A Two-Stage Simultaneous Optimization of NPV and Throughput in Production Planning of Open Pit Mines by Introducing Multi Range Stockpiles[1]

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ABSTRACT

Open-pit mines are complex businesses with lifelong profits of millions and in large mines billions of dollars. These mines consist of a minimum of one discrete (mining) and one continuous (processing) subsystem working subsequently to deliver input raw material to several downstream industries. The inherent difference between these two subsystems causes operational challenges in the production process leading to nonoptimal NPV and quality and quantity of throughput from discrete to continuous subsystem. In this paper, we present a two-stage clustering-MILP algorithm for long-term production planning in open-pit mines incorporating multi range stockpiles in the decision-making process that leads to determining the optimum number of stockpiles required to maximize the discounted value of the mine as well as balancing the quality and quantity of throughput. We evaluated our developed model in a real open pit mine case study. Results show that with a four-bin stockpile we can maximize the discounted value of the mine by minimizing head-grade deviation to 5.1% and maximizing the reclaimed material up to 10.7% of the total ore delivered to the plant.

1. Introduction

Among all mechanical and non-mechanical mining methods, open-pit mining method is the most common ore extraction method being applied for exploitation of more than 80% of raw material delivered to the market [1]. As multi-million/-billion-dollar expenditures with the same profit scale, open-pit mines usually integrate a discrete system (mining operation) with a continuous system (processing) to mine and deliver raw material to the market. Having the discrete and the continuous systems synchronized and at the same time decoupled, so that the material flows through the production chain with consistent quantity and quality is one of the major challenges of mining operations. This challenge is delt with by the introduction of stockpiles to the open pit mining operations.

Adding stockpiles to the open pit mining operations, subsequently leads to requirement of including them in the open pit mine production scheduling (OPMPS). Two approaches are common in the integration of stockpiles into OPMPS: making stockpiling decisions during the long-term OPMPS stage or postponing stockpiling decisions to the short-term OPMPS stage. The long-term OPMPS models must make decisions on the combination of millions of mining blocks and several

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time spams to maximize net present value (NPV) which already is a challenging and time-consuming task [2]. Thus, adding stockpiling decisions to the long-term OPMPS will make it even more challenging. Herein, we developed a two-stage aggregation-stockpiling decision-making framework that solves the long-term OPMPS in presence of stockpiles in a few seconds while creating a higher NPV compared to the same operation without any stockpile.

OPMPS problems have attracted several researchers since Johnson's introduction of mathematical programming to the field of mine planning [3] over 50 years ago. The literature of OPMPS until 2010 have been reviewed critically by Osanloo and Newman and are presented in [4] and [5], respectively. Since then, three streams of research can be tracked in OPMPS. The first is the direction where researchers have tried to reduce the solution time. Moreno et al. and Samavati et al. developed a multi-step algorithm that deals with the OPMPS as a multi-period precedence-constrained knapsack problem [6], [7]. Implementation of their multi-step algorithm on Marvin and other datasets showed approximately five times improvement in the solution time with a confidence interval of 94%. Most of the other works in this stream focus on the implementation of different relaxation approaches to cut the solution time for the OPMPS problems [8]–[11]. None of these proposed solution methodologies include stockpiling in their procedure.

In the second trackable stream of OPMPS literature, researchers integrated in-pit crushing and conveying (IPCC) systems location optimization problems with OPMPS and tried to solve both problems at the same time. As IPCC material handling systems have been practically proven to be economically premier to the regular truck and shovel system [12] especially over the recent two decades, researchers have proposed different algorithms to incorporate IPCC into OPMPS solution procedure [13]–[16]. Same as the first stream, none of the solution methodologies that integrate IPCC with OPMPS explicitly incorporate stockpiling in their procedure.

In the third traceable stream in the literature, researchers of introduced the stockpiling option into the OPMPS problem and tried to propose solution methodologies for that. The main challenge of adding the stockpiling option to the OPMPS problem is nonconvexity and nonlinearity of the produced optimization models [17]. Bley et al. relaxed the nonlinear constraints and received a linear outer approximation and introduced a branching method and a primal heuristic that generates feasible solutions [17]. The model proposed by Bley et al. [17] has the capability to track the material flow from aggregates to stockpile and plant. In another attempt, Gholamnejad and Kasmaee defined a block model over one low grade and one high grade stockpile in [18] and developed a goal programming model for selective rehandling of material from each of the two stockpiles to provide optimum blend for the plant. Although they targeted optimality of the blend, their model ignores material from the mine.

Grade or quality of material delivered to and rehandled from stockpile have been addressed by Dimitrakopoulos works with Asad in [19] and his work with Ramazan in [20]. In the former, researchers model the stockpile by dividing grade ranges on the grade-tonnage curve and determining material from each grade range. In the later, however, the model has a constant grade for the stockpile that is determined prior to scheduling which is used for the reclamation of material from the stockpile. Predetermined grade for stockpiles has also been considered by Mousavi [21] and Kumar [22] in their proposed models for OPMPS. The researchers in [21] implemented non-exact solution methodology to solve their model. Despite the novelty of their work, they have not evaluated its performance in a large-scale case study and neither they evaluated the possible errors caused by fixed stockpile reclamation grade. The researchers in [22] apply the same logic in an open pit coal mine. In another grade-based attempt, Smith and Wicks proposed an OPMPS model for a copper mine where low grade material was stored in a stockpile for future use in case needed [23]. Finally, in a series of studies, Moreno, Rezakhah, and Newmann classify the production scheduling and stockpiling models in the literature and propose new modeling approaches in [2], [24].

Lack of OPMPS models that consider more than one of the abovementioned streams (speed, IPCC, and stockpiles) convinced us to develop a multi-step algorithm to tackle the first stream (speed) and the third stream (stockpiling) by clustering mining blocks and introducing a mixed-integer linear model that incorporates stockpiling in the OPMPS solution procedure. Thus, in the following sections, we first explain how we developed the algorithm. Then, we discuss its implementation in a series of scenarios in a case study. Finally, we present the results of our evaluation on improvement of the run time and the net present value of the project.

2. Materials and Methods

2.1. Two-step algorithm

As mentioned earlier, we integrated a clustering algorithm with a mixed-integer linear model to schedule open pit production in presence of stockpiles. Our two-step algorithm helped us to incorporate the stockpiling decisions with the OPMPS decisions without sacrificing the solution time. In the first step of this two-step algorithm, we implemented the agglomerative hierarchical clustering method to aggregate blocks of the same bench (bench-phase) into larger units called mining-cuts. This clustering method merges the closest pairs of inputs respecting satisfaction of pre-defined similarity indices [25]. We defined distance, grade, rock type, destination, and under-cluster which tracks the ore beneath each cluster for developing a better schedule by accessing the higher-grade ore faster.

From the five defined indices, grade and distance have numeric values. Thus, we implemented Minkowski distance method [26], [27] to calculate similarity of those indices in our blocks and clusters as presented in equation (1).

$$d\,(j,k) = \left(\sum_{i=1}^{n} \left|x_{ji} - x_{ki}\right|^{r}\right)^{\frac{1}{r}} \tag{1}$$

Where *r* is always greater or equal to one. If *r=1* then it turns into Manhattan distance and if *r=2* the equation turns into Euclidean distance. For each dimension (*i)* in *n* possible dimensions, *d(j,k)* is dissimilarity index value of index *d* between variables (in our case blocks) x_{ji} and x_{ki}.

Among our indices, we have rock type, under-cluster, and destinations which are categorical indices and do not take numerical values. So, implementing Minkowski distance method in these cases are not possible. Thus, to evaluate similarity of our clusters in terms of these categorical indices, we implemented the method proposed by Dosea and his colleagues [28] based on integration of simple matching method offered by Huang [29], [30] and calibrated penalties for extent of dissimilarity between values. As the similarity indices are not of the same measurement units, we implemented linear scaling technique to normalize the numeric indices with the maximum value to be able combine them with the categorical indices for which we have chosen zero for not being similar and one for being similar to one another. The process has been explained in the Algorithm 1.

