# Pit Optimization Design Considering Minimum Mining Width for the Production Scheduling Problem

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## ABSTRACT

Mine planning leads to prioritizing the extraction of blocks, which covers scheduling, and determining pit limits. Therefore, powerful equipment with substantial space requirements is employed to extract the materials. In recent studies, a main solution is the automatic design of functional sections, known as pushbacks. A practical design considers desirable mining or bench width at the bottom of the mine and between successive pushbacks. Therefore, this paper presents an optimization mathematical model including minimum mining width and constraints based on mixed integer linear programming. The model has two main objectives. The first one is to maximize total profit. The second objective is to decrease greenhouse gas emissions from transportation and production operations in the mine. Simultaneously, the introduced model investigates production scheduling problems for extraction, mining, and processing steps. Also, implemented approaches to solve the programming models from meta-heuristic to exact solution methods have been compared, and based on that, an exact solution methodology in Python using Gurobipy is chosen. For solving the problems on a large scale, a Simulated Annealing metaheuristic algorithm and a Genetic algorithm are applied. After analyzing the results, it is concluded for large-scale problems, the SA algorithm, and for short-scale problems the exact method is more efficient.

Keywords: Pit optimization, production scheduling, push-back design, minimum mining width

### 1. Introduction

In the pit mine design problem, the ore deposit is better to be determined by a three-dimensional array of blocks which has properties such as grade and tonnage. This 3-D array with different properties is named the block model that is the core concept in mining studies. The fundamental data for the mine planning problem is the block model with operational and geotechnical constraints, economic attributes and costs. Based on these initial data, the production scheduling problem is able to be designed for extraction, mining and processing steps. The main objective in all of these problems is to maximize profit and economic value of the project. Therefore, the base of mine planning problems is optimization using the proposed algorithms [1]. To calculate the economic value of the mine, it is necessary to define the final pit and its limitations which can indicate the ore and waste blocks [2].

Decision makers in a mining project should be aware to define extended and developed parts to make shovels operate easier around the mine. The best way is to consider a minimum bottom width and then exclude the lowest width to reach more width in the next levels. Also, this is important for mine planners to determine mining or bench width among distinct pushbacks of the project providing the required access for large shovels and other equipment. There are different tools to propose a mine planning, production scheduling and push-back design problem: Mixed Integer Linear Programming [1, 3, 4], Integer Programming [5], Dynamic Programming [6], Meta-heuristic Algorithms [1, 7-9], Stochastic Optimization [9-11]. This research aims to propose a comprehensive reference to assess and compare the existing studies and applied methods of modeling and solving the related problems on production scheduling and pit optimization which have considered minimum mining width and push-back design. Also, in this study an optimization mathematical model is proposed in which the pit and pushback design constraints are followed and considers minimum mining width limitation. To solve the MILP model in large-scale problems, a maximum flow algorithm is applied with a genetic algorithm. Based on the study of Paithannkar [7], the graph structure for maximum flow is created for multiple periods under uncertainty, and the flow in the arcs is controlled by a genetic algorithm to develop a production schedule. Also, another metaheuristic method of Simulated Algorithm which is used by Christian Both [9] is implemented. The model proposed in this research has two main objective functions. The first one is to maximize total profit which is total income minus capital costs and operational costs. The second objective is to decrease greenhouse gas emissions from transportation and production operations in the mine. Carbon emissions are produced from different sources when extracting, mining, and processing. Also, GHG from facilities such as trucks, and shovels and energy consumption affect the amount of carbon produced in the mine environment [6].

## 2. Literature Review

In this section, a comprehensive review of the research topic, production scheduling with minimum mining width is proposed and then some of the main studies about pit optimization and pushback design is discussed.

