Block-cave Operations Optimization using Linear Programming with Absolute Values

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Abstract

For any block cave mining operation it is important to maintain a uniform extraction profile for the life of the mine. A uniform extraction profile reduces horizontal movements of the material between drawpoints in the same neighborhood and as a result minimizes the dilution. Achieving such an extraction profile while satisfying the constraints could be a challenge for the block caving operations. This research uses mathematical programming as a strong tool to model the operation in block cave mining with the objective function in which minimizes the deviation of extraction from drawpoints. The problem was first formulated as a quadratic programming model then the problem was converted to a linear programming with absolute values. Technical and operational constraints such as mining capacity, average grade for production, continuous mining, drawpoint's life, draw control and number of active drawpoints are considered for the operations. Testing both the quadratic and the linear model with absolute values for a real case mining project shows that the linear model with absolute values is easier and faster to solve.

1. Introduction

In this research, production scheduling in block-cave mining is optimized with respect to the objective function and the constraints using mathematical programming. First the background and a brief literature of this research is reviewed, then the model and related parameters are discussed. A case study is presented to verify the model and the results are presented. The problem is modeled in MATLAB and solved using CPLEX.

2. Background

A mining project usually deals with high rates of material movement, capital and operation costs, and in most cases, high rates of returns. Production scheduling is one of the sensitive decision making processes for this type of operations in which there is a huge difference between an operations with optimal or non-optimal production schedule. An optimum production schedule can significantly increase the profitability of the project while a project could be even failed because of a poor schedule. Since the surface mining has been going deeper, the stripping ratios have significantly increased. Bigger open-pits will result in more environmental concerns and more expensive closure plans. In such a situation, underground mining with less waste movement and environmental issues can be more reasonable. Among underground mining methods, block cave mining with high rates of production and low operation cost is a good alternative for open-pit mining. In block cave mining, after determining the extraction level, an undercut is carried on at the bottom of the orebody so that the rock starts breaking down, then the fragmented material can be extracted by the constructed drawpoints. A schematic view of block cave mining is shown in Fig.1.

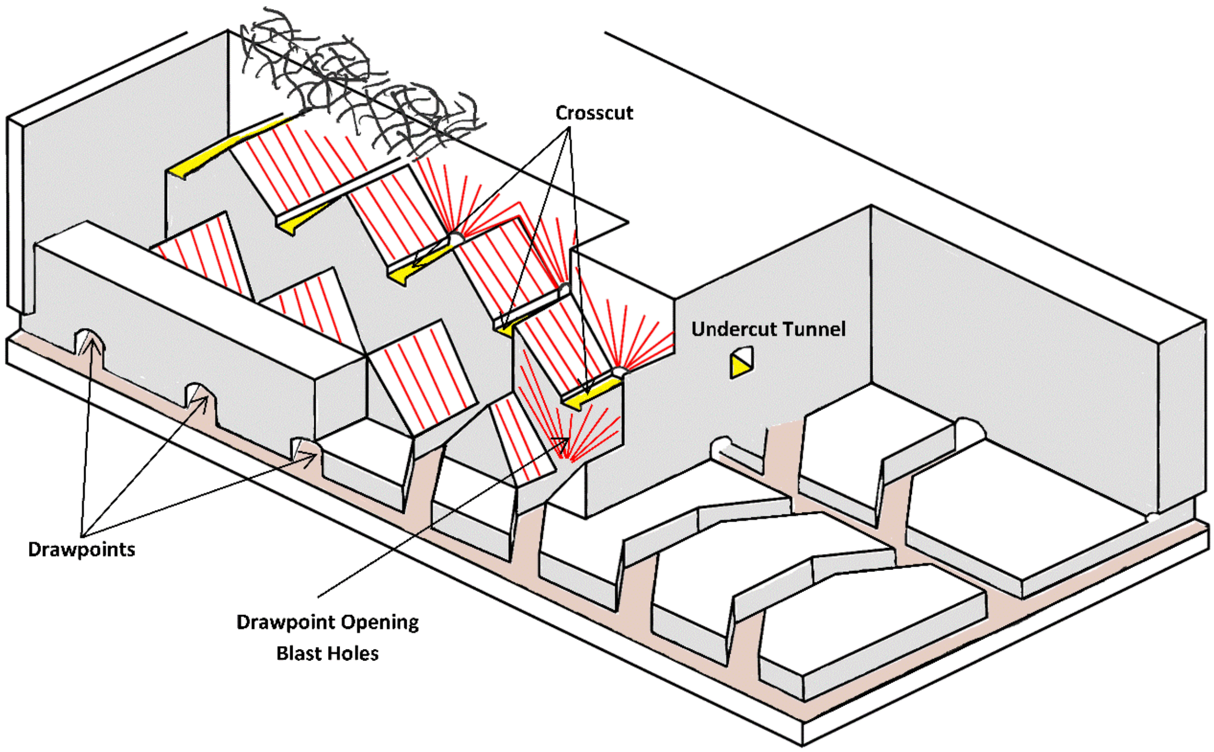

Fig.1. Block cave mining

3. Literature review

There is a significant amount of research for application of mathematical programming in production scheduling for mining projects. Researchers have used linear programming (LP), mixed–integer programming (MIP, MILP), and quadratic programming (QP). The models have mostly been proposed for open-pit operations (Newman, Rubio, Caro, Weintraub, & Eurek, 2010). Some researchers have been looking at production scheduling optimization for block cave mining such as; Song (1989), Chanda (1990), Guest, Van Hout, and Von Johannides (2000), Rubio (2002), Rahal, Smith, Van Hout, and Von Johannides (2003), Diering (2000), Rubio and Diering (2004), Rahal (2008), Pourrahimian, Askari-Nasab, and Tannant (2012), Pourrahimian, Askari-Nasab, and Tannant (2013), Khodayari and Pourrahimian (2014). A detailed literature review of production scheduling in block-cave mining can be found in (Khodayari & Pourrahimian, 2015).