We designed the clustering algorithm to only cluster best matches blocks in a single bench since each shovel mining face will only be consisted of one bench of material. The clustering procedure is presented in Algorithm 1.

Algorithm 1: generating bench-phase mining cuts from the block model

Inputs

Block model; maximum number of possible clusters; maximum length of possible clusters;

Begin

NC ← Total number of blocks

NC_{max} ← Maximum number of possible clusters

LC_{max} ← Maximum length of possible clusters

A ← zeros [N]

S ← zeros [N]

$D_{max} \leftarrow \left(\sum_{d=1}^{2} \left|x_{id} - x_{jd}\right|^2\right)^{\frac{1}{2}} \ \forall\, i \,\&\, j \in \{1,\ \dots,\ N\}$

$G_{max} \leftarrow g_i \ \forall\, i \in \{1,\ \dots,\ N\}$

for i = 1 to N **do**

  for j = 1 to N **do**

    if i = j **then**

      $S_{ij} \leftarrow 0$

      $A_{ij} \leftarrow 0$

    else

      $R_{ij} \leftarrow \{1\ if\ i \,\&\, j\ same\ rock\ type\ r\ otherwise$

      $C_{ij} \leftarrow \{1\ if\ i \,\&\, j\ above\ same\ cluster\ c\ otherwise$

      $D_{ij} \leftarrow \{1\ if\ i \,\&\, j\ same\ destination\ d\ otherwise$

      $L_{ij} \leftarrow \frac{\left(\sum_{d=1}^{2} \left|x_{id} - x_{jd}\right|^2\right)^{\frac{1}{2}}}{D_{max}}$

      $G_{ij} \leftarrow \{\frac{\left(\left|x_{id} - x_{jd}\right|^2\right)^{\frac{1}{2}}}{G_{max}}\ if\ \left|x_{id} - x_{jd}\right|^2 \neq 0\ \varepsilon\ otherwise$

      $S_{ij} \leftarrow \frac{R_{ij} \times C_{ij} \times D_{ij}}{L_{ij} \times G_{ij}}$

      if i & j are adjacent **then**

        $A_{ij} \leftarrow 1$

      else

        $A_{ij} \leftarrow 0$

      endif

while NC > NC_{max}

(i,j) ← $\{\{A_{ij}\}\}$

if $(LC_i + LC_j) \le LC_{max}$ **then**

$S_{i:} \leftarrow \{(S_{it} | \, t \in \{1, \ldots, N\}) \& (S_{jt} | \, t \in \{1, \ldots, N\})\}$

$S_{j:} \leftarrow 0$

$A_{i:} \leftarrow \{(A_{it} | \, t \in \{1, \ldots, N\}) \& (A_{jt} | \, t \in \{1, \ldots, N\})\}$

$A_{j:} \leftarrow 0$

$C_i \leftarrow C_i + C_j$

$NC \leftarrow NC - 1$

else

$A_{ij} \leftarrow 0$

endif

endwhile

After the clustering process following Algorithm 1, two post-processing steps are performed to deal with the geometrical constraints such as shape of the cluster and mining precedence. Then, the mathematical model explained in the next subsection is implemented on the practical representation of the deposit with a reduced number of variables and constraints.

2.2. Open pit mine production scheduling model

Introducing stockpiles imposes nonlinearity to the OPMPS models. To practically deal with this nonlinearity, we introduced operationally approved stockpiling method. With this stocking method, ore is stored in a divided area with a range of acceptable grades to be able to assign fixed reclamation grades to each stockpile. The storing grades and the reclaiming grades of this multi bin stockpiling method are determined based on the grade-tonnage graph prior to any OPMPS model implementation. After this step, we implement the mathematical formulation presented here to develop production schedule for the mine. Following we present the our developed OPMPS model.

- Sets

S^m	For each bench-phase m, there is a set of bench-phases (S^m) that have to be extracted prior to extracting bench-phase m to respect slope and precedence constraints
U^m	Each bench-phase m is divided into a set of clusters. U^m is the set of clusters that are contained in bench-phase m

- Indices

$d \in \{1, \ldots, D\}$	Index for material destinations
$m \in \{1, \ldots, M\}$	Index for bench-phases

$p \in \{1,...,P\}$	Index for clusters
$c \in \{1,...,C\}$	Index for processing plants
$e \in \{1,...,E\}$	Index for elements
$t \in \{1,...,T\}$	Index for scheduling periods

- Parameters

D	Number of material destinations (including processing plants and waste dumps)
M	Total number of bench-phases
P	Total number of clusters
E	Number of elements in the block model
T	Number of scheduling periods
$\overline{MC}^t$	Upper bound on the mining capacity in period t
$\underline{MC}^t$	Lower bound on the mining capacity in period t
$\overline{PC}_c^t$	Maximum tonnage allowed to be sent to plant c in period t
$\underline{PC}_c^t$	Minimum tonnage allowed to be sent to plant c in period t
$\overline{G}_c^{t,e}$	Upper limit on the allowable average grade of element e at processing plant c in period t
$\underline{G}_c^{t,e}$	Lower limit on the allowable average grade of element e at processing plant c in period t
s_m	Number of predecessors of bench-phase m (members of S^m)
o_m	Total ore tonnage in bench-phase m
w_m	Total waste tonnage in bench-phase m
o_p	Total waste tonnage in cluster p
w_p	Total waste tonnage in cluster p

c_m^t	Unit discounted cost of mining material from bench-phase m in period t
$r_{p,c}^t$	Unit discounted revenue of sending material from processing unit p to processing destination c in period t minus the processing costs
$r_c^{t,e}$	Unit discounted revenue of processing one unit of element e from stockpile in processing destination c in period t minus the processing and rehandling costs
g_p^e	Average grade of element e in cluster p

- Decision Variables

$y_m^t \in [0,1]$	Continuous decision variable representing the portion of bench-phase m extracted in period t
$x_{p,c}^t \in [0,1]$	Continuous decision variable representing the portion of ore tonnage in cluster p extracted in period t and sent to processing plant c
$b_m^t \in \{0,1\}$	Binary decision variable indicating if all the predecessors of bench-phase m are completely extracted by or in period t
f_c^t	Continuous decision variable representing the tonnage reclaimed from the stockpile and sent to processing plant c in period t
$G^{t,e}$	Continuous decision variable representing the reclamation grade of element e in period t

With the abovementioned parameters and variables, now we define our multi destination objective function. To do so and for the purpose of enhancing the solution procedure we defined $y_m^t \in [0, 1]$ as a set of variables to monitor the portion of the bench that is mined in each period *t* and $b_m^t \in \{0, 1\}$ to control the mining precedence in each period *t*. This will help the solver to solve the model faster as the variables are reduced compared to the number of blocks. To avoid non-linearity in the model, as we discussed earlier, we define operationally approved S number of stockpiles within acceptable grade range. This will lead to adding S destinations to the list of ore destinations in the model. Then, we determine average reclamation grade of element *e*, $G_s^{e,t}$, for each stockpile *s* to be delivered to the processing plant in period *t*. We also need to incorporate revenue and cost of stockpiling and rehandling from each stockpile in the OPMPS model. Thus, we define $r_{s,c}^t$ as the discounted profit from processing the reclaimed material of the stockpile *s* in the plant *c* during the period *t*. For each destination of mined material in the range of stockpiles ($d = C + s$), we define $\underline{G}_d^{e,t}$ as the upper bound and $\bar{G}_d^{e,t}$ as the lower bound of acceptable *e* grade range for material being delivered to stockpile *s*. Finally, we define a set of variables, $f_{s,c}^t \geq 0$, representing tonnage of material reclaimed from *s* and processed in *c* during the period *t*. That being elaborated, the OPMPS model we formulated is as followed.

