modify the model to be able to solve it with mixed integer linear programming solvers.

Mousavi et al. [36] also considered stockpiling with a predetermined grade and used a non-exact approach to deal with the problem. They compared their results against solutions obtained via exact method and showed how close to the optimum solution their solutions were. However, they did not study the errors caused by assuming a fixed reclamation grade for the stockpile and their largest case study had 2,500 blocks which is a relatively small number. Tabash and Askari-Nasab [49] proposed a mathematical formulation that uses aggregated units for making mining, processing and stockpiling decisions while respecting various mining and processing constraints. They first propose a non-linear model that estimates stockpile grade and controls the head grade of material sent to the processing plants. Next, they use the idea of piecewise linearization to modify the model to be able to solve it with mixed integer linear programming solvers. Afterwards, they illustrated how the model compares against other linear stockpiling models in the literature and finally, the model was tested on a small dataset to evaluate the performance of the model and show the errors introduced by linearization.

Kumar and Chatterjee [22] proposed and applied a mathematical formulation for production scheduling with stockpiling in a coal mine. Their formulation follows the same approach and assumes a fixed predetermined reclamation grade for the stockpile and show that the observed element head grades are within the required boundaries. Moreno et al. [35] classified the production scheduling and stockpiling models in the literature and proposed a new modeling approach. Moreover, they provided extensive computational results for the models they studied and developed. We will discuss their proposed model in the next chapter in more details.

Majority of studies so far illustrate the benefits of using the stochastic approach to incorporate grade uncertainty in open-pit production scheduling optimization. Notwithstanding the complexities associated with the development and implementation of underground mining methods, some efforts have been made in incorporating grade uncertainty in underground mining production scheduling optimization [14; 25; 19; 15]. However, the application of stochastic optimization is relatively recent in underground mining and the methods used in the literature remains to be verified when applied to different types of deposits and underground mining methods. Generally, stockpiles can be considered as buffers of material for future use or as sources of high or low-grade ore for controlling blending requirements. Again, the presence of a stockpile allows optimization to extract extra ore at early stages of the mine-life and the extra ore reduce the chances of short falls at later years [20; 49; 39].

The use of stockpiling in the optimization process has been extensively used in open pit mining but less considered in underground mining [3; 44; 50; 36; 35; 49]. The proposed optimization approach is to consider grade uncertainty for underground open stope extraction with capital development, ventilation development, operational development, ore pass development, backfilling, and stockpiling management[1].

This research seeks to develop a risk-based optimization framework using stochastic mixed integer linear programming (SMILP) that effectively integrates grade uncertainty into the optimization of long-term production scheduling in open stope underground mining, by developing a stochastically simulated equally probable representations of a deposit for underground production scheduling optimization. The objective function of the proposed SMILP model is to maximize the net present value and balance the risk associated with grade uncertainty simultaneously, while meeting all technical and operational requirements.

2. Objectives of the Study

The objective of this study is to investigate techniques to:

a) quantify and access the impact of grade uncertainty on the output parameters of underground mine optimization such as: NPV, ore tonnage, head grade, annual targeted production; and
b) propose a stochastic mixed integer linear programming (SMILP) formulation for optimal production scheduling that aims at maximizing the net present value while minimizing the deviations from targeted production, caused by grade uncertainty.

3. Methodology

An assessment of the proposed approach and results will be illustrated through a case study corresponding to a gold deposit. Fig. 1 is a schematic representation of the proposed approach.

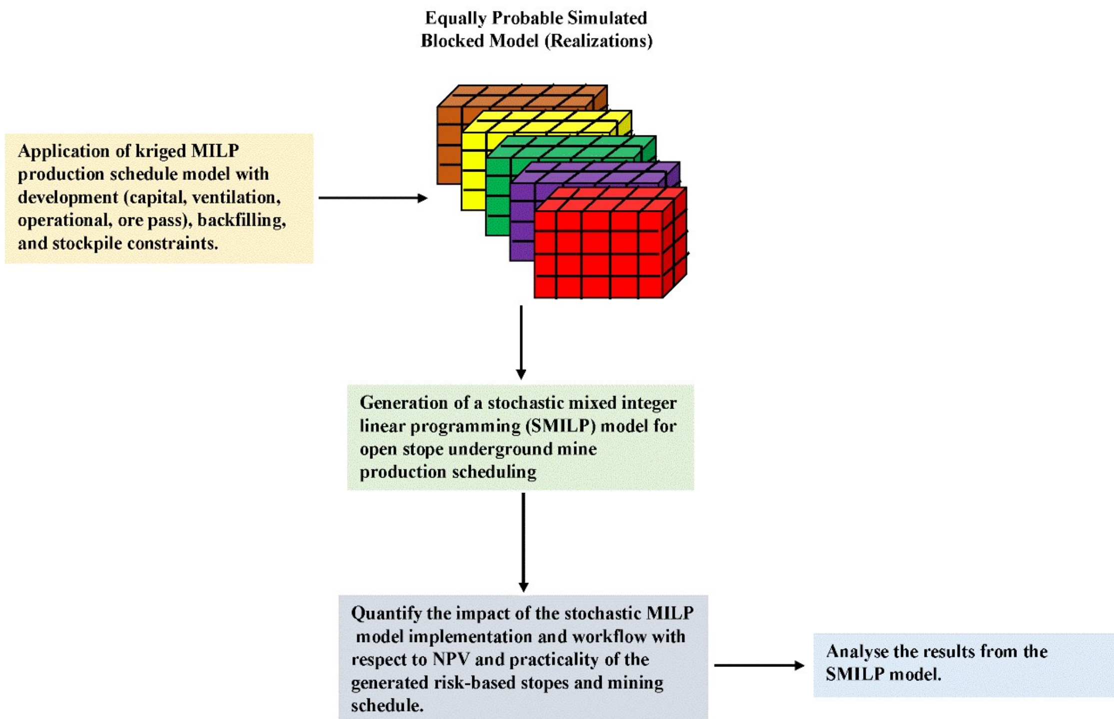

Fig. 1. Illustration of proposed approach.

Studies on the effect of grade uncertainty on NPV in long-term production plans have been reported for underground mining projects. It has been identified that there are differences between the actual and expected production targets especially in the early years of production [14; 4; 28]. To address the issue of grade uncertainty, instead of using a single estimated block model for production scheduling, 50 simulated realizations, that are representative of ore grade variability will be used as input to the SMILP model. A conventional model which was based on Ordinary Kriging (OK) estimation was considered as the base case model. Both models in addition to the E-type model were then compared to illustrate the impact of grade uncertainty on the integrated mine development and production scheduling problem.

The proposed workflow used in this study to generate production schedule under grade uncertainty from the SMILP framework are as follows:

a. Design stopes from an economic gold block model using Promine AutoCAD [41]. The gold grades in the block model are estimated using OK and serves as the base case model.
b. Implement geostatistical modelling using Sequential Gaussian Simulation (SGS) algorithm to map out gold ore grade uncertainty in the designed stopes. In this step, Stanford Geostatistical Modeling Software (SGeMS) [45].
c. Select all the stopes for all the realizations, E-type, and Ordinary Kriging. Save the selected stopes in ASCI file format.
d. For each case study, define the input scheduling parameters in MATLAB to formulate the problem.
e. Implement the developed mathematical programming formulations in MATLAB. TOMLAB/CPLEX [16] is used as the solver for the defined optimization problem
f. Perform production scheduling optimization and comparative analysis based on the generated results from the OK, E-type and SGS block models.

4. Statistical Analysis of Gold Data

To start geostatistical modelling, it is necessary to perform preliminary statistical analysis including compositing, recognizing outliers, identifying trends, and data transformation. This is necessary because data collection practices in general focuses on portions of the study area that are most important. For this reason, the element of interest is gold grades since its variability in estimation creates uncertainty which potentially impacts the overall net present value of the mining project [15]. Presented in Fig. 2 is a 3D map of the 120 stopes with gold grades.

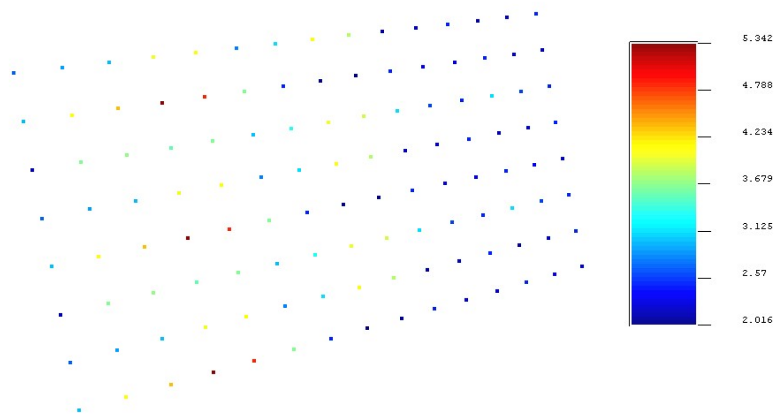

Fig. 2. Location of stopes with gold grade distribution.

4.1. Spatial Correlation Analysis using Variogram

The measurement of spatial continuity was employed to understand the correlation between the observations of the univariate sample at different locations. This analysis is useful to detect the presence of general trends in the data. Geostatistical techniques were used to analyze spatial variability and distribution of sample data to estimate parameters at unsampled locations in three main steps [17]:

1. Assumption of stationarity,
2. Spatial modelling of sample data, and
3. Estimation of variable value at unsampled location.

The analysis of spatial correlation can be undertaken using Stanford Geostatistical Modeling Software, (SGeMS) [45].

The original data set containing gold grades for 120 stopes were transformed to a gaussian space using standard normal score transformation applied in geostatistical analyses. Transformation of data to normal score distribution satisfies the assumption of stationarity of data. The transformed normal score data is also useful as input data in the stochastic gaussian simulation technique [32] as shown in Fig. 3 for the gold grades.

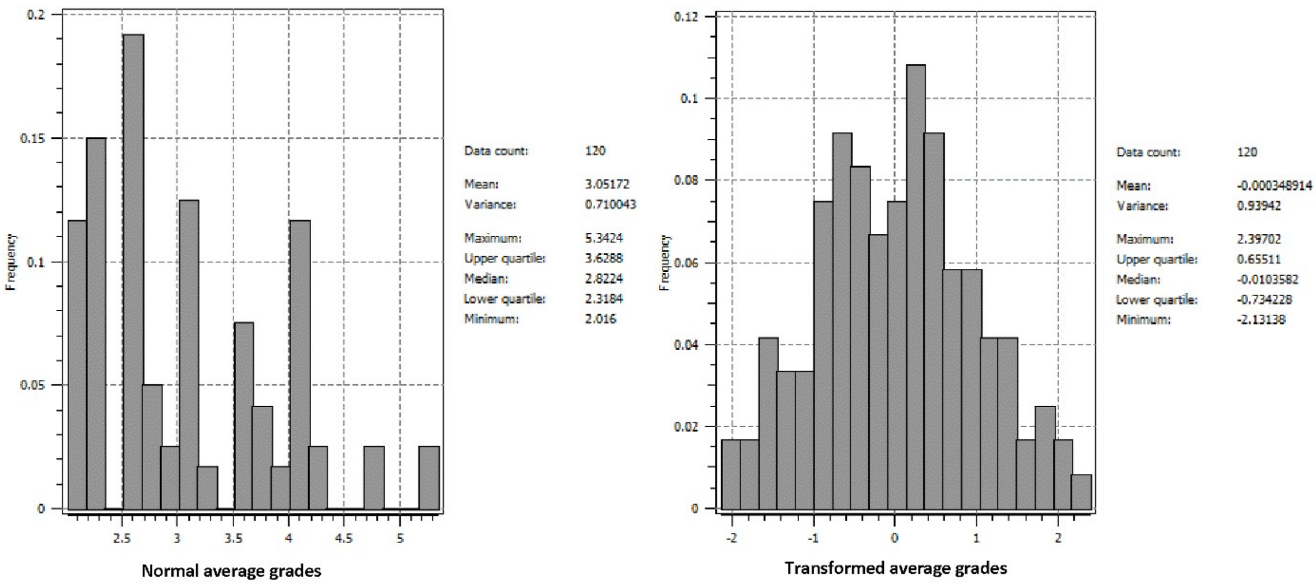

Fig. 3. Histogram and transformed normal score for gold grades.

Variogram analysis, which allows for examination of data correlation with respect to distance, was done for gold grades. Omnidirectional variogram for the grades was first computed to identify the sill while vertical variograms were used to identify the nugget effect. Primary variogram maps were calculated to determine the orientation of the major axis in the presence of anisotropy. Directional experimental variograms were calculated and theoretical variogram models were fitted to the experimental variograms. The parameters used to model the experimental variogram are presented in Table 1.

Table 1. Parameters used for variogram modelling.

Direction	Azimuth	Variogram model	Sill contribution	h_{min} (m)	h_{max} (m)	Nugget
Vertical	0.0	Gaussian	0.5	3.5	5.9	0.5
Minor	112.5	Gaussian	0.5	3.5	5.9	0.5
Major	67.5	Gaussian	0.5	3.5	5.9	0.5

A gaussian variogram model was fitted to the experimental variograms in the horizontal major direction and the horizontal minor direction. Fig. 4 shows the experimental and fitted variogram models in the major and minor directions, and in the vertical direction. The general equation for gaussian variogram model is as shown in Eq. (1).

$$\gamma(h) = C.\left(1 - \exp\left(\frac{-3h^2}{a^2}\right)\right) \tag{1}$$

Where C is the structure variance and a is the effective range.