### 2.1. Production Scheduling with Minimum Mining Width

The minimum mining width is an important constraint that makes mine planning more profitable and practical. For the first time, this concept was introduced by the research of Wharton et al. [12] in which the effect of minimum mining width on net present value was studied. They investigated the application of an automated approach to include minimum mining width for Four-X pit outlines. The final NPV of the project has an optimized value since this reaches a reduction in NPV. The reason for achieving a less NPV is due to the waste stripping inclusion and a space for mining extra in case of a pit development. Pourrahimian et al. [3] developed two distinct mixed integer linear programming models for production scheduling, taking into account various linear constraints such as minimum mining width. The researchers also considered the accessibility and mobility of equipment as part of the problem. In addition, they employed block clustering prior to optimizing the schedule. The outcomes of their study demonstrated that the second model successfully generated a realistic mining schedule, allowing sufficient room for equipment maneuverability and preventing the scattering of excavation sequences within the designated scheduling period. Yarmuch et al. [8] proposed a model focused on maximizing the value of pushbacks in mining operations. In their model, they introduced a closeness factor as a measure of the mineability of a design. The researchers considered that pushbacks should possess sufficient mining width and be composed of connected blocks. Furthermore, each pushback was required to have a ramp connecting its bottom to the surface. Upon solving the model, they concluded that it effectively captured the mineability conditions and provided appropriate guidelines for pushback design. The optimized design achieved through their proposed method exhibited a notable improvement of 15.517 USD million (6.71%) in the net present value (NPV) of the case study.

Another study was conducted by Yarmuch et al. [13] about a novel optimization model for semipractical pushbacks, taking into consideration operational conditions such as minimum operational width and connectivity among the blocks forming the pushbacks. The primary objective of the model was to maximize the approximate net present value (NPV) of mining extraction. To address this problem, the researchers proposed an algorithm incorporating a Sliding Window Heuristic, variable bounding, and other preprocessing routines. By utilizing the proposed algorithm, they were able to obtain high-quality solutions within significantly less amount of time compared to directly implementing and solving the model formulation using MILP solvers. Malharkumar et al.[14] conducted a study on an iron deposit, focusing on the uncertainty of ore grade. They formulated a production scheduling problem to evaluate and generate a production schedule for the iron ore deposit. The researchers specifically examined a case study in India. The operational requirements such as minimum mining width at pit bottom and maximum vertical depth are considered for planning the mine project. Their findings indicated that proper planning and production scheduling have the potential to enhance the profitability of iron ore projects by minimizing in-situ grade variability and uncertainty regarding the spatial distribution of ore. Additionally, in the case study, out of the five production scenarios generated, the mining direction towards the north was preferred due to its ability to meet specified quality targets.

Nancel-Penard et al. [1] conducted a study focusing on the minimum mining width, aiming to enhance profitability by considering geospatial and design limitations. Their primary objective was to optimize the pushback design for strategic planning in an open-pit mining scenario. To achieve this, they developed an integer linear programming model to generate optimal ultimate pits based on three designated block models. To solve the model efficiently, they employed a modified Lerchs and Grossman algorithm alongside a preprocessing heuristic approach. The findings of their research indicated that the proposed preprocessing technique reduced computation time by a factor of 14 in one of the block models. Also, Moradi-Afrapoli et al. [15] proposed a study in which advanced analytics for surface mining is investigated. They studied in-pit crushing and equipment selection in their research and applied multi criteria decision-making methods.

### 2.2. Pit Optimization and Pushback Design

Tabesh et al. [16] put forth a multi-step approach to address the long-term production planning problem in open-pit mining. Their approach encompassed three essential components: controlled optimal phase-design, selective mining-unit characterization, and long-term production scheduling optimization. To tackle this problem, the researchers utilized a combination of a greedy heuristic, a local search algorithm, and a clustering algorithm. The pushback design algorithm they developed provided the mine planner with the ability to regulate the mineralized material and rock tonnage within each pushback. Furthermore, the results obtained from the pushback design procedure exhibited tonnage uniformity. Kaydim et al.[5] introduced an integer programming model aimed at optimizing the long-term production scheduling of open-pit mines by maximizing the net present value (NPV). The model considered both minimum and maximum achievable mining and processing capacity values. By optimizing the model, the researchers were able to maximize the NPV for an eight-year production period, achieving a negligible optimality gap. The results indicated that metal prices were the most influential parameter on the resulting NPV value, even in the optimized production scenario.

