An Innovative Optimization Model for Long-term Production Scheduling of Sublevel Caving Mines Using Mixed Integer Linear Programming

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ABSTRACT

As a factory-like mining method, sublevel caving includes a range of activities such as development, stockpiling, and production, each occurring independently on different levels. In order to achieve more reliable production scheduling, it is essential to consider the coordination of operations from the development phase to the plant. This research paper introduces a mixed-integer linear programming (MILP) framework designed for the long-term production scheduling optimization of sublevel caving mines. The primary objective of this framework is to maximize the net present value while addressing various operational constraints such as development activities, mining and processing capacities, continuous mining, the allowable number of active mining units, grade blending, and vertical and horizontal sequencing. We have developed a Jupyter Notebook-based model to implement this framework and employed the powerful CPLEX solver to obtain efficient solutions. The proposed MILP model offers systematic schedules of all development activities, including vertical shaft and its ventilation, man-way-raises and ventilations, orepasses, and operational developments while ensuring optimal sequences of ore extraction and maximizing the operation's profitability in the scope of long-term planning. The efficacy of the proposed model is demonstrated through the implementation of the model on an illustrative sublevel caving case study. The results highlight the ability of the framework to generate robust schedules that account for the simultaneous occurrence of development activities and ore production throughout the mine life.

1. Introduction

The mining sector is currently focused on finding more cost-effective extraction methods and enhancing the efficiency of existing underground mining techniques [20]. Consequently, sublevel caving (SLC) methods have become increasingly popular in mining hard rock masses due to their potential for high production rates and economical operation [7]. Notably, significant progress in drilling and blasting techniques has led to a remarkable reduction in the development-to-production ratio within SLC mines [6].

As a caving mining method, SLC relies on the gravity flow of materials, resulting in a random and irregular flow pattern. Moreover, this unpredictability is further compounded by various factors, including mine layout, the chaotic material flow, blasting performance, hang-ups, fleet size, number of active production faces, multiple-level ore recovery, rate of cave propagation, and surface subsidence. Consequently, the intricate interplay of these factors makes the design of an SLC mine a considerably complex task.[21].

The advancements and complexities of SLC require a well-designed mine plan that improves overall performance and considers essential SLC components. A high-performance SLC mine plan is one

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that has a holistic view and aligns all operations with a strategic approach. As a critical component of mine planning, production scheduling determines the most beneficial mining sequence over the life of the mine. Developing a schedule that meets all mining aspects can substantially reduce costs and increase profitability. Therefore, providing an exhaustive mathematical model that can integrate all activities, including development, mining, and processing, simultaneously increases the model's practicality and strength [10; 4].

Underground production scheduling is subject to several discrete and continuous decisions that indicate the location, destination, extraction time, and mining units' extraction portion over the mine's life, along with vertical and horizontal precedence relations [20; 3]. A production schedule aims to define the most profitable extraction sequence of the material that produces the desired market specification while meeting the demanded quantities of run-of-mine ore at each period and satisfying a set of physical and operational constraints [15; 18]. Compared to the manual and heuristic methods, the exact algorithms created by mathematical programming models would be an excellent alternative to provide an operationally optimal multi-time-period schedule in mines [19; 11].

Although linear- and mixed-integer programming models have significant potential for optimizing production scheduling in both open-pit and underground mines, they often overlook the pre-extraction material flow [1; 3]. In the context of SLC mining, development activities are not fully integrated into the optimization models. This paper presents a mathematical programming framework based on mixed-integer linear programming (MILP) formulation for long-term SLC production scheduling. The MILP model maximizes the operation's NPV. The model's primary goal is to integrate the schedules of all development activities, including the capital, ventilation, orepass, and operational developments, into the mining and processing schedules. The model further controls the mining and processing capacities, continuous mining, the allowable number of active production areas, grade blending, and vertical and horizontal sequencing. The provided model is more comprehensive and includes more details than previous models, which only focus on the extraction sequences in SLC production scheduling.

2. Relevant Literature Review

Only a few studies have been conducted on SLC production scheduling, with the Kiruna mine in northern Sweden being the primary focus of successive efforts in this area. Khazaei and Pourrahimian [9] presented a comprehensive review and summarized the researchers' attempts in the SLC production scheduling optimization field, which mainly focused on the extraction sequencing of mining units.

Almegren [2] presented a long-term production scheduling of Kiruna mines using the application of Lagrangian relaxation, but this method was not used in the final formulation due to the gap phenomenon and the assumption of complete mining of blocks in each time period. Topal [23] formulated a combined model that solved one-year sub-problems with a monthly resolution to provide a five-year schedule. Deglegen et al. [5] set up a model to minimize the deviation from planned quantities by breaking the whole model into one-year sub-problems to achieve production plans for a seven-year time horizon. Kutcha et al. [12] used the aggregation method and the earliest and the latest start date to improve the tractability of their MILP model. However, more robust methods required to determine the latest start date for each machine placement were still required.

Newman et al. [16] designed a heuristic-based algorithm to directly solve the production scheduling problem of the Kiruna mine both at the machine placement level and production block level. Despite reducing the solution time as well as deviations from planned production quantities, it can only be applied for time horizons of 2 years or fewer. Newman et al. [17] reduced deviations in the total demand by developing an optimization-based decomposition heuristic. The formulation incorporates both short- and long-term production scheduling decisions to align production and demanded quantities. Shenavar et al. [22] optimized the stope boundary and applied the IP model to formulate the long-term production scheduling for a 2D representation of a real SLC mine.

The problem of obtaining an optimal mine plan for large-scale surface mining projects is often computationally challenging due to complex sequencing constraints and ore quality requirements. As a result, metaheuristics and intelligent computing methods have been widely used to approximate good solutions within a reasonable amount of time for the open-pit scheduling problem Mousavi et al. [14]. A review of metaheuristic approaches for the specific problem of long-term open-pit planning is provided by Franco-Sepulveda et al. [8].

Although metaheuristics techniques are popular in the mining industry due to their adaptability and the short time required to find good quality sub-optimal solutions, the exact methods are more reliable because of the ability to obtain the true optimal solution than metaheuristic techniques. Furthermore, there has been no holistic-viewed schedule that simultaneously considers all activities like development, mining, and processing. All existing SLC schedules focus on the extraction sequencing of ore material. This paper's primary focus is to provide a comprehensive model that integrates development activities into the production sequencing in SLC mines.

3. Problem Definition

Manual planning methods and heuristic algorithms found in commercial software do not ensure achieving an optimal schedule in SLC mining. Thus, a mathematical programming model offers a promising approach to generating an operationally feasible multi-time-period schedule for the SLC mine. This paper leverages operations research applications to optimize production scheduling using a MILP model.

In an SLC mine, the production schedule specifies the extracted tonnage and grades of scheduling units over the life of the mine to achieve a strategic objective. The proposed model generates the long-term production schedule at a mining unit level. A mining unit is where the Load-Haul-Dump unit (LHD) operates, which contains one production drift mining starts from the end of the drift and continues backward (Figure 1). Each mining unit consists of attributes like coordination, tonnage, grade, percentage of dilution, and economic data. The model determines which mining unit starts being mined in each period over the horizon to maximize the NPV. Furthermore, the model satisfies constraints such as development activities, mining, processing capacities, continuous mining of scheduling units, restrictions on the allowable number of active production faces, grade blending, and vertical and horizontal sequencing.