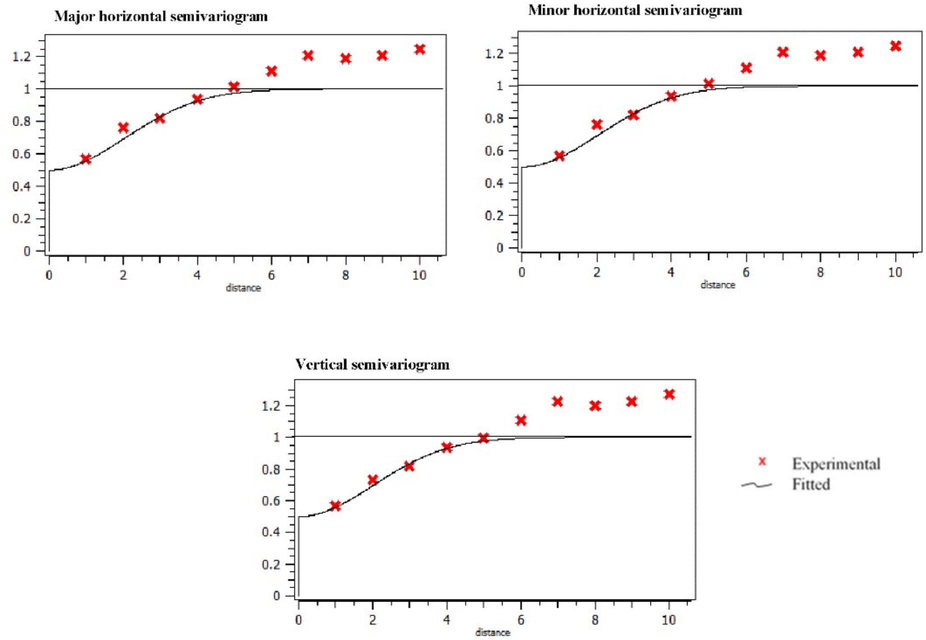

Fig. 4. Experimental directional variogram and fitted variogram models for gold grades (distance in meters).

5. Definitions and Assumptions

Open stope mining is an underground mining system applied in competent ores and mostly with no support provided for the wall or back of stopes after the ore is extracted. Stope dimensions may be the whole of the orebody (if small) or subdivided into blocks if the orebody is large. In theory, open stopes have no supports inside them. In practice, however, casual pillars of low-grade ore may be left or roof bolting (cable bolting) and timber cribs or posts may be used to support occasional patches of weak roof especially in orebodies of low dip angles. Open stope mining is comprised of horizontal development levels connected to the primary access (decline or shaft) at regular vertical intervals along the orebody (Fig. 5). Stopes may be designed as rectangular or square shapes according to dimensions with respect to the general outline of the deposit. A level access or drive representing a level is developed in the foot wall to serve as access for movement of broken materials from stopes on a level by haulage equipment directly out of the mine or into an ore pass. A crosscut which is a horizontal opening is driven perpendicular to the orebody and connects a stope to the drive.

A major drawback of open stope mining is that the span of the stope is limited by the rock strength. Thus, weaker rocks may limit stope width to about 20 m. According to Potvin and Hudyma [40], open stope mining is characterized by relatively small, single lift stopes ranging from 20,000 to 100,000 t per stope, and generally have fast stope turnaround times. For open stopes, host rock mass conditions are the controlling parameter in stope dimensioning. Dilution control and mining recovery are also a high priority.

Stoping sequence in open stope mining is predominantly driven firstly by the grade of ore (high grade), secondly by operational requirements such as access, ventilation, and backfilling, and lastly by rock mechanics considerations (stress management). Essentially, producing the best possible grade early in a mining project, along with consistent tonnage output throughout the project life, are the fundamentals of a successful underground mining operation. Options for extraction sequence in open stope mining include bottom-up and top-bottom, with bottom-up being the most common due to its advantages as a geotechnically effective means of stope stress management. When mining in high-stress conditions, extraction usually progresses from the bottom of a mine shaft or mine decline toward its top. As extraction progresses and stress concentrates, the extraction horizon moves towards the shallower levels of the mine to areas of lower pre-mining stresses. As a result, excessively induced stress and deteriorating ground conditions are better managed by the bottom-up approach.

For most open stope mines, backfilling may be done to achieve improved recovery of the mineral deposit. Backfilling helps to provide regional stability and local support in deep mining conditions. It also serves as a floor to work on in short lifts, bottom-up extraction sequences. Backfilling options mostly employed in open stope mining are rock fill, cemented rock fill, hydraulic fill and paste fill technology [40].

The scope of this formulation considers capital development (decline), ventilation development, operational development (drive and crosscut), backfilling, ore pass development and stockpile management. These developments are often completed at one time as a permanent facility for subsequent development and operation. Fig. 5 illustrates a schematic representation of open stope mining layout.

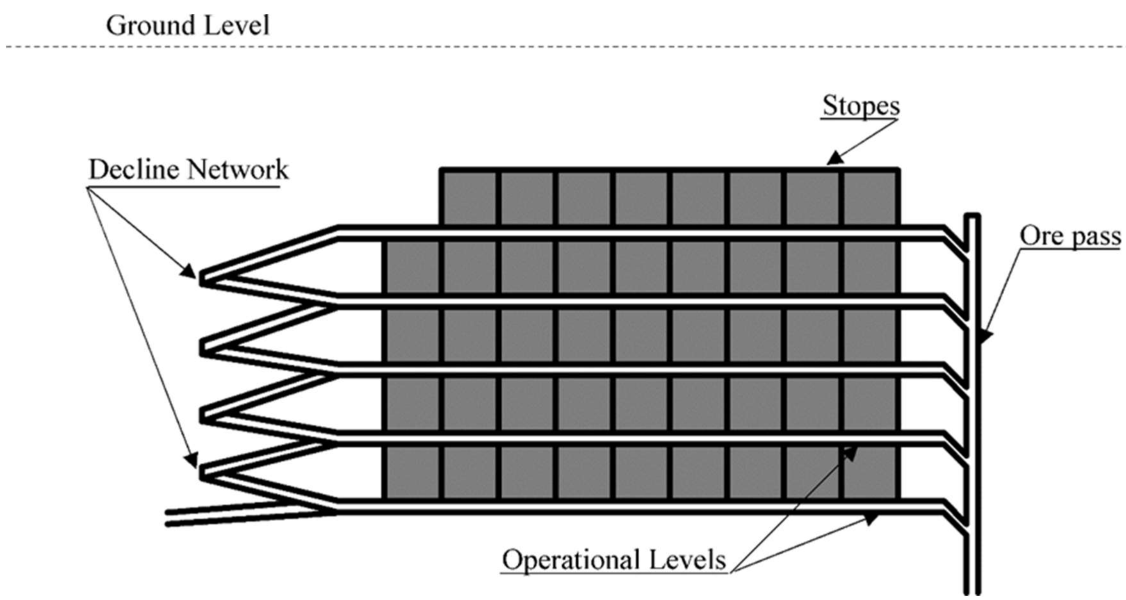

Fig. 5. Schematic representation of open stope mining layout.

5.1. Notation

We present a SMILP formulation for the open stope underground mining production scheduling problem. The notation for indices, decision variables, sets and parameters are as follows:

t index for schedule time periods: $t = 1, 2, 3, \ldots, T,$ where T is the schedule duration.

k index for stope identification: $k = 1, 2, 3, \ldots, K,$ where K is the total number stopes.

l index for level identification: $l = 1, 2, 3, \ldots, L$, where L is the total number of levels.

5.2. Decision variables

$x^{t}_{k,\mathrm{m}} \in [0,1]$ continuous variable representing the portion of stope k to be extracted as ore and processed in period t directly from the mine m through the ore pass. k is linked to the stope extraction variable b and only provides a non-zero value when b is non-zero.

$j_{k,si}^{t} \in [0,1]$ continuous variable representing the portion of stope k to be extracted as ore and sent to the stockpile si in period t through the ore pass. si is linked to the stope extraction variable b and only provides a non-zero value when b is non-zero.

$u_{k,so}^{t} \in [0,1]$ continuous variable representing the portion of stockpile so to be reclaimed and processed in period t from the stockpile. Stockpile so is linked to stope k and extraction variable b and only provides a non-zero value when b is non-zero.

$y_{k}^{t} \in [0,1]$ continuous variable representing the portion of the stope k to be mined in period t; fraction of y characterizes both ore and waste in the stope.

$f_{k}^{t} \in [0,1]$ continuous variable representing the portion of stope k to be backfilled in period t.

$d_{c}^{t} \in [0,1]$ continuous variable representing the portion of capital development length c to be completed in period t.

$d_{a}^{t} \in [0,1]$ continuous variable representing the portion of operational development length a to be completed in period t.

$d_{v}^{t} \in [0,1]$ continuous variable representing the portion of ventilation development v to be completed in period t.

$d_{p}^{t} \in [0,1]$ continuous variable representing the portion of ore pass development p to be completed in period t.

$x_{k,l}^{t} \in [0,1]$ continuous variable representing a portion of stope extraction k on a level l in period t.

g_{so}^{t} a continuous decision variable representing the predetermined grade of ore from the stockpile to processing plant in period t.

$b_{k}^{t} \in \{0,1\}$ binary integer variable; equal to one if the extraction of stope k is to be scheduled in time t, otherwise it is zero.

$b_{c,l}^{t} \in \{0,1\}$ binary integer variable controlling the precedence of capital developments. $b_{c,l}^{t}$ is equal to one if capital development cd has started by or in period t, otherwise it is zero.

$b_{a,l}^{t} \in \{0,1\}$ binary integer variable controlling the precedence of operational developments. $b_{a,l}^{t}$ is equal to one if operational development a on level l has started by or in period t, otherwise it is zero.

$b_{v,l}^{t} \in \{0,1\}$ binary integer variable controlling the precedence of ventilation development (installations). $b_{v,l}^{t}$ is equal to one if ventilation development (installation) v has started on level l by or in period t, otherwise it is zero.

$b_{p,l}^{t} \in \{0,1\}$ binary integer variable controlling the precedence of ore pass development. $b_{p,l}^{t}$ is equal to one if ore pass development p has started on a level l by or in period t, otherwise it is zero.

$lc_{l}^{t} \in \{0,1\}$ binary integer variable; when it is equal to one, it implies operational development activities are in progress on level l in period t.

$le_l^t \in \{0,1\}$ binary integer variable; when it is equal to one, it implies stope extraction activities are in progress on level *l* in period *t.*

5.3. Sets

$K = \{1,...,K\}$ set of all stopes in the model.

$K_l = \{1,...,K_l\}$ set of all stopes on a level in the model.

$C = \{1,...,C\}$ set of all capital development in the model.

$C_l = \{1,...,C_l\}$ set of all capital development on a level in the model.

$A = \{1,...,A\}$ set of all operational development in the model.

$A_l = \{1,...,A_l\}$ set of all operational development on a level in the model.

$V = \{1,...,V\}$ set of all ventilation development in the model.

$V_l = \{1,...,V_l\}$ set of all ventilation development on a level in the model.

$P = \{1,...,P\}$ set of all ore pass development in the model.

$P_l = \{1,...,P_l\}$ set of all ore pass development on a level in the model.

$A_k(J)$ for each stope *k*, there is a set $A_k(J) \subset K$ defining the adjacent stopes that cannot be mined simultaneously with the extraction or backfilling of stope *k*, where *J* is the total number of stopes in the set *A(J)*.

$G_k(S)$ for each stope *k*, there is a set $G_k(S) \subset K$, defining the immediate predecessor stopes that must be extracted prior to stope *k*, where *S* is the total number of stopes in the set $G_k(S)$.

$D_a(L)$ for each level, there is a set $D_a(L) \subset OD_l$, defining the number of operational developments on that level that must be done before a stope and an ore pass can be extracted; where L is the total number of operational developments on a level in the set $D_a(L)$.

$D_c(L)$ for each level, there is a set $D_c(L) \subset CD_l$, defining the number of capital developments that must be started before operational developments on that level can be started, where L is the total number of capital developments in set $D_c(L)$.

$D_v(L)$ for each level, there is a set $D_v(L) \subset VD_l$, defining the number of ventilation developments (installations) that must be started before operational developments on that level can be started; where L is the total number of ventilation development (installations) in set $D_v(L)$.

$L_l(M)$ for each level l , there is a set $L_l(M)$ defining all stopes on this level, where M is the total number of stopes in set $L_l(M)$.

$L_l(D)$ for each level l , there is a set $L_l(D)$ defining all operational developments on this level, where D is the total number of development activities in set $L_l(D)$.

5.4. Parameters

$R_{k,p}^{t}$ revenue generated by selling the final mineral commodity in stope k in period t from the ore pass.

$R_{k,so}^{t}$ revenue generated by selling the final mineral commodity in stope k in period t from the stockpile.

i discount rate.

o_k ore tonnage in stope k.

w_k waste tonnage in stope k.

g_k average grade of mineral in ore portion of stope k.

r^t processing recovery: the portion of mineral recovered in stope k in period t.

sp^t selling price in value terms obtainable per unit of mineral commodity.

sc^t selling cost in value terms per unit of mineral commodity.

mp_k^t extra cost in value terms per tonne of ore for mining and processing of stope k in period t.

rc^t mining recovery of stopes in period t.

di_k mining dilution of stopes k.

C_k^t total cost in value terms of capital development for stope k in period t.

c_k^t variable cost in value terms per length of capital development for stope k in period t.

H_k^t total cost in value terms of ventilation development for stope k in period t.

h_k^t variable cost in value terms per length of ventilation development for stope k in period t.

E_k^t total cost in value terms of operational development for stope k in period t.

e_k^t variable cost in value terms per length of operational development for stope k in period t.