Xu et al.[6] presented an optimization problem for production scheduling in open-pit metal mines, considering ecological costs that incorporated the carbon emission cost of energy consumption. The researchers generated a series of geologically optimal pushbacks within the ultimate pit and arranged them using a Dynamic Programming (DP) model to obtain the optimal production schedule. When incorporating ecological costs, the model demonstrated a reduction of 2.8% in the total present value of ecological costs. On the other hand, in scenarios where ecological costs were not considered, the model showcased a 2.5% increase in the overall net present value. Gu et al. [17] introduced a pushback sequencing model to address the production planning problem in open-pit coal mining. The primary objective of their study was to develop a dynamic method for sequencing pushbacks and implementing a moving cone elimination algorithm to optimize coal production and waste stripping

rates. The proposed method also aimed to optimize the mining sequence and mine life of the coal mine. By applying this approach, the researchers obtained the best production schedule with the highest net present value (NPV). This schedule provided information on the optimal quantities of mined coal and waste for each year, the optimal mining zones, and the optimal mine life for the final pit.

Paithankar et al. [7] introduced a production schedule optimization method for open-pit mines, aiming to generate a long-term schedule that maximizes flow in multiple periods while considering uncertainty. Their approach utilized a hybrid of maximum-flow and genetic algorithms to solve the optimization problem. The resulting solution exhibited robustness, with an average gap of less than 6% compared to the upper bound solution, except for one case. The researchers also evaluated the method on deterministic instances and observed that it did not perform as well as certain existing methods. Nevertheless, the method can be easily adapted to handle deterministic models, incorporating various uncertainty modeling approaches, and accommodate additional scheduling constraints.

Pierre Nancel-Penard et al. [1] introduced a recursive time aggregation-disaggregation heuristic for addressing the multi-dimensional and multi-period precedence-constrained knapsack problem. They specifically applied this heuristic to the open-pit mine block sequencing problem to validate its effectiveness in maximizing profit while satisfying minimum and maximum resource consumption constraints. The proposed method utilizes a binary tree structure to recursively aggregate and disaggregate scheduling periods, solving two-period sub-problems sequentially. The researchers implemented sliding time window heuristics, highlighting the challenges encountered when solving scheduling instances with conflicting knapsack constraints. The application of this heuristic provides insights into optimizing resource allocation and scheduling in open-pit mining, considering complex constraints and multi-period considerations.

There are some other important studies on pit-optimization in which quantitative analysis of nearface stockpile mining is proposed. For example, Gong et al. [18] presented a study in which a mixed integer linear programming model was presented for integrated simulation and optimization problem. They implemented the quantitative analysis of near-face stockpile mining. Also, for validation of their model, they applied a case study of oil sand mine. Gong et al. [19] also, used near-face stockpile mining in the other study in which a method was implemented to increase net present value and quality of the plant throughput.

In Table 1, there is a comprehensive summary of the related studies about production scheduling and pushback design problems including minimum mining width assumption. After investigating these studies and their shortcomings, we can propose the most important contributions of the current research.