At the mining unit level, the schedule of each mining unit is controlled by the completion of the development activities in the production area and the level that the mining unit is located while considering the vertical and horizontal precedence relationships among mining units. The mine planner also controls the number of new mining units that need to be started in each period to meet the mining and processing plant capacities, the allowable number of active mining units, and the average production grade sent to the processing plant in each period.

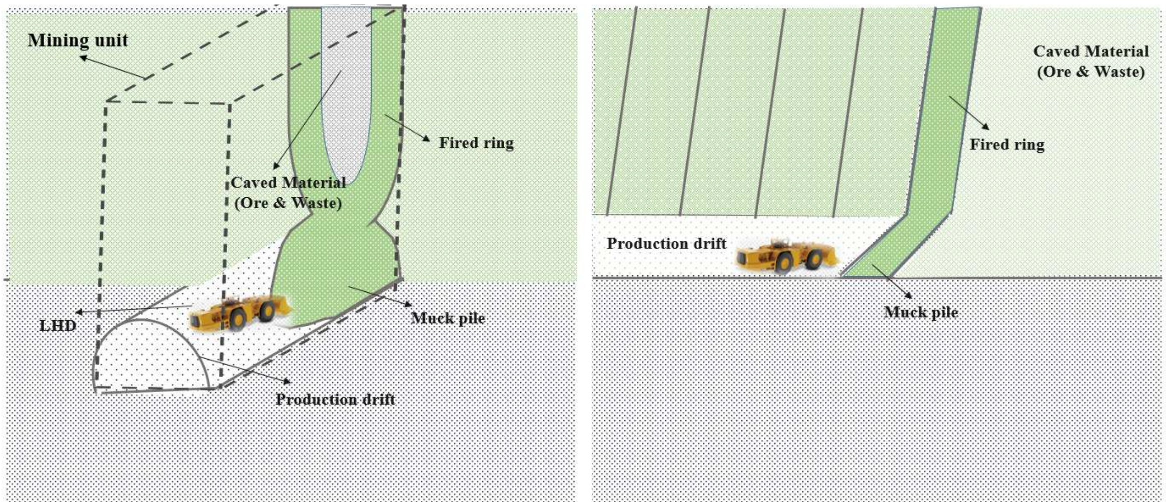

Figure 1. Typical view of mining unit in an SLC mine.

Figure 2 illustrates the schematic layout used to develop the model. The vein-like deposit with stable surrounding rock that caves in a controlled manner after drilling and blasting operations contributes to the use of the SLC mining method. On the largest scale, the mine is divided into a number of production areas, each of which includes horizontal sublevels, two orepasses, and one man-way-raise. Miners first drill vertical shafts, haulage level drift, and access routes driven along the strike of the orebody and provide access to all production areas. The development plan for each specific production area consists of capital development (man-way-raise), orepass development, and operational development (perimeter drifts and production drifts). A man-way-raise is extracted due to transferring workers and ventilation installations to supply fresh air in each sublevel. In addition, two orepasses are drilled for transferring material from each active sublevel to the haulage level. Finally, operational developments, including perimeter and production drifts, are mined to create horizontal sublevels for mining operations, including drilling, blasting, and loading extracted material from drawpoints to the orepasses. A perimeter drift in a sublevel is driven along to the strike of the orebody and is offset from the ore–waste contact on the footwall side. The perimeter drift's role is to provide access to the production drifts for ore transportation, services, and ventilation. The production drifts are staggered between the levels to provide optimal coverage for drilling and allow for the downward flow of caved material. Fan-shaped rings are drilled from the production drifts. In each sublevel, the ore is drilled and blasted in mining units. Depending on the regularity of the orebody, train systems or tuck haulages are utilized to transport ore from the orepasses to the haulage level. Then the ore is hoisted to the surface through the vertical shaft.

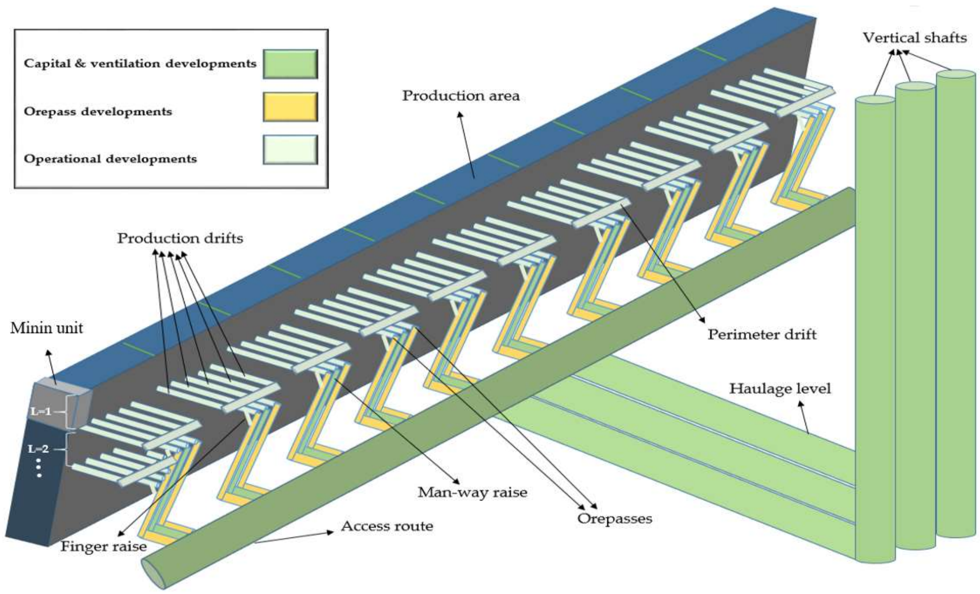

Figure 2. Schematic layout of Sublevel caving developments.

Figure 3 shows the considered mining and development activities in the model. The development activities are divided into four categories, including vertical shaft development, ventilation facilities, man-way-raise development, and ventilation requirements, in addition to orepass and operational developments. SLC is generally used as the primary mining method in mechanized mines with independent unit operations. All unit operations, including drilling, charging, blasting, loading, and transportation, are performed separately, resulting in a standardized procedure and safe operation [13].