4. Modeling

The model's aim is to optimize the long-term production schedule for a block-cave mine so that the deviation of the extracted tonnages (*extton*) of drawpoints from the defined initial tonnages (*expton*) is minimized with respect to the technical and operational constraints. The indices, variables, and parameters which are used in the models are defined as below:

4.1. Notation

- Indices

$t \in \{1,...,T\}$ Index for scheduling periods

$n \in \{1,...,N\}$	Index for individual drawpoints
g_n	Average copper grade of draw column *n* associated with drawpoint *n*
ton_n	Ore tonnage of draw column *n* associated with drawpoint *n*

- Variables

$ext\,\mathrm{ton}_n^t$	The tonnage of extraction for drawpoint n at period *t* based on the solution of the production scheduling problem (the optimum tonnages that we are looking for, considering the problem's constraints)
Z_n^t	The new set of continuous variables which connects the quadratic model to linear
$X_n^t \in [0,1]$	Continuous decision variable that represents the portion of draw column *n* which is extracted in period *t*
$(Y1)_n^t \in \{0,1\}$	First set of binary variables which determines whether drawpoint *n* in period *t* is active [$(Y1)_n^t = 1$] or not [$(Y1)_n^t = 0$]
$(Y2)_n^t \in \{0,1\}$	Second set of binary variables which determines whether drawpoint *n* till period *t* (periods 1, 2,..,t) has started its extraction [$(Y2)_n^t = 1$] or not [$(Y2)_n^t = 0$]

- Parameters

$\exp ton_n^t$	The expected tonnage of extraction for drawpoint *n* at period *t* which is defined based on the production goals
$DP_n^t \in [0,1]$	Depletion Percentage, the portion of draw column *n* which has been extracted till period *t*
$M_{\min}$	Minimum mining capacity based on the capacity of mining equipment
$M_{\max}$	Maximum mining capacity based on the capacity of mining equipment
$G_{\min}$	Minimum production grade
$G_{\max}$	Maximum production grade
$ActMin$	Minimum number of active drawpoints in each period
$ActMax$	Maximum number of active drawpoints in each period
M	An arbitrary big number
MaxDrawLife	Maximum life of drawpoints

4.2. Mixed integer quadratic programming model

This model is defined based on a mixed integer quadratic objective function and the linear constraints. The objective function is as following:

$$Minimize \quad \sum_{t=1}^{T}\sum_{n=1}^{N}(ext\,\mathrm{ton}_n^t - \mathrm{exp}\,ton_n^t)^2 \tag{1}$$

$$= \sum_{t=1}^{T}\sum_{n=1}^{N}(ext\,\mathrm{ton}_n^t)^2 - (2 \times ext\,\mathrm{ton}_n^t) \times \mathrm{expton}_n^t$$

$$= \sum_{t=1}^{T}\sum_{n=1}^{N}(ton_n \times X_n^t)^2 - (2 \times \mathrm{expton}_n^t \times ton_n) \times X_n^t$$

This quadratic formulation can be modeled based on the following basic problem:

$$Minimize \quad 0.5 \times \mathrm{X'} \times \mathrm{H} \times \mathrm{X} + \mathrm{f} \times \mathrm{X} \tag{2}$$

Where *X* is the matrix of variables, *H* is the coefficient matrix of the quadratic part, and *f* is the coefficient matrix for the linear part of the formulation. There are both continuous and binary variables, so the model is called mixed integer quadratic programming (MIQP). This formulation will be converted to a mixed integer linear programming with absolute values at the end of this section.

4.3. Constraints

In any real mining operation, many constraints can control the operations and force the management team to make decisions based on those conditions. This research has been trying to consider the key constraints which limit the operations and the production plan.

Binary variables and constraints

To define some of the constraints, it was needed to add several binary constraints in order to make sure that the binary variables work and can be properly used for the other constraints. Two sets of binary variables are used to be able to define the related constraints in the model:

- Set 1: $(Y1)_n^t \in \{0,1\}, \begin{Bmatrix} n \in N \\ t \in T \end{Bmatrix}$

This set contains N×T variables, which means for each drawpoint there is one variable per each period. Variables (N×T) +1 to (2×N×T) in the model are allocated to this set. This set determines whether drawpoint *n* is active in period *t* or not; if any extraction from drawpoint *n* at period *t* occurs then the drawpoint is active (x>0) and $(Y1)_n^t = 1$. If there is no any extraction (x=0) then it is inactive and $(Y1)_n^t = 0$. The mathematical formulation for this set of constraint includes two parts of equations:

$$\forall t \in T \;\&\; n \in N \rightarrow \quad (Y1)_n^t - M.x_n^t \leq 0 \tag{3}$$

$$\forall t \in T \;\&\; n \in N \rightarrow \quad x_n^t - (Y1)_n^t \leq 0 \tag{4}$$

- Set 2: $(Y2)_n^t \in \{0,1\}, \begin{Bmatrix} n \in N \\ t \in T \end{Bmatrix}$

This set also contains N×T variables. Variables (2×N×T) +1 to 3×N×T in the model are allocated to this set. This set determines whether the depletion percentage of drawpoint *n* in period t is 0 or not. Depletion percentage (DP) is the summation of the x values for drawpoint *n* from period *1* to period *t* based on the draw rate curve.

$$\forall n \in N \rightarrow \quad DP_n^t = \sum_{t=1}^{t} x_n^t \tag{5}$$

Two equations are defined for this set:

$$\forall t \in T \ \& \ n \in N \rightarrow \quad DP_n^t - (Y2)_n^t \leq 0 \tag{6}$$

$$\forall t \in T \ \& \ n \in N \rightarrow \quad (Y2)_n^t - M.DP_n^t \leq 0 \tag{7}$$

Mining capacity

Based on the mining space, the equipment and the required feed for the processing plant, we are limited to a certain amount of extraction for the mine in different periods of production during the life of the mine:

$$\forall t \in T \rightarrow \quad M_{\min} \leq \sum_{n=1}^{N} ton_n \times X_n^t \leq M_{\max} \tag{8}$$

Average mining grade

The processing plant is defined based on the quantity and quality of the material which is sent by the mine, the mining capacity (as it was mentioned above) takes care of the quantity and the constraint for average grade of production is defined to guarantee the quality which is the production grade:

$$\forall t \in T \rightarrow \quad G_{\min} \times (\sum_{n=1}^{N} ton_n \times X_n^t) \leq \sum_{n=1}^{N} g_n \times ton_n \times X_n^t \tag{9}$$

$$\forall t \in T \rightarrow \quad \sum_{n=1}^{N} g_n \times ton_n \times X_n^t \leq G_{\max} \times (\sum_{n=1}^{N} ton_n \times X_n^t) \tag{10}$$

Reserve

This constraint makes sure that the mine reserve which is calculated based on the best height of draw (BHOD) is extracted during the life of the mine:

$$\forall n \in N \rightarrow \quad \sum_{t=1}^{T} X_n^t = 1 \tag{11}$$

Active drawpoints

This constraint limits the number of the active drawpoints in different periods during the life of the mine. This will facilitate the decision making process for the management team.

$$\forall t \in T \rightarrow \quad ActMin \leq \sum_{n=1}^{N} (Y1)_n^t \leq ActMax \tag{12}$$

Mining direction

In block-cave mining, the drawpoints are designed in a layout; the extraction from drawpoints is started from one location and then expands continuously through the layout to a point where the whole ore body is extracted. Because of the economic value of the material and the geotechnical parameters, it is important where to start and which direction to expand. The mining direction constraint, finds the optimum direction based on the economic values of the draw columns, and then the best direction is manually defined. Fig. 2 shows an example of the mining direction for a block-cave layout (the mining starts from the South-East and then expands toward the North-West of the deposit).