- Objective Function

$$\sum_{t=1}^{T}\left(\sum_{p=1}^{P}\sum_{c=1}^{C}\left(r_{p,c}^{t}\times o_{p}\times x_{p,c}^{t}\right)-\sum_{m=1}^{M}\left(c_{m}^{t}\times\left(o_{m}+w_{m}\right)\times y_{m}^{t}\right)+\sum_{s=1}^{S}\sum_{c=1}^{C}\left(f_{s,c}^{t}\times r_{s,c}^{t}\right)\right) \quad (2)$$

- Constraints

$$\underline{MC}^{t}\leq\sum_{m=1}^{M}\left(\left(o_{m}+w_{m}\right)\times y_{m}^{t}\right)\leq\overline{MC}^{t} \qquad \forall t\in\{1,...,T\} \quad (3)$$

$$\sum_{p\in U^{m}}\sum_{d=1}^{D}\left(o_{p}\times x_{p,d}^{t}\right)\leq\left(o_{m}+w_{m}\right)\times y_{m}^{t} \qquad \forall t\in\{1,...,T\},\forall m\in\{1,...,M\} \quad (4)$$

$$\underline{PC}_{c}^{t}\leq\sum_{p=1}^{P}\left(o_{p}\times x_{p,c}^{t}\right)+\sum_{s=1}^{S}f_{s,c}^{t}\leq\overline{PC}_{c}^{t} \qquad \forall t\in\{1,...,T\},\forall c\in\{1,...,C\} \quad (5)$$

$$\underline{G}_{c}^{t,e}\leq\frac{\sum_{p=1}^{P}\left(o_{p}\times g_{p}^{e}\times x_{p,c}^{t}\right)+\sum_{s=1}^{S}\left(f_{s,c}^{t}\times G_{s}^{t,e}\right)}{\sum_{p=1}^{P}\left(o_{p}\times x_{p,c}^{t}\right)+\sum_{s=1}^{S}f_{s,c}^{t}}\leq\overline{G}_{c}^{t,e} \quad \forall t\in\{1,...,T\},\forall c\in\{1,...,C\},\forall e\in\{1,...,E\} \quad (6)$$

$$\underline{G}_{d}^{t,e}\leq\frac{\sum_{p=1}^{P}\left(o_{p}\times g_{p}^{e}\times x_{p,d}^{t}\right)}{\sum_{p=1}^{P}\left(o_{p}\times x_{p,d}^{t}\right)}\leq\overline{G}_{d}^{t,e} \quad \forall t\in\{1,...,T\},\forall e\in\{1,...,E\},\forall d\in SC|d=C+s \quad (7)$$

$$\sum_{i=1}^{t}\sum_{c=1}^{C}f_{s,c}^{i}\leq\sum_{i=1}^{t-1}\sum_{p=1}^{P}\left(o_{p}\times x_{p,d}^{i}\right) \quad \forall s\in\{1,...,S\},\forall t\in\{2,...,T\},\forall d\in SC|d=C+s \quad (8)$$

$$\sum_{i=1}^{t}\sum_{c=1}^{C}G_{s}^{t,e}\times f_{s,c}^{i}\leq\sum_{i=1}^{t-1}\sum_{p=1}^{P}\left(o_{p}\times g_{p}^{e}\times x_{p,d}^{i}\right) \quad (9)$$

$$\forall s\in\{1,...,S\},\forall t\in\{2,...,T\},\forall e\in\{1,...,E\},\forall d\in SC|d=C+s$$

$$\sum_{t=1}^{T}y_{m}^{t}=1 \qquad \forall m\in\{1,...,M\} \quad (10)$$

$$\sum_{i=1}^{t}y_{m}^{i}\leq b_{m}^{t} \qquad \forall m\in\{1,...,M\},\forall t\in\{1,...,T\} \quad (11)$$

$$s_{m}\times b_{m}^{t}\leq\sum_{i\in S^{m}}\sum_{j=1}^{t}y_{i}^{j} \qquad \forall m\in\{1,...,M\},\forall t\in\{1,...,T\} \quad (12)$$

$$b_m^t \leq b_m^{t+1} \qquad \forall m \in \{1,..., M\}, \forall t \in \{1,..., T-1\} \qquad (13)$$

The model optimizes discounted net present value from processing ore sent to the plant either directly from the mine or by rehandling the stockpiled material in equation (2). We introduced equations (3) for controlling the extraction capacity of the mine and equation (5) for controlling the processing capacity of the plant for each period of the mine life. We cap the maximum amount of material sent to be processed from a bench to the total amount available in the same bench by equation (4). In case the total tonnage mined from the bench and processed from the same bench vary, the difference is the amount of the waste from that bench and is sent to the dumping location. The blending is controlled by equation (6) where our model calculates the weighted average of the material sent to the plant controls the average head grade of the summation material sent to processing plants from both the mine and the stockpile in each period and keeps this weighted average between the minimum and the maximum acceptable head grade. To avoid nonlinearity here, we adjust the equation prior to matrix creation. We also appended stockpiles to the blending control constraint so that the model controls its input quality and quantity using the same constraint. The reclamation grade for element e in period t, $G^{t,e}$, is determined using equation (7). By defining constraint (8) we make sure that total amount of material reclaimed from stockpile will not exceed the total amount have been stockpiled since day one of the operation. Using the equation (9) the model guarantees content adjustment in case the predetermined average grade is higher than the grade of material currently available in the stockpile. To make sure that all the material inside the optimal pit limit is mined, we implement equation (10) in the model formulation. We also ensure the geotechnical practicality of our production schedule using precedence constraints presented in equations (11) to (13).

To implement the two-stage algorithm presented here, we used the Gurobi engine of Matlab software. For the case study presented in the following section, it takes the software five seconds to perform the first stage and 15 seconds to do the second stage.

3. Case Study

The case study we implemented our two-stage algorithm is an iron ore deposit. The final pit consists of 19561 blocks with a tonnage of 430 million tonnes from three ore rock types and four waste rock types. The ore zones contain desired iron trackable through mass precent of magnetic weight (MWT). The zones also contain deleterious elements including Sulfur (S) and Phosphor (P). The final pit includes four production pushbacks with a total of 40 bench-phases where the clustering algorithm can be implemented. The mining of this pit will be done with a fleet of trucks and shovels with a maximum capacity of 32 million tonnes of material movement which will incrementally decrease to eight million tonnes by the end of the mine life. The mine will have a processing plant with a capacity of seven million tonnes of ore per year which will start its operation from year four of the mine life. The plant will accept ore with a minimum MWT grade of 78% and a maximum S grade of 1.7% and a maximum P grade of 0.14%.

The OPMPS of the case study without incorporating the two-step algorithm we developed in this paper is presented in Figure 1. As shown in Figure 1 the mine has 20 years of life to mine and process all the material located inside the optimum final pit limit.