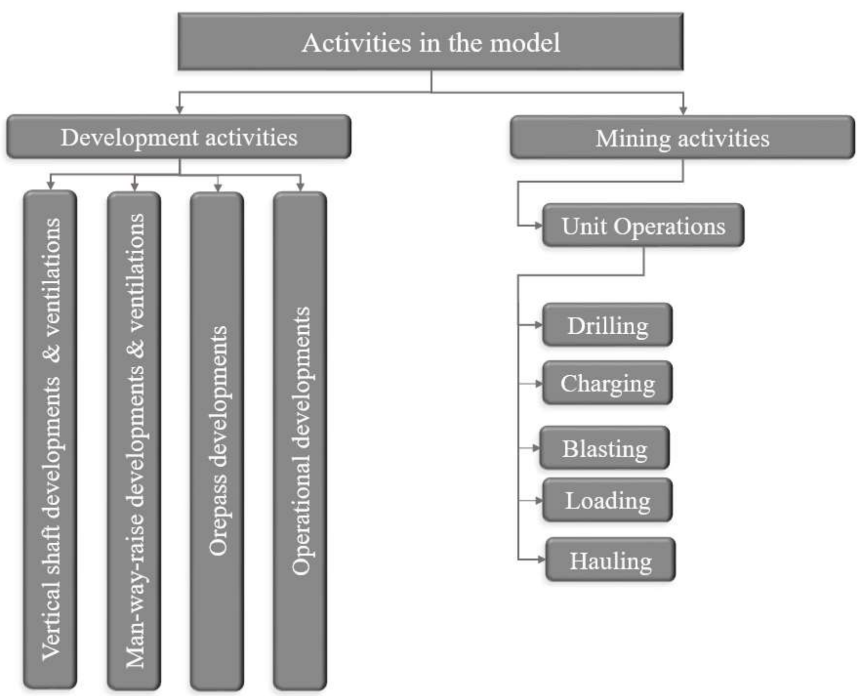

Figure 3. Mining and development activities in the SLC model.

4. Preliminary Mathematical Model for SLC Long-Term Production Scheduling

The initial model focuses on long-term decisions and constraints at the mining unit level, serving as the central scheduling unit. The formulated model aims to harmonize all development schedules with mining and processing schedules. It also manages the number of active mining units, extraction proportions per period, continuous mining, and the mining duration throughout the mine's life. Continuous decision variables track development activities and mined material for each mining unit in each period, while binary variables govern the order of development activities and whether a mining unit initiates extraction in a specific period, t.

The SLC production scheduling formulation involves defining sets, parameters, and decision variables for V production areas, L levels, and M mining units to be mined over T periods.

4.1. Sets

S_{M_v}	Set of mining units m s in the production area v.
S_{M_l}	Set of mining units in all production areas on the level l.
$S_{M_{v,l}}$	Set of mining units in the production area v on the level l.
$S_{M_m}^{V}$	Set of mining units whose start period is restricted vertically by mining unit m.
$S_{M_m}^{H}$	Set of mining units whose start period is forced by adjacency to mining unit m.

4.2. Decision Variables

$x_m^t \in [0,1]$	Continuous variable representing the portion of the mining unit to be mined at period t.
$x_{m,p}^t \in [0,1]$	Continuous variable representing the portion of mining unit m sending directly to the processing plant at period t.
bs_m^t	Binary integer variable equals one if mining of mining unit m is started at period t; otherwise, it is zero.
as_m^t	Binary integer variable equals one if mining unit m is active at period t; otherwise, it is zero.
$b_{sd_l}^{\ t} \in \{0,1\}$	Binary integer variable controlling the precedence of vertical shafts developments. It is equal to one if vertical shaft development on level l has started by or in period t; otherwise, it is zero.
$b_{vds_l}^{\ t} \in \{0,1\}$	Binary integer variable controlling the precedence of ventilation development in vertical shafts.
$b_{mwr_{v,l}}^{\ t} \in \{0,1\}$	Binary integer variable controlling the precedence of man-way-raise development in each production area. It is equal to one if man-way-raise development in production area v on level l has started by or in period t; otherwise, it is zero.

$b_{vd_{v,l}}^{t} \in \{0,1\}$ Binary integer variable controlling the precedence of ventilation development in man-way-raise in each production area.

$b_{opd_{v,l}}^{t} \in \{0,1\}$ Binary integer variable controlling the precedence of orepass development in each production area.

$b_{od_{v,l}}^{t} \in \{0,1\}$ Binary integer variable controlling the precedence of operational developments in each production area.

$d_{sd_{l}}^{t} \in [0,1]$ Continuous variable represents the vertical shaft development activities to be completed on level l at period t.

$d_{vds_{l}}^{t} \in [0,1]$ Continuous variable represents the ventilation development activities in vertical shafts to be completed on level l at period t.

$d_{mwr_{v,l}}^{t} \in [0,1]$ Continuous variable represents the man-way-raise development activities to be completed in production area v on level l at period t.

$d_{vd_{v,l}}^{t} \in [0,1]$ Continuous variable represents the ventilation development activities in the man-way-raise to be completed in production area v on level l at period t.

$d_{opd_{v,l}}^{t} \in [0,1]$ Continuous variable represents the orepass development activities to be completed in production area v on level l at period t.

$d_{od_{v,l}}^{t} \in [0,1]$ Continuous variable represents the operational development activities to be completed in production area v on level l at period t.

4.3. Parameters

csd_{l}^{t} Variable cost per length of vertical shafts development on level l at period t.

$cvds_{l}^{t}$ Variable cost per length of ventilation development in vertical shafts on level l at period t.

$cmwr_{v,l}^{t}$ Variable cost per length man-way-raise development in production area v on level l at period t.

$cvd_{v,l}^{t}$ Variable cost per length of ventilation development in man-way-raise in production area v on level l at period t.

$copd_{v,l}^{t}$ Variable cost per length of orepass development in production area v on level l at period t.

$cod_{v,l}^{t}$ Variable cost per length of operational development in production area v on level l at period t.

o_{m} Ore tonnage in mining unit m.

g_{m} Average grade of ore in mining unit m.

dil_{m} Mining dilution of mining unit m.

r_m	Mining recovery of mining unit m.
rp^t	Processing recovery: the portion of mineral recovered in mining unit m at period t.
sp^t	Selling price in present value terms obtainable per unit of the mineral commodity at period t.
sc^t	Selling cost in present value terms obtainable per unit of mineral commodity.
ec_m^t	Mining and processing cost per ton of ore extracted from mining unit m at period t.
p^t	Penalty cost per ton associated with tonnage deviations at period t.
l_a	The uppermost level in each production area assumed the same for all of them $(l=1)$.
l_v	The lowest level in each production area assumed the same for all of them $(l=L)$.
dl_{sd_l}	Vertical shaft development length on the level l.
dl_{vds_l}	The length of ventilation development in the vertical shaft on the level l.
$dl_{mwr_{v,l}}$	The length of the man-way-raise development in the production area v on the level l.
$dl_{vd_{v,l}}$	The length of ventilation development in man-way-raise in production area v on the level l.
$dl_{opd_{v,l}}$	Orepass development length in production area v on the level l.
$dl_{od_{v,l}}$	Operational development length in production area v on the level l.
$\underline{Dev_{sd}^t}$	Lower bound on vertical shaft development at period t.
$\overline{Dev_{sd}^t}$	Upper bound on vertical shaft development at period t.
$\underline{Dev_{vds}^t}$	Lower bound on ventilation development in vertical shafts at period t.
$\overline{Dev_{vds}^t}$	Upper bound on ventilation development in vertical shafts at period t.
$\underline{Dev_{mwr}^t}$	Lower bound on man-way-raise development for all production areas at period t.
$\overline{Dev_{mwr}^t}$	Upper bound on man-way-raise development for all production areas at period t.