Q_k^t total cost in value terms of mining from stope k in period t.

q_k^t variable cost in value terms per tonne of mining from stope k in period t.

F_k^t total cost in value terms of backfilling stope k in period t.

f_k^t variable cost in value terms per unit volume of backfilling stope k in period t.

Z_k^t total cost in value terms of ore pass development for stope k in period t.

z_k^t variable cost in value terms per length of ore pass development for stope k in period t.

cs_k^t	variable cost in value terms per tonne for stockpiling ore from stope k in period t.
$L_{c,lb}^t$	lower bound on capital development (decline) in period t.
$L_{c,ub}^t$	upper bound on capital development (decline) in period t.
d_{cl}	capital development length $_{cl}$.
$L_{v,lb}^t$	lower bound on ventilation development in period t.
$L_{v,ub}^t$	upper bound on ventilation development in period t.
d_{vl}	ventilation development length $_{vl}$.
$L_{a,lb}^t$	lower bound on operational development for period t.
$L_{a,ub}^t$	upper bound on operational development for period t.
d_a	operational development length $_a$.
$T_{m,lb}^t$	lower bound on available mining capacity in period t (tons).
$T_{m,ub}^t$	upper bound on available mining capacity in period t (tons).
$T_{pr,lb}^t$	lower bound on ore processing capacity in period t (tons).
$T_{pr,ub}^t$	upper bound on ore processing capacity in period t (tons).
$L_{p,lb}^t$	lower bound on ore pass development length in period t (tons).
$L_{p,ub}^t$	upper bound on ore pass development length in period t (tons).
$T_{sp,lb}^t$	lower bound on stockpile capacity in period t (tons).
$T_{sp,ub}^t$	upper bound on stockpile capacity in period t (tons).
$gr_{pr,lb}^t$	lower bound on acceptable average grade of mineral at processing plant in period t.
$gr_{pr,ub}^t$	upper bound on acceptable average grade of mineral at processing plant in period t.
$gr_{si,lb}^t$	lower bound on acceptable average grade of mineral at stockpile in period t.
$gr_{si,ub}^t$	upper bound on acceptable average grade of mineral at stockpile in period t.
$V_{f,lb}^t$	lower bound on backfilling for period t.
$V_{f,ub}^t$	upper bound on backfilling for period t.
d_{fv}	backfilling volume fv.
d_p	ore pass development length p.

N_{lc}^{t} the maximum number of active levels available for operational development in period t.

N_{le}^{t} the maximum number of active levels available for stope extraction in period t.

N_{xd} the maximum length of stope extraction duration for stopes.

$pntCT_{g,-}^{t}$ per-unit penalty cost in terms of the lower grade target deviation in period t.

$pntCT_{g,+}^{t}$ per-unit penalty cost in terms of the upper grade target deviation in period t.

$gdev_{s,-}^{t}$ continuous variable; the shortage from the grade lower bound in period t for realization s.

$gdev_{s,+}^{t}$ continuous variable; the excess from the grade upper bound in period t for realization s.

$pntCT_{o,-}^{t}$ per-unit penalty cost in terms of the lower ore tonnage target deviation in period t.

$pntCT_{o,+}^{t}$ per-unit penalty cost in terms of the upper ore tonnage target deviation in period t.

$odev_{s,-}^{t}$ continuous variable; the shortage to the ore tonnage lower bound in period t for realization s.

$odev_{s,+}^{t}$ continuous variable; the excess from the ore tonnage upper bound in period t for realization s.

6. Stochastic Mixed Integer Linear Programming (SMILP) Model formulation

The SMILP optimization framework for generating long-term production schedule in the presence of grade uncertainty was modeled using multiple realizations from Sequential Gaussian Simulation (SGS). The revenue is calculated for each stope in each realization. The revenue of a stope is generated by selling the final product less all the costs involved in developing, extracting, and processing the stope. The mining cost per stope is a function of the distance between its location and its destination (processing plant and stockpile). Since the long-term production plan is a multi-period optimization problem and stopes are extracted in different periods, a discount rate is applied to calculate the present value of the revenue and the costs. Grade uncertainty causes shortfalls from target production levels. The objective function of the SMILP model in Eq. (2) is to maximize the expected NPV and at the same time minimize the deviations from the production targets. The first part of the objective function comprises the total discounted cashflow from direct mining to the processing plant and from stockpiling to the processing of ore material for all realizations. The second part of the function minimizes the costs associated with deviating from the ore grade operating targets for all realizations. The third part minimizes the total costs associated with deviating from the ore tonnage operating targets. Thus, the second and third parts aim to control grade uncertainty by minimizing the variability from the ore grade and ore tonnage targets. The per-unit penalty costs are denoted by $pntCT_{g}^{t}$ and $pntCT_{o}^{t}$ in Eq. (2). Deviations are discounted in terms of Geological Risk Discounting (GRD) [23]. The application of GRD helps to control the grade and ore tonnage risk distribution over time and ensures the extraction of lower-risk and higher-grade stopes in the early periods and leaves the higher-risk stopes for later periods. It is essential for the operation to mine less risky parts of the deposits in the early stages with limited geological information. As more geological information become available in the form of operational exploration, new schedules will be produced for later periods. The per-unit value of penalty costs is defined based on the impact of one unit of deviation from the related production target on the overall mining strategy and they are defined based on trial and error with consideration of the magnitude of different production targets. Therefore, the per-unit value of the penalty can be altered to prioritize the most critical operational targets according to the optimization requirements in the mining complex. When the GRD is set to zero, a balanced-risk profile is produced. The concept presented in Appianing et al. [2] was used as the starting point of this work.

The objective function expressed in Eq. (2) is composed of the discounted revenues considering ore recovery rc^t and mining dilution di_t factors in Eqs. (3) and (4), discounted cost of stope extraction Eq. (5), discounted cost of capital development Eq. (6), discounted cost of ventilation development Eq. (7), discounted cost of operational development Eq. (8), discounted cost of ore pass development Eq. (9) and discounted cost of backfilling Eq. (10). There are four binary decision variables that indicate the time of capital development, b_c, ventilation development, b_v, operational development, b_a, and ore pass development, b_p to be scheduled. There are also seven continuous decision variables that indicate the portion of a stope mined y_k, the portion of the stope processed x_k, the portion of capital development completed d_c, ventilation development d_v, operational development d_a, ore pass development d_p, backfilling f_k, and stockpiling $u_{k.so}$ in each period. Therefore, to obtain an optimum schedule, NPV must be maximized and the deviation from target productions must be minimized simultaneously among all simulation realizations. Therefore, the revenue for the stochastic model is calculated using Eqs. (3) and (4). By this uncertainty integrated objective function, the SMILP model can reduce the risk of not meeting the planned production targets and provide a feasible schedule.

$$Max \frac{1}{S}\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{k=1}^{K}\left[\frac{\begin{matrix}(R_{k,p}^{t}\times x_{k,\mathrm{m}}^{t})+(R_{k,so}^{t}\times u_{k,so}^{t})-(Q_{k}^{t}\times y_{k}^{t})-(C_{k}^{t}\times d_{cd}^{t})\\ -(H_{k}^{t}\times d_{vd}^{t})-(E_{k}^{t}\times d_{od}^{t})-(Z_{k}^{t}\times d_{ops}^{t})-(F_{k}^{t}\times f_{k}^{t})\end{matrix}}{(1+i)^{t}}\right]$$
$$-\frac{1}{s}\sum_{s=1}^{S}\sum_{t=1}^{T}\left[\frac{(pntCT_{g,+}^{t}\times gdev_{s,+}^{t})+(pntCT_{g,-}^{t}\times gdev_{s,-}^{t})}{(1+g)^{t}}\right] \tag{2}$$
$$-\sum_{t=1}^{T}\left[\frac{(pntCT_{o,+}^{t}\times odev_{+}^{t})+(pntCT_{o,-}^{t}\times odev_{-}^{t})}{(1+g)^{t}}\right]$$

$$R_{k,so}^{t}=(g_{k,s}\times o_{k}\times r_{k}\times rc^{t}\times(sp^{t}-sc^{t}))-(o_{k}\times(1+di_{k})\times mp_{k}^{t}) \tag{3}$$

$$R_{k,p}^{t}=(g_{k,s}\times o_{k}\times r_{k}\times rc^{t}\times(sp^{t}-sc^{t}))-(o_{k}\times(1+di_{k})\times(mp_{k}^{t}+cstp_{k}^{t})) \tag{4}$$

$$Q_{k}^{t}=q_{k}^{t}\times(o_{k}+w_{k}) \tag{5}$$

$$C_{k}^{t}=c_{k}^{t}\times d_{c} \tag{6}$$

$$H_{k}^{t}=h_{k}^{t}\times d_{v} \tag{7}$$

$$E_{k}^{t}=e_{k}^{t}\times d_{a} \tag{8}$$

$$Z_{k}^{t}=z_{k}^{t}\times d_{p} \tag{9}$$

$$F_k^t = f_k^t \times d_{fv} \tag{10}$$

6.1. Constraints

Constraints considered for the formulation follow the underground mining sequence of operation. Thus, for a typical underground mining system, capital development precedes ventilation development before operational development. Extraction then precedes operational development and finally backfilling. Illustrated in Fig. 6 are the constraint dependencies for the open stope mining production schedule formalism. In summary, the primary contribution of this research is the integration of mine development, grade uncertainty, stockpiling and production schedules in a stochastic optimization framework that generates a practical mine-plan.

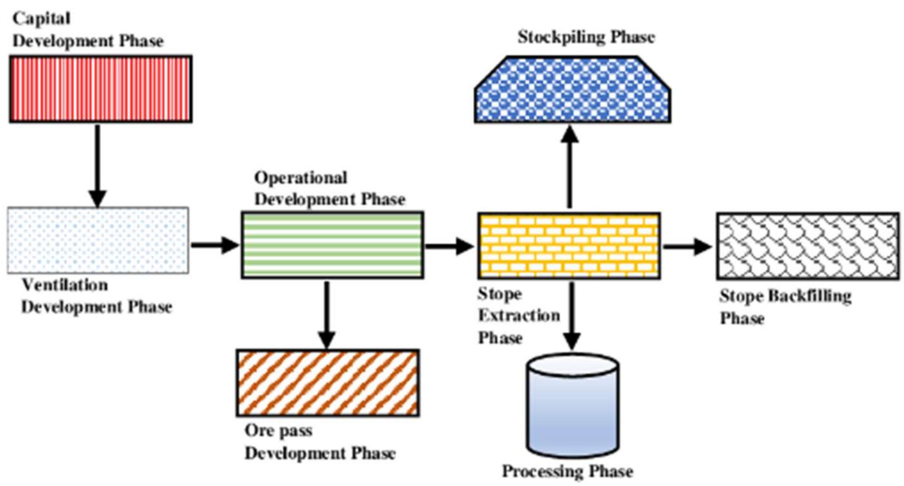

Fig. 6. Constraints dependencies.

6.1.1. Capital Development Constraints

Eq. (11) defines the capital development capacity constraint for the mine. The inequality ensures that the total length of capital development required in a period is within the stated lower and upper limits of the total available equipment capacity for developing the mine. Eqs. (12) to (15) control the precedence relationships between the sections of capital development leading to each mining level and the operational development on each level.

Eqs. (12) to (14) ensure that the set of capital development representing a section above a level must be completed before the capital development $d_{c,l}^t$ of the level commences. Eq. (15) ensures that the ventilation development $b_{v,l}^t$ on a level linking the capital development can only commence after completion of a set of required capital developments $\sum_{t=1}^{t} d_{c,l}^t$ above and on that level.

$$L_{c,lb}^t \le \sum_{c=1}^{C} \left(d_{cl} \times d_c^t \right) \le L_{cd,ub}^t , \ \forall t \in \{1,...,T\}; \tag{11}$$

$$b_{c,l}^t - \sum_{t=1}^{t} d_{s,l}^t \le 0 , \ \forall l \in (1,...,L), s \in D_c(L); \tag{12}$$

$$\sum_{t=1}^{t} d_{c,l}^t - b_{c,l}^t \le 0, \quad \forall l \in \{1,...,L\}, c \in \{1,...,C\}; \tag{13}$$

$$b_{c,l}^{t} - b_{c,l}^{t+1} \leq 0, \quad \forall l \in \{1,...,L\}, t \in \{1,...,T-1\}, c \in \{1,...,C\}; \tag{14}$$

$$b_{v,l}^{t} - \sum_{t=1}^{t} d_{c,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, c \in D_c, (L), v \in D_v(L); \tag{15}$$

6.1.2. Ventilation Development Constraints

Ventilation is required to provide airflow into development and workings for the comfort of workers and functioning of equipment. Eqs. (16) to (20) present the set of constraints defining the ventilation requirements to enhance stope extraction. Eq. (16) defines the ventilation development capacity constraint for the mine. The inequality ensures that the total length of ventilation development required in a period is within the stated lower and upper limits of the total available equipment capacity for ventilation development. Eqs. (17) to (20) control the precedence relationships between the sections of ventilation development leading to each mining level and the operational development on each level.