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Year/Author	Main goal	Methodology/tools	Main Findings	Future Research
2022/ Nancel- Penard et al. [1]	• Determining the extracted volumes to maximize profit with minimum width at the mine.	<ul> <li>A MILP model</li> <li>The modified Lerchs and Grossman</li> <li>Preprocessing heuristic approach</li> </ul>	<ul> <li>Lower computation time in the preprocessing</li> <li>A decrease in NPV between original and practical pushbacks.</li> </ul>	<ul> <li>Using a spatial aggregation/disaggregation heuristic approach in Jélvez et al. (2016).</li> <li>Using a model able to generate production plans considering minimum width.</li> </ul>
2009/ Pourrahimian et al. [3]	<ul> <li>Two alternative MILP production scheduling with minimum mining width</li> <li>To maximize the total discounted economic value, or NPV.</li> </ul>	TOMLAB/CPLEX optimization solver	<ul> <li>Model 1 with higher discounted economic value and more run time than model 2.</li> <li>Model 2 with a practical schedule and enough space for equipment.</li> </ul>	_
2021/ Yarmuch, J. L [8]	generate maximum value practical pushbacks with a closeness factor to quantify the design's mine ability.	<ul> <li>Python and Gurobi MIPsolver</li> <li>Commercial mine planning software</li> <li>Traditional methodology.</li> </ul>	<ul> <li>Closeness factor is 89.34% in contrast to the 48.00% obtained from the traditional methodology.</li> </ul>	<ul> <li>Aggregation technique to reduce the number of variables.</li> <li>Designing the pushbacks sequentially, in a greedy way.</li> </ul>
2014/ Tabesh et al. [16]	A multi-step methodology for open- pit mine planning with controlled optimal phase-design, characterization of selective mining- units, and long-term production scheduling optimization.	<ul> <li>A methodology using integer programming (CPLEX).</li> <li>A greedy heuristic with a local search algorithm and clustering (MATLAB).</li> <li>The Lerchs and Grosmann, parameterization approach (Whittle).</li> </ul>	<ul> <li>The push-back design algorithm is to control the mineralized material and rock tonnage in each push-back.</li> <li>The results from the pushback design are uniform in terms of tonnage.</li> </ul>	-

Table 1. Summary of the related research to minimum mining width in production scheduling and pushback design.

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Year/Author	Main goal	Methodology/tools	Main Findings	Future Research
2022/ Kaydım, H. E. [5]	An integer programming model to optimize the long-term production scheduling of open-pit mines by maximizing the NPV.	AMPL CPLEX module of the NEOS Server.	<ul> <li>Maximizing the NPV of eight-year production with a negligible optimality gap.</li> <li>The cost has the least effect on the NPV.</li> <li>Metal prices are the most influential parameter on the resultant NPV value.</li> </ul>	<ul> <li>The two-dimensional model can be extended to a three-dimensional model.</li> <li>Variations in cost and revenue can be evaluated by sensitivity analysis.</li> <li>Multi-stockpile option can be included into the model.</li> </ul>
2018/ Xu, X. C et al. [6]	<ul> <li>A method of open pit production scheduling to consider ecological costs as internal costs.</li> <li>A series of optimum pushbacks is generated and sequenced using a Dynamic Programming (DP) to obtain the best production schedule.</li> </ul>	<ul> <li>The moving cone exclusive method.</li> <li>The method of generating geologically optimum pushbacks.</li> <li>The opportunity cost method to estimate the economic value of conserving soil.</li> </ul>	<ul> <li>The schedule with ecological costs has lower production rates and a longer mine life than that without ecological costs.</li> <li>A reduction in NPV of ecological costs when using ecological costs and a gain in NPV in cases without ecological costs.</li> </ul>	-
2011/ Gu, X. W. [17]	A dynamic sequencing method to optimize the coal production rate, waste stripping rate, mining sequence, and mine life of an open- pit coal mine.	<ul> <li>Dynamic pushback sequencing</li> <li>A moving cone elimination algorithm.</li> </ul>	The highest NPV in the best production schedule, with the best quantities of coal and waste, the best zone to be mined in each year, and the best number of years to mine the entire final pit.	<ul> <li>Post-processing to adjust to the optimum solution.</li> <li>Solid programming</li> </ul>
2020/ Malharkumar, P. et al. [14]	The production schedule for an iron ore deposit in India.	Lerchs and Grossmann algorithm (Geovia Surpac software and Whittle software)	<ul> <li>To increase the profitability of iron ore projects by minimizing in-situ grade variability and uncertainty.</li> <li>Significant quantity of overburden is dumped into the internal dump of the mine to reduce re-handling of overburden during the reclamation.</li> </ul>	-