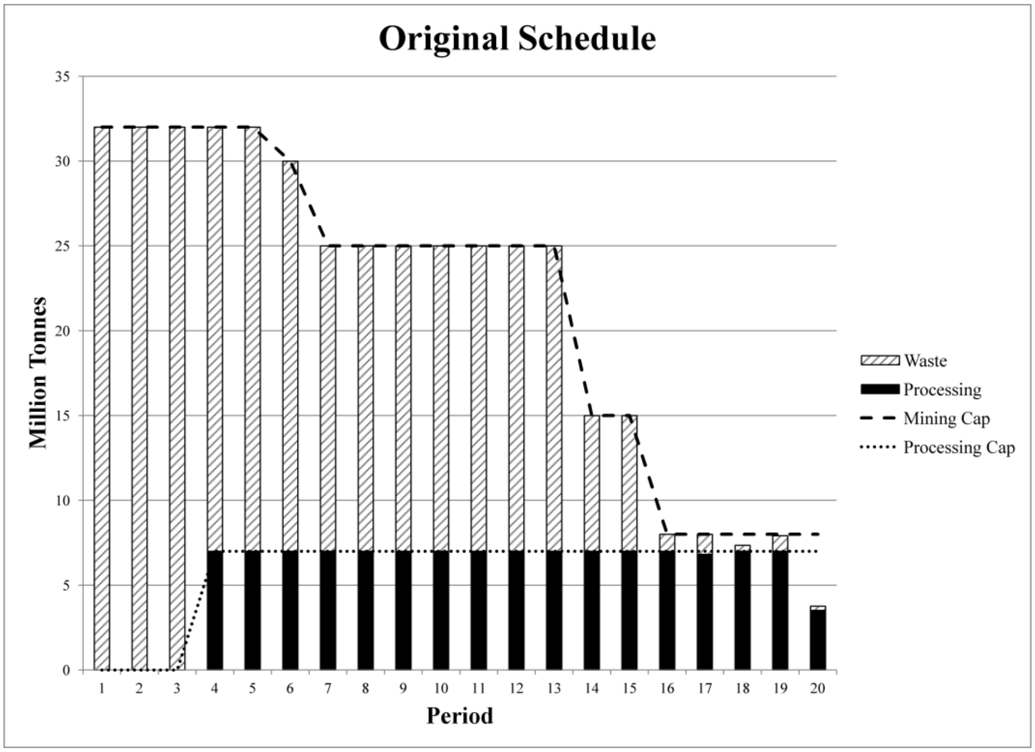

Figure 1. Life of mine production schedule of the case study prior to incorporating grade control in the production process.

Although taking a glance on the production schedule shows no issues in the production, investigating the grade control over the course of the mine life reveals issues with the quality of material delivered to the processing plant (Figure 2). As depicted in Figure 2 between the year four and eight as well as the year 12 and the year 15 if the mine life, the desired minimum MWT head grade has not been met. The same is true for the maximum P content that does not follow the plant requirement for the first four years and the year 11 of the mine life. The main reason for this problem is that the original model does not incorporates the blending constraints into the OPMPS procedure. It is worth noting that as the Sulfur content of the ore deposit is below 1.7% in all the collected samples, we do not present its impact on the schedule in Figure 2 for readability of the figure.

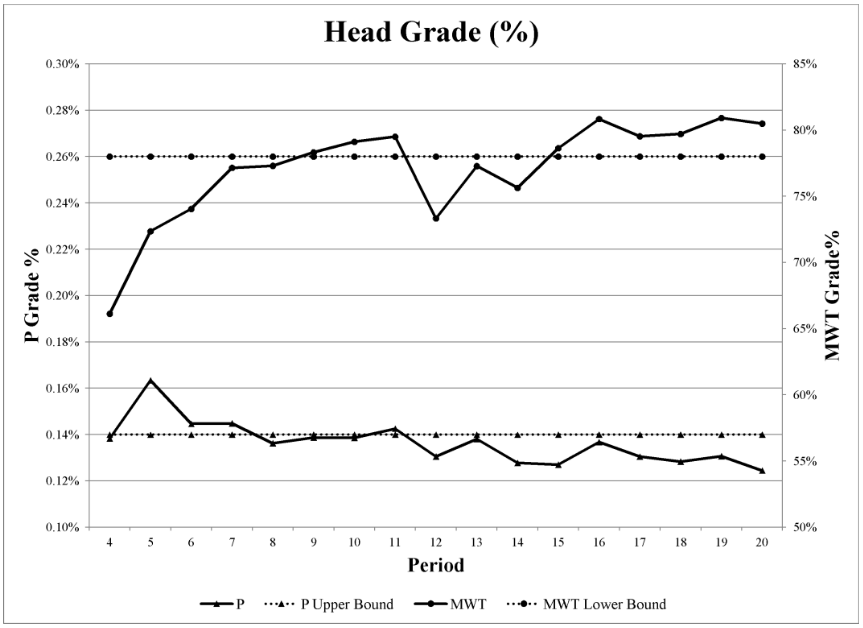

Figure 2. A comparison between desired and achieved iron and phosphor head grades without grade control.

3.1. Head grade constraints

First of all, the algorithm converted the case study to 1870 mining cuts in its first stage. Then, we ran the second stage of the algorithm (OPMPS) without introducing any stockpile option for the operation. This schedule generates 2109 million dollars of NPV. However, as presented in Figure 3, due to grade control enforcement, the plant is not fed to its maximum capacity for approximately 60% of the mine life. Moreover, despite availability of the plant in period four of the mine life, as the mining fleet could not extract material with desired processing grade, no ore has been sent to the plant.

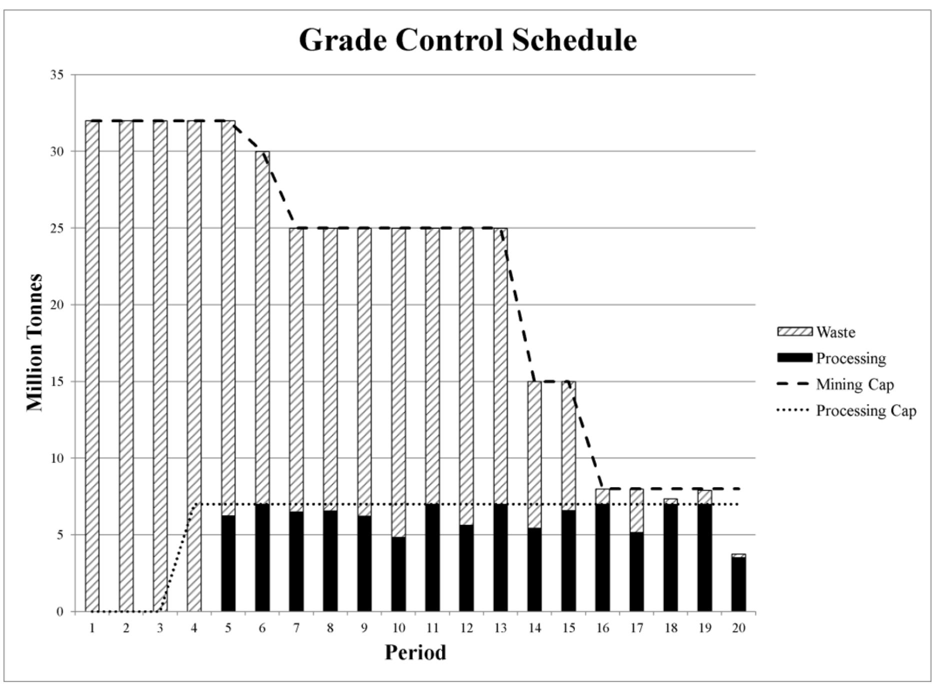

Figure 3. Life of mine production schedule after implementing grade control constraints in the optimization process.

We then add the stockpiling component to the OPMPS model. To analyze the impact of stockpiling in the production schedule of the open pit mine, we defined four different stockpiling scenarios in the operationally practical ranges of material quality as listed in Table 1. In the following subsections we will explain the effects of each scenario on the OPMPS of the case study.

Table 1. Operational scenarios defined for practical stockpiling options.