$\underline{Dev_{vdv}^{t}}$	Lower bound on ventilation development in man-way-raise for all production areas at period t.
$\overline{Dev_{vdv}^{t}}$	Upper bound on ventilation development in man-way-raise for all production areas at period t.
$\underline{Dev_{opd}^{t}}$	Lower bound on orepass development for all production areas at period t.
$\overline{Dev_{opd}^{t}}$	Upper bound on orepass development for all production areas at period t.
$\underline{Dev_{od}^{t}}$	Lower bound on operational development for all production areas at period t.
$\overline{Dev_{od}^{t}}$	Upper bound on operational development for all production areas at period t.
$\underline{Ton^{t}}$	Lower bound on mining capacity at period t.
$\overline{Ton^{t}}$	Upper bound on mining capacity at period t.
$\underline{Ton_{p}^{t}}$	Lower bound on ore processing capacity at period t.
$\overline{Ton_{p}^{t}}$	Upper bound on ore processing capacity at period t.
$\underline{g_{p}^{t}}$	Lower bound on an acceptable average grade by processing plant at period t.
$\overline{g_{p}^{t}}$	Upper bound on an acceptable average grade by processing plant at period t.

4.4. Objective Function

The optimization model maximizes the NPV of caving operations in Eq. (1), subtracting the cost of all development activities from the revenue obtained from ore extraction.

$$
\begin{aligned}
&Max\left[\sum_{t=1}^{T}\sum_{m=1}^{M}\frac{([o_m\times g_m\times r_m\times rp^t\times(sp^t-sc^t)-o_m\times(1+dil_m)\times ec_m^t]\times x_{m,p}^t)}{(1+i)^t}\right] \\
&-\left[\begin{array}{l}
\displaystyle\sum_{t=1}^{T}\sum_{l=1}^{L}\frac{(csd_l^t\times dl_{sd_l})\times d_{sd_l}^{\,t}+(cvds_l^t\times dl_{vds_l})\times d_{vds_l}^{\,t}}{(1+i)^t} \\
+ \\
\displaystyle\sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{l=1}^{L}\left[\frac{(cmrw_{v,l}^t\times dl_{mwr_{v,l}})\times d_{mwr_{v,l}}^{\,t}+(cvd_{v,l}^t\times dl_{vd_{v,l}})\times d_{vd_{v,l}}^{\,t}+(copd_{v,l}^t\times dl_{opd_{v,l}})\times d_{opd_{v,l}}^{\,t}+(cod_{v,l}^t\times dl_{od_{v,l}})\times d_{od_{v,l}}^{\,t}}{(1+i)^t}\right]
\end{array}\right]
\end{aligned}
\tag{1}
$$

4.5. Capital Development Constraints

Capital development is divided into two phases of vertical shafts and man-way-raises. That is developed in production areas. Eq. (2) defines the vertical shaft development capacity. The inequality ensures that the total length of capital development required in each period is within the stated lower and upper limits of the whole available equipment capacity for developing the mine. Eqs. (3) to (5) control the precedence relation between the sections of vertical shaft developments and the ventilation developments on each level. Eq. (6) ensures that the ventilation development on the level l_a starts after completing the vertical shaft developments on the lowest level.

$$\underline{Dev_{sd}^{t}} \leq \sum_{l=1}^{L} (dl_{sd_l} \times d_{sd_l}^{\ t}) \leq \overline{Dev_{sd}^{t}} \qquad \forall t \in \{1,2,...,T\} \tag{2}$$

$$b_{sd_{l+1}}^{\ t} - \sum_{t' \leq t} d_{sd_l}^{\ t'} \leq 0 \qquad \forall l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \tag{3}$$

$$\sum_{t' \leq t} d_{sd_l}^{\ t'} - b_{sd_l}^{\ t} \leq 0 \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{4}$$

$$b_{sd_l}^{\ t} - b_{sd_l}^{\ t+1} \leq 0 \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \tag{5}$$

$$b_{vds_{l_a}}^{\ t} - \sum_{t' < t} d_{sd_{l_v}}^{\ t'} \leq 0 \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{6}$$

Eq. (7) defines the man-way-raise development capacity constraint for the mine. Eqs. (8) to (10) control the precedence relation between the sections of man-way-raise developments on each level within a production area. Eq. (11) and (12) prevent man-way-raise ventilation and orepass development on the level l_v before completing the man-way-raise developments on the top level in the production area v.

$$\underline{Dev_{mwr}^{t}} \leq \sum_{v=1}^{V}\sum_{l=1}^{L} (dl_{mwr_{v,l}} \times d_{mwr_{v,l}}^{\ t}) \leq \overline{Dev_{mwr}^{t}} \qquad \forall t \in \{1,2,...,T\} \tag{7}$$

$$b_{mwr_{v,l}}^{\ t} - \sum_{t' \leq t} d_{mwr_{v,l+1}}^{\ t'} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \tag{8}$$

$$\sum_{t' \leq t} d_{mwr_{v,l}}^{\ t'} - b_{mwr_{v,l}}^{\ t} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{9}$$

$$b_{mwr_{v,l}}^{\ t} - b_{mwr_{v,l}}^{\ t+1} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \tag{10}$$

$$b_{vd_{v,l_v}}^{\ t} - \sum_{t' < t} d_{mwr_{v,l_a}}^{\ t'} \leq 0 \qquad \forall v \in \{1,2,...,V\}, t \in \{1,2,...,T\} \tag{11}$$

$$b_{opd_{v,l_v}}^{\ t} - \sum_{t' < t} d_{mwr_{v,l_a}}^{\ t'} \leq 0 \qquad \forall v \in \{1,2,...,V\}, t \in \{1,2,...,T\} \tag{12}$$

4.6. Ventilation Development Constraints

Ventilation development consists of two phases of ventilation installation in both vertical shaft and man-way-raises in production areas. Eq. (13) defines the ventilation development capacity constraint for the mine. Eqs. (14) to (16) control the precedence relation between the sections of ventilation developments and the ventilation developments on each level. Eq. (17) ensures that no man-way-raise development can start before completing the ventilation development in the vertical shaft on the level l_v.

$$\underline{Dev_{vds}^{t}} \leq \sum_{l=1}^{L} (dl_{vds_l} \times d_{vds_l}^{\ t}) \leq \overline{Dev_{vds}^{t}} \qquad \forall t \in \{1,2,...,T\} \tag{13}$$

$$b_{vds_{l+1}}^{\ t} - \sum_{t'\le t} d_{vds_l}^{\ t'} \le 0 \qquad \forall l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \tag{14}$$

$$\sum_{t'\le t} d_{vds_l}^{\ t'} - b_{vds_l}^{\ t} \le 0 \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{15}$$

$$b_{vds_l}^{\ t} - b_{vds_l}^{\ t+1} \le 0 \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \tag{16}$$

$$b_{mwr_{v,l_v}}^{\ t} - \sum_{t'<t} d_{vds_{l_v}}^{\ t'} \le 0 \qquad \forall v \in \{1,2,...,V\}, t \in \{1,2,...,T\} \tag{17}$$

Eq. (18) checks the man-way-raise development capacity for the mine. Eqs. (19) to (21) dictate the precedence relationships between the stages of ventilation facility installation in man-way-raise developments on each level within a production area. Eq. (22) enforces the operational development on the uppermost level in the production area v to complete the man-way-raise ventilation developments on that level and production area.