Eqs. (17) to (19) ensure that the set of ventilation development representing a section above a level must be completed before the ventilation development $d_{v,l}^{t}$ of the level commences. Eq. (20) ensures that the operational development $d_{a,l}^{t}$ on a level linking the ventilation development $b_{vd,l}^{t}$ can only commence after completion of a set of required ventilation developments $\sum_{t=1}^{t} d_{v,l}^{t}$ above and on that level.

$$L_{v,lb}^{t} \leq \sum_{v=1}^{V} \left(d_{vl} \times d_{v}^{t} \right) \leq L_{v,ub}^{t}, \quad \forall t \in \{1,...,T\}; \tag{16}$$

$$b_{v,l}^{t} - \sum_{t=1}^{t} d_{s,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, s \in D_v(L); \tag{17}$$

$$\sum_{t=1}^{t} d_{v,l}^{t} - b_{v,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, v \in \{1,...,V\}; \tag{18}$$

$$b_{v,l}^{t} - b_{v,l}^{t+1} \leq 0, \quad \forall l \in \{1,...,L\}, t \in \{1,...,T-1\}, v \in \{1,...,V\}; \tag{19}$$

$$b_{a,l}^{t} - \sum_{t=1}^{t} d_{v,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, a \in D_a(L), v \in D_v(L); \tag{20}$$

6.1.3. Operational Development Constraints

If a stope is scheduled to be mined in a period, a set of operational development must be ready ahead or in that period. Eqs. (21) to (25) present the set of constraints defining the operational development and lateral precedence relationships for stope extraction sequence. This includes the type and length of each operational development (level drives and crosscuts). Eq. (21) defines the operational development capacity constraints for the mine. This equation ensures that the total length of operational developments required in each period is within the defined lower and upper limits of the total available equipment capacity for developing the mine.

Eqs. (22) to (24) control the lateral precedence relation of the operational development required for each stope. Eq. (25) ensures that for each stope, there is a set of operational developments $\sum_{t=1}^{t} d_{a,l}^{t}$ that must be completed before mining the stope $b_{k,l}^{t}$.

$$L_{a,lb}^{t} \leq \sum_{a=1}^{A} \left(d_{a} \times d_{a}^{t} \right) \leq L_{a,ub}^{t}, \quad \forall t \in \{1,...,T\}; \tag{21}$$

$$b_{a,l}^{t} - \sum_{t=1}^{t} d_{s,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, s \in D_a(L); \tag{22}$$

$$\sum_{t=1}^{t} d_{a,l}^{t} - b_{a,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, a \in \{1,..., A\}; \tag{23}$$

$$b_{a,l}^{t} - b_{a,l}^{t+1} \leq 0, \quad \forall l \in \{1,...,L\}, t \in \{1,...,T-1\}, a \in \{1,..., A\}; \tag{24}$$

$$b_{k,l}^{t} - \sum_{t=1}^{t} d_{a,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, k \in \{1,...K_l\}, a \in D_a(L); \tag{25}$$

6.1.4. Ore Pass Development Constraints

Eq. (26) defines the ore pass development capacity constraint for the mine. The inequality ensures that the total length of ore pass developed in a period is within the stated lower and upper limits of the total available equipment capacity for developing the ore pass for the mine. Eqs. (27) to (30) control the precedence relationships between the sections of ore pass development leading to each mining level and the operational development on the level.

Eqs. (27) to (29) ensure that the set of ore pass developments representing a section above a level must be completed before the ore pass $d_{p,l}^{t}$ of the level commences. Eq. (30) ensures that the development of the ore pass $d_{p,l}^{t}$ linking the operational level can only commence after completion of a set of required operational developments $\sum_{t=1}^{t} d_{a,l}^{t}$ above and on that level.

$$L_{p,lb}^{t} \leq \sum_{p=1}^{P} \left(d_p \times d_p^{t} \right) \leq L_{p,ub}^{t}, \quad \forall t \in \{1,...,T\}; \tag{26}$$

$$b_{p,l}^{t} - \sum_{t=1}^{t} d_{s,l}^{t} \leq 0, \quad \forall l \in \{1,...,L\}, \ s \in D_p(L); \tag{27}$$

$$\sum_{t=1}^{t} d_{p,l}^{t} - b_{p,l}^{t} \leq 0, \quad \forall l \in (1,...,L), \ p \in \{1,...,P\}; \tag{28}$$

$$b_{p,l}^{t} - b_{p,l}^{t+1} \leq 0, \quad \forall l \in (1,...,L), t \in \{1,...,T-1\}, \ p \in \{1,...,P\}; \tag{29}$$

$$b_{p,l}^{t} - \sum_{t=1}^{t} d_{a,l}^{t} \leq 0, \quad \forall p \in \{1,...,P\}, a \in D_a(L), l \in (1,...,L); \tag{30}$$

6.1.5. Mining and Processing Capacity Constraints

Eqs. (31) and (32) define the mining capacity constraint. Mining is controlled by the continuous decision variable y_k^t. This inequality ensures that the total tonnage of rock material extracted in each period is within the acceptable lower and upper limits of the total available equipment capacity for the mining operation. Eqs. (33) and (34) are the processing capacity constraints that control the quantity of mill feed for the mine in each period and in each realization. We introduce continuous variables $odev_{s,+}^{t}$ and $odev_{s,-}^{t}$ which serve as buffers to allow deviations. They are however penalized in the objective function. Penalizing such deviations ensure that the proportion of ore processed at the plant is within the required targets as much as is feasible. The constraint also ensures that the processing capacity does not exceed a pre-defined limit. Ore extraction is controlled by the continuous decision variables $x_{k,\mathrm{m}}^{t}$ and $u_{k,so}^{t}$ respectfully.

$$T_{m,lb}^{t} \le \sum_{k=1}^{K} \left[(o_k + w_k) \times y_k^t \right] \le T_{m,ub}^{t}, \quad \forall t \in \{1,...,T\}; \tag{31}$$

$$T_{m,lb}^{t} \le \sum_{k=1}^{K} \left[(o_k + w_k) \times y_k^t \right] \ge T_{m,ub}^{t}, \quad \forall t \in \{1,...,T\}; \tag{32}$$

$$T_{pr,lb}^{t} \le \sum_{k=1}^{K} \left[((o_k \times x_{k,\mathrm{m}}^{t}) + \sum_{so=1}^{SO} u_{k,so}^{t}) - odev_s^t \right] \le T_{pr,ub}^{t}, \forall t \in \{1,...,T\}; \tag{33}$$

$$T_{pr,lb}^{t} \le \sum_{k=1}^{K} \left[((o_k \times x_{k,\mathrm{m}}^{t}) + \sum_{so=1}^{SO} u_{k,so}^{t}) + odev_s^t \right] \ge T_{pr,ub}^{t}, \forall t \in \{1,...,T\}; \tag{34}$$

6.1.6. Grade Blending Constraints

The grade blending constraints are presented in Eqs. (35) to (38). The ore grade schedule per period ensures that the material extracted satisfies the ore quality specification of the processing plant, stockpile and ore pass bin. Based on the cut-off grades or head grade of the plant, limiting grade requirements for appropriate ore blending are defined within lower and upper grade targets for the mining operation. Like the processing quantity constraints, the excess and shortage of the desired grade range in each period across all geological realizations are penalized in the objective function.

$$\sum_{k=1}^{K} \left[(g_k - gr_{pr,ub}^{t}) \times (o_k \times x_{k,m}^{t}) + \sum_{so=1}^{SO} (g_{k,so}^{t} - gr_{pr,ub}^{t}) \times (u_{k,so}^{t} \times o_k) - gdev_{s,+}^{t} \right] \le 0; \forall t \in \{1,...,T\}, \forall p \in \{1,...,SO\} \tag{35}$$

$$\sum_{k=1}^{K} \left[(gr_{pr,lb}^{t} - g_k) \times (o_k \times x_{k,u}^{t}) + \sum_{so=1}^{SO} (gr_{pr,lb}^{t} - g_{so}^{t}) \times (u_{k,so}^{t} \times o_k) + gdev_{s,-}^{t} \right] \ge 0; \forall t \in \{1,...,T\}, \forall p \in \{1,...,SO\} \tag{36}$$

$$\sum_{k=1}^{K} ((g_k - gr_{si,ub}^{t}) \times (o_k \times j_{k,si}^{t}) - gdev_{s,+}^{t}) \le 0; \quad \forall t \in \{1,...,T\}, \forall si \in \{1,...,SI\} \tag{37}$$

$$\sum_{k=1}^{K} ((gr_{si,lb}^{t} - g_k) \times (o_k \times j_{k,si}^{t}) + gdev_{s,-}^{t}) \ge 0; \quad \forall t \in \{1,...,T\}, \forall si \in \{1,...,SI\} \tag{38}$$

6.1.7. Stope Extraction Precedence Constraints

Eqs. (39) to (41) control the lateral precedence relationships of stope extraction on each level of the mining operations. The mining sequence for the deposit is implemented in an advancing and top-down approach. Thus, mining generally starts from the upper level to the lower level exploiting the deposit from the centre of the mine towards to the periphery of the mine.

$$b_k^t - \sum_{t=1}^{t} y_s^t \le 0, \quad \forall s \in G_k(S), k \in \{1,...,K\}; \tag{39}$$

$$\sum_{t=1}^{t} y_k^t - b_k^t \le 0, \quad \forall k \in \{1,...,K\}; \tag{40}$$

$$b_k^t - b_k^{t+1} \le 0, \quad \forall t \in \{1,...T-1\}, k \in \{1,...,K\}; \tag{41}$$

6.1.8. Backfilling Management Constraints

The backfilling constraints include backfilling capacity and sequencing constraints and stope status constraints. Eq. (42) presents the backfilling capacity constraint. The total volume of material backfilled in each period cannot exceed the capacity of the backfill plant and equipment in the period.

The stope status constraints are defined to monitor geotechnical and ground quality conditions. These constraints control where to start operational development and extraction for a level and progress sequentially. Eq. (43) ensures that each active stope can only be in either extraction or backfilling phase during any specific period. The two activities cannot be in progress at the same time for a stope. Eq. (44) ensures that stope ore extraction precedes backfilling. Eq. (45) ensures that each stope is in either the extraction or backfilling phase immediately after a non-zero stope extraction period. The stope status and backfilling sequencing constraints work together to ensure each stope progresses through discrete mining activities sequentially from operational development to extraction and then backfilling. These constraints further limit the exposure time of the void area after mining to reduce potential excessive geotechnical stresses prior to backfilling. The type of backfill material (uncemented rock fill, cemented paste fill (curing time), or cemented rock fill) is not considered during the optimization process for backfilling.

$$V_{f,lb}^{t} \leq \sum_{k=1}^{K}\left(d_{fv} \times f_{k}^{t}\right) \leq V_{f,ub}^{t}, \quad \forall t \in \{1,...,T\}; \tag{42}$$

$$b_k^t + f_k^t \leq 1, \quad \forall t \in \{1,...,T\}, k \in \{1,...,K\}; \tag{43}$$

$$f_k^t - \sum_{t=1}^{t}(x_{k,\mathrm{m}}^t + j_{k,si}^t) \leq 0, \quad \forall t \in \{1,...,T\}, k \in \{1,...,K\}; \tag{44}$$

$$b_k^t \leq b_k^{t+1} + f_k^{t+1}, \quad \forall t \in \{1,...,T-1\}, k \in \{1,...,K\}; \tag{45}$$

6.1.9. Non-Adjacent Stope Extraction Constraints

These are geotechnical constraints that focus on limiting the size of unsupported void areas. To demonstrate the functioning of these constraints, the nine adjacent stopes depicting the two kinds of spatial constraints required to facilitate geotechnical stability of the stoping area in extraction and backfilling scenarios can be seen in Fig. 7. When a stope is being extracted or backfilled, the adjacent stopes cannot be mined. As illustrated in Fig. 7, if the central stope Sp5 is in the extraction state, all operational activities are forbidden in the near adjacent stopes (Sp2, Sp4, Sp6, Sp8). Also, when a stope is being backfilled, the adjacent stopes cannot be exploited. Eqs. (46) and (47) ensure that all forms of adjacent stope extraction or backfilling activities are avoided to prevent excessively large unsupported voids.

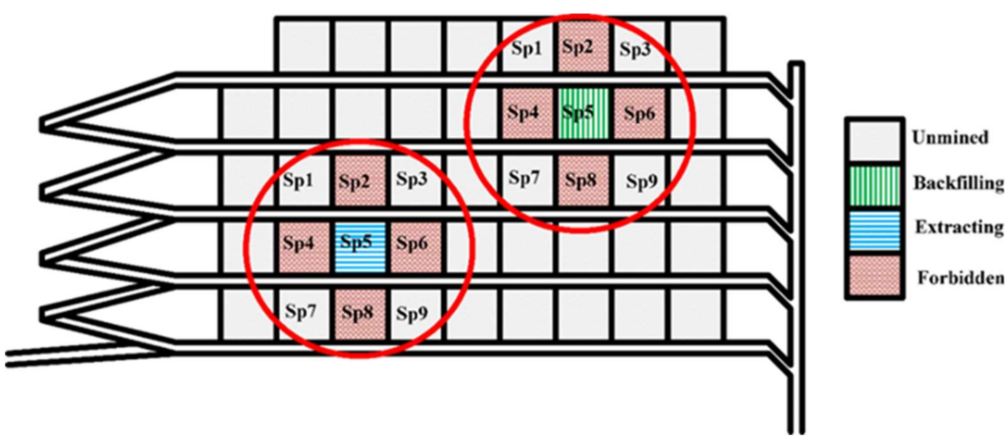

Fig. 7. Schematic illustration of non-adjacent stope extraction and backfilling.

$$b_k^t + b_j^t \leq 1, \; \forall t \in \{1,...,T\}, k \in \{1,...,K\}, \; j \in A_k(J); \tag{46}$$

$$f_k^t + b_j^t \leq 1, \quad \forall t \in \{1,...,T\}, k \in \{1,...,K\}, \; j \in A_k(J); \tag{47}$$