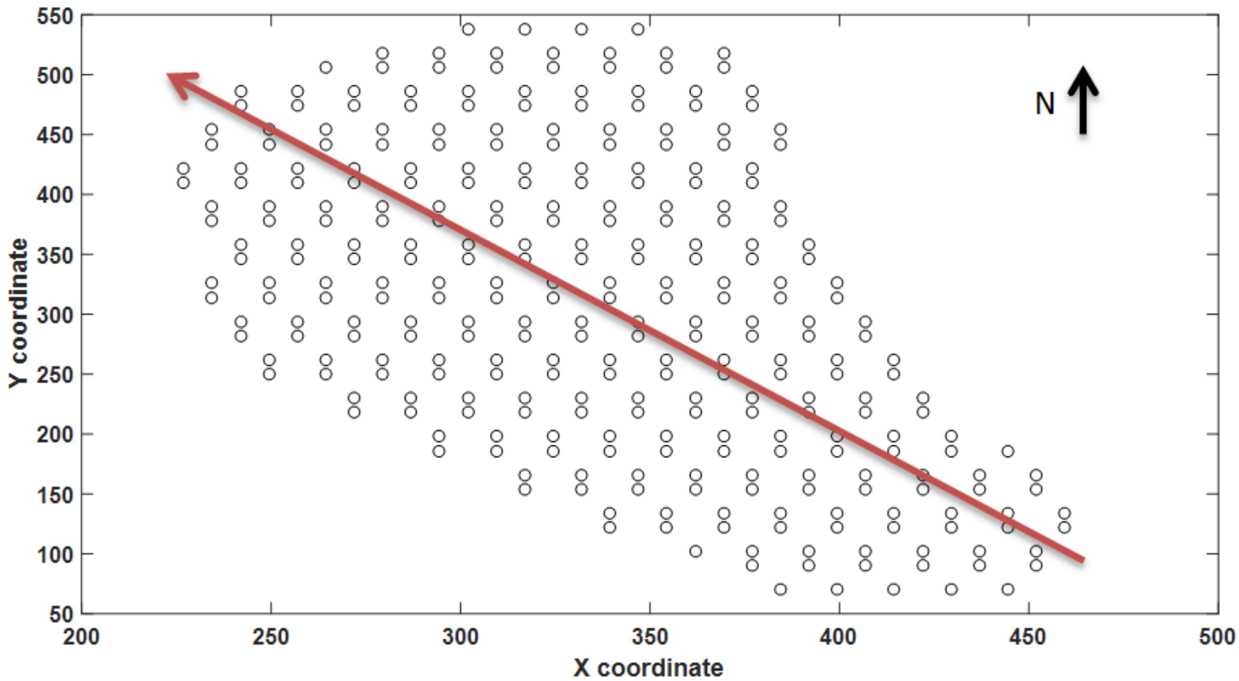

Fig. 2. Mining direction for a block-cave layout

For doing that; first the drawpoints which are located in the neighborhood of each drawpoint are found using a defined radius, the combination of a drawpoint and its neighbors is called production block, then the economic values of production blocks are calculated and compared, the production block with maximum economic value is where the extraction should start, and the expansion follows the direction toward the production blocks with lower economic values. More details about mining direction determination can be found in one of the author's paper (Khodayari & Pourrahimian, 2014).

Mining precedence

When the direction is determined, the precedence can be defined based on the direction so that for each drawpoint the extraction will be started if the extraction of its neighbors which are located ahead has already been started. The following equation defines this constraint:

$$\forall n \in N \ \& \ t \in T \rightarrow \qquad S * (Y2)_n^t \le \sum_{m=1}^{S} (Y2)_m^t \tag{13}$$

Where *S* is the number of drawpoints in the neighborhood of drawpoint *n* which are ahead (based on the defined direction) and *Y2* is the second set of binary variables.

Continuous mining

When extraction from a drawpoint is started, it should be continued till end of its life. This constraint limits the model to maintain the continuity of operations for drawpoints during the life of the mine:

$$\forall n \in N \& t \in T \rightarrow \qquad (Y1)_n^t + DP_n^{t-1} \ge (Y1)_n^{t-1} \tag{14}$$

Draw control

The tonnage of material which is going to be extracted from each of drawpoints in each period (or draw rate) during the life of the mine is controlled by this constraint. This will also control the height of extraction from each of draw columns.

$$\forall n \in N \,\&\, t \in T \rightarrow \qquad ton_n \times X_n^t \geq \min(ton) \times (Y1)_n^t \tag{15}$$

Where *min(ton)* is the minimum tonnage of draw columns based on the best height of draw (BHOD).

Draw life

For an optimum production schedule, the life of each drawpoint is important, in other words, a drawpoint produces for a certain time during the life of the mine.

$$\forall n \in N \rightarrow \qquad \sum_{t=1}^{T} (Y1)_n^t \leq MaxDrawLife \tag{16}$$

4.4. Conversion to mixed integer linear model with absolute values

It is not easy to solve the MIQP model and solving process is usually time consuming; it takes days to get answers with the MIP gap close to 5%. Even powerful computers can easily go to the situation which the RAM is not enough for solving the problem. The proposed quadratic problem can be converted to a linear programming problem with the absolute values. The conversion would be as following:

$$Minimize \; \sum_{t=1}^{T}\sum_{n=1}^{N} (ext\,\text{ton}_n^t - \text{exp}\,ton_n^t)^2 \rightarrow Minimize \sum_{t=1}^{T}\sum_{n=1}^{N} \left| ext\,\text{ton}_n^t - \text{exp}\,ton_n^t \right| \tag{17}$$