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Year/Author	Main goal	Methodology/tools	Main Findings	Future Research
2019/ Paithankar, A. et al.[7]	The long-term production schedule for maximum flow in multiple periods under uncertainty.	The maximum flow algorithm with a genetic algorithm.	The method can be adapted to solve a deterministic model or any uncertainty modeling.	• Adding other sources of uncertainty, additional operational constraints, multiple destinations, introducing stockpiles and cut-off grade optimization.
2021/ Yarmuch, J. L et al. [13]	<ul> <li>A new optimization model for semi practical pushbacks with minimum operational width and connectivity within the blocks.</li> <li>To maximize NPV of the mining extraction.</li> </ul>	An algorithm using Sliding Window Heuristic, variable bounding, and other preprocessing routines.	To obtain solutions with less than 10% optimality gap for mining instances ranging between 30,000 to 50,000 blocks.	<ul> <li>A general approach to model mining width to generalize rectangular templates to different shapes.</li> <li>The root relaxation offers a good approach to the final integer solution for large- size instances.</li> </ul>
2020/Aref Alipour [20]	<ul> <li>A 3D Genetic Algorithm array is employed to reflect the 3D feature of the real mine block model.</li> <li>To maximize the NPV under sequencing and capacity constraints.</li> </ul>	<ul> <li>Genetic Algorithm.</li> <li>The penalty and normalization methods</li> <li>SimSched commercial package.</li> </ul>	<ul> <li>The computational time of GA was short in relation to the complexity of the OPPS<sup>1</sup> problem.</li> <li>GA could produce better solutions rather than SimSched DBS.</li> </ul>	• Considering a more direct integration of block economic values and other operational constraints' uncertainty.
2021/ Gonzalo Nelis [21]	• propose an optimization mathematical model for operational issues.	<ul> <li>cut-off grade approach</li> <li>incorporate horizontal precedence constraints to ensure the existence of a feasible path from the bench access to each block.</li> </ul>	<ul> <li>The model was able to generate mining cuts and an extraction sequence fulfilling mining, processing and operational constraints.</li> <li>The mining cut design captured most of the profit.</li> </ul>	<ul> <li>Study the best location of the representative SMUs and their influence on production schedule.</li> <li>Appling heuristic algorithms</li> <li>Considering uncertainty in the parameters</li> </ul>

<sup>&</sup>lt;sup>1</sup> Open-Pit Production Scheduling

According to the previous studies and their shortcomings, the contributions and objectives of this work are as follows:

- 1. We develop a mixed integer linear programming mathematical model in which production scheduling constraints and optimizing the objective functions are investigated simultaneously.
- 2. We consider one important assumption, minimum mining width and related constraints in the mathematical model.
- 3. The proposed model includes two goals. The first objective function is to maximize the net present value. The second objective function is to decrease greenhouse gas emission from the operations in the mine.
- 4. In addition to production scheduling problems, the model aims at designing pushbacks, which is to digitize pushback considering the ramps and roads within the specified bench height, width and batter angle.

## **3.** Mathematical Formulation (MILP)

In this section, we want to describe the mathematical optimization model which is used for production scheduling problem- with minimum mining width. Before introducing the model, we have presented the indices, parameters and variables.