$$\underline{Dev_{vdv}^t} \le \sum_{v=1}^{V}\sum_{l=1}^{L}(dl_{vd_{v,l}} \times d_{vd_{v,l}}^{\ t}) \le \overline{Dev_{vdv}^t} \qquad \forall t \in \{1,2,...,T\} \tag{18}$$

$$b_{vd_{v,l}}^{\ t} - \sum_{t'\le t} d_{vd_{v,l+1}}^{\ t'} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \tag{19}$$

$$\sum_{t'\le t} d_{vd_{v,l}}^{\ t'} - b_{vd_{v,l}}^{\ t} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{20}$$

$$b_{vd_{v,l}}^{\ t} - b_{vd_{v,l}}^{\ t+1} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \tag{21}$$

$$b_{od_{v,l_a}}^{\ t} - \sum_{t'<t} d_{vd_{v,l_a}}^{\ t'} \le 0 \qquad \forall v \in \{1,2,...,V\}, t \in \{1,2,...,T\} \tag{22}$$

4.7. Orepass Development Constraints

Eq. (23) dictates the total orepass development length between the lower and upper bounds of the total available equipment capacity. Eqs. (24) to (26) control the precedence relationships between the stages of orepass developments on each level within a production area. Eq. (27) enforces the operational development on the uppermost level in each production area to complete orepass developments on level l_a.

$$\underline{Dev_{opd}^t} \le \sum_{v=1}^{V}\sum_{l=1}^{L}(dl_{opd_{v,l}} \times d_{opd_{v,l}}^{\ t}) \le \overline{Dev_{opd}^t} \qquad \forall t \in \{1,2,...,T\} \tag{23}$$

$$b_{opd_{v,l}}^{\ t} - \sum_{t'\le t} d_{opd_{v,l+1}}^{\ t'} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \tag{24}$$

$$\sum_{t'\le t} d_{opd_{v,l}}^{\ t'} - b_{opd_{v,l}}^{\ t} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{25}$$

$$b_{opd_{v,l}}^{\ t} - b_{opd_{v,l}}^{\ t+1} \le 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \tag{26}$$

$$b_{od_{v,l_a}}^{t} - \sum_{t'<t} d_{opd_{v,l_a}}^{t'} \leq 0 \qquad \forall v \in \{1,2,...,V\}, t \in \{1,2,...,T\} \tag{27}$$

4.8. Operational Development Constraints

If a mining unit is scheduled to be mined in a period, a set of operational development must be ready ahead or in that period. Eq. (28) defines the operational development capacity constraints. Eqs. (29) to (31) control the lateral precedence relation of the operational development required for mining each mining unit. Eq. (32) ensures that the operational developments in the production area v, and on level l must be completed before any of the mining units in that production area, and on that level starts being mined.

$$\underline{Dev_{od}^{t}} \leq \sum_{v=1}^{V} \sum_{l=1}^{L} (dl_{od_{v,l}} \times d_{od_{v,l}}^{t}) \leq \overline{Dev_{od}^{t}} \qquad \forall t \in \{1,2,...,T\} \tag{28}$$

$$b_{od_{v,l+1}}^{t} - \sum_{t'\leq t} d_{od_{v,l}}^{t'} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L-1\}, t \in \{1,2,...,T\} \tag{29}$$

$$\sum_{t'\leq t} d_{od_{v,l}}^{t'} - b_{od_{v,l}}^{t} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{30}$$

$$b_{od_{v,l}}^{t} - b_{od_{v,l}}^{t+1} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T-1\} \tag{31}$$

$$bs_{m}^{t} - \sum_{t'\leq t} d_{od_{v,l}}^{t'} \leq 0 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\}, t \in \{1,2,...,T\}, m \in S_{M_{v,l}} \tag{32}$$

4.9. Mining and Processing Capacity

Eq. (33) enforces the mining capacity, using continuous decision variable x_m^t, between the acceptable lower and upper limits of the total available equipment capacity in each period. Eq. (34) controls the quantity of mill feed using continuous decision variable $x_{m,p}^t$.

$$\underline{Ton^{t}} \leq \sum_{m=1}^{M} \left[(o_m \times (1+dil_m) \times x_m^t \right] \leq \overline{Ton^{t}} \qquad \forall t \in \{1,2,...,T\} \tag{33}$$

$$\underline{Ton_p^{t}} \leq \sum_{m=1}^{M} (o_m \times (1+dil_m) \times x_{m,p}^t) \leq \overline{Ton_p^{t}} \qquad \forall t \in \{1,2,...,T\} \tag{34}$$

4.10. Active Mining Units Constraints

Crew and LHD availability limit the number of active mining units in each period. LHD restrictions in each level, each production area, and each period within all production areas are controlled by Eq. (35) to Eq. (37), respectively.

$$\sum_{m\in S_{M_l}} as_m^t \leq LHD_l \qquad \forall l \in \{1,2,...,L\}, t \in \{1,2,...,T\} \tag{35}$$

$$\sum_{m\in S_{M_v}} as_m^t \leq LHD_v \qquad \forall v \in \{1,2,...,V\}, t \in \{1,2,...,T\} \tag{36}$$

$$\sum_{m=1}^{M} as_m^t \le LHD_t \qquad \forall t \in \{1,2,...,T\} \qquad (37)$$

4.11. Continuous Mining Constraints

Each mining unit must be continuously extracted after opening until closing. Eq. (38) forces variable as_m^t to be zero if no portion of the mining unit m is extracted at period t, while Eq. (39) changes the value as_m^t to 1 when a portion of the mining unit m is extracted at period t. Eq. (40) ensures that if extraction from the mining unit m is started during or after period two, at least a portion of the mining unit is extracted until all of the material within that mining unit has been extracted; otherwise, the mining unit must be closed. Eq. (41) ensures that if extraction from the mining unit m is started in period one, the related variable as_m^t for the mining unit is equal to 1.

$$as_m^t \le M \times x_m^t \qquad \forall t \in \{1,2,...,T\}, m \in \{1,2,...,M\} \qquad (38)$$

$$x_m^t \le as_m^t \qquad \forall t \in \{1,2,...,T\}, m \in \{1,2,...,M\} \qquad (39)$$

$$as_m^t - as_m^{t-1} \le bs_m^t \qquad \forall m \in \{1,2,...,M\}, t \in \{2,...,T\} \qquad (40)$$

$$as_m^1 - bs_m^1 \le 0.5 \qquad \forall m \in \{1,2,...,M\} \qquad (41)$$

4.12. Processing Plant Grade Control Constraint

Eq. (42) meets the ore quality specification of the processing plant within the predefined limits.

$$\underline{g_p^t} \le \left[\frac{\sum_{m=1}^{M} (g_m \times o_m \times (1+dil_m) \times x_{m,p}^t)}{\sum_{m=1}^{M} (o_m \times (1+dil_m) \times x_{m,p}^t)} \right] \le \overline{g_p^t} \qquad \forall t \in \{1,2,...,T\} \qquad (42)$$

4.13. Vertical and Horizontal Sequencing Constraints

Some strict operational rules must be satisfied when extracting the ore body using the SLC method. Figure 4 illustrates six mining units. Considering the operational rules, vertical sequencing requires MU5 to start being mined only after MU2 is returned to a safe distance. Furthermore, horizontal sequencing dictates that MU2 starts being mined once after MU1 retreats past enough its neighbour on the same level. Eqs. (43) and (44) enforce vertical and horizontal sequencing between mining units modelled with the long-term resolution, respectively. These operational rules are satisfied using two sets of $S_{M_m}^V$ and $S_{M_m}^H$ the required extraction percentage for the predecessor mining units, which is assumed constant for all mining units δ_m.

$$bs_m^t - \sum_{t'=1}^{t} x_{m'}^{t'} \le \delta_m \qquad \forall m \in \{1,2,...,M\}, t \in \{1,2,...,T\}, m' \in S_{M_m}^V \qquad (43)$$

$$bs_m^t - \sum_{t'=1}^{t} x_{m'}^{t'} \le \delta_m \qquad \forall m \in \{1,2,...,M\}, t \in \{1,2,...,T\}, m' \in S_{M_m}^H \qquad (44)$$

Figure 4. A depiction of six mining units.