The new model is mixed integer linear programming with absolute values. To solve this model, we need to add a new set of continuous variables (*Z*) and two new constraints (equations 19 and 20) which connect those variables to the objective function. Therefore the new objective function is:

$$Minimize \; \sum_{t=1}^{T}\sum_{n=1}^{N} Z_n^t \tag{18}$$

And the following two new constraints are added to the model:

$$\forall n \in N \,\&\, t \in T \rightarrow ext\,\text{ton}_n^t - \text{exp}\,ton_n^t \leq Z_n^t \tag{19}$$

$$\forall n \in N \,\&\, t \in T \rightarrow -(ext\,\text{ton}_n^t - \text{exp}\,ton_n^t) \leq Z_n^t \tag{20}$$

Which could be combined as: $-Z_n^t \leq (ext\,\text{ton}_n^t - \text{exp}\,ton_n^t) \leq Z_n^t$

5. Case study

The model is tested on a block-cave mining project which is designed based on 298 drawpoints. The best height of drawn (BHOD) is already calculated for the draw columns in which the total tonnage for the mineral reserve is 36.7 million tonnes of copper with the average grade of 1.24%. The total tonnage for draw columns fluctuates from 27,762 to 233,286 tonnes. The layout and the locations of the drawpoints for this mine is shown in Fig. 2. The constraints were implemented using the controlling parameters which are presented in Table 1.

The reserve will be extracted in 20 years with the mining capacity of 1 to 4 million tons of production per year in average grade of 1% to 1.4% of CU. Because of the operational considerations, 20 to 120 drawpoints can be active for each year of production. The model was run by a computer with configuration of i5-3470 CPU @ 3.20GHz and 8 GB installed memory (RAM). It took 42,732 seconds (11 hours, 52 minutes and 12 seconds) to find the integer optimal solution for the model at MIP gap of zero percent. Production during the life of mine is shown in Fig. 3.

Table 1. Controlling parameters for the constraints and the model

Parameter	Value	Unit	Description
T	20	Year	Number of periods (life of the mine)
G_{min}	1	%	Minimum allowable production average grade for CU per each period
G_{max}	1.4	%	Maximum allowable production average grade for CU per each period
M_{min}	1	Mt	Minimum production rate or mining capacity
M_{max}	4	Mt	Maximum production rate or mining capacity
ActMin	20	-	Minimum number of active drawpoints per period
ActMax	120	-	Maximum number of active drawpoints per period
MIPgap	0	%	Sets a relative tolerance on the gap between the best integer objective and the objective of the best node remaining
Radius	8.2	m	The average radius of the drawpoints
Density	2.7	t/m^3	The average density of the material
M	1E+12	-	An arbitrary big number
MaxDrawLife	7	Year	Maximum life of drawpoints
$expton_n^t$	40,000	tonne	Expected tonnage of extraction for drawpoint ***n*** at period ***t***

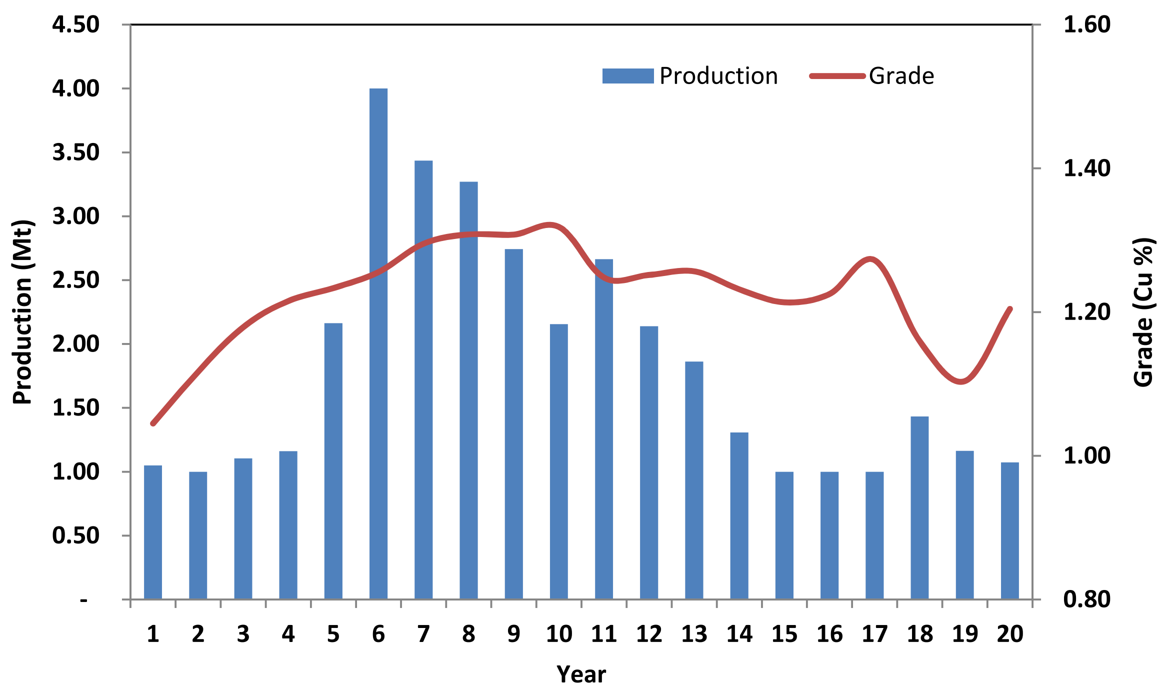

Fig. 3. Production and average grade during the life of mine

It can be seen that the production starts with the minimum capacity at the beginning of the project, increases to the maximum capacity in the years after and again close to minimum at the end of the life of mine. The resulted average grade for production can reasonably feed the processing plant (Fig. 3). Number of active drawpoints for different years during the life of mine falls in the boundary of the related constraint (Fig. 4).