### 3.1. Indices

- T The scheduling periods set
- N The set of blocks which are scheduled
- K The set of mining cuts which are scheduled

### 3.2. Parameters

- $gu^t$  Maximum acceptable average head grade of ore being transported to the mill during time period "t".
- $gl^t$  Minimum acceptable average head grade of ore being transported to the mill during time period "t".
- $g_k$  Ore grade average within mining cut "k".
- $Ot_k$  Amount of ore extracted in mining section "k"
- $W_k$  Quantity of waste material in mining cut "k".
- $PC_{max}^{t}$  Maximum limit on the capacity for processing ore during time period "t
- $PC_{min}^{t}$  Minimum limit on the capacity for processing ore during time period "t
- $MC_{max}^{t}$  Maximum limit on the capacity for mining ore during time period "t
- $MC_{min}^{t}$  Minimum limit on the capacity for mining ore during time period "t
  - $q_k^t$  Discounted cost of mining all the material in mining cut k as waste and in the period t

- $v_k^t$  Net present value of revenue obtained from selling the end product within mining section "k" during period "t," subtracting the discounted additional expenses of extracting all material from mining cut "k" as ore and subsequently processing it.
- $\alpha$  Pollution factor for extraction operation
- $\beta$  Pollution factor for processing operation

#### 3.3. Variables

- $s_k^t$  The proportion of mining cut "k" to be extracted as ore and subjected to processing during time period "t."
- $y_k^t$  The proportion of mining cut k intended for mining during period t.
- $X_n^t$  The sequence of extraction of blocks, equal to 1 if block n is to be mined in period t, otherwise 0.
- $b_k^t$  A binary variable that indicates the priority of extracting mining cut k in period t.

The production scheduling with minimum mining width model (PSMW) in this problem has two objective functions. The first one is maximization of total profit which is total revenue minus total costs. The second one is minimization of total greenhouse gas emission from mining and processing operations. The PSMW model developed from pourrahmanian et al. [3] is formulated by merging concepts from Caccetta et al. [22] and Boland et al. [23]. In this model, both mining and processing are operated on the mining-cut level. Before optimizing the schedule, the blocks are categorized into clusters, and two continuous variables present ore processing and mining activities. Within the mining bench, blocks are grouped into clusters, considering their attributes, spatial coordinates, rock composition, and grade distribution.

The PSMW model is proposed as below:

$$\sum_{t} \sum_{k} v_k^t \times s_k^t - q_k^t \times y_k^t \tag{1}$$

$$\sum_{t} \sum_{k} (\beta \times s_{k}^{t} - \alpha \times y_{k}^{t})$$
<sup>(2)</sup>

Equations (3) through (5) manage constraints related to grade blending, processing capacity, and mining capacity on the mining-cut level while considering fractional extraction from mining cuts.

$$gl^{t} \leq \frac{\sum_{k} g_{k} \times Ot_{k} \times s_{k}^{t}}{\sum_{k} Ot_{k} \times s_{k}^{t}} \leq gu^{t} \qquad \forall t \in \{1, \dots, T\}$$

$$(3)$$

$$PC_{min}{}^{t} \leq \sum_{k} Ot_{k} \times s_{k}^{t} \leq PC_{max}{}^{t} \qquad \forall t \in \{1, \dots, T\}$$

$$(4)$$

$$MC_{min}{}^{t} \leq \sum_{k} (Ot_{k} \times W_{k}) \times y_{k}^{t} \leq MC_{max}{}^{t} \qquad \forall t \in \{1, \dots, T\}$$
(5)

Equation (6) guarantees that the quantity of ore extracted and processed from a specific mining cut during any given period remains equal to or less than the amount of rock extracted from that same mining cut.

$$s_k^t \le y_k^t \qquad \qquad \forall t \in \{1, \dots, T\} \qquad \forall k \in \{1, \dots, K\}$$
(6)

Equations (7) and (8) verify the group of directly preceding cuts that need to be extracted before mining cut k can be extracted.

$$\sum_{i} y_{k}^{i} - b_{k}^{t} \leq 0 \qquad \forall t \in \{1, \dots, T\} \quad \forall k \in \{1, \dots, K\}$$

$$(7)$$

$$b_k^t - b_k^{t+1} \le 0 \qquad \forall t \in \{1, \dots, T-1\} \quad \forall k \in \{1, \dots, K\}$$
(8)

$$0.4(\text{Int}(m))X_{Wb}^{t} - \sum_{n=1}^{m} X_{n}^{t} \le 0 \quad \forall t \in \{1, \dots, T\} \quad \forall Wb \in \{1, \dots, N\}$$
(9)