4.14. Variable Control Constraints

Eq. (45) enforces the extracted material from each mining unit to be sent to the processing plant in each period. Eq. (46) to Eq. (47) ensure that the total fraction of material mined and sent to the processing plant is less than one over the scheduling periods. Eq. (48) to Eq. (53) ensure that all developments will be completed once over the life of the mine. Eq. (54) prevents a mining unit from being mined more than once. Eq. (55) ensures that all continuous decision variables in the model are between zero and one. Eq. (56) guarantees that all binary variables are non-negative and integers.

$$x_{m,p}^{t} = x_{m}^{t} \qquad \forall t \in \{1,2,...,T\}, m \in \{1,2,...,M\} \tag{45}$$

$$\sum_{t=1}^{T} x_{m}^{t} \le 1 \qquad \forall m \in \{1,2,...,M\} \tag{46}$$

$$\sum_{t=1}^{T} x_{m,p}^{t} \le 1 \qquad \forall m \in \{1,2,...,M\} \tag{47}$$

$$\sum_{t=1}^{T} d_{sd_{l}}^{t} \le 1 \qquad \forall l \in \{1,2,...,L\} \tag{48}$$

$$\sum_{t=1}^{T} d_{vds_{l}}^{t} \le 1 \qquad \forall l \in \{1,2,...,L\} \tag{49}$$

$$\sum_{t=1}^{T} d_{mwr_{v,l}}^{t} \le 1 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\} \tag{50}$$

$$\sum_{t=1}^{T} d_{vd_{v,l}}^{t} \le 1 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\} \tag{51}$$

$$\sum_{t=1}^{T} d_{opd_{v,l}}^{t} \le 1 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\} \tag{52}$$

$$\sum_{t=1}^{T} d_{od_{v,l}}^{t} \le 1 \qquad \forall v \in \{1,2,...,V\}, l \in \{1,2,...,L\} \tag{53}$$

$$\sum_{t \in T_m} bs_{m}^{t} \le 1 \qquad \forall m \in \{1,2,...,M\} \tag{54}$$

$$x_{m}^{t}, x_{m,p}^{t}, d_{sd_{l}}^{t}, d_{vds_{l}}^{t}, d_{mwr_{v,l}}^{t}, d_{vd_{v,l}}^{t}, d_{opd_{v,l}}^{t}, d_{od_{v,l}}^{t} \in [0,1] \tag{55}$$

$$bs_m^t, as_m^t, b_{sd\,l}^{\;t}, b_{vds\,l}^{\;t}, b_{mwr\,v,l}^{\;t}, b_{vd\,v,l}^{\;t}, b_{opd\,v,l}^{\;t}, b_{od\,v,l}^{\;t} \in \{0,1\} \tag{56}$$

5. Implementation of the MILP Model on an Illustrative Example

The effectiveness of the proposed models was assessed by evaluating NPV, mining production, and the practicality of the resulting schedules. The objective is to maximize NPV at a 15% discount rate while ensuring all constraints are met throughout the mine's lifespan. Additionally, the model assumes zero dilution for all mining units.

The proposed formulation is tested using a small-scale illustrative case. The orebody, resembling a vein, is situated 300 meters beneath the surface and comprises 18 mining units, each measuring 25×25×50. The mining activities occur at depths between 300 and 375 meters, distributed across three distinct levels, with each level having a height of 25 meters. The SLC mine is organized into three production areas, each spanning 50 meters and including three horizontal sublevels. Additionally, two vertical orepasses and one man-way-raise provide access to the sublevels, which are progressively deeper cuts in the earth's surface. Within each sublevel, the blasted ore is loaded and hauled by LHDs from the production face to the ore passes, where it is then directly transported to the main haulage level.

This paper relies on several key assumptions in the MILP formulations. The vertical shafts are considered the primary exit point for all materials, and a first-in-first-out approach is used with no material mixing. Development activities in production areas begin only after the vertical shaft development and its ventilation installations are completed. To avoid potential damage, orepasses and man-way-raise are not allowed to be mined simultaneously. As a result, man-way-raise is drilled first before commencing orepass mining in each production area. Furthermore, no operational developments are permitted to begin until man-way-raise ventilation and orepass developments in each production area are completed.

The ore extracted from the mine is sent directly to the surface-based processing plant. To mitigate stress and explosive damage on neighboring mining units, a predetermined number of LHDs operate simultaneously. This approach also prevents congestion and potential damage resulting from LHDs' operation. Once an LHD starts mining a mining unit, it must continue until completion; otherwise, the unit will be closed. According to the vertical and horizontal sequencing rules, at least 50% of a mining unit should be mined before starting to mine units below and adjacent to it.

All the development activities boundaries that are applied to the model are provided in Table 1. These annual boundaries have been assumed the same throughout the mine life.

Table 1. Development activities boundaries (m) over the life of mine (LB: Lower bound, UB: Upper bound).

Vertical Shaft		**Vertical Shaft Ventilation**		**Man Way Raise**		**Man Way Raise Ventilation**		**Orepass**		**Operational**	
LB	**UB**	**LB**	**UB**	**LB**	**UB**	**LB**	**UB**	**LB**	**UB**	**LB**	**UB**
0	250	0	500	0	300	0	300	0	300	0	600

Figure 5 provides a schematic layout of production areas, levels, mining units, and development activities through two cross-sections (i) along the strike of the orebody and (ii) along the dip of the orebody. Figure 6 shows the tonnage and the average grade of each mining unit. In Figure 6, the available tonnage of each mining unit, along with their average grade, is provided. In the horizontal axis, mining units are recognized based on their location (the production area and the level). For instance, MU9 and MU 10 are in Level 2 and Production Area 2.