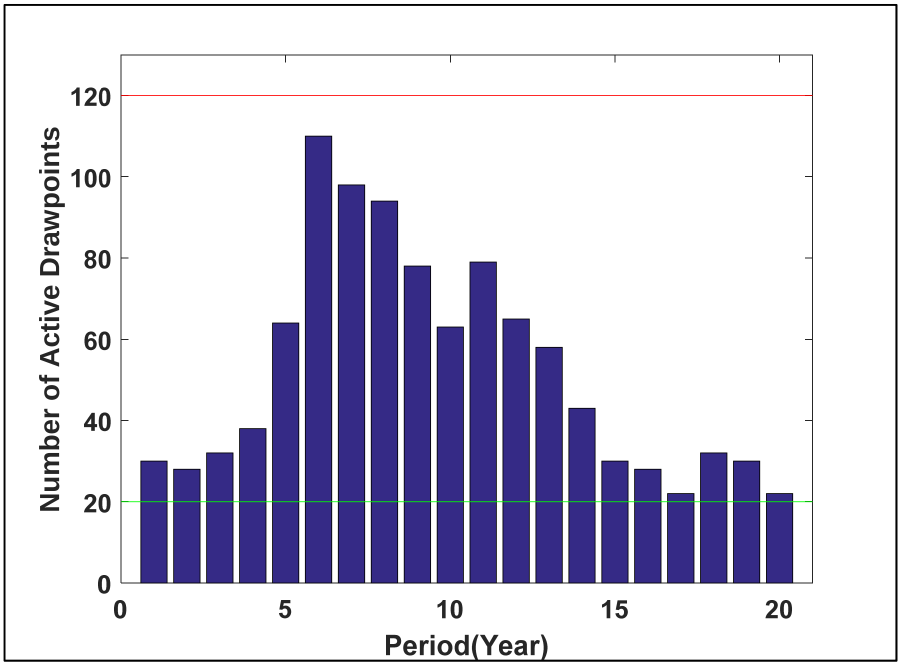

Fig. 4. Number of Active drawpoints during the life of the mine

The period (year) in which production from each drawpoint is started is shown in Fig. 5. It can be seen that the defined development direction and precedence of extraction is followed in the mine plan, extraction starts from the south-east and continues to the north west of the layout. This pattern can be seen by looking at the active and non-active drawpoints in each year as well (Fig. 6). Cumulative height of extraction (extraction profile) shows the cumulative height of extraction from drawpoints during the life of the mine (Fig. 7 to Fig. 10). The extraction moves smoothly from South-East to North-West by considering the neighborhood concept and reducing the horizontal mixing.

6. Discussion

Mathematical programming with absolute values can be used as a powerful tool for optimizing a problem when the goal is to minimize (maximize) a variance in the objective function. The original model for this research was a quadratic programming problem; the model was converted to a linear programming with absolute values. The resulted mine plan maintains the production rates as close as possible to the expected production which is defined in the objective function. This will reduce the horizontal movements of the material and as a result the dilution during the life of the mine. As the horizontal mixing reduces, the cave management and grade control is improved; in other words the operation is more predictable. The solution time for the converted model is significantly less than the original quadratic programming model. Results shows that although the size of the model will increase because of the conversion, the optimal mine plan can be achieved in a reasonable CPU time.