6.1.10. Active Levels Control Constraints

Many active levels in a mine might lead to serious operational problems. A controlled maximum number of simultaneous active levels allow mine planners to adjust extraction to different application scenarios depending on available infrastructure, production rates, equipment capacities, and the stability of the surrounding rocks. The constraints expressed in Eqs. (48) to (51) control the maximum number of simultaneous active levels in each period in the schedule. In Eqs. (48) and (49), when operational development activity is ongoing, or a portion of a stope is being extracted in period *t,* on level l, the relevant binary variable lc_l^t and le_l^t must be equal to one. In Eqs. (50) and (51), N_{lc}^t and N_{le}^t are provided as allowable input in the formulation for which the maximum number of simultaneous active levels must not exceed in each period. The active levels control constraints guide the model to generate a more operationally feasible mining pattern, thus ensuring a practical maximum NPV.

$$\sum_{t=1}^{t}\left(\frac{1}{D}\sum_{d=1}^{D}d_{a,l}^{t} - lc_l^t\right)\le 0, \quad \forall t\in\{1,...,T\}, l\in\{1,...,L\},\ \text{a}\in L_l(D); \tag{48}$$

$$\sum_{t=1}^{t}\left(\frac{1}{M}\sum_{m=1}^{M}x_{k,\text{m}}^{t} - le_l^t\right)\le 0, \quad \forall t\in\{1,...,T\}, l\in\{1,...,L\},\ k\in L_l(M); \tag{49}$$

$$\sum_{l=1}^{L} lc_l^t \le N_{lc}^t, \quad \forall t\ \in\{1,...,T\}; \tag{50}$$

$$\sum_{l=1}^{L} le_l^t \le N_{le}^t, \quad \forall t\ \in\{1,...,T\}; \tag{51}$$

6.1.11. Stope Extraction Duration Constraints

The length of stope extraction depends on notably, stope size, rate of production, and mining plan. Eq. (52) ensures that the period of extraction of a stope cannot exceed a pre-defined number of periods. This inequality enables the model to implement continuous stope extraction, mirror the mining environment, and regulate the allowed extraction period. This adds practicality and strengthens the functionality of the formulation.

$$\sum_{t=1}^{T} y_k^t \le N_{xd}, \quad \forall t\ \in\{1,...,T\}; \tag{52}$$

6.1.12. Stockpile Management Constraints

Eqs. (53) and (54) are the stockpile capacity constraints that control the quantity of ore feed to the stockpile. The constraint also ensures that the stockpile capacity does not exceed a pre-defined limit. The continuous decision variable $j_{k,si}^t$ controls ore to the stockpile. Eq. (55) ensures that the quantity of ore sent to the stockpile exceeds the quantity sent out of the stockpile to the plant, thus, at the end of the mine-life, there could be some amount of ore left in the stockpile and Eq. (56) ensure that reclamation from the stockpile stops if the predetermined average grade exceeds the actual grade of the material sent to the stockpile the ore sent to the stockpile must be reclaimed by the end of the mine-life.

$$T_{si,lb}^t \le \sum_{k=1}^{K}\left[(o_k\times j_{k,si}^t)\right]\le T_{si,ub}^t, \forall t\in\left\{1,...,T\right\}; \tag{53}$$

$$T_{si,lb}^t \le \sum_{k=1}^{K}\left[(o_k\times j_{k,si}^t)\right]\ge T_{si,ub}^t, \forall t\in\left\{1,...,T\right\}; \tag{54}$$

$$\sum_{i=1}^{t}\sum_{k=1}^{K}(u_{k,so}^{i}\times o_k)\leq \sum_{i=1}^{t-1}\sum_{k=1}^{K}(o_k\times j_{k,si}^{i}), \forall t\in\{1,...,T\}; \tag{55}$$

$$\sum_{i=1}^{t}\sum_{k=1}^{K}(u_{k,so}^{i}\times o_k\times g_{k,so}^{t})\leq \sum_{i=1}^{t-1}\sum_{k=1}^{K}(o_k\times g_k\times j_{k,si}^{i}), \forall t\in\{1,...,T\}; \tag{56}$$

6.1.13. Variables Control Constraints

Eq. (57) defines the relationship between mining, ore extraction, and stockpiling variables controlling the stope mining, ore stockpiling, and waste decisions. Thus, the continuous variables $x_{k,\mathrm{m}}^{t}$ and $u_{k,so}^{t}$ representing ore extraction and stockpiling plus amount of the waste dilution should always be smaller than or equal to the continuous variable y_k^t representing stope mining in each period. Eqs. (58) and (59) ensure that the total fractions of stopes and backfilling volumes over the scheduling periods sum up to one. Eqs. (60) to (61) ensure that the total fractions of ore sent to stockpile from the mine and sent from stockpile to processing plant from stockpile over the scheduling periods sum up to one. Thus, ore in each circumstance may be scheduled once. Eqs. (62) to (65) ensure that all developments (capital, ventilation, operational, and ore pass) are going to be constructed once in the life of the mine. The variable control constraints define the logics and inter-relations of the binary and continuous variables that define extraction, backfilling, and capital, ventilation, operational, and ore pass development to ensure they are within desired ranges. Eq. (66) ensures that all continuous decision variables are non-negative. Eq. (67) defines all binary variables as non-negative and integers.

$$x_{k,\mathrm{m}}^{t}+u_{k,so}^{t}\leq y_k^t, \forall t\in\{1,...,T\}, k\in\{1,...,K\}; \tag{57}$$

$$\sum_{t=1}^{T} f_k^t \leq 1, \forall k\in\{1,...,K\}; \tag{58}$$

$$\sum_{t=1}^{T} x_{k,\mathrm{m}}^{t} \leq 1, \forall k\in\{1,...,K\}; \tag{59}$$

$$\sum_{t=1}^{T} j_{k,si}^{t} \leq 1, \forall k\in\{1,...,K\}; \tag{60}$$

$$\sum_{t=1}^{T} u_{k,so}^{t} \leq 1, \forall k\in\{1,...,K\}; \tag{61}$$

$$\sum_{t=1}^{T} d_c^t \leq 1, \forall c\in\{1,...,C\}; \tag{62}$$

$$\sum_{t=1}^{T} d_v^t \leq 1, \forall v\in\{1,...,V\}; \tag{63}$$

$$\sum_{t=1}^{T} d_a^t \leq 1, \forall a\in\{1,...,A\}; \tag{64}$$

$$\sum_{t=1}^{T} d_p^t \leq 1, \forall p\in\{1,...,P\}; \tag{65}$$

$$x_{k,\mathrm{m}}^{t}, j_{k,si}^{t}, u_{k,so}^{t}, y_k^t, f_k^t, d_c^t, d_v^t, d_a^t,\ d_p \geq 0 \tag{66}$$

$$b_k^t, b_{c,l}^t, b_{v,l}^t, b_{a,l}^t, b_{p,l}^t \geq 0 \text{ and integers} \tag{67}$$

7. Case Studies

Two case studies are presented. For Case study 1, the MILP model is implemented without the stockpiling management constraints and for Case study 2, the MILP model is implemented with a grade controlled stockpiling management constraints.

In both case studies, the proposed MILP formulation model was applied to 120 stopes designed for a gold deposit. To assess the model's effectiveness, we incorporated active levels and extraction duration constraints as specified. Table 2 provides details about the gold deposit's characteristics and the designed stopes. Additionally, Fig. 8 displays a cross-section of the block model, illustrating the distribution of gold grades within the deposit. For a visual representation of the 120 designed stopes, please refer to Fig. 9**,** showing the sectional layout created using Promine software [41]. IBM CPLEX Optimization Studio V12.6.3 [16] was integrated within a MATLAB 2023a [31] environment to define the modelled framework and solve the optimization problem at a gap tolerance of 5%. The model was tested on an Intel (R) Core^{TM} i7-6500U CPU at 2.50 GHz with 64 GB of RAM.

Table 2. Characteristics of gold deposit and designed stopes.

Description	Value
Total mineralized material (Mt)	2.88
Maximum grade value of Au (g/t)	5.34
Minimum grade value of Au (g/t)	1.40
Average grade value of Au (g/t)	3.05
Number of levels	8
Stope height (m)	30
Stope length (m)	25
Stope width (m)	10-12

The orebody exhibits scattered high-grade mineralization throughout the deposit, while certain areas in the upper region contain prominent low-grade sections. The surrounding waste rock is composed of medium to low strength rock, making open stoping with backfilling the preferred mining method.

Table 3 contains the economic and technical criteria used to assess the case study. On the basis of the total amount of material to be extracted and the proposed plant capacity, respectively, annual capacities for mining and processing were established. Cost projections in Canadian dollars were gathered from industry professionals and technical mining company reports [29; 33; 13].

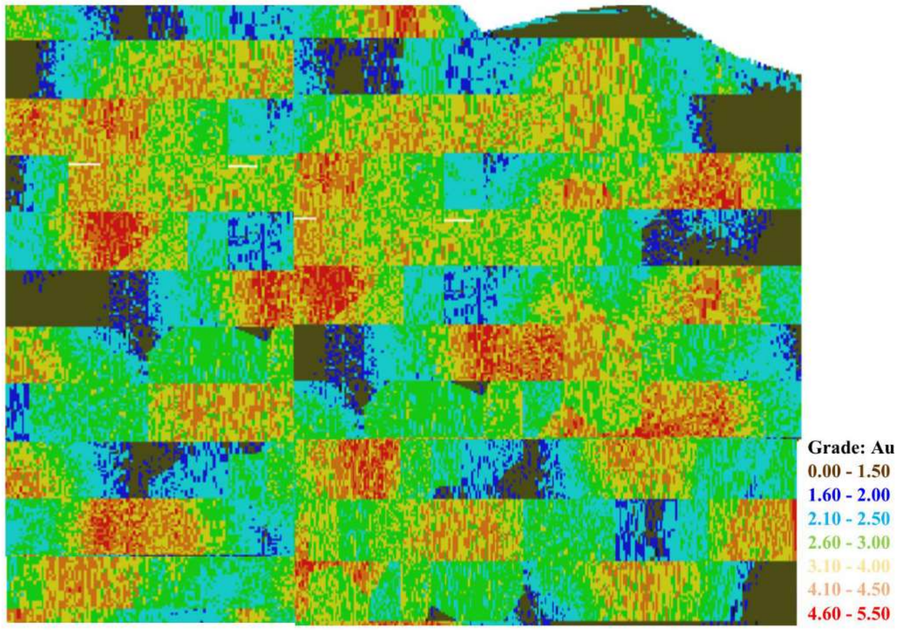

Fig. 8. Cross-section of the block model depicting the distribution of gold grades (Not drawn to scale).

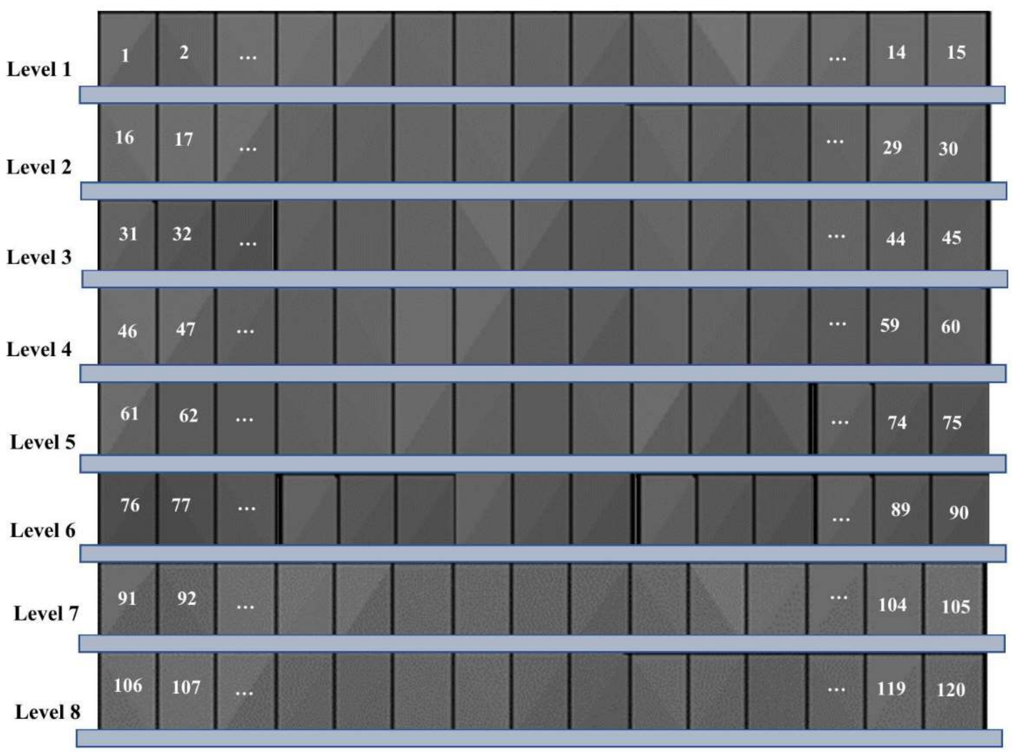

Fig. 9. A sectional layout of the mineralized stopes.

Table 3. Technical data used.

Parameter	Value
UG mining cost ($/t)	96.04
Processing cost ($/t)	15.21
Backfilling cost ($/t)	15.00
Operational development cost ($/m)	4,500
Capital development cost ($/m)	6,500
Ventilation cost ($/m)	3,000
Selling price of gold ($/oz)	2,000
Discount rate (%)	8.00
Mining recovery (%)	93.00
Variable Mining dilution* (%)	5.00 / 10.00
Maximum mining capacity (Kt/period)	140.0
Maximum processing capacity (Kt/period)	110.0
Maximum backfilling capacity (m^3/period)	50,000
Maximum operational development (m/period)	200
Maximum capital development (m/period)	500
Maximum ventilation development (m/period)	500
Maximum stope extraction duration (Periods)	2
Maximum ore pass development (m/period)	143
Maximum active levels*	7

Mining dilution factor of 10% was applied to stopes at the periphery of the orebody, and mining dilution factor of 5% was applied to the remaining stopes. All the lower boundaries equal zero in this case study. Due to the implementation of backfilling of the stopes, which enhances ground control and increases flexibility in operating multiple active mining levels, the optimizer was allowed more flexibility in operating multiple active mining levels.