Stockpile Type	Bin Number	Element	$\underline{G}_{d}^{t,e}$ (%)	$\overline{G}_{d}^{t,e}$ (%)	$G_{s}^{t,e}$ (%)
Single	1	P	0.10	0.15	0.13
		S	1.00	2.00	1.59
		MWT	70.00	80.00	76.55
Double	1	P	0.10	0.13	0.12
		S	1.00	2.00	1.59
		MWT	70.00	75.00	72.42
	2	P	0.13	0.15	0.14
		S	1.00	2.00	1.59
		MWT	75.00	80.00	79.49
Triple	1	P	0.10	0.11	0.10
		S	1.00	2.00	1.59
		MWT	70.00	74.00	71.83
	2	P	0.11	0.13	0.12
		S	1.00	2.00	1.59
		MWT	74.00	78.00	76.47
	3	P	0.13	0.15	0.14
		S	1.00	2.00	1.59
		MWT	78.00	82.00	80.34
Quadruple	1	P	0.10	0.13	0.12
		S	1.00	2.00	1.59
		MWT	75.00	80.00	77.75
	2	P	0.13	0.15	0.14
		S	1.00	2.00	1.59
		MWT	75.00	80.00	77.75
	3	P	0.10	0.13	0.12
		S	1.00	2.00	1.59
		MWT	70.00	75.00	72.24
	4	P	0.13	0.15	0.14
		S	1.00	2.00	1.59
		MWT	70.00	75.00	72.24

3.2. Scenario I single bin stockpile

Now we will add a stockpile to help the operation balance the head grade. In the single bin stockpile type, as shown in Table 1 only one specific range of grades can be piled in the stockpile. Based on information presented on Table 1, we plotted the acceptable range of MWT and P grades on Figure 4. Based on this acceptable range, the model calculates the reclamation grade by taking weighted average over the material. Using the rehandling cost of $0.5/tonne and the calculated average reclamation grade, revenue added to the project from the stockpiling is calculated. Figure 5 shows that, using Scenario I, we are able to feed the plant at its maximum input capacity for all the ore producing years except for year four when the ore was stored in the stockpile due to not meeting the desired plant quality and for further reclamation in the later periods. This resulted in a

9% increase in the NPV of the project (2,291 million dollars) compared to the no stockpile scenario. Keeping track of material delivered to the stockpile in this scenario, Figure 6 compares the grade of material stored in the stockpile with the predetermined reclamation grade for each period and plots the deviation. As shown in this figure, the model tried to store loads with lower MWT and higher P content in stockpile and reclaim with higher grade in the same period to increase the NPV. Over the mine life, the model reclaims 12 million tonnes of material with an average grade difference of 11.6% between stored and reclaimed grades in each period.

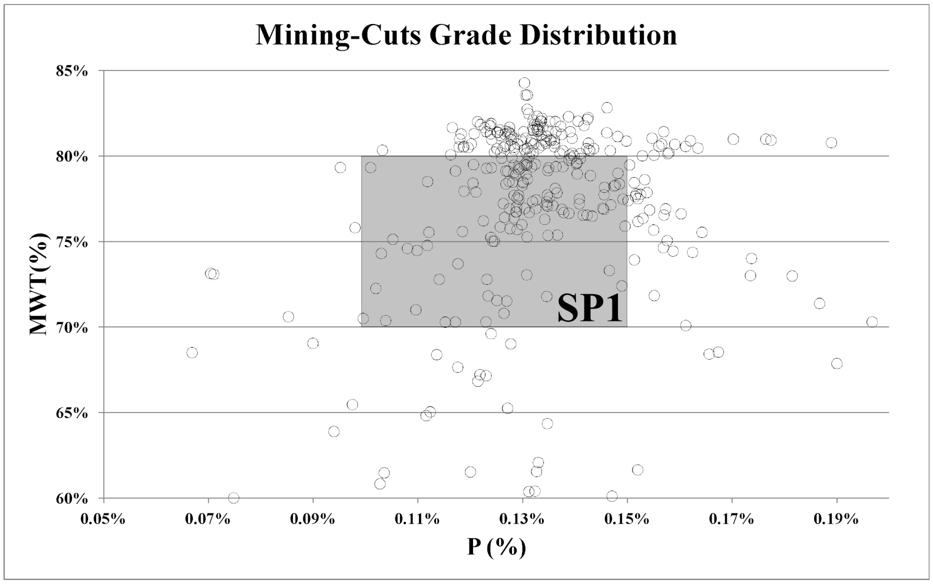

Figure 4. Grade range for MWT and P to be delivered to the stockpile in Scenario I.

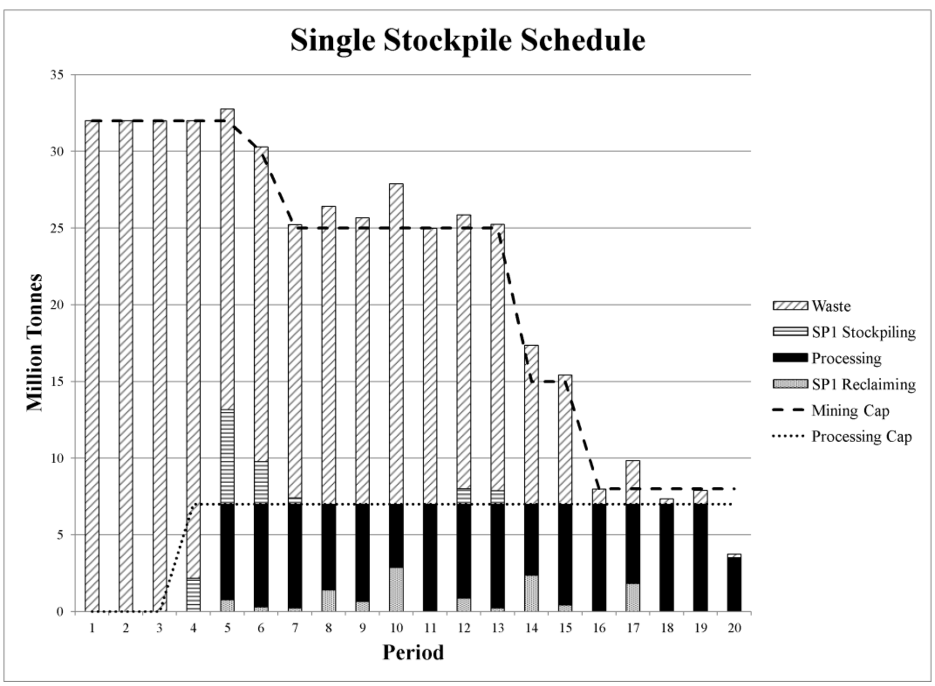

Figure 5. Life of mine production schedule of the deposit in Scenario I.

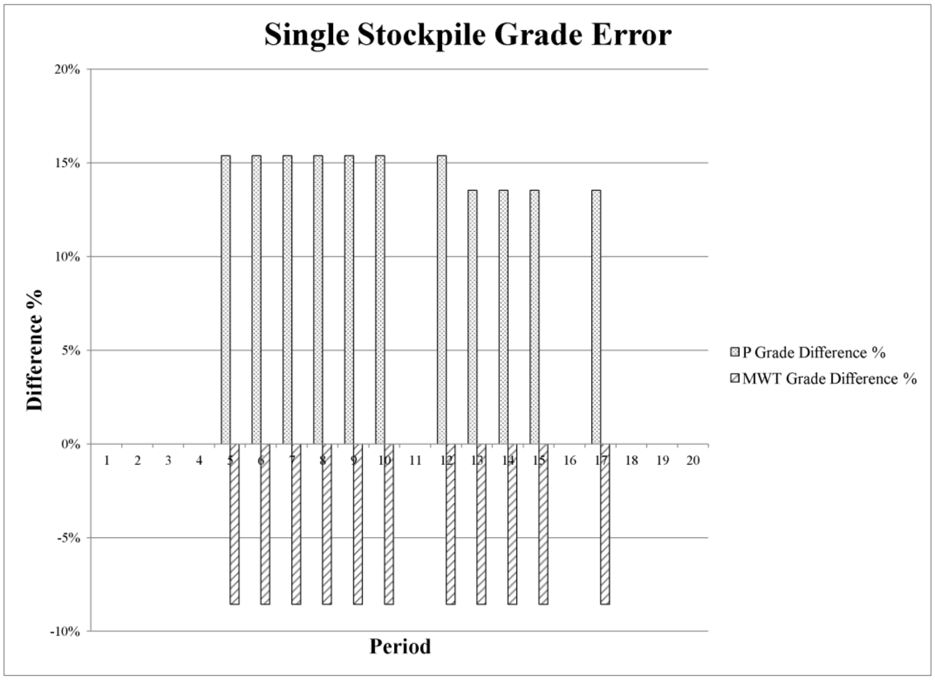

Figure 6. Stockpiling and reclamation grade difference in Scenario I.