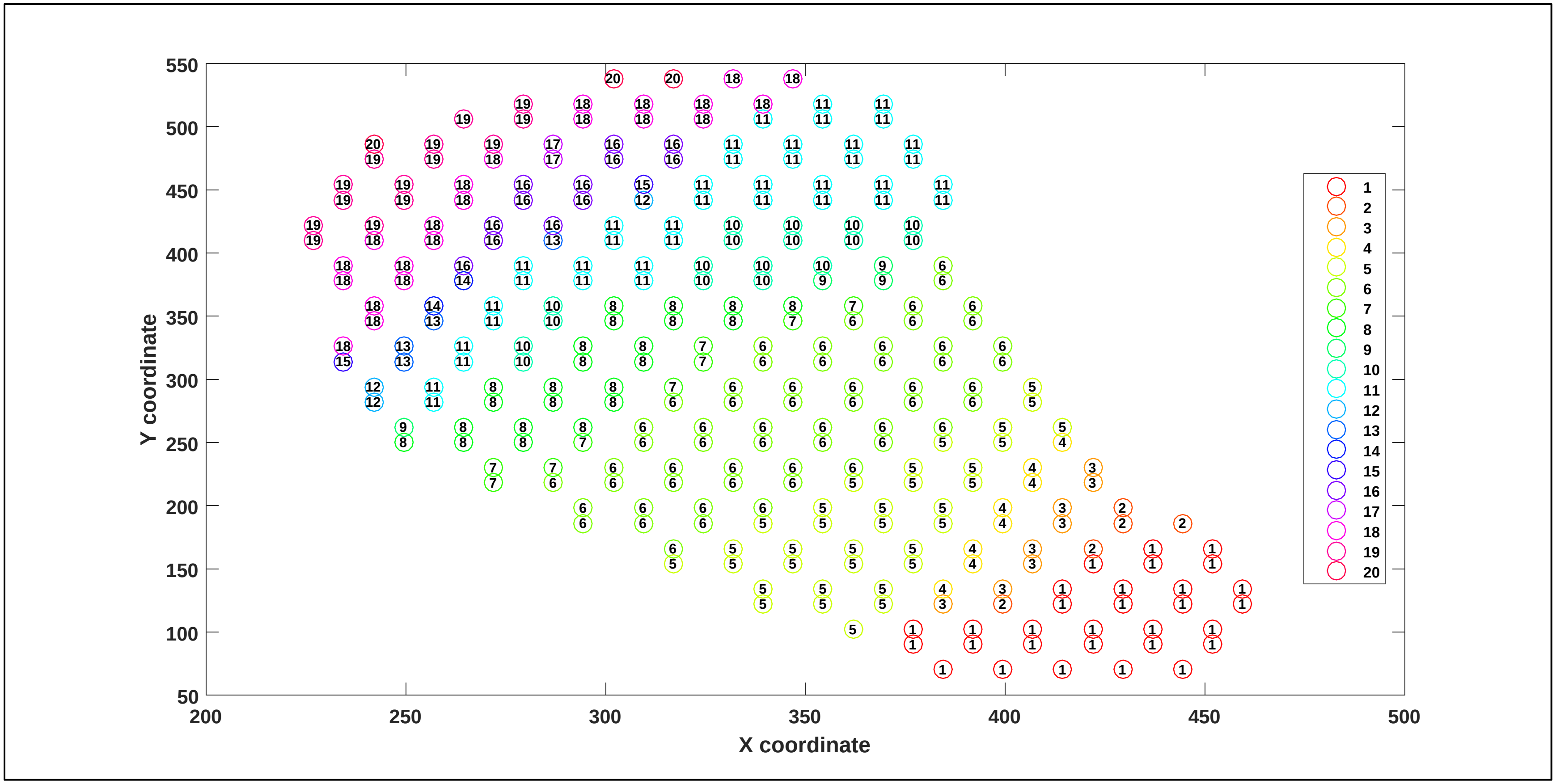

Fig. 5. Starting period (year) of production for drawpoints

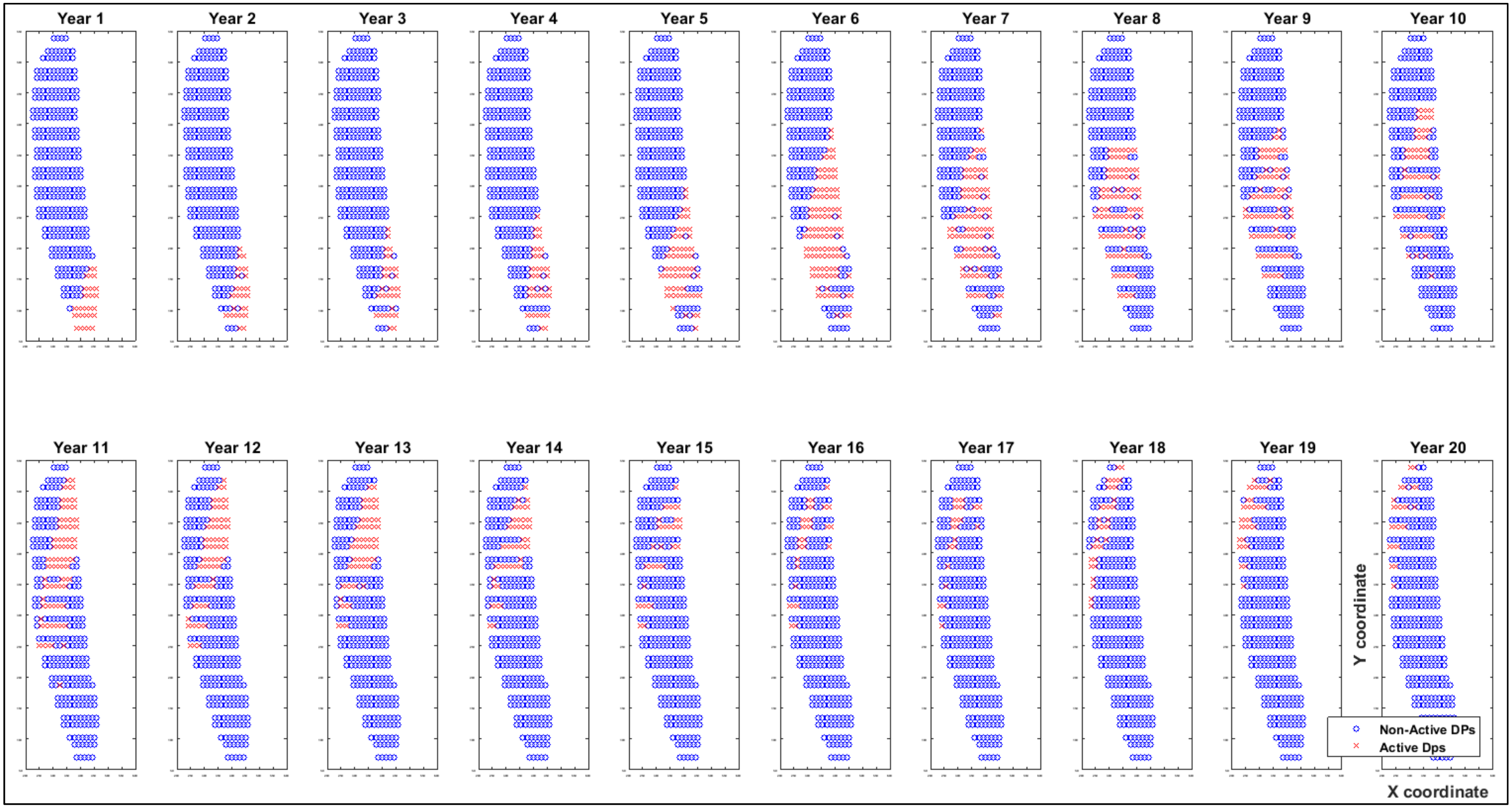

Fig. 6. Active drawpoints in the layout

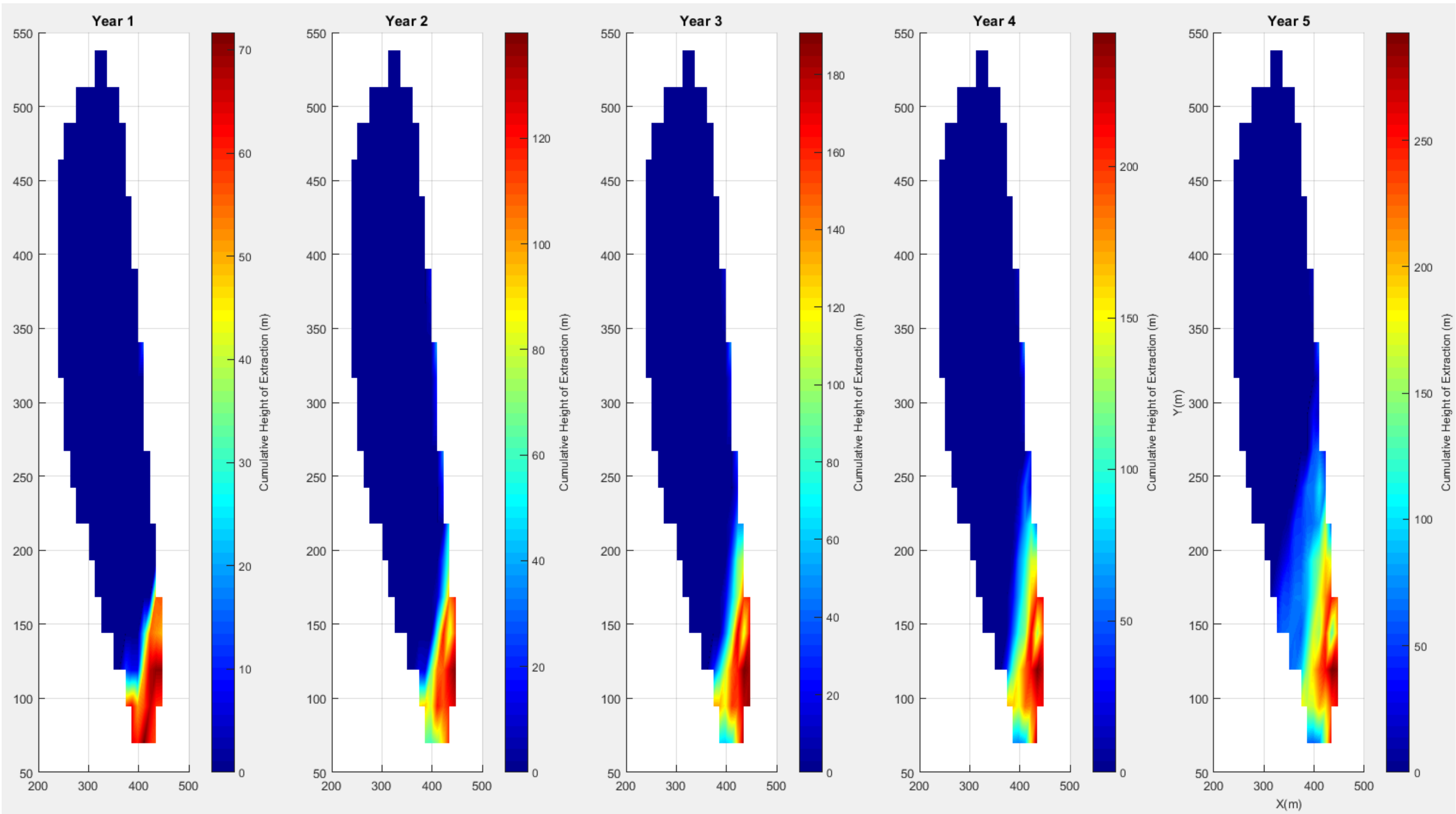