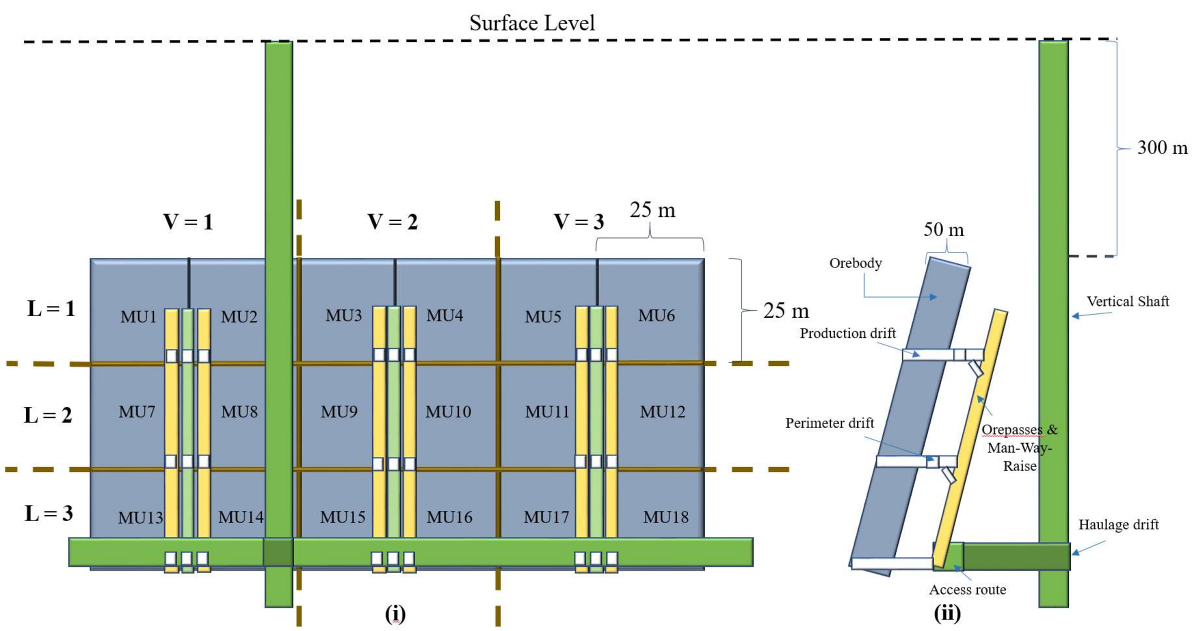

Figure 5. A schematic representation of production areas, levels, mining units, and development layout: (i) is a cross-section along the strike of the orebody, and (ii) is a cross-section along the dip of the orebody.

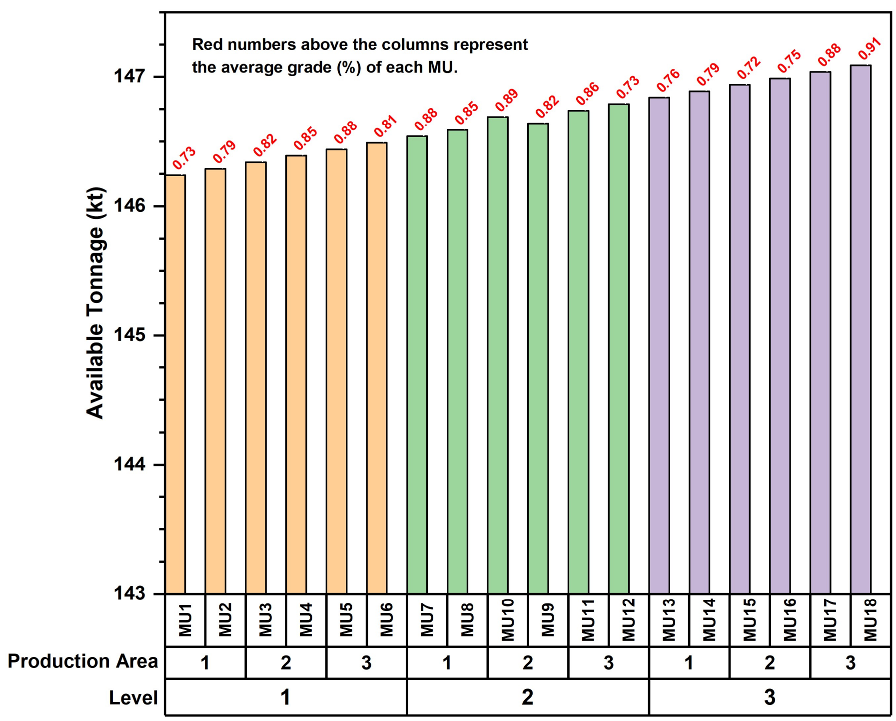

Figure 6. Summary of the tonnage and average grade of mining units in different levels and production areas.

6. Results and Discussion

Table 2 shows the components of the model. Figure 7 represents the development activities' schedules obtained from the model.

Table 2. Mathematical model components.

Model components	Number
Sets	7
Binary decision variables	1248
Continuous decision variables	1248
Constraints	4568

Figure 8 illustrates the annual mine production and processing plant feed, as well as the headgrade of the processing plant. It can be seen that the processing plant capacity limits production each year. Also, the first production takes place in year 7, which indicates that the vertical shaft and its ventilations, as well as all the developments in the first production area, including the man-way-raise and its ventilations, the orepass, and the operational development, need to be completed before commencing any ore extraction. Figure 9 shows the active periods and the extracted portion of each mining unit over the life of the mine. It can be seen that from MU12 to MU18, only 90% of them are extracted.

The developed model has been coded in Jupyter notebook while taking advantage of CPLEX Python API to solve and optimize the model on a PC Intel Core i7, 2.60 GHz, with 16 GB of RAM, running Windows 11. The gap tolerance is considered zero while running the model.

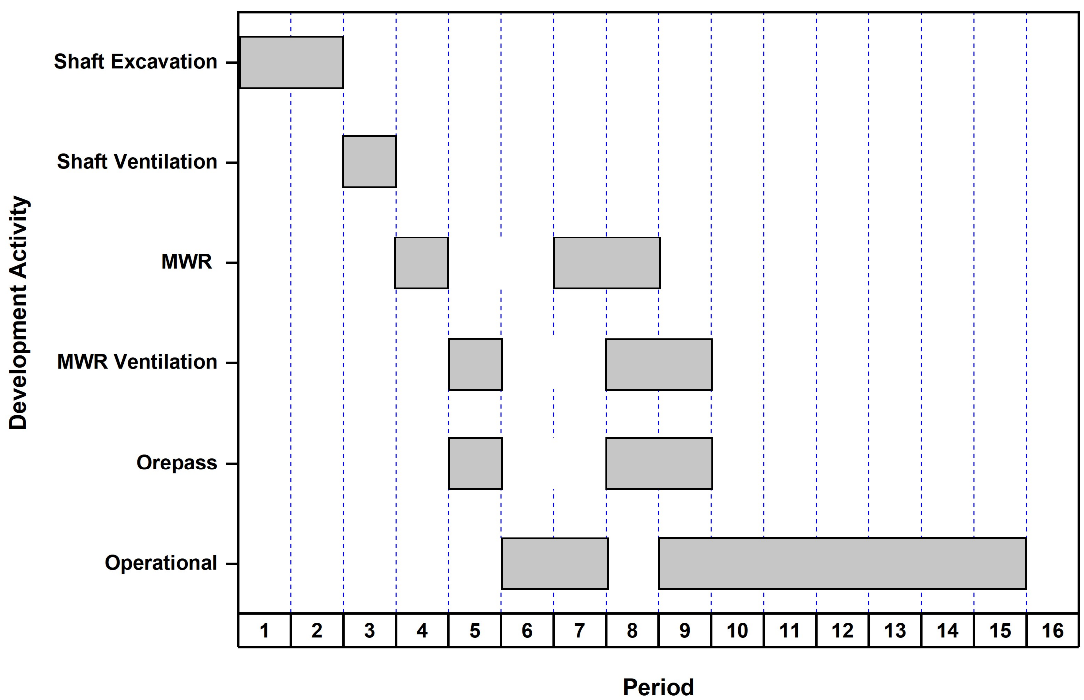

Figure 7. Development activities over the life of mine.