3.3. Scenario II double bins stockpile

Now we want to increase selectivity of stockpiling by adding another bin with more strict boundaries for storing and reclaiming grades as listed under double stockpiling type in Table 1. Figure 7 shows the range grades in the deposit that Scenario II will try to store in bin 1 and 2. It worth noting that adding a new bin to the stockpile will not have any cost associated with it. With a comparison between Figure 4 and Figure 7 we can navigate impact of double bin stockpiling on the mining units. Running the OPMPS model under Scenario II conditions generates production schedule, Figure 8, with 2234 million dollars NPV. Adding a new bin cuts the NPV by 2.6% as it increases the selectivity of the stockpiling and reclamation process with an actual to predetermined reclamation grade difference of 5% in each period, Figure 9.

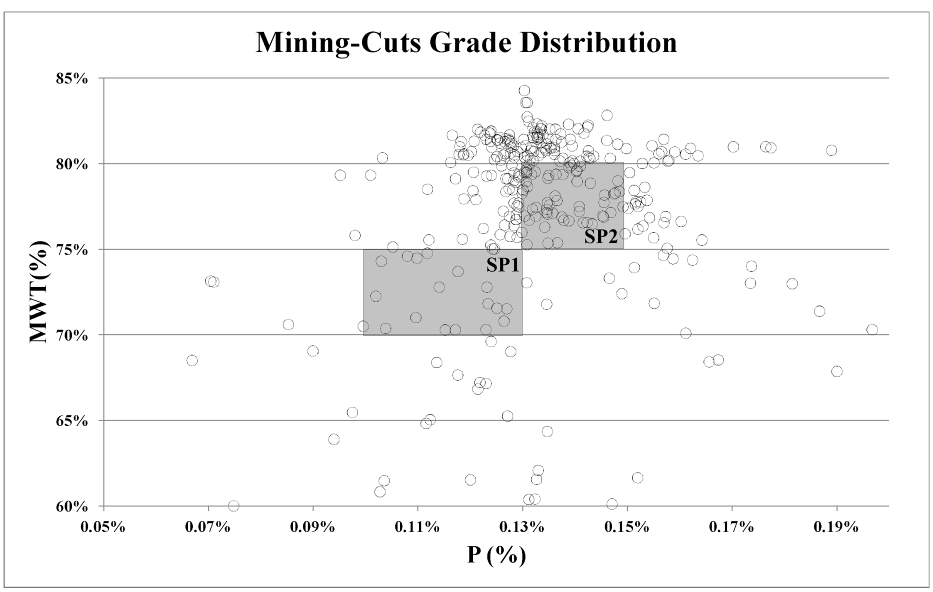

Figure 7. Grade range for MWT and P to be delivered to the stockpile in Scenario II.

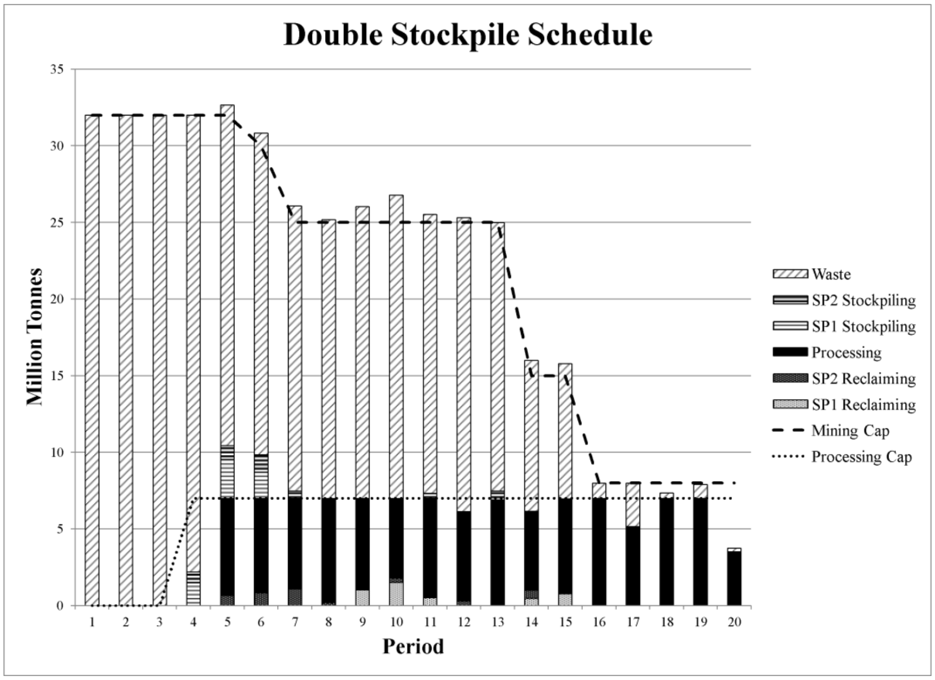

Figure 8. Life of mine production schedule of the deposit in Scenario II.

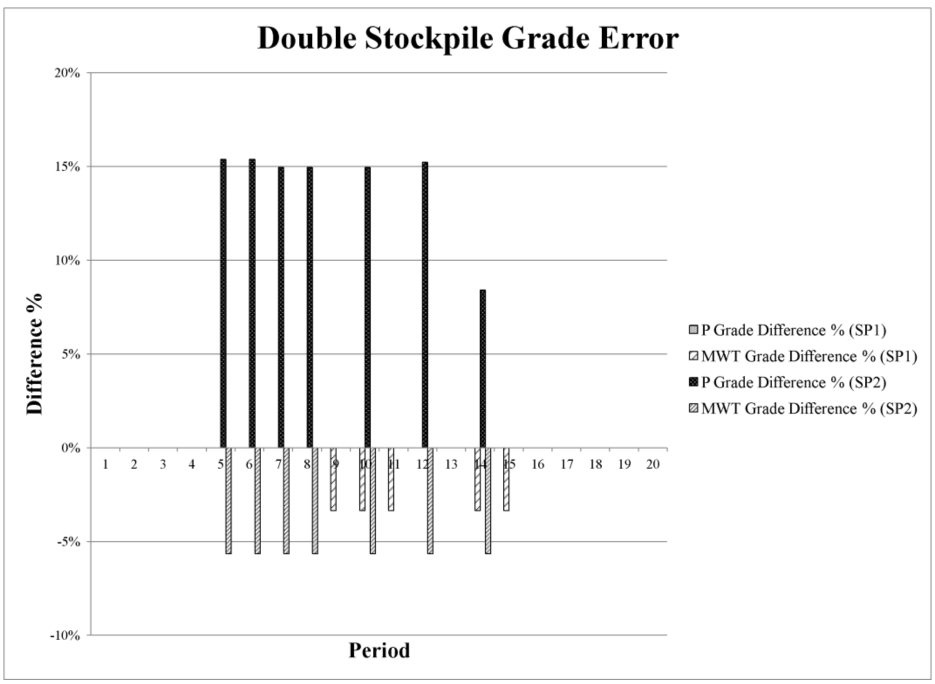

Figure 9. Stockpiling and reclamation grade difference in Scenario II.

3.4. Scenario III triple bins stockpile

In Scenario III we want to tighten the grade range for stockpile bins to investigate its effects on the production. Thus, we ran the OPMPS model with the parameters and grade ranges as shown in triple stockpile section of Table 1 and Figure 10. dividing the stockpile into three different grading bins drops the NPV for 3.4% to 2,155 million dollars compared to the Scenario II double bin stockpile. However, it helps in reducing the actual to planned grade deviation for each period to around 3% (Figure 12) with a total reclamation of 6 million tonnes of stockpiled ore by the end of the mine life. The resulted life of mine schedule is presented in Figure 11.