7.1. Results and Discussion - Case Study 1

Upon implementing the MILP model formulation without stockpiling, the open-stope underground gold project benefited from optimized schedules for capital, ventilation, operating, and ore pass development, as well as mining, processing, and backfilling production schedules for a 25-year mine-life. The case study involved 22,600 decision variables, consisting of 13,600 binary integer variables and 9,000 continuous variables. Furthermore, 57,412 constraints were generated to achieve the desired optimization. The solution time was 4.2 hours.

Fig. 10 illustrates the schedules for capital, ventilation, and ore pass development. Capital and ventilation development activities took place during the initial 5 years of the mine-life to facilitate the commencement of operational development. In addition to the optimized schedules for capital investment, ventilation, operational development, mining, processing, and backfilling, the MILP model also factored in ore pass development for the open-stope underground gold project. Ore pass development took place during two specific periods: Between Years 2 to 4: During this time frame, ore pass development activities were carried out to establish efficient and safe channels for transporting mined ore from the stopes to the processing plant, waste dump and later stockpile. Between Years 17 and 18: A second phase of ore pass development occurred in this later period, aiming to further optimize the transport system for mined ore to support the mining operation during this phase of the mine's life. As depicted in Fig. 11 throughout the mine's life, operational development occurred at varying rates based on production requirements.

The MILP model resulted in an impressive NPV (Net Present Value) of 636.98 million dollars. During the project's lifespan, a total of 2.49 Mt of material were mined and 2.20 Mt were processed.

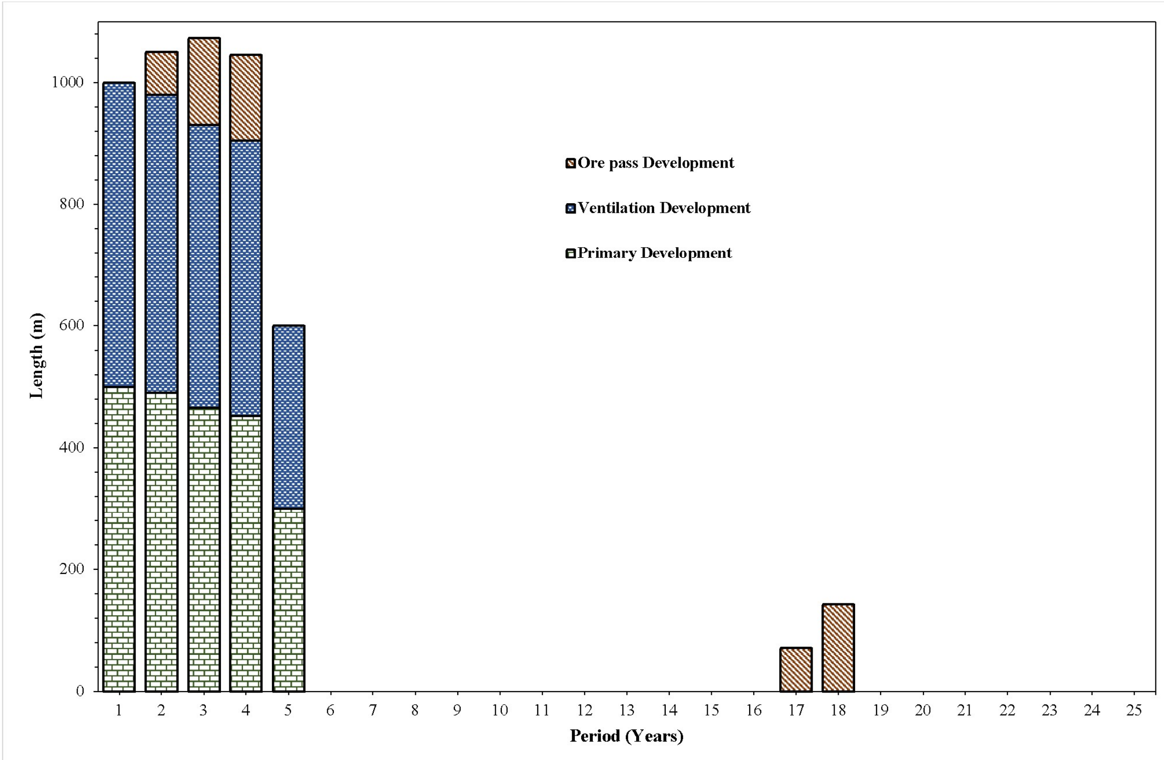

Fig. 10. Capital, ventilation, and ore pass development schedule.

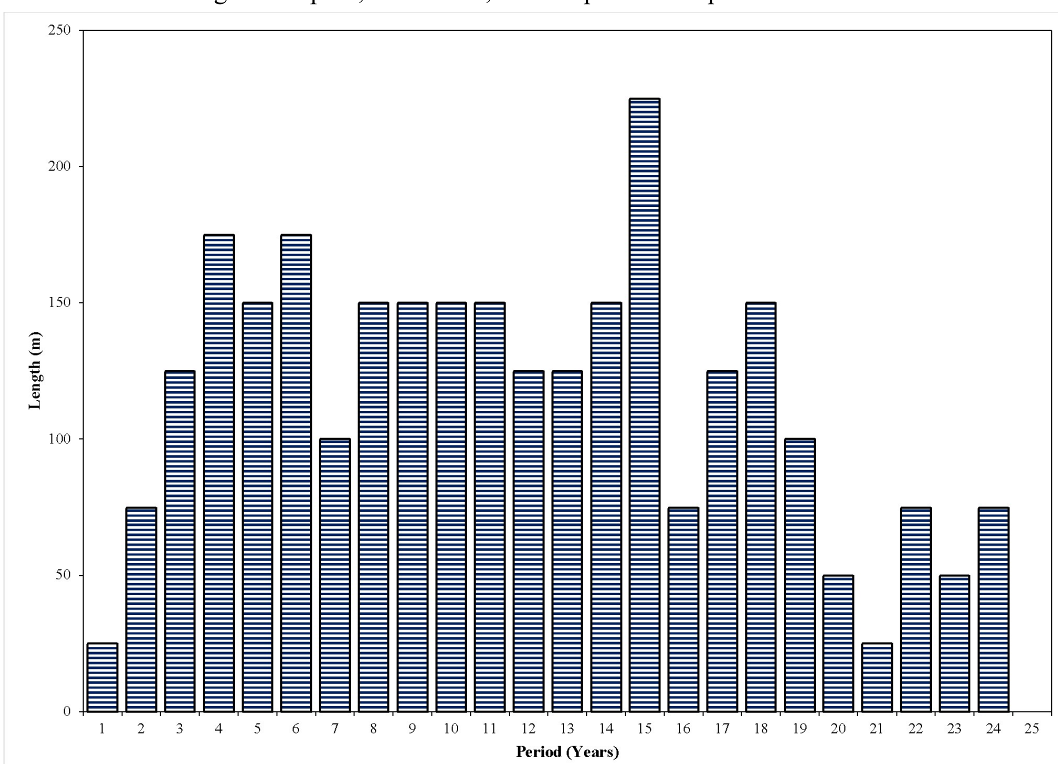

Fig. 11. Operational development schedule.

Fig. 12 showcases the schedule, of both the mined material, which includes waste dilution material. The mining operations commenced in Year 1 ramping up following the completion of capital, ventilation, and some initial operational development. From Year 4 to Year 14, the mining capacity was 140 Kt maximizing output during this period and is gradually decreased to 120 Kt from Year 15 to Year 19 and to 80 Kt from Year 20 to Year 25 as part of the mine's planned ramp-down strategy to complete its 25-year life cycle.

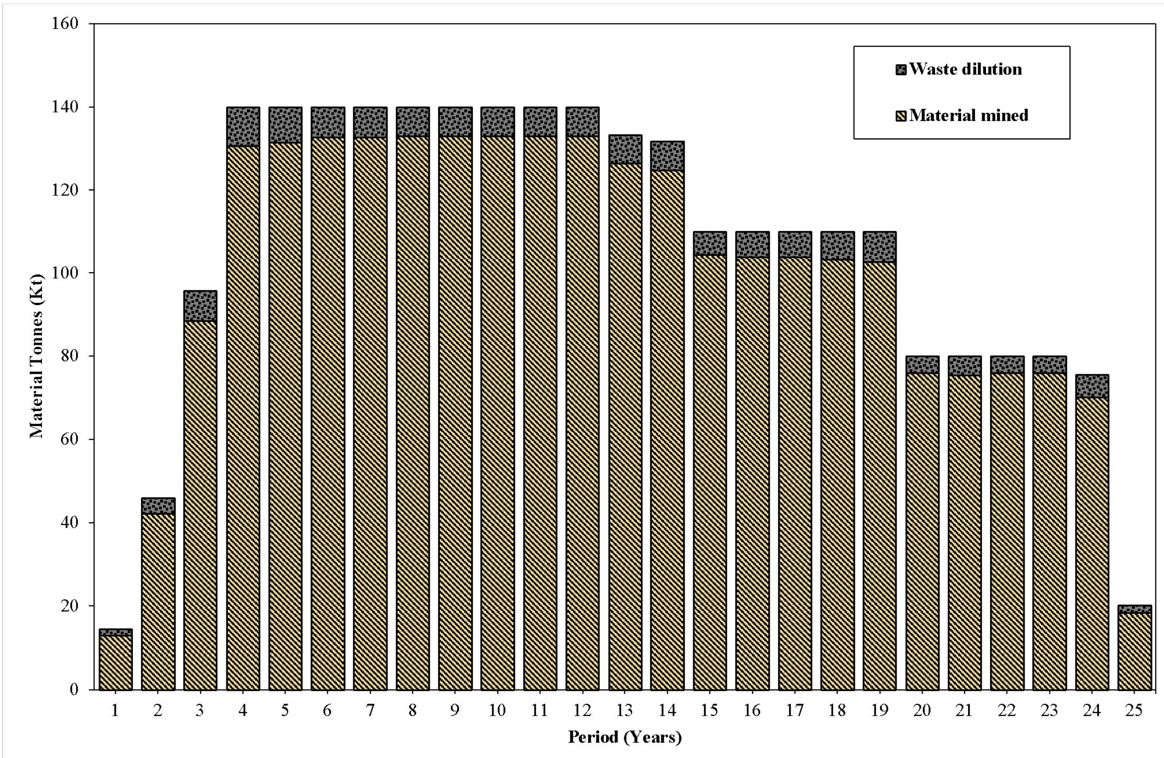

Fig. 12. Mined material with estimated waste dilution.

An essential aspect displayed in the schedule is the consideration of variable waste dilution during the optimization process. Stopes situated at the periphery of the orebody contribute more waste dilution compared to those located within the core of the orebody. By incorporating this variable waste dilution factor into the optimization, the model ensures a more accurate representation of the real-world mining scenario, leading to improved resource management and better decision-making throughout the mine's operational life.

Displayed in Fig. 13 is the material processed and the average grade distribution curve. Based on the average grade profile, the MILP model prioritizes targeting high-grade areas during the early years of mining to maximize the Net Present Value (NPV) of the project. However, during the initial three years, due to the development phase, only a limited number of stopes are available for extraction. As a result, the mining system is constrained to lower grades that are immediately accessible on the upper levels of the deposit. As the development progresses and more stopes become available for extraction, the processing plant's head grade experiences an improvement from Year 1 to Year 7 before starting to decline gradually. Consequently, medium to high-grade ores are primarily extracted during the early years of mining, from Year 1 to Year 10. Subsequently, from Year 11 to Year 25, the mining operation shifts towards extracting medium-grade ores as the higher-grade areas become depleted. This approach allows for strategic resource management and efficient extraction of the higher-grade material when it can contribute the most to the project's overall financial success. As the mine matures and the higher-grade zones are depleted, the focus shifts towards mining medium-grade ores to sustain the operation over the remaining mine-life.