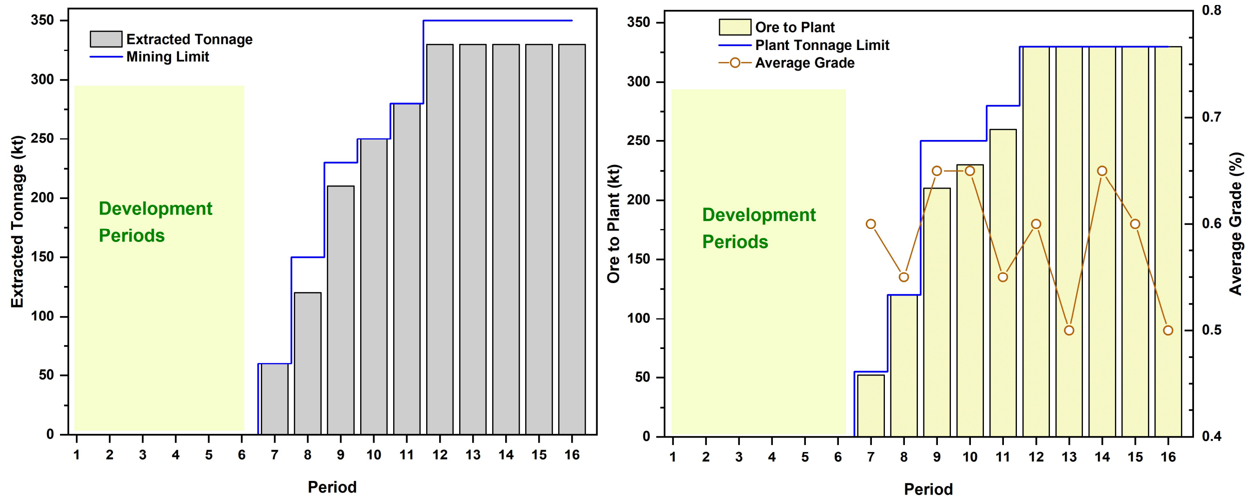

Figure 8. Mine extraction and processing plant feed over the life of mine.

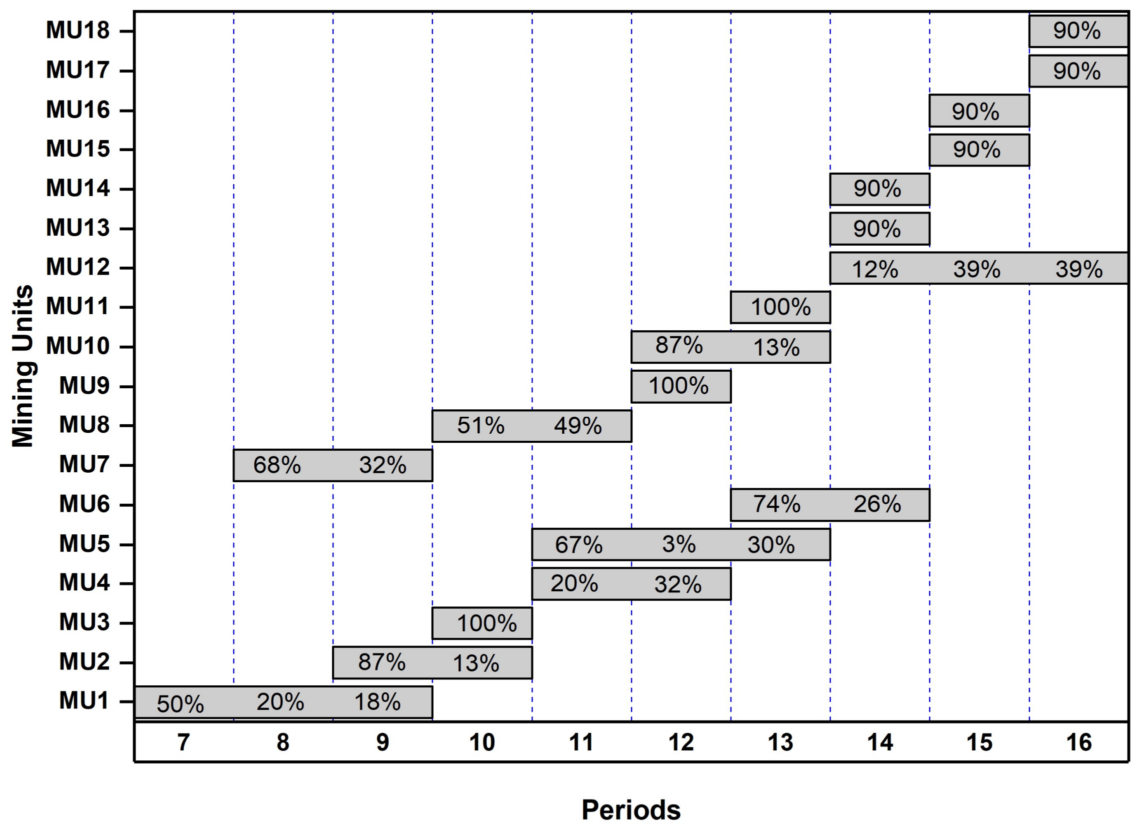

Figure 9. Mining units scheduling (extraction start time, duration, and portion (%) of each mining unit).

To examine the influence of the number of mining units on the running time of the model, it was evaluated for three different quantities of mining units, specifically 18, 36, and 54, resulting in respective running times of 11.9, 98.38, and 768.63 seconds. To ensure a constant mine life, the mine production and processing plant capacities were increased in proportion to the number of mining units. The outcomes indicate an exponential increase in the running time with the addition of mining units. It is noteworthy to mention that our analysis was performed on a schematic SLC mine, and the subsequent phase of our research will involve applying the model to an actual case study.

7. Conclusion and future works

This paper introduces a comprehensive mathematical model aimed at optimizing sublevel caving (SLC) production scheduling to maximize NPV. The model utilizes a mixed-integer-linear programming formulation implemented in the Jupyter Notebook with the CPLEX Python API for solving and optimization. The scheduling of each mining unit is governed by the completion of development activities in the production area and the respective level while considering vertical and horizontal precedence relationships between mining units. It also manages the number of new mining units to be mined in each period to meet production requirements, the active mining unit count per period, and the average production grade. Additionally, the model integrates pre-extraction material flow and development activities into mining unit schedules.

Future research will focus on adapting the model to handle post-extraction material flow, which involves incorporating a stockpiling system to manage processing plant capacity and material flows from the mine to a stockpile, mine to plant, and stockpile to plant. Based on mining unit schedule results, more detailed scheduling at the production ring level will be explored since each mining unit encompasses multiple production rings across the orebody's width, resulting in a 3D-dimensional plan. Moreover, efficient mathematical techniques will be investigated to reduce the number of decision variables and execution time for large-scale SLC production scheduling.

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