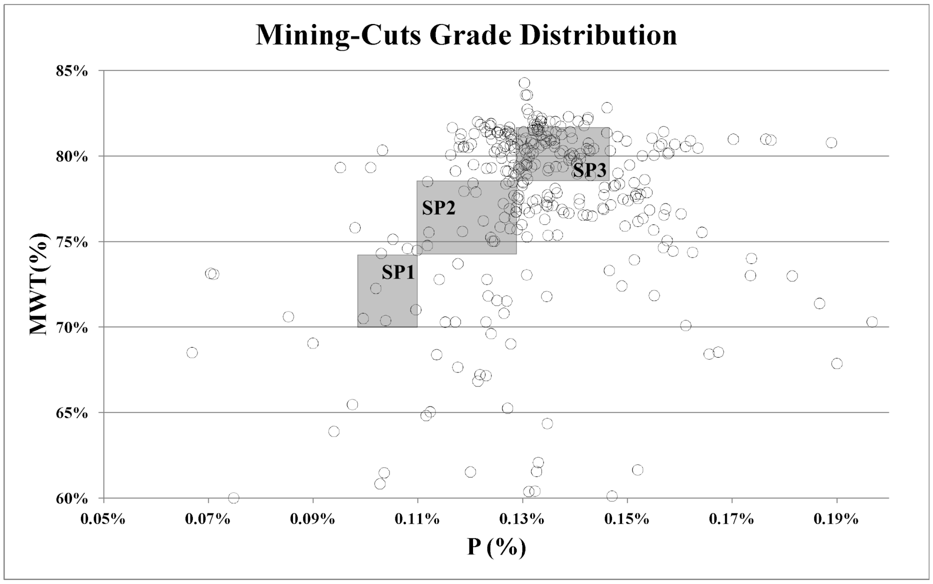

Figure 10. Grade range for MWT and P to be delivered to the stockpile in Scenario III.

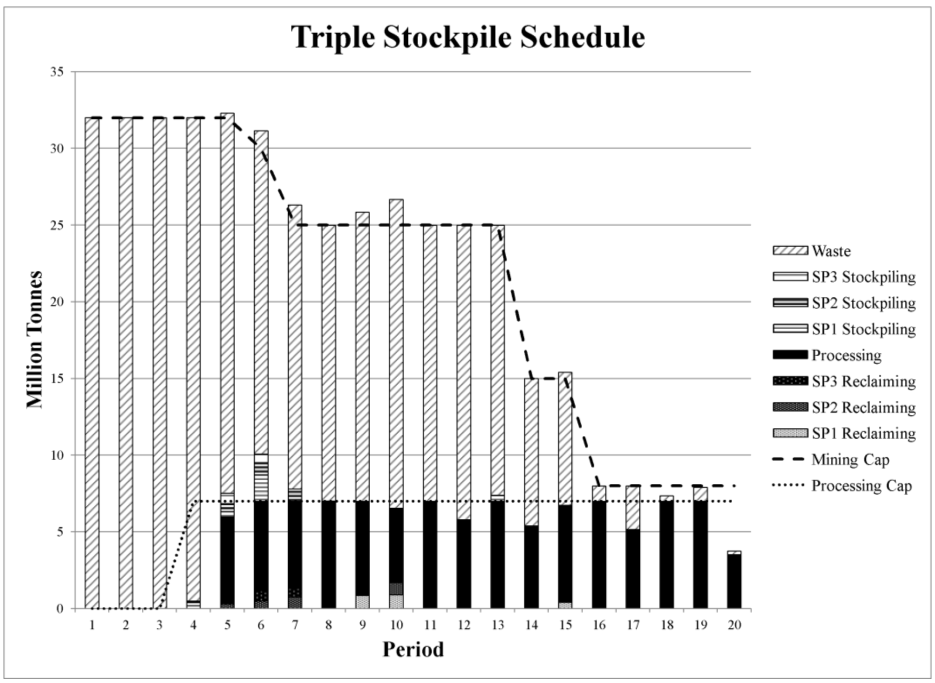

Figure 11. Life of mine production schedule of the deposit in Scenario III.

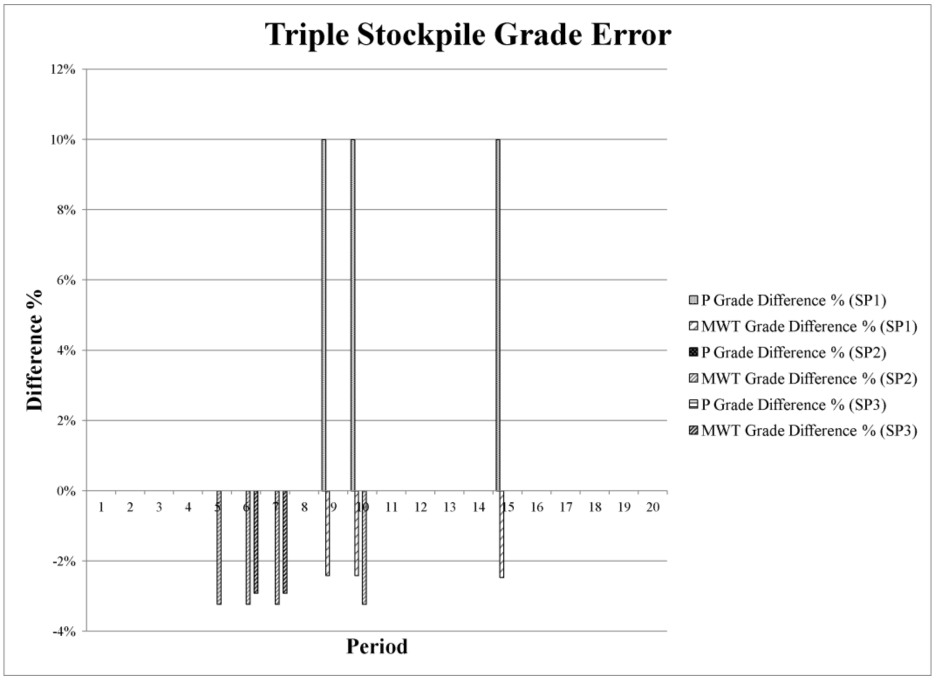

Figure 12. Stockpiling and reclamation grade difference in Scenario III.

3.5. Scenario IV quadruple bin stockpile

In this scenario we divide the grade range of Scenario I into four distinctive grade bins as presented in quadruple scenario type section of Table 1 and Figure 13. AS we need to divide two grades (MWT and P) we divide the bin range in Scenario I into four bins. Running the OPMPS model with the new grade ranges we can generate 2,331 million dollars NPV from the deposit, with the schedule presented in Figure 14, which is 40 million dollars higher than Scenario I and 10.6% higher than when we did not define any stockpiling option. This scenario also reduces the actual to predetermined reclamation grade difference from 11.6% in Scenario I to 5.1% (Figure 15).