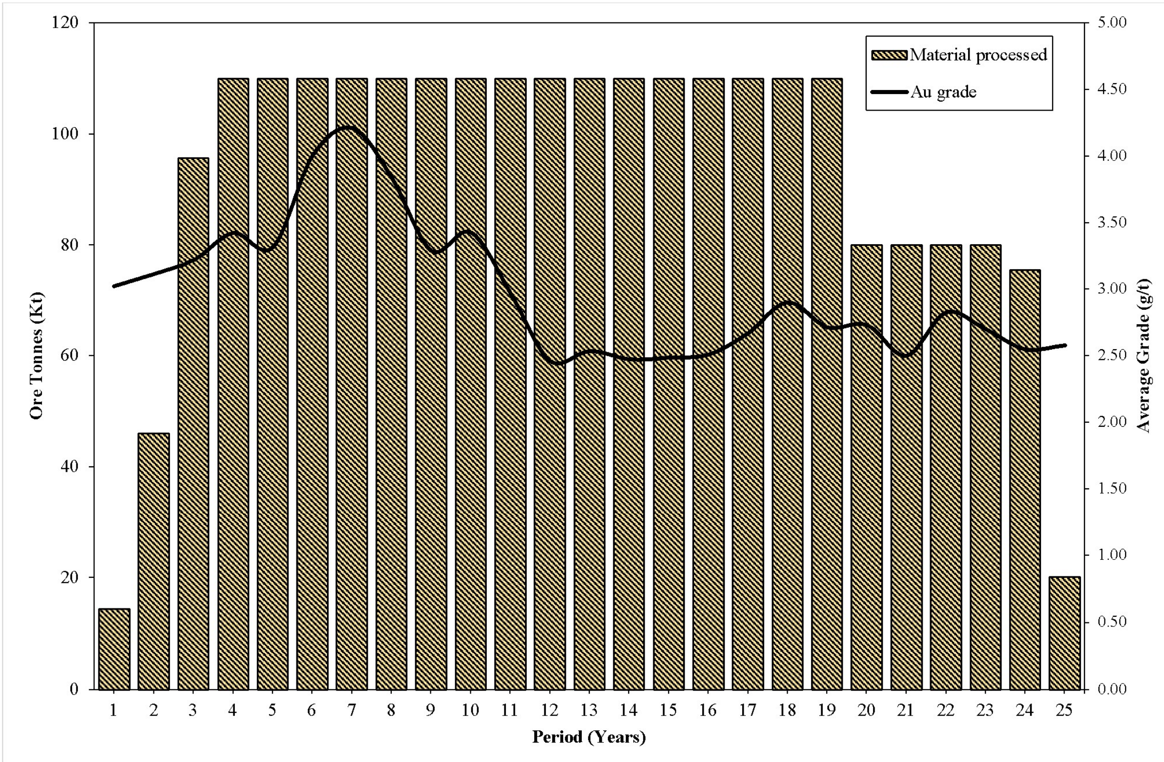

Fig. 13. Material processed with average grade curve.

The plant's operation started in Year 1 processing all available material from mining at a fixed capacity of 110 Kt. After ramping up from Year 1 to Year 3, full plant capacity was achieved from Year 4 to Year 19 and gradually ramps down at 80 Kt from Year 20 to the end of mine-life.

The stope production phases are presented in a sequential manner in Fig. 14. The schedule demonstrates the order in which each phase is completed, ensuring a systematic mining process: Operational Development: This phase involves the construction of drives and crosscuts within the mine, creating access to the different areas of the orebody. Operational development is prioritized and completed before moving on to stope extraction. Stope Extraction: Once the operational development is finished, the stope extraction phase begins. This is the main phase of mining, where ore is excavated from the stopes. The order of stope extraction follows the planned sequence, ensuring optimal resource utilization. Backfilling: After stope extraction, the backfilling phase takes place. The mined stopes are filled with suitable material to provide ground support and stability. This backfilling process enhances ground control and safety in the underground mining operation.

By following this sequence of production phases, the mining operation can maintain an organized and efficient workflow. Operational development is completed first to facilitate access to the orebody, ensuring a smooth transition to stope extraction. Once the extraction is completed, backfilling is implemented to enhance ground control and create a safer working environment. This systematic approach contributes to the overall success and productivity of the open stope underground gold project over its 25-year mine-life.

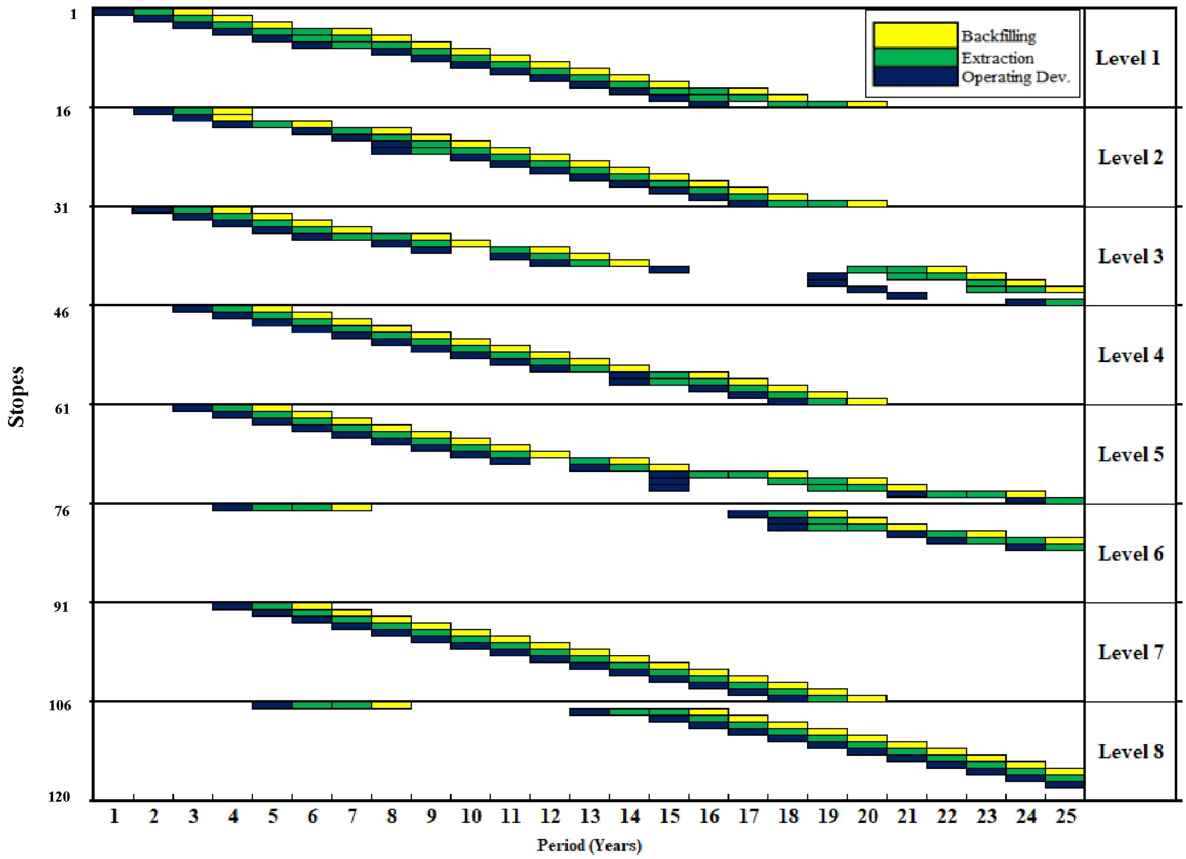

Fig. 14. Annual operational development, stope extraction and backfilling schedule level.

In the mining operation, there were some specific instances where the planned sequence of operational development and extraction was altered due to the availability of higher-grade stopes on other levels. These variations resulted in breaks in extraction, which affected certain stopes.

On Level 6, after the extraction and backfilling of Stope 76 in Period 7, there was no immediate stope operational development or extraction until Period 17 when Stope 77 development started. This deviation was caused by the presence of higher-grade stopes on Level 7, which were prioritized for extraction. Additionally, on Level 8, there was a break in extraction between Periods 9 and 12. Stopes 84 to 90 on level 6 and stope 120 on Level 8, which had average grades between 1.1 g/t to 1.38 g/t, were not extracted before the end of the mine-life.

Furthermore, at the end of the mine-life, some stopes were not backfilled due to the absence of other available stopes for extraction on those levels. Specifically: Stopes 45 on Level 3, 75 on Level 5, 82 on Level 6, and Stopes 118 and 119 on Level 8 respectively. Also, on Level 8, operational development, and extraction for Stope 119 occurred in Periods 25. These variations and decisions were made based on the optimization process and the availability of different ore grades in the deposit, aiming to maximize the project's NPV while maintaining efficient resource utilization and safety considerations.

7.2. Results and Discussion – Case Study 2

The implementation of the MILP model formulation with grade controlled stockpiling management constraint and its associated cost proved to be highly advantageous for the project maintaining all original constraints and associated parameters. The optimization process resulted in well-optimized schedules for various aspects of the mining operation, contributing to its successful operation over a 25-year mine-life. The MILP model involved a total of 28,600 decision variables among which, 13,600 were binary integer variables, used for making discrete decisions, and 15,000 were continuous variables, used for continuous optimization. To ensure the optimization adhered to all the project's requirements and limitations, a total of 57,702 constraints were generated and incorporated into the model. The optimization process was carried out effectively, achieving the desired results within a reasonable time frame of 3.2 hours. The optimized schedules for capital investment, ventilation, operational development, ore pass development, mining, stockpiling, processing, and backfilling allowed for the efficient utilization of resources and maximizing a resulting NPV of 718.00 million dollars which is approximately 10% higher than case study 1. Presented in Table 4 are the stockpile parameters used for the implementation.

Table 4. Stockpile parameters.

Stockpile	Value
Maximum grade value of Au (g/t)	1.40
Minimum grade value of Au (g/t)	0.10
Average grade value of Au (g/t)	0.75
Rehandling cost ($/t)	0.5

Fig. 15 the material mined schedule is presented with the implementation of grade control stockpile management constraints. These constraints were introduced to regulate the stockpile capacities over the mine-life. The maximum mining capacity was set at 140 Kt from Year 1 to Year 14, 120 Kt from Year 15 to Year 19, and 80 Kt from Year 20 until the end of the mine-life. Throughout the mining operation, the stockpile management constraints were effectively maintained, allowing for a controlled accumulation of ore. As a result, the total material mined from the open stope underground gold project amounted to 2,522,912.50 t. This optimized material mined schedule, combined with the grade control stockpile management, contributed to efficient resource utilization, smooth production, and improved project economics over the 25-year mine-life.

Presented in Fig. 16 is the schedule of material sent to and from the stockpile. The implementation of the grade control strategy is evident in this schedule. Low-grade material was sent to the stockpile during specific periods: from Year 4 to Year 11 and Years 14 and 15. This allowed for the selective mining of higher-grade material from the mine while stockpiling the lower-grade material for future use. From Year 20 until the end of the mine-life, grade-blended stockpiled material is sent from the stockpile to the processing plant as needed. This demonstrates the effective utilization of the stockpile to provide a continuous supply of material to the plant, ensuring efficient processing and resource management in the later stages of the mining operation. A total of 191,987.50 tons of material were stockpiled, contributing to the overall strategic planning and grade management of the project.

In Fig. 17, a comprehensive schedule is displayed, encompassing both the material processed directly from the mine and that from the stockpile. This integrated schedule demonstrates how the material is efficiently managed and processed throughout the 25-year mine-life, optimizing the production and economic performance of the open stope underground gold project. The implementation of the grade control strategy, stockpiling, and selective blending of material ensures a consistent supply of high-grade ore to the processing plant, maximizing the project's overall financial returns.

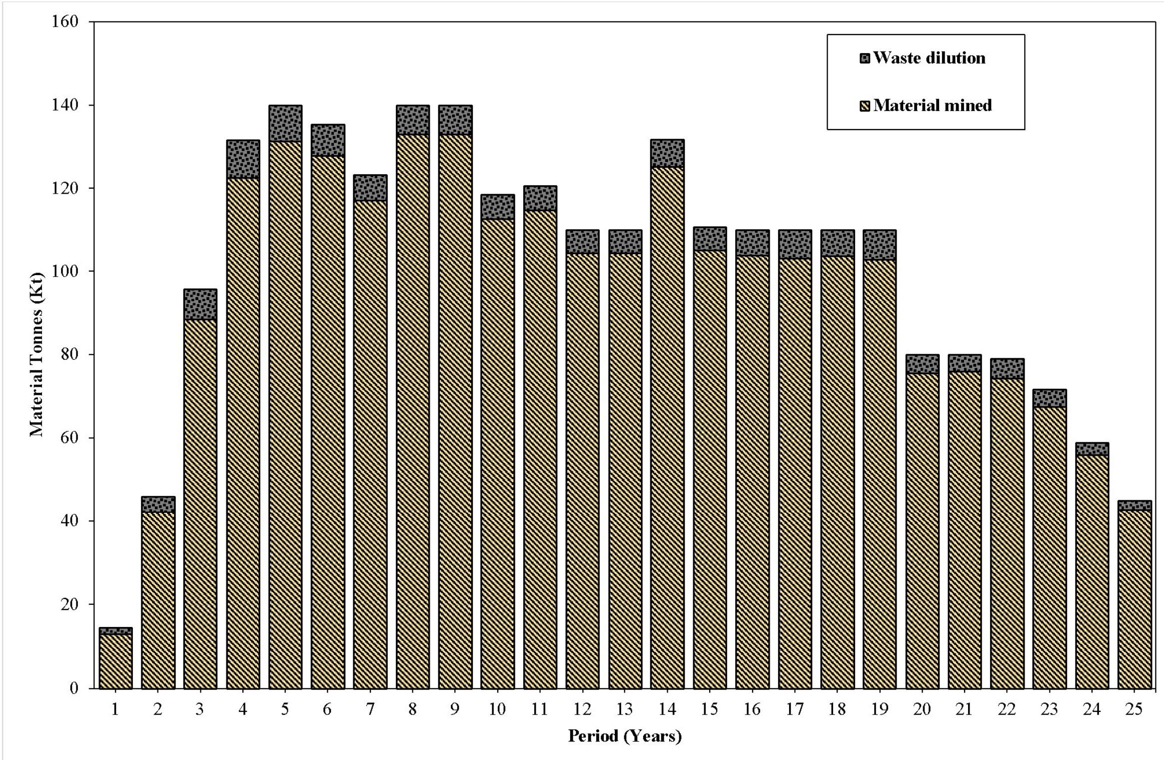

Fig. 15. Material mined considering grade control stockpile management.