Fig. 7. Cumulative height of extraction or extraction profile during the life of the mine (Year 1 to 5)

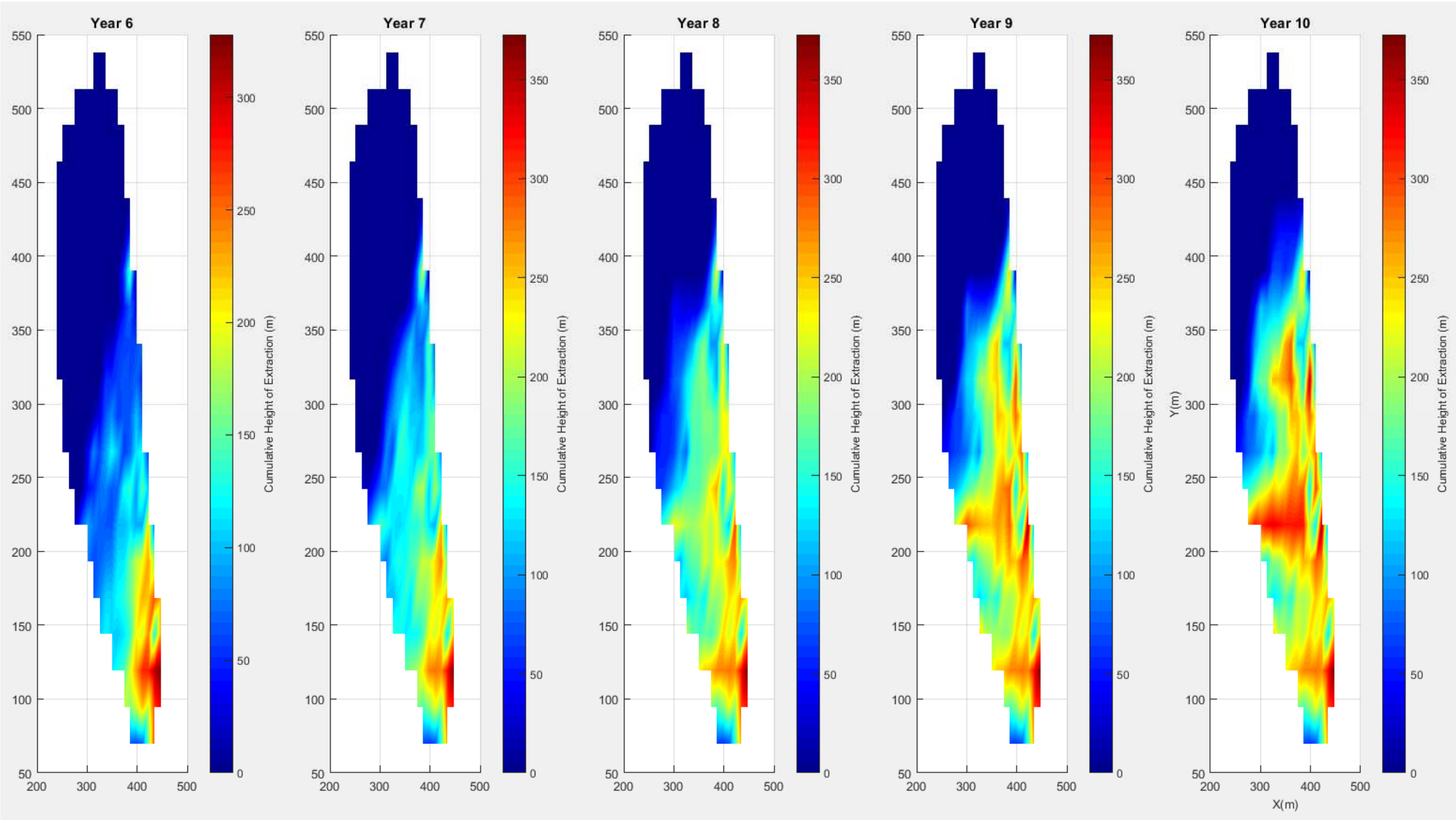

Fig. 8. Cumulative height of extraction or extraction profile during the life of the mine (Year 6 to 10)