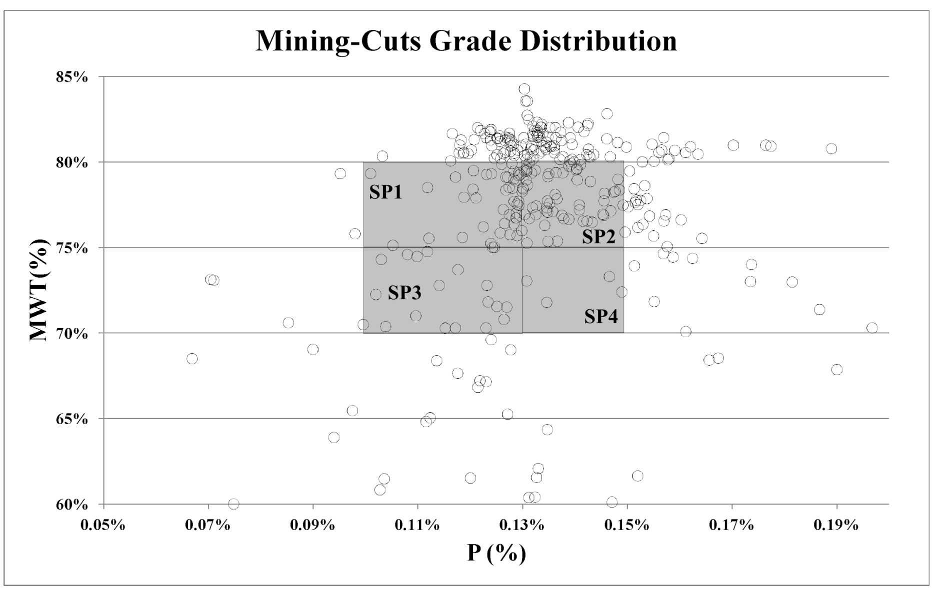

Figure 13. Grade range for MWT and P to be delivered to the stockpile in Scenario IV.

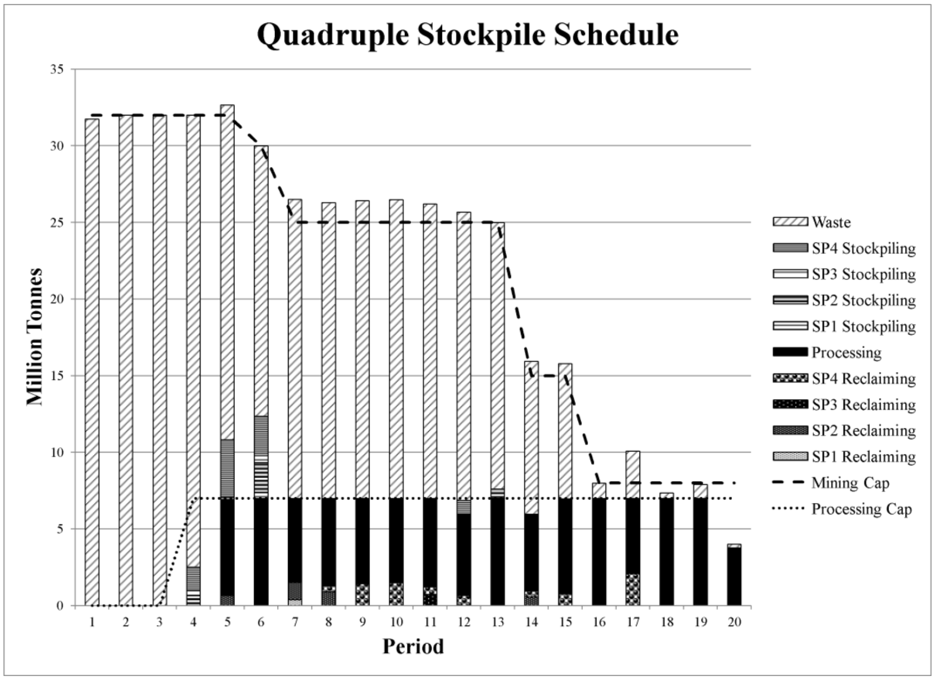

Figure 14. Life of mine production schedule of the deposit in Scenario IV.

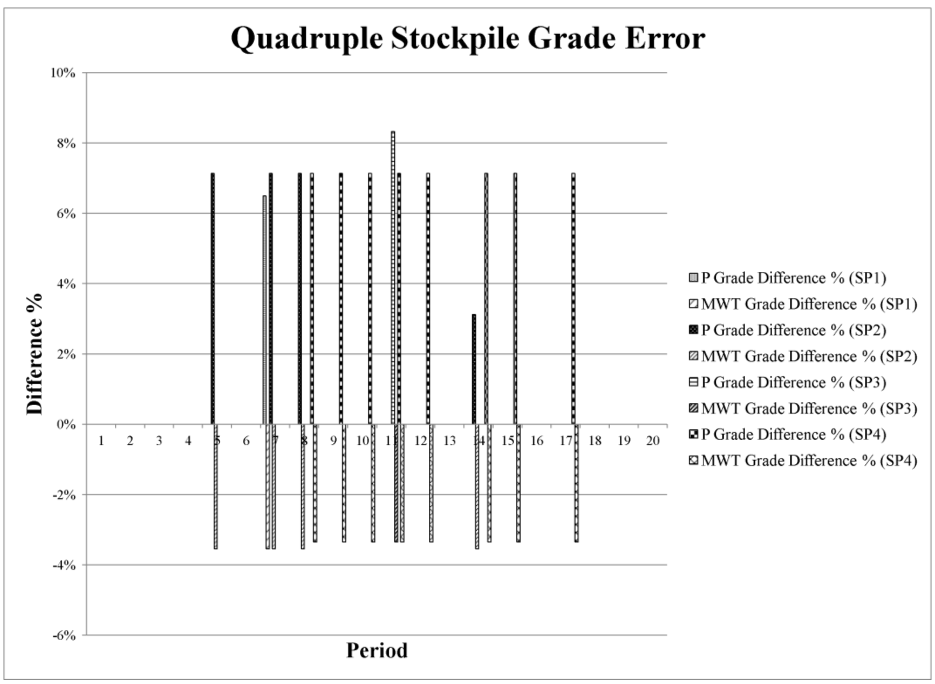

Figure 15. Stockpiling and reclamation grade difference in Scenario IV.

3.6. Summary of the results

We summarized the main Key Performance Indicators (KPIs) resulted from the five investigated scenarios in Table 2. The results show that the base scenario with no stockpile definition generates the least possible NPV as directly delivers all the mined ore to the processing plant without any consideration of the best possible timing. We added stockpiles to increase the NPV and balance the plant feed rate. Referring to the life of mine production schedule (Figure 3, Figure 5, Figure 8, Figure 11, and Figure 14) delivery to the plant was balanced for all stockpiling scenarios except for year four and in some cases year 14 of the mine life. Moreover, the NPV generated from the project was improved by a minimum of 2.2% and a maximum of 10.6%. Based on the results listed in Table 2, choosing Scenario IV with four bins will generate the highest NPV, 223 million dollars higher than the base case, and lowest grade difference of %5.1 in the mining of the deposit in hand.

Table 2. Comparison on the key performance indicators over the five scenarios.

Scenario	NPV ($M)	NPV Improvement (%)	Reclaimed Tonnage (MT)	Average Grade Difference (%)	CPU Time (s)
Base Case: No Stockpile	2108	-	-	-	2.17
Scenario I: Single Bin	2291	8.6%	12.0	11.6	4.17
Scenario II: Double Bins	2234	5.9%	8.2	6.5	8.43
Scenario III: Triple Bins	2155	2.2%	5.7	3.0	8.82
Scenario IV: Quadruple Bins	2331	10.6%	12.0	5.1	15.86

4. Conclusions

This paper presents a two-stage algorithm for open pit mine production scheduling (OPMPS) problem. The algorithm generates mining units by clustering blocks on the same bench to reduce the run time. Then in its second stage it implements a new mixed integer linear programming model to maximize net present value (NPV) of the project while controlling the quality and quantity of the throughput of the processing plant. It controls the quantity (tonnage) by utilizing stockpile option in the production scheduling process and quality (head grade) by defining different bins in the stockpile. Implementation of our developed algorithm on an iron ore case study shows that it needs less than five seconds to reduce the size of the problem from more than 19000 blocks to 1870 mining units in its first stage and less than 16 seconds to generate a production schedule for the same deposit. Examining different scenarios with different number of piling bins based on predetermined grade ranges show that the best possible stockpiling option for a deposit with two important material, iron and phosphor in this case, is a stockpile with four bins. This will lead to the highest NPV and the best quality and quantity control in the plant feed.

Data Availability

The data that support the findings of this study are available from the corresponding author, HA, upon reasonable request.

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