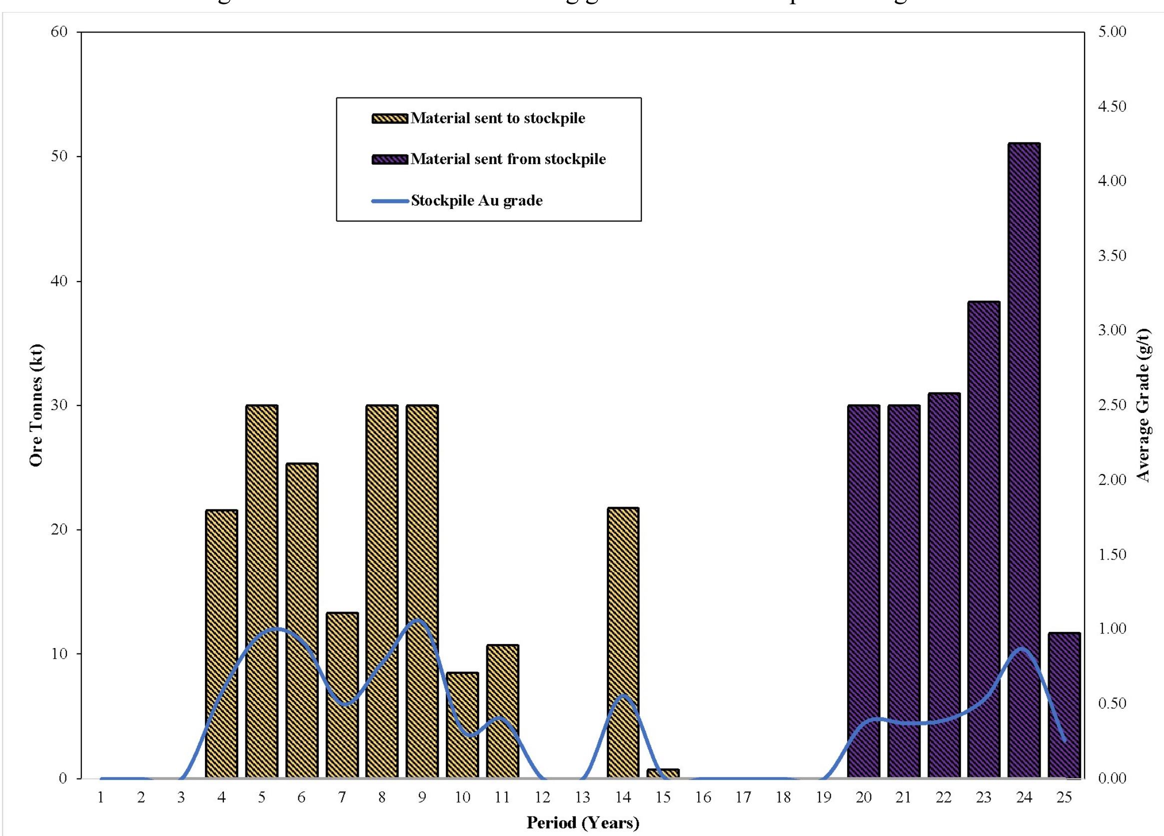

Fig. 16. Stockpiled material schedule and grade curve.

In Fig. 17, the comprehensive schedule for material processing is presented, which includes both material processed directly from the mine and material sourced from the stockpile. The implementation of the optimizer's strategy is evident in this schedule, resulting in various operational phases: After gradual ramping up from Year 1 to Year 3, the processing plant achieved full capacity from Year 4 to Year 19. During this period, the mining operation supplied sufficient high-grade material directly from the mine to meet the plant's processing capacity. From Year 20 to the end of the mine-life (Year 25), the processing plant's full capacity was not achieved due to various factors, such as declining ore grades and depletion of higher-grade areas in the mine. To ensure consistent plant operations and maximize output, the optimizer efficiently utilized the available grade blended material from the stockpile to meet the plant's processing capacity. The material processing statistics from the entire mine-life are as follows: 2,330,925.00 t of material were processed directly from the mine. 191,987.50 t of material were sourced from the stockpile and processed in the plant.

Overall, a total of 2,522,912.50 t of material were mined and processed, reflecting an optimized approach that effectively utilized available resources and improved the project's economic performance. The blended grade curve, comprising both direct processed grades and stockpile grades, ensured a consistent supply of material to the plant, contributing to the project's success over the 25-year mine-life.

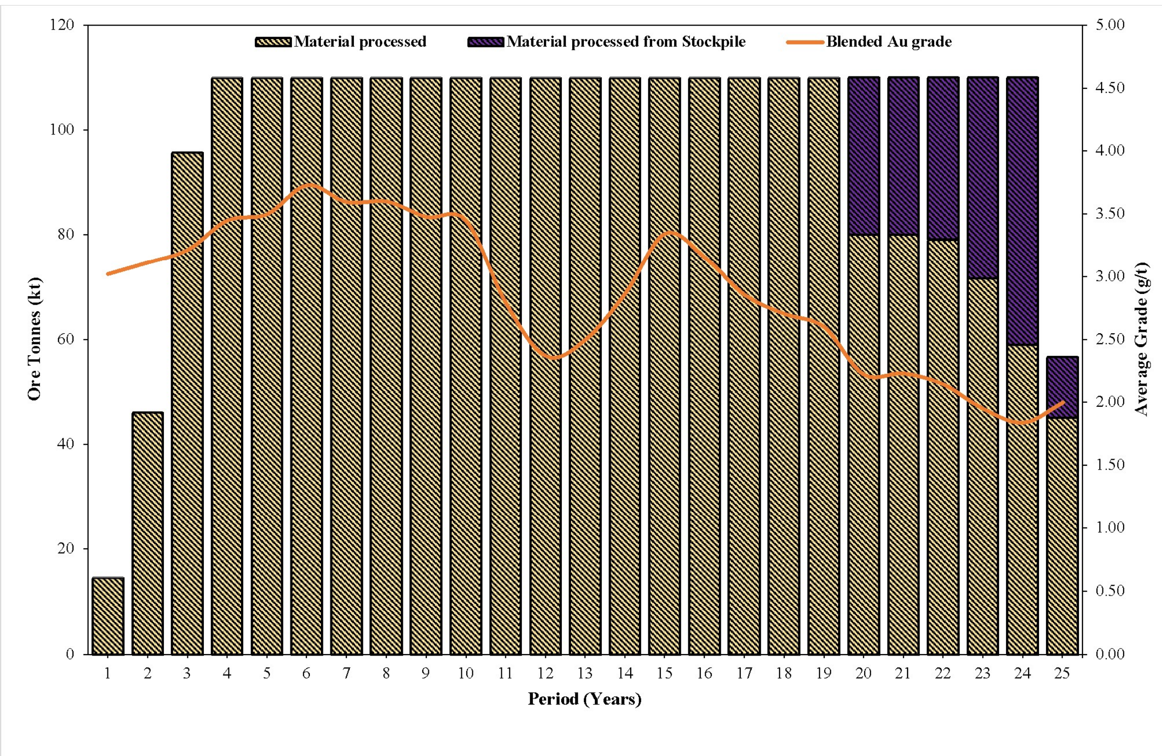

Fig. 17. Direct processed material, material from stockpile and grade blended curve schedule.

In Fig. 18, the stockpile inventory profile is presented, showcasing the accumulation and depletion of material over the mine-life. The profile demonstrates the following trends: From Year 4 to Year 11 and again from Year 14 to Year 15, the stockpile inventory gradually increased. During these periods, lower-grade material was selectively sent to the stockpile while higher-grade material was processed directly from the mine. This strategic stockpiling ensured a steady supply of material to the processing plant during periods of lower-grade mining, thereby maintaining a consistent production rate. From Year 20 to the end of the mine-life (Year 25), the stockpile inventory shows a gradual decline. As the mining operation entered its later stages and higher-grade areas in the mine became depleted, the stockpile was utilized to supplement the plant's processing needs. This utilization led to the gradual depletion of material from the stockpile. Overall, the stockpile inventory profile demonstrates how the optimizer effectively managed material storage, strategically using the stockpile to maintain steady plant operations and optimize the overall material processing. By carefully controlling the stockpile and using it to complement the direct mining process, the project could adapt to varying ore grades, ensuring continuous production, and maximizing the project's economic performance throughout its 25-year mine-life.

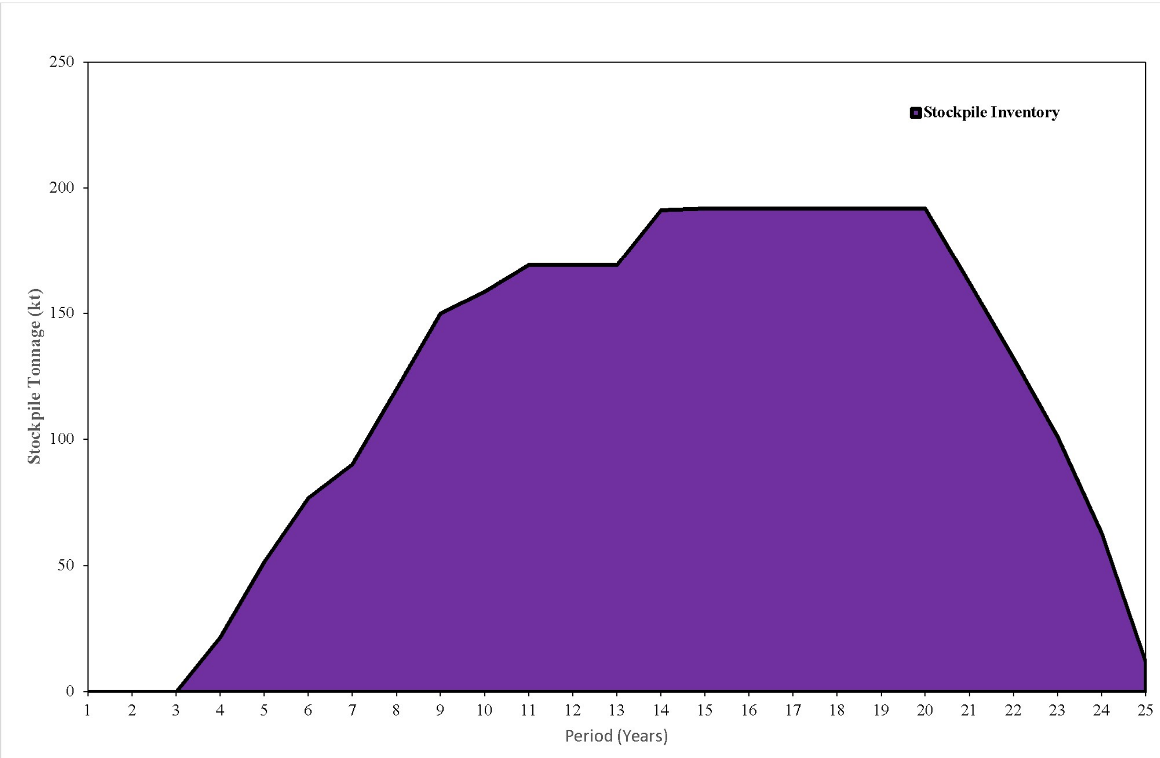

Fig. 18. Stockpile inventory profile.

8. Conclusion and Future work

In this paper, a MILP optimization framework which includes a grade control stockpiling management constraint for open stope mining production scheduling has been presented. The original model without stockpile was first implemented and results discussed and the model with stockpile was subsequently presented, and results discussed as well. The model's objective function maximizes the NPV of the mining operation while generating a practical stope extraction sequence. In addition to the mining and processing productivity constraints, capital development (decline), ventilation development, operational development, backfilling, active levels control, and maximum stope extraction duration constraints have been modelled and implemented. Thus, the identified limitations of existing underground production scheduling models in relation to generating global life-of-mine plans that integrate development like decline, ventilation, operational development, and ore pass resolved with this MILP framework.

The incorporation of grade control stockpile management has proven to be a significant improvement to the model, resulting in a substantial increase in the Net Present Value (NPV) of the open-stope underground gold project. By efficiently managing stockpiles and strategically blending grades, the model was able to optimize material utilization and ensure a continuous supply of high-grade ore to the processing plant when needed. This enhanced approach contributed to the overall profitability of the project. The approximate 10% increase in NPV is a testament to the effectiveness of the grade control stockpile management strategy. The model's ability to adapt to varying ore grades, store low grade material in the stockpile during high-grade periods and utilize the stockpile during low grade periods allowed for a more stable and profitable operation throughout the mine's life. By strengthening the model with grade control stockpile management, the mining operation achieved better financial outcomes and improved resource utilization. This optimization provided a competitive advantage, allowing the project to realize its full potential and enhance its economic viability over the 25-year mine-life.

The ongoing implementation of the Stochastic Mixed Integer Linear Programming (SMILP) framework is an exciting phase of the research. The stochastic stage focuses on documenting the case study and analyzing the optimization results obtained through the model. The documentation process will involve carefully detailing the methodology, data inputs, constraints, and objectives used in the SMILP framework for the case study. The researchers will explain the various components of the model, such as the decision variables, objective functions, and constraints, as well as any specific modifications made to address the unique characteristics of the open-stope underground gold project.

This analysis will involve evaluating the performance of the SMILP framework in achieving its goals, such as maximizing the Net Present Value (NPV), optimizing production schedules, and efficiently managing grade control and stockpiling. The impact of different parameters and constraints on the results will be assessed and any other areas for further improvement will be identified. The analysis of the optimization results will play a crucial role in validating the effectiveness and practicality of the SMILP framework for real-world mining projects. It will provide valuable insights into the model's capabilities and limitations, allowing us to draw meaningful conclusions and recommendations for its implementation in other mining scenarios. It is envisaged that findings and insights from the case study and analysis will likely contribute to the advancement of selective mining optimization techniques and their application in the mining industry. The documentation and analysis will also pave the way for potential future research, refinement of the model, and its broader adoption in the field of mining and mineral resource management.

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