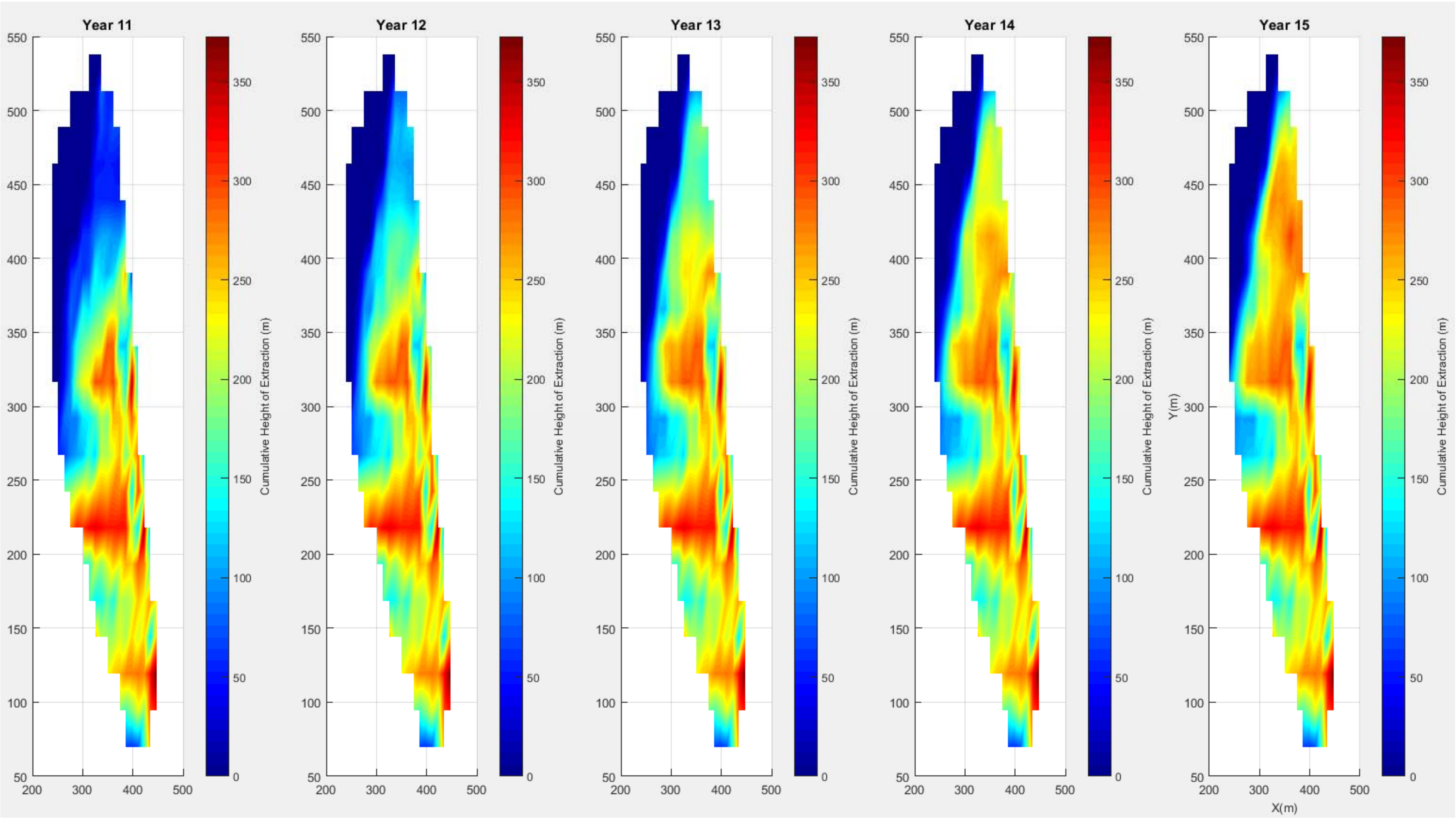

Fig. 9. Cumulative height of extraction or extraction profile during the life of the mine (Year 11 to 15)

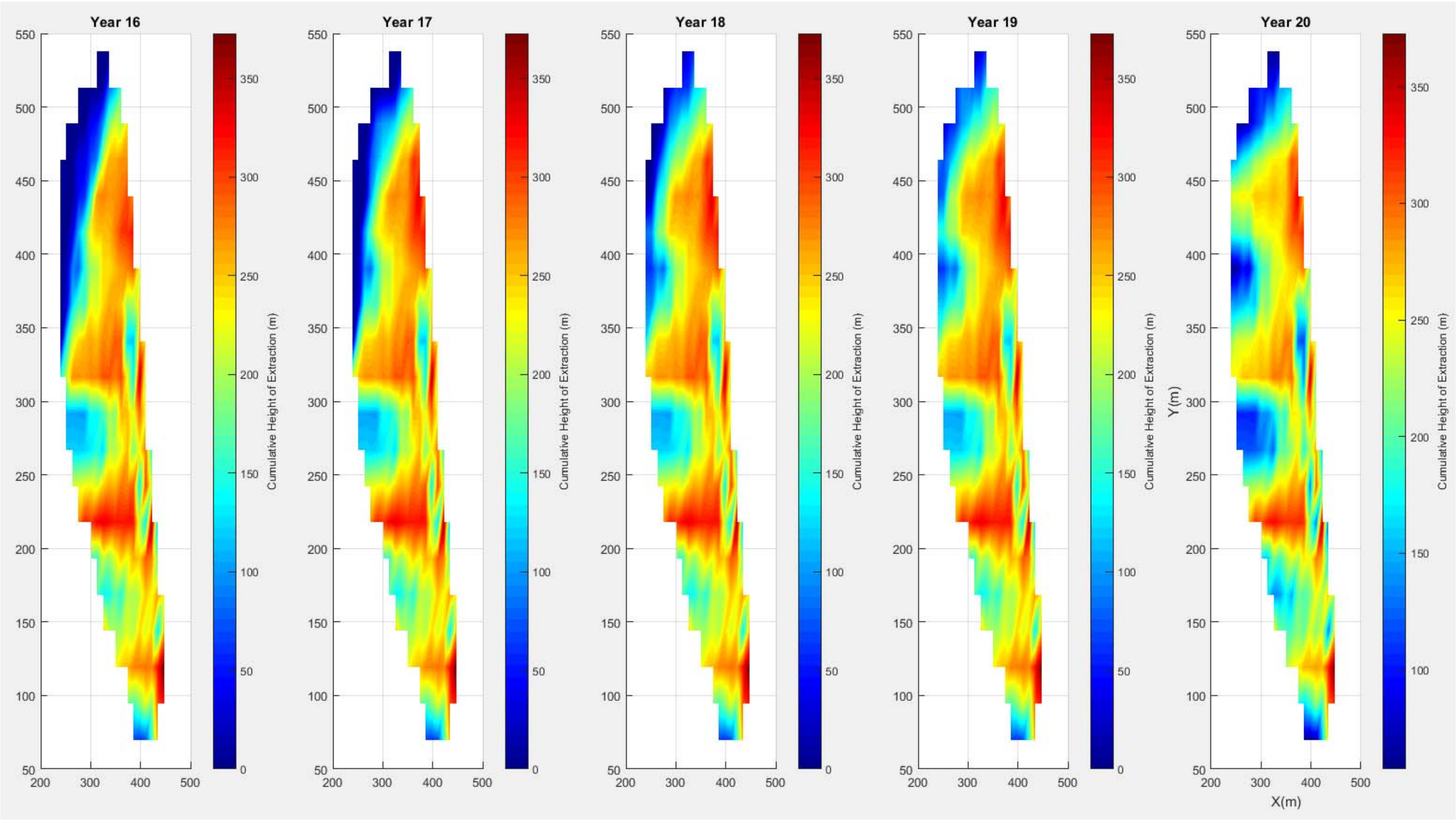

Fig. 10. Cumulative height of extraction or extraction profile during the life of the mine (Year 16 to 20)

7. References

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