Equation (9) indicates binary and continuous variables in the model.

$$b_k^t, X_n^t \in \{0, 1\}, \qquad s_k^t \ge 0 \quad y_k^t \ge 0$$
 (10)

#### 4. **Problem Solution**

In this section, the suggested solution methods are presented. Since the problem is presented as a mixed integer multi-objective optimization model, the exact epsilon-constraint method is used to solve the small numerical experiments and is solved in Python software. Then, to solve the model with medium and large-scale numerical examples, the meta-heuristic algorithm of simulated annealing (SA) and genetic optimization of non-dominant ranking (NSGA-II) are used.

When working with meta-heuristic algorithms, setting the parameters of the algorithms is very important. These parameters are performed through numerical experiments. There are different methods to design the numerical experiments. One of the methods is to adjust the parameters by Taguchi including a series of experiments with a fractional factor. According to Taguchi [24], the factors affecting parameter setting are generally divided into two groups of controllable factors and uncontrollable factors. In this method, the objective is to reach optimal levels of controllable factors and reducing the effects of uncontrollable factors. This is necessary to measure the qualitative characteristics of the tests as S/N ratio. Where S represents the value of the signal and N represents the noise. This ratio indicates deviations in the response variable (objective function). In this way, each algorithm is parameterized based on the target value of the problem. The parameter setting is performed two times.

After setting the parameters of algorithms, we compare the performance of two non-dominant genetic algorithms and simulated annealing. We use the method of relative performance increase, which is defined as the following formula.

$$RPI_s = \frac{f_s - f_b}{f_b} \times 100 \tag{11}$$

In the above relation,  $f_s$  means the value of the objective function, calculated by the applied algorithm.  $f_b$  is equal to the most optimal objective value obtained from the algorithms. To ensure more efficiency of the algorithms, each numerical example should be solved at least10 times. Based on the result and RPI calculated of each objective function, this is expected that the meta-heuristic algorithms have the higher performance for large-scale problems (n>17). Also, for small-scale problems, this is expected that the exact solution method has more efficiency.

Figure 1 and 2 show that for numerical examples with scale<17, exact solution method is capable to obtain optimal solutions and less running time, while for problems with scale>17, meta-heuristic algorithms are able to give more efficiency and less running time.



Figure 1. Running time for two solution methods.



Figure 2. The efficiency of two solution methods.

## 5. Conclusion

The mixed-integer linear programming formulation for the PSMW problem was introduced in the mathematical formulation section. This model aims to control extraction, processing, and the sequence operations of blocks at the mining-cut level, offering a more efficient problem size and computational time. The proposed model was evaluated using block model data in the studied problem of Nancel-Penard et al. [1]. The PSMW model has two main objective functions. The first one is to maximize total profit which is total income minus investment and operational costs. The second objective is to decrease greenhouse gas

emissions from transportation and production operations in the mine. Simultaneously, the introduced model investigates production scheduling problems for extraction, mining and processing steps. The MILP is applied for a case study to indicate the validation and ability of the model in generating practical solutions. Also, implemented approaches to solve the programming models from meta-heuristic to exact solution methods have been compared, and based on that, an exact solution methodology for pit design to solve the mathematical mixed integer linear programming model with the exact solution method in Python using Gurobipy is chosen. For solving the problems on a large scale, a Simulated Annealing metaheuristic algorithm and a Genetic algorithm are applied. After analyzing the final solutions from different methods, it is concluded that for large-scale problems SA and NSGA-II algorithms and for short-scale problems the exact method (epsilon-constraint) is more efficient in terms of running time, efficiency and optimal solutions.

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